Monica Ahlstedt

Analysis of Financial Risks in a GARCH Framework

SUOMEN PANKKI
Bank of Finland

BANK OF FINLAND STUDIES E:11 • 1998

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Abstract

This study uses GARCH modelling to estimate and forecast conditional variances and covariances of returns calculated from a set of financial market series: twelve markka exchange rates, twelve corresponding short-term euro interest rates and the Finnish short-term interest rate, the Finnish long-term interest rate, the Finnish all-share index and real estate prices.

The variances are specified through univariate estimation and the analysis is then extended to a portfolio of assets by presenting and applying two alternative methods for covariance modelling. The first method is based on the assumption of identical autocorrelation structure for variances and covariances. The other method is based on the assumption of constant correlation. Both methods are flexible and enable the extension of the analysis to a large number of return series.

The study then derives a forecast function from the models estimated from pooled data for variances and covariances of exchange rates and interest rates and from individual data for the other rates, in the form of a weighted moving average of past squared residuals. GARCH forecasts for the variances of individual return series as well as portfolios are compared in an ex post context, on the one hand, to two alternative forecasts based on piecewise homoscedastic variance models and, on the other, to actual data on squared returns.

The empirical results in the study show that the estimated variance-covariance models display a high degree of similarity both across the variables and across subsamples (ie across exchange rate regimes); GARCH(1,1) seems to represent the underlying conditional variance process fairly well. In terms of persistence in the variance processes, which is nearly IGARCH(1,1), the estimated models are also remarkably similar both for the individual variables and for pooled data. Hence parsimony suggests using an integrated process to represent volatility in the sample. The study also argues that the estimated GARCH models represent a methodological and empirical improvement over those estimates typically used eg in value-at-risk calculations.

Keywords: time-dependent volatility, GARCH estimation, value-at-risk models

Sammandrag

I denna studie används GARCH-modeller för att estimera och prognostisera såväl konditionella varianser som kovarianser för en avkastning beräknad på ett antal finansiella serier: tolv växelkurser för den finska marken, tolv motsvarande korta euroräntor, den finska korta räntan, den finska långa räntan, den finska fondbörsens generalindex och fastighetspriser.

Varianserna specificeras och estimeras individuellt och analysen utvidgas därefter till en värdepappersportfölj med en presentation och tilllämpning av två alternativa metoder för modellering av kovarianserna. Den första metoden är baserad på antagandet om en identisk autokorrelationsstruktur för varianser och kovarianser. Den andra metoden är baserad på antagandet om en konstant korrelation. Båda metoderna är flexibla och tillåter en utvidgning av analysen till att omfatta ett stort antal avkastningsserier.

Från de modeller som estimerats utifrån paneldata för valutakurserna och de korta räntorna och utifrån individuella data för de övriga serierna härleds sedan en formel för prognostisering uttryckt som ett vägt glidande medeltal av kvadraterna på tidigare perioders residualer. GARCH-prognoserna såväl för de individuella avkastningsseriernas varianser som för portföljerna jämförs i en ex post-uppföljning med å ena sidan två alternativa prognoser baserade på periodvisa homoskedastiska variansmodeller och å andra sidan aktuella data uttryckta som kvadraten på avkastningarna.

De empiriska resultaten i denna studie visar att i de estimerade varians-kovariansmodellerna föreligger en höggradig likformighet både mellan olika variabler och mellan olika delsampel (dvs. mellan olika valutakursregimer); GARCH(1,1) förefaller att väl representera den bakomliggande konditionella variansprocessen. Vad gäller graden av persistens i variansprocessen, nära IGARCH(1,1), är de estimerade modellerna också anmärkningsvärt lika. Följaktligen stöder parsimoniprincipen användandet av en integrerad process för att återge volatiliteten i urvalet. Arbetet argumenterar också för att de estimerade GARCH-modellerna representerar metodologiska och empiriska förbättringar jämfört med de estimat som typiskt används i s.k. value-at-risk-analyser.

Nyckelord: tidsberoende volatilitet, GARCH-estimering, value-at-risk-modeller

Acknowledgements

The subject of this study emerged from the need in my daily work to obtain a theoretical, methodological and empirical overview of risk measurement in banking activities. I am deeply grateful to my superiors at the Financial Supervision Authority and at the Bank of Finland for giving me the opportunity to carry out this project at the Research Department at the Bank of Finland. I sincerely hope that the end results will prove beneficial both for the further development of risk measurement methodologies applied in supervision and for evaluating the internal systems of supervised entities.

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Although it is a great privilege to be able to concentrate on scientific research, it is not always like being in a rose garden. It is sometimes more like a valley of tears. Without the support of my friends and colleagues –

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Helsinki, May 1998

Monica Ahlstedt

Contents

1 Introduction

The increased importance of risk measurement of portfolio investments can be seen in the emergence of a special class of variance -based models for financial applications, the value-at-risk (VaR) models. The VaR framework provides a means to measure and forecast the amount of value, as a function of variance, that can be lost or gained in a portfolio due to changes in market prices of the underlying assets. The VaR models have become popular among firms and financial institutions, and even supervisory bodies have accepted them as a tool for calculating risk-based capital requirements.

There are however methodological shortcomings in the main VaR applications. The most aggravating are that (i) the sample variance formula for the returns on assets and portfolios is mostly not based on statistical inference but rather on an ad hoc procedure; (ii) the assumption of a normal distribution is typically not supported by the data; and (iii) the use of the sample variance in forecasting has no methodological justification.

High frequency data on financial return series exhibit deviation from normality in the frequency distribution, mostly in the form of leptokurtosis. This feature can be modelled in the GARCH framework as a non-constant time-dependent variance. There is a vast amount of literature on GARCH applications, but almost all of it deals with the univariate approach and only a few series. In this literature, portfolio applications are rare; the number of parameters to be estimated in the general multivariate model is too large to be of any empirical use beyond the bivariate case.

The purpose of this study is to provide modifications to the VaR framework that correct for the aforementioned methodological shortcomings by estimating the variance of portfolios of the main financial assets in a GARCH framework. We first apply univariate GARCH modelling to a number of financial return series, including exchange rates, short- and long-term interest rates and equity and real estate prices. The analysis is then extended to a portfolio application by presenting and applying two alternative methods for covariance modelling. The first method is derived in this study and is based on the assumption of identical autocorrelation structures for variances and covariances. The other method is based on the assumption of constant correlation, and here we reconstruct an originally iterative estimation process as a two-stage procedure, which enables extension of the analysis to a large number of return series. Statistical testing supports the assumptions underlying both methods.

The GARCH methodology provides models of the variance which not only coincide with the data but are also well suited for forecasting since the GARCH model for the time-dependent variance is itself in the form of one-step-ahead forecast.

We then compare forecasts based on our estimated time-dependent GARCH variances to sample variance-based forecasts for two types of portfolios: one with weights roughly mirroring average portfolios of Finnish banks and the other with the weights of a global minimum-variance portfolio. A VaR application for these portfolios completes the comparison.

The main contributions in this study are that

- through GARCH modelling we have achieved the methodologically best estimates of the variance in the maximum likelihood context by accounting for the non-normal features in a large number of financial return series

- we end up with standardized GARCH residuals, which are theoretically normal, thus allowing the use of normal confidence statements in VaR applications

- we have extended the univariate approach that is generally applied in the literature to a portfolio approach by developing two flexible and easily applicable methods for covariance estimation for a large number of asset returns

- the estimated parameter structure for the individual series and the pooled data allows for an integrated GARCH interpretation of most series covered in this study. This greatly simplifies the forecasting formulas for the portfolio variance. Although the integrated GARCH model has complicating features in the long-run context, it appears to perform well in the short run

- our VaR-model comparisons show that the use of sample variances overestimates the risk in portfolios compared to GARCH variances, which is especially important when a VaR model is used for calculating capital requirements

- our estimated GARCH model, which is also the forecasting formula, is very close to the more ad hoc exponentially weighted sample variance formula used in the RiskMetrix by J.P. Morgan. This application is widely used and forms a benchmark in financial applications. Our results thus provide a methodological basis for the use of their approximating formula for calculating the variance and also for using it as a forecast. The RiskMetrix has been criticized for applying the same formula for all instruments. Also in this respect, our results support the approach in the RiskMetrix, since we were able to show that the estimated parameter structure for the integrated variance is also relevant for returns from changes in the main series, ie exchange rates, short-term interest rates and equity prices, but not for changes in the long-term interest rate.

In addition to serving the prime purpose of providing the methodological basis for estimating and forecasting the portfolio variance in VaR models, the results of this study are also of general interest and can be utilized in other areas of financial economics, where an appropriate estimate of variance, ie coinciding with the data, is of utmost importance.

Although the methodological reasoning in this study has general applicability, the empirical part is estimated from the viewpoint of a Finnish investor. We deal with twelve markka exchange rates, the long-term Finnish interest rate, the Finnish all-share stock index and Finnish property prices. Of the thirteen short-term interest rates included, one is the Finnish short-term rate and the others are Euro rates and thus relevant also for investors in other countries. Our results on exchange rates and equity prices are largely in line with results from univariate studies on dollar and sterling exchange rates and with results for share indices in other stock markets. We therefore have reason to believe that the findings in this study can contribute to further development of variance-based risk-measuring portfolio models for use by global investors.

1.1 General background

Increased volatility of financial asset prices during the 1980s and 1990s has raised important questions regarding both the measurement of volatility and its causes. Causes have been sought specifically in such factors as deregulation, internationalization of portfolios, new hedging instruments, and macroeconomic policies. The increased risk due to asset price volatility has in turn both micro- and macroeconomic consequencies, as it effects the allocation of financial resources via investment decisions of both individual and institutional investors and can threaten the stability of financial markets.[1]

Increased volatility underscores the importance of an appropriate and well-defined measure of risk that can be priced by asset markets. Defining risk as the variance of a probability distribution of returns is a central statistical concept in financial economics. Some of the key areas where variance is used as the measure of risk are derivatives pricing, evaluation of hedging strategy and risk premium identification. The increasing importance of risk measurement can be seen in the emergence of a special class of variance-based models for financial applications, the value-at-risk (VaR) models. The VaR framework provides a means to measure and forecast the amount of value, as a function of the variance, that can be lost or gained in a portfolio due to changes in market prices of the underlying assets.[2] The VaR models[3] have become popular among firms and financial institutions, and even supervisory bodies have accepted them as a tool for calculating risk-based capital requirements for supervised entities.[4]

If we choose to adopt variance as the measure of risk, we need to identify the distributions of financial returns. The standard way to model stochastic processes of price changes for financial assets, ie return series, in VaR models and in financial applications generally, is based on the assumption of a random walk process. This assumption implies that the price changes are independent and identically distributed, that their expected value is zero and that the variance is constant over time. The auxiliary assumption, that the changes are normally distributed, is usually added to the others, thus making the generating process a Brownian motion (Mill 1993, p. 1–2). The normality assumption is in general not necessary from a theoretical standpoint. It is adopted rather for statistical convenience, since it considerably simplifies both the estimation and the

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[1] See BIS (1996) for an extensive collection of papers on volatility in the main financial market instruments and countries.

[2] Simons (1996) provides a good description of the methodology and main applications in the vast literature on VaR models.

[3] There are three main methodological approaches: analytical, historical and simulation. These approaches are represented in the commercially distributed packages: J.P. Morgan's Riskmetrics, Bankers Trust's RAROC2020 and Chase's RISK$. The internal applications developed within corporations and financial institutions are essentially variations on these approaches. The estimation of variance in these approaches is not based on statistical inference but rather on ad hoc procedures. These points are elaborated further in section 8.5.

[4] Proposal of the Basle Committee on Banking Supervision (1995) and the Capital Adequacy Directive (1995) adopted by the European Commission.

forecasting. It justifies the use of multiples of the standard deviation around the expected mean to get the confidence level for the point forecast. Combined with the assumption of independence, it also enables calculation of the forecasted cumulative standard deviation over a finite holding period by multiplying the base-period standard deviation by the square root of the length of the holding period.

The assumption of a zero mean is in general confirmed by empirical findings suggesting that the true mean is close to zero (Figlewski 1994). This is also consistent with the hypothesis of market efficiency, which states that the random movements in returns cannot be forecasted. However, the constant variance and independence assumptions of the random walk process are contrary to the stylized facts found in empirical realizations of many financial return series. Typical stylized facts that are found in high frequency data but which tend to recede with time aggregation are fat tails and peakedness around the mean (leptokurtosis) in the unconditional distribution. These features carry over to the time series interpretation as clustering of small and large changes. This clustering means that the price changes are not independent. Deviation from normality and time dependence in the first and second moments render inappropriate the generally accepted measurement and forecasting based on the normally distributed random walk.

The family of generalized autoregressive heteroscedastic (GARCH) models has been developed (the seminal papers are Engle 1982 and Bollerslev 1986) to cope with these features. In these models, the return formation process is defined as a martingale, which means that the stochastic realizations are uncorrelated but not necessarily independent. The dependency is expressed in the conditional variance equation, which models conditional variance as a function of past variances and squared returns.

The need for an estimation method that accounts for the special features of the probability distributions of return series is widely recognized and the use of GARCH has increased extensively in academic research.[5] For practical applications, as in VaR models, the method has however been considered to be too laborious (J.P. Morgan 1995) or to generate unsolvable problems, above all in connection with covariance estimation, for rate sets beyond the bivariate case (Alexander 1995).[6]

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[5] Surveys by Bollerslev, Chou and Kroner (1992), Bera and Higgins (1993) and Bollerslev, Engle and Nelson (1994) already report more than a hundred papers on GARCH estimation of the variance of exchange rates, share prices and interest rates.

[6] The difficulties in achieving convergence (ie positive definiteness of the matrices) in a multivariate approach increase with the number of parameters estimated.

In this study, we apply univariate GARCH modelling to a number of financial return series, including exchange rates, short- and long-term interest rates and equity and real estate prices, to account for the stylized facts. We show that the appropriate GARCH methodology actually simplifies practical application since it replaces the usual ad hoc parameter selection with a uniform selection method derived from the data itself. The analysis is then extended to a multivariate framework to include the covariances. We show that the problems with the multivariate approach can be solved in a parsimonious and pragmatic way. We present two multivariate covariance estimation methods, both of which provide a flexible format that allows for easy addition or removal of instruments to/from the portfolio, with no need for re-estimation.

With the GARCH methodology, we not only take account of serial correlation but we also solve the problem of fitting a normal distribution to the data. GARCH parameterization produces standardized stochastic processes, which by definition are normal and in empirical work much closer to normal than are the raw data. This means above all that the standard confidence intervals can be attached to the point forecasts. The GARCH interpretation is also well suited for forecasting, since the parameterization of the volatility is expressed in the form of a one-step-ahead prediction.

In order to limit the scope of financial rates to be covered in this study, we have chosen to restrict the discussion to risks that are relevant to banking.[7] There are two main reasons for this choice. First, banks are the main users of VaR models and, second, there is an ongoing discussion among regulatory bodies about the suitability of these models for calculation of risk-based capital requirements, and the results from this study can serve as input to this discussion.

The ideal situation for banks would be to have an integrated risk management procedure, in which all risk categories would be measured using the same VaR methodology. VaR models have so far been used only for market risk measurement. In this study, we extend the methodology to real estate risk. Attempts to measure credit risk[8] (Oda and Muranga 1997) and operational risk (Wilson 1995, Ho 1997) in the VaR framework have already appeared in the literature.

[7] The framework is as such also directly applicable to insurance companies.

[8] J.P. Morgan has recently (April 1997) launched a VaR application for credit risk measurement. See also Ho (1997) for applying a VaR model to operational risk.

The main issue in the ongoing discussion between regulators and financial institutions[9] is how the measurement of risk exposures and calculation of capital requirements can be uniformly parameterized to yield results that are as similar as possible across the various VaR model applications. The VaR models differ in approach but also in the parameterization. Our results can be used to improve the theoretical and empirical foundation of VaR applications in regard to the distributional properties of the relevant return series. They also provide for a uniform way of selecting parameters, which are identified from the data itself by statistical inference and applicable to different models, as an alternative to the usual more or less ad hoc approach. The key parameters are the sample length and weight structure of the historical observations used in forecasting variances and covariances of returns on various instruments.[10] Consequently, the results from this study will be utilized in the VaR model developed at the Bank of Finland (Ahlstedt 1990) for use by the supervisory authority in the measurement of market risk in Finnish banks.

Although the risk areas considered are limited to those relevant to banking and the main application is with VaR models, the results found here on risk measurement as applied to financial return series are of general interest and can be utilized in other areas of financial economics, where an appropriate estimation of variance as a risk measure is of prime importance.

1.2 Banking risks

In the traditional literature on the theory of intermediation,[11] optimal behaviour of a bank is defined as the attempt to maximize an objective function in terminal wealth. The general form of the objective function can be specified according to three distinct perspectives on the balance sheet: asset allocation modelling, liability choice modelling and two-sided balance sheet modelling. According to the approach of the two-sided

[9] The main ideas are presented in a proposal by the Basle Committee on Banking Supervision published in April 1993. The Committee has, since receiving comments on this proposal, continued the discussion of parameterization and comparability of model-based risk calculations in a number of consultative papers.

[10] See eg Jackson, Maude and Perraudin (1995) for an evaluation of the sensitivity of VaR calculations to selected window length and weighting schemes.

[11] See eg surveys by Pyle (1971), Baltensperger (1980) and Santomero (1984). See also Yanelle (1988, 1989) for a strategic analysis of intermediation.

nature of the banking activity, a bank collects funds from lenders in order to redistribute them among final borrowers. In doing so, the bank exploits economies of scale by reducing risks via diversification and reducing transaction and information costs. Summarizing the functions, it can be said that a typical bank serves as both an asset transformer and a broker, ie the main functions in banking are intermediation and investment. The risks inherent in these two basic functions can, for our purposes, be broken down into five main categories: strategic risk, operational risk, market risk, real estate risk and credit risk.[12]

Strategic risk is the risk of selecting the wrong strategy in trying to maximize shareholders wealth. For example, strategic risk at the company level might involve choosing the wrong strategy in overall asset-liability management, in the sense that it causes so much uncertainty as to future asset values and profits that the bank's solvency is threatened.

Operational risk covers such areas as deficiencies in information or other systems or in internal routines that may result in unexpected losses. Additional categories of operational risk are legal risk, management risk, and the risk of fraud or theft or of overstepping of authority or competence by employees.

Market risk can be broken down into exchange rate risk, interest rate risk and equity price risk. In our framework, liquidity risk is also included in this risk category, the reason being that since funding for short-term liquidity needs is always available to banks either in the money markets or from the central bank, the risk is related to the price. This means that liquidity risk gets transformed into interest rate risk.

Credit risk is the uncertainty regarding borrowers' ability to fulfil commitments to repay loans. Insolvency can be caused by individual circumstances concerning borrowing companies or households or by an economic recession resulting in a deterioration of a bank's entire loan portfolio.

Analysis of strategic risk and operational risk fits naturally into the two-sided specification of the general objective function while market risk, real estate risk and credit risk fit into the portfolio risk-return modelling within the asset allocation approach. These risk categories

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[12] The common breakdown of banking functions into different types of risks does not include real estate risk. Although investment in real estate is not a typical business activity in banking, it is included in our analysis due to the growing stock of real estate holdings in banks' balance sheets. This stock consists mainly of redeemed collateral acquired in cases of realized credit risk. Another reason for including real estate prices in the risk categories is that in credit risk assessment estimation of future collateral value of the outstanding stock of loans is important.

differ from each other not only in this applied analytical approach but also in their measureability. Risk is strictly defined as randomness with a known probability distribution (Knight 1921). If the probability distribution is not known, we talk about uncertainty. Randomness in the case of banks' activities lies in future asset values and financial returns. Market risk, real estate risk and credit risk are measurable as outcomes of a statistical probability distribution and hence fulfil the strict definition of risk. Strategic risk, on the other hand is considered as uncertainty because probabilities cannot be attached to future outcomes. Operational risk can be considered as partly measurable risk, partly uncertainty.

The general form of the objective function for terminal wealth of either shareholders or management allows for distinct types of strategic behaviour.[13] The main aspect of strategy selection can be seen to be the allocation of assets or exposures to market risk, real estate price risk and credit risk. The allocation of assets can be based on the mean-variance strategy, which is one type of behaviour within the asset allocation modelling approach that is allowed for by the general objective function. The mean-variance approach utilizes the quantitative characteristics of the economic variables that generate the risks and allows for risk measurement with statistical distributional properties by linking the concept of risk-return to the concept of mean-variance. The expected return (ie the change in financial rates) is measured by the mean and the risk by the variance of the probability density distribution. This is the framework adopted in this study to measure market risk and real estate risk. Credit risk and operational risk, are not considered since the application of the mean-variance methodology to these risks categories has not yet been developed.

The mean-variance approach in the theoretical explanation of banks' behaviour connects portfolio theory to our framework. In order to measure the mean-variance composition of market risk and real estate exposures in banks' portfolios, the mean and variance parameters of the statistical univariate distributions of exchange rates, interest rates, share prices and real estate prices must be estimated. Because economic theory suggests that these variables are interrelated,[14] so too are the risks. Changes in the variables not only affect market risk and real estate price risk but also credit risk and strategic risk. An assessment of the total risk in a bank's portfolio should therefore also consider interactions not only

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[13] An alternative approach, based on game theory, has been suggested by Yanille (1988, 1989).

[14] Relevant theories are monetary theory, portfolio theory and purchasing power parity theory (see Dornbusch 1980).

within risk categories but also between risk categories. For those risk categories that can be measured by the variances, ie market risk and real estate risk, the interactions between risks can be measured by the covariances. A necessary requirement for identification of these covariances is that the economic variables behind the risks display behaviour within and between groups that is sufficiently uniform to fit them into a single type of stochastic process model.

The theoretical evaluation of the interaction between risk categories other than market risk and real estate risk, ie between strategic risk, operational risk and credit risk, must be based on methods other than the mean-variance approach.

Correlations between risks can also be viewed in a purely practical way. For example, one might note a correlation between credit risk and market risk. If the exchange rate risk in lending in a foreign currency is hedged by borrowing in the same currency, the realization of the credit risk, when a counterparty fails to meet his obligations, also nullifies the currency hedge. The same interdependence can be found between credit risk and interest rate risk. The realization of operational risk in the form eg of mispricing affects all other risks. The effects of realized strategic risk can nullify all measures taken to hedge or manage market risks.

The mean-variance approach in banking theory justifies the use of the variance as a measure of risk. Via the connection to portfolio theory, not only is the covariance concept included but also the portfolio composition,[15] ie the portions invested in the different risk categories. The available investment strategies in the mean-variance framework are all the portfolios on the efficient frontier. The global minimum-variance portfolio is a unique point on the frontier and thus a possible choice.

In the portfolio approach in this study, we choose the portfolio weights in two ways. First, we estimate the variance of a fictive portfolio with weights given by minimum-variance optimization. Secondly, we estimate the variance of a portfolio containing the same instruments but with weighting that roughly mirrors the average shares in the trading portfolios of Finnish banks.

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[15] Portfolio composition decisions can be based on capital asset pricing theory or portfolio theory. Portfolio theory considers how an optimizing investor would behave, whereas capital asset pricing theory is concerned with economic equilibrium assuming that all investors optimize in that particular manner (Markowitz 1991).

1.3 Modelling financial returns

There is a methodological dichotomy in the literature on modelling financial returns. Economic theory-based models are used to explain structural dependencies between macroeconomic variables, and time series analysis is used to explain the historic behaviour of the variables themselves.[16]

In the first approach, relationships among economic variables, the possible changes, which are the causes of the financial risks under consideration here, can be modelled according to alternative economic theories. These models express stationary, long-run equilibrium relationships among economic variables. According to Dornbusch (1980)[17] there are essentially three approaches to exchange rate determination, which also include the links to interest rate determination

- the monetary approach, which treats the exchange rate as the relative price of money
- the portfolio balance approach, which treats the exchange rate as the relative price of bonds
- the purchasing power parity approach, which treats the exchange rate as the relative price of goods.

Each of these approaches taken alone provides an incomplete theory since each gives only a partial picture of the exchange rate determination mechanism. A broader picture can be built on the integrated application of the models.

The monetary approach to exchange rate determination asserts that exchange rates are determined by supplys and demands for national money stocks. The major content of this approach is that national monetary policies exert the primary influence on exchange rate movements. Like the monetary approach, the portfolio balance approach asserts that exchange rates are determined by the interaction of supply and demand for financial assets. The portfolio approach broadens the more narrow monetary approach by extending the set of financial assets that influence exchange rates beyond relative money supplies to include relative bond supplies. The two approaches also differ in their assumptions regarding the degree of asset substitutability. The monetary

[16] In this context we apply the narrow interpretation of time series analysis, where no exogenous variables are included. The typical example is the ARIMA process.

[17] See also Pentecost (1991) and Taylor (1995).

approach assumes perfect substitutability while the portfolio balance approach admits imperfect substitutability between domestic and foreign bonds. These asset market exchange rate models pay little attention to the goods market. They assume that output is fixed at its full employment level and that price levels are fully flexible. The assumption of price flexibility leads to the connection with the purchasing power parity theory, ie that exchange rates adjust instantaneously to equate the relative prices of domestic and foreign goods, which is an equilibrium condition in asset market theories.

The macroeconomic structural models built on the monetary asset theories are estimated at low frequency, ie quarterly or annually, in line with the available frequencies of macroeconomic data such as GDP and inflation. Therefore these models are well suited for long-horizon analyses and forecasting, eg for banks' strategy planning. Financial time series, eg exchange rates and interest rates, are recorded daily or even hourly. In structural models (by definition medium- or long-horizon models) in which data on macroeconomic variables and financial rates are combined, a lot of financial data remains unused.

The second methodological approach, time series analysis, utilizes the available high frequency data. Time series analysis is used for modelling short-run movements in financial rates along the short-run adjustment path toward equilibrium. In the short run, exchange rates, interest rates and share prices are determined largely by expectation-based speculative flows, rather than by gradually evolving changes in macroeconomic fundamentals. Therefore, the short-run analysis applied in this study concentrates on time series modelling of nominal rates characterized by noisy changes caused mainly by news arrival processes.

In economic models the first moment (mean) of the levels or changes in economic variables is modelled and forecasted. The time series approach allows for analysis of the special stylized facts regarding the second moment in high frequency data on financial time series. These features fade with temporal aggregation and are therefore lost in low frequency economic models. Although we know that the linear random walk model does not fit observed data perfectly, these models have been found to dominate both structural marcoeconomic models and more sophisticated time series models in accuracy in predicting the mean (Meese and Rogoff 1983, Nerlove, Diebold, van Beeck and Cheung 1988, Marsh and Power 1996). Modelling of the second moment has produced strong empirical evidence that autoregressive conditional heteroscedasticity models are adequate to capture the leptokurtosis in the unconditional distribution (Nerlove et al 1988). The family of autoregressive conditional heteroscedasticity (ARCH) models was introduced by Engle (1982) and later generalized by Bollerslev (1986) to GARCH. Further applications, such as IGARCH, GARCH-M and EGARCH, have been developed to capture both linear and nonlinear dependencies in first and higher moments.

The body of academic research on ARCH has grown extensively since the seminal paper of Engle.[18] Although the implementation of ARCH represents a giant methodological step forward, there have been doubts about its ability to capture all the nonlinearities in time series. The alternative approach to modelling the stylized facts in financial returns is to use stochastic variance (SV) models (Hull and White 1987, Taylor 1993).[19] The GARCH and SV methodology can be combined by including an unobserved stochastic component in the conditional variance equation. Hsieh (1988), for example, concludes that time-varying means and variances in a GARCH model are not sufficient to fully account for the leptokurtosis in exchange rates but that a stochastic variance GARCH model with time-varying parameters can explain the nonlinearity of the data (Hsieh 1991). Andersen (1996) developed a 'mixture of distribution hypothesis' (MDH) model by combining a stochastic volatility process and generalized standard ARCH specifications and found that the model can be useful in analysing the economic factors behind the volatility clustering that has been observed in returns.

It is possible that even the ARCH and SV models and mixtures of them are too simple to capture the nature of the stochastic processes driving the financial markets. This leads to the methodology of complexity and chaos, which has been applied in numerous scientific fields.[20]

Abrupt and extreme changes in financial time series in particular, such as those associated with the stock market crash of 19 October 1987, have fostered the idea of extending the methodology of explaining time-dependence in volatility in financial data to deterministic chaotic dynamics. In time-series models such as Box-Jenkins and the GARCH family, the economy has a stable momentary equilibrium but is

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[18] See Bollerslev et al (1992, 1994) for an extensive survey.

[19] See Harvey, Ruiz and Shephard (1994) for a comparison of SV and GARCH models. The properties in the two types of models are shown to be very close. Nelson (1991) expounds the view that the stochastic volatility specifications can be interpreted as continuous time analogues of exponential GARCH models. See also Spanos (1994) for a derivation of a student's t autoregressive model with heteroscedasticity (STAR).

[20] See Tong (1994) for views on how the field of deterministic chaos has become a multi-disciplinary area of research and how this new paradigm has much to offer also to the statistical analysis of nonlinear time series in economics. Examples of applications to financial markets are Hsieh (1991) and Sengupta and Zheng (1995).

constantly being perturbed by external shocks. The behaviour of economic time series comes about as a result of these external shocks. In chaotic models the time series follow nonlinear dynamics, which are self-generating and never die out. The fact that the fluctuations in financial time series, according to this theory, can be internally generated is very appealing, especially since it has been very difficult to find a theoretical framework in economics that will explain the GARCH approach to modelling nonlinearity.

One can search for chaos using the method of correlation dimension proposed by Grassberger and Procaccia (1983). This method requires large data sets, which are available in the natural sciences but not to the same extent in economics and finance. The method also lacks a statistical theory for hypothesis testing (Hsieh 1991). Brock, Dechert and Sceinkman (1987) have developed a related method from the correlation dimension, called the BDS statistic. It tests the null hypothesis that a time series is IID against an unspecified alternative using a nonparametric technique. This statistic has been shown to have good asymptotic and finite sample properties and good power to detect chaotic behaviour and most types of nonlinear structures. The BDS test will be used in this study as part of the routine used to test for misspecifications of the estimated models.

1.4 Outline of the study

This study applies time series techniques developed for modelling heteroscedasticity in the conditional variance to obtain measures of risk in the main financial return series from the viewpoint of a Finnish investor.

Two techniques have been particularly successful in modelling time-dependent variance: stochastic variance models (SV) and autoregressive conditional heteroscedasticity models (GARCH). Studies in SV modelling have only been done for a few financial series. The identification generally results in different models for different instruments and no solution to the problem of covariance estimation has been provided (Harvey, Ruiz and Shephard 1994, Lopez 1995). However, one of the aims of this study is to try to fit the same model to different financial return series, above all because this allows for covariance calculations. Empirical studies[21] show that the GARCH(1,1) process is able to represent the majority of financial time series and

[21] Surveys by Bollerslev et al (1992, 1994) and Bera and Higgins (1993).

therefore the GARCH(1,1) framework is used in this study to model the stochastic processes driving the variances and covariances of preselected financial return series. The estimated model also turns out to work well for market risk at daily frequency and real estate risk at monthly frequency.

The GARCH(1,1) model is identified for the individual variances for a group of twelve markka exchange rates, a group of thirteen short-term interest rates, the Finnish long-term interest rate and the Finnish all-share index.[22] The covariances are calculated using two alternative methods. With the first method, the estimated parameter structure of the univariate conditional variances is extended to the conditional covariances. The applicability of this method is founded on statistical tests of dependence between the autocorrelation structures of the variances and covariances. The second method is a two-stage version of the restricted multivariate estimation procedure proposed by Bollerslev (1990).

Although the theory of general multivariate GARCH estimation is developed for simultaneous assessment of the full covariance matrix, the large number of return series included in this study means that there are too many parameters to be estimated to allow for general multivariate estimation.[23] In fact, the number of parameters in the general model is so large as to obviate its empirical usefulness, even with a small number of variables. Methods of restricting the parameter space include requiring diagonality of the parameter matices (Bollerslev, Engle and Wooldridge 1988) or carrying over the GARCH effects into common factors affecting both variances and covariances (Diebold and Nerlove 1989, Engle, Granger and Kraft 1984). In the restrictive multivariate approach (one of the two approaches applied here), which was developed by Bollerslev (1990), the estimation and inference procedures are simplified by the assumption of constant correlations between the stochastic processes. Although this seems like a strong assumption, numerous studies have shown it to be empirically reasonable.[24] In this study, we apply a two-stage version of the Bollerslev method in estimating conditional

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[22] The Helsinki Stock Exchange Index (HEX) is a price index (excluding dividends). See Hernesniemi (1990) on the composition of the index.

[23] See chapter 2 on the dependence between the number of variables and parameters to be estimated.

[24] See eg Baillie and Bollerslev (1990), Schwert and Seguin (1990), Cecchetti, Cumby and Figlewski (1988), Kroner and Classens (1991), McCurdy and Morgan (1991), Ng (1991), Malliaropulos (1997) and a number of unpublished manuscripts mentioned in the review of Bollerslev et al (1992). Sheedy (1997) shows that the assumption of constant correlation is plausible when autocorrelation in variance is adequately accounted for.

variances. The assumption of constant correlations within structurally homogeneous time periods, ie periods with the same exchange rate regime, will also be shown to pass statistical tests.

An alternative solution would have been to select only a few exchange rates and corresponding interest rates, which might have enabled use of a general multivariate estimation method.[25] However, the aim was to derive results for real life implementation, which means considering investments in more than a few major currencies and interest rates. The number of selected exchange rates and corresponding interest rates is already limited in that only the main currencies used by banks are included. The number of rates within the groups is however large enough to enable comparison of patterns within the European ERM, the group of Nordic rates and the freely floating rates in the US and Japan. The selected multivariate applications for the rates in our analysis produce two alternative solutions as to how the covariances can be estimated from univariately-determined variances. The loss in efficiency in our two-stage applications has to be weighed against the advantage that both these methodologies are very flexible, compared to the other restricted multivariate solutions, in that the number of return series included in the system or portfolio can be increased or decreased case by case without requiring re-estimation.

In the univariate estimation the same methodology, GARCH(1,1), is applied to daily changes in all rates, since empirical studies show that the GARCH(1,1) process is the best model in the sense of maximum likelihood for virtually all financial rates.[26] The estimation period, 1 January 1987 – 31 December 1995, was divided into three nonoverlapping subperiods to account for structural changes triggered by realignments in the Finnish markka. Prefiltering of the data was applied when necessary to remove linear dependence. Prior to model specification, unit root tests were applied to ensure stationarity of the mean. Next the mean equation identification was performed and the parsimonious GARCH(1,1) model was estimated for all rates. The goodness of fit was evaluated using BDS statistics and the usual statistical tests. The method of principal components was used to detect common factors driving the high-frequency stochastic processes. Spectral analysis was performed to identify length and regularity in the cyclical behaviour

[25] To our knowledge, illustrative empirical applications have been done only for the bivariate case.

[26] The GARCH-M(1,1) model was also tested but was rejected on statistical grounds. The outcomes of the statistical testing are presented in connection with each group of included rates.

of the estimated conditional variances and their principal components. Finding a dominant frequency would justify, at least for the purpose of comparison, the use of a random walk model with constant variance in a sample with the number of observations depending on the frequency. Since there turned out to be a pronounced likeness in the univariate estimation results within groups of rates, GARCH estimation on pooled data was applied to force the rates within groups into the same model.

The estimation procedure for monthly equity risk was much like that used for daily market risk. However, because of the lower frequency, neither division of the time period nor BDS testing were feasible.

Next a portfolio application was done to produce an empirical example of how the separately estimated risk categories are to be treated when they are combined in a portfolio. Since the portfolio is constructed for illustrative purposes, we included only four instruments, but the methodology can be generalized to more extensive portfolios, especially since empirical risk measures are derived in this study for twenty-seven instruments. The instruments included in the portfolio are two short-term money market instruments, entailing both interest rate risk and exchange rate risk, one long-term bond and Finnish equities.

Following the introduction, the study proceeds as follows. The alternative applications of ARCH modelling of the stylized facts of financial time series are described in chapter 2. Chapters 3 and 4 cover the market risk-generating return series at daily frequency. In chapter 3 we deal with markka exchange rates and in chapter 4 with short-term interest rates, the long-term interest rate, the all-share index and real estate risk. Chapter 5 summarizes the variance estimation results.

In chapter 6 the multivariate nature of the system of market rates is accounted for by estimating the covariances within and between groups of rates. Based on the estimation results with pooled data, formulas for conditional ex post and ex ante forecasting are developed in chapter 7 for variances and covariances, both within and between groups for the individual financial return series. Forecasts generated by the estimated heteroscedastic GARCH models are compared to forecasts generated by homoscedastic models with different window lengths. A portfolio is then composed so as to include the main risk areas, with weights determined first via minimum-variance optimization and second using reported historical average shares in Finnish banks' trading portfolios. The forecasting results are compared between models at monthly frequency. In chapter 7 we also clarify how the estimation results in this study can be used to improve the methodological foundation and unify the parameter selection in value-at-risk applications. Chapter 8 includes the summary and concluding remarks.

2 ARCH modelling of financial data

The stylized facts of the observable behaviour of financial return series are well documented. In the graphical interpretation of the time series, a typical feature is that large and small changes are each clustered over time. The clustering is reflected in the frequency distribution as fat tails, resulting from outliers of either sign and leptokurtosis due to the centring of small changes around the mean. In time series analysis the family of autoregressive conditional heteroscedasticity (ARCH) models have been developed, starting with the seminal paper of Engle (1982), to account for the clustering by explicitly modelling time variation in the second and higher moments of the conditional frequency distribution, which is assumed to be normal.[27] The assumption of a normal density function is convenient in that it enables probability statements about the conditional variance. In the ARCH models heteroscedasticity is treated as an intrinsic quality of the data, which has to be modelled, in contrast to econometric analysis, where heteroscedasticity is interpreted as a sign of model misspecification.

The ARCH approach has been used not only in modelling the time series interpretation of key financial return series, such as changes in exchange rates, interest rates and share prices, but also to test financial theories by introducing the concept of time-variation. ARCH modelling of time-dependence in risk premiums has been used in testing for market efficiency as well as in modelling time-variation in asset betas in testing capital asset pricing theories.

Bollerslev et al (1992 and 1994) and Bera and Higgins (1993) provide excellent overviews of the state of the art in ARCH modelling, which will not be repeated here. Only the main areas applied in the empirical part of this study are highlighted below.

In the seminal paper by Engle (1982) a discrete time stochastic process (ε_t) is defined as an ARCH model of the form

$$\begin{aligned} &\varepsilon_t = z_t \; h_t^{1/2} \\ &z_t \text{ IID}, \; E(z_t)=0, \; \text{var}(z_t)=1, \end{aligned} \qquad (2.1)$$

[27] See chapter 5 for a discussion of nonnormal distributions.

where h_t is a time-varying, positive and measurable function of the information set at time $t-1$. By the IID assumption, ε_t is serially uncorrelated with the zero mean. The conditional variance of ε_t equals h_t and may change over time. Engle (1982) suggests a time-dependent parameterization for h_t:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \tag{2.2}$$

where $\alpha_0 > 0$ and $\alpha_i > 0$ for all i. Constraining the parameters to non-negative values is necessary to ensure that the conditional variance, h_t, is always positive. The variance h_t is expressed as a linear function of past squared values of order q in the ARCH(q) model. From the ARCH parameterization of h_t, it follows that the stochastic process in the mean equation (2.1) is not a random walk but is a martingale, which rules out correlation but allows for dependence in ε_t. The time-dependent formula for the conditional variance captures the tendency toward volatility clustering that is found in financial data. The α_i parameters measure the persistence of shocks in the system.

The order of the process, q, can be based on model selection tests such as that based on the autocorrelation function of the squared residuals, and many applications of the linear ARCH model use a long length. With large q, estimation will often lead to violation of the nonnegativity constraints on the α_i's. Bollerslev (1986) introduced the generalized ARCH (GARCH) model to provide an alternative and more flexible lag structure.

