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DEVALUATION EXPECTATIONS AND SPECULATIVE ATTACKS ON THE CURRENCY

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CONTENTS page

1 INTRODUCTION

There is general agreement that a regime of fixed exchange rates can be sustained only if domestic economic policy is consistent with it. An excessively expansive monetary policy, for instance, leads to the depletion of foreign reserves, eventually forcing the central bank to either devalue the currency or allow it to float. However, speculative attacks on the currency have typically been preceded by a change in the foreign exchange target.

It has been shown in the literature on balance-of-payments crises that speculative attacks on the currency are not in contradiction with the assumption of rational behaviour and that the timing of such attacks is foreseeable.[1] These results, however, are derived under the assumption that there is some binding threshold level of foreign reserves, known by everyone, below which foreign reserves are not allowed to be depleted. As long as foreign reserves are above that level the central bank adheres with certainty to its fixed exchange rate target but after reserves have been depleted to the critical threshold level the central bank either devalues the currency or allows it to float.

One can, however, doubt whether any such binding minimum level of foreign reserves exists. A central bank facing a perfect capital market can, at least in principle, create foreign reserves by borrowing. Thus negative foreign reserves are also feasible.[2] However, even if such a binding threshold level of foreign reserves did exist and was known by everyone, it is very likely that the exchange rate target would be changed well before reserves had been

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[1]See e.g. Krugman (1979), Flood and Garber (1984), Connolly and Taylor (1984), Grilli (1986), Obstfeld (1984, 1986a,b), Wyplosz (1986) and Buiter (1986).

[2]See the discussion by Obstfeld (1986a).

depleted to the binding minimum level of reseves. This implies that the actual threshold level can be much higher than the binding minimum level.[3] Moreover, its size is not known by the public either.

In this paper speculative behaviour associated with devaluation expectations is studied in a framework in which the threshold level of foreign reserves is unknown to public. Its value is determined in three alternative ways: a new value is drawn from a known probability distribution at the end of each period, a value for the threshold level is drawn from the distribution only once and, as a combination of these two, a new value for the threshold level is drawn with a probability greater than zero but smaller than one at the end of each period. Investors are alternatively risk neutral or risk averters. The foreign exchange risk is one-sided, i.e. there is only risk of a devaluation.

The paper is organized as follows. In section 2 an equation for the domestic interest rate incorporating the risk premium is derived. In section 3 the processes determining the probability of devaluation are defined and in section 4 a simple macromodel is specified. The perfect foresight version of the model is studied in section 5 and in section 6 the speculative behaviour arising from devaluation expectations is studied. Because of the highly nonlinear nature of the model, in the latter section the model is studied utilizing a numerical simulation technique.

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[3]Krugman (1979) and Wyplosz (1986) discuss, in a heuristic way the case in which there is a series of possible threshold levels of foreign reserves with probabilities associated with each of them. The analysis in this paper can be thought of as a formalization and extension of their discussion.

2 THE DETERMINATION OF THE DOMESTIC INTEREST RATE

Assume a perfect international bond market in which domestic and foreign bonds differ only in one respect: their currency denomination. The foreign exchange risk is the only source of uncertainty affecting the portfolio allocation between domestic and foreign bonds. The utility function possessed by investors is assumed to be of the form

$$u(W_{t+1}) = u_0 - u_1 \exp(-aW_{t+1}); \qquad u_0, u_1, a > 0 \tag{1}$$

where W_{t+1} is the non-monetary wealth at the beginning of the period t+1, and a is the measure of absolute risk aversion. The larger is the value of a, the more risk averse investors are. Wealth at the beginning of period t+1 is

$$W_{t+1} = (1+r_t)H_t + (1+r_t^* + \Delta_{t+1})F_t \tag{2}$$

where r_t and r_t^* denote interest rates on domestic and foreign bonds, respectively. H_t is the amount of domestic bonds and F_t the amount of foreign bonds in the portfolio of a domestic investor during period t. Both of them are expressed in domestic currency. Δ_{t+1} denotes the percentage change in the exchange rate between periods t and t+1.[4]

The maximizing problem for expected utility in period t is now

$$\max_{H,F} E_t\{u_0 - u_1 \exp[-a((1+r_t)H_t + (1+r_t^*+\Delta_{t+1})F_t)]\}$$

where E_t refers to expectations formed at the beginning of period t. The first order condition of (4) implies

$$E_t[(1+r_t^*+\Delta_{t+1})\exp(-aW_{t+1})] = E_t[(1+r_t)\exp(-aW_{t+1})] \tag{5}$$

[4]To be precise Δ_{t+1} is $(1+r_t^*)$times the devaluation percentage.

Typically, under a regime of fixed exchange rates, the foreign exchange risk is not symmetric. If foreign reserves are continuously depleting and are expected to do so in the future as well, then devaluation will be much more likely than revaluation. In the following we assume that the probability of revaluation is practically zero and hence we specify

$$\Delta_{t+1} = \begin{cases} \delta \text{ with probability } \pi_t \\ 0 \text{ with probability } 1-\pi_t \end{cases} \tag{6}$$

where $0 \leq \pi_t \leq 1$ and δ is the devaluation percentage, if the central bank decides to devalue.

With perfect information about r_t and r_t^*, equations (5) and (6) imply

$$\begin{aligned} &\pi_t(r_t^*+\delta-r_t)\exp\{-a[(1+r_t)W_t + (r_t^*+\delta-r)F_t]\} = \\ &(1-\pi_t)(r_t-r_t^*)\exp\{-a[(1+r_t)W_t + (r_t^*-r_t)F_t]\} \end{aligned} \tag{7}$$

which gives for F_t

$$F_t = (1/a\delta)\{\log[\pi_t(r_t^*+\delta-r_t)] - \log[(1-\pi_t)(r_t-r_t^*)]\} \tag{8}$$

We see that F_t is determined only with values of r_t such that $r_t^* < r_t < r_t^* + \delta$. This results from the fact that the foreign exchange risk exists only in the direction of devaluation. Independently of how risk averse investors are, if $r_t < r_t^*$ there is no motivation for investors to hold domestic bonds, because the real return on foreign bonds is, with certainty, greater than that on domestic bonds. In the case where $r_t > r_t^* + \delta$ the situation is, of course, reversed. In both of these cases the risk relevant from the point of view of portfolio diversification is eliminated and hence only the values of r_t in the interval $[r_t^*, r^* + \delta]$ are admissible.

For the purposes of the rest of this paper we prefer equation (8) to be normalized with respect to the domestic interest rate. We obtain

$$r_t = r_t^* + \delta\pi_t - Z_t \tag{9}$$

where

$$Z_t = \pi_t\delta[\exp(a\delta F_t)-1]/[\exp(a\delta F_t) + \pi_t/(1-\pi_t)] \tag{10}$$

Z_t is the risk premium. It is easy to see that Z_t can obtain values only in the interval $[0, \delta\pi_t]$. In the risk neutral case ($a \to 0$) $Z_t \to 0$ and hence equation (9) implies uncovered interest parity. We also see that if $\pi_t \to 0$ or $\pi_t \to 1$ then $Z_t \to 0$. Hence, equation (9) also collapses to the equation of uncovered interest parity in the limiting case of perfect foresight (i.e. π_t can only obtain the values 0 or 1).

