Heikki Hella

On robust ESACF identification of mixed ARIMA models

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Abstract

Statistical data sets often contain observations that differ markedly from the bulk of the data. These outlying observations, 'outliers', have given rise to notable risks for statistical analysis and inference. Unfortunately, many of the classical statistical methods, such as ordinary least squares, are very sensitive to the effects of these aberrant observations, ie they are not outlier robust. Several robust estimation and diagnostics methods have been developed for linear regression models and more recently also for time series models.

The literature on *robust identification* of time series models is not yet very extensive, but it is growing steadily. Model identification is a 'thorny issue' in robust time series analysis (Martin and Yohai 1986). If outliers are known or expected to occur in a time series, the *first stage* of modelling the data should be done using robust identification methods. In this thesis, the focus is on following topics:

1. The development of a robust version of the extended autocorrelation function (EACF) procedure of Tsay and Tiao (1984) for tentative identification of univariate ARIMA models and comparison of non-robust and robust identification results.
2. Simulation results for the sample distributions of the single coefficients of the extended sample autocorrelation function (ESACF) table, based on classic and robust methods, both in outlier-contaminated and outlier-free time series.
3. Simulation results for two basic versions of the sample standard error of ESACF coefficients and the results of the standard error calculated from simulation replications.

Robust designing concerns two parts of the ESACF method: iterative autoregression, AR(p), and an autocorrelation function to obtain less biased estimates in both cases.

Besides the simulation experiments, robust versions of the ESACF method have been applied to single generated and real time series, some of which have been used in the literature as illustrative examples.

The main conclusions that emerge from the present study suggest that the robustified ESACF method will provide

a) A fast, operational statistical system for tentative identification of univariate, particularly mixed ARIMA(p, d, q), models

b) Various alternatives to fit the robust version of AR(p) iteration into a regression context and use of optional robust autocorrelation functions to handle both isolated and patchy outliers
c) Robust procedures to obtain more normal-shape sample distributions of the single coefficient estimates in the ESACF two-way table
d) The option of combining OLS with a robust autocorrelation estimator.

Simulation experiments of robust ESACF for outlier-free series show that, since the robust MM-regression estimator is efficient also for outlier-free series, robust ESACF identification can always be used with confidence.

The usefulness of the method in testing for unit roots is obvious, but requires further research.

Key words: robust tentative identification, robust extended autocorrelation function, outliers, robust regression estimation, Monte Carlo simulations, time series models

Tiivistelmä

Tilastoaineistossa on usein joitakin havaintoja, jotka poikkeavat merkittävästi aineiston muusta osasta. Nämä poikkeavat havainnot (outlierit) aiheuttavat huomattavia ongelmia tilastollisessa analyysissä ja päättelyssä. Valitettavasti monet klassisen tilastotieteen menetelmät, kuten tavallinen pienimmän neliösumman menetelmä, ovat hyvin herkkiä näiden poikkeavien havaintojen vaikutuksille, eli ne eivät ole robusteja. Lineaarisille regressiomalleille ja viime aikoina myös aikasarjamalleille on kuitenkin kehitetty useita robusteja estimointi- ja diagnostiikkamenetelmiä. Aikasarjamallien robustia täsmentämistä koskeva kirjallisuus ei ole vielä kovin laajaa, mutta kasvaa nopeasti. Mallien täsmentäminen on ”hankala juttu” robustissa aikasarja-analyysissä (Martin ja Yohai 1986). Jos aikasarjan tiedetään tai oletetaan sisältävän poikkeavia havaintoja, mallintamisen ensi vaihe pitäisi suorittaa robustein täsmentämismenetelmin.

Tämän tutkimuksen tavoitteita ovat:

1. Robustin version kehittäminen niin sanotusta laajennetusta autokorrelaatiofunktiomenetelmästä (EACF-proseduuri), jonka alun perin kehittivät Tsay ja Tiao (1984) yhden muuttujan ARIMA-mallien alustavaksi täsmentämiseksi, ja robustin menetelmän tulosten vertailu perinteisen menetelmän antamiin tuloksiin.
2. Laajennetun autokorrelaatiofunktion kertoimien (eli ESACF -matriisin elementtien) otosjakaumien simulointi klassisin ja robustein menetelmin sekä puhtaiden että outliereillä saastuneiden aikasarjojen tapauksissa.
3. Laajennetun autokorrelaatiofunktion kertoimille simuloitujen keskivirheiden vertaaminen teoreettisiin estimaatteihinsa.

Robustointi koskee kahta ESACF-proseduurin vaihetta: iteratiivista autoregressiota, AR(p), ja autokorrelaatiofunktiota, jota käytetään vähemmän harhaisten estimaattien tuottamiseksi. Simulointikokeiden lisäksi robusteja versioita ESACF-proseduurista sovelletaan työssä eräisiin synteettisiin ja aitoihin aikasarjoihin, joista muutamia on kirjallisuudessa käytetty havainnollisina esimerkkeinä.

Tutkimuksen tulokset viittaavat siihen, että robusti ESACF-proseduuri

a) on nopea ja joustava tilastollinen menetelmä yhden muuttujan ARIMA(p, d, q) -mallien alustavaa täsmentämistä varten

b) antaa useita robusteja vaihtoehtoja erilaisten iteratiivisten autoregressioiden ja autokorrelaatiofunktioiden sovittamiseksi aineistoon
c) tarjoaa menetelmiä, joilla saadaan läheisemmin normaalijakaumaa muistuttavia otosjakaumia yksittäisille ESACF-kertoimille
d) antaa mahdollisuuden käyttää pienimmän neliösumman menetelmää ESACF-proseduurin ensimmäisessä vaiheessa, jolloin vasta toinen vaihe perustuu robustiin autokorrelaatiofunktioon.

Simulointikokeet osoittavat, että koska robusti MM-estimaattori on tehokas myös outliereitä sisältämättömien aikasarjojen tapauksissa, tätä robustia ESACF-proseduuria voidaan aina soveltaa. Menetelmästä saatava tuki robustiin yksikköjuuritestaukseen on ilmeinen, mutta vaatii lisää tutkimusta.

Avainsanat: robusti täsmentäminen, robusti laajennettu autokorrelaatiofunktio, outlieri, robusti regressioestimointi, Monte Carlo -simulointi, aikasarjamallit

Foreword

Many years ago, the world of robust statistics began to appeal to the author of this thesis. The occurrence of outlying observations in different data sets was surprising and their destructive effects on many traditional statistical estimators and procedures indicated new challenges to outcome problems caused by outliers. As robust regression analysis contributed increasingly to time series modelling, opportunities to robustify these models improved. If the data are known or expected to include outliers, it is important to use robust tools in the first step of the modelling process to reduce the risk of producing a misspecified model. This is the benchmark and the basis of my thesis.

I wish to acknowledge those who have assisted and encouraged me during this study. Especially I wish to express my gratitude to Professor Ruey S. Tsay, another developer of the original ESACF method, who sent me, years ago, his message via internet: 'Robustifying ESACF is a fine ideal.' I thank Professor Erkki Liski for his encouragement in different phases of my study. I am indebted to my official examiners, Professors Antti Kanto and Seppo Pynnönen, for their expertise, useful discussions and guidance. Professor Jukka Nyblom receives my thanks for his comments and suggestions for focusing my work.

I wish also to thank my colleagues at the research department of the Bank of Finland and at the Research Institute of the Finnish Economy, ETLA. Especially I wish to thank the research department for use of the simulation program. This program was developed by Lic. Phil. (mathematics) Marko Laine, who worked inexhaustibly and created new and practical solutions. Many thanks, Marko. Special thanks go to Ms Riitta Repo for her skilful editing and writing earlier versions of this work.

I am grateful for the professional staff of the Bank of Finland for their efforts in the publication process. Mr Glenn Harma revised the language of my study, and Päivi Nietosvaara finalised the layout of the publication.

Finally, I want to warmly thank my dear wife, Riitta, and my sons, Otto-Juhana and Lauri, for their patience, great empathy and care of me during the long project.

Helsinki, November 2003
Heikki Hella

Contents

1 Introduction

1.1 The objective and summary of this thesis

Statistical data sets often contain observations that differ widely from the rest of the observations. These 'outliers' give rise to notable risks for statistical analysis and inference. Hampel et al (1986, p. 26–28) argue that the occurrence of outliers in routine databases is the rule rather than the exception, ie about 1–10% of observations. Nowadays one can apply many kinds of statistical procedures to handle these observations. With modern methods we are able to detect and estimate outliers, and protect against them.

The development of linear regression theory supported by fast and efficient computers has created the benchmark for a great number of new robust statistical procedures. Several robust estimation and diagnostics methods have been developed for linear regression models and more recently also for time series models.

The literature on *robust identification* of time series models is not yet extensive, but it is growing steadily. Model identification is still a 'thorny issue' in robust time series analysis (Martin and Yohai 1986). As Chang (1982, p. 232–233) remarks: 'We need to protect not only the parameter estimation process against the adverse effect of exogenous interventions but also the model identification process so that appropriate model forms for the underlying time series can be specified in the very first place.' The author of this thesis would add: and we can *always* begin to model data by robust methods, since we have available the robust regression estimation method, which appears to perform efficiently also with outlier-free data. Here our goal is to find at the start of the modelling process the most appropriate candidates for modelling ARIMA processes.

The objectives of this thesis are to:

1. develop the robust version of the extended autocorrelation function (EACF) procedure of Tsay and Tiao (1984) for identification of univariate ARIMA models
2. study simulation results on sample distributions of the single coefficients of the extended sample autocorrelation function (ESACF) table, based on standard and robust methods, both in outlier-contaminated and outlier-free time series
3. analyse simulation results of sample standard errors of single ESACF coefficient estimates

Robust designing concerns the two stages of the ESACF method: autoregressive AR(p) fitting and the autocorrelation function. Besides the simulations, the robust versions of the ESACF method have been applied to the real time series, some of which have been used as illustrative examples in the literature.

Our goal is to have the robustified ESACF method provide

a) a fast, operational system for identification of ARIMA(p, d, q) models
b) some robust versions which are able to encounter both isolated and patchy outliers
c) an improved procedure to obtain more symmetric and closer-to-normal sample distributions of single coefficient estimates of the ESACF two-way table.

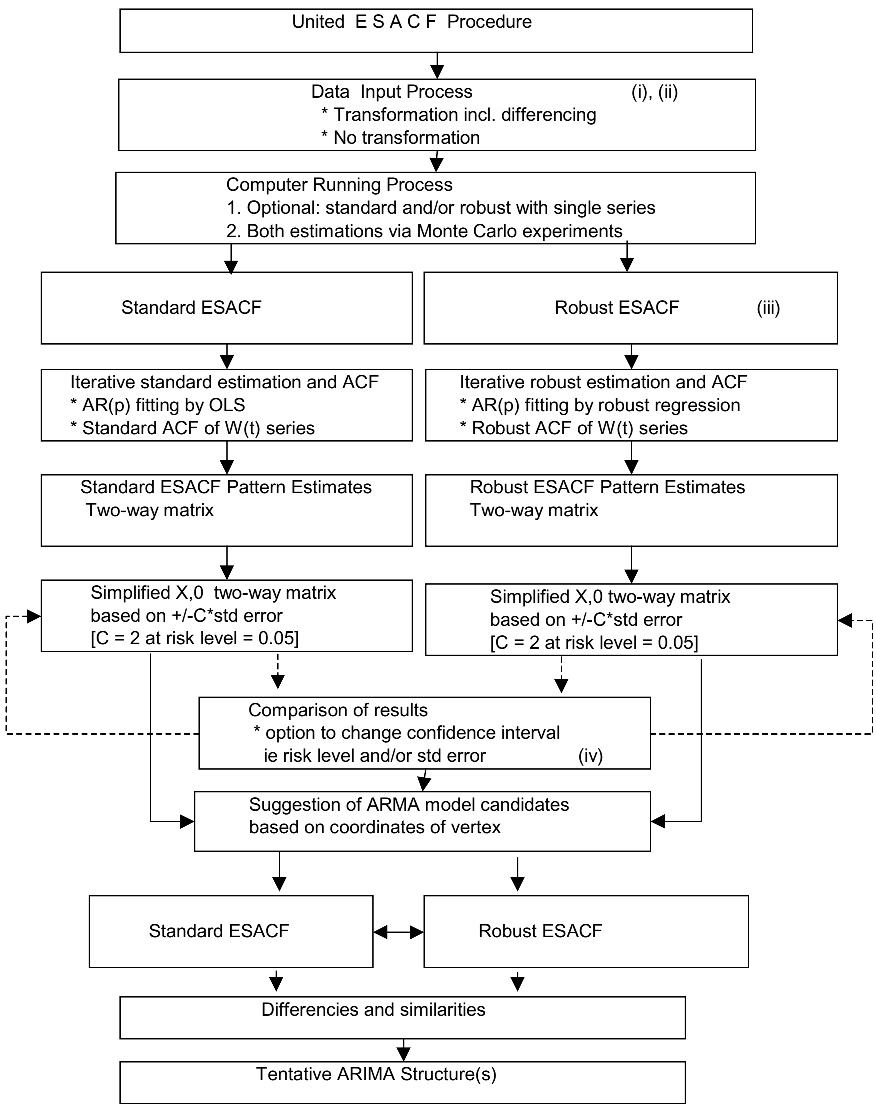
Main results

The main results that emerge from the present study show that the robustified ESACF method provides

e) a fast, operational system for identification of univariate ARIMA models (flow chart of united robust and standard ESACF procedure given in Figure 1) that is able to handle both isolated and patchy outliers.
f) robust procedures to achieve more normal-shaped sample distributions of the single coefficient estimates for the ESACF table; then the most important estimate has the vertex of a triangle of asymptotic zeros; some normalising effect is also found in the sample distributions of other coefficient estimates of the ESACF table
g) new and valuable information about the sample standard errors of the single ESACF coefficient estimates; generally the robust coefficients based on simulation repetitions have the greatest standard errors and the asymptotic estimator, $(n-k-j)^{-½}$ gives underestimates
h) different robust estimates of the ordinary SACF as the first row of the robust ESACF table; the standard errors (based on simulations) of these robust SACF estimates

The outline of the thesis is following: Chapters 1 and 2 contain the introduction and definitions of the main concepts of the statistical

Figure 1. **Flow chart of the United ESACF identification procedure**



(i) real or generated series: choice of robust methods and estimators
(ii) simulation experiments: choice of robust methods and estimators, summaries and graphics
(iii) robust alternatives of regression estimation, robust autocorrelation functions and scale estimators
(iv) single series case: not necessarily symmetric (+ -) intervals, may be different in standard and robust case

robustness theory, ARIMA models and two basic outlier types in time series. The theory of the standard (original) ESACF procedure is presented in Chapter 3. Treatment of outliers and the main influences of outliers on time series are considered in Chapter 4. The theoretical design of the robust ESACF method with different robust regression and autocorrelation configurations is presented in Chapter 5. Simulation experiments for the standard and robust ESACF procedures and their applications to single generated and real time series are considered in Chapters 6 and 7. The main conclusions and suggestions for further research are presented in Chapter 8.

2 Some basic concepts and definitions

2.1 Statistical robustness

The concept of robustness is old in the history of statistics. However, the word robust, which by dictionary definition means ‘strong, vigorous’, is a relatively young term in statistics. Box (1953) first gave the word a statistical meaning. Since then, rapid development of the theory of robustness has given rise to alternative approaches to robust statistical theory.

The earliest discussions and applications of robustness go back as far as the eighteenth-nineteenth century (Hampel et al 1986, Barnett & Lewis, 1994). For instance, as regards astronomical observations, there was concern about ‘unrepresentative’, ‘rogue’ or ‘outlying’ observations, ie *outliers*. It is noteworthy that already in those days, in the early 1800s, scientific debate was concerned with the effects of outlying observations on estimates calculated by ordinary least squares (OLS)[1].

Robustness, in general, refers to the ability of a procedure or an estimator to produce results that are insensitive to departures from ideal assumptions. This definition of robustness covers all scientific research. When the ideal assumptions pertain to statistical distributions, we speak about *distributional robustness* and in the context of outliers we refer to *outlier robustness*. The practicality aspect is always an essential part of robustness. As Lucas (1996, p. 1) remarks, robustness to researchers is an intriguing subject both from a theoretical and practical point of view. In reference to general robustness theory, Hampel et al (1986, p. 6) define statistical robustness as follows:

> In a broad informal sense, robust statistics is a body of knowledge, partly formalized into ‘theories of robustness’, relating to deviations from idealized assumptions in statistics.

---

[1] A.M. Legendre developed the method of least squares (1805) and published it in the book ‘Nouvelles pour la Determination des Orbites des Comets’, Courcier, Paris. Besides Legendre, C.F. Gauss contributed crucially (1809) to the theory of this method.

In time series analysis, the essential concept is *resistance*, which is related to the notion of robustness from a data-oriented point of view. This concept has the important advantage that, on the one hand, it can be applied without special assumptions about the model and, on the other hand, observations can be dependent or non-identically distributed (see Martin and Yohai 1985, Stockinger and Dutter 1987). A more detailed definition of resistance is presented below in section 2.3.

2.2 Outlier robustness

In this thesis we focus on outlier robustness. Outliers can be classified in statistics as outlying observations in linear regression, time series analysis, survey, directional and contingency table data (Barnett and Lewis 1994). Outliers always entail both theoretical and practical problems. Usually, depending on our goal(s), we need one or more procedures that are robust, to protect against and detect outlying observations in the data. For instance, in the case of a forecasting model, it is of utmost importance to be able to detect, estimate the effects of, and interpret outliers. In some cases, outliers in a residual series may indicate omission of an explanatory variable from the model. Furthermore, the robust regression estimates are less biased than OLS and provide estimates of outliers that are more strikingly seen in residual series.

The concept of an outlier is generally used rather informally. The literature provides a rich menu of informal definitions, but we still lack a mathematically strict, unique and generally accepted definition. Recently some researchers have attempted to construct a more formal definition of an outlier (eg Gather and Becker 1997). One of the main features (and 'problems') of outlier robustness in a time series context is that outliers are 'model dependent'. As a consequence, it may be that an observation is considered an outlier in respect of one model but a regular observation in respect of another.

Outlier analysis can be divided into three distinct parts: the i.i.d. (independent, identically distributed observations) regression context, the time series context, and survey data. The basic theory of robustness is developed in the i.i.d. context. In time series modelling, accounting for (the effects of) outliers is complicated because of the structure of the adjacent correlated observations. In addition, in time series, the influences of outliers can be shown to depend on their type, relative position, number and magnitude and – as mentioned before –

on the model structure underlying the time series. In the regression context, outliers are classified as y- and x-outliers. Outlier types occurring in survey data are considered eg by Barnett and Lewis (1994). In considering the features which robust estimators should possess, Huber (1972) presents an interesting approach: 'to view robustness as a kind of insurance problem'[2].

2.3 Main concepts of statistical robustness

In statistical theory, robustness can be classified into two main parts: qualitative and quantitative. Hampel (1971, Theorem1) introduced the concept of qualitative robustness. The main idea is to complement the notion of differentiability (influence function, see below) with continuity conditons, with respect to the Prohorov distance (definition in Hampel 1971, p. 1888). Hampel et al 1986 (Section 2.2b) considered the relationship of continuity to qualitative robustness. Two other concepts for judging the robustness performance of an estimator are efficiency robustness and min-max robustness, which are also directly applicable to time series data (see eg Martin and Yohai 1985, Huber 1996). Quantitative robustness builds on the concept of a breakdown point, whereas infinitesimal robustness incorporates the influence function as the critical concept.

Influence function

The influence function (IF) is a *local* robustness measure. By definition, an IF measures the change in the value of an estimator when outliers are added to the sample. Following Hampel et al (1986), the influence function is defined as follows. Let Δ_x denote the probability measure which puts the unit mass at the point x, ie the c.d.f. with a point mass at x. The influence function of an estimator H at F is given by

$$\mathrm{IF}(x,H,F)=\lim_{\eta\downarrow 0}\frac{H[(1-\eta)F+\eta\Delta_x]-H(F)}{\eta} \tag{2.1}$$

[2] '…I am willing to pay a premium (a loss of efficiency of, say, 5 to 10% at the ideal model) to safeguard against ill effects caused by small deviations from it; although I am happy if the procedure performs well also under large deviations…' (Huber 1972, p. 1047).

at those x where the limit exists. In (2.1) H(F) denotes the value of the estimator of the original distribution F. Similarly, $H[(1-\eta)F+\eta\Delta_x]$ denotes the value of this estimator of the slightly contaminated distribution $(1-\eta)F+\eta\Delta_x$. η refers to the fraction of the perturbation, $0<\eta<1$. The influence function IF is the first derivative of a statistic H of an underlying distribution F. If the IF is *bounded*, then the effects of a small number of outliers are also bounded. As for its interpretation, one can say that IF measures the asymptotic (standardised) bias of the estimator H caused by contamination of F.

There are also some studies in the literature on finite-sample versions of the influence function, ie the empirical influence function (EIF) and sample influence function (SIF) (see eg Lee 1990). Not all estimators have an influence function, but all of them have a breakdown point.

Breakdown point

The breakdown point (BP) is a *global* measure of the reliability of a statistic. The BP is also an asymptotic concept. As such, this measure is complementary to the IF. The finite-sample breakdown point of the estimator Hn for the sample $(x_1,...,x_n)$ is given by

$$\varepsilon_n^*(H_n;x_1,...,x_n): = \frac{1}{n}\max\{m:\max_{i_1,...,i_m}\sup_{y_1,...,y_m}|H_n(z_1,...,z_n)|<\infty\} \qquad (2.2)$$

where the sample $(z_1,...,z_n)$ is obtained by replacing the m data points $x_{i_1},...,x_{i_m}$ by the arbitrary values $y_1,...,y_m$ (Hampel et al 1986, Section 2.2a, definition 2.)[3]. In general, taking the limit of ε_N^* as $n\to\infty$ we obtain the asymptotic breakdown point. The breakdown point takes values between 0 and 1. The literature has introduced some variants of this definition of the breakdown point definition and of its finite-sample versions (Hampel et al 1986, p. 97).

The breakdown point is essentially the largest fraction of contamination which does not ruin an estimate. Note that the definition of the BP contains no probability distributions. For example, the BP of the arithmetic mean, standard error and OLS

[3] Professor Hannu Oja has remarked that in (2.2) definition the $|H_n(z)|$ is usually replaced by the |bias| of the $H_n(z)$. In the literature, see eg Huber (1996, p. 9).

estimator is of value zero, for the median it is 0.5 and for the α-trimmed mean α. A zero value for the BP reflects extreme sensitivity of an estimator to outliers.

In practice, the influence function and breakdown point provide complementary bits of information about the estimator. The bias can be approximated by the IF, and the neighbourhood in which this approximation is useful can be measured by the BP. In addition, the gross-error sensitivity of an estimator can be defined in terms of the IF. The *gross-error sensitivity* γ^* is defined as the supremum of the IF with respect to x:

$$\gamma^*(H,F) = \sup_x |IF(x;H,F)| \tag{2.3}$$

which contains information on the maximum relative change in the value of the estimator caused by a small change in x. A finite value of γ^* is desirable.

Maximal bias curve

The essential property of a good robust estimator is that the resulting bias is quantified and is under control. Then the problem in most cases is to evaluate the consistency, as well as the bias, of an estimator. In practice, we must always put up with some amount of bias in an estimator.

The bias curve of an estimator plots its maximun bias against the fraction of contamination, given the underlying distribution. Following Hampel et al (1986), the maximal bias curve of the estimator H is given by

$$\sup_G |H((1-\eta)F + \eta G) - H(F)| \tag{2.4}$$

where G is an arbitrary distribution.

The main advantage of the bias curve over the BP is that the bias curve is more 'operational', ie it provides more information. Given some degree of contamination, we can read from the curve the corresponding maximum bias of the estimator. The form of the bias curve in the neighbourhood of the breakdown bound of an estimator can be quite informative for analysts (for the bias curve of the median, see Lucas 1996, Fig. 2.2). We can compare the robust sensitivity of different estimators via their bias curves (see eg Martin and Yohai 1991 in an AR(1) model).

Outlier robustness in time series context

The influence function, the breakdown point and the bias curve comprise the main practical concepts (tools) of statistical robustness. However, we have no complete or general theory of robustness for time series. The literature contains many different suggestions for these concepts as regards specific stochastic processes and estimators. The research activity has been surprisingly brisk, as the following list shows: Martin (1979, 1980, 1981), Künsch (1984), Papantoni-Kazakos (1984), Martin and Yohai (1984, 1985, 1986), Boente, Fraiman and Yohai (1987) and Genton and Lucas (2000). These contributions include both influence functions and breakdown points of different time series processes. Martin and Yohai (1986) can be considered the epoch-making (milestone) comprehensive article on robust time series analysis and a 'robust bridge' between the classic i.i.d. and time series contexts. There is a steady growth of theoretical, practical and experimental results, eg Rousseeuw and Yohai 1984, Hampel et al 1986, Rousseeuw and Leroy 1987, Chan 1989, Chen 1994, Lucas 1996, Meintanis and Donatos 1999, You 1999 and Ma and Genton 2000.

Quality robustness seems to be the most important challenge for theoretical research. Martin and Yohai (1985) argue that the previously mentioned resistance and qualitative robustness are the most important forms of robustness in the time series context. They define (p. 121) resistance as follows:

> An estimate T_n is called resistant, if 'small' changes in the data result in only small changes in T_n, where 'small' changes in the data means (i) *large* changes in a *small* fraction of the data and/or *small changes in all* the data.

Boente, Fraiman and Yohai (1987) proposed a new approach to qualitative robustness, based on the concept of resistance. These concepts are developed mostly for autoregressive processes and their structure and effects are strongly dependent on the type of outlier configuration and the structure of the time series model. For example, in the case of stationary autoregressive models, the GM-estimator has the breakdown point of 1/(p+1) where p is the order of an AR(p) process (Rousseeuw and Leroy 1987). The breakdown point of the M-estimator is zero while the MM-estimator is 0.5 (Yohai 1987). It should be noted that the GM- and MM-estimators are not qualitatively robust in case of the ARMA(p, q; q>0) processes (eg Martin and Yohai 1985, p. 136 and 139). The reason is that each residual depends on all the previous observations and hence the effect of an outlier propagates, in principle, to all the residuals.

2.4 Definition of outlier

The literature contains many informal definitions of an outlier but only a few, recently published, more formal ones. One commonly used informal definition (Barnett and Lewis 1994) is:

> An outlier in a set of data is an observation or a patch of observations which appears to be inconsistent with the remainder of that set of data.

The inconsistency refers to the case where this outlying (aberrant) observation (or group of observations) is generated by some mechanism other than that of the rest (ie the majority) of the data[4]. Recently, some formal, mathematical definitions of an outlier have been proposed (see eg Becker & Gather 1999). Becker and Gather consider a wide range of outlier generating models, outlier identification rules (outlier identifier) and performance criteria for this identification in the i.i.d. context. In the time series context, Fox (1972) first developed two basic forms (models) of aberrant observations.

I.i.d. regression and time series models have their own types of outliers. Furthermore, an extreme observation of a set of data need not be an outlier and vice versa. So, since an outlier need not be large relative to the scale of the X_t process, it is important to compare an outlier to the scale of the residuals (ideally uncorrelated $N(0, \sigma^2)$) innovations of the estimated model. In the following we consider briefly the basic forms of outliers in an i.i.d. and in a time series context. The focus will be on the two basic outlier types of time series.

[4] For more on outliers, their history, handling and examining, see Barnett and Lewis (1994, Chapters 2 and 3). On outliers in time series, see the literature mentioned in Section 2.6. of this thesis.

2.5 Outliers in an i.i.d. context

In the classification of regression outliers we follow the presentation of Hampel et al (1986). Regression outliers can be classified into a) 'gross errors' and b) outliers due to model inadequacy and failure. Gross errors occur in the form of different recording errors due to technical difficulties or partly completed questionnaires, misinterpreted questions etc. The second main group includes outliers due to econometric or statistical model failure (see Hampel et al 1986, Section 1.2, Lucas 1996, Section 2.1.2).

We can classify y- and x-regression outliers as follows:

a) vertical outlier (outlier in y-direction),
b) good leverage point (outlier in x-direction)
c) bad leverage point (outlier in x-direction).

More on the classification, structure and effects of these regression outliers can be found in Chatterjee & Hadi (1986), Rousseeuw and Zomeren (1990) and Ryan (1997). We do not consider here 'influential observations' (Peña 2001) and interactions between them and outliers.

2.6 ARIMA models and basic types of outliers in time series

In this section we first introduce the ARIMA models and basic types of outliers in the time series context. After this we consider briefly the distributional forms of outliers and the distribution of outlier contaminated time series. The effects of outliers on the sample autocorrelation function of ARIMA processes are considered in Section 5.2.4.

Autoregressive integrated moving average (ARIMA) models

ARIMA processes are the most common family of time series models. Box and Jenkins (1970, 1976), in their seminal book, presented a systematic presentation of the structure and use of these time series models.

The general class of univariate ARIMA models takes the form

$$\Phi(B)Y_t = C + \theta(B)a_t \quad t = 1,...,n \tag{2.5}$$

where

$\Phi(B) = U(B)\phi(B) = 1 - \Phi_1 B - ... - \Phi_p B^p, U(B) = 1 - U_1 B - ... - U_d B^d$ is a nonstationary factor; $\phi(B) = 1 - \phi_1 B - ... - \phi_{p-d} B^{p-d}$ is the autoregressive and $\theta(B) = 1 - \theta_1 B - ... - \theta_q B^q$ the moving average polynomial of degree p–d and q in B; and the characteristic equations $\phi(B) = 0$ and $\theta(B) = 0$ have all roots outside the unit circle. The autoregressive part of order p, AR(p), is then stationary and the moving average part of order q, MA(q), is invertible, ie it can be written in terms of AR(∞) representation (Box and Jenkins 1976, p. 52–53, 67).

All the roots of U(B) are on the unit circle, d is here assumed to be a non-negative integer (in practice often 1). If U(B) = 1, ie d = 0, we have a stationary ARMA(p, q) process; Φ(B) and θ(B) are assumed to have no common roots; $\{Y_t\}$ is the observable time series, $\{a_t\}$ is a Gaussian white noise process N(0, σ_a^2) and C is a constant. B is the backshift operator, ie $BZ_t = Z_{t-1}$.

The ARIMA family consists of non-seasonal and multiplicative seasonal models (for the basic structure of the seasonal model, see Box and Jenkins 1976, Chapter 9). We focus in this thesis only on non-seasonal ARIMA models.

The classic autocorrelation function

Suppose a stationary stochastic process Z_t has mean μ, variance σ^2 and autocovariance γ(τ) where τ is the lag. The theoretical autocorrelation function (ACF) of Z_t is then

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)} = \frac{\gamma(\tau)}{\sigma^2} \tag{2.6}$$

Note that $\rho(0) = 1$.

The autocorrelation function has the following properties: $\rho(\tau) = \rho(-\tau)$ and $|\rho(\tau)| \le 1$. The sample autocorrelation function

(SACF), $\hat{\rho}(\tau)$, is calculated via the sample statistics $\hat{\gamma}(\tau)$ and $\hat{\sigma}^2$. It is well known that the classic autocorrelation function is very non-robust, and its breakdown point is zero. This means that just one clearly outlying observation is able to destroy the information content of the SACF.

Two basic types of outliers in time series

Fox (1972) first investigated outlier types and their modelling in the time series context. He defined two types of outliers: Type I and II, which are now known as additive (AO) and innovational (IO) outliers. The basic structures of these outliers are as follows:

Let $\{Y_t\}$ be a time series following an ARIMA model: $Y_t = \frac{\theta(B)}{U(B)\phi(B)} a_t \quad T = 1,...,N,$

where the polynomials $\theta(B)$ and $\phi(B)$ have roots outside the unit circle and all roots of U(B) are on the unit circle.

1. Additive outlier (AO)

An additive outlier is an event that affects a series for one time period only; its effects are independent of the ARIMA model. If we assume that an outlier occurs at time t = T, we have the AO model

$$Z_t = Y_t + \mathbf{o_A} P_t^{(T)} \tag{2.7}$$

where $P_t^{(T)}$ is a pulse function (ie $P_t^{(T)} = 1$ when $t = T$, 0 otherwise; see eg Mills 1991) and $\mathbf{o_A}$ is the magnitude of the isolated, additive outlier. Note that here Z_t is the observed, contaminated time series and Y_t the uncontaminated core process (here unobserved at time point $t = T$).

2. Innovational outlier (IO)

An innovational outlier is an event whose effect is propagated according to the structure of the ARIMA model of Y_t. The IO model is thus

$$Z_t = Y_t + U(B)^{-1}\frac{\theta(B)}{\phi(B)}\mathbf{o_I}P_t^{(T)}, \quad \text{or}$$
$$Z_t = \frac{\theta(B)}{U(B)\phi(B)}\left(a_t + \mathbf{o_I}P_t^{(T)}\right) \tag{2.8}$$

where $P_t^{(T)}$ is a pulse function and $\mathbf{o_I}$ the magnitude of a single innovational outlier at time $t = T$.

