

# The long-term risk caused by the stock market bubble

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ABSTRACT. In this paper we quantify the risk caused by the crash of a pricing bubble in the US stock market by utilizing a recently introduced econometric bubble model. The skewness and kurtosis are shown to vary widely with the price-dividend ratio. Simulation experiments quantify how the moments and VaR of the predictive distribution depend on the holding period, the price-dividend ratio and inflation. This information is useful in deciding on market timing and needed risk capital. In addition the analysis of higher moments support the old wisdom that stocks are a more attractive investment in the long run than in the short run.

KEYWORDS. Dividend-Price Ratio, Market Risk, Market Timing, LMARX Model, Risk Capital, Stock Return Distribution, Time Varying Risk, Value-at-Risk.

# 1 Introduction

The aim of this paper is to study the implications of pricing bubbles for long-term risk management in the stock market. A conventional wisdom in investment practice is that the longer horizon is the more investors benefit from investing in stocks. Usually, this is based on an assumption that stock returns are mean reverting over the long horizon, thus the variance of the cumulative log returns increases slower than the length of the holding period. The mean reversion of stock returns also plays an important role in many actuarial investment models (see, e.g. Wilkie (1995)). While most actuarial investment models are based on an assumption of mean reversion, empirical evidence of mean reversion is controversial (see, e.g. Campbell et al. (1997), Lanne (2002), Jorion (2003) and Campbell and Yogo (2006)). Using a variance ratio for thirty countries Jorion (2003) cannot find evidence that stocks are mean reverting.

A pitfall of empirical analysis which is based on variance ratio test is that it concentrates only on the two lowest moments. It is a plausible method when the conditional distribution of investment returns is log-normal. In the presence of stock market bubbles this assumption does not necessary hold even in the long run. Bubbles can generate a small probability of very high investment returns, thus the conditional distribution of cumulative log returns can be positively skewed.

We refer by the term "bubble" to a time period when the formation of asset price cannot be explained by rational expectation of forthcoming cash flows. A typical property for bubbles is rapidly increasing asset price with a subsequent crash. Economists have provided different explanations for stock market bubbles. In a behavioral model of Shleifer and Vishny (1997) the stock prices are determined by interaction between irrational noise traders and rational arbitrageurs. Noise traders are individuals who have erroneous beliefs about future returns of risky assets. Rational arbitrageurs cannot fully eliminate the impact of noise traders on stock prices because the resources of arbitrageurs are limited due to risk aversion, short horizon, an agency problem (Shleifer and Vishny) and synchronization risk (Abreu and Brunnermeier (2003)).

The analysis in this paper uses a version of the empirical bubble model proposed by Kaliva and Koskinen (2007). The model is based on the logistic mixture autoregressive model with an exogenous variable (LMARX model) (Wong and Li (2001)), where the probability of regime is a direct function of an observable variable. The process has two regimes: a bubble regime and a fundamental (correction) regime. This approach includes the assumption that the risk of the stock investment

is not constant over time. This kind of dynamics can be interpreted in the context of the aforementioned behavioral model of Shleifer and Vishny (1997).

An important assumption behind the model of Kaliva and Koskinen (2007) is that noise traders participation in the stock market is negatively related to inflation rate. Modigliani and Cohn (1979) have advanced a hypothesis that people suffer from the so-called "inflation illusion". According to this hypothesis, market participants discount real dividends by a nominal interest rate, which has a strong dependence on inflation. Also recently Ritter and Warr (2002) have found evidence supporting that hypothesis. Secondly, Shiller (1997) has studied public attitudes toward inflation using a survey method. He concluded that people associate high inflation rate with economic disarray and lower purchasing power. These studies imply that inflation is negatively related to the log dividend-price ratio, and that a low inflation rate has a tendency to create optimistic expectations, or even outright bubble behavior. Kaliva and Koskinen emphasize the importance of inflation and price-dividend ratio when assessing investment risk.

Previously, van Norden and Shaller (1999) have presented an econometric model for bubbles and crashes where the probability of a crash depends positively on the size of bubble. Psaradakis et al. (2004) have studied bubbles by a Markov error-correction model when stock prices and dividends are cointegrated in one regime and move independently in the other regime. These models are based on an assumption of rationality of bubbles.

