INTERNAL MODELS AND ARBITRAGE–FREE CALIBRATION

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There is a major trend in the insurance sector towards arbitrage-free valuation of insurance liabilities and assets. The assumption of no-arbitrage is fundamental in financial modelling. This paper surveys assumptions of arbitrage-free modelling and studies their consequences for the use of internal model in insurance.

The model uncertainty arises as a particularly severe problem under the assumption that the conditions of arbitrage-free complete market theory do not hold and all participants in the market are not fully rational. We argue that the approximation errors of these idealistic assumptions are generally larger in insurance applications than elsewhere in the financial sector. Hence, the model uncertainty plays a particularly important role in the use of internal models. This should be taken into account in the development of the models and in risk management practice. Finally, we present some known Bayesian methods that might be useful for managing the model risk.

Key Words: Bayesian method, Financial modelling, Model risk, Risk management, Solvency II, Stochastic modelling.

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1. INTRODUCTION

There is a major trend going on internationally in the insurance sector where the standard-setters and regulators are moving towards market consistent valuation of insurance liabilities and assets. Main examples of these projects include the International Accounting Standards (IFRS) developed by the IASB, and the solvency regulation developments under way such as Solvency II in the EU and the global insurance solvency project of the IAIS, see e.g. Sandström (2006).

The basic idea is to use similar methods for the valuation of insurance liabilities as the financial markets are using when forming the market prices for various traded assets. It is thought that this approach will increase consistency and transparency in the balance sheets of insurance companies and will lead to more advanced risk management practices, and ultimately make the insurance and capital markets more efficient. In the Solvency II framework firms should be allowed to use their own internal models to determine their regulatory capital requirements, subject to appropriate controls over the adequacy of those models. One of the key objectives is to establish a solvency system that will encourage and provide incentive for insurance companies to measure and manage their risks better, see e.g. Ronkainen et al. (2007).

The assumption of no-arbitrage is also fundamental in financial modelling. Real world arbitrage is the practice of taking advantage of a price differential between two or more markets. A combination of matching deals is struck that capitalize upon the imbalance, the profit being the difference between the market prices. Mathematical arbitrage, when used by academics, an arbitrage is a transaction that involves no negative cash flow at any probabilistic or temporal state and a positive cash flow in at least one state; in simple terms, a risk-free profit.

In finance, the efficient market hypothesis (EMH) asserts that financial markets are "informationally efficient", e.g., stocks, bonds, or property, already reflect all known information and therefore are unbiased in the sense that they reflect the collective beliefs of all investors about future prospects (e.g. Fama (1970)). An alternative approach to the EMH is behavioral finance. The key forces by which markets are supposed to attain efficiency, such as arbitrage, are likely to be weaker and more limited than the efficient market theorists have supposed. Behavioral finance starts with an observation that the assumptions of investor rationality and perfect arbitrage are overwhelmingly contradicted by psychological and institutional evidence (see e.g. Shleifer, 2000). The central argument of behavioral finance states that real-world arbitrage is risky and therefore limited.

If the market is sufficiently rich in liquid assets, then any future financial claim can be hedged (replicated) by an investment portfolio that is self-financing, which means that that the portfolio is initiated with a single investment and thereafter dynamically rebalanced with no infusion or withdrawal of capital. The amount needed to initiate this strategy is arbitrage-free price of the claim. Any other price would lead to arbitrage opportunity. In an incomplete market not all claims can be hedged (perfectly) by using a self-financing strategy.

For instance, in many pension products, where typical contracts have very long durations, there are very limited possibilities of hedging. More generally, hedging has been slow to catch on in insurance. Although the theory of incomplete market derivative models is contemporary, the idea is not new. Already in Keynes (1936) it was argued that the limited ability of economic agents to cope with the uncertain future lead to missing markets, rigidities in prices, and non-rational decision making.

A basic theme in internal modelling is that the applied model and the risk management principles are sound. The requirement of having sound theoretical principles is ambiguous. What is meant by sound and acceptable will evolve over time, but it also has other problems. Efficient market theory and behavioral finance offer sometimes conflicting opinions. Moreover, actuarial and financial economics modelling practices differ, see e.g. Embrechts (2000). Here we review the consequences of these facts for internal modelling.