3 PROBABILITY OF DEVALUATION

In the literature on balance-of-payments crises, it is normally assumed that there is a fixed threshold level of foreign reserves known by everybody below which the central bank does not allow the reserves to be depleted. The attainment of this threshold level implies either devaluation or a permanent shift from a fixed exchange rate regime to a floating rate regime.[5]

We preserve the assumption of a threshold level of foreign reserves. However, to take into account the fact that investors are uncertain about how much of its potential reserves the central bank is willing to use to defend its fixed exchange rate target, we assume that the threshold level of foreign reserves is drawn from a probability distribution. Investors know the probability distribution and its moments but they do not know the size of the threshold level drawn from that distribution. They only know that once the foreign reserves have been depleted below the threshold level the central

[5] Willman (1986) studies balance-of-payments crises by taking into account the possibility that the consistency between the fixed exchange rate target and economic policy is restored through a change in monetary policy.

bank will devalue the currency by δ per cent at the beginning of the next period.

We study the speculative behaviour of investors in three informationally different cases. In the simplest case the central bank makes a draw from the same probability distribution at the end of each period. If the threshold level drawn is greater than or equal to the level of the foreign reserves attained in the period in question, then there is a devaluation of the currency at the beginning of the next period. Otherwise, there is a new draw at the end of the next period. The process continues until the threshold level drawn is below the level of reserves attained.

Assume that the probability distribution from which draws are made is truncated so that the threshold level can obtain values only in the interval $[-\infty, R^u]$. This implies that during period t the probability attached to the occurrence of devaluation at the beginning of period t+1 is

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R^u \\ 1-G(R_t)/G(R^u), & \text{if } -\infty < R_t \leq R^u \end{cases} \tag{11}$$

where G refers to the distribution function and R_t is the level of the foreign reserves in period t. With $R_t < R^u$ we see that the closer R_t is to R^u the closer is π_t to zero, and the further below R^u is R_t the closer π_t is to unity. In the case $R^u \to \infty$ equation (11) reduces to $\pi_t = 1 - G(R_t)$.

In the second case it is assumed that a threshold level is drawn only once (i.e. at the beginning of the game) from a known probability function. With reserves diminishing and without the occurrence of devaluation, this allows investors to learn that the threshold level drawn by the central bank is below the lowest level of the foreign reserves attained up till the beginning of the present period. From the point of view of investors, the situation is same as if, at the end of each period, the central bank made a new draw from a probability distribution in which the truncation point changes with respect to time so that

$$R_t^u = \min(R_{t-1}, R_{t-1}^u) \tag{12}$$

Equation (12) states that, if during the previous period, the foreign reserves have been depleted below the level corresponding to the truncation point at the beginning of the previous period, then in the present period the truncation point equals the level of foreign reserves at the end of the previous period. Otherwise, the truncation point is the same as in the previous period.

The probability that the currency will be devalued at the beginning of the next period is now

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R_t^u \\ 1 - G(R_t)/G(R_t^u), & \text{if } -\infty < R_t \leq R_t^u \end{cases} \tag{13}$$

where R_t^u is determined by (12).

There is an important difference between these two cases. In the first case the threshold level is stochastic and hence depletion of foreign reserves does not give investors any information about the size of the threshold level. In the second case, changes in foreign reserves supply this kind of information. This results from the fact that now investors know that the threshold level is a fixed figure although, as in the first case, its actual size is unknown to them.

Perhaps more realistic than either of these two cases is the case in which investors do not know with certainty if the threshold level adopted by the central bank is fixed or reconsidered at the end of each period. This case is examined more closely in section 6.4.

4 THE MODEL

The following single-good, full-employment small open economy model is specified

$$M_t/p_t = b_0 - b_1 r_t \qquad ; b_0, b_1 > 0 \tag{14}$$

$$M_t = R_t + D_t \tag{15}$$

$$D_t = D_0 + \mu t \qquad ; \mu > 0 \tag{16}$$

$$p_t = p_t^* s_t \tag{17}$$

$$r_t = r_t^* + E_t(s_{t+1}/s_t - 1) - Z_t \tag{18}$$

$$E_t(s_{t+1}/s_t - 1) = \pi_t \delta \qquad ; \delta > 0 \tag{19}$$

$$Z_t = \pi_t \delta[\exp(a\delta F_t) - 1]/[\exp(a\delta F_t) + \pi_t/(1-\pi_t)] \quad ; a \geq 0 \tag{20}$$

$$R_t = R_{t-1} + p_t T_t - (F_t - F_{t-1}) \tag{21}$$

$$T_t = c_0 + c_1[r_t - E_t(p_{t+1}/p_t - 1)] \quad ; \quad c_1 > 0 \tag{22}$$

$$\pi_t = \begin{cases} 0, & \text{if } R_t < R_t^u \\ 1 - G(R_t)/G(R_t^u), & \text{if } -\infty < R_t \leq R_t^u \end{cases} \tag{23}$$

$$R_t^u = R^u \tag{24a}$$

or

$$R_t^u = \min(R_{t-1}, R_{t-1}^u) \tag{24b}$$

where M is the domestic money stock, r the domestic nominal interest rate, p the domestic price level, R the stock of foreign exchange reserves, D domestic credit, p* the foreign price level, s the spot exchange rate, r* the foreign nominal interest rate, Z the risk premium, π the probability that the currency will be devalued at the beginning of the next period. F the stock of foreign assets held by domestic residents and T the trade balance in real terms.

Equation (14) defines the demand for money and equation (15) the supply of money. Equation (16) states that domestic credit always grows at the positive constant rate μ and (17) defines purchasing power parity. Equations (18) and (20) are the interest rate and risk premium equations derived in section 2 and equation (19) defines the unconditional expected rate of devaluation. The balance-of-payments identity defines the change in the foreign assets as the difference between the trade balance surplus and the change in the stock of foreign assets. Equation (22) defines the trade balance as a function of the real interest rate and equation (23) defines the probability of devaluation with the truncation point of the distribution function G determined alternatively by equation (24a) or (24b).

We assume that p* and r* are constant and that as long as the central bank does not devalue $s_t = \bar{s}$. By solving R_t from equations (14) - (19), we obtain

$$R_t = \beta\bar{s} - D_0 - \alpha\delta\bar{s}\pi_t + \alpha\bar{s}Z_t - \mu t \tag{25}$$

where $\beta = b_0p^* - b_1p^*r^*$ and $\alpha = b_1p^*$. We assume that both β and α are positive. Equations (17) - (19) and (21) - (22) imply the following relation for the stock of foreign assets

$$F_t = \gamma\bar{s} - \eta\bar{s}Z_t - (R_t-R_{t-1}) + F_{t-1} \tag{26}$$

where $\gamma = c_0p^* + c_1p^*r^*$ and $\eta = c_1p^*$. The behaviour of the model in the fixed exchange rate regime $s_t = \bar{s}$ is now determined by equations (20), (23), (25), (26) and (24a) or (24b).

5 THE CASE OF PERFECT FORESIGHT

The model (14) - (24) reduces to the case of perfect foresight, if the threshold level of the foreign reserves is known with certainty i.e. all the probability mass of the distribution function G is concentrated on a single point. For simplicity we assume that the treshold level is zero. The interest rate equation (18) reduces to the form

$$r_t = r^*_t + (s_{t+1}-s_t)/s_t \tag{27}$$

After substituting equations (16) and (27) into (14) the model can be presented in the form

$$M_t = \beta s_t - \alpha(s_{t+1}-s_t) \tag{28}$$

$$M_t = R_t + D_t \tag{29}$$

$$D_t = D_u + \mu_t \tag{30}$$

The balance-of-payments equation (21) and the trade balance equation (22) form the post-recursive part of the model with no feedback to equations (28) - (30). Equations (28) - (30) imply the following relation for R_t

$$R_t = \beta s_t - D_0 - \mu t - \alpha(s_{t+1}-s_t) \tag{31}$$

Assume that $s_t = \bar{s}$ until period t* and that at the beginning of period t* + 1 there is a devaluation so that $s_{t+1} - s_t = \delta\bar{s}$. From (31) we see that in period t* there is a speculative attack on the currency the size of which is $\alpha\delta\bar{s}$. The attack is timed to occur in the first possible period in which (31) can obtain values smaller than zero. Hence it follows from (31) that

$$\frac{R_0}{\mu} - \frac{\alpha\delta\bar{s}}{\mu} \leq t^* < \frac{R_0}{\mu} - \frac{\alpha\delta\bar{s}}{\mu} + 1 \tag{32}$$

where $R_0 = \beta\bar{s} - D_0$. There is only one integer in the interval defined in (32). It is this integer which defines the period when the speculative attack occurs. We see that the greater is the size of devaluation $\delta\bar{s}$ or the faster the domestic credit expansion the earlier the devaluation occurs.