Additive outliers are in practice more common than innovational outliers. For statistical analysis, the AOs are more dangerous and their influence on parameter estimates can be very destructive. For instance, a single large AO outlier may destroy the information content of the sample autocorrelation function or sample partial autocorrelation function.

In the literature, the following types of outliers have also been proposed: level change (LC), transient change (TC), variance change (VC), reallocation (RE) and seasonal outlier (SLS); see Tsay (1988), Chen and Liu (1993), Wu et al (1993), Maddala & Yin (1997) and Kaiser & Maravall (1999).

Besides isolated outliers, a time series will often contain a patch of outliers of one or several types. Thus, a long time series may include a combination of outliers of different types and time configurations. In that case, modelling of the time series is usually a complicated and laborious task. In practice, the usual situation is that the ARIMA model and possible outliers must be estimated simultaneously, based solely on the information in the data. For this, an iterative modelling procedure has been developed (Chang 1982, Chang, Tiao and Chen 1988 and Chen & Liu 1993). If the exact timing of the different outliers is known, the intervention models can be estimated (eg Mills 1991, Chapter 12).

On the distributional form of outliers
Time series generated by mixture models

A. The distributions of the additive and innovational outliers are presented in the following (see Stockinger and Dutter 1987, p. 18).

(AO) The independently and identically distributed $\mathbf{o_A}$ have an arbitrary distribution, eg the Gaussian mixture distribution, 'contaminated normal':

$$\mathrm{CND}(\kappa, \sigma_3) = (1-\kappa)\delta_0 + \kappa \mathrm{N}(0, \sigma_3^2) \tag{2.9}$$

where δ_0 denotes the degenerated distribution having all its mass at the origin and κ is the fraction of the normal distribution. In practice, κ is in the range from 0.01 to 0.25. The AOs are independent of the underlying ARMA process, Y_t in (2.7).

(IO) In the case of an innovational outlier, the distribution of the innovation process of Z_t can be a t-distribution or a contaminated normal:

$$\mathrm{CN}(\nu, \sigma_1, \sigma_2) = (1-\nu)\mathrm{N}(0, \sigma_1^2) + \nu \mathrm{G}(0, \sigma_2^2) \tag{2.10}$$

where $\sigma_2^2 >> \sigma_1^2$ and ν is the (small) fraction of IOs. G is an arbitrary distribution and may be eg $\mathrm{N}(0, \sigma_2^2)$.

B. Time series generated by the mixture models represent chance contamination. Each observation Y_t comes with high probability, 1–ε, from the normal distribution N(.,.) but with small probability from the contaminating distribution H(.,.). Thus we can write

$Y \sim (1-\varepsilon)N + \varepsilon H$, with $0 < \varepsilon < 1$

Thus, a mixture distribution is a weighted average of two distributions, with weights (1–ε) and ε. For large n, roughly the proportion ε of the observations $Y_1,\ldots,Y_n$ will be contaminants (usually outliers). The frequency distribution of Y is then a heavy-tailed distribution. Chen (1979) examined several real data sets and found that the contaminated normal distributions 'appear to be a strong candidate for representing real world situations' (op cit p. 15 and 145).

3 EACF method

3.1 Pattern identification methods

For identifying an ARIMA process, many procedures are available in the literature. Choi (1992) classifies the identification methods into five groups[5]. The extended autocorrelation function (EACF) procedure of Tsay and Tiao (1984) belongs to the pattern identification methods. Other methods of this group are the R- and S-array methods, the corner method, the three generalised partial autocorrelations (GPAC), and the smallest canonical correlation method (SCAN), as well as some others (Choi 1992). Common to these procedures is that they utilise the extended Yule-Walker equations. One advantage of pattern recognition methods is that model fitting is not required at the model identification stage.

The Box-Jenkins approach is useful for pure stationary AR(p) and invertible MA(q) processes, while for mixed ARMA(p, q; q>0) processes these sample autocorrelations and partial autocorrelations are generally more complicated and hard to interpret. The reason is that their correlograms tail off to infinity rather than cut off at a particular lag (eg Tiao 1985, p. 98; Brockwell and Davis 1991, p. 296 and Gómez and Maravall 2001, p. 173). Also the sample variance confuses the interpretation, especially when a unit root is expected to occur in the time series (eg Box and Jenkins 1970, 1976, Section 6.2; Mills 1991, Sections 5.5 and 6.3).

Since the mixed ARIMA processes are quite common in econometrics and in some engineering sciences (eg quality control systems), there have been many proposals on methods of identifying these models. As an appropriate identification method for mixed ARIMA models, Gómez and Maravall (2001) mention the EACF and the SCAN methods of Tsay and Tiao (1985). Also other pattern identification methods have been proposed for mixed ARIMA models (Tiao 2001).

[5] The groups are: 1) autocorrelation methods, 2) penalty function methods, 3) innovation regression methods, 4) pattern identification methods, and 5) hypothesis testing methods.

3.2 Standard EACF method

3.2.1 Introduction

The aim of Tsay and Tiao (1984) was to develop an identification tool for ARIMA processes, particularly for two reasons:

1. to overcome most of the problems encountered in identifying mixed ARIMA(p, d, q; q>0) processes
2. to handle the direct identification of nonstationary mixed ARIMA(p, d, q; and q>0) processes.

From (2.5) we have

$$\phi(B)(1-B)^d Y_t = C + \theta(B)a_t \quad t = 1,...,n \tag{3.1}$$

where the zeros of the autoregressive polynomial, $\phi(B)$, are $p^* = p-d$, all lying outside the unit circle. The model (3.1) is the usual ARIMA(p^*, d, q) nonstationary process, where $(1-B)^d = U(B)$. In (3.1) we can also have a pair of complex roots on the unit circle. These rare, exceptional cases are not considered in this thesis.

The ESACF (sample EACF) method consists of two main steps:

1. find consistent estimates of the autoregressive parameters in order to transform Z_t into a moving average process
2. make use of the 'cutting-off' property of the autocorrelation function of this transformed series (which is a residual series after AR fitting) for model identification via ESACF table.

With the ESACF method, $\{a_t\}$ need not be a Gaussian process. It is required that the a_t's are independently and identically distributed continuous random variables with finite fourth moments (Tsay and Tiao 1984, p. 95).

3.2.2 Iterative, consistent OLS autoregression

Tiao and Tsay (1983a, 1984, hereafter referred to as TT83 and TT84) developed the EACF procedure. The basic idea of the EACF method is to identify, via an autoregression fitting, an AR(p) and an MA(q) part from a time series and hence to obtain the orders p and q for the ARMA(p, q) model. The AR regression parameters are estimated by

OLS. Because the directly calculated OLS estimates are not consistent for ARMA(p, q) with $\phi(B) \neq 1$ and q>0, (see Tiao and Tsay 1981 and TT83), TT84 proposed (in Section 2) the following iterated regression approach.

Assume a realisation of n observations from an ARMA(p, q) process

$$Z_t = \sum_{l=1}^{p} \Phi_l Z_{t-l} - \sum_{j=1}^{q} \theta_j a_{t-j} + a_t \tag{3.2}$$

The aim is to obtain consistent least squares estimates of the AR parameters Φ_l. In the case of the ordinary AR(p) fitting the data the OLS estimates, $\hat{\Phi}^{(0)}_{l(p)}$, are inconsistent (ie there is an MA(q) with q>0). The estimated residuals

$$\hat{e}^{(0)}_{p,t} = Z_t - \sum_{l=1}^{p} \hat{\Phi}^{(0)}_{l(p)} Z_{t-l} \tag{3.3}$$

are not white noise (even for large n) and the lagged values $\hat{e}^{(0)}_{p,t-j}$, j>0, will contain some information about the series Z_t. Therefore we define the first iterated AR(p) regression

$$Z_t = \sum_{l=1}^{p} \Phi^{(1)}_{l(p)} Z_{t-l} + \beta^{(1)}_{l(p)} \hat{e}^{(0)}_{p,t-1} + e^{(1)}_{p,t}, \quad t = p+2,\ldots,n \tag{3.4}$$

where the superscript (1) indicates the first iterated regression and $e^{(1)}_{p,t}$ denotes the corresponding error term. It is shown that the OLS estimates $\hat{\Phi}^{(1)}_{l(p)}$ of this regression are consistent, that is, $\hat{\Phi}^{(1)}_{l(p)} \to \Phi_l$ in probability as $l = 1,\ldots,p$ if $q \leq 1$ or $\phi(B) = 1$.

Similarly, if the $\hat{\Phi}^{(1)}_{l(p)}$ are inconsistent, the lagged values of the estimated residuals $\hat{e}^{(1)}_{p,t}$ of (3.4), where

$$\hat{e}^{(1)}_{p,t} = Z_t - \sum_{l=1}^{p} \Phi^{(1)}_{l(p)} Z_{t-l} + \beta^{(1)}_{l(p)} \hat{e}^{(0)}_{p,t-1}$$

also retain some useful information about the process Z_t. The second iterated AR(p) regression is then defined as

$$Z_t = \sum_{l=1}^{p} \Phi_{l(p)}^{(2)} Z_{t-l} + \beta_{1(p)}^{(2)} \hat{e}_{p,t-1}^{(1)} + \beta_{2(p)}^{(2)} \hat{e}_{p,t-2}^{(0)} + e_{p,t}^{(2)}, \quad t = p+3,\dots,n \qquad (3.5)$$

and it can be shown that $\hat{\Phi}_{l(p)}^{(2)} \to \Phi_l$ in probability as l = 1,…,p if q≤2 or $\phi(B) = 1$, and so on. The jth iterated AR(k) regression of a time series Z is defined as

$$Z = \sum_{l=1}^{k} \Phi_{l(k)}^{(j)} Z_{t-l} + \sum_{i=1}^{j} \beta_{i(k)}^{(j)} \hat{e}_{k,t-i}^{(j-i)} + e_{k,t}^{(j)}, \qquad (3.6)$$
$$t = k + j + 1,\dots,n,\ j = 0,\dots; k = 1,2,\dots$$

where

$$\hat{e}_{k,t}^{(i)} = Z_t - \sum_{l=1}^{k} \hat{\Phi}_{l(k)}^{(i)} Z_{t-l} - \sum_{h=1}^{i} \hat{\beta}_{h(k)}^{(i)} \hat{e}_{k,t-h}^{(i-h)}$$

is the estimated residual of the ith iterated AR(k) regression and the $\hat{\Phi}_{l(k)}^{(i)}$ and $\hat{\beta}_{h(k)}^{(i)}$ are the corresponding OLS estimates. In practice, the iterated estimates $\hat{\Phi}_{l(k)}^{(j)}$ satisfy the recursion

$$\hat{\Phi}_{l(k)}^{(j)} = \hat{\Phi}_{l(k+1)}^{(j-1)} - \frac{\hat{\Phi}_{l-1(k)}^{(j-1)} \hat{\Phi}_{k+1(k+1)}^{(j-1)}}{\hat{\Phi}_{k(k)}^{(j-1)}} \qquad (3.7)$$

where $\hat{\Phi}_{0(k)}^{(j-1)} = -1$; l = 1,…,k; k≥1; j≥1.

The implication of (3.7) is that the AR estimates of the jth iterated AR(k) regression can be recursively computed from the OLS estimates of stepwise AR(k), AR(k+1), …, AR(k+j) fittings.

Based on some consistency results for OLS estimates of autoregressive parameters for nonstationary and stationary ARMA(p, q) models, TT83 (see below) show that for k = p

$\hat{\Phi}^{(j)}(p) \to$ in probability $\Phi(p)$, when j≥q

where $\hat{\Phi}^{(j)}(p) = (\hat{\Phi}_{1(p)}^{(j)},\dots,\hat{\Phi}_{p(p)}^{(j)})$, (see Tiao 1985, p. 99).

For stationary ARMA models, (3.7) is the same as the generalised Durbin-Levinson algorithm (Piccolo and Tunnicliffe-Wilson 1984; Choi 1992; Brockwell and Davis 1991).

The theoretical consistency properties of the iterated AR estimates (3.7) are different for stationary and nonstationary processes. For the stationary case, the proof is found in Gersch (1970) and in TT84 (Theorem 5.1 and Lemmas 5.5 and 5.6). For the nonstationary process, the proof of consistency of iterated AR estimates is given in TT83 (Theorems 3.2 and 4.1, Corollary 2.6). The consistency properties of stationary and nonstationary processes are summarised in the following two theorems (TT84, p. 86).

Theorem 2.1

Suppose that Z_t follows the nonstationary ARIMA(0, d, q) model in (3.1). Then

$$\hat{\Phi}^{(j)}_{l(d)} = U_l + O_p(n^{-1}), \quad l = 1,\dots,d; \ j \geq 0 \tag{3.8}$$ [6]

The proof of (3.8) is presented in TT84, p. 91–92.

The practical implication of Theorem 2.1 is that for the nonstationary ARMA process $\{Z_t\}$ in (3.1), the OLS estimates $\hat{\Phi}^{(j)}_{l(d)}$ of any jth iterated AR(d) regression are superconsistent for the nonstationary AR coefficients U_l with the fast convergence rate $O_p(n^{-1})$. We are thus able to specify the nonstationary factor in practical modelling.

Theorem 2.2

Suppose that Z_t is an ARIMA(p, d, q) process, stationary or not. Then

$$\hat{\Phi}^{(j)}_{l(k)} = \Phi_l + O_p(n^{-½}), \quad l = 1,\dots,k \tag{3.9}$$

[6] Different rates of convergence of random variables are described in terms of *order in probability*. We say h_n is *at most of order* k_n and write $h_n = O_p(k_n)$ if there exists a real number M such that $k_n^{-1}|h_n| \leq M$ for all n; we say h_n is *of smaller order* than k_n and write $h_n = o_p(k_n)$ if $\lim_{n\to\infty} \frac{h_n}{k_n} = 0$. Here $\{h_n\}_{n=1}^{\infty}$ is a sequence of real numbers and $\{k_n\}_{n=1}^{\infty}$ is a sequence of positive real numbers (see eg Fuller 1996).

if $k \geq p$ and $j = q$, or $k = p$ and $j > q$, where it is understood that $\Phi_l = 0$ for $l > p$. Also, $O_p(n^{-1/2})$ becomes $O_p(n^{-1})$ if $\phi(B) = 1$ and $k = p$. The proof of Theorem 2.2 is given in TT84, p. 91–92.

It is important to note that theorem 2.2 covers both nonstationary and stationary ARMA processes. Note that in the nonstationary case these iterated AR estimates can be computed via the Durbin-Levinson algorithm (see also Choi 1992, p. 24–25 and 126).

These estimated AR coefficients are then used to define the extended sample autocorrelation coefficients, $r_{j(k)}$ ($p = 1,...,p_0$ and $j = 1,...,q_0$), from which the orders p and q of the ARIMA model are determined.

This procedure provides *consistent iterated* OLS estimates and hence ESACF coefficients, $r_{j(k)}$, for both stationary and nonstationary ARIMA processes (TT83).

This is the lag j sample autocorrelation of the transformed (AR filtered) series

$$W_{k,t}^{(j)} = Z_t - \sum_{l=1}^{k} \hat{\phi}_{l(k)}^{(j)} Z_{t-1} \tag{3.10}$$

For any finite k, the kth ESACF coefficient is defined as

$$r_{j(k)} = r_j(W_{k,t}^{j}) \tag{3.11}$$

From (3.10) and (3.11) we see that each $r_{j(k)}$ is a function of its own $W_{k,t}^{(j)}$. So each $r_{j(k)}$ is derived from a different set of estimates of the autoregressive parameters. These extended autocorrelation estimates are arranged in a two-way table from which the orders p and q can be determined at the position of the vertex[7] (see Table A and B below). Note that in the case of an $I(d;\ d = 0, 1)$ time series, a corresponding $W_{k,t}^{(j)}$ series is always an $I(0)$ series containing isolated and/or patchy outliers. The first row of the ESACF table ($k = 0$) displays the ordinary sample autocorrelations of Z_t. We have marked this first row by a shaded area or dotted line in the ESACF tables and different figures of this thesis. As TT84 (p. 86) stresses, the ESACF (beginning

[7] A vertex is mathematically defined as a point at which two or more lines or line segments meet on the boundary of a geometric figure (the edges of a polygon or polyhedron, etc.).

at the second row, k = 1, 2,…) is not the SACF of any transformed series of Z_t. This can be seen from (3.10) and (3.11).

Table A. **Original ESACF two-way table**

Autoregressive order	Moving average order				
	0	1	2	3	…
0	$r_{1(0)}$	$r_{2(0)}$	$r_{3(0)}$	$r_{4(0)}$	…
1	$r_{1(1)}$	$r_{2(1)}$	$r_{3(1)}$	$r_{4(1)}$	…
2	$r_{1(2)}$	$r_{2(2)}$	$r_{3(2)}$	$r_{4(2)}$	…
3	$r_{1(3)}$	$r_{2(3)}$	$r_{3(3)}$	$r_{4(3)}$	
…					

Iteration guarantees consistency and in each iteration round there is a check to see whether the new transformed series, $W_{k,t}^{(j)}$, is a white noise process. When it is, the order of MA(q) can be determined from the ESACF table.

3.2.3 Asymptotic EACF coefficients

TT84 showed that for a stationary ARMA(p, q) model, with $k \geq p$

$$\begin{aligned} r_{j(k)} &= c(k-p, j-q), \quad 0 \leq j-q \leq k-p \\ &= 0, \quad\quad\quad\quad\quad\;\; j-q > k-p \leq 0 \end{aligned} \tag{3.12}$$

TT84 proved that, for a general ARMA(p, q) process (3.1), stationary or nonstationary, and $k > p$, the ESACF coefficients have the following asymptotic property:

$$\begin{aligned} r_{j(k)} &\approx c(k-p, j-q), \quad 0 \leq j-q \leq k-p \\ &\approx 0, \quad\quad\quad\quad\quad\;\; j-q > k-p \leq 0 \end{aligned} \tag{3.13}$$

(here ≈ means convergence in probability).

Here c(k–p, j–q) is some nonzero constant or continuous random variable bounded in the interval [–1, 1] (see also Tiao 1985, formulas 2.27 and 2.54, and Tiao 2001, formulas 3.41 and 3.55, p. 63 and 67). It is important to note that in practice the asymptotic property of (3.12) and nonstationary model of (3.13) are not common in general (Tiao 1985, p. 98 and Tiao 2001, p. 65). One distinction will be in the distributional properties of estimates, as is known from the theory of stationary and nonstationary linear processes (see eg Tsay 1984, p. 123 and references therein).

As is shown by (3.12) and (3.13), the vertex is the crucial tool of the ESACF procedure in searching for the 'cutting-off' position in the two-way table. Thus the ESACF procedure possesses the 'cutting-off' property for mixed ARMA(p, q) models. Tiao (2001, p. 61) remarks that the 'cutting-off' property of the ordinary ACF function holds with respect to pure MA(q) models and that of the PACF function with respect to pure AR(p) models and that they do not hold for mixed ARMA(p, q) models. This 'cutting-off' property is proved by TT83 (Theorem 3.1, and Lemma 2.5) and by TT84 (Lemma 6.1).

Our simulation results show that in the case of a given ARMA(p, q) the ESACF coefficient estimates $r_{j(k)}$ outside the triangle of asymptotic 'zero values' usually have a very non-normal sample distribution (often bimodal, non-symmetric); these anomalies occur regularly in the lower-left part of the ESACF table. This feature can be seen, for instance, from the results of our example model in the Appendix 1.

In practice, the values of both p and q may be unknown, and complications arise when both the fitted AR order, k, is greater than the true AR order, p, and simultaneously the number of iterations j is greater than the unknown true MA order, q. This is a so-called *overfitting* problem. In ESACF calculations, this overfitting problem can be handled as a part of the estimation procedure and does not require any special actions or extraordinary conditions (TT84, p. 87 and Theorem 6.1; Tiao 2001, p. 67). An overfit of the AR(p), order k–p > 0, will generally lead to an increase in the order of the MA polynomial of the transformed series, $W_{k,t}^{(j)}$, and the number of additional terms is given by $\min\{k-p, j-q\}$.

Table B. **The simplified, asymptotic ESACF two-way table**

Autoregressive order	Moving average order				
	0	1	2	3	…
0	*	X	X	X	…
1	*	**0**	0	0	…
2	*	X	0	0	…
3	*	0	0	0	…
…		………………………………………..			

Since each individual element of the ESACF table below the first row is a sample autocorrelation of a transformed series, its sampling properties can be obtained from the known large-sample results of the SACF function. The approximate variance of $r_{j(k)}$ can be calculated using Bartlett's (1946) formula. Tsay and Tiao used the simple variance approximation of $(n–k–j)^{-1}$ under the hypothesis that the transformed series $W_{k,t}^{(j)}$ is white noise. This estimate of variance is utilised to calculate the confidence intervals for $r_{j(k)}$ for determining the X and 0 indicator symbols in ESACF table. These indicator symbols are used so that X refers to values greater than ±2 standard errors of $r_{j(k)}$ and 0 for values within ±2 standard errors. The sign * (denoting an arbitrary value between –1 and 1) means there are no pattern terms in the asymptotic and theoretical ESACF table of a certain order (p, q) (see eg TT84, Table 2 and Tsay 2002, Table 2.4). In practice, we calculate estimates over the whole matrix and then form the simplified X, 0 matrix, ie * may be X or 0.

3.2.4 Vertex of EACF table

The large-sample property (3.12) and (3.13) is used in ESACF identification. In the two-way table the rows are numbered 0, 1, 2, ... to indicate AR order and the columns in a similar way for MA order. We determine the values of p and q by searching for the upper left vertex of a triangle of asymptotic 'zero' values of the ESACF table. These 'zero' values have the boundary lines $k = c_1 \geq 0$ and $l - k = c_2 \geq 0$, where k refers to the kth extended sample autocorrelation and l refers to the lag of the kth extended sample autocorrelation. In general, we tentatively identify $p = c_1$ and $q = c_2$ (see Tiao 1985). The order of the model is given by the row and column coordinates of the vertex. So the vertex determines the ARMA(p, q) 'cutting-off' position in the EACF table (Tiao 2001, p. 61). The triangular cutting-off characteristic of the ESACF table may in practice become rectangular or trapezoidal shaped (TT84, p. 95; see eg Tiao 2001, p. 76 and 85).

In Table B (above) the vertex of an ARMA(1, 1) process is highlighted in bold. The theoretical vertex for some ARIMA processes is displayed in Appendix 1. As our null hypothesis for the 'vertex system' in the ESACF table, we are looking for a statistically significant zero value of a parameter estimate instead of a parameter estimate significantly different from zero, as in the usual null hypothesis.

3.2.5 EACF and I(d) nonstationary processes

The consistency properties of iterated AR estimates are derived separately for the stationary and nonstationary cases. In the asymptotic sample, the ACFs of nonstationary models are dominated by those nonstationary roots with the highest multiplicity, and thus the estimated generalised Yule-Walker equations cannot always provide consistent estimates of AR coefficients (Quinn 1980 and TT83, Corollary 2.6). Thus we need different solutions and conditions for iterated consistent AR estimates, as is shown by TT83 (Theorems 3.2 and 4.1) and TT84. In general, iterated AR(d) estimates provide the nonstationary part U(B) of (3.1), by the Theorems 2.1 and 5.1 and Lemma 5.7 of TT84. In model (3.1) we have a nonstationary case if $U(B) \neq 1$.

To study whether a series is nonstationary, TT84 suggest that for given specified values of p and q, the iterated AR estimates can be examined to see whether the AR polymial contains a nonstationary factor, ie a root on the unit circle[8]. This can also be found in Tsay (1985, Sections 2.4 and 3). Thus, in table B, the vertex of a triangle of the asymptotic 'zero' values provides information on the *maximum* orders of polynomials, ie in the form of an ARMA(p* + d, q).

This means that for nonstationary processes we are able directly to estimate the ESACF table without first specifying the order of differencing. Then we obtain the 'maximum' value of $p = p^* + d$, where d is the number of unit roots. Once p and q are determined we can analyse p* (= stationary AR order) and the nonstationary factor, d. In our simulation experiments we verified (in ESACF tables) this $p = p^* + d$ property for ARIMA(1, 1, 1) processes. Note that when we avoid differencing we are also precluding problems associated with overdifferencing (Lee and Park 1988; Tsay 1985, p. 236). The avoidance of differencing a time series is particularly important with AO outliers, as we see later in this thesis.

3.2.6 EACF in the literature

The structure of the ESACF procedure is illustrated in de Gooijer et al (1985), Tiao (1985, 2001), Tsay (1986), Kendall and Ord (1990), Wei

[8] A nonstationary time series can be transformed by the differencing operator (1–B). This is similar to the AR operator $(1-\phi B)$, with $\phi = 1$. We refer to our robust analysis later in this thesis.

(1990, 1994) and Pankratz (1991). The ESACF procedure can be used with both pure time series models and traditional regression models to model residual series (eg Tsay 1984, 1985). Kendall and Ord consider the ESACF procedure in tackling the problem of whether or not to difference a series in identifying a time series model. Tiao and Tsay (1983b) developed the multivariate version of this method, ie the extended sample cross-correlations (ESCC), which has been available for vector time series identification in the statistical software package SCA[9].

Two versions of the ESACF procedure have been developed, which are based on a vector autocorrelation function[10]. Jeon and Park (1986) developed a simple version and applied it to the example series of Tsay and Tiao (1984). Another more advanced version is developed in Paparoditis and Streitberg (1992) and Paparoditis (1993). Lee and Park (1988) applied the ESACF pattern in time series modelling of a decision support system for management planning. In the engineering literature, this iterative ESACF approach is known to be useful in ARIMA modelling (eg Li & Dickinson 1988).

Interest in the ESACF procedure has recently increased. Oliveira and Müller (2000) developed a generalised version of EACF to contribute to the identification of transfer function models. Mélard and Pasteels (2000), in their automatic ARIMA modelling program, incorporate an identification procedure related to the ESACF method. Hella (2002) gives a robust ESACF procedure and studies it via simulations including sample distributions of single ESACF coefficients for both the standard and robust cases.

The EACF procedure has also been criticised in the literature. According to Mareschal and Mélard (1988) the corner method is faster and probably more accurate than ESACF. They also criticise the ESACF for requiring many passes over the data and providing very crude statistical limits. Wei (1990, 1994) notes that, with few exceptions, the task of identifying nonstationarity via the ESACF method is generally difficult. Wei also mentions that the real advantage is for the identification of p and q of mixed ARMA models and that the ESACF can be used much more with properly transformed stationary series. Koreisha and Yoshimoto (1991) conducted simulation experiments with different methods including the EACF procedure and report its fairly poor performance. They

[9] The standard ESACF procedure is included in at least in the following statistical software packages: SCA (Scientific Computing Associates, user manual: Liu and Hudak 1992), Autobox, and SAS (see Yaffee and McGee 2000).

[10] This function is related to the canonical correlation function.

consider the ‘simple and crude’ variance estimate $(n–k–j)^{-1}$ of the ESACF coefficients. Koreisha and Yoshimoto tried to improve the performance of the method, experimenting with wider confidence intervals, of ±3 standard errors. They obtained results where the number of the correctly identified trials increased markedly.

4 Outliers in time series modelling

In this chapter we analyse outliers in a time series context, taking up the main reasons for the occurrence of outliers and the available ways of handling them. Then we consider outliers in the context of building ARIMA models and identify three groups of problems encountered in practice.

4.1 Treatment of outliers

Outliers can result for many external or internal reasons. Measurement (recording or typing) errors, classification mistakes in sampling or some non-repetitive exogenous interventions can have effects in the form of outliers, isolated or patchy. Economic and business time series are sometimes subject to the influence of strikes, outbreaks of wars, sudden change in the market structure of some group of commodities, technical change or new equipment in a communication system, or simply unexpected pronounced changes in weather etc. The classic reason for an AO is a typing or measurement error at the level of the Tth observation. An IO is typically caused by some external shock at time T that influences observations x_T, x_{T+1},… via the memory (ie ratio of lag-polynomials, the transfer function) of the underlying core model of the time series.

In modelling and analysing time series the researcher must decide how to handle potential and known outliers. There are three ways to deal with outlying observations:

a) deleting
b) accommodation (robust estimation of model)
c) detection, modelling and interpretation.

Earlier, outliers were usually thrown out, but nowadays this is not usually recommended. In careful modelling, the outlying observations are replaced by some robust estimates. If the identification and estimation of isolated or patchy outliers is not necessary for analysis and the goal is to construct the core model for the bulk of observations, the accommodation approach is relevant. This means that robust estimation methods (case b) are used in the modelling process: ie in the identification, estimation and diagnostic checking phases. Use of robust methods provides (optimal) protection against the damaging effects of outliers. It is important to know that *one* remarkable outlying observation can ruin OLS estimates (see the breakdown point concept in Chapter 1). We can, of course, use robust methods for detection and estimation of outliers. If our goal is to obtain a useful, high-quality forecasting or explanatory model, c) is the relevant approach. This approach is especially important in the linear regression context (eg Donald & Maddala 1993, Lucas 1996, Section 2.1.2). In practice it will often be relevant to use a combination of b) and c). When robust techniques are used the profile of outliers may become even more obvious (sharper) because robust regression estimation provides less biased parameter estimates and thus leads to residuals that enhance the visibility of possible outliers (see eg Kleiner et al 1979, Martin 1980, Section 5 and Levenbach 1982)[11].

4.2 Masking, swamping and smearing

It is well known that in the regression context, especially multivariate regression, the identification and detection of outliers is troublesome. Various test and estimation procedures have been developed in the literature (see eg Barnett & Lewis 1994, Chatterjee and Hadi 1986, Rousseeuw and van Zomeren 1990). In time series we encounter more difficulties due to the serial correlation between adjacent observations. In addition, there are various types of outliers with different effects on observations. In both frameworks, testing has been developed mainly for isolated outliers, whereas patches of outliers still cause severe problems. The influences of outliers include masking, swamping and smearing effects. We do not consider here sets of influential observations containing both outliers and normal observations (eg Barnett & Lewis 1994, Penã 2001).

Masking

Most severe are the masking effects. Masking means that an outlier covers, ie masks, the effects of one or more other outliers, so that the statistical estimation method and outlier test procedure fails to detect

[11] Chang (1982) presents a list of alternative approaches which an analyst can use to handle problems of outliers: 1) graphical methods 2) tests of hypotheses 3) premium-protection approach 4) robust estimators and 5) Bayesian approach.

any outlier. Usually this happens in OLS regression, because the parameter estimates, as well as the residuals are biased. Masking effects are especially severe for patchy outliers, which are quite common in time series. Robust estimation, diagnostic tools, and robust outlier testing (use of robust distance measures) provide more efficient results (see eg Rousseeuw & Leroy 1987, p. 81–84 and 282). Bruce & Martin (1989) proposed 'leave-k-out' diagnostics to deal with outlier patches and an iterative deletion procedure for the effects of masking. The problem remains how to specify the value of k.

Swamping

Swamping is the converse of masking and is also often difficult to detect. A group of true outliers may cause a good observation or group of good observations to be erroneously specified (tested) as an outlier or patch of outliers. In the literature various suggestions have been made to quantify and test for masking and swamping effects (see Bartlett & Lewis 1994). Also in the regression context, masking and swamping may be potentially difficult problems (eg Ryan 1997). The methods for treating isolated outliers are not valid for outlier patches, with either masking or swamping, and the development of procedures specifically for patches is still in its infancy.

Smearing

The effects of smearing are encountered in time series modelling, not in the i.i.d. framework. The smearing effect is the influence of an outlier on adjacent observations due to the serial correlational structure of a time series. Thus the strength of the different smearing effects will depend on the type of outlier and the dynamics of the underlying time series model (order of lag polynomials) and the absolute values of its parameter coefficients (Bruce & Martin 1989). Bruce & Martin (1989) presented quantitative results of smearing effects for the AR(1) model with AO and IO outliers.

When we use robust identification and estimation methods in modelling (accommodation principle discussed above) and carry out robust tests, we can avoid the majority of masking and swamping problems. The smearing effects can also be evaluated more accurately. We should, however, carefully choose the proper robust tools, separately in each modelling case.

4.3 ARIMA modelling and outliers

Assume a univariate ARIMA process of the form (2.5):

$$\Phi(B)Y_t = C + \theta(B)a_t \quad t = 1,...,n \tag{4.1}$$

For ARIMA modelling, Box and Jenkins (1970, 1976) have proposed a model building strategy based on three main stages:

1. tentative specification or identification of a model
2. estimation of model parameters
3. diagnostic checking of fitted model for further improvement.

Model building is in practice an iterative process and these phases may have to be repeated many times.

The original Box-Jenkins univariate ARIMA modelling has been criticised for a lack of robustness (eg Durbin 1979, Funke 1992). In recent years several articles, academic dissertations and research papers have been published on robustifying ARIMA models. In the following we briefly present notes and comments on the state of robust ARIMA modelling, ie model identification, parameter estimation and residual diagnostic checking.