In a GARCH(p,q) model the h_t follows the process

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}, \tag{2.3}$$

where $\alpha_0 > 0$, $\alpha_i > 0$ and $\beta_i > 0$ for all i. The conditional variance, h_t, depends linearly on the past behaviour of the squared values in an autoregressive AR(q) process and on past values of the conditional variance itself in a moving average MA(p) process. The persistence of shocks in the model is given by the sum of the parameters α_i and β_i.

By repeated substitution, it can be shown the GARCH model is simply an infinite-order ARCH model with exponentially decaying weights for large lags. A high-order ARCH can therefore be substituted by a low-order GARCH model, thus diminishing the problem of estimating many parameters subject to nonnegative constraints. GARCH(1,1) corresponds to a high-order ARCH of the form

$$h_t = \frac{\alpha_0}{(1-\beta_1)} + \alpha_1 \sum_{j=1}^{\infty} \beta_1^{j-1} \varepsilon_{t-j}^2. \tag{2.4}$$

Finding the optimal values for p and q can be facilitated through time series testing procedures. The parsimonious GARCH(1,1) model has proven to be an adequate representation for most financial series (Lamourex and Lastrapes 1990).

The conditional variance formula (2.3) can be interpreted as a one-step-ahead forecast expression. For the GARCH(1,1) model, the s-step-ahead forecast can be written as

$$E_t(h_{t+s}) = \sigma^2 + (\alpha_1 + \beta_1)^{s-1} \{h_{t+1} - \sigma^2\}. \tag{2.5}$$

Thus the GARCH model not only captures the clustering feature in the data but also, as this equation shows, encompasses the mean-reverting process also found in empirical observations on financial returns.

A common finding in empirical studies where the GARCH model has been applied to high-frequency data is that the shocks to the variance are highly persistent: ie the sum of the parameters α_i and β_i is close to one. Engle and Bollerslev (1986) define the class of models in which the parameters sum to one as being integrated in variance or IGARCH. Nelson (1990b) has provided an explanation for the empirical results by showing that the limit of the sum of the parameters in the GARCH(1,1) model converges to one as the sampling frequency declines.

If in the GARCH model in (2.3) the polynomial

$$1 - \sum_{i=1}^{q} \alpha_i z^i - \sum_{i=1}^{p} \beta_i z^i = 0 \tag{2.6}$$

has d > 0 unit roots and $\alpha_0 = 0$, the process is said to be integrated in variance of order d. If $\alpha_0 > 0$ the process is integrated in variance of order d with a trend. Integration in variance is analogous to a unit root in the mean equation, which for financial data is defined as a martingale. In a model that is integrated or persistent in variance, the current information remains important for the forecasts of the conditional variance for all horizons (Engle and Bollerslev 1986).

The problem with IGARCH is that, unlike the martingale of the mean equation for asset prices, it lacks theoretical motivation (Lamoureux and Lastrapes 1990b). A possible explanation is that the movements in volatility are driven by latent unobservable common factors, which themselves are integrated. Engle, Ito and Lin (1990) have investigated the possibility of such common factors in the news arrival process. An alternative explanation for the high persistence in daily data is given by Lamoureux and Lastrapes (1990b), who claim that this characteristic is due to time-varying GARCH parameters, especially the trend parameter, α_0.

Another problem with the IGARCH model is that the unconditional variance for the simple IGARCH(1,1),

$$\frac{\alpha_0}{(1-(\alpha_1+\beta_1))}, \tag{2.7}$$

does not exist. Nelson (1990b) has however shown that, in spite of the infinite unconditional variance, the IGARCH model is strictly stationary and ergodic, though not covariance stationary. Lumisdaine (1991) proves that standard asymptotic inference is valid even in the presence of IGARCH effects.

An extension of the GARCH model that incorporates the fundamental tradeoff in financial theory between risk and return is the GARCH-in-mean (GARCH-M) model (Engle, Lilien and Robins 1987). This model allows the conditional variance to be a determinate of the conditional mean:

$$\begin{aligned} z_t &= \gamma_0 + \gamma_1\sqrt{h} + \varepsilon_t \\ h_t &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}. \end{aligned} \tag{2.8}$$

The parameter γ_1 gives the time-varying risk premium and the level of risk aversion. Economic theory allows for a wide range of alternative risk measures to be incorporated into the mean equation. Engle, Lilian and Robins (1987) state that in general one might expect the mean to increase less than in proportion to the variance, thus supporting the choice of the standard deviation in the conditional mean equation. Log h_t has also been used in empirical work.

Nelson (1990a) has developed the EGARCH model, ie the exponential GARCH, to capture the asymmetric impact of shocks on the conditional variance. This asymmetry is found particularly in share price data. Negative innovations increase volatility more than positive innovations. The linear GARCH model is not able to capture this dynamic pattern since the sign of the shocks plays no role in the conditional variance model. In EGARCH, the leverage effects are modelled in the conditional variance as an asymmetric function of past ε_t's:

$$\log h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i(\phi z_{t-i} + \gamma[|z_{t-i}| - E|z_{t-i}|]) + \sum_{i=1}^{p} \beta_i \log h_{t-i}. \tag{2.9}$$

In this model the variance depends on both the magnitudes and signs of past shocks.

Empirical findings from the basic GARCH models indicate that the technique removes leptokurtosis for the most part from standardized residuals, but the remaining part often turns out to be significant. Solutions (Engle and Bollerslev 1986) have been sought in nonlinear GARCH models other than the EGARCH and, for time-dependent parameters, in stochastic conditional volatility (Hsieh 1991) as well as in nonnormal conditional densities (Bollerslev 1987, Hsieh 1989).

The univariate ARCH model only measures the conditional variances in individual rates. Portfolio theory implies that if nonzero covariances exist between rates, they should be incorporated in the risk-return evaluation. In the multivariate ARCH model by Nerlove, Diebold, van Beeck and Cheung (1988) parameterization of the time-dependent variances and covariances enables full efficiency in estimation.

The multivariate model of an N x 1 vector of stochastic process

$$\begin{aligned} &\varepsilon_t = z_t \Omega_t^{1/2} \\ &z_t \ \text{IID}, E(z_t) = 0 \ \ \text{var}(z_t) = I, \end{aligned} \tag{2.10}$$

given in Kraft and Engle (1983), is a direct generalization of the univariate ARCH(q) model, except that the entire N x N conditional variance-covariance matrix, Ω_t, must be modelled. Bollerslev, Engle and Wooldridge (1988) derived the properties of a general multivariate linear, GARCH(p,q), model:

$$\text{vech}(\Omega_t) = A_0 + \sum_{i=1}^{q} A_i \, \text{vech}(\varepsilon_{t-1}\varepsilon'_{t-i}) + \sum_{i=1}^{p} B_i \, \text{vech}(\Omega_{t-i}). \tag{2.11}$$

In an unrestricted parameterization of the general model, the number of parameters to be estimated is too large for it to be of much empirical use. The model has $(N(N + 1)/2)[1 + (p + q)N(N + 1)/2]$ parameters, which makes numerical maximization of the likelihood function extremely difficult, even for low values of N, p and q (Diebold and Lopez 1995). Several restrictions have been imposed to reduce the parameter space to a manageable size. Bollerslev, Engle and Wooldridge (1988) force parsimony by requiring the A_i and B_i matrices to be diagonal, thus reducing the number of parameters to $(N(N + 1)/2)[1 + p + q]$. However, the parsimony of this diagonal model may come at a high cost because much of the potential cross-variable volatility interaction, which is a prime concern in multivariate analysis, is assumed away (Diebold and Lopez 1995). Alternative models with reduced parameter spaces are the latent factor interpretation of a ARCH(q) model by Diebold and Nerlove (1989) and the k-factor GARCH(p,q) model by Engle, Granger and Kraft (1984). In these models, movements in the N time series are driven by a set of $k < N$ common shocks of 'factors' displaying GARCH effects. In the Diebold and Nerlove model, the factor is an unobserved latent factor, while in the k-factor GARCH model, the factors are linear combinations of the residuals. The number of parameters in a k-factor model is $N(k + 1) + k^2(1 + p + q)$. In the one-factor model, which is of practical importance, the number of parameters is $2N + (1 + p + q)$, which is a notable reduction from the general multivariate case, but still not small enough not to cause difficulty in ensuring the positive definiteness of the variance vector, again for dimensions exceeding the bivariate case.

In the multivariate generalized ARCH model of Bollerslev (1990) the estimation and inference procedures are simplified by the assumption of constant conditional correlations between the N stochastic processes. The GARCH(1,1) structure for the conditional variances and covariances is expressed as

$$
\begin{aligned}
\varepsilon_{i,t} &= z_{i,t} h_t^{1/2} \\
h_{ii,t} &= \alpha_i + \alpha_{i1} \varepsilon_{i,t-1}^2 + \beta_{i1} h_{ii,t-1} \qquad (2.12) \\
h_{ij,t} &= \varrho_{ij} (h_{ii,t} h_{jj,t})^{1/2},
\end{aligned}
$$

where ϱ_{ij} is the correlation between ϵ_i and ϵ_j.

In this study the GARCH(1,1) univariate process is estimated for exchange rates, interest rates, equity prices and real estate prices. An attempt to identify the GARCH-M model is also made for all series in order to test the theory of a time varying risk premium. A two-step procedure developed in this study for the multivariate version of Bollerslev is used as one of two applied methods to measure the covariance structure within and between groups. The forecasting formulas are based on the autoregressive interpretation in (2.4).

3 Exchange rate risk

In order to measure the exchange rate risk in typical bank portfolios, twelve currencies are included in the analysis. The reason for the large number is that we want to cover the main currencies in which the banks continuously have considerable exposures. This selection also enables comparison of movements in Scandinavian, continental European, US and Japanese exchange rates.

3.1 Markka exchange rates

Most studies that model probability distributions of foreign exchange rates concentrate on dollar exchange rates. Bollerslev et al (1992) provide a good survey of empirical results for model identification and parameter estimation involving US dollar exchange rates. Common results found by several authors are that the GARCH(1,1) model is best suited to capture the stylized facts for exchange rates. Hiesh (1988) found that an ARCH(12) model with a linearly declining lag structure captures most of the stochastic nonlinearities present in the conditional variance, but later (1989) he found that the simple GARCH(1,1) does a better job of describing the data. Ballie and Bollerslev (1989) showed that their daily data on six dollar-based currencies confirm the suitability of the GARCH(1,1) model.

This study deals with markka exchange rates expressed as markka values of foreign currencies. Results for non-markka exchange rates are not necessarily applicable to markka exchange rates due to the Finnish institutional structure, which affects the exchange rate generating process, and to the smallness in scale and scope of the Finnish markets. To our knowledge, only three studies deal with markka exchange rates (Ahlstedt 1990 and 1995, Sulamaa 1995). Some of these earlier results (Ahlstedt 1995), though not repeated here, are referred to in the sections on the empirical work.

3.2 Frequency

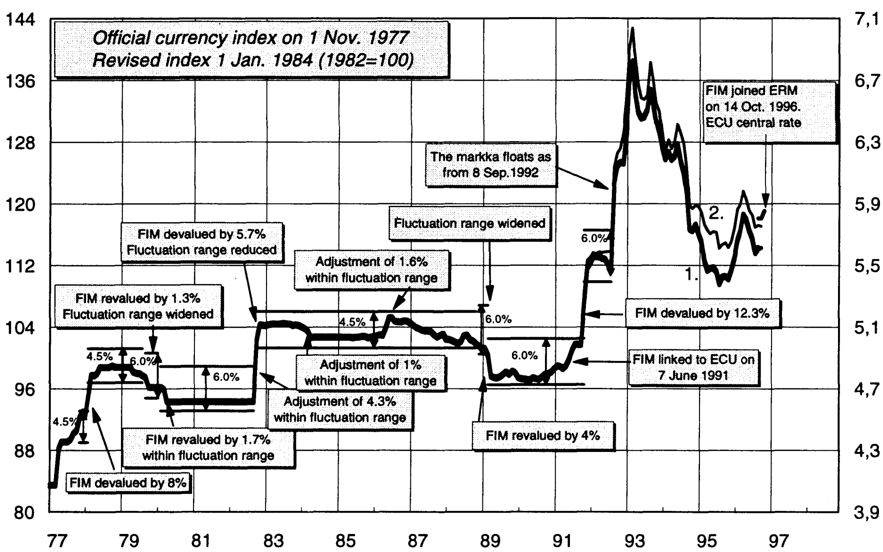
It is a well documented empirical fact that certain distributional properties of financial time series, such as heteroscedasticity and leptokurtosis, decrease as frequency decreases. Ballie and Bollerslev (1989) show that the stylized facts for daily spot rates carry over to weekly, forthnightly and monthly data in which the degrees of leptokurtosis and time-dependent heteroscedasticity decrease as the length of the sampling interval increases. Also the high persistence in the daily data diminishes with temporal aggregation. Convergence to unconditional normality occurs with temporal aggregation (Nerlove et al 1988), so that one-month changes are closer to normality than one-week changes (on a monthly level), which are closer to normality than daily observations. For exchange rates, even intraday prices are quoted. In intraday quotations, the volumes and prices of exchange rates are determined at points where supply and demand are in balance. These momentary equilibria are reached at numerous discrete points in time during ongoing trading. The price quotes on the way toward the long-term equilibrium mirror traders' reactions to the arrival of news. An attempt to explain these stylized facts in foreign exchange rate movements has been sought in common factors. For low frequency data, international economic variables have been tested and for high frequency data, the source of the pattern of variability has been sought in the news arrival process in the form of either meteor showers or heat waves (Engle, Ito and Lin 1990).

The purpose of this study, however, is to quantify, using time series techniques to model time-varying conditional variances, the inherent riskiness of short-term changes in the values of banks' portfolios, which are marked to market daily. Hence daily data is used. High frequency common factors will be tested for using principal components analysis on estimated daily variances.

3.3 Structural changes

Since this study also deals with the interaction between exchange rates and interest rates, the data is extended backward to cover the longest possible common interval for these rates in the Bank of Finland's database. This period is 1 January 1987 – 31 December 1995. The period (Figure 3.1) includes a 4 per cent revaluation of the Finnish markka against a currency basket on 17 March 1989, a 12.30 per cent devaluation on 15 November 1991 and a floating rate regime from 8 September 1992 onwards. A period of markka depreciation at the start of the float is followed by a period of appreciation. To account for possible structural shifts caused by changes in the exchange rate regime, the sample period is divided first into two periods: one covering the exchange rate band regime, 1 January 1987 – 5 September 1992, and the other covering the floating-rate regime, 8 September 1992 – 31 December 1995 (Figure 3.1). As it is not our aim to explain the effects or transmission mechanism of structural shocks or to forecast turning points, pre- and post-data around shifts in the exchange rate regime are excluded.

Figure 3.1 **External value of the markka**



1. Bank of Finland currency index (left scale)
2. Markka value of the ECU from 7 Jun 1991 (right scale)

The data consist of daily observations on log changes of closing rate bid-ask midpoints. Weekends and holidays are omitted. Monday is treated as the day after Friday. Relevant weekend or weekday effects have not been found (Ahlstedt 1990) in the exchange rates or interest rates. First differences are used in light of numerical studies on dollar rates that show that higher order differencing is not necessary to achieve stationarity (Chappell and Padmore 1995). For the markka's exchange rate band period, this is most certainly true, since the aim of the intervention mechanism is exchange rate stability.

The exchange rates included in this study are those of the twelve major currencies, USD, GBP, SEK, NOK, DKK, DEM, NLG, BEF, CHF, FRF, ITL and JPY, against the markka. The markka-values of these currencies display some special features during the period under consideration, which are reported in Table 3.1. These features may well have affected the final variance-covariance estimates presented in this study.

Table 3.1 **Special volatility features in the exchange rates 1 Jan 1987 - 31 Dec 1995**

Rates	Special volatility features
ALL	All series have huge peaks at the 15 Nov 1991 devaluation and 8 Sep 1992 shift to the floating-rate regime.
USD	The magnitude of the changes in the USD exchange rate is bigger than those of the ERM currencies.
GPB	Peaks at joining the ERM 8 Oct 1990 and shows turbulence and excess volatility at exit from the system 16 Sep 1992.
SEK	Increased volatility at the time preceding transition to the floating-rate regime in Sep 1992.
NOK	Increased volatility at the time preceding transition to the floating-rate regime in Sep 1992.
DKK	High volatility in connection with the turbulence of the Sep 1992 crisis involving the ERM and the other Nordic currencies; stayed in the ERM band.
DEM	3 % revaluation on 12 Jan 1987.
NLG	3 % revaluation on 12 Jan 1987; high volatility in connection with the ERM crisis in Sep 1992.
BEF	2 % revaluation on 12 Jan 1987; increased volatility in September 1992.
CHF	Turbulence in Sep 1992.
FRF	Turbulence in Sep 1992 as with the other ERM currencies: DKK, BEF, NLG and CHF.
ITL	The lira's band was narrowed from 6 % to 2.25 % on 5 Jan 1990, the effects of which can be clearly seen in reduced volatility; 3.5 % devaluation on 14 Sep 1992 and exit from the system and float as from 16 Sep 1992.
JPY	Increased volatility at the time of instability among the Nordic currencies and ERM in early autumn 1992 but for a longer period.

Preliminary statistical testing of the hypothesis of a normal distribution with zero mean and constant variance for the twelve exchange rates was conducted for both main periods. Since the test values are extremely sensitive to even a single outlier, the observations of the revaluation and devaluation trading days within this period strongly affect the descriptive statistics.

For the first main period, 1 January 1987 - 5 September 1992, the hypothesis of zero mean could not be rejected at the 5 per cent significance level. The skewness and leptokurtosis measures are high. To control for possible effects of structural breaks, the exchange rate band period is further divided into two subperiods with the dividing date being the revaluation date, 17 March 1989.

The full series was accordingly split into three nonoverlapping sub-series.

Exchange rate band period	1 Jan 1987 - 16 Mar 1989 21 Mar 1989 - 5 Sep 1992
Floating rate period	8 Sep 1992 - 31 Dec 1995

The realignments are treated in two alternative ways: either the data is divided into three periods or their effects are captured by dummy variables (in the second exchange rate band period).[28] The skewness figures are neglible in all periods except the second exchange rate band period, which includes the realignment.

3.4 Exchange rate band period

3.4.1 First subperiod: 1 January 1987 - 16 March 1989

Summary statistics for the two subperiods, 1 January 1987 - 16 March 1989 and 21 March 1989 - 5 September 1992, show that the data for the first period is much closer to a normal distribution than the data for the second period. In most cases, the skewness measure does not differ significantly from that of the theoretical distribution. The mean percentage change of spot exchange rates is significantly different from zero only for the SEK, NOK and ITL.

[28] Using dummy variables to account for the regime switch, identified ex post, the estimated variances will undervalue the risks when used for ex ante forecasting. This problem can be solved using the switching-regime Markov model developed by Hamilton (1989) and Lam (1990), which can accommodate the dynamics of conditional heteroscedasticity. See also Gray (1996) for an application to short-term interest rates.

While the magnitude of the excess kurtosis for the first subperiod is only a fraction of that for the later subperiod, it is significant for all currencies.

The later subperiod for the first main period, 21 March 1989 – 5 September 1992, includes the devaluation of 15 November 1991. Although the devaluation day and nearby days are excluded from the data, spillover effects from the devaluation remain. The subperiod ends with the volatile market activity preceding the switch from an exchange rate band regime to a float for the FIM, SEK and ITL. It also covers the joining of the GBP to the ERM and the period preceding its exit from the system. This turbulence can be seen in higher variances for all currencies except the FRF and ITL. The skewness measures differ significantly from zero for all currencies except the JPY. The figures for excess kurtosis are generally very high in this set of the data. The USD and JPY display less but still statistically significant kurtosis. This means that during this period extreme values occurred more often than in the theoretical distribution.

For the floating rate period, 8 September 1992 – 31 December 1995, the hypothesis of a zero mean rate of depreciation is rejected only for the ITL. The skewness measures differ significantly from zero for all currencies except the DKK, NLG and BEF. All excess kurtosis measures differ significantly from zero. Thus most of the empirical unconditional distributions appear to display asymmetries and to have fatter tails and more peakiness around the mean than the normal distribution.

The magnitudes of the variances are, as expected, largest for the floating rate period. Also as expected, currencies within the ERM tend to have lower variances for all periods than do the USD and JPY, which float freely.

As kurtosis in the unconditional distribution may be considered an indication of conditionality in the moments, we next proceed to model the mean and variance with the ARCH process.

Estimation of the ARCH process starts with specification of the conditional mean equation. Since first differencing produces stationarity, log changes of the exchange rates can be initially expressed as

$$R_t - R_{t-1} = \alpha + \varepsilon_t, \tag{3.1}$$

where $R_t = \ln(X_t)$ denotes the natural log of the original series, X_t, α is a constant and ε_t is a zero-mean error term. Under a serially uncorrelated and homoscedastic error process, R_t follows a random walk, possibly with drift. The results for each series R_t reveal the constant to be insignificantly different from zero, confirming the absence of a deterministic trend or drift. There is no evidence of serial correlation in the residuals, with the exception of the ITL. The ITL Ljung-Box test statistics for linear serial correlation for lags up to five are highly significant. The Jarque-Bera normality test statistic is significant for all currencies except the CHF, which lends support to earlier results showing deviation from normality in the form of leptokurtosis and skewness.

These deviations from normal errors may be evidence that the ε_t's are not independently distributed over time, although as such these non-normalities do not run counter to the assumption of a martingale process for exchange rates. The graphs of the logarithmic differences show clustering, which on the balance of the evidence is typical for high frequency dollar exchange rate data. Thus there is a tendency for large daily exchange rate changes to be followed by large changes and small changes by small changes, with the sign unpredictable in either case. This time-dependence, as well as various other sources of heteroscedastic behaviour, are modelled in the conditional variance equation in the ARCH processes. The variance equation explicitly allows for temporal dependence by parameterizing the conditional variance as a function of the past squared residuals and past conditional variances themselves.

Bollerslev et al (1992) suggests that the inclusion of a one-period lag for the squared innovations, ε_t^2, and conditional variance, h_t, in the variance functions, ie the GARCH(1,1) model, is usually sufficient to capture most of the conditional heteroscedasticity in financial market return data. This is also confirmed by previous results for markka exchange rates and interest rates (Ahlstedt 1990, 1995) and by the ARCH(1) test (Table 3.2).[29] Consequently a GARCH(1,1) structure for the mean and variance equations was assumed:

$$\begin{aligned} & R_t - R_{t-1} = \varepsilon_t \\ & \varepsilon_t \sim N(0,h_t) \\ & h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}. \end{aligned} \qquad (3.2)$$

If the value of the parameter β_1 is not significantly different from zero, then the process can be represented by an ARCH model. If α_1 is zero, the process depends only on its past history. If both α_1 and β_1 are zero, ε_i is white noise.

[29] The LM test for ARCH(1) reported in Table 3.2 is actually the same as the LM test for GARCH(1,1) (Bollerslev, Engle and Nelson 1994). The ARCH(1) test statistic is distributed according to student's t.

The GARCH model was estimated by the method of maximum likelihood, assuming conditional normality, although the Jarque-Bera normality test statistic strongly rejects the null hypothesis of normal errors. Conditional normality is however not necessary for consistency and asymptotic normality of the estimators. Weiss (1986), Bollerslev and Wooldridge (1992) and West and Cho (1995) have shown that when normality is inappropriately assumed the resulting quasi-maximum likelihood estimators are nonetheless asymptotically normally distributed and consistent if the conditional mean and variance functions are specified correctly. Moreover, Bollerslev and Wooldridge (1992) derive asymptotic standard errors for the quasi-maximum likelihood estimators that are robust to conditional nonnormality and readily calculated as functions of the estimated parameters and first derivatives of the conditional mean and variance functions (Diebold and Lopez 1995).

The results of the GARCH estimation are shown in Table 3.2. The models required a large number of iterations in order to achieve convergence. Based on the Ljung-Box test statistics for the ITL, the lagged endogenous variable was included in the mean equation for this currency. The drift parameter in the variance equation, α_0, was statistically significant for all currencies. Both the ARCH parameter α_1 and the GARCH parameter β_1 were significant in all equations.

Table 3.2 **GARCH estimation of the volatility of foreign exchange rates, 1 Jan 1987 - 16 Mar 1989 (t-statistics in parentheses)**

	α_0	α_1	β_1	ARCH(1) test	Ljung-Box test statistics				
					LAG(1)	LAG(2)	LAG(3)	LAG(4)	LAG(5)
USD	0.1341·E-5 (2.47)	0.0883 (3.29)	0.8661 (23.37)	2.20	1.72	1.83	1.83	2.83	3.98
GBP	0.1764·E-6 (2.04)	0.0305 (2.97)	0.9473 (52.78)	5.46	1.01	2.45	4.24	4.33	4.68
SEK	0.3315·E-7 (2.23)	0.0733 (4.35)	0.9011 (43.49)	8.75	6.97	7.70	8.83	9.03	12.80
NOK	0.1363·E-6 (2.64)	0.0757 (3.51)	0.8868 (27.81)	2.30	2.87	6.86	8.63	8.69	8.71
DKK	0.5654·E-6 (3.17)	0.1271 (3.11)	0.7070 (9.89)	11.32	0.10	2.79	4.35	4.40	4.41
DEM	0.2490·E-6 (4.04)	0.1468 (4.67)	0.7896 (23.86)	30.83	0.92	2.96	3.37	4.19	5.27
NLG	0.2207·E-6 (3.16)	0.1479 (4.50)	0.7967 (21.78)	25.77	0.57	1.64	1.64	2.12	3.89
BEF	0.1489·E-6 (2.62)	0.0914 (3.84)	0.8601 (26.25)	32.51	0.48	4.57	4.99	6.63	9.75
CHF	0.3854·E-6 (1.62)	0.0756 (2.85)	0.8659 (17.33)	21.96	0.41	4.61	4.90	5.41	5.45
FRF	0.1630·E-6 (2.62)	0.1054 (3.99)	0.8355 (20.95)	18.30	0.41	0.54	0.87	4.29	5.34
ITL	0.4655·E-5 (9.82)	0.2284 (4.30)	0.3007 (4.69)	47.95	57.44	59.95	64.02	64.41	66.54
JPY	0.1208·E-5 (2.62)	0.0925 (3.18)	0.8094 (14.12)	2.62	0.51	2.81	3.44	3.56	3.72

The effect of the squared surprises (shocks) on the variance is measured by the parameter α_1. The magnitude of the impact was quite similar for the freely floating currencies, USD and JPY, and the European currencies except for the ITL, which has a unique pattern. The sum of the parameters α_1 and β_1 is close to one,[30] indicating a GARCH process integrated in variance or a GARCH process with persistence in the sense of Engle and Bollerslev (1986).[31] In such a persistent variance model,

[30] The formal statistical ML-based testing of the hypothesis $\alpha_1 + \beta_1 = 0$ was not allowed for in the computational package. This shortcoming seems to be common, since testing cannot be found in the existing empirical GARCH studies either.

[31] See Nelson (1990b) for a general analysis of persistence and convergence in GARCH(1,1) models.

current information remains important for the forecasts of the conditional variance at all horizons.

An extension of the GARCH model to the regression framework is the GARCH-in-mean (GARCH-M) model proposed by Engle, Lilien and Robbins (1987). In financial applications, the GARCH-M model is employed to capture a linear relationship between return and variance (risk) according to the intertemporal capital asset pricing model of Merton (1973) (Mills 1993, p. 137):

$$\begin{aligned} R_t - R_{t-1} &= \gamma_0 + \gamma_1\sqrt{h_t} + \varepsilon_t \\ h_t &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}. \end{aligned} \tag{3.3}$$

The conditional standard deviation (or variance) is included as an explanatory variable in the mean equation. The impact of the standard deviation on returns is interpreted as a time-varying risk premium.

To test for the existence of time-varying risk premia in the foreign exchange market and ensure that the GARCH(1,1) is the correct model, a GARCH(1,1)-M model was also tested for comparitive purposes. The results showed that the parameter values γ_1 for the risk premium are not statistically significant. These results agree with the outcomes of other studies dealing with non-markka exchange rates (eg Chappel and Padmore 1995) where no risk premium was found when modelling the return in excess of the riskless yield.

According to portfolio theory the risks in a portfolio are reflected not only by the variances of the individual currencies but also by the covariances. One way to estimate the covariances would be to switch from univariate to multivariate modelling. Theoretically, the multivariate model is a direct generalization of the univariate model, except that an entire variance-covariance matrix is modelled. The problem, as discussed earlier, is that the number of parameters in the general form may be so large as to render the approach infeasible.

An alternative method to the general multivariate GARCH for accounting for simultaneous dependencies between rates is the use of principal component techniques to test for common factors driving the individual exchange rate variances, h_{it}. If significant common factors are found, they can be used in a factor GARCH model such as that of Diebold and Nerlowe (1989). The underlying assumption here is that exchange rate movements depend on a common set of international variables observable only at certain frequencies (Bollerslev 1990). If the common factors are macroeconomic variables, they are relevant only at low frequencies. If the common factors are to be found in the news arrival process, they are relevant only at high frequencies. Hence, using the GARCH model, one can predict how exchange rates react to shocks or news, whereas with the principal component method one tries to identify the shocks.

In this study the principal components were calculated for the conditional variances, $h_{i,t}$, for the twelve rates.[32] The use of principal component analysis serves two main purposes. First, it provides a means of comparing uniform behaviour in the individiual variances on a wider scale than is possible in parewise comparison. Secondly, the method allows for measurement of the degree of homogeneity within the groups of rates. The eigenvalues and cumulative fractions of variance explained are shown in Table 3.3. When the variables are highly correlated and form a homogeneous group, the first principal component explains more than 90 per cent of the total variation. This is usually the case for a set of macroeconomic variables. The results presented in Table 3.3 indicate that the variances within the group of exchange rates are more heterogeneous, and the total variance cannot be concentrated into a few common factors as it can for the macroeconomic variables. The fraction of explained variance for the exchange rates starts at 50 per cent for the first principal component and rises by about 10 percentage points with each additional component. The factor loading values of the individual variances show that the variance for the USD dominates the first principal component with a value of 0.815. The GARCH estimation results concerning the exceptional behaviour of the ITL are confirmed also in the principal component calculations. The factor loading values for the ITL are only 0.152 and 0.036, thus indicating practically no correlation with any of the first two principal components. Removal of this currency would increase the fraction explaned by the first few components for the remaining currencies.

[32] Note, however, that the factors in the k-factor model of Diebold and Nerlove (1989) are linear combinations (principal components) of the residuals, whereas we look at the factors as principal components of the variances.

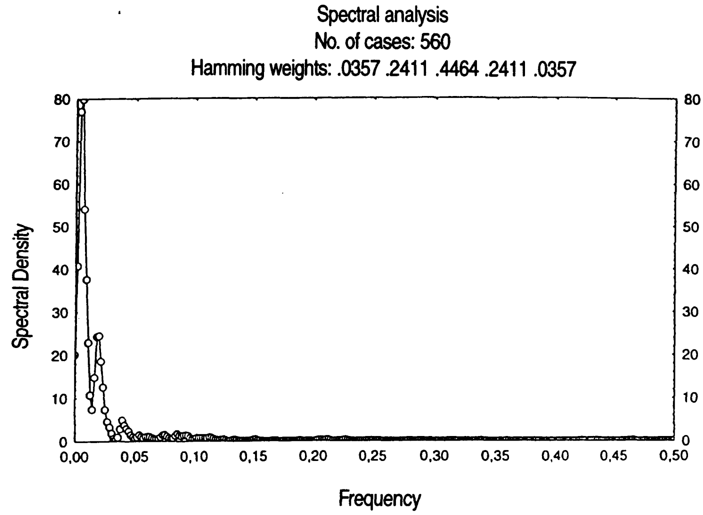
Table 3.3 **Principal components of conditional variances, eigenvalues and cumulative fraction explained, foreign exchange rates for 1 Jan 1987 - 16 Mar 1989**

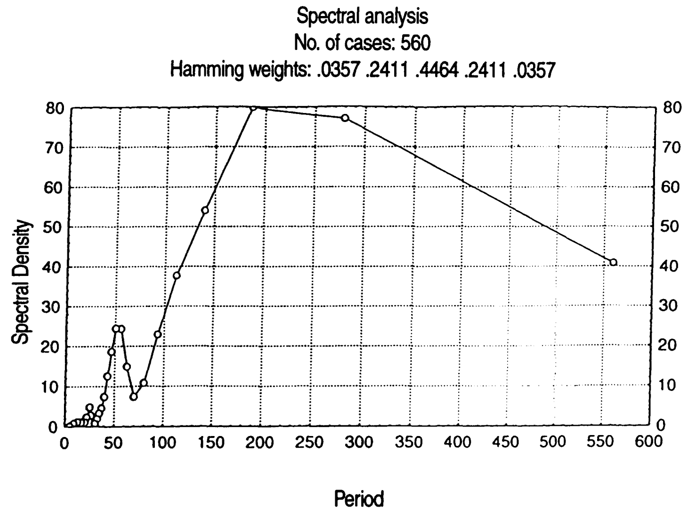
Component	Eigenvalue	Cumulative R-squared
1	5.5140	0.4595
2	1.5249	0.5865
3	1.3212	0.6966
4	0.9697	0.7775
5	0.8193	0.8457
6	0.5164	0.8888
7	0.4753	0.9284
8	0.0118	0.9294
9	0.0767	0.9358
10	0.1658	0.9496
11	0.2974	0.9744
12	0.3068	1.0000

Using spectral analysis on both the individual conditional variances and the principal components, one can decompose the observed time-variability of the conditional variances or principal components into contributions from periodic cycles at different angular frequencies (and hence for different cycle lengths). Furthermore, visual inspection of the power spectra provides a potentially powerful tool for identifying the autocorrelation structure of the underlying process generating the observed time variability of the conditional variances and ultimately the process itself. Finally, spectral analysis may prove useful in constructing optimal filters to remove specific cycles of a given length from the data.

Now, the overall shape of the power spectrum of the first principal component (Figure 3.2) of the conditional variances, which is very similar to the power spectrums of the individual variances themselves, provides evidence of persistence in the component, ie the general shape of the spectrum resembles that of a positively autocorrelated process. Furthermore, additional contributions to the time-variability of the conditional variances come from cycles with frequencies in the range 0.0224-0.0561 radians or 0.0036-0.0089 cycles per day (corresponding to wavelengths between 70 and 280 days). Given the shape of the spectrum, however, cycles within this range need not be highly regular.

Figure 3.2 **Spectral density function of the first principal component of conditional variances of foreign exchange rates for 1 Jan 1987 - 16 Mar 1989**





3.4.2 Second subperiod: 21 March 1989 – 5 September 1992

The results of the GARCH(1,1) estimation for the second subperiod, 21 March 1989 – 5 September 1992, of the fixed-rate regime are shown in Table 3.4. This period includes a 12.3 per cent devaluation of the Finnish markka on 15 November 1991. The effects of this realignment of the markka are modelled by three dummy variables, which take the value one for the actual devaluation day and two following days. The estimated coefficients for the dummy variables show a devaluation effect of 13 per cent for the actual devaluation day, a strengthening of 4 per cent on the following day and a weakening of 1 per cent on the third day. The cumulative effects of the three days amount to a 10 per cent weakening of the markka against the currency basket.

The ARCH coefficient is significant for all currencies. The GARCH parameter is not significant for the GBP and CHF. The change in the pattern of the variance of the GBP is explained by the fact that the period includes the GBP's entry to and exit from the ERM. The sum of α_1 and β_1 is less than one, suggesting that the underlying variance processes are weakly stationary, though in most cases the sum of the parameters is very close to one.

The values of the estimated principal components appear in Table 3.5. The fractions explained are almost identical to those of the earlier subperiod of the band regime. However, the dominant currency is not the USD but the DEM (and DEK and NLG because of their high correlation with the DEM).

The spectral density functions of the individual conditional variances are very similar to those of the first band period. The pattern is also confirmed in the spectral density function of the first principal component. The overall picture is that the spectrum shows strong persistence although there are visible local peaks for the period at about 180 days and its harmonic 340 days.

Table 3.4 **GARCH estimates of the volatility of foreign exchange rates for 21 Mar 1989 – 5 Sep 1992 (t-statistics in parenthesis), estimation with 3 dummy variables**

	α_0	α_1	β_1	D_1	D_2	D_3	ARCH(1) test	Ljung-Box test statistics				
								LAG(1)	LAG(2)	LAG(3)	LAG(4)	LAG(5)
USD	0.7865·E−6 (2.21)	0.0679 (5.02)	0.9187 (59.2)	0.1299 (18.93)	−0.0495 (7.21)	0.9870·E−2 (1.44)	0.66	2.48	3.45	7.02	7.09	7.80
GBP	0.9137·E−5	0.2004 (5.02)	0	0.1280 (38.31)	−0.0364 (10.91)	0.8440·E−2 (2.52)	12.77	0.18	3.99	4.39	10.57	12.38
SEK	0.5207·E−7 (5.16)	0.0506 (6.53)	0.9280 (100.68)	0.1285 (83.22)	−0.0346 (22.39)	0.0131 (8.48)	6.88	6.07	6.07	8.69	9.49	9.57
NOK	0.2631·E−7	0.0619 (6.78)	0.9286 (120.12)	0.1291 (86.40)	−0.0357 (23.86)	0.0113 (7.57)	9.66	8.36	8.40	14.55	29.14	30.77
DKK	0.1645·E−6 (4.40)	0.0590 (5.83)	0.8975 (57.18)	0.1302 (66.06)	−0.0361 (18.34)	0.0116 (5.90)	7.52	3.64	4.64	6.23	6.23	6.24
DEM	0.6303·E−7 (3.00)	0.0615 (7.34)	0.9229 (91.51)	0.1303 (66.16)	−0.0348 (17.67)	0.0102 (5.18)	4.23	0.89	1.20	2.95	6.19	8.42
NLG	0.8201·E−7 (3.71)	0.0701 (7.22)	0.9095 (81.73)	0.1303 (65.95)	−0.0348 (17.63)	0.0103 (5.22)	10.65	2.13	3.12	4.10	9.60	12.84
BEF	0.1212·E−5 (5.07)	0.1023 (5.40)	0.5968 (8.64)	0.1300 (63.63)	−0.035 (17.30)	0.0114 (5.59)	68.48	8.02	9.64	10.10	11.24	12.11
CHF	0.1113·E−4 (24.61)	0.1061 (5.33)	0	0.1310 (37.16)	−0.0342 (9.71)	0.9660·E−2 (2.74)	2.55	0.21	4.57	8.10	8.56	9.40
FRF	0.1139·E−6 (4.52)	0.0585 (5.94)	0.9050 (66.03)	0.1299 (72.98)	−0.0352 (19.76)	0.1173 (6.59)	3.03	1.26	2.79	4.20	5.04	5.38
ITL	0.9295·E−6 (2.95)	0.1101 (4.26)	0.7120 (9.18)	0.1297 (56.39)	−0.0358 (15.55)	0.0111 (4.84)	7.48	33.03	33.73	33.93	34.51	35.12
JPY	0.7420·E−6 (2.49)	0.0790 (3.96)	0.9009 (39.09)	0.1288 (23.10)	−0.0428 (7.68)	0.5930·E−2 (1.06)	0.03	0.74	0.74	0.80	1.66	1.67

Table 3.5 **Principal components of conditional variances, eigenvalues and cumulative fraction explained, foreign exchange rates for 21 Mar 1989 – 5 Sep 1992**

Component	Eigenvalue	Cumulative R-squared
1	5.5139	0.4594
2	1.6188	0.5944
3	1.2500	0.6985
4	0.9555	0.7781
5	0.8184	0.8464
6	0.5995	0.8963
7	0.4758	0.9360
8	0.5208	0.9794
9	0.1007	0.9878
10	0.0200	0.9894
11	0.0717	0.9954
12	0.0543	1.0000

3.5 Floating-rate period

The markka's floating rate regime is analysed here more thoroughly, because forecasting will be based on estimates of conditional variances for this period, whereas estimates for the exchange rate band period are used to compare volatility estimates across regimes. These comparisons may prove useful, since formally the markka's free float come to an end on 14 October 1996, when it was joined to the ERM. The institutional circumstances and obligations of ERM membership are however presently closer to those of the floating rate period than to the earlier band periods.