Kaliva and Koskinen (2007) used a model where bubbles are caused by irrational noise traders (Shleifer & Vishny (1998)). They applied the bubble model to monthly U.S. stock market data. Instead, here we estimate a version of their model with quarterly data since our goal is to analyse long-term risks and monthly variation is not our interest. The used quarterly version is especially suitable for risk management because its simplicity compared to the monthly version (and many other econometric bubble models) makes its use and interpretation easy. The importance of simplicity for risk modelling is emphasized by e.g. Down (2005, page 224 ) who argues that one should choose the simplest reasonable model for combating model risk in market risk measurement. We show analytically that a characteristic property of the model is its time-varying conditional moments for return distribution. The model is used to perform simulations for quantifying the long-term risk.

The paper is organized as follows. Section 2 introduces the used bubble model, section 3 presents empirical results, section 4 discusses simulation results. The final section concludes.

## 2 The Bubble Model

We model the bubble phenomenon by a version of the two-regime model proposed by Kaliva and Koskinen (2007), where like in the model of Psaradakis et al. (2004) in one regime the changes in the stock price are independent of dividends, and in the other regime the stock price change depends on the log price-dividend ratio.

The first regime can be interpreted as a state where the stock price is determined mainly by noise traders. In that regime the stock price does not react to information about price-dividend ratio (market fundamental) at all. In the other regime the stock price changes towards its equilibrium keeping the price-dividend ratio stable in the long run. A switch from the bubble regime to the fundamental regime can generate a stock market crash.

In order to control the weak short-term autocorrelation of monthly stock returns Koskinen and Kaliva (2007) assume that the log price change depends also on its lagged value. Because we use here quarterly data this term can be omitted and a simpler model is obtained. In regime 1 the log difference of the stock price  $p_t$  is independent of dividends but depends on inflation  $i_t$  ( $\varepsilon_t \sim NID(0, 1)$ )

$$(1) \quad \Delta p_t = a_1 + c_1 i_t + \sigma_1 \varepsilon_t, \quad (\text{Bubble Regime})$$

and in regime 2, the change in the log stock price depends on inflation and log price-dividend ratio  $y_{t-1}$  through the linear form

$$(2) \quad \Delta p_t = a_2 - b y_{t-1} + c_2 i_t + \sigma_2 \varepsilon_t. \quad (\text{Fundamental Value Regime})$$

In the model of Kaliva and Koskinen (2007) the probability of being in a fundamental value regime depends on inflation. Based on these results we suggest a model where the probability of being in the fundamental regime  $\pi_t$  is a function of inflation.

$$(3) \quad \pi_t = \Phi(\lambda_0 + \lambda_1 (\sum_{k=0}^3 i_{t-k})^2).$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

Traditionally risk management concentrates on the changes of the conditional volatility (see e.g. Alexander (2001), but for instance McNeil et al. (2005) have demonstrated the importance of higher moments and extremal events. The relative simplicity of the model (1-2) makes it easy to show how higher central moments vary.

Let  $\Omega_t = \{i_{t-j}, y_{t-k} \mid 0 \leq j < \infty, 1 \leq k < \infty\}$  be the information set consisting of the data on inflation up to time  $t$ , and of the price-dividend ratio up to time  $t-1$  and let  $\pi_t$  be the probability of fundamental regime

at time  $t$ . For the model (1-2) the conditional density of price difference  $\Delta p_t$  is a mixture of two normal distributions

$$f(\Delta p_t|\Omega_t) = (1 - \pi_t)\phi(\Delta p_t, \mu_{t1}, \sigma_1^2) + \pi_t\phi(\Delta p_t, \mu_{t2}, \sigma_2^2),$$

where  $\mu_{t1} = a_1 + c_1i_t$ ,  $\mu_{t2} = a_2 + c_2i_t - by_{t-1}$  and  $\phi(x, \mu, \sigma^2)$  is the density of  $N(\mu, \sigma^2)$ -variable. Hence, the conditional expectation of variable  $\Delta p_t$  based on information set  $\Omega_t$  is

$$E(\Delta p_t|\Omega_t) = (1 - \pi_t)\mu_{t1} + \pi_t\mu_{t2}$$

and the conditional variance is

$$\text{Var}(\Delta p_t|\Omega_t) = (1 - \pi_t)\sigma_1^2 + \pi_t\sigma_2^2 + 2\pi_t(1 - \pi_t)(\mu_{t1} - \mu_{t2})^2.$$

