
Eelis Hein

Deposit Insurance: Pricing and Incentives

SUOMEN PANKKI
Bank of Finland



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Deposit Insurance: Pricing and Incentives

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Abstract

This study analyzes the valuation and bank risk incentive effects of deposit insurance using an approach based on options theory.

While the value of deposit insurance can obviously be set under existing regulatory measures such as capital adequacy and reserve requirements, the actual and expected behaviour of the regulator is shown to exert an effect on bank risk policy, and thus, on the stability of the banking sector. The following factors are identified as possible causes of increased preference for risk on the part of banks:

- an expectation that in the event of insolvency the deposit insurance will cover claim holders not otherwise initially insured;
- an expectation on the part of shareholders that they are not threatened with losing their position; and
- underpricing of deposit insurance premium in relation to a bank's market-valued capital adequacy.

These expectations increase preference for higher risk because they remove both the need for debt holders to require any risk premium for their investment and the threat that shareholders might lose their participation in the bank's future earnings. Thus, banks are not "penalized" for taking on risk. Instead, the costs of higher risk are borne by the deposit insurer, which in Finland's case, is ultimately the government and taxpayers. A related issue is that the efficiency of the bank inspection authority seems to affect the risk-taking behaviour of banks (i.e. if a bank believes that the bank inspection authority is incapable of determining its true financial condition and actual risk exposure, it has incentive to take a riskier position).

Using bank stock prices, point estimates of the value of deposit insurance are calculated for listed Finnish banks between 1987-1993. The results indicate that the value of the insurance has varied among banks and over time. Generally, charged deposit premia have been underpriced in comparison to the risk position of the studied banks. Thus, one consequence of the shakeout in Finland's banking sector appears to be that a sizable wealth transfer from the government to bank shareholders has taken place.

Keywords: Banking, Deposit Insurance, Risk Incentives, Option Pricing, Regulatory Behaviour

Tiivistelmä

Tutkimuksessa analysoidaan talletussuojan arvon muodostumista sekä tämän pankeille aiheuttamia riskinottoinsentivejä käyttämällä optioteoreettista lähestymistapaa.

Talletussuojan arvoa voidaan tutkimuksen mukaan kontrolloida sellaisilla välineillä, jotka ovat jo nykyisin viranomaisten käytössä kuten kassavarantovelvoitteella sekä vakavaraisuusvaatimuksella. Viranomaisten käyttäytymisellä ja odotetulla käyttäytymisellä on myös vaikutus pankkien riskikäyttäytymiseen ja siten pankkijärjestelmän stabiliteettiin. Pankkien riskinottohalukkuutta näyttäisi kasvattavan seuraavat tekijät: 1) odotukset talletussuojan kattavuuden laajentamisesta pankkien konkurssitilanteessa koskemaan pankkien niiden velkojien saatavia, jotka eivät alunperin kuuluisi talletussuojan piiriin, 2) odotukset siitä, ettei pankin vanhojen osakkaiden omistusta nollattaisi missään tilanteessa, ja 3) talletussuojamaksujen alihinnoittelu suhteessa riskeihin annetulla markkinahintaisella vakavaraisuudella. Riskinottohalukkuuden lisääntyminen perustuu siihen, että em. odotukset poistavat toisaalta pankkien velkojilta tarpeen vaatia pankeilta riskipreemioita ja toisaalta uhkan pankin osakkeenomistajilta menettää osallistumisen pankin mahdollisiin tulevaisuuden tuottoihin. Tällöin pankeille ei aiheudu lisäkustannuksia riskin lisäyksestä jolloin riskin kasvattaminen tapahtuu talletussuojan tarjoajan, Suomessa viimekädessä valtion kustannuksella. Myös pankkien riskejä valvovan viranomaisen tehokkuudella näyttäisi olevan vaikutus pankkien riskikäyttäytymiseen siten, että uskomukset pankkitarkastusviranomaisen kyvyttömyydestä selvittää pankin taloudellinen tilanne ja liiketoiminnan riskit kasvattaisivat riskinottohalukkuutta.

Tutkimuksen empiirisessä osassa lasketaan estimaatteja suomalaisten pörssilistattujen pankkien talletussuojan arvolle vuosina 1987–1993 käyttämällä osakekursseihin sisältyvää informaatiota. Tulokset osoittavat, että talletussuojan arvo on vaihdellut sekä eri aikoina että eri pankkien välillä. Lisäksi tulosten mukaan suomalaisilta pankeilta on peritty niiden riskisyyteen nähden liian alhaisia talletussuojamaksuja. Siten varallisuudensiirtoa näyttää tapahtuneen valtiolta pankkien osakkeenomistajille.

Asiasanat: Pankkitoiminta, Talletussuoja, Riski-insentivit, Optiohinnoittelu, Viranomaisten käyttäytyminen

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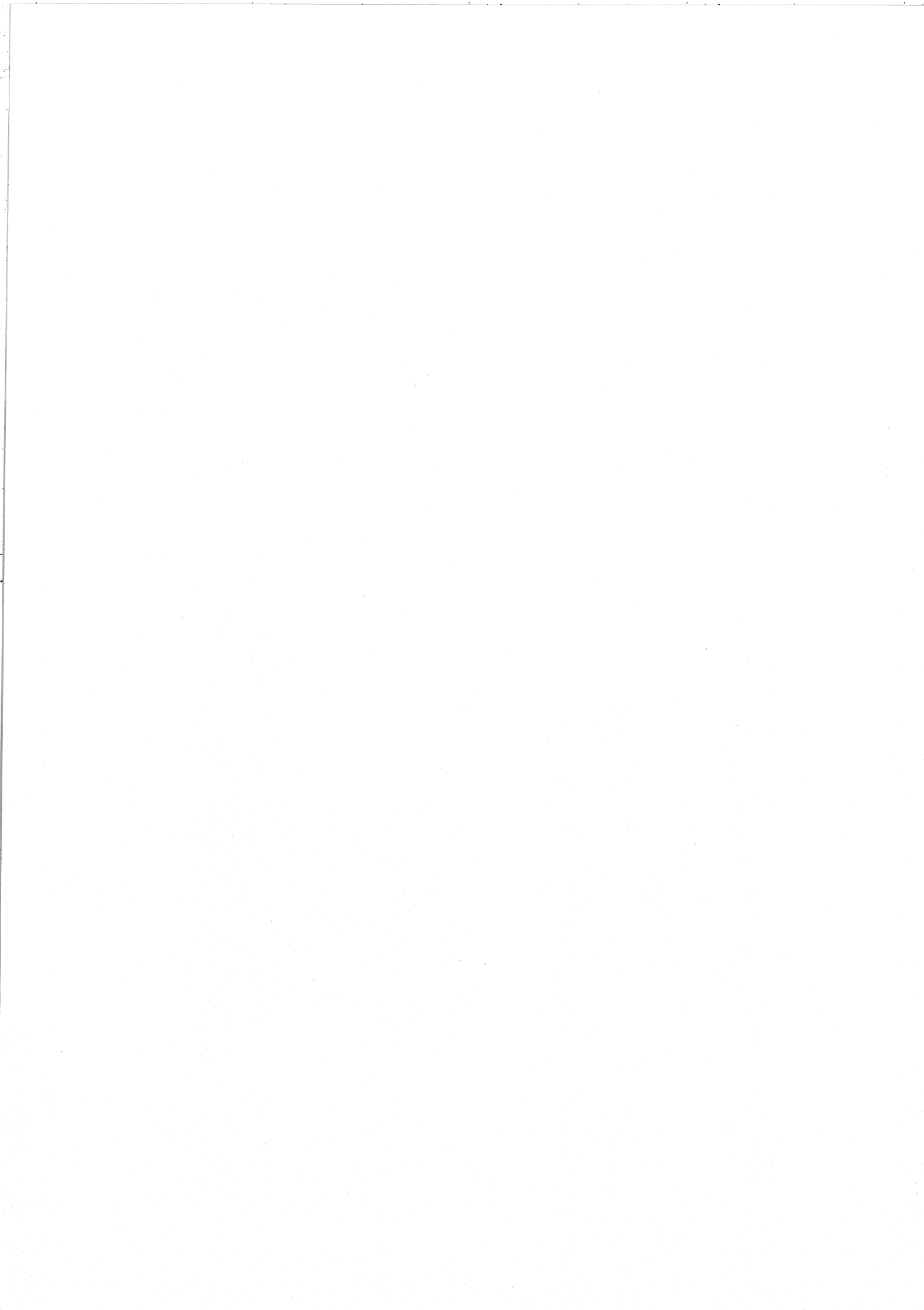
Robert C. Merton's pioneering work in deposit insurance pricing and the innovative research of George Pennacchi captured my original theoretical interest in a subject I have since come to understand is painfully relevant to present-day banking policy in Finland. Most of the present report, which also serves as my licentiate thesis for the Helsinki School of Economics and Business Administration, was written during my 1993-1994 stay at the Bank of Finland's Research Department.

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1 The Emergence of Deposit Insurance

1.1 Introduction

Instability in the banking system can inflict serious damage to a national economy. It may impair the payments mechanism, reduce the savings rate, diminish financial intermediation or bring grave harm to small savers. To prevent such adverse effects, governments around the world intervene in banking markets with a variety of arrangements, which include laws, regulations, supervisory activities and lender-of-last-resort facilities. Many countries also have set up deposit insurance systems. In a complete market setting, where information is perfect and symmetrically shared by all agents in the economy, bank deposit insurance is, of course, redundant and provides no social benefit. But modern banking systems are not perfect, and thus there is virtually universal agreement among regulators, academics and market participants that deposit insurance systems are generally in society's interest.

The purpose of most deposit insurance schemes is twofold. First, they seek to make small depositor savings risk free, since, as argued by Merton (1977a), it is unreasonable to force small depositors to analyze the riskiness of their deposits due to the large information processing capability and surveillance costs needed to do so. Providing deposit guarantees to small savers is equitable because the cost of obtaining information about the solvency of a financial institution is higher for small depositors than for a deposit insurer. The second, and perhaps more important, role of deposit insurance is to protect the banking sector from bank runs and banking panics and thus, help stabilize the banking system.¹

Indeed, bank runs have added a good deal of spice to the work of monetary historians. Most people have at least passing knowledge of the bank runs in the United States during the Great Depression and the subsequent disruption to the economy's payment systems and money

¹ Some scholars argue, however, that deposit insurance is not needed to prevent bank runs because an effective lender of last resort can handle such runs if they occur. Talley and Mas (1993) refer to Anna Schwarz (1987) and George Kaufman (1987), whose writings are based on the assumption that bank runs take the form of deposit transfers from weak banks to strong ones and thus, no change in aggregate bank reserves, money supply or interest rates result. They claim that lender of last resort would only have to prevent the bank subject to a bank run from becoming illiquid and offset the resulting bank reserves e.g. through open market operations.

supply. Nor are bank panics a particularly new phenomenon: in 33 AD, for example, the Roman empire was afflicted by a massive bank panic.²

The “classic” bank run occurs when depositors rush to withdraw their claims because they expect the bank to fail. Depositors, because they lack relevant information, may have a rational incentive to participate in bank runs whether or not the bank is actually solvent. Another common type of bank run emerges when bank investors refuse to roll over their claims on the bank, and simultaneously, access to alternative funding sources is blocked.

Banks almost by necessity face exposure to the threat of bank runs because it is the nature of their business to offer short-term and demandable claims which are then invested into longer-term assets. When crisis hits, the bank’s assets are generally too illiquid to realize quickly without having to settle for an amount worth less than the overall claims on the bank.

Diamond and Dybvig (1983) analyze the economics of banking and associated policy issues, including deposit insurance, within an economy consisting of a single demand deposits issuing bank. They show that without a deposit insurance contract to prevent bank runs, there is always a possible equilibrium at which a bank run is precipitated. Once the run starts, all depositors panic and immediately attempt to withdraw their assets or exercise their claims. In a world of many banks, this equilibrium implies real economic damage – even economically sound banks are forced to fail as a result of a run. As bank failures snowball, loan recalls increase. Inevitably, productive investments financed by the bank must be terminated. Finally as the bank loses its ability to provide a source of transacting balances, nearly everybody in the society sees their ability to transact and operate effectively impaired.

The awareness of the adverse effects of a bank run on the economy, combined with painful experiences during times of disruption, has led many countries, especially industrially advanced nations, to establish deposit insurance systems. Some systems date back to the early 1800s.²

1.2 Organization of deposit insurance

Given that the concerns of both banks and regulators are essentially the same, deposit protection schemes vary to a surprising extent from country to country. The reason for this is that while deposit insurance as such is a simple concept, deposit insurance systems can be complex. Further, not all deposit insurance systems are exclusively the domain of the government, but instead have been created by the banks themselves.

² See e.g. Calomiris (1990) and Mas and Talley (1993).

A deposit insurance system can take a variety of forms with regard to sponsorship, administration and financing. It may be purely public, purely private, or a public-private sector mix. In some countries, membership in a system is compulsory and required by law; in others, banks can choose whether they want to participate in a deposit insurance scheme.

By far the most common way to finance a deposit insurance system is the creation of a deposit insurance fund financed through periodic premium payments from member banks. These funds are typically administered by the representatives of the banks themselves. Whether a particular deposit insurance fund works under the supervision of the regulators, or alternatively without guidance, depends on the legislation of the country in question. Countries using a fund system include Norway, Germany, Spain, Belgium and Finland.

Another common way to organize a deposit insurance is based on multilateral coinsurance among banks. Under such systems, no annual contributions are collected from member banks, rather the realized (or potential) losses of a failed bank are divided *ex post* among participants. The Netherlands, Italy, Austria, France and Switzerland have systems based on *ex post* funding.

Finally, there is the American model of deposit insurance, whereby a deposit insurance corporation collects periodic insurance premiums from banks. The deposit insurance systems of Japan, Canada and the United States apply such a system.

No matter what approach is used, most countries place the cost burden of deposit insurance largely or entirely on the banking system in the form of periodic premium payments. Usually the premium assessment is based on the amount of insured or total deposits. The assessment rate applied to the assessment base can be fixed (and thus the same for all banks irrespective of the riskiness of any particular bank's operations), or alternatively, based on the bank's overall risk.

1.3 Coverage

The amount of protection that a deposit insurance extends to depositors depends on the maximum insurance coverage specified in the statute and whether the scope of the insurer's authority to resolve failing bank situations extends to *de facto* protection of uninsured depositors. Coverage schemes can be limited, full or discretionary.

The *limited coverage* scheme is designed primarily to protect small depositors when banks fail. All deposit accounts are insured up to a certain maximum amount, so that when the bank fails, the insurer is authorized to pay off insured depositors up to a maximum amount, or arrange for all the

failed bank's insured deposits to be transferred to another bank. With a truly limited coverage scheme, the insurer does not rehabilitate banks or arrange financially assisted mergers, because to do so would extend *de facto* protection to uninsured depositors by preventing failures.

Full coverage scheme represent the other end of the protection spectrum. All deposit accounts are fully insured and the insurer has a broad range of devices to resolve failing bank situations, including insured deposits payoffs or transfers, financially assisted mergers and rehabilitations.

A *discretionary coverage* scheme implies that all deposit accounts are insured up to certain amount. However, unlike a limited coverage system, the insurer is authorized under certain circumstances to extend *de facto* coverage to uninsured depositors using a purchase and assumption transaction to resolve the failure, or by arranging a financially assisted merger or rehabilitation to prevent failure. An example of special circumstances that might have to prevail before the insurer is authorized to extend *de facto* protection to uninsured depositors would be a situation where the entire banking system of a country is imperiled by a massive loss of public confidence that could lead to widespread bank runs. Coverage extension might also evolve out of situation where the costs of providing protecting against banking runs outweigh the erosion of market discipline that extending *de facto* protection to uninsured depositors would entail. Therefore, a discretionary coverage scheme functions as a limited coverage arrangement when the banking system is not threatened, but can be converted into a *de facto* full insurance system if the threat is perceived as so grave as to cause erosion of market discipline. Given the instability and banking concentration often found in banking systems, a discretionary coverage system, as noted by Talley and Mas (1993), easily leads to policies where uninsured deposits are essentially guaranteed.

1.4 Forms of deposit insurance

The basic features of the deposit insurance systems presented above provide a framework within which most *explicit* deposit insurance systems can be nested. An explicit insurance is a formal, legally enforceable *de jure* guarantee. However, as Kane (1986) and others have noted, the coverage of deposit insurance may actually be far more extensive than suggested by formal limitations on which balances are guaranteed. Actual practice has shown that during times of distress in banking systems, deposit insurance coverage in many countries has been extended far beyond the formal coverage of the insurance. This means that beyond explicit deposit

insurance, many countries provide *implicit* deposit insurance. In most cases, the provider is the government.

In implicit deposit insurance, the protection of depositors, other liability holders, and even equity holders, is totally discretionary. The government (in most cases) offers such protection, not because it is obliged to do so by law, but because it believes that such action will achieve certain public policy goals or it considers it cheaper in the long run to do so. Moreover, the determination of the amount and form of the protection is based on *ad hoc* decisionmaking within the government. No pre-existing rules and procedures guide the decisionmaking process, although earlier experiences may influence the choices.

The government can extend implicit deposit insurance protection by:

- making direct payments to depositors or arrange for the failed bank's deposits to be assumed by another bank when an insolvent bank is closed;
- arranging and financially supporting the merger of the problem bank with another bank (i.e. avoiding failure of the bank, and thereby protecting all depositors as well as other liability holders);
- preventing the failure by rehabilitating the financially insolvent bank through regulatory *forbearance*. The government may provide help as a direct equity capital injection, or it may acquire some or all of the failing bank's non-performing assets at book value (which is essentially equity injection). In any case, the aim is to give the bank a fresh start with a portfolio of performing assets. Through such a rehabilitation, the government may end up as the dominant shareholder, and thus nationalizing the bank.

1.5 Effects and problems

1.5.1 Depositor and creditor discipline

A deposit insurance guarantee is an odd sort of contract – the *de facto* beneficiaries and payers of the insurance enjoy the benefits of the contract inversely to the *de jure* parties. This peculiarity results from the fact that the purchasers of the insurance contract, i.e. the banks, pay the insurer an explicit premium. By doing so, they make the depositors the beneficiaries of the contract. If the deposit insurance system is perfect in the sense that the insuring organization can meet its obligation in all circumstances, then the deposits are made risk-free in all circumstances.

The risk-free nature of deposit accounts, however, removes any need for depositors to monitor bank risk, and thus no market discipline is

imposed on them. One consequence of a loss of market discipline means that depositors do not require any premium above the risk-free rate paid on their deposits. In a limited coverage case, market discipline is removed only from those depositors whose deposits do not exceed the formal maximum coverage. In the case of full coverage, no depositor, small or large, has any incentive to monitor their investment at the bank. If, however, the coverage is discretionary (i.e. depositors face uncertainty as to whether their deposits are actually insured in the event of bank insolvency), depositor expectations with regard to insurer behaviour during a bank insolvency determine depositor market discipline.

Market discipline may also be lost by the bank's other liability holders and creditors who are not formally insured. For example, such creditors may be given reason to believe that they are, in fact, implicitly insured. This implicit insurance can take the form of a *"too-big-to-fail" guarantee* which according to Berlin, Saunders and Udell (1991) has been one of the major weaknesses of the US deposit insurance system. Too-big-to-fail policies are undertaken by the government when it fears that collapse of a large bank would cause widespread havoc in the banking sector, and thus, the economy as a whole. As the insurer or regulator will not let the bank fail, it must guarantee the bank's liability holders and creditors in order to prevent them from forcing the bank into liquidation. Too-big-to-fail policies were invoked, for example, in 1984 by the Federal Deposit Insurance Corporation (FDIC) in its rescue of Continental Illinois – no creditor lost money. Too-big-to-fail policies, as we shall see, have also been invoked in Finland.

At the theoretical level, whether liability holders not formally insured have incentive to monitor or distinguish among banks according to the banks' inherent quality depends on their assumption as to whether they enjoy the same implicit failure guarantees as those covered by explicit guarantees. In their study of large US banks, Gorton and Santomero (1990) found that a too-big-to-fail guarantee may have even eliminated the risk premium from long-term and subordinated debt of bank holding companies, which indicates a severe loss of market discipline.

1.5.2. Fixed versus variable insurance premia and moral hazard

Deposit insurance can cause erosion of market discipline almost immediately because it reduces funding costs to the banks and thus permits them to fund risky projects at lower interest rates than would otherwise be possible without insurance. Further, the bank is free to ignore depositor behaviour as a decision variable when choosing its risk management strategy. The question therefore becomes whether the banks are charged insurance premia by the insurer which appropriately compensate for the

reduction of their funding costs caused by depositor protection. If the deposit insurer charges a deposit insurance premium equal to the risk premium the market would require to provide the same level of risk-bearing services, then the deposit insurance can be said to be appropriately priced, i.e. a "fair" insurance premium. This further implies that the loss of market discipline might be offset by imposing risk-adjusted insurance rates.

Most academics and policy makers have, in fact, long argued for implementing variable premium rate schedules.³ A risk-based premium would impose a marginal penalty on banks involved in risk taking and thus, encourage them to exercise greater discipline when considering loans on risky projects.

Bank assets often include private information which is both asymmetric and costly to acquire. The deposit insurer might, therefore, be tempted to assume the role of market disciplinarian by seeking to discover the true riskiness of the bank and "penalizing" the bank accordingly. As an all-knowing enforcer, it would learn the riskiness of the bank's asset portfolio through audits and examinations and would be able to determine a fair premium for the bank. Certainly this notion appeals to some circles, but in reality it must be admitted that no bank supervision entity can ever reach a level of perfect information symmetry with the banks it supervises. Giammarino, Lewis and Sappington (1993) are more direct, noting that bank monitoring is only imperfectly informative, and indeed, regulators (or the insurer) are occasionally unable or unwilling to act even on the information they already have. Moreover, Chan, Greenbaum and Thakor (1992) have argued that risk-sensitive deposit insurance pricing imposes great informational demands on the deposit insurer that can be both costly and difficult to apply.

Thus, the private information possessed by the banks introduces the element of *moral hazard*; i.e. the bank's management or stockholders may have an incentive to misrepresent their asset risks in order to obtain more favourable insurance pricing. Moreover, when information is asymmetric or the insurer has less-than-perfect control over the bank's choice of risk strategy then, even if the overall risk of the bank could be accurately determined and a premium would be set to be fair *ex ante*, the insurer faces a problem of moral hazard since the bank can *ex post* unilaterally revise its overall risk. Therefore, a risk-sensitive pricing scheme may run afoul of

³ Goodman and Santomero (1986) provide a fascinating discussion (unfortunately, somewhat outside the scope of this study) of the use of variable-rate premia to avoid adverse risk incentives. They show that a variable-rate system may change the equilibrium behaviour of both financial and real sectors by raising the cost of funds made available to the real sector by financial firms. This, in turn, is claimed to reduce the overall availability of funds and increase the probability of bankruptcy of firms. The overall result is a dead-weight loss to society.

both measurement and implementation problems owing much to observability considerations. Problems due to measurement errors have been studied by e.g. Pyle (1983), who claims that modest errors in measuring the critical risk-related pricing parameters of deposit premia may overwhelm the desirable properties of risk-adjusted deposit insurance premia. The threat of *ex post* discipline has been analyzed by Merton and Bodie (1992) and Kane (1986).

Thus, fairly priced deposit insurance is, if not altogether infeasible, difficult to establish. This is probably the reason most countries have traditionally based deposit insurance systems on fixed rates. Only recently have some countries, including the United States and Finland, attempted to move closer to a fair deposit premia schedule. In these systems, deposit insurance premia are based on the riskiness of individual banks. The range of these variable rates which are determined by risk-related factors such as capital adequacy and asset quality is, however, narrow and premia are not perfect in terms of risk sensitivity, and thus not fair in the strict sense.

As noted earlier, in a fixed deposit insurance premium schedule all banks are charged the same percentage of some assessment base regardless of the bank's probability of failure, the riskiness of its portfolio or the potential cost to the insurer should the institution fail. Of course, a fixed premium can thus only be fair when all banks have identical overall risk. Many studies (including this one) have found that there are significant differences in risk taking from bank to bank. For example, Marcus and Shaked (1984), Giammarino, Schwartz and Zechner (1989), Lucey (1993), Fries, Mason and Perraudin (1993) have found differences in bank risk within various countries using option pricing models. In light of this evidence, it is likely that in a fixed premium schedule banks are either overcharged or undercharged for their deposit protection. This further implies that if the insurer would fulfil its objective function of insuring the banking sector subject to zero profit/loss constraint by aiming at an actuarially fair level of total banking industry deposit premium, then healthier banks would end up subsidizing riskier ones.

One of the most problematic pecuniary externalities caused by fixed rate insurance premium schedules is that they can contribute to *moral hazard* by increasing the incentive for banks to choose a risky strategy. This aspect of moral hazard has received a great deal of attention in deposit insurance literature. Many studies, e.g. Marcus (1984), have shown that there is a risk-taking subsidy inherent in a fixed-rate deposit insurance assessment. Marcus has also shown that this incentive increases as the health of the bank wanes.

The incentive for choosing a riskier strategy is based on a notion that for paying a fixed cost, the insurer banks receive an asset (deposit

insurance), the value of which is maximized by maximizing the riskiness of the bank. A bank can increase risk in many ways, including

- increasing the institution's leverage by reducing its ratio of capital to assets.
- increasing total portfolio risk by altering the composition of assets and/or liabilities.
- increasing total asset risk by decreasing its degree of portfolio diversification.
- increasing total portfolio risk by mismatching asset maturity and liability maturity (or mismatching interest rate sensitivity of assets and liabilities).

All of these measures increase the value of the deposit insurance which will be shown for the first two cases in this study. The last case is analyzed by Kefriden and Rochet (1993). However, as will be shown later in this study, the shortcomings of fixed premium rates can be somewhat reduced by regulatory measures which include capital adequacy and reserve requirements.

Moral hazard problems are, therefore, not only present in the case of a fixed insurance premium schedule but also in the case of a risk-based premium when the bank possesses private information or the insurer lacks full control over the bank's choice of risk or alternatively is not able to commit to some kind of *ex post* "adjustment" of charged premia.

Risk-taking incentives may also be created by the *de facto* or *expected* behaviour of the insurer in the case of a bank insolvency. Recently, regulators in many countries have permitted insolvent banks to remain in operation. In some cases, even though the formal *ex ante* explicit deposit insurance has not covered the shareholders, the *ex post* implicit insurance has. In other words, the shareholders have been bailed out along with the insolvent bank without losing their right to participate in the possible future earnings of the bank. If shareholders have good reason to expect such actions from the insurer, they may be subject to moral hazard by their ability to bet with their insurer's (in many cases the government's) money. In such a case, the bank's shareholders can earn higher returns without facing the potential losses associated with higher risk strategies. Accordingly, they enjoy an ongoing subsidy from the insurer.

1.6 Motivation and organization of this study

There is a growing consensus among regulators, banks and economists that the deposit insurance system somehow contributed to the recent crisis in the

Finnish banking sector, which has subsequently made the design of deposit insurance systems a topic of much interest. This study aims to look at the economics of a deposit insurance system using option valuation models.

