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**Olli Castrén**

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# **Fiscal-Monetary Policy Coordination and Central Bank Independence**

SUOMEN PANKKI  
Bank of Finland



**BANK OF FINLAND STUDIES E:12 • 1998**

**Fiscal-Monetary  
Policy Coordination  
and Central Bank  
Independence**

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**SUOMEN PANKKI  
BANK OF FINLAND  
P.O. Box 160  
FIN - 00101 HELSINKI  
FINLAND**

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The views expressed in this study are those of the author and do not necessarily reflect the views of the Bank of Finland.

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# Abstract

This study contains four essays in the areas of fiscal-monetary policy coordination, public finance and optimal monetary institutions.

Essay 1 analyses inflation targeting in an economy with decentralised monetary and fiscal policies and centralised wage setting. Depending on the specification of the trade unions' utility functions, both fiscal and monetary policy can be subject to time-inconsistency problems. Inflation targeting can achieve society's optimal outcome in this model only when the trade unions do not have an employment target which is lower than full employment. The result is robust to uncertainty about the monetary authority's preferences.

Essay 2 studies inflation targeting in the context of a monetary union. The setup resembles the Maastricht treaty where a politically representative council delegates monetary policy to an independent central bank. The optimal delegation decision is shown to include an inflation target and a central banker with conservative preferences. It is shown that fiscal discipline in the union increases under such optimal delegation. Moreover, if the voting rules for the delegating council are designed optimally, the council's incentives to renegotiate *ex post* the central bank's target can be eliminated.

Essay 3 focuses on Central Bank (CB) institutions and fiscal-monetary policy coordination under debt stabilisation programmes. When the government and the CB cooperate, a less inflation-averse CB induces faster debt reduction. Under non-cooperative strategies, the opposite result holds. In the presence of political instability, the government shifts fiscal adjustment to the future. Additional adjustment time does not alleviate the situation, but electoral incentives can induce earlier adjustment.

Essay 4 looks at optimal fiscal policy in the presence of foreseeable shocks. When the government cares about the future, the deficit is optimally set lower before the arrival of the shock and more adjustment effort is shifted from *ex post* to *ex ante*. In EMU, fiscal policy will be constrained by the Stability and Growth Pact, which penalises excessive deficits. Thus, in the presence of shocks, fiscal policy before the shock can become highly restrictive under the pact.

**Key words:** fiscal-monetary policy coordination, optimal institutions, inflation targeting, dynamic budget constraint, debt stabilisation.

# Tiivistelmä

Tämä tutkimus koostuu neljästä esseestä, jotka käsittelevät raha- ja finanssipolitiikan yhteensovittamista, julkisen talouden rahoitusproblematiikkaa ja optimaalisia rahapolitiikan instituutioita.

Ensimmäisessä esseessä sovelletaan inflaatiotavoitetta instituutiona talouteen, jossa raha- ja finanssipolitiikka on eriytetty toisistaan ja palkanmuodostus on keskitettynä. Ammattiliittojen hyötyfunktioista riippuen sekä raha- että finanssipolitiikka voivat kärsiä aikaepäjohdonmukaisuus-ongelmasta. Inflaatiotavoite tuottaa yhteiskunnan kannalta optimaalisen tuloksen vain, mikäli ammattiliitoilla ei ole erillistä työllisyystavoitetta joka olisi alhaisempi kuin täystyöllisyystaso.

Toisessa esseessä tarkastellaan inflaatiotavoitteen asettamista rahaliitossa. Tarkasteltava tilanne muistuttaa Maastrichtin sopimusta, jossa edustuksellinen poliittinen elin, neuvosto, delegoi rahapolitiikan itsenäiselle keskuspankille. Optimaalinen delegointi käsittää sekä inflaatiotavoitteen että konservatiiviset preferenssit omaavan keskuspankkijohdon valinnan. Optimaalinen delegointi lisää myös finanssipolitiikan kurinalaisuutta rahaliiton alueella. Jos delegeoivan neuvoston äänestysäännöt suunnitellaan optimaalisesti, houkutus inflaatiotavoitteen poistamiseksi jälkikäteen voidaan eliminoida.

Kolmas essee käsittelee keskuspankki-instituutiota ja raha- ja finanssipolitiikan koordinoitua velkasopeutusohjelman yhteydessä. Kun hallitus ja keskuspankki toimivat yhteistyössä, vähemmän inflaationvastainen keskuspankki mahdollistaa nopeamman velan sopeutuksen. Kun keskuspankki on itsenäinen, tulos on päinvastainen. Poliittisen epävakauden vallitessa hallitus optimaalisesti siirtää velan sopeutusta tulevaisuuteen. Sopeutusajan pidentäminen ei poista ongelmaa, mutta vaalit voivat tarjota hallitukselle kannustimen aikaisempaan sopeutukseen.

Neljännessä esseessä analysoidaan finanssipolitiikkaa ja velan dynamiikkaa tilanteessa, jossa talouteen kohdistuu sokkeja. Jos hallitus on kaukonäköinen, optimaalinen budjettivaje ennen sokkia on pienempi kuin tilanteessa, jossa sokkia ei odoteta. EMUn vakaussopimus, joka rankaisee ylisuurista vajeista, pakottaa sokkien olemassa ollessa myös *ex ante* -vajeen alhaisemmalle uralle kuin mitä ilman vakaussopimusta olisi optimaalista valita.

Asiasanat: finanssi- ja rahapolitiikan koordinointi, optimaaliset instituutiot, inflaatiotavoite, dynaaminen budjettirajoite, velkasopeutus

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Helsinki, April 1998  
Olli Castrén





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# General Introduction

## 1 Motivation

During the past decade, the global economic-political environment has experienced institutional reforms of unprecedented scale. One of the leading ideas behind these reforms has been the desire to eliminate the so called political business cycle, according to which electoral gains can be achieved by exploiting the short-run Phillips curve.<sup>1</sup> As a result, Central Bank independence, or formal separation of monetary and fiscal powers, has by now been widely accepted as a way of achieving sustainable, non-inflationary growth.

However, the academic research focusing on Central Bank independence tends to ignore the fiscal response on the proposed changes in the monetary regimes. The common theme of this thesis is to fill a part of this gap by analysing optimal fiscal and monetary policy, and the interaction between them, under different institutional frameworks. However, due to the general complexity of macroeconomic policy, and the many different issues that can be studied, the individual chapters of this thesis show a relatively large variety. More specifically, the concepts that we have chosen to cover in a more detailed manner can be classified as follows. (i) Essays 1 and 2 analyse optimal inflation targets for Central Banks in the presence of endogenous fiscal policy. (ii) Essays 3 and 4 study macroeconomic policy under a dynamic government budget constraint that allows the policymakers to trade off present policy to future policy by adjusting debt accumulation. Before we turn to review the literature in these topics, we take a brief look at the past research in macroeconomic policy in more general terms.

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<sup>1</sup> The theoretical analysis of political business cycles was formalised by Nordhaus (1975) and Alesina (1987).

## 2 General Literature Overview

The standard economic treatment of macroeconomic policy takes monetary and fiscal policy as exogenous to the economic system. In contrast, the analysis of policy that was initiated by Tinbergen (1952) and further developed by Brainard (1967) and Poole (1970) studied the behaviour of the policymaker in the face of economic constraints and uncertainties. In other words, this literature endogenised the policy process in the economic analysis. Building on the work by Poole, the possibility of conflict between the fiscal and monetary policymakers was formally modelled by Pindyck (1976) and Ribe (1980) who dealt with the impact of coordination and lack of coordination on the efficiency of policy. Soon thereafter, the studies in rules versus discretion (Kydland and Prescott, 1977 and Barro and Gordon, 1983) revolutionised the research in the area. Focusing on monetary policy as a game between the government and the private sector, these models included the rational expectations hypothesis in an analysis of credibility and dynamic consistency.<sup>2</sup> Finally, the important link between the literatures in credibility and policy coordination was provided by Tabellini (1986) and Alesina and Tabellini (1987), who were the first ones to model the fiscal-monetary conflict using rational expectations and non-cooperative game theory.<sup>3</sup> These two influential papers form the foundations to the analysis in this thesis.

### 2.1 Inflation Targeting and Fiscal-Monetary Policy Coordination

During the past few years, inflation targeting has become an increasingly popular means of achieving price level stability among the OECD countries. In particular, the benefits of inflation targeting have often been linked with increased monetary policy account-

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<sup>2</sup> Persson and Tabellini (1990) provide an excellent discussion about the implications of this approach to the research in monetary policy.

<sup>3</sup> More recent extensions to Alesina and Tabellini's model, and on the effects of budgetary policy on inflation control more generally, include Debelle and Fischer (1994), Nordhaus (1994), Woodford (1994) and Canzoneri and Diba (1996).

ability.<sup>4</sup> Many Central Banks who have recently been granted higher levels of independence in the day-to-day conduct of monetary policy have simultaneously been subjected to publicly announced inflation targets, which nail down the medium and long term goals of monetary policy. In that way, the public has been made aware of the ultimate objectives of the monetary authorities, while the Central Bank still can use its discretion when responding to short-run disturbances in the economy.

The theoretical link between inflation targeting and credible low-inflation policy was provided by Svensson (1997), who showed that an optimal inflation target can achieve the so called contractual equilibrium if it is enforced by a simple dismissal rule for the Central Bank head. The foundations of the contractual approach to monetary policy rest on the ideas of Walsh (1995a) and Persson and Tabellini (1993). According to Walsh, a contract which specifies the income of the Central Bank head as a function of realised inflation can mimic the optimal monetary policy rule of Barro and Gordon (1983). In particular, the optimal contract can induce the Central Bank to generate lower inflation without any adverse stabilisation effects. Therefore, by affecting the Central Bank's *targets* rather than *preferences* the contract avoids the credibility-flexibility trade-off which generally arises when policy is delegated to an inflation-averse, or "conservative", central banker (Rogoff, 1985).<sup>5</sup>

However, the literature in contracts and inflation targets has ignored the endogenous response of fiscal policy to the proposed changes in the Central Bank's status. Indeed, in a model where the fiscal authority (the government) chooses the tax rate and fiscal and monetary policies are made interdependent through the government budget constraint, Alesina and Tabellini (1987) found that policy rules are not necessarily welfare improving if the two authorities have

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<sup>4</sup> The academic literature has roughly categorised the institutions for increased monetary stability as follows. The legislative route aims at lower inflation with a Central Bank (CB) law that explicitly states the objectives of monetary policy. The famous example in this tradition is the German Bundesbank. The targeting approach, which has recently become popular in several countries, states that the CB can be granted formal independence if its long run policy objectives are specified by society and it is held accountable *ex post* for its actions.

<sup>5</sup> Lohmann (1992) introduced an escape clause -type alternative dismissal rule, that induces the conservative central banker to stabilise more after a particularly severe realisation of the output shock.

asymmetric preferences between their policy objectives.<sup>6</sup> These results were extended by Jensen (1992) who showed that in the presence of two time inconsistency problems, commitment by one policymaker has positive externalities in that it alleviates the other player's problem.

The only attempt to provide a link between the contractual approach and the literature in decentralised fiscal and monetary policy has so far been provided by Huang and Padilla (1995). They included a Walsh contract in a stylised version of the Alesina and Tabellini model and found that in the presence of endogenous fiscal policy, a contract cannot induce the optimal policy rule. In Essays 1 and 2, we provide a critical analysis of this result.

## 2.2 Optimal Policy under a Dynamic Government Budget Constraint

During the 1980's many countries, both in the developed and in the developing world, experienced substantial increases in government indebtedness. Therefore, debt stabilisation has become a prominent policy issue. In particular, recent interest in debt stabilisation has been related to the fiscal entrance criteria of the European Economic and Monetary Union (EMU) and the subsequent conduct of national fiscal policy once the union has taken to the air.<sup>7</sup> Another topical area are the strong fiscal austerity requirements which have become commonly associated with international lending to sovereign governments. A crucial issue which is closely related with both these topics concerns the optimal design of the economic and political institutions during the stabilisation programmes.

In the dynamic interaction context, the policymakers generally have a choice between adjusting today and shifting the burden till tomorrow. This issue has been analysed in slightly different contexts

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<sup>6</sup> More specifically, the lower inflation rate, which is due to the Central Bank's commitment to an optimal monetary policy rule, can produce an excessive loss to the government in terms of other objectives.

<sup>7</sup> According to the Maastricht treaty, a country can enter EMU only if its government debt is below 60% of GDP or if the debt-GDP ratio is approaching the target level at sufficient speed. On the other hand, EMU Stability and Growth Pact states that in EMU, governments who run deficits higher than 3% of GDP will be penalised by a proportional fine not greater than 0.5% of the GDP.



by Barro (1979), Lucas and Stokey (1983) and Blanchard (1985). In addition, fiscal and monetary authorities typically face a conflict about whether fiscal or monetary instruments should be employed in such adjustment. The theoretical analysis of dynamic interdependence in economic policy was initiated by Sargent and Wallace (1981) in the famous "unpleasant monetarist arithmetic" argument, and further developed by Drazen and Helpman (1990). Another branch of research has explored the interaction in a game-theoretic framework. Petit (1989) and Huges Hallett and Petit (1990) consider the open-loop equilibria in which the private sector plays a non-strategic role. Levine and Pearlman (1992) and Levine and Brockiner (1994) study the interaction between fiscal authorities in a monetary union. Other papers which utilise dynamic budget constraints in fiscal-monetary interaction include Obstfeld (1991), Jensen (1994) and van der Ploeg (1995).

Perhaps the most insightful analysis was provided by Tabellini (1986), who employed a differential game and a dynamic government budget constraint to model the strategic interaction between fiscal and monetary authorities.<sup>8</sup> In the context of EMU, van Aarle, Bovenberg and Raith (1995a,b) extended Tabellini's work in several dimensions by applying more rigorous analytical treatment. Essays 3 and 4 follow this tradition.

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<sup>8</sup> As was discussed above, in a static analysis Alesina and Tabellini (1987) include the private sector as third player in the fiscal-monetary game.

## 3 Brief Outline of the Four Essays

### 3.1 Inflation Targeting, Wage Formation and Fiscal-Monetary Policy Coordination

This Essay analyses monetary and fiscal policy interaction in a closed economy where a monopoly trade union faces a trade-off between employment and real wages. The natural rate of employment and the equilibrium policy outcomes are shown to be contingent upon the trade union's wage negotiation strategy. In addition, we find that depending on the trade union's targets and preferences, both fiscal and monetary policy can be subject to time-inconsistency problems in the model. In such a framework, the properties of the optimal inflation target are more general than in the original model of Svensson (1997). In particular, the optimal target is the lower the more weight the trade union assigns on its real wage objective. The main result is, however, that the performance of the inflation targeting institution in inducing the society's optimal outcome is sensitive to the trade union behaviour. If both fiscal and monetary policies are subject to time-inconsistencies, inflation targeting as a single instrument cannot be successful in eliminating both problems. However, inflation targeting can solve the problem which arises from the asymmetry between the fiscal and monetary preferences. In the absence of a trade union employment target, which would be lower than full employment, the fiscal time-inconsistency problem does not arise, and inflation targeting can implement the optimal outcome. This result holds even in the presence of uncertainty about the policymakers' preferences.

### 3.2 Inflation Targeting in a Monetary Union

In this Essay, inflation targeting is applied in the context of the European Monetary Union (EMU). We explore the case where a representative institution, such as the EU Council, has delegated common monetary policy to a Central Bank (like the ECB). More specifically, we study two countries who together form the Council but who retain their national tax-setting powers. The Council, who is assumed to represent the social welfare in the monetary union,

delegates monetary policy to the CB. However, due to the independent nature of the CB, it stabilises the output deviations in the whole currency area, while the Council is interested in the sum of individual output deviations. The optimal delegation decision, which minimises the Council's welfare loss, is then shown to include both inflation targeting and conservative CB preferences. The optimal inflation target, which is contingent upon the Council's preferences, is shown to increase fiscal discipline in the union, because lower seignorage revenues induce the governments to choose higher taxes. Therefore, in our one-period framework, inflation targeting can work as a partial substitute to the proposed EMU Stability Pact. In addition, if the voting rules of the Council are designed optimally, so that the individual Council members enter in a staggered order, the Council's incentives to renegotiate the ECB's inflation target can be eliminated and the institution becomes *ex post* time-consistent.

### 3.3 Optimal Fiscal and Monetary Policy under Debt Reduction Programmes

This Essay looks at optimal Central Bank institutions and policy coordination between the government and the Central Bank under a dynamic government budget constraint. Optimal fiscal and monetary policies are derived under the constraint that government debt must be reduced under a finite time horizon. The time paths for debt, budget deficits and money supply are evaluated under different assumptions about the policymakers' static and dynamic preferences. It turns out, that under policy cooperation a higher rate of interest, or a less inflation-averse Central Bank, can induce faster debt reduction. The cooperative equilibrium is evaluated using the phase diagram analysis, and the results from the deterministic analysis are shown to be robust to stochastic disturbances. Under non-cooperative strategies, we find that a more inflation-averse Central Bank induces faster debt adjustment by choosing lower inflation and forcing the government to more substantial deficit reductions. Political instability shifts adjustment to the future under both regimes, thereby causing debt to accumulate over time. It is shown how prolonged adjustment does not alleviate the situation, while simple electoral incentives can induce the policymakers to adjust earlier.

### 3.4 Optimal Fiscal Policy in the Presence of Debt Shocks and the EMU Stability Pact

Continuing the analysis under a dynamic government budget constraint, this Essay studies how a future shock in debt affects the government's optimal choice of deficit before and after the shock. Lucas and Stokey (1983) and Flemming (1987) found that if the government expects debt to go up at a certain future date, it optimally runs surpluses before the event takes place. In our stylised debt adjustment framework, we find a similar pattern. When the government cares about the future, deficit is optimally set lower before the arrival of the shock. In other words, more adjustment will be shifted from *ex post* to *ex ante*. This effect is the stronger the higher is the economy's rate of interest or the higher is the weight assigned by the government on lower debt relative to the higher deficit. We then turn to analyse how the results change if the government's choice of deficit is constrained by a mechanism, like the proposed EMU Stability Pact, which penalises the government from running excessive deficits. It turns out, that before the shock fiscal policy becomes even more restrictive when the government is threatened by a penalty as a result of deficit exceeding the critical level after the shock. In a world where large and regular shocks are possible, the sanctions of the Pact can thus reduce society's welfare significantly especially in the cases where the government is short-sighted or it has significant spending commitments.

# Inflation Targeting, Wage Formation and Fiscal-Monetary Policy Coordination

## 1 Introduction

During the 1990's, inflation targeting has become an increasingly popular way of achieving price stability in the OECD countries.<sup>1</sup> However, in relation to the literature on rules versus discretion, the theoretical foundations of inflation targeting were only recently laid by Svensson (1997). His analysis extended the work on Central Bank contracts, which was originally initiated by Walsh (1995a). This Essay represents an attempt to generalise the properties of inflation targeting in a framework where fiscal policy responds endogenously to changes in the monetary regime. In particular, our aim is to investigate how the standard welfare results change when the fiscal and monetary policies are made interdependent, and to what extent the outcomes are sensitive to assumptions about the private sector behaviour and uncertainty on the policymakers' preferences.

The analysis is based on the time-consistent policy framework, originally developed by Kydland and Prescott (1977) and Barro and Gordon (1983). The general conclusion from this field of research is that because commitments to optimal policy rules are not time-consistent, monetary policy under discretion shows an inflationary bias. Alesina and Tabellini (1987) generalised this result to a fiscal-monetary framework. They found that if the fiscal and monetary policymakers assign different relative weights on the policy objectives, commitments are not necessarily welfare improving even if the time-inconsistency problem could be somehow resolved. This happens because in the presence of fiscal objectives, the model is characterised by two types of distortions: those which arise from the

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<sup>1</sup> Explicit inflation targeting regimes have been introduced at least in New Zealand, Canada, the U.K., Sweden, Finland, Austria and Spain.

time-inconsistency problems, and those which originate to the asymmetry in preferences.

In this Essay, we model the private sector as a continuum of trade unions (henceforth: TUs), each of which has monopoly power in its own sector. A typical TU faces a trade-off between real wages and employment, and it is willing to accept some unemployment in exchange to higher real wages. Because the TU's real wage claims and the TU's too low employment target render the natural rate of employment below the policymakers' target level, the monetary authority (the Central Bank, henceforth: CB) attempts to increase employment through inflation surprises, while the fiscal authority (henceforth: FA) attempts to stimulate the economy by cutting a distortionary tax. The policies are made interdependent through the government budget constraint, which requires that government expenditures are financed through tax returns as well as seignorage. However, because the FA and the CB assign different weights on the different policy objectives, the two authorities face a conflict about the relative burden of financing the government expenditures.<sup>2</sup>

Our results illustrate that if the FA subjects the CB to an optimal inflation target, the FA can improve its welfare compared to the discretionary outcome. However, the inflation targeting regime does not result in the society's (the FA's) optimal outcome, unless the TU puts no weight on its employment objective. Therefore, the performance of the institution is contingent upon the TU's preferences and targets, which constitute the source of unemployment in the model.<sup>3</sup> When we relax the assumption that the private sector faces no uncertainty about the policymakers' objective functions, we find that the distortions which are characteristic for discretionary policymaking are made worse. Nevertheless, uncertainty does not have to undermine the performance of the inflation targeting institution, if the target is designed in a way that correctly takes into account the aspects of uncertainty.

The rest of this Essay proceeds as follows. Section 2 specifies the model with two policymakers and centralised wage setting. In

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<sup>2</sup> Throughout the Essay, the social welfare is assumed to coincide with the fiscal authority's welfare, reflecting the fact that the fiscal authority (the government) generally represents the view of the majority of the society.

<sup>3</sup> A related study is Huang and Padilla (1995), who applied the Walsh contract in a framework where the private sector is atomistic and where only the CB is subject to a time-inconsistency problem. However, the fact that we use the inflation targeting approach alters Huang and Padilla's conclusions substantially, as they show how the contract can not induce the optimal outcome in the presence of endogenous fiscal policy.

section 3, we study how different assumptions about the TU's wage negotiation strategy affect the economy's natural rate of employment. In section 4, we derive the equilibrium policy outcomes when both FA and CB operate under discretion. Section 5 studies the welfare effects of commitments to policy rules under decentralised policymaking. In section 6, we study the optimal inflation targeting institution under different assumptions about the TU behaviour, and discuss the implications of uncertainty about the policymakers' preferences. Section 7 concludes.

## 2 The Model

Our model has the following agents: a fiscal authority, FA; a central bank, CB; and a private sector, which is composed of a large number of trade unions, TUs, and firms. Following Herrendorf and Lockwood (1997), we assume that each TU has monopoly power over wage setting in a particular sector, and that a sector is composed of one firm and one trade union. Moreover, there is a continuum of identical TUs and firms. Fiscal policy (i.e. the tax rate) and monetary policy (i.e. the inflation rate) are set simultaneously by the two authorities, who take each others' and the TU's actions as given. We make the simplification which is common in two-authority games, namely we abstract from shocks and thus from short-run stabilisation policies.<sup>4</sup> Under these assumptions, a typical firm's labour demand results from profit maximisation under a Cobb-Douglas technology with labour as the only variable input (see Alesina and Tabellini, 1987, for the detailed microfoundations):

$$(2.1) \quad l = p - w - \tau$$

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<sup>4</sup> In other words, in the current analysis the fiscal-monetary interaction occurs solely through the effects of taxation on long-run output. Clearly, there are different ways of characterising these interactions, most notably by assuming that the policymakers are also involved in stabilising stochastic shocks in the short run. For a discussion of stabilisation policy in a related model, the reader is referred to Debelle and Fischer (1994), Beetsma and Bovenberg (1997) and Essay 2 of this thesis.

In (2.1),  $l$  = employment,  $p$  = price level,  $w$  = nominal wage and  $\tau$  = distortionary tax levied on firms' total revenues (all in logs). With  $p_{t-1}$  and  $w_{t-1}$  normalised to zero,  $p$  also equals inflation and  $w$  equals the growth rate of nominal wages. Employment falls with an increase in the real wage,  $w-p$ , but increases when taxes are cut.

Fiscal and monetary policies are made interdependent through the government budget constraint. We ignore here the dynamic aspects of the budget constraint, and therefore with no public debt, it is required that expenditures equal tax revenues plus seignorage financing.<sup>5</sup> In steady state, the latter approximately equals the inflation rate in *per capita* terms:

$$(2.2) \quad g = \tau + p$$

In (2.2),  $g$  denotes the ratio of public expenditures to output. Taxes are assumed to have no demand effects, i.e. any tax induced change in government expenditures is exactly offset by an equivalent change in private expenditures. Therefore, taxes affect the economy only through their distortionary effects on firms' decisions.

Nominal wages are negotiated by the private sector in the labour market. In the literature where the FA is not explicitly included, the private sector is usually modelled as atomistic. However, we apply a more general framework along the lines of new-Keynesian theories of unemployment. In particular, we consider a representative monopoly TU who cares about two objectives, employment and real wages (Jensen, 1992; Schaling, 1995; Herrendorf and Lockwood, 1997). In particular, we assume that the TU trades off these objectives according to the following standard loss function:

$$(2.3) \quad \frac{1}{2} \{ (l - l^*)^2 + \chi (w - p - u^*)^2 \}$$

In (2.3),  $\chi$  is the relative weight assigned by the TU on the real wage objective. We call this the TU's "wage negotiation parameter." The TU wishes to stabilise real wages around a target level  $u^*$ , and it dislikes deviations of employment from its target level  $l^*$ , which is

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<sup>5</sup> See Beetsma and Bovenberg (1997) for an analysis of two-period debt dynamics in a related model.



assumed to correspond to the level where all union members are employed.<sup>6</sup> However, because not all the participants of the labour force are union members, the model employs a variant of insider-outsider approach to the labour market (Blanchard and Summers, 1986, Lindbeck and Snower, 1986). In such a framework, for the TU an employment level above its own target level may result in a welfare loss because it increases the demand for labour from non-union members. The formulation of (2.3) has two particular implications, which are discussed in more detail below. First, there are two sources for unemployment, the TU's real wage target  $u^*$  and the TU's employment target  $l^*$ , which is too low from the social point of view. Second, whenever  $\chi$  is finite the TU forms expectations of both taxes and inflation.

Policies are performed by two independent authorities. More specifically, the FA chooses the tax rate, whereas the CB chooses the rate of growth of money supply. In the latter case, it is assumed that inflation can be directly set by choosing money growth. Both authorities care about employment, public expenditures and inflation. Therefore, their loss functions,  $L^{FA}$  and  $L^{CB}$ , respectively, can be specified as follows:

$$a) L^{FA} = \frac{1}{2} \{ p^2 + \delta_1 l^2 + \delta_2 (g - g^*)^2 \} \quad (2.4)$$

$$b) L^{CB} = \frac{1}{2} \{ p^2 + \mu_1 l^2 + \mu_2 (g - g^*)^2 \}$$

Policymakers dislike deviations of actual employment ( $l$ ) from full employment, which is normalised to zero. Policymakers also dislike deviations of public spending *per capita* ( $g$ ) and inflation ( $p$ ) from their respective targets  $g^*$  and zero.<sup>7</sup> The assumption that  $g^*$  is positive is crucial for the results that follow, because it implies that the policymakers will tolerate some inflation and some tax distortion to finance a positive amount of public expenditures. Furthermore, the

<sup>6</sup> Alesina and Tabellini implicitly assume that the TU gives no weight on the employment objective and only cares about stabilising real wages, i.e. they set  $\chi = \infty$ .

<sup>7</sup> More specifically, zero corresponds to the optimal inflation rate if the weight given to government expenditures were zero and the CB faced no tradeoff between inflation and expenditures. This target rate will generally differ from the explicit inflation target  $p$ , which will be introduced below.

two authorities disagree about the relative importance of the different targets:  $\mu_i \neq \delta_i$  for  $i = 1, 2$ . Specifically, we assume that  $\delta_i \geq \mu_i$ ,  $i = 1, 2$ , so that the CB assigns a higher relative weight on inflation than the FA does. Finally, the FA is a benevolent government who represents the interests of the majority in the society. Therefore, we can evaluate society's welfare under the different regimes simply by using the FA's loss function (2.4a). We now turn to study the wage formation process in detail.

### 3 Trade Union Behaviour and the Natural Rate of Employment

The purpose of this section is to study how different assumptions about the TU behaviour affect the TU's optimal wage negotiation strategy. In particular, we will consider aggressive and cooperative TU strategies, and their respective implications to the economy's natural rate of employment. The natural rate in turn will affect the equilibrium policy outcomes which will be derived later on.

The objective of the representative monopoly TU is to stabilise employment and real wages around their respective target levels. In so doing, the TU chooses the nominal wage to minimise its loss (2.3), subject to (2.1), taking as given expected inflation and taxes.<sup>8</sup> This gives the following TU reaction function:

$$\begin{aligned}
 w &= p^e - \gamma \tau^e - \gamma v^*, \\
 (3.1) \quad v^* &\equiv l^* - \chi u^* \\
 \gamma &\equiv \frac{1}{1 + \chi}
 \end{aligned}$$

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<sup>8</sup> This follows because each TU is small relative to the size of the economy.

In (3.1), the superscript  $e$  denotes an expected value. We impose the standard assumption that  $u^* > l^*/\chi$ . This means that the union's real wage target exceeds the real wage that, *ex ante*, would be consistent with full employment, if there were no tax distortions (see Alesina and Tabellini, and Herrendorf and Lockwood, for further discussion). When  $\gamma > 0$  (i.e. when  $\chi$  is finite), the TU sets wages conditional on expectations of both policymakers' control variables. This follows directly from the specifications of the employment identity (2.1) and the TU loss function (2.3). A direct implication of (3.1) is that because employment can be increased by both inflation and taxes when the natural rate is below the policymakers' target level, both FA and CB face incentives to generate policy surprises to boost employment.

Next, we turn to the factors that determine the natural rate of employment in the model. We define the natural rate as the average level of employment around which the actual level fluctuates due to policy surprises or shocks (the latter are not present in our model). Therefore, in our model the natural rate can be obtained by substituting (3.1) into (2.1), and eliminating all policy surprises, by setting  $p^e = p$  and  $\tau^e = \tau$ . This gives:

$$(3.2) \quad \tilde{l} = -(1-\gamma)\tau + \gamma w^*$$

In (3.2),  $\tilde{l}$  denotes the natural rate of employment. When considering how the TU's wage setting decision affects the natural rate, we first focus on the the term  $\gamma w^*$  in equation (3.2). This term illustrates the equilibrium level of employment that results when the TU optimally trades off employment to real wages, if there were no tax distortions. Because the TU targets full employment for its members only, the economy's unemployment rate net the tax distortions can be defined as the difference between total employment, which is targeted by the policymakers and is normalised to zero, and  $\gamma w^*$ . Therefore, it follows that

$$(3.3) \quad n = 0 - \gamma w^* = \frac{1}{1+\chi}(\chi u^* - l^*)$$

In (3.3),  $n$  denotes the economy's unemployment rate that results from the TU's optimal wage-setting decision, in the absence of tax distortions.<sup>9</sup> Studying (3.3) reveals how the TU's preferences *per se* affect the unemployment rate via  $n$ . Taking the first and second order conditions of (3.3) with respect to the wage negotiation parameter  $\chi$  on one hand, and the first-order condition with respect to the TU's employment target  $l^*$  on the other hand, gives:

$$(3.4) \quad \frac{\partial n}{\partial \chi} = \frac{u^* + l^*}{(1 + \chi)^2} > 0 \quad \frac{\partial^2 n}{\partial \chi^2} = \frac{-2(1 + \chi)(u^* + l^*)}{(1 + \chi)^4} < 0,$$

$$\frac{\partial n}{\partial l^*} = -\frac{1}{1 + \chi} < 0$$

It follows that the higher is the wage negotiation parameter  $\chi$ , the higher is unemployment. This is because a TU that pushes for higher real wages drives down labour demand and hence increases unemployment, as can be seen from (2.1). On the other hand, the higher is the TU's employment target  $l^*$ , the lower is unemployment. Equation (3.2) can now be written as:

$$(3.5) \quad \tilde{l} = -(1 - \gamma)\tau - n$$

Clearly, the natural rate of employment is decreasing in  $n$ . In addition, from (3.5) it can be seen that the extent to which the distortionary taxes affect the natural rate also depends on the wage negotiation parameter  $\chi$ , via  $\gamma$ . Further manipulations of (3.5) yield the following result:

**Proposition 1.** If the TU's preferences and target levels satisfy  $\chi = l^*/u^*$  and  $l^* = 0$ , the natural rate of employment is at the policymakers' target level, independent on the tax rate.

**Proof.** From (3.5), in order to obtain  $\tilde{l} = 0$  it is required that  $n = 0$  and  $\gamma = 1$ . From (3.3), it follows that  $n = 0$  iff  $\chi = (l^*/u^*)$ . On the

<sup>9</sup> Another potential source of fiscal-monetary interactions is the case where distortionary tax policies do not affect the natural rate, but labour taxation impacts on the capital labour ratio, leading to different types of inefficiencies. However, because our model is a single-factor model, it does not allow such an additional channel to operate.

other hand,  $\gamma = 1$  iff  $\chi = 0$ . Therefore, if the TU's preferences satisfy  $\chi = (l^*/u^*)$  and its target level of employment equals the policymakers' target level so that  $l^* = 0$ , it follows that  $\chi = (0/u^*) = 0$  and  $\gamma \equiv u^*/(u^*+0) = 1$ . Under such parameter values, the TU's reaction function and the implied natural rate of employment are given as follows:

$$(3.6) \quad \begin{aligned} w &= p^e - \tau^e \\ \Rightarrow \tilde{l} &= 0. \end{aligned}$$

Clearly, the natural rate is independent on taxes and it is at the policymakers' target level.

*Q.E.D.*

We call a TU with  $\chi = (l^*/u^*)$  and  $l^* = 0$  *cooperative*, as the policymakers' target level of employment is not jeopardised by such TU's optimal choice of nominal wages.

However, if the TU has "cooperative" preferences while it targets full employment only for the TU members (so that  $\chi = (l^*/u^*)$  but  $l^* < 0$ ), we obtain the following outcome:

$$(3.7) \quad \begin{aligned} \chi &= \frac{l^*}{u^*} \Rightarrow \gamma = \frac{u^*}{u^* + l^*} \equiv \gamma^o \Rightarrow n = 0 \\ \Rightarrow w &= p^e - \gamma^o \tau^e \\ \Rightarrow \tilde{l} &= -(1 - \gamma^o) \tau \end{aligned}$$

In a model with no endogenous fiscal policy (e.g. Schaling, 1995) such TU would already be fully cooperative, as the natural rate of employment would be on the policymaker's target level. However, in our model it holds that whenever  $\chi$  is positive (so that  $\gamma < 1$ ), the TU negotiates wages that are too high to be compatible with a zero employment rate. Consequently, we say that a TU with  $\chi = l^*/u^*$ ,  $l^* < 0$  is only *imperfectly cooperative*.

Finally, whenever  $\chi > l^*/u^*$ , the term  $\gamma^*$  in (3.1) is positive and the TU chooses wages that result in positive union unemployment and thereby lower natural rate of employment. In the extreme case, where  $\chi = \infty$  (which implies that  $\gamma = 0$ ), the TU has no employment objective at all, and the economy's unemployment rate net of tax

distortions equals  $u^*$ , the TU's real wage target.<sup>10</sup> The TU's reaction function and the natural rate of employment are then given by:

$$(3.8) \quad w = p^e + u^*$$

$$\Rightarrow \tilde{l} = -\tau - u^*$$

We call a TU with  $\chi = \infty$  *aggressive*, as its real wage choice reduces the natural rate of employment more than in the other two cases, thereby imposing strong incentives to the CB to generate inflation surprises. Note that when the TU is aggressive, it will not form any expectations about taxes, and consequently the FA is not subject to a time-inconsistency problem. The natural rate is then decreasing in the tax rate, independent on the TU's preferences.