Model identification

Surprisingly, only a few research papers on robust identification procedure have been presented in the literature. Martin and Yohai (1986, p. 849) call model identification 'a thorny issue' in robust time series analysis. In the 1990s activity in this area picked up. Several versions of robust autocorrelation functions have been published in the literature, eg Masarotto (1987a), Polasek and Mertl (1990), Chan (1989, 1992), Chen (1994), Chan and Wei (1992) and Wang and Wei (1993). As for other classic tools of identification, such as partial autocorrelation and inverse autocorrelation functions, only a small number of robust studies have been published to date (eg Chan 1989 and Chen 1994). The situation is quite similar in the case of estimators that serve as robust model selection criteria (see Martin 1980, Ronchetti 1997). In recent years, activity has clearly increased in the area of robustifying autocovariance and canonical correlation (eg Ma and Genton 2000).

Parameter estimation

As in the regression context and partly dependent on it, robustifying focuses on the estimation phase also in time series modelling. The most well known robust regression estimators are the M-, GM-, S-, RA- and MM-estimators (eg Martin 1979, 1980, 1981, Martin and Yohai 1985 and 1986, Franke et al 1984, Yohai 1987, Maddala and Yin 1997). High breakdown parameter (HBP) estimators have been used more with econometric than with time series models (eg Lucas 1996). In time series modelling, the methodological research has traditionally focused more on parameter estimation and diagnostic methods than on identification tools and algorithms.

Diagnostic checking

The robustified autocorrelation, partial autocorrelation and cross-correlation functions and robust location and scale estimators of residual series can be used also in model diagnostics. In addition, some applications have been done with robust versions of the portmanteau test statistic (eg Li 1988, Li & Hui 1994). A new approach for testing goodness of fit is presented in Gerlach et al (1999), in which testing model adequacy in a Bayesian framework is based on Monte Carlo simulation of Markov chains applied to an autoregressive model with outliers.

Huber (1991) has considered the relationship between diagnostics and robustness. As the purpose of robustness is to safeguard against deviations from assumptions, the purpose of diagnostics is to find and identify these deviations. As regards outlier robustness, a procedure should be insensitive to outliers; outlier detection/rejection is part of diagnostics, not robustness. Huber remarks that robustness and diagnostics are complementary and for important and large deviations we need robust diagnostics (see also this study, Section 4.1). However, as Lucas (1996, Section 2.5.1) notes, there may still be problems concerning robust model selection.

5 Robust EACF procedure

As mentioned above outliers are quite common in routine statistical data sets. Therefore, one should always carry out a robust exploratory analysis of the data, particularly if data quality is suspect. When the data are anticipated to contain outliers, it is important to use robust methods in the first step of data handling (see eg Rousseeuw and Leroy 1987, Donald and Maddala 1993, Maddala and Yin 1997, Lucas 1996). Thus if one is doing ARIMA modelling, the robust identification tool(s) should be used first. Thereafter the traditional tools can be used in a complementary fashion. Robust procedures are emphasised in the time series context because, in practice, one usually does not know in advance the type, number, relative position, magnitude and time configuration of outliers in time series data. Furthermore, as outliers are model dependent in the time series context, it is crucial to identify the 'best' candidate(s) for the model structure at the start of modelling. Using robust methods, one can also reduce the risk that the modelling will produce spurious outliers.

The standard ESACF seems to be robust to some degree. Tsay (1986, p. 139) remarked that the ESACF procedure may be robust to some degree if the number of outliers is small, the outliers are of moderate size and the sample size is relatively large[12]. Surprisingly, the literature does not contain any response or research experiments concerning Tsay's remark on robustness. On the contrary, Lee (1989) and Wei (1990, 1994) carried out only a single AO experiment of the classic extreme decimal point error for which the ESACF is naturally non-robust. For ARIMA models we usually assume a normal distribution for the $\{a_t\}$ series (Section 3.2). This assumption is not crucial for the ESACF approach based on the note of (TT84, p. 95): 'All that is needed is that the a_t's are independently and identically distributed continuous random variables with finite fourth moments'. This may be open to some interpretation of distributional robustness of the standard ESACF procedure.

In the well known iterative joint 'estimation-detection-correction-estimation' procedure (see eg Chen and Liu 1993, Tsay 1988, Chang

[12] Tsay carried out a small-scale simulation experiment for an ARMA(1, 1) model with a single AO outlier and n = 100 observations and 400 repetitions.

et al 1988)[13], identification begins *subjectively* with the OLS regression, and with the assumption that the orders p and q of an ARMA model are known. However, in practice, we usually do not know p and q beforehand. Thus we may obtain biased regression estimates in iterations and perhaps an incorrectly identified ARMA model and therefore possible spurious outliers.

5.1 Robustifying the EACF procedure

Due to the common occurrence of outliers, the robustified EACF procedure is a reasonable tool for identifying ARIMA models. If there are outliers in the data and we use standard ESACF, we obtain very biased OLS estimates of the autoregression coefficients also in recursion estimation. This in turn leads to biased estimates of the transformed series $W_{k,t}^{(j)}$ and the ESACF coefficients $r_{j(k)}$ (see formulas 3.10 and 3.11). The literature contains examples of such destructive effects on ESACF table coefficients (eg Wei 1990, 1994 and Lee 1989). Thus the ordinary OLS method in autoregression and the sample ACF in calculating autocorrelations of the $W_{k,t}^{(j)}$ series must be replaced by their robust counterparts.

For some special cases, we did simulation experiments using OLS regression combined with robust ACF. The conditions for this combination are a low degree of contamination and outliers of small size in a time series. In simulations and for some real series, we used the combination of OLS and weighted ACF. The results are promising.

5.1.1 Iterative, consistent robust autoregression

For robustifying the OLS method we replace the minimising function in a straightforward way:

[13] This procedure is criticised in the literature (eg Lee 1990, p. 72, Tatum 1991, p. 36, Maddala and Yin 1997, p. 241), mainly because it starts with an ARMA model of the observed series as if no outliers were present in the data. Furthermore, this procedure estimates outliers only one by one in iterations. In practice, time series often contain patchy outliers. The widely used iterative programs TRAMO/SEATS and X-12-ARIMA also search for outliers and it is somewhat unclear how these programs handle patchy outliers. It seems that these procedures are most useful for low and medium contaminated series.

instead of minimising the sum of squares,

$$\sum_i \left(Y_i - \sum X_{ij}\phi_j\right)^2 \tag{5.1}$$

we minimise the sum of a weighting function of the residuals,

$$\sum_i \rho\left(Y_i - \sum X_{ij}\phi_j\right) \tag{5.2}$$

(see Huber 1981, p. 156 and 162).

In robust regression theory many different weight functions, ρ(.), are used. These functions must fulfil certain mathematical conditions: the function ρ(.) is assumed to be convex, nonmonotone, and to possess bounded derivatives of sufficiently high order (approximately four). Particularly the first derivative, $\psi(u) = \frac{d\rho(u)}{du}$, should be continuous and bounded. To make the function $\psi(u)$ scale invariant, a robust scale estimator, σ, is introduced into $\psi(u)$, as seen in formulas (5.4) and (5.5) of this thesis (eg Huber 1981, Martin and Yohai 1985 and 1986, Hampel et al 1986, Rousseeuw & Leroy 1987).

For robustifying the ESACF procedure we replace (5.1) by (5.2) in the regression of (3.4)–(3.7) and continue the procedure according with this 'accommodation approach' of robust regression analysis (eg Barnett and Lewis 1994). The robust regression estimators selected for this thesis are presented with detail in Section 5.2 and in Appendix 7. In our robust ESACF procedure, we use the 'double' iterative approach: iteratively calculated robust regression estimator inside the iterative autoregression.

5.1.2 Robust autocorrelation function

The basic reason for robustifying the classic ACF function is mentioned in Section 5.1. As is known, both classic estimators OLS and ACF have the breakdown point of zero. In the presence of outliers, the residual (transformed) series, $W_{k,t}^{(j)}$, contains estimates of these possible isolated and/or patchy outliers. Using robust ACF, one safeguards against these aberrant observations, obtaining ESACF estimates, $r_{j(k)}$, as unbiased as possible. Using robust autoregression, we try to safeguard against outliers as best as we can (here we are not interested in observing them) and then use robust ACF to try to provide the core information of the series $W_{k,t}^{(j)}$ for identification of the MA(q) part of an ARIMA process. We use three different kinds of robust versions: weighted, trimmed and rank-based ACF. These robust versions are introduced in Section 5.2.

5.1.3 Vertex of the robust ESACF table

In the robustified ESACF table the vertex is based on the robust autoregression and autocorrelation estimators. Both of these estimators have the established state in robust statistical theory. Hence the vertex can be expected to show quite reliably the cutoff point also for a nonstationary process. It is important to note that the first row of the ESACF table displays the robust version of the ordinary sample autocorrelation function. Thus we have available both robust SACF and ESACF estimates. In some of our simulation results we found a polarisation feature around the vertex, especially concerning the $r_{j(k)}$ preceding the vertex point in the same row. For the unit root analysis, this feature is promising also when the HBP regression estimator, MM, is available for the autoregression.

5.2 Designing the robust ESACF procedure

In robustifying the ESACF procedure we have two main goals: *first*, to robustify the iterative AR(p) regression estimation phase, and *second*, to implement a robust autocorrelation function for every iteration round. Our aim in this study is to use the three common regression estimators which are next in increasing degree of robustness: the M-, GM- and MM-estimator. In the following we first consider the robust autoregression estimation which has the strongest theoretical connections between traditional regression models and time series models and, second, three different types of robust autocorrelation functions.

In our ESACF system we are able to handle single generated or real time series or conduct Monte Carlo simulation experiments. We are able to use both standard and robust procedures with optional estimators with different confidence intervals for ESACF estimates. Figure 1 (p. 15) displays the flow chart for the proposed united ESACF system in the ‘Tsrob’ program (see also section 6.1 and a brief illustration in Appendix 7).

5.2.1 M- and GM-estimator

The M-estimator

Huber (1964, 1973) introduced the class of the Maximum Likelihood type (M) estimators in linear regression theory. For the linear regression model

$$y_i = x_i^{'}\beta + u_i \tag{5.3}$$

the minimising function for M estimator of β is

$$\sum \rho\left(\frac{y_i - x_i^{'}\beta}{\sigma}\right) \tag{5.4}$$

where $\rho(.)$ is a function defined on $\Re$ and σ^2 is the variance of u_i. For $\rho(u) = u^2$ we obtain the OLS estimator. The first order condition for the minimisation (5.4) is

$$\sum \psi\left(\frac{y_i - x^{'}\beta}{\sigma}\right)x_i^{'} = 0 \tag{5.5}$$

where $\psi(u) = \dfrac{d\rho(u)}{du}$.

The M-estimates are obtained via the iterative solution of (5.5).

Various functions for $\psi(.)$ have been developed in the literature (see eg Marazzi 1993 and Rousseeuw & Leroy 1987).

Huber (1981, Section 7.3) considers robustifying the least squares regression in a straightforward way. In autoregressive AR(p) models OLS may be replaced by the M-estimator, where $\rho(.)$ is a symmetric robustifying loss function and σ is a robust scale estimator of the innovations. In the AR(p) models the M-estimator is shown to be robust against IO outlier(s), but not if the data contain AO outlier(s). The M-estimator is not qualitatively robust; it has an unbounded influence function and an empirical breakdown point of zero.

Since the robustness of the M-estimator is not satisfactory, the generalised M-estimator (GM) (also first used in regression; see Rousseeuw and Leroy 1987) for autoregressive models has been proposed and studied in the literature (see Denby and Martin 1979, Martin 1979, 1980, 1983, and Masarotto 1987b). Bustos (1982) proved the consistency and asymptotic normality of the GM-estimator for contaminated p_{th} order autoregressive processes (see also the survey article of Martin and Yohai 1985). The basic idea of GM-estimator is to modify the minimisation problem so that the summands of the estimating equation (5.5) are bounded and continuous functions of the data. For example, in the AR(1) model

$$y_t = \phi y_{t-1} + u_t \tag{5.6}$$

the GM-estimate is obtained via iterative solution of

$$\sum \eta\left(\frac{y_t - \phi y_{t-1}}{\sigma}, y_{t-1}\right) y_{t-1} = 0 \tag{5.7}$$

where $\eta(.,.)$ is a robustifying weight function.Various versions of this weight function have been presented in the literature (eg Hampel et al 1986, Rousseeuw and Leroy 1987). The following types of weight functions are used most in practice: the Mallows and Schweppe types[14]. The GM-estimator is shown to be robust against AO outliers in AR(1) models (Denby and Martin 1979). Increasing the order p of an AR model reduces the performance of the GM estimator. As Rousseeuw and Leroy (1987) remark, the fitting of an AR(p) model in the presence of an isolated AO outlier yields one vertical outlier and p leverage points in time series similar to outliers in the regression context. It is shown that the breakdown point of the GM-estimators is at most 1/(p+1), where p is the number of regressors (order p in AR(p) model). Martin and Yohai (1985, section 4) consider briefly the cases where the GM-estimator is qualitatively robust in ARMA models.

[14] For an AR(1) process with AO outliers, Stockinger and Dutter (1987) recommend the Mallows type when $\phi_1 = 0.5$ and the Schweppe type when $\phi_1 = 0.8$. Schweppe type is important since then the GM estimator only downweights vertical outliers and bad leverage points and fully exploits the correct signal in good leverage points.

5.2.2 The MM-estimator

The robust MM-estimator of Yohai (1987) is developed for the regression context and belongs to the class of high breakdown point (HBP)[15] estimators. The MM-estimator is defined in the following procedure:

Assume a standard linear regression model

$$y_i = x_i'\beta + u_i, \quad i = 1,2,...,n \tag{5.8}$$

where $y_1, y_2, \ldots, y_n$ are response values and $x_1, x_2, \ldots, x_n$ are p-dimensional regressors, β is a p-dimensional vector of unknown parameters to be estimated, and $u_1, u_2, \ldots, u_n$ are i.i.d. random errors with mean zero and constant variance σ_0^2. The three stages in constructing the MM-estimator are

1. compute as an initial regression estimator, $\hat{\beta}_n$, the S-estimator (Rousseeuw and Yohai 1984), which is a consistent HBP estimator ($\varepsilon_n^* = 0.5$), and get the residuals $\hat{u}_i$
2. compute the M-scale estimate, $\hat{\sigma}_S$, of the residuals based on $\hat{\beta}_n$
3. find the MM-estimate $\tilde{\beta}_n$, defined as any solution of

$$\sum_{i=1}^{n} \rho_1'((y_i - x_i'\tilde{\beta}_n)/\hat{\sigma}_S)x_i' = 0 \tag{5.9}$$

with $S(\tilde{\beta}_n) \le S(\hat{\beta}_n)$, where $S(\beta) = \sum_{i=1}^{n} \rho_1((y_i - x_i'\beta)/\hat{\sigma}_S)$.

The estimate $\hat{\sigma}_S$ is obtained from the equation $1/n\sum_{i=1}^{n} \rho_0((y_i - x_i'\hat{\beta}_n)/\hat{\sigma}_S) = b$, where, for constant b, the ratio $b/\sup\rho_0(u) = 0.5$ (Yohai 1987, p. 644); ρ_0 is the same rho function as in the S-estimator and ρ_1 is another rho function.

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[15] Intuitively, the breakdown point measures the largest possible proportion of outliers in the data which an estimator can tolerate before its estimate collapses to a nonrelevant value.

These loss functions, ρ_0 and ρ_1, are assumed to satisfy the known regularity conditions (see below) and $\rho_1(u) \leq \rho_0(u)$ and $\sup \rho_1(u) = \sup \rho_0(u)$ for all $u \in \Re$ (see also Salibian-Barrera 2000, Section 4.1 and You 1999, Section 2).

The values of the tuning constants, $c_0 = 1.56$, $c_1 = 4.68$ and $b = 0.0833$, are the conditions that the breakdown point of the MM-estimator is 0.5 and that the MM-estimator is asymptotically 95% efficient in normal error case (see Yohai 1987, p. 648 and Yohai, Stahel and Zamar 1991, p. 367–369).

Regularity conditions for the loss function ρ(.)

The MM-estimator of Yohai (1987) contains a loss function $\rho:\Re\rightarrow\Re_+$ that satisfies the following regularity conditions (see Salibián-Barrera 2000, p. 141):

R.1 $\rho(-u) = \rho(u)$ for all $u \in \Re$, and $\rho(0) = 0$
R.2 ρ is continuously differentiable
R.3 $\sup_x \rho(x) = 1$
R.4 if $\rho(u) < 1$ and $0 \leq v < u$, then $\rho(v) < \rho(u)$.

The computing algorithm is a modified version of the IWLS procedure. With the known constant values, the MM-estimator has an asymptotic efficiency of 95% with respect to the maximum likelihood estimator, and the breakdown point is $\varepsilon^* = 0.5$ through the initial S-estimator (Yohai 1987, p. 644–648). The influence function of the MM-estimator is not bounded. Lucas (1996, p. 105) remarks that, with a small positive fraction of contamination, the bias of the MM-estimator standardised by this fraction is bounded for strictly positive amounts of contamination. Hence the inference is that this unboundedness is not a serious practical problem. Recently, Salibian-Barrera (2000) studied the MM-estimator (and the S-estimator) extensively in linear regression models and presented some illustrative examples of robust confidence intervals based on a bootstrap technique.

The MM-estimator has default settings of different tuning and control parameters and so it is suitable for modern routine use[16] (see eg Yohai, Stahel and Zamar 1991). Most of the default settings can be changed through proper functions. The MM-estimator has been successfully applied to macroeconomic time series (eg Lucas 1995a, 1996).

It is important to investigate the performance of the MM-estimator also in the robust design of the ESACF procedure. Although this estimator itself contains three stages of calculation and an AR(p) fitting in the ESACF procedure has its own rounds, a modern computer with a high quality R program can successfully run this challenging estimation process, as the results of our simulation experiments show.

5.2.3 OLS replaced by robust regression

In the AR(p) fitting of the ESACF procedure the OLS estimator can be replaced by the proper equations of the M-, GM- or MM-estimator. While OLS gives the explicit solution, we now have only iterative estimation solutions. We do not write these iteration equations here, but refer to Martin and Yohai (1985) and to our simulation program, 'Tsrob', which contains equations in which OLS estimators are replaced by robust regression estimators. The robust regression estimates are obtained by the iterative weighted least squares (IWLS) algorithm.

The M-, GM- and MM-estimator were developed in the i.i.d. regression context. In time series modelling they have been used with stationary time series. For instance, Masarotto (1987b) studied the GM-estimator for ARMA(0, 1) models quite successful by simulation with isolated and patchy outliers (IO and AO). His criteria for selection of a parameter estimation method were that it should be both consistent and insensitive to outliers in the data.

The nonstationary time series context is a largely open area for this kind of robust estimation in ARIMA time series. Martin (1980, p. 241) gave an example of an artificial sixth-order, near nonstationary AR process with OLS and GM-estimation results. Martin (1983, p. 198)

[16] As Kelly (1992) remarks the lack of proper standardisation of various robust regression procedures can still be problematic for users both in econometrics and statistics, although some progress has been made in recent years. It is remarkable that the robust regression techniques still are quite rarely used, for instance, in applied econometrics; many reasons can be found (see eg Zaman et al 2001).

refers to this example and notes that the GM-estimator is not '…guaranteed to correspond to stationary autoregressions. However, this has not proved to be a practical limitation in our applications,...'. Martin and Yohai (1986) and Künsch (1984) have considered nonstationarity problems in the context of outlier-robust time series modelling. As the discussion in Martin and Yohai (1986) shows, the effects and problems of nonstationarity depend on the type of contamination and structure of the ARMA model. Stockinger and Dutter (1987) also refer to the problems of nonstationarity in robust time series modelling and the need for further research. There exists a relationship between these problems and robust unit root methods. A good survey article on robust nonstationary analysis is Maddala and Yin (1997).

The estimation results of the standard ESACF has been compared with results for single time series published in the literature (eg ESACF estimates of series C of Box and Jenkins 1976, Tiao 1985 and the Canadian lynx pelt data of Wei 1990, 1994). Comparison of the standard and robust ESACF simulation results also shows that the iterative robust estimation of our 'Tsrob' program works well in the ESACF procedure, both in the stationary and nonstationary cases. As mentioned earlier, here the robust estimation in a nonstationary case is a largely open question theoretically. Lucas (1996, Chapter 5.5) used the MM-estimator in outlier-robust unit root analysis estimating the regression equation which contains a constant, an AR(8) autoregression part, and a trend component. Lucas remarks that the robust high breakdown estimation for ARMA models is, however, still a largely open area (op cit p. 118).

Choi (1992, p. 25) noticed that most of the identification methods for stationary ARMA processes (reviewed in his book) can be applied to nonstationary processes if the iterated least squares (ILS) estimates are used instead of the extended Yule-Walker estimates and maximum likelihood estimates. Personal communication with Choi (1997) inspired the author to apply robust estimation methods in place of the ILS method with the ESACF procedure[17].

[17] I would like to thank Professor Choi for his encouraging communication concerning this important methodological question.

5.2.4 Three robust alternatives of ACF

Outliers and ACF

There are a variety of studies in the literature which show, theoretically and via applications, how outliers destroy the estimates of autocorrelation and partial autocorrelation coefficients (see eg Chang 1982, Chang and Tiao 1983, Burn and Ord 1984, Deutsch, Richards and Swain 1990 and Chan 1995). One sizable AO alone can ruin the idenfication outcome of ARMA modelling[18]. The main damaging effects found in the literature can be summarised as follows:

1. destroying effects depend on the magnitude, time position, number and type of outliers
2. effects depend also on the outlier configuration of the time series (isolated and/or patchies)
3. problems are more striking when ARMA coefficients are large in absolute value
4. a large signal-to-noise ratio, $(\sigma^2_{\text{outl}} - \sigma^2_{\text{core}})/\sigma^2_{\text{core}}$, indicates high risk of misidentification of the model
5. with an AO outlier, if the length of time series and the magnitude of AO are fixed, ARMA processes with small variance, σ^2_{core}, are more susceptible to identification errors
6. the shorter the time series, the more sensitive it is to misidentification.

In practice, the most important factors seem to be the type and relative magnitude of an outlier[19]. AOs are more damaging than IOs. Unfortunately, AOs are more common in practice. The results above concern isolated outliers, one or more in a time series. If there are isolated and one or more patches of outliers in a series, their effects on autocorrelation estimates may be very complicated.

Chan (1992) showed that a sizable single AO can destroy all information about the underlying core process. The effects of AO outliers on estimates of SPACF coefficients are as disturbing as on the

[18] As we know, identification may be a difficult task in practice also in the case of an ordinary ARMA model without outliers.

[19] Chang and Tiao (1983, p. 536) concluded, based on their experiments (AR model and AO outlier), that '...fitting a model of inappropriate order may cause more problems than it usually does when there is an intervention.'

SACF while the effects of an IO are in general not so strong in either case (Chang 1982, Chan 1992).

Chang (1982) found that effects of AOs and IOs on autocorrelation estimates depend on their position in a series. She also found clear distinctions in behaviour between AOs and IOs. If the last observation of a series is an outlier, one naturally cannot identify its type based on the time series' own information (Chen and Liu 1993). When the number of isolated outliers in a series increases, information for tentative identification of an underlying model is rapidly destroyed. This also happens if the differencing transformation is taken from a series including isolated AOs. Consequently, we have in the literature some reports on experiments to develop a robust version of autocorrelation and partial autocorrelation functions.

For the robust ESACF procedure, we chose three different types of robust autocorrelation function: weighted, trimmed and rank-based. With the weighted ACF, we can select the different weight functions of observation; in the trimmed case we can select the percentage of extreme observations to delete. In the rank-based case, the autocorrelation estimates are based on ranked observations. Brief descriptions of robust variants follow.

The weighted wacf

Wang and Wei (1993) proposed a robustified autocorrelation function in which each observation has its own weight, w_i. The weighting function is constructed iteratively (see Maronna 1976, Gnanadesikan & Kettenring 1972, p. 95–96).

The weighted sample autocorrelation function (*wacf*) is then defined as

$$\hat{\rho}_w(k) = \frac{\hat{\gamma}_w(k)}{\hat{\gamma}_w(0)} \qquad (5.10)$$

where

$$\hat{\rho}_w(k) = \frac{\sum_{t=k+1}^{n}(z_{t-k} - \overline{z}_w)(z_t - \overline{z}_w)w_{t-k}w_t}{\sum_{t=k+1}^{n} w_{t-k}w_t} \qquad (5.11)$$

and

$$\bar{z}_w = \frac{\sum_{t=1}^{n} z_t w_t}{\sum_{t=1}^{n} w_t} \tag{5.12}$$

Wang and Wei (1993) show, based on Dunsmuir and Robinson (1981, Theorem 2), that under the white noise model, the $\sqrt{n}\hat{\rho}_w(k)$, k≥1, are asymptotically independent normal random variates with asymptotic variance $(\upsilon(k))^{-1}$, where

$$\upsilon(k) = \lim_{n\to\infty} \frac{1}{n} \sum_{t=k+1}^{n} w_{t-k} w_t \tag{5.13}$$

The approximated standard error (s.e.) of the wacf is then

$$\text{s.e.}\left[\hat{\rho}_w(k)\right] \approx \frac{1}{\sqrt{n\hat{\upsilon}(k)}} \tag{5.14}$$

where $\hat{\upsilon}(k) = \frac{1}{n} \sum_{t=k+1}^{n} w_{t-k} w_t$

Note that $\text{s.e.}[\hat{\rho}_w(k)] > \text{s.e.}[\hat{\rho}(k)]$ for a white noise model in an outlier-free situation (see also Chan and Wei 1992, p. 153). Different kinds of weight functions are available. The common principle is to give smaller weights to extreme observations. Wang and Wei used five weight functions, ie those of Tukey, Huber, Hampel, Andrews and Maronna. Wang and Wei (1993) remark that with a proper choice of a weight function a robust SACF performs as well as an ordinary SACF with an outlier-free time series. In this study the default choice is Huber's function. Some robust regression-based versions of ACF have also been developed (Masarotto 1987a, Chen 1994).

Robust tacf

In robustifying the autocorrelation function, the trimming principle (with variants) has been dominant because trimming has many strong statistical properties (Huber 1972, Stigler 1973).

The Polasek and Mertl (1990) estimator is based on the trimming variance principle (see Gnanadesikan and Kettenring 1972). The basic idea is to approximate the covariance of a series by a linear combination of transformed variances which can be estimated robustly more easily. We can transform the moment estimator of ACF into the following variance-ratio of differences and sums (D/S variance estimator) (Gnanadesikan and Kettenring 1972, p. 90–91, Gnanadesikan 1977, p. 273–274):

$$r_k = \frac{s_+^2(k) - s_-^2(k)}{s_+^2(k) + s_-^2(k)}, \quad k = 1,...,K \tag{5.15}$$

with variances

$$s_+^2(k) = \mathrm{Var}(S^k x_t) \quad \text{and} \quad s_-^2(k) = \mathrm{Var}(D^k x_t), \quad k = 1,...,K \tag{5.16}$$

where

$S^k x_t$ and $D^k x_t$ denote the k–th lagged sums and differences of the original time series x_t:

$D^k x_t = x_t - x_{t-k}$ and $S^k x_t = x_t + x_{t-k}$, $t = k+1,...,n$ and $k = 1,...,K$.

Polasek and Mertl (1990) defined the α%-trimmed ACF-estimator as

$$r_k = \frac{\mathrm{Var}_\alpha(S^k r_k) - \mathrm{Var}_\alpha(D^k r_k)}{\mathrm{Var}_\alpha(S^k r_k) + \mathrm{Var}_\alpha(D^k r_k)}, \quad k = 1,...,K \tag{5.17}$$

where $\mathrm{Var}_\alpha(x_t)$ denotes the α%-trimmed variance of the time series x_t. Since the formula (5.15) is a ratio of robust variance estimators, any adjustment (normalising) factors cancel out.

The main results of their simulations show that we should use robust ACF, especially if additive outliers are suspected in a time series. Polasek and Mertl found hardly any differences in the results for seasonal and non-seasonal processes.

Two variants of robust trimmed ACF

Chen (1994) has proposed the ‘generalised α-trimmed SACF’ because outliers are not necessarily the extreme values in a time series. Chen, however, assumes that the outliers have already been detected. He proposes also a variant, ‘cross-product-based α-trimmed SACF’ in which the idea is to remove the effects of the observations that cause extreme cross products (cross-product of mean-corrected y_i and y_{i-k}; see Chen 1994, Section 5.5).

Rank-transformation based rkacf

In statistics, many rank methods are found to be robust (eg Conover 1980). By replacing the values of the observations by their ranks, the influence of some extreme outliers may be reduced. Kelley and Noel (1982) estimated the conventional autocorrelation and partial autocorrelation coefficients based on unranked and ranked time series to identify the AR(1) and ARMA(1, 1) processes in outlier-free and outlier cases. The results were surprisingly promising[20]. Our simulation program includes the simple option of rank-based autocorrelation by Kelley and Noel (1982). Our experience with the rank transformation is similar to that of Kelley and Noel. If we know that a time series contains only a small number of moderate, isolated outliers, the rkacf may be a clear-cut and quick alternative tool.

5.2.5 Combination of OLS and robust ACF

The OLS method is to some extent robust in regression and time series modelling (eg Bustos 1982, p. 492, Lucas 1995a, 1995b and 1996). It is known that if a time series contains only IO outliers, OLS provides consistent but not efficient estimates. With AOs, the OLS estimator breaks down, and we need a robust estimator.

In the EACF framework one robust version of this procedure could be the combination of OLS in AR(p) fitting and a robust autocorrelation function. In simulations and for some real series, we used the combination of OLS and weighted ACF. The results are promising. However, we can expect that if the parameter coefficients of the ARIMA model are high in modulus and/or the standard deviation of the outlier distribution in contamination is large, OLS may produce sufficiently biased AR(p) estimates to reduce the performance of the OLS/wacf combination.

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[20] Drawing on the robustness literature, we can consider the simulation experiments of Abdullah (1990) with the Spearman rho and Kendall tau estimators. The results show that these estimators are robust to some extent, but not sufficiently robust when the percentage of outliers is high.

5.3 Standard error of the ESACF coefficients

5.3.1 Standard error based on the white noise assumption

Tiao and Tsay used the simple approximation, $(n–k–j)^{-1/2}$, as the asymptotic standard error of the single ESACF coefficients. This is based on the assumption that the transformed $W_{k,t}^{(j)}$ series is white noise. As they remark, this estimator may underestimate the standard errors of single ESACF coefficients. The results of our simulations seem to be in line with their suggestion. According to our simulation results of Tables 4 and 5 concerning the vertex, this asymptotic estimate (1/D) is on average 61% of the sample *robust* estimate calculated directly from the results of 1000 replications of 16 models. For the *non-robust* case, the percentage is greater, as can be seen from the results of column A of these Tables.

5.3.2 Bartlett's asymptotic formula

In this study the well-known Bartlett (1946) formula for the variance of the estimated autocorrelation coefficients of a stationary Gaussian process

$$\mathrm{Var}(r_k) \approx \frac{1}{n}\left[1 + 2\sum_{j=1}^{q} r^2(j)\right], \quad k > q \tag{5.18}$$

is applied in the ESACF table, on the assumption that the transformed $W_{k,t}^{(j)}$ series is generated by an MA(q) model. However, as is known in the literature, the application of Bartlett's formula can produce unsatisfactory results. The adequacy of different approximations to the distributions of the sample autocorrelations has been examined (eg Ali 1984). Pukkila (1984) did a simulation study of Bartlett's (1946) asymptotic formula for certain ARMA models and parameter values and *all* lags (here 1–20). His results show that the outcomes are highly model dependent and can be very different for seemingly similar models (see also the conclusions of Mélard and Roy 1987). Aczel and Josephy (1992, p. 72) remark that the use of Bartlett's formula in confidence intervals, ie estimated autocorrelations with two standard error bounds, may also be problematic in the traditional (normality of observations) case, and lead to incorrect conclusions. They note that the symmetric interval, $r_k \pm C * \hat{\sigma}$, may not be appropriate and at least some correction should be made. Berlinet and Francq (1997) show that the estimate given by this formula can deviate markedly from the true asymptotic value when the requirements (linearity and vanishing of fourth-order cumulants) on the underlying process are not satisfied. They remark that this is the case for a large class of models and investigate the behaviour of smoothed empirical estimates.

A new and still more problematic situation occurs in our simulation experiments, since in the ESACF procedure we use a robust autocorrelation function because the transformed series, $W_{k,t}^{(j)}$, contains information on outliers. Thus conditions for Bartlett's classic formula are violated: a series is contaminated-normal (non-Gaussian) and we use a robust autocorrelation function. Therefore we should develop a robust version of Bartlett's approximation. In addition, we should use something other than the traditional t-values for robust 95% confidence intervals in a robust ESACF table. Various approaches and combinations are available in the literature on constructing robust confidence intervals (see Birch and Martin 1981, Wilcox 1997 and Field & Welsh 1998).