The paper is structured as follows. In section 2 the implications of the results of behavioral finance are studied. In Section 3 we discuss the incompleteness of insurance markets. Section 4 presents techniques for accounting model risk and section 5 concludes.

2. BEHAVIORAL FINANCE

"The key forces by which markets are supposed to attain efficiency, such as arbitrage, are likely to be much weaker and more limited than the efficient market theoretists have supposed."

Andrei Shleifer (2000)

Behavioral finance assumes that all individuals are not fully rational and are subject to cognitive biases in the decision-making. Psychology has exhibited many systematic deviations from rationality in the human judgement. The field of behavioral finance is very broad. Good surveys about the discussion and empirical evidence of this area are e.g. Barberis & Thaler (2003) and Shleifer (2000). The central argument of behavioral finance states that real-word arbitrage is risky and therefore limited.

An example of a behavioral model in the financial market is Kirman's (1993) model for herding behaviour. This model is based on entomologists' observations of the behaviour of ants, when they are faced with two identical food sources. They have a tendency to concentrate more on one of these, but after a period they would turn their attention to the other. The ant model will generate bubbles and crashes, which are characteristics properties of the stock market. The behavioral finance theory rests on two major foundations. The first is the limited (real word) arbitrage and second is investor sentiment.

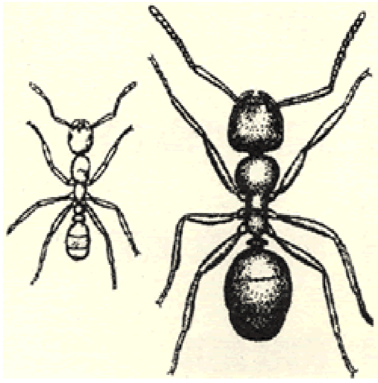

FIG. 1. Even ants generate bubble and crash patterns in their herding behavior.

2.1. The limit of real-world arbitrage

Arbitrageurs are investors, who have rational expectations about the future cash flows of risky asset and try to exploit so called noise traders' incorrect beliefs. They are usually professional portfolio managers, who manage money for other people and their investment horizon is usually short. An investment strategy which tries to exploit mispricing due to irrational noise traders can be very risky, because noise traders' beliefs can change to even more extreme in the short run and an arbitrageour has to liquidate his position before the stock price changes towards its equilibrium.

In the model of Shleifer & Vishny (1997) there are two kinds of investors: noise traders and arbitrageurs. Noise traders are individuals who have erroneous beliefs about the future returns of risky asset. Their demand for risky assets is determined by short-run performance of these assets.

According to the model of Shleifer & Vishny, resources of arbitrageours are most limited in the situation where mispricing is largest in the asset market. This consequence implies that mispricing can grow very severe, which is essential for risk management.

The conditions of arbitrage-free pricing do not hold perfectly even in a the financial market where transaction costs are relatively small and information is easily available. Shleifer & Vishny have shown that arbitrage by rational investors cannot fully eliminate influence of irrational noise traders in the market prices because resources of arbitrageours are limited due to risk aversion, short horizon and agency problems. According to their model, noise traders have a substantial impact on the market price and investment risk.

There is a further important source of risk for an arbitrageur, which he faces even when securities do have perfect substitutes. Even with two securities that are fundamentally identical, need not be perfect substitutes from the point of view of the arbitrageur who cannot literally covert one security to another. This risk comes from the possibility that mispricing becomes worse before it disappears.

2.2. Investor sentiment

Investor sentiment is a theory of how real-world investors actually form their demands for securities. Combined with limited arbitrage, a theory of investor sentiment may help to generate preditictions about security prices and returns. Risk created by the unpredictability of investor sentiment significantly reduces the attractiveness of arbitrage.

Daniel et al. (1998) have proposed an interesting model for price formation in the asset market. They assume that investors overreact to private information and underreact to public information. These assumptions are based on two well-known psychological biases: overconfidence and biased self-attribution. In their model investors are overconfident about the precision of their private information.

Biased self-attribution means that people overreact to information that confirm their existing beliefs and underreact to information that does not. In the model of Daniel et al. (1998) individuals suffer from biased self-attribution when they process public information: they emphasize more public information which is consistent with their private information than public information which contradicts their private information.