The conditional skewness and the conditional kurtosis can be written as

$$\text{Skewness}(\Delta p_t|\Omega_t) = \left\{ \pi_t(1 - \pi_t)(\mu_{t1} - \mu_{t2}) \left[ 3(\sigma_1^2 - \sigma_2^2) \right. \right. \\ \left. \left. (\pi_t^2 - (1 - \pi_t)^2)(\mu_{t1} - \mu_{t2})^2 \right] \right\} \text{Var}(\Delta p_t|\Omega_t)^{3/2}$$

and

$$\text{Kurtosis}(\Delta p_t|\Omega_t) = \left\{ 3(1 - \pi_t)\sigma_1^4 + 3\pi_t\sigma_2^4 \right. \\ \left. + (1 - \pi_t)\pi_t(\mu_{t1} - \mu_{t2})^2 [6(\sigma_1^2\pi_t + (1 - \pi_t)\sigma_2^2) \right. \\ \left. + (\pi_t^3 + (1 - \pi_t)^3)(\mu_{t1} - \mu_{t2})^2] \right\} \text{Var}(\Delta p_t|\Omega_t)^2.$$

These formulae tell us that the moments vary with inflation and the price-dividend ratio through the quantities  $\pi_t$  and  $\mu_{t1} - \mu_{t2}$ . This is a feature where model differs fundamentally from traditional stock return models.

### 3 Empirical Results

We apply the bubble model (1-2) to the US post-war stock data period from 4/1945 to 4/2007. The data is from Standard and Poor's and it is available on the home page of Shiller (<http://www.econ.yale.edu/~shiller>). The US inflation is measured as the log difference of consumer price index CPI-U (Consumer Price Index-All Urban Consumers) published by the US Bureau of Labor Statistics. The quarterly data is used instead of more frequent sampling since we concentrate on long-term risk. The estimation results and the diagnostic checks are reported in Appendix A.

### 3.1 Inflation and dividends

In order to investigate total real returns we also need a model for inflation and dividends. According to Campbell and Shiller (1988), the log price-dividend ratio has no power to forecast the growth of dividends. To test a null hypothesis that price-dividend ratio does not Granger-cause the real dividend growth we estimate a VAR(4) -model for the log price-dividend ratio, the real dividend growth and inflation. The test results reported in Table 1 show that the price-dividend ratio has no power to forecast real growth of dividends or inflation.

Table 1: Cranger causality Wald test statistics

| Hypothesis                                | Test statistics | DF | P-value |
|---|-----------------|----|---------|
| P/D -ratio $\Rightarrow$ real div. growth | 5.211           | 4  | 0.266   |
| P/D -ratio $\Rightarrow$ inflation        | 4.954           | 4  | 0.292   |
| real div. growth $\Rightarrow$ inflation  | 9.906           | 4  | 0.042   |
| inflation $\Rightarrow$ real div. growth  | 7.753           | 4  | 0.101   |

Due to some problems with quarterly data and since we think that quarterly patterns of inflation and dividend payment are not most crucial factor in risk management, the model for the real dividend growth is estimated on an annual basis. In the simulation we assume that inflation and dividends in each year are equally distributed. For inflation we arrived at the AR(4) -model where the mean of annual inflation is approximately 4%. The real dividend growth is modelled by a MA(1) -model with positive MA(1) -coefficient. In this model the average growth rate of real dividends is approximately 2%. The correlation coefficient of residuals by these model is  $-0.18$ , which is insignificant at 5% level.

### 3.2 Estimation, testing and diagnostics

We estimate stock price process  $\Delta p_t$  using the log-likelihood function and standard numerical estimation techniques. The model to be estimated can be written as

$$\begin{aligned}\Delta p_t &= a_1 + c_1 i_t + \sigma_1 \varepsilon_t | s_t = 0 \\ \Delta p_t &= a_2 - b y_{t-1} + c_2 i_t + \sigma_2 \varepsilon_t | s_t = 1 \\ \varepsilon_t &\sim NID(0, 1),\end{aligned}$$

where  $s_t$  is the nonobservable Bernoulli( $\pi_t$ )-distributed regime indicator variable. The probability of being in the fundamental value regime,  $\pi_t$ , is observable and determined by formula  $\pi_t = \Phi(\lambda_0 + \lambda_1(\sum_{k=0}^3 i_{t-k})^2)$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

The influence of the change of inflation on the stock price is the same in both regimes, thus  $c_1 = c_2 = c$ . The proposed model is tested against more general alternative models where a) a switching probability depends on inflation and the log dividend price ratio  $y_{t-1}$  b) a switching probability depends on inflation the lagged change of the stock price  $p_{t-1}$  c) the change of the log stock price depends on the log price-dividend ratio in both regimes. In each case the likelihood-ratio statistics is lower than the critical value 3.84 of the  $\chi^2(1)$ -distribution at the 5% level and the null hypothesis of the simpler model cannot be rejected. Thus we arrived at a model where the probability of being in a particular regime depends only on the inflation rate.