Such models are useful for evaluating deposit insurance because:

1. It is possible to assign specific values to the claims of each of the interested parties involved in the deposit insurance system (the insurer, the financial institutions and the various liability holders). These valuations can be used, for example, to estimate a fair price that a bank should pay for its depositor protection. Such estimates can be compared to the actually charged premia to determine whether the banks are being subsidized by the deposit insurance provider.⁴
2. Option pricing models can be used for finding ways to control the value of the insurance i.e. the insurer's (and many times the government's) liability. This analysis is useful since the deposit insurance scheme is usually based on fixed, non-risk-sensitive premia which do not provide the insurer with a basis for discriminatory treatment of banks with different degrees of riskiness.
3. It is possible to analyze the incentive structure of the deposit insurance scheme. As the economic agents' behaviour is based on their expectations concerning the future, and as history does have an effect on such expectations, it is necessary to investigate how the actual and expected regulator action might affect the economic incentives and thus, the behaviour of the different interest parties of the banks. These analyses are of particular interest in the case of the Finnish banking sector, not least because the handling of the Finnish banking crises involved heavy government intervention.

It is the aim of this study to construct a model which enables analysis of insurer and regulator behaviour. By comparing insurance valuations with different model parameters, one can investigate the system incentives for bank shareholders and depositors under various regulatory schemes. One of the main motivations of this study is to show from the angle of the deposit

⁴ Earlier studies of Finnish deposit insurance pricing calculating point estimates for the value of deposit insurance for Finnish banks have used one-period option pricing models together with information included in the historical market prices of bank stocks. Such models fail to account for the effect of the actual and expected behaviour of the regulator on the valuation of the bank stocks. This may explain the low point estimates for "fair" deposit insurance, which in hindsight of the Finnish banking crisis, look unrealistic. This study will present a model which includes various parameters of regulatory behaviour which better filter the information content of stock market prices, and thus, achieve more credible point estimates for deposit insurance valuations of individual Finnish banks.

insurance system, how important the role the regulator is in banking dynamics.

This study is organized as follows: Section 1 is a general presentation of deposit insurance systems. Section 2 gives both a brief history of the Finnish deposit insurance system and a description how it has functioned and changed in the environment of the recent economic and banking crises. Section 3 is a review of earlier academic studies of deposit insurance which have used option valuation models. Sections 4 and 5 present a one-period European-type put option model of deposit insurance. After the model derivation, comparative static analysis are conducted to identify the basic determinants in the value of a deposit insurance contract and their interactive effects. The section ends with an analysis of deposit insurance coverage as far as the various liability holders are concerned. Section 5 uses the one-period model for estimating the value of deposit insurance for those Finnish banks, whose stock price information is available. Sections 6, 7 and 8 analyze deposit insurance with a multiperiod American-style put option model. Section 6 starts with a discussion of the weaknesses of a one-period model after which a multi-period model is derived. This model is used in Section 7 for analyzing bank risk incentives under various regulatory schemes. In Section 8 point estimates of the value of deposit insurance premia are calculated under various insurance schemes and assumptions of stock market's expectations concerning the regulator's behaviour. Section 9 contains conclusions.

2 Deposit Insurance, Banking Supervision and Crisis in the Banking Sector⁵

2.1 Bank guarantee funds

Although, it was not until 1931 that a compulsory annual premium payment was actually required of savings banks, systematic attempts to protect the claims of Finnish depositors date back to 1924 when a savings bank deposit guarantee fund was established by special act.

A similar cooperative bank guarantee fund was started in 1929, and commercial banks established their first guarantee fund in 1966. Both were based on voluntary membership.

With comprehensive banking laws reform in 1969, membership in a fund became mandatory, and administration of funds became the duty of member banks. The new deposit insurance scheme was based on *full coverage*, i.e. the funds were obliged to assure repayment of 100% of depositors' claims in the event of bank insolvency. This obligation, however, was to be invoked only after the entire bankruptcy estate had been realized and exhausted. Moreover, the guarantee funds were given the right to grant subsidies and loans to member banks when necessary.

All the banks were required to pay their own fund a premium which according to the law *had to be adequate with respect to the obligations and duties of the fund*. These compulsory annual premium payments, collected until 1992, were to represent at minimum 0.01% and at maximum 0.5% of the total amount of a bank's assets.⁶ The decision on the exact premium to be charged was set by the fund itself, and the premium had to be confirmed by the quasi-governmental Banking Supervision Office (BSO). The BSO had the right to require a higher premium when it had legitimate cause to consider the charged premium inadequate for protecting depositor claims. Prior to 1992, the premiums charged were *fixed*, and thus, not based on any

⁵ For a more detailed analysis of Finnish banking crisis see e.g. Koskenkylä and Vesala (1994) and Nyberg and Vihriälä (1994).

⁶ Starting in 1993, the new banking laws require a premium payments to guarantee funds totalling a minimum 0.05% and maximum 1% of the total book value of member bank assets. The undisclosed premium division between the individual banks is based on each bank's riskiness.

risk measure within each guarantee fund. The savings and cooperative bank guarantee funds reviewed their rates annually.

Both the savings and cooperative bank guarantee funds had their own supervision bodies. These functioned under the supervision of the BSO, which had the final responsibility for prudential supervision of Finnish banks until October 1993. The larger savings and cooperative banks as well as the commercial banks were, on the other hand, directly supervised by the BSO. The BSO was subordinate to the Ministry of Finance but was mainly self-financed through the supervision contributions of the banks. These contributions were fixed payments based on the book values of bank assets.

The Bank of Finland has also performed certain supervisory functions. These have been based on the Foreign Exchange Act, which assigns to the Bank of Finland (BOF) the role of supervisor of the currency risks of financial institutions. Moreover, the law stipulates that the Bank of Finland's mission is to maintain the stability of the currency and financial markets, and it has the authority to act as a lender of last resort.

2.2 Macroeconomic developments behind the banking crisis

Although, technically speaking, deregulation of Finnish financial and banking markets started in 1980, the big moves took place in 1983 (deregulation of interest rates) and 1986-87 (liberalization of external capital movements). Deregulation coincided with rapid growth in national income. Riding a wave of optimism generated by extremely favourable terms of trade and growth in all Finland's major export markets, consumers began to expect greater disposable income and companies began to expect higher profits. Credit demand skyrocketed. In the meantime, banks pursued competitive strategies based on increasing market share. An unprecedented credit expansion followed.

By mid-1989 the Finnish economy was severely overheated. When the inevitable monetary tightening did arrive, asset prices, household and firm debt levels topped out. Interest rates, however, continued to rise as the economy cooled, and soon debt servicing was taking a large share of the cash flows of many firms and households. As domestic demand declined sharply, asset values weakened further and profits deteriorated. Export performance declined due to lost competitiveness and a gloomy international market situation. Adding insult to injury, a severe export demand shock was caused by the collapse of trade between Finland and the Soviet Union in 1991. By year's end, Finland found itself in the midst of its

worst economic recession of the century. Real GDP declined about 15% from 1991 to 1993.

2.3 The banking crisis and changes in the deposit insurance system

Having reached the record levels in 1988, bank profits started to decline as household and firm credit demand dropped and capacity to service newly-raised debt worsened. In 1991, banks started to show losses from a rapidly increasing number of non-performing loans. In retrospect, as noted by Nyberg and Vihriälä (1994), it is clear that neither banks nor their customers took adequate account of the changing financial environment. Banks continued to operate with standards established during the times of stringent regulation. Credit standards were low, inadequate attention was paid to the quality of collateral, and credit risk was not appropriately priced. Moreover, interest rate risks were widely overlooked. It is now evident, as shown by Koskenkylä and Vesala (1994), that Finland's banking supervision authorities also lacked the resources and operative procedures to cope with a deregulated banking environment. Most existing banking regulations and examination approaches still focused on judicial compliance rather than business risk evaluation.

As the Finnish economy slid deeper into recession, the risks taken by the banks during the boom years started to materialize. The Finnish banking crisis emerged into the public consciousness in September 1991, when the BOF took control of the failing Skopbank, a commercial bank that had acted as a "central bank" for the savings banks.⁷ This historically unprecedented action was conducted by the BOF on a purely *ad hoc* basis since no other regulatory authority was equipped to deal with this scale of operation. The action was not based on the BOF's *de jure* responsibilities. It sent a clear signal to the markets that *too-big-to-fail* policies would be implemented by the authorities when necessary. In the end, not only were Skopbank's depositors *explicitly insured*, but also the other liability holders' claims proved to be *implicitly insured*.

By the end of the year, the total accumulated capital of the banks' own guarantee funds was FIM 387 million. By contrast, in its first phase alone, the BOF's Skopbank operation required a nearly FIM 4 billion capital injection. Meanwhile, the stock of non-performing assets and credit losses at other large Finnish banks was increasing rapidly. It was becoming quite

⁷ The BOF transferred its ownership of Skopbank to the newly-established Government Guarantee Fund (see Section 2.3 below) in June 1992.

evident to the authorities that the bank guarantee funds could neither handle problems of this size nor could they alone meet their obligations of backing depositors' claims.

The first direct action from the government's side was the provision of a capital injection totalling FIM 8 billion to all banks regardless of each individual bank's financial condition. The capital injected started in March 1992 and was in the form of preferred capital certificates carrying no cumulative return. The money was to be included in banks' primary (Tier 1) capital and could be written down to cover losses. The government retained to option to convert the certificates to voting shares if either the interest, set slightly above the market rate, was not be paid for three succeeding years or, the bank's capital adequacy dropped below the legally required minimum.

In August 1992, the Government declared that the stability of the Finnish banking system would be secured under all circumstances. The Parliament reaffirmed the Government's promise with unanimous approval in February 1993 of the following resolution:

"Parliament requires the State to guarantee that Finnish banks will be able to meet their commitments on a timely basis under all circumstances. Whenever necessary, Parliament shall grant sufficient appropriations and powers to be used by the Government for meeting such commitments."

These commitments clearly manifest, according to Nyberg and Vihriälä (1994), that the entire political system stands behind the commitments made by the Finnish banks. In essence, Finland's politicians had promised to insure all the liabilities of Finnish banks.

In line with the Government's declaration and the Parliament's Resolution, the Government Guarantee Fund (GGF) was established by law in April 1992 and reorganized in March 1993. The GGF's function was to ensure the stability of the banking system and secure the claims of both domestic and foreign depositors. It could provide support to banks either through the banks' own guarantee funds or directly (the latter having been the norm⁸). The GGF had a wide array of methods to choose from when carrying out its objectives; e.g. acquisition of bank shares, provision of other types of equity capital, loans guarantees, or other types of support. The GGF collects an annual payment from the banks, which may total a maximum of 0.1% of the aggregate amount of all banks' book value of assets. The premiums may range from bank to bank according to the riskiness of the

⁸ For the general principles in all the support provided by the GGF, see e.g. Nyberg and Vihriälä (1994).

bank in question. In the first premium collection in 1992, the maximum premium was applied.

The banking crisis showed that the banking supervisory authorities had failed to detect the risks built into the banks' balance sheets during the years of credit expansion. Therefore, the bank supervision was reorganized and the duties of the BSO were transferred to the Financial Supervision Authority (FSA). The FSA was established in October 1993 as an autonomous unit in conjunction with the Bank of Finland. Like the former BSO, the FSA is mainly funded through the "supervision contributions" of banks. These contributions are based on the book value of bank assets.

2.4 Bank support

2.4.1 Direct support

By the end of 1993, the total net amount for various types of direct support to banks committed by the Government, GGF and the Bank of Finland was close to FIM 40 billion. Besides the government's pre-emptive general capital injection and the BOF's Skopbank operation, bank support has also been administered through the Government Guarantee Fund.

A total of FIM 16.5 billion, corresponding 41% of these committed funds have been granted to the publicly listed Skopbank, Bank of Finland's share being FIM 12.2 billion. The GGF has provided capital injections in many different forms to Skopbank totalling FIM 4.3 billion.

The second biggest share, 37%, of the net public bank support by the end of 1993 was given to the savings banks. In order to resolve the deteriorating financial situation of a number of savings banks, the GGF decided in June 1992 to support these banks on condition that 41 would merge and establish a new bank, the Savings Bank of Finland (SBF). By the end of 1993, this newly established bank had received support from the GGF in various forms totalling FIM 14.5 billion. Even so, the GGF concluded that there was little hope that the SBF would overcome its difficult economic situation and thus, in October 1993, the performing assets of the SBF were sold off to four of the bank's competitors for a total price of FIM 5.6 billion. Simultaneously, an asset-holding company, Arsenal Ltd (100%-owned by the GGF) was established for the purpose of managing the non-performing assets of the SBF. Banks which had bought the SBF's performing assets were given an option written by Arsenal Ltd. This option gave a right to sell assets acquired from the SBF at their initial acquisition price, even though it was clear that such assets would eventually have to be classified as "non-performing".

At the end of 1992, a small publicly listed commercial bank, the STS-Bank, also fell into financial distress. As its shareholders were unable to provide new capital, it was partially merged to Kansallis-Osake-Pankki (KOP), one of the largest publicly listed commercial banks. In the merger, KOP bought the majority of STS-Bank shares at the market rate, but only put the performing assets of the STS-Bank on its balance sheet. The problem loans remained on the books of the former STS-bank, which was renamed Siltapankki. A 10-90 split (10% to KOP, 90% to the GGF) of responsibility for Siltapankki's loan losses was agreed on. By the end of 1993, the GGF had injected capital into Siltapankki totalling FIM 3 billion, i.e. 7% of total bank support paid out by that time.

The remaining 15% of the net amount of direct bank support constitutes of the Government's general direct capital injection in 1992 to banks such as the Union Bank of Finland (UBF), KOP, Ålandsbanken, OKO bank and the cooperative banks.

In addition to direct support, the Government has guaranteed the funding of the Arsenal Ltd totalling FIM 28 billion. Moreover, guarantees worth FIM 4 billion for the banks to be used when raising new risk capital in international markets was granted. These guarantees have so far not been used.

Even though the specific terms of each support programme are considered separately, they should, according to the GGF's guidelines fulfil certain criteria. One of the essential criteria is the shareholders' responsibility of the supported bank. It is required that their economic responsibility should be as wide as possible. This responsibility has, however, realized only through stock dilution in the case of Skopbank where the number of shares has increased through the GGF's equity capital injection. Otherwise, generally no initial shareholder of any bank did lose the right to participate to the potential future earnings. Most probably the bank shares' future prospects have been enhanced by the public support programmes as stated in the Communication from the Government to Parliament on Bank Support (1993).

2.4.2 Indirect support

Bank asset quality has been enhanced by the government through support programmes targeted at households and companies with debt servicing problems. These programmes include loan guarantees and interest payment supports.

3 The Use of Option Pricing Models in Analyzing the Economics of Deposit Insurance

3.1 Introduction

Most studies of deposit insurance economics apply a theory of option pricing based on two seminal works of Robert C. Merton: "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees" (1977) and "On the Cost of Deposit Insurance When There are Surveillance Costs" (1978). While the second study is essentially an extension of the first, each has spawned its own distinct line of further theoretical and empirical academic investigation. Subsequent students of the subject have tried flesh out the theory with more intricate modelling and real-world solutions.

In the following, the academic work on option theoretical analysis of deposit insurance is reviewed. Studies are discussed within the context whether the model applied 1) European-style put option pricing based on Merton's 1977 paper or 2) American-style put option pricing based on the 1978 paper. At the end of the section, a theoretical framework of this study is presented.

3.2 European-style put option models

Deposit insurance guarantees impose a liability on the deposit insurer in the form of a potential financial loss in the future and, therefore, have an economic present value. Merton (1977a) noted that a deposit insurance contract could also be modelled as a put option written by the insurer and held by the shareholder of the bank.⁹ As such, the option holder would have a right, but not the obligation, to sell the underlying assets to the insurer at a predetermined price and time. The model supposes that the maturity of the contract is fixed and interpreted as the length of time until the deposit insurer's next audit of the bank's assets. The predetermined price in the contract is the value of the bank's liabilities. If, during the future audit (i.e. on the expiration/exercise day of the contract) the bank is found to have negative net worth (i.e. its assets are worth less than its liabilities), the

⁹ The derivation of a 1977 Merton model is detailed in Section 4.

contract will have a value exacting this shortfall. Otherwise, the unexercised contract expires and becomes worthless.

As long as there is a positive probability that the value of the bank's assets at the maturity date might be less than the value of the bank's liabilities, there is a positive probability of the contract having positive value (i.e. being in the money) in the future. The present value of this contract will then depend on the probability distribution of the value of the bank's assets. Merton (1977) argued that as the features of the deposit insurance contract are similar to the European-style common stock put option, an application of the Black-Scholes (1973) option valuation formula could be used for valuing the contract and thus establishing a "fair" insurance premium. If we accept this premise, the model for calculating fair deposit insurance premia would need only five parameter values as inputs: i.e. interest rate, market value of the bank's liabilities as the exercise price, the current market value of the assets as the underlying instrument of the put option, the maturity of the contract and the variance rate of the assets. The Black-Scholes type of a model, however, uses geometric Brownian motion as an assumption for the returns process for the bank's assets. A different process assumption naturally results in a different pricing equation for deposit insurance. Thus, the choice of the returns process for modelling bank's assets becomes a salient factor in the insurance's value.

Merton's one-period model assumes a limited term¹⁰ insurance contract. It was used empirically for making point estimates of deposit insurance premia by Marcus and Shaked (1984) for publicly listed US banks. The estimation of Merton's 1977 model is, however, complicated by the fact that the market value of the bank's assets and their variance rate are not directly observable. Marcus and Shaked solve this problem by using stock market information. They exploit the fact that the value of the bank's equity can be modelled as a call option on the value of the assets with a strike price equal to the value of the bank's liabilities. If the maturity of this call option is interpreted as coinciding with the put option deposit insurance contract, then put-call parity can be used. Using this parity together with the relation between the variances of the equity and the bank's assets, two simultaneous equations with two input parameters not directly observable are achieved. Using these two simultaneous equations, in turn, they then go on to estimate the "fair" deposit insurance premia for 40 publicly listed banks. Their results not only indicated that fair premia vary substantially among banks (which supports the argument in favour of risk-related deposit insurance premia), they also showed that the calculated values of the deposit insurance are, with rare exception, below the actual premia charged by the

¹⁰ A limited-term contract expires at the end of each insurance period. The terms of a subsequent period's contract have to be renegotiated.

Federal Deposit Insurance Corporation (FDIC). They note, however, that McCulloch's (1981, 1983) estimates of insurance values derived from a paretian-stable distribution greatly exceeded their own.

Ronn and Verma (1986) also demonstrated that the value of the deposit insurance premium can be calculated using an application of the Black-Scholes option pricing model when time-series data on the market value of the bank's equity and the book value of its debt are available. Their approach differs from Marcus and Shaked (1984) in a number of aspects, the most significant being the explicit modelling of the deposit insurer's policies when aiding distressed banks. Ronn and Verma argue that equity prices should reflect these policies; by ignoring them Marcus and Shaked end up understating the cost of deposit insurance.

In order to include the deposit insurer's willingness to bail out banks of negative net worth, Ronn and Verma add a forbearance parameter into their equation. This parameter denotes the value of the bank's equity and gives the fraction by which a bank's liabilities are allowed to exceed its assets before it is closed.¹¹ They estimate the fair premia for 43 publicly listed US banks using a forbearance parameter of 97% (i.e. the critical value of assets is 97% of the liabilities, below which the deposit insurer finds it optimal to force bankruptcy). Since this forbearance parameter is chosen so that the aggregate estimated deposit-insurance-weighted average roughly equals the premia actually charged from the sample banks, they avoid discussion of whether deposit insurance is under- or overpriced. Instead, they achieve a rank ordering of the banks on the basis of their risk to the deposit insurer. Their results showed that the distribution of the estimated fair premia was skewed: most banks' "fair" premia were below those actually charged, while the insurance seemed to be substantially underpriced in the case of a few banks. Thus, Ronn and Verma conclude that in the fixed deposit insurance premium system low-risk banks are, in fact, subsidizing banks with higher risk. Ronn and Verma also show that neither random interest rates nor non-stationary equity returns significantly affect their insurance valuations.

Ronn and Verma's work was followed by a number of empirical studies that used their technique of allowing the bank to have slight negative net worth without forcing closure. For example, Giammarino, Schwartz and Zechner (1989) estimated the deposit insurance premia for eight Canadian banks with a forbearance parameter of 97% and found significant interbank differences in deposit insurance value. Hence, they suggest that the fixed premia system induces substantial cross-subsidization (at least among Canadian banks).

¹¹ For more details and criticism see Section 6.1.

Sato, Ramachandran and Kang (1990) estimated the value of deposit insurance premia for 35 Japanese banks. They, too, found large interbank variation in "fair" premia. Their results also show that the change in the forbearance parameter may cause instability in the risk ranking of banks. Fries, Mason and Perraudin (1993) estimated "fair" insurance premia for a sample of 16 Japanese banks along the lines of Ronn and Verma. They show that the chosen value for the forbearance parameter greatly affects the results. They also claim that as the forbearance parameter can only be guessed, the interest one might have in the absolute values of the estimated premia is significantly reduced.

The results of Fries et al. do not correspond with those of Sato et al. (1990). The reason, they claim, is that Sato et al. estimated their parameters over a much shorter period. The results of Fries et al. indicate that insurance values vary over time.

Räsänen (1994) used the technique of Ronn and Verma for estimating the deposit insurance values for eight publicly listed Finnish banks. He estimates fair premia by giving different values for the forbearance parameter. The findings are similar to Fries et al. (1993), i.e. that the achieved results vary significantly in accordance with the choice of forbearance parameter value. He also uses point estimates for insurance value by choosing a forbearance parameter of 97% when inferring whether Finnish deposit insurance is fairly priced. He claims that for most of the period between 1982-1992, Finnish banks were undercharged for their deposit insurance.

The 1977 Merton model has also been used to analyze the potential case of moral hazard built into the system of fixed deposit insurance. The standard view is that a fixed deposit insurance premium introduces an incentive to the banks to increase the riskiness of their assets or to reduce their capital since by doing this, higher expected yields to bank shareholders could be achieved by exploiting the non-risk-rated insurance contract. Marcus (1984) showed that, in the context of a 1977 Merton model of deposit insurance, the smaller the charter value of the bank, the larger the shareholders' incentive to obtain wealth from the deposit insurer by increasing the riskiness of the bank.¹²

3.3 American-style put option models

Merton (1977) as well as others who have applied his model describe the deposit insurance contract as a limited-term European put option contract with maturity at audit date. In practice, however, deposit insurance is

¹² Keeley (1990) has developed similar results using a state preference model.

usually not granted for a limited term. Moreover, bank audits do not necessarily follow regular schedules. Recognizing these inconsistencies in the Black-Scholes-type model, Merton followed up with a second work in 1978, wherein deposit insurance was modelled as unlimited in nature.

According to his 1978 paper, Merton reasoned that despite the finite time between audits due to the cost of the insurer of "continuous surveillance", no explicit recognition of these cost was included in his 1977 model. Thus, he includes auditing costs and explicitly takes into account random auditing times. He then constructs a multiperiod model wherein the bank is charged a one-time premium to insure all its deposits in perpetuity provided that during random, Poisson-distributed audits the bank is found to be solvent.¹³ The audit cost component of Merton's 1978 model causes (in some cases) surprising comparative statics for the parameters of the model since an increase in the asset-value-to-deposit ratio results in an increase in the deposit insurance premium and an increase in volatility of the bank's assets results in a decrease in equity value. (See Section 7, footnote 4.)

Marcus (1984) analyzes optimal bank policy under the 1978 Merton model. He shows that the results are qualitatively similar to those achieved with a 1977 Merton model (i.e. when banks are sufficiently solvent, both increases in capital and decreases in asset volatility increase bank shareholders' wealth; when banks reach a certain degree of insolvency, capital withdrawals and increases in portfolio riskiness increase bank shareholders' wealth). Therefore, the value of capitalization of a bank is a decreasing function of the spread between market and bank interest rates. Further, this spread is an increasing function of the bank's monopoly power, so a fall in charter value may induce extreme risk-taking behaviour.

Pyle (1986) used a variation of Merton's 1978 model in analyzing the effects in the deposit insurer's liability when the bank closure rule is allowed to deviate systemically from the economic insolvency condition used in most option pricing models of deposit insurance. He showed that the failure to close banks on a timely basis has a profound effect on increasing the insurer's liability. He also claimed that the often-used regulatory practice of using book value capital-to-asset ratios may easily lead to deposit insurance renewal at fixed rates, despite deterioration in the economic condition of the bank.

Pennacchi (1987b) generalized the 1978 Merton model in order to consider additional characteristics of bank financial structure and alternative policy assumptions concerning deposit insurance pricing and methods for handling bank closures. First, he showed that the bank's incentive for leverage is greater with a fixed-rate deposit insurance scheme than in a

¹³ The complete derivation of a Merton 1978-type model appears in Section 6.

variable-rate scheme. Second, he confirmed the findings of Merton (1978) and Marcus (1984), (i.e. that when the deposit insurer follows a policy of resolving bank failures by making direct payments to insured depositors, sufficient charter value induces banks to prefer less leverage). Third, he found that when the deposit insurer handled bank failure by arranging a merger (essentially an effort by the insurer to recover any charter value of the failed bank), the bank would always prefer a high-risk strategy in a variable premium schedule no matter how large its degree of monopoly power.

Pennacchi (1987a) used his model (1987b) to calculate "fair" deposit insurance premia for 23 US banks under alternative assumptions concerning the deposit insurer's regulatory control. Following the lead of Marcus and Shaked (1984), he used information contained in bank share prices to derive estimates of parameters. His results indicated that if the extent of the insurer's control over the banks was such that deposit insurance could be viewed as "limited term" or variable-rate insurance, then nearly all the sample banks had likely been overcharged for their deposit insurance. If, however, the deposit insurer followed a policy which may be interpreted as "unlimited term" or fixed-rate insurance, sample banks appeared to have been considerably undercharged for their deposit protection.

A common assumption in all the above studies was that a bank would be closed if (at the time of audit) its asset-to-deposits ratio had dropped below a prespecified minimum. Allen and Saunders (1993), therefore, allowed the decision of a bank closure to be discretionary and not dependent on any fixed net worth level. They modelled the deposit insurance contract as a perpetual American put option (following Merton, 1973) with a deposit insurer's call provision that allows exercise of the put option at any point in time, thereby foreclosing on the bank. By showing that this call provision has value if the insurer-induced closure policy is stricter than the bank's self-closure policy, forbearance could thus be defined as the delay in implementing the deposit insurer's optimal closure policy. They further showed that the value of the deposit insurance can be seen as equalling the value of the noncallable put option minus the value of the insurer's call provision. According to their comparative statics analysis, when the deposit insurer uses a closure rule that rewards lower risk banks with a less stringent closure policy, then expanding risk beyond a certain point may decrease the bank shareholders' wealth.