For the simulation results, we have presented the ratio of simulation sample std. error to Bartlett's formula, robust case (column C in Tables). We can use these numerical results in future studies in constructing the confidence intervals for single robust ESACF coefficient estimates. For a single real and simulated series, we can use the asymptotic standard error $(n–k–j)^{-1/2}$, in addition to Bartlett's approximation, to obtain the simplified (X,0)-indicator version of the ESACF pattern. We are also able to set different values of C in calculating the confidence intervals ±C*std.error (C is a decimal number). Further, we can experiment with different constants (t values), C_{low} and C^{up}, in calculating the confidence intervals (see Birch and Martin 1981).

5.4 Robust confidence intervals

The factors of the traditional confidence interval, ie a parameter estimate, its standard error and t values, are all non-robust. For robust confidence intervals, one should have a robust version of each of these factors. For robust standard errors and t values, we have only single, separate research results in the literature. Next, we look at some results from the robustness literature.

Short review of robust confidence intervals research

There is no comprehensive review of the distributions of robust estimators and their confidence intervals in the statistical literature. The main reason is that the exact distributions of robust estimators are often difficult to derive, and simulation studies are usually needed to assess and compare the features and performance of various types of robust regression estimators and other robust estimators.

In a manner analogous to the classical Gaussian confidence intervals one can construct the robust confidence intervals of the form $\left[T \pm t_r^* w / \sqrt{n}\right]$. Here T is some robust location estimator,[21] w is its scale estimator, and t_r^* is a chosen percentage point of the t distribution with n–1 degrees of freedom. The core problem is how to choose (or construct) t_r^* and w for various robust estimators.

Regression model context

Huber (1968) first considered the robust intervals ('robust confidence limits') problem. Gross (1977) examined various robust confidence intervals for regression coefficients using Monte Carlo methods for different small samples. In particular he studied the problems of finding t* values (95 per cent t values) by simulations and found that t* values depend on various factors such as the underlying distribution of errors, scale estimator and robust estimator of regression coefficients. From recent studies of this problem area of different location estimators, we would single out Wilcox (2001, especially Chapters 5 and 9).

Hoaglin et al (1983) consider robust confidence intervals for selected location and scale estimators and present some examples from the literature. They present the percentage points of the t distribution with 0.7(n–1) degrees of freedom for a robust biweight pair of estimators. Rousseeuw and Leroy (1987, p. 41, 59–60, 81) consider the effects of outliers on t-based significance levels and on the construction of confidence intervals. They also give (p. 48–49 and 59–60) some simple examples of fitting the classic and robust

[21] Among earlier studies, Cox and Hinkley (1974, p. 353–354) considered the asymptotic and simulated variances of some classical and robust location estimators under four different underlying distributions. Their results (Table 9.1 op cit) show how the variance of the trimmed mean increases clearly if the underlying distribution is contaminated normal instead of normal. For recent studies of the trimmed mean, see Wilcox (2001).

regression with coefficient estimates (the outliers are assigned zero weights in the estimation) and their standard errors and 95% confidence intervals. Robust estimates have smaller standard errors and narrower confidence intervals compared with ordinary OLS. Staudte and Sheather (1990) consider t testing and the classic and robust confidence intervals. They note that robust regression schemes generally should include procedures for constructing confidence intervals and hypothesis tests. Ryan (1997) remarks that this important area is still relatively unexplored and requires much further study.

In their review article Field and Tingley (1997) report that bootstrap techniques have become useful tools in constructing robust confidence intervals in linear regression. Wilcox (1997, 2001) considers the percentile bootstrap technique and deems it the best method for computing confidence intervals when using M-regression. But, as Maddala and Rao (1997) remark, there are many problems ahead (see Stromberg 1997, Amado & Pires 2000) and further study is needed in applying bootstrap methods in the presence of outliers and high leverage points. Salibian-Barrera (2000) also studies bootstrap and robust bootstrap confidence intervals with MM-location and -regression estimators and presents some examples from real data. Yohai and Zamar (2001) briefly review recent robust inference. They also consider the bootstrap and robust bootstrap techniques and refer to the concept of globally robust confidence intervals.

Time series framework

There are few studies on robust confidence intervals in the context of time series models. Hampel et al (1986, p. 422) refer to the problem. So far, the most robust theoretical simulation study is Birch and Martin (1981). They used Monte Carlo experiments to investigate robustness properties of confidence intervals of the AR(1) parameter based on the OLS and GM-estimator. The separate analyses were carried out for innovational (IO) and additive (AO) outliers. They found that GM-estimators possess desirable confidence interval robustness properties in terms of robustness of validity and robustness of efficiency concepts (which they defined for their experiments). Birch and Martin present the results for 95% confidence intervals (Table V: maximum and minimum t values, t^* and t_*, based on 250 repetitions) of the GM-estimator with three values of ϕ_1 of an AR(1) process and three sample sizes. It is remarkable that the t values obtained indicate asymmetric robust confidence intervals in every case; for instance, when $n = 100$ and $\phi_1 = 0.50$, conservative 95%

t values (for the combination of GM/biweight) obtained maximum and minimum upper and lower 2.5% points $t^* = 2.23$ and $t_* = -2.58$. The pair of t values obtained vary more due to differences in sample size than to differences in value of ϕ_1.

The bootstrap technique has also been applied to time series models. Two basic types of bootstrap can be used: parametric and moving block bootstrap. Künsch developed the latter technique for the time series context (Künsch 1989). Aczel and Josephy (1992) applied the Künsch procedure successfully to a nonstationary foreign exchange rate series. Paparoditis & Streitberg (1992) applied the parametric bootstrap to vector autocorrelation. Glendinning (1998) used a special version of bootstrap in the case of the corner method in ARMA identification of outlier-contaminated data. Jeong and Chung (2001) applied a recursive bootstrap in testing for autocorrelation in regression models.

Potential use in ESACF approaches

We applied a conservative t value principle in our standard and robust ESACF estimation. At the five per cent risk level, a confidence interval of ±2 std. errors of estimator was used. The main question is how valid this harring t value is in the case of robust estimators. How should we construct some t_r^* values for our design of robust confidence intervals? In our study we encounter the robust confidence interval problem as regards both AR(p) iterative regression and coefficients of the robust ESACF table.

From our simulation experiments we obtained results for standard errors of the ESACF coefficients. We should try to combine the influences of the robust iterative AR(p) and robust autocorrelation on the robust confidence intervals of ESACF coefficients. The work of Birch and Martin (1981), Aczel and Josephy (1992), Glendinning (1998) and Wilcox (1997) provides results and optional approaches for examining and experimenting with various robust confidence intervals. One potential bootstrap is a recursive bootstrap (see Jeong and Chung 2001).

It seems that our conservative practice of using 2 as the value of t in every confidence interval for 2.5% upper and lower points might be a starting-point for most robust cases. An argument for this is our result that in robust cases the shape of an approximate sample distribution of a single ESACF coefficient is often more normal than in non-robust cases.

In experiments with a single series, we are able to input various symmetric and non-symmetric t values for the standard and robust ESACF coefficients to display the simplified (X,0) version of the ESACF table (see an example in Appendix 7). However, in our Monte Carlo experiments, we used only symmetric t values for standard and robust ESACF estimates.

5.5 Robust ESACF complementing robust unit root testing

In statistics and econometrics there is a comprehensive literature on unit root testing procedures for time series. This high degree of activity is understandable. A great majority of economic time series exhibit nonstationary behaviour. Additionally, mixed ARIMA(p, d, q; q>0) processes are quite common in economics[22]. As is known, the MA(q) part can cause severe problems in unit root testing (eg Schwert 1989). So, it is important to ensure the 'most correct' specification of the ARIMA model before testing for the existence of a unit root. In practice, this means the use of all three identification tools, SACF, SPACF and ESACF, and comparing the results.

If outliers are known to occur in a time series, unit root testing encounters many difficulties. Thus, in unit root econometrics, certain robust methods are already incorporated into both the estimation of parameters and the diagnostic testing of econometric and time series models. Of recent studies and reviews, we would mention Lucas (1995a and 1995b, 1996), Maddala and Yin (1997), Maddala and Kim (1998), Yin and Maddala (1998) and Yin (1995).

It is well known that, if one takes first differences of a time series in which AO outliers are known to occur, then, for each isolated outlier, an immediately consecutive, new spurious outlier of the same magnitude is obtained (Chang 1982, p. 118). The situation may become distorted if there are other original isolated and/or patchy AO outliers. The tolerance limit for the number of outliers may be reached quite quickly, even if robust methods are used (eg HBP regression). So, if we can avoid differencing and at the same time use a robust procedure, eg a robust ESACF, we are better able to find a correct solution for existence of a unit root. Yin and Maddala (1998) have

[22] An invertible MA part is often used due to measurement errors which usually are assumed to be stationary.

shown that the existence of isolated AO outliers leads to a biased rejection of the unit root hypothesis in the case of an I(1) time series (ie generate spurious stationarity). According to Yin and Maddala (1997, 1998), unit root testing is differently sensitive to outliers, depending on the type of outlier. For the case of patchies of AO outliers, we have no results in the statistical or econometric literature, and so the contribution of the robust ESACF procedure may be important.

With outlier-free time series, the standard ESACF can be used as a simple complementary, regression-based method in addition to the standard unit root testing. With outliers, the robust version of the ESACF can be used as a complementary tool in unit root testing[23]. Additionally, we may obtain lacking (hidden) information eg on the problem involved in difference-stationary or trend-stationary series[24] (eg Mills 1991, Chapter 11, Tsay 1993). As is known in the literature, the handling of nonstationarity and outliers simultaneously, particularly in ARIMA(p, d, q; q>0) modelling, has been a real challenge to researchers (eg Fieller 1979, Stockinger & Dutter 1987 and Maddala and Yin 1997). The best performance should be obtained with the combination of a robust ESACF and robust unit root testing, in which ordinary least squares is replaced by the MM-regression estimator.

Some simple examples

In Section 6.3 we have generated three I(1) processes ARIMA(1, 1, 0), ARIMA(1, 1, 1) and ARIMA(0, 1, 1) and isolated AO outliers. We see, for instance, that for the first-differenced form of an I(1) series it is difficult to identify a correct model when the original series contains AO outliers. Only the robust ESACF procedure (most often based on MM-estimator) is able to produce correct results. For the ARIMA(1, 1, 1) process we found that, with added AO outliers, the

[23] Another way could be first to estimate the outliers and their effects on the time series and then to apply unit root testing for the outlier-adjusted series. Because the widely used combined outlier and ARIMA modelling procedure (eg Chen and Liu 1993) is not adequately robust, this other way may perform poorly. We need further research and applications of different ARIMA models and types of outliers and outlier configurations.

[24] DeJong et al (1992) concluded in their study that it is difficult to discriminate between the two models using classical testing methods. McCulloch and Tsay (1994) proposed a Bayesian test procedure for distinguishing between trend-stationarity and difference-stationarity of a linear time series. As is well-known, a long economic time series may contain both kinds of stationarity.

original ESACF results were ARMA(1, 1), ie outliers destroy the unit root. This result is in line with the results of Yin and Maddala (1998).

For practical work, one can sketch a simple example of the use of ESACF in the case of a nonstationary ARIMA(1, 1, 1) process. In the ESACF table we have the result in the form of an ARMA(2, 1) process (see p. 39 of this thesis); we can check the nonstationary part of the AR-polynomial using iterated AR coefficients once the values of p and q are specified (TT84, p. 95). In the case of known or expected outliers (isolated and/or patchy) or structural breaks in the time series, we provide robust estimates of AR coefficients and check the non-stationarity part based on these parameter estimates. Then we may obtain a decisive bit of additional information to determine whether the underlying process contains a unit root. Similarly, two known processes, an ARMA(1, 1) and an ARIMA(0, 1, 1), may have a common robust ESACF result and if, in the ordinary model fitting, the robust AR estimate $\hat{\phi}_R \approx 1$, the process is an I(1) process ARIMA(0, 1, 1). However, the two fitted models may be very close (see eg Tiao 2001, p. 77 and 80)[25]. We can quite safely estimate the $\hat{\phi}$ with the robust MM-estimator and obtain information about the possible existence of a unit root. This can also be especially interesting but difficult in the extreme situation where both estimates, $\hat{\phi}$ and $\hat{\theta}$, are near in value to 1 and we may interpret this as a white noise series. In particular, the size of the MA parameter θ is shown to have a clear influence on unit root testing and hence on identification results (eg Schwert 1989).

[25] Tiao did this in the estimation stage of modelling and checked the stationarity condition (see also Pankratz 1991, p. 64 and 75).

6 Monte Carlo experiments

6.1 Objectives and design of simulations

The objectives of the simulation experiments are: a) to study the performance of the various versions of the robust ESACF procedure, b) to compare the results with the standard ESACF method and c) to study the sample distributions of single estimates of the ESACF table, especially their standard errors. The experiments are classified as the following runs:

1. general experiments of stationary and nonstationary ARMA(1, 1) and ARIMA(1, 1, 1) series with randomly placed, isolated AO and IO outliers and outlier-free time series
2. experiments based on the combination of OLS estimation and the weighted autocorrelation function, *wacf*
3. three experiments of the most common nonstationary I(1) time series, ARIMA(1, 1, 0), ARIMA(1, 1, 1) and ARIMA(0, 1, 1), to study the effects of first-order differencing on AO outliers and ESACF identification for the original and transformed series.

For each model and outlier combination, both standard and robust ESACF estimation was carried out, for comparison purposes. Three different versions of the standard error were estimated: a standard asymptotic $(n-k-j)^{-½}$ (white noise assumption), Bartlett's formula (MA(q) assumption), and a standard error calculated from simulated values. In addition to the routine output, the sample frequency histogram and box plot of each single coefficient estimate of the first 4×4 sub-matrix (first four rows and columns) can be calculated for each ARMA model. This was carried out with the special R code command 'plot.simu.esacf' from the saved simulation results files. These histograms are displayed with the following descriptive statistics: location[26], scale, root-mean-square-error (*rmse*), minimum, maximum, skewness, kurtosis and Jarque-Bera test value. An example

[26] As usual, we use the mean, but in this exploratory case we could experiment also with the median and (after some programming) with the mode (see Bickel 2002 and Rousseeuw and Leroy 1987). Bickel´s article was available after our simulation runs. In the case of a mode, comparison of simulation results between standard and robust ESACF could be 'a creative process'.

of the basic simulation results with these 16+16 histograms is given in Appendix 1.

The inspection of each ESACF table as to the whole pattern of estimates is essential in evaluating the performance of the method. The idea is to search the vertex of a triangle of asymptotic 'zero' values from the ESACF table. As is mentioned earlier, the coordinates of this point describe the maximum order of the AR and MA parts of a model also containing a unit root[27]. In simulation results, the focus is on the estimate of theoretical vertex and on its neighbour coefficient estimates. The first row of the matrix (here AR0) contains the estimates of the SACF. Due to lack of space, it is possible to report only a part of the comprehensive simulation results here. The 'truncated' ESACF pattern (a 3×6 matrix) estimates of the 16 models are presented in Appendices 2–3. The total number of different Monte Carlo simulation models was 56.

Design of Monte Carlo simulations

We focused the simulation experiments on the mixed models ARMA(1, 1) and ARIMA(1, 1, 1). The main reasons for the choice are:

1. to study the ESACF approach in mixed ARMA(1, 1) and ARIMA(1, 1, 1) models which most often occur in economics and engineering sciences; as considered in Sections 3.1–3.2 and 5.1, the original ESACF was developed for a mixed ARMA(p, q; q > 0) scheme, which often is an appropriate candidate also for residual series of econometric models; robust modelling of these residual series may help us to detect 'hidden' outliers (isolated and/or patchy) and so we may uncover an inadequacy of our econometric model

2. the nonstationary ARIMA(1, 1, 1) model is challenging, especially for iterative robust regression estimation; furthermore, robust identification and estimation of the AR(1) regression parameter, ϕ_1, may provide important additional information for unit root testing (see section 5.5 of this thesis).

[27] For instance, in the case of an ARIMA(1, 1, 1) process, a unit root is presented as an AR(1) with parameter $\phi_1 = 1$ and a tentative model in the ESACF table is found in the vertex of an ARMA(2, 1) model (Pankratz 1991, p. 63–64 and 75).

The common parsimony principle (smallest possible number of parameters) in time series modelling and experiences of 'real life', especially in economic and engineering time series, affected our choice of models. The principle of parsimony is widely considered in the literature (see eg Box and Jenkins 1970, 1976; Priestley 1981, p. 140–141 and Chatfield 1996, p. 41).

All the simulation results are based on 1000 replications. The sample sizes are n = 50, 100 and 200 (not common to all the models). The outlier types used in simulations were the additive (AO) and innovational (IO) outlier (see section 2.6). As is common in the literature, outliers are generated so that every observation is an outlier with a certain probability, here 0.02 and 0.05. From the general simulations, we used in 38 models the contamination proportions, $\gamma = 0.02$ and 0.05. The standard deviation of the outlier distribution applied was $\sigma_o = 3$, 5 or 10, depending on the form and parameter values (ie on degree of autocorrelation) of the selected ARMA models. The reason for selecting the standard deviation $\sigma_o = 10$ for contamination in our experiments was to study the robustness power of the ESACF in the heavily contaminated time series. The choices made in the literature are also taken into account.[28] In special experiments, the outlier patches of two or three fixed AOs were used, both at fixed places and randomly (see Appendix 4).

The simulation experiments were run via the program 'Tsrob',[29] which includes the standard and robust ESACF procedure programmed with the modern R code (see Appendix 7). The following robust regression estimators were used: M, GM and MM. The great majority of the experiments reported in this thesis were carried out with the MM-estimator, which is the default estimation method. The M-estimator was used in case of IOs. The GM-estimator was used in the examples of single real and generated series. Two alternative weight functions are available here for the GM-estimator: the Mallows- and Schweppe-type (eg Stockinger and Dutter 1987, p. 40–41).

Three robust versions of the classisc sample autocorrelation function were used: weighted ACF (*wacf*), variance trimmed ACF

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[28] Allende & Heiler (1992) used contamination proportions $\gamma = 0.05$ and 0.10; as a variance of the outlier distribution they used 9 var (x_t) and 100 var (x_t). Stockinger and Dutter (1987) used the contamination proportions $\gamma = 0.05$ and 0.10; as the variance of outlier distribution they used the quantity 9 var (x_t) and $\sigma^2_{IO} = 121$.

[29] With this program we are able to insert isolated AOs and IOs *simultaneously*, both randomly and at fixed points. Additionally, the patch of AOs can be scattered *simultaneously* at fixed points and randomly.

(*tacf*) and rank-based autocorrelation (*rkacf*). In the *wacf* procedure the default weighting function is Huber's psi-function. In the *tacf* the default for trimming is 0.05. The robust *rkacf* is based on a ranked time series. All these autocorrelation estimators are available with each of the robust regression estimators. We used mostly the wacf version.

For the robust autocorrelation functions no robust standard error formulas are available, so we used the classic formulas, ie $(n-k-j)^{-½}$ and Bartlett's formula. For comparison, we also calculated the standard errors of the robust estimates based directly on the replications. The robust estimators of scale, the MAD (median of absolute median deviations) and Q_n (see Croux and Rousseeuw 1992) are optionally available for the M-, GM- and MM-estimator. The MAD is the default choice.

The flow chart, 'United ESACF identification procedure', based on the Tsrob program, is displayed in Figure 1 (p. 15). More detailed technical information about the robust scale estimators, autocorrelation functions and robust regression estimators is given in Appendix 7.

6.2 Main results

General simulations[30]

The simulation results show that the new robust ESACF procedures work technically well and fast alongside the standard ESACF method. This is a good starting-point since the robust estimation methods for the AR(p) parameters are themselves iterative, multi-stage procedures. Only in some cases did the message appear that a convergence of an algorithm was not reached at the maximum number, 20 (or 10), of iteration steps[31]; this seems not to have had any noticeable effect on the estimation results. The selected main results of the general simulations are presented in Tables 1–6 and Figures 2–10. Only parts of the ESACF tables can be displayed in Appendices 1–3, due to lack of space. Monte Carlo results for some common ARIMA models (also random walk model) in economics and engineering sciences are displayed in Appendix 4; there are also three experiments with special

[30] The main Monte Carlo simulations were run in two sets of models in a network of 18 simultaneously running PCs.
[31] This kind of message was signalled also in estimating some single time series.

outlier configurations. All the robust ESACF results of Appendix 4 are based on the MM/wacf combination.

6.2.1 ARMA(1, 1) and ARIMA(1, 1, 1) processes

ARMA(1, 1) process

Here we apply standard and robust ESACF to the ARMA(1, 1) models with two parameter structures. With the parameter values $\phi_1 = 0.6$, $\theta_1 = -0.4$ and $\sigma_o = 5$, the performance of standard and robust ESACF is similar, independent of length of series, but when $\phi_1 = 0.8$, $\theta_1 = -0.7$ and $\sigma_o = 10$ robust ESACF outperforms the standard ESACF.

Besides the estimate of the theoretical vertex of the triangle in the ESACF table we need to inspect the vertex's neighbouring estimates, $r_{j(k)}$, especially the immediately preceding one. In general, it is important to inspect how 'polarised' the ESACF pattern is in the neighbourhood of the vertex. The inspection is especially important for the mixed models. The distinctions between robust and standard ESACF estimates can be found in Appendix 1.

In simulation results for the ARMA(1, 1) models the mean vertex value is marked by an asterisk if the model is correctly identified at the 5% level. Table 1shows that 9/16 of standard and 15/16 of robust estimation models are correctly identified.

Table 1. **Summary of simulation results: mean of theoretical vertex[1)] of ESACF table for an ARMA(1, 1) model, 1000 replications**

Model ARMA(1, 1)	n	Percentage: vertex value beyond ±2 standard errors[2)], % A	B	Mean of theoretical vertex (expected value = 0) A	B	Outlier type[3)] and contamination proportion (%)
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 5$						
1	50	3.0*	2.8*	0.057	0.043	AO2
2	50	4.4[4)]	3.8*	0.054	0.037	AO5
3	50	2.6*	3.1*	0.061	0.048	IO2
4	50	2.8*	2.6*	0.046	0.025	IO5
5	200	3.6*	4.3*	0.022	0.010	AO2
6	200	4.6*	4.1*	0.019	0.004	AO5
7	200	4.5*	3.7*	0.024	0.018	IO2
8	200	4.3*	3.6*	0.023	0.014	IO5
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 10$						
9	50	4.1[4)]	2.4*	0.041	0.026	AO2
10	50	6.4[4)]	4.1*	0.033	0.016	AO5
11	50	3.1*	2.0*	0.055	0.028	IO2
12	50	3.1*	2.5*	0.070	0.033	IO5
13	200	6.2[4)]	4.5*	0.018	–0.011	AO2
14	200	6.3	6.9	0.021	–0.030	AO5
15	200	5.5	3.7*	0.029	0.019	IO2
16	200	5.7	3.9*	0.026	0.004	IO5

1) Point at which row and column coordinates of the vertex of a triangle of asymptotic 'zero' values correspond to AR order p and MA order q, respectively.
2) Standard error based on 1000 replications.
3) AO = additive outlier, IO = innovational outlier.
A = OLS estimation, B = Robust estimation (MM-estimator: AO; M-estimator: IO)
σ_o = standard deviation of outlier distribution in contamination
* = correct identification at 5% level
4) = ESACF identifies an AR(1) model.
n = sample size

As can be seen from Table 1, for models 1–8, the results of standard ESACF are good independent of sample size and when $\sigma_o = 5$. Only when the AOs share is 5% are the results incorrect. For robust ESACF, the results are very good. In models 9–16, when σ_o is 10, the robust method clearly outperforms the standard one. As may be expected, the standard ESACF fails especially in the case of 5% AO contamination.

The sample distributions of single standard ESACF estimates may be quite non-Gaussian, and robustifying seems to give them more normal shapes or normal distributions. Figure 2 dispays an example of the ARMA(1, 1) process. The values of the Jarque-Bera test statistics show the difference between robust and non-robust case. Another example which concerns the ARMA(1, 1) model with two patches of outliers in time series is given in Figure 3. We find both normalising of sample distribution and decreasing of bias in the ESACF vertex estimates. We return to outlier-free ARMA(1, 1) processes in Section 6.2.2. The results of the combination OLS/wacf in case of ARMA(1, 1) models are considered in Section 6.2.3.

Figure 2. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARMA(1, 1), phi = 0.80, theta = –0.70; n = 200; st.dev of AO = 10; simulations: 1000 replications; 2% isolated AO outliers; robust (MM/wacf) and standard (OLS/acf) autoregression/autocorrelation are used in ESACF estimation.

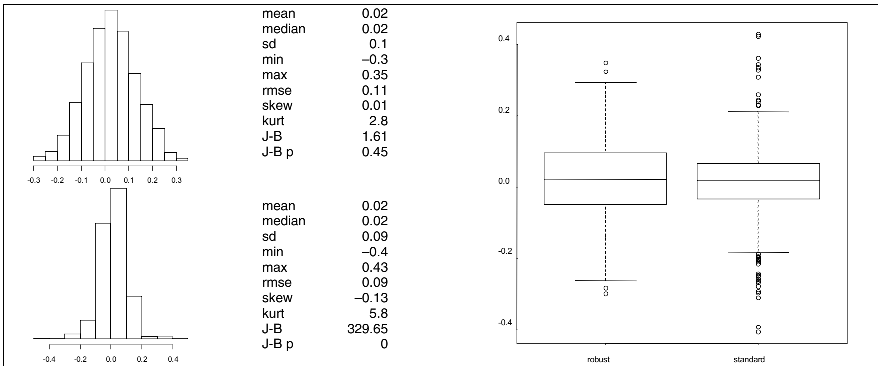


Figure 2a: robust (upper) and standard histogram

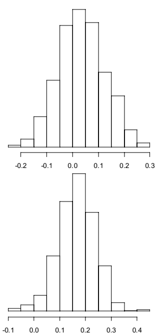
Figure 2b: robust and standard box plot

In general, the sample variance based on 1000 simulation replications is greater than that for simple asymptotic and Bartlett's approximation. Another property is that the **rmse** of the single coefficient estimates in the ESACF pattern, is slowly convergent with increasing value of q for both robust and standard estimates, as can be seen in the example model of Appendix 1. This can also be seen in Figures 11 and 12.

In Appendix 4 we have an ARMA(1, 1) model (model 10) where we examine the ESACF and special outlier configuration: two patches of fixed AOs, one at the end of series and another randomly. The results show that the robust ESACF clearly outperforms the standard one.

Figure 3. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARMA(1, 1), phi = 0.70, theta = 0.40; n = 200; simulations: 1000 replications; a fixed 3-AO patch at end and a fixed two-AO patch placed randomly; robust (MM/wacf) and standard (OLS/acf) autoregression/ autocorrelation are used in ESACF estimation.



mean	0.04
median	0.03
sd	0.09
min	–0.24
max	0.28
rmse	0.1
skew	0.05
kurt	2.75
J-B	3.12
J-B p	0.21
mean	0.17
median	0.17
sd	0.07
min	–0.1
max	0.43
rmse	0.18
skew	–0.24
kurt	3.69
J-B	29.72
J-B p	0

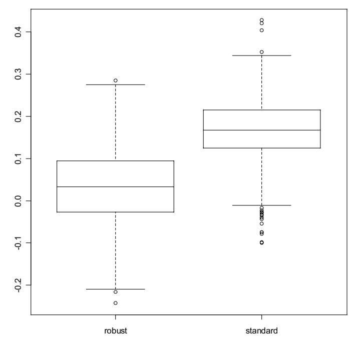


Figure 3a: robust (upper) and standard histogram

Figure 3b: robust and standard box plot

ARIMA(1, 1, 1) processes

In simulations of the nonstationary ARIMA(1, 1, 1) models, we used robust regression estimators ‘without theory’ in the AR(p) iteration phase of the ESACF procedure. As expected, OLS estimation and standard ESACF encounters fail, especially in the AO case. Of the robust estimation cases, 6/16 were exactly correctly identified, while of the standard cases 4/16 were exactly correct (see Table 2). For models 17–24 the mean of ‘vertex-%’ of standard ESACF (column A) is 7.4%, while for the robust ESACF (column B) the corresponding mean is 4.3%. The standard ESACF performs well with IOs, but fails with AOs. The robust ESACF performs quite successfully with both types of outliers, except for the cases of AO5. The results seem not to depend on length of time series. For the extreme models, 25–32, the results are not so good. The case of AO5 is the most difficult, as might be expected. Here too, the results do not depend on length of time series. The results in Table 2 indicate the need for further simulations and a sensitivity analysis with different ARIMA parameter combinations and different σ_o values. In particular, the design of different values of σ_o, relative to different size and sign combinations of ARIMA coefficient parameters, will be needed.

In Appendix 4 we examine the unit root model 11, [ARIMA(0, 1, 0)], with the outlier configuration: a patch of two fixed AOs located randomly in time series. The identification results show that the robust ESACF performs quite well (5.5% in 5% theoretical confidence interval, 2.4% on the Bartlett-based interval) while the standard ESACF clearly fails. The similar outlier configuration with model 12 [ARIMA(0, 0, 0)] produces similar results. In this model the robust Bartlett-based result for the vertex interval is 4.2%.

Table 2. **Summary of simulation results: mean of theoretical vertex[1)] of ESACF table for an ARIMA(1, 1, 1) model, based on 1000 replications**

Model ARIMA (1, 1, 1)	n	Percentage: vertex value beyond ±2 standard errors[2)], % A	B	Average of theoretical vertex value (expected value = 0) A	B	Outlier type[3)] and contamination proportion, %
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 5$						
17	50	3.8[4)]	3.2*	0.036	0.000	AO2
18	50	6.6[4)]	3.7[4)]	0.066	0.025	AO5
19	50	3.9*	3.8*	–0.035	–0.039	IO2
20	50	3.8*	3.6*	–0.041	–0.043	IO5
21	200	11.4[4)]	5.6	0.082	0.038	AO2
22	200	26.5[4)]	7.1[4)]	0.117	0.044	AO5
23	200	3.6*	3.6*	0.001	–0.001	IO2
24	200	3.6*	4.2*	–0.009	–0.018	IO5
$\phi_1 = 0.85$ $\theta_1 = 0.75$ $\sigma_o = 10$						
25	50	6.1[4)]	5.7[4)]	–0.015	0.024	AO2
26	50	6.3[4)]	6.2[5)]	–0.053	0.039	AO5
27	50	6.5[4)]	4.5*	0.021	0.023	IO2
28	50	6.6[4)]	6.2[4)]	0.016	0.019	IO5
29	200	4.9[4)]	9.2[4)]	–0.015	0.054	AO2
30	200	9.1[6)]	9.3[4)]	–0.108	0.066	AO5
31	200	6.8[4)]	5.9	0.026	0.023	IO2
32	200	6.8	4.6*	0.020	0.014	IO5

1) Point at which row and column coordinates of the vertex of a triangle of asymptotic 'zero' values correspond to AR order p and MA order q, respectively.
2) Sample simulation std's.
3) AO = additive outlier, IO = innovational outlier
A = OLS estimation, B = Robust estimation: MM-estimator
σ_o = the standard deviation of outlier distribution in contamination
* = correct identification at 5% level.
4) = ESACF identifies an AR(2) model.
5) = ESACF identifies an AR(1) model.
6) = ESACF identifies an ARMA(1, 1) model.
n = sample size

The shape of the sample distribution of single ESACF estimates is interesting in the case of a nonstationary ARIMA process[32]. The results given in Figure 4 show that robustifying normalises the sample distributions of single ESACF estimates. Here we have used the MM-estimator in the robust regression part. We consider outlier-free ARIMA(1, 1, 1) models in Section 6.2.2. The case of OLS/wacf in robust regression in ARIMA(1, 1, 1) models is considered in Section 6.2.3. The potential use of robust ESACF in robust unit root analysis is briefly considered in Section 5.5.

As mentioned in Section 6.1 we generated the outliers so that every observation is an outlier with a certain probability. We also carried out a small experiment on the condition that every realisation has exactly a given number of outliers, in this case 5% (2 outliers in 50 observations). As Figure 5 shows, the results for robust and standard ESACF are quite different. The bias of the robust ESACF estimate is clearly smaller but standard error greater compared with the standard ESACF estimate. Jarque-Bera test results show the normalising effect of the robust ESACF procedure.

In most cases robust standard error for the vertex is greater than in the standard case. This can be seen from Table 6 (p. 86). Reasons for this property are the robust AR(p) fitting (see residuals of robust regression versus OLS regression in examples of Rousseeuw & Leroy 1987, eg p. 48 and 59) and the structure of the standard error of the weighted autocorrelation (see Chan & Wei 1992, p. 153–154). Most clearly this property can be seen in ratio D where we also find the effects of the asymptotic formula $(n-k-j)^{-1/2}$, which underestimates the standard error (eg TT84, p. 87; Tiao 2001, p. 68). The combination of values of ARIMA coefficients, outlier type and variability of outliers, σ_o, has an impact on these ratios; the length of time series does not. The degree of contamination also seems to affect this ratio in some cases, eg ratios A and B for IOs.

[32] To our knowledge, no research on this has been published in the literature.