In the following sections we investigate how the agents' preferences and incentives interact to determine the equilibrium policy outcomes under different institutional regimes.

## 4 Monetary and Fiscal Policy under Discretion

Let us start with considering the regime where no commitments can be made by any policymaker. The three players then act like Nash players, all taking everyone else's actions as given, and the outcome is therefore time-consistent by construction. The policy game proceeds as follows: (i) In the first stage, the TU rationally chooses the nominal wage. (ii) In the second stage, the FA chooses the tax rate and the CB simultaneously chooses the inflation rate, both taking each others' and the TU's choice as given. (iii) Finally, employment results from the firms' labour demand (2.1) and government expenditures result from the budget constraint (2.2).

The two policymakers' first-order conditions are obtained by minimising (2.4a,b), with respect to (2.1) and (2.2):

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<sup>10</sup> We then obtain the same specification than Alesina and Tabellini.

$$a) \frac{\partial L^{FA}}{\partial \tau} = -\delta_1(p^e - w - \tau) + \delta_2(p^e + \tau - g^*) = 0$$

(4.1)

$$b) \frac{\partial L^{CB}}{\partial p} = p + \mu_1(p - w - \tau^e) + \mu_2(p + \tau^e - g^*) = 0$$

Next, we substitute the TU's optimal choice of nominal wages, equation (3.1), into (4.1):

$$a) -\delta_1(p^e - p^e + \gamma\tau^e - n - \tau) + \delta_2(p^e + \tau - g^*) = 0$$

(4.2)

$$b) p + \mu_1(p - p^e + \gamma\tau^e - n - \tau^e) + \mu_2(p + \tau^e - g^*) = 0$$

After taking expectations through this equation, and rearranging, we obtain the following expressions for expected taxes and expected inflation under discretion:

$$a) \tau^e = \frac{1}{\delta_1(1-\gamma) + \delta_2} [\delta_2(g^* - p^e) - \delta_1 n]$$

(4.3)

$$b) p^e = \frac{1}{1 + \mu_2} \{ \mu_1[(1-\gamma)\tau^e + n] + \mu_2(g^* - \tau^e) \}$$

It turns out, that the more weight the TU puts on the real wage objective (meaning that  $\chi$  is high and, consequently, that  $\gamma$  is low, i.e., the TU is more aggressive), the lower are taxes and the higher is inflation. Clearly, as the natural rate of employment is rendered further below zero by the TU's more aggressive behaviour, the policymakers face higher incentives to use their instruments to boost employment. These incentives are correctly anticipated by the rational TU. In the second stage of the game, the policymakers observe the expectations and set the inflation and tax rates accordingly. Note that as the model is completely deterministic, actual inflation and taxes coincide with expected inflation and taxes (so that  $p = p^e$  and  $\tau = \tau^e$ ). The time consistent Nash equilibrium can then be obtained by solving (4.3a) and (4.3b) together, and using the employment and government expenditures relations (2.1) and (2.2), respectively:

$$\begin{aligned}
a) p^D &= \frac{\mu_1 \delta_2 + \delta_1 \mu_2}{B} [(1-\gamma)g^* + n] \\
b) \tau^D &= g^* - \frac{1}{B} \{(\delta_1(1+\mu_2) + \mu_1 \delta_2) [(1-\gamma)g^* + n]\} \\
(4.4) \quad c) l^D &= p^D - w - \tau^D = -\frac{\delta_2}{B} [(1-\gamma)g^* + n] \\
d) g^D &= \tau^D + p^D = g^* - \frac{\delta_1}{B} [(1-\gamma)g^* + n] \\
B &\equiv \delta_2 + (1-\gamma)[\delta_1(1+\mu_2) + \mu_1 \delta_2]
\end{aligned}$$

It turns out, that inflation is above zero whenever the CB assigns some weight either on the employment objective or on the expenditure objective (i.e. when  $\mu_1 > 0$  and/or  $\mu_2 > 0$ : See Appendix 1 for the comparative statics). A positive weight on employment means that the CB has an incentive to generate surprise inflation, whenever the natural rate of employment is below zero. In the equilibrium, however, these incentives are correctly anticipated by the TU in the first period, and therefore the CB is not fully successful in its attempt to boost employment. We look at this problem in more detail below. In addition, a positive weight on expenditures means that the CB is willing to tolerate some excess inflation in exchange for higher expenditures.

On the other hand, the time-consistent tax rate depends on the respective weights the FA puts on the employment and expenditure objectives ( $\delta_1$  vs.  $\delta_2$ ). A high (low) relative weight on employment implies a low (high) tax rate. Furthermore, a higher weight assigned by the CB on the expenditure objective ( $\mu_2$ ) means that taxes can be set lower, as the CB is willing to finance a larger share of the expenditure objective through seignorage.

The equilibrium tax rate is increasing and the inflation rate is decreasing in  $\gamma$ . Intuitively, if the TU assigns a higher weight on its employment objective relative to the real wage objective ( $\gamma$  is high), the policymakers face lower incentives to generate policy surprises. From (4.4) we can also see that the equilibrium inflation rate is increasing and the equilibrium tax rate is decreasing in the share of unemployment which is captured by  $n$ . Clearly, the optimal distortionary tax is lower the higher is unemployment, implying that inflation must be set higher to finance a greater share of the government expenditures. Finally, the time consistent employment rate is further away from its target level zero when the TU's weight



assigned on employment ( $\gamma$ ) is low, or the FA's weight on the expenditures objective ( $\delta_2$ ) is high. Similarly, expenditures are further away from the target when the FA's weight on the employment objective ( $\delta_1$ ) is high. The last two results illustrate the trade-off the FA faces between the employment and expenditure objectives.

Next, by substituting (4.4 a,c,d) into (2.4a), we can obtain the society's equilibrium loss under discretion, which can be conveniently expressed in terms of  $p^D$ , the equilibrium inflation rate under discretion:

$$(4.5) \quad L^{FA,D} = (p^D)^2 \left[ 1 + \frac{\delta_1 \delta_2 (\delta_1 + \delta_2)}{(\mu_1 \delta_2 + \mu_2 \delta_1)^2} \right]$$

Evaluating (4.5) reveals that in terms of inflation, the loss is increasing in  $\mu_1$  and  $\delta_1$ , the weights assigned by the policymakers on the employment objective (the first term in the squared brackets). However, in terms of employment and government expenditures, the loss is decreasing in these parameters (the second term in the squared brackets). Therefore, even with no stabilisation objectives, the society faces a trade-off if it wants to achieve a lower equilibrium inflation rate, as in our model employment and expenditures are not invariant to changes in the monetary and the fiscal regimes. We now turn to study this trade-off in detail.

## 5 Commitments, Wage Setting and the Social Welfare

In this section we look at the hypothetical case where the policymakers can commit to optimal policy rules, before expectations are formed. In so doing, they can internalise the effects of their decision rules on expectations. In models with no endogenous fiscal policy, it is by now well understood that a CB who can commit towards the private sector constitutes a genuine welfare improvement compared to the discretionary outcome (Barro and Gordon, 1983, and the subsequent literature in rules versus discretion). However, as was

shown by Alesina and Tabellini, in the presence of a fiscal objective commitments may become counterproductive. We study how this result is affected by different assumptions about the TU wage setting behaviour.

The commitment regime is characterised by the following timing of policy actions: (i) The two authorities commit themselves to *ex ante* optimal policy rules. (ii) The TU rationally chooses nominal wages. (iii) Employment results from the firms' labour demand (2.1), and government expenditures result from the budget constraint (2.2). Under commitments, the two policymakers' first order conditions can be obtained by minimising (2.4) subject to (2.1), (2.2), (3.1) and the restrictions that  $p=p^e$  and  $\tau=\tau^e$ . Under these conditions, the two authorities' first-order conditions are given as follows:

$$(5.1) \quad \begin{aligned} a) \frac{\partial L^{FA}}{\partial \tau} &= \delta_2(p + \tau - g^*) + \delta_1(1 - \gamma)(\tau(1 - \gamma) + n) = 0 \\ b) \frac{\partial L^{CB}}{\partial p} &= p + \mu_2(p + \tau - g^*) = 0 \end{aligned}$$

Rearranging (5.1) yields:

$$(5.2) \quad \begin{aligned} a) \tau &= \frac{1}{(1 - \gamma)^2 \delta_1 + \delta_2} [\delta_2(g^* - p) - (1 - \gamma)\delta_1 n] \\ b) p &= \frac{\mu_2}{1 + \mu_2} (g^* - \tau) \end{aligned}$$

Under commitments,  $\gamma$  does not enter into the CB's best response function (recall that  $\gamma \equiv 1 / (1 + \chi)$ , and  $\chi$  is the weight the TU assigns on the real wage objective). However, the FA's best response is affected by the TU's employment and real wage targets whenever  $\gamma > 0$ , which will have implications on the equilibrium outcomes below. The inflation and tax rates in (5.2) clearly differ from the values under discretion. This can be verified by studying the difference between (4.3) and (5.2), after setting  $p=p^e$  and  $t=t^e$  in (4.3):

$$a) \tau^D(p) - \tau^C(p) = -\frac{\delta_1 \delta_2 \gamma \{(1-\gamma)(g^* - p) + n\}}{[(1-\gamma)\delta_1 + \delta_2][(1-\gamma)^2 \delta_1 + \delta_2]} < 0$$

(5.3)

$$b) p^D(\tau) - p^C(\tau) = \frac{\mu_1}{1 + \mu_2} [(1-\gamma)\tau + n] > 0$$

Obviously, (5.3a) is non-positive and (5.3b) is non-negative for all parameter values, as long as the natural rate of employment is below zero (recall that in equation 3.5, the natural rate of employment was defined as  $-(1-\gamma)\tau - n$  and that the government budget constraint gives  $g^* - p = \tau$ ). Under discretion, and with a forward-looking TU, the CB reaction function implies a positive inflation bias while the FA reaction function implies that taxes are set too low. This happens because the TU correctly anticipates the policymakers' incentives to increase employment by higher inflation and lower taxes. The tax and inflation distortions under discretion are decreasing in the natural rate of employment, because lower employment means that the policymakers have higher incentives to use their instruments in order to boost employment.

Because the natural rate is determined by the TU's wage negotiation strategy, the distortions as specified in (5.3) are directly caused by the TU's behaviour. In particular, if the TU is assumed to be fully cooperative, so that it has preferences  $\chi = (l^*/u^*)$  and an employment target which is identical to the policymakers' target level ( $l^* = 0$ ), the natural rate of employment is equal to zero. From (5.3) it then follows that both taxes and inflation are identical under the two regimes. Clearly, the policymakers face no incentives to generate policy surprises when the natural rate is equal to the target rate, and therefore commitments have no welfare effects. On the other hand, if the TU is only imperfectly cooperative, so that it has preferences  $\chi = (l^*/u^*)$  but an employment target which is lower than the policymakers' target, the natural rate of employment is rendered below zero. Consequently, both policymakers face incentives to generate policy surprises, which gives rise to discretionary losses in both fiscal and monetary policies. Finally, if the TU is aggressive, i.e., it puts no weight on the employment objective ( $\chi = \infty$  so that  $\gamma = 0$ ), the FA's reaction function will be the same under commitments and under discretion. Hence, the FA is not subject to a time-consistency problem and equation 5.3a becomes equal to zero. However, compared to an imperfectly cooperative TU, an aggressive TU negotiates nominal wages which push the natural rate further below

zero, and hence the CB's time consistency problem is made more severe under an aggressive TU. Formally, this follows because (5.3b) is increasing in  $\chi$ , the TU's wage negotiation parameter.

We now return to equation 5.2. Solving (5.2a) and (5.2b) together gives the equilibrium inflation and tax rates under commitments. These values in turn determine the equilibrium values of employment and government expenditures:

$$\begin{aligned}
 (5.4) \quad a) p^c &= \frac{\mu_2 \delta_1 (1-\gamma)}{A} [(1-\gamma)g^* + n] < p^D \\
 b) \tau^c &= g^* - \frac{\delta_1 (1-\gamma)(1+\mu_2)}{A} [(1-\gamma)g^* + n] > \tau^D \\
 c) l^c &= (p^c - w - \tau^c) = -\frac{\delta_2}{A} [(1-\gamma)g^* + n] \\
 d) g^c &= \tau^c + p^c = g^* - \frac{(1-\gamma)\delta_1}{A} [(1-\gamma)g^* + n] \\
 A &\equiv \delta_2 + (1-\gamma)^2 \delta_1 (1+\mu_2)
 \end{aligned}$$

Compared to discretion, under commitments the two authorities collect less revenue in form of inflation and more revenue in form of taxes, unless the TU is fully cooperative. Therefore, the discretionary policy mix is generally ineffective. However, under commitments, employment and government expenditures are further away from their respective targets than under discretion. This can be verified if we study the differences between the respective equilibrium levels:

$$\begin{aligned}
 (5.5) \quad l^D - l^c &= \frac{\delta_2 (1-\gamma) [\mu_1 \delta_2 + \gamma \delta_1 (1+\mu_2)] [(1-\gamma)g^* + n]}{AB} > 0 \\
 g^D - g^c &= \frac{\mu_1 \delta_1 \delta_2 [\gamma^2 + (1-\gamma)] [(1-\gamma)g^* + n]}{AB} > 0
 \end{aligned}$$

Intuitively, the higher is the unemployment rate which is measured by  $n$ , and/or the higher is the government expenditure target  $g^*$ , the higher is the loss from commitments in terms of employment and expenditures. This happens because under discretion, higher inflation and lower taxes help the policymakers to shoot closer to their employment and expenditure targets. In other words, the policy-

makers are partially successful in “cheating” the TU, as the levels of the state variables are not invariant to changes in the fiscal and monetary regimes. However, if the TU is fully cooperative, government expenditures can be completely financed through taxes, without any adverse effects on employment. Consequently, (5.5 a,b) are equal to zero, and the policymakers are able to hit all their targets under both regimes.

Finally, substituting (5.4 a,c,d) into (2.4a), yields the society’s (the FA’s) equilibrium welfare loss under commitments:

$$(5.6) \quad L^{FA,C} = (p^c)^2 \left[ 1 + \frac{\delta_1 \delta_2 (\delta_1 (1-\gamma)^2 + \delta_2)}{\mu_2^2 \delta_1^2 (1-\gamma)^2} \right]$$

The social net *gain* from commitments can be obtained by studying the difference between the benevolent FA’s equilibrium loss under discretion (equation 4.5) and commitments (equation 5.6). Under discretion, the first term in the squared brackets is generally higher whereas the second term is lower. This illustrates the fact that in terms of inflation, commitments are generally welfare improving, while in terms of employment and expenditures they imply a welfare loss. If the FA would alone control both policy instruments (i.e.  $\mu_i = \delta_i$ ), the overall welfare effect from commitments would be positive, because under centralised decision making commitments take correctly into account the trade-off between different policy objectives. However, when monetary and fiscal policies are decentralised, the overall outcome from the FA’s point of view is contingent upon the CB’s preferences. In particular, as was found by Alesina and Tabellini, a commitment by a CB who puts a low relative weight on the employment and expenditure objectives means that the FA must raise taxes more than it is optimal. The gain in reduced inflation can then be more than compensated by the loss in employment and expenditures. This is the essence of the policy coordination problem in our model. In particular, the problem arises because the two authorities disagree about the optimal mix of financing the government expenditure objective.

In sum, if the policymakers face a coordination problem, commitments are not necessarily successful in improving social welfare. On the other hand, even if commitments were welfare improving, they do not constitute time-consistent solutions in the first place. We now turn to investigate whether the society (which is

represented by the FA) can design a CB institution that is able to improve upon the discretionary outcome. In our framework it is apparent that the optimal institution should address two issues. It should alleviate the time-inconsistency problems by providing a commitment technology for the CB, but at the same time the CB should be given incentives to generate optimal seignorage so that the regime would indeed constitute a welfare improvement from society's point of view.

## 6 Inflation Targeting and Trade Union Behaviour

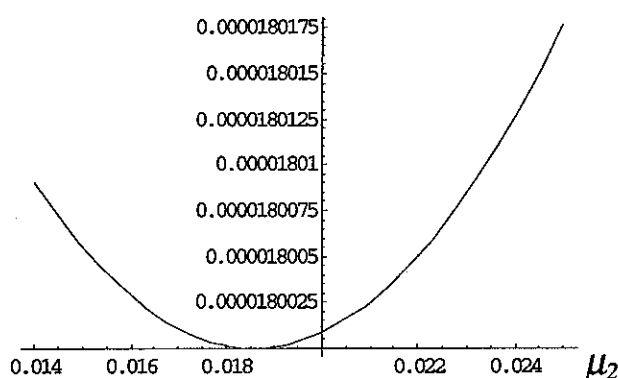
In the previous section we found that in a fiscal-monetary game, the policy mix under discretion is inefficient for two reasons. First, the policymakers are generally not able to commit to optimal policy rules. Second, there can exist a dispute between the policymakers about the relative importance of the government expenditure objective. As a solution to the commitment problem, academic research has so far suggested three different ways in which the discretionary outcome can be improved. Barro and Gordon (1983) argued that in a repeated game setting, reputational incentives can help the policymakers to sustain the commitment equilibrium over time. Rogoff (1985) found that by delegating monetary policy to a central banker who assigns a high relative weight on the inflation objective, society can achieve an outcome where the inflationary bias is credibly reduced.<sup>11</sup> Clearly, when applied to our two-authority framework, a central banker who puts a zero weight on the employment objective ( $\mu_1 = 0$ ) would be successful in eliminating the inflationary bias as specified in equation (5.3b). However, in the presence of an expenditure objective, such delegation would not correctly take into account the fiscal response, and from the FA's point of view the low inflation outcome might therefore create a welfare loss in terms of the other two objectives, employment and expenditures.

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<sup>11</sup> However, Rogoff also found that such a "conservative" CB shows insufficient response to stochastic output shocks, and therefore the model gives rise to a credibility-flexibility trade-off.

In Figure 1.1, we have used numerical simulation methods to illustrate how the FA would optimally delegate monetary policy to a more inflation averse CB. As an alternative to the standard Rogoff case, we study how changes in  $\mu_2$ , the CB's weight on expenditures, affect the FA's welfare.<sup>12</sup> The vertical axis shows the FA's loss under discretion (equation 4.5), and the horizontal axis measures  $\mu_2$ . In the calibration,  $\delta_2$ , which denotes society's weight on expenditures, is set  $\delta_2 = 0.02$ . It can be seen that the loss is minimised when  $\mu_2$  is slightly below  $\delta_2$ . Under an overly conservative CB, who puts too little weight on the expenditure objective, the gain from lower inflation would be more than offset by losses in terms of the other objectives.

Figure 1.1 **Optimal Choice of Central Bank Preferences under Endogenous Fiscal Policy**



More recently, Walsh (1995a) has suggested that by subjecting the central bank's head to a contract that specifies her income as a linear function of observed inflation, the inflationary bias can be eliminated, with no adverse effects on shock stabilisation. Svensson (1997) showed that in a static model, a Walsh contract is equivalent to giving the CB an explicit inflation target.<sup>13</sup> An inflation target can be

<sup>12</sup> See Lossani et al (1997), who consider similar institutional solutions and their effects on stabilisation policy.

<sup>13</sup> Svensson also found that under output persistence, inflation target must be supplemented by a conservative CB in order to yield the same outcome than a Walsh contract. Herrendorf and Lockwood (1997) show that the same holds in a static model if the inflation bias is stochastic rather than deterministic.

enforced by a suitable dismissal rule, and therefore there is no need for monetary transfers to the CB governor.<sup>14</sup>

In general, the policy delegation models of Rogoff, Walsh and Svensson all constitute a principal-agent relationship between the government as the principal and the CB as the agent. However, these models do not consider fiscal objectives. Huang and Padilla (1995) applied the Walsh contract under endogenous fiscal policy. With atomistic wage setting and no explicit policy coordination problem, they concluded that if the tax rate is fixed, a monetary transfer as defined by the contract can successfully implement the optimal outcome. However, if taxes are determined endogenously, the contract fails to implement the targeted equilibrium.

We extend Huang and Padilla's analysis in several ways. In particular, we use the inflation targeting approach and employ a framework where both policymakers can face a time-inconsistency problem. In addition, because the policymakers have asymmetric preferences, our model also features a policy coordination problem between the two authorities. Our purpose is to find out whether the two time-inconsistency problems *and* the policy coordination problem can simultaneously be alleviated by inflation targeting, and to what extent the success of such an institution depends on the TU's wage-setting behaviour. Finally, we investigate briefly the implications of uncertainty about the CB's preferences.

## 6.1 Society's Optimal Outcome

To start with, it is convenient to explicitly specify the optimal equilibrium targeted by society. Clearly, such hypothetical outcome requires that (i) both fiscal and monetary time-inconsistency problems are eliminated and (ii) the policy coordination dispute is resolved optimally from the society's point of view. In general, the optimal outcome can be obtained when the benevolent FA alone chooses both policy instruments *and* is at the same time able to commit to fiscal and monetary policy rules. Under this scenario, the FA's gain from reduced inflation more than offsets its loss from lower employment and government expenditures.

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<sup>14</sup> See Canzoneri, Nolan and Yates (1997) and Walsh (1995b) for a discussion about the design and implementation of such dismissal rules.



Analytically, we can derive the optimal outcome by modifying (5.4a,b), the equilibrium inflation and tax rates under commitments, and (5.6), the equilibrium welfare loss under commitments, by replacing  $\mu_2$  by  $\delta_2$ :

$$(6.1) \quad \begin{aligned} a) p^{OPT} &= \frac{\delta_2 \delta_1 (1-\gamma)}{\delta_2 + (1-\gamma)^2 \delta_1 (1+\delta_2)} [(1-\gamma)g^* + n] \\ b) \tau^{OPT} &= g^* - \frac{\delta_1 (1+\delta_2)(1-\gamma)}{\delta_2 + (1-\gamma)^2 \delta_1 (1+\delta_2)} [(1-\gamma)g^* + n], \\ c) L^{FA,OPT} &= (p^{OPT})^2 \left[ 1 + \frac{\delta_1 \delta_2 (\delta_1 (1-\gamma)^2 + \delta_2)}{\delta_2^2 \delta_1^2 (1-\gamma)^2} \right] \end{aligned}$$

In equation (6.1), the superscript “*OPT*” refers to the socially optimal outcome. We now turn to study how the CB’s inflation target should be chosen so that the FA, who is assumed to represent society’s preferences, can move from the decentralised discretionary outcome to the outcome as specified in (6.1).

## 6.2 Derivation of the Optimal Inflation Target

In this section, we study how the FA can choose the CB’s inflation target so that the equilibrium outcome will minimise the FA’s own loss. Following Svensson (1997), we modify the CB’s loss function (2.4b) by including an explicit inflation target  $\bar{p}$ , which is set by the FA in an initial “institution design stage.” This means that the optimal target is chosen and made public before the TU negotiates the nominal wages, and hence the regime provides a “commitment technology” for the CB.<sup>15</sup>

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<sup>15</sup> McCallum (1995) argued that institutions like conservative CB’s, Walsh contracts or inflation targets do not in fact solve the underlying time-inconsistency problem, because it becomes optimal for the government to override (renegotiate) the institution after the wages have been set. However, Jensen (1997) showed that if overriding the institution is costly, credibility can be increased. See Essay 2 for a formal treatment of the renegotiation proofness problem in the context of a monetary union.

$$(6.2) \quad L^{CB} = \frac{1}{2} [(p - \bar{p})^2 + \mu_1 l^2 + \mu_2 (g - g^*)^2]$$

The policymakers operate under discretion, and the resulting equilibrium is a Nash equilibrium as before. However, the CB now optimises under the inflation target. This means that the CB enjoys instrument independence, but not goal independence, in its choice of inflation rate (terminology by Fischer, 1995). Next, following the steps which lead to the discretionary equilibrium above, we obtain the equilibrium outcomes under inflation targeting, for a given  $p$ :

$$(6.3) \quad \begin{aligned} a) p^T &= \bar{p} + \frac{\mu_1 \delta_2 + \delta_1 \mu_2}{B} [(1 - \gamma)(g^* - \bar{p}) + n] \\ b) \tau^T &= g^* - \frac{1}{B} \{ \delta_2 \bar{p} + (\delta_1 (1 + \mu_2) + \mu_1 \delta_2) [(1 - \gamma)g^* + n] \} \\ c) l^T &= p^T - w - \tau^T = -\frac{\delta_2}{B} [(1 - \gamma)(g^* - \bar{p}) + n] \\ d) g^T &= \tau^T + p^T = g^* - \frac{\delta_1}{B} [(1 - \gamma)(g^* - \bar{p}) + n] \end{aligned}$$

$$B \equiv \delta_2 + (1 - \gamma) [\delta_1 (1 + \mu_2) + \mu_1 \delta_2]$$

The optimal  $\bar{p}$ , which is chosen by the FA before expectations are formed, can be found as follows. First, we substitute the equilibrium outcomes (6.3 a,c,d), which include  $p$ , into the FA's loss function (2.4a), where  $p$  does *not* enter directly. Then, minimising this function with respect to  $\bar{p}$ , and rearranging, gives the optimal  $\bar{p}$  as follows:

$$(6.4) \quad \bar{p} = -\frac{(\delta_1 (1 - \gamma) + \delta_2)(\mu_1 \delta_2 + \mu_2 \delta_1) - \delta_1 \delta_2 (\delta_1 + \delta_2)}{[\delta_1 (1 - \gamma) + \delta_2]^2 + \delta_1 \delta_2 (\delta_1 + \delta_2)(1 - \gamma)} [(1 - \gamma)g^* + n]$$

$$\frac{\partial \bar{p}}{\partial \gamma} > 0 \Rightarrow \frac{\partial \bar{p}}{\partial \chi} < 0, \frac{\partial \bar{p}}{\partial n} < 0, \frac{\partial \bar{p}}{\partial g^*} < 0, \frac{\partial \bar{p}}{\partial \mu_i} < 0$$

The sign of (6.4) is negative, unless  $\mu_1$  and  $\mu_2$  are very low. Obviously, in our framework the properties of the optimal inflation target are more general than in the original model by Svensson (1997). In particular, unlike in Svensson, the optimal target is not

designed to eliminate the inflation rate which is in excess to zero, but it constitutes an optimal solution to the trade-off between the gain from lower inflation and the loss from lower employment and government expenditures. Evaluating (6.4) further (in Appendix 1) reveals that the optimal target is the lower the higher is the TU's wage negotiation parameter. Clearly, if the TU is more aggressive nominal wage claims tend to go up. The FA will then optimally induce the CB not to accommodate such demands, and hence the inflationary pressures are dampened before they can emerge. Moreover, the higher is  $n$ , the unemployment rate net the tax distortions, the lower must the inflation target be. This happens because a higher  $n$  reduces the natural rate of employment, thereby increasing the CB's incentives to generate surprise inflation.<sup>16</sup> A high target level for government expenditures also means that the inflation target must be lower, because a higher target level increases the CB's incentives to generate more seignorage, thereby raising the inflationary pressures. Finally, the lower are the weights assigned by the CB on other objectives than inflation ( $\mu_i$ ), the higher is the optimal target. Intuitively, a more inflation averse CB works as a partial substitute for inflation targeting in achieving a low time-consistent inflation rate.

In order to find the final equilibrium outcomes for inflation and taxes, we substitute (6.4) back into (6.3 a,b). Using these values, we can then determine the equilibrium values of employment and government expenditures. The equilibrium values for inflation, employment and government expenditures can then be substituted back to (2.4a), in order to determine the social welfare loss under inflation targeting. However, in order to avoid excessively tedious calculations, we evaluate the outcomes directly under imperfectly cooperative and aggressive TU behaviour. This is sufficient to demonstrate how the performance of the inflation targeting institution depends on different TU wage setting strategies. Note that under fully cooperative TU behaviour, the discretionary outcome is identical with the optimal outcome, and inflation targeting clearly makes no sense in that case.

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<sup>16</sup> This result is in line with Svensson, who showed that the higher is the CB's target level of output, and thus the higher is the temptation to generate inflation surprises, the lower is the optimal inflation target.

## 6.3 The Outcome under Imperfectly Cooperative TU Behaviour

If the TU is imperfectly cooperative, its has *preferences* that are optimal from the FA's point of view while its *employment target* renders the natural rate of employment below the FA's target level. Under such TU behaviour, both fiscal and monetary authorities face a time-inconsistency problem, and the FA's optimal outcome as specified in (6.1) reduces to the following form:

$$(6.5) \quad \begin{aligned} a) p^{OPT} \Big|_{\gamma=\gamma^o} &= \frac{\delta_2 \delta_1 (1-\gamma^o)^2}{\delta_2 + (1-\gamma^o)^2 \delta_1 (1+\delta_2)} g^* \\ b) \tau^{OPT} \Big|_{\gamma=\gamma^o} &= \frac{\delta_2}{\delta_2 + (1-\gamma^o)^2 \delta_1 (1+\delta_2)} g^* \\ c) L^{FA,OPT} \Big|_{\gamma=\gamma^o} &= (p^{OPT} \Big|_{\gamma=\gamma^o})^2 \left[ 1 + \frac{(\delta_1 (1-\gamma^o)^2 + \delta_2)}{\delta_2 \delta_1 (1-\gamma^o)^2} \right] \end{aligned}$$

In (6.5),  $\gamma^o = u^*/(u^*+l^*)$ . Under an imperfectly cooperative TU, the expressions for actual inflation and taxes under a given  $p$ , and the expression for the optimal  $p$ , are given as follows:

$$(6.6) \quad \begin{aligned} a) p^T \Big|_{\gamma=\gamma^o} &= \bar{p} + \frac{\mu_1 \delta_2 + \delta_1 \mu_2}{B^o} (1-\gamma^o) (g^* - \bar{p}) \\ b) \tau^T \Big|_{\gamma=\gamma^o} &= g^* - \frac{1}{B^o} \{ \delta_2 \bar{p} + (1-\gamma^o) (\delta_1 (1+\mu_2) + \mu_1 \delta_2) g^* \} \\ c) \bar{p} \Big|_{\gamma=\gamma^o} &= \frac{\delta_1 \delta_2 (\delta_1 + \delta_2) - (\delta_1 (1-\gamma^o) + \delta_2) (\mu_1 \delta_2 + \mu_2 \delta_1)}{[\delta_1 (1-\gamma^o) + \delta_2]^2 + \delta_1 \delta_2 (\delta_1 + \delta_2) (1-\gamma^o)} (1-\gamma^o) g^* \\ B^o &\equiv \delta_2 + (1-\gamma^o) (\delta_1 (1+\mu_2) + \mu_1 \delta_2) \end{aligned}$$

Substituting (6.6c) into (6.6 a,b), and rearranging, yields the following equilibrium values for inflation and taxes under inflation targeting:

$$(6.7) \quad \begin{aligned} a) p^T \Big|_{\gamma=\gamma^o} &= \frac{\delta_1 \delta_2 (\delta_1 + \delta_2) (1 - \gamma^o)}{C} g^* \\ b) \tau^T \Big|_{\gamma=\gamma^o} &= \frac{\delta_2 (\delta_1 (1 - \gamma^o) + \delta_2)}{C} g^* \end{aligned}$$

$$C \equiv (\delta_1 + \delta_2)(\delta_1 + \delta_2 + \delta_1 \delta_2 (1 - \gamma^o)) + \delta_1 \gamma^o (\delta_1 \gamma^o - 2(\delta_1 + \delta_2))$$

Comparing these with (6.5 a,b), reveals that the two are not identical in general. Only in the case where one of the parameters  $\delta_1$ ,  $\delta_2$  or  $\gamma^o$  is equal to zero would inflation targeting generate the optimal outcome.<sup>17</sup> However, the last possibility would require that  $\gamma^o \equiv u^*/(u^*+l^*) = 0 \Leftrightarrow u^* = 0$ , which is not possible since the preferences of an imperfectly cooperative TU are not defined when  $u^* = 0$  (recall that an imperfectly cooperative TU has the following wage negotiation parameter:  $\chi \equiv l^*/u^*$ ). Finally, the FA's equilibrium loss which follows from (6.7 a,b) is given by:

$$(6.8) \quad L^{FA,T} \Big|_{\gamma=\gamma^o} = (p^T \Big|_{\gamma=\gamma^o})^2 \left[ 1 + \frac{\delta_2 M + \delta_1 N}{\delta_1 \delta_2 [(1 - \gamma^o)(\delta_1 + \delta_2)]^2} \right]$$

$$M \equiv [\delta_2 + \delta_1 (1 - \gamma^o)]^2 (1 - \gamma^o)^2$$

$$N \equiv [\delta_2 (1 - \gamma^o) + \delta_1 - \delta_1 \gamma^o (2 - \gamma^o)]^2$$

Clearly, (6.8) is not the same as the optimal loss under imperfectly cooperative TU (equation 6.5c). However, it is straightforward to show that:

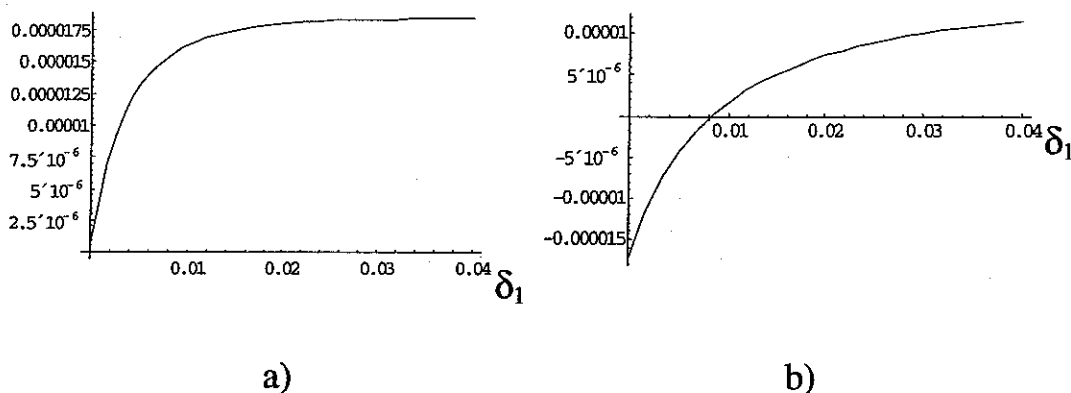
**Proposition 2.** Under imperfectly cooperative TU behaviour, inflation targeting solves the problems associated with asymmetric FA and CB preferences optimally from the FA's point of view.

<sup>17</sup> Note that when  $\delta_1$ ,  $\delta_2$  or  $\gamma^o$  change, both the optimal outcome and the actual outcome will change.

**Proof.** Comparing (6.8) with (6.5c) shows that the CB's preferences are replaced by the FA's preferences in the equilibrium outcome, and hence the policy coordination problem is eliminated. *Q.E.D.*

To find out whether the inflation targeting institution is successful in alleviating the time-inconsistency problems, we refer to numerical simulation. The first result, illustrated in Figure 1.2, shows that under an imperfectly cooperative TU, society's (the FA's) gain from subjecting the CB to an inflation target is the higher the more severe is the FA's time-inconsistency problem. More specifically, Figure 1.2a shows how the FA's loss under discretion ( $L^{FA,D}$ , on the vertical axis) is increasing in the FA's preferences for employment ( $\delta_l$ , on the horizontal axis). Figure 1.2b shows that the gain from inflation targeting (defined as  $L^{FA,D} - L^{FA,T}$ , on the vertical axis) is also increasing in  $\delta_l$ , albeit with a decreasing rate. Because the FA's incentive to generate policy surprises, i.e., its time-inconsistency problem, is the higher the higher is  $\delta_l$ , the result follows.