The domestic credit expansion equation (28) implies that the new exchange rate regime $s_t = (1+\delta)\bar{s}$ cannot survive for ever. By assuming that the size of all successive devaluations is $\delta\bar{s}$ we can ask how great δ has to be in order for the new exchange rate to be viable even for one period.

Because the size of all future devaluations is $\delta\bar{s}$, we know that the size of speculative attacks connected with these devaluations is also $\alpha\delta\bar{s}$. The viability of the new exchange rate requires now that

$$M_{t*+1} > D_{t*+1} + \alpha\delta\bar{s} \tag{33}$$

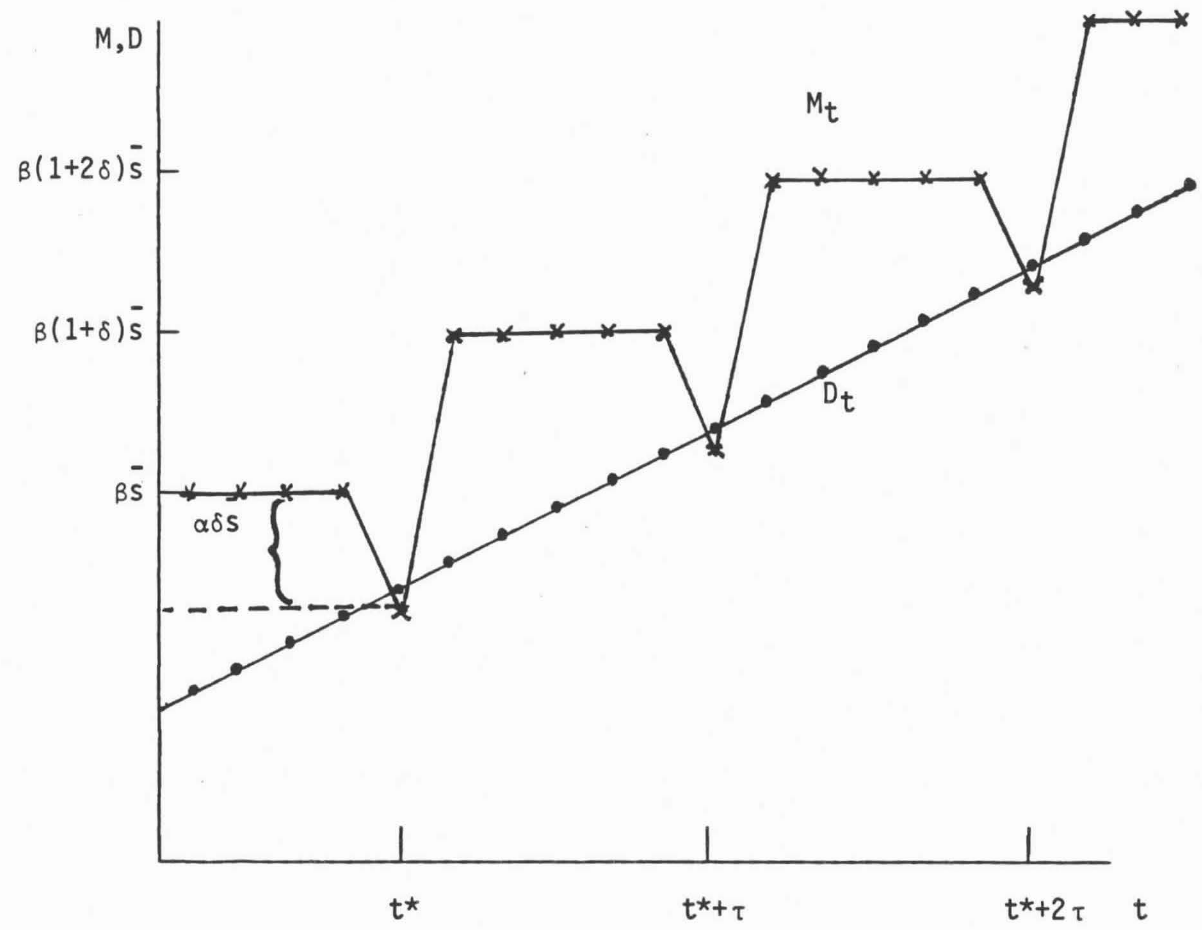
If inequality (33) does not hold, the exchange rate is floating in the period t*+1. If it holds, (28) implies that $M_{t*+1} = \beta(1+\delta)\bar{s}$, which, together with (33) and (32), results in

$$\delta\bar{s} > \mu/\beta \tag{34}$$

As shown by Flood and Garber (1984), μ/β is the rate of depreciation in the case where there is a permanent shift from the fixed exchange rate regime to the floating rate regime. Hence in order for the new exchange rate to be viable even for one period, the size of the devaluation has to be greater than the rate of depreciation in the case of a freely floating exchange rate.

It is easy to see that the model (27)-(30) produces a devaluation cycle (see figure 1). By assuming that the size of all successive devaluations is $\delta\bar{s}$, the length of this cycle τ can be calculated from equation (33). We find that τ is the integer in the interval $\beta\delta\bar{s}/\mu \leq \tau < \beta\delta\bar{s}/\mu + 1$. This is the time needed for domestic credit to expand by the amount corresponding to the increase in the demand for money caused by the devaluation.

Figure 1. A devaluation cycle



6 SPECULATIVE BEHAVIOUR WITH AN UNKNOWN THRESHOLD LEVEL OF FOREIGN RESERVES

In this section the threshold level of foreign reserves is assumed to be unknown to investors, i.e. it is drawn from a probability distribution. The distribution function is assumed to be normal with parameters m and σ^2. Because of nonlinearities contained in the model, it can only be solved numerically. In our numerical simulation experiments, the normal distribution is approximated by Hastings' best approximation formula.[6] To preserve comparability with our perfect foresight example as far as possible, we assume that $m = \alpha\delta + \mu$, i.e. the threshold level is distributed around the level of reserves, which in our perfect foresight example is the level of foreign reserves in the period preceding the attack on the currency. In all of our simulation experiments, we assumed that $\bar{s} = 1$, $\beta - D_0 = 110$, $\alpha = 500$, $\delta = 0.1$ and $\mu = 1$, implying that $m = 51$. The trend variable t obtained values 1, 2, 3, etc., except in section 6.4 where it obtained values 2, 3, 4, etc.

We start our analysis by assuming first that investors are risk neutral. In section 6.1 the threshold level is stochastic and in section 6.2 fixed but unknown to investors. The speculative behaviour of risk averse investors is examined in section 6.3. In section 6.4 the analysis is extended so that investors are uncertain if the new threshold level is drawn from a known normal distribution or if the threshold level in force at the beginning of the previous period is still in force.