Figure 4. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARIMA(1, 1, 1), phi = 0.85, theta = 0.75; n = 50;st. dev of AO = 10; simulations 1000 replications; 5% isolated AO outliers; robust (MM/wacf) and standard (OLS/acf) autoregression/autocorrelation are used in ESACF estimation

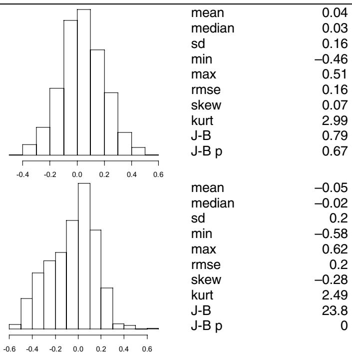


Figure 4a: robust (upper) and standard histogram

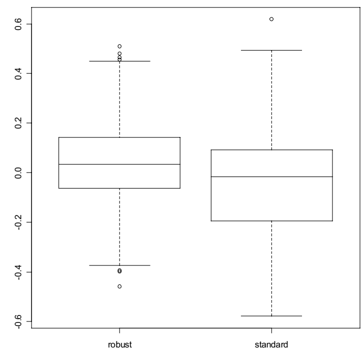


Figure 4b: robust and standard box plot

Figure 5. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARMA(1, 1, 1), phi=0.60, theta=-0.40; n=50; st.dev of AO= 10, simulations: 1000 replications; *5% isolated AO outliers in every generated realisation*; robust (MM/wacf) and standard (OLS/acf) autoregression/ autocorrelation are used in ESACF estimation

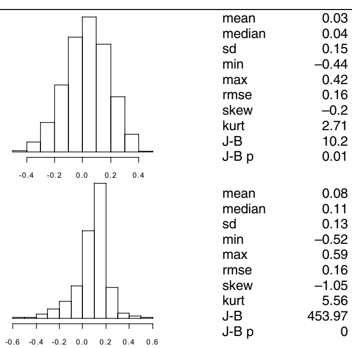


Figure 5a: robust (upper) and standard histogram

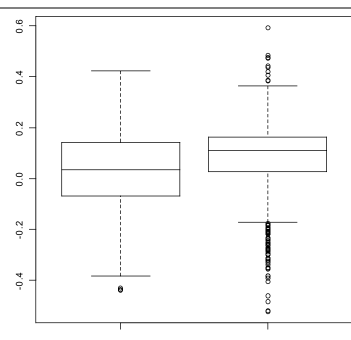


Figure 5b: robust and standard box plot

6.2.2 Outlier-free series

The standard and robust ESACF estimations were carried out for the outlier-free series corresponding to the general simulation models. The results of Table 3 show that, generally, there are no essential differences in performance between robust and standard ESACF method with outlier-free data. This inference is supported by the fact that the sample distributions of the vertices in ESACF pattern are quite similar in shape, as can be seen from Figure 6 (see also Section 6.2.4).

Table 3.

Summary of simulation results: mean of theoretical vertex[1)] of the pattern triangle in the ESACF table for an ARMA(p, d, q) model, 1000 replications. The <u>outlier-free</u> models corresponding to the ARMA(1, 1) and ARIMA(1, 1, 1) models of Table 1 and Table 2.

Model ARMA(1, 1)		Percentage: vertex value beyond ±2 standard errors[2)], %		Mean of theoretical vertex value (expected value = 0)	
	n	A	B	A	B
$\phi_1 = 0.6$ $\theta_1 = -0.4$					
a	50	2.5*	2.2*	0.058	0.057
b	50	2.3*	3.1*	0.063	0.061
a	200	3.8*	4.2*	0.024	0.025
b	200	4.7*	4.3*	0.024	0.022
Model ARIMA (1, 1, 1)					
	n	A	B	A	B
$\phi_1 = 0.6$ $\theta_1 = -0.4$					
a	50	3.5*	2.9*	–0.034	–0.033
b	50	3.3*	3.4*	–0.029	–0.029
a	200	3.4*	3.9*	–0.016	–0.017
b	200	4.2*	3.5*	–0.009	–0.008
$\phi_1 = 0.85$ $\theta_1 = 0.75$					
a	50	6.3[3)]	6.0[3)]	0.019	0.017
b	50	5.1[3)]	5.4[3)]	0.023	0.025
a	200	8.3[3)]	8.0[3)]	0.031	0.029
b	200	7.7[3)]	6.8[3)]	0.029	0.030

[1)] The point at which row and column coordinates of vertex of a triangle of asymptotic 'zero' values, correspond to AR order p and MA order q, respectively.
[2)] Standard error based 1000 replications.
[3)] = ESACF pattern identifies an AR(2) model.
A = OLS estimation is carried out with MM- and M-estimator
B = Robust estimation: a. MM-estimator, b. M-estimator
* = correct identification at 5% level
n = sample size

In practice this result is important, since it removes the risk of using these robust methods when the data do not contain outliers; in the case of the MM-estimator we are safeguarded against possible outliers[33]. However, further research on distinct ARMA structures, with combinations of different parameter values and robust ESACF versions, is needed to obtain more general results. As reported in the literature, a non-robust estimation method often outperforms a robust method in the case of outlier-free data.

Figure 6. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARIMA(1, 1, 1), phi = 0.85, theta = 0.75; n = 50; simulations: 1000 replications; outlier-free series; robust (MM/wacf) and standard (OLS/acf) autoregression/autocorrelation are used in ESACF estimation

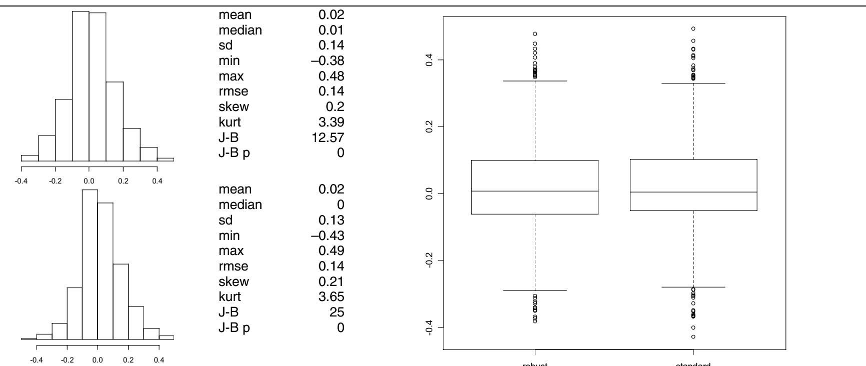


Figure 6a: robust (upper) and standard histogram

Figure 6b: robust and standard box plot

6.2.3 The OLS/wacf combination

In section 5.2.5 we considered the ESACF procedure with AR(p) regression estimation by the combination of OLS and a robustified autocorrelation function. We used the weighted autocorrelation function, wacf. The simulation results show that this combination is

[33] Franses and Lucas (1995, p. 4) refer to some of Lucas's robust unit root estimation results and note that, if there are no outliers in the data, the MM-estimator is nearly as efficient as the OLS estimator. You (1999, p. 210) reports, for his Monte Carlo results with high breakdown point estimators, that when the error (of a model) distribution is N(0,1), the OLS outperforms all the other robust estimators (in his study) except the MM-estimator.

robust to some degree also in short time series. The performance of this ESACF version is important to study, especially for the ARMA(1, 1) and ARIMA(1, 1, 1) models (see Tables 4–5). In Table 4 the correct identification ratio is 4/8 for standard ESACF but 8/8 for the robust ESACF case (see columns A and B).

The ratios of different standard errors contain important results for both ARMA(1, 1) and ARIMA(1, 1, 1) models. The ratio of the simulation-based standard errors are similar for the robust and standard estimates (ratio A), and similar results were obtained with Bartlett's formula (ratio B). More differences between robust directly simulation-based and robust Bartlett-based results (ratio C) are displayed. The results for ratio D in Tables 4 and 5 are similar to those for ratio D of models 17–24 in Table 6.

The OLS/wacf combination provides downward biased estimates of the AR(p) fitting, with AOs. Thus, in ARMA(1, 1) models, when there are AOs, the OLS/wacf combination normalises the sample distribution of single ESACF coefficient estimates only to some extent (see Jarque-Bera test values), as the results in Figure 7 show. [GM- and MM-based AR(p) estimates are also biased to some extent as AOs occur.] Note that OLS is consistent with IOs.

Figure 7. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARMA(1, 1), phi = 0.8, theta = –0.70; n = 200; st.dev of AO = 10, simulations: 1000 replications; 2% AO outliers; robust (OLS/wacf) and standard (OLS/acf) autoregression /autocorrelation are used in ESACF estimation

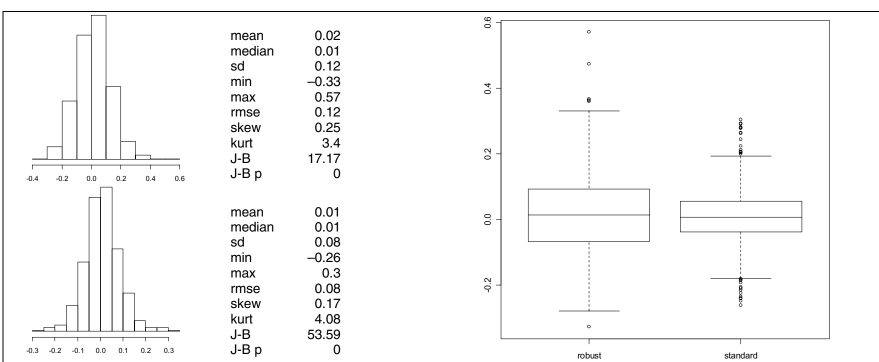


Figure 7a: robust (upper) and standard histogram

Figure 7b: robust and standard box plot

Table 4.

Summary of simulation results: mean of theoretical vertex[1)] of pattern triangle in standard and OLS/wacf based ESACF table for ARMA (p, q) model, 1000 replications

Model ARMA(1, 1)	n	Percentage: vertex value beyond ±2 standard errors[2)], % A	B	Average of theoretical vertex value (expected value = 0) A	B	Outlier type[3)] and contamination proportion (%)
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 5$						
1	50	2.7	2.1*	0.054	0.048	AO2
2	50	3.3	2.5*	0.047	0.055	AO5
3	50	2.7*	2.9*	0.063	0.068	IO2
4	50	1.8*	2.5*	0.053	0.058	IO5
5	200	4.7*	3.8*	0.022	0.023	AO2
6	200	5.1	4.1*	0.028	0.035	AO5
7	200	3.9*	4.5*	0.023	0.023	IO2
8	200	5.1	4.8*	0.022	0.027	IO5

[1)] The point at which row and column coordinates of vertex of a triangle of asymptotic 'zero' values, correspond to AR order p and MA order q, respectively.
[2)] Standard error based on 1000 replications.
[3)] AO = additive outlier, IO = innovational outlier.
A = OLS estimation
B = Robust estimation: **OLS/wacf**
σ_o = standard deviation of outlier distribution in contamination
* = correct identification at 5% level (immediately preceding value > 5 in ESACF table)
n = sample size

Summary of simulation results: mean of ratio of standard errors of theoretical vertex in ESACF estimation[1)] for an ARMA(p, q) model, 1000 replications

Model ARMA(1, 1)	n	Ratios for different cases of standard error A	B	C	D	Outlier type[2)] and contamination proportion (%)
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 5$						
1	50	1.091	1.043	1.139	1.398	AO2
2	50	1.152	1.068	1.076	1.321	AO5
3	50	1.039	1.039	1.120	1.408	IO2
4	50	1.072	1.054	1.129	1.433	IO5
5	200	1.181	1.089	1.597	1.857	AO2
6	200	1.351	1.096	1.688	1.944	AO5
7	200	1.035	1.043	1.504	1.794	IO2
8	200	1.114	1.061	1.585	1.918	IO5

[1)] OLS, **OLS/wacf** estimation.
[2)] AO = additive outlier, IO = innovational outlier.
A = Simulation sample std (based 1000 replications): robust and non-robust estimate
B = Bartlett's formula: robust and non-robust estimate
C = Robust estimates: simulation sample std and Bartlett's formula
D = Ratio simulation sample std, robust estimate and asymptotic standard error $(n-k-j)^{-1/2}$
σ_o = Standard deviation of outlier distribution in contamination

Table 5.

Summary of simulation results: mean of theoretical vertex[1)] of pattern triangle in standard and OLS/wacf based ESACF table for ARIMA(p, d, q) model, 1000 replications

Model ARIMA (1, 1, 1)		Percentage: vertex value beyond ±2 standard errors[2)], %		Mean of theoretical vertex value (expected value = 0)		Outlier type[3)] and contamination proportion (%)
	n	A	B	A	B	
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 5$						
1	50	3.6	2.1*	0.029	0.003	AO2
2	50	6.0	4.5*	0.066	0.057	AO5
3	50	2.2*	2.1*	–0.021	–0.013	IO2
4	50	3.4*	2.4*	–0.020	0.005	IO5
5	200	9.2	4.1*	0.077	0.029	AO2
6	200	27.1	9.2	0.117	0.081	AO5
7	200	4.2*	4.1*	–0.011	–0.005	IO2
8	200	4.9*	4.6*	–0.002	0.013	IO5

[1)] The point at which row and column coordinates of vertex of a triangle of asymptotic 'zero' values correspond to AR order p and MA order q, respectively.
[2)] Standard error based on 1000 replications.
[3)] AO = additive outlier, IO = innovational outlier.
A = OLS estimation
B = Robust estimation: **OLS/wacf**
σ_o = standard deviation of the outlier distribution in contamination
* = correct identification at 5% level (immediately preceding value > 5 in ESACF table)

Summary of simulation results: mean of ratio of standard errors of theoretical vertex in ESACF simulation[1)] for ARIMA(p, d, q) model, 1000 replications

Model ARIMA (1, 1, 1)		Ratio for different cases of standard error				Outlier type[2)] and contamination proportion (%)
	n	A	B	C	D	
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 5$						
1	50	1.091	1.018	1.184	1.383	AO2
2	50	1.246	1.015	1.104	1.297	AO5
3	50	1.034	1.040	1.236	1.485	IO2
4	50	1.083	1.061	1.239	1.515	IO5
5	200	1.392	1.042	1.696	1.925	AO2
6	200	1.784	0.978	1.761	1.995	AO5
7	200	1.041	1.046	1.535	1.798	IO2
8	200	1.115	1.065	1.626	1.951	IO5

[1)] OLS, **OLS/wacf** simulation.
[2)] AO = additive outlier, IO = innovational outlier.
A = Simulation sample std (based 1000 replications): robust and non-robust estimate
B = Bartlett's formula: robust and non-robust estimate
C = Robust estimates: simulation sample std and Bartlett's formula
D = Ratio simulation sample std, robust estimate and asymptotic standard error $(n-k-j)^{-½}$
σ_o = standard deviation of outlier distribution in contamination
n = sample size

Table 6.

Summary of simulation results: mean values of ratio of standard errors of theoretical vertex in ESACF estimation[1)] for ARIMA(p, d, q) model, 1000 replications

Model ARIMA (1, 1, 1) n:o	n	A	B	C	D	Outlier type[2)] and contamination proportion (%)
		Ratio of different standard errors				
$\phi_1 = 0.6$ $\theta_1 = -0.4$ $\sigma_o = 5$						
17	50	1.057	1.015	1.103	1.290	AO2
18	50	1.123	1.011	1.017	1.188	AO5
19	50	1.016	1.038	1.233	1.474	IO2
20	50	0.999	1.044	1.188	1.435	IO5
21	200	1.090	1.037	1.300	1.472	AO2
22	200	1.072	0.965	1.095	1.220	AO5
23	200	0.945	1.043	1.425	1.676	IO2
24	200	0.928	1.053	1.352	1.597	IO5
$\phi_1 = 0.85$ $\theta_1 = 0.75$ $\sigma_o = 10$						
25	50	0.880	1.045	0.822	0.975	AO2
26	50	0.802	1.078	0.871	1.070	AO5
27	50	1.148	1.055	0.778	0.926	IO2
28	50	1.220	1.073	0.767	0.932	IO5
29	200	0.500	1.048	0.878	1.054	AO2
30	200	0.499	1.106	1.063	1.320	AO5
31	200	1.175	1.057	0.916	1.103	IO2
32	200	1.231	1.096	0.936	1.173	IO5

[1)] OLS/acf and MM/wacf simulations.
[2)] AO = additive outlier, IO = innovational outlier.
A = Simulation sample std (based on 1000 replications): robust and non-robust estimate
B = Bartlett's formula: robust and non-robust estimate
C = Robust estimates: simulation sample std and Bartlett's formula
D = Simulation sample std, robust estimate and asymptotic standard error $(n-k-j)^{-½}$
σ_o = Standard deviation of outlier distribution in contamination

Figure 8 shows the results of the OLS/wacf combination when the outlier type is IO. Standard ESACF gives a better result than the robust one, in terms of the Jarque-Bera test. The reason for the inferior robust estimate is the wacf function. Thus the experiments with different weight functions were useful (Wang & Wei 1993). Note that here the rmse of the robust ESACF is greater than the standard one. The results in Figure 8 are remarkable eg for nonstationary business cycle time series, which quite often are known to contain, after first differencing, contamination of the original IOs (see Chang 1982, p. 217).

Figure 8. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARMA(1, 1), phi = 0.80, theta = –0.70; n = 200; st.dev of IO = 10, simulations: 1000 replications; 2% IO outliers; robust (OLS/wacf) and standard (OLS/acf) autoregression / autocorrelation are used in ESACF pattern

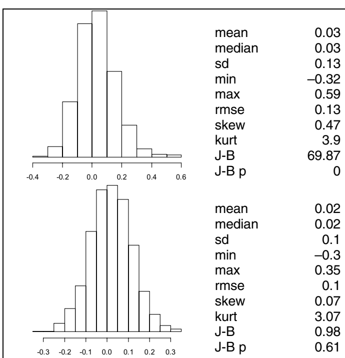


Figure 8a: robust (upper) and standard histogram

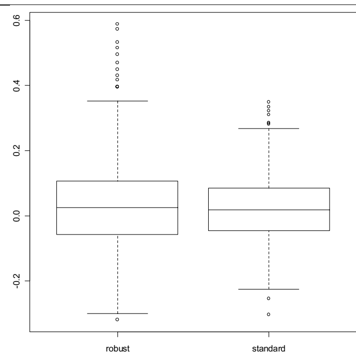


Figure 8b: robust and standard box plot

The ESACF procedure based on the OLS/wacf combination is a theoretically valid method in case of a nonstationary time series; here we have used an I(1) series. The results for the ARIMA(1, 1, 1) model are displayed in Table 5 and Figure 9. The robust OLS/wacf performs well and fails only in the AO5 case (Table 5). The results for the ratios of different standard errors are in our ARIMA(1, 1, 1) example quite similar to those for the ARMA(1, 1) (see Table 4). The histogram and box plot of the sample distribution of the standard and robust vertex in our ARIMA(1, 1, 1) model are displayed in Figure 9; as shown by the Jarque-Bera test values, the differences in shape between the standard and robust distributions are remarkable. The results of this 'extreme' model (high parameter values) indicate the power of the OLS/wacf combination with unit root and AOs in time series (here 2%). For more general results, further simulations are needed.

The results for the ARIMA(1, 1, 1) model, where AR and MA parameters, ϕ_1 and θ_1, are of medium size, but with high variability of the AO distribution, are displayed in Figure 10. The results show the 'power' of the MM/wacf combination also for the ARIMA(1, 1, 1) model. Note that the degree of AO contamination is 5% and the length of each simulated time series is only 50.

Further experiments are needed especially to study the biasedness of AR(p) estimates in cases of high contamination and patchy outliers (of selected types of outliers) with different ARIMA models. A future study might also include sensitivity analysis of certain model/ contamination/outlier type combination(s) in searching for the highest proportion of contamination(s), where such combination(s) are still robust.

Figure 9. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARIMA(1, 1, 1), phi = 0.85, theta = 0.75; n = 200; st.dev of AO = 10; simulations: 1000 replications; 2% isolated AO outliers; robust (OLS/wacf) and standard (OLS/acf) autoregression/autocorrelation are used in ESACF estimation

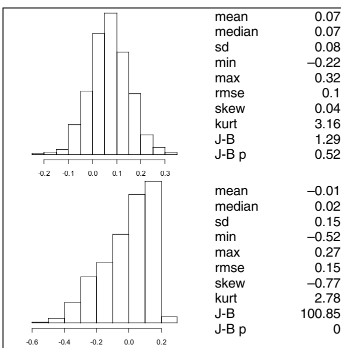


Figure 9a: robust (upper) and standard histogram

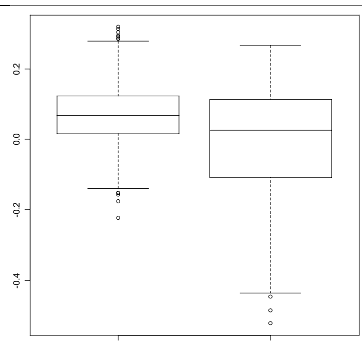
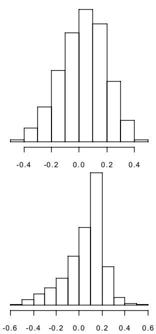


Figure 9b: robust and standard box plot

Figure 10. **Histograms and box plots of theoretical vertex in ESACF table**

Model: ARMA(1, 1, 1), phi = 0.60, theta = –0.40; n = 50; st.dev of AO = 10, simulations: 1000 replications; 5% isolated AO outliers; robust (MM/wacf) and standard (OLS/acf) autoregression/autocorrelation are used in ESACF estimation



mean	0.03
median	0.04
sd	0.16
min	–0.47
max	0.43
rmse	0.16
skew	–0.27
kurt	2.74
J-B	14.46
J-B p	0
mean	0.05
median	0.1
sd	0.16
min	–0.51
max	0.52
rmse	0.17
skew	–0.91
kurt	3.83
J-B	165.73
J-B p	0

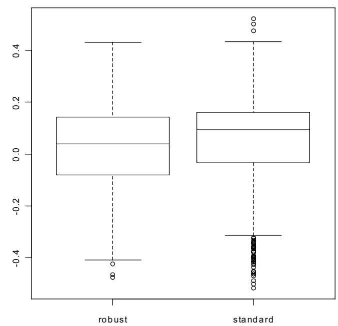


Figure 10a: robust (upper) and standard histogram

Figure 10b: robust and standard box plot

6.2.4 Sample distributions for single ESACF coefficients

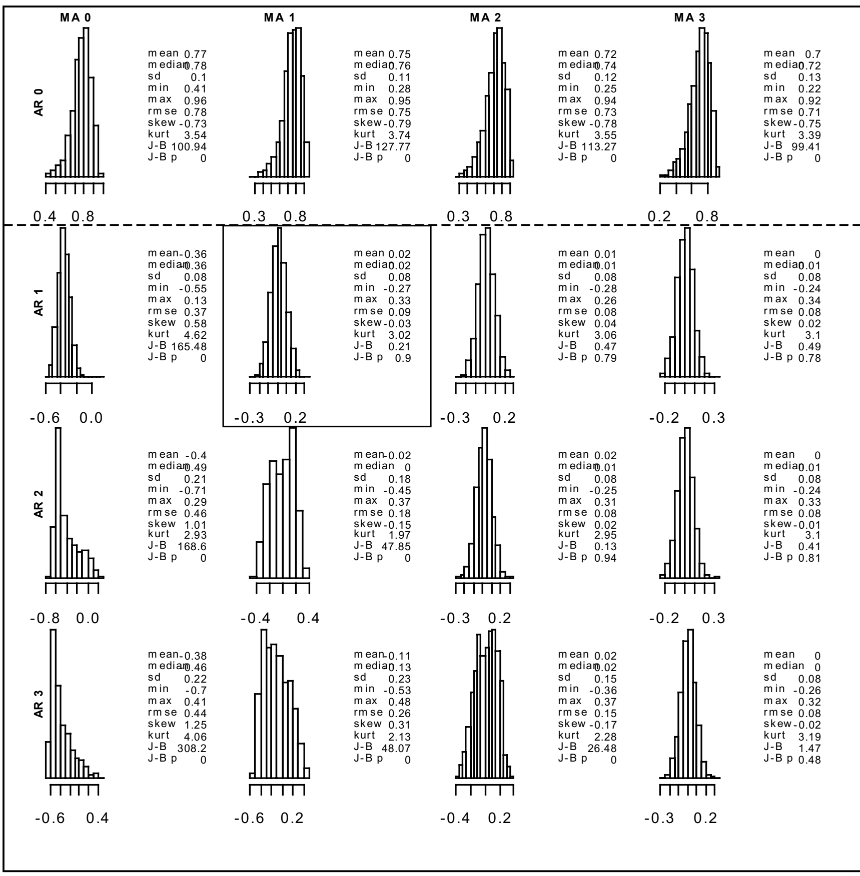
As previously mentioned, it seems there are no studies in the literature on the theoretical or sample distributions of single ESACF coefficient estimates. Because these coefficients are the sample autocorrelations of the transformed series $W_{k,t}^{(k)}$ in iterations, we can expect sample distributions analogous to those for the regular sample autocorrelation coefficients. Surprisingly, our simulation results show that most of these sample distributions of the standard ESACF are quite skewed and not clearly platykurtic[34] or leptokurtic, as can be seen from the multiple small figures of the ARMA(1, 1) model in Appendix 1. Most important is the sample distribution of the vertex. Its sample distribution is quite symmetric (see the special frame display). In general, skewness of sample distributions decreases as the length of a series increases. The sample distributions of the coefficients in the first *column* of the ESACF table (variables, see ESACF theory,

[34] The symmetric platykurtic distribution, with $b_2 < 3$ ($b_2 = 3$ in normal distribution), is characterised by a flatter top and more abrupt tails than the normal curve; the symmetric leptokurtic distribution, with $b_2 > 3$, has a sharper peak at the mean and more extended tails.

Figure 11. **Histograms of first 16 coefficient estimates of upper-left part of ESACF matrix**

Calculations based on simulation, 1000 replications. Area of SACF coefficients (first row) separated by a dotted line.

Robust ESACF

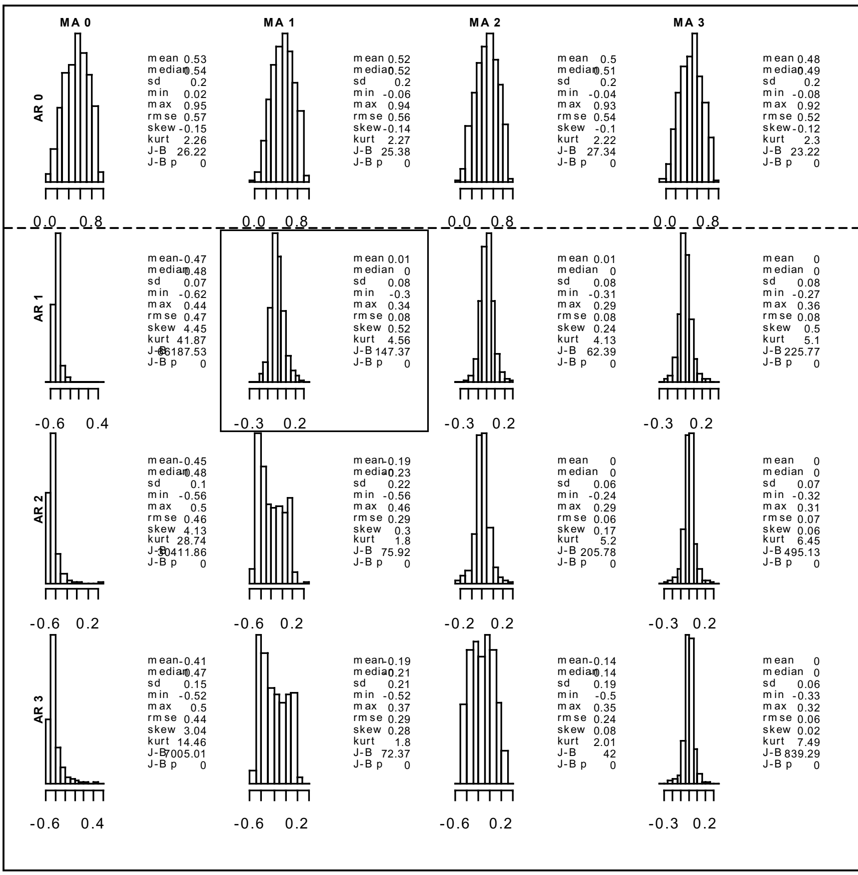


Theoretical vertex frame

Model: ARIMA(0, 1, 1), theta = 0.50
5% AO outliers
Autoregression: MM-estimator
Autocorrelation: wacf
St.dev. of outliers = 10
n = 200
ndiff = d = 1 (I(1) series)

cont.

Standard ESACF



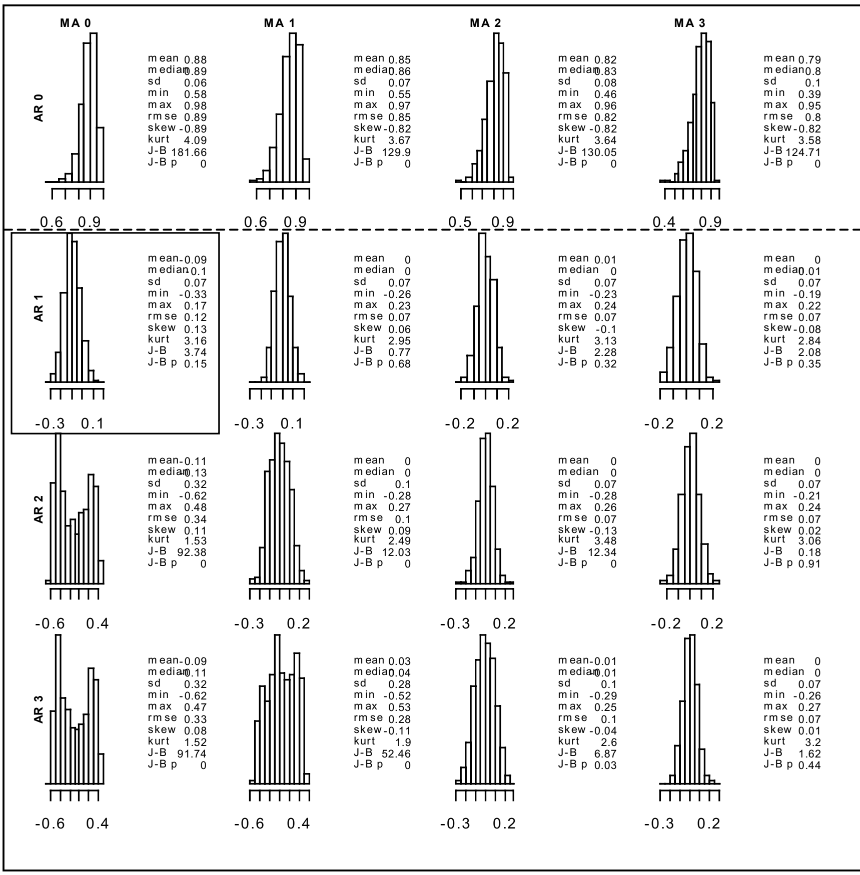
Theoretical vertex frame

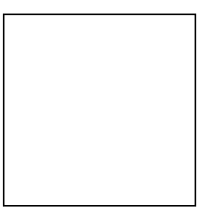
Model: ARIMA(0, 1, 1), theta = 0.50
5% AO outliers
Autoregression: OLS-estimator
Autocorrelation: standard acf
St.dev. of outliers = 10
n = 200
ndiff = d = 1 (I(1) series)

Figure 12. **Histograms of first 16 coefficient estimates of upper-left part of ESACF matrix**

Calculations based on simulation, 1000 replications. Area of SACF coefficients (first row) separated by a dotted line.