3.4 Models and methodology used in this study

This study is divided into two parts. The first part takes Merton 1977 approach, i.e. deposit insurance is modelled as a European-style put option. Even though this model fails to capture many complexities of the real world, it distills a host of economic factors down to a few relevant parameters whose interaction can be readily analyzed. Since these parameters represent either policy instruments readily available in the bank regulators' tool box or decision variables of the bank, useful comparative static analysis can be conducted.

The 1977 Merton model is extended so that the bank's balance sheet can be divided into different classes of assets and liabilities. With the help of a more detailed breakdown of the bank's balance sheet, we may analyze each liability classes' contribution to the cost of deposit insurance as well as the issue of deposit insurer's forbearance when the insurance is extended to cover other senior debt and subordinated debt. The model can also be used for calculating point estimates of the value for the deposit insurance as well as insurance or guarantees for other liabilities. The results can be used for risk ranking of the sample banks. The methodology used closely follows the lead of Marcus and Shaked (1984).

While the 1977 Merton model is useful for analysis of interdependencies between factors affecting the insurance value, it does not correspond to the reality that deposit insurance is not a limited term contract, but is granted for an unlimited term. Therefore, we also analyze deposit insurance in a multiperiod setting. The model used is an extended version of the models of Merton (1978) and Pennacchi (1987a,b) and allows the bank closure to be discretionary regardless of the bank's assets-to-liabilities ratio. The model can be used for analyzing bank incentives within alternative deposit insurance structures and schemes as well as various expectations concerning the deposit insurer's behaviour. Further, stock market information can be better filtered in the point estimations of the insurance values. In such estimations, Pennacchi's methodology is applied.

4 Black and Scholes -Type Model of Deposit Insurance

4.1 Derivation of the model

In addition to the assumptions given later in the text, the following is assumed:

- 1) Financial markets are assumed to be complete in the sense that any financial claim can be replicated in the market place by a combination of other financial assets so that the price of any asset equals the value of the replicating portfolio.
- 2) There are no transaction costs or taxes.
- 3) All assets are perfectly divisible and short sales are allowed.
- 4) The Modigliani-Miller theorem, that the value of the firm is invariant to its capital structure obtains.
- 5) Markets are efficient in the sense that all available information are fully reflected in the stock prices.
- 6) Bank auditing, operation of the insurance fund and bankruptcy procedures are all free of costs.
- 7) Any forbearance of the insurer is ruled out. This means that the insurer follows the insurance contract without any flexibility regarding to the bank closure rule.

Although the model is based on Merton's (1977a) one-period model, the bank's balance sheet is divided into various claims and assets, and is assumed at the beginning of a deposit insurance period to have the following structure:

Assets		Liabilities	
Risky assets	V	Deposits	D
Risk-free assets	\hat{V}	Other senior debt	B
		Subordinated debt	S
		Equity	E

(1)

All the above balance sheet items are market values. Deposits and other senior debt are assumed to equal seniority hold; subordinated debt is junior. For the sake of simplicity it is assumed that all three

classes of debt are issued at the risk-free rate of interest.¹ Also, the risk-free assets earn a risk-free rate.²

The maturities of D, B, S and \hat{V} do not have to be the length of time until the deposit insurer's next audit of the bank's assets, which is equated to the length of the insurance contract. The three classes of debt and the risk-free assets, therefore, are not necessarily homogenous in their maturities. Following Merton (1977a), their maturities can be, however, reinterpreted as having the same maturity as the insurance contract, which is done in the following.

Assuming the insurance contract gives no depositor protection, holders of deposits at the maturity date (i.e. at the time of the insurer's audit) are entitled either the future value of their deposits (face value plus accrued interest) or a prorated fraction of the value of the bank's assets, should the total value of the assets be less than the future value of the bank's total senior debt (D+B).³

Depositors will thus receive

$$\min \left\{ De^{rT}, (V_T + \hat{V}e^{rT}) \frac{De^{rT}}{De^{rT} + Be^{rT}} \right\} \quad (2)$$

where T is the time to maturity and r the risk-free rate. Therefore, the value of deposits at maturity is

$$D_T = \begin{cases} De^{rT} & \text{if } V_T \geq e^{rT}(D + B - \hat{V}) \\ (V_T + \hat{V}e^{rT}) \frac{D}{D + B} & \text{if } V_T < e^{rT}(D + B - \hat{V}) \end{cases} \quad (3)$$

¹ This is not a necessary assumption and the analysis that follow could be carried out by using the *promised* payment according to the terms of the insurance contract.

² The risk free rate is certainly valid for the risk free assets, which are mostly government liabilities or reserves held in the Bank of Finland. Between May 1990 and May 1993 paid the 6 month interbank money market rate minus one per cent. Since July 1993 no interest has been paid on required reserves). In Finland demand and time deposit rates have mostly been below the rates paid by the government. In the case of deposits, r can be, however, considered as including the rate paid in the form of bank services. Other senior debt and subordinated debt have been issued at and above the market rates respectively.

³ Note that depositors begin to relinquish their claims on the bank only after the subordinated debt holders have lost their entire position.

Now, it is assumed that there exists a one-period deposit insurance contract according to which the insurer charges a fair premium from the bank at the beginning of the insurance period. The premium is based on measures which characterise the riskiness of the bank.⁴ The insurer is assumed to know perfectly what the magnitudes of these measures are going to be during the insurance period.⁵ The contract is assumed to be provided by such a guarantor, whose capability and willingness to meet its obligations is not in doubt.⁶

At the end of the insurance period, the insurer audits the bank, i.e. calculates the market value of the bank's net worth. If it is found to be negative, i.e. $V + \hat{V} < D + B + S$, the bank is declared insolvent, closed, and its shareholders' position is closed out. Finally, the bank is liquidated, the liability holders are paid their claim according to their seniority, and the deposit insurer covers the possible losses of the depositors. However, if during the audit the bank is found to be solvent, the bank will be allowed to remain in business after the deposit insurance premium is adjusted to a new fair level based on the agreed bank's riskiness during the next insurance period.⁷

If the deposit insurance guarantee covers both the face value and the interest of the deposits,⁸ the maturity value of the deposit insurance is:⁹

⁴ How riskiness is measured is discussed later.

⁵ This assumption excludes the possibility for moral hazard.

⁶ Prior to 1993, depositors in Finland were protected by the banks' own guarantor funds. As practice has shown, however, the government has in fact backed these funds, which makes the assumption realistic.

⁷ In the model, shareholders receive the difference between the audit day value of the bank's assets and its liabilities. The shareholders can then refinance the bank and continue its operations.

⁸ This assumption is in line with the Government Guarantor Fund Act, 1 §.

⁹ The model requires that $\hat{V} < D + B$. Otherwise, the value of the deposit insurance contract, modelled as a put option, can not be calculated.

$$\begin{aligned} & \max \left\{ 0, D e^{rT} - (V_t + \hat{V} e^{rT}) \frac{D}{D+B} \right\} \\ & = \max \left\{ 0, D \left(1 - \frac{\hat{V}}{D+B} \right) e^{rT} - V_T \frac{D}{D+B} \right\} \end{aligned} \quad (4)$$

Presuming that the log of the market value of the bank's risky assets, V , follows a diffusion-type stochastic process, its dynamics can be described by the following stochastic differential equation:

$$dV = (\alpha V - \delta V) dt + \sigma_V V dz \quad (5)$$

where α is the instantaneous expected rate of the total return on the bank's risky assets per unit of time; δ the bank's constant dividends per assets pay-out ratio, DD/V (dividends are assumed to be paid continuously); σ_V is the instantaneous standard deviation of the return on the bank's risky assets and dz a standard Wiener process.

Moreover, constant interest rates are assumed.¹⁰ Therefore, D , B , S and \hat{V} all are assumed to follow the same deterministic processes, which are instantaneously perfectly correlated. Thus, we may write:

$$d(S + B + D - \hat{V}) = r(S + B + D - \hat{V}) dt \quad (6)$$

The market value of the deposit insurance guarantee contract, or in other words the insurers liability, I_D , at any point of time can be,

¹⁰ An explicit interest rate risk of the assets could be allowed by assuming stochastic interest rates. Here all sources of asset risk, including the interest rate risk, are assumed to be embodied in σ_V . The present analysis, however, neglects the effects of interest rate risk due to potential bank's asset-liability duration mismatch on the price of the deposit insurance. Rindell (1993) has shown that using deterministic instead of stochastic interest rates when pricing European style options with a Black and Scholes (1973) model as developed here, may lead to significant pricing errors if the *negative* correlation between the returns of the underlying asset (bank assets) and bonds (bank liabilities) is large. Here, it is however, assumed that banks are effectively immunized implying high *positive* correlation. This assumption is supported by Pennacchi (1987a), who finds positive correlation coefficients of the order of over 0.9 between asset returns and deposit returns of 23 large US banks. Moreover, Ronn and Verma (1986), who estimate the fair deposit premia for 32 US banks with both constant and stochastic interest rates. They find that interest rate risk makes only a very small difference in the overall risk of a bank and thus, does not have a significant impact on the point estimates of "fair" deposit premia.

therefore, written as a function of the value of the bank's risky assets and time i.e. $I_D = I_D(V,t)$. By using Itô's lemma, the dynamics of the insurer's liability is presented by

$$dI_D = \frac{\partial I_D}{\partial V} dV + \frac{\partial I_D}{\partial t} dt + \frac{1}{2} \frac{\partial^2 I_D}{\partial V^2} (dV)^2$$

$$= \left[\frac{1}{2} \frac{\partial^2 I_D}{\partial V^2} \sigma_V^2 V^2 + \frac{\partial I_D}{\partial V} (\alpha V - \delta V) + \frac{\partial I_D}{\partial t} \right] dt + \frac{\partial I_D}{\partial V} \sigma_V V dz$$
(7)

Following the lines of the Black and Scholes' (1973) derivation of their option pricing model, the equilibrium conditions of the expected return of the deposit insurance contract is derived by forming a zero-investment hedge portfolio containing the type of risky assets held by the bank,¹¹ the deposit insurance guarantee contract and a risk-free bond, B_{rf} , with the same maturity as the insurance contract. The value of the zero investment portfolio, P , is expressed as

$$P \equiv Q_1 V + Q_2 I_D - \left(\frac{Q_1 V + Q_2 I_D}{B_{rf}} \right) B_{rf}$$
(8)

where Q_1 and Q_2 are the quantities of the risky asset and the deposit insurance contract respectively. To assure zero net investment, the quantity of the risk-free bond must be $[(Q_1 V + Q_2 I_D)/B_{rf}]$. The market value of B_{rf} is assumed to be non-stochastic and follow the process $dB_{rf} = rB_{rf}dt$, r being the risk-free interest rate.

The instantaneous markka return to the portfolio is given by¹²

¹¹ Or alternatively a portfolio with the same CAPM beta as the bank's asset portfolio. This is possible due to the fact that according to CAPM the idiosyncratic risks are not priced in the market.

¹² Note that the instantaneous per markka return on the risky assets is here $dV = \alpha V dt + \sigma_V V dz$. For notational consistency the correction term $\delta V dt$ is thus needed in equation (9).

$$dP \equiv Q_1(dV + \delta V dt) + Q_2 dI_D - \left(\frac{Q_1 V + Q_2 I_D}{B_{rf}} \right) dB_{rf} \quad (9)$$

which after substitution becomes

$$dP = \left[Q_1(\alpha - r)V + Q_2 \left[\frac{1}{2} \frac{\partial^2 I_D}{\partial V^2} \sigma_v^2 V^2 + \frac{\partial I_D}{\partial V} (\alpha - \delta)V + \frac{\partial I_D}{\partial t} - rI_D \right] \right] dt + \left[Q_1 + Q_2 \frac{\partial I_D}{\partial V} \right] \sigma_v V dz \quad (10)$$

The quantities of the risky assets, insurance contract and risk-free bond are chosen so that the systematic risk of the hedge portfolio is zero i.e.

$$\text{Var}(dP) = E[(dP)^2] = \left[Q_1 + Q_2 \frac{\partial I_D}{\partial V} \right]^2 \sigma_v^2 V^2 dt = 0 \quad (11)$$

which¹³ implies that

$$\frac{Q_1}{Q_2} = - \frac{\partial I_D}{\partial V} \quad (12)$$

Dividing (10) by Q_2 , using (12) and the arbitrage condition that the expected return of the zero investment portfolio must be equal to zero i.e. $E[dP] = 0$, we get¹⁴

$$\frac{\partial I_D}{\partial T} = \frac{1}{2} \frac{\partial^2 I_D}{\partial V^2} \sigma_v^2 V^2 + \frac{\partial I_D}{\partial V} (r - \delta)V - rI_D \quad (13)$$

which is the partial differential equation (PDE) for the total value of the deposit insurance contract.

¹³ $E[\cdot]$ is here the expectation operator.

¹⁴ The relationship $\partial I / \partial t = -(\partial I / \partial T)$ is used here.

The differential equation is finally solved subject to the boundary conditions given by (3) and (4) by using the theorem reviewed by Smith (1976 page 16). The equilibrium solution to the value of the deposit insurance contract per-markka of deposits, $i_D = I_D/D$, is then given by

$$i_D = \left(1 - \frac{\hat{V}}{D+B}\right) N(y + \sigma_V \sqrt{T}) - e^{-\delta T} \frac{V}{D+B} N(y) \quad (14)$$

where

$$y = \frac{\ln[(D+B - \hat{V})/V] + [\delta - (\sigma_V^2/2)]T}{\sigma_V \sqrt{T}} \quad (15)$$

and $N(\cdot)$ is the cumulative normal distribution.

On the right hand side of equation (14), the first term is the difference between the bank's risk-adjusted present value of the proportional net obligations to its depositors and senior debt holders beyond its risk-free assets. The second term is the proportional dividend exempt risk-adjusted present value of the banks assets. The value of the deposit insurance contract, i , is expressed as the difference between these two, $N(y + \sigma_V^2 \sqrt{T})$ and $N(y)$ being the risk-adjustment factors respectively.¹⁵

Equation (14) is identical to the formula for the value of a put option with exercise price $[1 - \hat{V}/(D+B)]$ on dividend paying risky assets with current value V divided by the face value of total senior debt $(D+B)$, and a modification of the Black and Scholes (1973) option pricing formula.¹⁶ It may, however, be noted that the risk-free interest rate does not appear in equation (14) in the factor with which the present value of the exercise price is calculated as in the Black and Scholes formula. This relies on the fact that D, B and \hat{V} are present values. Thus, in this model the value of the per-markka deposit

¹⁵ Even though σ_V is in reality a decision variable of the bank, it is considered as exogenous. In the model σ_V is fixed at the beginning of the insurance period and moral hazard is assumed not to exist.

¹⁶ For a thorough derivation of the Black and Scholes (1973) model see e.g. Duffie (1988).

insurance premium does not directly depend on the level of the risk-free rate of interest.

4.2 Determinants of the value of the deposit insurance contract

Applying the insights of the Black and Scholes formula, $N(y + \sigma_v \sqrt{T})$ can be interpreted as an approximation of the risk-adjusted probability for the event that the bank is unable to meet the claims of the depositors and other senior debt holders. In other words, it is the probability that the deposit insurer will have to provide funds to pay for a shortfall between the value of the depositors' share of total senior debt and the value of the bank's assets.

Now, if we let $L \equiv D + B + S$, i.e. the total liabilities, implying that $L - S = D + B$, then (15) can be rewritten as

$$y = \frac{\ln[(L - S - \hat{V})/V] + [\delta - (\sigma_v^2/2)]T}{\sigma_v \sqrt{T}} \quad (16)$$

Since $N(y + \sigma_v \sqrt{T})$ is a strictly increasing function, from (16) it is easy to see that the probability of the event that the bank is unable to meet the depositors' and other senior debt holder claims increases if the risk-free assets-to-risky assets ratio and/or the subordinated debt-to-risky assets ratio increases. Also, a decrease in the total senior debt-to-risky assets and an increase in equity-to-risky assets both decrease this probability. Moreover, if the amount of risk-free assets equals the amount of total senior debt, which is the case of a "narrow bank", then this probability will be zero. This relies on the fact that if $\hat{V} = D + B$, then $(y + \sigma_v \sqrt{T}) = -\infty$ and therefore

$$N(-\infty) = \int_{-\infty}^{-\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = 0 \quad (17)$$

Finally, it can be seen from equation (16), that an increase of σ_v (i.e. the bank's business risk) causes an increase in this probability.

Naturally, the value of the deposit insurance contract changes along with the above illustrated probability changes. This is seen in the following comparative static analysis:

First, it is clear that the value of the deposit insurance contract must increase along with the increase in the riskiness of the bank's assets, which is measured by σ_V . This can be seen from the following non-negative partial derivative of i_D with respect to σ_V , given by:

$$\frac{\partial i_D}{\partial \sigma_V} = e^{-\delta T} \frac{V}{D+B} n(y) \sqrt{T} \geq 0 \quad (18)$$

where $n(\cdot)$ is the standard normal density function.

According to the model, the value of deposit insurance is non-linear and very sensitive to changes in σ_V , as can be seen from figures 1, 2 and 3.

Second, let $l = (D+B)/V$ i.e. the total senior debt-to-risky assets ratio, or in other words, the leverage of the bank.¹⁷ The partial derivative of the per-markka value of deposit insurance with respect to l is non-negative and given by:

$$\frac{\partial i_D}{\partial l} = \frac{e^{-\delta T}}{l^2} N(y) \geq 0 \quad (19)$$

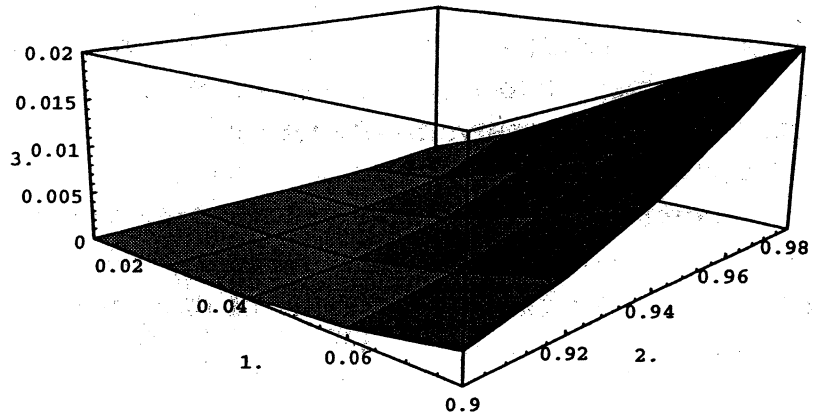
Hence, the deposit insurers's liability increases as the leverage of the bank increases. Accordingly, the value of the insurance contract decreases as the equity's and subordinated debt's share of the bank's total funding of risky assets (i.e. the bank's capital adequacy) increases. This also shows the rationale behind capital adequacy regulation, which is the reduction of the bankruptcy risk of the bank and therefore, reduces the deposit insurer's risk. Since the deposit insurance transfers the bank's risk to the insurer, the argument of Crouhy and Galai (1991), that when deposits are all fully insured, capital regulation should be applied in order to protect the deposit insurer rather than the depositors, makes considerable sense.

Figure 1 shows the interactive effect of volatility and leverage on the per-markka of deposits value of deposit insurance. From the figure it can be seen, *that the deposit insurer's liability to a bank with a high business risk can be reduced by imposing a higher capital requirement.* Or, alternatively, if the bank is not adequately capitalized,

¹⁷ In the model, $(D+B)/V$ is generally in accordance with BIS standards for calculation of capital adequacy so that 1) subordinated debt and reserves are to a great extent treated as capital and 2) risk free assets are not subject to capital requirements. However, the model assumes, opposite to BIS standards, that mortgage loans and assets due from other banks are subject to a 100 % capital requirement.

its deposit insurance value can be reduced by decreasing its business risk.

Figure 1. The interactive effect of changes in bank asset volatility and bank leverage, $(D + B)/V$, on the per-markka value of deposit insurance. In the figure $\delta = 0$, $T = 1$ and $\hat{V} = 0$



1. Volatility of the bank's assets
2. Leverage
3. Deposit insurance value (per-markka of deposits)

Third, the change in the value of the deposit insurance contract to an increase in the risk-free assets-to-senior debt ratio is, on the other hand, non-positive. This is shown by letting $q = \hat{V}/(D + B)$ and taking the partial derivative of i_D with respect to q , given by:

$$\frac{\partial i_D}{\partial q} = -N(y + \sigma_v^2 \sqrt{T}) \leq 0 \quad (20)$$

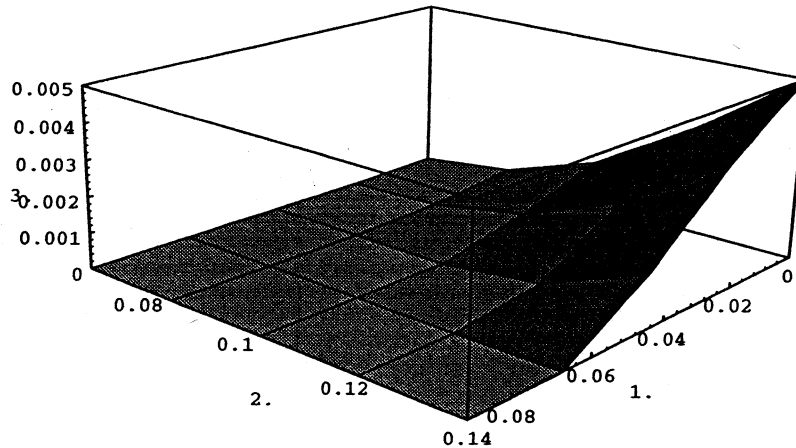
\hat{V} can be the bank's own decision variable and/or represent the obligatory reserve requirement set by bank regulators, in the case of which the parameter q could be interpreted as the chosen reserve requirement policy.¹⁸ In the framework of this model, the latter case would mean a forced possession of risk-free assets earning a risk-free rate as reserve funds. *Therefore, the deposit insurer's liability can be,*

¹⁸ Here the bank would have to provide a reserve requirement for both deposits and other senior debt. However, even if the bank would have to hold reserves based only on the amount of the deposits, i.e. q would be equal to \hat{V}/D , the sign of the partial derivative, $\partial i/\partial q$, would be negative.

ceteris paribus, controlled by imposing a reserve requirement to the bank. This is also seen in figure 2 where given the riskiness of the bank's assets, the insurer's liability is decreased by increasing the reserve requirement.

Figure 2.

The interactive effect of changes in bank asset volatility and required reserves, $\hat{V}/(D + B)$, on the per-markka value of deposit insurance. In the figure $\delta = 0$, $T = 1$ and $(D + B)/V = 0.82$.



1. Required reserves
2. Volatility of the bank's assets
3. Deposit insurance value (per-markka of deposits)

Fourth, the partial derivative of i_D with respect to time to the deposit insurer's audit of the bank's assets, T , is taken. The relation between the per-markka value of the deposit insurance contract and time to maturity is positive, as seen in the following:

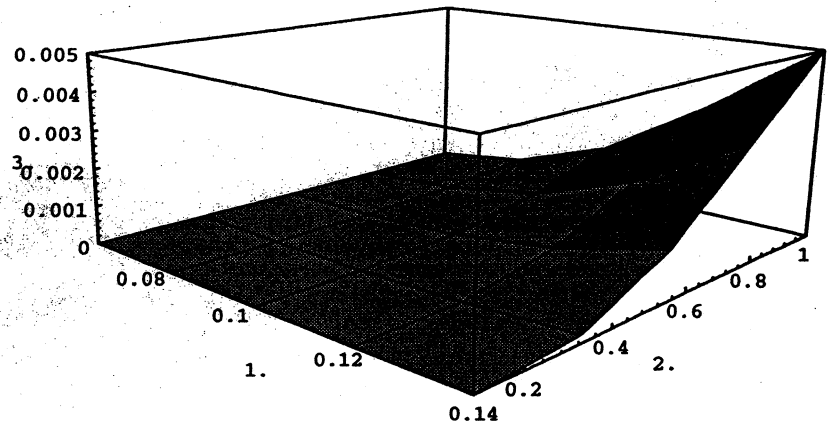
$$\frac{\partial i_D}{\partial T} = \delta e^{-\delta T} \frac{V}{D+B} N(y) + e^{-\delta T} \frac{V}{D+B} n(y) \frac{\sigma_V}{2\sqrt{T}} \geq 0 \quad (21)$$

Therefore, the deposit insurer's liability can be decreased by increasing the frequency of bank audits. Marcus and Shaked (1984) also point out that since σ_V is always multiplied by \sqrt{T} in the model, the value of the deposit insurance is equally sensitive to the audit interval as to the business risk measure, σ_V . *Recalling, that the model is very sensitive to σ_V , frequent monitoring seems to be an efficient way of*

reducing the insurer's liability also in case of a risky bank.¹⁹ This can be seen from figure 3, where given the riskiness of the bank's assets, the insurers liability decreases along with the inspection frequency.

Figure 3.

The interactive effect of changes in bank asset volatility and time to next bank inspection, T, on the per-markka value of deposit insurance. In the figure $\delta = 0$, $\hat{V} = 0$ and $(D + B)/V = 0.82$.



1. Volatility of the bank's assets
2. Time to next inspection of the bank (years)
3. Deposit insurance value (per-markka of deposits)

Finally, figures 1–3 provide an additional perspective, namely the dualities between the insurance premium and 1) capital-asset ratio, 2) reserve requirements and 3) bank inspection frequency. From the insurer's point of view, a regulatory policy mix of a risk-based "fair" insurance premium together with a fixed capital requirement, a fixed reserve requirement and a fixed inspection frequency is equal to a policy mix with a fixed deposit insurance premium together with a risk-based capital-asset ratio, or a risk-based reserve requirement, or a risk-based inspection frequency, or a (risk-based) combination of the latter three.

¹⁹ Given of course that the insurer is able to determine the true value of the bank's net worth.

4.3 Deposit insurance coverage and insurer forbearance

If, on the audit date, the bank's net worth is found to be negative, the uninsured debt holders will have the right to file a bankruptcy petition and demand liquidation of the bank. It may however be, as suggested by Kane (1986), that there will be some political, legal or other constraints, which do not allow the government (i.e. the deposit insurer) to close the bank. If the deposit insurer wants to let the bank to continue its business, it must either buy out the claims of the initially uninsured liability holders or insure their position in order to keep them from forcing a liquidation. This means relaxation of the assumption 7 in section 4.1.

Following Thompson (1987) it is assumed that $b(\kappa)$ and $s(\kappa)$ are functions, which give the probabilities²⁰ that the deposit insurer insures the other senior debt and subordinated debt holders' positions respectively, conditional that the bank's net worth is found to be negative during an audit. These probabilities can be larger than zero if the true insurance coverage is discretionary (i.e. not formally set by law but offered by the insurer because of political, legal or some other constraint which controls the deposit insurer's ability to close the bank). This is the insurer's or in the case of a governmental deposit insurance, the government's implicit insurance.

The single index parameter κ comprises all imaginable constraints. Values of $b(\kappa)$ and $s(\kappa)$ can be thought as being formed by market participants' expectations of the optimal future bank closure policy decision of the insurer (i.e. the government), in case of a bank insolvency. Both $b(\kappa)$ and $s(\kappa)$ are hence measures of the deposit insurer's expected forbearance.