2.2.1. Implications for internal modelling

An implication of behavioral finance is so-called price bubble, in which prices go up without much news just because noise traders are chasing the trend (so-called positive feedback investment strategy). The following crash constitutes a considerable risk. A characterization of large crash on the stock market is that its financial impact is considerable, hence, by definition, bubbles and crashes are of profound importance to risk management of investment portfolios.

Several practicable statistical risk models have been presented for this phenomenon. For instance, Kaliva & Koskinen (2007) have presented a simple statistical model for bubbles and crashes in the stock market. They have proposed a two-regime model of stock price dynamics in which the process behaves like a random walk in one regime while it is like an error-correcting process in the other. In their model the probability to prevail in the bubble regime is an increased function of the inflation rate.

There is no single unifying model in behavioral finance. All deviations from rationality in the human judgement, what psychology is exhibited, cannot be incorporated into a single well-specified model. We agree with Fama (1998) that behavioral finance cannot offer a single specific alternative to the EMH. But we also suggest that in investment management it is not necessary to operate only by a single model such as in the case of typical physical problem. A more reasonable approach is to combine several different models.

When there is no arbitrage opportunity the central proposition in corporate finance is the Modigliani-Miller (1958) theorem, which implies that the financial structure is irrelevant. One can deduce e.g. that investment in stocks do not reduce pension cost (see e.g. Exley, 2003). Instead, when arbitrage is risky the Modigliani - Miller theorem does not apply directly.

3. INCOMPLETENESS OF MARKET

"The parameter estimation of the pricing models represents the real heart of insurance pricing."

Jon Holtan (2007)

The main examples of idealised arbitrage-free complete market models in financial applications are the Black-Scholes model and its discrete counterpart, the Cox-Ross-Rubinstein model. These models include many restrictions, e.g. unrealistic distribution of asset returns, no market frictions, no default risk, short selling is allowed without restrictions, competitive markets, rational agents, no arbitrage. However, more complex and realistic approaches, which for example allow for transaction costs, or stochastic volatility or interest rates, or 'jumps', generally lead to incomplete market models, see e.g. Embrechts and Meister (1997).

Derman and Taleb (2005) give examples why the dynamic replication principle underlying the Black-Scholes approach cannot always be realistic in practice. They also point out the risk involved when instruments that do not allow for dynamic replication are still priced using the standard financial derivative models and software.

3.1. Incompleteness of insurance market

In insurance applications we face incompleteness for instance when trying to find a replicating portfolio, i.e. a portfolio of assets giving similar cash-flows to the liabilities, for the financial parts of certain life insurance products. One such line of business would be a long-term savings contract with death benefit, surrender option and profit-sharing feature. There are several reasons that may lead customers to act differently than the efficient market and rational behavior hypotheses would expect, for example:

• long-term product structures with profit participating features are quite common, and the market consistent valuation of these complex options can be difficult. For instance in Joshi (2003, chapter 15.9) it is noted that the valuation of complex (exotic) options can be very much model dependent.

• Information that would allow credible comparisons is not easily and cheaply available (products are different and the final analysis of the price can in many cases be made only afterwards because the profit-sharing rules are not stated explicitly). Furthermore, most individuals are not in a position to analyse and deal insurance products effectively.

• Market is far from frictionless as there are restrictions, tax issues, and transaction costs involved when changing insurance company. For example in Finland pension insurances policies have certain tax benefits which do not allow a client to take his accumulated savings (surrender value) out before the retirement age. In ordinary life assurance products this is possible but there are penalties or other restrictions in most cases. These frictions are not easy to value.

• Individuals seem to have different behaviour in reality (as can be observed from empirical surrender-rates) than theoretical utility-maximisation etc models assume (e.g. the pricing models for American options).

• Insurance risk processes include jumps that cannot typically be fully hedged.

According to Holtan (2007) the non-life insurance market compared the with financial derivatives markets is most often characterized by: higher transaction costs, higher and differentiated risk aversion, less price rationality and sensitivity, less or no dynamic hedging, less decision speed and fewer players.

From the above we can conclude that approximation errors of the idealistic, no-arbitrage and complete market assumption based models have to be generally larger in insurance applications than in finance. The difficulties relating to incomplete market can be quite serious. The first problem is the lack of observable market prices for most insurance liability portfolios. Therefore the value of these liabilities has to be based on a suitable model.