Stationarity tests were performed on the data for the floating rate period, 8 September 1992 – 31 December 1995. The Weighted Symmetric τ test, the augmented Dickey-Fuller τ test and the Phillips-Perron Z test were employed with both logs of exchange rates and log differences. The estimated test statistics for the levels imply that the hypothesis of a unit root cannot be rejected, even at the 1 per cent significance level. The only value close to the 1 per cent critical value is the Dickey-Fuller τ test for the USD; the other two statistics for this currency do not support rejection. Pantula (1985) has shown that the asymptotic distribution of the Dickey-Fuller statistic is invariant to ARCH, meaning that the test is asymptotically robust to autoregressive conditional heteroscedasticity. The Phillips-Perron test, on the other hand, has good finite sample properties and may thus be more reliable here. Based on all the test statistics for the first differences, the hypothesis of a unit root can thus be rejected. The presence of a trend, which is detected for the levels, cannot be found in the differences. The tests support the presence of one, and only one, unit root in the levels of the series. Thus each series is appropriately made stationary by taking first differences.

The results of the GARCH(1,1) estimation are presented in Table 3.6. The initial turbulent days of the floating rate regime are omitted so that the estimation period begins on 14 September 1992. The ARCH parameter α_1 is zero for DEM and JPY and 1 for BEF. These values are determined in the iteration process when the estimated values approach the boundaries of 0 and +1 for the parameters. The constant α_0 is significant for all currencies but very small in magnitude. The sum $\alpha_1 + \beta_1$ is close to one for most currencies; forcing the α_1 parameter to its boundary value in the iteration causes the sum of the coefficients to be much greater than one for the BEF. The parameter values for the ITL indicate nonstationarity in variance.[33]

[33] The large values of the standard t-statistics, found here for ITL and later on for some interest rates and especially for the pooled models, cast doubt on the distributional properties of the statistics. The empirical evidence in this study and in others (see eg Heynen and Kat 1994 for exchange rates) seems to be that the value of the statistic increases at a very high rate as the value of the parameter β_1 is approaches unity, ie the process approaches the unit root property. The standard t-statistics are commonly used in GARCH literature. We have also calculated corrected t-statistics by using robust standard errors as described by Bollerslev and Wooldridge (1992) and found them more reasonable.

Table 3.6 **GARCH estimation of the volatility of foreign exchange rates for 14 Sep 1992 – 31 Dec 1995 (t-statistics in parentheses)**

	α_0	α_1	β_1	ARCH(1)	Ljung-Box test statistics				
					LAG(1)	LAG(2)	LAG(3)	LAG(4)	LAG(5)
USD	0.3189·E−5 (4.57)	0.0792 (4.87)	0.8608 (39.79)	63.71	0.84	1.12	1.61	2.03	2.03
GBP	0.1134·E−5 (3.53)	0.0393 (4.20)	0.9225 (59.51)	64.60	0.51	2.20	18.04	20.16	22.65
SEK	0.4929·E−8 (4.27)	0.0549 (5.03)	0.9196 (58.09)	22.55	22.45	23.32	36.55	41.28	47.71
NOK	0.2793·E−8 (4.43)	0.0670 (7.42)	0.9292 (139.94)	30.06	8.12	8.40	9.35	11.54	22.94
DEK	0.3186·E−5 (4.03)	0.4586 (14.77)	0.5686 (13.24)	0.02	0.93	5.50	12.05	14.17	14.20
DEM	0.1653·E−6 (7.78)	0	0.9859 (699.75)	26.20	4.85	4.87	14.68	14.93	14.94
NLG	0.6442·E−7	0.1025 (7.59)	0.8623 (77.52)	18.76	8.53	11.93	12.41	15.64	66.44
BEF	0.1568·E−6 (6.20)	1	0.4677 (36.17)	6.93	10.79	26.02	43.04	44.87	52.81
CHF	0.4272·E−5 (7.72)	0.5814 (24.93)	0.3996 (12.87)	70.94	4.29	4.62	6.71	7.21	7.33
FRF	0.2884·E−5 (5.61)	0.2109 (6.91)	0.6612 (14.86)	74.82	0.14	3.54	7.18	7.35	7.60
ITL	0.8532·E−7 (17.41)	0.4615 (17.60)	0.6914 (55.29)	41.70	17.26	85.97	86.29	97.85	98.02
JPY	0.9024·E−9 (11.98)	0	0.9898 (1563.21)	26.51	1.32	4.54	9.57	9.58	14.10

The principal components are presented in Table 3.7. Compared to the band period, the fraction explained is 10 per cent higher for the three first components. This indicates greater homogeneity in the variance structures. The dominant currencies seem to be the GBP and DEM. The first component can perhaps be interpreted as a general underlying factor, strongly correlated with the dominant currencies in the ERM.

Visual inspection of the spectrum of the first principal component of the conditional variances once again strongly suggests that the variance processes are persistent. The spectrum decreases almost monotonically from its value at the lowest frequency of 0.00754 radians per day or 0.0012 cycles per day (corresponding to a wavelength of 834 days) to its value at the Nyqvist frequency (π radians or ½ cycles per day, wavelength 2 days).

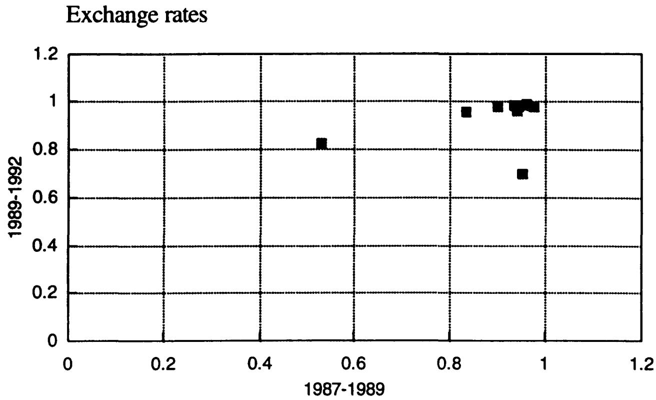
Table 3.7 **Principal components of conditional variances, eigenvalues and cumulative fraction explained, foreign exchange rates for 14 Sep 1992 - 31 Dec 1995**

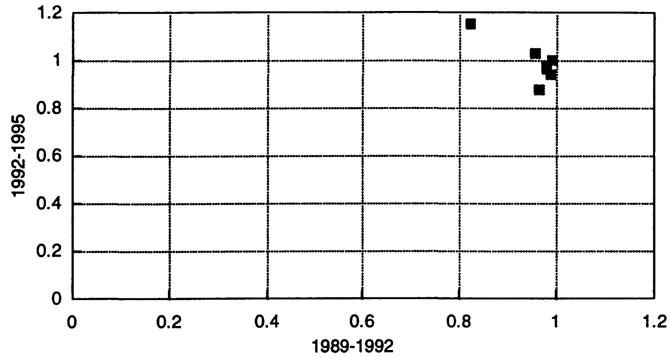
Component	Eigenvalue	Cumulative R-squared
1	6.5713	0.5476
2	1.5919	0.6802
3	1.3205	0.7903
4	0.9224	0.8671
5	0.5081	0.9095
6	0.4612	0.9479
7	0.2909	0.9722
8	0.1673	0.9861
9	0.0773	0.9926
10	0.0463	0.9964
11	0.0381	0.9996
12	0.0043	1.0000

3.6 Pooled data

The results of the GARCH estimation for the individual currencies during both the band period and the float show that there is a close similarity in the estimated parameter values of the variance processes within the periods. To evaluate the similarity between the individual conditional variance models, the sum $\alpha_1 + \beta_1$ for the variances for different pairs of periods were plotted against each other. In the upper part of Figure 3.3, the sum for the first band period is plotted against the second band period. In the lower part, the second band period is plotted against the floating-rate period. The figures show pronounced clustering, which is interpreted as similarity between the individual parameter structures and so justifies pooling of the data.

Figure 3.3 **The sum $\alpha_1 + \beta_1$ for pairs of periods**





The next step was to force the conditional variances for all the currencies into the same model by identifying a GARCH model on the pooled data. In the estimation, the log differences for the twelve individual currencies were pooled separately for each period and a GARCH(1,1) model was estimated on this data. The pooled data within periods was constructed by simply connecting the data on the individual currencies. While this implies incorrect data at the connecting points, given the huge amount of data, the impact of such a few data points is considered to be negligible.

The GARCH(1,1) estimates for the first fixed-rate period, 1 January 1987 - 16 March 1989, are

$$h_t = \underset{(7.69)}{0.2692 * E\text{-}7} + \underset{(37.73)}{0.0567} \varepsilon_{t-1}^2 + \underset{(953.01)}{0.9406} h_{t-1}. \tag{3.4}$$

To compare the goodness of fit of the pooled model to the individual models, maximum values of the likelihood functions were calculated. The sum of the individual maximum likelihood values is 31 794; the maximum likelihood value for the pooled model is 31 582; and the corresponding test statistic ($\chi^2_{(44)}$) for the null of the same GARCH(1,1) model is 420. This is highly significant, thus confirming the expectation that forcing leads to an inferior model.

The impact of news given by the parameter $\alpha_1 = 0.0567$ is not very strong. The persistence parameter is $\beta_1 = 0.9406$, which implies that the estimated mean lag of the variance expression, $1/(1-\beta_1)$, equals 16.7, and that it takes more than 3 weeks for shocks to come through in the model. The sum $\alpha_1 + \beta_1 = 0.9973$, which indicates that the process is integrated. One way to measure how long shock effects remain in the process (ie the persistence) is to use the half-life figure, λ, which gives the number of days over which a shock to volatility diminishes to half its original size (Lamoureux and Lastrapes 1990b). The half-life figure depends only on the sum of $\alpha_1 + \beta_1$ and is given by

$$\lambda = 1 - \left(\frac{\log 2}{\log(\alpha_1 + \beta_1)} \right). \tag{3.5}$$

For an integrated process, $\log(\alpha_1 + \beta_1)$ approaches zero from below and so the λ value is infinity. This is another way of expressing the typical feature of an integrated process: impacts of shocks on the variance never die out. For the pooled data, the sum $\alpha_1 + \beta_1$ gives a half-life value of $\lambda = 257$ days.

For the second subperiod of the fixed exchange rate regime, 21 March 1989 – 5 September 1992, the GARCH(1,1) estimates on pooled data were

$$\begin{aligned} h_t = {} & \underset{(13.41)}{0.3813*E\text{-}7} + \underset{(71.22)}{0.1295}D_1 - \underset{(32.37)}{0.0361}D_2 + \underset{(9.17)}{0.0109}D_3 + \underset{(32.19)}{0.0621}\varepsilon^2_{t-1} \\ & + \underset{(532.95)}{0.9353}h_{t-1}. \end{aligned} \tag{3.6}$$

The estimated values of the ARCH and GARCH parameters are almost the same as for the earlier subperiod. The models indicate a rather weak reaction of the conditional variance to shocks but a strong persistence. Even the values of the variance drift parameters are very close to each other. Hence one can surmise that the behaviour of the exchange rates is homogeneous throughout the band period when the effects of realignments are eliminated.

The sum of the values of the individual maximum likelihood functions was 48 089 and the value for the pooled data model was 47 969. The $\chi^2_{(79)}$ was 240. Although this test statistic is also highly significant, it is clear that the violence done to the data by forcing the same model on the individual exchange rates is much less during this period than during the other periods.

The GARCH(1,1) estimation on pooled data for the floating rate period, 14 September 1992 – 31 December 1995 produced the following results:

$$h_t = \underset{(24.79)}{0.2642*E\text{-}5} + \underset{(71.53)}{0.1883\varepsilon^2_{t-1}} + \underset{(169.51)}{0.7556h_{t-1}}. \tag{3.7}$$

The sum of the maximum likelihood functions for the individually estimated currencies is 47 249 and for the pooled data 38 013. The test statistic $\chi^2_{(44)} = 18\ 471$ is highly significant, which indicates the inferiority of the forced model estimated from the pooled data compared to the freely estimated individual models and even more so for this period than for the band period.

The value of the α_1, 0.1883, shows that the impact of news on the variance is much greater than during the band period. The impact of the lagged conditional variance dies considerably faster in this period than during the band period. The estimated mean lag of the variance expression, $1/(1-\beta_1)$, equals 4.17 or about four days. The sum $\alpha_1 + \beta_1$ is 0.9439, which means that the model is highly persistent but strictly speaking not integrated. The half-life, λ, equals 13 days. The value of the estimated drift parameter for the variance, α_0, is much higher for the floating rate period than for the band period.

Looking at Figure 3.1 we see a clear turning point in the currency index in the middle of March 1993. From the start of the floating rate regime on 8 September 1992, there is a strong positive trend in the level of the index up to 10 March 1993, followed by a similarly strong negative trend. This kind of trend change could have implications for the estimation results that should be taken into account. Perron (1989) has suggested that widespread evidence of a unit root in the univariate representation of a time series may be due to the presence of important structural changes in the trend function. Such changes can occur in the intercept, in the slope or in both. Similarly, ARCH effects may occur because of misspecification of the mean of the process or, more precisely, of the markka's trend during the float. The trend reversal itself may be an indication of the markka overshooting its long-term value or of a shift in the intervention policy pursued by the central bank. In any case, the observed trend reversal point is taken as exogenously given, and to account for its possible effects on estimated volatility, the sample is split into two subsamples at that point.

With the currency index, the hypothesis of an exogenously chosen break point is preferable, especially because the slope change occurred not slowly but immediately after reaching a certain level (probably reflecting overshooting), which may have triggered intervention by the central bank.[34] There is a clear break point in the data, found ex post, that can be interpreted as a sign of nonstationarity, eg an unpredictable regime change. To account for this change in the regime, the floating rate period was divided into two parts and new pooled estimations were carried out: one covering the period of the uptrend in the currency index (markka depreciation) and the other the downtrend.

GARCH(1,1) estimation for pooled data covering the period of markka depreciation, 14 September 1992 – 10 March 1993, produced the following results:

$$h_t = \underset{(8.54)}{0.2958*E\text{-}2} + \underset{(7.88)}{0.3176}\varepsilon_{t-1}^2 + \underset{(7.30)}{0.4455}h_{t-1}. \tag{3.8}$$

The results of the GARCH(1,1) estimation on pooled data for the period starting with the break date 10 March 1993 and ending at 31 December 1995 were

$$h_t = \underset{(14.87)}{0.9189*E\text{-}6} + \underset{(18.16)}{0.0809}\varepsilon_{t-1}^2 + \underset{(152.92)}{0.8847}h_{t-1}. \tag{3.9}$$

The maximum value of the log-likelihood function for the whole floating rate period based on pooled data was 38 013. The sum of the maximum values of the log-likelihood function for the subsamples was 38 303. The value of χ^2 was 580, which is highly significant, thus indicating that splitting the floating rate period results in a superior model. Nonstationarity within the original full floating rate period is imbedded in α_0. Accounting for the trend beak by allowing the constant to be freely

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[34] See Hung 1995 on the effects of intervention strategies on exchange rate volatilities in the US.

estimated for the subperiods, we obtained widely differing values for α_0. Also the values of α_1 and β_1 differ between subsets. The identified model for the uptrend period is far from being integrated, with $\alpha_1 + \beta_1 = 0.7731$. For the downtrend period, the sum is 0.9656 and the half-life figure $\lambda = 21$ days.

The pooled model for the first band period, 1 January 1987 - 16 March 1989, is

$$h_t = \underset{(7.69)}{0.2692*E\text{-}7} + \underset{(37.73)}{0.0567}\varepsilon_{t-1}^2 + \underset{(953.01)}{0.9406}h_{t-1} \tag{3.10}$$

and for the second band period, 21 March 1989 - 5 September 1992

$$\begin{aligned} h_t = {} & \underset{(13.41)}{0.3813*E\text{-}7} + \underset{(71.22)}{0.1295}D_1 - \underset{(32.37)}{0.0361}D_2 + \underset{(9.17)}{0.0109}D_3 + \underset{(32.19)}{0.0621}\varepsilon_{t-1}^2 \\ & + \underset{(532.95)}{0.9353}h_{t-1}. \end{aligned} \tag{3.11}$$

These have very similar α_1 and β_1 parameter values, which suggests that the same model is applicable for the whole band period. The estimated model for the downtrend floating rate period,

$$h_t = \underset{(14.87)}{0.9189*E\text{-}6} + \underset{(18.16)}{0.0809}\varepsilon_{t-1}^2 + \underset{(152.92)}{0.8847}h_{t-1}, \tag{3.12}$$

is also very close to the model identified for the band period. The F-test of equality of the coefficients estimated for different periods turned out to be highly significant, thus rejecting the null hypothesis. Given the large number of observations in the pooled data, however, this formal rejection of the null is perhaps not surprising. We must therefore rely on common sense to justify the conclusion that the conditional volatility of exchange rates can be modelled as the same integrated process regardless of the exchange rate regime. The assumption of equality simplifies the multivariate analysis considerably.

3.7 Testing for deviation from IID

The abrupt large changes that occur in financial time series, especially in share prices, has fostered the idea that even GARCH modelling is too simple to capture the dynamics of the stochastic processes driving the financial markets. This has led to attempts to apply the method of complexity and chaos to financial market data.

Most applied studies on chaotic behaviour of financial time series deal with share returns. The results have been mixed. Chaos is found in some papers in US share returns, while others dispute the claim. Chaos as a general model of German stock returns is also rejected (Booth et al 1992). In an extensive study, Hsieh (1991) rejects the hypothesis that weekly share returns are IID. He tests various explanations for the rejection: linear dependence, nonstationarity, chaos and nonlinear stochastic processes. The cause cannot be found either in regime changes or chaotic dynamics but rather in conditional heteroscedasticity. Similar results are reported in a study by Booth et al (1992) on Finnish share returns. The paper concludes that the share returns exhibit nonlinear dependence but that the form of dependence is not chaotic. The nonlinear behaviour in their data is best explained by a GARCH model.

Although the evidence found so far for the presence of deterministic chaotic generators in economic and financial time series has not been very strong, the search for such generators has led to the development of new statistical tests (Brock et al 1991) of which the most commonly used one is the Brock, Dechart and Scheinkman BDS test (Brock et al 1987).

The BDS statistic provides a general test for model misspecification. It is a diagnostic test where a rejection of the null hypothesis of IID innovations is consistent with some type of dependence in the data. Dependence may result from a linear stochastic system, a nonlinear stochastic system, or a nonlinear deterministic system, ie chaos. Additional diagnostic tests are therefore needed to determine the source of the rejection (Mills 1993, p. 125).

The asymptotic distribution of the BDS statistic, N(0,1), can be used to approximate the finite sample distribution for 500 or more observations. The approximation appears unaffected by skewness or heavy tails. Simulations by Hsieh (1991) confirm that neither the asymptotic nor the finite sample distribution of the BDS test is altered by using residuals instead of raw data linear models. This is not the case, however, when the test is applied to residuals from GARCH and EGARCH models. For these conditional variance models, the BDS test may reject too infrequently. Hsieh (1991) gives simulated critical values for the BDS statistic, for use at the 2.5 per cent and 97.5 per cent confidence levels for GARCH and EGARCH residuals.

The BDS statistics for standardized residuals of the mean equation of the log differences are reported in Table 3.8 for the entire floating rate period. For this data, the N(0,1) assumption for the distribution of the test statistic is applicable. The test statistics provide strong evidence against the null hypothesis of IID for all the series. Simulations done by Hsieh (1991) show that the BDS test has good power to detect at least four types of non-IID features: linear dependence, nonstationarity, nonlinear stochastic processes and low-dimensional chaos. In our case, prefiltering of the data rules out linear dependence. Nonstationarity caused by structural changes is accounted for by dividing the estimation period into three intervals. What is left then is nonlinearity in the mean and variance. To capture nonlinearity in the mean, the GARCH-M(1,1) model was tested. The results showed that the MEAN parameter is not statistically significant for any currency. The GARCH(1,1) model was postulated to capture nonlinearity in the variance. If the GARCH model is correctly specified, the standardized residuals should be IID in large samples. To determine whether any remaining nonlinear structure is present in the model, the BDS test was applied to the standardized GARCH(1,1) residuals (Table 3.8). For five currencies, the null of IID could not be rejected when we used the simulated critical value of Hsieh, which is 2.11 for m=2 and $\epsilon/\sigma = 0.5$. For the SEK, DKK, DEM, NLG, BEF, CHF and ITL the test finds evidence of remaining nonlinearity or deterministic chaos. These findings are well in line with results reported in the literature for dollar exchange rates.

Table 3.8 **BDS statistics for exchange rates**
9 Sep 1992 - 31 Dec 1995, m = 2, $\epsilon/\delta = 0.5$

	Standardized residuals	GARCH-residuals
USD	5.32	0.88
GBP	4.66	0.64
SEK	9.55	7.99
NOK	9.16	1.52
DKK	7.22	2.79
DEM	7.32	5.49
NLG	13.92	2.90
BEF	15.19	4.29
CHF	8.19	3.13
FRF	6.85	1.23
JPY	16.26	1.00
ITL	7.79	6.34

3.8 Summary for exchange rates

So far we have shown that the stylized facts found in the FIM bilateral exchange rates can be modelled with a GARCH(1,1) process. Log-changes in the spot exchange rates are martingales, since conditional means are zero and there is no serial correlation. The results indicating a unit root in levels, no linear dependence and high persistance are in conformity with results of studies on daily exchange rates.[35]

The ARCH and GARCH parameters are significant for all exchange rates. The sum of the estimated parameters in the conditional equation for the individual currencies is close to one, indicating an integrated variance process. This is also seen in the model estimated on pooled data, which turned out to be integrated for all periods.

The principal component analysis applied to the estimated conditional variances was used to detect a common set of variables generating exchange rate movements. The reason for the relatively low value of the fraction explained by the first principal components may be related to the ITL, the inclusion of which reduces the homogeneity within the group.

Spectral analysis was performed on the estimated principal components to assess and measure common cyclical behaviour for the variances. There is a local peak in the spectral density functions of the individual variances and the first principal component at 180 days for both band periods. The spectral density function of the first principal component for the floating period shows a peak at 420 days, but the overall interpretation of the density function is that of an at-least-persistent conditional variance process, perhaps even an integrated one.

In financial applications such as VaR models, the most common assumption for the stochastic process in first differences of financial rates is that of a normal distribution of a random walk generating process with a constant unconditional variance. Although we know that this model does not fit observed data as well as autoregressive conditional variance models, this does not necessarily mean that its average performance is inferior to the time-varying models. The estimated integrated conditional model for exchange rates derived in this study indicates that a constant variance forecast may be a good approximation of the time-varying model. The random walk model can be considered as a benchmark against which the more sophisticated changing volatility models can be compared (Heynen and Kat 1994).

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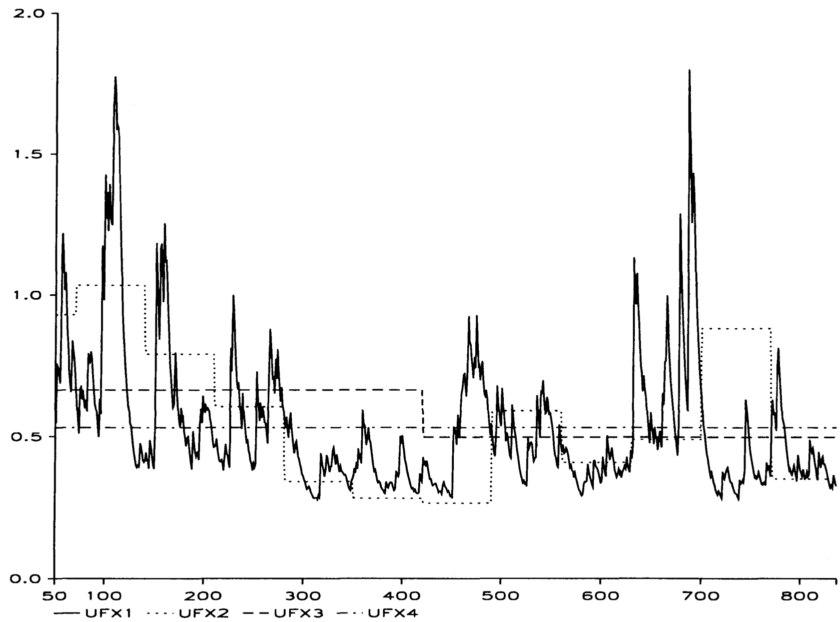
[35] See eg Harvey, Ruiz and Shephard (1994) for dollar exchange rates, Chappel and Padmore (1995) for the sterling-Deutschemark exchange rate and McKenzie (1997) for Australian bilateral exchange rates.

The alternative measures of the conditional, unconditional and sample variance of movements in the individual exchange rates can be summed up in a performance evaluation as follows:

- GARCH(1,1) conditional variance $h_{i,t}$.
- GARCH(1,1) unconditional variance $\alpha_0/(1-(\alpha_1 + \beta_1))$, which also is the convergence limit for the conditional variance h_t.
- sample variance constant for the local peak frequency evaluated on the cyclical behaviour of the individual conditional variances, h_t, and their principal components; 180 days for the band periods and 420 days for the floating rate period.
- sample variance calculated on quarterly data; frequency selection is based on previous results (Ahlstedt 1990) where a subsample of 70 observations was found to be large enough to yield reasonable statistical efficiency but still small enough to make it likely that the sample variance remains constant. The power spectrum of the variances also gives some support to this period.

Figure 3.4 **Conditional, unconditional and sample variance comparison: USD in the floating rate period**

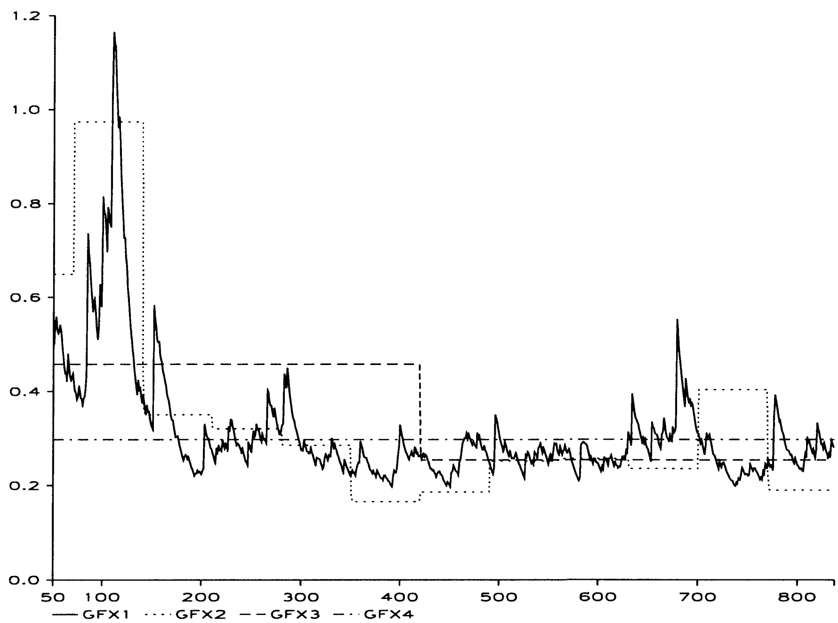


UFX1 GARCH(1,1) conditional variance
UFX2 GARCH(1,1) unconditional variance
UFX3 Homoscedastic variance; 420-day window
UFX4 Homoscedastic variance; 70-day window

Figure 3.4 gives a comparison of the four methods for the USD in the floating rate period. The GARCH unconditional variance can be visually interpreted as a mean approximation of the conditional variance. The dominant frequency for the floating rate period, 420 days, appears twice in the sample size. This two-step function also gives a good visual approximation of the mean of the conditional variance. The step function formed by the 70-day sample period, ie quarterly frequency, smooths out the wide swings in the conditional variance and seems to capture the basic pattern of fluctuations in the variance.

The corresponding variance measures are displayed in Figure 3.5 for the GBP in the same floating rate period. The outcome of the comparison for this exchange rate is generaly the same as for the USD exchange rate.

Figure 3.5 **Conditional, unconditional and sample variance comparison: GBP in the floating rate period**



GFX1	GARCH(1,1) conditional variance
GFX2	GARCH(1,1) unconditional variance
GFX3	Homoscedastic variance; 420-day window
GFX4	Homoscedastic variance; 70-day window

4 Interest rate risk, equity risk and real estate risk

In this chapter we will consider short- and long-term interest rate risk, the equity risk measured by the volatility of the all-share index and real estate risk.

4.1 Conditional variance of interest rates

ARCH methodology has been applied to interest rate data mainly to model the term structure with a time-varying risk premium.[36] The time-series variables modelled in these studies have been measures of excess returns of long-term yields over short-term yields or yields on corporate bonds over yields on credit-risk-free Treasury bonds.

The ARCH-M and GARCH-M models in particular have been used in these studies. In these models, a function of the conditional variance is included as a regressor in the mean equation as a measure of the risk premium. These models have however not been very successful. Inclusion of a MEAN term usually makes variables that have previously been found significant no longer so. As a result, the usefulness of the model has also been challenged both on theoretical grounds by Backus, Gregory and Zin (1989) and on empirical grounds by Mehra and Prescott (1985), who showed that the ARCH effects are more closely related to forecast errors than to risk premiums.

Since most studies involving interest rates have nonetheless adopted the GARCH(1,1) or GARCH-M(1,1) specifications, these models are also used here.[37] Most of the studies concentrate on yields, which are measured separately for individual bonds. However, since the aim of this study is to find a measure for interest rate risk in banks' portfolios

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[36] See eg Shiller and Singleton (1980), Engle, Lilien and Robins (1987), Bollerslev, Engle and Wooldridge (1988) and other papers mentioned in the survey of Bollerslev et al (1992, 1994).

[37] The GARCH-M(1,1) model was tested both for the band and floating rate periods. For the floating rate period, the MEAN variable was statistically significant only for ERUSD. Inclusion of MEAN, however, makes the GARCH parameter insignificant, thus confirming the results from other studies.

without knowing the individual bond holdings,[38] publicly quoted interest rates were used instead of yields.

Inclusion of the entire term structure of interest rates for all currencies in the study was not feasible. One way of reducing the number of variables while still including the behaviour of the entire term structure would be to use the method of principal components. Using this method, the variances of all interest rates for one currency can be transformed into three main variables describing changes, respectively, in the overall level of the term structure and changes in the slope and curvature of the term structure (Kärki and Reyes 1994). If the principal component method is used, then forecasting should accordingly concentrate on these types of changes in the curve. Since the objective of this study is to construct an estimate for the future behaviour of the rates themselves, we decided not to use the principal component method in this context. Instead, the solution to the problem of term structure coverage was sought by selecting one rate to represent all short rates up to one year. We calculated the correlation matrix for one-, three-, six- and twelve-month rates. Since the three-month rate was tested to have the highest correlation with the other short-term rates, it was selected to represent the term structure up to one year. The domestic three-year rate was selected to represent the longer rates. Selection of the three-year term is based on the historical data (reported to the supervisors) on the average duration of bonds in Finnish banks' trading portfolios.

The main time periods were the same as for the exchange rates. Although the structural changes on which the division is based are not as clear as for the exchange rates, the division is defendable (Figure 4.1). Daily changes in interest rates are expressed as differences relative to levels:[39]

$$\frac{R_t - R_{t-1}}{1 + R_t}.$$

The order of differencing is dictated by the requirement of stationarity in the mean equation. To this end, the Weighted Symmetric τ test, Phillips-Perron Z-test and the augmented Dickey-Fuller test were applied to levels and differences, both for the band period, 1 January 1987 – 5 September 1992, and the floating rate period, 9 September 1992 – 31 December

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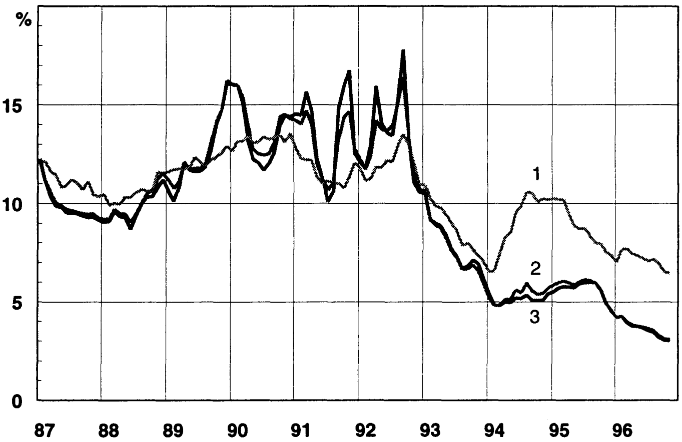
[38] Supervisors do not have such detailed information, but the banks themselves do.

[39] Multiplying a change defined in this way by the duration of the interest rate instrument gives the change in a bond price, ie the capital gain.

1995. The hypothesis of a unit root in levels was not rejected, but was strongly rejected in first differences by all three tests for both periods and all interest rates. The estimated p-values for differences for Type I error are always zero for the band period. The largest p-value for the floating period is 0.005 per cent, for ERGDP. Based on the results of the augmented Dickey-Fuller test, we conclude that there is no trend or constant in the unit root process generating observed interest rates.

Figure 4.1 **Key interest rates**



1 Long-term bond rate (close to ten-year)
2 3-month HELIBOR
3 1-month HELIBOR

The interest rate differentials, unlike the exchange rates, reveal strong linear serial correlation according to the Ljung-Box test. The mean equation therefore is expressed in the following form:[40]

$$\frac{R_t - R_{t-1}}{1+R_t} = \Delta R_t = \sum_{i=1}^{p} \phi_i \Delta R_{t-i} + \varepsilon_t,$$

where ϕ_i is the autocorrelation coefficient of order i.

Prior to specifying the variance equation in the GARCH model for the interest rate series, the rates had to be filtered from this linear dependence. AR(p) models with $p \leq 5$ were accordingly identified. The

[40] Based on the test results, no constant term was included in the mean equation.

selection of the order p (Table 4.1) is based on the 1 per cent probability level for the Jung-Box test statistic. It would have been very convenient to use the same order of AR filtering for all series. However, it was found that over-filtering for some interest rates removed the significant GARCH effects in the data under lower-order filtered data. In order to avoid the harmful effects on the data of over-filtering, the orders of the linear autoregressive filtering models were chosen individually for all thirteen series. The test values for the residuals of these pre-filtered models show that the filtering process produced linearly independent data for all interest rates except ERGBP. For ERGBP, even using twelve lags is insufficient to remove serial correlation during the first band period.

Table 4.1 **Selected order of pre-filtering, 3-month interest rates**

	Lags up to order AR(p)		
	1.1.87–16.3.89	21.3.89–5.9.92	8.9.92–31.12.95
ERUSD	AR(1)	AR(1)	AR(3)
ERGBP	AR(12)*	AR(2)	AR(3)
ERSEK	AR(2)	AR(2)	AR(5)*
ERNOK	AR(1)	AR(3)	AR(5)
ERDKK	AR(2)	AR(1)	AR(3)
ERDEM	AR(1)	AR(2)	AR(3)
ERNLG	AR(1)	AR(1)	AR(1)
ERBEF	AR(4)	AR(1)	AR(1)
ERCHF	AR(1)	AR(2)	AR(3)
ERFRF	AR(5)	–	AR(4)
ERITL	AR(2)	AR(2)	AR(5)*
ERJPY	AR(1)	AR(1)	AR(1)
ERFIM	AR(1)	AR(2)	AR(4)

* Linear dependence remaining in the pre-filtered data.

4.2 Short-term interest rate risk

4.2.1 Exchange rate band period

In this section we deal with short-term interest rate risk, measured by the volatility of thirteen three-month money market rates.

4.2.1.1 First subperiod: 1 January 1987 - 16 March 1989

Table 4.2 shows the results of GARCH(1,1) estimation for the first band period, 1 January 1987 - 16 March 1989, on the prefiltered interest rate differences for ERUSD, ERGBP, ERSEK, ERNOK, ERDKK, ERDEM, ERNLG, ERBEF, ERCHF, ERFRF, ERITL, ERJPY and ERFIM. In the iterative estimation, the ARCH parameter for ERDKK was set at its lower boundary value, zero, which means that news has no impact on the variance process. The GARCH parameter for ERGBP and ERCHF were also set at the lower boundary value, zero. Thus for these two interest rates, past conditional variance does not help in forecasting future conditional variances.

Table 4.2 **GARCH (1,1) estimation of the volatility of 3-month interest rates, 1 Jan 1987 - 16 Mar 1989 (t-statistics in parentheses) (data multiplied by 100)**

	α_0	α_1	β
ERUSD	0.4374E-2	0.2885	0.1442
	(6.12)	(4.53)	(1.27)
ERGBP	0.0294	0.1105	0
	(73.42)	(2.55)	
ERSEK	0.1371E-2	0.2400	0.7390
	(4.37)	(6.47)	(25.79)
ERNOK	0.1177E-2	0.1253	0.8481
	(2.49)	(8.25)	(41.37)
ERDKK	0.8462E-5	0	0.9957
	(0.31)		(1200.28)
ERDEM	0.1516E-3	0.0927	0.8871
	(2.70)	(3.85)	(31.55)
ERNLG	0.7243E-4	0.0947	0.8888
	(2.92)	(4.68)	(40.72)
ERBEF	0.4927E-3	0.0930	0.8359
	(4.99)	(5.60)	(41.33)
ERCHF	0.8482E-2	0.0892	0
	(19.93)	(1.59)	
ERFRF	0.1237E-2	0.1766	0.7299
	(3.77)	(4.39)	(15.32)
ERITL	0.5298E-2	0.2976	0.5605
	(5.07)	(6.42)	(9.56)
ERJPY	0.1143E-3	0.0583	0.9080
	(2.14)	(3.20)	(38.19)
ERFIM	0.6582E-4	0.2949	0.7559
	(4.83)	(10.33)	(38.97)

The estimated models are (weakly) stationary in variance[41] with the exception of ERFIM, for which the sum $\alpha_1 + \beta_1$ is 1.0508. Even though this value is probably not significantly different from one, the Finnish interest rate was excluded from the pooled data. In forecasting, the domestic interest rate will be forced to follow the model that is estimated on the pooled data.

In order to detect common factors driving the conditional variances of interest rates, principal components were estimated for the band periods. The eigenvalues and cumulative fraction explained by the components for the first band period are shown in Table 4.3. The fractions explained by the first components are generally, relatively small compared to macroeconomic data and clearly lower than those for the exchange rates. The conditional variance of ERBEF has the strongest factor loading on the first principal component, followed by ERITL and ERJPY. The factor loading for the US interest rate is practically neglible. Graphical analysisis of the principal components also indicates that it is not possible with this method to identify strong common factors that could be used in an application of the k-factor multivariate GARCH model proposed by Engle, Granger and Kraft (1984).

[41] In the absence of autocorrelation, $\alpha_1 + \beta_1 < 1$ is sufficient for weak stationarity. Serial autocorrelation requires a correction term that is a function of the autocorrelation coefficients. Bera, Higgins and Lee (1990) give the stationary condition for an ARCH(q) process in the presence of first-order serial correlations as $\frac{1}{1-\phi^2}\sum_{i=1}^{q}\alpha_i < 1$, as pointed out in Bera and Higgins (1993).