In the following analysis it is assumed that information concerning the index κ is symmetric and "common knowledge" to all agents. This means that the investors always have the "correct" assessments concerning the probabilities $b(\kappa)$ and $s(\kappa)$. This is a strong assumption, but made here in order to neutralize the effects of any dynamically inconsistent closure rule announcement of the insurer²¹ and thus to avoid game theory complexities, which would make the empirical

²⁰ Both $b(x)$ and $s(x)$ are numbers between zero and one.

²¹ For example the insurer would face credibility problems when trying *ex ante* to announce some dynamically inconsistent insurance coverage rule. Such an announcement would, therefore, not have any effects on the behaviour of the banks' debt holders and shareholders.

estimations of deposit insurance premia extremely cumbersome, if not impossible.

With insurer forbearance, rather than being I_D , the deposit insurer's total liability, I_L , will now be

$$I_L \equiv I_D + b(\kappa) * I_B + s(\kappa) * I_S, \quad (22)$$

where I_B and I_S are the "fair" total (market) values of the other senior debt's and subordinated debt's insurance guarantees, respectively.

The derivation of the equilibrium solution to the value of I_L needs solutions for I_B and I_S . The solution of I_B is identical to that of I_D . The partial differential equation (13)²² is solved to the boundary conditions

$$I_{B(\text{at maturity})} = \begin{cases} 0 & \text{if } V_T \geq e^{rT}(D+B-\hat{V}) \\ B(1 - \frac{\hat{V}}{D+B})e^{rT} - V_T \frac{B}{D+B} & \text{if } V_T < e^{rT}(D+B-\hat{V}) \end{cases} \quad (23)$$

The equilibrium solution for the value of the deposit insurer's liability for insuring only other senior debt will be:

$$I_B = B \left(1 - \frac{\hat{V}}{D+B} \right) N(y + \sigma_v \sqrt{T}) - e^{-\delta T} B \frac{V}{D+B} N(y) \quad (24)$$

where y is given by equation (15). The "fair" per-markka insurance premium for the other senior debt, i_B , is achieved by dividing the expression (24) by B .

If $s(\kappa)$ is assumed to be equal to zero, the deposit insurers liability is:

²² With the replacement of I_D by I_B .

$$I_D + b(\kappa)I_B = (D + b(\kappa) * B) \left[\left(1 - \frac{\hat{V}}{D+B} \right) N(y + \sigma_V \sqrt{T}) - e^{-\delta T} \frac{V}{D+B} N(y) \right], \quad (25)$$

where y is given by equation (15). Equation (25) gives the present value of the insurer's expected negative cash flow. Interestingly, no matter what the value of $b(\kappa)$, equation (25) is identical to equation (14) if divided by $D + b(\kappa)B$. *This means that the riskiness of the bank and therefore, the "fair" deposit insurance premium of per-markka of insured liabilities, does not depend on the depositors' and other senior debt investors' percentage shares of the bank's total senior debt funding, as long as deposits and other senior debt are of equal seniority.*²³

If, however, $s(\kappa)$ is non-zero (i.e. the probability that the deposit insurer is willing to cover the possible losses of subordinated debt holders is more than zero), then the insurer's liability is as given by equation (22). In order to derive the equilibrium value for I_S , the solution for I_L is needed. This is obtained by using the bank audit date value of the insurance, which covers all the bank's liabilities given by:

$$I_{L(\text{at maturity})} = \begin{cases} (L - \hat{V})e^{rT} - V_T & \text{if } V_T - (L - \hat{V})e^{rT} < 0 \\ 0 & \text{if } V_T - (L - \hat{V})e^{rT} \geq 0 \end{cases} \quad (26)$$

Solving the PDE (13)²⁴ for the boundary conditions of equation (26), the following equilibrium solution for the value of deposit insurance, which covers all liabilities in all circumstances ($b(\kappa) = s(\kappa) = 1$), is obtained:

$$I_L = (L - \hat{V})N(z + \sigma\sqrt{T}) - Ve^{-\delta T}N(z) \quad (27)$$

where

²³ Here, asset-liability maturity mismatch is ruled out. Therefore, this is not exactly true if one source of funding is durationwise superior to another in terms of asset-liability matching. Here it is, however, assumed, that the availability of funds is perfect if only the bank is willing to pay the appropriate interest rate.

²⁴ With I_L replacing I_D .

$$z = \frac{\ln[(L - \hat{V})/V] + [\delta - (\sigma_v^2/2)]T}{\sigma_v \sqrt{T}} \quad (28)$$

Using the above results and the relation (30), the equilibrium solution for the value of I_s will be the difference of I_L and $(I_D + I_B)$ and given by

$$I_s = (D + B - \hat{V})[N(z + \sigma\sqrt{T}) - N(y + \sigma\sqrt{T})] + SN(z + \sigma\sqrt{T}) - e^{-\delta T}V[N(z) - N(y)], \quad (29)$$

where z and y are as in equations (15) and (28) respectively. Again, the "fair" per-markka insurance premium for subordinated debt, i_s , is calculated by dividing expression (29) by S .

After substitution, equation (22) can be written as:

$$I_L = (D + b(\kappa) * B) \left[\left(1 - \frac{\hat{V}}{D + B} \right) N(y + \sigma_v \sqrt{T}) - e^{-\delta T} \frac{V}{D + B} N(y) \right] + s(\kappa) \{ (D + B - \hat{V}) [N(z + \sigma\sqrt{T}) - N(y + \sigma\sqrt{T})] + SN(z + \sigma\sqrt{T}) - e^{-\delta T} V [N(z) - N(y)] \}, \quad (30)$$

From equation (30) as well as from equation (22) it may be seen that an increase in the probabilities that the deposit insurer fails to liquidate the bank in cases of a default in its subordinated debt, measured by $s(\kappa)$, and further in its other senior debt, measured by $b(\kappa)$, both increase the expected present value of the insurer's liability.

5 Empirical Estimation of Point Estimates of Deposit Premia by Using Stock Market Information

5.1 Estimation methodology

Two complications of using equations (14), (24), (27), and (30) for calculating point estimates of different banks' $i_{D:S}$, $i_{B:S}$, $i_{S:S}$, and $i_{L:S}$ arise. Firstly, neither the market values of banks' assets, V , nor their assets' instantaneous standard deviations, σ_V , can be directly observed, and secondly, the insurer's forbearance, measured by $b(\kappa)$ and $s(\kappa)$ is not observable.

The former complication can be managed. Instead of the market values of the underlying assets of the deposit insurance contract, the market value of the bank's equity¹ together with the book values of D , B , S and \hat{V} can be observed. The motivation for using stock market data is based on the assumption 5, which presupposes that the investors' information about the banks' assets is contained in the stock prices. On the other hand, the book values of the bank's liabilities can be, according to Marcus and Shaked (1984), considered as fairly good estimates of market values if they are to a sufficient extent of short maturities. With D and \hat{V} this can be considered to be the case, since they are both free of default risk. Cases B and S are different, because both subject to default risk, their market values may differ from their book values. Thus, direct use of book values for B and S could lead to biased estimates of insurance values.

This problem can be, however, solved by using the expressions (24) and (29): The total value of risk-bearing services, provided together by both the deposit insurer and the uninsured liability holders, is the same irregardless of how this burden is allocated. Therefore, equations (24) and (29) give not only the "fair" premiums the insurer *should* charge from the bank but also the risk premiums the senior and subordinated debt holders *would* require if they would not be insured.² This is to say that if the insurer does not formally insure B and S , then the senior and subordinated debt holders will "implicitly

¹ if publicly traded

² Naturally, this is also true for the deposit holders in cases where the risk premium is as given by equation (14).

insure" their claims by charging risk premiums as given by (24) and (29).³

Assuming that in case of no insurance the liability holders would charge an up-front premium payment according to (14), (24) and (29),⁴ then according to the Modigliani-Miller Theorem, regardless of whether all or no liabilities are insured, the following equilibrium value of equity must hold:⁵

$$\begin{aligned} E &= V + \hat{V} - D(1 - i_D) - B(1 - i_B) - S(1 - i_S) \\ &= V + \hat{V} - (D + B + S) + (I_D + I_B + I_S) \end{aligned} \quad (31)$$

Since $I_S = I_L - (I_D + I_B)$ the equilibrium expression may also be written as

$$E = V - (1 - i_L) \cdot L + \hat{V} \quad (32)$$

Solving equation (32) for V, using the relationship

$$\delta = \frac{DD}{V} = \frac{DD}{E + (1 - i_L) \cdot L - \hat{V}} \quad (33)$$

and substituting into equation (27), an implicit solution for i_L , in terms of observable values of L (equalling D+B+S), DD and \hat{V} , is obtained and given by

³ In fact, the senior and subordinated debt holders would charge a risk premium equal to $b(x) \cdot i_B$ and $s(x) \cdot i_S$. The analysis that follows would be consistent with any values of $b(x), s(x) \in [0, 1]$.

⁴ This means that the senior and subordinated debt are issued at a price below their face value, the difference being paid out of the bank's assets on the issue date.

⁵ Ronn and Verma (1986) claim that this equation is not needed if the bank is charged a fair deposit insurance premium, which, they claim would prevent any accretion in the value of the bank's assets. Equation (31) would in this case look like a usual balance sheet equation. Equation (31), however, represents the put-call-parity, where equity is seen as a call option on the assets of the bank with strike price equal to the face value of the bank's total debt. Therefore, the argument of Ronn and Verma is a violation against this no-arbitrage condition used e.g. by Markus and Shaked (1984) and Pennacchi (1987).

$$i_L = \frac{(L - \hat{V})N(y + \sigma_V \sqrt{T}) - e^{-T \left(\frac{DD}{E + (1 - i_L) \cdot L - \hat{V}} \right)} (E + L - \hat{V})N(y)}{L \left(1 - e^{-T \left(\frac{DD}{E + (1 - i_L) \cdot L - \hat{V}} \right)} N(y) \right)}$$

where

(34)

$$y = \frac{\ln[(L - \hat{V}) / (E + (1 - i_L) \cdot L - \hat{V})] + \left[\frac{DD}{E + (1 - i_L) \cdot L - \hat{V}} - (\sigma_V^2 / 2) \right] T}{\sigma_V \sqrt{T}}$$

The other unobservable variable, σ_V , is captured using stock market data: if the value of the bank's equity, E , is seen as a function of the value of the bank's risky assets and time, i.e. $E = E(V, t)$, its dynamics can be described using Itô's lemma as

$$dE = \frac{\partial E}{\partial V} dV + \frac{1}{2} \frac{\partial^2 E}{\partial V^2} (dV)^2 + \frac{\partial E}{\partial t} dt \quad (35)$$

By substituting $dV = (\alpha - \delta)Vdt + \sigma_V Vdz$ in equation (35) and manipulating, the relation between the standard deviation rate of the bank's assets and the observable standard deviation of the bank's equity price, σ_E , is given by

$$\sigma_E = \frac{\partial E}{\partial V} \frac{V}{E} \sigma_V \quad (36)$$

$(\partial E / \partial V)$ is solved from equation (32) and substituted in (36). Solving for σ_V yields

$$\sigma_V = \sigma_E \left\{ 1 - \frac{(L - \hat{V})N(-y - \sigma_V \sqrt{T})}{(E + (1 - i_L) \cdot L - \hat{V}) \left(1 - e^{-T \left(\frac{DD}{E + (1 - i_L) \cdot L - \hat{V}} \right)} N(y) \right)} \right\} \quad (37)$$

Equations (37) and (34) comprise two simultaneous equations in the two unknowns σ_V and i_L , which can be numerically solved. From equation (32) it is then easy to get the solution for the other unobservable variable V . Point estimates of i_D , i_B and i_S (and/or I_D , I_B and I_S) can be now calculated by plugging the estimated values of σ_V and V together with the observable parameters D , B , S , \hat{V} and DD into the formulas (14), (24) and (29) respectively.

The other complication is far more inconvenient. An endogenous solution for $b(\kappa)$ and $s(\kappa)$ would need a characterization of the government's utility function together with a full description of the constraints, which dictate the value for the parameter κ . Such an enormous task is not undertaken here and the values for $b(\kappa)$ and $s(\kappa)$ are treated as exogenous.

5.2 Estimation results

As was described in the section 2, the Parliament of Finland approved the following resolution on 23 February 1993:

"Parliament requires the State to guarantee that Finnish banks will be able to meet their commitments on a timely basis under all circumstances. Whenever necessary, Parliament shall grant sufficient appropriations and powers to be used by the Government for meeting such commitments."

In the context of the model in this paper, the above resolution means that $b(\kappa) = s(\kappa) = 1$ can be seen as a realistic assumption and, therefore, the appropriate model for estimating the value of the deposit insurance for year 1993 is given by equation (27). It is obvious, that the values for $b(\kappa)$ and $s(\kappa)$ can change both across different banks and different times. Despite the limited explicit coverage of the deposit insurance, the above resolution, given during times of banking crisis makes, however, the assumption realistic, that actually an implicit insurance covers all liabilities for all banks, for all times i.e.

$s(\kappa) = b(\kappa) = 1$. In the estimation which follows this assumption is used.⁶

An estimation of deposit insurance premiums was performed for a sample of 7 Finnish banks during the period 1987–1993. Monthly logarithmic stock yield time series, calculated from a sample of one year prior to the beginning of each insurance period, was used for the estimation of σ_E . During the estimation period, some of the sample banks had more than one series of shares traded on the Helsinki Stock Exchange. Therefore, let Y represent the monthly yield of the total traded equity of a single bank and $Y = a_1X_1 + \dots + a_nX_n$, where a_i is each stock serie's, i , share of the total market capitalization of the traded equity at the beginning of each insurance period and X_i each serie's, i , monthly yield. The needed estimate for the standard deviation of Y is then given by

$$\text{STD}(Y) = \sqrt{\sum_{i=1}^n a_i^2 \sigma_i^2 + 2 \sum_{i < j} a_i a_j \sigma_{ij}}, \quad (39)$$

where, $\sigma_i^2 = \text{Var}(X_i)$ and $\sigma_{ij} = \text{Cov}(X_i, X_j)$ so that $(i, j=1, \dots, n; i \neq j)$. The standard error of the estimated σ_E is approximately

$$\frac{\sigma_E}{\sqrt{2n}}, \quad (40)$$

where n is the number of share price observations.

The empirical counterpart of D is represented by the current accounts and other deposits of the public. B constitutes other liabilities except subordinated debt, S . \hat{V} includes cash in hand, assets due from Bank of Finland and the Republic of Finland and finally, Finnish Government bonds held by the parent company (foreign affiliates excluded). All values were taken from consolidated balance sheets reported in annual reports except those for the government bonds, which were collected from the Bank of Finland archives. The bank's total market capitalization was used as the market value of equity, E . All this data is dated at the end of the year prior to each insurance period. DD is, however, represented by the actual dividends paid at the

⁶ Note, that any other values of $s(x)$, $b(x) \in [0,1]$ could be used here. Thus, the point estimates of I_b and I_s listed in table 1 can be justified to any forbearance level by simply multiplying them by the appropriate values of $b(x)$ and $s(x)$ respectively.

year during the insurance period. For year 1993, it was assumed that no dividends are paid for any bank except Ålandsbanken and Interbank, in case of which their year 1990 and 1991 dividends were used respectively. Finally, T was set to be equal to one⁷ and equations (24) and (27) were solved simultaneously for two unknowns, i and σ_v by using Mathematica-program's "FindRoot"-tool. The results are reported in table 1.

Firstly, the results in table 1 are interesting, because the value of the deposit insurance, I_D , seems to be, with very few exceptions virtually zero for all sample banks for all sample periods. This result is, however, somewhat in line with the results of Ronn and Verma (1987) and Marcus and Shaked (1984).

Secondly, even though the point estimates of i_D are small, they show that the banks' riskiness varies not only across banks but also across different times. *This result speaks against a fixed rate deposit insurance premium.*

Table 1 also shows the obvious result that when all liabilities are insured in all circumstances, the per-markka-of-liabilities value of deposit insurance, i_L , is larger than the estimated values of the per-markka-of-deposits value of deposit insurance i_D . This relies on the fact that now subordinated debt no longer protects the deposit insurer, rather it imposes extra liability.

At first sight, it might be difficult to believe in the estimated values of the banks' asset volatilities when knowing the developments of both the financial condition and the level of bad loans of Finnish banks during the sample period. Another reason for doubt is the result that bank asset volatilities, with the exception of SKOP,⁸ seem to have a downward trend towards the end of the sample period, even though the economic environment has all that time been deteriorating.

⁷ Even though most empirical studies of option theory deposit insurance use option theory and one year as the periodicity of bank audits, the choice is somewhat arbitrary. Ronn and Verma (1986) have, however, shown that the cross-sectional comparison of estimated fair premia among banks is robust to changes in T . Moreover, using one year, annualized deposit premia are yielded.

⁸ Skopbank has been technically bankrupt in 1993 (which agrees with the estimation results; i.e. the market capitalization is less than the value of I_D). Thus, its share price has probably reflected the uncertainty of whether the initial stockholders would lose their position in the bank rather than the riskiness of its assets. This may bias the estimates severely. These issues are discussed later.

Table 1.

Point estimates of deposit and other liability insurance values and bank data

	σ_E % p.a. (std error)	σ_V % p.a.	E mill.	E/V	i_D, i_B	I_D thous.	I_B thous.	i_S	I_S thous.	i_L	I_L thous.
Interbank											
1991	70.11 (14.3)	5.25	47	0.067	$2.7 \cdot 10^{-3}$	975	821	-	-	$2.7 \cdot 10^{-3}$	1,796
1992	56.41 (11.5)	1.65	55	0.028	$1.1 \cdot 10^{-4}$	200	18	$3.1 \cdot 10^{-2}$	372	$3.0 \cdot 10^{-4}$	590
1993	47.43 (9.7)	0.75	51	0.016	$6.8 \cdot 10^{-10}$	<1	<1	$3.0 \cdot 10^{-3}$	192	$5.6 \cdot 10^{-5}$	192
Kansallis-Osake-Pankki (KOP)											
1987	21.19 (4.3)	0.97	4,447	0.046	0	0	0	$1.3 \cdot 10^{-7}$	<1	$5.4 \cdot 10^{-9}$	<1
1988	22.95 (4.7)	1.05	5,104	0.046	0	0	0	$9.1 \cdot 10^{-7}$	4	$3.5 \cdot 10^{-8}$	4
1989	18.41 (3.8)	1.16	8,798	0.063	0	0	0	$2.0 \cdot 10^{-9}$	<1	$8.2 \cdot 10^{-11}$	<1
1990	22.48 (4.6)	1.19	7,836	0.053	$5.2 \cdot 10^{-17}$	0	0	$3.3 \cdot 10^{-7}$	2	$1.2 \cdot 10^{-8}$	2
1991	27.55 (5.6)	0.97	5,392	0.035	$5.3 \cdot 10^{-15}$	0	0	$8.6 \cdot 10^{-6}$	38	$2.5 \cdot 10^{-7}$	38
1992	28.89 (5.9)	0.60	2,953	0.021	0	0	0	$1.3 \cdot 10^{-5}$	50	$3.5 \cdot 10^{-7}$	50
1993	84.69 (17.2)	1.08	1,673	0.010	$1.7 \cdot 10^{-8}$	<1	2	$3.2 \cdot 10^{-2}$	191,340	$1.2 \cdot 10^{-3}$	191,342
Osuuspankkien Keskusosakepankki (OKO)											
1991	24.46♦ (6.5)	0.89	1,932	0.037	$9.7 \cdot 10^{-14}$	0	0	$3.2 \cdot 10^{-6}$	3	$6.7 \cdot 10^{-8}$	3
1992	23.71 (4.8)	0.71	1,537	0.030	$3.3 \cdot 10^{-17}$	0	0	$8.8 \cdot 10^{-7}$	1	$2.0 \cdot 10^{-8}$	1
1993	29.50 (6.0)	0.53	948	0.018	$2.2 \cdot 10^{-10}$	<1	<1	$5.1 \cdot 10^{-5}$	22	$4.0 \cdot 10^{-7}$	22
Säästöpankkien Keskusosakepankki (SKOP)											
1991	25.34 (5.2)	0.68	2,496	0.027	$3.0 \cdot 10^{-15}$	0	0	$2.5 \cdot 10^{-6}$	4	$4.9 \cdot 10^{-8}$	4
1992	48.16 (9.8)	1.24	2,079	0.025	$6.9 \cdot 10^{-5}$	116	5,417	$1.8 \cdot 10^{-2}$	1,861	$9.2 \cdot 10^{-5}$	7,395
1993	149.84 (30.6)	11.02	1,646	0.029	$6.1 \cdot 10^{-2}$	105,190	3,478,780	0.642	67,404	$6.2 \cdot 10^{-2}$	3,561,370
STS-Pankki											
1992	22.97 (5.7)	0.81	522	0.035	$7.2 \cdot 10^{-13}$	0	0	$6.0 \cdot 10^{-7}$	<1	$7.6 \cdot 10^{-9}$	<1

	σ_E % p.a. (std error)	σ_V % p.a.	E mill.	E/V	i_D, i_B	I_D thous.	I_B thous.	i_S	I_S thous.	i_L	I_L thous.
UNITAS											
1987	20.82 (4.3)	1.37	6,711	0.067	$8.0 \cdot 10^{-14}$	0	0	$1.4 \cdot 10^{-7}$	<1	$3.3 \cdot 10^{-9}$	<1
1988	16.25 (3.3)	1.06	7,150	0.065	0	0	0	$3.1 \cdot 10^{-11}$	<1	$1.1 \cdot 10^{-12}$	<1
1989	21.30 (4.4)	1.97	11,972	0.092	$9.5 \cdot 10^{-13}$	0		$1.3 \cdot 10^{-7}$	<1	$3.4 \cdot 10^{-9}$	<1
1990	16.36 (3.4)	1.10	9,403	0.067	0	0	0	$2.3 \cdot 10^{-11}$	<1	$6.6 \cdot 10^{-13}$	<1
1991	17.49 (3.6)	0.76	5,578	0.044	0	0	0	$1.5 \cdot 10^{-11}$	<1	$5.5 \cdot 10^{-12}$	<1
1992	34.64 (7.1)	1.03	4,063	0.030	$8.2 \cdot 10^{-12}$	0	0	$1.8 \cdot 10^{-4}$	688	$5.0 \cdot 10^{-6}$	688
1993	64.76 (13.2)	0.88	1,707	0.012	$7.4 \cdot 10^{-18}$	0	0	$6.4 \cdot 10^{-3}$	44,998	$3.2 \cdot 10^{-4}$	44,998
ÅLANDSBANKEN											
1987	24.29 (5.0)	3.41	245	0.14	$5.1 \cdot 10^{-8}$	<1	<1	-	-	$5.1 \cdot 10^{-8}$	<1
1988	27.96 (5.7)	3.79	366	0.136	$7.7 \cdot 10^{-7}$	1	1	-	-	$7.7 \cdot 10^{-7}$	2
1989	23.13 (4.7)	3.95	498	0.171	$1.3 \cdot 10^{-8}$	<1	<1	-	-	$1.3 \cdot 10^{-8}$	<1
1990	24.52 (5.0)	2.60	393	0.106	$9.4 \cdot 10^{-8}$	<1	<1	-	-	$9.4 \cdot 10^{-8}$	<1
1991	28.47 (5.8)	2.40	282	0.084	$1.9 \cdot 10^{-6}$	2	4	-	-	$1.9 \cdot 10^{-6}$	6
1992	27.30 (5.6)	1.70	227	0.062	$1.1 \cdot 10^{-6}$	2	2	-	-	$1.1 \cdot 10^{-6}$	4
1993	34.30	2.01	204	0.059	$9.7 \cdot 10^{-6}$	18	17	$2.2 \cdot 10^{-3}$	14	$1.4 \cdot 10^{-5}$	49

♦ May-December

The assets volatility *levels* may not, however, be unrealistic when one bears in mind that bank assets include many loans which are themselves debt claims on underlying risky assets. Being debt claims, their rate of return volatility is lower than the underlying asset in which the borrowed funds are invested. Secondly, since banks hold a portfolio of loans and other assets, portfolio diversification lowers σ_V , relative to the volatility of any single bank asset.

The coincidence of deteriorating economic recession with a fall in asset volatilities may also be perfectly reasonable, since banks may react to increasing loan losses with more conservatism and decrease the proportion of relatively risky projects that they were previously undertaking.⁹ Moreover, in the estimation the risk measure σ_V is derived by using stock market data and thus, reflects the investors'

⁹ This may be optimal to the bank due to charter value (see e.g. Markus (1984)) or due to incentives created by the interplay between banks and the government in case of a high probability of bankruptcy. At this point the issue is not developed further.

assessment for the uncertainty of the development of the value of banks' risky assets. Investors may very well have a clear view in which direction the value of banks' assets are developing and therefore, the investors' assessment for the asset value uncertainty i.e. the value of σ_v may be fairly small. Formally, this would mean that an increase in bad loans and thus a decrease in the value of assets is captured by the drift term α in equation (5) the variation term σ_v at the same time being small in value. The assumptions in section 2 permit the insurer to hedge against losses caused by this deterministic part of the asset value process. Thus, the expected rate of the total return on a bank's risky assets, α , does not appear in equation (13) and therefore does not affect the value of the insurance premium. Estimates can in such an environment turn out to be surprisingly small as seen above.

The above argumentation may be regarded, however, as controversial. The diminutive numbers obtained for most banks' point estimates may be due to weaknesses of the implemented model, discussed in the following.

The problem of the used model is, firstly, the limited-term insurance contract assumption. As mentioned by Pennacchi (1987a), this one-period modelling would be correct only, if the insurance premia were being adjusted at each audit to a new "fair" rate and thus, would revert the insurer's liability to zero. Since in reality it is known *ex post* that this is not the case, the limited term assumption is not justified. Using the limited term assumption leads to downward biases in the point estimates. This is shown by Pennacchi (1987a), whose point estimates of deposit insurance premia, with an explicit modelling of an unlimited-term contract, substantially exceed those of Markus and Shaked (1984) and Ronn and Verma (1987), their models having the limited-term insurance as an underlying assumption. Also, setting the insurance period equal to one year may not be a perfect description of the reality. Thus the point estimates may be biased in this respect as well.

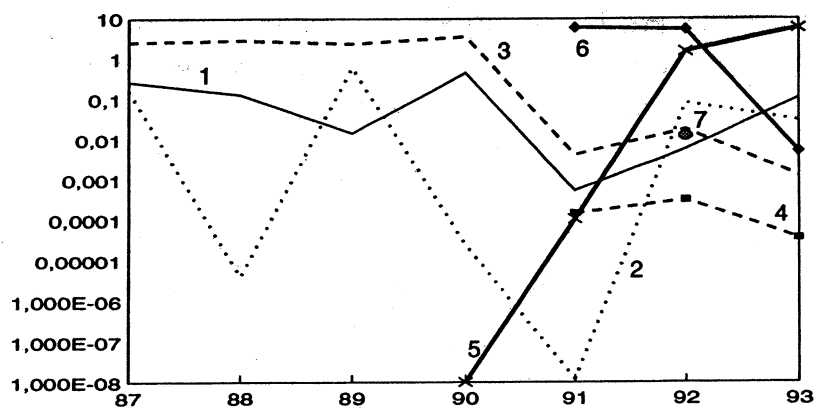
Also, one further possibility for a source of biases in the estimates may arise from the fact, that the model does not include the government's potential implicit insuring of the share holders. If this exists in practice, then it is obviously reflected in the prices of banks' stocks and therefore, affects the used values of volatility and market capitalization of bank equity which are inferred from stock market data. Since these inputs by far determine the values of asset volatility and market value of assets, on which the model is very sensitive, point estimates in table 1 may well be underestimated. The explicit modelling of shareholder subsidy would, therefore, be necessary in

order to get more accurate estimates of "fair" deposit premia. This issue is discussed in the subsequent section.