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[6] If $x \sim N(0,1)$ and $y = |x|$, then Hastings' best approximation formula for the distribution function of normal distribution is

$$\phi(x) = \begin{cases} 1-F(y) & \text{if } x \geq 0 \\ F(y) & \text{if } x < 0 \end{cases}$$

where $F(y) = (1/\sqrt{2\pi})\exp[-y^2(0.31938153t - 0.356563782t^2 + 1.78147937t^3 - 1.821255978t^4 + 1.330274429t^5)/2]$ and $t = 1/(1+0.2316419y)$.

6.1 The case of a stochastic threshold level

In this section we assume that investors are risk neutral (the risk premium $Z_t = 0$) and that the central bank draws a new threshold level of foreign reserves from a known normal distribution at the end of each period. In this case the behaviour of the model in the fixed exchange regime $s_t = 1$ is determined by equations (23), (24a) and (25). After defining $G(R_t) = \phi[(R_t-m)/\delta]$, where ϕ refers to the normal distribution function with mean m and standard error σ, the system can be written in the form

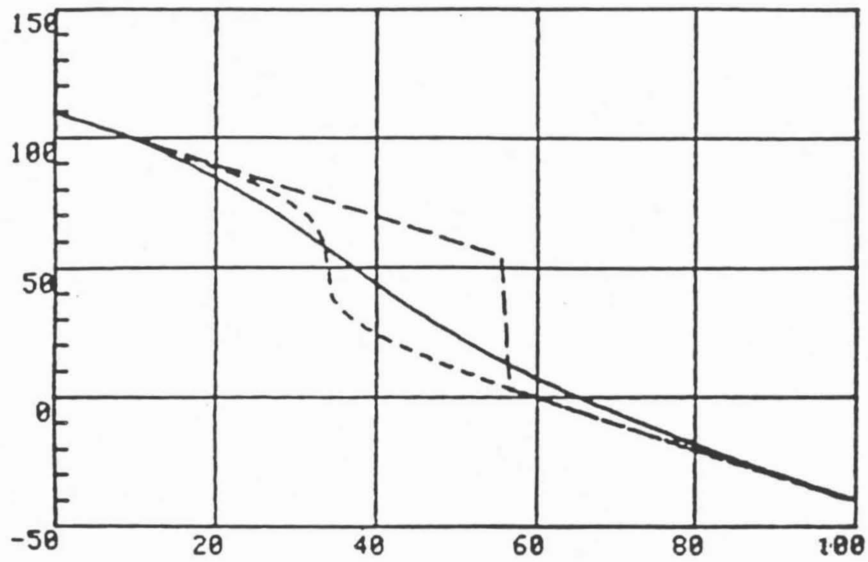
$$R_t = \beta - D_0 - \alpha\delta\pi_t - \mu t \tag{25a}$$

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R^u \\ 1 - \phi[(R_t-m)/\sigma]/\phi[(R^u-m)/\sigma], & \text{if } -\infty < R_t \leq R^u \end{cases} \tag{23a}$$

The model was simulated with alternative values of σ, i.e. with $\sigma = 40$, $\sigma = 20$ and $\sigma = 1$. R^u was set to equal 100, implying that as long as foreign reserves are above that level the probability of devaluation is zero. We see from figure 2 that, independently of the size of σ, the cumulative capital outflow caused by devaluation expectations is of equal size, which in our examples is 50. However, the smaller is σ the shorter is the period over which speculative behaviour is concentrated. In the case where $\sigma = 40$, the speculative outflow is distributed quite evenly over a long time period. As can be seen in table 1, when $\sigma = 20$ almost half of, and in the case when $\sigma = 1$, practically all of the speculative capital outflow is

Figure 2. Currency speculation with risk neutral investors (stochastic threshold level)

——— Foreign reserves with σ = 40
------- Foreign reserves with σ = 20
— — — Foreign reserves with σ = 1



——— Probability of devaluation with σ = 40
------- Probability of devaluation with σ = 20
— — — Probability of devaluation with σ = 1

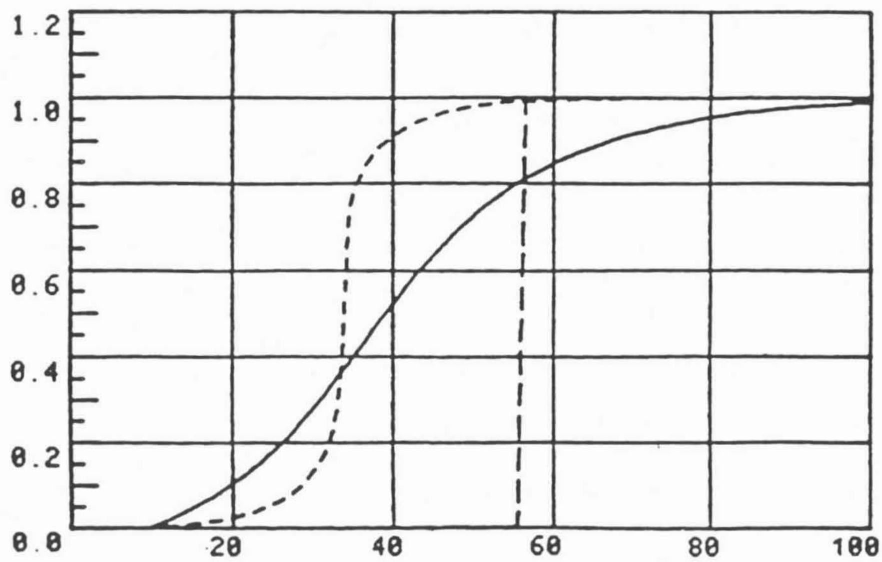


Table 1. The maximum size of the depletion of the foreign reserves and its timing with different values of σ

σ	period	the size of the capital outflow
40	38	2.3
20	35	21.2
1	57	50.9
0[1]	60	51

[1] The case in which $\sigma = 0$ indicates the case of perfect foresight. In this case the timing of the speculative attack is calculated directly from (35) after setting $\pi_t = 1$. The size of the depletion of the foreign reserves in the period of the attack is $\alpha\delta + \mu = 51$.

concentrated in a single period. The speculative capital outflow is greatest in the period in which the increase in the probability of devaluation is greatest. This can be seen from figure 2b, which shows the time paths of the probability of devaluation with alternative values of σ. As investors are risk neutral, figure 2b also shows the time paths of the interest rate differential $r_t - r_t^*$, so that with $\pi_t = 0$ also $r_t - r_t^* = 0$ and with $\pi_t \to 1$ the differential $r_t - r_t^* \to \delta$.

6.2 The case of a fixed but unknown threshold level

In this section, we assume that, instead of being stochastic, the threshold level of foreign reserves is fixed. Its value is drawn by the central bank from a known truncated normal distribution before the foreign reserves have been depleted below the truncation point of that distribution. Hence, investors know that the threshold level is fixed but they do not know its values. If investors are risk neutral, equations (23), (24a) and (25) determine the behaviour of the model. With $s_t = 1$ and $G(R_t) = \phi[(R_t-m)/\sigma]$, the model can be written in the form

$$R_t = \beta - D_0 - \alpha\delta\pi_t - \mu t \tag{25a}$$

$$\pi_t = \begin{cases} 0, & \text{if } R_t < R_t^u \\ 1 - \phi[(R_t-m)/\sigma]/\phi[(R_t^u-m)/\sigma], & \text{if } -\infty < R_t \leq R_t^u \end{cases} \tag{23a}$$

$$R_t^u = \min(R_{t-1}, R_{t-1}^u) \tag{24b}$$

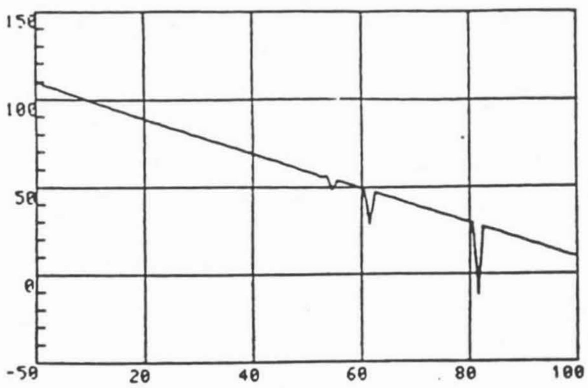
Simulation experiments were made with the same values of σ as in the previous section i.e. with $\sigma = 40$, $\sigma = 20$ and $\sigma = 1$. The initial value of R_{t-1}^u was set to equal 100. The results are shown in figure 3.

