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OUTPUT AND INFLATION IN A FLEXPRICE-FIXPRICE ECONOMY

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1. INTRODUCTION*

The purpose of this essay is to explain inflation and output using a disaggregated model in which both classical or Marshallian price adjustment and Keynesian price rigidity are present in the goods market. The model can also be used in an open economy context employing the distinction between traded and non-traded goods.

The ideas behind the model can be traced to Hicks (1965), who introduced the concepts "flexprice and fixprice" markets, where the former refers to the Marshallian price adjustment mechanism and the latter to the Keynesian assumption of price rigidity. Hicks (1974) has developed the flexprice-fixprice analysis further, and the following sentences summarize his view fairly well: "The fact surely is that in modern (capitalist) economies there are, at least, two sorts of markets", and "What we need is a theory which will take account of both sorts of markets, a theory in which both fixprice and flexprice markets have a place" (Hicks 1974, p. 23-4).

Other authors have also considered these two market mechanism. Okun (1975) explained inflation by the interaction of "auction and customer" markets. Kaldor (1976) disaggregated the economy in primary, secundary and tertiary sectors, and according to him the main reason for the recent inflation and unemployment is the strong variability of prices of primary products, which might be eliminated by building up buffer stocks. Scarfe (1977, Ch. 11) used the terms "Marshallian and Keynesian" price adjustment to stress the point that prices are also variable in the latter case. In the following, however, the Hicksian terms are used.

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* The first draft of this essay (chapters 1,4-7) was prepared for the Macroeconomic Course arranged by the Yrjö Jahnsson Foundation in 1977-78. I am grateful for the comments of the teachers, prof. J. Tobin and Dr. Janet Yellen, and of the participants of the course. Sixten Korkman read the manuscript and made useful suggestions, but he is not of course responsible for the remaining errors in the analysis. Dr. Gavin Bingham checked the English text.

An empirical study of the behaviour of Finnish foreign trade prices showed that the prices of raw materials and related goods, which constitute about a half of Finland's total trade, depend on the world commodity prices as well as on world demand, while the prices of manufactured goods depend on unit labor and raw material costs (the estimation results are reported in the appendix). In the disaggregated price equations estimated by Eckstein and Wyss (1972), demand effects were significant in more than half the industries studied. The empirical evidence thus indicates that both types of price adjustment occur in the goods market.

In the following two sections a graphical analysis of a flexprice-fixprice economy is developed. First, a flow equilibrium model is presented omitting the stocks of flexprice goods, and then a stock-flow version of the model is presented including the stocks of flexprice goods. In the stock-flow model rational speculation is assumed on the market for flexprice goods.[1] In the latter part of the study, the model is expanded to take account of the expectations augmented Phillips-curve in the determination of fixprices, and the stability and the dynamic properties of the model are studied and compared to those of the one sector model developed by Tobin (1975).

The two sector model of this study, suggest, that there is a trade-off between the terms of trade between the two sectors and employment when one sector produces raw materials whose prices are determined by supply and demand and the other produces finished goods whose prices are determined by labour and raw material cost. If the terms of trade are fixed or if the supply of raw materials is restricted, full employment output cannot be reached in the long run unless raw materials can be substituded in the finished goods' production. If the terms of trade are allowed to vary, shocks in raw material production or autonomous changes in the demand for finished goods produce long cyclical oscillations until full equilibrium is reached.

1. The model is similar to recent models of exchange rate dynamics, see e.g. Dornbusch (1976).

2. A DYNAMIC FLOW EQUILIBRIUM FLEXPRICE-FIXPRICE MODEL

In the combined flexprice-fixprice model, the economy is disaggregated into two sectors, the flexprice sector denoted by a "1" and the fixprice sector denoted by a "2". The following three equations describe the behaviour of the flow equilibrium flexprice-fixprice economy:

$$(1) \quad E^2(p^2,\ Y^2-k(E^1-Y^1)) + G = Y^2$$

$$(2) \quad E^1(Y^2,k) = Y^1(k)$$

$$(3) \quad p^2 = p^2(p^1)$$

$$(4) \quad k = p^1/p^2$$

In the first equation, private demand for fixprice goods E^2 depends on own prices[1)] p^2 and the real income of both sectors $Y = Y^2-k(E^1-Y^1)$, which is constructed by subtracting the raw material input E^1 from the gross output of fixprice goods Y^2 and adding the real income or output of the flexprice sector Y^1 multiplied by the price ratio of flexprice and fixprice goods $k = p^1/p^2$. Total real income Y is equal to the real gross output of fixprice goods because the flows of Y^1 and E^1 are always in equilibrium. G describes the autonomous government expenditure.