Finally, however, if measurement errors based on above mentioned sources are systematic both across all banks and all time periods, then the above estimates can be used for cross-sectional comparison of riskiness of different banks for each estimated time period. The comparisons should be, however, done by bearing in mind, that the bank's volatility and its financial position are assumed to stay unchanged during the insurance period. Shifts in these parameters during insurance periods may well change the risk ordering among banks.

Figure 4 shows a risk comparison based on the results in table 1. The figure plots the various banks' i_L 's divided by each year's cross-sectional mean of premia estimates. An observation equal to one represents a bank with business risk of cross-sectional average. The scaling is logarithmic and the lines portray the development of risk ordering among banks.

Figure 4. **Comparison of riskiness among publicly traded Finnish banks**



- 1 KOP
- 2 Unitas
- 3 Ålandsbanken
- 4 OKO
- 5 SKOP
- 6 Interbank
- 7 STS (one observation)

6 A Random Audit Model of the Value of Deposit Insurance

6.1 Insurer forbearance and shareholder subsidy

Implicit insurance of bank shareholder equity can in practice take many forms. These might e.g. be capital infusions of the government which either are *not* paid back by the bank or do *not* bear an appropriate interest rate and ultimately do not result in government takeover of the bank. Another form of shareholder subsidy is government overpayment for non-performing loans from of a distressed bank. All such approaches assure that the value of the banking firm, i.e. the value of the shareholders' equity, will not decline even as loan losses are occurring.

When implicit insurance forms are involved to provide shareholder subsidy, or alternatively, when market participants are given reason to assume that such measures *will* be invoked to support a distressed bank, the share price of the bank is affected. Again, since the point estimates in the table 1. are achieved using stock market data without taking the possible government subsidy into account, they are potentially biased downwards. It may also be suggested that the low estimates of bank asset volatilities in table 1 may in fact be attributable to such implicit insurance of bank assets.

Therefore, as mentioned in section 5, an explicit modelling of the insurer's (i.e. the government's) forbearance acting as an implicit insurance of shareholder equity of distressed banks needs to be performed to get more realistic results.

A natural starting point for the inclusion of these type of governmental policies in the model seems to be the modelling of the equity value. There are several attempts at getting a handle on this problem, the most popular, perhaps, is the idea presented by Ronn and Verma (1986).¹ They include the deposit insurer's (i.e. the government's) forbearance in the equation, which they use to model the bank's equity as a call option on the bank's assets with strike price equal to the present value of the bank's liabilities. Ronn and Verma use a parameter ρ to be the fraction of the value of liabilities below

¹ See e.g. Giammarino, Schwartz, Zechner (1989) or Kang, Ramachandran, Sato (1990).

which the bank's assets must fall before the bank is closed. Their equation for the value of bank equity is:

$$E = VN(x) - \rho BN(x - \sigma_v \sqrt{T}) \quad (41)$$

where B represents total senior debt and ρ is a number between zero and one. Equation (41) assumes that the equity holders will get a pay-off $\text{Max}[V - \rho B, 0]$ at the end of a insurance period. As noted by Kluester and O'Brien (1991), this is clearly incorrect for solvent banks, which according to the pay-off function would be paid a lump sum equal to $B - \rho B$ at the end of the period.

Thomson's (1987) approach to modelling the governmental forbearance is based on the equity holder's end of period one payoff function $\text{Max}[y(\kappa)C_e, V - B]$, where $y(\kappa)$ is the conditional probability that at the bank audit date the stockholders retain their position in the bank conditional that the bank is insolvent. The value of $y(\kappa)$ is again assumed to depend on political, legal and other constraints, which prevent the insurer from closing an insolvent bank. C_e represents the equity position of the investor modeled as a call option for the second (next) period. His modelling of the payoff function is problematic, because of a time horizon inconsistency of the two pay-offs, contingent on the bank's end of period one solvency condition.

Thomson assumes that when a bank is solvent and it is liquidated,² the equity holders receive the difference between the market value of the bank's assets and liabilities. This is a one-period set-up and the equity holders do not have the possibility to continue their call option holdings.³ If, however, the bank is insolvent, the equity holders receive nothing but with some probability the bank is not liquidated, and the equity holders by maintaining their position in the bank are entitled the bank's potential positive net worth sometime in the future. *This, in turn, is a multi-period setting.*

In the context of the balance sheet given by (1), Thomson's ideas may, however, be developed into the following equity holder's conditional end of period one payoff function:

² Note that liquidation here does not mean bankruptcy. The liquidation happens here because of the one-period nature of the model.

³ Recall that they may refinance the bank. In this context, it is equivalent to purchasing a new call option on the bank's assets.

$$C_{e,t1(\text{at maturity})} = \begin{cases} y(\kappa)C_{e,t2(\text{at begin})} & \text{if } V_T - (L - \hat{V})e^{rT} < 0 \\ C_{e,t2(\text{at begin})} & \text{if } V_T - (L - \hat{V})e^{rT} \geq 0 \end{cases} \quad (42)$$

where the indexes t1 and t2 refer to periods 1 and 2 respectively.

In (42), $y(\kappa)C_{e,t2}$, multiplied with the probability, p , that the bank is found insolvent at the end of the insurance period gives the deposit insurers', i.e. the government's subsidy to bank shareholders. The total present value of the deposit insurance guarantees and subsidies, I_U , is then given by

$$I_U = I_L + p \cdot y(\kappa)C_{e,t2} \quad (43)$$

Therefore, the total value of the deposit guarantee increases with the probability of the shareholders' position not being closed in case of a bank insolvency.

Finally, since C_e includes also the future periods' payoff functions, *the appropriate model for analyzing and estimating the value of deposit guarantees, which would include the equity holder subsidy, should have a multi- or infinite-period set-up.* In the following, a multiperiod model, which allows the insurer's potential subsidy towards the bank's equity holders, is presented.

6.2 Insurer power and liability

6.2.1 Insurer's control over capital and forbearance

As mentioned in the section 5.2, the one-period model used earlier in this study as well as in the other empirical studies of deposit insurance value, based on Merton's (1977a) paper (e.g. Marcus and Shaked (1984), Ronn and Verma (1986)), assume implicitly that the deposit insurer's net liability is zero after every bank audit or more precisely, at the time of the start of each new insurance period. As shown in section 4, this could be achieved through an implementation of a risk-based fixed-variable combination of insurance premium, capital-asset ratio and reserve requirements at the time of every bank inspection. Both the deposit insurance premia and the reserve requirements of Finnish banks have been fixed in their nature. Thus, the usage of a Merton -type model in estimating "fair" deposit premia would be

justified only if in actual practice after each audit the capital-asset ratio would be set at exactly such a (risk-based) level, which together with the fixed insurance premium and fixed reserve requirement would cause the insurer's net liability to be equal to zero.

The only regulation which controls the capital level of the Finnish banks follows closely the BIS (Bank of International Settlements) 8 % capital adequacy rule. Even though this rule is an attempt to match the banks' capital levels to their asset risk, it is not to be confused with the risk-based capital-asset ratio which together with a fixed deposit premium would release the deposit insurer from net liability: Firstly, despite of the duality between the insurance premium and the capital level, which as shown by Sharpe (1978), should be the starting point of the capital adequacy standards, the BIS's 8 % level of capital is in no means synchronized with the current deposit insurance system. Secondly, the BIS standard is determined in book value terms, whereas the critical element in determining the insurer's liability is the market value capital-asset ratio. As is implicit in Ronn and Verma (1989), given a fixed deposit insurance together with a fixed book value capital-asset ratio (which may comply with the BIS 8 % standard), the needed market value of capital-asset ratio for the insurer's zero net liability depends on the riskiness (volatility) of the bank's assets. Since the latter is a decision variable of the bank management, the BIS rule can be argued to leave the bank management equipped with considerable flexibility in controlling whether the market valued i.e. risk-based capital-asset ratio sets the value of the insurer's liability to a zero level. It follows that the capital-adequacy regulation being less than perfect, the one-period set up will not succeed in capturing the value of the bank's ability to renew its deposit insurance after the audit at a cost which is possibly below the "fair" premium.

The probability of a "coincidental" occurrence of the deposit insurer's net liability being zero after a bank audit is very petite. Therefore, given the fixed deposit insurance premium and reserve requirements, it is necessary to consider, whether the banks would tend to hold capital in excess or too little relative to such a level, which would impose a zero net liability to the insurer. As suggested by Pringle (1974), it is reasonable to suppose that since banks are private economic units, shareholder interest will influence, if not control, managerial decisions. Thus, it is logical to tackle the question of the optimal bank capital structure from the perspective of the equity-holder's wealth maximization.

The theory of corporate capital structure provides a useful framework for analyzing this issue. There are, however, differences

between banking firms and non-financial firms, regulatory environment being of fundamental importance. The work by Modigliani and Miller (1958) established that the total equilibrium, arbitrage-free value of a firm is, in the absence of taxes, constant across all degrees of financial leverage. Since then, numerous studies have detected "imperfections", which make the equity-holder's maximization problem with respect to the capital structure non-trivial. Without any banking regulation, the following standard results of corporate finance literature can be directly applied to banking firms:

Firstly, as shown by Modigliani and Miller (1963), the deductibility of interest expenses (deposit interest) provides an economic incentive for firms (banks) to maximize their use of debt (deposit) financing. Miller (1977) adds into this model another two types of taxes: the personal income taxes on equity returns and on interest. Whether in equilibrium there is any advantage in having more or less leverage, depends on the interplay between the relative levels of these different taxes. In Finland the tax on deposit interest income has in the past been lower than both the tax on capital income and the corporate tax. According to Miller, this would give support for the case of more leverage preferred.

An incentive for holding more capital i.e. less leverage is in turn provided by potential non-trivial bankruptcy costs as initially suggested by Baxter (1967). More leverage increases the probability⁴ of bankruptcy (see e.g. Santomero and Watson (1977)) and provokes the debt holders (depositors) for requiring higher rate of return, making debt (deposit) financing less attractive. Comparably, agency costs discourage debt financing due to required compensation for lenders' monitoring costs as demonstrated by Jensen and Meckling (1976).

Both the bankruptcy costs and the agency costs of a banking firm are, however, intimately bound to the system of bank regulation and deposit insurance. If deposits as well as other liabilities of the bank are perfectly⁵ insured, as seems to be the situation in Finland, depositors and other liability holders will not require compensation for increased leverage. The attention then is focused on the question, whether the insurance premium charged from banks is fixed or "fairly priced" i.e. reflects the bank's leverage as much as the risk. Following the lines of Buser, Chen and Kane (1981), the deposit insurance system would have the same influence on the bank capital decision as bankruptcy costs (and agency costs) if it would mimic the market's (liability

⁴ See e.g. Santomero and Watson (1977).

⁵ Perfect insurance means here, that the insurer is able to meet its obligations in all possible states of the world.

holders') behaviour in the form of insurance premia, operating restrictions and other supervisory costs imposed to banks, the latter two classified as implicit insurance premia by Buser, Chen and Kane. Thus, the fixed deposit insurance premium schedule of the Finnish system can be thought as favouring leverage. Furthermore, as shown by Marcus (1984), the fixed deposit insurance premium may lead to moral hazard problems by generating further incentive for maximizing the equity-holders' wealth through greater leverage.

Finally, despite of the conclusion of Greenbaum and Taggart (1978), that a reserve requirement, set proportional to the amount of deposits (and/or other liabilities), would create a disincentive for more leverage, it is assumed in the following that these effects would cause the equity value-maximizing bank's management to reach an interior unconstrained optimum at a low capital-asset ratio.

As shown in section 4.2, a lower capital-asset ratio leads to a higher equilibrium value of deposit insurance premium i.e. to a higher insurer's liability. Therefore, given a fixed premium charged from banks and a fixed reserve requirement, it follows that, first, the low capital-asset ratio chosen by the bank's management is apt to lead to a positive net liability of the deposit insurer i.e. the premium charged would be too small relative to the risk. Second, only by imposing an appropriate risk-based minimum required capital-asset ratio constraint, can the insurer adjust its net liability to zero and simultaneously "make" the insurance premium "fair". Since it is assumed, that the minimum capital requirement is likely to exceed the bank's optimal unconstrained capital-asset ratio, the bank's management would then presumably achieve the maximum value of shareholders' equity at a point equal to the minimum required capital-asset ratio.

The model presented earlier in section 4 is now embedded in a multi-period setting following closely the lines of Pennacchi (1987a). Once again, it is assumed that the deposit insurer examines the bank at the end of each insurance period and finds out the bank's market value net worth. To begin with, it is assumed that at the audit the bank is found to be solvent, in case of which the following two types of events might result:

First, if the bank's capital-assets ratio would be found above the minimum required level, which would set the insurer's liability to zero, the bank would now be above its constrained optimal capital level. Therefore, the optimal capital structure would be obtained by either issuing more debt or by paying greater dividends.

Second, if the bank's capital-assets ratio would be below the minimum required level imposing a net liability to the insurer, assume that the deposit insurer would exert regulatory power on the bank in

order to close the capital shortfall. The insurer's power over controlling the bank's capital level can be considered as being a function of how much capital the bank could be obliged to contribute at once. Two polar cases measuring regulatory power control follow:

(a) *full control*, in which the bank can be charged a higher insurance premium or forced to i) instantaneously fill the gap between the required capital level and its contemporary level, or ii) cut its dividends or iii) reduce its asset risk;

(b) *no control*, in which the insurer would have no power to force the bank to bring up its capital-asset ratio.

Next, assume that during an audit the bank is found to be insolvent i.e. its net worth is discovered to be negative. Two scenarios may ensue: *First*, as the most valuation models of deposit insurance premium assume, the bank will in such a case be closed and the equity holders lose their position for good (see e.g. Merton (1977a, 1978), Marcus and Shaked (1984), Pennacchi (1987a)). The insurer's liability will then be the bank's negative net worth. However, as discussed earlier, the probability of an implementation of such a stringent closure rule in case of an actual insolvency may be, due to political and/or other restrictions, less than 100 per cent. Thus, in the *second* potential scenario the insurer is not able to close neither the bank nor the equity-holders' position, and again two polar cases comparable to above (a) and (b) of regulatory power follow:

(c) *full control*, where the insurer has to first provide the shareholders with an amount of capital equal to the negative net worth of the bank, which will now be the insurer's liability. Since the equity-holders' liability is limited, this is of course necessary, in order to be able to assume forcing the old shareholders to come up with the minimum required amount of capital.

(d) If the case of *no control* prevails, the insurer "does nothing" and is left with a liability equal to the total amount of capital, which would have to be provided in order to achieve the minimum required capital-assets ratio.

Thus, since in both cases (c) and (d) the shareholders keep their position in the bank, they are in fact subsidized by the insurer.

It is evident that, *ceteris paribus*, the deposit insurer prefers more power to less in forcing the capital-deficient banks to move straightaway back to the minimum required capital level. Consequently, in the case of full control the "fair" insurance premium would be smaller than in the case of less than full control. Therefore, it is obvious that the "fair" deposit insurance premia cannot be calculated without considering first, the degree of deposit insurer's control over the banks' capital level as argued by Pennacchi (1987a),

and secondly, the potential inability of the insurer to close an insolvent bank.

6.2.2 Insurer's audit frequency and efficiency

Given both a fixed insurance premium and a fixed reserve requirement together with the minimum required capital-assets ratio defined above, there is yet another fundamental element in the audit models of deposit insurance, which may be interpreted as measuring the deposit insurer's power over controlling its liability: the frequency and the efficiency of the bank audits. In the one-period model presented in section 4 of this paper, it was assumed that the bank inspection takes place at a given time with certainty. Moreover, a critical assumption that the audits are efficient i.e. the insurer is able to calculate the true market value net worth of the bank, was made. The reality, however, is not so immaculate. In the Finnish system, banks have typically been subject to an inspection every second year, which would seem to be compatible with the assured, predetermined audit interval assumption. The inspections have, however, mostly concentrated on legal aspects and thus, can not be thought as satisfying the criteria of an efficient audit. It follows, that since there seems to be no systematic timing of *efficient* bank audits, it is more realistic to assume there being significant uncertainty with respect to their frequency. In other words, even though an inspection would take formally place at a given time with certainty, nobody knows *ex ante* for sure, how efficient the audit will at that time be.

The uncertainty may be thought as being to a fair extend conditional on the organization, capacity, skills, incentives and other institutional attributes of the insurer. These factors, which then establish the foundation for the insurer's ability to perform efficient audits and thus, the power to control its liability, can possibly be managed to some extend, yet not perfectly, since these can be either rigid in the short or long run, or entirely exogenous.

Since there is a (negative) correlation between the (efficient) audit frequency and the insurer's liability as demonstrated in section 3, this uncertainty will definitely have an influence on the insurer's liability. Therefore, the "fair" value of the deposit insurance can not be calculated without taking also this dimension of the deposit insurer's power into consideration.

6.3 Derivation of the model

This section develops a continuous time valuation model, which will extend the model presented by Pennacchi (1987a, b), which in turn is an extension of the Merton's (1978) deposit insurance pricing model. These models have assumed, that the bank closure rule would not be discretionary and a bank which would be found having negative net worth would immediately be closed and a nullification of the bank share owners' position would follow. As discussed earlier, the regulators may in a case of bank insolvency be reluctant to let a bank fail and even the share owners may turn out to be *implicitly* insured. The extension suggested here, which will be the expected insurer's forbearance in the case of bank insolvency, has two motives: Firstly, we may analyze if and how this type of forbearance influences the banks risk taking incentives. Secondly, since this forbearance would have economic value, it would be reflected in the bank's share prices and therefore should be included in the modelling when calculating point estimates of "fair" deposit insurance premia by using the information contained in the bank share prices.

Also, a perhaps new content for the uncertainty of the bank audit interval is suggested according to section 6.2.2 allowing us to investigate, whether the institutional and operational properties of the insurer do have an effect on the bank's risk behaviour.

In addition to the assumptions given later in the text together with assumptions 1–5 given in section 2, the following is assumed:

Assumption 1. It is assumed that the value of the bank's assets, V , follow the continuous time diffusion-type stochastic process:

$$\begin{aligned}dV &= (\alpha V - C)dt + \sigma_V V dz, \quad V > 0 \\ &= 0, \quad V = 0\end{aligned}\tag{44}$$

where α is the instantaneous expected rate of the total return on the bank's risky assets and C the total net payments out of bank assets per unit of time. σ_V is the constant instantaneous standard deviation of the return on the bank's risky assets and dz a standard Wiener process. C is the sum of dividend payments to shareholders, net growth in deposits and deposit insurance premium payments, i.e. $C = \delta V - nD + hD$.

Assumption 2. There exists a riskless-in-terms-of-default discount bond, the value of which follows a deterministic process:⁶

$$dB_{rf} = rB_{rf} dt \quad (45)$$

where r is the risk free interest rate.

Assumption 3. Banks issue deposits, which are assumed to earn a rate of return equal to the risk free rate minus the bank's interest rate margin, m . The constant m is assumed to be positively correlated with the bank's monopoly power. The total deposits, D , which are assumed to grow by a (perhaps negative) constant percentage rate of n ,⁷ thus, follow the deterministic process

$$dD = (r_D + n)D dt \quad (46)$$

where $r_D = r - m$.

Assumption 4. The deposit insurer is assumed at random intervals to perform costly bank audits, each c markkas per markka of deposits. The event of an audit is assumed to be Poisson distributed with an intensity parameter (mean) λ . The probability of an audit, which is assumed to be uncorrelated with any non-diversifiable (market) risk,⁸ is λ over the time interval dt . The intensity parameter λ , however, is here assumed to depend on the institutional properties of the insurer as explained in the previous section 6.2.2. Thus, the more "powerful" the insurer, the larger the characteristic parameter of the Poisson distribution λ . Since λ is a pricing parameter in the model derived here, its above interpretation allows us to analyze the effects of the insurer's characteristics on the insurer's liability.

⁶ See section 4 footnote 2.

⁷ The condition $n < m$ must hold in order to avoid the bank's treatment of deposit from becoming a "Ponzi game".

⁸ This assumption may be somewhat inaccurate since the insurer would ignore almost costless available information e.g. the stock market index printed daily in newspapers as criticised by Fries, Mason and Perraudin (1993). Pennacchi (1987), however argues, that the insurer's resources may be limited in the short run and thus, it would fail to increase the number of (efficient) audits per unit of time in response to negative market returns. This argument seems to describe the Finnish reality quite well.

Assumption 5. If the bank is audited and is found to be solvent, it is allowed to continue its business. If, however, the bank's net worth is being found negative, it will be closed with probability $1-y(\kappa)$.⁹ Thus, $y(\kappa)$ measures the degree of regulatory forbearance.

Assumption 6. The banks pay to the deposit insurer a continuous insurance premium h per unit of time per markka of bank deposits. Thus, the premium payments are not tight to the beginning or end of an insurance period as assumed in the one-period model in section 2 of this paper.

The deposit insurer's contingent claim on bank assets equal to minus its liabilities is denoted by G . A larger negative value of G will mean greater liability to the insurer, while a positive G means that, given the current risk and the capital-assets ratio, the bank is charged a payment which exceeds the "fair" deposit insurance premium.

Assume, that at the time of an audit, the bank's insurance premium is adjusted to the "fair" level, or alternatively, the bank's capital will be adjusted to the minimum required level as in the cases (a) or (c) in the preceding section. Following Pennacchi (1987a), this is interpreted as a "limited-term" deposit insurance contract since the insurer's liability will be reverted to zero i.e. the change in the insurer's liability will be $-G$ due to the adjustment in the capital-assets ratio. Or, given the duality between a risk-based insurance premium and a risk-based capital requirement, this may be alternatively interpreted as representing the case where the insurer implements a risk-based insurance premium schedule. This "limited-term" (variable rate) case coincides with the one-period model presented earlier in this study.

On the other hand, if it is assumed that the capital-assets ratio or the insurance premium is not calibrated at the time of an audit, corresponding to the above cases (b) and (d), the deposit insurer's net liability will not revert to zero. This represents the case of an "unlimited-term" or, alternatively, a fixed rate insurance contract. Now, considering both the "limited-term" and "unlimited-term" deposit insurance contracts, the change in the value of the deposit insurer's liability derived from the adjustment of the deposit premium or capital level will be denoted by $-\gamma_1 G$ if the bank is found solvent and $-\gamma_2 G$ if

⁹ $y(\kappa)$ is, as defined earlier, a function, which gives the probability that the insurer is not able to close neither the bank, nor the shareholders' position when the bank is found to be insolvent. The single index parameter κ embodies all the conceivable political, legal and other constraints.

found to be insolvent. $\gamma = 1$ corresponds to the limited-term case and $\gamma = 0$ to the unlimited-term insurance contract. Therefore, γ_1 and γ_2 can be interpreted as the fractions of the pre-audit insurer's liability eliminated by a capital adequacy improvement cases where the bank continues its operations.

Finally, it is assumed that the value of the insurer's liability can be written as a twice differentiable function $G(V,D)$ with continuous first derivatives. A negative $G(V,D)$ means positive net liability for the insurer. Using the above assumptions and a generalized version of Itô's lemma for both diffusion and Poisson processes, the stochastic process for $G(V,D)$ may be written as

$$dG(V,D) = \left[\frac{\partial G}{\partial V} (\alpha V - C) + \frac{\partial G}{\partial D} (r_D + n)D + \frac{1}{2} \frac{\partial^2 G}{\partial V^2} \sigma_V^2 V^2 \right] dt + \frac{\partial G}{\partial V} \sigma_V V dz - I_{A \cap P} (cD + \gamma_1 G) + I_{A \cap N \cap S} (V - (1+c)D - G) + I_{A \cap N \cap O} (-cD + \gamma_2 (V - D - G)) \quad (47)$$

where

$$I_{A \cap P} = \begin{cases} 1 & \text{if an audit occurs and } V \geq D \\ 0 & \text{otherwise} \end{cases}$$

$$I_{A \cap N \cap S} = \begin{cases} 1 & \text{if an audit occurs and } V < D \text{ and the bank is closed} \\ 0 & \text{otherwise} \end{cases}$$

$$I_{A \cap N \cap O} = \begin{cases} 1 & \text{if an audit occurs and } V < D \text{ and the bank is not closed} \\ 0 & \text{otherwise} \end{cases}$$

The third to last term of equation (47) represents the jump in the value of the insurer's liability if during an audit the bank is found to be solvent. First, an audit cost is spent. Second, the insurer's liability may change if the insurer is able to force the bank either to acquire more capital or alternatively to sell its assets i.e. $\gamma_1 \neq 0$. If $\gamma_1 = 1$, the deposit insurer has full control over the bank's capital and a positive net worth audit will always follow with the banks' capital ratios being adjusted

to the minimum required level. Thus this limited-term contract case corresponds perfectly to the case (a) in the section 6.2.1. Moreover, the unlimited-term case i.e. $\gamma_1 = 0$ matches with the case (b) in the section 6.2.1 in the sense that the insurer is unable to force the capital deficient bank to expand their capital base. There is, however, a dissimilarity with respect to the case (b) which originates from the fact that the closer γ_1 is to zero, the less active will the banks with "excess" capital be in returning to the minimum required level. Consequently, since there is no fall in the value of G in this respect, the model developed here will slightly understate the insurer's liability relative to the case (b).

The second to last term of equation (47) represents the jump in the value of the insurer's liability if during an audit the bank is found to be insolvent and closed. In this case the indicator function $I_{A \cap N \cap S} = 1$, the insurer's liability will terminate, and the insurer will end up with a negative cash flow equal to the bank's negative net worth plus the audit cost cD . Finally, the last term of equation (47) describes the case where the bank is allowed to continue its operations despite it being insolvent i.e. $I_{A \cap N \cap O} = 1$. Now, the insurer is again incurred with a negative cash flow worth the bank's negative net worth plus the audit cost cD but also with a continuing claim, the value of which depends on the insurer's control over the bank measured by γ_2 . Here, both the limited-term insurance ($\gamma_2 = 1$) and the unlimited-term insurance ($\gamma_2 = 0$) correspond exactly the section's 6.2.1 cases (c) and (d) respectively.