Unlike in the case of the stochastic threshold level, the probability of devaluation does not grow continuously as reserves diminish. Rather, there is a series of speculative attacks the duration of which is one period. Between the successive attacks, the probability of devaluation is zero. Only before the first attack does the probability of devaluation deviate from zero for a longer time period. However, during this period, its size is quite small (in our examples less than 0.1 at its maximum) and is the more clearly observable the greater is the standard error σ. It can also be seen that the sizes of the successive attacks and the probabilities of devaluation associated with their occurrence grow as foreign reserves diminish. The maximum size of the attack is $\alpha\delta$, which equals 50 in our examples. In addition the time interval between successive attacks grows as the size of the attack grows. This is due to the fact that the following attack occurs after the period in which the foreign reserves have attained the level $R_t < R_t^u + \mu$ for the first time. The attack does not occur earlier because investors know that the threshold level of reserves is below R_t^u, i.e. the level to which reserves were depleted during the preceding attack. The increase in the magnitude of the successive attacks is due to the fact that the lower the truncation point of the probability distribution, the more concentrated the probability mass is in the neighbourhood of the truncation point. Figure 3 also shows that the smaller is σ the earlier speculative attacks attain a size very close to their maximum size $\alpha\delta$.

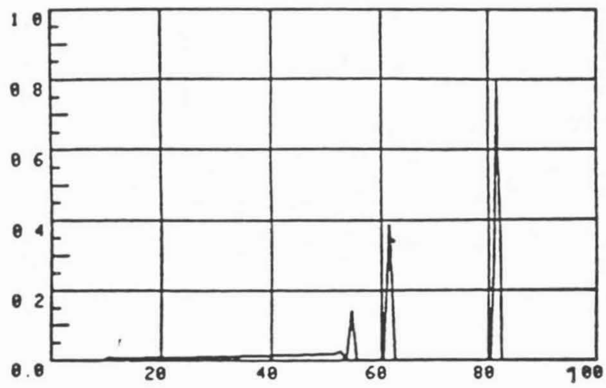
Figure 3. Currency speculation with risk neutral investors (fixed but unknown threshold level)

a) σ = 40

Foreign reserves

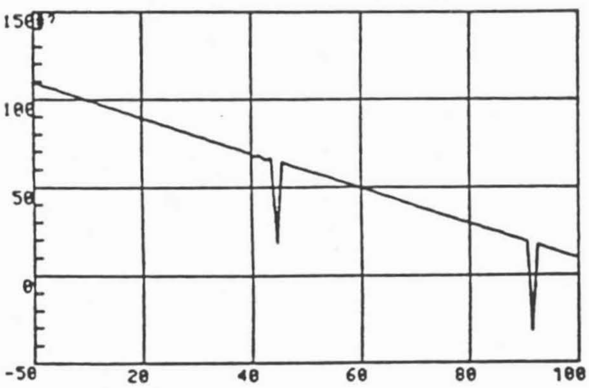


Probability of devaluation

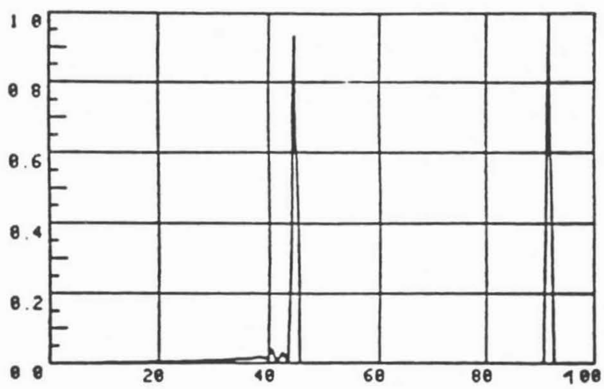


b) σ = 20

Foreign reserves

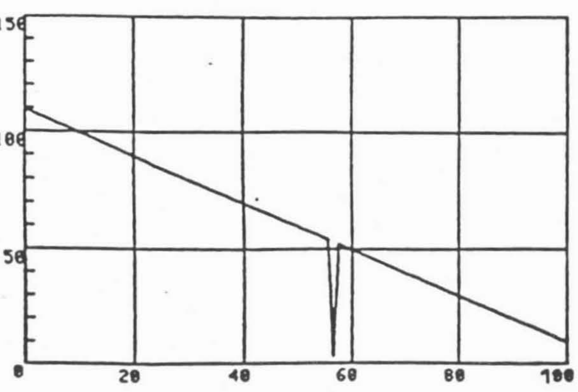


Probability of devaluation

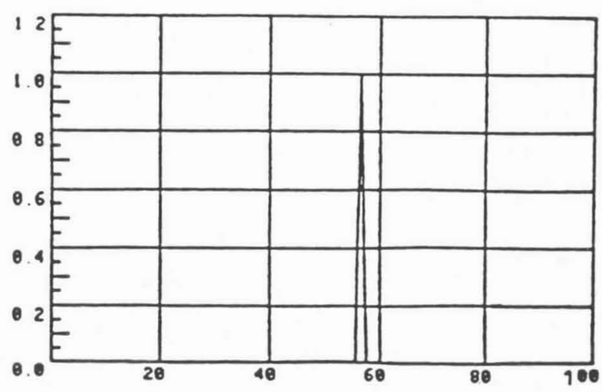


c) σ = 1

Foreign reserves



Probability of devaluation



6.3 Speculative behaviour with risk aversion

In this section the assumption of risk neutrality is relaxed. This implies that the risk premium equation (20) and equation (26) determining the stock of foreign assets held by domestic residents are needed. With $s_t = 1$ the model can now be written in the form

$$R_t = \beta - D_0 - \alpha\delta\pi_t + \alpha Z_t - \mu t \tag{25}$$

$$Z_t = \delta\pi_t[\exp(a\delta F_t)-1]/[\exp(a\delta F_t)+\pi_t/(1-\pi_t)] \tag{20}$$

$$F_t = \gamma - \eta Z_t - (R_t - R_{t-1}) + F_{t-1} \tag{26}$$

$$\pi_t = \begin{cases} 0, & \text{if } R_t > R_t^u \\ 1 - \phi[(R_t-m)/\sigma]/\phi[(R_t^u-m)/\sigma], & \text{if } -\infty < R_t \leq R_t^u \end{cases} \tag{23a}$$

$$R_t^u = R^u \tag{24a}$$

or

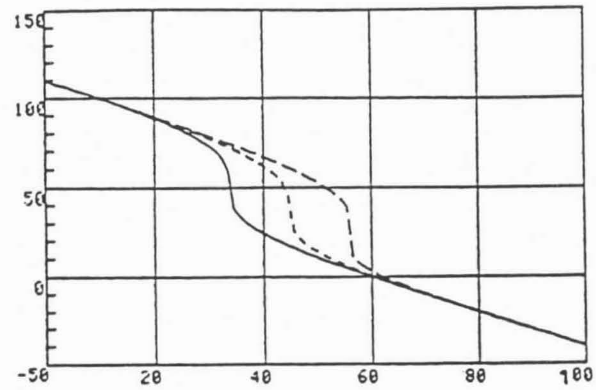
$$R_t^u = \min(R_{t-1}, R_{t-1}^u) \tag{24b}$$