Figure 1.2 **Gain from Inflation Targeting under an Imperfectly Cooperative Trade Union**

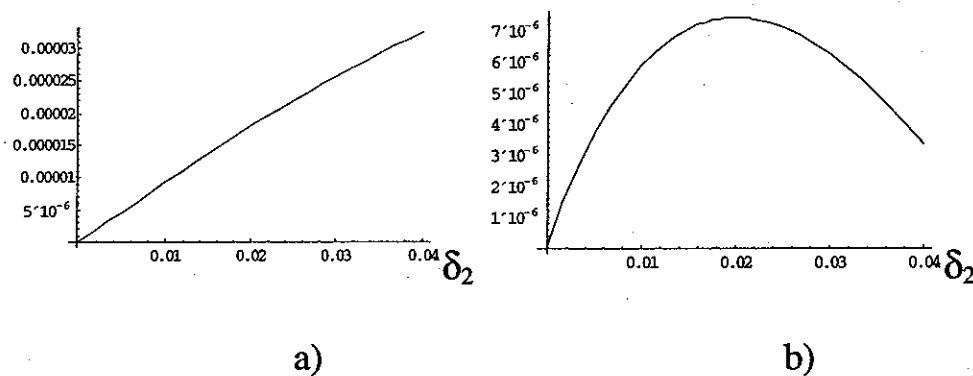


Note that according to Figure 1.2b, inflation targeting creates a loss if  $\delta_l$  is sufficiently low. However, the value for  $\mu_l$  used in the simulations was 0.01, and in section 2 we assumed that  $\delta_l > \mu_l$ . Under this assumption, it follows from Figure 2b that the gain is positive for all feasible parameter values.

The result that inflation targeting, which provides a commitment technology for the CB, is successful in increasing the FA's welfare even if the FA's own time-inconsistency problem remains, is in line with Jensen (1992). He found that in the presence of two time-

inconsistency problems, commitment by one authority constitutes a positive externality for the other authority. Therefore, by alleviating one time-inconsistency problem the other is moderated as well, and the overall welfare is improved. We can also show that the gain from inflation targeting is contingent upon the weight assigned by the FA on its employment objective relative to the expenditure objective ( $\delta_1$  vs.  $\delta_2$ ). This can be seen in Figure 1.3. In Figure 1.3a, the FA's discretionary loss ( $L^{FA,D}$ , on the vertical axis) is increasing in the FA's preferences for expenditures ( $\delta_2$ , on the horizontal axis). Figure 1.3b shows that the gain from inflation targeting ( $L^{FA,D} - L^{FA,T}$ , on the vertical axis) is first increasing in  $\delta_2$ , as the marginal gain from lower inflation exceeds the marginal loss from lower expenditures. However, if the value of  $\delta_2$  exceeds the value of  $\delta_1$  (in the calibration,  $\delta_1 = 0.02$ ), the marginal loss from lower expenditures exceeds the marginal gain from lower inflation.

Figure 1.3      **Fiscal Preferences and the Gain from Inflation Targeting**



Therefore, if the FA's time-inconsistency problem is moderate relative to its desire for higher expenditures ( $\delta_1 < \delta_2$ ), the disadvantages of inflation targeting in terms of lower expenditures start to become more obvious. Note however, that the gain in Figure 1.3b does not become negative unless  $\delta_2$  is very large. Hence, under imperfectly cooperative TU behaviour inflation targeting is welfare improving, even if it does not result in the socially optimal outcome.

## 6.4 The Outcome under Aggressive Trade Union Behaviour

In section 3, an aggressive TU was defined as one who assigns no weight on the employment objective when negotiating the nominal wages. In other words, an aggressive TU is characterised by  $\gamma = 0$ . Under such TU preferences, the FA's optimal outcome as specified in (6.1) changes as follows:

$$(6.9) \quad \begin{aligned} a) p^{OPT} \Big|_{\gamma=0} &= \frac{\delta_2 \delta_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} (g^* + u^*) \\ b) \tau^{OPT} \Big|_{\gamma=0} &= \frac{\delta_2 g^* - \delta_1 (1 + \delta_2) u^*}{\delta_1 + \delta_2 + \delta_1 \delta_2} \\ c) L^{FA, OPT} \Big|_{\gamma=0} &= (p^{OPT} \Big|_{\gamma=0})^2 \left[ 1 + \frac{\delta_1 + \delta_2}{\delta_2 \delta_1} \right] \end{aligned}$$

The rates of inflation and taxes for a given  $\bar{p}$ , and the expression for the optimal  $p$ , are now given by:

$$(6.10) \quad \begin{aligned} a) p^T \Big|_{\gamma=0} &= \frac{[(\delta_1 + \delta_2) \bar{p} + (\mu_1 \delta_2 + \mu_2 \delta_1)] (g^* + u^*)}{\delta_1 + \delta_2 + \mu_1 \delta_2 + \mu_2 \delta_1} \\ b) \tau^T \Big|_{\gamma=0} &= \frac{(g^* - \bar{p}) \delta_2 - (\delta_1 (1 + \mu_2) + \mu_1 \delta_2) u^*}{\delta_1 + \delta_2 + \mu_1 \delta_2 + \mu_2 \delta_1} \\ c) \bar{p} \Big|_{\gamma=0} &= \frac{1}{\delta_1 + \delta_2 + \delta_1 \delta_2} (\delta_1 \delta_2 - \mu_1 \delta_2 - \mu_2 \delta_1) (g^* + u^*) \end{aligned}$$

Following the same steps than in the previous case, we arrive at a result which we summarise in the following proposition:

**Proposition 3.** If the TU is aggressive, an inflation target which is optimally chosen by the FA can implement the FA's optimal outcome as specified in (6.9).

**Proof.** Substituting (6.10c) into (6.10 a,b), and rearranging, yields the following equilibrium values for inflation and taxes under inflation targeting:



$$(6.11) \quad \begin{aligned} a) p^T|_{\gamma=0} &= \frac{\delta_1 \delta_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} (g^* + u^*) \\ b) \tau^T|_{\gamma=0} &= \frac{\delta_2 g^* - \delta_1 (1 + \delta_2) u^*}{\delta_1 + \delta_2 + \delta_1 \delta_2} \end{aligned}$$

Comparing these with (6.9 a,b) reveals that the two are identical. Therefore, and because the equilibrium levels of employment and government expenditures are determined by the equilibrium rates of inflation and taxes only, the result follows. *Q.E.D.*

Our findings can be summarised as follows. Independent of the TU's and the CB's preferences, inflation targeting always solves the problem which originates from the asymmetric preferences in an optimal way from the FA's point of view. However, the solution to the policymakers' commitment problems depends on the TU's wage setting behaviour. If the model is characterised by one time consistency problem only, i.e., the TU is aggressive, inflation targeting can take the FA to the optimal outcome, because one instrument (inflation targeting) is used in an attempt to achieve one target (to eliminate the CB's time inconsistency problem). However, when the TU is imperfectly cooperative, both FA and CB are subject to time-inconsistency problems, and consequently inflation targeting as a single instrument can not achieve both targets (i.e., to eliminate both time inconsistency-problems). Put in another way, by optimally choosing the inflation target the principal (the FA) can eliminate the agent's (the CB's) time-inconsistency problem, but it clearly cannot eliminate its own time-inconsistency problem if such a problem exists. Therefore, some additional instrument is needed in this case. For example, a FA who assigns no weight either on the employment objective or on the expenditure objective could be one such "instrument".

## 6.5 The Implications of Uncertainty

In this section we briefly extend our model by incorporating uncertainty about the policymakers' preferences. More specifically, and following Nolan and Schaling (1996) and Schaling (1997), we assume that after the TU has negotiated the nominal wages, but before inflation and taxes are set by the policymakers, there are stochastic

shocks to the CB's and the FA's preferences. The TU cannot observe the shocks, but it can use information about the distributions of the shocks when it forms expectations. Clearly, because monetary and fiscal policies are interdependent in our model, uncertainty about monetary policy will affect the optimal fiscal policy and *vice versa*. In what follows, we first derive the equilibrium tax and inflation rates under uncertainty. We then discuss the implications of uncertainty on the performance of the inflation targeting institution.

For analytical tractability, we consider here only the case where the TU is aggressive, so that it puts no weight on the employment objective and its reaction function is given by equation (3.8). The policymakers' objective functions are now modified as follows:

$$(6.12) \quad \begin{aligned} a) L^{FA} &= \frac{1}{2} \{ p^2 + \widehat{\delta}_1 l^2 + \widehat{\delta}_2 (g - g^*)^2 \} \\ b) L^{CB} &= \frac{1}{2} \{ (p - \bar{p})^2 + \widehat{\mu}_1 l^2 + \widehat{\mu}_2 (g - g^*)^2 \} \end{aligned}$$

The stochastic nature of the preference parameters can be specified as follows:

$$(6.13) \quad \begin{aligned} \widehat{\delta}_i &= \delta_i + v_{i,t}, & \widehat{\mu}_i &= \mu_i + \vartheta_{i,t}, \\ E_{t-1}[v_{i,t}] &= 0, \text{VAR}[v_{i,t}] = \sigma_v^2 & E_{t-1}[\vartheta_{i,t}] &= 0, \text{VAR}[\vartheta_{i,t}] = \sigma_\vartheta^2 \end{aligned}$$

We assume that both shocks are independently and identically distributed over time and that there is no correlation between them. The TU expects the CB's preferences to be  $\mu_i$  and the FA's preferences to be  $\delta_i$ . However, at any particular date the true preferences will fluctuate according to the distributions of  $v$  and  $\vartheta$ , respectively.<sup>18</sup> Next, taking the first-order conditions of (6.12a,b) with respect to taxes and inflation, respectively, and substituting for the

<sup>18</sup> The society still evaluates its welfare according to the "standard" loss function (2.4a). The uncertainty about monetary preferences comes from "delegation uncertainty" (see Muscatelli, 1996). On the other hand, the uncertainty about the fiscal preferences can be motivated by electoral uncertainty (for example, the society may not expect the preferences to remain constant over the electoral cycle).

TU's choice of nominal wages (equation 3.8) yields the following expressions:

$$(6.14) \quad \begin{aligned} a) & -\widehat{\delta}_1(p - p^e - \tau - u^*) + \widehat{\delta}_2(p + \tau - g^*) = 0 \\ b) & p + \widehat{\mu}_1(p - p^e - u^* - \tau) + \widehat{\mu}_2(p + \tau - g^*) - \bar{p} = 0 \end{aligned}$$

Because the TU is assumed to be aggressive, it forms expectations on inflation only. However, taking expectations through (6.14b) is now slightly more complicated than in the absence of uncertainty. Following the method developed by Nolan and Schaling (1996), we obtain (see Appendix 2 for derivation):

$$(6.15) \quad p^e = X\{(\mu_1 - \mu_2)\tau + \mu_1 u^* + \mu_2 g^* + \bar{p}\},$$

$$X \equiv \frac{1}{1 + \mu_2} \left[ 1 + \frac{\sigma_{\theta 2}^2}{(1 + \mu_2)^2} \right] \frac{\partial X}{\partial \sigma_{\theta 2}^2} > 0.$$

Under aggressive TU behaviour, equation (6.15) is identical to (4.3b) where  $\gamma$  has been set to zero, except to the term  $\frac{\sigma_{\theta 2}^2}{(1 + \mu_2)^2}$ . This term reflects the fact that the TU has had to "guess" how the shock to the CB's preference parameter  $\mu_2$  affects the actual inflation rate. More specifically, increased uncertainty about the future seignorage revenues threatens the risk averse TU's real wages, and therefore the TU will build in an inflation rate hedge in the nominal contracts. Clearly, under the fiscal-monetary game framework such TU strategy also has implications on fiscal policy and the distortionary policy mix, through the government budget identity. We summarise the result in the following proposition:

**Proposition 4.** The CB's optimal inflation rate is increasing and the FA's optimal tax rate is decreasing in the uncertainty about the weight the CB assigns on the government expenditure objective. Therefore, for given FA and CB preferences, the discretionary policy mix is more inefficient in the presence of uncertainty.

**Proof.** Substituting (6.15) back into (6.14b), and rearranging, gives the CB's optimal choice of inflation, for given taxes:

$$(6.16) \quad p = \frac{1}{1+\mu_2} \left[ 1 + \frac{\mu_1 X(1+\mu_2) - \mu_1}{(1+\mu_2)(1+\mu_1+\mu_2)} \right] \{(\mu_1 - \mu_2)\tau + \mu_1 u^* + \mu_2 g^* + \bar{p}\},$$

$$\Rightarrow \frac{\partial p}{\partial \sigma_{\theta 2}^2} > 0$$

Clearly, setting  $\sigma_{\theta 2}^2 = 0$  in equation (6.16) again yields an expression which is identical to (4.3b) under aggressive TU behaviour. However, subtracting (6.15) from (6.16) yields:

$$(6.17) \quad p - p^e = \frac{1 - X(1+\mu_2)}{1+\mu_1+\mu_2} \{(\mu_1 - \mu_2)\tau + \mu_1 u^* + \mu_2 g^* + \bar{p}\} \neq 0.$$

Under uncertainty, the realised inflation generally differs from expected inflation, unless  $\sigma_{\theta 2}^2 = 0$ . Next, substituting (6.17) into equation (6.14a), and rearranging, gives the optimal tax rate which is chosen by the FA:

$$(6.18) \quad \tau = g^* - \frac{1}{Y} \{[\delta_1(1+\mu_1+\mu_2) - \mu_1 \delta_1(1-X(1+\mu_2))]g^* + \delta_1(1+\mu_2)(1+\mu_1 X)u^* + \delta_2(1+\mu_1+\mu_2)p - \delta_1(1-X(1+\mu_2))\bar{p}\}$$

$$Y \equiv (\delta_1 + \delta_2)(1+\mu_1+\mu_2) - \delta_1(1-X(1+\mu_2))(\mu_1 - \mu_2)$$

$$\Rightarrow \frac{\partial \tau}{\partial \sigma_{\theta 2}^2} < 0$$

From (6.18), it can be seen that the optimal distortionary tax is the lower the higher is the uncertainty about the CB's preferences. Because in the discretionary equilibrium taxes are too low and inflation is too high even *with no uncertainty*, the distortion is made worse in the presence of uncertainty. *Q.E.D.*

Intuitively, because the inflation rate is higher due to the additional hedge in the TU's expectations, the FA can choose lower

taxes since a greater share of expenditures is actually financed via seignorage. Obviously, Proposition 4 suggests that in the presence of uncertainty there is even greater need for institutions that can correct for the discretionary outcome. The question is then whether the performance of the inflation targeting regime is sensitive with respect to preference uncertainty. This issue has been investigated by Schaling (1997) in a model without fiscal policy. He found that if the additional distortions are associated with the preference shock, inflation targeting can induce the optimal outcome under uncertainty if the target is appropriately specified. In particular, because uncertainty leads to increased inflationary expectations, the optimal target is the lower the higher is uncertainty. In our framework this result suggests that, given that the TU is aggressive, inflation targeting allows the FA to reach the optimal outcome if the target is made contingent upon uncertainty about the CB's preferences.

## 7 Conclusion

In this Essay, we have applied inflation targeting in a model where fiscal policy responds endogenously to changes in the monetary regime. The analysis was carried out in a framework where the trade union optimises under a trade-off between real wages and employment, and where the fiscal and the monetary authorities have different preferences for their objectives inflation, employment and public expenditures. In such a set-up, a policy coordination problem arises in addition to the traditional time-inconsistency problems because the authorities disagree about the optimal mix of financing the government expenditure objective.

The equilibrium outcomes were evaluated under different scenarios about the TU's wage setting behaviour. The main findings were as follows. Due to the formulation of the TU loss function, a positive weight assigned by the TU on the employment objective generally implies that both fiscal and monetary policies are subject to time-inconsistency problems. However, if the TU's preferences and employment target are compatible with the policymakers' employment objectives (i.e., the TU is fully cooperative), no time-inconsistency problems arise. If, on the other hand, the TU assigns no

weight on the employment objective (i.e., the TU is aggressive), the model shows only a monetary time-inconsistency problem.

In the cases where the TU is not fully cooperative, we generalised the results from previous research that in the presence of a policy coordination problem, commitments by the policymakers are not necessarily welfare improving. In addition, we showed that the outcome depends not only on the degree of asymmetry between the policymakers' preferences but also on the TU's preferences, as these determine the natural rate of employment and therefore the seriousness of the time-inconsistency problems in the model. Building on these results, we studied how the benevolent FA could increase the discretionary welfare by imposing an inflation target on the CB. It turned out, that inflation targeting is generally welfare improving in that it solves the policy coordination problem. However, in solving the time-inconsistency problems the "performance" of the inflation targeting institution is sensitive to the TU's preferences. In particular, under aggressive TU behaviour, inflation targeting can implement the society's optimal outcome, but if the TU has an employment objective, the optimum cannot be achieved. This result reflects the fact that more than one instruments are needed when the model features two time-inconsistency problems. Nevertheless, and in contrast to the results of Huang and Padilla (1995), inflation targeting can improve social welfare under endogenous fiscal objectives.

Finally, we analysed the case where an aggressive TU faces uncertainty about the CB's preferences. It turned out, that the distortions that characterise the discretionary policy mix in the absence of uncertainty are more serious in the presence of uncertainty. However, the performance of the inflation targeting institution does not necessarily suffer from the uncertainty if the target is designed so that it takes into account the additional inflationary expectations.

# Appendix 1.

## Comparative Statics for Chapter 1

Equation (4.4a):

$$\frac{\partial p^D}{\partial \mu_1} = \frac{(1-\gamma)(\delta_2^2 \mu_1 + \delta_1 \delta_2 (1-\mu_1)) + \delta_2^2}{(\delta_2 + (1-\gamma)(\delta_1(1+\mu_2) + \mu_1 \delta_2))^2} > 0;$$

$$\frac{\partial p^D}{\partial \mu_2} = \frac{\delta_1 \delta_2 + (1-\gamma)((\delta_1 - \delta_2)(\delta_1 \mu_2 + \delta_2 \mu_1) + \delta_1^2)}{(\delta_2 + (1-\gamma)(\delta_1(1+\mu_2) + \mu_1 \delta_2))^2} > 0, \text{ iff}$$

$$\frac{\delta_1 \delta_2 + \delta_1^2 (1-\gamma)}{(1-\gamma)(\delta_1 \mu_2 + \delta_2 \mu_1)} > \delta_2 - \delta_1;$$

$$\frac{\partial p^D}{\partial (1-\gamma)} = \frac{(\delta_1 \mu_2 + \mu_1 \delta_2) \delta_2 g^* - (\delta_1(1+\mu_2) + \mu_1 \delta_2) n}{(\delta_2 + (1-\gamma)(\delta_1(1+\mu_2) + \mu_1 \delta_2))^2} > 0, \text{ iff}$$

$$\delta_2 g^* > \frac{\delta_1(1+\mu_2) + \mu_1 \delta_2}{\delta_1 \mu_2 + \mu_1 \delta_2} n;$$

$$\Rightarrow \frac{\partial p^D}{\partial \gamma} < 0 \Rightarrow \frac{\partial p^D}{\partial \chi} > 0$$

Equation (4.4b):

$$\frac{\partial \tau^D}{\partial \delta_1} = -\frac{\delta_2(1+\mu_2)}{(\delta_2 + (1-\gamma)(\delta_1(1+\mu_2) + \mu_1 \delta_2))^2} < 0;$$

$$\frac{\partial \tau^D}{\partial \delta_2} = \frac{\delta_1(1+\mu_2)}{(\delta_2 + (1-\gamma)(\delta_1(1+\mu_2) + \mu_1 \delta_2))^2} > 0;$$

$$\frac{\partial \tau^D}{\partial (1-\gamma)} = -\frac{\delta_2(\delta_1(1+\mu_2) + \mu_1 \delta_2) g^* + (\delta_1(1+\mu_2) + \mu_1 \delta_2)^2 n}{(\delta_2 + (1-\gamma)(\delta_1(1+\mu_2) + \mu_1 \delta_2))^2} < 0, \text{ iff}$$

$$\delta_2 g^* > \frac{(\delta_1(1+\mu_2) + \mu_1 \delta_2)^2}{\delta_1(1+\mu_2) + \mu_1 \delta_2} n$$

$$\Rightarrow \frac{\partial \tau^D}{\partial \gamma} > 0 \Rightarrow \frac{\partial \tau^D}{\partial \chi} < 0$$

Equation (6.4):

$$\frac{\partial \bar{p}}{\partial \gamma} = \frac{((\delta_1(1-\gamma) + \delta_2)^2 + \delta_1\delta_2(\delta_1 + \delta_2)(1-\gamma))(\delta_1(\mu_1\delta_2 + \mu_2\delta_1) + g^*)}{(\cdot)^2}$$

$$-\frac{2\delta_1(\delta_1(1-\gamma) + \delta_2) + \delta_1\delta_2(\delta_1 + \delta_2)}{(\cdot)^2} > 0;$$

$$\Rightarrow \frac{\partial \bar{p}}{\partial \gamma} < 0 \Rightarrow \frac{\partial \bar{p}}{\partial \chi} > 0$$

$$\frac{\partial \bar{p}}{\partial \mu_1} = -\frac{\delta_2(\delta_1(1-\gamma) + \delta_2)^2}{(\cdot)^2} < 0;$$

$$\frac{\partial \bar{p}}{\partial \mu_2} = -\frac{\delta_1(\delta_1(1-\gamma) + \delta_2)^2}{(\cdot)^2} < 0;$$



## Appendix 2.

### Derivation of equation (6.15).

Taking expectations across equation (6.14b):

$$(A1) \quad \begin{aligned} p^e + \hat{\mu}_1(p^e - p^e - u^* - \tau) + \hat{\mu}_2(p^e + \tau - g^*) - \bar{p} &= 0 \\ \Rightarrow p^e &= E\left[\frac{1}{1 + \mu_2}(\mu_1(\tau + u^*) + \mu_2(g^* - \tau) + \bar{p})\right] \end{aligned}$$

Following Nolan and Schaling (1996), taking expected value of ratios of random variables can be achieved through Taylor series expansion. Assuming that the problem at hand is to expand  $\theta(z) = X/Y$  about the respective means, and that the first two moments of  $E(X/Y)$  exist, they use the following second-order approximation:

$$(A2) \quad E\left(\frac{X}{Y}\right) \approx \frac{m_X}{m_Y} - \frac{1}{m_Y^2} COV[X, Y] + \frac{m_X}{m_Y^3} VAR[Y]$$

In (A2),  $m_X$  and  $m_Y$  denote the respective means of variables  $X$  and  $Y$ . Therefore, after factorising (A1), we can write

$$(A3) \quad \begin{aligned} \text{a)} \quad E\left(\frac{\hat{\mu}_1}{1 + \hat{\mu}_2}\right) &\approx \frac{\mu_1((1 + \mu_2)^2 + \sigma_{\mu_2}^2)}{(1 + \mu_2)^3}, \\ \text{b)} \quad E\left(\frac{\hat{\mu}_2}{1 + \hat{\mu}_2}\right) &\approx \frac{\mu_2((1 + \mu_2)^2 + \sigma_{\mu_2}^2)}{(1 + \mu_2)^3} \text{ and} \\ \text{c)} \quad E\left(\frac{1}{1 + \hat{\mu}_2}\right) &\approx \frac{\sigma_{\mu_2}^2(1 + \mu_2)^2}{(1 + \mu_2)^3}. \end{aligned}$$

Substituting (A3 a,b,c) into (A1), and rearranging, gives equation (6.15) in the text.

# Inflation Targeting in a Monetary Union

## 1 Introduction

In this Essay, we extend the inflation targeting framework to a two-country case. In particular, we study fiscal-monetary interaction and the role of optimal institutions in monetary unions. In so doing, our aim is to shed light on the structure of the European Economic and Monetary Union (EMU).

In the previous Essay, it was shown that by subjecting the Central Bank to an optimal inflation target, the government can eliminate the Central Bank's time-inconsistency problem without creating excessive losses in terms of lower employment and government expenditures. A crucial feature of the analysis was an explicit principal-agent framework, where the government (the principal) delegates monetary policy to the Central Bank. In EMU, we argue that this particular principal-agent game was defined when the participating countries, by ratifying the Maastricht Treaty, delegated the common monetary policy to the European Central Bank (ECB). According to the treaty, the independent ECB's sole responsibility is to guarantee price stability. In practice, "price stability" has been interpreted as low (around 2%) rate of inflation. However, formally the ECB itself is responsible for the technical definition of price stability.

In this Essay, we propose a general model of such multinational delegation where a representative institution, here called the Council, works as an intermediary between the national governments and the Central Bank (the CB). The role of the Council could be interpreted as modelling the setup in the Maastricht Summit, where the decision to delegate the common monetary policy to the ECB was made by the EU Council in the composition of heads of states or governments. More formally, building on the work by Alesina and Tabellini (1987), Alesina and Grilli (1993) and Debelle and Fischer (1994), we use a framework where monetary policy delegation is assumed to take place in two stages. In the first stage, the national governments have agreed

upon a "supra-national" institution, the Council, whose objectives reflect the social welfare in the monetary union. In the second stage, the Council optimally delegates the common monetary policy to an independent agent, the CB. The CB's independence is demonstrated by the fact that it stabilises output deviations in the union level, while the Council is assumed to care about output in each country separately. We show that without commitment to price stability through inflation targeting, the CB's independent nature *per se* can create additional distortions. However, if the Council's delegation decision is made optimally, an equilibrium can be achieved where the social welfare is increased under the delegation regime.

Two different delegation regimes are considered. By appointing a sufficiently inflation-averse ("conservative") CB, the Council can achieve a lower time-consistent inflation rate, with a cost in terms of lower equilibrium output and insufficient response to shocks (as in Rogoff, 1985). On the other hand, if the CB is given an optimal inflation target, the inflation bias can be reduced further, with some costs in terms of average output but with no costs in terms of stabilisation. In the case of inflation targeting, optimal delegation is illustrated to include conservative CB preferences as well. Furthermore, because of the direct link between the fiscal and monetary policy in the model, it turns out that inflation targeting works as a voluntary commitment to increased fiscal discipline in the monetary union. In the context of EMU, this clearly suggests that subjecting the ECB to an optimal inflation target could, at least to a certain extent, work as an alternative to the proposed EMU "Stability Pact".<sup>1</sup> The final parts of this Essay illustrate how the Council's voting rules can be designed in a way that makes the inflation targeting regime an *ex post* renegotiation proof institution.

The rest of this Essay proceeds as follows. In section 2, we derive the benchmark case where monetary policy is formulated by the Council. Section 3 studies the outcome when monetary policy is set by an independent CB. The optimal delegation regimes and the

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<sup>1</sup> However, this result is conditional upon the assumption that the fiscal authority is not able to postpone adjustment by using debt financing. If debt financing were possible, then supplementing the inflation target with a stability pact set on debt ratios could still provide an alternative to a pact set on fiscal deficit ratios. As it stands, the purpose of the Pact is to enforce fiscal discipline inside EMU by strengthening the excessive deficit procedure of the Maastricht treaty. In particular, the pact imposes sanctions on countries who run excessive deficits. See Essay 4 for a more detailed analysis of the potential implications of the Pact.

conditions for renegotiation proofness are derived in Section 4. Section 5 concludes.

## 2 Equilibrium Policy under the Council

In our model, the monetary union is organised as follows. The participating governments form a supra-national institution, the Council, which consists of one representative from each government. The Council in turn can delegate monetary policy to a common monetary institution, the CB. Optimal delegation implies that the Council chooses the CB's objectives in order to maximise the community's welfare, which in the monetary union is assumed to coincide with the Council's welfare.<sup>2</sup>

This section derives a benchmark case for monetary policy in the monetary union. Before the optimal institutional relationships between the Council and the CB can be analysed, we have to study what the (hypothetical) equilibrium policy outcome would be if the monetary policy were set directly by the Council. In this way, we can identify the potential distortions that are associated with the policy under the Council.

### 2.1 The Model and the Discretionary Equilibrium

Assume that the monetary union consists of two countries of equal size. The economic environment in both participating countries, 1 and 2, is specified following DeBelle and Fischer (1994).<sup>3</sup> In particular, the two countries are characterised by the following supply functions,

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<sup>2</sup> The specification of the monetary union follows Alesina and Grilli (1993), who studied the feasibility of a monetary union in a model with two asymmetric countries in the absence of fiscal policy. However, in contrast to Alesina and Grilli, we assume that the monetary union already exists and focus on the interaction between the different institutions in the union.

<sup>3</sup> Note that the model in Essay 1 is based on a different literature, and therefore the notation followed in this Essay is not completely identical.

government budget constraints and objective functions for the fiscal authority (the government):

$$\begin{aligned}
 (2.1) \quad & \text{a) } \begin{aligned} & y_1 = \pi - \pi^e - \tau_1 + z_1 \\ & g_1 = \pi + \tau_1 \\ & L^1 = \frac{1}{2} E\{\pi^2 + a(y_1 - \bar{y})^2 + b(g_1 - \bar{g})^2\} \end{aligned} \\
 & \text{b) } \begin{aligned} & y_2 = \pi - \pi^e - \tau_2 + z_2 \\ & g_2 = \pi + \tau_2 \\ & L^2 = \frac{1}{2} E\{\pi^2 + c(y_2 - \bar{y})^2 + d(g_2 - \bar{g})^2\} \end{aligned}
 \end{aligned}$$

In (2.1),  $y_i$  denotes output in country  $i$  ( $i = 1, 2$ ), with a target level  $\bar{y} > 0$ . The common inflation rate is denoted by  $\pi$ , and  $\pi^e \equiv E_{t-1}(\pi)$  represents inflationary expectations which are rationally formed by the private wage setters one period earlier. The private sector is modelled as atomistic, so that it is not capable of co-ordinating the wage setting. The government of country  $i$  chooses the distortionary taxes,  $\tau_i$ , which are levied on production. The presence of distortionary taxes renders output below the natural rate (see Alesina and Tabellini, 1987).<sup>4</sup> Monetary policy is then subject to a time-inconsistency problem, as the monetary authority faces an incentive to boost output above the natural rate by generating inflation "surprises" ( $\pi > \pi^e$ ). Finally, the stochastic output shock  $z_i$  has a zero mean and  $\text{VAR}(z_i) = \sigma_{z_i}^2$ , and  $E$  is an expectations operator.

The formulation of the government budget constraint states that government expenditures, denoted by  $g_i$  and with a target level  $\bar{g}$ , must be financed either by taxes or by seignorage revenues.<sup>5</sup> In addition, the budgets must be balanced in each period, i.e., no debt can be issued. Finally, the objective functions state that both governments care about the common inflation rate, as well as output and government expenditures in their own countries. However, they do not necessarily assign equal weights ( $a, b$  vs.  $c, d$ ) on these

<sup>4</sup> In contrast to the private sector objective function which was applied in Essay 1, the private sector has no employment objective here.

<sup>5</sup> We assume that there has been sufficient convergence in the economies before the monetary union, so that the target levels for the various objectives are the same in both countries.

objectives.<sup>6</sup> The governments are assumed to be completely benevolent, so that their objective functions represent the social welfare functions in the respective countries. In the benchmark case, the two governments delegate their monetary policies to the Council. The Council consists of representatives of both participating governments. However, to simplify the analysis, we assume that the Council has no explicit government expenditure objective (see Debelle and Fischer, 1994 who make a similar assumption about the monetary authority's objective function).<sup>7</sup> However, in the Council, the representative of each country is assumed to share her own government's preferences between the inflation and output objectives, and every representative wants to maximise the welfare of her own country in the monetary union, with respect to these two objectives.

The two countries receive equal shares of the total seignorage revenues generated in the monetary union and have equal weights in the Council's decision-making process. In that case, the Council chooses the inflation rate so as to minimise the following loss function:

$$(2.2) \quad L^{CL} = \frac{1}{2} E \left\{ \pi^2 + \phi \frac{1}{2} [(y_1 - \bar{y})^2 + (y_2 - \bar{y})^2] \right\}$$

In (2.2), the superscript "CL" refers to the council. The Council minimises inflation and simply a weighted sum of the two participating governments' output objectives. More specifically, the formulation of (2.2) implies that the Council cares about output in each country separately, and it prefers seignorage revenues to be distributed separately between the two governments. The weight assigned by the Council on the aggregate output objective relative to the inflation objective,  $\phi$ , is jointly chosen by the participating governments as a linear combination of their own weights,  $a$  and  $c$ .<sup>8</sup>

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<sup>6</sup> Clearly, assuming that the target levels are symmetric while preferences are asymmetric may not be completely consistent. However, this assumption greatly simplifies the algebra.

<sup>7</sup> Because the expenditure objective is likely to be politically very sensitive for individual governments, the assumption that the participants omit that objective from the aggregate (the council's) objective function is quite realistic.

<sup>8</sup> See Alesina and Grilli (1993), who study the bargaining process between the participating countries about the choice of the CB's preferences in a related framework but in the absence of endogenous fiscal policy.

In general, the lower is  $\phi$ , the more the Council cares about the price stabilisation objective, in which case we say that the Council is "conservative". Finally, it is assumed that when the participating governments agree upon the parameters of (2.2), they also implicitly agree that the Council's loss function will constitute a "supra-national" social welfare function in the monetary union.

When choosing the community's monetary policy, the Council is not able to commit to *ex ante* optimal monetary policy rules, i.e., it chooses the inflation rate under discretion. More specifically, the Council chooses the monetary union's inflation rate and simultaneously the two governments choose their national taxes, all taking each others' decision and the private sector's inflationary expectations as given. The precise timing of events in the game between the Council, the two governments and the private sectors in countries 1 and 2, is specified as follows:

(i) The governments of the two participating countries, 1 and 2, decide to join the monetary union and send their representatives to the Council. (ii) The private sectors set the nominal wages, i.e., they rationally form inflationary expectations. (iii) The supply shocks  $z_1$  and  $z_2$  are realised. (iv) The Council chooses the inflation rate in the monetary union and governments 1 and 2 choose their respective national taxes. (v) Output and government spending in countries 1 and 2 are determined.