Table 4.3 **Principal components of conditional variances, eigenvalues and cumulative fraction explained 3-month interest rates 1 Jan 1987 - 16 Mar 1989**

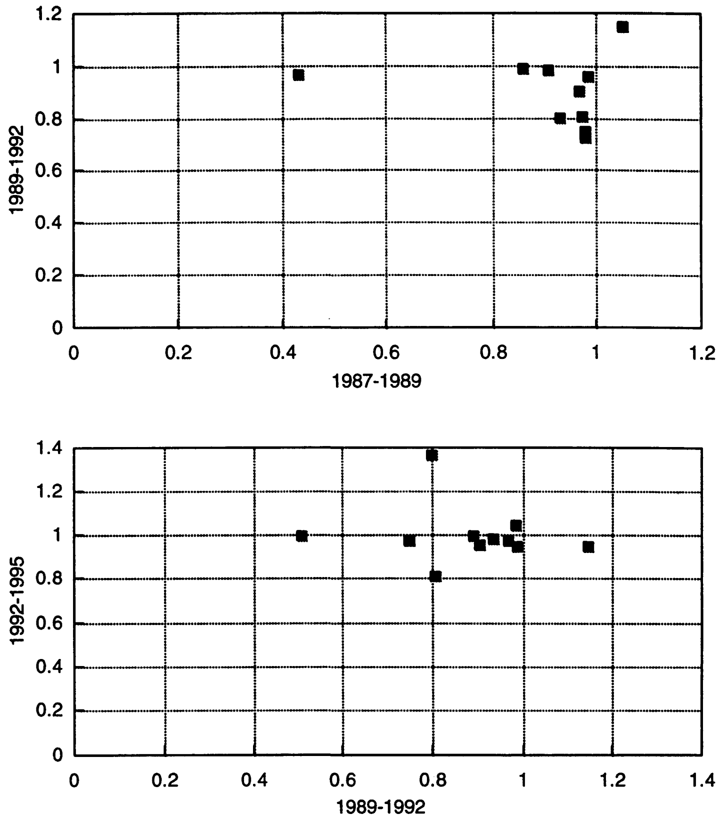
Component	Eigenvalue	Cumulative R-Squared
1	4.2484	0.3268
2	1.9312	0.4753
3	1.4134	0.5840
4	0.9874	0.6600
5	1.0417	0.7401
6	0.6315	0.7887
7	0.8260	0.8523
8	0.7940	0.9133
9	0.4775	0.9501
10	0.3746	0.9789
11	0.1327	0.9891
12	0.1082	0.9974
13	0.0328	1.0000

Spectral analysis of the individual variances, h_t, and of the first principal component once again strongly suggest persistence in the underlying factors affecting the time variability of the conditional variances. Moreover, a cycle corresponding to a period of 280 days (second harmonic and 0.0224 radians or 0.0036 cycles per day) is seen as a local peak in the spectral density function.

As in the case of exchange rates, the use of pooled data would impose the same structure on all the interest rates. In the same way as with exchange rates, we graphically tested the similarity between the estimated individual interest rate models by plotting the sum of the ARCH and GARCH coeffients for the three main periods against each other. Figure 4.2 displays strong clustering and in this sense sustains the analysis on the pooled data.

Figure 4.2 **Pairs of sums $\alpha_1 + \beta_1$ for different periods**

Interest rates



The following GARCH(1,1) model was estimated on the pooled data for the first band period:

$$h_t = \underset{(6.36)}{0.3770 * E\text{-}6} + \underset{(49.08)}{0.0669\varepsilon_{t-1}^2} + \underset{(1526.19)}{0.9418h_{t-1}}. \qquad (4.1)$$

The sum of the estimated ARCH and GARCH parameters is 1.0087, which indicates that the conditional variance process of the interest rates is integrated. The mean lag, $1/(1-\beta_1)$, equals 17 days and the half-life frequency for the integrated process, λ, is infinite.

4.2.1.2 Second subperiod: 21 March 1989 – 5 September 1992

Prior to GARCH estimation, the data for the second band period, 21 March 1989 – 5 September 1992, were prefiltered. The selected order, p, based on the Box-Ljung statistic is presented in Table 4.1 for each interest rate.

Table 4.4 presents the results from the GARCH(1,1) estimation for the second band period. Both the ARCH and GARCH parameters are significant for all interest rates. With the exception of ERFIM, all interest rates are stationary in variance. For the first band period, the sum $\alpha_1 + \beta_1$ for the ERFIM was 1.0508 and for this second period 1.0476. Neither sum differs significantly from one, and so we conclude that there is a unit root in the conditional variance process for both band periods for the Finnish interest rate. This second band period includes a 12.3 per cent devaluation of the Finnish currency on 15 November 1991. This realignment is accounted for in the GARCH estimation for foreign exchange rates on the corresponding period by using dummy variables. In the Finnish interest rate data, there is a high spike at the devaluation date. An alternative model was tested for ERFIM with a dummy variable for the crucial date. There was no change in the estimated ARCH and GARCH parameter values compared to the model estimated without the dummy variable.

The results from the principal components analysis on the second band period are shown in Table 4.5. The cumulative fraction explained by the components grows very slowly with the number of components included. There is an even stronger heterogenity in this group of conditional variances than for the previous period. For this period, we find the same dominant interest rates in the factor loadings of the first principal component as were found for the first band period.

Whereas the spectral density function of the first principal component also in this subperiod clearly provides evidence of persistent factors underlying the conditional variance processes, the contributions from higher frequencies can be seen, most notably from those corresponding to cycle lengths of 62–174 days (0.0362–0.1014 radians or 0.0058–0.0161 cycles per day, ie 5–14 harmonics).

Table 4.4 **GARCH(1,1) estimation of the volatility of 3-month interest rates, 21 Mar 1989 - 5 Sep 1992 (t-statistics in parenthesis) (data multiplied by 100)**

	α_0	α_1	β_1
ERUSD	0.2160E-3	0.0878	0.8772
	(4.39)	(5.80)	(44.77)
ERGBP	0.6923E-2	0.4098	0.0983
	(15.50)	(6.40)	(1.81)
ERSEK	0.8382E-2	0.2768	0.4710
	(14.39)	(11.03)	(13.26)
ERNOK	0.3710E-2	0.2058	0.6001
	(10.97)	(5.94)	(17.60)
ERDKK	0.1520E-2	0.2788	0.6092
	(9.23)	(8.77)	(21.71)
ERDEM	0.1387E-2	0.1862	0.5381
	(4.68)	(5.74)	(6.67)
ERNLG	0.1504E-3	0.1037	0.8551
	(3.52)	(5.68)	(33.57)
ERBEF	0.9879E-3	0.0743	0.7244
	(3.24)	(3.51)	(9.81)
ERCHF	0.8691E-3	0.1359	0.7958
	(2.98)	(7.02)	(22.55)
ERFRF	0.8490E-4	0.0565	0.9258
	(3.44)	(5.26)	(80.28)
ERITL	0.7565E-3	0.2001	0.7874
	(6.32)	(10.72)	(45.44)
ERJPY	0.4072E-3	0.1881	0.7140
	(10.37)	(7.36)	(37.19)
ERFIM	0.1788E-2	0.5929	0.5547
	(10.05)	(19.10)	(45.05)

Table 4.5 **Principal components of conditional variances, eigenvalues and cumulative fraction explained 3-month interest rates, 21 Mar 1989 – 5 Sep 1992**

Component	Eigenvalue	Cumulative R-squared
1	2.8366	0.2182
2	2.0931	0.3792
3	0.2627	0.3994
4	0.3257	0.4244
5	0.4213	0.4568
6	1.1914	0.5485
7	0.6008	0.5947
8	0.6849	0.6474
9	0.7879	0.7080
10	0.8417	0.7728
11	0.9601	0.8466
12	1.0010	0.9236
13	0.9923	1.0000

The GARCH(1,1) model for the pooled data on the second band period is

$$h_t = \underset{(58.15)}{0.1497*E\text{-}5} + \underset{(61.76)}{0.0958}\varepsilon_{t-1}^2 + \underset{(1444.69)}{0.9005}h_{t-1}. \quad (4.2)$$

The sum of the ARCH and GARCH coefficients is 0.9963, which indicates an integrated variance process even for this second band period. Although the sum of the coefficients is the same for both band periods, the estimated values differ between the individual coefficients. The impact of news, α_1, is bigger for the second period and thus the impact of the past conditional variance is smaller. Since the value of β_1 determines the mean lag of shocks, this lag measure will also differ between the two periods. The mean lag for the first period is 17 days and for the second period 10 days. The half-life statistic, λ, is 188 days for the second period.

4.2.2 The floating rate period

The results from prefiltering of order p are shown in Table 4.1. Despite the prefiltering, ERSEK and ERITL still showed linear dependence according to the Ljung-Box test.

The results from the GARCH(1,1) estimation for the floating rate period, 9 September 1992 - 31 December 1995, are shown in Table 4.6. The GARCH coefficient is significant for all interest rates. For ERDEM and ERNLG, the ARCH parameter was set at its lower boundary value, zero. News has no impact on these interest rates. The sum $\alpha_1 + \beta_1$ is less than one for most interest rates. The sum is exactly one for ERGBP, thus indicating an integrated model. The models for ERNOK, ERDKK, ERBEF and ERFRF are however nonstationary in variance.

Graphs of the individual prefiltered interest rate data display increasing volatility during the turbulent times at the start of the floating rate period; interest rates reacted strongly to the perceived uncertainty in the currencies while they were approaching equilibrium after fierce speculative attacks against them. This period must however be regarded as exceptional and inappropriate as a basis for forecasting. Thus, the turbulent period can be removed from the estimation period for those currencies whose sum $\alpha_1 + \beta_1$ exceeds one. For these currencies, the estimation period was chosen in accord with the requirement that the conditional variance process be at most integrated. In the case of ERNOK, this was constructed by dropping the first 100 data points; for DKK, by dropping the first 250 observations. The nonstationarity in ERBEF could not be eliminated by selection of a subperiod, since the nonstationary features in its variance are distributed over the entire period. ERFRF shows clear nonstationarity at the beginning and end of the period. The middle period is too short to be used for identification of the model. Even though several subperiods were tested, stationarity was not achieved.

The GARCH estimation results for stationary periods for ERNOK and ERDKK appear in Table 4.7.

Table 4.6 **Garch (1,1) estimation of the volatility of the 3-month interest rate, 8 Sep 1992 – 31 Dec 1995 (t-statistics in parenthesis) (data multiplied by 100)**

	α_0	α_1	β_1
ERUSD	0.4320·E−4	0.0460	0.9298
	(4.10)	(5.01)	(66.52)
ERGBP	0.2766·E−4	0.0663	0.9289
	(4.52)	(7.51)	(144.17)
ERSEK	0.4519·E−3	0.0610	0.9098
	(8.74)	(7.48)	(114.59)
ERNOK	0.2272·E−2	0.9826	0.4215
	(6.06)	(59.82)	(17.69)
ERDKK	0.8906·E−3	0.4743	0.6849
	(18.12)	(18.75)	(57.47)
ERDEM	0.8772·E−5	0	0.9902
	(11.35)		(1550.77)
ERNLG	0.3544·E−5	0	0.9941
	(5.57)		(1731.46)
ERBEF	0.1576·E−2	0.8218	0.5428
	(14.36)	(16.34)	(40.95)
ERCHF	0.5007·E−4	0.0393	0.9420
	(3.70)	(5.17)	(93.02)
ERFRF	0.2938·E−4	0.1705	0.8687
	(6.90)	(27.10)	(361.04)
ERITL	0.1029·E−6	0.0944	0.8501
	(8.24)	(6.08)	(52.53)
ERJPY	0.1038·E−7	0.1447	0.8113
	(6.72)	(10.30)	(45.28)
ERFIM	0.2387·E−3	0.0711	0.8768
	(14.87)	(14.99)	(131.87)

Table 4.7 **GARCH (1,1) volatility estimation, interest rates, subperiod 9 Sep 1992 – 31 Dec 1995**

	α_0	α_1	β_1	
ERNOK	0.2520E−2	0.1319	0.6772	(−100)
	(3.49)	(4.41)	(8.83)	
ERDKK	0.1380E−2	0.4133	0.5829	(−250)
	(12.92)	(11.29)	(22.77)	

Principal components are presented in Table 4.8. The figures for the cumulative fraction explained by the principal components show a much higher degree of homogeneity for this period than for the band period. The same pattern is also present in the factor loadings of the first components.

This time the power spectrum of the first principal component conforms well to the spectrum of a highly persistent component process, although there is a local peak at 420 days and at its harmonic, 840 days. The overall interpretation of the power spectrum for interest rates during the floating rate regime is the same as for the exchange rates during the corresponding regime, ie the spectrum is typical for an integrated stochastic process.

Table 4.8 **Principal components of conditional variances, eigenvalues and cumulative fraction explained, 3-month interest rates, 8 Sep 1992 - 31 Dec 1995**

Component	Name	Eigenvalue	Cumulative R-squared
1	P1	7.3626	0.5663
2	P2	1.4651	0.6790
3	P3	1.0793	0.7620
4	P4	0.8284	0.8258
5	P5	0.9202	0.8966
6	P6	0.5711	0.9405
7	P7	0.3500	0.9674
8	P8	0.1778	0.9811
9	P9	0.1012	0.9889
10	P10	0.0674	0.9941
11	P11	0.0516	0.9981
12	P12	0.0205	0.9996
13	P13	0.0041	1.0000

A pooled series was formed from the stationary estimation period for the individual interest rates, which was the full period for ERUSD, ERGBP, ERSEK, ERDEM, ERNLG, ERCHF, ERITL, ERJPY and ERFIM. Sub-periods were used for ERNOK and ERDKK. Due to the nonstationarity of its conditional variance process over the entire period, ERBEF is excluded from the pooled data. Since no stationary subperiod was found for ERFRF, this interest rate is also excluded from the pooled series. In the forecasting, ERBEF and ERFRF as well as the other interest rates, were forced to follow the process estimated from the pooled data.

The estimated GARCH(1,1) model for the pooled data for this period is

$$h_t = \underset{(173.49)}{0.4223*E\text{-}8} + \underset{(112.32)}{0.0881}\varepsilon^2_{t-1} + \underset{(2683.54)}{0.9367}h_{t-1}. \quad (4.3)$$

The sum $\alpha_1 + \beta_1$ is 1.0247, which suggests nonstationarity in variance. The null of an integrated variance model, $\alpha_1 + \beta_1 = 1$, would most likely pass statistical testing.

At the start of the empirical part of this study, the data were divided into three separate subperiods to account for exogenously identified structural changes in the exchange rate regime. In the third period, there is a clear change in the trend of exchange rate levels. Consequently this subperiod was divided into two parts: one covering the uptrend and the other the downtrend. The same partition was then adopted for the interest rates, and a GARCH(1,1) model was estimated on the pooled data for the second half of the third period, 11 March 1993 – 31 December 1995, to detect possible effects of the trend break. The results are very similar to those for the model covering the full period:

$$h_t = \underset{(136.96)}{0.1981*E\text{-}9} + \underset{(95.07)}{0.0793}\varepsilon^2_{t-1} + \underset{(2699.19)}{0.9399}h_{t-1}. \quad (4.4)$$

As the sum $\alpha_0 + \beta_1 = 1.0192$ does not differ significantly from 1, we conclude that the inclusion of a structural change has a neglible impact on the estimated parameter values and that the resulting model is approximately IGARCH(1,1).[42] The estimated mean lag for the floating rate period is about 17 days, the same lag as was estimated for the first band period.

The similarity between the estimated models on pooled data for the three main periods is not as strong as for the exchange rates. The estimated GARCH(1,1) model on the pooled data for the first subperiod of the band period is

$$h_t = \underset{(6.36)}{0.3770*E\text{-}6} + \underset{(49.08)}{0.0669}\varepsilon^2_{t-1} + \underset{(1526.19)}{0.9418}h_{t-1}. \quad (4.5)$$

The GARCH(1,1) model for the pooled data on the second band period is

[42] See Lamourex and Lastrapes (1990) on the effects of structural changes on persistence parameters.

$$h_t = \underset{(58.15)}{0.1497*E\text{-}5} + \underset{(61.76)}{0.0958}\varepsilon_{t-1}^2 + \underset{(1444.69)}{0.9005}h_{t-1} \tag{4.6}$$

and for the second part of the floating rate period

$$h_t = \underset{(136.96)}{0.1981*E\text{-}9} + \underset{(95.07)}{0.0793}\varepsilon_{t-1}^2 + \underset{(2699.19)}{0.9399}h_{t-1}. \tag{4.7}$$

Due to the large number of observations, the equality of the coefficients in a formal F-test is rejected; differences in the estimated parameter vectors are however fairly small, so that the conditional variance of the interest rate process is assumed to be the same across exchange rate regimes.

4.2.3 Testing for deviation from IID

The BDS statistics for the standardized residuals of the prefiltered raw data are reported in Table 4.9 for the full period and for shorter periods. There is strong evidence against the null hypothesis of IID for all series. The values of the test statistics for ERNOK and ERDKK are reduced with the shortening of the period but remain significant. After controlling for linear dependence, ie nonstationarity due to possible structural changes, deviations from the null of IID residuals could be due to nonlinearity in either the mean or variance. To capture nonlinearity in the mean, the GARCH-M(1,1) model was tested. The results showed that the MEAN parameter is not statistically significant for most interest rates. In one case where it is significant, it makes the GARCH parameter insignificant. The GARCH(1,1) was postulated to capture nonlinearity in variance. If the GARCH model is correctly specified, the standardized residuals should be IID in large samples. To determine whether any remaining nonlinear structure is present in the model, the BDS test was applied to the standardized GARCH(1,1) residuals (Table 4.9). Although the figures are smaller than those for the prefiltered raw data, a trace of nonlinearity still appears in most of the residual processes.

Table 4.9 **BDS statistics for filtered interest rates, 9 Sep 1992 – 31 Dec 1995, m = 2, $\epsilon/\delta = 0.5$**

	Long period	Shortened period	GARCH-residuals
ERUSD	9.97		7.84
ERGBP	7.71		3.74
ERSEK	11.94		8.39
ERNOK	14.90	1.96	-0.98
ERDKK	17.89	11.62	2.66
ERDEM	8.32		7.66
ERNLG	4.17		2.70
ERBEF	12.66		2.99
ERCHF	4.74		
ERFRF	18.07		
ERJPY	8.40		7.46
ERITL	10.75		6.07
ERFIM	11.55		10.10

4.2.4 Summary for short-term interest rates

For the 13 three-month interest rates, GARCH(1,1) models were estimated for three intervals selected to account for possible structural changes triggered by realignments of the domestic currency. The interest rates differed from the foreign exchange rates in that they reveal strong linear dependence in the raw data, which required prewhitening of the data. Also, nonstationarity conditional variance tends to be more typical of the interest rates than the exchange rates.[43] We were not able to identify the whole model for some interest rates since either the ARCH parameter or the GARCH parameter was set to its lower boundary value in the iteration process, regardless of the selection of initial values. The models estimated on pooled interest rate data turned out to be integrated in variance. The same parameter values are valid for all periods regardless of the current exchange rate regime. The same result was found for the exchange rates.

Spectral analysis of individual conditional variances and first principal components also suggests that the conditional variance processes are highly persistent; large contributions to the time-variability

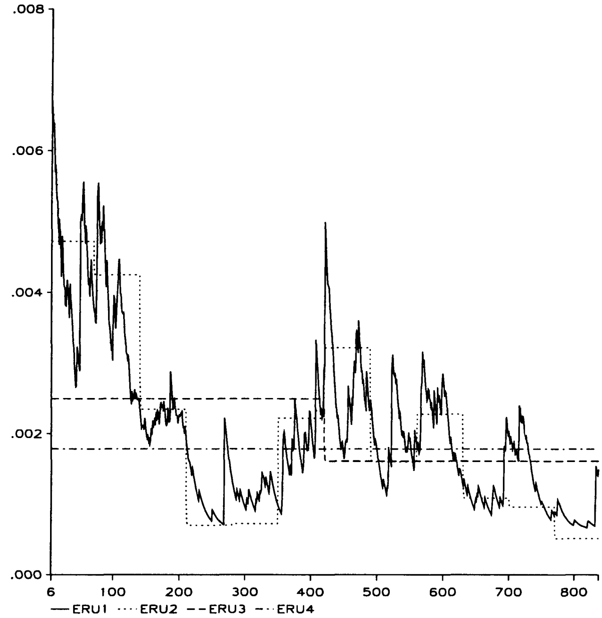
[43] Other studies also report problems with short interest rate modelling in that the conditional variance process is not covariance-stationary (Gray 1996). Examples of studies where the sum of the estimated ARCH and the GARCH parameters exeeds one are Engle, Ng and Rothschild (1990), Hong (1988) and Engle, Lilien and Robins (1987).

of the conditional variances or their first principal components also come from shorter cycles, especially during the later band period. Indications were found of a common local cyclical period of 180 days for both band periods and 420 days for the floating rate period. Even in this feature, the results coincide with corresponding results for exchange rates.

Next we compare alternative measures of interest rate volatility derived from the estimation results. The expressions for the conditional, unconditional and sample variance measures are the same as for the foreign exchange rates:

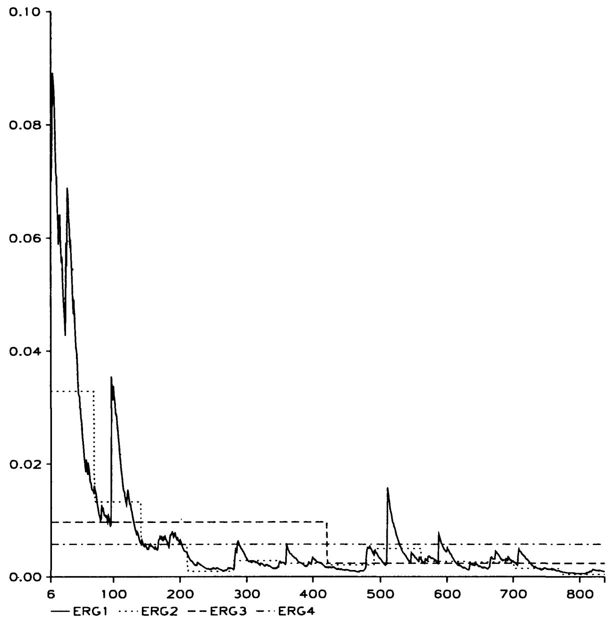
- GARCH(1,1) model conditional variance, $h_{i,t}$
- GARCH(1,1) unconditional variance, $\alpha_0/(1-(\alpha_1+\beta_1))$, the convergence limit for the conditional variance, h_t
- sample variance constant for the local peak frequency evaluated on the cyclical behaviour of individual conditional variances and their principal components; 180 days for the band period and 420 days for the floating rate period.
- sample variance calculated on quarterly data; frequency selection based on previous results (Ahlstedt 1990, 1995). A subsample of about 70 observations was found to be large enough to yield reasonable statistical efficiency yet small enough to make it likely that the sample variance remains constant.

Figure 4.3 **Conditional, unconditional and sample variance, measure comparison, ERGBP, floating rate period**



ERU1	GARCH(1,1) conditional variance
ERU2	GARCH(1,1) unconditional variance
ERU3	Homoscedastic variance, 420-day window
ERU4	Homoscedastic variance, 70-day window

Figure 4.4 **Conditional, unconditional and sample variance, measure comparison, ERUSD, floating rate period**



ERG1	GARCH(1,1) conditional variance
ERG2	GARCH(1,1) unconditional variance
ERG3	Homoscedastic variance, 420-day window
ERG4	Homoscedastic variance, 70-day window

The four alternative volatility measures are plotted in Figure 4.3 for the ERUSD and in Figure 4.4 for ERGBP for the floating rate period. The sample variances work as smooth mean values for the rough fluctuations in the daily conditional volatility, $h_{i,t}$.

4.3 Long-term interest rate risk

To capture the interest rate risk inherent in the bonds in banks' trading portfolios, we need an estimate of the variance of the long-term rate. The bonds in those portfolios are mainly markka-denominated, so we focus on the three-year markka bond rate as a proxy for the interest rate term structure for the entire bond portfolio. Selection of the three-year term is based on historical data on the average duration of bonds in the trading portfolios of individual Finnish banks.

Figure 4.1 shows the long-term bond rate for the period under consideration. The data were divided into two periods: the band period, 1 January 1990 – 5 September 1992, and the floating rate period, 8 September 1995 – 31 December 1995.

Next we look at the stationarity of the long-term rate. The unit root test statistics for the weighted symmetric τ test, the Dickey-Fuller τ test and the Phillips-Peron Z test on first-differenced series are all statistically significant, which means that the hypothesis of a unit root is rejected. Differencing once produces a mean stationary series.

To detect linear dependency, the Ljung-Box test was performed on the differences. The test statistics reveal strong autocorrelation. AR filtering of order one for the first period and order three for the second period are sufficient to remove the linear dependence in the mean.[44] ARCH effects are detected for both periods.

The GARCH(1,1) model was estimated for both periods and the results are shown in Table 4.10. All estimated parameter values are statistically significant. The process is not integrated for either period: for the band period the sum $\alpha_1 + \beta_1$ is 0.8870 and the mean lag 1.1 days; for the floating rate period the sum is 0.6183 and the mean lag 1.2 days. Low persistence is also measured by the half-life statistic, λ, which is 7 days for the band period and 2.5 days for the floating rate period. The low persistence also means a strong mean-reverting process in the time path of the conditional variance, ie the effects of shocks on the current conditional variance of the forecast of the future variance die out relatively quickly.

Table 4.10 **Long Rate, Differences**

GARCH(1,1) estimation of volatility (t-values in parentheses)

		α_0	α_1	β_1
1 Jan 90 – 5 Sep 92	AR(1)	0.9206E–7 (19.50)	0.7800 (10.50)	0.1070 (3.84)
8 Sep 92 – 31 Dec 95	AR(3)	0.2030E–6 (14.93)	0.4420 (7.02)	0.1763 (3.19)

[44] Estimation results for the drift parameter in the mean equation showed variability in statistical significance and very small values. The constant was excluded from the GARCH estimation following Figlewski (1994), showing that since the true mean in most financial time series is both close to zero and prone to estimation errors, estimates of the volatility are often made worse by including noisy estimates of the mean.

These results conform with results on bond rates in other countries. Fischer (1996), for example, finds that the volatility of daily data on Swiss government bond rates is well modelled by GARCH(1,1) process. The rates show significant first-order autocorrelation in the mean equation but the variance model is closer to an integrated process than that for the Finnish data.

BDS statistics to detect remaining nonlinearity are presented in Table 4.11. Conditional variance modelling reduced the values of the test statistics to half the value for the filtered raw data, but they were still high enough to reject the null of IID.

Table 4.11 **BDS statistics for the standardized residuals for the 3-year interest rate and the all-share index**

	Residuals from AR filtered data	GARCH(1,1) residuals
Band period		
Long-term rate	8.71	4.17
Stock index	1.74	1.14
Floating rate period		
Long-term rate	8.57	4.22
Stock index	4.97	1.71

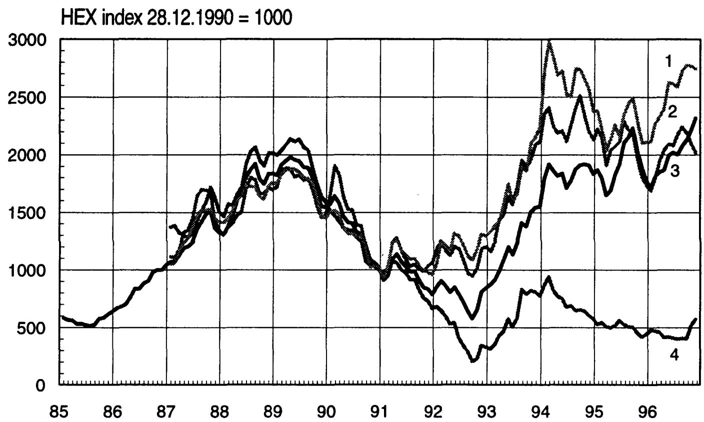
To test the hypothesis of a time-varying risk premium in the long-term rate, the GARCH-M(1,1) model was estimated for both periods. For the band period, inclusion of the standard deviation with a coefficient of γ_1 in the mean equation resulted in a statistically significant γ_1 parameter, but at the same time β_1 lost its significance. For the floating rate period, the estimated γ_1 parameter was set at its boundary value, zero, in the iteration process.

4.4 Equity risk

Market risk includes also uncertainty about future values of traded shares in the portfolios. To measure this risk, we need an estimate of volatility in share prices. Assuming that banks and investors in general behave as enlightened traders who diversify their portfolios to remove idiosyncratic risk, the variance of the all-share index can be used as an estimate of the remaining systematic risk in the equity portfolio.

The data employed are daily log changes of the all-share index of the Helsinki Stock Exchange. This index transformation is used to measure capital gains in equity investments (excluding dividends). Figure 4.5 displays the all-share index in levels.

Figure 4.5 **Helsinki Stock Exchange share prices by sector**



1 Metal industries
2 Forest industries
3 All-share index
4 Banks and finance

Latest observation: Oct 1996

Empirical studies have shown that Finnish index series have one unit root, ie stationarity is achieved by transformation into first differences (Malkamäki 1993). To check if first differencing is enough to produce stationarity, unit root tests were performed on log differences of the stock index for the band period, 1 January 1987 – 5 September 1992, and the floating rate period, 14 September 1992 – 31 December 1995. Based on the results of the weighted symmetric τ test, Dickey-Fuller τ test and Phillips-Perron Z test, the hypothesis of a unit root was rejected. Thus, the logarithmic transformation of the stock index is integrated of order one.

Descriptive statistics for the stock index were then calculated. The null hypothesis of zero mean cannot be rejected for the band period but is rejected for the floating rate period. While skewness for the band period is huge, removal of the 'black Monday' observation substantially reduces it. Again, we see that a single outlier can considerably affect the value of a test statistic.

In empirical studies, skewness has been found to be a much stronger feature in share prices than in exchange rates and interest rates. This is not the case with the Finnish stock index.

Most empirical implementations of GARCH(p,q) models for stock indices have adopted low orders for the lag lengths, p and q. Typically, GARCH(1,1), GARCH(1,2) or GARCH(2,1) models have been selected. However, a limitation in GARCH models is the assumption that only the magnitude, and not the sign, of unanticipated returns determines volatility (Mills 1992, p. 140). Nelson (1990a) presented an alternative to the GARCH model, the exponential GARCH, ie EGARCH, which encompasses the observed feature that changes in share return volatility are negatively correlated with the returns themselves, ie volatility tends to rise in response to 'bad news' and fall in response to 'good news'.

In this study we hope to be able to use a GARCH model with the same orders p and q for all the market risks and therefore the GARCH(1,1) process, which was selected for exchange rates and interest rates, is selected for the Finnish stock index as well.

Prior to the GARCH identification, the data were prefiltered to remove linear dependence. An AR(3) process was selected for the band period and AR(1) for the floating rate period. The selection was based on the Ljung-Box test statistics.

The estimated parameter values for the GARCH(1,1) model for both periods are presented in Table 4.12. The values of the parameters α_1 and β_1 differ between periods, while the estimated α_0 parameters are very similar indeed. For the band period, the sum is 0.8584, the mean lag 7 days and the half-life 6 days. For the floating period, the sum of the two parameters is 0.9475, the mean lag 7 days and the half-life 14 days.

Table 4.12 **All-share index, compound yield, log differences**

GARCH(1,1) estimation of the volatility (t-values in parentheses)

		α_0	α_1	β_1
1 Jan 87 – 5 Sep 92	AR(3)	0.1208E–4 (11.43)	0.3275 (10.99)	0.5309 (15.96)
8 Sep 92 – 31 Dec 95 (15 Nov 92 included)	AR(1)	0.9196E–5 (2.37)	0.0966 (4.31)	0.8509 (21.97)

Results from applications of ARCH to share return data are reported in Bollerslev et al (1992). Highly significant test statistics for ARCH have been found both for individual share returns (Engle and Mustafa 1992) and for index returns (Akgiray 1989). Akgiray also reports a strong first-order autocorrelation process in the mean equation for the CRSP stock index, which is also found in this study using a Finnish stock index.

The BDS statistic was originally developed to test for deterministic chaos as a form of nonlinearities in share returns. For share returns, most studies find that standardized ARCH residuals exhibit very little evidence of nonlinear dependence (see eg LeBaron 1988, 1989, Hsieh 1991).

BDS statistics for the residuals in the Finnish stock index prior to and after GARCH estimation are displayed in Table 4.11. The test statistics are significantly reduced by accounting for GARCH(1,1) effects in the conditional variance and the hypothesis of IID residuals cannot be rejected for the resulting standardized returns to the stock index.

The GARCH-M(1,1) was also estimated for the stock index to detect a possible time-varying risk-return relationship in the mean equation. The γ_1 parameter was not significant for either of the two data periods.

Prior to selecting the maintained volatility models, the odds for the general idea of imposing the same GARCH structure on all rates affecting banks' portfolio returns were perceived to be least favourable for share returns, since evidence from other sources strongly favoured an EGARCH model these returns.[45] Yet, the empirical estimation in this study revealed that GARCH(1,1) was best suited to capture heteroscedasticity in share returns. For exchange rates and interest rates, the BDS test statistic still detects deviations from IID in the standardized GARCH residuals.

4.5 Real estate risk

Since market risk-sensitive assets are marked to market daily, we have in previous chapters looked at daily changes in exchange rates, interest rates and the all-share index in connection with the measurement of market risk. In this section we deal with real estate risk where changes in asset values, because of the thinner markets involved, are more reliably measured at lower frequencies. The frequency differences between real

[45] Although the EGARCH model was especially developed for share returns, Hsieh (1991) has shown by applying the BDS test to the residuals from a EGARCH(1,1) model for share indices and portfolios that not even the EGARCH model can completely account for all deviation from IID in share returns.

estate risk and market risk enable us to determine empirically the effects of temporal aggregation on the distributional properties of financial series.

Real estate is included in banks' activities in three different categories: fixed assets, real estate acquired as redeemed collateral from defaulting borrowers, and real estate required as collateral for the loan portfolio. The first category of real estate, which banks hold for long periods and is reported at book value, is thus disregarded in this study. The market value of collateral for the loan portfolio is important for assessing the true credit losses in case of a realization of credit risk. Since the supervisors for whom this real estate risk evaluation model is developed do not have continuous information on loan-specific collateral, the real estate price risk for this category can only be measured by the banks themselves. What the regulators can measure then is the uncertainty or variance of the market values of real estate acquired either as redeemed collateral or in some cases for investment purposes.

The aim is to model the real estate risk in the same way as the market risk in previous chapters. While the methodology will be the same, there will be a difference in the frequency of the observed data. Market risk is measured in previous chapters at daily frequency. Data on real estate prices are available only on quaterly or monthly bases. That is not necessarily a limitation because changes in market values of real estate may react more slowly to new information and changes in the economic environment. Therefore the optimal frequency for the measurement of real estate risk may be monthly.

The bulk of real estate owned by Finnish banks is in domestic residental properties. Statistics on housing prices in Finland are available from two sources: a real estate agency and Statistics Finland (SF). The (monthly) data provided by the real estate agency are regional prices on existing two-room apartments, and the (quarterly) series published by SF covers the prices of all apartments and properties sold throughout the country, as reported for tax purposes. The real estate agency series entail a great deal of stochasticity and noise due to occasionally thin markets, which limits their credibility. The SF series are used in this study because they are smoother and more reliable due to the inclusion of a greater number of sales.

Although the SF series is officially published on a quaterly basis, it was possible to get monthly figures from Statistics Finland. Figure 4.6 shows the monthly SF series in levels for the relevant period, 1 January 1987 – 31 December 1995.

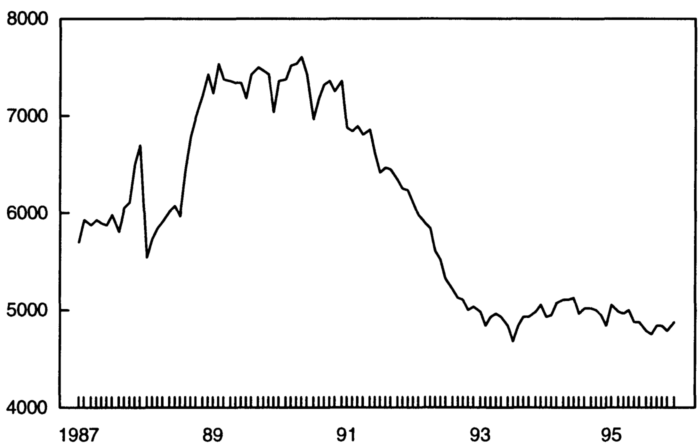
In order to estimate the GARCH(1,1) model, we start with the identification of the mean equation. The following equation was estimated for the seasonally adjusted log SF series:

$$\log SF_t = \underset{(0.29)}{0.0186} + \underset{(68.32)}{0.9954} \log SF_{t-1}. \tag{4.8}$$

Hence the mean equation was statistically confirmed as a martingale process without drift.

Stationarity in the mean was tested both for seasonally adjusted log levels and log differences. The same unit root tests were performed for the monthly real estate price series as for the daily market rates. The weighted symmetric τ test, the Dickey-Fuller τ test and the Phillip-Peron Z test were not significant for the log levels series but were significant for the log difference series. Thus differencing once produces a mean stationary series.

Figure 4.6 **Monthly housing prices in levels 1 Jan 1987 - 31 Dec 1995, FIM**



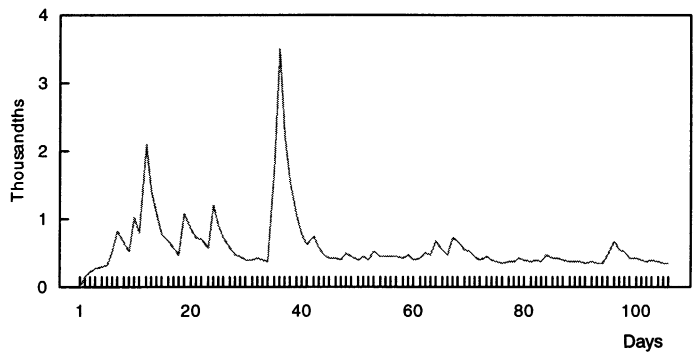
The descriptive statistics for log differences of SF (Table 4.13) show that the skewness measure of 0.37 with a standard error of 0.23 is not significant but that the kurtosis measure of 5.80 with standard error of 0.46 is highly significant. Leptokurtosis is thus found also in monthly series. The efficient market hypothesis is confirmed by the fact that the mean does not differ significantly from zero.

To detect linear dependence, the Ljung-Box test for up to five lags was performed. The test statistics reveal strong autocorrelation, which was removed via AR(1) filtering. The ARCH test statistic was significant and so the next step was to estimate an AR(1)-GARCH(1,1) model. The following parameter values resulted:

$$h_t = \underset{(2.25)}{0.13024*E\text{-}5} + \underset{(2.76)}{0.1969\varepsilon_{t-1}^2} + \underset{(4.20)}{0.5904h_{t-1}}. \tag{4.9}$$

All estimated parameter values are statistically significant. The sum α_1 + β_1 is 0.7873, which means that the process is far from being integrated but strongly mean reverting. Compared to the results for the models estimated in this study with daily data, this result can be seen as confirmation of empirical findings of other studies indicating that the degree of persitence increases with frequency.

Figure 4.7 **Conditional variance of housing prices in log differences, GARCH (1,1) model**



In the conditional variance shown in Figure 4.7, two clearly separate periods are found, in terms of oscillation: high volatility in the early part of the estimation period and a smoother path in the latter part. To test the stability of the parameters in the conditional variance equation, the estimation period was divided into two non-overlapping periods, the first covering 50 months and the other 57 months. GARCH estimation was applied separately to each subperiod. The number of observations in the subperiods clearly puts us at the lower limit for degrees of freedom for identification of a GARCH model, which affects the efficiency of the estimation results.

The estimated model for the first subperiod for the mean equation turned out to be an AR(1) process with a conditional variance equation consisting only of a constant and an MA(1) part:

$$h_t = \underset{(1.59)}{0.1637 \cdot E\text{-}3} + \underset{(6.80)}{0.8419} h_{t-1}.$$

For the later subperiod the model turned out to be an AR(1) process in mean with a constant variance.