Merton (1976) has shown, that if jumps in the movement of a security price generated by the Poisson process are uncorrelated across securities, they represent nonsystematic, i.e. diversifiable risk and will not be priced in the market. Given the assumption 4 that the event of an audit is independent of the market returns, the equilibrium return to the insurer's claim G can be derived by using the standard Black and Scholes hedging techniques as was done in section 4. Thus, by forming a zero-net-investment portfolio containing the type of assets held by the bank, the deposit insurer's claim and a risk free bond with the same maturity as the bank's deposits, and using the arbitrage condition that the expected return and the systematic risk of such a portfolio must be zero, the insurer's claim must satisfy the partial differential equation (as shown in Appendix 1)

$$\frac{1}{2} \frac{\partial^2 G}{\partial V^2} \sigma_V^2 V^2 + \frac{\partial G}{\partial V} ((r - \delta)V + nD) + \frac{\partial G}{\partial D} (r + n - m)D + hD \left(1 - \frac{\partial G}{\partial V}\right) - rG - \lambda(cD + \gamma_1 G) + I_N \lambda [(1 - (1 - \gamma_2)y(\kappa))(V - D - G) + \gamma_1 G] = 0 \quad (48)$$

where

$$I_N = \begin{cases} 1 & \text{if } V < D \\ 0 & \text{otherwise} \end{cases}$$

Equation (48) can be transformed into an ordinary differential equation system by the change in variables $g \equiv G/D$ and $x \equiv V/D$, where x is the asset-deposit ratio and g the deposit insurer's claim per markka of deposits. By denoting $g_1 = g(x)$ for $x \geq 1$ and $g_2 = g(x)$ for $0 \leq x < 1$, equation (48) can be rewritten as

$$\frac{1}{2} \sigma^2 x^2 \frac{\partial^2 g_1}{\partial x^2} + [(m - \delta - n)x + (n - h)] \frac{\partial g_1}{\partial x} + (n - \lambda \gamma_1 - m)g_1 + (h - \lambda c) = 0, \quad x \geq 1 \quad (49a)$$

$$\frac{1}{2} \sigma^2 x^2 \frac{\partial^2 g_2}{\partial x^2} + [(m - \delta - n)x + (n - h)] \frac{\partial g_2}{\partial x} + [n - m - \lambda(1 - (1 - \gamma_2)y(\kappa))]g_2 + [h + \lambda((1 - (1 - \gamma_2)y(\kappa))(x - 1) - c)] = 0, \quad x < 1 \quad (49b)$$

The boundary conditions, similar to those in Merton (1978), which the insurer's claim must meet are

$$g_1(1) = g_2(1) \quad (50a)$$

$$\frac{\partial}{\partial x} g_1(1) = \frac{\partial}{\partial x} g_2(1) \quad (50b)$$

$$g_2(0) = 1 + c \quad (50c)$$

$$\lim_{x \rightarrow \infty} |g_1(x)| < \infty \quad (50d)$$

Conditions (50a) and (50b) are continuity requirements. As Modigliani-Miller theorem (1958) obtains even in the presence of bankruptcy, and $x=1$ represents this situation, (50a) is as shown by Merton (1977b) necessary to avoid arbitrage. Condition (50b) follows from the assumption that the solution for $g(x)$ is twice-differentiable function with continuous first derivatives. Condition (50c) covers the situation where the value of assets go to zero and the bank will not be able to pay the promised return on its deposits and, thus, will immediately be audited and declared bankrupt. Merton (1978) has shown that if this case is formally treated by letting λ jump to infinity, then the insurer's liability can be proved to be worth the full amount of deposits plus the audit cost which is the message of the condition (50c). Finally, condition (50d) takes care of the boundedness of the insurer's liability in case where the value of the bank's assets grow large relative to its deposit.

The differential equation system (49) is solved subject to the boundary conditions (50a-d) as shown in Appendix 2. The equilibrium solution for the value of the deposit insurer's claim per markka of deposits is then given by

$$g_1(x) = a_{12} x^{r_{12}} F \left(-r_{12}, 1 + \mu_1, \frac{-2(h-n)}{\sigma_v^2 x} \right) + (h - \lambda c) / (m + \gamma_1 \lambda - n), \quad x \geq 1 \quad (51a)$$

$$g_2(x) = a_{21} x^{r_{21}} F \left(-r_{21}, 1 - \mu_2, \frac{-2(h-n)}{\sigma_v^2 x} \right) + a_{22} x^{r_{22}} F \left(-r_{22}, 1 + \mu_2, \frac{-2(h-n)}{\sigma_v^2 x} \right) \quad (51b)$$

$$+ \frac{\lambda(1 - (1 - \gamma_2)y(\kappa))x}{\delta + \lambda(1 - (1 - \gamma_2)y(\kappa))} + \frac{\delta(\lambda c - h) + \lambda(1 - (1 - \gamma_2)y(\kappa))(\delta + \lambda c - n + \lambda(1 - (1 - \gamma_2)y(\kappa)))}{\delta(n - m) + \lambda(1 - (1 - \gamma_2)y(\kappa))(n - m - \lambda(1 - (1 - \gamma_2)y(\kappa)))}, \quad x < 1$$

where

$F(\cdot)$ is the confluent hypergeometric function

$$\mu_1 = [(1 - (2/\sigma_v^2)(m - \delta - n))^2 + 8(m + \gamma_1 \lambda - n)/\sigma_v^2]^{1/2},$$

$$\mu_2 = [(1 - (2/\sigma_v^2)(m - \delta - n))^2 + 8(m + \lambda(1 - (1 - \gamma_2)y(\kappa)) - n)/\sigma_v^2]^{1/2},$$

$$r_{11} = [1 - (2/\sigma_v^2)(m - \delta - n) + \mu_1]/2,$$

$$r_{12} = r_{11} - \mu_1$$

$$r_{21} = [1 - (2/\sigma_v^2)(m - \delta - n) + \mu_2]/2$$

$$r_{22} = r_{21} - \mu_2$$

a_{11} , a_{12} , a_{21} , a_{22} are expressions given in appendix 2.

The first term in (51a) and the first two terms in (51b) are the option values i.e. the insurance components of the insurer's contingent claim. The second term in (51a) is the present value of the insurer's collected premium minus expected auditing costs, given that the bank remains solvent. In (51b) it is given that the bank is having negative net worth. The last two terms give the present value of the insurer's collected premium minus the sum of paid dividends and auditing costs for different degrees of regulatory power and forbearance. These terms also give the fraction by how much the insurer's liability is covered by the bank's current assets. If the bank is not paying out dividends, this will be equal to x and the insurer's liabilities are covered by the whole value of the banks assets. If, however, the bank is paying dividends out of its assets, the insurer's liability grows depending on the magnitudes of the insurer's forbearance and regulatory power. The intuition here is that if, for example, it would not be probable, that the initial

shareholders would lose their position in the case of negative net worth and at the same time the regulator could not force them to refinance the bank, a reduction in the assets in the form of a dividend payment would go to the initial shareholders and thereby increase the insurer's liability.

7 Risk Incentives under Various Degrees of Regulatory Forbearance and Regulatory Power

The equilibrium value of the shareholders' claim (i.e. the bank's equity), $E(V,D)$, could be developed similarly to that of the deposit insurer's liability. A much less complicated way is to apply the suggestion of Merton (1974), that the value of the bank's assets, V , must equal the sum of the values of the deposit insurer's, depositors' and shareholders' claims on the assets i.e.

$$V = E + D + G \quad (52)$$

Dividing both sides of expression (52) by the amount of deposits, D , and letting $e = E/D$ i.e. the equilibrium value of the shareholders' equity per markka of deposits, (52) may be rewritten as

$$\begin{aligned} e &= x - 1 - g \\ &= x - 1 - a_{12} x^{r_{12}} F \left(-r_{12}, 1 + \mu_1, \frac{-2(h-n)}{\sigma_{VX}^2} \right) \\ &\quad - (h - \lambda c) / (m + \gamma_1 \lambda - n), \quad x \geq 1, \end{aligned} \quad (53)$$

where $F(\cdot)$ is again the confluent hypergeometric function and other parameters as in section 6.3. Formula (53) can be used to analyze the bank's preference for more or less leverage. A contribution of equity capital will increase the bank's assets and thus increases the shareholders' wealth at a rate of $\partial E / \partial V = \partial e D / \partial V = \partial e / \partial x$. The change in the shareholders' wealth with respect to one unit of contributed equity capital is thus given by

$$\frac{\partial e}{\partial x} = 1 - \frac{\partial g}{\partial x} \quad (54)$$

By assuming that the bank managers maximize the value of shareholders' equity and following Marcus (1984), we assume that a

bank prefers less leverage and thus less risk if a one markka of contributed capital increases the shareholders' wealth by more than one Markka i.e. $\partial e/\partial x > 1$. Accordingly, a bank may be assumed to prefer more leverage and thus more risk if $\partial e/\partial x < 1$.

After substitution, expression (54) becomes

$$\frac{\partial e}{\partial x} = 1 - a_{12} r_{12} e^{-2(h-n)/\sigma_v^2 x} x^{r_{12}-1} F\left(r_{11}, 1 + \mu_1, \frac{2(h-n)}{\sigma_v^2 x}\right) \quad x \geq 1 \quad (55)$$

The sign and magnitude of the expression (55) cannot be unambiguously determined and thus, a numerical analysis is needed.

Table 2 presents numerical values for the equation (55) calculated under various degrees of regulatory forbearance and regulatory power. The parameter values characterizing the bank are similar to those estimated in section 5.2.¹ A smaller number implies larger incentive for choosing a low capital (high risk) strategy for the bank since one markka of provided new capital will increase the equity-holders' wealth by less and less than one markka.

The results in Table 2 show that the incentive for the bank management to increase leverage and thus risk is negatively correlated with the frequency of *efficient* audits measured by the parameter λ .² This is seen if Tables 2a, 2b and 2c are compared to each other. Take for example Table 2a and table 2b, which present the cases where the expected number of *efficient* audits within a one year period are 1.5 and 1 respectively. The partial derivatives, $\partial e/\partial x$, i.e. the incentives for

¹ See Table 1.

² Note that in Table 2 the result that the frequency of efficient audits would matter in cases either a) there is no control over capital (deposit insurance being risk-insensitive) or b) when the probability is zero that the shareholders will lose their money if the bank is found to have a negative net worth, i.e. $\gamma_2 = 0$ and $y(\kappa) = 1$, is not reasonable: In such cases it should not matter how often the bank is efficiently audited i.e. no matter how large the value of λ , the insurer is unable to do anything about it anyway. If the bank is confirmed after an audit to have a negative net worth, the insurer would just pay the shortfall and would continue the insurance contract of a zero capital bank with the same fixed insurance premium. The shareholders' wealth would not be affected by the magnitude of the negative net worth, implying no effect on incentives through changes in λ . The reason why the incentive analysis is not carried over with values γ_2 and $y(\kappa)$ being simultaneously 0 and 1 respectively relies on the fact that this point introduces an unremovable discontinuity for the solution $g(x)$, at which $g(x)$ appears to have poles and thus is not twice-differentiable (the prerequisite of the whole model derivation). The same situation emerges if $\lambda=0$. Therefore, in the analysis the points studied are chosen to be remote enough from these "black holes" in order to achieve interpretable results.

the bank to decrease capital adequacy (or increase risk) are smaller in the latter case compared to the former across all the different levels of regulatory forbearance and capital control, measured by $y(\kappa)$ and $\gamma_{1,2}$ respectively. Recalling the earlier interpretation of λ , this result indicates that the banks' eagerness to choose a riskier strategy depends (indirectly) on the institutional properties of the insurer. An insurer vulnerable to such strategies could, e.g. be staffed by unskilled personnel, equipped with low audit capacity, or have poor audit efficiency. Moreover, the frequency of *efficient* audits may depend on the incentives of the bank inspection organization, which makes the case where the bank examinations are conducted by an organization somehow influenced by the bank, interesting, since risk incentives may be affected by the degree of independency of the bank inspection organization. Also, if flexibility both in accounting standards as well as in reporting loan losses, asset values and non-performing loans allows the banks to effectively hide problems from the insurer, embodied formally in a lower λ , incentives may be driven towards more risk taking through lower intensity of *efficient* audits.³

Second, Tables 2a–2c show that the incentives for more risk-taking and/or high leverage grow along the loss of insurer's control over the capital level of a solvent bank i.e. as γ_1 moves toward zero. The intuition here is quite straightforward; if under a fixed insurance premium schedule the capital-adequacy regulation is not perfect, i.e. $\gamma_1 < 1$, banks have an incentive to exploit their ability to after the audit be able to continue their deposit insurance at a rebate premium. The larger the gap between the level of capital which sets the insurers liability to zero and the *de facto* level, the larger the subsidy gained by the bank, and thus, the larger the incentive to decide for a high leverage strategy. Or, recalling that the cases of $\gamma_1 = 1$ and $\gamma_1 = 0$ may be interpreted as representing the cases of variable risk-adjusted and fixed insurance premium schedules respectively, the results in Table 2, which are in line with Pennacchi (1987b), demonstrate a uniformly greater incentive for higher leverage as the deposit insurance premium becomes less sensitive to changes in bank risk.

³ Technically speaking the decrease in λ increases the value of the option part of the solution $g_1(x)$ corresponding to the case in section 4 where changes in the time to the next audit, T , changed the value of the equilibrium solution value in the one-period model. Here, we stress the factors, which might be behind the changes in the frequency of *efficient* audits in order to analyze the importance of institutional arrangements.

Table 2.

**Bank leverage incentives under various degrees
of regulatory power and regulatory forbearance**

Table 2a.

$\lambda = 1,5$

$\frac{\partial e}{\partial x}$	$y(\kappa)$	$\gamma_2 = 1$			$\gamma_2 = 0,5$			$\gamma_2 = 0^{**}$		
		γ_1			γ_1			γ_1		
		1	0,5	0	1	0,5	0	1	0,5	0
	0	0,9834	0,9623	0,7152	0,9834	0,9623	0,7152	0,9834	0,9623	0,7152
	0,5	0,9834	0,9623	0,7152	0,9822	0,9591	0,7135	0,9806	0,9548	0,7109
	1	0,9834	0,9623	0,7152	0,9806	0,9547	0,7108	0,9709	0,9237	0,6438

Table 2b.

$\lambda = 1$

$\frac{\partial e}{\partial x}$	$y(\kappa)$	$\gamma_2 = 1$			$\gamma_2 = 0,5$			$\gamma_2 = 0^{**}$		
		γ_1			γ_1			γ_1		
		1	0,5	0	1	0,5	0	1	0,5	0
	0	0,9689	0,9410	0,7128	0,9689	0,9410	0,7128	0,9689	0,9410	0,7128
	0,5	0,9689	0,9410	0,7128	0,9667	0,9361	0,7108	0,9637	0,9294	0,7076
	1	0,9689	0,9410	0,7128	0,9636	0,9292	0,7075	0,9469	0,8846	0,6347

Table 2c.

$\lambda = 0,5$

$\frac{\partial e}{\partial x}$	$y(\kappa)$	$\gamma_2 = 1$			$\gamma_2 = 0,5$			$\gamma_2 = 0^{**}$		
		γ_1			γ_1			γ_1		
		1	0,5	0	1	0,5	0	1	0,5	0
	0	0,9292	0,8941	0,7075	0,9292	0,8941	0,7079	0,9292	0,8941	0,7075
	0,5	0,9292	0,8941	0,7075	0,9243	0,8856	0,7047	0,9176	0,8737	0,7004
	1	0,9292	0,8941	0,7075	0,9174	0,8734	0,7002	0,8907	0,8138	0,6269

$$x = 1.02, \sigma, = 0,01, m = 0,001, n = 0, \delta = 0,00105, a = 0, h = 0,0001$$

*) Because $g(x) = g(x; \gamma_2, y(K))$ is not continuous if γ_2 and $y(K)$ are simultaneously equal to zero, the value of $\gamma_2 = 0,01$ is used to approximate the polar case $\gamma_2 = 0$ (see footnote 2 in section 7).

Third, the effects of the control over the capital level in case of an insolvent bank, measured by the parameter γ_2 , on the bank's risk incentives are similar to those of the parameter γ_1 . It is clear, that the value of γ_2 affects the bank's incentives only if the shareholders give a non-zero ($y(\kappa) > 0$) probability for the regulator's ability to *not* close an insolvent bank. In that case, if the bank is found insolvent and $\gamma_2 < 1$, the insurer will after paying the negative net worth of the bank offer the shareholders a deposit insurance contract at a rebate rate. Thus, if the bank's target is to maximize the shareholders' wealth, it is in its interest to maximize the value of this rebate by increasing the bank's leverage and/or asset risk.

Concerning to the expectations of the regulator's ability to close a bank, which during an audit is found to have negative net worth, the results are as expected. The incentives are uniformly addressed towards more leverage as the (shareholders') expectations are firmed that it is not optimal for the regulators to close either the bank or the shareholders' position, i.e. as $y(\kappa)$ moves towards unity. One exception is, however, present. If the capital adequacy control is perfect in the case of insolvency or alternatively, if the deposit insurance premium is perfectly risk-sensitive, i.e. γ_2 is equal to 1, there seems to be no changes in leverage (risk) incentives along the changes in the expected regulatory forbearance $y(\kappa)$. The reason is clear, since here the bank would have to pay virtually in full for the increased risk-taking and thus no subsidy from the insurer would take place regardless of the regulators forbearance.

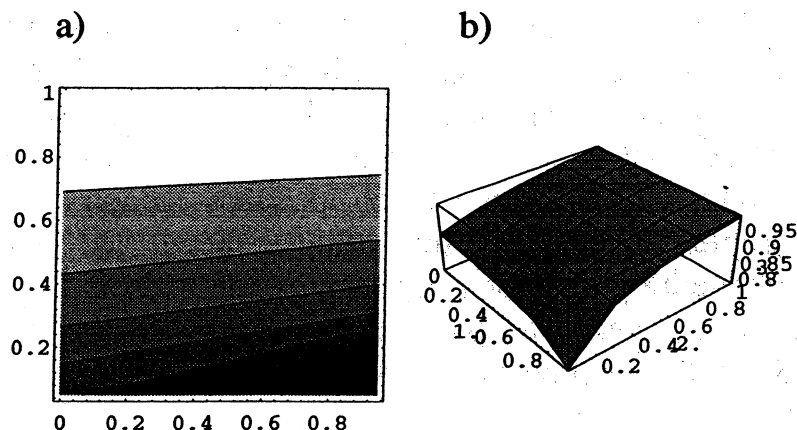
As the incentives for higher risk seem to be positively correlated with the market's (shareholders') expectations of the deposit insurer's ability to close a bank conditional that it is found to have a negative net worth, and moreover, as this probability $y(\kappa)$ depends on political legal and other constraints which the insurer is subject to, the interaction of the regulator and bank is dynamic and thus time inconsistency problems may cause moral hazard problems in the deposit insurance system. If for example it is not optimal for the regulator to close a bank due problems e.g. caused in financial intermediation and financial market stability, and if the market knows this, expectations concerning the regulatory forbearance may produce incentives toward more risk-taking. Accordingly, a dynamically inconsistent closure rule would not be taken into account when choosing the risk strategy of the bank. It follows that at certain circumstances the restrictions embodied in the single index parameter κ may in some circumstances lead to incentives which make the banking system unstable.

The above analysis is depicted in the figure 5 for the case where $\lambda = 1$. The monotonic increase in the incentives for risk is clearly seen in the figure 5a, which is the profile of the derivative $\partial e/\partial x$ presented as a "topographic map". In the figure the areas separated by the lines are joined points on the surface that have the same height in figure 5b. By choosing any point of $y(\kappa)$ to be held constant, moving vertically any distance in figure 5a causes monotonic changes in the height i.e. in the leverage incentives.⁴ This property is robust across the different values of λ , as can be seen from Table 2. It may be, therefore, worth pointing out that even though a perfect risk-sensitive insurance premium schedule (or capital adequacy standard) would be impossible to implement, there is something to be gained for the regulators from implementing at least a less than perfect capital regulation and/or risk-sensitive deposit insurance policy, as the value of $\partial e/\partial x$ is monotonically increasing with increases in γ_1 .

⁴ When looking at equation (54), it is worth noting Merton's (1978) findings that the insurer's liability is not universally a monotonically decreasing function of the asset-to-deposit ratio. This is because there are two sources of the insurer's liability: 1) the guarantee of deposits, which is a monotonically decreasing function, and 2) the audit cost which is a monotonically increasing function (i.e. a high capital adequacy will decrease the probability of an audit with a negative net worth and will increase the expected number and thus, the present value, of the upcoming audits). For a very small x , the guarantee component will dominate the audit component, and the insurer's liability is expected to decline in response to a increase in x . This dominance will, however, decline as x increases until, for very large x , further increase in x will cause only a tiny reduction in the deposit guarantee but a larger increase in the expected audit costs. The comparative static analysis above were made with extreme values of x , which are also probably more realistic.

Figure 5.

The interactive effects of changes in expected forbearance and capital control on the bank leverage (risk) incentives. In the figure $\lambda=1$, $x=1.035$, $\sigma_v=0.01$, $m=0.005$, $n=0$, $\delta=0.00105$, $h=0.0001$, $a=0.00001$ and $\gamma_1=\gamma_2$.



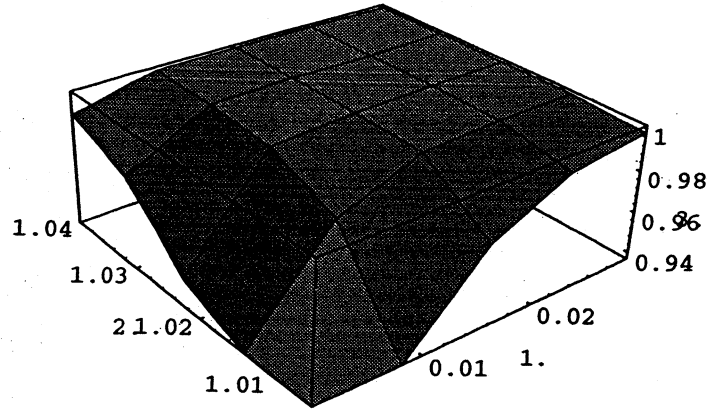
1. Expected regulatory forbearance, $y(\kappa)$
2. Degree of capital control, $\gamma_{1,2}$
3. Bank leverage incentive, $\partial e/\partial x$

Finally, it must be pointed out that the results change either if the bank is assumed to have large monopoly power or a large asset-to-deposit ratio, i.e. m and x are given large numbers. In both cases the partial derivate $\partial e/\partial x$ approaches to unity as m and x grow larger. The intuition is similar to both cases; the larger the value of equity capital and/or the charter value, the more the shareholders have to lose and the greater their incentives to undertake risky investment strategies. These results appearing in Figure 6 have also been presented by Marcus (1984), who demonstrated that as the value of the bank charter and/or asset-to-deposit ratio rises, a risk-averse strategy starts to dominate.⁵

⁵ Marcus (1984) also points out in the study that deregulation of the banking industry may through lost charter value hold potential for increases in risk taking of banks unless offsetting policies are established.

Figure 6.

The interactive effects of changes in bank monopoly power and bank leverage on the bank leverage (risk) incentives. In the figure $\gamma_1=\gamma_2=y(\kappa)=0.5$, $\sigma_v=0.01$, $\delta=0.00105$, $h=0.0001$, $a=0.00001$, $n=0$.



1. Bank leverage, x (assets/deposits)
2. Monopoly power, m
3. Bank leverage incentive, $\partial e/\partial x$

8 Empirical Estimation of Deposit Insurance Premia

8.1 Empirical methodology

If one has values for all the other parameters of the equation, it is possible to implicitly solve the "fair" value for the insurance premium, h , for a particular bank by setting the right hand side of equation (51a, b) equal to zero. As in Section 5.1., neither the market value asset/deposit ratio x , nor the bank asset variance, σ_v^2 , can be observed. Moreover, the values attributable to the insurer's behaviour, γ , λ and $y(\kappa)$, are not available.

The estimation must be done in two steps. In the first step, by exogenously giving values for these three insurer characteristic parameters, values for x and σ_v^2 are derived by using the information contained in the bank share prices as was done in the estimation of Section 5. Therefore, by again applying the Modigliani-Miller Theorem, the equilibrium value for the shareholder's equity-per-markka of bank deposits is given by¹

$$e(x) = x - 1 - g(x) \quad (56)$$

$e(x)$ follows a stochastic process, which depends on the process of x . Thus, by using Itô's Lemma the instantaneous variance of $e(x)$ is parallel to equation (36) given by

$$\sigma_e^2 = \sigma_v^2 \left(\frac{\partial e}{\partial x} \frac{x}{e} \right)^2 \quad (57)$$

Now, firstly by replacing from (51a), the first of the two simultaneous equations is obtained from (56) and given by

¹ The Modigliani-Miller Theorem is a reasonable assumption in the case of Finnish banks as far as bankruptcy costs are concerned — most troubled Finnish banks have been either merged or kept in operation by the regulators.

$$e(x) = x - 1 - a_{12} x^{r_{12}} F \left(-r_{12}, 1 + \mu_1, \frac{-2(h-n)}{\sigma_v^2 x} \right) - (h - \lambda c) / (m + \gamma_1 \lambda - n), \quad x \geq 1 \quad (56')$$

Secondly, by taking the first derivative of (53) and replacing in (57), the other simultaneous equation becomes

$$\sigma_e^2 = \sigma_v^2 (x/e)^2 \left\{ 1 - a_{12} r_{12} e^{-2(h-n)/\sigma_v^2 x} x^{r_{12}-1} F \left(r_{11}, 1 + \mu_1, \frac{2(h-n)}{\sigma_v^2 x} \right) \right\}^2 \quad (57')$$

Finally, the equations (56') and (57') can be numerically solved for the two unknowns x and σ_v^2 , representing the case where $x \geq 1$. In the second step, the right hand side of the equation (51a) characterizing the deposit insurer's liability is set equal to zero, after which the "fair" value of the deposit insurance, h , can be implicitly solved.

Accordingly, if $x < 1$, then the two unknown parameters x and σ_v^2 can be solved from equations

$$e(x) = x - 1 - a_{21} x^{r_{21}} F \left(-r_{21}, 1 - \mu_2, \frac{-2(h-n)}{\sigma_v^2 x} \right) - a_{22} x^{r_{22}} F \left(-r_{22}, 1 + \mu_2, \frac{-2(h-n)}{\sigma_v^2 x} \right) - \frac{\lambda(1 - (1 - \gamma_2)y(\kappa))x}{\delta + \lambda(1 - (1 - \gamma_2)y(\kappa))} - \frac{\delta(\lambda c - h) + \lambda(1 - (1 - \gamma_2)y(\kappa))(\delta + \lambda c - n + \lambda(1 - (1 - \gamma_2)y(\kappa)))}{\delta(n - m) + \lambda(1 - (1 - \gamma_2)y(\kappa))(n - m - \lambda(1 - (1 - \gamma_2)y(\kappa)))}, \quad x < 1 \quad (56'')$$

and

$$\sigma_e^2 = \sigma_v^2 (x/e)^2 \left\{ 1 - a_{21} r_{21} e^{-2(h-n)/\sigma^2 v x} r_{21}^{-1} F\left(r_{22}, 1 - \mu_2, \frac{-2(h-n)}{\sigma^2 v x}\right) - a_{22} r_{22} e^{-2(h-n)/\sigma^2 v x} r_{22}^{-1} F\left(r_{21}, 1 + \mu_2, \frac{-2(h-n)}{\sigma^2 v x}\right) - \frac{\lambda(1 - (1 - \gamma_2)y(\kappa))}{\delta + \lambda(1 - (1 - \gamma_2)y(\kappa))} \right\}^2 \quad (57'')$$

The values obtained can be used when implicitly solving the equation (51b) for such a value of h which sets the deposit insurer's liability equal to zero.