In IAA (2004), a distinction has been made between 'Type A' and 'Type B' risks. It is proposed that to the extent possible the value of policy liabilities (or technical provisions) should be based on a replicating portfolio of traded assets (these are termed 'type A risks'). The remaining parts, e.g. insurance underwriting risks and non-hedgeable risks (which are called 'type B risks'), should be valued by marking to model. This proposal however leaves undetermined how to choose the approach when marking to model Type B risks, but further development work regarding this issue is under way in the IAA. In Solvency II this issue has been addressed by requiring that non-hedgeable risks should be valued using a best estimate plus risk margin approach, for details see e.g. CEIOPS (2006a, b).

3.2. Implication for internal modelling

Incomplete market models by definition include some contingent claims that cannot be replicated from the other basic securities of the model, and therefore by the fundamental theorems the prices are not unique as there are several (in fact infinite) risk-neutral measures. Consequently the process of choosing the risk-neutral measure amongst many possible alternatives is a key issue. Another one is the construction of hedging strategy for the non-attainable contingent claims in a reasonable way.

Unfortunately there is not yet a decisive approach and method to deal with these problems, but this is an area of active current research. Several approaches have been suggested, for example based on optimizing quadratic or other types of risk functions or utility functions, or using other techniques such as super-replication or embedding in complete markets. For an overview and references see Bingham and Kiesel (2004).

An interesting alternative is the so-called good deal approach in which bounds for option prices are derived by restricted optimization; the algorithm searches for positive (stochastic) discount factors that have limited volatility (see Cochrane 2001 for discussion and references).

In Möller-Steffensen (2007) diversifiable and non-diversifiable mortality risk and certain incomplete market methods such as super-hedging and variance minimization are discussed. We can conclude that mark-to-model valuation seems unavoidable in insurance applications, and that it is very important to also assess modelling errors.

In Hardy (2003) the issue of unhedged liability has been addressed by adding a separate capital requirement to the hedge cost. This includes amounts for discrete hedging error, transaction costs and model error. Thus a combination of financial engineering and actuarial approaches is recommended to deal with incomplete market valuation problems.

As Norberg (2006) remarks in pensions there are numerous unsolved issues that cannot be resolved by financial economics, but will require actuarial modelling: fluctuations and trends in longevity, population dynamics and state pensions etc.

4. MODEL RISK

"For the purpose of forecasting risk all models are subjective and all data incomplete."

Carol Alexander (2005)

The model risk arises in a situation where the results and decisions emerging from an analysis are sensitive to the choice of model and the there is uncertainty about the suitable model. In finacial modelling it arises as a particularly severe problem under the assumption that the conditions of arbitrage theory do not hold and all participants in the market are not fully rational. Hence the model uncertainty plays an important role in the incomplete insurance market. Cairns (2001) states that most actuarial problems related to model uncertainty fall into a situation where there is a range of models which may provide a proxy for a more complex reality about which the modeller has little prior knowledge. A possible remedy for model risk is a Bayesian approach, which provides a coherent framework to make inferences in the presence of model uncertainty.

4.1. Bayesian averaging approach

Internal modelling should be forward-looking, because assessing solvency is mainly about forecasting the future based on past experience and available information. The idea that forecast performance can be improved by combining forecasts from different models dates back at least to the seminal paper of Bates and Granger (1969). Bayesian model averaging (e.g. Hoeting et al. (1999)) provides plausible and statistically well-founded techniques for accounting for this model uncertainty. In this technique each considered model is weighted by its posterior probability.

For financial modelling the Bayesian model averaging is employed e.g. by Avramov (2002). He has found that incorporating model uncertainty can substantially weaken stock return predictability and makes stocks less attractve for longer-horizon investors. He has also found that model uncertainty appears to be more significant than parameter estimation uncertainty in financial risk management.

4.2. Bayesian averaging of classical estimates (BACE)

The pure Bayesian approach is a theoretically coherent way to account for model uncertainty. A drawback of this approach is that it requires specification of the prior distribution of all the relevant parameters conditional on each possible model, as Sala-i-Martin et al. (2004) have pointed out. From the perspective of the practical actuary perhaps a more feasible technique for accounting for model uncertainty is a Bayesian Averaging of Classical Estimates (BACE) of Sala-i-Martin et al. (2004). This method is used only in the case of linear regression models, but it is easy to extend to the case which includes also non-linear models.