Quantile residuals (Dunn and Smyth (1996)),  $u_t$ , are based on the fact that the inverse normal distribution transformations of standard uniform variables  $u_t = \Phi^{-1}(v_t)$  are standard normal variables themselves. We tested normality, autocorrelation and conditional heteroskedasticity of quantile residuals. The normality is tested by the Jarque-Bera test. The value of the test statistics 2.55 is lower than the critical value 5.99 at the 5% level. Neither there is any evidence of serial correlation or conditional heteroskedasticity of quantile residuals (see Appendix A).

### 3.3 Model interpretation

The ex-post probabilities to stay in the fundamental regime are plotted in Figure 1 and the probability of fundamental regime with respect to the inflation rate in Figure 2. It can be seen that the process is much more often in the bubble regime than in the fundamental value regime. This means that most of the time the stock price does not react to the information on dividends. This is consistent with the short-term unforecastability of the stock price.

A regime switch from the bubble regime to the fundamental regime can cause a jump in the stock price. The sign and magnitude of the jump is mainly determined by the log price-dividend ratio of the previous quarter  $y_{t-1}$ . When annual inflation is (typical) 4 per cent formula (2) implies that if for the log price-dividend ratio is large enough i.e.  $y_{t-1} > 3.2$ , a jump is more likely negative than positive (and vice versa). In addition, formula (1) implies that  $a_1 = 0.03 > E(\Delta d_t) = 0.02$ . Hence, the stock price grows faster than the dividend most of the time generating a bubble. It means that the jump of the process after a switch from the bubble regime to the fundamental value regime is more likely to be negative (causing market crash or bubble burst) than positive. A bubble usually starts slowly, and gradually builds up to the peak over



a period of several years. After it has peaked, prices almost always fall during short time period.

## 4 Simulation Results

We study by simulation the conditional distributions of the cumulative log real total returns in different holding periods: 1/4, 5 and 20 years. All simulations were repeated  $10^4$  times. Dividends are assumed to be reinvested at the end of each year. The quarterly returns are calculated by assuming that the dividends of each year are equally distributed.

### 4.1 Risk Analysis

Simulations are done in three typical cases: low valuation with high inflation, moderate valuation with moderate inflation and high valuation with low inflation. Three initial values (12%, 4%, 0%) for annual inflation and three initial values (3.0, 3.5, 4.0) for log price-dividend ratio are considered.

In the case of low valuation ( $\log(p_0/d_0) = 3$ ) and high inflation (12%) simulation results are represented in Table 1. Stocks are in this situation much more attractive for a long-term investor than a short-term investor. The annualized geometric mean of real return is  $-2.0\%$  in the quarterly year holding period,  $5.2\%$  in the 5-year holding period and  $8.6\%$  in the 20-year holding period. The mean reversion of stock returns is also quite strong: the variance ratio is  $32.5\%$  in the 5-year holding period and  $34.3\%$  in the 20-year holding period.

Table 2: Logarithmic cumulative real returns: low valuation and high inflation

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | -0.005   | 0.254   | 1.642    |
| Std            | 0.084    | 0.214   | 0.440    |
| Skewness       | -0.008   | -0.008  | 0.084    |
| Kurtosis       | 3.000    | 3.110   | 3.079    |
| VaR(5%)        | -0.143   | -0.096  | 0.930    |
| VaR(1%)        | -0.202   | -0.249  | 0.627    |

For moderate valuation ( $\log(p_0/d_0) = 3.5$ ) and moderate inflation (4%) simulation results are represented in Table 2. Also in this case stocks are more a attractive investment in a long holding period than a short holding period. According to our simulations there is weak mean reversion in the 20-year holding period but not in the 5-year holding

period. The variance ratio is 109.8% in the 5-year holding period and 68.2% in the 20-year holding period. However, skewness of log returns is significantly negative in the case of the quarterly holding period and slightly positive in the case of a longer holding period. These results imply that risk management which concentrates only on the two lowest moments can underestimate investment risk in the short run and overestimate investment risk in the long run.