Analytically, the discretionary equilibrium can be derived as follows. Minimising (2.2) with respect to inflation, and (2.1a,b) with respect to taxes, gives the following three first order conditions (reaction functions):

$$(2.3) \quad \begin{aligned} \text{a)} \quad & \pi = \frac{\phi}{1+\phi} \left\{ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} - \frac{z_1 + z_2}{2} \right\} \\ \text{b)} \quad & \tau_1 = \bar{g} + \frac{a-b}{a+b} \pi - \frac{a}{a+b} (\pi^e + \bar{y} + \bar{g} - z_1) \\ \text{c)} \quad & \tau_2 = \bar{g} + \frac{c-d}{c+d} \pi - \frac{c}{c+d} (\pi^e + \bar{y} + \bar{g} - z_2) \end{aligned}$$

From the reaction functions, using repeated substitution and rational expectations ( $\pi^e = \pi$ ), we can solve the Nash equilibrium rates of inflation and taxes. However, before reporting these, we define the following notation to simplify the algebra:

Table 2.1      **Definitions of Notation**

$$(i) A \equiv \frac{a}{a+b}, B \equiv -\frac{b}{a+b}, C \equiv \frac{c}{c+d}, D \equiv -\frac{d}{c+d};$$

$$(ii) E(\pi^{CL}) \equiv -\frac{\phi(B+D)}{2-\phi(B+D)}(\bar{y} + \bar{g}) > 0;$$

$$(iii) U^{CL} \equiv -\frac{\phi B}{2(1+\phi) - \phi(A+B+C+D)} > 0;$$

$$(iv) V^{CL} \equiv -\frac{\phi D}{2(1+\phi) - \phi(A+B+C+D)} > 0;$$

The equilibrium inflation and tax rates can now be written as follows:<sup>9</sup>

$$(2.4) \quad \begin{aligned} a) \pi^{CL} &= E(\pi^{CL}) - U^{CL} z_1 - V^{CL} z_2 \\ b) \tau_1 &= \bar{g} - \frac{\phi(B+D) - 2A}{\phi(B+D)} E(\pi^{CL}) \\ &+ \frac{\phi(B+2AD) - 2A}{\phi B} U^{CL} z_1 + (A+B) V^{CL} z_2 \\ c) \tau_2 &= \bar{g} - \frac{\phi(B+D) - 2C}{\phi(B+D)} E(\pi^{CL}) \\ &+ \frac{\phi(D+2CB) - 2C}{\phi D} V^{CL} z_2 + (C+D) U^{CL} z_1 \end{aligned}$$

From equation (2.4) and Table 2.1, it can be seen that the average<sup>10</sup> part of the expected inflation rate ( $E(\pi^{CL})$ ) and the variance of inflation ( $U_{CL}^2 \sigma_{z1}^2$ ,  $V_{CL}^2 \sigma_{z2}^2$ ) are both increasing in the weight the Council assigns on the output objective ( $\phi$ ).<sup>11</sup> A higher weight assigned on output means that under discretion, the Council has a larger incentive to generate policy surprises, and consequently the

<sup>9</sup> See Appendix 1 for derivation of the equilibrium outcomes under the Council's discretion.

<sup>10</sup> From now on, we refer to "average" outcomes as the state-independent parts of the respective equilibrium outcomes.

<sup>11</sup> See Appendix 2 for derivation of the comparative statics in this section.



Council faces a more severe time-inconsistency problem. The time-inconsistency problem implies that in equilibrium, monetary policy shows an inflationary bias. Higher inflation in turn means that a higher share of the government expenditure objective can be financed through seignorage, which allows the national governments to choose lower taxes. Under discretion, therefore, too much revenues are created in form of seignorage and too little are collected in form of taxes. In other words, under discretion the policy mix is inefficient.

The expressions for the two policy instruments, taxes and inflation, can be used to determine the equilibrium levels of the state variables, output and government expenditures, in countries 1 and 2:

$$\begin{aligned}
 (2.5) \quad a) y_1^{CL} &= \bar{y} - \frac{2B}{\phi(B+D)} E(\pi^{CL}) + \frac{2(1-\phi D)}{\phi} U^{CL} z_1 + 2BV^{CL} z_2 \\
 b) g_1^{CL} &= \bar{g} + \frac{2A}{\phi(B+D)} E(\pi^{CL}) + \frac{2A(\phi D - 1)}{\phi B} U^{CL} z_1 - 2AV^{CL} z_2 \\
 c) y_2^{CL} &= \bar{y} - \frac{2D}{\phi(B+D)} E(\pi^{CL}) + \frac{2(1-\phi B)}{\phi} V^{CL} z_2 + 2DU^{CL} z_1 \\
 b) g_2^{CL} &= \bar{g} + \frac{2C}{\phi(B+D)} E(\pi^{CL}) + \frac{2C(\phi B - 1)}{\phi D} V^{CL} z_2 - 2CU^{CL} z_1
 \end{aligned}$$

It turns out, that output and expenditures in country 1 are the lower the higher is the positive shock hitting country 2's economy (recall that  $B, D < 0$ ). This is because the Council's deflationary response to 2's shock reduces the seignorage revenues received by both countries, and the government in country 1 must therefore raise taxes in order to compensate for this spillover. However, as the government faces a trade-off between the output and expenditure objectives, in equilibrium there is a reduction in the levels of both objectives. Furthermore, in both countries, the average levels of output and expenditure are increasing in  $\phi$ . This means that a less conservative Council (i.e., a Council with a high  $\phi$ ) is partially successful in raising output closer to the natural rate, and hence the state variables are not invariant to changes in the monetary regime.<sup>12</sup> The variances in output and expenditures are decreasing in  $\phi$ , illustrating the fact that a less conservative Council allows inflation to vary more (see equation 2.4),

<sup>12</sup> Note that in models where fiscal policy is not endogenously determined, higher inflation is never successful in raising output above the natural rate if the private sector has rational expectations.

in order to stabilise the other objectives around their respective average values.

The Council's equilibrium loss, which constitutes the social welfare loss in the monetary union, will form the benchmark with respect to the other institutional arrangements. The equilibrium loss can be obtained by substituting the outcomes for inflation and the two countries' outputs, from equations (2.4) and (2.5), respectively, into equation (2.2).

$$(2.6) \quad L^{CL} = E(\pi^{CL})^2 \left[ 1 + \frac{2(B-D)}{\phi(B+D)} \right] (\bar{y} + \bar{g})^2 \\ + (U^{CL})^2 M \sigma_{z1}^2 + (V^{CL})^2 N \sigma_{z2}^2 - 2\sigma_{z1}\sigma_{z2}\rho_{1,2} U^{CL} V^{CL} [MN]^{\frac{1}{2}}$$

In (2.6), we have written

$$M \equiv \left[ 1 + 2\phi \left( \frac{(1-\phi D)^2}{\phi^2} + D^2 \right) \right] \\ N \equiv \left[ 1 + 2\phi \left( \frac{(1-\phi B)^2}{\phi^2} + B^2 \right) \right].$$

In (2.6),  $\rho_{1,2}$  is the correlation coefficient between the two shocks. The top row in (2.6) illustrates the average part of the Council's expected loss. More specifically, the first term in the squared brackets shows the expected average inflation rate, and the second term shows the expected average output in the monetary union. As was seen above from equations (2.4) and (2.5), all these entries are increasing in  $\phi$ . On the other hand, the bottom row in (2.6) shows the expected loss from the Council's optimal response to the two output shocks. The terms which are associated with the variance of *inflation* (the first terms in the expressions for  $M$  and  $N$ , respectively) are increasing in  $\phi$ , while the terms which are associated with the variance of *output* (the second terms in the expressions for  $M$  and  $N$ ) are decreasing in  $\phi$ . Hence, the more conservative is the Council, the less it allows inflation to vary in order to stabilise output in the monetary union. Finally, the higher is the positive correlation between the two shocks,

the lower is the Council's loss. This is because positive correlation means that stabilising one shock partially reduces the effort that otherwise should be devoted to stabilise the other shock.

In the following sections, we compare the benchmark outcome (2.6) to the discretionary equilibrium where union's monetary policy is set by an independent CB. To start with, we derive the outcomes under a general CB loss function. We then turn to analyse the optimal delegation regimes, where the CB's objectives are specified by the Council.

### 3 Equilibrium Policy under the Independent Central Bank

Assume now that monetary policy in the monetary union is set by one common Central Bank, who has no particular links neither to any of the member states nor to the Council. We argue that due to its independent nature, the CB's objective function has a different structure than the Council's objective function (2.2). In this section, we focus on the details in the equilibrium policy outcomes that arise solely from this formulation. The optimal delegation decision, where the Council can manipulate the CB's objective function, is covered in the next section.

In the literature, the main argument in favour of Central Bank independence is that an independent CB, which has price stability as its primary objective, can be better insulated from the political pressures to generate higher inflation. However, in the context of monetary unions, there has been suggestions that an independent common monetary authority could undermine fiscal discipline in the union (see van Aarle, Bovenberg and Raith, 1995a, for a review of the studies in this area). We argue that the source to such distortions must lie in the monetary authority's objective function. In particular, we suggest that the ECB formulates monetary policy according to the following loss function:

$$(3.1) \quad L^{CB} = \frac{1}{2} E \left\{ \pi^2 + \gamma \frac{1}{2} (y_1 - \bar{y} + y_2 - \bar{y})^2 \right\}$$

In (3.1),  $\gamma$  denotes the weight assigned by the CB on the output objective. In this section, we take  $\gamma$  as given, and focus on the council's optimal choice of  $\gamma$  in the next section. The CB minimises inflation and the deviation of the union's output from its target, and it does not individualise the participating governments' output objectives. The distinction between (3.1) and the Council's objective function (2.2) illustrates the fact that the Council is concerned about the deviation of output in the two countries separately, while the CB stabilises across the whole currency union.<sup>13</sup> Minimising (3.1) with respect to  $\pi$ , yields the CB's first-order condition:

$$(3.2) \quad \pi = \frac{2\gamma}{1+2\gamma} \left\{ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} - \frac{z_1 + z_2}{2} \right\}$$

The national output functions, the budget constraints, as well as the objective functions and the first-order conditions of the two governments, are all identical to the previous case. After repeated substitution, and using rational expectations, we get the following equilibrium rates of inflation and taxes when monetary policy is set by the CB:<sup>14</sup>

(3.3)

$$a) \pi^{CB} = E(\pi^{CB}) - U^{CB} z_1 - V^{CB} z_2$$

$$b) \tau_1^{CB} = \bar{g} - \frac{\gamma(B+D) - A}{\gamma(B+D)} E(\pi^{CB}) + \frac{\gamma(B+2AD) - A}{\gamma B} U^{CB} z_1 + \frac{\gamma D(A+B)}{\gamma D} V^{CB} z_2$$

$$c) \tau_2^{CB} = \bar{g} - \frac{\gamma(B+D) - C}{\gamma(B+D)} E(\pi^{CB}) + \frac{\gamma(D+2CB) - C}{\gamma D} V^{CB} z_2 + \frac{\gamma B(C+D)}{\gamma B} U^{CB} z_1$$

<sup>13</sup> In a dynamic analysis, van Aarle et al. (1995a) use a similar distinction between the loss functions when analysing the evolution of deficit and debt in a monetary union under two alternative designs for the CB.

<sup>14</sup> See Appendix 3 for derivations of the equilibrium outcomes under the independent ECB.

In (3.3), we have used the following notation:

$$E(\pi^{CB}) \equiv -\frac{\gamma(B+D)}{1-\gamma(B+D)}(\bar{y} + \bar{g}) > 0;$$

$$U^{CB} \equiv -\frac{\gamma B}{1+2\gamma-\gamma(A+B+C+D)} > 0;$$

$$V^{CB} \equiv -\frac{\gamma D}{1+2\gamma-\gamma(A+B+C+D)} > 0;$$

The equilibrium levels of output and expenditures can be solved using (3.3). They are given by:

$$(3.4) \quad \begin{aligned} a) y_1^{CB} &= \bar{y} - \frac{B}{\gamma(B+D)} E(\pi^{CB}) + \frac{1-2\gamma D}{\gamma} U^{CB} z_1 + 2BV^{CB} z_2 \\ b) g_1^{CB} &= \bar{g} + \frac{A}{\gamma(B+D)} E(\pi^{CB}) + \frac{A(2\gamma D-1)}{\gamma B} U^{CB} z_1 - 2AV^{CB} z_2 \\ c) y_2^{CB} &= \bar{y} - \frac{D}{\gamma(B+D)} E(\pi^{CB}) + \frac{1-2\gamma B}{\gamma} V^{CB} z_2 + 2DU^{CB} z_1 \\ d) g_2^{CB} &= \bar{g} + \frac{C}{\gamma(B+D)} E(\pi^{CB}) + \frac{C(2\gamma B-1)}{\gamma D} V^{CB} z_2 - 2CV^{CB} z_1 \end{aligned}$$

Compared to the outcome where monetary policy is set by the Council, the differences that arise from the CB's different output objective can be summarised as follows:

**Proposition 1.** For given preferences, the common inflation rate is higher and the national tax rates are lower under an independent CB than under the Council.

**Proof.** The average inflation rate under the CB can be decomposed as follows:

$$(3.5) \quad \begin{aligned} \pi^{CB} &= -\frac{\gamma(B+D)}{2-\gamma(B+D)}(\bar{y} + \bar{g}) \\ &\quad - \frac{\gamma(B+D)}{2-\gamma(3(B+D)-\gamma(B+D)^2)}(\bar{y} + \bar{g}). \end{aligned}$$

If we replace  $\gamma$  with  $\phi$  in (3.5), the top row becomes identical to the average inflation rate under the Council. However, discretionary monetary policy under the CB shows an additional positive bias (the bottom row of equation 3.5), which is due to the CB's "failure" to

separate between the national output objectives. Because a higher share of the national expenditure objectives is being financed by seignorage, taxes are optimally set lower under the CB than under the council. *Q.E.D.*

Note that because the equilibrium shows higher inflation and lower taxes, monetary policy under the CB results in higher output in both participating countries. Nevertheless, because an independent CB creates an opportunity for the individual governments to behave in an undisciplined manner, the discretionary policy mix is more inefficient than under the Council's discretion.

Finally, the Council's equilibrium welfare loss when the monetary policy is set by the CB can be obtained by substituting the equilibrium outcomes for inflation (from 3.3a) and output (from 3.4 a,c) into the Council's objective function (2.2). This gives:

$$L^{CL,CB} = E(\pi^{CB})^2 \left[ 1 + \frac{1}{2} \phi \left( \frac{(B-D)}{\gamma^2(B+D)} \right) \right] (\bar{y} + \bar{g})^2 + U_{CB}^2 I \sigma_{z1}^2 + V_{CB}^2 J \sigma_{z2}^2 - 2\sigma_{z1}\sigma_{z2}\rho_{1,2} U^{CB} V^{CB} [IJ]^{1/2} \quad (3.6)$$

$$I \equiv \left[ 1 + \frac{1}{2} \phi \left[ \left( \frac{1-2\gamma D}{\gamma} \right)^2 + (2D)^2 \right] \right],$$

$$J \equiv \left[ 1 + \frac{1}{2} \phi \left[ \left( \frac{1-2\gamma B}{\gamma} \right)^2 + (2B)^2 \right] \right]$$

In (3.6), the notation  $L^{CL,CB}$  means that the Council now evaluates its loss using the CB's inflation rate. Due to the sub-optimal policy mix, the Council is made worse off compared to the benchmark case in (2.6) if the independent CB shares the Council's preferences.

However, there is no reason why the preferences should be identical. Consequently, having identified the distortions that can arise from the CB's independent nature *per se*, we now turn to study the case where the CB is subordinated to the Council. More specifically, we explore an organisational form where the Council acts as the principal for the CB and seeks a common monetary policy formulated in a way that maximises the union's social welfare. Clearly, in order to make policy delegation incentive compatible for the Council, the resulting outcome must not generate a welfare loss higher than the benchmark case, equation 2.6.

## 4 Optimal Design of the Central Bank's Objectives

In the previous section we saw that monetary policy under an independent CB can lead to excessive inflation and lower fiscal discipline. However, if a principal-agent relationship is established between the Council and the CB, the Council can manipulate the CB's objectives in order to reduce such distortions. More importantly, and in order to make the policy delegation meaningful, the Council should specify the CB's objectives so that the Council's own welfare is increased compared to the discretionary equilibrium (equation 2.6).

Applications of principal-agent relationships in monetary policy games go back to Rogoff (1985). In a closed-economy model where fiscal policy was not included he illustrated how, by delegating monetary policy to an inflation averse (conservative) CB, society can become on the average better off. However, a conservative CB shows an insufficient response to shocks, and the institution therefore generates a trade-off between credibility and flexibility.<sup>15</sup> Debelle and Fischer (1994) extended Rogoff's idea into a framework where taxes are chosen by the government. They found that a conservative CB forces the government to choose higher taxes, and therefore policy delegation improves the discretionary policy mix. More recently, in a model with no endogenous fiscal policy Walsh (1995a) suggested that a simple linear incentive contract for the CB governor can improve the discretionary outcome with no adverse stabilisation effects. Building on these results, Svensson (1997) argued that in a static model and with no uncertainty, the "Walsh equilibrium" can be reached if the CB is subjected to an optimal inflation target.

In what follows, we analyse how the Council as the principal can design the CB's (the agent's) objectives before delegation in a way that could improve the Council's own welfare under discretion. In particular, we compare the "performance" of two different institutions, a conservative CB and an inflation target, in relation to the benchmark outcome.

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<sup>15</sup> Lohmann (1992) complemented Rogoff's analysis by deriving a solution to the trade-off which included an escape clause to the central bank head, inducing her to stabilise more after a particularly severe realisation of the shock.

## 4.1 Optimal Choice of the Central Bank's Preferences

In this section, we study how the Council can improve upon the discretionary outcome (2.6), by choosing the CB's preference parameter  $\gamma$  optimally. To start with, a careful investigation of the equilibrium outcomes (2.6) and (3.6) reveals that if the CB has preferences  $\gamma = (1/2)\phi$ , the Council's loss under the CB becomes identical to the loss in the benchmark outcome (2.6).<sup>16</sup> However, following Rogoff, the Council can do better by nominating a CB who is even more conservative. The following proposition generalises Rogoff's result in the current framework:

**Proposition 2.** The Council can improve its welfare by delegating monetary policy to a CB with  $\gamma < (1/2)\phi$ , if (i) the Council's time inconsistency problem is severe and (ii) the shocks are not too large.

**Proof.** The optimal choice of the CB's preferences ( $\gamma$ ) can be obtained by minimising equation (3.6), i.e., the Council's loss when monetary policy is set by the ECB, with respect to  $\gamma$ . Assuming for simplicity that  $\rho_{1,2} = 0$ , this gives:

$$(4.1) \quad \frac{\partial L^{CL,CB}}{\partial \gamma} = \frac{2(B+D)^2(\gamma - \phi(B+D))}{(1 - \gamma(B+D))^3} (\bar{y} + \bar{g})^2 + \frac{(2\gamma B^2 - P)\sigma_{z1}^2 + (2\gamma D^2 - Q)\sigma_{z2}^2}{[1 + \gamma(2 - (A+B+C+D))]^3} = 0$$

In (4.1), we have written:

$$P \equiv 4B\gamma[A(\phi + B) - \phi(A + B + C + D)D^2] + 2D^2[\phi((A + B + C + D) - 2(1 + B))],$$

$$Q \equiv 4D\gamma[C(\phi + D) - \phi(A + B + C + D)B^2] + 2B^2[\phi((A + B + C + D) - 2(1 + D))].$$

<sup>16</sup> This is, of course, rather strong result and it implies that the aggregation bias due to the CB's "multinational" objectives results in inefficiencies *per se*. In fact, this makes the case for institution design particularly strong in monetary unions.



Clearly, whenever (4.1) is positive, the loss is increasing in  $\gamma$ . If we replace  $\gamma$  with  $(1/2)\phi$  in (4.1), and rearrange (4.1), it follows that the Council is better off by delegating monetary policy to a CB with  $\gamma < (1/2)\phi$  when the following inequality holds:

$$(4.2) \quad \left[ \frac{2\phi(B+D)^2(1-B-D)}{(2-\phi(B+D))^3} \right] (\bar{y} + \bar{g})^2 > \frac{2[(\hat{P} - \phi B^2)\sigma_{z_1}^2 + (\hat{Q} - \phi D^2)\sigma_{z_2}^2]}{[2 + \phi(2 - (A+B+C+D))]^3},$$

In (4.2), we have written

$$\begin{aligned} \hat{P} &\equiv 2B\phi[A(\phi+B) - \phi(A+B+C+D)D^2] \\ &+ 2D^2[\phi(A+B+C+D-2(1+B))], \end{aligned}$$

$$\begin{aligned} \hat{Q} &\equiv 2\phi D[C(\phi+D) - \phi(A+B+C+D)B^2] \\ &+ 2B^2[\phi(A+B+C+D-2(1+D))]. \end{aligned}$$

From (4.2), we get

$$\begin{aligned} \frac{\partial LHS}{\partial \phi} &= \frac{2(2-\phi(B+D))^2(B+D)^2(1-B-D)[(2-\phi(B+D))-3\phi(B+D)]}{(2-\phi(B+D))^6} > 0, \\ \frac{\partial RHS}{\partial \sigma_{z_1}^2} &= \frac{2(\hat{P} - \phi B^2)[2 + \phi(2 - (A+B+C+D))]}{[2 + \phi(2 - (A+B+C+D))]^3}, \\ \frac{\partial RHS}{\partial \sigma_{z_2}^2} &= \frac{2(\hat{Q} - \phi D^2)[2 + \phi(2 - (A+B+C+D))]}{[2 + \phi(2 - (A+B+C+D))]^3}. \end{aligned}$$

The RHS is increasing in  $\sigma_{z_1}^2$  and  $\sigma_{z_2}^2$ , if  $\hat{P} - \phi B^2 > 0$  and  $\hat{Q} - \phi D^2 > 0$ , respectively. In that case, and because the Council's time-inconsistency problem is positively linked to  $\phi$ , inequality (4.2) is more likely to hold when the time-inconsistency problem is more severe and  $\sigma_{z_1}^2$  and  $\sigma_{z_2}^2$  are low. *Q.E.D.*

Note that in our model the average part of the expected output is not invariant to changes in the monetary regime. More specifically, output is the lower the lower is inflation, and therefore, in contrary to Rogoff's model, delegating monetary policy to a conservative CB constitutes a cost both in terms of average output and variance of output. The welfare results for the *individual governments* from this

institution design can be summarised as follows. Compared to the benchmark case where the monetary policy is set by the Council, a conservative CB generates lower average inflation, output and expenditures. In addition, the variance of inflation is lower and the variances of output and expenditures are higher under the CB. Therefore, governments who assign high weights on the inflation objective are the relative winners, whereas governments who put high weights on output and government expenditures are the relative losers from the delegation to a conservative CB.

## 4.2 Inflation Targeting

As an alternative to the Rogoff model, we consider the contractual approach to monetary policy delegation. According to Walsh (1995a), a contract that includes a linear monetary transfer to the CB and penalises her from exceeding a pre-specified inflation rate can reduce the average inflation rate, without any adverse stabilisation effects. Under such a regime, the CB is granted *instrument independence*, but not *goal independence*, in the conduct of monetary policy (Fischer, 1995). Recently, Svensson (1997) has shown that in a static context with symmetric information structure a Walsh contract is analytically equivalent to giving the CB an explicit inflation target.<sup>17</sup> An inflation target can be enforced by a dismissal rule, or some other sanction that makes deviations from the policy rule costly for the CB, which is triggered whenever the target is not hit.

Under inflation targeting, the CB's objective function includes an explicit inflation target  $\pi$ , which is chosen and made public by the Council before the game starts:

$$(4.3) \quad L^{CB,TA} = \frac{1}{2} E \left\{ (\pi - \bar{\pi})^2 + \gamma \frac{1}{2} (y_1 - \bar{y} + y_2 - \bar{y})^2 \right\}$$

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<sup>17</sup> However, inflation targeting should be supplemented by a conservative CB if the model includes asymmetric information (Canzoneri & al., 1997, Herrendorf and Lockwood, 1997) or output persistence (Svensson, 1997). This is because in these cases the inflation bias becomes state-dependent.

As in Walsh, the inflation target acts to raise the marginal cost of inflation to the CB by the same amount across all states of nature. In other words, inflation targeting affects only the non-state contingent part of the loss, and the CB will respond to the shocks precisely in the same way as if there were no target. The Council's task is then to optimally choose both  $\pi$  and  $\gamma$  in the CB's objective function. We look at the choice of  $\pi$  first.

Because the inflation target affects only the average loss, we can ignore the shocks when deriving the optimal target. Under inflation targeting, the average inflation rate generated by the CB and the average tax rates generated by the two governments, all operating under discretion, are similar than in (3.3), except that they now include a term in  $\pi$ :<sup>18</sup>

$$(4.4) \quad \begin{aligned} a) E(\pi^{TA}) &= -\frac{\gamma(B+D)}{1-\gamma(B+D)}(\bar{y}+\bar{g}) + \frac{1}{1-\gamma(B+D)}\bar{\pi} \\ b) E(\tau_1^{TA}) &= \bar{g} - \frac{\gamma(B+D)-A}{\gamma(B+D)}E(\pi^{TA}) + \frac{B}{1-\gamma(B+D)}\bar{\pi} \\ c) E(\tau_2^{TA}) &= \bar{g} - \frac{\gamma(B+D)-C}{\gamma(B+D)}E(\pi^{TA}) + \frac{D}{1-\gamma(B+D)}\bar{\pi} \end{aligned}$$

From (4.4), we can solve for the average levels of output and expenditures for the two countries in the monetary union:

$$(4.5) \quad \begin{aligned} a) E(y_1^{TA}) &= \bar{y} + \frac{B}{1-\gamma(B+D)}(\bar{y}+\bar{g}) - \frac{B}{1-\gamma(B+D)}\bar{\pi} \\ b) E(y_2^{TA}) &= \bar{y} + \frac{D}{1-\gamma(B+D)}(\bar{y}+\bar{g}) - \frac{D}{1-\gamma(B+D)}\bar{\pi} \\ c) E(g_1^{TA}) &= \bar{g} - \frac{A}{1-\gamma(B+D)}(\bar{y}+\bar{g}) + \frac{A}{1-\gamma(B+D)}\bar{\pi} \\ d) E(g_2^{TA}) &= \bar{g} - \frac{C}{1-\gamma(B+D)}(\bar{y}+\bar{g}) + \frac{C}{1-\gamma(B+D)}\bar{\pi} \end{aligned}$$

<sup>18</sup> See Appendix 4 for derivations of the equilibrium outcomes under inflation targeting.

The optimal inflation target can now be derived as follows. First, we substitute the expressions for average inflation (4.4a) on one hand and the two countries' average outputs (4.5a,b) on the other hand into the Council's objective function (2.2), where  $\pi$  does not enter directly (in other words, the Council's optimal inflation rate in the absence of an output objective is zero):

$$(4.6) \quad L^{CL,TA} = \frac{1}{2} \left\{ \left[ -\frac{\gamma(B+D)}{1-\gamma(B+D)}(\bar{y} + \bar{g}) + \frac{(1+2\gamma)}{1-\gamma(B+D)}\bar{\pi} \right]^2 + \frac{1}{2}\phi[R+S] \right\}.$$

In (4.6), we have written:

$$R = \left( \frac{B}{1-\gamma(B+D)}(\bar{y} + \bar{g}) - \frac{B}{1-\gamma(B+D)}\bar{\pi} \right)^2,$$

$$S = \left( \frac{D}{1-\gamma(B+D)}(\bar{y} + \bar{g}) - \frac{D}{1-\gamma(B+D)}\bar{\pi} \right)^2.$$

In its optimal choice of the target, the Council trades off the gain from lower inflation to the loss from lower average output. A reduction in the average inflation rate will imply a fall in average output, because the national governments will respond to reduced seignorage revenues by raising distortionary taxes. Therefore, as output is not invariant to the monetary regime in place, depending on the parameter values a target that would simply induce the CB to generate zero average inflation may not be optimal if the participating governments assign positive weights on the expenditure objective.<sup>19</sup> Next, minimising (4.6) with respect to  $\pi$  gives the inflation target that is optimal from the Council's point of view:

$$(4.7) \quad \bar{\pi} = \frac{2\gamma(B+D) + \phi(B^2 + D^2)}{2 + \phi(B^2 + D^2)}(\bar{y} + \bar{g})$$

<sup>19</sup> However, if these weights ( $b$  and  $d$  in equation 2.1) are low, the optimal inflation rate is close to zero in this model. Note that Walsh (1995a) and Svensson (1997) did not consider fiscal policy, and therefore the *ex ante* optimal average inflation rate is zero in these models.

Note that in contrast to the literature with no endogenous fiscal policy, the optimal inflation target now depends not only on the CB's weight on output and the target level of output, but also on the weights assigned by the participating governments on their respective expenditure objectives. In particular, the higher are the parameters associated with the expenditure objective ( $b$  and  $d$ , included in  $B$  and  $D$ ), the lower the target has to be. This is because the governments with higher  $b$  and  $d$  are more keen to raise taxes to finance expenditures, thereby curbing output and increasing the CB's incentives to generate inflation surprises. We are now ready to state:

**Proposition 3.** The optimal inflation target minimises the Council's average loss, independent on the CB's preferences. In contrast to Rogoff delegation, the Council can therefore choose  $\gamma$  so that the CB will stabilise optimally from the Council's point of view. The optimal choice of  $\gamma$  implies conservative CB preferences.

**Proof:** Substituting (4.7) into (4.4a,b) and (4.5a,c), rearranging and adding the CB's optimal responses to the output shocks, gives the final equilibrium levels of inflation, taxes, output and expenditures for country 1 in the monetary union with inflation targeting (country 2's outcomes can be obtained in a similar way):

$$\begin{aligned}
 \pi^{TA} &= E(\pi^{TA}) - U^{CB} z_1 - V^{CB} z_2 \\
 \tau_1^{TA} &= \bar{g} - \frac{2A + \phi(B^2 + D^2)}{\phi(B^2 + D^2)} E(\pi^{TA}) \\
 &\quad + \frac{\gamma(B + 2AD) - A}{\gamma B} U^{CB} z_1 + \frac{\gamma D(A + B)}{\gamma D} V^{CB} z_2 \\
 (4.8) \quad y_1^{TA} &= \bar{y} + \frac{2B}{\phi(B^2 + D^2)} E(\pi^{TA}) + \frac{B(1 - 2\gamma D)}{\gamma B} U^{CB} z_1 + 2BV^{CB} z_2 \\
 g_1^{TA} &= \bar{g} - \frac{2A}{\phi(B^2 + D^2)} E(\pi^{TA}) + \frac{A(2\gamma D - 1)}{\gamma B} U^{CB} z_1 - 2AV^{CB} z_2, \\
 E(\pi^{TA}) &\equiv \frac{\phi(B^2 + D^2)}{2 + \phi(B^2 + D^2)} (\bar{y} + \bar{g})
 \end{aligned}$$

Assuming again, for simplicity, that  $\rho_{1,2} = 0$ , the Council's equilibrium loss under inflation targeting is given by:

$$\begin{aligned}
(4.9) \quad L^{CL,TA} &= E(\pi^{TA})^2 \left[ 1 + \frac{2}{\phi(B^2 + D^2)} \right] (\bar{y} + \bar{g})^2 \\
&+ U_{CB}^2 \left[ 1 + \frac{1}{2} \phi \left( \frac{B(1-2\gamma D)}{\gamma B} \right)^2 + (2D)^2 \right] \sigma_{z1}^2 \\
&+ V_{CB}^2 \left[ 1 + \frac{1}{2} \phi \left( \frac{D(1-2\gamma B)}{\gamma D} \right)^2 + (2B)^2 \right] \sigma_{z2}^2
\end{aligned}$$

In (4.9), the top row illustrates the optimal solution to the trade-off between average inflation and average output. To find the optimal  $\gamma$ , recall first that the Council stabilises optimally when it sets the inflation rate under its own discretion (equation 2.6). On the other hand, recall that the outcomes (2.6) and (3.6) are identical if  $\gamma = (1/2)\phi$ . Therefore, appointing a CB with preferences  $\gamma = (1/2)\phi$ , and subjecting it to inflation targeting, yields the optimal outcome from the Council's point of view. *Q.E.D.*

From Proposition 3 it follows that with inflation targeting, the Council's optimal delegation decision shows a CB who is less conservative than the CB that would be optimally chosen in the absence of inflation targeting. Compared to the conservative CB, inflation targeting allows for more stabilisation and hence from the participating countries' point of view, the design is less biased in favour of the inflation-averse governments of the union. However, compared to the outcome under the council's discretion, inflation targeting will imply a loss in terms of lower average output and expenditures, and therefore the regime is incentive compatible for the participating governments only if these do not assign too high weights on other objectives than inflation.

Furthermore, using (4.8), we can derive the following result:

**Proposition 4.** An inflation target for the CB works as a voluntary restraint for the fiscal authorities. Therefore, in a monetary union, such an agreement can become a (partial) substitute to arrangements such as the the proposed EMU Stability Pact.

**Proof.** Comparing the average outcomes under inflation targeting to the case where the Council operates under discretion reveals:

$$\begin{aligned}
(4.10) \quad E(\pi^{CL}) - E(\pi^{TA}) &= -\frac{2\phi(B^2 + B + D^2 + D)}{(2 + \phi(B^2 + D^2))(2 - \phi(B + D))} > 0 \\
E(\tau_1^{CL}) - E(\tau_1^{TA}) &= \frac{2\phi(1 - A)(B^2 + B + D^2 + D)}{(2 + \phi(B + D)^2)(2 - \phi(B + D))} < 0 \\
E(y_1^{CL}) - E(y_1^{TA}) &= \frac{2\phi B(B^2 + B + D^2 + D)}{(2 + \phi(B + D)^2)(2 - \phi(B + D))} > 0 \\
E(g_1^{CL}) - E(g_1^{TA}) &= -\frac{2\phi A(B^2 + B + D^2 + D)}{(2 + \phi(B + D)^2)(2 - \phi(B + D))} > 0
\end{aligned}$$

In (4.10),  $(B^2 + B + D^2 + D) \equiv -\frac{ba(c+d)^2 + dc(a+b)^2}{(a+b)^2(c+d)^2} < 0$  so that the indicated signs follow. Inflation targeting indeed reduces inflation *and* increases taxes, therefore inducing fiscal discipline and a more efficient policy mix. *Q.E.D.*

Proposition 4 formalises a point which has clearly received too little attention in the discussion about optimal CB institutions. Because fiscal and monetary policies are interdependent even when they are formally decentralised, institutions that are successful in reducing equilibrium inflation will automatically imply increased fiscal discipline. Therefore, in the context of EMU, it is possible that the current proposal where the ECB is subjected to a strict inflation target while the participating governments are constrained by the Stability Pact, can lead to a disciplinary overkill in terms of the national fiscal policies.

We now turn to study how the inflation targeting institution can become a renegotiation proof institution in a monetary union.

### 4.3 Renegotiation Proofness in the Monetary Union

In this section, we study how the CB's optimal inflation target can be implemented after wages have been set and the output shocks have been realised. Generally speaking, the target is not implemented if all parties of the game, i.e., the private sectors in the two countries, the Council and the CB all would benefit from an agreement which does not respect the delegation equilibrium. In that case, we say that the inflation targeting regime is not renegotiation proof.

In a closed economy set-up, al-Nowaihi and Levine (1996) argued that in the Walsh model, the opportunity for renegotiation arises because the CB's response to a positive shock implies a "surprise" deflation.<sup>20</sup> Due to the standard linear-quadratic form of the objective functions, surprise deflation is equally bad than surprise inflation in policy games. Therefore, after the realisation of the shock all parties involved in the game would be better off by adopting a higher average inflation rate. The Council, and the individual Council members, value output above the natural rate and prefer low inflation to deflation, while the private sector values low inflation to a deflationary surprise. The CB is indifferent and implements any renegotiated policy if the dismissal rule is scrapped.

The way to overcome the problem is to ensure that at least one party of the game will always prefer the inflation targeting equilibrium. Because the Council as the CB's principal is the institution that is responsible for implementing the target, we focus on the Council's incentives.