The estimation results for the two subperiods differ so sharply from each other that there is no need to apply a formal statistical test on parameter stability. The two subperiods with differing volatility patterns correspond closely to the pattern for the price series in Figure 4.6. The period of high price levels during the first subperiod of the time interval is characterized by higher volatility compared to the second subperiod with lower prices. This fact gives reason to test the fit of a model in which the price level or volume of sales is treated as an exogenous variable in the conditional variance equation, in what can be called a GARCH-X model. This kind of model has been used with daily data on financial return series in order to capture the source of the ARCH effects. The use of an exogenous volume variable in a GARCH model has been tested for daily share return data (Lemourex and Lastrapes 1990a). The authors start with the hypothesis suggested by Diebold (1986) and Gallant, Hsieh and Tauchen (1988) that returns are generated by a mixture of distributions, in which the daily arrival of information is a stochastic mixing variable for which GARCH modelling might capture the time series properties. The hypothesis was tested for twenty actively traded shares on the CBOE by estimating a GARCH(1,1) model, using daily trading volumes as a proxy for the mixing variable. The results show that the strongly persistent GARCH effect in the GARCH(1,1) without the volume variable becomes negligible in the GARCH-X(1,1) model with the volume variable included. They thus conclude that there is no need for a contemporaneous explanatory volume variable at daily frequency, since the tested hypothesis is correct and the GARCH framework captures the behaviour of share prices, ie on the market microstructure level the volume is driven by exactly the same factors that generate return volatility (Andersen 1996). Sharma et al (1996) extended the work of Lemoureux and Lastrapes from the implied micro level of individual stocks to the macro level described by the NYSE index, a share price indicator. They showed that the inclusion of volume as a proxy for news arrival in the conditional variance dampens but does not eliminate the GARCH effects. Andersen (1996) ends up with results confirming the work by Sharma et al for five stocks commonly traded on the NYSE. The decay in persistence when return-volatility analysis is expanded to return-volume analysis can be explained by the hypothesis that the variance generating process cannot be completely captured by volume as a proxy for news.

To test the joint return-volume hypothesis, a GARCH-X(1,1) model with volume of sales, Z, included as an explanatory variable in the conditional variance equation of the following form was estimated over the whole period:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma Z_t, \tag{4.10}$$

and the result was

$$h_t = \underset{(3.26)}{0.1599 \cdot E\text{-}3} + \underset{(2.03)}{0.1157}\, \varepsilon_{t-1}^2 + \underset{(15.58)}{0.8654}\, h_{t-1} - \underset{(-3.24)}{0.3057 \cdot E\text{-}4}\, Z_t. \tag{4.11}$$

The Z variable, ie the volume of sales, is highly significant. Compared to the GARCH model estimated for the whole period but without exogenous variables in the variance equation, the impact of news as measured by the parameter α_1 has diminished but the persistance parameter, β_1, has increased in size. The sum $\alpha_1 + \beta_1$ is 0.9811 so that the process is close to being integrated. This suprising result shows that the unit-root-in-variance proposition can also appear in lower frequency data.

The conditional variance estimated in the GARCH-X model is shown in Figure 4.8. The signs of constancy present in the later period in the GARCH model disappear in the GARCH-X interpretation.

In our application with monthly real estate prices, the results for volatility persistence in moving from a GARCH model to a GARCH-X model are the reverse of those with daily return data on individual shares and stock indices. Inclusion of a volume variable in the real estate variance model does not cause the GARCH effects to disappear or decay but instead strengthens the GARCH-type persistence in the conditional variance. In the estimated GARCH-X model, the persistence expressed as the sum $\alpha_1 + \beta_1$ was much higher (nearly integrated) than in the GARCH model. The significant parameter estimate for the volume variable shows that both GARCH variables and the volume variable are needed to accurately model the variance process.

Figure 4.8 **Conditional variance of housing prices in log differences, GARCH-X model**

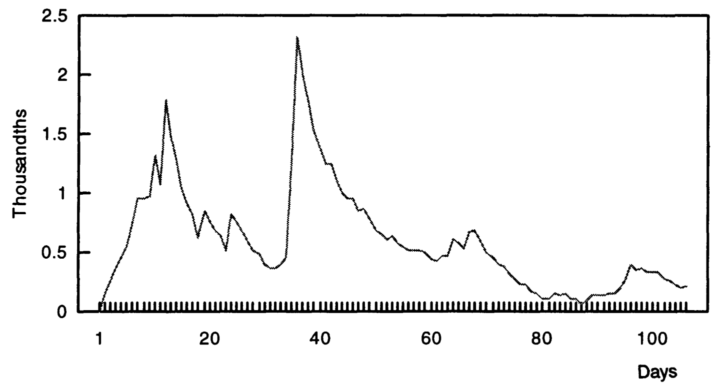
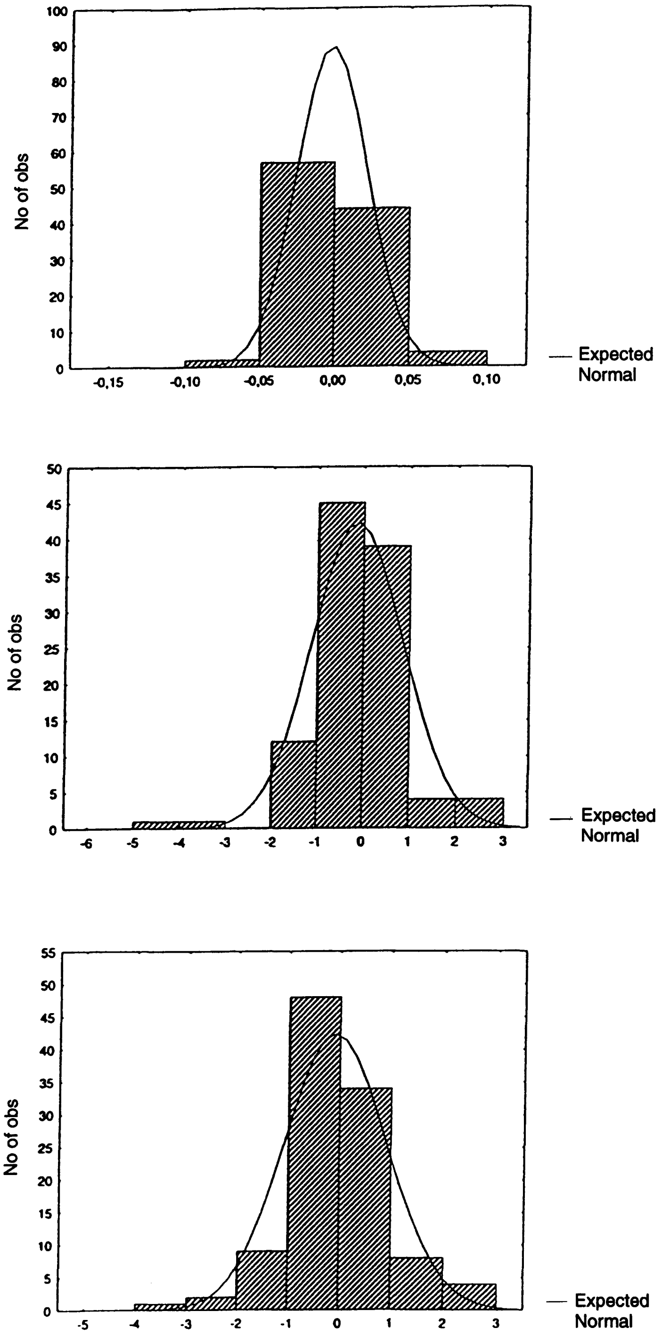


Table 4.13 **Descriptive statistics for housing prices in log differencies**

Skewness			Kurtosis		
Raw data	GARCH residuals	GARCH-X residuals	Raw Data	GARCH residuals	GARCH-X residuals
0.37	-0.41	-0.077	5.80	3.97	2.72
Stand error 0.23			Stand error 0.46		

The limited number of observations, 107, does not allow the use of the BDS statistics for testing the hypothesis of IID. What can be done, however, is to compare the deviation from normality in terms of skewness and kurtosis in the residuals of the raw data to the GARCH standardized residuals. Figure 4.9 includes the histograms of the raw data and the standardized GARCH and GARCH-X residuals for the whole period. Table 4.13 gives the descriptive statistics for the residuals of the raw data and the GARCH and GARCH-X standardized residuals.

Figure 4.9 **Housing prices: graphical comparison of raw data and GARCH and GARCH-X standardized residuals to the theoretical normal distribution**



Temporal aggregation into monthly series seems to lead to lower kurtosis figures compared to the daily data delt with in pevious chapters. This is consistent with both the theory and empirical findings and can be explained in respect of the housing price series by the fact that the effects of outliers are smoothed out partly because the data is recorded at a lower frequency and parly because a large amount of sales is included. The conclusions that can be drawn are that a trace of kurtosis still remains after GARCH modelling of the variance but that the standardized conditional residuals are closer to normality than are the residuals from the raw data. The GARCH-X modelling reduces the kurtosis figure even more.[46]

4.5.1 Forecasting the exogenous volume variable

The final estimated interpretation of real estate price volatility is an integrated model with an explanatory volume variable, Z, included in the conditional variance equation. In order to be able to forecast changes in real estate values with the estimated GARCH-X(1,1) model, we need a forecast for Z.[47] To evaluate forecasting possibilities, the autocorrelation function was estimated for the Z series. Based on this estimate, the following AR(1) interpretation was identified:

$$Z_t = \underset{(3.75)}{1.1018} + \underset{(12.94)}{0.7670}\, Z_{t-1},$$

which enables one-period-ahead forecasting.

[46] The only publication found on GARCH modelling of real estate data is by Fischer (1996). His results on monthly data for the Swiss housing market are that the prices of office buildings and rental units follow a GARCH process with a high positive moving average component. Prices of apartments and single family homes do not show any GARCH effects. The results, according to the author, must be treated with caution because of the limited sample size.

[47] An alternative model with the level variable log SF as a more easily forecastable exogenous variable was estimated, but it proved to be statistically inferior to the selected model.

5 Evaluation of variance estimation results

The objective of the first part of the study has been to model the time-varying variances in twelve exchange rates, thirteen short-term interest rates, one long-term rate, and the stock index on a daily basis. The variance of real estate prices is modelled on a monthly frequency. A GARCH structure was used in order to account for the observed heteroscedasticity in the rates, and GARCH(1,1) turned out to perform reasonably well for all the rates. The evidence on the significance of GARCH-M effects remains inconclusive and so the study argues against its use here. Hence, it is concluded that no significant time variability can be observed in the risk-return relationship in the selected data set.

One of the strongest conclusions of the present study is that the conditional variance model for individual exchange rates and short-term interest rates is at least approximately the same across exchange rate regimes. The model for long-term interest rate volatility, on the other hand, displays less persistence with floating exchange rates than with fixed rates, although the estimated conditional variance process appears (weakly) stationary under both regimes.

Furthermore, results from the pooled data suggest that changes in markka exchange rates and short-term interest rates have a time-varying conditional variance that can be modelled as an identical IGARCH process. Perhaps suprisingly, observed volatility in the stock index also seems to follow the same IGARCH(1,1) process, while the long-term rate exhibits strong mean-reverting behaviour. The finding of an IGARCH process is consistent with the common finding that when a GARCH model is applied to high-frequency data, shocks to variance are strongly peristent; that is, the sum of the ARCH and GARCH parameters is very close to one. One possible explanation for integration in the conditional variance can be found in Nelson (1990b), who derives the stationary distribution of the GARCH conditional variance process in continuous time. This underlying diffusion model, which is close to IGARCH, provides accurate approximations to high frequency data. Furthermore, the distribution of the diffusion limit, and hence of the approximating process in high frequency data, displays some interesting properties; the GARCH innovation process is conditionally normal (ie given the conditional variance), but unconditionally its distribution is approximately Student's t. Also, in the special case of the diffusion limit of the IGARCH(1,1) model, the Student's t has an infinite variance.

Lamoureux and Lastrapes (1990b) suggest that the persistence is overstated when the estimation is based on long series. The resulting IGARCH could as well be due to the existence, but failure to take into account, of deterministic structural shifts in the model or to time-varying parameters. Structural shifts may result in instability of the drift parameter, α_0, over the sample period, ie nonstationarity of the conditional variance and high persistence in α_1 and β_1. The reason for the division of the full data into subsets in this study was to account for the possibility of such structural shifts due to changes in the exchange rate regime. In the GARCH estimation on the pooled data, the same model was forced on the individual rates and the individual drift parameters, α_0, were also composed into a single constant in the estimation for each period. The drift parameters in the individual models are very small in magnitude but differ between rates and so can be interpreted as structural shifts. This feature might have had the effect on the estimation results of the model identified on the pooled data of giving the appearance of extremely strong persistence in variance. However, the average sum of the ARCH and GARCH parameters of the individual models, where the structural changes were accounted for, is close to one, thus supporting the hypothesis of an integrated variance process.

In the following we evaluate the variance estimation results in two ways. First, we discuss the assumption of normally distributed standardized residuals. Secondly, we test for asymmetry in the volatility.

As a result of the GARCH estimation, it was possible to construct new variables by standardizing the raw data with the estimated standard deviations. Through this procedure we should theoretically end up with series that are normal. Table 5.1 and Figure 5.1 compare the descriptive statistics for measuring skewness and kurtosis between the raw data and the GARCH residuals for USD, ERUSD, DEM, ERDEM, ERFIM, the FIM long-term rate and the stock index HEX. Skewness is found in the raw data only for ERFIM. The kurtosis figures are also in most cases substantially reduced via conditional variance modelling, with ERLONG being the exception, whereas for ERFIM a substantial amount of kurtosis remains after filtering with the estimated GARCH model. The BDS statistics, given in Tables 3.7, 4.9 and 4.11, however show a considerable reduction for the GARCH residuals compared to the raw data for all daily rates considered, although some of them show traces of deviation from IID normality.

Hence, while the GARCH(1,1) model is able to track the own temporal dependencies, the assumption of conditionally normally distributed innovations may require further study with the present data. As a reference, under the null of IID normally distributed standardized residuals, the sample skewness should be the realization of a normal distribution with a mean of 0 and a variance of $6/831 = 0.085^2$, while the sample kurtosis is asymptotically normally distributed with a mean of 3 and a variance of $24/831 = 0.17^2$.

The traces of remaining conditional leptokurtosis in the standardized residuals indicate that the distribution of the conditionally normal GARCH process is not sufficiently heavy tailed to account for the all the excess kurtosis found in the return series.[48] The rejection of the conditional normality assumption is also frequently reported in applications of ARCH family models (Bera and Higgins 1993). Several methods have been proposed to account for this recorded inability of the standardized residuals to pass the diagnostic normality test by introducing a nonnormal conditional frequency distribution.[49] Nonnormal alternatives, which allow for leptokurtosis, are above all the Paretian family and variates of the Student's t distribution. The selection of the Paretian distribution has however proved to be unsatisfactory because of the fact that the distribution of financial data under temporal aggregation exhibit diminishing leprokurtosis and convergence toward normality, which is inconsistent with the Paretian distribution. On the other hand, on either the normal or Student's t assumption for the conditional distribution, the unconditional distribution modelled by an ARCH process not only exhibits leptokurtosis but also convergences to normality under addition over time. The normal and t distibutions are therefore the distributions which are most consistent with the observed behaviour of financial data.

Bollerslev (1987) proposed a conditional-t distribution that allows for heavier tails than the normal distribution and, as the degrees of freedom goes to infinity, approaches the normal distribution in the limit. Studies of Bollerslev (1987), Engle and Bollerslev (1986), Baillie and Bollerslev (1989) and Hsieh (1989) show that employing a conditional-t distribution succeeds in accounting for the excess kurtosis in daily rates of return on a number of exchange rates.

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[48]Another reasonable explanation is the presence of outliers, the effects of which are not removed in the GARCH estimation. Both the skewness and kurtosis figures have been found to be very sensitive to even a single extreme observation.

[49]This describtion of the handling of nonnormal conditional distributions is based mainly on the presentation in Bera and Higgins (1993).

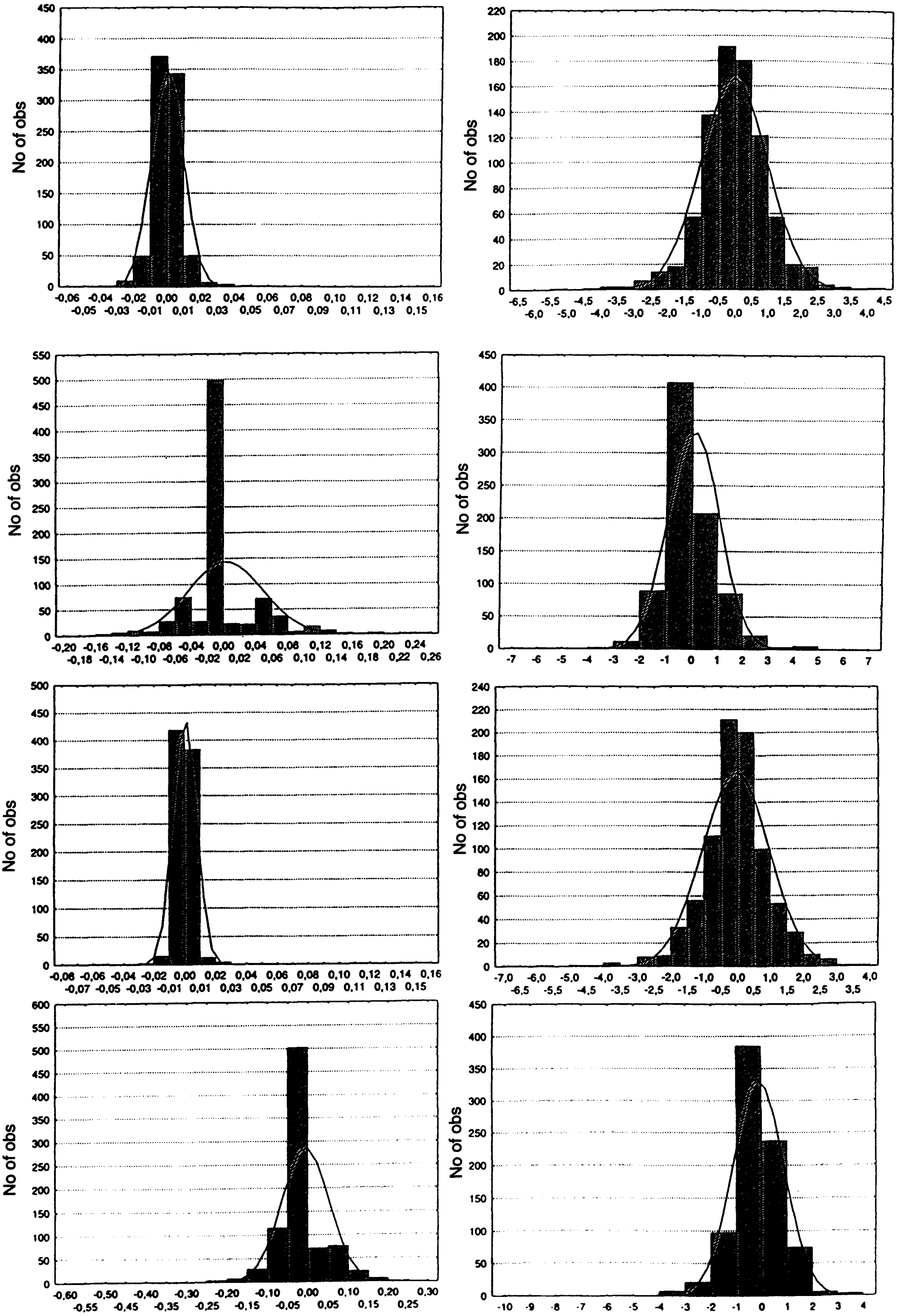
Table 5.1

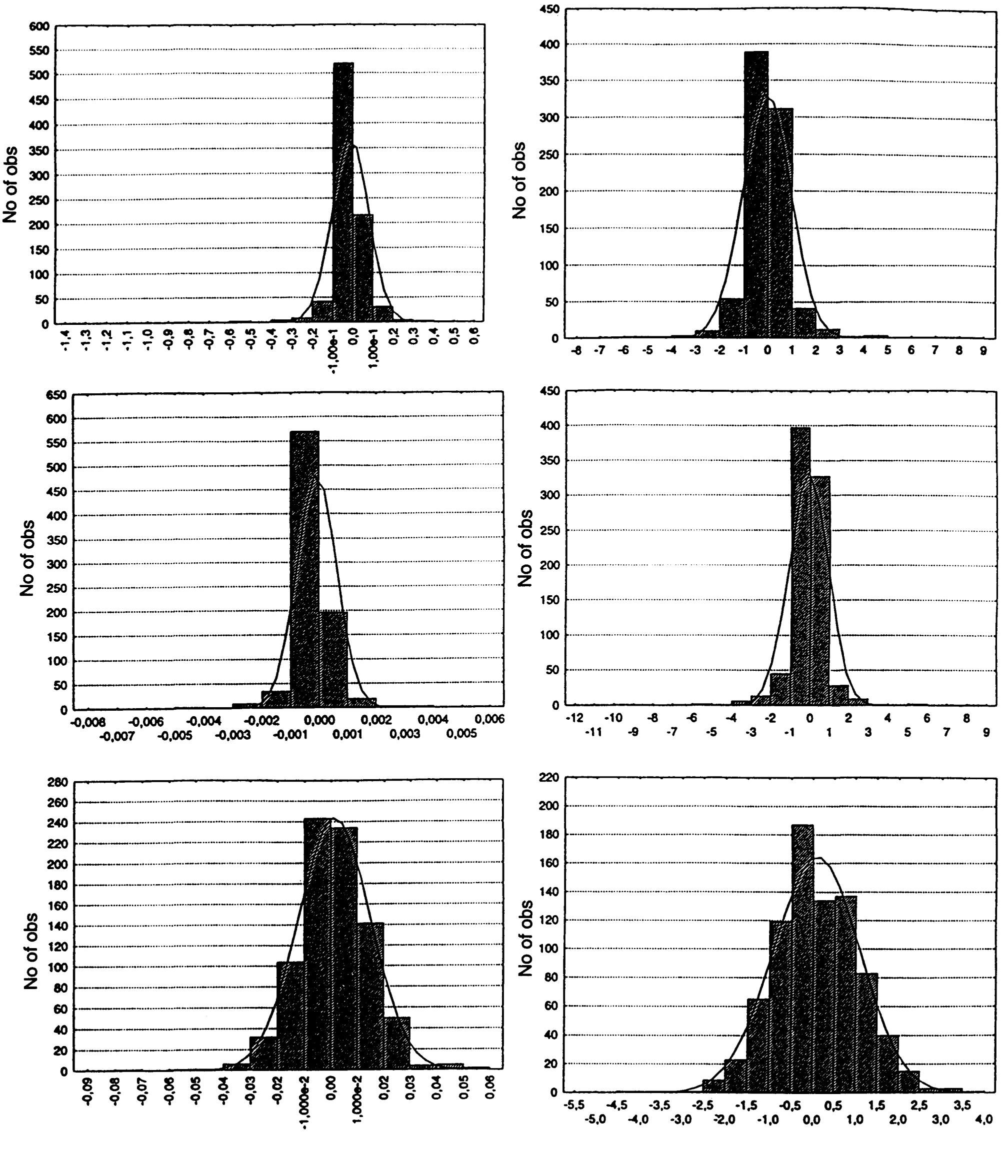
Skewness and Kurtosis statistics for raw data and GARCH(1,1) residuals for USD, ERUSD, DEM, ERDEM, ERFIM, FIM long-term rate and all-share index for the floating rate period, 14 Sep 1992 - 31 Dec 1995

	Skewness figures	
	Raw data	GARCH(1,1) residuals
USD	0.34	−0.23
ERUSD	0.60	0.75
DEM	0.11	−0.52
ERDEM	−1.33	−0.78
ERFIM	−3.27	0.59
ERLONG	−0.45	−1.55
HEX	−0.07	0.04

	Kurtosis figures	
	Raw data	GARCH(1,1) residuals
USD	78.11	2.40
ERUSD	3.42	5.73
DEM	240.28	3.17
ERDEM	14.24	6.74
ERFIM	42.74	16.02
ERLONG	23.49	27.35
HEX	2.36	0.71

Figure 5.1 **Raw data in the left column for USD, ERUSD, DEM, ERDEM, ERFIM, FIMlong and HEX, figures for corresponding GARCH residuals in the right column**



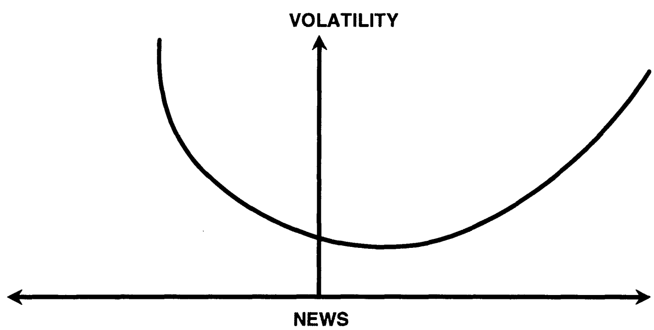

No of obs
No of obs
No of obs
No of obs
No of obs
No of obs

Other specifications of a nonnormal error distribution are given by Nelson (1991) and Lee and Tse (1991). Nelson employed a generalized error distribution (GED), which encompasses distributions with tails both thicker and thinner than the normal, and includes the normal as a special case. Lee and Tse used a distribution based on the first three terms of the Gram-Charlier series, allowing for both thick tails and skewness. Hansen (1992) introduced an autoregressive conditional density (ARCD) model with a conditional-t distribution and time-varying degrees of freedom. McCulloch (1985) suggested an infinite variance leptokurtic stable Paretian distribution in his so-called adaptive conditional heteroscedasticity (ACH) model.

Empirical evidence on the suitability of the conditional distributions described above is contradictory. Applications where none of the conditional distributions is totally satisfactory for modelling conditional heteroscedasticity have been reported.[50] This leaves the door open for further research on more adequate methods of implementation.

We continue the evaluation of the variance estimation results by testing for asymmetry. Asymmetry is found in empirical studies, particularly in share price data where downward movements in the market are followed by higher volatilities than upward movements of the same magnitude. Figure 5.2 shows an asymmetric news impact curve as described by Engle and Ng (1991).

Figure 5.2 **Asymmetric news impact curve**



[50]Hsieh (1989) found, for example, that a GARCH(1,1) model with either a conditional-t or conditional GED distribution could not adequately represent daily returns on the Brittish pound nor Japanese yen. Our work also shows that the GARCH(1,1) is not suitable for the Japanese yen for the floating rate period for the Finnish markka.

If asymmetry is found in volatility, the forecasting procedure will be complicated since the outcome of the forecast depends on the sign of the future changes in the financial rates.

TARCH and EGARCH models will be used here to describe and test for asymmetry. The TARCH model or the Threshold ARCH has been introduced, among others, by Glosten, Jaganathan, and Runkle (1994). The model for the variance is

$$h_t = \alpha_0 + \alpha\varepsilon_{t-1}^2 + \gamma_1\varepsilon_{t-1}^2 d_{t-1} + \beta h_{t-1},$$

where $d_t = 1$ if $\varepsilon_t < 0$ and 0 otherwise.

Good news has an impact of α, while bad news has an impact of $\alpha + \gamma_1$. If γ_1 is significantly different from zero, then the null of symmetry is rejected.

The EGARCH model, which is the second model to be used here for testing for asymmetry, was proposed by Nelson (1991) and specifies the variance in the form

$$\log(h_t) = \alpha_0 + \beta\log(h_{t-1}) + \alpha\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| + \gamma_2\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}.$$

The log transformation smooths out the effects of exceptionally large shocks but there is some evidence that this smoothing may go too far. The null hypothesis of symmetry is rejected if γ_2 is significantly different from zero.

The estimated parameter values γ_1 and γ_2 along with their t-statistics for exchange rates and interest rates are presented in Table 5.2. The t-values, calculated in a quasi maximum likelihood estimation procedure, are corrected by using robust standard errors as described by Bollerslev and Wooldridge (1992). The t-values for γ_1 in the TARCH model show that the hypothesis of symmetry remains valid for all exchange rates and almost all interest rates. The only exception showing asymmetry is the ERNLG interest rate. As a rule, the γ_2 coefficient in the EGARCH model is also insignificant. Significance is found in these parameter values only for BEF and ITL exchange rates and for ERDKK, ERBEF and ERITL interest rates. The asymmetry coefficients γ_1 and γ_2 are in general of opposite sign. This is because standardized residuals are used for the EGARCH model and a dummy for negative shocks in the TARCH model. When the sign is interpreted according to these models, a general conclusion is that if the γ_1 parameters are significant, then the negative values found for almost all rates indicate that there are higher volatilities related to upward movements than to downward movements.

The results from the testing show that traces of asymmetry are found only for a few small country exchange rates and interest rates. The hypothesis of symmetry cannot be rejected for the big countries. Since the dominant parts of portfolio investments are made in major country financial instruments, we can use the asymmetry assumption in the forecasting framework presented in this study.

Table 5.2 **Testing for asymmetry of the conditional distribution. γ_1 corresponds to the TARCH model and γ_2 to the EGARCH model (t-statistics in parentheses)[1)]**

	Exchange rates		Interest rates	
	γ_1	γ_2	γ_1	γ_2
USD	0.0004	-0.0184	-0.0358	0.0385
	(0.005)	(-0.437)	(-0.786)	(0.893)
GBP	-0.0039	0.0148	-0.0209	-0.0212
	(-0.063)	(0.456)	(-0.720)	(-0.508)
SEK	0.0531	0.0002	-0.0772	0.0583
	(0.516)	(0.003)	(-0.542)	(0.702)
NOK	0.1531	-0.0198	-1.5337	0.4285
	(0.790)	(-0.474)	(-1.051)	(2.730)
DKK	-0.4048	0.1665	-0.5059	0.2005
	(-0.863)	(1.193)	(-1.780)	(3.233)
DEM	-0.0796	0.0636	-0.0101	0.0028
	(-1.075)	(1.479)	(-0.772)	(0.198)
NLG	-0.0967	0.0643	-0.0202	-0.0015
	(-0.678)	(0.775)	(-2.049)	(-0.057)
BEF	-1.4982	0.4041	-1.0891	0.2696
	(-1.054)	(2.715)	(-1.535)	(2.330)
CHF	-0.5242	0.2035	0.0091	-0.0079
	(-0.970)	(1.330)	(0.268)	(-0.213)
FRF	-0.0742	0.0526	-0.1598	0.1987
	(-0.663)	(0.8575)	(-1.844)	(2.146)
ITL	-0.6740	0.1848	-0.3395	0.1276
	(-1.334)	(3.101)	(-1.524)	(1.994)
JPY	0.0418	0.0049	-0.0059	-0.0229
	(0.517)	(0.335)	(-0.058)	(-0.293)
ERFIM			-0.0287	0.0391
			(-0.892)	(1.042)
FIMLONG			-0.0395	-0.0141
			(-1.635)	(-0.389)

[1)] The t-values are calculated using Bollerslev-Wooldridge robust standard errors.

6 Covariances in market risk

Next, two multivariate methods will be suggested and used to measure the covariances of returns to allow for measurement of the total portfolio variance. The first method is developed in this study and is based on the assumption of identical autocorrelation structures for variances and covariances between rates. The assumption allows us to extend the univariate estimation results for the conditional variances to the conditional covariances. The other method is developed here as a two-stage version of the originally iterative Bollerslev method (1990), which is based on the assumption of constant correlation between rates. Both methods simplify the estimation procedure by enabling use of the results for individual conditional variances in the calculation of conditional covariances. The first method can be applied only to covariances within groups of rates, unless the parameter structures of the different groups can be proven to be sufficiently similiar. The other can be applied also to covariances between groups of rates.

6.1 Conditional covariances: identical autocorrelation structures

In the first method for covariance estimation, we test for dependence between the autocorrelation structure of the variances and covariances. If dependence is found to exist, then the conditional covariances can be modelled with the same parameter structure as their conditional variances.

By analogy to the conditional variance formula

$$h_{i,t}=\sigma_{i,t}^2=\alpha_0+\alpha_1\varepsilon_{i,t-1}^2+\beta_1\sigma_{i,t-1}^2. \tag{6.1}$$

The conditional covariances can be expressed as

$$\sigma_{ij,t}=\alpha_{0,ij}+\alpha_{1,ij}(\varepsilon_{i,t-1}\varepsilon_{j,t-1})+\beta_{1,ij}\sigma_{ij,t-1}, \tag{6.2}$$

where ϵ_i and ϵ_j are the innovations in the mean equation of returns series i and j.

Since the terms $\sigma_{i,jt-1}$ are not observable, the covariances cannot be estimated using a univariate GARCH method.

The null hypothesis of independence is tested using the non-parametric Kendall coefficient of concordance, W (Siegel 1956, p. 227–239), which expresses the degree of association among a set of ranked variables. The variables to be ranked in this case are the autocorrelation functions of variances and covariances within the two groups of rates.

In the case of dependence between the autocorrelation structure for the univariate conditional variance (6.1) and the conditional covariance (6.2), the parameter values for the β_1's univariately estimated for the variances, can also be used for covariances. Most conditional variances in interest rates in this study were estimated to follow an integrated process in which the ARCH parameter, α_1, and the autocorrelation or GARCH parameter, β_1, sum up to one. If we can show that the autocorrelation structure measured by β_1 is not independent between variances and covariances, it then follows from the unit root proposition that there is dependence between the α_1 parameters for variances and covariances as well.

By repeated substitution, the conditional variance formula in (6.1) can be written as

$$h_t = \frac{\alpha_0}{(1-\beta_1)} + \alpha_1 \sum_{j=1}^{\infty} \beta_1^{j-1} \varepsilon_{t-j}^2, \qquad (6.3)$$

where the conditional variance is expressed in the form of a geometrically weighted average of past squared residuals, so that the parameter β_1 gives the decay rate.

For the IGARCH process, formula (6.3) takes the form

$$h_t = \frac{\alpha_0}{\alpha_1} + \alpha_1 \sum_{j=1}^{\infty} (1-\alpha_1)^{j-1} \varepsilon_{t-j}^2. \qquad (6.4)$$

The expression for the covariances corresponding to formula (6.4) for the variances is then

$$\sigma_{ij,t} = \frac{\alpha_0}{\alpha_1} + \alpha_1 \sum_{s=1}^{\infty} (1-\alpha_1)^{s-1} \varepsilon_{i,t-s} \varepsilon_{j,t-s}. \qquad (6.5)$$

Hence, in the empirical implementation of the derived formula for the conditional covariances, we use the parameter estimates of the α_i's and

β_i's from the pooled data within groups and periods and for the ε_i's and ε_j's the observations on innovations in the individual returns.

6.2 Conditional covariances: constant correlations

In the second method for covariance estimation, the assumption of time-varying variances and covariances with constant conditional correlation between the N stochastic processes, following Bollerslev (1990), allows the univariate GARCH estimation to be extended to a multivariate framework via a simplified estimation and inference procedure. The GARCH(1,1) structure for the conditional variances and covariances is expressed as

$$\varepsilon_{i,t} = z_{i,t} h_t^{1/2}$$

$$h_{ii,t} = \alpha_i + \alpha_{i1} \varepsilon_{i,t-1}^2 + \beta_{i1} h_{ii,t-1} \tag{6.6}$$

$$h_{ij,t} = \varrho_{ij} (h_{ii,t} h_{jj,t})^{1/2}.$$

In the original application, the correlation coefficients, ρ_{ij}, of the standardized residuals are estimated simultaneously with the conditional moments. In our application we use a two-step method: first we calculate the correlation coefficients on the univariately estimated standardized GARCH(1,1) residuals and then we calculate the covariances from the joint information on the correlation coefficients and the estimated conditional univariate variances. In our application we assume constant correlation within periods of homogeneous exchange rate regimes but allow for time-variation between periods.

6.3 Covariances between exchange rates

Nontrivial covariation of the exchange rates and interest rates is highly likely, not only because of the arrival of new information affecting all the rates but also because of intervention policies of central banks.

In the first method of measuring the covariances between exchange rates, we test for the possibility of encompassing the coherence between rates in the analysis by extending the estimated parameter structure from the conditional variances to the conditional covariances. In order to do so, we must test for dependence between these conditional moments.

The null hypothesis of independence was tested using the Kendall coefficient of concordance, W, (Siegel 1956) which expresses the degree of association among sets of ranked variables. The variables to be ranked are the sample autocorrelation functions of variances and covariances. The test was performed separately for the group of twelve exchange rates and the group of thirteen interest rates.

Autocorrelations in variances and covariances up to the fifth order were calculated from the exchange rate data separately for the band and floating rate periods. The numerical autocorrelation values were then ranked. The test statistic, W, was calculated to test the null hypothesis that the rankings are unrelated. The numerical value of the coefficient of concordance was 0.691 for the band period and 0.469 for the floating rate period. The coefficient W is in this case approximately distributed as $\chi^2_{(4)}$ and the corresponding test statistics are 215.59 and 146.33. These test statistics are highly significant, which means that the null of independence can be rejected for both periods. Based on the outcome of the Kendall-W test procedure, which indicated that the variances and covariances were not independent, we modelled the conditional covariances between exchange rates using the same parameter structure, ie the same values of $\alpha_{1,i}$ and $\beta_{1,i}$, as for their conditional variances.

The Kendall-W test was performed on ranked autocorrelation values of variances and covariances. The numerical values of the autocorrelation function can also be used to approximate the similarity between the variances and covariances. Table 6.1 gives the means of the numerical values of the autocorrelation functions up to order five for variances and covariances. For the exchange rates, we can conclude that the structures are very similar.

Table 6.1 **Autocorrelation mean values of variances and covariances**

Exchange rates

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
Exchange rate band period					
Variances	0.1279	0.0202	0.0322	0.0214	-0.0050
Covariances	0.0908	0.0421	0.0248	0.0057	0.0144
Floating rate period					
Variances	0.1964	0.0830	0.0851	0.0301	0.0612
Covariances	0.2190	0.1247	0.1165	0.0383	0.0687

Interest rates

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
Exchange rate band period					
Variances	0.1863	0.0619	0.0874	0.0422	0.0273
Covariances	0.0143	0.0534	0.0105	0.0013	0.0123
Floating rate period					
Variances	0.1509	0.0987	0.1083	0.0926	0.1001
Covariances	0.0049	0.0160	0.0009	0.0031	0.0185

A third method of evaluating the dependence between conditional variances and covariances is based on principal components analysis. Principal components were calculated separately for the sample variances and covariances for the second band period, which in the estimation were found to be identical to those of the first band period, and for the floating rate period. The correlation coefficient, R, was then determined by regressing the first principal component of the variances on the first principal component of the covariances. The correlation coefficient is 0.54 for the band period and 0.87 for the floating rate period. A strong dependence is thus found in this way between variances and covariances.

The outcome of the Kendall-W test, the visual interpretation of the mean values of the autocorrelation functions, and the high degree of correlation between principal components all support the use of the same parameter structure for variances and covariances of exchange rates.

The estimated conditional variance model of the pooled data can therefore be used as the basic model for the conditional covariances between exchange rates. The estimated pooled model for the band period is

$$h_t = 0.3813*E\text{-}7 + 0.0621\varepsilon_{t-1}^2 + 0.9353h_{t-1} \tag{6.7}$$

and for the floating rate period

$$h_t = 0.9189*E\text{-}6 + 0.0809\varepsilon_{t-1}^2 + 0.8847h_{t-1}. \tag{6.8}$$

The sum $\alpha_1 + \beta_1$ does not differ significantly from one, so we can conclude that the conditional variance for the exchange rates follows a GARCH process integrated in variance, which appears to apply across exchange rate regimes.

Rewriting formula (6.4) we get the following weight structure for the floating rate period when $\alpha_1 = 0.08$ and $\beta_1 = 1 - \alpha_1$:

$$\begin{aligned} h_t = \frac{\alpha_0}{0.08} &+ 0.08\varepsilon_{t-1}^2 + 0.08(0.92)\varepsilon_{t-2}^2 + 0.08(0.92)^2\varepsilon_{t-3}^2 \\ &+ 0.08(0.92)^3\varepsilon_{t-4}^2 + \ldots + 0.08(0.92)^{n-1}\varepsilon_{t-n}^2 + \ldots \, . \end{aligned} \tag{6.9}$$

The series of lagged squared residuals to be included in the formula is truncated at 28, as the weight of the subsequent observation is less than 10 per cent of the weight of the first observation.

Table 6.2 gives the numerical values of the weights.