Using the methodology explained above, estimates of bank asset volatility, assets-to-liabilities ratio and finally of "fair" deposit insurance premium were calculated on a sample of six publicly listed Finnish banks for years 1987–1993. As in the section 5, it was assumed that all liabilities of these banks including subordinated debt are *de facto* insured by the government. This is a reasonable assumption in view of the government's handling of the Finnish banking crises.²

The observable data used in the estimations was collected as explained in the section 5.2. The frequency of the expected rate of efficient bank audits per unit time was assumed to be once per year.³ Thus, in the estimation $\lambda = 1$. The banks' "supervision contributions" actually charged by the Banking Supervision Office were used to calculate the cost of the audit per markka liabilities, c . The used values for h in the first step of the estimation were the actual insurance premiums charged by the banks' own guarantee funds during each year of the estimation period added with the those additional premiums charged by the Government Guarantee Fund during 1992–1993. Following Pennacchi (1987a), the net growth in

² See section 4.

³ Once again, the assumption of one efficient audit per one year is arbitrary but made here in order to achieve results comparable firstly, with the point estimates in the section 5 and secondly with those actual premia annually charged from the banks.

deposits, n , was set to be zero and the interest rate margin, m , was assumed to be equal to λc .⁴

The estimations were performed under four assumptions concerning the regulatory power and stock market's expectations of regulatory forbearance. These assumptions appear in the four cases explained below, each representing a polar case of a combination of regulatory power and expected regulatory forbearance. Thus, the parameters which represent these, γ_1 , γ_2 and $y(\kappa)$, were given either the value zero or one.⁵

It must be noted that realistically the deposit insurer exerts some intermediate level of regulatory control over the banks' capital levels and that the stock market's expectations concerning the bank closure rule lie somewhere between the extreme polar cases. Moreover, these may vary from bank to bank (e.g. some banks being more subject to expectations of too-big-to-fail -policies than others) and as time passes since the market adjusts its expectations in light of new information concerning e.g. the regulators behaviour. Therefore, upper and lower bounds of "fair" deposit insurance premia which the insurer should be charging from banks can be found from the estimation results of the four different cases. Finally, even though it is difficult to say with certainty, which one of the following cases would be closest to the

⁴ Merton (1978) shows that $m = \lambda c$ is the equilibrium deposit margin condition if there are no barriers to entry into banking. This means that the equivalent maturity government bond rates must be subtracted by the average auditing cost in order to achieve the equilibrium deposit rate. Moreover, Pennacchi (1987b) notes that in order for $m = \lambda c$ to hold, it must be assumed that the deposit insurer charges a fair insurance premium. Therefore, if either there is an entry barrier to the industry implying monopoly power or the deposit insurance is overpriced, then in equilibrium $m > \lambda c$.

⁵ As there are three parameters, which are given two extreme values i.e. zero or one in the estimations, there are altogether $2^3 = 8$ different combinations. However, four of these are redundant as far as the deposit insurance value estimations are concerned. *First*, take the case where $y(\kappa) = 0$. This means that the probability that the deposit regulator will close a bank with negative net worth is one. Therefore, as the parameter value γ_2 is relevant only on the condition that $y(\kappa) > 0$, it can be ignored. Thus, for $y(\kappa) = 0$ we have only two alternative relevant polar cases of regulatory power i.e. $\gamma_1 = 1$ and $\gamma_1 = 0$. This can be easily seen from the equation (51), where γ_2 is always multiplied by $y(\kappa)$. *Second*, take the case where $y(\kappa) = 1$ i.e. the shareholder's subjective probability of the regulator closing an insolvent bank is zero. If, during an audit the bank is found insolvent and not closed, the critical parameter will be γ_2 . If this is equal to one, i.e. the regulator can force the old shareholders to fully recapitalize the bank, then the expected wealth effects to both the insurer and the bank (shareholders) correspond exactly the case where $y(\kappa) = 0$. Thus, for $y(\kappa) > 0$ we only need to consider those polar case combinations where $\gamma_2 = 0$. This too can be easily seen from the equation (51), where $y(\kappa)$ is always multiplied by $(1-\gamma_2)$. It follows, therefore, that the estimation of the value of the deposit insurance contract for the eight polar case combinations can be conducted by four calculations.

actual realism, they may be mirrored against the actual form of deposit insurance and the handling of distressed banks, described in Section 2. The results of the estimations will also be used when considering, which ones of the following cases would have been most realistic during the various years of the estimation period as far as their assumptions are concerned.

CASE 1:

In the CASE 1, the parameter values used for describing the deposit insurer's regulatory power and market's expectations of forbearance are $\gamma_1 = 1$ and $y(\kappa) = 0$ with γ_2 taking any value.⁶ This case corresponds perfectly with the Black and Scholes -type one-period model used in Section 5 for the following two reasons: First, the underlying assumption is that the deposit insurer has perfect control over the bank's capital level or, alternatively, that the charged insurance premium is variable i.e. risk-based and adjusted after each bank audit. In other words, after each audit the insurer can either force the bank to adjust its capital adequacy or the insurance premium or both to such a level which makes the net present value of the insurer's liability worth zero. Second, it is assumed that the stock market gives zero likelihood for such an event where the bank would not be closed if found insolvent.

As described in section 2.1., Finnish banks have during the estimation period 1987–1993 been paying deposit premiums which have not perfectly reflected the riskiness of their operations. Generally speaking the deposit insurance premium has been fixed, and not granted for a limited term, but instead has been unlimited in its nature. Therefore, the contract has continuously been "renewed" at possibly below "fair" rates. It follows, that the assumption $\gamma_1 = 1$ most probably does not correspond to the reality.

It is also most probable that the assumption $y(\kappa) = 0$ does not conform with the stock market's perception. Even though the formal explicit deposit insurance has not covered the shareholders, the handling of the Finnish banking crises showed that too-big-to-fail policies were implemented and the regulatory forbearance translated into full protection of the bank shareholders either directly or indirectly. Thus, the bank share owners have had good reason to believe that they have been implicitly insured. The strength of this belief has naturally varied conditional on the regulator's actions and got probably stronger towards the end of the estimation period

⁶ See footnote 5 in Section 8.

1987–1993 as it became clear that the initial bank shareholders' right to participate to the potential future earnings would be reduced only through stock dilution.⁷

For the above reasons, the assumptions in CASE 1 do probably not match with the reality and therefore not with the information contents of the bank share prices. However, the estimation results of CASE 1 are interesting, since they are directly comparable with those achieved with the often used Black and Scholes -type model of this and other studies.

CASE 2:

In this case, the deposit insurance contract is assumed to have been granted for an unlimited time period and the charged premia (or the capital level) are assumed to not have been adjusted according to the riskiness of the bank. Thus, $\gamma_1 = 0$, which probably corresponds with the stock market information used in the estimations. The regulatory forbearance assumption is the same as in CASE 1 i.e. $y(\kappa) = 0$ (and thus γ_2 can take any value). As discussed above, actual practice has shown that this has not been the case. However, the assumptions of CASE 2 may be valid for smaller banks which may have not been expected to be subject to too-big-to-fail policies, at least not before the evidence of the regulator's handling of the banking crises. The assumption of no forbearance may have from time to time more closely described the stock market's perception for bigger banks as well, than the assumption of $y(\kappa) = 1$. Especially during the times prior the banking crises and during times of the early stages of the banking crises, as the nullification of the initial shareholders' position was subject to public discussion, the stock market's perceptions were probably more towards no forbearance i.e. $y(\kappa)$ would be closer to zero than one.

CASE 3:

This case represents a potential situation, where firstly, the deposit insurance premium would be risk-based and/or the insurer would have perfect control over the bank's capital-to-assets ratio in the case of solvent banks i.e. $\gamma_1 = 1$. Secondly, regardless of the formal explicit coverage of the deposit insurance, the stock market would not give much probability for a scenario where both an insolvent bank would be closed, i.e. $y(\kappa) = 1$, and the initial share owners would lose their

⁷ See sections 2.4.1 and 2.4.2.

position even if they would not recapitalize the bank, i.e. $\gamma_2 = 0$. In fact, γ_2 is the parameter, which measures the regulator's degree of shareholder subsidy and therefore is critical when attempting to capture the share owners' expectations concerning their wealth effects conditional on the solvency condition of the bank during an audit. Again, this case description does not match with the reality, since the deposit insurance premia actually charged are not "fair" in the strict sense. However, the results of CASE 3 are interesting, because when compared to the results in CASE 1 we may get evidence about the stock market's expectations regarding the forbearance during the estimation period.

CASE 4:

In order for this case to be correct for estimating the "fair" insurance premium the stock market should believe first, that even though the bank would be found being insolvent, the political, legal or other constraints would prevent the regulators from closing the bank i.e. $y(\kappa) = 1$ and second, that the initial share owners could not be forced to bring the capital adequacy of an insolvent bank to such a level which would make the insurer's net liability after premium contribution worth zero (or, alternatively, the insurance premium would not be adjusted) i.e. $\gamma_2 = 0$. Third, as in CASE 2, the charged insurance premia for an unlimited term contract should be fixed also in case of a solvent bank i.e. $\gamma_1 = 0$. When comparing these assumptions to the description of the Finnish deposit insurance system and the handling of the banking crises of section 2, it appears that they seem to be not far from the actual practice, not at least during the last years of the estimation period 1987–1993. Thus, most probably, the fair premia should be checked from the estimations of CASE 2 and CASE 4.

8.2 Estimation results

The estimation results are collected into Tables 3 and 4. Table 3 presents the parameter estimates of the asset-to-liabilities ratios, x , and the asset volatilities, σ_v , for the four above explained cases. Also, the estimated volatilities of the bank share returns used in the estimations can be found in table 3.

Comparing the estimates of the standard deviations of banks' asset returns in Table 1 of section 5.2 and CASE 1 in Table 3, it appears that they are mostly very close to each other. This is not a surprising

result since the assumptions in the CASE 1 perfectly match with those behind the model used for producing the results in Table 1. The second observation is that the estimated values of σ_v are uniformly greater as we move from CASE 1 to CASE 2 and to finally CASE 4, whereas CASE 3 seems to produce estimates of σ_v which are mostly below those of the CASE 1. In spite of being small, these differences are important since the calculation of the fair premium is very sensitive to changes in the value of σ_v . Moreover, referring to the discussion in section 8.1, the most realistic banks' asset variation estimates for the estimation period lie probably somewhere between those done under the assumptions of CASE 2 and CASE 4.⁸ Finally, the results indicate that the volatility of banks' assets varies not only across different times but also interbank differences are present. It also seems, that the volatility of Ålandbanken's asset returns would have been constantly larger than those of the other sample banks. This may perhaps rely on the fact that the business of the relatively small Ålandbanken has been geographically concentrated in a very limited part of Finland and thus, their asset portfolio may not have been as diversified as the portfolios of other banks. Another potential reason for these results may lie in the information content of the bank's share prices which may be somewhat blurred because of the high degree of concentration in Ålandbanken's ownership and the low trading activity in its share.

Table 3 reports also the estimated values for each bank's asset-to-liabilities parameter x , which is the other critical parameter in the premium estimations. The results show, that the estimated leverage depends greatly on the regulatory assumptions under which the calculation is performed. Generally, banks appear to be more levered under assumptions of less regulatory power and more forbearance.

Once again the results do vary across banks. The most striking results are found under CASE 4 of Ålandbanken and Interbank. Under this set of assumptions, these banks seem to have negative net worth through the years 1987–1991 and 1991–1992 respectively.⁹ However, these unrealistic results indicate that the assumptions of CASE 4 do not match with the information contents of

⁸ For discussion concerning the absolute level of the estimates for σ_v , see section 5.2.

⁹ The first step of the estimations under CASE 4 of Ålandbanken and Interbank produced a result for years 1987–1991 and 1992–1993 respectively that $x < 1$. Thus, an alternative estimation was tried by numerically solving equations (56'') and (57'') but no convergence occurred. However, even though x is equal to less than unity but not too far from unity, equations (56') and (57') can be used since the solution is continuous at point $x = 1$ (see boundary conditions (50a) and (50b)).

Ålandsbanken's and Interbank's share prices for the above estimation periods. Indeed, since the estimation results correspond to the reality only if the assumptions match with the stock market's perceptions, the results of Ålandsbanken and Interbank may indicate, that the shareholders have not strongly believed that these banks would be subject to too-big-to-fail -policies and/or that the shareholders' position would be implicitly insured during these years. An opposite argument may be constructed in light of the KOP's asset-to-liabilities estimation result under CASE 4 for the year 1993. At that time, the initial shareholders had good reason to believe that the regulators would not nullify their position and the bank would not be allowed to fail. Thus, the assumptions of CASE 4 may well reflect the stock market's perceptions. It follows, that the value of x being 0.997, i.e. under the solvency level, may reflect the reality and therefore, in market value terms, KOP may have been kept viable only by the help of either regulator's direct and indirect pecuniary contributions or other guarantee -type commitments and promises or both.

Finally, the results in table 3 suggest, that the banks' leverage has increased towards the end of the estimation period. This finding is in line with the economic developments of the banking sector, described in section 2. It is, however, interesting since inspite of the fact that the banks have met the BIS 8% capital adequacy criteria in book value terms, the values of x , which only narrowly exceed 1 suggest that in market value terms this criteria may not have been fulfilled.¹⁰

¹⁰ Note that x includes only equity capital and reserves and not Tier 2 capital and thus, is not directly comparable with the banks book valued capital adequacy, which are calculated by using the BIS rules.

Table 3.

	$y(\kappa) = 0, \gamma_1 = 1$ CASE 1				$y(\kappa) = 0, \gamma_1 = 0$ CASE 2				$y(\kappa) = 1, \gamma_1 = 1, \gamma_2 = 0$ CASE 3				$y(\kappa) = 1, \gamma_1 = 0, \gamma_2 = 0$ CASE 4			
	σ_E	E/V	σ_v	x	E/V	σ_v	x		E/V	σ_v	x		E/V	σ_v	x	
Interbank																
1991	70.11	0.065	4.96	1.066	0.066	5.48	1.048		0.066	6.11	1.058		0.091	16.21	0.342	
1992	56.41	0.026	1.59	1.027	0.023	1.79	1.023		0.027	1.66	1.027		0.029	4.59	0.937	
1993	47.43	0.015	0.71	1.015	0.015	0.83	1.015		0.015	0.73	1.015		0.015	0.96	1.012	
KOP																
1987	21.19	0.044	0.93	1.047	0.0445	1.03	1.044		0.044	0.83	1.052		0.046	1.51	1.021	
1988	22.95	0.044	1.012	1.047	0.044	1.12	1.043		0.044	0.94	1.053		0.046	1.71	1.011	
1989	18.41	0.061	1.12	1.066	0.061	1.24	1.063		0.060	0.88	1.078		0.062	1.77	1.048	
1990	22.48	0.051	1.16	1.055	0.0513	1.26	1.052		0.051	1.08	1.057		0.0519	1.60	1.039	
1991	27.55	0.034	0.94	1.036	0.0342	1.00	1.035		0.034	0.94	1.036		0.0343	1.06	1.033	
1992	28.89	0.020	0.59	1.021	0.0203	0.63	1.021		0.020	0.59	1.021		0.0203	0.68	1.019	
1993	84.69	0.010	0.97	1.010	0.0101	1.18	1.007		0.010	0.94	1.010		0.010	1.54	0.997	

Table 3 continued

	$y(\kappa) = 0, \gamma_1 = 1$ CASE 1		$y(\kappa) = 0, \gamma_1 = 0$ CASE 2		$y(\kappa) = 1, \gamma_1 = 1, \gamma_2 = 0$ CASE 3		$y(\kappa) = 1, \gamma_1 = 0, \gamma_2 = 0$ CASE 4							
	σ_E	E/V	σ_v	x	E/V	σ_v	x	E/V	σ_v	x				
SKOP														
1991	25.34	0.027	0.68	1.029	0.028	0.71	1.027	0.027	0.68	1.028	0.027	no convergence	0.76	1.032
1992	48.16	no convergence ¹⁵	no convergence	no convergence	no convergence	no convergence	no convergence	no convergence	no convergence	no convergence	no convergence	no convergence	no convergence	no convergence
1993	149.84	0.027	4.07	1.028	0.028	6.23	0.999	0.029	9.60	0.9561	no convergence	no convergence	no convergence	no convergence

¹⁵ The reason for non-convergence is the large parameter value for the actually paid insurance premium. See footnote 14.

¹⁶ Infinite expressions were encountered and non-convergence occurred for SKOP in CASE 4 in 1993. The reason was discovered to be the high value for equity volatility which brought the parameter value set outside the convergence range of the solution. It may be that in case of SKOP, the bank had in fact negative net worth i.e. $x < 1$. If at the same time it had small charter value (m being small), no premium might ever compensate the insurer for its liability in covering the SKOP's negative net worth. The intuition here is, that if the bank would be kept open, allowing at the same time the equity holders to retain the possibility in some future state of the world to receive a positive payment without ever contributing more assets to the bank, then no premium, h, would turn the value of the bank positive, i.e. the insurer's liability would be infinite.

Table 3 continued

	y(κ) = 0, $\gamma_1 = 1$ CASE 1			y(κ) = 0, $\gamma_1 = 0$ CASE 2			y(κ) = 1, $\gamma_1 = 1, \gamma_2 = 0$ CASE 3			y(κ) = 1, $\gamma_1 = 0, \gamma_2 = 0$ CASE 4			
	σ_E	E/V	σ_v	x	E/V	σ_v	x	E/V	σ_v	x	E/V	σ_v	x
UNITAS													
1987	20.02	0.065	1.35	1.071	0.065	1.52	1.065	0.063	1.28	1.094	0.066	2.03	1.047
1988	16.25	0.063	1.00	1.069	0.063	1.13	1.066	0.062	0.66	1.087	0.064	1.67	1.053
1989	21.30	0.089	1.99	1.099	0.089	2.20	1.090	0.087	1.85	1.122	0.0940	3.21	1.044
1990	16.36	0.064	1.05	1.070	0.064	1.15	1.069	0.064	0.87	1.074	0.064	1.37	1.068
1991	17.49	0.042	0.73	1.044	0.042	0.76	1.045	0.042	0.70	1.044	0.042	0.81	1.044
1992	34.64	0.029	1.00	1.030	0.029	1.08	1.029	0.029	1.01	1.030	0.029	1.15	1.027
1993	64.76	0.012	0.79	1.013	0.012	0.94	1.013	0.012	0.80	1.013	0.012	1.00	1.010

continued...

Table 3 continued

	$y(\kappa) = 0, \gamma_1 = 1$ CASE 1				$y(\kappa) = 0, \gamma_1 = 0$ CASE 2				$y(\kappa) = 1, \gamma_1 = 1, \gamma_2 = 0$ CASE 3				$y(\kappa) = 1, \gamma_1 = 0, \gamma_2 = 0$ CASE 4			
	σ_E	E/V	σ_v	x	E/V	σ_v	x		E/V	σ_v	x		E/V	σ_v	x	
ÅLANDSBANKEN																
1987	24.29	0.181	5.24	1.224	0.194	5.77	1.142	0.174	5.61	1.271	0.263	6.33	0.841			
1988	27.96	0.129	4.04	1.149	0.135	4.44	1.102	0.126	4.26	1.179	0.184	6.79	0.808			
1989	23.13	0.165	4.45	1.201	0.173	4.91	1.143	0.156	4.95	1.271	0.249	7.91	0.795			
1990	24.52	0.100	2.63	1.114	0.102	2.91	1.093	0.095	3.02	1.168	0.115	4.82	0.967			
1991	28.47	0.081	2.46	1.091	0.083	2.70	1.069	0.081	2.41	1.088	0.099	5.46	0.888			
1992	27.30	0.061	1.68	1.065	0.060	1.83	1.059	0.061	1.76	1.064	0.061	1.91	1.056			
1993	34.30	0.053	1.85	1.057	0.054	2.08	1.053	0.053	1.97	1.056	0.054	2.18	1.049			

Table 4 shows the results of the second step of the estimation where the fair value of deposit insurance is implicitly solved by setting the insurer's net liability equal to zero i.e. $g(x) = 0$. The "fair" premia are solved for each set of the underlying regulatory assumptions of CASEs 1 to 4. Table 4 reports also the actual commitments paid by the banks ("Actual") and the results of the estimation of section 5 (I_L). The actual commitments include the insurance premia collected by the GGF and the banks' own guarantee funds added with the "supervision contributions" collected by the BSO. The figures not in parenthesis are Finnish markkas per 1000 markkas of liabilities whereas the figures in parenthesis are absolute values in thousand markkas.

The first observation from the results in Table 4 is, that it matters a great deal under which assumptions the premium estimates are calculated. Also, within each CASE, the premium levels seem to vary not only over the time but also across banks. This result, similar to that in the section 5, again speaks against a fixed rate deposit insurance scheme.

In table 4 it can be seen, that the insurance premia estimates of CASE 1 and those under the column I_L deviate an order of magnitude from each other even though they have been produced under the same regulatory assumptions. Only for KOP and Unitas for the year 1993 do these two different models produce premia close to each other. The difference may come from the different values of x and σ_V used in the estimations. However, the reason why generally the estimates of CASE 1 exceed those of I_L can not unambiguously be found there; for CASE 1 the values for σ_V are smaller than those used in I_L while the order is the opposite for the asset-to-capital ratios as can be seen from the equity-to-assets ratios, E/V , in tables 1 and 3. For an insignificant part, the difference can be explained by the supervisory contributions, which are present in CASE 1 but not in I_L . The main reason for the difference lies therefore probably in the random audit assumption of the CASE 1.

Differences notwithstanding, the results of CASE 1 and of the estimation in section 5 (column I_L) are generally consistent with each other in the sense that both suggest that the banks would have been overcharged for their deposit insurance. This can be seen when comparing columns CASE 1 and I_L with the column under "Actual". The results of CASE 1 are similar to those in Pennacchi (1987a) produced under the same regulatory assumptions for a sample of 23 American banks in the sense that his point estimates are of the same

order of magnitude and also, in the sense that they are below those actually charged by the insurer.¹¹

The regulatory assumptions behind these two estimations do not match with the reality and thus an overcharge hypothesis is not supported. As discussed above, the Finnish deposit insurance and regulatory behaviour are probably better represented by CASE 2 and 4. The "fair" premium estimates produced under both of them suggest, that the banks have generally been undercharged for their deposit insurance, OKO-bank being the exception. This result corresponds with Räsänen (1994). Moreover, as in CASE 1 the results under CASE 2 are generally in line with those unlimited-term insurance premium estimates of Pennacchi (1987a) in both of the above mentioned respects. However, as far as the order of magnitude of the premium estimates is concerned, the results of Ålandbanken for the whole sample period, of Interbank for years 1991–1992 and of OKO-bank for the year 1991 significantly deviate from those of Unitas and KOP for both CASE 2 and CASE 4. They do also deviate from the estimated premium levels in Pennacchi (1987a) for CASE 2. It may thus be, that the most reliable results are produced for the bigger banks (KOP and Unitas), whose stocks are more actively traded in the exchange.

The results in table 4 may also provide some insight into the question, whether it is the CASE 2 or the CASE 4 that better reflects actual regulatory practice. When comparing the columns under CASE's 2 and 4, one observation is striking. For KOP and Unitas, the estimated premia for the CASE 4 during the years 1987–1990 generally seem to be out of hand whereas the estimates under CASE 2 for the same period do not. Subsequently, from the outset of the Finnish banking crisis in 1991 (and the bail-out of Skopbank¹²), CASE 2 produces probably too low premium estimates, whereas CASE 4 begins to look more realistic. The same phenomenon emerges for OKO and Ålandsbanken after the year 1991 and for Interbank after 1992.

This may be evidence of a shift in stock market expectations towards banks' initial shareholders positions being implicitly insured. Further confirmation for the shift is found when comparing the premium estimates under CASE 1 and CASE 3. What makes CASE 1 different from CASE 3 is that the latter includes regulatory forbearance whereas the former does not. Therefore, the premium estimates under CASE 3 should exceed those of CASE 1. However, before the year

¹¹ See section 3.3 for a brief review of Pennacchi (1987a).

¹² See section 2.3.

1991 the results suggest that the converse prevails for Unitas, KOP, and Ålandbanken. It follows that the assumption of shareholders' expectations of regulatory forbearance seems to match with the reality starting from the year 1991.

Finally, the reason why the shareholders' expectations seem to have changed earlier in the case of the bigger banks' (KOP and Unitas) than in the case of the smaller ones (Ålandsbanken and Interbank) may be the following: As soon as Skopbank was bailed out by the regulators, it became evident that in the case of larger banks, too-big-to-fail policies would be implemented in order to safeguard stability of the Finnish banking sector. The smaller banks' share owners, however, may have felt as being implicitly insured only after 1992 as it became evident that the Finnish political system would be backing all banks' commitments.¹³

¹³ See section 2.3.

Table 4.

	Actual	CASE 1	CASE 2	CASE 3	CASE 4	i_L
INTERBANK						
1991	0.1257 (85)	3.880 (2,623)	6.434 (4,349)	9.478 (6,407)	55.610 (37,596)	2.700 (1,796)
1992	0.2028 (401)	0.6020 (1,189)	1.173 (2,320)	1.178 (2,329)	26.436 (52,263)	0.300 (590)
1993	0.3652 (1,252)	0.283 (970)	0.3030 (1,039)	0.277 (951)	1.220 (4,182)	0.056 (192)
KOP						
1987	0.0994 (9,536)	0.0351 (3,352)	0.4565 (43,643)	0.0486 (4,644)	5.790 (553,593)	$5.4 \cdot 10^{-6}$ (-0)
1988	0.1016 (11,164)	0.0478 (5,265)	0.592 (65,263)	0.0847 (9,333)	8.734 (962,608)	$3.5 \cdot 10^{-5}$ (4)
1989	0.1042 (14,120)	0.0245 (3,312)	0.3559 (48,124)	0.0198 (2,6716)	4.491 (607,124)	$8.2 \cdot 10^{-8}$ (-0)
1990	0.1117 (15,534)	0.0317 (4,423)	0.3355 (46,814)	0.0435 (6,069)	2.904 (405,179)	$1.2 \cdot 10^{-5}$ (2)
1991	0.1114 (16,952)	0.0355 (5,179)	0.1549 (23,578)	0.0454 (6,906)	0.4075 (62,032)	0.00025 (38)
1992	0.1825 (26,067)	0.0344 (4,915)	0.1201 (17,154)	0.0443 (6,327)	0.3152 (45,005)	0.00035 (50)
1993	0.5778 (95,244)	1.1239 (185,226)	1.6231 (267,508)	1.4354 (236,563)	6.7884 (1,118,800)	1.200 (191,342)
OKO						
1991	2.294 (118,050)	0.0359 (1,852)	0.2425 (12,501)	0.2763 (14,243)	1.9510 (100,584)	$6.7 \cdot 10^{-5}$ (3)
1992	3.109 (162,561)	0.0236 (1,235)	0.0916 (4,788)	0.0314 (1,643)	0.2987 (15,621)	$2.8 \cdot 10^{-5}$ (1)
1993	3.732 (209,797)			no convergence		$4.0 \cdot 10^{-4}$ (22)
SKOP						
1991	0.715 (62,248)	0.0259 (2,255)	0.0462 (4,422)	0.0292 (2,538)	0.1161 (10,105)	$4.9 \cdot 10^{-5}$ (4)
1992	4.351 (349,409)			no convergence		0.092 (7,395)
1993	1.280 (75,889)	8.3896 (497,394)	95.9061 (5685,990)	13.5823 (805,252)	no conv.	62.000 (3561,370)

continued...