R_t^u is determined by equation (24a) if the threshold level of foreign reserves is stochastic and by equation (24b) if it is fixed but unknown to investors. As above we assume that R^u in (24a) and the initial value of R_{t-1}^u in equation (24b) equal 100. In the simulation experiments shown in figures 4 and 5, parameters γ, η and σ were set to equal 0, 1 and 20, respectively. The initial value of F_{t-1} was set to equal 110 and the risk aversion parameter a obtained, alternatively, values of 0.05 and 0.1. Simulations were also run with different values of η, the size of which depends positively on the size of the interest sensitivity of the trade balance. Our simulation results showed that the size of η is significant for the timing of speculative behaviour. The greater was η, the earlier speculative capital movements started.

Figure 4. Currency speculation with risk averse investors (stochastic threshold level)

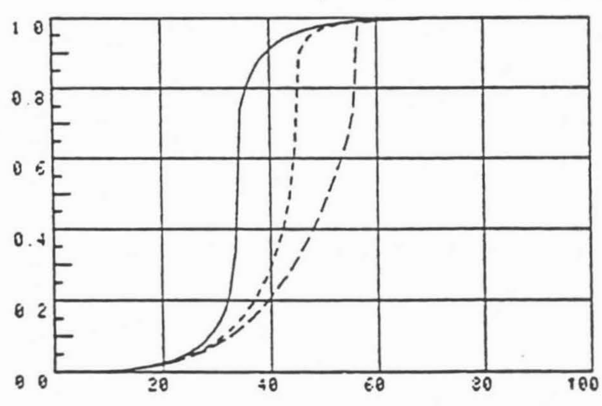
Foreign reserves

a = 0 ————
a = 0.05 - - - - - - -
a = 0.1 — — —



Probability of devaluation

a = 0 ————
a = 0.05 - - - - - - - -
a = 0.1 — — —



Risk premium

a = 0.05 ————
a = 0.1 - - - - - - -

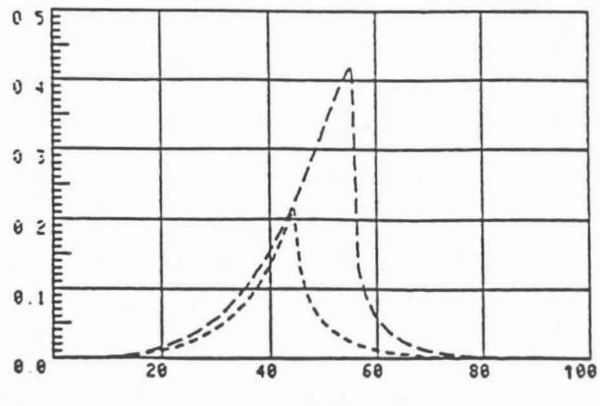
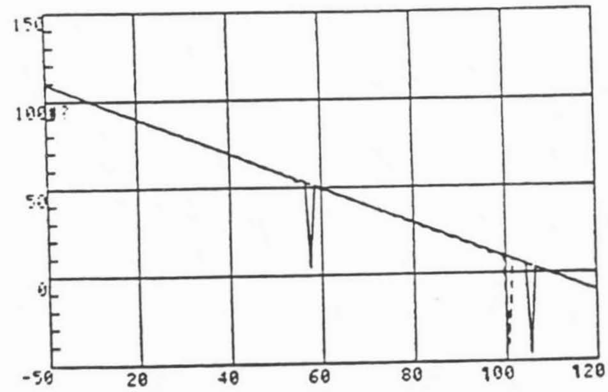


Figure 5. Currency speculation with risk averse investors (fixed but unknown threshold level)

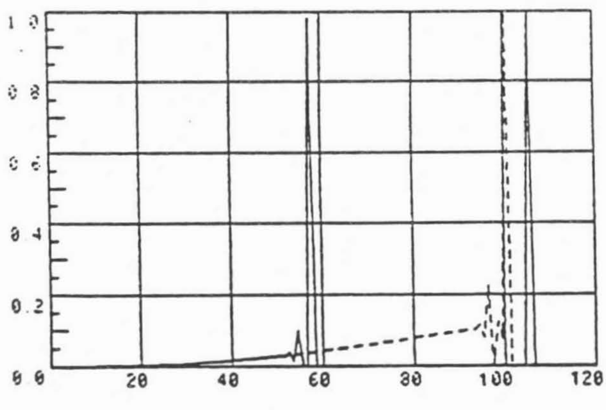
Foreign reserves

a = 0.05 ————
a = 0.1 - - - - - - -



Probability of devaluation

a = 0.05 ————
a = 0.1 - - - - - - -



Risk premium

a = 0.05 ————
a = 0.1 - - - - - - -

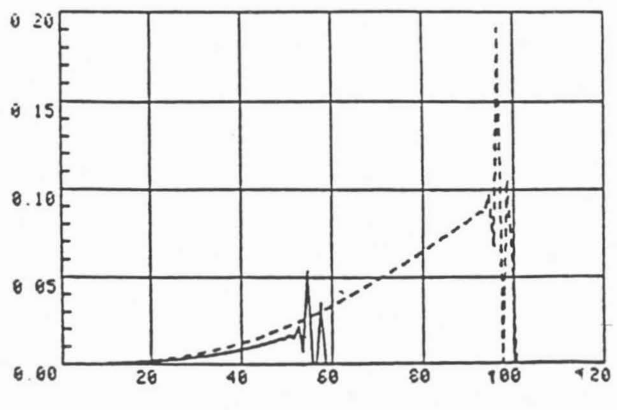


Figure 4 shows the case with a stochastic threshold level of foreign reserves. We see that the more risk averse investors are (i.e. the greater is a), the later the speculative capital outflow occurs. Also, the shape of the time paths associated with the probability of devaluation deviates from that of the risk neutral case (a = 0).

The lowest panel of figure 4 shows developments in the risk premium divided by the size of devaluation δ. As expected, we see that the greater is parameter a, the greater is the risk premium. It can also be seen that the risk premium grows at first but, after attaining its maximum value, it suddenly decreases and starts to converge towards zero. This kind of time pattern is due to the fact that, with probabilities of devaluation close to zero and close to unity, uncertantities concerning the occurrence of devaluation are small. In our examples the maximum values of the risk premium are associated with probabilities of devaluation greater than 0.5. With a = 0.05 and a = 0.1 these probabilities are 0.62 and 0.73, respectively.

As regards the domestic interest rate this kind of time pattern for the risk premium implies that, up to the point the risk premium attains its maximum value, the interest rate differential between domestic and foreign interest rates grows more slowly than the probability of devaluation but thereafter it approaches the size of the expected devaluation $\delta\pi_t$ fairly rapidly.