Table 3: Logarithmic cumulative real returns: moderate valuation and moderate inflation

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | 0.011    | 0.206   | 1.041    |
| Std            | 0.067    | 0.314   | 0.495    |
| Skewness       | -0.793   | 0.192   | 0.254    |
| Kurtosis       | 4.255    | 2.847   | 3.146    |
| VaR(5%)        | -0.118   | -0.292  | 0.262    |
| VaR(1%)        | -0.191   | -0.457  | -0.021   |

The simulation results for high valuation ( $\log(p_0/d_0) = 3.5$ ) and low inflation (0%) are given in Table 3. In this case stocks are more attractive in the short holding period than in the long holding period. The annualized geometric mean of real return is 11.9% in quarterly year holding period, 5.9% in the 5-year holding period and 2.7% in the 20-year holding period. The variance ratio of log returns is 177.2% in the 5-year holding period and 99.5% in the 20-year holding period. Thus, log returns are not mean reverting even in the 20-year holding period. In the 5-year holding period these are actually strongly mean averting.

Table 4: Logarithmic cumulative real returns: high valuation and low inflation

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | 0.028    | 0.285   | 0.540    |
| Std            | 0.064    | 0.381   | 0.571    |
| Skewness       | -1.105   | -0.144  | 0.442    |
| Kurtosis       | 5.622    | 2.795   | 3.388    |
| VaR(5%)        | -0.093   | -0.375  | -0.321   |
| VaR(1%)        | -0.190   | -0.614  | -0.646   |

The results are consistent with studies that variance of stock returns is proportional to the length of the holding period: a mean reversion of the proposed process is quite weak in the case of moderate and high

valuation ratios. A second interesting result is that the shape of the predictive distribution depends heavily on the investment horizon. The substantial positive skewness in the twenty-year predictive distribution is a characteristic feature for high and moderate valuation. Usually the stock return model attains normality in a few months but the bubble model may produce nonnormal predictive log return distribution for several years.

## 4.2 Sensitivity Analysis

In order to investigate how sensitive the results are to parameter uncertainty we have generated new parameter values from a multivariate normal distribution  $N(\mu, \Sigma)$ , where parameter  $\mu$  is estimated values of the bubble model and covariance matrix  $\Sigma$  is inverse of Hessian. This experiment has been repeated five times for moderate valuation and inflation (3.5, 4). Resulted parameter values are give in Table 5 and the conditional moments of cumulative log real returns and value-at-risk measures in the three holding periods are represented in Appendix B.

Table 5: Parameter values in scenarious

| Scenario    | 1      | 2      | 3       | 4      | 5      |
|-------------|--------|--------|---------|--------|--------|
| $a_1$       | 0.0400 | 0.0329 | 0.0432  | 0.0442 | 0.0359 |
| $\sigma_1$  | 0.0513 | 0.0532 | 0.0459  | 0.0470 | 0.0486 |
| $a_2$       | 0.4082 | 0.5164 | 0.3182  | 0.6024 | 0.3888 |
| $b$         | 0.1267 | 0.1666 | 0.0968  | 0.1826 | 0.1172 |
| $\sigma_2$  | 0.0863 | 0.0741 | 0.0891  | 0.0969 | 0.0804 |
| $c$         | -0.928 | -0.078 | -1.491  | -1.615 | -0.962 |
| $\lambda_0$ | -1.281 | -1.537 | -1.6362 | -1.781 | -1.257 |
| $\lambda_1$ | 214.1  | 377.1  | 636.2   | 520.3  | 492.0  |

The experiment shows that value-at-risk measures are slightly sensitive to the influence of inflation. The investment risk is smallest in the case of the first scenario where the influence of inflation on the probability of fundamental regime is smallest. In this scenario the influence of inflation within regimes is smaller than in the case of the estimated model. Value-at-risk measur of the investment risk is largest in the scenario 3 where the influence of inflation on the probability of fundamental regime is largest and inflation has large influence within regimes.

The conditional skewness is relatively sensitive to parameter uncertainty. The dependence between the length of the holding period and the conditional skewness is strongest in scenario 4. In this case the probability of the fundamental value is lowest during low inflation. The

coefficient of log dividend price ratio in the fundamental regime is highest in this scenario. Thus, crashes occur quite rarely but these can be quite large. The conditional skewness in the 20-year holding period seems to be large in those scenarios where the variance of 20-year cumulative returns are also large.