Table 4 continued

	Actual	CASE 1	CASE 2	CASE 3	CASE 4	i_L
UNITAS						
1987	0.0927 (9,005)	0.0362 (3,518)	0.587 (56,977)	0.0265 (2,572)	7.9547 (772,203)	$3.3 \cdot 10^{-6}$ (-0)
1988	0.1016 (10,721)	0.0193 (2,049)	0.327 (34,784)	0.0161 (1,713)	4.2457 (451,113)	$1.1 \cdot 10^{-9}$ (-0)
1989	0.1063 (13,054)	0.038 (4,609)	0.5878 (71,886)	0.0281 (3,437)	6.8080 (832,566)	$3.4 \cdot 10^{-6}$ (-0)
1990	0.1046 (14,330)	0.0170 (2,323)	0.1676 (22,910)	0.0161 (2,204)	0.8071 (110,358)	$6.6 \cdot 10^{-10}$ (-0)
1991	0.1124 (14,297)	0.0169 (2,153)	0.0851 (10,819)	0.0179 (2,282)	0.2630 (33,426)	$5.5 \cdot 10^{-9}$ (-0)
1992	0.1900 (25,874)	0.0805 (10,982)	0.2729 (37,212)	0.1187 (16,179)	0.7108 (96,919)	0.0050 (688)
1993	0.7026 (97,184)	0.3586 (49,961)	0.5662 (78,889)	0.6013 (83,779)	1.473 (205,277)	0.3200 (44,998)
ÅLANDSBANKEN						
1987	0.1151 (179)	0.1410 (219)	2.4540 (3,816)	0.174 (271)	33.7126 (52,423)	$5.1 \cdot 10^{-5}$ (-0)
1988	0.1010 (249)	0.1880 (464)	2.1609 (5,329)	0.259 (638)	35.4435 (87,411)	0.0007 (2)
1989	0.1221 (307)	0.1001 (251)	1.786 (4,490)	0.095 (236)	35.4377 (89,098)	$1.3 \cdot 10^{-5}$ (-0)
1990	0.1077 (380)	0.0850 (299)	1.174 (4,143)	0.071 (250)	24.886 (87,824)	$9.4 \cdot 10^{-5}$ (-0)
1991	0.1211 (388)	0.2064 (658)	2.104 (6,702)	0.993 (3,163)	31.175 (99,297)	0.0019 (6)
1992	0.1176 (704)	0.1648 (576)	1.393 (4,868)	1.179 (4,120)	2.240 (7,828)	0.0011 (4)
1993	0.6336 (2,289)	0.2665 (963)	1.416 (5,116)	1.609 (5,813)	2.209 (7,979)	0.0140 (49)

9 Conclusions

This study presents two closed form solutions for the value of bank deposit insurance derived by applying option price modelling techniques. The first model is a Black and Scholes (1973) -type one-period model, which assumes that a deposit insurance contract is limited-term in its nature. The second model is a multi-period model, which in turn builds on the assumption that deposit insurance is granted for an unlimited term. Both models can be used to analyze the economics of the deposit insurance system. However, the assumptions behind the one-period model do not adequately mirror the actual reality and thus the model fails to capture many of the important features involved in an insurance scheme, e.g. the actual and expected regulatory behaviour. Thus, the more general multiperiod model is needed to achieve more reliable point estimates of the value of deposit insurance and to clarify the interplay between the regulatory behaviour and the bank incentive structure.

In light of the one-period model, it is shown that the value of the deposit insurance and thus the insurer's liability can be controlled by using regulatory measures such as capital adequacy rules and reserve requirements. Also, increasing the frequency of bank inspections seems to be an efficient way to manage the deposit insurance liability.

As practice in Finland shows, it is reasonable to believe that in case of banking crises the government may extend the deposit insurance protection to cover not only the deposit holders but also the other claimants, (including shareholders) of the bank. The government does not intervene because it is obliged by law, but because it considers that such action will achieve certain public policy goals. This study shows, that such an implicit insurance increases the value of the insurance contract and, thus, the insurer's liability. It follows, that even if the depositors were the only ones formally insured, the expectation of the government coming to the rescue of distressed banks could encourage other claimants of the bank to behave differently. Thus, both the deposit insurer's and/or regulator's actual and expected behaviour seem to have an effect on the bank's risk-taking incentives.

In section 7 it is shown that risk incentives of a bank depend on a number of characteristics and behaviour of the regulator and/or the insurer. Also, whether the insurance premia charged are fixed or risk-sensitive seem to play an important role in the bank's optimal risk policy. First, uncertainty in the frequency of efficient audits is included in the multi-period model. All audits are not necessarily efficient i.e.

the true financial condition is not always discovered. The efficiency of the audits can be considered as depending on the methods, capacity and skills of the deposit insurer and/or the bank inspection authority. It is shown that the bank has an economic incentive towards more risk taking the less capable the inspection authority is. Therefore, if deposits (and other liabilities) are insured, a strong bank inspection authority is needed to avoid excess risk taking of the banks. Second, it is shown that the incentives for more risk-taking are increased if either the deposit insurance premia are not perfectly risk-sensitive or the banks capital adequacy is not kept at such a level, which make a certain fixed insurance premium actuarially fair. Third, it is shown, that the bank's incentives are biased towards more risk-taking as the expectations are firm that, in the case of insolvency, it is not optimal for the regulator either to close the bank or to nullify the initial shareholders' claims on the bank.

Both the one and multiperiod models of this study are used for estimating bank specific "fair" insurance premia. The unobservable pricing parameters of the models are derived by using the stock market price information on traded bank equity. The estimates can be used for cross sectional comparison of the riskiness different of banks. The interbank variation of the point estimates indicates that a fixed premium schedule can lead either to unfair penalization or subsidization of banks and also that the riskiness varies among banks. Hence, the results strengthen the argument in favour of risk-based deposit insurance premia.

The multi-period model of this study allows to estimate fair deposit premia under different assumptions concerning regulatory behaviour and stock market's expectations of regulatory forbearance. Estimation results from the one-period model and the multiperiod one under the assumptions underlying the one-period model suggest that Finnish banks have mostly been overcharged for their deposit protection. The overcharge hypothesis is, however, not supported since these assumptions (i.e. limited term, or variable rate, insurance contract and no forbearance) do not match reality. If more realism is added by modelling the deposit insurance as an unlimited-term (fixed rate) contract, the estimation results suggest that the Finnish banks have been paying too little and thus been subsidized by the insurer. Moreover, if regulatory forbearance is assumed in the estimations, Finnish banks seem to have been undercharged even further. Finally, the estimation results give some evidence that stock market expectations shifted towards more regulatory forbearance at the time of the outbreak of the Finnish banking crisis. These expectations seem

to have been strengthened by the Parliament's and Government's promises to secure the commitments of the Finnish banks.

The results of this study stress the importance of banks' expectations concerning regulatory behaviour. If the bank management is maximizing shareholders' wealth, its risk taking behaviour may be affected by the expectations concerning the regulatory behaviour. Thus, it is not only the *explicit* deposit insurance but also the *implicit* deposit insurance, which potentially has an effect on the perceived risk policies in the business of banking. Therefore, the deposit insurer's and/or regulator's reputation in handling distressed banks probably indirectly influences the stability of the banking sector.

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Appendix 1

In order to derive the partial differential equation (48), which the value of the insurer's claim must satisfy, standard Black and Scholes hedging techniques are used. A zero-net-investment portfolio containing the type of assets held by the bank, V , the deposit insurer's claim, G , and a risk free bond B_{rf} is formed. The value of this portfolio, H , is expressed as

$$H \equiv q_1 V + q_2 G - \left(\frac{q_1 V + q_2 G}{B_{rf}} \right) B_{rf} \quad (A1.1)$$

where q_1 and q_2 are the quantities of the risky asset, V , and the insurer's claim, G , respectively. To assure zero net investment, the quantity of the risk-free bond, B_{rf} , must be $[(q_1 V + q_2 G)/B_{rf}]$. The market value of B_{rf} is assumed to follow the process $dB_{rf} = rB_{rf}dt$, where r is the risk-free rate. The instantaneous return to the portfolio H is give by¹

$$dH = q_1(dV + Cdt) + q_2(dG + hDdt) - \left(\frac{q_1 V + q_2 G}{B_{rf}} \right) dB_{rf} \quad (A1.2)$$

which after substitution becomes

¹ For notational consistency the correction term Cdt is needed in equation A1.2. Moreover, since the insurer receives an instantaneous premium of hD , this is added to the middle term of A1.2.

$$\begin{aligned}
dH = & \left[q_1(\alpha - r)V + q_2 \left[\frac{1}{2} \frac{\partial^2 G}{\partial V^2} \sigma_v^2 V^2 + \frac{\partial G}{\partial V}(\alpha V - C) + \frac{\partial G}{\partial D}(r_D + h) + hD - rG \right] \right] dt \\
& + \left[q_1 + q_2 \frac{\partial G}{\partial V} \right] \sigma_v V dz \\
& + q_2 \left[-I_{A \cap P}(cD + \gamma_1 G) + I_{A \cap N \cap S} V - (1+c)D - G \right] \\
& + I_{A \cap N \cap O}(-cD + \gamma_2(V - D - G)) \quad (A1.3)
\end{aligned}$$

The quantities q_1 and q_2 are selected to eliminate all risk from the portfolio which implies that

$$\frac{q_1}{q_2} = - \frac{\partial G}{\partial V} \quad (A1.4)$$

Using the assumptions 4 and 5, dividing (A.1.3) by q_2 , using (A.1.4) and the fact that the arbitrage condition that the expected return of a zero-net-investment portfolio must be equal to zero i.e. $E[dH] = 0$, we get

$$\begin{aligned}
& \frac{1}{2} \frac{\partial^2 G}{\partial V^2} \sigma_v^2 V^2 + \frac{\partial G}{\partial V}(rV - C) + \frac{\partial G}{\partial D}(r_D + n)D + hD - rG \\
& + \lambda \left[-I_p(cD + \gamma_1 G) + I_N(1 - y(K))(v - (1+c)D - G) \right. \\
& \left. + I_{N \cap y(K)}(-cD + \gamma_2(V - D - G)) \right] \quad (A1.5)
\end{aligned}$$

$$\text{where } I_p = \begin{cases} 1 & \text{if } V \geq D \\ 0 & \text{otherwise} \end{cases}$$

$$I_N = \begin{cases} 1 & \text{if } V < D \\ 0 & \text{otherwise} \end{cases}$$

Using the fact, that $I_p \equiv (1 - I_N)$, A1.5 can be easily shown to be equal to the equation (48).

Appendix 2

The derivation of the general solution (51a) for the deposit insurer's liability is shown in the following.

Erderlyi et al. (1957, p. 251–252) have shown that by the substitution of $w = x^{-1}$, $\beta_1 = (2/\sigma_v^2)(h-n)$, $\beta_2 = 2(1-(m-\delta-n)/\sigma_v^2)$, $\beta_3 = (2/\sigma_v^2)(n-\gamma_1\lambda-m)$, the homogenous part of the differential equation (49a) is reduced to

$$\frac{d^2g}{dw^2} + \left(\beta_1 + \frac{\beta_2}{w} \right) \frac{dg}{dw} + \frac{\beta_3}{w^2} g = 0 \quad (\text{A2.1})$$

As shown by Buchholtz (1969, p. 35), (A2.1) has two solutions

$$g_{1,2}(w) = w^{-\beta_2/2} \cdot e^{-\beta_1 \cdot w/2} \cdot W_{\zeta, \pm\mu/2}(\beta_1 \cdot w), \quad (\text{A2.2})$$

where $\zeta = -\beta_2/2$, $\mu = [(\beta_2-1)^2 - 4\beta_3]^{1/2}$ and $W_{\zeta, \pm\mu/2}$ is a transformation of the *Whittaker's function* evaluated at the point $\beta_1 w$ given, in accordance with Buchholtz (ibid. p. 12), by

$$W_{\zeta, \pm\mu/2}(\beta_1 w) = e^{-\beta_1 w/2} (\beta_1 w)^{\frac{1 \pm \mu}{2}} F\left(\frac{1}{2} \pm \frac{\mu}{2} - \zeta, 1 \pm \mu, \beta_1 w\right) \frac{1}{\Gamma(1 \pm \mu)}, \quad (\text{A2.3})$$

where $F(\cdot)$ is the confluent hypergeometric function, often known as the *Kummer's function* and $\Gamma(\cdot)$ is the *Euler's gamma function*. Therefore, the solutions can be rewritten as

$$g_{1,2}(w) = \beta_1^{\frac{\beta_2}{2} - A} w^{-A} e^{-\beta_1 w} F(1 + A, \beta_2 - 2A, \beta_1 w) \cdot \frac{1}{\Gamma(\beta_2 - 2A)} \quad (\text{A2.4})$$

where

$$A = \left(\beta_2 - 1 \pm \sqrt{(\beta_2 - 1)^2 - 4\beta_3} \right) / 2 = r_{11}, r_{12}$$

The particular solution to (49a) is easily found to be $g(x) = (h - \lambda c) / (m + \gamma_1 \lambda - n)$. Then, using the fact that the solutions (A2.4) are linearly independent since μ is a non-integral (Buchholtz, *ibid.* p. 10–11), and re-transforming the variables in (A2.2), the general solution to (49a) reduces to²

$$\begin{aligned} g_1(x) = & a_{11} x^{r_{11}} e^{-2(h-n)/\sigma_v^2 x} F \left(1 + r_{12}, 1 - \mu_1, \frac{2(h-n)}{\sigma_v^2 x} \right) \\ & + a_{12} x^{r_{12}} e^{-2(h-n)/\sigma_v^2 x} F \left(1 + r_{11}, 1 + \mu_1, \frac{2(h-n)}{\sigma_v^2 x} \right) \\ & + \frac{h - \lambda c}{m + \gamma_1 \lambda - n} \end{aligned} \quad (\text{A2.5})$$

where a_{11} and a_{12} are arbitrary constants.

Correspondingly, by making the changes $w = x^{-1}$, $\beta_1 = (2/\sigma_v^2)(h-n)$, $\beta_2 = 2(1 - (m - \delta - n)/\sigma_v^2)$, $\beta_3 = (2/\sigma_v^2)(n - m - \lambda(1 - (1 - \gamma_2)y(\kappa)))$ in the differential equation (A2.1), the solution for the homogenous part of the differential equation (49b) can be found. Combining this with the particular solution found by applying the *method of undermined coefficients*, the general solution to (49b) becomes

² The first and the last terms of the two independent solutions (A2.4) are not functions of x and thus, do not appear explicitly in the general solution. Their information is included in the constants a_{11} and a_{12} .

$$\begin{aligned}
g_2(x) &= a_{21} x^{r_{21}} e^{-2(h-n)/\sigma_v^2 x} F\left(1+r_{22}, 1-\mu_2, \frac{2(h-n)}{\sigma_v^2 x}\right) \\
&+ a_{22} x^{r_{22}} e^{-2(h-n)/\sigma_v^2 x} F\left(1+r_{21}, 1+\mu_2, \frac{2(h-n)}{\sigma_v^2 x}\right) \\
&+ \frac{\xi x}{\delta + \xi} + \frac{\delta(\lambda c - h) + \xi(\delta + \lambda c - n + \xi)}{\delta(n-m) + \xi(n-m-\delta-\xi)}
\end{aligned} \tag{A2.6}$$

where $\xi = \lambda(1-(1-\gamma_2)y(\kappa))$. The constant terms a_{11} , a_{12} , a_{21} , and a_{22} can be recovered from the boundary conditions (50a-d):

The condition (50d) states that when $x \rightarrow \infty$, $|g_1(x)| < \infty$. Since $\lim_{x \rightarrow \infty} F(a, b, (c/x)) = 1$ and it can be proven that $r_{11} > 1$, $r_{12} < 0$,³ this condition implies that a_{11} must equal zero, because $\lim_{x \rightarrow \infty} x^{r_{11}} = \infty$. With $a_{11} = 0$, condition (50a) implies

$$\begin{aligned}
&a_{12} e^{-2(h-n)/\sigma_v^2} F\left(1+r_{11}, 1+\mu_1, \frac{2(h-n)}{\sigma_v^2}\right) \\
&- a_{21} e^{-2(h-n)/\sigma_v^2} F\left(1+r_{22}, 1-\mu_2, \frac{2(h-n)}{\sigma_v^2}\right) \\
&- a_{22} e^{-2(h-n)/\sigma_v^2} F\left(1+r_{21}, 1+\mu_2, \frac{2(h-n)}{\sigma_v^2}\right) \\
&= \frac{\xi}{\delta + \xi} + \frac{\delta(\lambda c - h) + \xi(\delta + \lambda c - n + \xi)}{\delta(n-m) + \xi(n-m-\delta-\xi)} \\
&\quad - \frac{h - \lambda c}{m + \gamma_1 \lambda - n}
\end{aligned} \tag{A2.7}$$

Condition (50b) implies

³ $r_{11} > 1 \Leftrightarrow -\gamma\lambda < \sigma_v^2 + \delta$, which is true since $\gamma\lambda \geq 0$, $\sigma_v^2 > 0$ and $\delta \geq 0$.
 $r_{12} < 0 \Leftrightarrow -\gamma\lambda < m-n$, which is true since $m-n > 0$ (see footnote 7 in chapter 6).

$$\begin{aligned}
& a_{12} r_{12} e^{-2(h-n)/\sigma_v^2} F\left(r_{11}, 1 + \mu_1, \frac{2(h-n)}{\sigma_v^2}\right) \\
& - a_{21} r_{21} e^{-2(h-n)/\sigma_v^2} F\left(r_{22}, 1 - \mu_2, \frac{2(h-n)}{\sigma_v^2}\right) \\
& - a_{22} r_{22} e^{-2(h-n)/\sigma_v^2} F\left(r_{21}, 1 + \mu_2, \frac{2(h-n)}{\sigma_v^2}\right) \\
& = \frac{\xi}{\delta + \xi}
\end{aligned} \tag{A2.8}$$

Kummer's function has the property $\lim_{x \rightarrow 0} F(a, b, (c/x)) = (\Gamma(b)/\Gamma(a))e^{c/x}(c/x)^{a-b}$. Therefore, according to condition (50c)

$$\begin{aligned}
& a_{21} \frac{\Gamma(1 - \mu_2)}{\Gamma(1 + r_{22})} \left(\frac{2(h-n)}{\sigma_v^2}\right)^{r_{21}} \\
& + a_{22} \frac{\Gamma(1 + \mu_2)}{\Gamma(1 + r_{21})} \left(\frac{2(h-n)}{\sigma_v^2}\right)^{r_{22}} \\
& = 1 + c - \frac{\delta(\lambda c - h) + \xi(\delta + \lambda c - n + \xi)}{\delta(n - m) + \xi(n - m - \delta - \xi)}
\end{aligned} \tag{A2.9}$$

Now, equations (A2.7) through (A2.9) constitute an equation system, $Da = e$, say, where $D = (d_{ij})$ is 3×3 matrix, $a = (a_{12}, a_{21}, a_{22})'$ and $e = (e_1, e_2, e_3)'$ are 3×1 vectors, which can be solved for a_{12} , a_{21} and a_{22} by using the *Cramer's rule*. After some manipulation we have the following elements of the equation system's matrix form,

$$d_{11} = e^{-2(h-n)/\sigma_v^2} F\left(1 + r_{11}, 1 + \mu_1, \frac{2(h-n)}{\sigma_v^2}\right),$$

$$d_{21} = r_{12} e^{-2(h-n)/\sigma_v^2} F\left(r_{11}, 1 + \mu_1, \frac{2(h-n)}{\sigma_v^2}\right),$$

$$d_{31} = 0,$$

$$d_{12} = -e^{-2(h-n)/\sigma_v^2} F\left(1 + r_{22}, 1 - \mu_2, \frac{2(h-n)}{\sigma_v^2}\right),$$

$$d_{22} = -r_{21} e^{-2(h-n)/\sigma_v^2} F\left(r_{22}, 1 - \mu_2, \frac{2(h-n)}{\sigma_v^2}\right),$$

(A2.10)

$$d_{32} = \frac{\Gamma(1 - \mu_2)}{\Gamma(1 + r_{22})} \left(\frac{2(h-n)}{\sigma_v^2}\right)^{r_{21}},$$

$$d_{13} = -e^{-2(h-n)/\sigma_v^2} F\left(1 + r_{21}, 1 + \mu_2, \frac{2(h-n)}{\sigma_v^2}\right),$$

$$d_{23} = -r_{22} e^{-2(h-n)/\sigma_v^2} F\left(r_{21}, 1 + \mu_2, \frac{2(h-n)}{\sigma_v^2}\right),$$

$$d_{33} = \frac{\Gamma(1 + \mu_2)}{\Gamma(1 + r_{21})} \left(\frac{2(h - n)}{\sigma_v^2} \right)^{r_{22}},$$

$$e_1 = \frac{\xi}{\delta + \xi} + \frac{\delta(\lambda c - h) + \xi(\delta + \lambda c - n + \xi)}{\delta(n - m) + \xi(n - m - \delta - \xi)} - \frac{h - \lambda c}{m + \gamma_1 \lambda - n},$$

$$e_2 = \frac{\xi}{\delta + \xi}, \text{ and}$$

$$e_3 = 1 + c - \frac{\delta(\lambda c - h) + \xi(\delta + \lambda c - n + \xi)}{\delta(n - m) + \xi(n - m - \delta - \xi)}.$$

Thus, the determinant of the coefficient matrix of the equation system is given by

$$\Delta = \begin{vmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{vmatrix} \quad (\text{A2.11})$$

The solution values obtained are

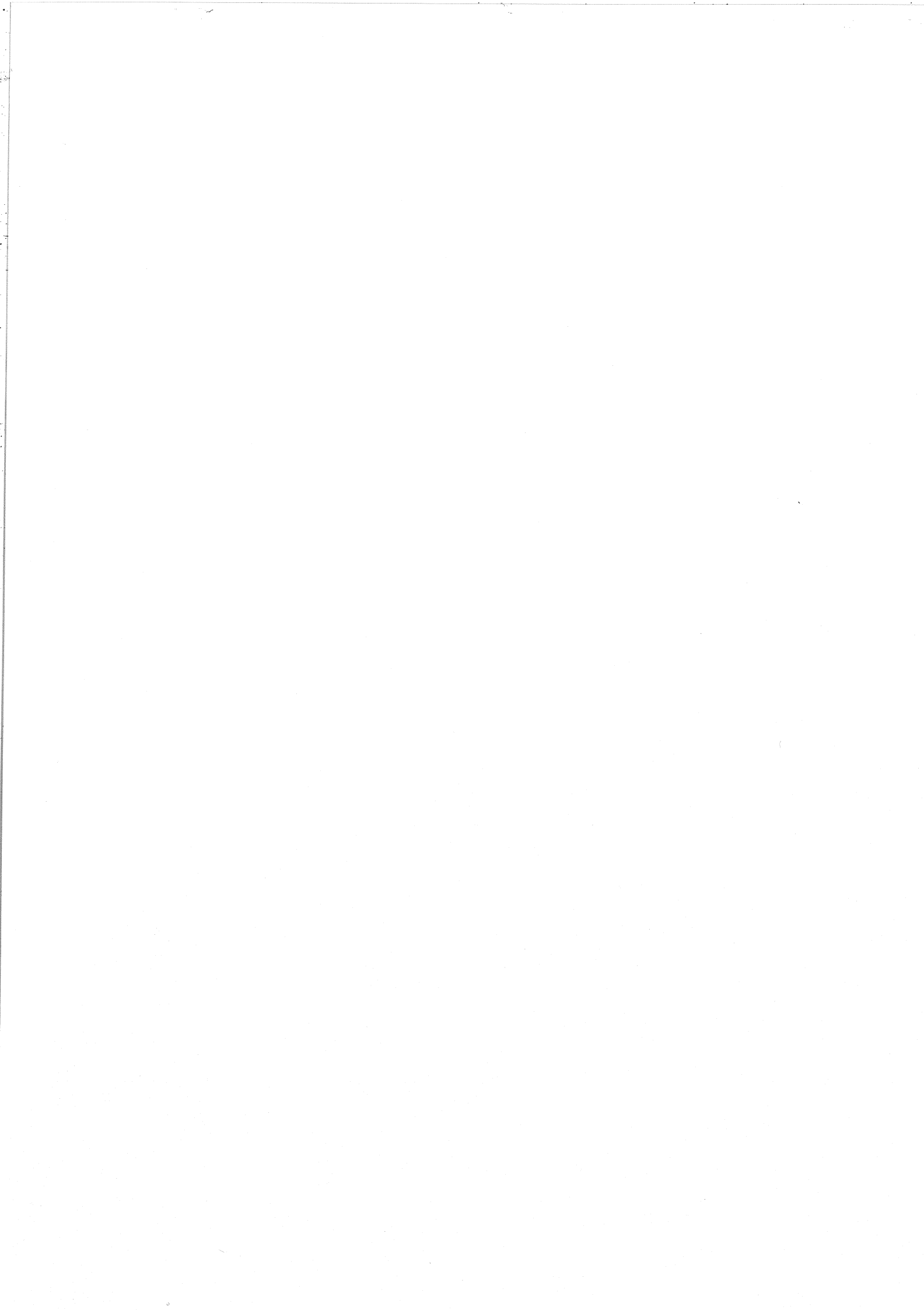
$$a_{12} = \frac{\begin{vmatrix} e_1 & d_{12} & d_{13} \\ e_2 & d_{22} & d_{23} \\ e_3 & d_{32} & d_{33} \end{vmatrix}}{\Delta}$$

$$a_{21} = \frac{\begin{vmatrix} d_{11} & e_1 & d_{13} \\ d_{21} & e_2 & d_{23} \\ d_{31} & e_3 & d_{33} \end{vmatrix}}{\Delta}$$

(A2.12)

$$a_{22} = \frac{\begin{vmatrix} d_{11} & d_{12} & e_1 \\ d_{21} & d_{22} & e_2 \\ d_{31} & d_{32} & e_3 \end{vmatrix}}{\Delta}$$

Finally, using the above results, and taking the *Kummer transformation* of (A2.5) and (A2.6), the closed form solution for the value of the deposit insurer's claim reduces to equation (51a) and (51b) in the main text.



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