Assume that the Council's constitution specifies the following issues. Both Council members (henceforth: CMs) serve one equally long period. For each CM, a period at office consists of two complete "games" against the private sector (see Figure 2.1). When entering office, a CM inherits a monetary union with inflation targeting, which is the *ex ante* most preferred design for the monetary union.<sup>21</sup> The two CMs enter the office in staggered order, i.e., if CM 1 enters first, CM 2 enters near the end of CM 1's period at office. Country 1's new representative in turn enters before the end of CM 2's period, and so forth *ad infinitum*. The Council votes about the implementation of the target twice during each CM's period at office, once at the beginning, and once at the end. The positive supply shock in a given CM's home country is realised during the CM's period at office, more specifically after the first voting round, but before the second voting round. The target can be renegotiated only if both CM's agree, otherwise it is implemented. Once the target has been renegotiated, it cannot be

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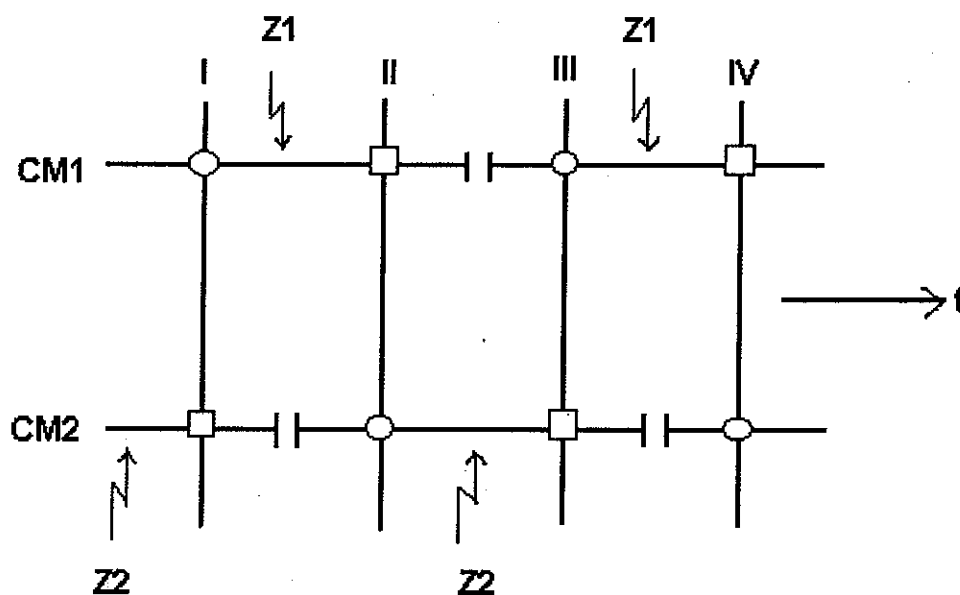
<sup>20</sup> The term "surprise" refers here to a different case than in the time-inconsistency literature in general. Surprise deflation arises due to the CB's optimal response to the output shock, whereas surprise inflation normally arises when the CB cheats on the private sector's inflationary expectations.

<sup>21</sup> This means that *ex ante* both countries are better off in the inflation targeting equilibrium than they are under any other design of the monetary union, or outside the union. Above it was shown that the first condition generally requires that the participating governments do not put too much weight on the output and expenditure objectives.



restored during the same period of office. Between the first voting and the realisation of the shock, the private sector forms inflationary expectations for the next voting. The private sector is assumed to have no prior beliefs about the CMs probability to renegotiate, i.e., the CMs face no signalling incentives. The structure of the voting game and the timing of events are graphically illustrated in Figure 2.1.

Figure 2.1 Voting About Inflation Targeting



O = First voting round for  $CM_i$

□ = Second voting round for  $CM_i$

Following Figure 2.1, in the first voting round (I) CM 1 will consider her *ex ante* payoff for the entire period at office (I+II). The first voting decision is made after the shock in country 2 ( $z_2$ ) has been realised, but under the presumption that  $E(z_1) = 0$ . To work out the optimal first-round voting strategy CM 1 will work backwards, first considering the vote she is expecting to cast in the second voting round (II). At the time of the first voting, CM 1 considers the optimal decision in the coming second voting by comparing the following two expected outcomes:

$$\begin{aligned}
a) L_{II}^{1,RNG} &= [E(\pi^{CL})]^2 + (U^{CL})^2 \sigma_{z_1}^2 + a \left[ \frac{4B - 2\phi(B+D)(A+B-D)}{(2-\phi(B+D))\phi(B^2+D^2)} E(\pi^{TA}) \right]^2 \\
&+ a \left[ \frac{2(1-\phi D)U^{CL}}{\phi} \right]^2 \sigma_{z_1}^2 + b \left[ \frac{2\{A(\phi(B+D)-2) - \phi(AB+CD)\}}{(2-\phi(B+D))\phi(B^2+D^2)} E(\pi^{TA}) \right]^2 \\
&+ b \left[ \frac{2A(\phi D-1)U^{CL}}{\phi B} \right]^2 \sigma_{z_1}^2
\end{aligned}$$

(4.15)

$$\begin{aligned}
b) L_{II}^{1,TA} &= [E(\pi^{TA})]^2 + (U^{CL})^2 \sigma_{z_1}^2 + a \left[ \frac{2B}{\phi(B^2+D^2)} E(\pi^{TA}) \right]^2 + a \left[ \frac{2(1-\phi D)U^{CL}}{\phi} \right]^2 \sigma_{z_1}^2 \\
&+ b \left[ \frac{2A}{\phi(B^2+D^2)} E(\pi^{TA}) \right]^2 + b \left[ \frac{2A(\phi D-1)U^{CL}}{\phi B} \right]^2 \sigma_{z_1}^2,
\end{aligned}$$

$$E(\pi^{TA}) \equiv \frac{\phi(B^2+D^2)}{2+\phi(B^2+D^2)} (\bar{y} + \bar{g}) < -\frac{\phi(B+D)}{2-\phi(B+D)} (\bar{y} + \bar{g}) \equiv E(\pi^{CL}).$$

In (4.15), subscript II refers to the second voting round. Outcome (4.15a) illustrates CM 1's expected loss after the second voting if the target is renegotiated as a result of that voting.<sup>22</sup> Outcome (4.15b) illustrates the expected loss after the second voting, if the target is implemented as a result of that voting. The expected optimal voting strategy in the second round can be characterised as follows.

**Proposition 5.** At the time of the first voting, CM 1 expects to vote for renegotiation in the second voting, if she will have the choice in the second voting.

**Proof.** If the target was renegotiated in the *first* voting, in the second voting the discretionary equilibrium will be automatically the outcome without any second voting necessary because it is not possible to restore the target during the same period of office. However, if the target was implemented as a result of the first voting, in the second voting CM 1 has the choice between voting for implementation and voting for renegotiation. Clearly, because the second

<sup>22</sup> Note that CM1 expects that renegotiation implies a one-period inflationary surprise, thereby raising output and expenditures to higher levels than under inflation targeting. In addition, because under inflation targeting the CB's preferences are chosen to be  $\gamma = (1/2)\phi$  in order to guarantee optimal stabilisation, the renegotiated inflation rate will be  $E(\pi^{CL})$  and not  $E(\pi^{CB})$ .

voting takes place at the end of CM 1's period at office, she chooses the "cheating" outcome (4.15a) that implies a lower one-period loss than the inflation targeting equilibrium (4.15b). *Q.E.D.*

We can now turn to CM 1's optimal voting strategy in the first round. In the first round, CM 1 faces the choice between the following two expected losses for the *entire period at office*:

$$\begin{aligned}
 a) L_{I+II}^{1,RNG} &= [E(\pi^{CL}) - V^{CL} z_2]^2 + a \left[ \frac{4B - 2\phi(B+D)(A+B-D)}{(2-\phi(B+D))\phi(B^2+D^2)} E(\pi^{TA}) + 2BV^{CL} z_2 \right]^2 \\
 &+ b \left[ \frac{2\{A(\phi(B+D)-2) - \phi(AB+CD)\}}{(2-\phi(B+D))\phi(B^2+D^2)} E(\pi^{TA}) - 2AV^{CL} z_2 \right]^2 \\
 &+ E(\pi^{CL})^2 + (U^{CL})^2 \sigma_{z_1}^2 + a \left[ -\frac{2B}{\phi(B+D)} E(\pi^{CL}) \right]^2 + a \left[ \frac{2(1-\phi D)}{\phi} U^{CL} \right]^2 \sigma_{z_1}^2 \\
 &+ b \left[ \frac{2A}{\phi(B+D)} E(\pi^{CL}) \right]^2 + b \left[ \frac{2A(\phi D - 1)}{\phi B} U^{CL} \right]^2 \sigma_{z_1}^2
 \end{aligned}$$

(4.16)

$$\begin{aligned}
 b) L_{I+II}^{1,TA} &= [E(\pi^{TA}) - V^{CL} z_2]^2 + a \left[ \frac{2B}{\phi(B^2+D^2)} E(\pi^{TA}) + 2BV^{CL} z_2 \right]^2 \\
 &+ b \left[ -\frac{2A}{\phi(B^2+D^2)} E(\pi^{TA}) - 2AV^{CL} z_2 \right]^2 \\
 &+ [E(\pi^{CL})]^2 + (U^{CL})^2 \sigma_{z_1}^2 + a \left[ \frac{4B - 2\phi(B+D)(A+B-D)}{(2-\phi(B+D))\phi(B^2+D^2)} E(\pi^{TA}) \right]^2 \\
 &+ a \left[ \frac{2(1-\phi D)}{\phi} U^{CL} \right]^2 \sigma_{z_1}^2 \\
 &+ b \left[ \frac{2\{A(\phi(B+D)-2) - \phi(AB+CD)\}}{(2-\phi(B+D))\phi(B^2+D^2)} E(\pi^{TA}) \right]^2 + b \left[ \frac{2A(\phi D - 1)}{\phi B} U^{CL} \right]^2 \sigma_{z_1}^2
 \end{aligned}$$

In (4.16), outcome *a*) illustrates the expected two-period loss if the target is renegotiated in the first voting, while outcome *b*) shows the expected two-period loss if the target is implemented as a result of the first voting. More specifically, the top two rows in *a*) and *b*), respectively, illustrate CM 1's loss after the first voting round, while the two bottom rows show her expected loss after the second voting round. Note that according to (4.16b), if CM 1 decides to vote against

renegotiation in the first round, she expects to vote for renegotiation in the second round.

To start with, we ignore the expected outcomes from the second voting. In other words, we consider CM 1's optimal strategy in the first voting, if she does not care about the problem in the second voting.

**Proposition 6.** If CM 1 does not consider the outcome from the second voting, and if the supply shock  $z_2$  is sufficiently large, voting for renegotiation in the first voting round is the optimal strategy both for CM 1 and for the private sector.

**Proof.** Renegotiating the target in the first round (the top two rows in equation 4.16a) implies higher output than implementing the target. In addition, the deflationary response to country 2's shock will be partially countered which will give the private sector compensation for the losses which are caused by the inflation surprise. Indeed, if  $z_2$  is large, it follows that  $[E(\pi^{CL}) - V^{CL}z_2]^2 < [E(\pi^{TA}) - V^{CL}z_2]^2$ , i.e., renegotiation yields lower deflation than implementation. Moreover, remembering from equation (2.5) that country 2's shock has a negative impact on country 1's output, renegotiation after a large  $z_2$  in fact renders output closer to its target as well. Hence, both CM 1 and the private sector can be better off if the target is renegotiated. *Q.E.D.*

However, the result will change if CM 1 takes the expected outcome from the second voting (the two bottom rows of equations 4.16a,b, respectively) into account when choosing the optimal strategy in the first voting:

**Proposition 7.** If CM 1 considers the expected outcome from the second voting round, she will never vote for renegotiation in the first round.

**Proof.** Voting against renegotiation in the first round requires that the expected loss for the entire period at office that results from renegotiating the target is higher than the expected loss for the entire period at office that results from implementing the target in the first round. From (4.16), it follows that this is true when the following inequality holds:

$$(4.17) \quad \begin{aligned} L_{I+II}^{1,RNG} &> L_{I+II}^{1,TA} \Leftrightarrow \\ L_I^{1,RNG} + E(L_{II}^{1,CL}) &> L_I^{1,TA} + E(L_{II}^{1,RNG}) \end{aligned}$$

Clearly, the RHS of (4.17) is lower than the LHS in so far as  $L^{CL} > L^{TA}$ .<sup>23</sup> But this is the same condition that is required for inflation

<sup>23</sup> Note that the terms from (4.16) that include  $\sigma_{z1}^2$  and  $\sigma_{z2}^2$  cancel out as the CB responds to the shocks optimally, irrespective of whether the target is renegotiated or not.

targeting to be incentive compatible *ex ante*, which we have assumed to hold. *Q.E.D.*

It remains to be shown that CM 1's actual voting behaviour in the *second round* also implies voting against renegotiation. The following proposition illustrates that this is indeed the case.

**Proposition 8.** In the second voting round, CM 1 will not vote for renegotiation even if this would make her and the private sector better off. Hence, before the second voting the private sector expects that the target will not be renegotiated and consequently it rationally forms low inflationary expectations.

**Proof.** When the second (final) voting round arrives, CM 1 wants to generate an inflationary surprise by renegotiating the target. Furthermore, by the time of the voting CM 1's own shock has realised and it could be optimal even for the private sector to renegotiate (see Proposition 6). But since the electoral periods are staggered, every time when there is voting only one of the CMs is at the end of her period at office. Then CM 1 knows that when she is at the end CM 2 is not, and that consequently CM 2 will vote against renegotiation for sure. Because the target can be renegotiated only when both CM's agree, CM 1 does not find it meaningful to vote for renegotiation and the result follows. *Q.E.D.*

Of course, when there are more members in the monetary union, staggered electoral periods guarantee that the Council (the principal) becomes even less vulnerable to some particular CM's incentives to renegotiate.

One can of course argue that staggering the CM's periods at office is a very abstract way to circumvent the problem of renegotiation. As was argued by von Hagen (1995), in real world reciprocal voting incentives between the individual CMs can easily destroy the equilibrium. Nevertheless, the idea of staggered periods is not new, and such institutions can at least partially increase the credibility of monetary policy institutions. For example, the constitution of the U.S. Federal Reserve explicitly states:

*"The board of governors consists of seven governors appointed by Congress on the nomination of the President, each serving for fourteen years, with one reappointment falling due every two years."*

Staggering the board members' periods at office obviously tries to avoid situations where a majority of board members would simultaneously face pressures to give in for short term political incentives.

## 5 Conclusion

In this Essay we analysed the optimal design of monetary institutions and the interaction between monetary and fiscal authorities in a monetary union. In so doing, we assumed that the participating governments first set up a common institution, the Council, and agree that the Council's objectives reflect the social welfare in the monetary union. The Council was then assumed to act as the principal for the Central Bank, so that it delegates the monetary policy to the CB in order to improve the union's social welfare. The setup resembles the Maastricht treaty whereby the EU governments agreed to establish the European Central Bank and formulated its monetary policy objective (price stability). However, the extent to which the true EMU institutions resemble the optimal ones derived here depends on whether one regards the monetary policy objectives of the Maastricht treaty as sufficiently precise. In particular, it is not clear whether the phrase "price stability" is unambiguously interpreted in a way which coincides with an optimal rate of inflation.

To start with, we derived a benchmark case where monetary policy is set directly under the Council's discretion. We then analysed how the Council could delegate the policy to the CB. First, it was illustrated how the CB's independent nature *per se* can lead to aggregation distortions from the Council's point of view. We then showed how the Council could improve upon the benchmark by delegating monetary policy to a sufficiently conservative CB, with the cost of suboptimal responses to output shocks. Finally, we found that if the delegation involves subjecting the CB to an optimal inflation target, the Council is able to increase its expected welfare further. However, because of the CB's aggregation bias, the optimal outcome also requires that the inflation target is supplemented by a nomination of a more modestly conservative CB.

Because of the interplay between the fiscal and monetary policies in the model, optimal delegation increases fiscal discipline compared to the benchmark. This leads us to conclude that a properly designed CB institution *per se* could, at least in a static model, work as an alternative to arrangements such as the proposed EMU "Stability Pact". Finally, we addressed the problem of non-renegotiation proofness which is generally associated with optimal delegation regimes. In the context of inflation targeting, we derived a voting-game equilibrium where implementing the target always becomes an optimal strategy for the Council.

# Appendix 1

## Derivation of the Equilibrium Inflation and Tax Rates under the Council

Consider the reaction functions (2.3). Substituting the Council's reaction function (2.3a) into the two governments' reaction functions (2.3b,c) yields:

(A1.1)

$$\begin{aligned} a)\tau_1 &= \bar{g} + \frac{\phi(A+B)}{1+\phi} \left[ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} - \frac{z_1 + z_2}{2} \right] - A(\pi^e + \bar{y} + \bar{g} - z_1) \\ b)\tau_2 &= \bar{g} + \frac{\phi(C+D)}{1+\phi} \left[ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} - \frac{z_1 + z_2}{2} \right] - C(\pi^e + \bar{y} + \bar{g} - z_2) \end{aligned}$$

Next, calculating  $(\tau_1 + \tau_2)$ , and rearranging, gives:

(A1.2)

$$\begin{aligned} \tau_1 + \tau_2 &= 2\bar{g} + \frac{2(\phi(B+D) - A - C)}{2(1+\phi) - \phi K} \pi^e + \\ &\frac{1}{2(1+\phi) - \phi K} [(\phi K - 2A(1+\phi))(\bar{y} + \bar{g} - z_1) + (\phi K - 2C(1+\phi))(\bar{y} + \bar{g} - z_2)], \end{aligned}$$

$$K \equiv A + B + C + D$$

Substituting (A1.2) into the Council's reaction function (2.3a), and rearranging, gives:

$$\begin{aligned} \pi &= \frac{(2 - A - C)\phi}{2(1+\phi) - \phi K} \pi^e + \\ (A1.3) \quad &\frac{2\phi}{2(1+\phi) - \phi K} \{(1-A)(\bar{y} + \bar{g} - z_1) + (1-C)(\bar{y} + \bar{g} - z_2)\} \end{aligned}$$

Next, impose rational expectations ( $\pi^e = \pi$ ) into (A1.3), and rearrange:

$$(A1.4) \quad \pi^e = \frac{\phi}{2 - \phi(B+D)}((1-A) + (1-C))(\bar{y} + \bar{g}) = -\frac{\phi(B+D)}{2 - \phi(B+D)}(\bar{y} + \bar{g})$$

Combining (A1.4), the expected inflation rate under the Council, and the terms of  $z_1$  and  $z_2$  from (A1.3), gives equation (2.4a) in the text. Next, substitute (A1.2) into (A1.1a,b), and rearrange, to get:

$$(A1.5) \quad \begin{aligned} \tau_1 = & \bar{g} + \frac{\phi(A+B) + \phi A(C+D) - 2A(1+\phi)}{2(1+\phi) - \phi K}(\bar{y} + \bar{g} - z_1) \\ & + \frac{\phi(A+B)(1-C)}{2(1+\phi) - \phi K}(\bar{y} + \bar{g} - z_2) + \frac{\phi B(2-C) - A(2-\phi D)}{2(1+\phi) - \phi K}\pi^e \end{aligned}$$

$$\begin{aligned} \tau_2 = & \bar{g} + \frac{\phi(C+D) + \phi C(A+B) - 2C(1+\phi)}{2(1+\phi) - \phi K}(\bar{y} + \bar{g} - z_2) \\ & + \frac{\phi(C+D)(1-A)}{2(1+\phi) - \phi K}(\bar{y} + \bar{g} - z_1) + \frac{\phi D(2-A) - C(2-\phi B)}{2(1+\phi) - \phi K}\pi^e \end{aligned}$$

Finally, substituting in the expected inflation, equation (A1.4), into (A1.5), and rearranging, gives equations (2.4 b, c) in the text.



## Appendix 2

### Comparative Statics for Equations (2.4) and (2.5)

**Equation (2.4):**

$$\frac{\partial E(\pi^{CL})}{\partial \phi} = -\frac{2(B+D)}{(2-\phi(B+D))^2} > 0$$

$$\frac{\partial U^{CL}}{\partial \phi} = \frac{2B}{(2(1+\phi)-\phi(A+B+C+D))^2} > 0$$

$$\frac{\partial V^{CL}}{\partial \phi} = \frac{2D}{(2(1+\phi)-\phi(A+B+C+D))^2} > 0$$

**Equation (2.5):**

$$\frac{\partial E(y_1^{CL})}{\partial \phi} = \frac{2B(B+D)}{(2-\phi(B+D))^2} > 0 \quad \frac{\partial E(g_1^{CL})}{\partial \phi} = -\frac{2A(B+D)}{(2-\phi(B+D))^2} > 0$$

$$\frac{\partial VAR(y_1^{CL}|z_1)}{\partial \phi} = \frac{2B(1-A-B)}{(2(1+\phi)-\phi(A+B+C+D))^2} < 0$$

$$\frac{\partial VAR(y_1^{CL}|z_2)}{\partial \phi} = -\frac{4BD}{(2(1+\phi)-\phi(A+B+C+D))^2} < 0$$

$$\frac{\partial VAR(g_1^{CL}|z_1)}{\partial \phi} = \frac{2A(A+B-1)}{(2(1+\phi)-\phi(A+B+C+D))^2} < 0$$

$$\frac{\partial VAR(g_1^{CL}|z_2)}{\partial \phi} = \frac{4AD}{(2(1+\phi)-\phi(A+B+C+D))^2} < 0$$

## Appendix 3

### Derivation of the Equilibrium Inflation and Tax Rates under the CB

Substituting the CB's reaction function (3.2) into the two governments' reaction functions (2.3b,c) yields:

$$(A3.1) \quad \begin{aligned} a) \tau_1 &= \bar{g} + \frac{2\gamma(A+B)}{1+2\gamma} \left[ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} - \frac{z_1 + z_2}{2} \right] - A(\pi^e + \bar{y} + \bar{g} - z_1) \\ b) \tau_2 &= \bar{g} + \frac{2\gamma(C+D)}{1+2\gamma} \left[ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} - \frac{z_1 + z_2}{2} \right] - C(\pi^e + \bar{y} + \bar{g} - z_2) \end{aligned}$$

Next, calculating  $(\tau_1 + \tau_2)$ , and rearranging, gives:

$$(A3.2) \quad \begin{aligned} \tau_1 + \tau_2 &= 2\bar{g} + \frac{2\gamma(B+D) - A - C}{1+2\gamma - \gamma K} \pi^e + \\ &\frac{1}{1+2\gamma - \gamma K} [(\gamma K - A(1+2\gamma))(\bar{y} + \bar{g} - z_1) + (\gamma K - C(1+2\gamma))(\bar{y} + \bar{g} - z_2)], \end{aligned}$$

$$K \equiv A + B + C + D$$

Substituting (A3.2) into the CB's reaction function (3.2), and rearranging, gives:

$$(A3.3) \quad \begin{aligned} \pi &= \frac{(2 - A - C)\gamma}{1+2\gamma - \gamma K} \pi^e + \\ &\frac{2\gamma}{1+2\gamma - \gamma K} \{(1-A)(\bar{y} + \bar{g} - z_1) + (1-C)(\bar{y} + \bar{g} - z_2)\} \end{aligned}$$

Next, impose rational expectations ( $\pi^e = \pi$ ) into (A3.3), and rearrange:

$$(A3.4) \quad \pi^e = \frac{\gamma}{1-\gamma(B+D)}((1-A)+(1-C))(\bar{y}+\bar{g}) = -\frac{\gamma(B+D)}{1-\gamma(B+D)}(\bar{y}+\bar{g})$$

Combining (A3.4), the expected inflation rate under the CB, and the terms of  $z_1$  and  $z_2$  from (A3.3), gives equation (3.3a) in the text. Next, substitute (A3.2) into (A3.1a,b), and rearrange, to get:

$$(A3.5) \quad \begin{aligned} \tau_1 = & \bar{g} - \frac{\gamma(A+B) + \gamma A(C+D) - A(1+2\gamma)}{1+2\gamma - \gamma K} (\bar{y} + \bar{g} - z_1) \\ & + \frac{\gamma(A+B)(1-C)}{1+2\gamma - \gamma K} (\bar{y} + \bar{g} - z_2) + \frac{\gamma B(2-C) - A(1-\gamma D)}{1+2\gamma - \gamma K} \pi^e \end{aligned}$$

$$\begin{aligned} \tau_2 = & \bar{g} - \frac{\gamma(C+D) + \gamma C(A+B) - C(1+2\gamma)}{1+2\gamma - \gamma K} (\bar{y} + \bar{g} - z_2) \\ & + \frac{\gamma(C+D)(1-A)}{1+2\gamma - \gamma K} (\bar{y} + \bar{g} - z_1) + \frac{\gamma D(2-A) - C(1-\gamma B)}{1+2\gamma - \gamma K} \pi^e \end{aligned}$$

Finally, substituting in the expected inflation, equation (A3.4), into (A3.5), and rearranging, gives equations (3.3 b,c) in the text.

# Appendix 4

## Derivation of the Equilibrium Inflation and Tax Rates under Inflation Targeting

Substituting (4.3), the CB's reaction function under inflation targeting, into the two governments' reaction functions (2.3b,c), and ignoring  $z_1$  and  $z_2$ , yields:

$$(A4.1) \quad \begin{aligned} a) \tau_1 &= \bar{g} + \frac{2\gamma(A+B)}{1+2\gamma} \left[ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} \right] - A(\pi^e + \bar{y} + \bar{g}) + \frac{A+B}{1+2\gamma} \bar{\pi} \\ b) \tau_2 &= \bar{g} + \frac{2\gamma(C+D)}{1+2\gamma} \left[ \pi^e + \frac{\tau_1 + \tau_2}{2} + \bar{y} \right] - C(\pi^e + \bar{y} + \bar{g}) + \frac{C+D}{1+2\gamma} \bar{\pi} \end{aligned}$$

Next, calculating  $(\tau_1 + \tau_2)$ , and rearranging, gives:

$$(A4.2) \quad \begin{aligned} \tau_1 + \tau_2 &= 2\bar{g} + \frac{2\gamma(B+D) - A - C}{1+2\gamma - \gamma K} \pi^e + \frac{K}{1+2\gamma - \gamma K} \bar{\pi} \\ &+ \frac{1}{1+2\gamma - \gamma K} [(\gamma K - A(1+2\gamma))(\bar{y} + \bar{g}) + (\gamma K - C(1+2\gamma))(\bar{y} + \bar{g})], \end{aligned}$$

$$K \equiv A + B + C + D$$

Substituting (A4.2) into the CB's reaction function under inflation targeting, and rearranging, gives:

$$(A4.3) \quad \begin{aligned} \pi &= \frac{(2-A-C)\gamma}{1+2\gamma - \gamma K} \pi^e + \frac{1}{1+2\gamma - \gamma K} \bar{\pi} \\ &+ \frac{2\gamma}{1+2\gamma - \gamma K} \{(1-A)(\bar{y} + \bar{g}) + (1-C)(\bar{y} + \bar{g})\} \end{aligned}$$

Next, impose rational expectations ( $\pi^e = \pi$ ) into (A4.3), and rearrange:

$$(A4.4) \quad \begin{aligned} \pi^e &= \frac{\gamma}{1-\gamma(B+D)}((1-A)+(1-C))(\bar{y}+\bar{g}) + \frac{1}{1-\gamma(B+D)}\bar{\pi} \\ &= -\frac{\gamma(B+D)}{1-\gamma(B+D)}(\bar{y}+\bar{g}) + \frac{1}{1-\gamma(B+D)}\bar{\pi} \end{aligned}$$

This is equation (4.4a) in the text. Next, substitute (A4.2) into (A4.1a,b), and rearrange, to get:

$$(A4.5) \quad \begin{aligned} \tau_1 &= \bar{g} - \frac{\gamma(A+B) + \gamma A(C+D) - A(1+2\gamma)}{1+2\gamma - \gamma K}(\bar{y} + \bar{g}) + \frac{A+B}{(1+2\gamma) - \gamma K}\bar{\pi} \\ &+ \frac{\gamma(A+B)(1-C)}{1+2\gamma - \gamma K}(\bar{y} + \bar{g}) + \frac{\gamma B(2-C) - A(1-\gamma D)}{1+2\gamma - \gamma K}\pi^e \end{aligned}$$

$$\begin{aligned} \tau_2 &= \bar{g} - \frac{\gamma(C+D) + \gamma C(A+B) - C(1+2\gamma)}{1+2\gamma - \gamma K}(\bar{y} + \bar{g}) + \frac{C+D}{(1+2\gamma) - \gamma K}\bar{\pi} \\ &+ \frac{\gamma(C+D)(1-A)}{1+2\gamma - \gamma K}(\bar{y} + \bar{g}) + \frac{\gamma D(2-A) - C(1-\gamma B)}{1+2\gamma - \gamma K}\pi^e \end{aligned}$$

Finally, substituting the expected inflation, equation (A4.4), into (A4.5), and rearranging, gives equations (4.4 b, c) in the text.

# Optimal Fiscal and Monetary Policy under Debt Reduction Programmes

## 1 Introduction

The Maastricht criteria for economic convergence specify the conditions for debt, deficits and inflation that a country must reach before it can qualify as a member of the Economic and Monetary Union (henceforth: EMU), scheduled to start in 1999. In October 1996, the Italian centre-left government, following the examples of the conservative governments in France and Spain, decided to introduce a tough fiscal programme in order to qualify among the first group of countries. These programmes involved not only promises of general fiscal austerity, but also plans of selling public sector enterprises and introduction of specific "Maastricht taxes", to quickly bring the public finances into order. Understandably, proposing such short-sighted fiscal policies raised protests on one hand among those countries who have managed to put their houses in order by conventional means, and on the other hand among those countries who are most concerned about the credibility of the long-run low inflation policy in EMU.

This Essay studies the analytical properties of debt reduction programmes. In so doing, we leave the static policy game framework and turn to study the dynamic interaction between fiscal and monetary authorities. More specifically, we evaluate the time paths of money, deficits and debt under different institutional arrangements between the policymakers. We also analyse how the length of the adjustment period affects the time paths and suggest institutional reforms that help the policymakers to reach a debt target without taking extreme measures. While the topical empirical connection is the EMU, the chapter develops a model that can have very general applications in designing economic and political institutions for countries where debt levels must be cut by some fixed terminal dates.

When operating under a dynamic budget constraint, fiscal and monetary authorities typically face a conflict about whether fiscal or monetary instruments should be adjusted to stabilise government debt. In a dynamic framework, strategic interaction between fiscal and monetary authorities implied by the government budget constraint was first formalised by Tabellini (1986), and extended by van Aarle & al. (1995b).<sup>1</sup> Using a differential game analysis, these models incorporate the time-inconsistency problem now familiar to macroeconomists from the models of Kydland and Prescott (1977) and Barro and Gordon (1983).<sup>2</sup>

In our model, the main modification to Tabellini's analysis is that a finite time horizon is assumed. Furthermore, the time paths for the different variables are explicitly solved. Although the finite horizon will change the way of solving the model and complicate the analysis, it will increase our understanding about the behaviour of the policymakers in a dynamic interaction framework. In particular, we consider two institutional set-ups between a fiscal authority (the government) and a monetary authority (the Central Bank, CB). A cooperative equilibrium refers to a hypothetical Pareto-optimal case. This equilibrium is derived both analytically and using the phase diagram approach. We then move to the non-cooperative case, where the two policy objectives are carried out separately by independent authorities, the government and the CB. We apply an open-loop information structure and, to enable a closed-form solution to the game with a fixed endpoint structure, we suggest that the terminal condition for debt binds only the government. As an extension to the standard case, we analyse a situation where the government's prospects for re-election are positively correlated with its success in the management of public finances. The final part of the chapter contains numerical simulations, which illustrate how different institutional reforms affect the equilibrium time paths.<sup>3</sup>

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<sup>1</sup> In a static context, Alesina and Tabellini (1987) included the private sector as a third player in the fiscal-monetary policy game.

<sup>2</sup> However, in the literature following Tabellini (1986), the time-inconsistency arises independent of private sector expectations, solely from the policymakers' inability to pre-commit themselves to time paths different from those which are implied by their own preferences and the initial- and terminal conditions of the game.

<sup>3</sup> A natural connection to the two previous Essays would have been provided by an analysis of the feasibility of time-varying inflation targets. However, we have chosen to leave this extension to future research. See Jensen (1994) for an analysis of time-varying credibility problems.

The rest of this Essay is organised as follows. Section 2 sets up the model. In section 3, we solve the model in the case where the government and the CB cooperate, and section 4 derives the non-cooperative outcome. In section 5, we introduce electoral incentives. The numerical simulations are performed section 6. Section 7 concludes.

## 2 The Setup of the Differential Game

Public debt has to be serviced by either base money creation or reduction of fiscal deficits. Following Tabellini (1986) and van Aarle et al. (1995b), we assume that decisions on primary fiscal deficits are taken by the government, while management of monetary policy is the responsibility of the CB. Tabellini uses the following dynamic government budget constraint which renders these two policies interdependent:

$$(2.1) \quad d' = rd(t) + u(t) - v(t) + k$$

In (2.1),  $d'$  denotes government debt accumulation,  $u \equiv f(t) - \bar{f}$ , where  $f(t)$  is fiscal deficit net of interest payments and  $\bar{f}$  is the government's target level of deficit, and  $v \equiv m(t) - \bar{m}$ , where  $m(t)$  is change in the monetary base and  $\bar{m}$  is the CB's target level of money growth. Furthermore,  $rd$  is interest payments on existing government debt, and  $r$  is defined as the (constant) difference between interest rate and growth rate of output. If the fiscal deficit,  $f(t) + rd(t)$ , exceeds seignorage from base-money creation, government debt accumulation allows policymakers to shift the debt adjustment to the future. The term  $k$ ,  $k \equiv \bar{f} - \bar{m}$  measures the tension between the desired deficit financing and desired monetary accommodation, and a large  $k$  thus illustrates an intensified conflict between the target values for the various objectives.

The strategic interaction between the two authorities is formalised by means of a differential game. The fiscal authority, or government, features the following intertemporal loss function:



$$(2.2) \quad L^G = \int_0^T \left[ \frac{1}{2} u^2(t) + \delta d(t) \right] e^{-\alpha t} dt$$

The government faces a trade-off between deficit *stabilisation* and debt *reduction*, as debt enters linearly into (2.2). The parameter  $\delta$  measures the relative weight assigned on the debt objective. Government discounts future losses at the rate  $\alpha$ , and it manages fiscal deficits to minimise the loss, subject to the government budget constraint (2.1), the initial stock of debt and given transversality conditions.

On the other hand, the monetary authority, or the CB, chooses the growth of base money, and hence the inflation rate, so as to minimise the following loss function:

$$(2.3) \quad L^{CB} = \int_0^T \left[ \frac{1}{2} v^2(t) + \mu d(t) \right] e^{-\beta t} dt$$

In (2.3),  $\mu$  is the weight the CB puts on the debt objective relative to inflation stabilisation, so that a low  $\mu$  reflects a “conservative” CB who cares more about monetary stability than low levels of debt.<sup>4</sup> The CB discounts future losses at rate  $\beta$ . We argue that central bankers’ generally longer periods at office can be reflected by the fact that  $\beta < \alpha$ , i.e., the CB is more far-sighted than the government. As will be seen below, the parameters  $\delta$  and  $\mu$  on one hand and the discount rates  $\beta$  and  $\alpha$  on the other hand are important determinants of how the adjustment burden is distributed between the two authorities. In particular, policy conflicts arise if fiscal and monetary policies are controlled by different institutions that assign different weights to various objectives.

In the following we derive the equilibrium time paths for the control variables money supply and budget deficits, as well as for the state variable debt, under two different institutional set-ups. We start with the case where the fiscal and monetary authorities coordinate their actions, and turn then to the case where the CB chooses its policy independent from the government.