Robust ESACF



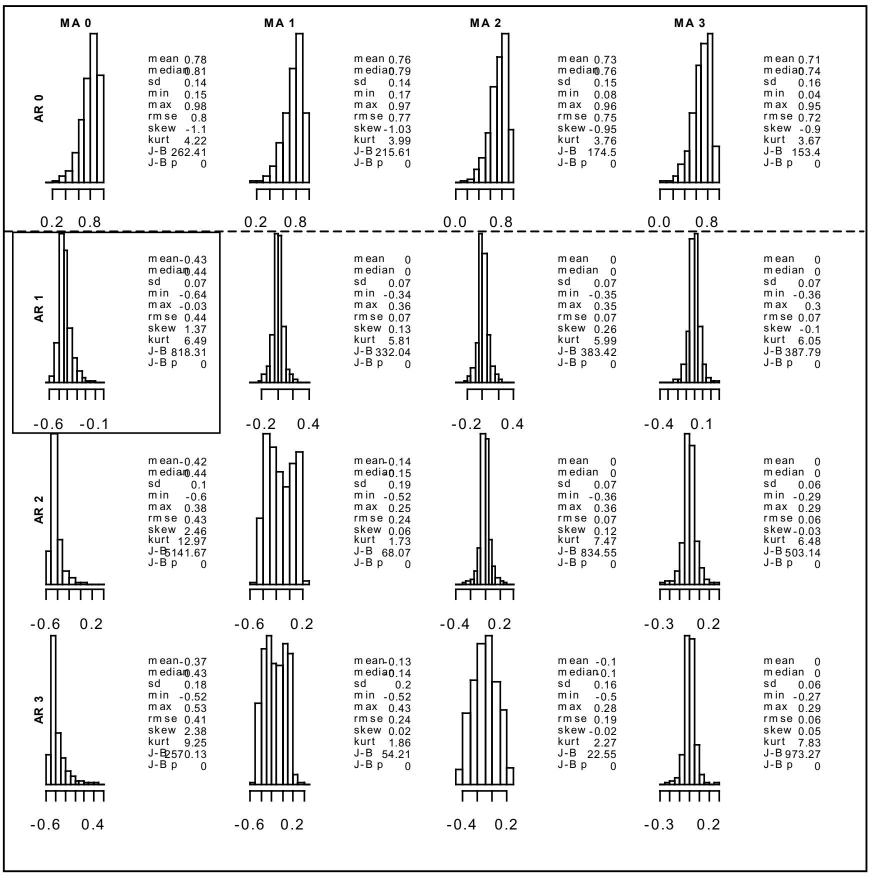


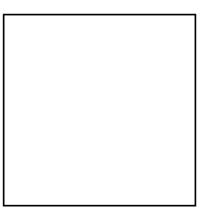
Theoretical vertex frame

Model: ARIMA(0, 1, 0)
5% AO outliers
Autoregression: MM-estimator
Autocorrelation: wacf
St.dev. of outliers = 10
n = 200
ndiff = d = 1 (I(1) series)

cont.

Standard ESACF





Theoretical
vertex
frame

Model: ARIMA(0, 1, 0)
5% AO outliers
Autoregression: OLS-estimator
Autocorrelation: standard acf
St.dev. of outliers = 10
n = 200
ndiff = d = 1 (I(1) series)

Section 3.2.3 of this thesis and TT84, p. 87) are extraordinary in form. In the first column the sample distributions of coefficient estimates may be bimodal from the second row on. The slow decrease in variability, with increasing lag q, can be found in general. This feature can also be seen in Figures 11 and 12.

The simulation results indicate that robustifying the ESACF procedure has direct effects on the sample distributions of the ESACF coefficients. Generally, the sample distributions are 'normalised' in case of robust ESACF. This means increased performance of ESACF procedure as an identification tool. We found no distinction between cases of isolated and patchy outliers, but further research is needed on this question. The normalising effect can be seen in the increased p value of the Jarque-Bera test. Skewness is diminished and/or kurtosis turned towards a value of 3, ie of normal distribution. However, the changes in these sample distributions are dependent on ARMA structure and sample size and therefore no general exact 'rules' can be obtained from our simulation results.

A striking feature of the robust cases is the increase in variability of ESACF coefficients compared with standard ESACF. The robust sample distributions have in most cases a more clear-cut form compared with the standard ones. If we compare the AO results of GM- and MM/wacf and OLS/wacf based ESACF, the skewness remains more often in the OLS/wacf case. This indicates greater bias for OLS estimates in the AR(p) fitting stage of the ESACF procedure. Often in an OLS/wacf case, either skewness or kurtosis changes towards the value of normal distribution, ie if skewness decreases, kurtosis increases and vice versa, compared with the standard ESACF. With outlier-free series, the sample distribution of the theoretical vertex is similar in shape to the standard and robust ESACF. Although we analyse here mainly the theoretical vertex points, the example of the ARMA(1, 1) model of Appendix 1 and the results of Figures 11 and 12 show how the sample distribution of the other ESACF coefficients changes in the robust case, particularly in a triangle area of zeros.

Finally, we would emphasise that, besides the results for the standard and robust ESACF estimates, we obtain in every case the standard and robust results for the ordinary SACF estimates. In the histograms of Figures 11–12 and in Appendix 1, we can see the sample distributions of the robust and standard coefficients, for both ESACF and ordinary SACF.

6.2.5 Summary of main results

The simulation results of the ESACF procedure support the general experience that AO outliers have more destroying effects on statistical estimates than do IO outliers. The results show that the standard ESACF procedure is robust to some extent, with low or moderate contamination of data.

It is useful to compare results for the different standard errors. The standard error of the robust estimator is larger than that of the standard one[35]. From the results of the ARMA(1, 1) model in Appendix 1 we see how the asymptotic standard error $(n-k-j)^{-1/2}$ ('Standard asymptotic STD') is clearly smaller than the Bartlett's formula- and the simulation-based standard error estimates. This result supports the suspicion of Tiao (1985, p. 101) that the simple approximation $(n-k-j)^{-1}$ underestimate the variance of ESACF coefficients, $r_{j(k)}$.

Another new aspect of this thesis is the calculation of the asymptotic distributions of single ESACF coefficients based on simulation experiments. There are no studies in the literature on the sampling properties of extended autocorrelations of TT84. In Appendix 1, the histograms of 16 single ESACF estimates for an ARMA(1, 1) model are displayed. The histograms are, in the standard case, highly kurtic and skewed.

In summary, the most striking findings of the main Monte Carlo results are:

1. the sample distributions of the standard ESACF coefficients generally show skewness and excess kurtosis
2. there were normalisation effects of robustifying on these sample distributions in most cases
3. the differences between sizes of the various standard error estimates
4. in outlier-free time series, the standard ESACF and robust ESACF based on MM/wacf perform equally well (see also Hella 2002)

Overall, the standard errors of robust ESACF estimates based directly on simulation replications are greater than those of a) similar estimates of standard ESACF, b) standard asymptotic estimates $(n-k-j)^{-1/2}$, and

[35] The confidence interval properties of robust estimators are in most cases an open question. Birch and Martin (1981) investigated confidence intervals of some robust estimators of AR(1) processes; Gross (1977) studied confidence intervals for bisquare regression estimates. The robust regression estimators and their standard error are considered in Staudte and Sheather (1990, Section 7.6).

c) estimates based on the Bartlett approximation formula in standard ESACF. Also in the Bartlett case, the robust version has slightly greater variability than in the standard one (see robust formulas 5.13 and 5.14). This feature is related to the robust confidence interval analysis of estimators (see Section 5.4). An open question arises: how can these results be utilised for constructing robust confidence intervals for single ESACF coefficient estimates.

6.3 Three examples: nonstationarity, outliers and differencing

A. ARIMA(1, 1, 0) model

For the generated ARIMA(1, 1, 0) model, with $\phi_1 = 0.8$, N = 200 and noise variable $\varepsilon_t = N(0,1)$, we applied the ESACF directly to the generated series. The standard ESACF pattern indicated correctly an AR(2) model (AR(1 + 1); see Pankratz 1991, p. 63). If we assign an AO outlier of value $\omega = 2.3$ at t = 121 (ie 4.0% of true value $y_{121} = 57.7$), the standard ESACF works well, but if we increase ω to 7.3 (12.7% of true value) the standard ESACF breaks down and identifies an ARIMA(1, 1, 1). The robust ESACF, based on the MM-estimator, performs well for $\omega = 7.3$.

```
The vertex of ESACF table is marked in bold.

Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.99	0.98	0.97	0.96	0.95	0.93
AR1	0.72	0.53	0.31	0.17	0.11	0.06
AR2	**-0.03**	0.20	0.07	-0.10	0.04	-0.08
AR3	0.12	0.20	0.11	-0.03	-0.01	-0.13

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	X	0	0	0
AR2	**0**	X	0	0	0	0
AR3	0	X	0	0	0	0

```
Values above marked with X are more than 2 * std errors away from zero,
using Bartlett type std error.


> y[121]<-60
> esacf(y)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.99	0.98	0.97	0.96	0.95	0.93
AR1	0.71	0.52	0.31	0.16	0.10	0.09
AR2	**-0.06**	0.18	0.10	-0.10	-0.03	0.06
AR3	0.28	0.18	0.13	-0.13	0.02	0.09

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	X	0	0	0
AR2	**0**	X	0	0	0	0
AR3	X	X	0	0	0	0

```
> y[121]<-65
> esacf(y)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.99	0.98	0.97	0.96	0.95	0.93
AR1	0.52	0.44	0.27	0.12	0.07	0.09
AR2	-0.38	**0.15**	0.10	-0.06	-0.06	0.01
AR3	0.14	0.23	0.15	-0.04	-0.10	0.02

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	X	0	0	0
AR2	X	**0**	0	0	0	0
AR3	0	X	0	0	0	0

```
Values above marked with X are more than 2 * std errors away from zero,
using Bartlett type std error.

> esacf(y,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.99	0.98	0.97	0.96	0.95	0.93
AR1	0.69	0.51	0.31	0.15	0.09	0.08
AR2	**-0.08**	0.16	0.10	0.02	-0.01	-0.14
AR3	0.34	0.15	0.06	0.01	0.03	-0.13

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	X	0	0	0
AR2	**0**	X	0	0	0	0
AR3	X	0	0	0	0	0

B. ARIMA(1, 1, 1) model

In Section 5.5 we considered the robust ESACF as a complementary tool in robust unit root testing. In this example we will show: 1) how the robust ESACF performs in the case of the generated outlier-free nonstationary series and 2) when the outliers are placed into this series, what is the outcome of the identification for the standard ESACF and various robust ESACF versions. Finally, we consider the outliers and identification when first differences are used.

The process ARIMA(1, 1, 1) is generated with parameters $\phi_1 = 0.7$ and $\theta_1 = 0.4$, sample size is $n = 200$ and noise variable $\varepsilon_t = N(0,1)$. The artificial series is given in Figure 13 below (the generated data can be obtained on request).

As can be seen from the following tables, the ESACF estimation is successful with both OLS and robust MM. In the ESACF table, we find the vertex in coordinates (AR2, MA1) ie an ARMA(p’, 1) model, where p = 1 and d = 1, so that p’ = p + d = 2 contains a unit root (see Pankratz 1991, p. 75).

The vertex of ESACF table is marked in **bold**.

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting
and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.97	0.94	0.91	0.87	0.84	0.81
AR1	0.35	0.27	0.15	0.01	0.02	-0.03
AR2	-0.40	**0.09**	0.14	-0.12	0.01	-0.04
AR3	-0.09	0.19	0.14	0.03	0.08	-0.02

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	0	0	0	0
AR2	X	**0**	0	0	0	0
AR3	0	0	0	0	0	0

Values above marked with X are more than 2 * std errors away from zero,
using Bartlett type std error.

```
> esacf(y,method=MM)
```

Extended Autocorrelation Table
Calculated using **MM** method for AR fitting
and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.96	0.93	0.90	0.87	0.83
AR1	0.33	0.30	0.17	0.02	0.05	0.00
AR2	-0.35	**0.12**	0.19	0.07	0.05	-0.03
AR3	0.05	0.19	0.19	0.07	0.09	0.02

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	0	0	0	0
AR2	X	**0**	X	0	0	0
AR3	0	0	X	0	0	0

Figure 13. **Original series**

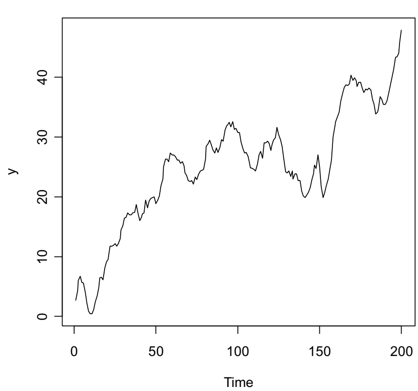
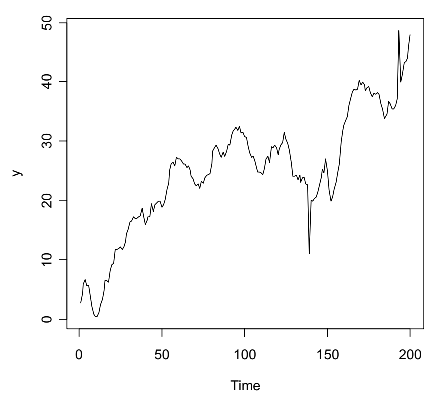


Figure 14. **Original series including two isolated AO outliers = series G**



Outliers

In the next case, two additive outliers are located at $t_1 = 139$, $\omega_1 = -10$ and $t_2 = 193$, $\omega_2 = 10$. The contaminated series is displayed in Figure 14. In identification, the standard ESACF breaks down and indicates an ARMA(1, 1) model while the robust versions, the MM- and GM-estimators and the combination of OLS/wacf indicate correctly an ARIMA(1, 1, 1) process.

```
> y[139]<-10.98
> y[193]<-48.68
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.96	0.93	0.90	0.87	0.84	0.81
AR1	-0.20	**0.10**	0.06	-0.02	0.03	-0.01
AR2	0.28	0.14	0.10	0.02	0.03	0.03
AR3	-0.47	0.32	0.15	-0.07	0.02	0.03

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

```
Values above marked with X are more than 2 * std errors away from zero,
using Bartlett type std error.
```

```
> esacf(y,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.95	0.92	0.89	0.86	0.83
AR1	0.25	0.25	0.14	0.02	0.06	-0.02
AR2	-0.44	**0.06**	0.07	-0.08	0.06	-0.05
AR3	-0.35	0.19	0.06	-0.12	0.03	0.00

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	0	0	0	0
AR2	X	**0**	0	0	0	0
AR3	X	0	0	0	0	0

First-order differencing

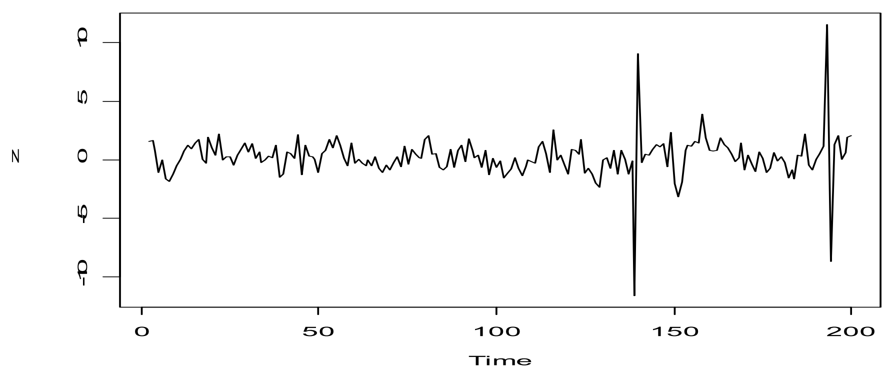
In the following phase the contaminated series, y, was differenced once. The transformed series, z, is displayed in Figure 15. We find that, due to the first-order differencing, the number of AO outliers is doubled (d+1; see Chang 1982, p. 118 and 217) to four[36]. If our series were a real nonstationary series, we would encounter the problem of how to interpretate these additional, spurious AO outliers. Note that with an isolated IO we still have this outlier in the differenced series (Chang 1982, p. 217).

The result of the standard ESACF for z indicates an MA(1) model (next ESACF table). The model is identified correctly as ARMA(1, 1) by the robust ESACF procedure.

[36] For each old isolated AO we have a new immediately following AO of the same size but opposite sign.

```
> z<-diff(y,lag=1)
> esacf(z)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.

       MA0   MA1   MA2   MA3   MA4   MA5          MA0 MA1 MA2 MA3 MA4 MA5
AR0  -0.20  0.10  0.05 -0.02  0.03 -0.01      AR0  X   0   0   0   0   0
AR1   0.27  0.14  0.10  0.02  0.02  0.03      AR1  X   0   0   0   0   0
AR2  -0.46  0.32  0.13 -0.07  0.02  0.03      AR2  X   X   0   0   0   0
AR3   0.01 -0.09  0.05 -0.06  0.02  0.00      AR3  0   0   0   0   0   0

Values above marked with X are more than 2 std errors away from
zero,using Bartlett type std error.


> esacf(z,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.

       MA0   MA1   MA2   MA3   MA4   MA5          MA0 MA1 MA2 MA3 MA4 MA5
AR0   0.26  0.25  0.14  0.02  0.06 -0.02      AR0  X   X   0   0   0   0
AR1  -0.44  0.06  0.07 -0.06  0.07 -0.02      AR1  X   0   0   0   0   0
AR2  -0.35  0.19  0.06 -0.11  0.04  0.00      AR2  X   0   0   0   0   0
AR3   0.09  0.27  0.17 -0.10  0.05  0.01      AR3  0   X   X   0   0   0
```

Figure 15. **First-order differences of G-series**



C. ARIMA(0, 1, 1) model

Lastly we generated a nonstationary ARIMA(0, 1, 1) series, often denoted an IMA(1, 1) series. This model is quite common in various process industries and in macroeconomics, especially in modelling business cycles.

We generated the series with parameter $\theta_1 = 0.6$, sample n = 200, and noise variable $\varepsilon_t = N(0,1)$. The artificial series is given in Figure 16 below. The standard and robust ESACF approach is applied directly. The results are the following.

```
The vertex of the ESACF table is marked in bold.

Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.72	0.68	0.64	0.63	0.58	0.49
AR1	-0.41	**-0.03**	-0.04	0.10	0.07	-0.19
AR2	-0.45	0.12	-0.04	0.08	0.05	-0.19
AR3	-0.49	-0.11	-0.19	0.03	0.00	-0.15

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	X
AR2	X	0	0	0	0	X
AR3	X	0	0	0	0	0

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.


> esacf(y,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.75	0.68	0.67	0.66	0.59	0.52
AR1	-0.35	**-0.06**	-0.05	0.12	0.05	-0.20
AR2	-0.46	-0.04	-0.06	0.08	0.06	-0.20
AR3	-0.47	0.13	-0.18	0.04	0.01	-0.12

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	X
AR2	X	0	0	0	0	X
AR3	X	0	0	0	0	0

As can be seen, the vertices show that both the standard and robust ESACF identified the correct model. The single estimates in the tables have quite similar values. It is notable that, due to the ESACF approach, we find directly the MA(1) part of the model. The SACF estimates in both of the ESACF tables (first row) indicate nonstationarity.

Two AO outliers

Next we added two isolated AO outliers to the original series at t = 74 and 144. In the first case, we introduced the classic decimal point error (original value 2.6954; see below) and in the second case the sign error (original value +6.1791). The new series (F) is displayed in Figure 17 below.

```
> y[74]<-26.954
> y[144]<--6.1791
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.32	0.31	0.27	0.30	0.32	0.31
AR1	-0.49	**0.02**	-0.05	0.02	0.02	0.03
AR2	-0.45	-0.24	-0.06	0.03	0.00	0.05
AR3	-0.49	-0.24	-0.37	0.03	0.00	0.03

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	X	0	0	0	0
AR3	X	X	X	0	0	0

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.
```

```
> esacf(y,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.71	0.65	0.63	0.64	0.60	0.54
AR1	-0.29	**-0.03**	-0.07	0.11	0.08	-0.17
AR2	-0.49	0.04	-0.05	0.11	0.10	-0.17
AR3	-0.45	0.18	-0.16	0.12	0.09	-0.17

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

The results are interesting: a) standard ESACF indicates an ARMA(1, 1) model, not a unit root, because the regular sample autocorrelations in the first row of the standard ESACF table are distorted by AOs; b) in the robust (MM/wacf) case, the sample autocorrelations in the ESACF table indicate the existence of a unit root and c) due to the robust ESACF approach we are able to identify the correct model (we should then estimate robust $\hat{\phi}_1$ and $\hat{\theta}_1$ to check the actual model). This identification illustrates the extreme case in which we have not checked the series before modelling and there occur two classic outliers; graphical inspection here reveals clearly the AOs and we adjust them before the identification task. However, in practice it may often be difficult to discover (or confirm) outliers eg by graphical methods before modelling, and the robust procedures can offer ‘insurance’.

Figure 16. **Original series**

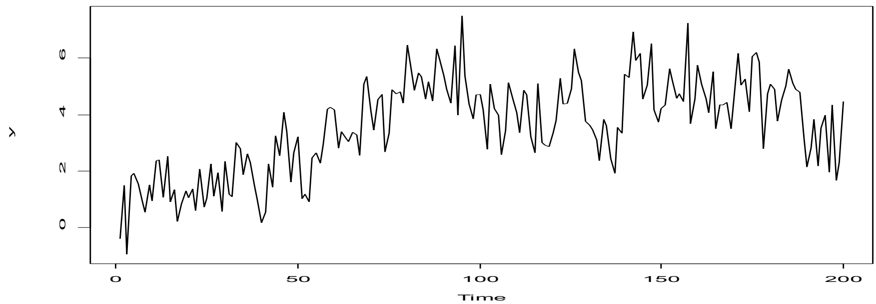
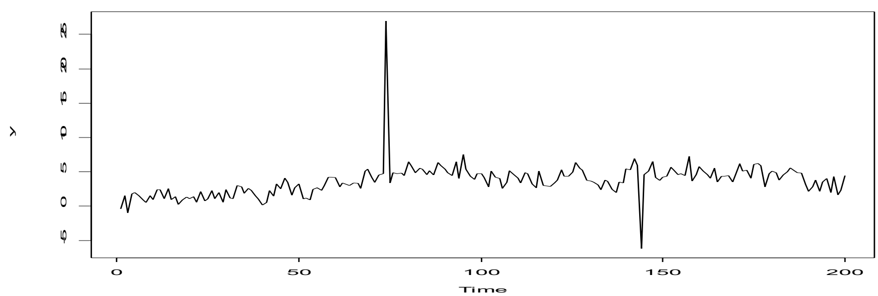


Figure 17. **Original series including two isolated AO outliers = F**



7 Applications of some real time series

7.1 Some illustrative examples from the literature

A. Spirit series

In the following we consider briefly the residual series of the famous spirit consumption model of Prest(1949). In the literature, Fuller (1996), Tsay (1986), Lee (1989), Chen (1994) and others have analysed this residual series (Figure 18 below in text).

Tsay (1986) applied to this residual series the ESACF identification procedure combined with an iterative outlier detection and estimation method. The ESACF analysis of the original residual series indicated an AR(1) model (Tsay 1986, Table 2). In the second phase Tsay conducted the iterative outlier detection process with AR(1) as the initial model structure of this residual series. After six iterations, he obtained the outlier-adjusted version of the residual series. The original ESACF analysis of this *adjusted* residual series resulted in an ARMA(1, 1) model (Tsay 1986, Table 4).

In the following we also applied the standard ESACF procedure to the spirit residual series and, in addition, three robust versions of the ESACF. The results of our estimations show that by a robust ESACF method we are able *directly* to find an ARMA(1, 1) model instead of the AR(1) process found by Tsay.

In the next table we report standard ESACF and robust ESACF results for spirit residual series. As a robust version, we use the combinations MM/wacf, GM/wacf and OLS/wacf.

ESACF pattern estimates:

The vertex of ESACF table is marked in **bold**.

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting
and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.72	0.46	0.26	0.15	0.01	-0.14
AR1	**0.14**	0.13	0.01	0.17	0.02	-0.16
AR2	-0.49	0.12	-0.01	0.14	0.09	-0.16
AR3	0.24	-0.15	-0.05	0.12	0.12	-0.15

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	0	0	0
AR1	**0**	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	0	0	0	0	0	0

Values above marked with X are more than 2 std errors away from zero, using Bartlett type std error.

> esacf(spirit,method=MM)
Extended Autocorrelation Table
Calculated using **MM** method for AR fitting
and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.77	0.51	0.29	0.17	0.03	-0.13
AR1	0.41	**0.17**	0.16	0.19	0.05	-0.16
AR2	-0.10	0.08	0.09	0.00	0.07	-0.15
AR3	-0.12	-0.47	-0.07	0.10	0.07	-0.04

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	0	0	0
AR1	X	**0**	0	0	0	0
AR2	0	0	0	0	0	0
AR3	0	X	0	0	0	0

> esacf(spirit,method=GM)
Extended Autocorrelation Table
Calculated using **GM** method for AR fitting
and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.77	0.51	0.29	0.17	0.03	-0.13
AR1	0.41	**0.25**	0.23	0.19	0.02	-0.17
AR2	-0.44	0.03	0.01	-0.02	0.07	-0.10
AR3	-0.56	-0.44	-0.05	0.07	0.10	0.01

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	0	0	0
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	X	0	0	0	0

> esacf(spirit,method=ols,acf.fun=wacf)
Extended Autocorrelation Table
Calculated using **ols** method for AR fitting
and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.77	0.51	0.29	0.17	0.03	-0.13
AR1	0.48	**0.28**	0.09	0.19	0.04	-0.17
AR2	-0.39	0.25	0.09	-0.02	0.11	-0.15
AR3	0.47	-0.14	0.02	0.08	0.09	-0.07

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	0	0	0
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

It is worth noting that all the robust ESACF versions identified the same ARMA structure.

Figure 18. **Residual series of the spirit model**

The preceding results show that, if Tsay had had the possibility of using the robustified ESACF method, he could have estimated an ARMA(1, 1) tentative structure directly to start the combined iterative ARMA modelling procedure for detecting and estimating outliers (Tsay 1986, Section 2.5). Perhaps he would have detected and estimated a different set of outliers.

It is well known that the detected set of outliers is dependent on the underlying core model (see eg Findley et al 1986, p. 141). As Stockinger and Dutter (1987, p. 87) remark, the inadequate model would declare a 'normal' observation to be atypical. In general, when we use robust identification and estimation methods, we obtain true outliers for which we usually can find a meaningful interpretation.

B. RESEX series

As the second example, we analyse the well-known RESEX monthly series (residence telephone extensions inward movement) analysed originally by Martin, Samarov and Vandaele (1983). Masarotto (1987a) applied this series as an experiment for his version of the robust autocorrelation and partial autocorrelation function (robust lattice procedure). Because of the seasonal component, Masarotto used the transformed 12-lag differenced series, RESEX12. In the following, we apply both standard and robust ESACF procedures to the series RESEX12.

ESACF pattern estimates:

```
> esacf(resex12)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.41	**0.03**	0.02	0.03	0.01	0.09
AR1	0.36	-0.31	-0.01	0.02	0.01	0.11
AR2	0.44	-0.30	-0.12	0.01	0.08	0.11
AR3	0.21	-0.09	-0.06	0.00	0.03	0.11

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	**0**	0	0	0	0
AR1	X	X	0	0	0	0
AR2	X	X	0	0	0	0
AR3	0	0	0	0	0	0

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.
```

```
> esacf(resex12,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.65	0.59	0.51	0.43	0.36	0.35
AR1	-0.36	**-0.01**	0.12	0.10	-0.04	0.11
AR2	-0.27	0.04	0.20	0.00	-0.10	0.10
AR3	0.41	-0.16	-0.02	-0.05	-0.15	0.07

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

In comparing the results of Masarotto (1987a) with our results, the following points can be made:

1. When we use the standard ESACF, an MA(1) model is suggested as in Masarotto (1987a, Table 3).

2. Our robust ESACF procedure (MM/wacf) suggests an ARMA(1, 1) model, as in Masarotto, on the grounds of his robust SACF and SPACF procedures (1987a, Table 2). Masarotto used the M-estimator in his robust lattice procedure.

3. The robust sample autocorrelation estimates found by Masarotto are nearly the same as the sample autocorrelation estimates in the first row of our robust ESACF table.

C. The Lydia Pinkham annual advertising data

The well-known Lydia Pinkham annual advertising data (Figure 22, Appendix 5) have been used by many authors in time series modelling. The literature includes reports of different research results. For instance, Chan (1989) carried out robust modelling and detected a set of outliers in the series. Wei (1990, 1994) used the series in constructing transfer function models. Chan and Wei obtained a different form of ARMA model. Hoek et al (1995) and Lucas (1996) estimated a unit root value for this series. Thus, it is proper to investigate the identification of this series also by standard and robust ESACF procedure.

Chan (1989, p. 58–62) identified, using his proposed trimmed (robust) SACF and SPACF, a tentative ARMA(1, 1) model and used his modified iterative modelling procedure to obtain the same form of final model with four significant outliers. Wei (1990, 1994, p. 304) examined the SACF and SPACF of the original series (X_t) and of its first differences (x_t) and concluded that X_t is nonstationary and that a stationary series x_t can be modelled as an AR(2) process.

We carried out a sequence of quick computer runs of ESACF estimation by standard and different robust estimators and with different versions of robust autocorrelation functions. The results, shown in Appendix 5, indicate that the original series, X_t, is a stationary AR(1) or nonstationary ARIMA(0, 1, 0) model, except with OLS/tacf, which indicated an ARMA(1, 1) model. For the series x_t, both standard and robust ESACF indicate a white noise model. Note that Chan (1989) found four outliers in the original series, X_t. Differencing the series X_t generated additional (spurious) outliers (Figure 23, Appendix 5).

Perhaps we would detect a different group of outliers than Chan (1989) if we began the iterative modelling procedure with an AR(1) or an ARIMA(0, 1, 0) scheme. Then comparison of the results concerning the final ARMA model and detected outliers might be useful.

The Lydia Pinham annual series is an example of a time series that is difficult to model. So it is important to use various identification tools to search for a possible underlying model. Here both standard and robust ESACF estimations encountered problems in this task.

7.2 Three monetary time series

Bond rate series

In practice a researcher often encounters the difficult problem of choosing between two or three potential candidates for a tentative model. In such a situation (often in the case of a nonstationary series), it is important to have available a quick procedure of both standard and robust identification to apply and to evaluate and compare results.

In the following we conduct experiments which should provide the tentative model structure of the monthly bond rate series[37]. We analyse the series by the standard ESACF and robust ESACF based on the MM/wacf and OLS/wacf combinations.

```
ESACF pattern estimates:

The vertex of ESACF table is marked in bold.

> esacf(bonds)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.94	0.90	0.86	0.81	0.77
AR1	0.39	**0.10**	0.05	0.06	0.14	0.09
AR2	0.16	-0.14	-0.04	-0.02	0.13	0.01
AR3	0.47	-0.13	0.03	-0.02	0.12	0.00

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.

> esacf(bonds,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.93	0.89	0.86	0.82	0.78
AR1	0.42	**0.17**	0.06	0.09	0.13	0.08
AR2	0.02	0.05	-0.10	0.01	0.10	0.00
AR3	-0.22	0.02	-0.19	0.01	0.11	0.00

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	0	0	0	0	0	0
AR3	X	0	X	0	0	0

```
> esacf(bonds,method=ols,acf.fun=wacf)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.93	0.89	0.86	0.82	0.78
AR1	0.42	**0.16**	0.06	0.08	0.14	0.08
AR2	0.20	-0.03	-0.12	-0.01	0.10	0.02
AR3	0.52	-0.10	-0.08	-0.02	0.10	0.01

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

The results of the standard and robust ESACF suggest that an ARMA(1, 1) model would be the relevant tentative structure for our bond rate series. If there are many series to model, this kind of quick procedure may help to classify the series into 'difficult and clear cases' and thus indicate which require more effort and time for explorative analysis. Tsay (2002) has applied the standard ESACF to

[37] Source: Rousseeuw and Leroy (1987).

the monthly stock returns series of the 3M company and obtained an ARMA(0, 0) model. Because this series seems to contain some outliers (Figure 2.7, op cit), it might be interesting to apply also a robust ESACF and compare the results.

Money stock M3: Finland

The standard and robust ESACF procedures were applied to the Finnish M3 money stock (January 1990 – December 2000). The results of the standard and robust identification indicate a tentative ARIMA(0, 1, 0) model. Instead of an MA(q) model there is a white noise component in the Finnish M3 monthly series. The standard and robust ESACF results are displayed below. The log transformation was applied to M3 before the computer runs.

ESACF pattern estimates:

```
> esacf(log(M3Fin))
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.97	0.94	0.91	0.88	0.85	0.81
AR1	**-0.15**	-0.07	0.01	-0.10	0.07	-0.08
AR2	-0.44	-0.09	-0.02	-0.08	-0.02	-0.08
AR3	-0.17	-0.15	0.05	-0.06	0.05	-0.05

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	0	0	0	0	0	0

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.
```

```
> esacf(log(M3Fin),method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.97	0.94	0.92	0.89	0.87	0.85
AR1	**-0.17**	-0.04	0.00	-0.17	0.04	-0.03
AR2	-0.49	-0.10	-0.05	-0.19	-0.08	-0.04
AR3	-0.06	-0.14	0.00	-0.18	0.05	0.00

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	0	0	0	0	0	0

```
> esacf(log(M3Fin),method=ols,acf.fun=wacf)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.97	0.94	0.92	0.89	0.87	0.85
AR1	**-0.17**	-0.04	0.00	-0.17	0.05	-0.03
AR2	-0.46	-0.07	-0.04	-0.18	-0.08	-0.04
AR3	-0.18	-0.14	0.03	-0.13	-0.01	0.00

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	0	0	0	0	0	0

First-order differencing[38]

We take the first-order differences of log (M3Fin). The standard and robust ESACF results indicate that the transformed series, M3Fin1, is white noise, as Figure 19 below and the following ESACF tables show.