Table 6.2 **Numerical values of weights for the truncated sequence of lagged squared innovations**

Lag number	weight
1	0.089
2	0.082
3	0.075
4	0.069
5	0.063
6	0.058
7	0.054
8	0.050
9	0.045
10	0.042
11	0.038
12	0.035
13	0.033
14	0.031
15	0.028
16	0.025
17	0.023
18	0.021
19	0.020
20	0.018
21	0.017
22	0.015
23	0.014
24	0.013
25	0.012
26	0.011
27	0.010
28	0.009

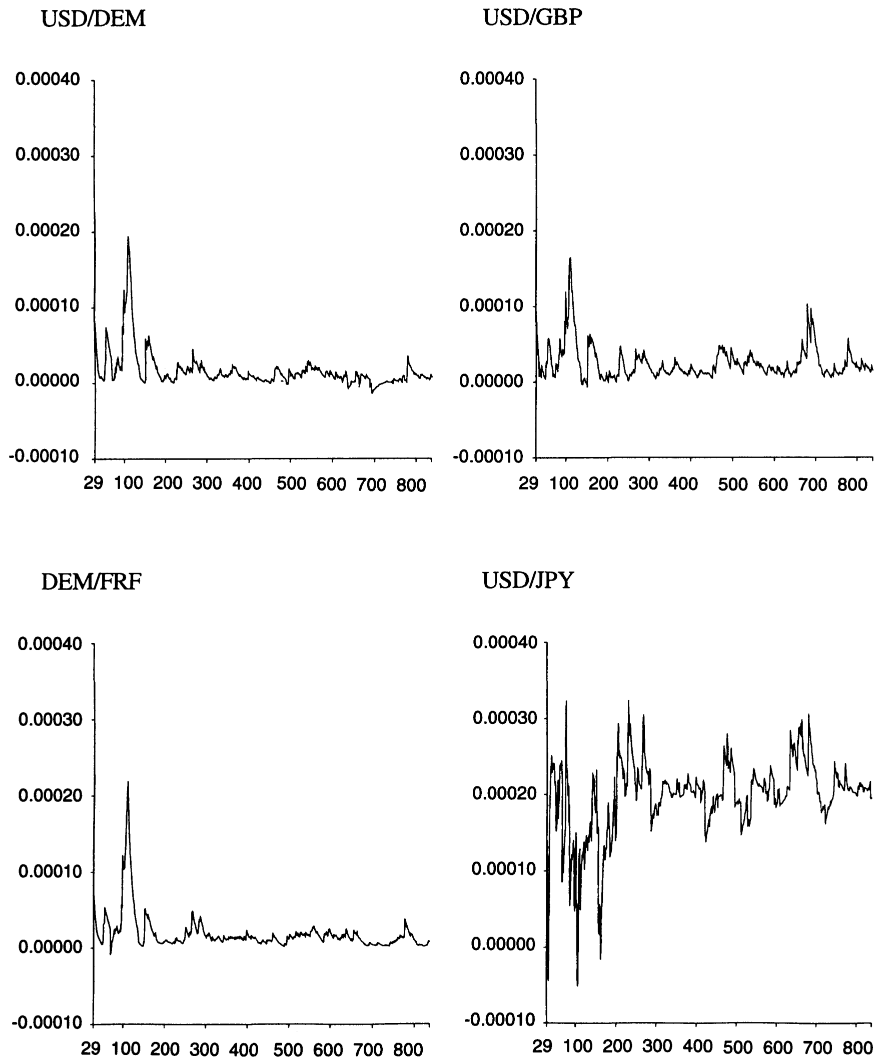
The expression for the covariances corresponding to formula (6.4) for the variances is then

$$\sigma_{ij,t}=\frac{\alpha_0}{0.08}+0.08\varepsilon_{i,t-1}\varepsilon_{j,t-1}+0.08(0.92)\varepsilon_{i,t-2}\varepsilon_{j,t-2}$$
$$+0.08(0.92)^2\varepsilon_{i,t-3}\varepsilon_{j,t-3}+0.08(0.92)^3\varepsilon_{i,t-4}\varepsilon_{j,t-4}+\ldots \quad (6.10)$$
$$+0.08(0.92)^{n-1}\varepsilon_{i,t-n}\varepsilon_{j,t-n}+\ldots\ .$$

In the empirical implementation of the derived formula for the conditional covariances, we thus use the parameter estimates of α_1 and β_1 from the pooled data within groups and periods. For the ε_i's and ε_j's, we use observations on the individual exchange rates.

The plots of the conditional covariances for the main currencies calculated according to the formula (6.10) for USD/DEM, USD/GBP, DEM/FRF, and USD/JPY are displayed in Figure 6.1. Visually, the variation in covariances is often very similar to that of the corresponding variances. The excessive time variability of the USD/JPY conditional covariance at the beginning of the period reflects increased volatility of the JPY at that time.

Figure 6.1 **Conditional covariances, 9 Sep 1992 – 31 Dec 1995, USD/DEM, USD/GBP, DEM/FRF and USD/JPY (weighted series)**



The alternative approach to calculating covariances is the Bollerslev method of constant conditional correlations. In applying this method, we assume that the correlations are constant within each of the three main periods but allow them to change between periods.

To evaluate the empirical correctness of the assumption of constant correlation in the Bollerslev method, a CUSUM test was applied to the standardized residuals to test the stability of the regression parameter in an OLS estimation where the exchange rate residuals were regressed one at a time on each of the other exchange rate residuals.[51] CUSUM test values for the floating rate period are presented in Appendix 2. The test values for the first band period were well below the critical values even at the 10 per cent significance level and consequently did not allow rejection of the null hypothesis that the regression parameters remain constant. Exceptions were the correlations ITL/JPY, ITL/DEM and DKK/JPY, for which the null was rejected at the 5 per cent level but not at the 1 per cent level. For the second band period the test statistics did not allow rejection for any correlation between exchange rates at any significance level. For the floating rate period the test statistics allowed rejection of the null at the 5 per cent level for almost all correlations with the GBP and for BEC/FRF, BEC/ITL and JPY/ITL. None of these test values were however significant at the 1 per cent level.

The evidence in the data in favour of the constant correlation assumption supports the use of the Bollerslev method. In applying this method, the conditional correlation coefficients of pairs of GARCH standardized residuals of individual exchange rates in (6.6) were first calculated using sample data from the three subperiods. The correlation coefficients for all 27 return series for the floating rate period are presented in Appendix 2. Theoretically, these correlation coefficients are approximately normally distributed under the null of either small or no correlation if, in the latter case, one uses the normal to approximate the exact Student's t distribution. The bigness of the sample sizes makes even low correlations statistically significant. For the first band period, the sample size is 558 and the critical value at the 5 per cent level for the correlation coefficient is 0.083. The corresponding figures for the second band period are 866 and 0.067 and for the floating rate period, 819 and 0.068.

[51] Since the CUSUM test is classified as a weak test, the emphasis in our testing is placed on the 10 per cent significance level. An alternative method of testing for constant correlation is given in Jennrich (1970).

The numerical values of within-group correlations are highest for the group of the exchange rates. This conforms with the theory that the arrival of information affects all markka rates instantaneously.

With the Bollerslev method, we assume constant conditional correlations within periods, but allow for changing correlations between periods reflecting different regimes. A significant difference in the level of the coefficients is also to be seen in the sample estimates.

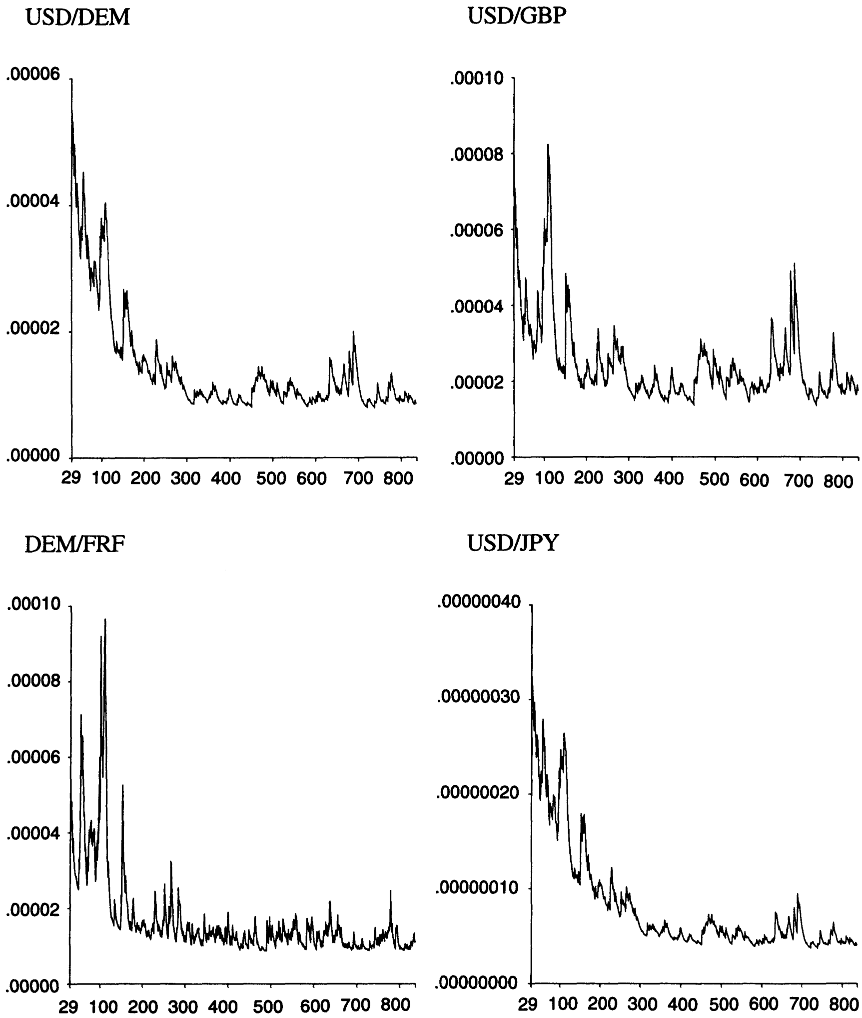
During the first band period, 1 January 1987 – 16 March 1989, the highest covariances are found between the European currencies, DKK, DEM, NLG, BEC, CHF and FRF. The ERM apparently affects these correlations. For the USD, the correlations are significant only with the GBP, SEK, NOK and JPY.

For the second band period, 21 March 1989 – 5 September 1992, the conditional correlations are significant for all pairs of exchange rates and much higher in value than during the first band period. An explanation of the phenomenon could be intensified central bank intervention activity aimed at dampening exchange rate movements during this turbulent period.

During the floating rate period, 8 September 1992 – 31 December 1995, the covariances between ERM currencies are again significant, although much lower than for the first band period. Correlations between the other currencies are generally insignificant. High correlation between the USD and GBP and the Nordic currencies is found for this period.

Figure 6.2 shows the conditional covariances for the floating rate period for USD/DEM, USD/GBP, DEM/FRF and USD/JPY, using the constant correlation method. These figures correspond to the figures in Figure 6.1, showing the conditional covariances for the same pairs of exchange rates calculated by assuming identical parameter structures for the variances and covariances.

Figure 6.2 **Conditional covariances, 9 Sep 1992 – 31 Dec 1995, USD/DEM, USD/GBP, DEM/FRF and USD/JPY (constant correlation)**



6.4 Covariances between interest rates

Similar calculations for determining covariances were then carried out for the group of thirteen interest rates as for the group of twelve exchange rates.

In the first method, we test the null hypotheses of linear independence of the autocorrelation structures of the variances and covariances, ie of the parameters β_{1j} and $\beta_{1,ij}$. If the null hypothesis is rejected, dependence between the $\alpha_{1,i}$ and $\alpha_{1,ij}$ parameters results from the assumption of IGARCH variance processes.

Ranking was done according to the numerical values of the autocorrelation functions of the sample variances and covariances for interest rates. The value of the Kendall coefficient W was 0.1014 for the band period and 0.0264 for the floating rate period. The corresponding $\chi^2_{(4)}$ test statistics are 36.504 and 9.502. For the first period, the test statistic is significant even at the 0.1 per cent level and for the second period at the 5 per cent level. The null hypothesis of independence can thus be rejected.

The test implies, both for interest rates and exchange rates, that the parameter values estimated for the conditional variances can also be used to calculate the conditional covariances.

As for exchange rates, the mean values of the numerical autocorrelation functions up to the fifth order were calculated separately for variances and covariances. The figures presented in Table 6.1 confirm the results of the Kendall-W in revealing a clearly weaker dependence between the conditional second moments for interest rates as compared to exchange rates. The assumption of idential parameter structures thus gets less empirical support with respect to interest rates.

As a third test of dependence between variances and covariances, a principal component-based analysis was used, as was the case for the exchange rates. The correlation coefficient, R, between first principal components of variances and covariances for the first band period was estimated to be 0.88, for the second band period 0.83 and for the floating rate period 0.58. This outcome supports the assumption of dependence between variances and covariances.

The estimated conditional variance for the pooled data was used as the basic expression for the conditional covariances between interest rates in the same way as for the exchange rates. The estimated pooled model for the band period for interest rates was

$$h_t = 0.1497 * E\text{-}7 + 0.0958\varepsilon^2_{t-1} + 0.9005h_{t-1} \qquad (6.11)$$

and for the floating period

$$h_t = 0.1981*E\text{-}9 + 0.0793\varepsilon_{t-1}^2 + 0.9399h_{t-1}. \tag{6.12}$$

For both periods, the sum $\alpha_1 + \beta_1$ for both the pooled exchange rate model and the pooled interest rate model does not differ significantly from one. Therefore, we conclude that the conditional variances of the rates follow a GARCH process integrated in variance.

For the floating rate period, we end up with the same weighted formula (6.9) for interest rates as for exchange rates and consequently the same values of weights as in Table 6.2.

The expression for the covariances corresponds to formula (6.10) for exchange rates. The conditional covariances for interest rates for the floating period are displayed in Figure 6.3 for ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ERFIM.

The CUSUM test statistics for constant correlation in standardized residuals within periods, which is the simplifying assumption in the second method of covariance estimation, were calculated by regressing the pairs of standardized residuals for the interest rate series on each other. The test statistics for the floating rate period are presented in Appendix 2. For the first band period, the null of constancy was rejected at the 1 per cent significance level for all correlations between ERUSD and the other interest rates. The null was rejected for ERNLG/ERFRF and ERBEC/ ERFRF also at the 1 per cent level and for ERCHF/ERFRF and ERBEC/ERFIM at the 5 per cent level. For the second band period, the null was rejected at the 5 per cent level only for the SEK/NOK correlation. For the floating rate period, there were no rejections at the 1 per cent significance level. At the 5 per cent level, some test statistics were significant, most of them between ERDKK and ERGBP and the other interest rates. For this period the same pattern of nonconstancy is seen for the United Kingdom, both for the exchange rate and the interest rate.

Compared to the CUSUM test results for the exchange rates, the test results for the interest rates cast some doubt on the hypothesis of constant correlation within periods. Since the main emphasis is placed here on the results for the floating rate period, we however conclude that the use of the Bollerslev method is justified.

In applying our two-stage application of the Bollerslev method as a second method for covariance estimation, we calculated the conditional correlation coefficients for GARCH standardized residuals for all pairs of interest rates for the three periods into which the data was divided in order to account for regime changes.

For all periods, the numerical values of the correlations are much smaller within the group of interest rates than within the group of exchange rates. For the first band period, there is significant contemporaneous correlation between ERUSD and almost all the other interest rates. Within the group of ERM currencies, only a few significant coefficients were found.

The pattern for the calculated coefficients is to a large extent the same for the second band period. The only difference is found in the greater dependence between ERGBP and the other interest rates.

For the floating rate period, more of the correlations were significant compared to the band period, but they still remain low in comparison to the correlations within the group of exchange rates (Appendix 2). Even ERJPY, which in the band periods appears to be completely uncorrelated with the other interest rates, shows significant correlations with almost all the other interest rates under the floating rate regime. The change in the correlation between the interest rates may be attributed to a degree of integration of financial markets, both within and outside Europe.

Covariances calculated according to the second method are displayed in Figure 6.4 for the floating rate period for the pairs of interest rates ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ ERFIM. These figures correspond to the outcomes for the first method of covariance estimation based on identical parameter structures for variances and covariances for the same pairs of interest rates and the same period as in Figure 6.3.

Figure 6.3 **Conditional covariances, 9 Sep 1992 – 31 Dec 1995, ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ERFIM (weighted series)**

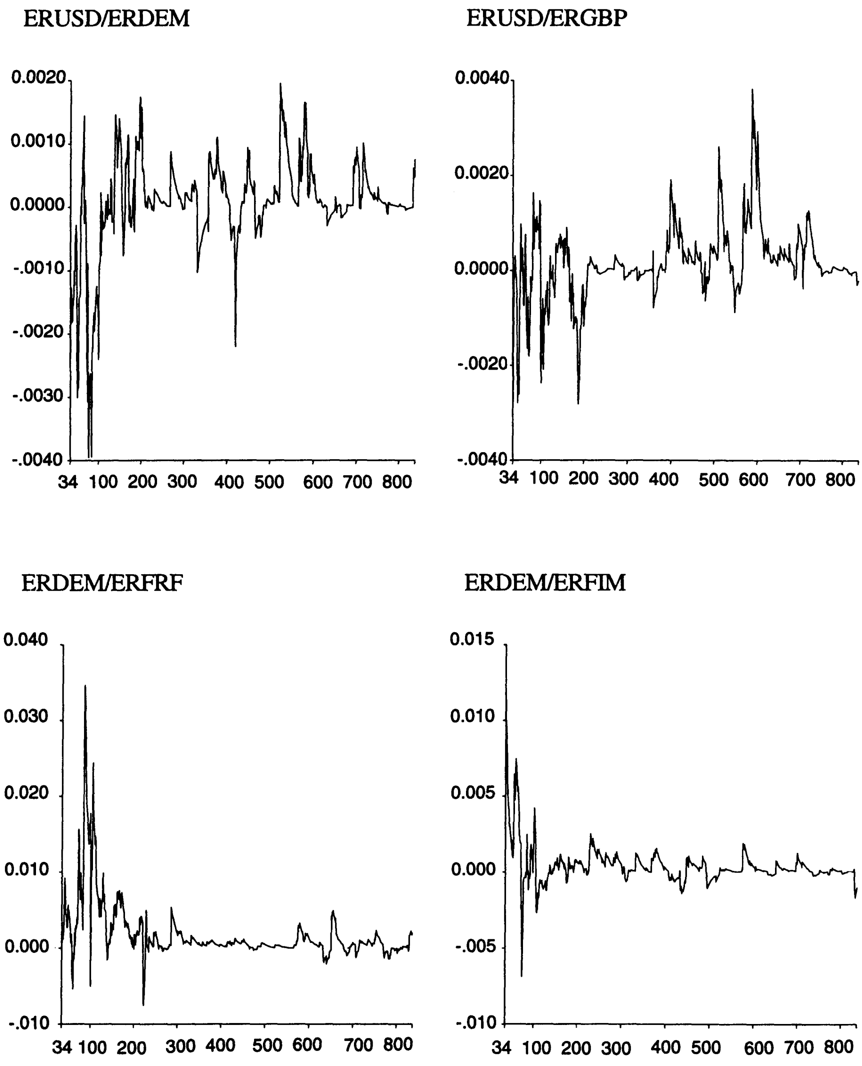
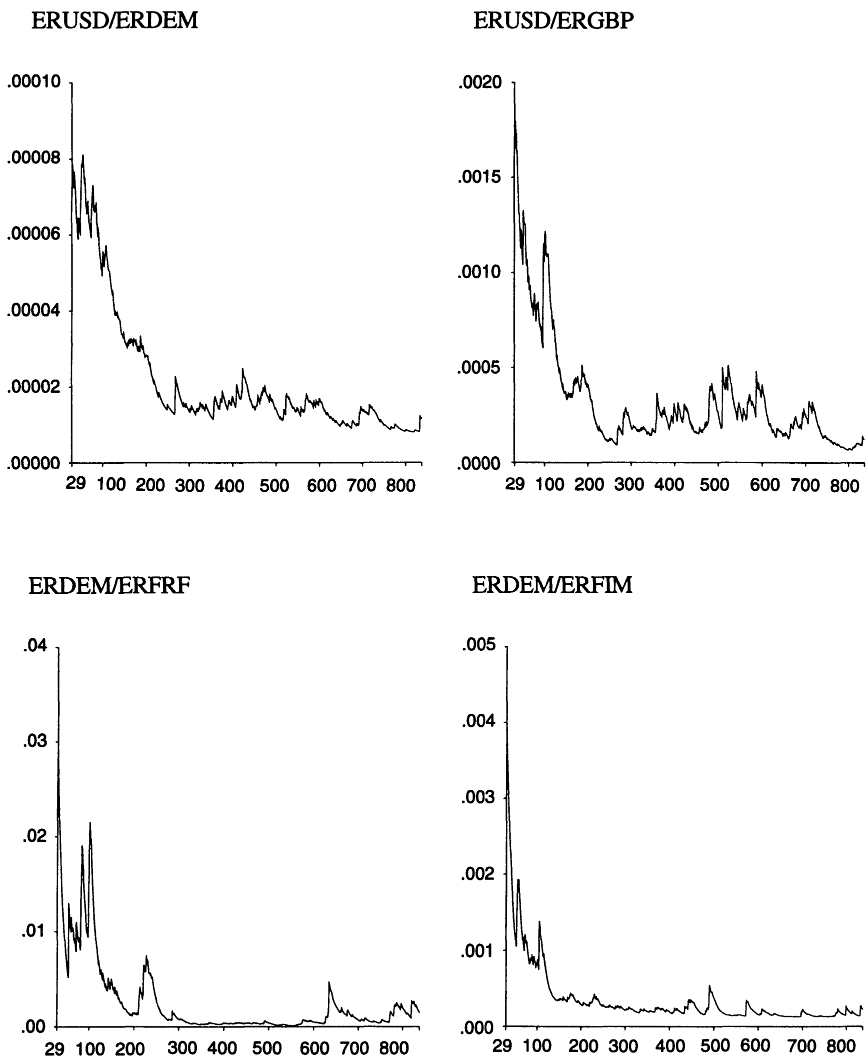


Figure 6.4 **Conditional covariances, 9 Sep 1992 – 31 Dec 1995, ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ERFIM (constant correlation)**



6.5 Covariances between groups of rates

It was also necessary to undertake the multivariate measurement of coherence between different groups.

Within groups of exchange rates and interest rates, the encompassment of conditional covariances is accomplished by showing that the autocovariance structures of variances and covariances are not independent. Thus the parameter estimates of the conditional variance processes for the pooled data are used to model the conditional covariance processes. However, this method is not applicable to covariances between groups unless the parameter structures are identical. The other method, based on Bollerlev (1990), can be used also to measure coherence between groups. The validity of the working assumption of time-dependent conditional variances and covariances with constant correlation is however more open to dispute for between-group than within-group relationships.

To measure the covariation between groups in the covariance estimation method of Bollerslev, the correlation matrix was calculated for all twenty-seven rates: twelve exchange rates, thirteen short-term interest rates, one long-term interest rate and the all-share index. The correlation coefficients were calculated for the GARCH residuals of the individual rates, which are assumed to be normal and IID. In this context, we assume the correlations to be constant within periods but allow them to change between periods with different regimes. The correlations were thus calculated from sample data for each of the three main periods. The correlation matrix for the floating rate period is presented in Appendix 2.

Simple correlations between exchange rates and interest rates are generally low compared to correlations within groups. Statistically significant correlations between exchange rates and interest rates are found only for ERFRF and ERITL. Although over half of the correlation coefficients between either of these two interest rates and one of the twelve exchange rates are significant, their numerical values are small compared to the intragroup correlations. It is difficult to find a theoretical rationale for this pattern of correlation. Of course, it could simply reflect data-specific features.

Strong correlations are found, as expected, between long-term and short-term interest rates. Surprisingly, movements in the all-share index are totally independent of contemporaneous movements in all other rates. Allowing for leads or lags might possibly uncover a significant correlation.

A strong correlation of $\rho = 0.330$ for the floating rate period is found between the Finnish long- and short-term rates. The corresponding conditional covariance is displayed in Figure 6.5.

The hypothesis of an impact of the US exchange rate or interest rate on European financial rates finds no support in the contemporaneous correlation coefficients. To visualize the comovements between the USD and the Finnish long rate, the conditional covariance calculated with the value $\rho = 0.139$ is presented in Figure 6.5.

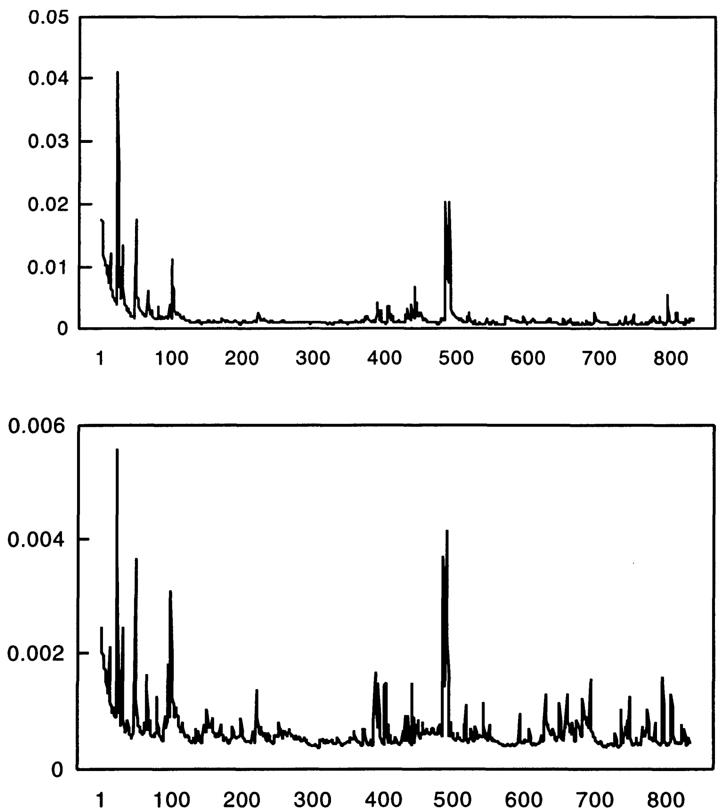
The negligible correlation values between exchange rates, short-term interest rates and the all-share index suggest that covariances between these groups of rates do not have a significant contemporaneous impact on the total risk of the portfolio. A stronger impact might be found if temporal adjustment processes were allowed for or the data were temporally aggregated.

For the sake of uniform measurement, the CUSUM tests were also performed between groups to test the assumption of constant correlation (test values are presented in Appendix 2).

For the first band period the assumption was violated at the 5 per cent level for correlations between the ITL exchange rate vs almost all interest rates and the all-share index. Critical values for the 5 per cent level were also exceeded for the correlation between BEC and ERUSD. For the second band rate period the null hypothesis of constant correlation was rejected at the 1 per cent level for the covariation between the JPY exchange rate and all rates outside the group of exchange rates ie all short-term interest rates, the long-term interest rate and the all-share index. The other test statistics were well below even the critical values at the 10 per cent level. For the floating rate period the test values were significant at the 5 per cent level within the group of exchange rates as were the values for correlations with all other rates. The null was also rejected for the covariation between SEK and six other currencies, as reported above, and the same pattern holds for correlations between SEK and each interest rate. The null was rejected at the 5 per cent level for correlations between SEK and eight out of the thirteen short-term interest rates. Some of the test statistics for correlation between BEC and short-term interest rates exceed the 5 per cent critical value. The test statistics for interaction between ITL and the other rates outside the exchange rate group are highly significant. For the long-term interest rate, significant values at the 5 per cent level were found only for correlations with GBP, ITL and ERGBP. Correlations between the all-share index and the other return series seemed to pass the constancy test, with the exception of the correlations with GBP, ITL, ERGBP, ERDKK, ERJPY and the long-term interest rate. The test statistics did not however have the power to reject at the 1 per cent level.

Summing up for the floating rate period, which is the most important period from the standpoint of forecasting, we can conclude that the test values indicate that variability is present (although not at the 1 per cent level) in correlations between GBP and all other return series, ie for exchange rates, interest rates and the all-share index. Correlations between JPY and all other rates outside the group of exchange rates also seem to be non-constant. We must therefore be cautious in implementing the Bollerslev method for calculating covariances for portfolios where the GBP and JPY are included.

Figure 6.5 **Conditional covariances, constant correlation, 9 Sep 1992 - 31 Dec 1995 FIM long-term rate/FIM short-term rate* and USD/FIM rate****



* Figures multiplied by 10^4.
** Figures multiplied by 10^7.

6.6 Covariation at monthly frequency

Covariances between groups of daily recorded financial rates were calculted in previous sections using the Bollerslev method, assuming constant correlation within periods. The correctness of the assumption is verified using the CUSUM test. Next we discuss the covariances between the real estate return and market risk return series at monthly frequency.

Theoretically there should be covariance also between exchange rate risk, interest rate risk, equity risk, and real estate risk. Hence we calculated correlation coefficients between these series. Correlations for the high frequency data in the previous chapters were measured in GARCH standardized residuals in order to ensure normality. The normality property is needed for evaluation of the statistical significance of calculated cross correlations. The temporaneous differences in frequency between the real estate price model and the other market rate models called for the aggregation of the daily data to obtain a monthly series. Both theory and empirical findings, also in this study, show a clear convergence toward normality with aggregation over time. The montly correlation coefficients are therefore calculated assuming normality for the raw data on monthly log differences for twelve exchange rates, the all-share index and housing prices and on differences relative to levels for fourteen interest rates. Tests showed no instantaneous correlation for any pairs of variables at monthly frequency. Therefore we decided to exclude the covariances between real estate risk and each of the individual market risks and to handle daily market risk separately from monthly equity risk. The separate treatment of real estate risk can also be founded on differences in the forecasting horizon. Market risks in portfolios are due to short-term fluctuations in the prices of the underlying assets. Equity prices are more rigid and ought to be treated from a longer-term perspective.

7 Forecasting conditional variances and covariances with portfolio applications

In this section we present the theoretical forecasting formulas for the variance in GARCH models and the corresponding estimated values for the variance of the return on instruments entailing exchange rate, interest rate and equity price risk. A hypothetical portfolio whose composition covers these main risks is formed in order to illustrate how the separately estimated risks are to be treated comprehensively. This is important since financial instruments are generally sensitive to more than one category of risk. An investment eg in a foreign bond is exposed to both exchange rate risk and interest rate risk.[52]

The asset weights in the portfolio are determined in two ways: first, using theoretically derived global minimum-variance portfolio weights and, second, using more realistic exogenously selected weights so as to replicate the average structure in Finnish banks' trading portfolios. These two types of weights are selected since neither requires information on expected returns[53] and thus they allow us to focus on risk as reflected by the variance-covariance matrix of the portfolio. Four types of forecasts and corresponding conditional confidence intervals are calculated for the ex post period, 1 January – 31 December 1996: two types of GARCH model forecasts and two types of homogeneous model forecasts for the individual variances and covariances in the portfolio. The four types of individual forecasts are compared to the actual values and the results are evaluated on the basis of graphical interpretation, regression analysis and loss function measurement. The forecasting methods are also evaluated for the total portfolio by comparing calculated portfolio variances for the four models and two types of portfolios and also in a value-at-risk framework.

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[52] Exchange rate risk can of course be separately present in a cash deposit in a foreign currency, but this form of investment is not very realistic for any longer horizon.

[53] Coupon returns on bonds and dividends on stocks are excluded from this study. These returns would have to be measured if the return dimension were to be incorporated in the portfolio analysis.

7.1 Forecasting formulas

If log differences of exchange rates, interest rates and stock market indices are unpredictable and follow a homoscedastic process (ie random walk), we can write

$$R_t - R_{t-1} \equiv \Delta R = \varepsilon_t, \tag{7.1}$$

where ε is IID(0,σ).

In this model, the unconditional variance, σ^2, is constant and equals the conditional variance. An unbiased estimator for the variance from a sample of size N is given by

$$\sigma^2 = \frac{1}{N-1} \sum_{t=1}^{N} (\Delta R_t)^2. \tag{7.2}$$

Based on the assumption of identically and independently distributed (IID) errors, ε_t, the volatility over a longer horizon can be estimated by multiplying the one-day volatility by the number of days as a scaling factor. The forecast of the volatility over a period of T days ahead is simply $\sigma^2 T$.

For financial time series, the assumption of IID for the disturbances is typically violated. The interpretation of a GARCH model is that the disturbances are uncorrelated but not independent. Current conditional variance is a function of past conditioning information. This means that the time-dependent conditional volatility can be forecasted. For the GARCH(1,1) model, the one-step-ahead forecast of the conditional variance is

$$\sigma_t^2 = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{7.3}$$

and the s-step-ahead forecast of the conditional variance can be written as

$$E_t(h_{t+s}) = \alpha_0 + (\alpha_1 + \beta_1) E_t(h_{t+s-1}). \tag{7.4}$$

Under the assumption of weak stationarity of foreign exchange rates, interest rates and the all-share index in GARCH(1,1) parameterization

($\alpha_1 + \beta_1 < 1$), the conditional variance will be near its unconditional mean at a sufficiently long horizon. This can be seen by writing (7.4) as

$$E_t(h_{t+s}) = \sigma^2 + (\alpha_1 + \beta_1)^{s-1}\{h_{t+1} - \sigma^2\}. \tag{7.5}$$

The forecast mean reverts to a constant volatility with a decay rate depending on ($\alpha_1 + \beta_1$).

Repeated substitution in (8.4) generates an expression for the time t forecast of the variance over the next s days, expressed on a daily basis:

$$E(h_{t,s}) = \frac{1}{s}\sum_{k=1}^{s} E_t(h_{t+k}) = \sigma^2 + (h_{t+1} - \sigma^2)\frac{1-(\alpha_1+\beta_1)^{s+1}}{s(1-(\alpha_1+\beta_1))}, \tag{7.6}$$

where σ^2 is the unconditional constant variance, which can be shown to be $\alpha_0/(1-(\alpha_1 + \beta_1))$.

For the stationary GARCH(1,1) process, the current information continues to be important even for large s, while the relevant importance decreases with the horizon length.

In the integrated process, IGARCH, α_1 and β_1 sum to one and the model can be expressed as

$$h_{t+1} = \alpha_0 + \alpha_1\varepsilon_t^2 + (1-\alpha_1)h_t. \tag{7.7}$$

If ($\alpha_1 + \beta_1$) = 1 in (7.5) we see from (7.6), applying L'Hôspital's rule, that the s-step-ahead forecast for this model is

$$E_t(h_{t+s}) = h_{t+1}. \tag{7.8}$$

This means that the forecast for the conditional variance s steps into the future is the same as the conditional variance one step ahead for any horizon, s, ie the conditional variance follows a driftless random walk. Thus information today retains its importance in forecasting indefinitely into the future and shocks to conditional variance are permanent. The forecast of the variance over the next s days is simply

$$E(h_{t,s}) = \Sigma E(h_{t+s}) = sh_{t+1}. \tag{7.9}$$

The prediction error variance for the IGARCH process does not converge as the forecast horizon lengthens but instead increases linearly with the length of the forecast horizon.

In the IGARCH(1,1) model with trend in (7.7), the forecast for time t+s is

$$E_t(h_{t+s}) = (s-1)\alpha_0 + h_{t+1} \tag{7.10}$$

and for the next s days

$$E_t(h_{t,s}) = s((s-1)\alpha_0 + h_{t+1}). \tag{7.11}$$

While formula (7.6) can be used for h_{t+1} in ex post forecasts when the model estimation period ends at time t, we also need a formula for ex ante forecasting for periods starting from points in time where the h_t is not known. For this purpose, the GARCH(1,1) model can be rewritten as (6.3), where the conditional variance is expressed in the form of a geometrically weighted average of past squared residuals so that the parameter β_1 gives the decay rate. For the IGARCH process, expression (6.3) reduces to (6.4).

The conditional variance of the pooled data estimated in this study for the floating rate period will be used as the forecasting formula. The estimated model for exchange rates is (6.8), ie

$$h_t = 0.9189*E\text{-}6 + 0.0809\varepsilon_{t-1}^2 + 0.8847h_{t-1}$$

and for interest rates (6.12), ie

$$h_t = 0.1981*E\text{-}9 + 0.0793\varepsilon_{t-1}^2 + 0.9399h_{t-1}.$$

The sum $\alpha_1 + \beta_1$ does not differ significantly from one and thus we conclude that the conditional variance of both exchange rates and three-month interest rates can be modelled as a GARCH process integrated in variance. Based on the outcome of the Kendall-W test procedure, we also conclude that the conditional covariances between exchange rates and conditional covariances between interest rates can be modelled with the same parametric structure as their conditional variances.

We consequently use the weight structure presented in (6.9) for forecasting variances when $\alpha_1 = 0.08$ and $\beta_1 = 1 - \alpha_1$ and formula (6.10) for forecasting the covariances. The series of lagged squared residuals to be included in actual calculations is truncated at 28 past observations. The weights of the observations thereafter are each less than 10 per cent of the weight of the first observation. Table 6.2 gives the numerical values of the weights.

The sample period is truncated at 28 observations and the decay factor in the weight structure is $(1 - \alpha_1) = 0.92$. The short period of 28 observations implies a rapid updating of the estimated volatility. It also means that in periods of increasing volatility, the heavy weighting of more recent observations leads to higher estimates of volatility compared to equally weighted observations and correspondingly lower estimates for periods of diminishing volatility.

In the quarterly ex post VaR model evaluation of market risks for supervised banks' portfolios (developed at the Bank of Finland), formulas (6.9) and (6.10) will be used to calculate, using daily data, the individual conditional variances and pairs of covariances for twelve exchange rates, thirteen short-term interest rates and the all-share index. Since the variance model for the long-term rate is not integrated, we must use a different structure to forecast this volatility. The point in time t is then the day for reporting the composition of the portfolio. In forecasting the inherent risk over the banks' planning horizon, which is assumed to be one year, one needs forecasted measures of variances and covariances for lower frequencies. Monthly, quarterly, semiannual and annual volatility forecasts are calculated using formula (7.11) for the $h_{ij,t}$'s derived from expressions (6.9) and (6.10).

7.2 Ex post and ex ante forecasting of market risk

In previous chapters we have compared, for the within-sample period, the estimated GARCH(1,1) univariate variances of exchange rates and money market interest rates to the unconditional constant variances calculated in two ways: over a sample period corresponding to the dominant frequency and over a sample period of three months. In this section, an ex post forecasting procedure for the out-of-sample period 1 January – 31 December 1996 is performed, in which GARCH one-step-ahead predictions of the conditional variance and homoscedastic variance with one- and three-month windows will be compared to the actual volatility. We assume that the results of this comparison are also valid for ranking the performance of the models in ex ante forecasting.

The diagram below illustrates the time horizon and the concepts that will be used in evaluating the forecasting ability of the estimated models.

estimation: within-sample period   forecast: out-of-sample period
I--I---------------------today------------------I
1987-1995   ex post forecast   ex ante forecast
1996

Evaluation of the predicting ability is performed for a fictional portfolio constructed so as to capture the return volatility (risk) for the exchange rate, the interest rate and equity investments. Predicton evaluations for the individual variances and covariances are carried out on a daily basis. The portfolio variance performance is dealt with on a monthly basis. Monthly performance evaluation allows for the inclusion of real estate risk in the fictive portfolio. However, we prefer not to include real estate because the risk is managed differently from the daily market risks for which the assets are constantly marked to market.

The portfolio consists of investments in

- three-month DEM CP-type instruments
- three-month USD Treasury bills
- three-year FIM bonds
- Finnish shares.

The unsystematic risk in share investment is assumed to be diversified away and thus the return on the investment will change according to changes in the all-share index. The total value of the portfolio will consequently change with changes in USD and DEM exchange rates, USD and DEM short-term interest rates, the three-year Finnish interest rate and the all-share index. The variance, σ_P, can be expressed as

$$\sigma_P^2 = w'\Sigma w = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}, \qquad (7.12)$$

where w is the vector of portfolio shares with $\Sigma w_i = 1$ and Σ is the variance-covariance matrix of returns. Thus to assess the total variance of the portfolio, the covariances must be included. However, from the formula,

$$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$$

it is clear that the contribution to the portfolio variance of covariances with ρ_{ij} close to zero is neglible.