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<sup>4</sup> Note that the simplest possible formulations of the authorities’ loss functions as featured in (2.2) and (2.3) exclude one authority’s policy instrument from the other authority’s objective function. See van Aarle & al. (1995b) for a model with more general objective functions.

### 3 Equilibrium Policy under Cooperation

In policy games, coordination of actions means that a player can internalise the positive externalities from the other player's actions. The cooperative equilibrium is thus Pareto-efficient and can serve as a benchmark to determine the inefficiencies associated with lack of cooperation. In the case of cooperation, we introduce a parameter  $\omega$ , which represents the relative weight attached to the objectives of the CB in the coalition's loss function. More specifically, the cooperative equilibrium is found by minimising the following current-value Hamiltonian:

$$(3.1) \quad H^c = \frac{1}{2} \{u^2(t) + \omega v^2(t)\} + (\delta + \mu\omega)d(t) + \lambda[rd(t) + u(t) - v(t) + k]$$

The optimisation is carried out with respect to both policy instruments  $\{u(t), v(t)\}$  and the following boundary conditions:

$$\begin{aligned} d(0) &= d_0 \\ d(T) &= \bar{d} \end{aligned}$$

The terminal level of debt ( $\bar{d}$ ) is predetermined, and we can think it as the Maastricht debt target. In other words, we are dealing with a fixed endpoint problem since both the terminal time *and* the terminal state are predetermined. In (3.1),  $\lambda$  is the co-state variable associated with the dynamic government budget constraint, and it represents the marginal cost of public funds for the coalition of policymakers. The first-order conditions of our dynamic optimisation problem amount to:

$$\begin{aligned}
& a) \frac{\partial H^c}{\partial u} = u + \lambda = 0 \Rightarrow u = -\lambda \\
(3.2) \quad & b) \frac{\partial H^c}{\partial v} = \omega v - \lambda = 0 \Rightarrow v = \frac{\lambda}{\omega} \\
& c) \frac{\partial H^c}{\partial d} = (\delta + \omega\mu) + r\lambda = -\lambda' + \gamma\lambda \Rightarrow \lambda' + (r - \gamma)\lambda = -(\delta + \omega\mu),
\end{aligned}$$

In (3.2),  $\gamma \equiv \alpha + \omega\beta$  measures the coalition's time preferences. Furthermore, in (3.2) and in what follows, we omit the time symbol associated with the variables  $d$ ,  $u$ ,  $v$  and  $\lambda$ . Substituting for  $u$  in the equation of  $d'$ , after setting  $v = -(u/\omega)$ , allows us to set up the following system of differential equations:<sup>5</sup>

$$(3.3) \quad \begin{bmatrix} d' \\ \lambda' \end{bmatrix} = \begin{bmatrix} r & -\frac{1+\omega}{\omega} \\ 0 & \gamma - r \end{bmatrix} \begin{bmatrix} d \\ \lambda \end{bmatrix} + \begin{bmatrix} k \\ -(\delta + \omega\mu) \end{bmatrix}$$

Using the method described in Appendix 1, the solution to the system (3.3) can be obtained as:

$$\begin{aligned}
(3.4) \quad & \begin{bmatrix} d \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{1+\omega}{\omega(2r-\gamma)} \\ 1 \end{bmatrix} K_1 e^{(\gamma-r)t} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} K_2 e^{rt} + \begin{bmatrix} \tilde{d} \\ \tilde{\lambda} \end{bmatrix} \\
& \tilde{d} = -\left[ \frac{(1+\omega)(\delta+\omega\mu)}{\omega r(r-\gamma)} + \frac{1}{r} k \right], \tilde{\lambda} \equiv -\frac{\delta+\omega\mu}{r-\gamma}
\end{aligned}$$

Furthermore, in Appendix 2, we analyse the behaviour of the system (3.3) graphically with a phase diagram. The last matrix in the RHS of (3.4) illustrates the steady-state values of the state and the costate variables, respectively. Remembering from the first-order conditions

<sup>5</sup> Because the game is linear-quadratic and symmetric, we can express  $v$  as a function of  $u$ . The cooperative case thus amounts to an ordinary optimal control problem where the coalition simultaneously controls both policy instruments, assigning a weight  $\omega$  on the monetary objective.

that  $\lambda \equiv -u \equiv v\omega$ , it also follows that  $\tilde{\lambda} \equiv -\tilde{u} \equiv \tilde{v}\omega$ . In other words, we can obtain the steady-state values for deficit and money growth as functions of  $\tilde{\lambda}$ . Furthermore, and following van Aarle & al (1995b), the steady-state values can be decomposed in two parts, which illustrate the distribution of the *intratemporal* and the *intertemporal* adjustment burdens between the policymakers.

(i) The terms  $\frac{(1+\omega)(\delta+\omega\mu)}{\omega}$  (for  $d$ ) and  $-(\delta + \omega\mu)$  (for  $\lambda$ ) illustrate the immediate (intratemporal) adjustment effort. In particular, the immediate debt adjustment is the higher the more weight the coalition of policymakers puts on the debt reduction objective relative to the other objectives, i.e., the higher are  $\delta$  and  $\mu$ . The intratemporal adjustment term for  $\lambda$  shows that the higher are the weights ( $\delta, \mu$ ) assigned by the coalition on the debt objective, the lower is the intratemporal cost in terms of the controls deviating from their target levels. For example, remembering that  $\tilde{\lambda} \equiv -\tilde{u} \equiv \omega\tilde{v}$ , it follows that a higher  $\mu$ , which implies a less "conservative" CB who is more concerned about reducing debt than holding money supply low, means that deficits can be set higher. This is because more seignorage is being generated to monetise debt. Similarly, the higher is  $\delta$ , the weight assigned by the government on the debt objective, the lower is the required money growth as a more substantial share of the adjustment is achieved by the government's efforts. In other words, under cooperation, there is a positive externality to the government from a high  $\mu$ , and vice versa there is a positive externality to the CB from a high  $\delta$ .

(ii) The terms  $\frac{1}{r(r-\gamma)}$  (for  $d$ ) and  $\frac{1}{r-\gamma}$  (for  $\lambda$ ) illustrate the postponed (intertemporal) adjustment effort. In particular, future debt is the higher the higher is the interest rate  $r$  to be paid on debt. Therefore, a high  $r$  implies lower deficit and higher money growth, i.e., more fiscal and monetary intertemporal adjustment as a response to the increased debt accumulation. However, with myopic policymakers (a high  $\gamma$ ), money growth is low and deficit is high. This illustrates the fact that myopic policymakers prefer to shift the adjustment to the future.

Returning back to equation (3.4), the remaining task is to determine the arbitrary constants  $K_1$  and  $K_2$ , which are needed to determine the exact solutions for  $u^*(t)$ ,  $v^*(t)$  and  $d^*(t)$  under the cooperative equilibrium. Normally, we would resort to the boundary conditions for the costate variable  $\lambda$  in order to solve out the constants. However, with the fixed terminal point problem, the

boundary conditions are linked to the state variable  $d$  instead of  $\lambda$ . Therefore, applying the boundary conditions for debt, i.e.  $d(0) = d_0$  and  $d(T) = \bar{d}$ , we can solve for  $K_1$  and  $K_2$ :

$$K_1 = \frac{\omega(2r - \gamma)}{(1 + \omega)(1 - e^{-(\gamma-2r)T})} [d_0 - \bar{d}e^{-rT} - \tilde{d}(1 - e^{-rT})] \quad (3.5)$$

$$K_2 = -\frac{1}{1 - e^{-(\gamma-2r)T}} [d_0 e^{(\gamma-r)T} - \bar{d} + \tilde{d}(1 - e^{(\gamma-r)T})] e^{-rT}$$

Substituting these into (3.4), and using the condition  $\lambda \equiv -u \equiv \omega v$ , we obtain the following equilibrium time paths for deficit, money and debt under cooperation:

$$\begin{aligned} a) u^*(t) &= \tilde{u} - \frac{(2r - \gamma)\omega}{(1 - e^{-(\gamma-2r)T})(1 + \omega)} [d_0 - \bar{d}e^{-rT} - \tilde{d}(1 - e^{-rT})] e^{-(r-\gamma)t} \\ b) v^*(t) &= \frac{\tilde{v}}{\omega} + \frac{(2r - \gamma)}{(1 - e^{-(\gamma-2r)T})(1 + \omega)} [d_0 - \bar{d}e^{-rT} - \tilde{d}(1 - e^{-rT})] e^{-(r-\gamma)t} \\ c) d^*(t) &= \tilde{d} + \frac{1}{1 - e^{-(\gamma-2r)T}} [d_0 - \bar{d}e^{-rT} - \tilde{d}(1 - e^{-rT})] e^{(\gamma-r)t} \\ &\quad - \frac{1}{1 - e^{-(\gamma-2r)T}} [d_0 e^{(\gamma-r)T} - \bar{d} + \tilde{d}(1 - e^{(\gamma-r)T})] e^{-r(T-t)} \end{aligned} \quad (3.6)$$

From (3.6 a,b) it can be seen that deficit is the lower and money supply is the higher the higher is the difference between the initial and the terminal levels of debt. Furthermore, a higher value of  $\omega$ , which measures the CB's bargaining power, implies that in equilibrium deficit and money supply are both lower as a higher share of the adjustment burden is shifted to the government. In Appendix 3, we show that these results remain robust even if the model is enriched by stochastic disturbances. Finally, rearranging (3.6c) gives:

$$\begin{aligned}
d^*(t) &= \frac{1}{1 - e^{(\gamma-2r)T}} \left\{ d_0 (e^{(\gamma-r)t} - e^{(\gamma-r)T+r(t-T)}) - \bar{d} (e^{-rT+(\gamma-r)t} - e^{r(t-T)}) \right. \\
(3.7) \quad & \left. + \bar{d} [(1 - e^{(\gamma-2r)T}) - (1 - e^{-rT}) e^{(\gamma-r)t} - (1 - e^{(\gamma-r)T}) e^{r(t-T)}] \right\}
\end{aligned}$$

By setting  $t=0$  and  $t=T$  in turn, it can be verified that  $d^*(0) = d_0$  and  $d^*(T) = \bar{d}$ . We now turn to analyse the equilibrium outcomes under non-cooperative strategies.

## 4 Equilibrium Policy under Non-cooperative Strategies

This section analyses the case where deficits are set by the government and money growth is chosen by the CB, but under the assumption that the two authorities optimise independently from each other. Compared to the cooperative case, which constitutes a Pareto-optimal outcome, the non-cooperative case suffers from inefficiencies. These result from the fact that a player's reaction to the other player's expected behaviour is different when the players are not able to coordinate their actions.

To study the non-cooperative equilibrium analytically, we focus on open-loop Nash equilibria.<sup>6</sup> In the Nash open-loop game, the players simultaneously pre-commit to a strategy for *the entire time path*, taking as given the opponent's entire time path of actions. However, under decentralised policymaking, the fixed-endpoint assumption creates problems when solving the equilibrium. To make the game analytically tractable, we have to assume that the two authorities face asymmetric boundary conditions. In particular, this means that the government alone is responsible for hitting the debt target, whereas the CB optimises under a free terminal state.<sup>7</sup>

<sup>6</sup> Closed-loop equilibria, which generally cannot be solved analytically, would only further intensify the contrast between cooperative and non-cooperative games. See van Aarle & al (1995b) for a discussion.

<sup>7</sup> This assumption is quite good description of the situation in most of the countries before the start of EMU, as they have introduced institutional reforms to increase the independence of their national CBs before the policy is delegated to the European Central Bank.

We concentrate here only on the analytical solution to the deterministic case, because with two separate policy instruments derivation of a phase diagram is complicated and solving a stochastic game is practically impossible. The current-value Hamiltonian of the CB is now given by:

$$(4.1) \quad H^{CB} = \frac{1}{2}v^2 + \mu d + \lambda^{CB}(rd + u - v + k)$$

The monetary authority is not responsible for hitting the debt target, and consequently it optimises with respect to  $v$  and subject to the following boundary conditions:

$$\begin{aligned} d(0) &\text{ given} \\ d(T) &= \text{free.} \end{aligned}$$

Equation (4.1) gives rise to following first-order conditions:

$$(4.2) \quad \begin{aligned} a) \quad \frac{\partial H^{CB}}{\partial v} &= v - \lambda^{CB} = 0 \Rightarrow v = \lambda^{CB} \\ b) \quad \frac{\partial H^{CB}}{\partial d} &= \mu + r\lambda^{CB} = -\lambda'^{CB} + \beta\lambda^{CB} \Rightarrow \lambda'^{CB} + (r - \beta)\lambda^{CB} = -\mu \end{aligned}$$

From (4.2a), it follows that  $v' = \lambda'^{CB}$ . Using this condition in (4.2b), we obtain a first-order differential equation for the CB's control variable:

$$(4.3) \quad \begin{aligned} v' + (r - \beta)v &= -\mu \\ \Rightarrow v(t) &= Ce^{-(r-\beta)t} - \frac{\mu}{r-\beta} \end{aligned}$$

Because the terminal state is free, and remembering that  $v = \lambda^{CB}$ , we can directly apply the appropriate boundary condition  $\lambda^{CB}(T) = 0$  to specify the arbitrary constant  $C$ :

$$\begin{aligned}
v(T) &= Ce^{-(r-\beta)T} - \frac{\mu}{r-\beta} = 0 \\
(4.4) \quad \Rightarrow C &= \frac{\mu}{r-\alpha} e^{(r-\beta)T} \\
\Rightarrow v^*(t) &= \frac{\mu}{r-\beta} (e^{(r-\beta)(T-t)} - 1).
\end{aligned}$$

Note that because  $r > \beta$ , the RHS of the bottom line of (4.4) is positive for all  $t < T$ . Equation (4.4) then yields the intuitive result that when the CB optimises with a free terminal state, the optimal initial level of money growth is the lower the more conservative is the CB (i.e., the lower is  $\mu$ ). In addition, (4.4) implies that money supply declines steadily over the time horizon.

The government's optimisation problem is given by:

$$(4.5) \quad H^G = \frac{1}{2}u^2 + \delta d + \lambda^G(rd + u - v + k)$$

The optimisation is carried out with respect to  $u$  and subject to the following boundary conditions:

$$\begin{aligned}
d(0) &= d_0 \\
d(T) &= \bar{d}
\end{aligned}$$

These boundary conditions illustrate the fact that the government is now alone responsible for hitting the debt target. The first-order conditions are:

$$\begin{aligned}
(4.6) \quad a) \quad \frac{\partial H^G}{\partial u} &= u + \lambda^G = 0 \Rightarrow u = -\lambda^G \\
b) \quad \frac{\partial H^G}{\partial d} &= \delta + r\lambda^G = -\lambda'^G + \alpha\lambda^G \Rightarrow \lambda'^G + (r-\alpha)\lambda^G = -\delta
\end{aligned}$$

Substituting for  $u$  (from 4.6a) and for  $v^*(t)$  (from 4.4) into equation (2.1) for  $d'$  on one hand, and rearranging (4.6b) on the other hand, gives the following dynamic system in the state and costate variables:



$$(4.7) \quad \begin{bmatrix} d \\ \lambda^G \end{bmatrix} = \begin{bmatrix} r & -1 \\ 0 & \alpha - r \end{bmatrix} \begin{bmatrix} d \\ \lambda^G \end{bmatrix} + \begin{bmatrix} -\frac{\mu}{r-\beta}(e^{(r-\beta)(T-t)} - 1) + k \\ -\delta \end{bmatrix}$$

Using the method described in Appendix 1, the general solution to the system (4.7) is given by:

$$(4.8) \quad \begin{bmatrix} d \\ \lambda^G \end{bmatrix} = \begin{bmatrix} \frac{1}{(2r-\alpha)} \\ 1 \end{bmatrix} L_1 e^{(\alpha-r)t} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} L_2 e^{\alpha t} + \begin{bmatrix} \frac{\mu}{r(r-\beta)}(e^{(r-\beta)(T-t)} - 1) + \tilde{d} \\ \tilde{\lambda}^G \end{bmatrix}$$

$$\tilde{d} \equiv -\frac{k}{r} - \frac{\delta}{r(r-\alpha)}, \quad \tilde{\lambda}^G \equiv -\frac{\delta}{r-\alpha}$$

We again evaluate the intratemporal and the intertemporal adjustment effects in turn.

(i) The last matrix in the RHS of (4.8) shows that the intratemporal component of the steady-state value of debt is the lower the more weight the government puts on the debt reduction objective, i.e., the higher is  $\delta$ . Clearly, this suggests that more immediate adjustment takes place when  $\delta$  is high. However, in contrary to the cooperative case, a *low* weight assigned by the CB on the debt reduction objective (i.e., a more "conservative" CB) now induces more immediate adjustment from the government's side. This happens because under non-cooperative Nash strategies, a conservative CB who is concerned with the money growth objective is free to choose a more restrictive time path for money growth, and the government must therefore carry a higher burden of the adjustment. In addition, in the non-cooperative case the government gets no positive externalities from the monetary adjustment via the  $\tilde{\lambda}$  term. A conservative CB therefore makes the government's short-run fiscal policy more disciplined, and acts as a partial substitute for cooperation in fastening the debt adjustment.

(ii) Turning to the intertemporal components, it turns out that the higher is the interest rate  $r$ , the higher is future debt and consequently the lower is deficit. On the other hand, it can be seen that the higher is  $\alpha$ , i.e., the more myopic is the government, the higher is the steady state level of deficit as the government favours to shift the adjustment to the future. Hence, a situation where  $\delta$  and  $\mu$  are both small and  $\alpha$  is large, is particularly unpleasant from the government's point of view: it must carry the adjustment burden more or less alone, while it is actually not willing to adjust, at least not in short run.

In order to determine the arbitrary constants  $L_1$  and  $L_2$ , which are needed to define the complete solutions for  $u^*(t)$  and  $d^*(t)$  under the non-cooperative equilibrium, we again apply the boundary conditions  $d(0) = d_0$  and  $d(T) = \bar{d}$ :

$$L_1 = \frac{2r - \alpha}{1 - e^{(\alpha - 2r)T}} \left[ d_0 - \bar{d}e^{-rT} + \frac{\mu}{r(r - \beta)} (1 - e^{(r - \beta)T}) - \tilde{d}(1 - e^{-rT}) \right] \quad (4.9)$$

$$L_2 = -\frac{e^{-rT}}{1 - e^{(\alpha - 2r)T}} \left[ d_0 e^{(\alpha - r)T} - \bar{d} + \frac{\mu}{r(r - \beta)} (1 - e^{(r - \beta)T}) e^{(\alpha - r)T} + \tilde{d}(1 - e^{(\alpha - r)T}) \right]$$

Substituting these into (4.8), and using the condition  $\lambda^G \equiv -u$ , we obtain the following equilibrium time paths for deficit, money and debt under the open-loop Nash game:

$$a) u^*(t) = \tilde{u} - \frac{(2r - \alpha)e^{(\alpha - r)T}}{(1 - e^{(\alpha - 2r)T})} \left[ d_0 - \bar{d}e^{-rT} - \tilde{d}(1 - e^{-rT}) - \frac{\mu}{r(r - \beta)} (e^{(r - \beta)T} - 1) \right]$$

$$b) v^*(t) = \frac{\mu}{(r - \beta)} (e^{(r - \beta)(T - t)} - 1)$$

(4.10)

$$\begin{aligned} c) d^*(t) = & \tilde{d} + \frac{1}{1 - e^{(\alpha - 2r)T}} \left[ d_0 - \bar{d}e^{-rT} - \tilde{d}(1 - e^{-rT}) - \frac{\mu}{r(r - \beta)} (e^{(r - \beta)T} - 1) \right] e^{(\alpha - r)t} \\ & + \frac{1}{1 - e^{(\alpha - 2r)T}} \left[ \bar{d} - d_0 e^{(\alpha - r)T} - \tilde{d}(1 - e^{(\alpha - r)T}) + \frac{\mu}{r - \beta} (e^{(r - \beta)T} - 1) e^{(\alpha - r)T} \right] e^{-r(T - t)} \\ & + \frac{\mu}{r(r - \beta)} (e^{(r - \beta)(T - t)} - 1) \end{aligned}$$

From (4.10a), it turns out that a more conservative CB indeed increases fiscal discipline, as deficit is increasing in  $\mu$ . Moreover, comparing the time paths for debt under cooperation and under open-loop strategies reveals that in the case where  $\mu \rightarrow 0$ , i.e., when the CB becomes extremely conservative, the cooperative and the non-cooperative cases become identical except for the weight parameter  $\omega$  in the former case. The additional terms of  $\mu$  in the non-cooperative case illustrate the general inefficiency problem due to incomplete

policy coordination. Finally, rearranging (4.10c) yields the following expression:

$$\begin{aligned}
 d^*(t) = & \tilde{d} + \frac{1}{1 - e^{(\alpha-2r)T}} \left\{ d_0 (e^{(\alpha-r)t} - e^{(\alpha-r)T+r(t-T)}) \right. \\
 & - \bar{d} (e^{(\alpha-r)t-rT} - e^{r(t-T)}) + \tilde{d} (e^{(\alpha-r)t-rT} - e^{(\alpha-r)t} - e^{r(t-T)} + e^{r(t-T)+(\alpha-r)T}) \\
 (4.11) \quad & + \frac{\mu}{r(r-\beta)} (e^{(\alpha-r)t} - e^{(r-\beta)T+(\alpha-r)t} - e^{r(t-T)+(\alpha-r)T} \\
 & \left. + e^{(r-\beta)T+r(t-T)+(\alpha-r)T} + (e^{(r-\beta)(T-t)} - 1)(1 - e^{(\alpha-2r)T}) \right\}
 \end{aligned}$$

By setting  $t = 0$  and  $t = T$  in turn, the reader can verify that  $d^*(0) = d_0$  and  $d^*(T) = \bar{d}$ . We now turn to analyse the model under the assumption that the government is concerned about its possibility of getting re-elected.

## 5 Optimal Policy under Electoral Incentives

How do the optimal time paths for money, deficit and debt change if the government faces an election at the end of the debt reduction period? Intuitively, it is obvious that electoral incentives, i.e., the government's desire to affect the outcome of the coming elections by appropriately choosing its policy actions before the voting takes place, should alter the equilibrium outcomes. In this section we study this issue by making an assumption that successful management of public finances increases the government's possibility of re-election. The intuition is as follows: the electorate penalises the government by ousting it from the office if at the time of the elections debt is too high and/or deficit is too much off the target. Elections are held at time  $t_l$ , and without loss of generality we assume that a government can be re-elected only once. By choosing its actions appropriately between the beginning of the game and time  $t_l$ , the government can increase its probability of being at office after  $t_l$ . On the other hand, the CB is not affected by electoral incentives, and it *minimises a loss function* exactly similar than (2.3).

We argue that the effects of the electoral incentives can be captured by the following formulation in the period  $0 < t < t_1$ :

$$a) \max_u V^G = - \int_0^{t_1} \left[ \frac{1}{2} u^2(t) + \delta d(t) \right] e^{-(\alpha - \Delta)t} dt$$

(5.1)

$$b) \min_v L^{CB} = \int_0^{t_1} \left[ \frac{1}{2} v^2(t) + \mu d(t) \right] e^{-\beta t} dt$$

The government now maximises a vote function (5.1a), which is assumed to coincide with the median voter's utility function. The solution to the optimisation problem associated with (5.1) proceeds in a similar way than in the case with no electoral incentives. However, in (5.1a), we have introduced the parameter  $\Delta$ ,  $\Delta > \alpha$ , to illustrate the fact that the government's time preferences change due to its incentives to affect the voters' behaviour, and therefore the result of the coming elections. In particular, because the sign of the exponent is now positive, the government applies "reversed" discounting. This means that the dates before the election become more important to a government who cares about its probability of getting re-elected.

Because it is hard to derive comparative statics results on the equilibrium time paths with respect to the policymakers' discount rates, we turn to numerical simulations to illustrate the time paths under different institutional set-ups, including electoral incentives.

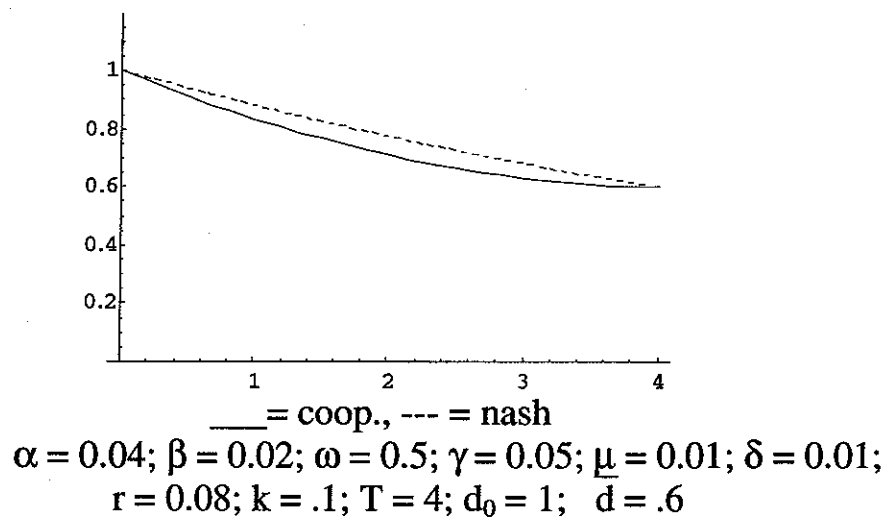
## 6 Simulation Results

In the previous sections, we briefly evaluated the equilibrium solutions for the time paths of money, deficits and debt. In this section, we use the methods of numerical simulation to study the particular shapes of the time paths under different institutional circumstances. These results will then allow us to derive some policy recommendations.

## 6.1 The Standard Case

We start with a “benchmark” case where the growth rate is moderate (so that the net interest rate  $r$  is high) and the policymakers are relatively far-sighted. We also assume that the policymakers assign relatively high weights on their own objectives, low money growth and high deficits. The outcome is shown in Figure 3.1, where we have time on the horizontal axis and debt on the vertical axis. The cooperative solution is illustrated by the solid line and the non-cooperative solution by the dashed line. In the spirit of the recent Maastricht game, we assume that debt must be reduced from an initial level of 100% per GDP to a target level of 60%. The policymakers have four years time to adjust. In this case the time paths show a steady, nearly linear reduction in debt over time. During the adjustment, debt is slightly lower under the cooperative strategies, meaning that the authorities adjust later in the non-cooperative case.

Figure 3.1 Debt Adjustment in the Benchmark Case

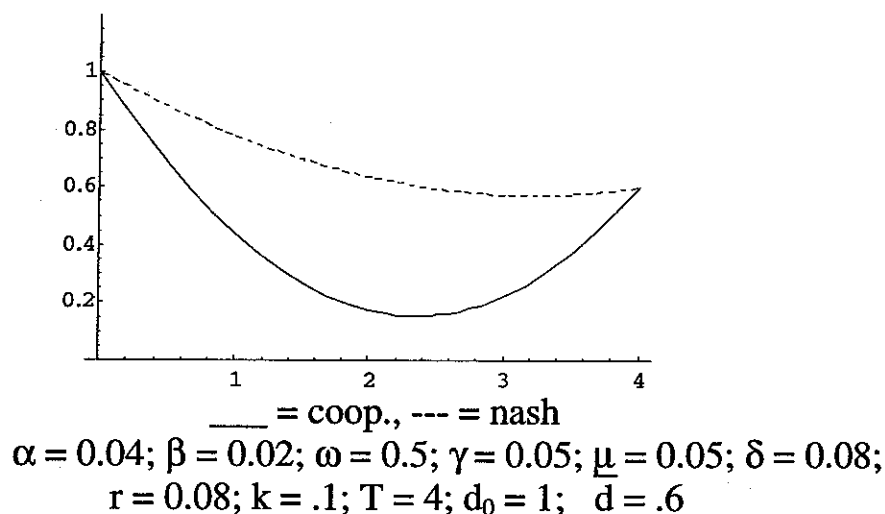


We now turn to study how different modifications in the institutional framework affect the benchmark outcome.

## 6.2 Debt Adjustment and the Policymakers' Preferences

In sections 3 and 4 it was shown that delegating monetary policy to a more conservative CB has different implications to the government's behaviour under the two regimes. In other words, the optimal choice of CB preferences is not invariant to the institutional regime at place. If the policymakers assign high weights on debt ( $\delta$ ,  $\mu$  are high), the time path under cooperation shows a "dive", undershooting the target level of debt as in Figure 3.2. This illustrates the fact that the policymakers prefer more immediate adjustment, as was shown in equation (3.4). The time path under non-cooperative strategies is less convex, because a high weight assigned by the CB on the debt objective imposes less fiscal discipline and therefore a more moderate adjustment over time. This suggests that making the CB independent should be associated with a nomination of a conservative central bank head, whereas under a cooperative regime a less conservative central banker would provide the government with positive externalities and enable earlier adjustment.

Figure 3.2 Debt Adjustment under High Values of  $\delta$  and  $\mu$



Considering the time path under cooperation, one could clearly ask why the policymakers would not stop after one year's adjustment when debt has first hit the target level. The reason is as follows. At the beginning of the game, the authorities optimally choose the entire time path for their control variables. The shape of the optimal control path depends on the initial and terminal levels of debt, the time available for adjustment and the authorities' preferences. Therefore, the authorities are not able to precommit themselves to paths that differ from the optimal path, even if that path would imply under-shooting. In other words, under given parameter values a pre-commitment to stop adjustment earlier is not a time consistent strategy.<sup>8</sup>

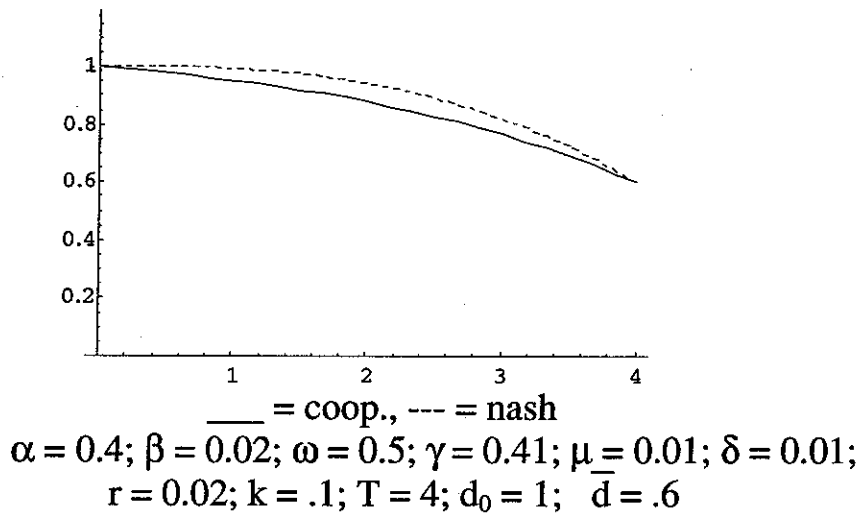
### 6.3 Debt Adjustment under Political Instability

We now turn to analyse the time paths of debt under political instability. With political instability we mean a situation where the governments are short-lived, i.e., the economy is characterised by repeated elections. The effects of short periods at office can be captured by assuming that the government discounts the future heavily. The government then prefers to postpone its adjustment, which means that debt accumulates in the early years and falls rapidly at the end of the period. Figure 3.3 shows how the time paths change compared to Figure 3.1 when the government is short-sighted.

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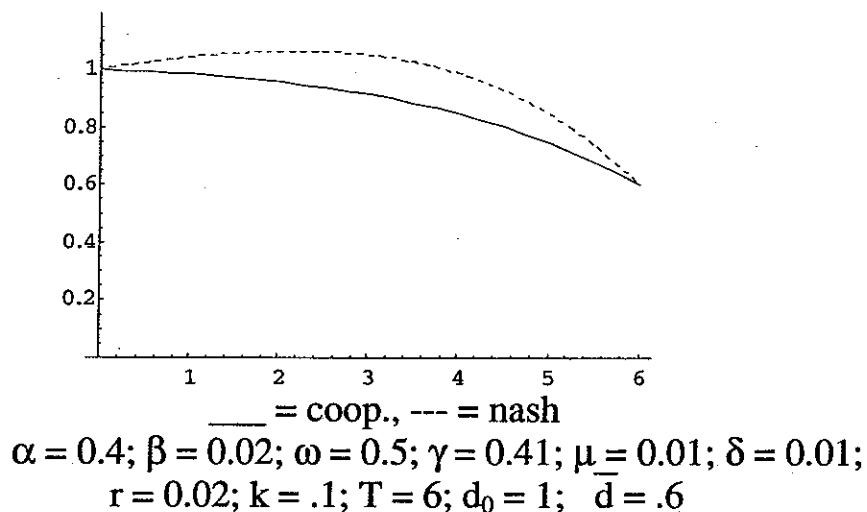
<sup>8</sup> From society's point of view, cooperation is beneficial if the society shares the government's preferences. However, if the government is more myopic than society, or it cares less about the current level of debt than society does (not unrealistic assumptions), then it can happen that an independent CB with suitable preferences improves the social welfare. See Tabellini (1990), who found that international fiscal policy coordination may lead to "perverse" social welfare results in terms of lower fiscal discipline.

Figure 3.3 Debt Adjustment under Political Instability



The outcome in Figure 3.3 could illustrate the situation faced by several European countries before the EMU, as they struggled to bring down high levels of debt. On the other hand, the fundamental political reforms which have been carried out e.g. in Italy could describe a movement from a situation in Figure 3.3 to a situation in Figure 3.1. Finally, in Figure 3.4 we show what happens if a country who faces political instability is granted two extra years adjustment time. If no institutional reform takes place, the situation is worsened because the authorities are unable to commit to a time path which is different than the original path. Giving more adjustment time *per se* is therefore an insufficient and time-inconsistent solution to the debt reduction problem.

Figure 3.4 Debt Adjustment with Extended Time Horizon

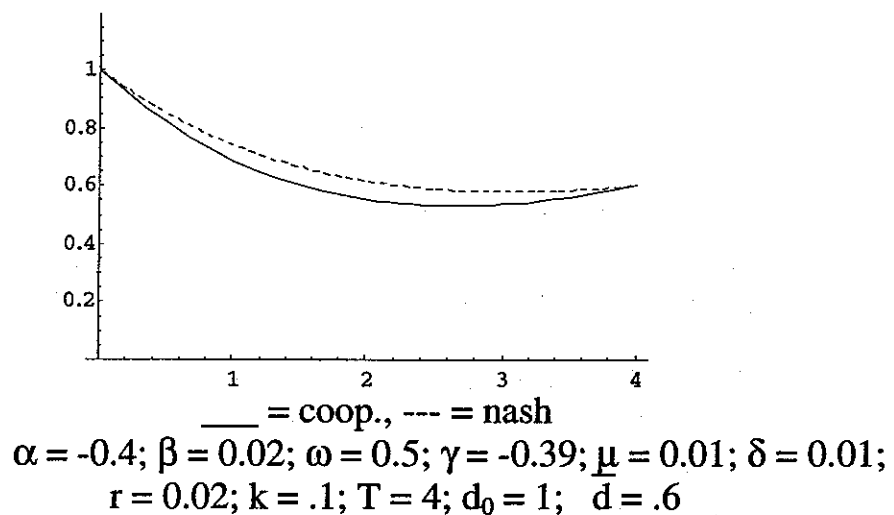




## 6.4 Electoral Incentives and Debt Adjustment

Finally, we study how simple electoral incentives can affect the shape of the time path under political instability (Figure 3.3). In Figure 3.5, everything else is as in Figure 3.3, except that electoral incentives change the sign of the government's discount rate. It turns out that this simple idea changes the outcome dramatically, as the government now prefers to adjust at the early stages of the game. This is because the dates before the election are more valuable now, and the government prefers to shift the bulk of the painful adjustment to the beginning of the period.