## 5 Conclusions

A simple econometric model has been used for quantifying the long-term risk caused by a pricing bubble in the stock market. Bearing in mind that conditional skewness is to some extent sensitive to parameter errors we have found evidence that higher moments depend strongly on the price-dividend ratio. We conclude that the price-dividend ratio is economically more significant than the studies on the linear predictability of the stock returns indicate.

The results have implications for risk management. One is that an investor who concentrates only on the two lowest moments may underestimate the investment risk in the presence of a pricing bubble. Second, both price-dividend ratio and inflation are potentially important in strategic investment timing decisions and in risk capital quantification.

Another interesting finding is that the conditional (log real) return distribution is clearly positively skewed with moderate and high valuation ratios over the long horizon. This result indicates that empirical studies (e.g. Jorion (2003)), which are based on the variance ratio test overestimate the long-term risk of the stock investment.

Our results give some support for the conventional wisdom that the longer the horizon is the more investors benefit from investing in stocks: the negative conditional skewness of short-term returns and positive conditional skewness of long-term returns imply that stocks are a more attractive investment in the long run than in the short run.

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## Appendix A: Estimation Results

Table 6: Estimates of dividend process  $\Delta d_t$ .

| Regression coefficient | Constant | MA(1)  | St.deviation |
|------------------------|----------|--------|--------------|
| Estimated value        | 0.0215   | 0.5685 | 0.0498       |
| Standard errors        | 0.0125   | 0.1009 | 0.0026       |

Table 7: Estimates of inflation process  $i_t$ .

| Regression coefficient | Constant | AR(1)  | AR(2)   | AR(3)   | AR(4)  | St.deviation |
|------------------------|----------|--------|---------|---------|--------|--------------|
| Estimated value        | 0.0392   | 0.6485 | -0.2499 | -0.1591 | 0.4798 | 0.0241       |
| Standard errors        | 0.0135   | 0.1087 | 0.1323  | 0.1403  | 0.1398 | 0.0024       |

Table 8: Estimates of the Bubble Model.

| Parameter   | Estimated value | Standard errors |
|-------------|-----------------|-----------------|
| $a_1$       | 0.0425          | 0.0058          |
| $\sigma_1$  | 0.0462          | 0.0041          |
| $a_2$       | 0.3795          | 0.1072          |
| $b$         | 0.1155          | 0.0339          |
| $\sigma_2$  | 0.0840          | 0.0081          |
| $c$         | -1.2037         | 0.5225          |
| $\lambda_0$ | -1.30           | 0.34            |
| $\lambda_1$ | 383.66          | 167.17          |

We test serial correlation of residuals by the Ljung-Box Q-test (Box (1994) pp. 314 – 317) and conditional heteroskedasticity by Engle’s test statistics (Engle (1982)). We start by testing in one lag, second including the first four lags, third including the first ten lags. Results of these tests are presented in the tables below.

**Appendix B: Risk simulations with alternative parameter values in the case of moderate valuation (log price-dividend ratio 3.5) and annual inflation (4)**

Table 9: Logarithmic cumulative real returns: scenario 1

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | 0.016    | 0.264   | 1.092    |
| Std            | 0.067    | 0.299   | 0.491    |
| Skewness       | -0.661   | 0.116   | 0.204    |
| Kurtosis       | 4.278    | 2.888   | 3.123    |
| VaR(5%)        | -0.104   | -0.212  | 0.312    |
| VaR(1%)        | -0.185   | -0.393  | -0.007   |

Table 10: Logarithmic cumulative real returns: scenario 2

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | 0.011    | 0.183   | 1.000    |
| Std            | 0.070    | 0.291   | 0.453    |
| Skewness       | -0.678   | 0.271   | 0.136    |
| Kurtosis       | 3.790    | 2.920   | 3.172    |
| VaR(5%)        | -0.124   | -0.264  | 0.271    |
| VaR(1%)        | -0.191   | -0.421  | -0.031   |

Table 11: Logarithmic cumulative real returns: scenario 3

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | 0.007    | 0.172   | 1.026    |
| Std            | 0.071    | 0.353   | 0.542    |
| Skewness       | -0.768   | 0.220   | 0.334    |
| Kurtosis       | 4.318    | 2.730   | 3.304    |
| VaR(5%)        | -0.127   | -0.375  | 0.179    |
| VaR(1%)        | -0.207   | -0.554  | -0.127   |