```
> M3Fin1<-diff(log(M3Fin),lag=1)

> esacf(M3Fin1)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**-0.16**	-0.07	0.01	-0.10	0.07	-0.08
AR1	-0.45	-0.09	-0.03	-0.10	-0.03	-0.09
AR2	-0.22	-0.14	0.05	-0.05	0.05	-0.07
AR3	-0.17	-0.44	-0.30	0.04	0.00	-0.08

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	0	0	0
AR1	X	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	0	X	X	0	0	0

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.


> esacf(M3Fin1,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**-0.17**	-0.06	0.00	-0.17	0.04	-0.03
AR1	-0.50	-0.10	-0.04	-0.18	-0.07	-0.03
AR2	-0.12	-0.15	0.01	-0.16	0.05	-0.01
AR3	-0.10	-0.43	-0.42	-0.14	0.00	-0.03

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	0	0	0
AR1	X	0	0	0	0	0
AR2	0	0	0	0	0	0
AR3	0	X	X	0	0	0

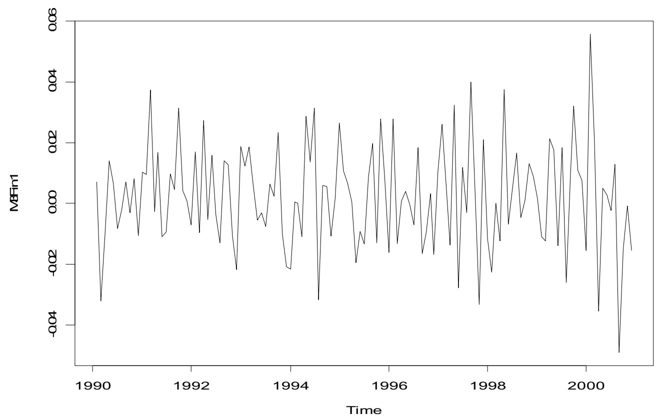
```
> esacf(M3Fin1,method=ols,acf.fun=wacf)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**-0.17**	-0.06	0.00	-0.17	0.04	-0.03
AR1	-0.46	-0.07	-0.03	-0.18	-0.07	-0.04
AR2	-0.23	-0.14	0.03	-0.11	0.00	-0.02
AR3	-0.18	-0.43	-0.34	-0.06	-0.03	-0.04

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	0	0	0
AR1	X	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	0	X	X	0	0	0

[38] The author refers to the concepts of difference-stationary and trend-stationary, which are important in econometric modelling (eg Chatfield 1996, p. 235). In our thesis we make the series stationary by differencing.

Figure 19. **First-order differences of log (M3Fin)**
y-axis: M3Fin1



Money stock M3: Euro area

The identification result for the euro area M3 is similar to the Finnish M3 series. All the ESACF analyses tentatively indicate an ARIMA(0, 1, 0) model.

```
ESACF pattern estimates:

> esacf(log(M3Euro))
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.97	0.94	0.91	0.88	0.84	0.82
AR1	**−0.16**	−0.16	0.04	−0.19	0.01	0.34
AR2	−0.50	−0.20	−0.03	−0.17	0.01	0.33
AR3	−0.12	−0.37	0.11	−0.12	0.02	0.23

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	0	0	X
AR2	X	X	0	0	0	X
AR3	0	X	0	0	0	X

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.


> esacf(log(M3Euro),method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.95	0.93	0.91	0.88	0.85
AR1	**0.03**	−0.20	−0.07	−0.26	0.12	0.42
AR2	−0.35	−0.20	−0.14	−0.22	−0.01	0.45
AR3	−0.06	−0.23	−0.07	−0.20	−0.02	0.16

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	X	0	X
AR2	X	X	0	0	0	X
AR3	0	X	0	0	0	0

```
> esacf(log(M3Euro),method=ols,acf.fun=wacf)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.95	0.93	0.91	0.88	0.85
AR1	**0.04**	−0.17	−0.01	−0.25	0.12	0.43
AR2	−0.42	−0.17	−0.12	−0.25	0.12	0.45
AR3	0.05	−0.27	0.06	−0.15	0.06	0.18

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	X	0	X
AR2	X	0	0	X	0	X
AR3	0	X	0	0	0	0

Difference transformations

For the M3Fin series, first-order differencing makes it white noise. In the M3Euro series we found by lag = 1 differencing the seasonal component (Figure 20 below) and then by lag = 12 differencing the outcome was white noise (Figure 21 below). The following standard and robust ESACF patterns also suggest white noise.

```
> M3Euro1<-diff(log(M3Euro),lag=1)
> M3Euro112<-diff(M3Euro1,lag=12)
```

Figure 20. **First-order differences of log(M3Euro)**
y-axis: M3Euro1

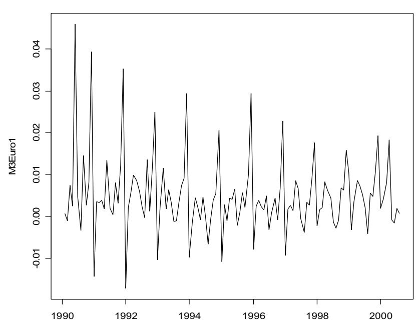
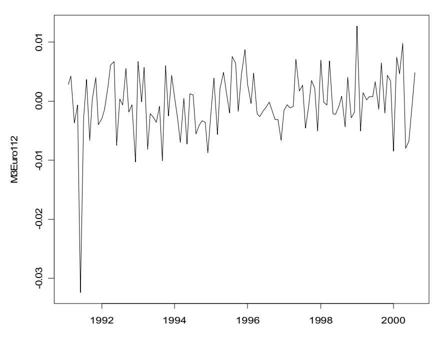


Figure 21. **(1–B)(1–B^{12}) transformation of log(M3Euro)**
y-axis: M3Euro112




```
> esacf(M3Euro112)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**−0.04**	0.02	0.03	0.04	0.00	0.11
AR1	0.33	0.03	0.00	0.07	0.00	0.11
AR2	−0.40	0.03	0.01	−0.01	−0.04	0.12
AR3	−0.46	0.03	0.10	0.02	−0.02	0.11

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	0	0	0
AR1	X	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

```
Values above marked with X are more than 2 std errors away from zero,
using Bartlett type std error.
```

```
> esacf(M3Euro112,method=MM)
Extended Autocorrelation Table
Calculated using MM method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**-0.06**	0.03	0.04	0.07	0.02	0.12
AR1	-0.48	0.03	0.02	0.10	0.06	0.11
AR2	0.26	0.45	-0.05	0.05	0.00	0.14
AR3	-0.54	0.41	-0.21	0.09	-0.01	0.14

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	0	0	0
AR1	X	0	0	0	0	0
AR2	X	X	0	0	0	0
AR3	X	X	0	0	0	0

```
> esacf(M3Euro112,method=ols,acf.fun=wacf)
Extended Autocorrelation Table
Calculated using ols method for AR fitting
and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**-0.06**	0.03	0.04	0.07	0.02	0.12
AR1	0.30	0.06	0.06	0.08	0.01	0.12
AR2	-0.44	0.05	0.06	0.02	-0.02	0.15
AR3	-0.49	0.05	0.14	0.08	-0.01	0.14

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	0	0	0
AR1	X	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

For both the Finnish and euro area series, an alternative way to start the modelling procedure is by carrying out a robust AR(p) filter analysis, for instance an AR(3) fitting based on the MM-estimator. Thereafter, for the filtered/cleaned series, it should be reasonable to apply the standard ESACF procedure. This kind of approach to ARMA modelling is suggested by Martin, Samarov and Vandaele (1983). We could add the robust ESACF estimation of the filtered/cleaned series (and the corresponding residual series) to this approach and compare the standard and robust ESACF results.

7.3 Five real exchange rate series

In the following standard and robust ESACF exercises, we used five long monthly time series of the real effective exchange rate, with 1995 = 100. The real rate is based on consumer prices. The following countries' series were studied: Unites States, Germany, United Kingdom, Sweden and Finland. The period of the time series is January 1972 – December 1999, totalling 336 observations. Before the ESACF runs, the series were transformed into natural logarithms.

The Finnish markka

For the Finnish markka series, the results of all standard and robust ESACF estimates based on the total period indicate an ARMA(1, 1) model with high values of sample autocorrelation estimates. This result is consistent with an alternative ARIMA(0, 1, 1) model, as the differencing operator (1–B) can be viewed as the AR operator (1 –ϕB) with $\phi \approx 1$. An MA(1) model was obtained for the first-order difference transform of the original series. In robust cases the correlation estimates of the ESACF tables were more clearly distributed to the separate 'X' and '0' sub-fields (in the ESACF matrix) compared with the standard OLS/acf estimation. If we divide the markka series into three sub-periods, we obtain the following results:

(a) *January 1972 – February 1989:* both the standard and robust estimates indicate an ARIMA(0, 1, 0) model, ie a random walk model of the original series and white noise for the first-order difference series.

(b) *March 1989 – August 1992:* includes three important monetary and foreign exchange policy measures: revaluation of the markka, linking the markka to the ECU and devaluation of the markka; both the standard and robust estimates produce an ARMA(1, 1) or ARIMA(0, 1, 1) model and an MA(1) model for the differenced series.

(c) *September 1992 – December 1999,* in which the markka was floated and Finland and ten other European countries adopted a single currency, the euro, from 1 January 1999; the results achieved are similar to those for period (b). More detailed ESACF results concerning total series are given in Appendix 6.

Results for the other real exchange rates

The results are the following for the series of the other four countries. For the US, UK and Sweden, the original series follow a stationary ARMA(1, 1) or a nonstationary ARIMA(0, 1, 1) model and the first-difference series follow an MA(1) model.

The result for Germany is an interesting exception: the original series follow an ARIMA(0, 1, 1) model based on the standard ESACF identification, but the robust ESACF based on the MM/wacf, GM/wacf, OLS/tacf and OLS/rkacf provided an ARIMA(0, 1, 2) ie an IMA(2) model. For the first-order difference series, the standard ESACF indicates an MA(1) model, while the robust ESACF based on the combinations GM/tacf, MM/tacf and OLS/tacf indicates an MA(2) model. This is the result which requires careful further research. More detailed results from identification of our German series are displayed in Appendix 6.

8 Concluding remarks and suggestions for further research

Concluding remarks

In this thesis we have designed a robustified version of the ESACF procedure for identification of ARIMA time series models. The different simulations and applications of the single real and artificial time series show that this design is operational with alternative statistical algorithms and estimators. A technically quick and flexible system is incorporated in an integrated statistical program: data generation, simulations with outlier modelling, ESACF estimations of simulated and real series with or without outliers. In addition, it is possible, using the saved results, to combine different summary results. In the program both outlier types, AO and IO, can be generated simultaneously for a time series, as isolated and patchy configurations; furthermore, the generated and fixed outliers can be simultaneously placed as determined and randomly.

The system described provides a benchmark for researchers in the identification of mixed ARIMA processes by standard and robust ESACF procedures. We also obtain the results for ordinary and robust autocorrelation functions, since the first row of the ESACF table displays the ordinary autocorrelation estimates of the series. One drawback of the standard and robust ESACF procedures is that they focus on small orders of p and q, ie on a small area of the ESACF table. Of course, parsimony in model building is itself a common and important goal. Another drawback is that the ESACF method is (in practice) suitable only for non-seasonal time series. We did not conduct robust experiments on ESACF for seasonal models, but we believe that this restriction can be replaced by the modern robust seasonal adjustment procedures (eg Findley et al 1998).

The key findings from the simulation experiments are:

– Identification of the ARMA(1, 1) and ARIMA(1, 1, 1) processes by the robust ESACF procedure was quite successful in cases of low contamination and low volatility of outliers; AOs were problematic, as is known in the literature; in cases of high contamination and volatility of outliers, the results were clearly less satisfactory.

- No great differences in performance between GM- and MM-estimators were found for series including outliers; in the case of single series, the GM-estimator was found to take notably more computer time compared with other robust regression estimators; the combination MM/wacf performs well in almost all the cases and is therefore recommended for use as the default combination.

- Due to outliers, the sample distributions of the standard ESACF coefficient estimates are generally quite skewed and characterised by excess kurtosis, and in the lower left part of the ESACF matrix some sample distributions of the coefficient estimates may be bimodal (Appendix 1).

- In the robust ESACF case, these sample distributions are more symmetric in shape and, according to the Jarque-Bera test values, also closer to a normal distribution, this occured particularly in the area of triangle of zeros, as seen in Appendix 1. We have here dealt with the results for the vertices, but in general these can be viewed as valid 'samples' of the triangle region of zeros in the ESACF table.

- The robust ESACF procedure based on MM/wacf performs well also in case of an outlier-free series; it is impossible to determine whether this depends more on the robust AR(p) fitting than on the robust autocorrelation function. In the literature, robust methods are often verified to be inferior to conventional methods in outlier-free cases.

- The robust ESACF results of mixed nonstationary time series are reasonable, but theoretically, these results remain open due to the robust autoregressions. However, the combination of OLS estimation of AR(p) iteration and a robust autocorrelation function (OLS/wacf) was shown to be useful in the majority of the nonstationary series with a low contamination level. Generally, the promising robust results with nonstationary series provide a possible contribution to robust unit root testing.

- In the case of ARMA(1, 1) models, the OLS/wacf combination was successful (Table 4).

The sensitive experiments with single generated series containing outliers showed the usefulness of the robust ESACF procedure, as did the example of a generated nonstationary time series and its differencing, with outliers. Also the applications to series studied earlier in the literature provided useful results, especially in the spirit model's residual series. In the case of telephone data, RESEX, the robust ESACF suggested the same model form as did Masarotto (1987a). For the real monetary series, money stock and real exchange rate series, the robust ESACF gave relevant suggestions on the existence of a unit root. The application of robust ESACF to sub-series of the Finnish markka time series provided suggestions for different model forms.

In summary, the results of our simulations and illustrative examples of single real series show how useful it would be to compute ESACF estimates by both robust and standard versions. This should always be done, since the robust regression method, MM, performs as well as the OLS method with pure, outlier-free data. In cases of high contamination of data, the robust ESACF also gives more false signals. In macroeconomic series, especially monetary series, structural breaks may occur as outliers and the United ESACF Identification Procedure (Figure 1) should be carried out as a routine procedure. Comparison of the results provides statisticians and econometricians with valuable suggestions for constructing explanatory or forecasting models. In econometric modelling, the combined robust and standard residual diagnostic methods are becoming more important, and in this area our combined procedure will make a definite contribution. In practical terms, the crucial finding is that, with outlier-free data, the use of the robust ESACF procedure based on MM regression entails no real risk of a false inference.

Suggestions for further research

We have used three types of the robust autocorrelation function in the robust ESACF procedure: weighted, trimmed and rank-based. For comparisons of results, experiments based on the *median-type* autocorrelation function might also be useful. Such a simple version is the autocorrelation function of Sen (see Yoshida et al 1984).

A second aim of future research might be to obtain robust ESACF estimates based on robust autocorrelation and partial autocorrelation functions formed by a *lattice* structure (see eg Li & Dickinson 1988, Masarotto 1987a). These estimates could be obtained from the transformed $W_{k,t}^{(j)}$ series in AR(p) iterations calculated via Durbin-Levinson recursions. This system with computer-intensive methods requires extensive calculations, but is not a problem with modern computer facilities.

A third aim might be to investigate the potential of the *robust vector autocorrelation* function for developing a robust ESACF procedure on the lines laid down by Jeon and Park (1986) and Paparoditis and Streitberg (1992).

Overall, however, the most acute open problem is the standard error of the robust ESACF coefficients with t_r^* values for constructing the *robust confidence intervals* (see the flow chart of the proposed united procedure in Figure 1). One could study a robust version of Bartlett's approximation. The use of the bootstrap technique would help in searching for a relevant solution to the confidence interval problem (see Paparoditis and Streitberg 1992, Aczel and Josephy 1992, Wilcox 1997, and Glendinning 1998). Currently, the most promising method for computing robust confidence intervals when using M regression (the M- and GM-estimator) seems to be the percentile bootstrap (Wilcox 1997). But, as Maddala and Rao (1997) remark, one should first investigate a robust version of the bootstrap. In the ESACF procedure proposed in this thesis, this problem is more complicated, since we use a robust regression in the AR(p) iteration of the ESACF procedure in the first stage and a robust autocorrelation function in the second stage.

Finally, the need for robust analysis of nonstationary time series, with and without difference transformation, will be a permanent topic of interest in statistics and econometrics. The robust ESACF procedure is a potential tool to deal with the above modelling problems (eg in a similar context as in Tsay 1984, 1985). We will encounter these problems with both stationary and nonstationary series and various types of outliers and their time configurations, ie isolated and patchy outliers. Furthermore, the effects of the different degrees of outlier contamination on robust procedures will be an important goal of further research.

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Appendices 1–7

Notes:

1. The vertex of each ESACF table is marked in **bold**.
2. Besides the *mean* of the robust and standard ESACF estimates for each model, the *percentage* of 1000 repetitions is given when the ESACF estimate is greater than its two standard errors in modulus (Appendices 1–3).
3. Standard error is calculated from simulated values (Sample s.e.).

Appendix 1

Example of the basic and an optional[39] output from the main simulation experiments

Model: ARMA(1, 1) process

Label: ARMA(1, 1) 5 % AO contamination
NSIMU: 1000 NTS: 50 AR: (0.8) MA: (–0.7) ndiff: 0 METHOD: MM, wacf OSD:10

NSIMU	=	number of repetitions in simulation
NTS	=	number of observations in time series
AR	=	value of phi coefficient of autoregressive part
MA	=	value of theta coefficient the moving average part
ndiff	=	value of d in ARIMA(p, d, q) process; d = 0 is a stationary process of order I(0); d = 1 is an integrated process of order I(1)
METHOD	=	regression method, type of autocorrelation function
OSD	=	standard deviation of outliers
s.e.	=	standard error

Vertex of the ESACF table and its corresponding statistics are marked in **bold**.

[39] See also Appendix 7, p. 156 and 158.

ESACF Tables (Patterns):

Robust ESACF
Mean[40] robust ESACF = ESACF based on robust method of AR(p) fitting and robust ACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.716	0.486	0.326	0.209	0.121	0.057
AR1	0.245	**0.028**	0.030	0.016	-0.005	-0.021
AR2	0.032	-0.076	0.011	0.015	0.014	-0.004
AR3	0.026	-0.053	0.024	0.012	0.007	-0.003

Standard ESACF
Mean standard ESACF = original ESACF based on ols AR(p) fitting and regular ACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.501	0.345	0.233	0.153	0.087	0.041
AR1	-0.072	**0.038**	0.030	0.025	0.005	-0.005
AR2	-0.021	-0.013	0.015	0.016	0.003	-0.001
AR3	-0.038	0.038	0.029	0.013	0.007	0.001

Sample s.e. of robust ESACF = calculated from *simulation* repetitions

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.123	0.165	0.193	0.209	0.213	0.213
AR1	0.172	**0.177**	0.156	0.150	0.148	0.145
AR2	0.272	0.169	0.148	0.140	0.134	0.130
AR3	0.291	0.242	0.160	0.136	0.134	0.132

Sample s.e. of standard ESACF = calculated from *simulation* repetitions

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.248	0.217	0.201	0.187	0.181	0.174
AR1	0.315	**0.137**	0.124	0.114	0.110	0.105
AR2	0.358	0.200	0.115	0.106	0.097	0.092
AR3	0.363	0.250	0.168	0.101	0.096	0.092

Asymptotic s.e.
$(n-k-j)^{-1/2}$

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.143	0.144	0.146	0.147	0.149	0.151
AR1	0.144	**0.146**	0.147	0.149	0.151	0.152
AR2	0.146	0.147	0.149	0.151	0.152	0.154
AR3	0.147	0.149	0.151	0.152	0.154	0.156

Robust ESACF
Mean Bartlett s.e. (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.151	0.153	0.155	0.156	0.158	0.160
AR1	0.169	**0.176**	0.189	0.201	0.211	0.217
AR2	0.170	0.174	0.184	0.190	0.199	0.205
AR3	0.174	0.179	0.192	0.199	0.206	0.212

Standard ESACF
Mean Bartlett s.e. (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.143	0.144	0.146	0.147	0.149	0.151
AR1	0.160	**0.163**	0.173	0.182	0.189	0.193
AR2	0.165	0.169	0.178	0.184	0.189	0.194
AR3	0.167	0.173	0.184	0.191	0.196	0.201

ESACF estimates greater than its two standard errors in modulus, % of 1000 repetitions

A. asymptotic s.e. $(n–k–j)^{-1/2}$

B. Bartlett s.e.

C. s.e. based on simulated values

A.

Percentage accepted > zero (2 asymptotic s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	99.7	87.6	57.9	36.0	23.5	17.1
AR1	41.1	**11.0**	5.4	3.8	3.8	3.4
AR2	34.8	10.9	4.6	3.0	3.4	1.7
AR3	41.0	26.1	6.7	2.8	2.1	2.3

Percentage accepted > zero (2 asymptotic s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	77.8	57.7	37.4	23.4	14.2	9.4
AR1	50.5	**5.3**	3.7	2.7	1.5	1.4
AR2	59.8	15.9	3.0	2.4	1.1	0.7
AR3	63.1	29.3	9.6	2.1	1.2	1.1

[40] Besides the arithmetic mean, we can list the median estimates from our simulation results.

B.

Percentage accepted > zero
(2 Bartlett s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	99.6	85.7	55.9	32.5	20.8	14.5
AR1	29.9	**3.1**	1.3	0.7	0.5	0.7
AR2	21.9	7.4	1.5	0.5	0.7	0.3
AR3	25.5	19.5	1.8	0.8	0.5	0.3

Percentage accepted > zero
(2 Bartlett s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	77.8	57.7	37.4	23.4	14.2	9.4
AR1	44.3	**2.4**	2.1	1.2	0.7	0.5
AR2	55.2	12.9	1.7	1.3	0.7	0.3
AR3	58.0	22.2	6.9	1.6	0.8	0.6

C.

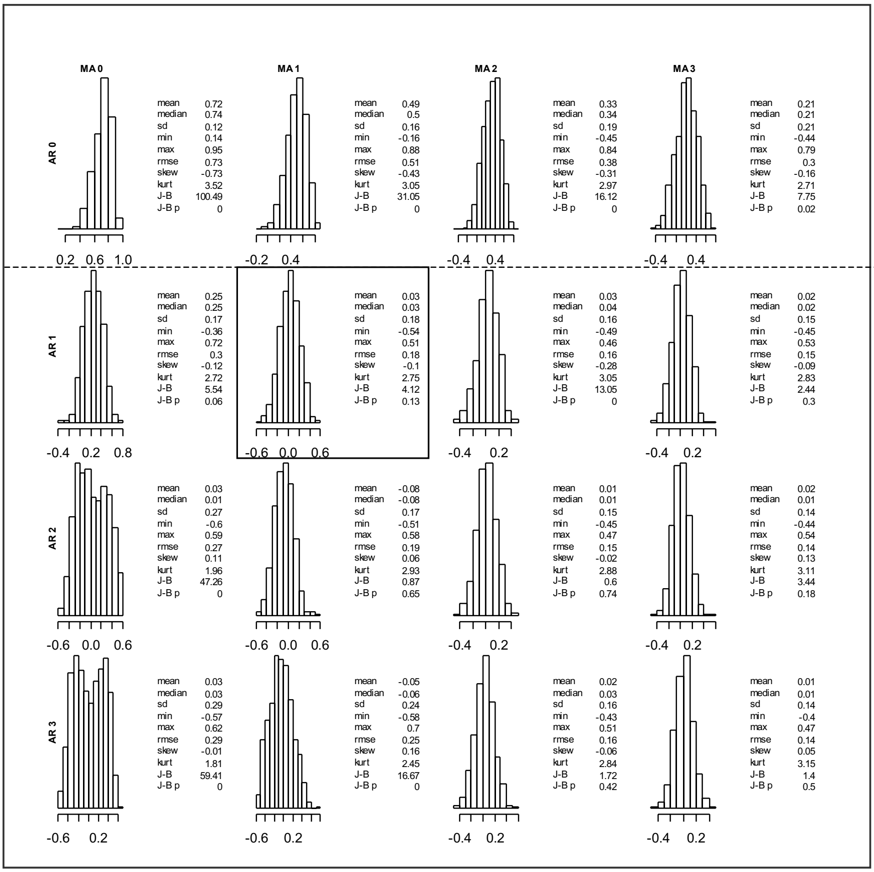
Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

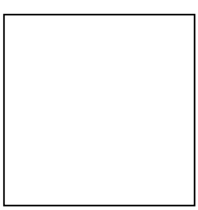
	MA0	MA1	MA2	MA3	MA4	MA5
AR0	99.8	82.8	40.9	17.3	8.4	6.1
AR1	30.4	**4.8**	4.2	3.6	4.1	4.4
AR2	1.3	7.1	4.7	4.6	5.2	4.7
AR3	0.2	3.3	5.1	5.3	5.6	4.8

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	51.2	36.0	21.7	13.2	8.0	6.4
AR1	0.7	**6.5**	6.0	6.3	6.2	6.1
AR2	0.0	5.4	7.0	6.7	7.1	6.6
AR3	0.0	1.1	6.8	6.4	7.1	6.6

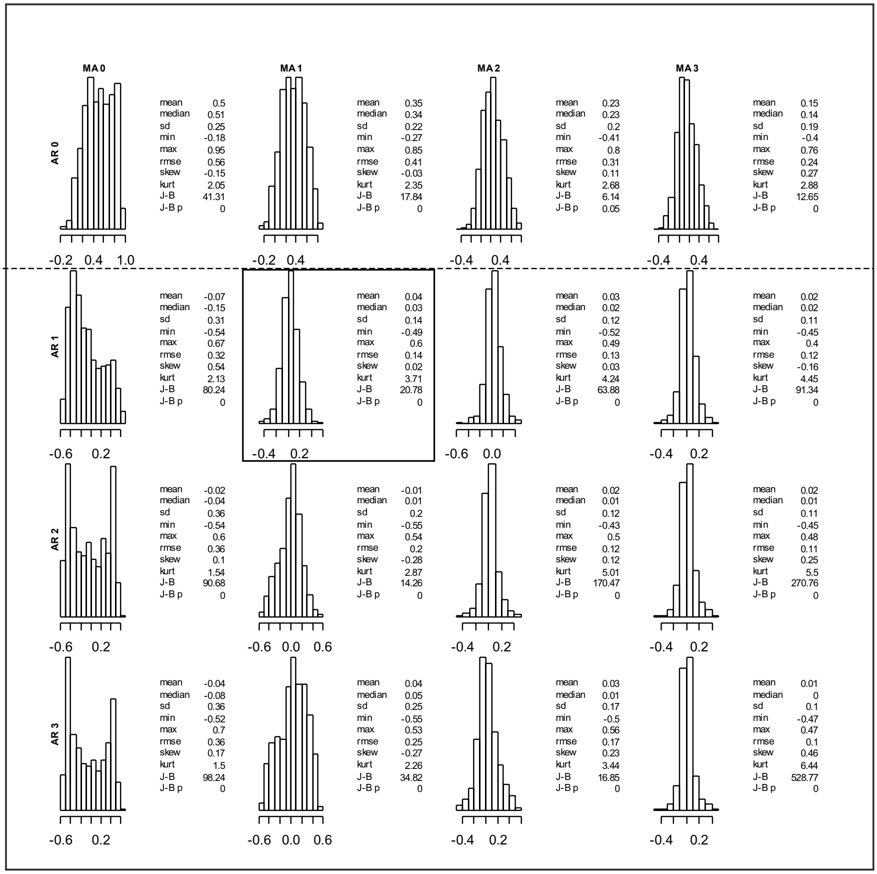
Model: ARMA(1, 1)
Histograms of <u>robust</u> ESACF 4×4 matrix elements
[AR0 ↔ AR3]×[MA0 ↔ MA3], 1000 repetitions.
Area of SACF coefficients (first row) separated by dotted line.





: Frame of theoretical vertex histogram

Model: ARMA (1, 1)
Histograms of <u>standard</u> ESACF 4×4 matrix elements
[AR0 ↔ AR3] ×[MA0 ↔ MA3], 1000 repetitions.
Area of SACF coefficients (first row) separated by dotted line.



: Frame of theoretical vertex histogram

Some theoretical forms of ESACF tables, asymptotic form

Indicator symbols: X = values *beyond* ± 2 standard errors
0 = values *within* ± 2 standard errors

Vertex is marked in **bold**. Ordinary SACF coefficients are marked by shaded area and a triangle of zero values by two lines. In practice, these ideal representations do not occur often, and the triangle may become rectangular or trapezoidal in shape.

A. ARMA(1, 1)

AR order	MA order 0	1	2	3	4
0	X	X	X	X	X
1	X	**0**	0	0	0
2	X	X	0	0	0
3	X	X	X	0	0
4	X	X	X	X	0

B. ARMA(2, 1)

AR order	MA order 0	1	2	3	4
0	X	X	X	X	X
1	X	X	X	X	X
2	X	**0**	0	0	0
3	X	X	0	0	0
4	X	X	X	0	0

C. ARMA(1, 2)

AR order	MA order 0	1	2	3	4
0	X	X	X	X	X
1	X	X	**0**	0	0
2	X	X	X	0	0
3	X	X	X	X	0
4	X	X	X	X	X

D. ARMA(1, 1, 1)

AR order	MA order 0	1	2	3	4
0	X	X	X	X	X
1	X	X	X	X	X
2	X	**0**	0	0	0
3	X	X	0	0	0
4	X	X	X	0	0

E. ARIMA(1, 1, 0) ⇒ ARI(1, 1)

AR order	MA order 0	1	2	3	4
0	X	X	X	X	X
1	X	X	X	X	X
2	**0**	0	0	0	0
3	X	0	0	0	0
4	X	X	0	0	0

F. ARIMA(0, 1, 1) ⇒ IMA(1, 1)

AR order	MA order 0	1	2	3	4
0	X	X	X	X	X
1	X	**0**	0	0	0
2	X	X	0	0	0
3	X	X	X	0	0
4	X	X	X	X	0

Appendix 2

Main simulations: ARMA(1, 1) models (1–8)

NSIMU= number of replications, NTS = sample size, AR: ( ) value of phi coefficient, MA: ( ) value of theta coefficient, ndiff: 0 stationary series, ndiff: 1 nonstationary I(1) series, OSD: st.dev of outliers, s.e. = standard error.