Figure 3.5 Debt Adjustment under Electoral Incentives



We have therefore shown that under political instability, institutional reforms which are targeted to the policymakers' time preferences can serve as temporary substitutes to more fundamental reforms. Such reforms would change the time preferences permanently, or alternatively affect the parameters  $\delta$  and  $\underline{\mu}$ .

## 7 Conclusion

In this Essay, we applied a linear-quadratic differential game to study the political economy of debt dynamics. In particular, we focused on the policy coordination problem between fiscal and monetary authorities in a set-up where the two authorities have conflicting private objectives, while they both care about debt. A Pareto-optimal case of fiscal-monetary cooperation was compared to a non-cooperative Nash equilibrium. Under a finite time horizon, we first showed that under cooperation higher weight assigned by the policymakers on the common debt objective results in earlier adjustment. This is because the positive externalities from the other authority's adjustment efforts can be internalised. With decentralised policymaking, a more conservative monetary authority who assigns high weight on her own policy objective increases fiscal discipline and brings around earlier adjustment. A more conservative CB thus works as an imperfect substitute for cooperation.

The simulation results showed that the shape of the time path of debt is greatly affected by the policymakers' preferences. In particular, political instability (myopic policymaking) can lead to a situation where extreme measures are necessary to bring the debt down shortly before the terminal date. This resembles the situation recently faced by many countries with respect to the Maastricht criteria. It can also illustrate the plight of several Latin American nations who faced strict debt targets during the 1980's crisis. In such circumstances, granting extra adjustment time merely worsens the situation unless some institutional reforms are introduced simultaneously. Finally, it was shown that even under political instability the policymakers can, at least temporarily, commit to earlier adjustment if the end of the adjustment period is associated with an election.

# Appendix 1

A general solution to the system of equations (3.3).

We have the following system of equations:

$$(A1.1) \quad \begin{bmatrix} d' \\ \lambda' \end{bmatrix} = \begin{bmatrix} r & -\frac{1+\omega}{\omega} \\ 0 & \gamma-r \end{bmatrix} \begin{bmatrix} d \\ \lambda \end{bmatrix} + \begin{bmatrix} k \\ -(\delta + \omega\mu) \end{bmatrix} \equiv y' = Ay + b$$

Next, define

$$(A1.2) \quad \begin{aligned} z &\equiv Q^{-1}y \\ \Rightarrow z' &\equiv Q^{-1}y' \end{aligned}$$

where  $Q$  is a matrix with the eigenvectors of  $A$  as its columns. Consider the complementary solution to (A1.2) first. Ignoring matrix  $b$ , we can re-write (A1.1) in the following form:

$$(A1.3) \quad \begin{bmatrix} z'_1 \\ z'_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix},$$

where  $X_1, X_2$  are the eigenvalues of  $A$ . Multiplying (A1.3) through by factors  $e^{-X_1 t}$  and  $e^{-X_2 t}$  yields:

$$(A1.4) \quad \begin{aligned} a) e^{-X_1 t} z'_1 - X_1 z_1 e^{-X_1 t} &= 0 \\ b) e^{-X_2 t} z'_2 - X_2 z_2 e^{-X_2 t} &= 0. \end{aligned}$$

Integrating (A1.4a,b) now yields:

$$(A1.5) \quad \begin{aligned} z_1 &= K_1 e^{X_1 t} \\ z_2 &= K_2 e^{X_2 t} \end{aligned}$$

Using the expressions for  $z_1$  and  $z_2$ , and remembering that  $z \equiv Q^{-1}y$ , gives the following solution:

$$(A1.6) \quad y = Qz \Rightarrow y = q_1 K_1 e^{X_1 t} + q_2 K_2 e^{X_2 t}$$

In (A1.6),  $q_1$  and  $q_2$  illustrate the first and the second columns of matrix  $Q$ , i.e., the first and the second eigenvectors of matrix  $A$ . All that remains is to define the eigenvalues and eigenvectors of matrix  $A$ , as well as the arbitrary constants  $K_1$  and  $K_2$ . To find the eigenvalues, we first set up the following matrix:

$$(A1.7) \quad A - XI = \begin{bmatrix} r - X & -\frac{1+\theta}{\omega} \\ 0 & \gamma - r - X \end{bmatrix}.$$

This gives rise to the following characteristic equation, the solutions of which constitute the eigenvalues of  $A$ :

$$(A1.8) \quad \begin{aligned} X^2 - \gamma X - r(r - \gamma) &= 0 \\ \Leftrightarrow X_1 = \gamma - r \wedge X_2 &= r \end{aligned}$$

To ensure the dynamic stability of the system, the negative eigenvalue must be chosen as the stable one. The no Ponzi game -condition which excludes the possibility of debt growing without a bound states that the discount rate must be less than the interest rate ( $r > \gamma$ ), and this constitutes  $X_1$  as the stable eigenvalue. Next, using  $X_1 = \gamma - r$  and  $X_2 = r$ , and (A1.7), allows us to solve for the eigenvectors:

$$\begin{aligned}
& \begin{bmatrix} r-\gamma+r & -\frac{1+\omega}{\omega} \\ 0 & \gamma-r-\gamma+r \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = 0 \\
& \Rightarrow (2r-\gamma)B_1 - \frac{1+\omega}{\omega}B_2 = 0 \\
\text{(A1.9)} \quad & \Rightarrow B_2 = 1, B_1 = \frac{1+\omega}{\omega(2r-\gamma)};
\end{aligned}$$

$$\begin{aligned}
& \begin{bmatrix} r-r & -\frac{1+\omega}{\omega} \\ 0 & \gamma-r-r \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = 0 \\
& \Rightarrow D_2 = 0, D_1 = 1
\end{aligned}$$

This yields the following solution to the matrix  $Q$ :

$$\text{(A1.10)} \quad Q = \begin{bmatrix} \frac{1+\omega}{\omega(2r-\gamma)} & 1 \\ 1 & 0 \end{bmatrix}.$$

Substituting the eigenvectors (the columns of  $Q$ ) into (A1.6), yields the complementary solution to the system.

To find the particular solution, we first need to invert matrix  $A$ . We then multiply the resulting matrix with the negative of matrix  $b$ :

$$\begin{aligned}
\text{(A1.11)} \quad & -\begin{bmatrix} r & -\frac{1+\omega}{\omega} \\ 0 & \gamma-r \end{bmatrix}^{-1} \times \begin{bmatrix} k \\ -(\delta+\omega\mu) \end{bmatrix} = \\
& -\begin{bmatrix} \frac{1}{r} & \frac{1+\omega}{\omega r(\gamma-r)} \\ 0 & \frac{1}{\gamma-r} \end{bmatrix} \times \begin{bmatrix} k \\ -(\delta+\omega\mu) \end{bmatrix} = \begin{bmatrix} -\frac{k}{r} - \frac{(1+\omega)(\delta+\omega\mu)}{\omega r(\gamma-r)} \\ -\frac{\delta+\omega\mu}{r-\gamma} \end{bmatrix}
\end{aligned}$$

The complete solution can then be written as follows:

$$\text{(A1.12)} \quad \begin{bmatrix} d \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{1+\omega}{(2r-\gamma)\omega} \\ 1 \end{bmatrix} K_1 e^{(\gamma-r)t} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} K_2 e^{rt} + \begin{bmatrix} -\frac{k}{r} - \frac{(1+\omega)(\delta+\omega\mu)}{\omega r(\gamma-r)} \\ -\frac{\delta+\omega\mu}{r-\gamma} \end{bmatrix}.$$

Finally, the arbitrary constants  $K_1$  and  $K_2$  can be solved using the initial and transversality conditions associated with the problem at hand.

# Appendix 2

## Phase Diagram Analysis

Because time does not enter the equation (3.3) as a separate function, the dynamic system is called autonomous. The evolution of the system can then be graphically illustrated with a phase diagram.

For illustrative purposes, we want to draw the diagram in the debt-deficit space. Therefore, we modify first equation (3.3), by using the first-order condition  $\lambda = -u$ :

$$(A2.1) \quad \begin{bmatrix} d' \\ u' \end{bmatrix} = \begin{bmatrix} r & \frac{1+\omega}{\omega} \\ 0 & \gamma - r \end{bmatrix} \begin{bmatrix} d \\ u \end{bmatrix} + \begin{bmatrix} k \\ \delta + \omega\mu \end{bmatrix}$$

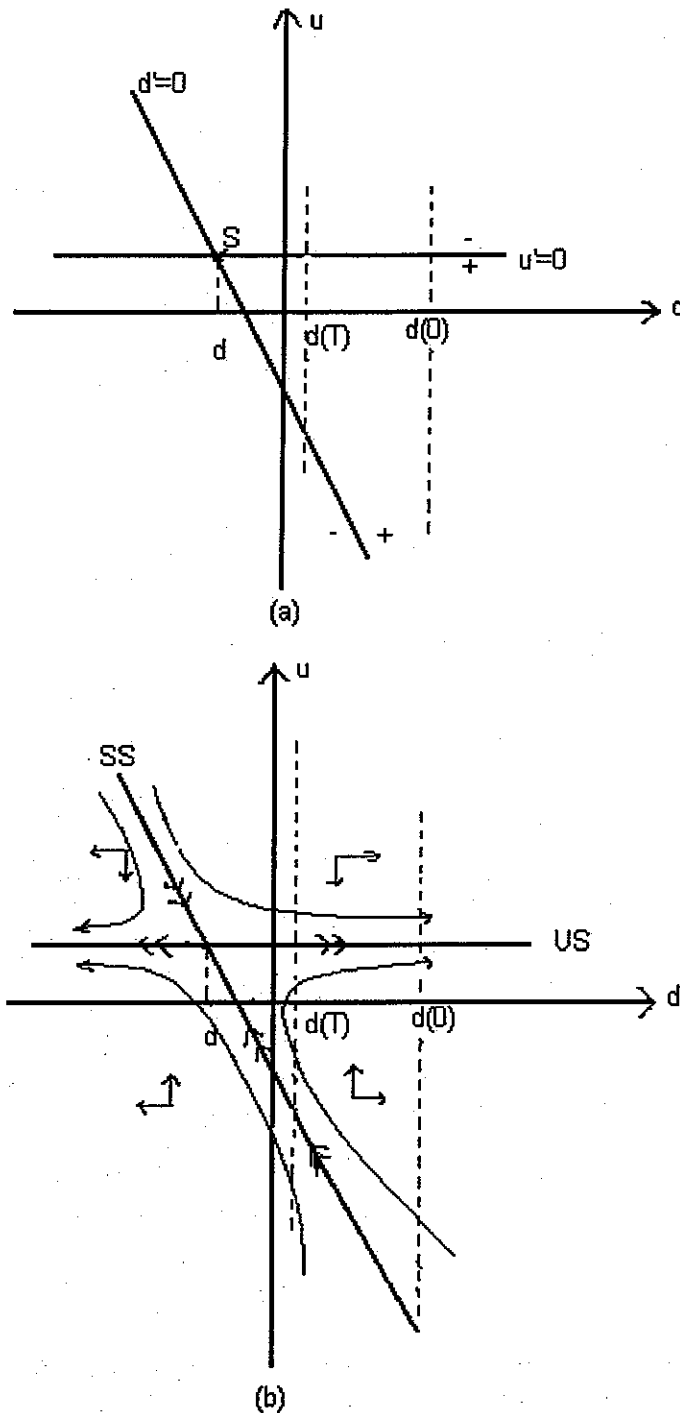
From (A2.1), we can calculate the steady state as follows:

$$(A2.2) \quad - \begin{bmatrix} r & \frac{1+\omega}{\omega} \\ 0 & \gamma - r \end{bmatrix}^{-1} \times \begin{bmatrix} k \\ \delta + \omega\mu \end{bmatrix} = \begin{bmatrix} -\frac{k}{r} - \frac{1+\omega}{\omega r(\gamma-r)}(\delta + \omega\mu) \\ \frac{1}{r-\gamma}(\delta + \omega\mu) \end{bmatrix} \equiv \begin{bmatrix} \tilde{d} \\ \tilde{u} \end{bmatrix}$$

We can now plot the steady-state points in Figure 3.6a. Note that when  $\mu$  increases, the steady state (point  $S$ ) moves towards north-west. This happens because higher monetary adjustment implies lower debt and less fiscal adjustment (higher deficit). The elements of finite time horizon are included in the figure in form of the dashed lines  $d(0) = d_0$  and  $d(T) = \bar{d}$ , which illustrate the initial- and endpoint conditions, respectively. According to these conditions, the system starts from some point along the  $d_0$  line and, after time  $T$  has elapsed, it must end up in a point on the  $\bar{d}$  line.

The next step is to plot the lines along which  $d'$  and  $u'$  are equal to zero, i.e., lines  $rd + k + (1+\omega)u/\omega = 0$  and  $(\gamma - r)u + \delta + \omega\mu = 0$ . According to these equations, the slope of  $d' = 0$  is negative in the debt - deficit space, while  $u' = 0$  is a horizontal line. Deficit remains constant over time if it is at any point in time chosen so that it equals the steady-state level. On the other hand, debt does not change when the combination of policy instruments (i.e., the term  $(1+\omega)u/\omega$  in the  $d' = 0$  equation) is chosen so that it covers the interest payments on existing debt plus the additional debt which is due to the conflict in target levels ( $k$ ).

Figure 3.6 A Phase Diagram for the System of Equations (A2.1)



Intuitively, if debt is high, deficit must be set sufficiently low (in fact, there must be a sufficiently high surplus) so that enough revenues can be generated to finance at least the interest payments. We now turn to analyse the behaviour of the dynamic system in the four different regions which are separated by the  $d' = 0$  and  $u' = 0$  lines. Because

$\frac{\partial u'}{\partial u} = \gamma - r < 0$ , the sign sequence of  $u'$  around the  $u' = 0$  line follows (+,0,-) when  $u$  increases (going northbound).<sup>9</sup> On the other hand, since  $\frac{\partial d'}{\partial d} = r > 0$ , around the  $d' = 0$  line the sign sequence for debt states (-,0,+) when  $d$  increases (going eastbound).<sup>10</sup>

In Figure 3.6b, we have used the sign sequences of  $u'$  and  $d'$  to draw the kinked arrows that show the directions of the system in the different regions of the diagram. The four trajectories illustrate the evolution of the system around the steady-state, obeying the kinked arrows. Finally, we have used the eigenvectors from the general solution (3.4) to sketch the stable and unstable manifolds ( $SS$  and  $US$ , respectively), which are illustrated by the series of arrows leading towards and away from the steady state, respectively.<sup>11</sup> The directions of the eigenvectors and the trajectories imply that in order to meet the transversality condition  $d(T) = \bar{d}$ , the coalition must choose a sufficiently low initial level of deficit. For example, if the initial deficit was chosen above the  $u' = 0$  line (which coincides with the unstable manifold), the coalition would not be able to pay even the interest of existing debt. On the other hand, if the time path of deficit started below but close to the  $u' = 0$  line, debt would initially be serviced but over time the desire for higher deficits would outweigh the incentives to cut debt. Therefore, choosing too high initial deficit means that sooner or later debt will end up on an explosive path.

In Figure 3.7, in the bottom panel we have plotted a family of trajectories ( $J_i$ ). Each  $J_i$  is associated with different initial deficit, but nevertheless they all start from low initial levels of deficits. We have also drawn what we call "equal time curves",  $T_i$ , which represent positions attained on the various trajectories  $J_i$  after time  $T_i$  has elapsed from the beginning of the adjustment.

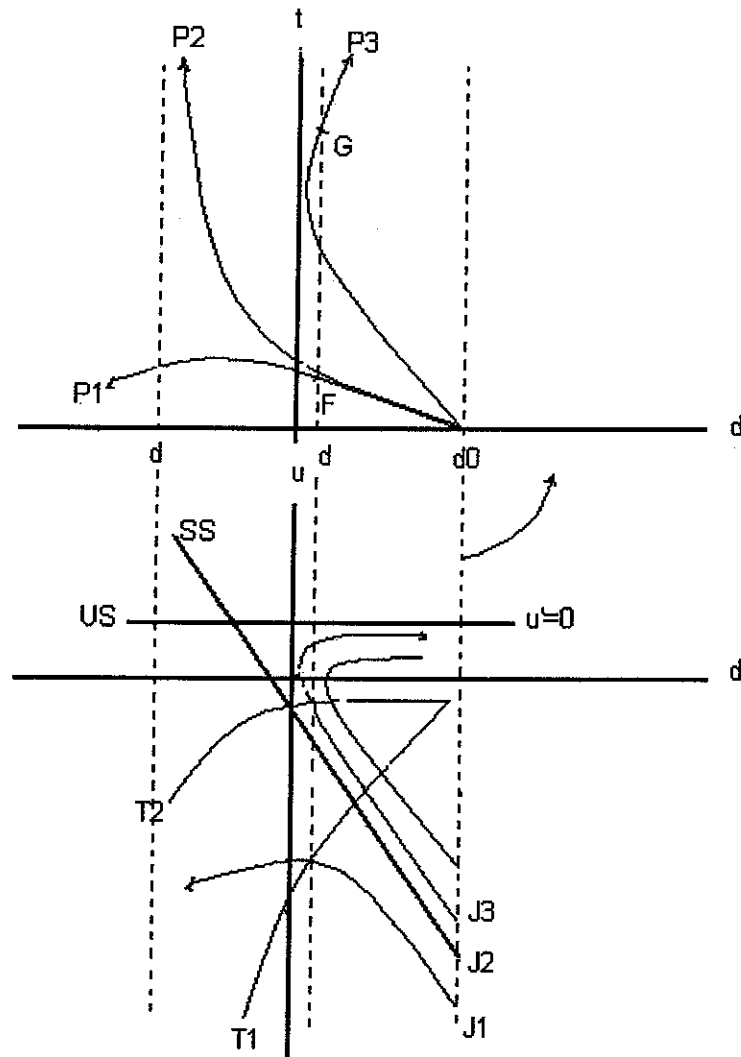
<sup>9</sup> The sign sequence illustrates whether a variable increases, remains constant or decreases over time in the different regions of the diagram.

<sup>10</sup> If it was the other way round, the boundary condition  $d(T) = \bar{d}$  could never be met because the dynamics would always push the actual debt towards the  $d' = 0$  line. In that case, the coalition would adjust just enough to service the interest payment, but no principal of the debt would be serviced.

<sup>11</sup> Note that because Figure 3.6 is drawn with  $u$  on the vertical axis, the first eigenvector has different sign than in equation (3.4), which illustrates the relationship between the state and the costate variables.



Figure 3.7 Graphical Derivation of the Equilibrium Time Paths



The interpretation of Figure 3.7 is as follows. Trajectory  $J_1$  is associated with lowest initial deficit and therefore it lies further away from the  $d' = 0$  curve than the other trajectories. Therefore, adjustment along  $J_1$  implies a large value of  $d'$ , because by choosing a low initial deficit the coalition can be more effective in reducing debt in  $T_1$  periods. If the coalition chooses the initial deficit which is associated with trajectory  $J_3$ , it ends up with less debt adjustment at time  $T_1$  (similarly if it chooses  $J_2$  which corresponds to the stable manifold). In other words, for a given time horizon there is a unique phase path (trajectory), along which the coalition can adjust towards the terminal level of debt.

In the top panel of Figure 3.7, each trajectory is reflected by a specific time path for the state variable debt. For example, for a problem with a planning horizon  $T_1$ , the relevant time path consists of the segment  $d_0F$  on path  $P_1$ , showing a rapid reduction in debt to the terminal level. However, if the time horizon is pushed forward to  $T_2$ , a higher initial deficit can be chosen at the outset. The relevant time path will then be the segment  $d_0G$  on path  $P_3$ , showing a more moderate adjustment over time. In general, the shorter the time horizon, or the higher the initial level of debt, the lower is the initial deficit which must be chosen in order to take the system to a stable adjustment path.

# Appendix 3

## Cooperation with Stochastic Debt Shocks

In this appendix we extend the cooperative model by assuming that the dynamics of the budget constraint is subject to stochastic noise.<sup>12</sup> In the presence of such uncertainty, the optimal choice of the policymakers' control variables will generally differ from the deterministic case, because the policymakers possess only indirect control over debt via their policy instruments. More specifically, in the presence of disturbances, the budget constraint (2.1) becomes:

$$(A3.1) \quad d' = [rd(t) + u(t) - v(t) + k]dt + \sigma dW(t)$$

In (A3.1),  $\sigma$  is a constant and  $W$  is a standard Brownian motion process. For simplicity, it is assumed that  $k = 0$ , i.e., there is no conflict between the target levels of the two policy instruments. We now define the following cooperative loss function, where we ignore discounting to simplify the analysis further:

$$(A3.2) \quad L = E \left\{ \int_0^T \frac{1}{2} \{u^2(t) + \omega v^2(t) + (\delta + \mu\omega)d^2(t)\} dt + \frac{1}{2} \Phi d^2 \pi \right\}$$

In (A3.2),  $E$  is the expectations operator. Unlike in the deterministic case, we assume that the loss is quadratic in debt. It will turn out that this modification makes the solution easier in the stochastic case, without affecting the nature of the results. The last term in (A3.2) is the scrap value function  $\Gamma(T, d)$ , which measures the value of (A3.2) at the terminal date  $T$ . We have introduced the coefficient  $\pi$  to measure the importance for the cooperating policymakers to reach the terminal debt target; a high value of  $\pi$  means a higher weight is assigned on this objective.

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<sup>12</sup> Because the theory of non-cooperative stochastic differential games is incomplete, we only consider the cooperative case.

Next, we define the Hamilton-Jacobi-Bellman (HJB) equation, for a given value function  $\psi(t, d)$ :

$$(A3.3) \quad 0 = \frac{\partial \psi}{\partial t} + \min_{u, v} \left\{ \frac{1}{2} [u^2(t) + \omega v^2(t) + (\delta + \mu \omega) d^2(t)] \right. \\ \left. + [rd(t) + u(t) - v(t) + k] \frac{\partial \psi}{\partial d} + \frac{1}{2} \sigma^2 \frac{\partial^2 \psi}{\partial d^2} \right\}$$

The value function, which illustrates the minimum cost, must satisfy the following terminal condition:

$$(A3.4) \quad \psi(T, d) = \Gamma(T, d) = -\frac{1}{2} \Phi d^2 \pi.$$

Following Oksendal (1985), and because the structure of the objective function is linear-quadratic, we propose the following value function as a solution to the HJB:

$$(A3.5) \quad \psi(t, d) = -\frac{1}{2} \phi(t) d^2 - a(t) \\ \frac{\partial \psi(t, d)}{\partial d} = -\phi(t) d, \quad \frac{\partial^2 \psi(t, d)}{\partial d^2} = -\phi(t) \\ \phi(T) = \Phi \pi \\ a(T) = 0$$

Substituting this into the HJB, and minimising the HJB with respect to the control variables (budget deficit  $u$  and money growth  $v$ ), and rearranging, yields the following expressions for deficit and money growth:

$$(A3.6) \quad a) u(t) = -\phi(t) d \\ b) v(t) = \frac{\phi(t) d}{\omega}$$

To derive the conditions under which these controls indeed are optimal, we substitute them back to the HJB and collect the terms of  $d^2$ ,  $d$  and constants. It turns out that (A3.6 a,b) are optimal, so that  $\psi(t,d)$  is the minimum cost, only if  $\phi(t)$  and  $a(t)$  are chosen so that the following two conditions are satisfied simultaneously:

$$(A3.7) \quad \begin{aligned} a)\phi' &= \frac{1+\omega}{\omega}\phi(t)^2 - 2r\phi(t) - (\delta + \omega\mu) \\ b)a' &= -\frac{1}{2}\sigma^2\phi \end{aligned}$$

The next step is to solve the value function's parameters,  $\phi(t)$  and  $a(t)$ . The particular (or steady-state) solution to (A3.7a) can be obtained by setting  $\phi'$  equal to zero and solving for  $\phi(t)$ . This yields:

$$(A3.8) \quad \phi = \frac{\omega}{1+\omega} \left[ r - \left( r^2 + \frac{1+\omega}{\omega}(\delta + \omega\mu) \right)^{\frac{1}{2}} \right]$$

The steady-state value of  $\phi$  differs from zero whenever the policymakers face a trade-off between the different policy objectives (i.e., when  $\delta > 0$  and  $\mu > 0$ ). The homogenous part can be obtained using Bernoulli equations. The solution is:

$$(A3.9) \quad \phi(t) = \frac{1}{Ae^{2\alpha t} + \frac{1+\omega}{2r\omega}}$$

Using the boundary condition  $\phi(T) = \Phi\pi$  in (A3.9), we can determine  $\phi(t)$  uniquely. Together with the particular solution (A3.8), this gives the complete solution to  $\phi$  as follows:

$$\begin{aligned}
\phi^*(t) &= \frac{1}{\frac{1+\omega}{2r\omega} [1 - e^{-2r(T-t)}] + \frac{1}{\Phi\pi} e^{-2r(T-t)}} \\
&+ \frac{\omega}{1+\omega} \left[ r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] \\
\text{(A3.10)} \quad &= \frac{\omega}{1+\omega} \left[ 2r(1 - e^{-2r(T-t)}) + r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] + \Phi\pi e^{-2r(T-t)}.
\end{aligned}$$

Setting  $t=T$  in (A3.10) reveals that at the terminal time  $T$ , the solution consists of the terminal condition  $\phi(T) = \Phi\pi$  and the steady-state solution (A3.8). Next, (A3.7b) gives:

$$\text{(A3.11)} \quad a = -\frac{1}{2} \sigma^2 \int_0^T \phi(t) dt$$

Plugging in  $\phi^*(t)$  from (A3.10) gives the following expression for  $a$ :

$$\begin{aligned}
a^*(t) &= -\frac{1}{2} \sigma^2 \left\{ \int_0^T \left[ \frac{2r\omega}{1+\omega} (1 - e^{-2r(T-t)}) + \Phi\pi e^{-2r(T-t)} \right] dt \right. \\
\text{(A3.12)} \quad &+ \left. \frac{\omega}{1+\omega} \left[ r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] \right\} \\
&= \frac{1}{2} \sigma^2 \left\{ \left( \frac{\Phi\pi}{2r} - \frac{\omega}{1+\omega} \right) (1 - e^{-2rT}) - \frac{\omega}{1+\omega} \left[ 3r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] \right\}
\end{aligned}$$

Therefore, when  $\phi^*(t)$  and  $a^*(t)$  satisfy (A3.10) and (A3.12), respectively, (A3.6a,b) will constitute the optimal controls and the minimum cost is given by:

(A3.13)

$$\begin{aligned} \psi(t, d) &= -\frac{1}{2} \phi^*(t) d^2 - a^* \\ &= -\frac{1}{2} \left\{ \left[ \frac{\omega}{1+\omega} \left[ 2r(1-e^{2r(T-t)}) + r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] + \Phi\pi e^{2r(T-t)} \right] d^2 \right. \\ &\quad \left. - \sigma^2 \left[ \left( \frac{\Phi\pi}{2r} - \frac{\omega}{1+\omega} \right) (1-e^{2rT}) - \frac{\omega}{1+\omega} \left[ 3r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] \right] \right\} \end{aligned}$$

Evaluating (A3.13) at  $t=T$  and comparing with (A3.4) reveals that there are two sources of additional cost in the model. First, like in the deterministic case,  $\delta > 0$  and  $\mu > 0$  imply that there are costs due to the trade-off between the different policy objectives. The trade-off creates costs because the policymakers are not able to stick completely to their private policy objectives, and consequently in equilibrium they will miss both the private policy target *and* the common debt target. Second,  $\sigma^2 > 0$  implies that there are costs due to the noise, as represented by (A3.12). In the absence of these distortions, at  $t = T$  equation (A3.13) collapses to (A3.4). Finally, substituting  $\phi^*(t)$ , the optimal  $\phi$  from (A3.10), into the expressions for the controls (A3.6a,b) completes the solution:

(A3.14)

$$\begin{aligned} u^*(t) &= - \left\{ \frac{\omega}{1+\omega} \left[ 2r(1-e^{2r(T-t)}) + r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] + \Phi\pi e^{2r(T-t)} \right\} d(t) \\ v^*(t) &= \left\{ \frac{1}{1+\omega} \left[ 2r(1-e^{2r(T-t)}) + r - \left( r^2 + \frac{1+\omega}{\omega} (\delta + \omega\mu) \right)^{\frac{1}{2}} \right] + \frac{\Phi\pi e^{2r(T-t)}}{\omega} \right\} d(t) \end{aligned}$$

$$\frac{\partial u^*(t)}{\partial \mu} > 0, \quad \frac{\partial v^*(t)}{\partial \delta} < 0, \quad \frac{\partial u^*(t)}{\partial \pi} < 0, \quad \frac{\partial v^*(t)}{\partial \pi} > 0,$$

$$\frac{\partial u^*(t)}{\partial \omega} < 0, \quad \frac{\partial v^*(t)}{\partial \omega} < 0, \quad \frac{\partial u^*(t)}{\partial r} < 0, \quad \frac{\partial v^*(t)}{\partial r} > 0$$

The solutions to the control variables are expressed as functions of the state variable, and thus the results in (A3.14) look fairly different than the equilibrium solutions to deficit and money in the deterministic case (equation 3.6a,b). However, the solutions show similar properties. In (A3.14), at any point in time deficit is the lower the higher is the level of debt. Moreover, the positive externalities which are characteristic for the cooperative regime imply that deficit can be set higher when  $\mu$ , the weight assigned by the CB on the debt objective, is high. In the same way, at any point in time money supply is the higher the higher is debt. On the other hand, money supply is the lower the higher is  $\delta$ , the government's weight assigned on the debt objective. When the CB's bargaining power  $\omega$  increases, money supply and deficit fall simultaneously, as the adjustment burden shifts to the government. Finally, an increase in the interest rate  $r$  implies that policy becomes more "cooperative" so that deficits fall and money supply increases to reduce  $d(t)$ . The results obtained in the deterministic case are therefore robust to stochastic disturbances. An additional result, due to the formulation of the terminal condition in the loss function (A3.2), is that an increase in  $\pi$  has the same effect than an increase in  $r$ . If achieving the debt target becomes more important, both policymakers prefer to adjust more in order to hit the common objective.



# Optimal Fiscal Policy in the Presence of Debt Shocks and the EMU Stability Pact

## 1 Introduction

In the previous Essay, we analysed a situation where the policymakers were able to optimally choose the entire time paths for their respective control variables at the beginning of the game. This choice was made on the basis of the initial conditions and the policymakers' own preferences. The features of time-inconsistency were incorporated by the fact that the policymakers were not able to commit themselves to follow any other adjustment strategy.

However, a more realistic analysis should allow for the possibility of government re-assessing the policy which was selected earlier, and continuing as it sees optimal given the new information that may have arrived. In this Essay, we make the choice of the control path contingent on a jump in the state variable. In particular, focusing on fiscal policy only, we study how the fact that the government is aware of a future positive shock in debt affects its optimal choice of deficit before and after the shock. We then analyse how the results change if the government's choice of deficit is constrained by a mechanism that penalises the government from running excessive deficits. Such arrangements are incorporated in the so called EMU stability pact, which acts to enforce fiscal discipline inside EMU by strengthening the "excessive deficit procedure" of the Maastricht treaty.<sup>1</sup>

The issue of optimal budget deficits in the presence of shocks has been studied by Lucas and Stokey (1983) and Flemming (1987), who build on the tax smoothing literature initiated by Barro (1979).<sup>2</sup> In

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<sup>1</sup> The Stability Pact was concluded, according to the schedule, at the EU Summit in Amsterdam in June 1997. However, as a concession to French demands a separate "Resolution on Growth and Employment" was adopted alongside the Pact.

<sup>2</sup> Other classical papers in the area of optimal taxation are Ramsay (1924) and Mirrlees (1971). Examples of dynamic treatment of government policy in a related framework include Sidrauski (1967) and Turnovsky and Brock (1980).

short, their results suggest that if the government expects a war to break out in a certain future date, it optimally runs surpluses before the war in order to be able to spend more during the war. In our stylised debt adjustment framework, we find a similar pattern. When the government cares about future, deficit is optimally set lower before the arrival of the shock and thus more adjustment will be shifted from *ex post* to *ex ante*. We also find that before the shock, fiscal policy is more disciplined when the government is threatened by a penalty from excessive deficits.

In a world where large shocks are possible, therefore, the implications of the proposed Stability Pact can be quite severe if the government has significant spending commitments. As a reaction to such possibilities, voices have recently been raised against the rationale of the sanctions which effectively work to increase the plight of countries who experience problems in their fiscal environment, often as a result of exogenous shocks. Although the sanctions will be applied upon discretion, a simple majority vote on one hand and fears of destroying the credibility of the Pact on the other hand makes it likely that the Pact will be applied strictly from the beginning.<sup>3</sup>

The rest of this Essay proceeds as follows. Section 2 sets up the model and analyses debt adjustment in the presence of shocks. In section 3, we introduce the Stability Pact and study how the government's optimal deficit path is affected by the fact that excessive deficits are penalised by a proportional fine. Section 4 concludes.

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<sup>3</sup> Some researchers have recently analysed the issues connected with the Stability Pact. Beetsma and Uhlig (1997) show that the Pact can help to solve the problem of excessive union-wide deficits and increased inflationary pressure towards the European Central Bank. Artis and Winkler (1997) argued that the Pact should mainly be seen in strengthening of the independence of the ECB. In addition, Essay 2 of this thesis suggested that an optimal inflation target designed by the participating governments for the ECB can be seen as a way of imposing "voluntary" fiscal discipline, with no Stability Pact necessary.

## 2 Fiscal Policy in the Presence of Foreseeable Shocks

In the previous Essay it was assumed that public debt can be cut either by money creation or by reducing the budget deficits. In this Essay, we focus on fiscal policy only. This can be motivated if the growth rate of money supply is chosen by an independent central bank (CB), who follows a fixed (zero) money supply rule. The government then has the following intertemporal budget constraint (see Tabellini, 1986):

$$(2.1) \quad d' = rd(t) + f(t)$$

In (2.1),  $d'$  denotes government debt accumulation (the state variable in the model),  $f(t)$  is fiscal deficit net of interest payments,  $rd(t)$  is interest payments on existing debt and  $r$  is defined as the (constant) difference between interest rate and growth rate of output. Because monetary policy is treated as exogenous, there is no formal conflict between the objectives of the government and the CB. In other words, we are dealing with a simple optimal control problem.

Debt can be subject to positive shocks, which are modelled as simple deterministic jumps in the state variable. Such jumps can occur for example as a result of a large future government construction programme, or indeed as a result of a preparation for a war (as was assumed by Lucas and Stokey).