It can be seen from figure 5 that, in the case of a fixed but uncertain threshold level of foreign reserves, an increase in risk aversion delays the occurrence of the first speculative attack on reserves. In our examples, when a = 0 (see figure 3) the first speculative attack occurs in period 40, while when a = 0.05 and a = 0.1 the first attack occurs in periods 58 and 101, respectively. We also see that with low probabilities of devaluation the time pattern of the risk premium is quite similar to that of unconditional expected devaluation $\pi_t\delta$. However, the risk premium drops permanently to zero in our examples in connection with the first speculative attack. This is due to the fact that, after the first attack, the probability of devaluation drops to zero and hence there is no uncertainty in the model. After foreign reserves have, without speculative capital movements, diminished to the level which they attained when the first attack occurred, the probability of devaluation suddenly jumps close to unity. In that case, there is practically no uncertainty in the model either, and hence the risk premium stays at zero.

6.4 The combined case

So far, speculative behaviour associated with one-sided foreign exchange risk has been studied in two polar cases, i.e. in the case in which the threshold level of foreign reserves is a stochastic variable and in the case in which the threshold level is fixed but unknown to investors. In this section, we combine these two cases by assuming that, with probability 1-k, the central bank chooses a new threshold level from a known normal distribution at the end of each period and that, with probability k, the threshold level is same as at the end of the preceding period.

Denote the probability of devaluation by $\pi(R_t)$ if the threshold level were drawn from a known normal distribution at the end of each period and by $\pi(R_t, R_t^u)$ if the threshold level were drawn from a known truncated normal distribution with R_t^u as the truncation point.

Assume that under the fixed exchange rate regime $s_t = \bar{s}$ there has not yet been any speculative attack on the currency. This implies that R_{t-1} is the minimum value of R until the beginning of period t. The probability that the currency will be devalued at the beginning of the next period is now simply

$$\pi(t) = (1-k)\pi(R_t) + k\pi(R_t; R_{t-1}) \tag{35}$$

After the first attack the situation becomes more complicated. Assume that there was an attack in the period t-1-n and that R_{t-1} is the minimum value R in the interval [t-n, t-1]. n is an integer such that $n > 0$. Now, with probability k^n, the threshold level is the same in period t as in period t-n and, with probability $(1-k^n)$, a new threshold level has been drawn. If the new value has been drawn, then at the beginning of period t investors know that its value is smaller than R_{t-1}. The probability that the currency will be devalued at the beginning of the next period is now

$$\pi(t) = (1-k)\pi(R_t) + k[k^{n_t}\pi(R_t, R_t^u) + (1-k^{n_t})\pi(R_t, R_{t-1})]$$

with

$$n_t = \begin{cases} n_{t-1} + 1, & \text{if } R_{t-1}^u < R_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

$$R_t^u = \min(R_{t-1}^u, R_{t-1}) \tag{36}$$

Formula (36) is not general, however, because it does not take into account the possibility that, between the time interval t-n-1 and t-1, there may have occurred smaller attacks on the currency associated with a level of reserves lower than R_{t-1} but greater than R_t^u. Denote by the absolute minimum level of foreign reserves by $R_t^{u_0}$ and all the successive local minima by $R_t^{u_i}$ such that $R_t^{u_{i-1}} < R_t^{u_i} < R_{t-1}$ $(i = 1, \ldots, q)$. This condition states that an earlier local minimum contains information which is not included in later local minima only as long as it is smaller than all later local minima until period t. If we denote by $n_{0,t}$ and $n_{i,t}$, the numbers of the periods elapsed since absolute and local minimum values were equal to the level of reserves at the beginning of the period, then the probability of devaluation can be written in general form as

$$\pi(t) = (1-k)\pi(R_t) + k\left[k^{n_{0,t}}\pi(R_t; R_t^{u_0}) + \sum_{i=1}^{q} k^{n_{i,t}}\pi(R_t; R_t^{u_i})\right.$$

$$\left.\prod_{j=1}^{i}(1-k^{n_{i-j,t}}) + \pi(R_t, R_{t-1})\prod_{i=0}^{q}(1-k^{n_{q-i,t}})\right]$$

with

$$R_t^{u_0} = \min(R_{t-1}^{u_0}, R_{t-1})$$

$$R_t^{u_i} = \begin{cases} \min(R_{t-1}^{u_i}, R_{t-1}), & \text{if } R_{t-1} < R_{t-2} \text{ or } R_{t-1}^{u_i} \neq R_t^{u_{i-1}} \\ R_{t-1}, & \text{if } R_{t-1} > R_{t-2} \text{ and } R_{t-1}^{u_i} = R_{t-1}^{u_{i-1}} \end{cases}$$

$$n_{0,t} = \begin{cases} n_{0,t-1} + 1, & \text{if } R_{t-1}^{u_0} \leq R_{t-1} \\ 0, & \text{if } R_{t-1}^{u_0} > R_{t-1} \end{cases}$$

$$n_{i,t} = \begin{cases} n_{i,t-1} + 1, & \text{if } R_{t-1}^{u_i} \leq R_{t-1} \\ 0, & \text{if } R_{t-1}^{u_i} > R_{t-1} \end{cases} \tag{37}$$

and q is any positive integer which is great enough to take into account the number of all possible successive local minimum values of reserves.

It is easy to see that with $R_t^{u_0} = R_t^{u_i} = R_{t-1}$ relation (37) reduces to (36) and that with $R_t^{u_0} < R^{u_i} = R_{t-1}$ it reduces to (35).

After defining

$$\pi(R_t) = 1 - \phi[(R_t-m)/\sigma] \tag{38}$$

$$\pi(R_t;\ R_t^{u_i}) = \begin{cases} 0, & \text{if } R_t < R_t^{u_i} \\ 1 - \phi[(R_t-m)/\sigma]/\phi[(R_t^{u_i}-m)/\sigma], & \\ & \text{if } -\infty < R_t \leq R_t^{u_i};\ i = 0, 1, \ldots, q \end{cases} \tag{39}$$

equations (37)-(39), together with (20), (25)-(26), determine the behaviour of the model in the fixed exchange rate regime. With k=0 and k=1, the model reduces to the case of the stochastic threshold level and the case of the fixed but unknown threshold level, respectively.

In simulations of the model the parameter k was set to equal 0.9, implying that a new threshold level is chosen with a probability of 0.1 at the end of each period and that the threshold level is the same as in the preceding period with a probability of 0.9. σ was set to equal 40 and the initial value of F_{t-1} was set to equal zero.

As in the case of a fixed but unknown threshold level, there are speculative attacks on the currency the duration of which is one period. After a speculative attack foreign reserves do not, however, return to the level corresponding to the paths of zero devaluation expectations. This is due to the fact that in equation (37) the term $(1-k)\pi(R_t)$ is always greater than zero.

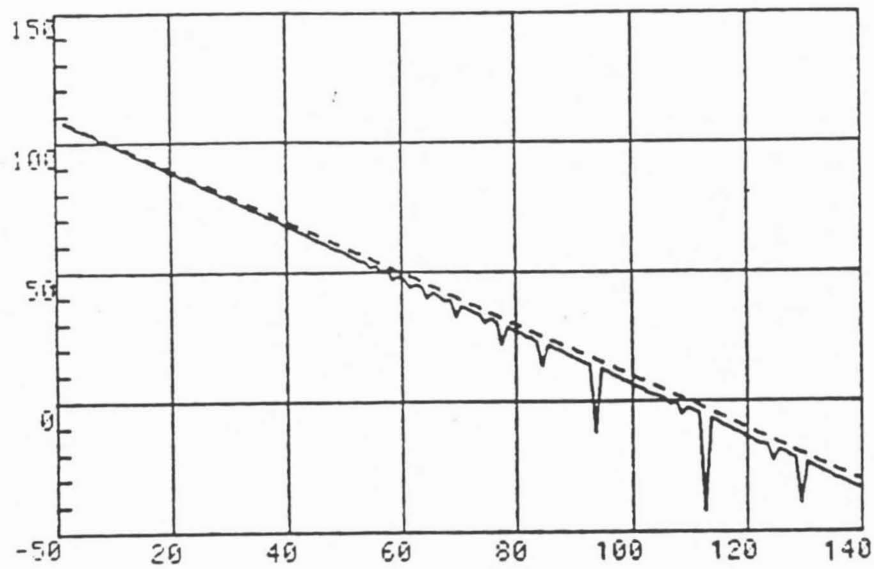
The sizes of successive attacks do not grow steadily. Between bigger attacks there can be smaller attacks. As a result, the timing of successive attacks is less regular than in the case of a fixed but unknown threshold level. When a speculative attack occurs, the pre-attack level of reserves can be well above the level which reserves attained during the previous attack. This is due to the fact that investors cannot be sure that the threshold level of reserves has not been changed since the reserves attained their previous absolute or local minimum level.