Table 12: Logarithmic cumulative real returns: scenario 4

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | 0.011    | 0.215   | 1.081    |
| Std            | 0.069    | 0.331   | 0.538    |
| Skewness       | -0.977   | 0.343   | 0.442    |
| Kurtosis       | 5.266    | 2.985   | 3.526    |
| VaR(5%)        | -0.118   | -0.288  | 0.269    |
| VaR(1%)        | -0.218   | -0.460  | -0.043   |

Table 13: Logarithmic cumulative real returns: scenario 5

| Holding period | 3 months | 5 years | 20 years |
|----------------|----------|---------|----------|
| Mean           | 0.005    | 0.152   | 0.989    |
| Std            | 0.068    | 0.284   | 0.446    |
| Skewness       | -0.585   | 0.269   | 0.130    |
| Kurtosis       | 3.810    | 2.983   | 3.137    |
| VaR(5%)        | -0.120   | -0.288  | 0.283    |
| VaR(1%)        | -0.187   | -0.457  | -0.026   |

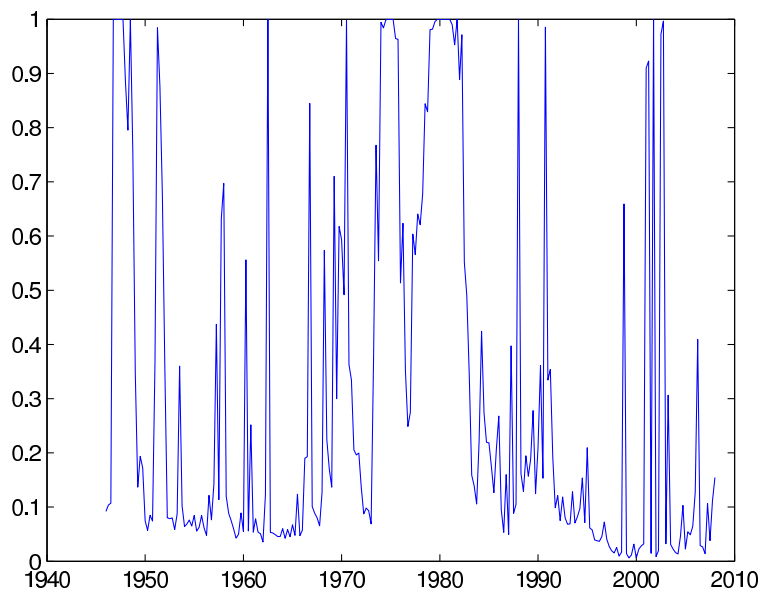


Figure 1: The ex-post probabilities to stay in the Fundamental Regime.

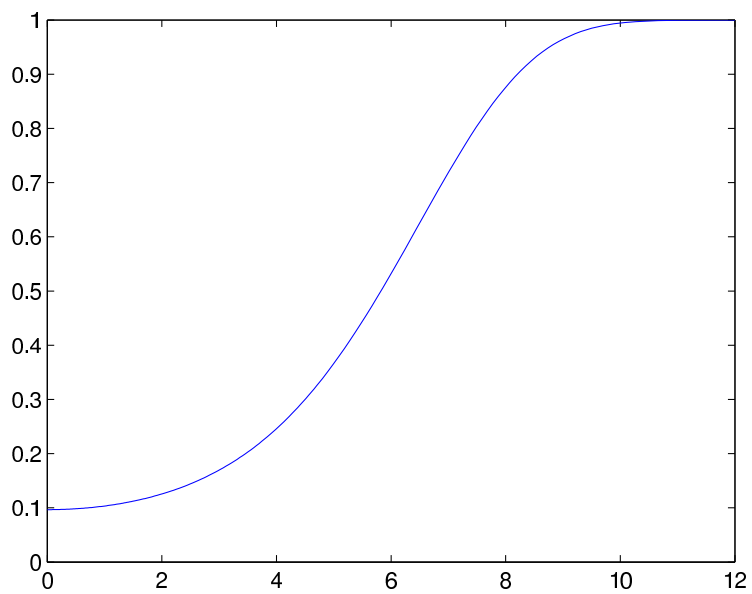


Figure 2: The probability of Fundamental Regime with respect to the inflation rate.