In the BACE technique the chosen models are estimated by the classical maximum likelihood methods and the weights of the models are based on the Bayesian Information Criterion (BIC) (Schwarz (1978))

$$\mathrm{BIC}(\beta) = -2\log(L(y;\beta)) + k\log(n),$$

where $L(y;\beta)$, k and n are the likelihood function, size and the number of observation of the model. The same model selection criterion can also be derivate from a non-bayesian base (Rissanen (1978). The mixing probability π_j of the model M_j from the class of models $M_{\{1,\dots,m\}}$ is determined by the formula

$$\pi_j = \frac{P(M_j)\exp(-\mathrm{BIC}(j)/2)}{\sum_{i=1}^{m} P(M_i)\exp(-\mathrm{BIC}(i)/2)}$$

where $P(M_i)$ is a prior probability of the model M_i. Sala-i-Martin et al. (2004) have shown that this weighting method can be derived as a limiting case of a standard Bayesian approach under a very general condition.

In the BACE approach the prior probability $P(M_i)$ of model M_i is determined by size k_i of the model: a simple model has higher prior than a complicated model. The prior weight of model M_i is $\bar{p}^{-k_i}$ where $0 < \bar{p} \leq 1$ is a hyperparameter, which is determined by the modeller's attitude toward to complexity. Its inverse $\frac{1}{\bar{p}}$ can be called the "complexity aversion coeffient". This is the only prior parameter which the BACE technique requires to specify. How close to the parameter $\bar{p}$ is to zero so high weight of the simple models have. In the case of a linear regression model the parameter $\bar{p}$ can be intepreted as a prior probability that a single regressor is being included in the model (Sala-i-Martin et al. (2004)).

Using the proposed prior probabilities the mixing probability π_j of the model M_j is determined by the formula

$$\pi_j = \frac{\bar{p}^{-k_j}\exp(-\mathrm{BIC}(j)/2)}{\sum_{i=1}^{m} \bar{p}^{-k_i}\exp(-\mathrm{BIC}(i)/2)}.$$

When $\bar{p} = 1$ the prior probabilites of each possible model is the same. Techniqally, the parameter $\bar{p}$ can be also be over one if the modeller want to favour complicated models.

In the simulation studies the BACE technique can be applied using the following procedure: it picks up randomly a process from the selected class of models with mixing probabilities in the case of each simulation path and then it simulates realization of this model with estimated parameters. The main difference between this method and the fully Bayesian approach is that this method does not take account of parameter estimation uncertainty.

5. CONCLUSIONS

"Abonding the established actuarial precautions and relying entirely on the financial markets to come to our rescue, is no responsible strategy. The way forward lies somewhere in the middle."

Ragnar Norberg (2006)

This paper surveyed the assumptions of arbitrage-free modelling and studied their consequences for the use of internal models in insurance. From this perspective, the key findings in the literature of behavioral finance are:

- Real-world arbitrage is risky;
- Investors are not fully rational;
- There is no single unifying model in behavioral finance;
- It is possible to make certain predictions about security prices;
- Pricing bubbles exist.

In the insurance market a special difficulty can be quite serious: the lack of observable market prices for most insurance liability portfolios, because of the slow securitization process.

The consequences for internal modelling include the following:

- It is common practice in the derivative market to calibrate the model by fitting the parameters to the market. It is assumed that an arbitrage-free market is a good approximation of reality. An incomplete market situation is quite common in insurance applications. Hence a combination insurance mathematics and financial economics is needed.
- Thus in many insurance applications it is not possible to find a unique model and price, which leads to more complex and subjective modelling techniques and assumptions.
- Pricing bubbles and a following crash constitutes a considerable risk. This risk can be analyzed by statistical models.
- From an actuarial and mathematical point of view one should pay suffcient attention to verification of the assumptions lying behind the financial models as Rantala (2006) remarks.

In the face of model risk, rather than to base decisions on a single selected "best" model, the modeller can base his inference on oan entire set of models by using model averaging.

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