We can also see that the behaviour of the risk premium is quite different from what it was in the cases studied in the previous section. In our present example the risk premium does not converge or drop to zero with low values of foreign reserves.

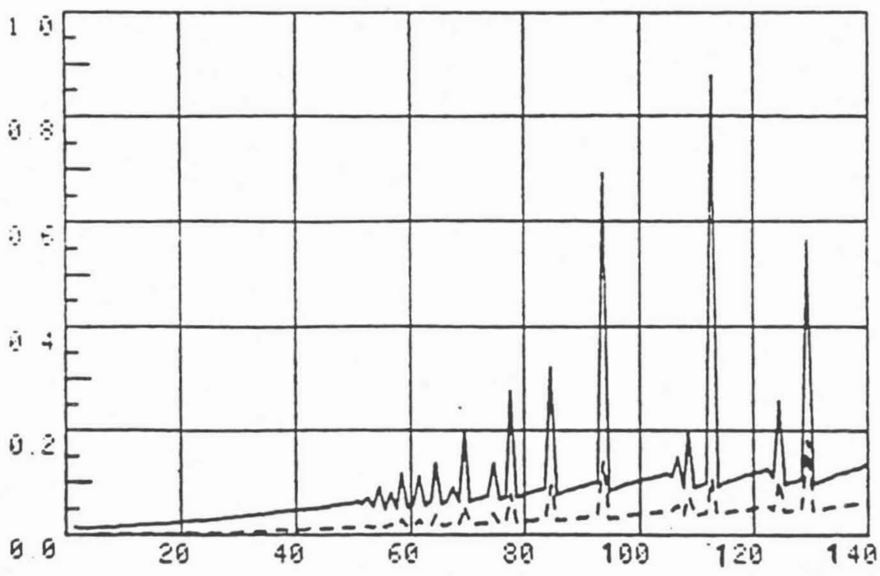
In the previous section we found that values of the probability of devaluation close to zero or close to unity were associated with values of practically zero for the risk premium. In the present case, the probability of devaluation never drops to zero and it can obtain values close to unity only during periods of speculative attack. This explains why the risk premium in figure 5 is above zero all the time. It can be seen that the risk premium also rises during the periods of speculative attack. This, in turn, reflects the fact

Figure 6. Currency speculation with risk averse investors (the combined case)

——— Foreign reserves with speculation
------- Foreign reserves without speculation



——— Probability of devaluation
------- Risk premium



that probabilities of devaluation associated with these attacks are not close enough to unity.

7 SUMMARY AND CONCLUDING REMARKS

Previous studies of balance-of-payments crises have shown that in a fixed exchange rate regime speculative attacks on the currency can result from rational behaviour. This result, however, was derived in a framework in which investors have perfect information about the threshold level of foreign reserves, the attainment of which implies that a central bank abandons with certainty its fixed exchange rate target and devalues the currency or, alternatively, allows it to float. Owing to this perfect information assumption, a speculative capital movement occurs only at the moment (or in the period) immediately preceding the shift in the exchange rate regime. Hence, it is not able to explain the empirical fact that speculative attacks on the currency quite oftern occur in anticipation of a devaluation which does not materialize or materializes much later.

In this paper we relaxed the assumption that investors have perfect information about how much of its reserves the central bank is willing to use to defend the exchange rate. What they know is the probability distribution from which the threshold level of foreign reserves is drawn. In our numerical examples the distribution function was assumed to be normal. In the case of the stochastic threshold level (i.e. a new value for the threshold level is drawn at the end of each period) we found that a speculative outflow rather than a speculative attack on the currency occurred in anticipation of a future devaluation. However, the smaller was the variance of the distribution function the shorter was the period over which the speculative outflow was concentrated and the more this outflow resembled a single period concentrated speculative attack. Because of devaluation expectations foreign reserves remain below the level at which they would be without devaluation expectations, until the devaluation materializes. We also found that as long as there are devaluation expectations, the domestic interest rate has to be above the foreign interest rate. As was shown analytically in section 2, this is the case even with risk averse investors. This results from the assumption that the foreign exchange risk is one-sided, i.e. there is only risk of a devaluation.

In the second case we assumed that the threshold level was fixed but unknown to investors, i.e. a value for the threshold level of reserves was drawn only once from the distribution function. This framework produced speculative attacks on the currency the length of which was one period. Between these attacks the probability of devaluation was zero. The interpetation is that, if devaluation does not materialize in connection with the attack, investors know with certainty that the threshold level is smaller than the level to which the attack depleted reserves. The probability of devaluation drops to zero and stays at that level until the reserves, without speculative capital movements, have been depleted to the level attained during the preceding attack. Then there is a new attack, the size of which is greater than the size of the preceding attack. This is due to fact that the probability of devaluation is also greater than the one associated with the preceding attack. This process continues until the currency is devalued in the context of some future attack.

The third case is a combination of these two cases. We assumed that, with a certain probability, the central bank draws a new value for the threshold level at the end of each period. If the new value is not drawn, then the threshold level is the same as at the beginnig of the preceeding period. In this case, too, there are speculative attacks associated with probabilities of devaluation which, at least relatively, are much greater than in the periods preceding them. If devaluation does not occur, then reserves are re-built in the next period. In this case, however, the probability of devaluation does not drop to zero between successive attacks. Nor do the successive attacks steadily increase in size. There may be several smaller attacks between two bigger attacks. A new attack may also occur well above the level to which foreign reserves were depleted by the previous attack.

How does risk aversion affect speculative behaviour? We found that the greater is risk aversion the longer it took, i.e. the lower was the level to which reserves fell, before speculative capital movements started to play a major role. In section 2 we showed that the risk premium converges to zero as the probability of devaluation aproaches zero or unity. In the polar cases of a stochastic threshold level and a fixed but unknown threshold level, this implied that after the reserves had diminished to a low enough level, the risk premium disappeared. In the mixed case, however, the risk premium did not disappear. This was due to the fact that, although the probability of devaluation fluctuated, it never diminished to zero and that only during the periods of speculative attack could it obtain values close to zero. Only during these periods could the risk premium drop to approximately zero.

In the case of perfect foresight we showed that, with the size of devaluation given an excessively expansive monetary policy rule resulted in a devaluation cycle the length of which was constant. This result also holds in the case in which the threshold level is unknown, except that then the length of the cycle is not constant. The length of the cycle varies because the threshold level of reserves associated with each devaluation changes stochastically. We can conclude that, for a devaluation of given size and an unchanged monetary policy rule, the lower is the threshold level of reserves associated with the preceding devaluation the earlier starts the next round of speculative capital movements. Or, one could equally well say the shorter the time the new exchange rate is viable.

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