To identify the significant covariances, we form the correlation matrix for the markka's floating rate period of the GARCH standardized residuals for the relevant return variables. The elements of the matrix are displayed in Table 7.1.

Table 7.1 **Correlation matrix for the floating rate period for the variables included in the portfolio**

	USD/FIM	DEM/FIM	ERUSD	ERDEM	FIMLONG	HEX
USD/FIM	1					
DEM/FIM	0.435	1				
EKUSD	−0.014	−0.012	1			
EKDEM	−0.041	−0.017	0.103	1		
FIMLONG	0.002	0.134	0.139	0.010	1	
HEX	0.005	−0.060	0.038	−0.008	−0.066	1

The 95 % critical value is 0.0685.

The out-of-sample period includes the change of regime from the floating rate regime to the ERM on 14 October 1996. This means that the parameters in the conditional variance and covariance models, as well as the correlation coefficients, must be re-estimated for the new regime as soon as sufficient data are available. Before undertaking this re-estimation, the dependencies estimated up to 31 December 1995 are assumed to hold also for the out-of-sample period, 1 January – 31 December 1996. For the GARCH parameter values, this assumption in not necessarily a short-coming, since we have shown that the parameter values are independent of the exchange rate regime in respect of both exchange rates and money market interest rates. The values and significance of the correlation coefficients, however, may change with the regime.

In the correlation matrix above, only the following correlations are statistically significant:

- USD/DEM
- ERUSD/ERDEM
- FIMLONG/DEM
- FIMLONG/EKUSD,

and thus the corresponding covariances are included in the volatility estimate for the total portfolio. Theoretically, every correlation less than one affects portfolio composition and should therefore be included in the portfolio calculations, but since a low value for the correlation coefficient makes the covariance very small, we decided for practical reasons to exclude covariances with insignificant correlations.[54]

As a result of the findings of the empirical part of this study, we claim that an IGARCH(1,1) model with approximately the same parameter values (neglecting constants) for exchange rates, interest rates and the all-share index has been estimated for the floating rate period. For exchange rates and interest rates, the parameter values even remain the same regardless of the exchange rate regime. The integrated model was identified for these rates for forecasting purposes with $\alpha_1 = 0.08$ and $\beta_1 = 0.92$ according to (6.9) with the weight structure given in Table 6.2.

The long-term interest rate variance does not follow the pattern found for exchange rates and interest rates but exhibits low persistence with $\alpha_1 + \beta_1 = 0.6183$, which means strong mean-reverting behaviour. The forecast of the long-term rate will therefore be done according to formula (6.3):

$$h_t = \frac{\alpha_0}{(1-\beta_1)} + \alpha_1 \sum_{j=1}^{\infty} \beta_1^{j-1} \varepsilon_{t-j}^2 .$$

No linear dependence was found in this study in the mean equation for exchange rates. The input data for the USD and DEM in forecasting formula (6.9) will consequently be in the form of log differences. For the interest rates ERUSD, ERDEM and FIMLONG, however, an AR(3) process was identified using the Ljung-Box test statistic as a criterion for selecting the prefiltering order. The autocorrelation parameters were estimated in the GARCH model for these three interest rates. For ERUSD, the autocorrelation structure was very peculiar, with $\phi(1) = -0.100$ and $\phi(3) = 0.098$ significant but $\phi(2)$ nonsignificant and very low (0.023). This peculiarity and the low values of the significant coefficients motivated exclusion of the autocorrelation correction for ERUSD in the forecasting procedure. Only for ERDEM was $\phi(3)$ statistically significant, but its value was low (0.065). The other autocorrelation parameters where both statistically nonsignificant and neglible in

[54] To evaluate the effects of neglecting statistically nonsignificant covariances (values set at zero), some of the calculations were performed with the entire variance-covariance matrix. The effects on the total portfolio variance were found to be covered by rounding errors on the accuracy level of two decimal places.

numerical value. Also for ERDEM, we decided to use unfiltered data in the forecasting formula.

For the long-term rate, the order AR(3) of prefiltering was selected in the GARCH model on the basis of the outcome of the Ljung-Box tests. When the lag structure was encompassed within the GARCH mean equation, only the first-order autocorrelation parameter was statistically significant. The two other parameters were not only nonsignificant but also negligible in numerical value. We therefore decided to use the AR(1) form for the long-term rate, with $\phi(1)= 0.527$.

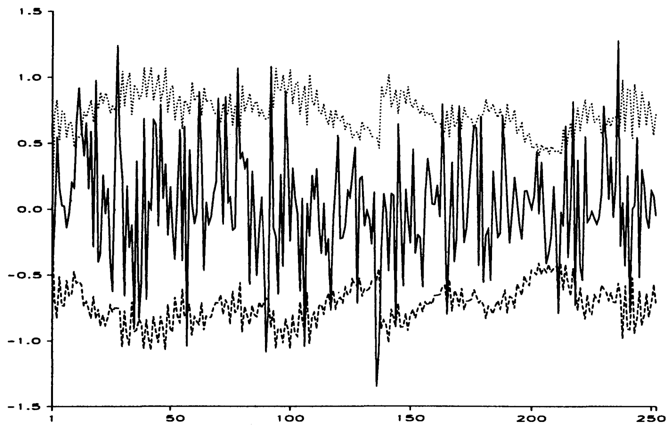
For the all-share index, we use the AR(1) filtered log differences with $\phi(1)= 0.227$.

The daily one-step-ahead GARCH forecasts can be exploited to obtain time-varying confidence intervals for point forecasts of changes in the rates under consideration. These intervals reveal a typical conditional variance feature: in periods of high (low) volatility, the intervals are large (small) (Diebold and Nerlove 1986). Figure 7.1 presents the realized returns, ie the series of first-differences for USD and DEM exchange rates and ERUSD and ERDEM short-term interest rates, as well as their GARCH-based $\pm 1.96\sigma$ confidence intervals for one-step-ahead forecast errors. In the corresponding figures for FIMLONG and HEX, the realized return series for the band periods are in the AR(1) form.[55] The 95 per cent probability statement can be attributed to the $\pm 1.96\sigma$ intervals due to the theoretical proposition that GARCH estimation produces normally distributed residuals. It can also be seen that the realized return figures stay within these confidence intervals most of the time.

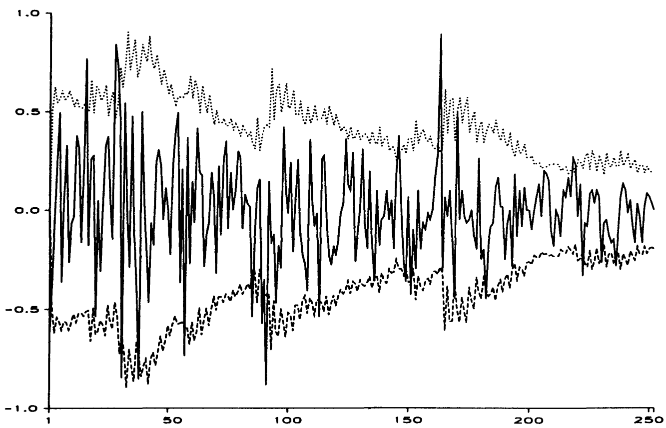
[55] We use the autocorrelation coefficients estimated from historical data and assume them to be constant over time.

Figure 7.1 **±1.96σ confidence intervals for changes in USD and DEM exchange rates, short-term interest rates, the long-term interest rate and the all-share index constructed using ex post forecasts on the conditional variance for the out-of-sample period, 1 January – 31 December 1996**

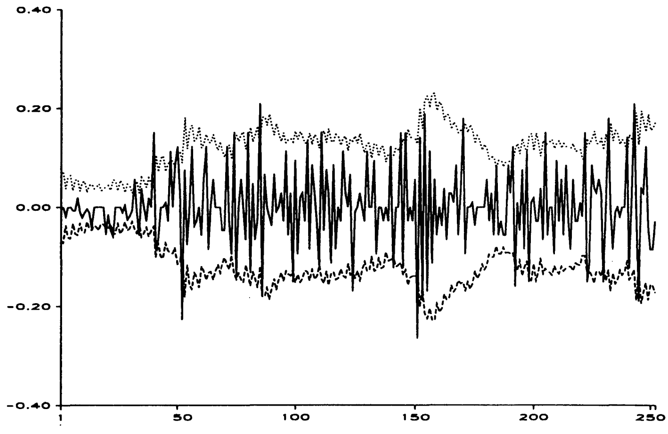
USD/FIM



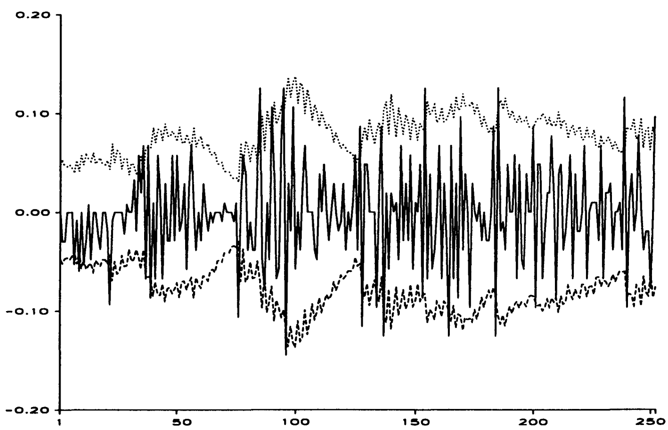
DEM/FIM



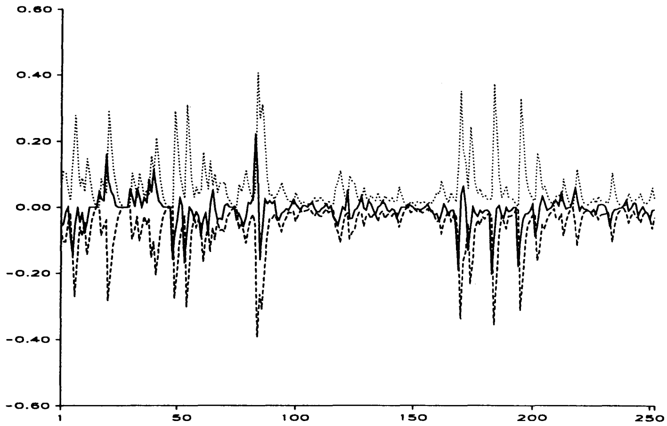
ERUSD


0.40
0.20
0.00
-0.20
-0.40
1
50
100
150
200
250

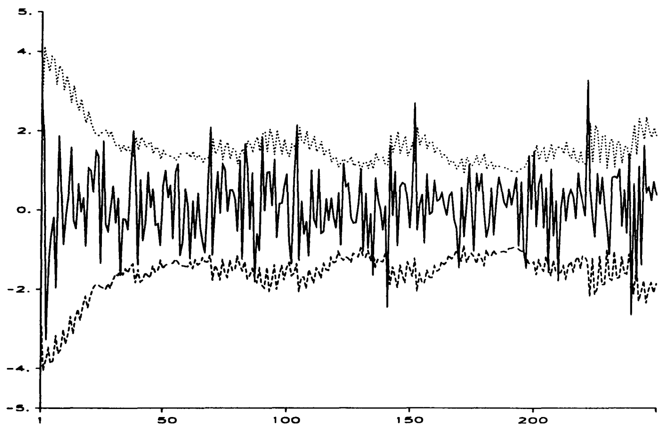
ERDEM


0.20
0.10
0.00
-0.10
-0.20
1
50
100
150
200
250

FIMLONG



HEX

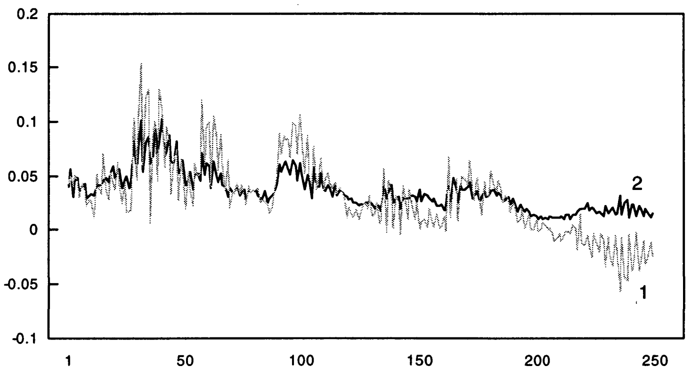


Covariances between USD and DEM exchange rates and ERUSD and ERDEM interest rates are calculated using the two methods presented above. In Figure 7.2 the series USD/DEM$_{cov1}$ is constructed using formula (6.10) and the series USD/DEM$_{cov2}$ is constructed assuming a constant correlation of 0.435 between these exchange rates, calculated on standardized residuals for the floating rate period. Since the calculation of the two covariances is based on different methods, the series are not expected to be identical. To measure the correlation between the two series, a linear regression was estimated giving the following parameter values:

$$USD/DEM_{cov1} = \underset{(27.66)}{0.0222} + \underset{(25.41)}{0.4403}\, USD/DEM_{cov2}$$

and $R^2 = 0.72$.

Figure 7.2 **Covariances of changes in USD and DEM exchange rates**



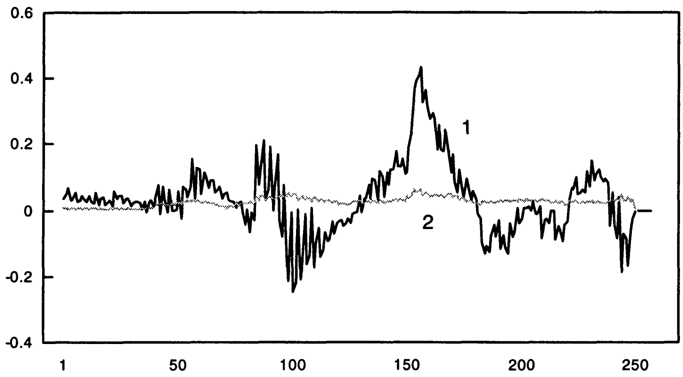
1 USD/DEM_{cov1}
2 USD/DEM_{cov2}

The same calculations were perfomed to obtain the two ERUSD/ERDEM covariances shown in Figure 7.3. The corresponding regression produced the following parameter values:

$$ERUSD/ERDEM_{cov1} = \underset{(0.83)}{-0.00008} + \underset{(3.06)}{1.1416}\, ERUSD/ERDEM_{cov2}$$

and $R^2 = 0.025$.

Figure 7.3 **Covariances of changes in USD and DEM short-term interest rates** (figures multiplied by 100)

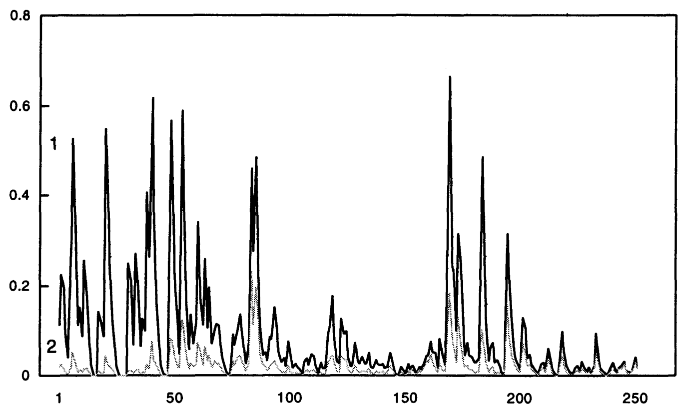


1 EKUSD/EKDEM$_{cov1}$
2 EKUSD/EKDEM$_{cov2}$

The covariances settle at quite different levels according to the scaling parameter ρ. The larger the correlation coefficient, the closer together the results from the two methods.

The other two significant covariances in the portfolio, FIMLONG/DEM and FIMLONG/ERUSD, are between groups, and can therefore only be calculated with the cov2 method, which assumes constant correlation. These covariances are shown in figure 7.4.

Figure 7.4 **Covariances of changes in FIMLONG/DEM and FIMLONG/ERUSD** (figures multiplied by 100)



1 FIMLONG/DEM$_{cov2}$
2 FIMLONG/EKUSD$_{cov2}$

When the cov1 and cov2 methods of covariance assessment are compared, the first method is found to be preferable since it has a higher degree of multivariate methodology. Thus in practical applications, the cov1 method should be used when dealing only with covariances within groups. When the risk of a portfolio with instruments from different groups is calculated, the cov2 method should be applied for the sake of methodological comparability and homogeneity.

7.3 Comparison of prediction performance

The volatility forecasting comparison will be based on out-of-sample monthly forecasts calculated from daily data covering the ex post period, 1 January – 31 December 1996, with four models: two GARCH models estimated in this study and two sample variance models. The comparison is made between models and relative to the actual values.

7.3.1 Results from prediction comparison in the literature

In the literature, volatility prediction comparisons are usually made between random walk models, GARCH models and stochastic volatility models. The outcome of the performance comparison has been found to depend on the frequency of the data, on the forecasting horizon, on the specific asset class under consideration and on the error statistic used. Heynen and Kat (1994) use daily data on seven different stock indices and five currencies to compare volatility predictions. Their results show that the volatility prediction peformance of different models depends heavily on the asset class. For the stock indices, the best volatility predictions are generated by the stochastic volatility model, for both short- and long-term horizons, followed by the EGARCH and GARCH predictors. The random walk predictor ranks last. For currencies, the ranking is GARCH, EGARCH, random walk and, lastly, the stochastic volatility model. Brailsford and Faff (1996) show in a study on daily Australian data that although no single model is clearly superior, the ARCH class of models and a simple regression model provide superior forecasts of volatility. However, the various model rankings are shown to be sensitive to the error statistics used to assess forecast accuracy.[56] Lopez

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[56] The main error statistics compared are: the mean error (ME), the mean absolute error (MAE), the root mean square error (RMSE) and the mean absolute percentage error (MAPE).

(1995) shows in a study on three currencies that forecasts cannot be evaluated with a single statistical loss function and that the selected loss function directly influences the evaluation results. West and Cho (1995) compare the forecasting performance of random walk, GARCH, autoregressive and nonparametric models for conditional variances, using weekly data on five dollar exchange rates. Their results are that for the one-period-ahead (one-week) horizon GARCH models tend to give slightly more accurate forecasts. For longer horizons they find it difficult to choose among the models. Alexander (1995) shows, using a likelihood function to distinguish between different models for forecasts of daily variances of dollar exchange rates and equity indices, that short-term, ie one-day-ahead, GARCH forecasts generally do better than weighted moving-sample-window forecasts, but that the superiority of both models over the unconditional random walk model declines as the forecast period lengthens. In all time series of exchange rates and stock indices studied, the equally weighted random walk estimate was the worst in forecasting.

To summerize these and other results from studies on forecasting evaluation, it can be concluded that the evidence leans in favour of the GARCH model, especially in models using high frequency data and a short forecasting horizon. The preferability of GARCH models is enhanced by the findings of Nelson (1992) and Nelson and Foster (1995), showing that GARCH models estimated on high frequency data are robust to misspecification. It is possible to make a good forecast with the wrong GARCH model. As the time interval between observations approaches zero, a sequence of GARCH models can consistently estimate the underlying conditional variance even if the GARCH models do not reflect the correct data-generating processes. This consistency holds not only for GARCH but for many other ARCH models as well and is unaffected by a wide variety of misspecifications.

The use of a GARCH model for forecasting has important advantages compared to unconditional variance estimation. The GARCH model was developed to capture the stylized facts in daily data, the frequency at which we want to operate. Only GARCH models can produce high frequency variance measures and forecasts. Although the GARCH model conditional variances estimated in this study are the obvious choice for use in the risk measurement framework developed here, we want to compare our estimated GARCH forecasts to certain unconditional variance models.

7.3.2 Model forecasts

The forecasting comparison will be based on out-of-sample predictions of monthly variances of returns calculated with four models: two GARCH models estimated in this study (Models I and II) and two common sample variance models (Models III and IV), all estimated on daily data.

As discussed in Christoffersen and Diebold (1997), there is no obvious horizon for risk management and consequently not for forecasting either. The relevant horizon will likely vary by position in the firm, by asset class and by industry. For financial institution trading desks, overnight forecasts are the most relevant since this frequency covers the period during which portfolios cannot be liquidated. Longer horizons may be relevant for strategic portfolio planning. For longer horizons, such as three months, six months or one year, the advantages of using conditional models estimated on high frequency data vanish, the forecastability weakens and the forecasts converge to constant unconditional variances.

Christoffersen and Diebold (1997) estimated that the forecastability of return volatility for equity indices, dollar exchange rates and a long-term bond ranges from one through twenty trading days. Of our models, Model I is especially well suited for overnight forecasts and therefore applicable to trading desk risk management. In our study we are however interested in risk management more generally and therefore choose the one-month horizon for the forecast comparisons.

Model I is the GARCH forecast of the variance. Prediction of h_{t+1} for an integrated process estimated in this study in exchange rates, short-term interest rates and the all-share index is calculated with the exponentially weighted formula (6.9):

$$h_{t+1} = 0.08\varepsilon_t^2 + 0.08(0.92)\varepsilon_{t-1}^2 + 0.08(0.92)^2\varepsilon_{t-2}^2$$
$$+ 0.08(0.92)^3\varepsilon_{t-3}^2 + \ldots + 0.08(0.92)^{n-1}\varepsilon_{t-27}^2,$$

with 28 lagged squared residuals, and the daily forecasts are summed within months to give the monthly forecasts:

$$E(h_{t+1,s}) = \sum_{i=1}^{N} h_{t+i},$$

where N is the number of days in the month.

For the nonintegrated process in the long-term rate, h_{t+1} is given by formula (6.3).

The forecasting equation corrected for first-order autocorrelation to be used for the conditional variance of the all-share index and the long-term rate is (Akgiray 1989)

$$E(h_{t,s}) = \sum_{t=1}^{N} \left(\frac{1-\phi^{t}(1)}{1-\phi(1)} \right)^{2} [(\alpha_1+\beta_1)^{N-t} h_t + \sum_{j=0}^{N-t} \alpha_0 (\alpha_1+\beta_1)^{j}],$$

where h_t is given by formula (6.9) or (6.3) respectively. The value of the drift parameter was estimated to be statistically significant but negligible in numerical value, and therefore the second sum in brackets is set equal to zero.

Model II is also a GARCH prediction of the variance based on the same one-step-ahead forecasting formula as in Model I. The difference between Model I and Model II is in the method used for aggregating daily volatility forecasts into monthly series.

One-month predictions in Model II are based on the information up to the first day of the predicted month:

$$E_t(h_{t+s}) = h_{t+1}$$

and the formula

$$E_t(h_{t,s}) = \Sigma E_t(h_{t+s}) = N h_{t+1},$$

where N is the number of days. h_{t+1} is calculated according to the same formula (6.9) as in Model I.

Model III is a forecast of a contant variance calculated with an equally weighted one-month window. The sample variance of the daily returns of month S multiplied by the number of trading days in month S+1 is used as a forecast for month S+1.

Model IV is a forecast with an equally weighted three-month window. The sample variance is calculated over daily return series for months S, S+1 and S+2 and multiplied by the number of trading days in month S+3 to produce the forecast for month S+3.

Table 7.2 summarizes the four models.

Table 7.2 **Variance forecasting model description**

Model I	GARCH(1,1) variance one-step-ahead daily forecast: $h_{t+1}=0.08\varepsilon_t^2+0.08(0.92)\varepsilon_{t-1}^2+0.08(0.92)^2\varepsilon_{t-2}^2$ $+0.08(0.92)^3\varepsilon_{t-3}^2+\ldots+0.08(0.92)^{n-1}\varepsilon_{t-27}^2,$ monthly forecast: $E(h_{t+1,s})=\sum_{i=1}^{N} h_{t+i}$
Model II	GARCH(1,1) variance one-step-ahead daily forecast: $h_{t+1}=0.08\varepsilon_t^2+0.08(0.92)\varepsilon_{t-1}^2+0.08(0.92)^2\varepsilon_{t-2}^2$ $+0.08(0.92)^3\varepsilon_{t-3}^2+\ldots+0.08(0.92)^{n-1}\varepsilon_{t-27}^2,$ monthly forecast: $E_t(h_{t,s})=\Sigma E_t(h_{t+s})=Nh_{t+1}$
Model III	Homogenous sample variance with one-month window (N = number of days in one month) one-day variance: $h_t=\frac{1}{N-1}\sum_{t=1}^{N}\varepsilon_t^2$ monthly forecast: $h_{t,s} = sh_t,$ where s is the number of days in the forecasted month
Model IV	Homogeneous sample variance with three-month window (N = number of days in three months) $h_t=\frac{1}{N-1}\sum_{t=1}^{N}\varepsilon_t^2$ monthly forecast: $h_{t,s} = sh_t,$ where s is the number of days in the forecasted month

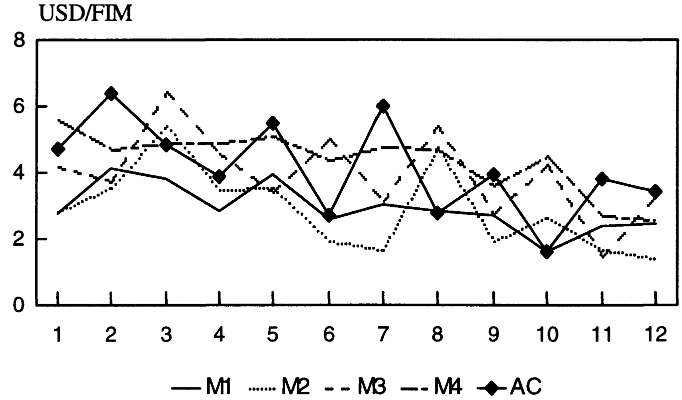
7.3.3 Comparisons between models

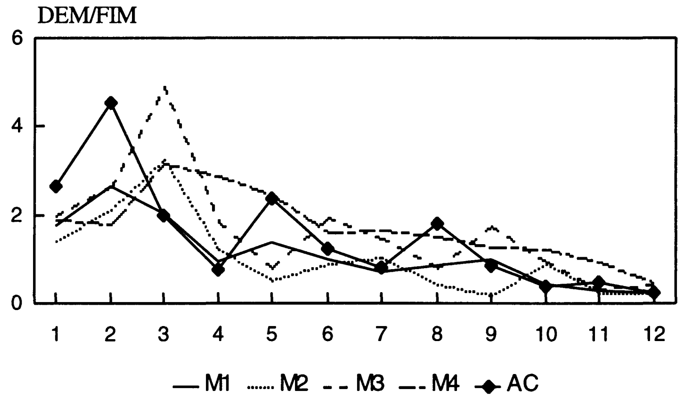
Comparison of forecasts with Models I and II gives the differences between the use of ex post realized daily returns in forecasting the monthly volatility and the use of an ex ante type of forecast, in which we do not know the realizations for the month to be forecasted but can calculate the volatility forecast for the first day of the month. Comparison of forecasts III and IV gives the sensitivity of the forecast to window length.

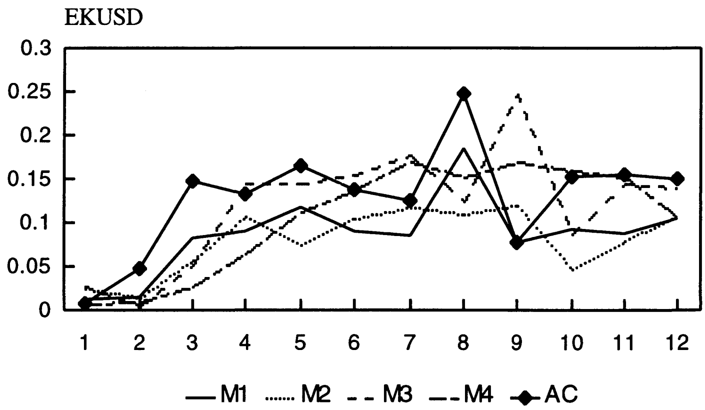
Figure 7.5 gives the four types of forecasts for USD and DEM exchange rates, USD and DEM money market interest rates, the long-term interest rate and the all-share index (HEX) respectively. A special feature in the figure for HEX is that the turbulent period in December 1995 produces high volatility forecasts for the first few months of the ex post period.

The same pattern is visually noted for all rates in the internal comparisons of different variance forecasts. The most volatile series are in Model III, the random walk generated variances with a one-month window. In Model IV the volatility is dampened with the use of the longer window. The lowest variance figures, and also the smoothest ones, are calculated with Model I.

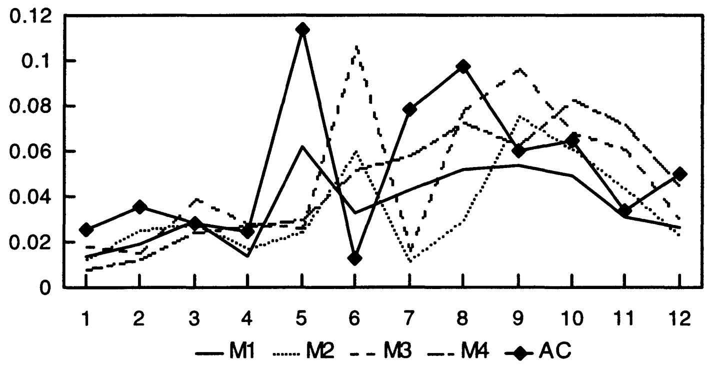
Figure 7.5 **Monthly model I–IV forecasts (M1, M2, M3 and M4) for variances of USD and DEM exchange rates and interest rates, long-term interest rate, FIMLONG, and all-share index, HEX, and the corresponding actual volatility (AC)**



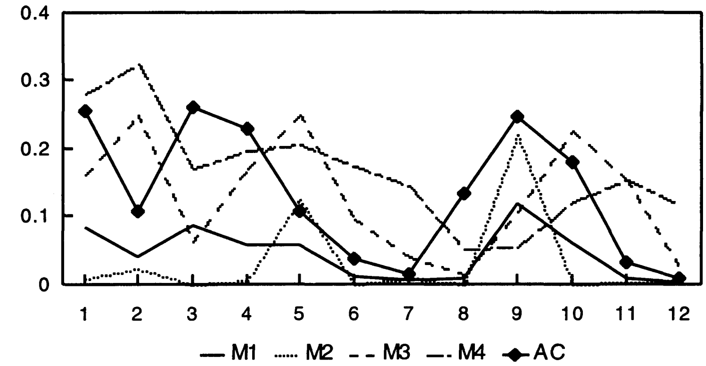




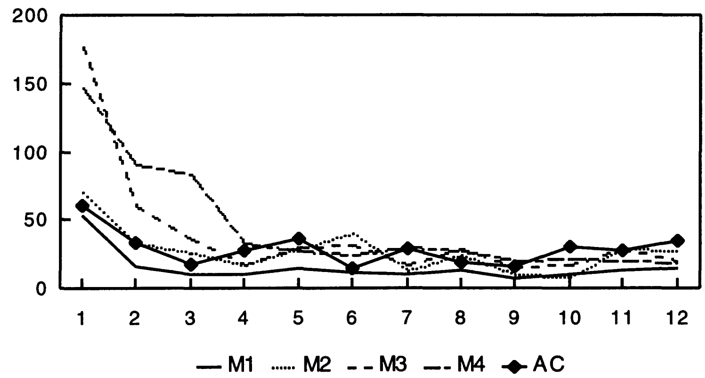
EKDEM



FIMLONG



HEX



7.3.4 Comparison with actual volatility

In order to enable comparison of prediction accuracy, we should define the true volatility that we are trying to predict. Under the null hypothesis that the estimated GARCH(1,1) model constitutes the correct specification, the true variance is by definition identical to the GARCH volatility forecast of Model I (Andersen and Bollerslev 1997). Therefore one means of comparison is to evaluate the forecast of a model according to its ability to trace the Model I forecast. In this comparison, Model II outperforms the random walk models.

An alternative, commonly used evaluation method is to compare forecasted variances to the actual volatility. Usually the actual monthly volatility is defined as the sum of daily realized squared returns of trading days within a month:

$$E_t(h_{t,s}(A)) = \sum_{i=1}^{N} R_{t+i}^2.$$

The actual variance when first-lag autocorrelation is present in the mean equation is calculated ex post (Akgiray 1989) as

$$E_t(h_{t,s}) = \sum_{i=1}^{N} R_{t+i}^2 \left[1 + \frac{2}{N} \sum_{j=1}^{N} (N-j)\phi(1)^j \right].$$

In this study we compare the prediction ability of different models in three ways: first, by graphical interpretation, secondly, by regressing the individual model variance forecasts on the actual variance and, thirdly, by ranking performance by calculating a likelihood based loss function.

The first method, graphical interpretation, is based on Figure 7.5 where the prediction outcomes of Models I–IV are compared with the actual values. A visual evaluation of the ability of the GARCH models (Models I and II) to trace the actual monthly variance is difficult. But what the figures tell quite clearly is that the sample variance is a poor predictor of the future variance. Although sample variances are commonly used as forecasts, there is no methodological basis for this. By comparing our sample variance models (Models III and IV) to the actual values, we can see that there is a clear lag in the graphs due to the use of the sample variance of the previous month(s) as a forecast.

With the second comparison method, we find the same similarity in the regression results for different models as was found by examining the graph. Model I has the highest prediction power as measured by R^2, ranging from 0.685 for ERDEM to 0.904 for ERUSD. The reason for this is obvious: Both the forecast and the actual variance are to a large extent based on the same frequency (higher compared to the other methods) information set on past residuals. Both sample variance-based forecasts, Models III and IV, perform poorly, as does the GARCH forecast Model II. However, if we want to use the GARCH forecasting framework, we should use Model II. Compared to Model I, Model II is more strictly a one-month-ahead prediction model and the only one that can be used in ex ante prediction at monthly frequancy.

With the third method of comparison, we calculate the log likelihood function for the ex post period January–December 1996 (Alexander 1995). The likelihood function, based on the normality assumption, can be transformed as

$$\sum_{t=1}^{12}\left[\frac{R_t^2}{\hat{\sigma}_t^2}+\log\hat{\sigma}_t^2\right],$$

where t is the number of months in the evaluation period and $\hat{\sigma}^2$ is the variance forecast given by Models I–IV.

The smaller the quantity calculated with the formula, the better the forecasting model. Table 7.2 shows the values of the loss functions for the six instruments and the four variance models. There is greater variability in the loss function values for the DEM exchange rate as compared to the USD exchange rate, but according to both, Model II is ranked slightly worse than the other models. For the USD short-term interest rate, Model I performs worse while the others perform equally well or badly. For the long-term interest rate, the outstandingly high value for Model II is explained by the fact that the estimated variance for certain months is very close to zero. For two months it actually equalled zero, and these months, for computational reasons, had to be left out of the loss function calculation. The loss function values give no unambiguous ranking of the forecasting ability of the models but confirm empirical results from other studies, ie that the rankings differ between instruments.

Table 7.2 **Loss-function values for forecasts with Models I–IV**

	Model I	Model II	Model III	Model IV
USD/FIM	29.18	31.65	30.17	29.04
DEM/FIM	13.83	20.38	15.99	15.44
EKUSD	-13.16	-10.78	-8.63	-9.48
EKDEM	-23.68	-17.22	-19.25	-21.36
FIMLONG	-24.72	88.35	-17.48	-14.73
HEX	47.49	48.18	48.81	49.68

The conclusion to be drawn from the different evaluation methods applied here is that the decision on which model should be used ought to be based on methodological adequacy. This means that the GARCH models are preferable to the unconditional models. Therefore Model I should be used in ex ante forecasting for the one-day-ahead horizon and Model II for one-month-ahead or longer horizons.

7.4 Portfolio composition

The variance of a portfolio, σ_P^2, consists of the variances and covariances of the individual instruments as given in formula (7.12). In the previous section the elements of the variance-covariance matrix were defined. In this section we calculate the variance of the total portfolio. This involves selecting the composition vector, w, and adding up the weighted variances and covariances. In our application the vector w will be determined in two ways. First, the portfolio shares are determined via minimum-variance optimization and secondly based on average shares in Finnish banks' portfolios.

7.4.1 Minimum-variance portfolio

In the standard mean-variance portfolio theory developed by Markowitz and Tobin, the class of potentially optimal portfolios for investors are those with the greatest expected return for a given level of variance or, equivalently, the smallest variance for a given expected return. Such portfolios are defined as mean-variance efficient. A larger class of portfolios is the set of minimum-variance portfolios, which includes as a special case the single portfolio with the smallest variance for any level of return, ie the global minimum-variance portfolio.

Since we do not deal with the returns of the portfolio, but rather concentrate on the variance, a general minimum-variance efficient portfolio could not be identified. But the global minimum-variance and the corresponding vector of portfolio weights are independent of expected returns. Based on the variance-covariance matrices for the four forecasting models, we calculated the weights and the variances for global minimum-variance portfolios for each of the twelve months in the ex post forecasting period.[57]

The selected instuments show extreme differences in the levels of volatility (Table 7.3). Since the global minimum variance of the portfolio is independent of the expected return, one solution to the composition optimization problem is to invest 100 per cent in the asset with the smallest variance. This asset is one of the interest rate instruments, either the three-month USD treasury bill or the corresponding DEM instrument or the Finnish three-year bond, depending on which has the smallest monthly variance forecast. When the diversification advantages reflected in the covariances are included, we end up with minimum-variance portfolios that are roughly evenly divided between the USD and DEM instruments. A few of the GARCH model portfolios allow up to 10 per cent of total value in Finnish long-term rate instruments. None of the global minimum-variance portfolio solutions include any investment in the equity index, which shows the highest volatility of all return series for the selected instruments. Table 7.5 gives the portfolio weights for Model II.

[57] The durations of the interest rate instruments, 0.25 for the money market instruments and 3 for the long rate instruments, were connected to the relevant covariances and variances to make the interest rate series comparable to the other return series. To simplify the calculations, we assumed the three-year instrument to be a zero-cupon bond. For this instrument, duration is identical to maturity.

Table 7.4 **Model II variances and covariances for variables included in the portfolio, per cent**

	USD/FIM	DEM/FIM	ERUSD	ERDEM	FIMLONG	HEX
1996M01	2.8173	1.4027	0.0257	0.0125	0.0078	71.0859
1996M02	3.5408	2.1155	0.0162	0.0254	0.0237	33.6271
1996M03	5.4235	3.2565	0.0569	0.0277	0.0014	25.9283
1996M04	3.4402	1.2164	0.1080	0.0168	0.0072	16.8564
1996M05	3.5099	0.5118	0.0754	0.0248	0.1247	28.5870
1996M06	1.8989	0.8865	0.1060	0.0600	0.0000	40.5970
1996M07	1.6469	1.0217	0.1182	0.0120	0.0090	13.3602
1996M08	4.7249	0.4462	0.1109	0.0295	0.0000	24.9557
1996M09	1.9020	0.1689	0.1208	0.0754	0.2208	9.9828
1996M10	2.6243	0.8952	0.0469	0.0610	0.0014	9.0471
1996M11	1.6701	0.2528	0.0794	0.0439	0.0039	28.3094
1996M12	1.4063	0.2246	0.1076	0.0225	0.0051	28.0029

	USD/DEM	ERUSD/ERDEM	FIMLONG/DEM	FIMLONG/ERUSD
1996M01	0.8648	0.0019	0.0019	0.0020
1996M02	1.1906	0.0021	0.0026	0.0027
1996M03	1.8281	0.0041	0.0012	0.0013
1996M04	0.8899	0.0044	0.0037	0.0039
1996M05	0.5830	0.0045	0.0130	0.0135
1996M06	0.5644	0.0082	0.0000	0.0000
1996M07	0.5647	0.0039	0.0044	0.0045
1996M08	0.6316	0.0059	0.0000	0.0000
1996M09	0.2466	0.0098	0.0219	0.0227
1996M10	0.6667	0.0055	0.0011	0.0011
1996M11	0.2827	0.0061	0.0024	0.0024
1996M12	0.2445	0.0051	0.0031	0.0033

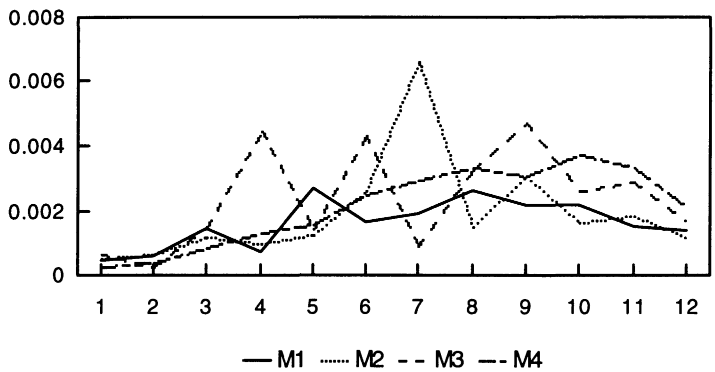
Table 7.5 **Portfolio weights**

Global minimum-variance portfolio weights for Model II, per cent

	DEM	ERUSD	ERDEM	FIMLONG
Jan		31.7	68.2	
Feb		62.9	37.4	-0.3
Mar		26.1	66.3	7.5
Apr		10.4	88.4	1.1
May	0.3	21.6	78.3	-0.1
Jun	0.3	35.0	64.7	
Jul		6.2	93.2	0.6
Aug	0.4	17.7	82.0	
Sept	0.2	37.0	61.2	-0.2
Oct	0.1	50.1	39.6	10.1
Nov	0.6	31.0	64.8	3.6
Dec	0.5	13.7	84.0	1.8

The monthly global minimum variances of portfolios with variances and covariances for the individual instruments calculated for Models I-IV are shown in Figure 7.6. Visual comparison of portfolio variances repeats the results from the comparison of the variance for the individual instruments; Model I gives the lowest and smoothest variance figure, which Model II closely replicates.