A rise in the price level E^2 reduces the demand for fixprice goods for the following reasons. A given nominal quantity of money will correspond to a larger real quantity at a lower price level, the interest rate will be lower and investment demand higher. Secondly a lower price level leads to higher net private wealth in nominal terms and to greater consumption demand. The marginal propensity to spend is as-

1. Prices of other goods are net included because it is assumed that only flexprice goods are consumed.

sumed to be positive and less than 1. For a discussion of the partial derivatives, see Tobin (1975, p. 196-8).

In the second equation, the demand for flexprice goods E^1 depends on the gross output of fixprice goods Y^2 and on the price ratio $k = p^1/p^2$ between flexprice and fixprice goods. Flexprice goods are used only as inputs in the production of fixprice goods and it is possible to find substitutes or to economize on raw material input if the relative price k changes. The partial derivatives are $E^1_Y > 0$ and $E^1_k < 0$. The output of flexprice goods is a function of the price ratio k and $Y^1_k > I$. If p^2 is assumed to describe the input cost of the production of flexprice goods, they are produced up to the point where marginal cost equals marginal revenue.

The third equation is a simplified version of an aggregate supply curve. The price level of fixprice goods depends on the price level of the raw material input or of the flexprice goods p^1. Equation (4) defines the terms of trade between the flexprice and fixprice sectors.

In order to study the effects of an exogenous increase in government expenditure G on the endogenous variables of the model, a dynamic graphical analysis similar to the IS-LM presentation is used. The normal comparative static results are also presented. The model is dynamized by lagging the input cost of flexprice goods in the price equation of fixprice goods. Thus the model presented in Figure 1 is as follows:

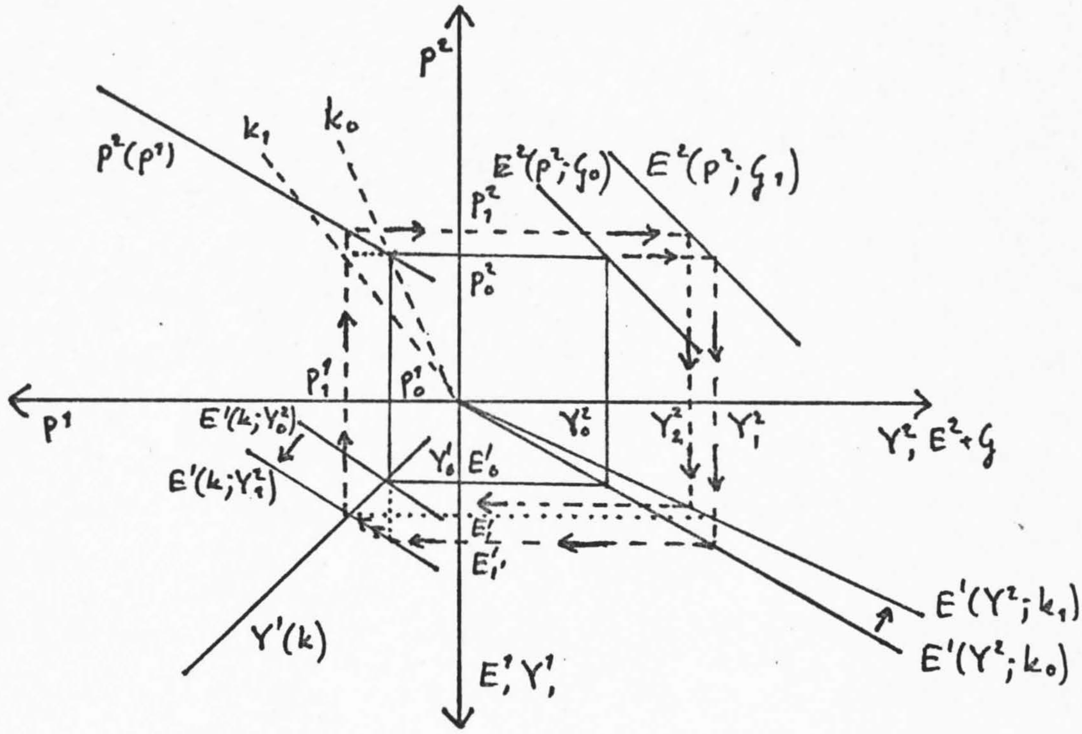
(1) $E^2(p^2, Y^2 - k(E^1 - Y^1)) + G = Y^2$