Robust ESACF **Robust ESACF**

Model 1

Label: ARMA(1, 1)2% AO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.648	0.320	0.141	0.043	-0.011	-0.039
AR1	0.292	**0.043**	-0.009	-0.034	-0.035	-0.028
AR2	0.068	-0.058	0.023	0.023	-0.009	-0.007

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.602	0.298	0.129	0.036	-0.014	-0.040
AR1	0.224	**0.057**	0.002	-0.027	-0.031	-0.025
AR2	0.046	-0.028	0.031	0.020	-0.007	-0.007

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	99.9	47.1	10.7	4.6	4.0	4.8
AR1	42.7	**2.8**	3.9	4.9	6.0	4.9
AR2	0.0	5.7	5.4	4.7	5.3	4.6

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	92.2	41.0	11.0	5.3	5.1	4.9
AR1	8.3	**3.0**	3.7	5.4	6.8	5.3
AR2	0.0	4.5	5.4	6.5	5.2	6.2

Model 2

Label: ARMA(1, 1)5% AO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.604	0.302	0.135	0.042	-0.010	-0.042
AR1	0.204	**0.037**	-0.004	-0.026	-0.025	-0.030
AR2	0.018	-0.031	0.022	0.001	-0.001	-0.023

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.493	0.246	0.105	0.030	-0.010	-0.039
AR1	0.086	**0.054**	0.000	-0.015	-0.016	-0.022
AR2	-0.004	0.012	0.019	0.002	-0.002	-0.019

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	99.5	47.4	11.3	4.7	4.8	5.2
AR1	17.9	**3.8**	4.2	4.8	5.0	4.3
AR2	0.2	5.0	4.5	4.9	4.3	5.0

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	73.1	31.7	10.0	4.5	5.2	5.1
AR1	0.2	**4.4**	4.9	5.6	5.9	6.1
AR2	0.0	5.9	6.0	6.1	7.3	6.9

Model 3

Label: ARMA(1, 1)2% AO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.688	0.352	0.164	0.056	-0.005	-0.038
AR1	0.358	**0.048**	0.000	-0.030	-0.038	-0.031
AR2	0.162	-0.074	0.023	0.019	-0.004	-0.007

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.698	0.349	0.156	0.049	-0.011	-0.041
AR1	0.369	**0.061**	0.005	-0.024	-0.038	-0.030
AR2	0.151	-0.076	0.032	0.025	0.000	-0.011

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	58.5	12.8	6.0	4.9	4.7
AR1	76.7	**3.1**	3.1	3.7	5.6	6.1
AR2	0.1	6.0	4.1	3.6	5.4	5.6

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	61.8	12.9	5.7	5.2	5.2
AR1	80.8	**2.6**	3.0	4.4	5.8	6.1
AR2	0.0	7.1	4.8	4.8	5.4	6.7

Model 4

Label: ARMA(1, 1)5% IO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.694	0.369	0.182	0.069	0.003	-0.032
AR1	0.350	**0.025**	0.007	-0.024	-0.038	-0.030
AR2	0.199	-0.095	0.024	0.018	0.005	-0.011

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.697	0.351	0.162	0.056	-0.003	-0.034
AR1	0.362	**0.049**	0.010	-0.025	-0.034	-0.029
AR2	0.173	-0.087	0.035	0.019	0.004	-0.009

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	62.8	15.2	7.0	4.8	4.8
AR1	76.3	**2.6**	3.5	4.8	4.2	5.0
AR2	2.8	5.6	4.4	3.6	3.6	5.0

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	65.4	13.8	5.6	5.2	4.5
AR1	77.8	**2.8**	3.9	4.8	6.1	6.0
AR2	0.0	7.1	5.2	4.3	5.1	6.4

Model 5

Label: ARMA(1, 1)2% AO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.694	0.394	0.223	0.122	0.064	0.029
AR1	0.257	**0.010**	0.012	0.003	-0.006	-0.009
AR2	0.103	-0.071	0.001	0.006	0.010	0.005

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.626	0.360	0.204	0.112	0.059	0.027
AR1	0.110	**0.022**	0.018	0.006	-0.002	-0.004
AR2	0.045	-0.022	0.007	0.006	0.009	0.004

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	99.8	62.4	21.0	9.4	5.0
AR1	82.1	**4.3**	3.8	4.3	5.5	3.9
AR2	0.6	8.4	5.7	5.0	5.6	5.1

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	97.1	51.5	19.3	8.9	5.4
AR1	8.0	**3.6**	3.3	3.0	4.8	4.9
AR2	0.0	6.0	5.7	5.2	5.0	5.2

Model 6

Label: ARMA(1, 1)5% AO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.650	0.370	0.209	0.115	0.061	0.030
AR1	0.174	**0.004**	0.009	-0.001	-0.005	-0.008
AR2	0.003	-0.040	0.004	0.003	0.009	0.003

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.512	0.293	0.166	0.090	0.048	0.023
AR1	-0.053	**0.019**	0.019	0.003	0.002	-0.003
AR2	-0.031	-0.003	0.012	0.010	0.007	0.000

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	99.3	57.6	18.6	7.9	5.0
AR1	48.1	**4.1**	3.9	4.2	4.3	3.8
AR2	0.0	6.7	5.9	5.5	4.3	4.3

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	99.9	86.7	42.4	14.2	8.2	4.6
AR1	6.3	**4.6**	4.0	4.4	4.8	5.7
AR2	0.0	5.5	6.7	6.1	4.9	6.1

Model 7

Label: ARMA(1, 1)2% IO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.730	0.423	0.243	0.136	0.072	0.033
AR1	0.343	**0.018**	0.016	0.003	-0.003	-0.007
AR2	0.287	-0.092	0.005	0.009	0.014	0.009

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.742	0.428	0.243	0.134	0.070	0.032
AR1	0.351	**0.024**	0.017	0.003	-0.006	-0.010
AR2	0.288	-0.088	0.007	0.008	0.012	0.006

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	99.6	69.4	24.1	10.3	5.2
AR1	99.6	**3.7**	3.3	3.8	3.7	5.0
AR2	53.3	7.2	4.6	5.4	5.7	4.6

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	99.8	70.4	23.9	9.8	5.7
AR1	99.5	**4.5**	2.7	4.2	4.2	4.8
AR2	47.8	6.9	4.7	5.4	5.7	5.9

Model 8

Label: ARMA(1, 1)5% IO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 0 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.738	0.442	0.259	0.147	0.080	0.040
AR1	0.349	**0.014**	0.012	0.004	-0.014	-0.012
AR2	0.301	-0.097	0.003	0.006	0.002	0.004

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.744	0.432	0.247	0.137	0.071	0.034
AR1	0.348	**0.023**	0.023	0.008	-0.011	-0.010
AR2	0.290	-0.093	0.014	0.014	0.006	0.007

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	71.4	25.9	9.4	6.8
AR1	99.9	**3.6**	3.5	3.8	4.5	3.9
AR2	63.2	5.7	4.9	4.2	4.2	4.1

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	99.9	74.0	25.4	10.0	5.8
AR1	99.5	**4.3**	2.8	4.2	4.8	6.0
AR2	46.0	7.8	6.7	5.6	5.0	6.0

Appendix 3

Main simulations: ARIMA(1, 1, 1) models (17–24)

The vertex of the ESACF table is marked in **bold**.
NSIMU= number of replications, NTS = sample size, AR: ( ) value of phi coefficient, MA: ( ) value of theta coefficient, ndiff: 0 stationary series, ndiff: 1 nonstationary I(1) series, OSD: st.dev of outliers, s.e. = standard error.

Robust ESACF **Robust ESACF**

Model 17

Label: ARIMA(1, 1, 1)2% AO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.932	0.839	0.735	0.629	0.527	0.433
AR1	0.602	0.353	0.214	0.137	0.089	0.056
AR2	0.160	**0.000**	-0.018	-0.017	-0.012	-0.007

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.923	0.828	0.722	0.616	0.515	0.422
AR1	0.466	0.322	0.194	0.123	0.079	0.052
AR2	0.073	**0.036**	-0.004	-0.009	-0.008	-0.006

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.7	93.3	74.0	55.0
AR1	98.5	58.9	19.8	8.8	5.9	4.8
AR2	10.8	**3.2**	4.0	3.7	5.3	4.5

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.5	93.8	75.5	55.3
AR1	45.1	47.4	19.4	8.5	6.3	5.6
AR2	0.1	**3.8**	4.9	5.7	5.8	6.3

Model 18

Label: ARIMA (1, 1, 1)5% AO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.922	0.832	0.730	0.628	0.529	0.437
AR1	0.486	0.311	0.194	0.126	0.083	0.054
AR2	0.005	**0.025**	-0.019	-0.002	-0.017	-0.003

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.909	0.817	0.715	0.612	0.513	0.423
AR1	0.240	0.253	0.159	0.104	0.066	0.046
AR2	-0.052	**0.066**	0.014	0.006	-0.003	0.002

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.6	93.4	75.7	55.2
AR1	81.0	49.0	17.0	8.7	6.0	5.1
AR2	2.5	**3.7**	4.1	4.7	4.7	4.8

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.6	92.4	74.5	54.4
AR1	13.1	30.7	15.3	9.2	6.3	5.7
AR2	0.0	**6.6**	6.4	6.2	6.6	6.3

Model 19

Label: ARIMA(1, 1, 1)2% IO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.942	0.850	0.747	0.644	0.544	0.451
AR1	0.689	0.384	0.226	0.137	0.080	0.039
AR2	0.294	**-0.039**	-0.028	-0.026	-0.020	-0.021

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.933	0.838	0.732	0.626	0.526	0.432
AR1	0.702	0.392	0.236	0.149	0.095	0.055
AR2	0.300	**-0.035**	-0.025	-0.020	-0.018	-0.016

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.5	94.3	79.3	60.7
AR1	100.0	64.8	20.9	9.1	5.1	3.9
AR2	47.6	**3.8**	4.9	5.1	4.6	4.4

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.5	93.6	77.9	58.0
AR1	99.9	66.8	24.2	10.4	5.1	4.4
AR2	50.7	**3.9**	4.6	4.9	5.0	4.3

Model 20

Label: ARIMA(1, 1, 1)5% IO contam.
NSIMU: 1000 NTS: 50 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.942	0.851	0.747	0.642	0.540	0.443
AR1	0.691	0.386	0.216	0.118	0.057	0.013
AR2	0.305	**-0.043**	-0.036	-0.031	-0.026	-0.029

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.932	0.836	0.730	0.623	0.521	0.426
AR1	0.700	0.389	0.234	0.149	0.095	0.051
AR2	0.307	**-0.041**	-0.030	-0.016	-0.009	-0.014

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.9	95.3	80.6	58.2
AR1	100.0	68.0	19.6	8.9	5.6	4.1
AR2	57.6	**3.6**	4.7	4.2	5.0	5.2

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	99.8	94.0	79.1	56.8
AR1	100.0	69.5	24.0	10.5	6.5	4.8
AR2	57.5	**3.8**	4.7	5.9	5.5	6.0

Model 21

Label: ARIMA(1, 1, 1)2% AO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.983	0.959	0.930	0.899	0.867	0.834
AR1	0.638	0.389	0.235	0.147	0.097	0.068
AR2	0.177	**0.038**	0.009	-0.004	-0.005	-0.004

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.982	0.957	0.928	0.897	0.865	0.832
AR1	0.429	0.331	0.201	0.126	0.082	0.057
AR2	-0.229	**0.082**	0.008	0.005	0.001	0.003

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	100.0	99.3	62.0	26.7	12.3	9.0
AR2	39.8	**5.6**	3.6	3.9	4.5	5.0

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	57.2	87.6	49.0	24.1	12.3	8.7
AR2	2.6	**11.4**	4.6	5.0	4.7	5.5

Model 22

Label: ARIMA(1, 1, 1)5% AO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.980	0.957	0.930	0.900	0.869	0.839
AR1	0.517	0.340	0.206	0.133	0.087	0.066
AR2	-0.002	**0.044**	-0.010	0.000	-0.004	0.001

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.979	0.955	0.927	0.897	0.866	0.835
AR1	0.164	0.239	0.144	0.095	0.060	0.048
AR2	-0.244	**0.117**	0.018	0.019	0.012	0.013

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	100.0	98.6	61.6	27.2	13.0	10.0
AR2	4.5	**7.1**	4.3	3.4	4.5	4.6

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	13.6	70.5	35.5	20.6	11.7	9.2
AR2	0.0	**26.5**	6.0	5.9	7.1	6.6

Model 23

Label: ARIMA(1, 1, 1)2% IO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.986	0.963	0.934	0.904	0.873	0.841
AR1	0.732	0.434	0.259	0.156	0.096	0.062
AR2	0.332	**-0.001**	-0.004	-0.009	-0.009	-0.011

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.984	0.960	0.931	0.900	0.869	0.837
AR1	0.745	0.441	0.265	0.164	0.107	0.074
AR2	0.338	**0.001**	-0.003	-0.009	-0.009	-0.007

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	100.0	100.0	75.1	31.4	13.0	8.1
AR2	98.6	**3.6**	3.0	4.5	5.1	4.6

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	100.0	100.0	79.0	34.5	14.4	9.1
AR2	99.2	**3.6**	2.8	4.5	5.0	5.0

Model 24

Label: ARIMA(1, 1, 1)5% IO contam.
NSIMU: 1000 NTS: 200 AR: (0.6) MA: (-0.4) ndiff: 1 METHOD: MM, wacf OSD: 5

Mean robust ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.986	0.962	0.934	0.903	0.872	0.840
AR1	0.735	0.442	0.266	0.157	0.091	0.052
AR2	0.333	**-0.018**	-0.009	-0.009	-0.010	-0.016

Mean standard ESACF

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.984	0.959	0.930	0.899	0.867	0.834
AR1	0.745	0.443	0.270	0.169	0.109	0.073
AR2	0.333	**-0.009**	0.001	-0.004	-0.005	-0.009

Percentage accepted > zero
(2 Sample s.e.) (robust ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	100.0	100.0	74.8	30.4	11.2	6.3
AR2	99.8	**4.2**	4.7	4.4	4.2	3.9

Percentage accepted > zero
(2 Sample s.e.) (standard ESACF)

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	100.0	100.0	100.0	100.0	100.0	100.0
AR1	100.0	99.9	75.8	33.0	15.2	9.5
AR2	99.1	**3.6**	3.3	3.1	4.5	4.2

Appendix 4

Simulation experiments with some common ARIMA models in econometrics and special configurations of outliers

Summary of simulation results: mean of theoretical **vertex**[1)] of triangle ESACF table for an ARIMA(p, d, q) model, 1000 replications

Model		Percentage: vertex value beyond ±2 standard errors[2)], %		Mean of the theoretical vertex value (expected value = 0)		Outlier type[3)] and contamination, proportion (%), isolated or {patch}
n:o	n	A	B	A	B	
ARIMA(0, 1, 1) θ_1 = 0.5			**σ_o = 10**			
1	200	4.7*	4.2*	0.002	0.007	AO2
2	200	6.2	4.8*	0.005	0.016	AO5
ARIMA(0, 1, 0)			**σ_o = 10**			
3	200	75.3[4)]	7.0	–0.329	–0.037	AO2
4	200	99.4	24.6[4)]	–0.431	–0.095	AO5
ARIMA(0, 0, 0)			**σ_o = 10**			
5	200	5.1	4.6*	–0.004	–0.004	AO2
6	200	5.9	3.9*	–0.004	–0.005	AO5
ARIMA(0, 1, 1) θ_1 = 0.5			**outlier-free**			
7	200	4.8*	4.5*	0.002	0.003	
ARIMA(0, 1, 0)			**outlier-free**			
8	200	4.9*	4.3*	0.003	0.003	
ARIMA(0, 0, 0)			**outlier-free**			
9	200	5.0*	4.2*	–0.007	–0.007	
ARMA(1, 1) ϕ_1 = 0.7 θ_1 = 0.4						
10	200	7.0	3.2*	0.046	0.012	AO{8, 7, -5} end and AO{8, -5} randomly
ARIMA(0, 1, 0)						
11	100	99.6	5.5 2.4Ba	–0.425	–0.036	AO{-5, 6} randomly
ARIMA(0, 0, 0)						
12	100	68.1	7.2 4.2Ba	–0.197	–0.040	AO{-5, 6} randomly

1) Point at which row and column coordinates of vertex of triangle of asymptotic 'zero' values correspond to AR order p and MA order q, respectively.
2) Standard error based on 1000 replications.
3) AO = additive outlier.
4) Identifies ARIMA(0, 1, 1) model.
A = OLS estimation
B = Robust estimation: MM-estimator
Ba = based on Bartlett's approximate standard error
* = correct identification at 5% level
n = sample size
σ_o = standard deviation of outlier distribution

Appendix 5

Standard and robust ESACF identification of Lydia Pinkham annual advertising data

Vertex is marked in **bold** only for robust MM/wacf case for the original series. This series is an example of a very difficult case of identification a possible underlying model.
std error = standard error

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.81	0.60	0.53	0.50	0.32	0.12
AR1	0.19	-0.37	-0.09	0.44	0.21	-0.07
AR2	0.39	-0.40	-0.07	0.49	0.21	0.03
AR3	0.02	0.44	0.07	0.33	0.21	-0.06

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	0
AR1	0	X	0	X	0	0
AR2	X	X	0	X	0	0
AR3	0	X	0	0	0	0

Values above marked with X are more than 2 std errors away from zero, using Bartlett type std errors.

Extended Autocorrelation Table
Calculated using **MM** method for AR fitting and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.83	0.63	0.52	0.46	0.33	0.16
AR1	**[-0.01]**	-0.17	0.05	0.39	0.13	-0.26
AR2	-0.24	-0.05	0.06	0.35	0.10	-0.30
AR3	0.35	0.23	-0.04	0.16	0.10	-0.25

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	0
AR1	**[0]**	0	0	X	0	0
AR2	0	0	0	X	0	0
AR3	X	0	0	0	0	0

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.83	0.63	0.52	0.46	0.33	0.16
AR1	0.22	-0.11	0.08	0.43	0.21	-0.02
AR2	0.37	-0.19	0.11	0.42	0.21	0.06
AR3	0.06	0.46	0.06	0.26	0.21	-0.02

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	0
AR1	0	0	0	X	0	0
AR2	X	0	0	X	0	0
AR3	0	X	0	0	0	0

Figure 22. **Original series**
y-axis: Pinkham

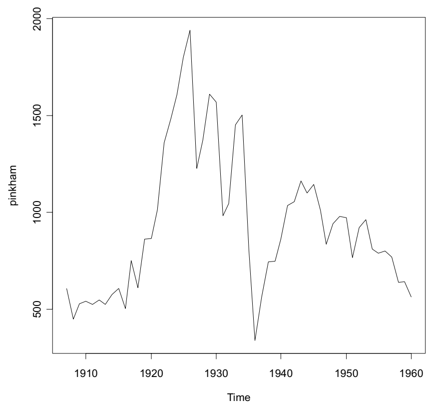
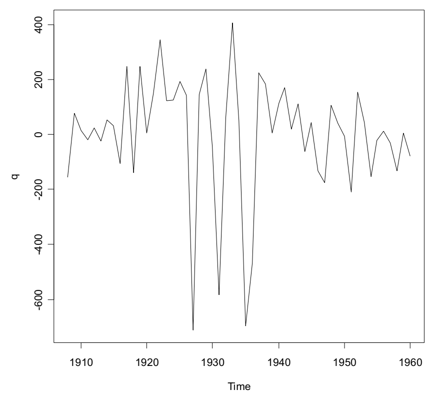


Figure 23. **First difference series**
y-axis: q



First-order differencing

```
Extended Autocorrelation Table
Calculated using ols method for AR fitting and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0.05**	−0.40	−0.09	0.43	0.04	−0.34
AR1	0.12	−0.40	−0.08	0.44	0.05	−0.35
AR2	−0.11	0.52	−0.06	0.11	−0.08	−0.16
AR3	0.13	0.49	0.20	0.03	−0.07	−0.19

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	X	0	X	0	X
AR1	0	X	0	X	0	0
AR2	0	X	0	0	0	0
AR3	0	X	0	0	0	0

```
Extended Autocorrelation Table
Calculated using MM method for AR fitting and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0.00**	−0.14	0.09	0.35	0.12	−0.27
AR1	−0.14	−0.16	0.08	0.33	0.11	−0.29
AR2	0.35	0.30	−0.03	0.08	0.04	−0.15
AR3	−0.14	0.23	−0.20	0.07	0.00	−0.17

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	X	0	0
AR1	0	0	0	X	0	0
AR2	X	0	0	0	0	0
AR3	0	0	0	0	0	0

```
Extended Autocorrelation Table
Calculated using ols method for AR fitting and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0.00**	−0.14	0.09	0.35	0.12	−0.27
AR1	0.09	−0.21	0.03	0.34	0.12	−0.27
AR2	−0.07	0.51	−0.01	0.08	−0.09	−0.18
AR3	0.13	0.52	0.31	0.10	−0.05	−0.19

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	**0**	0	0	X	0	0
AR1	0	0	0	X	0	0
AR2	0	X	0	0	0	0
AR3	0	X	0	0	0	0

Appendix 6

Standard and robust ESACF identification of real exchange rate series: FIM and DEM

Period: January 1972 – December 1999 = 336 observations; std error = standard error. Vertex of the ESACF table is marked in **bold**.

FIM (ffirecm)

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.96	0.94	0.92	0.90	0.87
AR1	0.26	**0.02**	0.01	0.13	0.21	0.11
AR2	0.19	−0.10	0.00	0.04	0.15	0.01
AR3	0.32	−0.01	−0.04	0.03	0.14	0.02

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	X	X	0
AR2	X	0	0	0	X	0
AR3	X	0	0	0	X	0

Values above marked with X are more than 2 std errors away from zero, using Bartlett type std errors.

Extended Autocorrelation Table
Calculated using **MM** method for AR fitting and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.96	0.94	0.92	0.89	0.86
AR1	0.27	**0.07**	0.03	0.07	0.09	0.07
AR2	0.09	−0.01	−0.02	0.06	0.07	−0.07
AR3	0.39	−0.02	−0.02	0.06	0.07	−0.06

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	0	0	0	0	0	0
AR3	X	0	0	0	0	0

Experiments with first sub-sample [1:206]: Jan 1972 – Feb 1989

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.97	0.93	0.90	0.87	0.84	0.80
AR1	**0.10**	−0.05	−0.04	0.08	0.13	0.02
AR2	0.44	−0.09	−0.03	0.05	0.12	−0.02
AR3	−0.41	0.31	0.04	0.02	0.09	−0.03

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	X	0	0	0	0

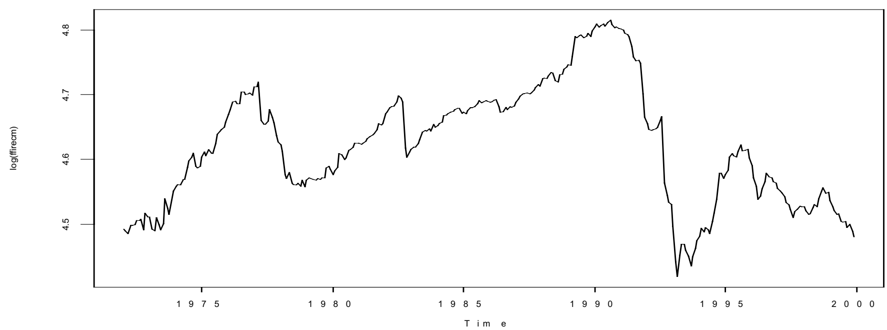
Values above marked with X are more than 2 std errors away from zero, using Bartlett type std errors.

Extended Autocorrelation Table
Calculated using **MM** method for AR fitting and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.97	0.94	0.91	0.88	0.85	0.82
AR1	**0.06**	−0.03	−0.04	0.08	0.16	0.06
AR2	0.32	−0.03	−0.06	0.03	0.12	0.03
AR3	0.06	0.14	−0.02	0.04	0.11	−0.06

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	**0**	0	0	0	X	0
AR2	X	0	0	0	0	0
AR3	0	0	0	0	0	0

Figure 24. **Log (ffirecm)**
y-axis: log(ffirecm)



First-order differencing

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.25	**0.01**	0.00	0.11	0.19	0.08
AR1	0.22	0.05	0.00	0.04	0.15	0.03
AR2	0.18	0.00	-0.02	0.03	0.14	-0.06
AR3	-0.09	0.30	-0.01	-0.06	0.14	0.01

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	**0**	0	X	X	0
AR1	X	0	0	0	X	0
AR2	X	0	0	0	X	0
AR3	0	X	0	0	X	0

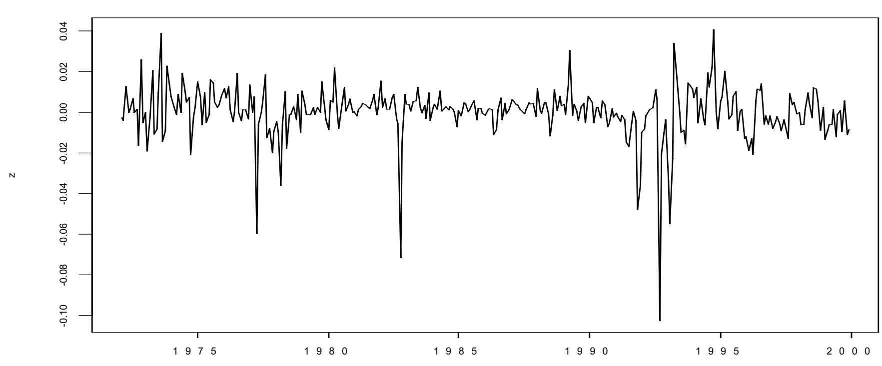
Values above marked with X are more than 2 std errors away from zero, using Bartlett type std errors.

Extended Autocorrelation Table
Calculated using **MM** method for AR fitting and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.27	**0.07**	0.03	0.07	0.09	0.07
AR1	0.09	-0.01	-0.01	0.06	0.07	-0.07
AR2	0.38	-0.01	-0.02	0.06	0.07	-0.06
AR3	0.13	0.03	-0.02	-0.02	0.07	-0.02

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	**0**	0	0	0	0
AR1	0	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

Figure 25. **First difference of log (ffirecm)**
y-axis: z



DEM (fgerecm)

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.95	0.92	0.89	0.86	0.83
AR1	0.32	**0.08**	0.05	-0.01	-0.06	-0.02
AR2	0.07	-0.11	0.05	-0.01	-0.06	-0.01
AR3	0.48	-0.01	0.11	0.03	0.01	0.00

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	0	0	0	0	0	0
AR3	X	0	0	0	0	0

Values above marked with X are more than 2 std errors away from zero, using Bartlett type std errors.

Extended Autocorrelation Table
Calculated using **MM** method for AR fitting and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.96	0.93	0.90	0.87	0.85
AR1	0.32	0.13	**0.05**	-0.01	-0.02	0.01
AR2	-0.03	-0.04	0.07	-0.01	-0.01	0.01
AR3	-0.44	-0.01	0.12	0.03	-0.02	-0.01

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	X	**0**	0	0	0
AR2	0	0	0	0	0	0
AR3	X	0	0	0	0	0

Figure 26. **Log (fgerecm)**
y-axis: log(fgerecm)

First-order differencing

Extended Autocorrelation Table
Calculated using **ols** method for AR fitting and **acf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.30	**0.06**	0.02	-0.04	-0.08	-0.04
AR1	0.13	-0.05	0.02	-0.02	-0.07	-0.04
AR2	0.34	0.02	0.10	0.01	-0.01	-0.03
AR3	0.27	-0.24	0.09	0.08	0.00	-0.01

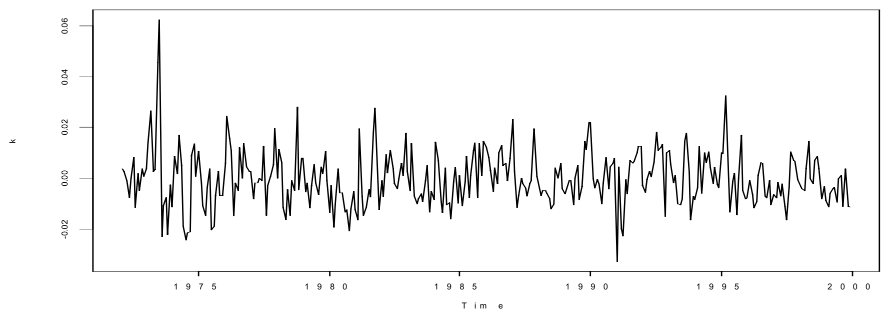
	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	**0**	0	0	0	0
AR1	X	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	X	0	0	0	0

Extended Autocorrelation Table
Calculated using **MM** method for AR fitting and **wacf** for ACF calculations.

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.31	**0.11**	0.02	−0.03	−0.04	−0.01
AR1	0.00	0.04	0.03	−0.02	−0.02	−0.02
AR2	0.39	0.01	0.10	0.01	−0.02	−0.01
AR3	−0.11	0.08	0.23	0.13	0.05	0.01

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	**0**	0	0	0	0
AR1	0	0	0	0	0	0
AR2	X	0	0	0	0	0
AR3	0	0	X	0	0	0

Figure 27. **First-order difference of log (fgerecm)**
y-axis: k



Appendix 7

‘Tsrob’ program: optional robust estimators, weight functions and default choices
An illustration of the simple experiment with the united standard and robust ESACF procedure

This appendix contains robust scale estimators, autocorrelation functions and robust regression estimators and their weight functions. We also refer to the use of information on the large set of result files through customised R functions. Finally, a simple example of the proposed united ESACF procedure is illustrated.

Scale estimators

The default value is the MAD, ie the median of absolute median deviations. The other highly robust scale estimator is Q_n (Croux and Rousseeuw 1992, Hampel et al 1986).

$\mathrm{MAD}(x_i) = 1.4826\ \mathrm{med}_i\{|x_i - \mathrm{med}_j(x_j)|\}$, where 1.4826 is the consistency factor.

$Q_n = 2.2219\ \{|x_i - x_j|; i < j\}_{(k)}$, where the factor 2.2219 is for consistency and $k \approx \binom{n}{2}/4$.

Autocorrelation functions

TACF:

The default of symmetric trimming is 0.05.

WACF:

The default weight scheme is Huber.psi function (Wang and Wei 1993).

Regression parameter estimators

M-estimator:

The initial estimator is that of ordinary least squares. The robust weight function is Huber.psi and the default scale estimator is the MAD (see ‘rlm’ function, eg Venables and Ripley 1996, p. 216). The maximum of iterations is 20, convergency accuracy is 1e-4.

GM-estimator:

The initial estimator for this method is the LMS (the least median of squares) (Rousseeuw and Leroy 1987). The MAD or Q_n can be used as the scale estimator. The robust weight functions are Huber.psi and redescending bisquare function, and we can use either the Schweppe or Mallows type (default) weight function (see Stockinger and Dutter 1987). Maximum iteration steps are 10 for Huber.psi and 2 for bisquare; convergency accuracy is 1e-4.

MM-estimator:

The high breakdown point MM-estimator contains three phases (Yohai 1987, You 1999). The initial estimator is the S-estimator (Rousseeuw and Yohai 1984). Maximum iteration steps is 20, and convergency accuracy is 1e-4. The estimator uses the Tukey bisquare weight function. The MM-estimator is the default estimation method in the ESACF AR(p) iterative fitting. Note that the MM-estimator procedure in ‘Tsrob’ does not contain a controlling test procedure for bias, as does the S-Plus program package (see also Yohai, Stahel and Zamar 1991). Based on our simulation results, this can be regarded only as a ‘small’ drawback of our MM-estimator procedure. In our simulations and in the case of single series, there sometimes occurred ‘warnings’, but they proved not to be serious.

The Huber and Tukey bisquare weight functions with certain tuning constants are commonly used in various robust regression estimators (see eg Franses and van Dijk 2000, p. 65–67, Marazzi 1993 and Yohai, Stahel and Zamar 1991). Marazzi (1993) gives the most comprehensive survey of the various weight functions, tuning contants and computational descriptions for various robust estimators and procedures.

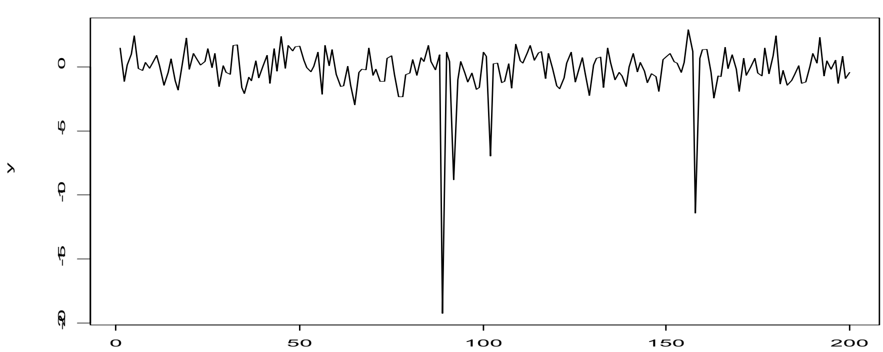
Tsrob R program and use of comprehensive simulation results:

The saved ESACF simulation results can be used through various 'summary.simu.esacf' R code commands to summarise the optional ESACF results. We can produce and print different matrices and plot curves, histograms and box plots (even the coefficients of variation of certain estimates) based on the results of each model from the large set of files named individually 'simutulos**xx**R.data' of single file **xx**.

I An example of the generated ARMA(1, 1) process with 5% isolated AOs

Model: $\phi = 0.5$, $\theta = 0.3$, $a = N(0,1)$, $n = 200$; AOs occur in 5% probability; standard deviation of AO distribution is 10.

```
> nts<-200
> aofun<-function(n){
+ osd<-10 #outlier standard deviation
+ ctnpr<-0.05 #per cent of data contaminated
+
+ a<-rep(0,n)
+ oloc<-runif(n)<ctnpr
+ ncont<-sum(oloc)
+ a[oloc]<-rnorm(ncont,0,osd)
+ a
+ }
> y<-arima.sim(nts,ar=0.5,ma=0.3,aofun=aofun)
> plot(y)
```



II An illustration of the simple experiment with the united ESACF procedure

Data BJA: Series A, Chemical process concentration readings (Box and Jenkins 1970, 1976).

Joint R command:
print(esacf(BJA,method=ols),se.crit=2.2);print(esacf(BJA,method= MM),se.crit=2.5)

```
Vertex is marked in bold. Note that the confidence limits are
calculated by different C values in the standard and robust cases.
std error = standard error

Extended Autocorrelation Table
Calculated using ols method for AR fitting and acf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.57	0.50	0.40	0.36	0.33	0.35
AR1	-0.39	**0.04**	-0.06	-0.01	-0.06	-0.01
AR2	-0.29	-0.27	-0.04	0.01	-0.05	-0.01
AR3	-0.50	-0.01	0.10	-0.01	-0.01	-0.03

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	X	0	0	0	0
AR3	X	0	0	0	0	0

```
Values above marked with X are more than 2.2 std errors away from zero,
using Bartlett type s.e.

Extended Autocorrelation Table
Calculated using MM method for AR fitting and wacf for ACF calculations.
```

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.58	0.50	0.41	0.39	0.35	0.36
AR1	-0.32	**0.05**	-0.08	-0.01	-0.04	-0.02
AR2	-0.27	-0.14	-0.05	0.02	-0.01	-0.01
AR3	-0.49	0.08	-0.06	0.03	0.00	0.00

	MA0	MA1	MA2	MA3	MA4	MA5
AR0	X	X	X	X	X	X
AR1	X	**0**	0	0	0	0
AR2	X	0	0	0	0	0
AR3	X	0	0	0	0	0

```
Values above marked with X are more than 2.5 std errors away from zero,
using Bartlett type s.e.
```

It is interesting to compare the corresponding results of the same series in TT84, Table 6. Note furthermore that the MM/wacf robust estimation gives the same result for this outlier-free series.

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