Figure 7.6 **Forecasted variances of minimum-variance portfolios for Models I-IV**



7.4.2 Exogenously given portfolio weights

As long as asset selection is independent of expected return, ie we are dealing with the global minimum-variance portfolio, instruments with higher volatility will not be included. Such a portfolio is interesting as a special case of investment selection but not a very realistic choice. The selected risk exposure with no exchange rate risk can be realized only by hedging the exchange rate risk inherent in money market instruments, for example using derivatives.

In the second portfolio composition application, we choose a more realistic approach by selecting portfolio weights according to the average composition of Finnish banks' trading portfolios, as reported to the supervisory body. The average weights are 40 per cent for each of the foreign short-term money market instruments, which then include both interest rate risk and foreign exchange rate risk, 15 per cent for the Finnish long-term instrument and 5 per cent for Finnish equities. If the banks base their investment shares in the trading portfolio on efficient mean-variance portfolio strategy, the fact that equities are included (albeit with a very small share) means that banks have implicit expectations of high returns on these securities.

In Figure 7.7 the portfolio variances for Models I–IV are displayed. By definition, the variances calculated for the exogenously weighted portfolios are higher than the variances of the global minimum-variance portfolios. Table 7.5 gives the numerical values of the monthly variances for both types of portfolio for the four models.

Comparison of the variances of the two portfolios shows that the variances of global minimum-variance portfolios are on average only one-hundredth the size of the variances of the exogenously weighted portfolio.

Compared to calculated variances of the minimum-variance portfolio, the distinction between the GARCH models and the random walk models is strengthened with the exogenously weighted portfolio. The variances of Models I and II are very close to each other as are the variances of Models III and IV, and there is a notable difference in the variances for the two pairs.

Table 7.6 **Forecasted portfolio variances for Models I–IV**

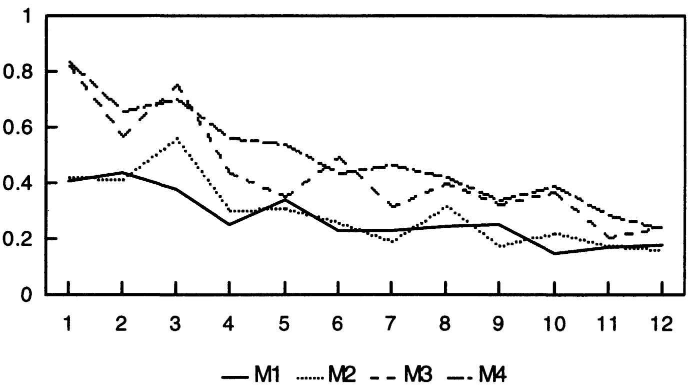
Global minimum-variance portfolio

Month	M I	M II	M III	M IV
Jan	0.00047	0.00058	0.00071	0.00032
Feb	0.00059	0.00066	0.00031	0.00042
Mar	0.00145	0.00120	0.00153	0.00089
Apr	0.00074	0.00100	0.00446	0.00132
May	0.00273	0.00124	0.00145	0.00162
June	0.00168	0.00263	0.00434	0.00252
July	0.00190	0.00660	0.00096	0.00296
Aug	0.00264	0.00155	0.00326	0.00335
Sept	0.00215	0.00310	0.00470	0.00312
Oct	0.00219	0.00165	0.00264	0.00372
Nov	0.00150	0.00187	0.00289	0.00335
Dec	0.00141	0.00122	0.00166	0.00214

Exogenously weighted portfolio

Month	M I	M II	M III	M IV
Jan	0.40687	0.41781	0.82418	0.83937
Feb	0.43293	0.41107	0.56961	0.65720
Mar	0.37408	0.55907	0.75762	0.70136
Apr	0.25327	0.30224	0.43841	0.56519
May	0.34174	0.30745	0.35639	0.53727
June	0.23070	0.25852	0.49192	0.43230
July	0.22828	0.18858	0.31619	0.46371
Aug	0.23947	0.32014	0.39982	0.42374
Sept	0.24685	0.17772	0.32360	0.33569
Oct	0.14630	0.21754	0.36994	0.39225
Nov	0.17128	0.17196	0.20594	0.28739
Dec	0.17520	0.15689	0.24547	0.23419

Figure 7.7 **Forecasted variances for Models I–IV with exogenously given weights**



7.5 A value-at-risk application

The results derived in this study on conditional variance estimation and forecasting are of general interest and can be applied in different areas of financial economics in which an appropriate estimation of variance as a measure of risk is crucial. Such an application is the value-at-risk (VaR) model, which has recently become popular among corporations and financial institutions and has even been accepted by regulators as a valuable risk assessment tool. In the following, we clarify how the findings of this study can be used to improve the methodological basis for VaR modelling.

The VaR framework is a statistical method for measuring and forecasting, at a certain confidence interval, the maximum amount of value that can be lost or gained in a portfolio due to changes in market prices of underlying assets.[58] Probability statements generating the confidence intervals are derived using estimated variances to measure expected changes either in the return series for instruments in the portfolio or in the return on the portfolio as a whole. Although value-at-risk models can be used in assessing credit risk,[59] the main area of

[58] There is an extensive literature on the use of VaR models. Simons (1996) provides a good description of the basic methodology and applications.

[59] The best known credit risk management tools are CreditMetrix and CreditManager developed by J.P. Morgan and Credit Monitor by KMV Corp. See also Oda and Muraga (1997) for a theoretical framework.

application is in evaluating market risk. Thus one needs estimates of variances of changes in exchange rates, interest rates and share prices.

As there is no generally accepted way of calculating the variance, there are a number of VaR models in use. Each yields results that mirror the underlying assumptions and methodological approach. There are three main approaches represented in the commercial risk management tool packages: the analytical, historical and simulation approaches.[60]

The analytical VaR (also known as the parametric or correlation method) is based on estimation of the variance-covariance matrix for historical returns on the individual instruments in the portfolio. By multiplying the vector of porfolio composition in the instruments by the variance-covariance matrix, we end up with a figure (expressed in the selected currency) that gives the maximum amount that can be lost or gained on the portfolio with a given probability. Based on the normality assumption, using multiples of one standard deviation of return gives the corresponding normal probability levels.

With the historical approach to VaR methodology, the distribution of portfolio returns is constructed from a series of changes in portfolio values generated by inserting in the portfolio the market prices of the individual instruments over a selected historical time period. From the distribution of the calculated portfolio returns, ie changes in the market value of the portfolio, corresponding to the selected market values of the instruments, the historical portfolio profit or loss can be calculated, again at a certain confidence level. The confidence level can be given in nonparametric form by ranking the return values of the portfolio from smallest to largest and then selecting the percentiles of interest. The alternative parametric form of the confidence statements requires the assumption of normality of the portfolio returns.

The historical method differs from the analytical in several ways. The most noticable difference in the assumptions is that the analytical approach assumes a normal distribution of returns on the individual instruments, which of course also leads to normality of the portfolio return, while the historical approach requires no explicit assumptions regarding variances of returns on the instruments or covariances between them,[61] nor does it make assumptions about the shape of the distribution itself (in the nonparametric version).

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[60] Examples are J.P. Morgan's Riskmetrics, Bankers Trust's RAROC2020 and Chase's RISK$.

[61] The covariance structure is implicitly included in the market prices used as input in pricing of the portfolio.

In the third type of methodology used in VaR models, the simulation approach, market prices generated randomly by numerous Monte Carlo simulations are used instead of historical prices to construct the empirical distribution of portfolio returns.

Most applications developed for internal use by corporations and financial institutions are essentially variations on these three main approaches. The BIS proposal[62] and the Capital Adequacy Directive (CAD) adopted by the European Commission impose a standard method for calulating market risk capital requirements, which can best be characterized as being representative of the analytical method.

Much effort has gone into comparison of portfolio VaR results from different models. Regulators have also participated in comparisons between models in an attempt to define standards for assessing capital positions based on financial institutions' internal VaR model calculations. Overall findings[63] indicate that VaR calculations differ significantly for the same portfolio because they are highly sensitive not only to the selected methodology but also to the assumptions made in parameterizing the data. All of the commonly used models make some unrealistic or simplifying assumptions, generally about the mean and variance of the return series frequency distribution, which contradict empirical realizations of financial time series and thus affect the outcome of the model calculations. The main assumptions are serial independence and normal distribution of returns on instruments or portfolios. In empirical calculations of the sample variance of the commonly used normally distributed random walk process, the selection of the sample time horizon and weight structure in variance estimation and forecasting are issues on which there is no widespread agreement as to parameterization.

For the user, the selection of the specific VaR methodological approach should be based on the purpose of this risk management tool. Each approach has its strengths and weaknesses, which must be weighed against each other in light of the purpose of the model. The purpose can relate to the size of the company and thus the complexity of the portfolio to be evaluated, the length of the evaluation horizon, or the evaluation frequency.

While the selection of the methodological approach to VaR calculations is dictated by the purpose of the intended use, it is possible to

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62 Basle Committee on Banking Supervision (1995), Planned Supplement to the Capital Accord to Intercorporate Market Risks, Basle.

63 See eg Beder 1995, Heindricks 1995, Liu 1996, Jorion 1996.

reduce the actual variability in parameterization between models by using estimation methods in which the parameters are chosen by the data itself.

The results derived in this study in applying generalized autoregressive heteroscedastic methodology to estimate conditional variances of financial return series are used to solve the problems of serial dependence, nomal distribution, sample length and decay factors in the VaR framework. With GARCH parameterization, we account for non-normalities in the raw return series and end up with standardized stochastic processes, which, by definition, are normal and empirically at least much closer to normal than the raw data on which normally distributed random walk processes are applied in current VaR applications. The assumption of linear dependence is met by using AR-filtered return series in the mean equation in estimating the conditional variance, when necessary.

The GARCH interpretation also makes it possible to forecast expected future variances in a more methodology-based way than with the current method of extrapolating historical sample variances. Based on the parameter structure of the estimated conditional variances, one can derive a theoretically and empirically based formula for selecting the sample period and the decay factors giving the weight vector for estimation of future conditional variances and covariances. This common parameterization technique can be used with any model.

The GARCH estimation in this study produced the following formula for forecasting one-step-ahead variance of returns on exchange rate, interest rate and equity instruments:

$$h_{t+1} = 0.08\varepsilon_t^2 + 0.08(0.92)\varepsilon_{t-1}^2 + 0.08(0.92)^2\varepsilon_{t-2}^2$$
$$+ 0.08(0.92)^3\varepsilon_{t-3}^2 + \ldots + 0.08(0.92)^{n-1}\varepsilon_{t-27}^2 .$$

The formula implies that the one-step ahead variance forecast ought to be calculated as a weighted moving average of past squared residuals. We end up with a lag number of 28 and the weights given in Table 6.2. In forecasting, the sample period is thus truncated to the 28 most recent observations[64] with a decay factor in the weight structure of $(1-\alpha_1)$ = 0.92.

In some VaR applications a weighted lag structure has been imposed on the historical observations in the sample, as an alternative to equally

[64] Note that although the forecasts are calculated on the relatively short time interval of 28 recent observations, the decay factor, which determines the lag length, was estimated over long periods of data and found to be similar regardless of the exchange rate regime.

weighted observations, without solid justification, although it is clear that different weight structures generate significant differences in the resulting volatility estimates. J.P. Morgan's RiskMetrics uses a decay factor of 0.94 for daily volatilities of all instruments to account for volatility clustering. Simons (1996) simulates the effects of a decay factor ranging between 0.94 and 0.97. These simulations however give only the sensitivity of the weight structure with respect to the volatility measure. They do not solve the problem of selecting the 'correct' stucture.

Our derived decay factor, 0.92, is very close to that of the RiskMetrics, 0.94, thus lending methodological and empirical support to the more or less ad hoc choice of J.P. Morgan. RiskMetrics has also been criticized for applying the same decay factor to all instruments. However, our results support the use of the same empirically identified lag structure and decay factor for most instruments. A uniform lag structure and decay factor are adequate for daily variances of exchange rates, short-term interest rates and the all-share index, all of which are modelled as an integrated process. The variance model for the long-term rate is strongly mean reverting, and this leads to a corresponding formula with only three lags of squared residuals.

To illustrate the practical VaR application, the results derived in this study for individual instruments are aggregated into the two types of hypothetical portfolios constructed in earlier chapters. One is the global minimum-variance portfolio and the other is the portfolio with weights roughly mirroring the average composition of Finnish banks' trading portfolios. To assess the VaR figures, we use monthly portfolio variances for the ex post period, January–December 1996, to calculate the monthly VaR figures for the two portfolios. We do not measure the effects of the selection of the methodological approach (analytical, historical and simulation), since we apply only the analytical framework, but the effect of GARCH models vs sample variance models can be evaluated by comparing Models I and II to Models III and IV. The impact of the window length can be evaluated by comparing the outcomes of Models III and IV.[65]

The relevant VaR figures for the illustrative portfolios are derived from the estimated variances of Models I–IV (presented in Table 7.6) as

$$\pm\alpha\sqrt{\hat{\sigma}_p^2},$$

[65] See Jackson, Maude and Perraudin (1995) for a comparison between different models of the effects of alternative window lengths, weighting schemes and holding periods.

where α indicates the normal confidence level and $\hat{\sigma}_p^2$ the portfolio variance. The monthly figures with $\alpha = 1.96$ giving the 95 per cent probability are presented in Figure 7.8. The corresponding average values of the twelve monthly figures are given in Table 7.7, both on monthly and annual bases.

The VaR interpretation of the figures in Table 7.7 is that for the global minimum-variance portfolio Model I it can be stated that with a probability of 95 per cent the portfolio profit will be in the interval ±0.08 per cent on a monthly level. If we focus on the loss tail of the distribution, we can say that the probability of a loss that is greater than 0.08 per cent of the portfolio value is 2.5 per cent. The corresponding 95 per cent probability interval for Model I with the exogenously weighted portfolio is ±1 per cent. The difference in levels of variance as between the two portfolios, which was found to be a factor of 100, is thus 10 for standard deviations.

The message from the outcomes of the VaR calculations is the same for both types of portfolios. Models I and II using the GARCH results for the window length of 28 recent observations with a weight structure based on a decay factor of 0.92 show lower forecasted maximum losses and profits than the equally weighted sample variance-based Models III and IV. The differences in variance level between the two groups of models is more pronounced for the exogenously weighted portfolio.

The outcome of the comparison of effects of lag length and weight structure is of course valid only for the portfolios on which the comparison is applied, and the results cannot be generalized. What is achieved, however, is a methodological and empirical basis for the data parameterization.

7.5.1 Practical cost-benefit views

We have shown in this study that estimating and forecasting the variance of individual financial return series as well as portfolios in a GARCH framework is superior to a sample variance-based procedure, both from a methodological viewpoint and in forecasting accuracy. These advantages have to be weighed against the required investments in personnel skills for implementing VaR models in practice.

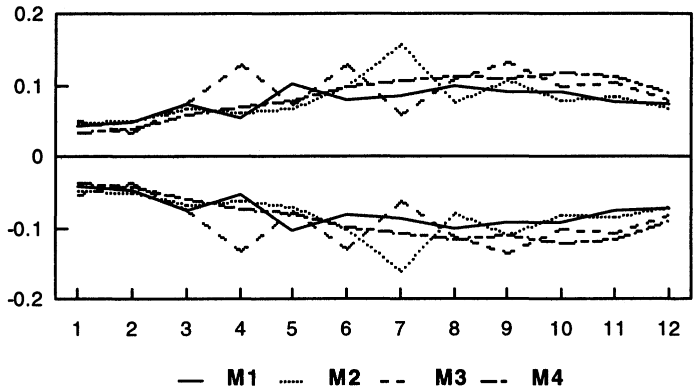
The estimation of covariances has been the main obstacle to the development of variance-based risk measuring models with more than a few instruments within the GARCH framework. We have in this study presented two flexible ways to identify covariances once the individual variances have been estimated. The development of sophisticated software has also made it possible to swiftly perform a GARCH estimation for a large number of series. This extends the possibility of using GARCH models beyond the academic world. Compared to the use of commercial packages or internal sample variance-based models, a GARCH model requires more knowledge on the part of the users, especially in the re-estimation of the model, than just the ability to push a button. It is therefore important to compare the differences in outcomes between the sophisticated conditional variance models and the more easily applicable unconditional variance models and then to consider the corresponding costs.

What we have shown, and others before us, is that there is no justification for the use of equally weighted sample variance models on high frequency data for measurement and forecasting of the variance of financial return series nor consequently of portfolio returns. The sample variance does not capture the stylized facts, and the forecasts perform poorly.

However, our results show that the GARCH forecasting formula estimated in this study is very similar to the exponentially weighted formula used in RiskMetrix, which is widely used and forms a benchmark for VaR applications. We ended up with a lag length of 28 days and a decay factor of 0.92 while RiskMetrix uses a lag length of 25 days and a decay factor of 0.94. RiskMetrix uses this structure for all instruments. According to our result, the same formula is valid for forecasting the variance of exchange rates, short interest rates and the stock market index but not for the long-term rate. The purpose of a VaR model is to provide an estimate of the risk due to short-term fluctuations in the value of investments in financial instruments, the main sources of which are exchange rates and short-term interest rates. The results found in this study therefore provide methodological basis for the use of RiskMetrix for forecasting the variances of the main financial series. This is beneficial to frequent users since continuous updating of the variances and covariances along with the data is provided by the developer, J.P. Morgan. Continuous checking of the RiskMetrix formula is however necessary, since re-estimation of the GARCH model may change the parameter values.

Figure 7.8 **Confidence intervals (95 %) for monthly forecasted profits for the two types of portfolios for Models I–IV, %**

Global minimum-variance portfolio



Exogenously weighted portfolio

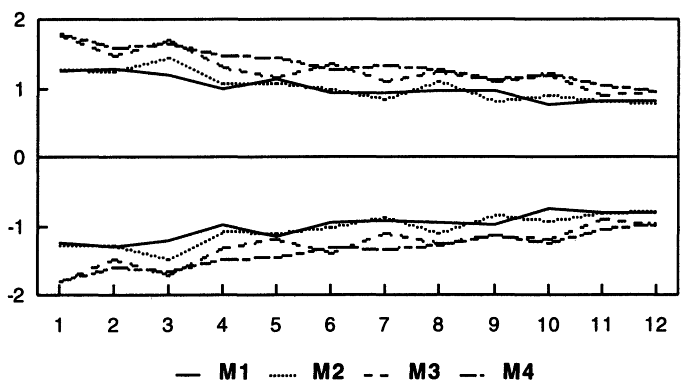


Table 7.7 **Average forecasted maximum loss or profit at 95 % confidence for the two types of portfolios for Models I–IV, %**

Global minimum-variance portfolio

	Monthly	Annual
Model I	0.0766	0.2654
Model II	0.0813	0.2816
Model III	0.0909	0.3149
Model IV	0.0863	0.2990

Exogenously weighted portfolio

	Monthly	Annual
Model I	1.0052	3.4821
Model II	1.0366	3.5909
Model III	1.2764	4.4216
Model IV	1.3498	4.6758

8 Conclusions

Increased volatility of financial asset prices has underscored the importance of having an adequate measure of variance for the purpose of evaluating the risk in portfolio investments. Variance-based methods of measuring portfolio risk include VaR applications that have been either developed internally by financial institutions or commercially distributed.

In this study GARCH modelling is used to measure the time dependence, ie clustering in variance, that is typically found in most high frequency financial return data. We manage to show that, in constrast to the commonly used homoscedastic method, this heteroscedastic method not only adequately captures the stylized facts but also produces a solution based on statistical inference to the sample length and weight structure in the forecast formula. The related parameters in the commercial VaR models are generally selected on a more or less ad hoc basis.

In the following, the methodological approach and main empirical findings of this study are summarized. We then conclude the paper with some remarks on possible further research.

8.1 Methodological summary

In the variance-based approach used in this study to evaluate measurable risks relevant in banking, we apply GARCH(1,1) modelling of the variance of returns on exchange rate instruments, short- and long-term interest rate instruments, and equity and real estate investments.

With the GARCH model, both the mean and the variance are estimated. In defining the mean equation, a unit root in levels was found for all rates and thus differencing once was sufficient to achieve stationarity. Changes in short-term interest rates and in the long-term rate exhibit high-order linear dependence. The well documented one-lag autocorrelation in changes in the general stock market index is also found in this study. The linear dependence is accounted for by prefiltering the data when necessary. For all the return series, we ended up with a mean equation without drift.

The parsimonious GARCH(1,1) model for conditional variances was found to be adequate for practically all the rates. The results of the real estate return volatility with the GARCH(1,1) model were greatly improved by including sales volume as an exogenous variable in the variance equation, in what we call a GARCH-X(1,1) model. The theory of a time-varying risk premium was tested with the GARCH-M(1,1) model but without success for any risk category.

Principal component analysis on the estimated univariate conditional variances was performed for each period within the groups of exchange rates and interest rates for two reasons: first, to measure and compare uniformity of behaviour for the individual rates on a wider scale than allowed for in pairwise comparison and, secondly, to detect common factors driving the rates. The low degree of homogeneity found within the groups of exchange rates and interest rates did not however support the use of a common factor-based multivariate approach.

Spectral analysis was then performed in order to measure cyclical regularity in the conditional variances and principal components. In the spectral density figures, local peak values were found on average for a period of 180 days and its harmonics for the band periods, both for exchange rates and interest rates. The power spectrum for all periods (most strongly for the floating rate period) for both groups of rates however confirmed the estimation results, indicating an integrated process.

The results of the univariate GARCH(1,1) estimation for exchange rates and short-term interest rates for the band and floating rate periods showed that there was much similiarity in the estimated parameter values within the groups of rates. Therefore GARCH models were estimated with pooled data, forcing the conditional variances within the group of exchange rates and within the group of interest rates into a single model for each group. The estimated models for the pooled data were found to be integrated in variance for both groups. The surprising results were that the parameter structure was independent of the exchange rate regime and that we ended up with almost the same parameter values in models estimated on pooled data for both exchange rates and interest rates.

The estimated GARCH(1,1) model for the long-term interest rate was dependent on the period. For the floating rate period, the persistence was considerable lower than for the band period, thus implying a strong mean-reverting process in the time path of the conditional variance. The all-share index showed low persitence for the band period but a nearly integrated process for the floating rate period.

BDS test statistics were applied to the standardized GARCH residuals to test for model misspecification. For all rates the applied GARCH(1,1) model produced lower test values compared to the raw data but as a rule evidence was found for some remaining deviation from IID. This result coincides with results of other empirical GARCH studies, and the causes have been sought in the presence of single outliers or in the nonnormality of the return distribution.

The large number of variables, both within groups of rates and even more so for all rates taken together, did not allow for the use of the general multivariate GARCH estimation to assess covariances in the system. The problem of estimating the significant covariances was handled in two restricted multivariate ways: first, by assuming the same parameter structure for covariances as was estimated univariately for the variances and, secondly, by assuming constant correlation between standardized residuals. In the first method, which was developed in this study, the dependence between the autocorrelation structures of variances and covariances was tested using the Kendall W test. Based on the outcome of this test and other dependence measures, the null of independence could be rejected within both groups of rates for all periods. The same estimated parameter structure was therefore used in forecasting variances and covariances.

In the second method, a two-stage extention of the Bollerslev method, the co-movements between groups were measured with a correlation matix for the three periods, including all twenty-seven daily financial rates under discussion. The CUSUM test had as a rule no power to reject the null of constant correlation within periods. Consequently, our version of the Bollerslev method was applicable for covariance estimation both within groups and between groups. Compared to the general multivariate analysis, the two methods applied here are inefficient since the estimation is done in two stages, but apparently these methods are the only alternatives with such a large number of variables.

After identification of the mean and variance-covariance generating GARCH processes, the results were used for forecasting. The GARCH model itself is a formula for a one-step-ahead forecast of the variance of the residuals. This interpretation can be expressed in the form of a geometric weighted average of past squared residuals to be used for ex post and ex ante forecasting.

According to our results, the same parameters for the integrated model could be used in the autoregressive formula with twenty-eight lags for forecasting the variances and covariances of exchange rates, short-interest rates and the all-share index. The small number of observations in the formula expresses the rapidly changing nature of the volatility, where the importance of information from past data dies out quickly. Although the forecasts are calculated from this relatively short period, the decay factor, which determines the lag length, is estimated on long periods of data and was found to be similar within structurally homogeneous periods for these rates. The mean-reverting behaviour of the conditional variance of the long-term rate was forecasted with a lower order autoregressive model, consisting of only three lags. To assess the forecasting ability, the GARCH generated forecasts were compared to the forecasts from homoscedastic models with different window lengths and equally weighted past squared residuals. The evaluation was perfomed by regressing individual model forecasts on actual values during the ex post period, using a log likelihood-based loss function. The outcome of the forecasting evaluation turned out to be dependent on the class of asset considered. The evaluation however showed unambiguously that as the use of the sample variance as a forecast for future variances lacks methodological relevance, it produces poor empirical results.

The next step was to combine the individual risk areas into a portfolio. We formed a fictive portfolio covering the main market risks, ie exchange rate risk, short- and long-term interest rate risk and equity risk as well as the covariances between them. The weights in the portfolio were determined in two ways: first, we used the equlibrium weights from the minimum-variance optimization solution and, secondly, we used the average weights in the trading portfolios found in the historical data reported to the supervisory body by the Finnish banks. These types of portfolios were selected since they do not require any information on the return of the portfolio. For the two types of portfolios, we evaluated the ex post forecasting. The main outcome of the comparison was that heteroscedastic GARCH models produce, at least for the forecasting period considered, clearly lower estimates of the variances than homoscedastic models.

8.2 Findings and suggestions for further research

The studies in existing literature on GARCH modelling generally cover equity or exchange rates and include only a few rates. One important contribution of this study is that the modelling is extended to a large number of return series for investments in different markets. The variances of the returns on investments in exchange rate instruments, short- and long-term interest rate instruments, equity and real estate – altogether twenty-seven return series – are analysed in a multivariate framework. The inclusion of several markets with a variety of rates means that not only the covariances within markets but also between markets can be analysed. The main finding of the study is that the estimated variance-covariance models display a high degree of similarity both across variables and across subsamples (ie across exchange rate regimes); the parsimonious GARCH(1,1) model seems to represent the underlying conditional variance process fairly well. In terms of the degree of persistence in the variance, the estimated models are also remarkably similar, ie near an IGARCH(1,1). Hence the same forecasting formula with identical parameter values for an integrated process can be used for forecasting the variance of changes in exchange rates, interest rates and equity prices. Due to the vast coverage of rates and the selected multivariate methods and even more to the identified similar parameter structure between markets and regimes, the results can be generalized to exchange rates and interest rates other than those covered in this study. In forecasting the variance of the return of portfolios, we ended up with a flexible system in which new instruments can be included or old ones removed without the need for re-estimation or new estimation (the only exception being the estimation of the correlation coefficient for new instruments if the Bollerslev method is used).

The findings can be used as input to the discussion on the accuracy of VaR models as portfolio risk assessment tools. In our portfolio risk comparison of GARCH models and fixed window models, which VaR models commonly implement, we show that statistically incorrect homoscedastic models overstate the portfolio variance. This has important consequences when the capital requirements for financial institutions' market portfolios are calculated using internal VaR models.

What we have shown, as a matter of fact, is that the theoretically adequate heteroscedastic variance model IGARCH(1,1) seems to represent the underlying process in the main financial return series remarkably well[66] and that this model is at least as easy to apply as the homoscedastic model in VaR applications.

The volume of research on ARCH methodology is growing rapidly and new areas of implementation are steadily surfacing. Some of the fields in which development is under way are identification of the nonnormal distribution, time-varying parameter models and unification of SV and ARCH methodologies. Cointegration and long memory are also noteworthy topics in recent developments in time series analysis that can be integrated with ARCH modelling.[67]

We feel that univariate analysis of individual assets or risk categories will benefit from futher reasearch above all on deepening and refining methods for modelling time-varying volatility. However, as studies

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[66] Although we have analysed markka exchange rates and the Finnish stock index, the results should be valid also for other markets. Sheedy (1997) has shown that the multivariate GARCH(1,1) results for a group of four exchange rates and a group of four stock indices are essentially the same, regardless of the currency of denomination.

[67] See Alexakis and Apergis (1996) for an example of ARCH effects and cointegration in the foreign exchange market.

already show, the more complex estimated GARCH models differ as between assets. This complicates the calculation of covariances and thus the handling of portfolios, which in order to be realistic include more than just a few assets and risk categories. We have shown in this work that the GARCH(1,1) model is a good parsimonious approximation for a large set of return series from the main ares of risk in market portfolios. For portfolios larger than the bivariate case, the simple GARCH model, as modified in this study, will likely be operational to the extent that it will be difficult to outperform or replace.

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Appendix 1

List of variables

Foreign exchange rates		Three-month interest rates
USD	US dollar	ERUSD
GBP	Pound sterling	ERGBP
SEK	Swedish krona	ERSEK
NOK	Norwegian krone	ERNOK
DKK	Danish krone	ERDKK
DEM	Deutschemark	ERDEM
NLG	Dutch guilder	ERNLG
BEF	Belgian/Luxembourg franc	ERBEC
CHF	Swiss franc	ERCHF
FRF	French franc	ERFRF
ITL	Italian lira	ERITL
JPY	Japanese yen	ERJPY

Domestic rates	
ERFIM	three-month interest rate
ERlong	three-year long rate
HEX	all-share index

Appendix 2

Correlation coefficients and CUSUM test statistics

Correlation matrix for the floating rate period

	USD	GBP	SEK	NOK	DKK	DEM	NLG	BEC	CHF	FRF	ITL	JPY	EKUSD	EKGBP	EKSEK	EKNOK	EKDKK	EKDEM	EKNLG	EKBEC	EKCHF	EKFRF	EKITL	EKJPY	EKFIM	FIML	HEX
USD	1.000																										
GBP	0.583	1.000																									
SEK	-0.003	0.019	1.000																								
NOK	-0.074	-0.043	0.104	1.000																							
DKK	0.291	0.232	-0.044	-0.013	1.000																						
DEM	0.435	0.628	0.009	0.030	0.220	1.000																					
NLG	-0.012	0.045	0.050	0.053	-0.159	0.131	1.000																				
BEC	-0.039	0.026	0.113	0.074	-0.122	0.097	0.261	1.000																			
CHF	0.353	0.487	0.010	0.053	0.187	0.786	0.131	0.094	1.000																		
FRF	0.324	0.513	0.015	0.039	0.081	0.837	0.196	0.185	0.789	1.000																	
ITL	0.016	0.018	0.057	0.020	-0.084	0.128	0.106	0.140	0.124	0.187	1.000																
JPY	0.023	0.028	0.112	0.156	-0.029	0.038	0.098	0.145	0.025	0.037	0.068	1.000															
EKUSD	-0.014	-0.047	0.065	0.055	-0.008	-0.012	0.037	0.021	-0.002	0.005	0.012	0.024	1.000														
EKGBP	-0.071	-0.027	0.000	0.011	-0.040	-0.017	0.023	0.041	0.003	0.013	0.009	0.025	0.108	1.000													
EKSEK	0.012	0.017	0.000	-0.023	-0.019	0.029	-0.109	0.017	-0.007	0.033	0.017	0.061	0.035	0.043	1.000												
EKNOK	0.011	0.051	-0.033	0.021	0.032	0.063	0.022	0.081	0.074	0.111	-0.029	0.029	0.115	0.086	0.272	1.000											
EKDKK	-0.003	-0.012	0.010	0.016	-0.161	0.057	0.035	0.074	-0.015	0.087	0.050	0.022	0.042	0.032	0.109	0.132	1.000										
EKDEM	-0.041	-0.002	0.038	0.043	0.021	-0.016	-0.080	-0.049	0.017	-0.033	-0.014	0.026	0.103	0.151	0.079	0.145	0.066	1.000									
EKNLG	0.023	0.066	0.030	0.002	-0.013	0.057	-0.016	-0.038	0.067	0.064	0.050	0.139	0.135	0.106	0.100	0.140	0.064	0.518	1.000								
EKBEC	0.001	-0.042	0.026	0.049	0.000	0.035	0.041	-0.001	-0.004	0.049	0.056	0.050	-0.011	0.151	0.286	0.267	0.207	0.211	0.231	1.000							
EKCHF	0.007	0.009	0.075	0.044	-0.044	-0.049	-0.007	-0.011	-0.041	-0.065	0.010	0.078	0.134	0.140	-0.001	0.025	0.030	0.326	0.286	0.077	1.000						
EKFRF	-0.066	-0.023	0.030	0.027	-0.090	0.036	0.108	0.085	0.030	0.102	0.108	0.086	0.136	0.135	0.129	0.163	0.342	0.226	0.170	0.275	0.079	1.000					
EKITL	-0.059	-0.001	0.019	-0.009	-0.119	0.033	0.179	0.097	0.039	0.080	0.041	0.048	0.091	0.179	0.151	0.138	0.212	0.179	0.161	0.153	0.139	0.385	1.000				
EKJPY	-0.041	-0.034	0.054	0.046	-0.004	-0.044	-0.012	0.003	-0.033	-0.040	0.045	0.022	0.068	0.006	-0.019	0.013	0.049	0.133	0.053	0.000	0.064	-0.008	0.013	1.000			
EKFIM	0.032	0.038	-0.001	0.046	-0.023	0.060	0.023	-0.088	0.039	0.047	-0.041	-0.003	0.096	0.036	0.212	0.163	0.126	0.096	0.061	0.197	0.074	0.124	0.110	0.070	1.000		
FIML	0.002	0.069	0.046	0.026	-0.099	0.134	0.230	0.069	0.113	0.186	0.025	0.030	0.139	0.044	0.129	0.170	0.155	0.009	0.042	0.126	0.005	0.149	0.213	0.027	0.330	1.000	
HEX	0.004	-0.012	0.015	-0.029	-0.031	-0.060	0.007	-0.032	-0.055	-0.057	-0.022	-0.039	0.037	0.001	-0.080	-0.054	-0.020	0.008	0.027	-0.035	0.075	-0.013	0.019	0.027	-0.033	-0.066	1.000

CUSUM test statistic for the floating rate period

	USD	GBP	SEK	NOK	DKK	DEM	NLG	BEC	CHF	FRF	ITL	JPY	EKUSD	EKGBP	EKSEK	EKNOK	EKDKK	EKDEM	EKNLG	EKBEC	EKCHF	EKFRF	EKITL	EKJPY	EKFIM	FIML	HEX
USD																											
GBP	0.531																										
SEK	0.622	1.113																									
NOK	0.550	1.059	0.916																								
DKK	0.555	1.186	0.939	0.555																							
DEM	0.546	1.175	0.968	0.563	0.636																						
NLG	0.522	1.143	0.964	0.522	0.658	0.469																					
BEC	0.632	1.075	0.921	0.574	0.609	0.622	0.631																				
CHF	0.553	1.140	0.957	0.530	0.615	0.421	0.514	0.845																			
FRF	0.600	1.141	0.958	0.557	0.448	0.437	0.745	1.048	0.450																		
ITL	0.553	0.942	0.841	0.400	0.655	0.484	0.579	0.968	0.405	0.983																	
JPY	0.572	1.166	0.961	0.582	0.649	0.387	0.607	0.788	0.413	0.867	1.169																
EKUSD	0.519	1.124	0.891	0.523	0.725	0.500	0.597	0.907	0.449	0.886	1.135	0.524															
EKGBP	0.583	1.088	0.924	0.562	0.619	0.485	0.709	1.002	0.438	0.858	1.115	0.610	0.834														
EKSEK	0.514	1.038	0.930	0.506	0.580	0.370	0.643	0.798	0.463	0.855	1.152	0.475	0.824	1.038													
EKNOK	0.522	1.121	0.967	0.518	0.686	0.473	0.746	0.996	0.497	0.885	1.149	0.415	0.757	1.025	0.724												
EKDKK	0.514	1.151	0.987	0.565	0.684	0.377	0.756	0.960	0.421	0.855	1.160	0.500	0.732	0.955	0.718	0.499											
EKDEM	0.531	1.093	0.965	0.616	0.703	0.449	0.738	0.944	0.459	0.872	1.140	0.537	0.784	0.990	0.730	0.503	0.998										
EKNLG	0.575	1.102	0.944	0.613	0.630	0.448	0.718	0.881	0.441	0.843	1.186	0.522	0.824	0.914	0.782	0.492	0.990	0.392									
EKBEC	0.563	1.166	1.060	0.579	0.692	0.355	0.628	0.805	0.463	0.903	1.142	0.441	0.767	1.073	0.654	0.516	1.006	0.360	0.623								
EKCHF	0.537	1.147	0.937	0.513	0.624	0.392	0.624	0.823	0.456	0.854	1.172	0.416	0.809	1.062	0.613	0.534	0.997	0.407	0.574	0.543							
EKFRF	0.579	1.102	0.965	0.662	0.619	0.460	0.804	1.039	0.465	0.809	1.185	0.610	0.813	0.776	1.021	0.508	1.022	0.366	0.881	0.538	0.424						
EKITL	0.558	1.101	0.967	0.659	0.630	0.464	0.781	0.986	0.442	0.811	1.196	0.580	0.834	0.762	0.884	0.494	0.992	0.352	0.687	0.584	0.426	0.761					
EKJPY	0.530	1.105	0.971	0.564	0.597	0.581	0.765	0.980	0.513	0.824	1.209	0.568	0.768	0.985	0.695	0.511	1.005	0.344	0.420	0.521	0.475	0.727	0.725				
EKFIM	0.560	1.114	0.978	0.551	0.692	0.397	0.722	1.003	0.467	0.766	1.161	0.523	0.808	1.003	0.738	0.498	1.021	0.358	0.545	0.501	0.466	0.737	0.711	0.547			
FIML	0.602	1.129	0.896	0.525	0.735	0.382	0.574	0.724	0.433	0.850	1.202	0.457	0.712	1.013	0.554	0.545	0.925	0.386	0.612	0.525	0.407	0.938	0.907	0.522	0.421		
HEX	0.424	1.169	0.848	0.366	0.752	0.419	0.432	0.647	0.468	0.928	1.119	0.346	0.474	1.101	0.720	0.567	1.111	0.328	0.730	0.490	0.732	1.001	1.068	0.536	0.423	1.050	

The critical values are
1.143 for 1 % significance
0.948 for 5 % significance
0.850 for 10 % significance

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