To solve the model in the case where the state variable can take discrete jumps, we proceed backwards, first deriving the optimal solution for the period after the shock has occurred. If the shock arrives at time  $t=t_1$ , and there are no later shocks, after the shock the government chooses the budget deficit in order to minimise the following infinite-horizon loss function:

$$(2.2) \quad L^G = \int_{t_1}^{\infty} \left[ \frac{1}{2} (f(t) - \bar{f})^2 + \delta d(t) \right] e^{-\alpha(t-t_1)} dt$$

The government faces a trade-off between stabilising deficit around the target value  $\bar{f}$ , and reducing debt. The parameter  $\delta$  measures the relative weight assigned on the debt objective. Government discounts future losses at the rate  $\alpha$ , and it manages fiscal deficits to minimise the loss, subject to the government budget constraint (2.1), the initial stock of debt and given transversality conditions. In the current case, the initial condition and the transversality condition are given as:

$$(2.3) \quad \begin{aligned} a) & d(t_1) = d_1 \\ b) & \lim_{t \rightarrow \infty} d(t)e^{-r(t-t_1)} = 0. \end{aligned}$$

The transversality condition (2.3b) ensures government solvency in the limit when time approaches infinity. Next, we set up the following current-value Hamiltonian:

$$(2.4) \quad H^G = \frac{1}{2}(f - \bar{f})^2 + \delta d + \lambda(rd + f)$$

In (2.4), we have omitted the time symbols associated with variables  $f$ ,  $d$  and  $\lambda$ . The variable  $\lambda$  measures the cost from an additional unit of government debt that requires low future deficits to pay for its amortisation. The first-order conditions of (2.4), with respect to the control variable deficit and state variable debt, are given as follows:

$$(2.5) \quad \begin{aligned} a) & \frac{\partial H^G}{\partial f} = f - \bar{f} + \lambda = 0 \\ b) & \frac{\partial H^G}{\partial d} = \delta + r\lambda = \alpha\lambda - \lambda' \end{aligned}$$

After substituting (2.5a) into (2.1), and rearranging (2.5b), we are ready to set up the following system of differential equations:

$$(2.6) \quad \begin{bmatrix} d' \\ \lambda' \end{bmatrix} = \begin{bmatrix} r & -1 \\ 0 & \alpha - r \end{bmatrix} \begin{bmatrix} d \\ \lambda \end{bmatrix} + \begin{bmatrix} \bar{f} \\ -\delta \end{bmatrix}$$

Following the steps described in Appendix 1 to Essay 3, the general solution to the system (2.6) can be written as follows:

$$(2.7) \quad \begin{bmatrix} d \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{1}{2r-\alpha} \\ 1 \end{bmatrix} K_1 e^{(\alpha-r)(t-t_1)} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} K_2 e^{r(t-t_1)} + \begin{bmatrix} \tilde{d} \\ \tilde{\lambda} \end{bmatrix}$$

In (2.7),  $K_1$  and  $K_2$  are arbitrary constants to be determined below,  $(\alpha - r)$  and  $r$  are the stable and unstable eigenvalues, respectively, and  $\tilde{d}$  and  $\tilde{\lambda}$  denote the steady-state values of the state and costate variables. Using the condition  $f = \bar{f} - \lambda$  from (2.5a), which implies that  $\tilde{f} = \bar{f} - \lambda$ , we obtain:

$$(2.8) \quad \begin{aligned} \tilde{d} &= -\frac{\tilde{f}}{r} - \frac{\delta}{r(r-\alpha)} \\ \tilde{f} &= \bar{f} + \frac{\delta}{r-\alpha} \end{aligned}$$

From (2.8), it can be seen that the higher is the weight  $\delta$  assigned by the government on the debt objective, the lower is the steady-state level of debt. On the other hand, the higher is the target level of deficit, the higher is the actual deficit. A higher interest rate  $r$  implies that future payments on debt are higher. Therefore, a high  $r$  induces the government to choose lower deficit, unless the government's discount rate  $\alpha$  is high. A high rate of discount means that the government is short-sighted, in which case it prefers to shift the adjustment effort to future periods.

We now return back to the general solution (2.7). From the fact that the time horizon is infinite it follows that  $K_2$  is zero. The initial condition for debt then allows us to solve for  $K_1$ :

$$(2.9) \quad \begin{aligned} d(t_1) &= \frac{1}{2r-\alpha} K_1 + \tilde{d} = d_1 \\ \Rightarrow K_1 &= (2r-\alpha)[d_1 - \tilde{d}] \end{aligned}$$

Substituting (2.9) into (2.7), after having set  $K_2=0$ , finally gives the equilibrium time paths for deficit and debt:

$$(2.10) \quad a) d(t) = \tilde{d} + [d_1 - \tilde{d}]e^{(\alpha-r)(t-t_1)}$$

$$b) f(t) = \tilde{f} - (2r - \alpha)[d_1 - \tilde{d}]e^{(\alpha-r)(t-t_1)}$$

By setting  $t=t_1$  and  $t=\infty$  in turn, we get that  $d(t_1) = d_1$  and  $d(\infty) = \tilde{d}$ , respectively. Therefore, the system converges to the steady state when time approaches infinity, which is to be expected given the autonomous character of the problem at hand. Finally, substituting (2.10a,b) back into (2.2), and integrating, gives the equilibrium loss function for the period after the shock, evaluated at time  $t=t_1$ :

$$(2.11) \quad L_1^G = \frac{1}{2} \left\{ \frac{[\tilde{f} - \bar{f}]^2}{\alpha} + 2 \frac{2r - \alpha}{r} [d_1 - \tilde{d}][\tilde{f} - \bar{f}] + (2r - \alpha)[d_1 - \tilde{d}]^2 \right\} + \delta \left( \frac{d_1 - \tilde{d}}{r} + \frac{\tilde{d}}{\alpha} \right)$$

The equilibrium loss is the higher the higher is  $\delta$ , which measures the size of the trade-off the government faces between stabilising deficit and reducing debt. Since the government has two targets but only one instrument, in equilibrium it will fail to hit any one of the two targets exactly. The loss is also increasing in the initial level of debt,  $d_1$ , which is inherited from the adjustment period before the shock.

We now turn to the optimisation problem before the arrival of the shock. In the first period, the government faces the following finite-horizon problem:

$$(2.12) \quad L_0^G = \int_0^1 \left[ \frac{1}{2} (f(t) - \bar{f})^2 + \delta d(t) \right] e^{-\alpha t} dt + L_1^G * e^{-\alpha t}$$

In (2.12), the scrap value function  $L_1^G * e^{-\alpha t}$ , which is added to the integral, represents the minimum value of the integral of the second-period flow of losses (as illustrated by the RHS of equation 2.11) starting from time  $t_1$  with an initial stock of debt  $d(t_1) = d_1$ .<sup>4</sup> The fact that the government attaches some value to the terminal stock of debt has a bearing on the transversality condition, which is given by:

<sup>4</sup> See Leonhard and van Long (1995) and Kamien and Schwartz (1995) for an analysis of dynamic optimisation under scrap value functions.

$$(2.13) \quad \frac{\partial L_1^G * e^{-\alpha t_1}}{\partial d_1} = \lambda(t_1)$$

The government now minimises (2.12) subject to (2.1), (2.13) and the initial condition  $d(0)=d_0$ . The Hamiltonian is similar to (2.4), the first-order conditions are given by (2.5) and the general solution to the system in the first period is given as follows:

$$(2.14) \quad \begin{bmatrix} d(t) \\ \lambda(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 2r - \alpha \\ 1 \end{bmatrix} K_1 e^{(\alpha-r)t} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} K_2 e^{rt} + \begin{bmatrix} \tilde{d} \\ \tilde{\lambda} \end{bmatrix}$$

Using the initial condition  $d(0)=d_0$ , we can immediately solve for the arbitrary constant  $K_2$ :

$$(2.15) \quad K_2 = d_0 - \tilde{d} - \frac{1}{2r - \alpha} K_1$$

However, in order to solve for the second arbitrary constant  $K_1$ , we have to refer to the transversality condition (2.13). At this stage, we incorporate the shock by assuming that at time  $t_1$  debt takes a discrete jump of size  $J$ , and define the level of debt *before* the jump as  $d_1^- = d_1 - J$ . Evaluating (2.11) before the jump, and using (2.13), then gives:

$$(2.16) \quad \frac{\partial L_1^G * e^{-\alpha t_1}}{\partial d_1^-} = \left[ (2r - \alpha)(d_1^- + J - \tilde{d}) + \frac{1}{r} [(2r - \alpha)(\bar{f} - \tilde{f}) + \delta] \right] e^{-\alpha t_1} = \lambda(t_1)$$

Rearranging (2.16) gives an expression for  $d_1^-$ :

$$(2.17) \quad d_1^- = \tilde{d} - J - M + \frac{\lambda(t_1)e^{\alpha t_1}}{2r - \alpha},$$

$$M \equiv \frac{\delta}{r(2r - \alpha)} + \frac{1}{r} [\bar{f} - \tilde{f}]$$

In (2.17),  $M$  consists of losses from the adjustment effort after the shock. On the other hand, from the general solution (2.14) we can derive an expression for  $d(t_1)$  before the shock has arrived. Equating this with (2.17) allows us to eliminate  $d_1^-$ . Rearranging the resulting expression gives a solution for  $\lambda(t_1)$ :

$$(2.18) \quad \frac{1}{2r-\alpha} K_1 e^{(\alpha-r)t_1} + K_2 e^{rt_1} + \tilde{d} = \tilde{d} - J - M + \frac{\lambda(t_1) e^{\alpha t_1}}{2r-\alpha}$$

$$\Rightarrow \lambda(t_1) = (2r-\alpha) e^{-\alpha t_1} \left[ J + M + \frac{1}{2r-\alpha} K_1 e^{(\alpha-r)t_1} + K_2 e^{rt_1} \right]$$

Next, we can derive the general solution to  $\lambda(t_1)$  from (2.14). Together with (2.18), this allows us to eliminate  $\lambda(t_1)$ . Rearranging the resulting expression finally gives a solution to  $K_1$  as a linear function of  $K_2$ :

$$(2.19) \quad K_1 e^{(\alpha-r)t_1} + \tilde{\lambda} = (2r-\alpha) e^{-\alpha t_1} \left[ J + M + \frac{1}{2r-\alpha} K_1 e^{(\alpha-r)t_1} + K_2 e^{rt_1} \right]$$

$$\Rightarrow K_1 = \frac{(2r-\alpha) e^{-\alpha t_1}}{(1-e^{-\alpha t_1}) e^{(\alpha-r)t_1}} \left[ J + M + K_2 e^{rt_1} - \frac{\tilde{\lambda} e^{\alpha t_1}}{2r-\alpha} \right]$$

Solving  $K_1$  (from 2.19) and  $K_2$  (from 2.15) together, gives the final solutions to the two arbitrary constants:

$$(2.20) \quad K_1 = \frac{(2r-\alpha) e^{-\alpha t_1}}{\Omega} \left[ J + M + (d_0 - \tilde{d}) e^{rt_1} - \frac{\tilde{\lambda} e^{\alpha t_1}}{2r-\alpha} \right]$$

$$K_2 = \frac{1}{\Omega} \left[ (d_0 - \tilde{d}) (1 - e^{-\alpha t_1}) e^{(\alpha-r)t_1} - \left( J + M - \frac{\tilde{\lambda} e^{\alpha t_1}}{2r-\alpha} \right) e^{-\alpha t_1} \right],$$

$$\Omega \equiv (1 - e^{-\alpha t_1}) e^{(\alpha-r)t_1} + e^{(r-\alpha)t_1}$$

Using (2.20), (2.14) and the first order condition (2.5a), we obtain the equilibrium time paths for debt and deficit for the period before the shock has arrived:



$$d(t) = \tilde{d} + \frac{1}{\Omega} \left\{ (d_0 - \tilde{d})(e^{(r-\alpha)(t_1-t)} + (1 - e^{-\alpha_1})e^{(\alpha-r)t_1+\pi}) \right. \\ \left. - (J+M)(e^{-\alpha_1+\pi} - e^{-\alpha_1+(\alpha-r)t}) - \frac{\tilde{f} - \bar{f}}{2r-\alpha}(e^{\pi} - e^{(\alpha-r)t}) \right\}$$

(2.21)

$$f(t) = \tilde{f} - \frac{2r-\alpha}{\Omega} \left[ J + M + (d_0 - \tilde{d})e^{\pi} + \frac{(\tilde{f} - \bar{f})e^{\alpha_1}}{2r-\alpha} \right] e^{-\alpha_1+(\alpha-r)t}$$

The higher is the predicted jump  $J$ , or the higher is the weight  $\delta$  assigned on the debt reduction objective, the lower is debt at any point in time (note that  $\tilde{f}$  and  $M$  are both increasing in  $\delta$ ). Evaluating the time paths at the points  $t=0$  and  $t=t_1$  gives:

$$d(0) = d_0$$

$$d(t_1) = \tilde{d} + \frac{1}{\Omega} \left\{ [d_0 - \tilde{d}]e^{\alpha_1} - (J+M)(e^{(r-\alpha)t_1} - e^{-\pi}) + \frac{\tilde{f} - \bar{f}}{2r-\alpha}(e^{\pi} - e^{(\alpha-r)t_1}) \right\}$$

(2.22)

$$f(0) = \tilde{f} - \frac{(2r-\alpha)e^{-\alpha_1}}{\Omega} \left[ J + M + (d_0 - \tilde{d})e^{\pi} + \frac{(\tilde{f} - \bar{f})e^{\alpha_1}}{2r-\alpha} \right] \\ f(t_1) = \tilde{f} - \frac{(2r-\alpha)e^{-\pi}}{\Omega} \left[ J + M + (d_0 - \tilde{d})e^{\pi} + \frac{(\tilde{f} - \bar{f})e^{\alpha_1}}{2r-\alpha} \right]$$

It turns out, that the higher is the initial level of debt ( $d_0$ ), the lower is the initial level of deficit  $f(0)$  chosen by the government. In addition, the initial deficit is decreasing in the predicted jump in debt at  $t=t_1$ , and in the predicted losses from the period after the shock (as measured by  $M$ ). All these components therefore have an effect of disciplining the government in the first period so that debt is reduced more before the arrival of the shock. Therefore, because the government cares about future, it is willing to spread the adjustment burden more equally over the whole period. This outcome shows similar logic than the results from the optimal deficit models by Lucas and Stokey (1983) and Flemming (1987). They show that when a government is preparing to a war, it optimally runs surpluses during

the periods before the war in order to be able to spend more when the war has broken out.

Finally, from the solution to  $f(t)$  in (2.21) it follows that because shocks call for additional adjustment, they are particularly painful for governments who have high deficit targets  $f$  (for example, due to significant public expenditure commitments), or who are generally short-sighted (with a high discount rate  $\alpha$ ). Such governments prefer higher deficits, and choosing low deficits is therefore unpleasant, especially in the early stages of the adjustment programme.

It remains to be checked that our solution is consistent, in other words, we have to determine the size of the jump  $J$  as a function of the parameters of the model so that after the jump, debt is at the level  $d_1$ . From (2.22) we get the level of debt,  $d(t_1) = d_1^-$ , before the jump at  $t=t_1$ . Setting  $d(t_1) + J = d_1$  and rearranging, gives the following expression for the jump:

$$(2.23) \quad J = e^{(r-\alpha)t_1} \left[ \Omega[d_1 - \tilde{d}] - [d_0 - \tilde{d}]e^{\alpha t_1} + M(e^{(r-\alpha)t_1} - e^{-rt_1}) + \frac{\tilde{f} - \bar{f}}{2r - \alpha} (e^{rt_1} - e^{(\alpha-r)t_1}) \right]$$

From (2.23), it can be seen that the higher is the initial level of debt  $d_0$ , the lower is the jump needed to reach  $d_1$ . On the other hand, the higher is  $\delta$  (incorporated in  $\tilde{f}$  and  $M$ ), the higher is the required jump. This is because a higher  $\delta$  means that debt is being reduced more in the first period, which implies a lower  $d(t_1)$  before jump. We now turn to analyse the behaviour of the model in the case where the government's choice of deficits is restricted by the Stability Pact.

### 3 The Implications of the Stability Pact

In the future European Economic and Monetary Union (EMU), fiscal discipline is considered to be an elementary part of the economic policy framework which is characterised by the common currency and the common central bank. To stress the importance of continuing budgetary prudence the participating countries have signed the so called Stability Pact. The essence of the Pact is to impose sanctions on governments who run excessive budget deficits. In this section, we study how the government's optimal choice of deficit is affected by such an arrangement.

We argue that the features of the pact can be incorporated by modifying the government's objective function in the following way:

$$(3.1) \quad \int_0^{\infty} \left[ \frac{1}{2} (f(t) - \bar{f})^2 + \delta d(t) + \kappa (f(t) - f^*) \right] e^{-\alpha t} dt$$

$$\Rightarrow H^G = \frac{1}{2} (f - \bar{f})^2 + \delta d + \kappa (f - f^*) + \lambda (rd + f)$$

In (3.1),  $f^*$  is the critical, or threshold, level of deficit determined in the pact. The formulation of (3.1) implies that a government whose deficit exceeds the level  $f^*$  will automatically be penalised by a linear fine of proportion  $\kappa$ .<sup>5</sup> The exact behaviour of the model around the level  $f(t) = f^*$  can be illustrated in terms of  $\kappa$  as follows:

$$(3.2) \quad \begin{aligned} \kappa > 0 &\Leftrightarrow f(t) > f^* \\ \kappa = 0 &\Leftrightarrow f(t) \leq f^* \end{aligned}$$

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<sup>5</sup> We make this simple interpretation of the penalty for analytical tractability. In reality, the Pact initially imposes a penalty in form of a non-interest bearing deposit, which only after 2 years becomes non-refundable. In addition, the penalty includes a small fixed component and an upper limit of 0.5% of GDP. See Artis and Winkler (1997) who make a similar simplification.

In other words, it is assumed that at the limit where  $f(t) = f^*$  the pact does not bind. We assume that  $f^* > \bar{f}$  i.e., the government's target level of deficit is below the threshold level. Under that condition, the government will not run excessive deficits unless there is a shock to the system. In EMU, such situations can arise if there are adverse country-specific disturbances triggering a sudden increase in debt, a deterioration in the conditions of the government bond market or a sudden depreciation of the common currency towards some other currency in which the government has issued significant amounts of debt. As a direct result of such debt disturbances, and because adjustments in the levels of government spending take time to get implemented, deficit can shoot to a level where the sanctions of the pact apply.

As in the absence of the pact, the loss is minimised with respect to (2.1) and (2.3). This gives the Hamiltonian in the bottom row of (3.1). However, under the pact, the first-order conditions and steady state levels for deficit and debt are different:

$$\begin{aligned}
 (3.3) \quad & \frac{\partial H^G}{\partial f} = f - \bar{f} + \lambda + \kappa = 0 \\
 & \frac{\partial H^G}{\partial d} = \delta + r\lambda = \alpha\lambda - \lambda' \\
 & \hat{d} = -\frac{\bar{f} - \kappa}{r} - \frac{\delta}{r(r - \alpha)} \\
 & \hat{f} = \bar{f} + \frac{\delta}{r - \alpha} - \kappa
 \end{aligned}$$

In (3.3), the third and fourth rows illustrate the steady state levels of debt and deficit, respectively. The disciplinary effect of the penalty is incorporated by the fact that deficit is decreasing in the severity of the penalty ( $\kappa$ ). Therefore, whenever  $\kappa > 0$ , the steady state of deficit is lower than in the absence of the pact.

We now turn to analyse the optimal time paths for deficit and debt, under the following scenario: the government starts with an initial level of debt which induces it to choose deficit below the threshold  $f^*$ . In normal circumstances, the time path would simply evolve towards the steady state staying below the threshold level over time. However, a debt shock can take the government temporarily to a fiscal regime where the penalties apply. After the shock, deficit will eventually cross the threshold back to the regime where the penalty is

not binding. To solve the model in this case, the optimisation problem must be divided in three different phases. Counting backwards, we must first find the optimal solution to the period which starts from the point where the government leaves the “penalty” regime (i.e., when it crosses the  $f(t) = f^*$  line) and which ends to the steady state. Second, we find the optimal path for the period which starts after the shock has occurred and which ends at the point of exit from the “penalty” regime. In the process the arbitrary time of switching the regime is to be chosen optimally. Finally, we can solve for the period from the beginning of the game to the point where the shock arrives.

In the last period, the government’s problem is given by:

$$(3.4) \quad L_2^G = \int_{t_2}^{\infty} \left[ \frac{1}{2} (f(t) - \bar{f})^2 + \delta d(t) e^{-\alpha(t-t_2)} \right] dt$$

Clearly, (3.4) resembles (2.2) except that the initial time is changed to  $t_2$ , which is the optimal time for the regime shift from the “penalty” regime to the normal regime. The last-period Hamiltonian is (2.4), the first order conditions are (2.5) and the steady states are (2.8). Following the steps leading to (2.10), the time paths for debt and deficit are now given by:

$$(3.5) \quad \begin{aligned} a) d(t) &= \tilde{d} + [d_2 - \tilde{d}] e^{(\alpha-r)(t-t_2)} \\ b) f(t) &= \bar{f} - (2r - \alpha)[d_2 - \tilde{d}] e^{(\alpha-r)(t-t_2)} \end{aligned}$$

Substituting (3.5a,b) back into (3.4), and integrating, gives the equilibrium loss at  $t=t_2$ , which looks similar than (2.11):

$$(3.6) \quad \begin{aligned} L_2^{G*} &= \frac{1}{2} \left\{ \frac{[\bar{f} - \tilde{f}]^2}{\alpha} + 2 \frac{2r - \alpha}{r} [d_2 - \tilde{d}] [\bar{f} - \tilde{f}] + (2r - \alpha) [d_2 - \tilde{d}]^2 \right\} \\ &+ \delta \left( \frac{d_2 - \tilde{d}}{r} + \frac{\tilde{d}}{\alpha} \right) \end{aligned}$$

The next step is to analyse the period under which the penalty binds, i.e., the period immediately after the shock has hit until the switch to the normal regime. The government’s problem is now:

$$L_1^G = \int_{t_1}^{t_2} \left[ \frac{1}{2} (f(t) - \bar{f})^2 + \delta d(t) + \kappa (f - f^*) e^{-\alpha(t-t_1)} \right] dt + L_2^G * e^{-\alpha(t_2-t_1)} \quad (3.7)$$

$$d(t_1) = d_1, d(t_2) = d_2$$

In (3.7),  $L_2^G *$ , which is illustrated by equation (3.6), enters as a scrap value function. This function represents the minimum value of the integral of the last-period flow of losses, starting from time  $t_2$ . Because  $t_2$ , the time of the regime switch, is to be chosen optimally, the loss (3.7) is minimised with respect to  $f$ , and subject to (2.1) as well as the conditions for the initial and terminal levels of debt and the relevant transversality condition. The transversality condition must now reflect the fact that we have a free terminal time problem with a scrap value function:

$$H_1^{Opt} + \frac{\partial L_2^G * e^{-\alpha(t_2-t_1)}}{\partial t_2} = 0 \quad (3.8)$$

In (3.8),  $H_1^{Opt}$  represents the value of the optimised Hamiltonian, associated with the problem (3.7), at time  $t = t_2$ . The role of the transversality condition (3.8) is to bind together the two optimisation problems. More specifically, (3.8) equates the optimised Hamiltonian from the period after the shock, evaluated at the endpoint  $t_2$ , with the derivative with respect to  $t_2$  of the last-period equilibrium loss function, evaluated at the starting point  $t_2$ . The essence of this condition is to say that if  $t$  is increased from  $t_2$  by a small amount, the value of the integral (3.7) can become higher (lower) than the value of the integral (3.4), in which case it is optimal to switch to the normal regime (to continue in the "penalty" regime) at  $t_2$ .

We are now ready to turn to the solution of (3.7). The first order conditions for (3.7) are given by (3.3). On the other hand, condition (3.8), which constitutes the transversality condition, can be written as follows (in present value terms):

$$(3.9) \quad \frac{1}{2} (f^* - \bar{f})^2 + \delta d_2 + \lambda(t_2)(rd_2 + f^*) - \alpha L_2^G * = 0,$$

$$d_2 \equiv \tilde{d} + \frac{1}{2r - \alpha} [\tilde{f} - f^*]$$

In the top row of (3.9) the first three terms in the LHS illustrate the value of the Hamiltonian after the shock at  $t=t_2$ , when deficit is at the threshold level and debt is at the terminal level  $d_2$  (i.e. we are at the point  $f = f^*$ ). The last term illustrates the derivative of the last-period equilibrium loss function (3.6), with respect to  $t_2$ . In the bottom row of (3.9), we have solved for the terminal debt  $d_2$  which illustrates the level of debt that corresponds to deficit being equal to  $f^*$ . This has been obtained from the last-period initial condition for deficit (equation 3.5b), by setting  $t = t_2$ ,  $f(t_2) = f^*$ , and rearranging. The solution to the control problem with a regime switch requires that the level of the state variable debt is the same before and after the switch. In other words, the terminal level of debt at the end of the period which starts immediately after the shock must be equal to the initial level of debt at the beginning of the last period, which starts from the point where the threshold  $f^*$  is crossed and the pact ceases to bind.

Solving (3.7) can now proceed by conventional means. First, we use the initial condition  $d(t_1) = d_1$  to solve for the arbitrary constant  $K_2$  in the general solution (2.7), after replacing  $\bar{f}$  and  $\bar{d}$  with  $\hat{f}$  and  $\hat{d}$ , respectively:

$$(3.10) \quad \begin{aligned} d(t_1) &= \frac{1}{2r-\alpha} K_1 + K_2 + \hat{d} = d_1 \\ \Rightarrow K_2 &= d_1 - \hat{d} - \frac{1}{2r-\alpha} K_1 \end{aligned}$$

However, the second arbitrary constant  $K_1$  must be solved by using the transversality condition (3.9). Substituting for  $\lambda(t_2)$ , from the general solution, into (3.9) gives:

$$(3.11) \quad \begin{aligned} \frac{1}{2}(f^* - \bar{f})^2 + \delta d_2 + [K_1 e^{(\alpha-r)(t_2-t_1)} + \hat{\lambda}](rd_2 + f^*) - \alpha L_2^G = 0 \\ \Rightarrow K_1 = \left[ \frac{1}{rd_2 + f^*} \left( \alpha L_2^G - \delta d_2 - \frac{1}{2}(f^* - \bar{f})^2 \right) - \hat{\lambda} \right] e^{-(\alpha-r)(t_2-t_1)} \end{aligned}$$

It turns out that  $K_1$  is a function of the optimal switching time  $t_2$ , which itself is a variable. Therefore, the solution to  $K_1$  is not complete until we have determined  $(t_2 - t_1)$ . To solve for  $(t_2 - t_1)$ , we substitute  $K_1$  (from 3.11) and  $K_2$  (from 3.10) into the general solution to the state variable (which can be obtained from equation 2.7), and evaluate at the endpoint  $t=t_2$ :

$$(3.12) \quad d(t_2) = \frac{1}{2r - \alpha} \hat{\Theta} + \left[ d_1 - \hat{d} - \frac{1}{2r - \alpha} \hat{\Theta} e^{-(\alpha - r)(t_2 - t_1)} \right] e^{r(t_2 - t_1)} + \hat{d} = d_2,$$

$$\hat{\Theta} \equiv \frac{1}{rd_2 + f^*} \left( \alpha L_2^G * - \delta d_2 - \frac{1}{2} (f^* - \bar{f})^2 \right) - \hat{\lambda}$$

Recall that the terminal state  $d_2$  is given by the bottom row of (3.9), because at time  $t_2$  debt must be equal to the next period's initial level. Substituting for  $d_2$ , and rearranging (3.12), yields:

$$(3.13) \quad \frac{\bar{f} - f^*}{2r - \alpha} + \tilde{d} - \hat{d} - \frac{1}{2r - \alpha} \hat{\Theta} = [d_1 - \hat{d}] e^{r(t_2 - t_1)} - \frac{1}{2r - \alpha} \hat{\Theta} e^{(2r - \alpha)(t_2 - t_1)}$$

Unfortunately, (3.13) is a transcendental function and therefore it is not possible to solve out  $(t_2 - t_1)$  explicitly. However, we can use numerical simulation to characterise the solution. In short, the procedure is as follows. First, we determine the value of the optimal switching time  $t_2$  from equation (3.13), under the given parameter values. This value can then be used to solve for  $K_1$  from (3.11), which together with the solution to  $K_2$  from (3.10) allows us to characterise the time paths for deficit and debt.<sup>6</sup> Using these, we can finally calculate the value of  $L_1^G *$  (equation 3.7), under the given parameter values.

Table 4.1 **Optimal Switching Time and the Equilibrium Loss under Different Penalty Levels**

	$\kappa = 0.01$	$\kappa = 0.04$
$t_2$	1.78	1.19
$L_1^G *$	0.78	5.90

The results are reported in Table 4.1. It turns out, that a higher penalty from exceeding the deficit threshold (a higher  $\kappa$ ) implies that the regime switch from the "penalty" regime to the normal regime will

<sup>6</sup> Note that because the solution to  $t_2$  (from 3.13) is a function of the initial state  $d_1$ , the initial state will enter the solution of  $K_1$  via  $t_2$ . For this optimisation problem, the initial state  $d_1$  is given. However,  $d_1$  results from the optimal choice of deficit in the first period (before the arrival of the shock), as a function of  $\kappa$ .



take place earlier. This happens because the government is, after the shock, initially in the region where the penalty binds. The penalty encourages the government to increase the adjustment effort, so that it will be able to switch to the normal regime earlier.

It remains to characterise the behaviour of the model before the shock arrives. Compared to the case in the previous section where the features of the Pact were not included, the only new aspect that arises here is how the optimal choice of deficit is affected by the size of the penalty which will apply after the predicted shock. To start with, from (3.7) we know that the equilibrium loss after the shock is a function of  $\kappa$  and  $d_1$ , the level of debt inherited from the previous period. However,  $d_1$  is itself indirectly a function of  $\kappa$ , because the choice of deficit before the shock (which in turn determines  $d_1$ ) is contingent on  $\kappa$  via the scrap value function. To find out how  $d_1$  varies when  $\kappa$  changes, we can use implicit differentiation:

$$(3.14) \quad \frac{\partial d_1}{\partial \kappa} = - \frac{\frac{\partial L_1^G * e^{-\alpha(t_2-t_1)}}{\partial \kappa}}{\frac{\partial L_1^G * e^{-\alpha(t_2-t_1)}}{\partial d_1}}$$

In (3.14),  $L_1^G *$  is the value of the equilibrium loss at time  $t_1$ . From Table 1, it can be seen that when  $\kappa$  is increased from 0.01 to 0.04,  $L_1^G *$  increases from 0.78 to 5.90. Hence,  $\frac{\partial L_1^G *}{\partial \kappa} > 0$ . On the other hand, under the same parameter values, increasing  $d_1$  from 1 to 2 induces an increase in  $L_1^G *$  from 0.78 to 1.48. Therefore,  $\frac{\partial L_1^G *}{\partial d_1} > 0$ .

Consequently, (3.14) is negative under the given parameter values. It then follows that the higher is the penalty from violating the constraint, the lower is the initial level of debt which is inherited from the first period. Because lower inherited debt implies that the government must have adjusted more before the shock, we can expect that  $\frac{\partial f(t)}{\partial \kappa} < 0$  holds in the first period. Hence, in the presence of jumps in debt, the stability pact increases fiscal adjustment before the arrival of the shock. This implies, in turn, that the jump which takes the government to a given level of debt must be the higher the more severe is the penalty (the higher is  $\kappa$ ), since the higher *ex ante*

adjustment effort, which results from a high  $\kappa$ , reduces the level of debt at which the jump takes place.

## 4 Conclusion

In this Essay we have applied continuous time constrained optimisation methods to study the optimal fiscal policy in the presence of positive debt shocks. In particular, we showed that the government's optimal choice of the control variable is contingent upon the shock even before the arrival of the shock. Therefore, a jump in debt at a certain future date tends to reduce fiscal deficits and increase debt adjustment from the beginning of the period. The additional costs which are imposed on the government due to the shock are higher for governments with high deficit targets and low rates of discount. In addition, if the government's choice of control variable is constrained, e.g. by an agreement like the EMU stability pact that penalises the government from excessive deficits, the government will optimally adjust even more before the arrival of the shock. In this way the government can minimise the time after the shock which is to be spent in the regime where the penalties apply. The pact thus works as an additional incentive for increased fiscal discipline, not only after deficit has exceeded the threshold but also before the event that causes the fiscal disturbance has taken place. The question is, then, whether the potential credibility gains from such a strict fiscal policy are sufficiently large to compensate for the loss in government expenditures. What is clear is that under the Pact the room for national fiscal policy can easily become extremely limited if the economy is characterised by large and regular shocks. Therefore, many observers have criticised the Stability Pact on the basis that it fails to account for necessary cyclical adjustments. An alternative formulation of the Pact, e.g. one which is framed around average debt levels over certain periods of time, could provide a more flexible incentive structure for the government. This issue would clearly provide an interesting avenue for future research.

# Discussion and Suggestions for Future Research

In this study we have used static and dynamic game theory, optimal control theory and principal-agent models to analyse macroeconomic policy and the optimal design of fiscal and monetary institutions.

In Essays 1 and 2, we studied the implications of inflation targeting in a model where fiscal policy responds endogenously to changes in the monetary regime. In Essay 1, the main finding was that the “performance” of the inflation targeting institution is sensitive to the private sector’s wage-setting behaviour. Clearly, this result stresses that the prevailing market structure of an economy must be well understood when the advantages and disadvantages of different institutional reforms are evaluated. Essay 2 generalised the model in a monetary union. We analysed the case where a Council, which represents the interests of countries with national fiscal authorities, delegates monetary policy to a common Central Bank (CB). If the delegation decision is characterised by an optimal inflation target, the regime encourages fiscal discipline without generating excessive welfare losses to the participating governments. In addition, we found that if the Council’s voting rules are optimally designed, inflation targeting can become an *ex post* renegotiation-proof institution. These results suggest that in monetary unions optimal institution design is likely to be more complicated than a simple delegation to an independent CB would suggest.

One obvious extension to the analysis in these Essays would be to set up a two-period model, where the fiscal authority can issue debt and thereby trade off future taxes to current output/employment (like in Beetsma and Bovenberg 1997). In particular, this would have interesting implications in the case where the trade union’s objectives give rise to a fiscal time-inconsistency problem. Another possible avenue would be to assume that the monetary authority is subjected to an intermediate policy target, e.g. a monetary aggregate which may not be fully observable to the fiscal authority, and analyse the additional information requirements that the fiscal authority faces under such a regime.

The last two Essays applied a continuous-time approach to analyse optimal policy under a dynamic government budget constraint. Essay 3 concluded that the optimal choice of the Central Bank’s preferences is contingent on the institutional regime between

the government and the Central Bank. We also found that electoral incentives can induce earlier debt adjustment if the economy is characterised by political instability. Essay 4 analysed the dynamics of fiscal policy in the presence of shocks. It turned out, that a debt shock at a certain future date tends to reduce fiscal deficits and increase debt adjustment even before the shock arrives. If the government faces penalties from excessive deficits, which may emerge as a result of the shock, it will optimally shift even more adjustment from the period after the shock to the period before the arrival of the shock.

The economic analysis in these Essays would benefit from more vigorous micro-foundations for the dynamic government budget constraint. In particular, the assumption that there is a fixed relationship between the interest rate and the growth rate of the economy is not particularly relevant in the real-world macroeconomic environment. Furthermore, it would be interesting to see how the "deficit smoothing" result in Essay 4 would change if monetary policy was included. Our prediction is that under non-cooperative strategies, a more conservative Central Bank would force the fiscal authority to even greater sacrifices in order to reduce debt before the shock arrives. Another possible extension would be to analyse the implications of different maturity structures of debt on the optimal policy.

Finally, the general tendency in the research in macroeconomic policy has recently been towards more detailed "micropolitical" foundations. For example, Persson and Tabellini (1996) and Persson, Roland and Tabellini (1997) study the implications of political processes and different political institutions on public finance. Adding elements of this literature could provide numerous interesting studies both in the static policy delegation context and in the dynamic debt adjustment framework.

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