(2) $E^1(Y^2, k) = Y^1(k)$

(3') $p^2 = p^2(p_{-1})$

(4) $k = p^1/p^2$

FIGURE 1



In Figure 1, prices of fixprice goods p^2 are measured on the vertical axis upwards and the output and expenditure of these goods on the horizontal axis to the right. Thus the economy's total demand curve $E^2(p^2; G)$ with given values of G is shown in the northeast quadrant. The vertical axis downwards measures expenditure on and output of flexprice goods, E^1 and Y^1. The southeast quadrant thus describes the expenditure on flexprice goods as a function of the output of fixprice goods, $E^1(Y^2;k)$. The horizontal axis to the left measures the prices of flexprice goods, p^1. In the southwest quadrant expenditure on and supply of flexprice goods are shown to be a function of the price level or of the price ratio of flexprice goods, $E^1(k;Y^2)$ and $Y^1(k)$. In the northwest quadrant the rays projecting from the origin are different price ratios k, but this quadrant also shows the prices of fixprice goods as a function of the price of flexprice goods.

To analyze the effects of a permanent increase in autonomous expenditure G, the model is assumed to be initially in full equilibrium so that the price level and the output of fixprice goods are p_0^2 , and the price level, demand and supply of flexprice goods are p_0^1 and $E_0^1 = Y_0^1$. An increase in G shifts the total demand curve rightwards to the point (p_0^2, Y_1^2). An increase in the output of fixprice goods requires an increase in the input of flexprice goods E_1^1,, according to the curve $E^1(Y^2;k_0)$ in the southeast quadrant. Simultaneously, the curve $E^1(k;Y_0^2)$ must shift downwards and pass through the point $(E_1^1,,p_0^1)$ in the southwest quadrant. At price level p_0^1 and price ratio k_0, the demand for flexprice goods exceeds their supply, which leads to a rise of the price level or in the price ratio to the p_1^1 or k_1, where the demand and supply of flexprice goods are equal. The decrease in expenditure on flexprice goods through the substitution effect $E^1(k)$ rotates the $E^1(Y^2;k)$-curve to a new position passing through the point $(E_1^1 = Y_1^1, Y_1^2)$. In the first period, the increased price level or price ratio of flexprice goods does not affect the price level of fixprice goods (the $p^2(p^1)$-curve is flat). The short-run equilibrium values of the endogenous variables are thus p_0^2, Y_1^2, $E_1^1 = Y_1^1$, p_1^1 and $k_1 = p_1^1/p_0^2$.

In the second period, the increased price ratio k_1 raises the price of fixprice goods as the $p^2(p^1)$ - curve in the northwest quadrant shows. The higher price of fixprice goods reduces expenditure on them to Y_2^2 and further spending on flexprice goods to E_2^1 according the new $E^1(Y^2; k_1)$-curve in the southeast quadrant. Now the supply of flexprice goods exceeds expenditure on them, which leads to a fall in their price ratio, and the $E^1(Y^2; k_1)$-curve rotates to a new position, which lies between the earlier curves $E^1(Y^2;k_0)$ and $E^1(Y^2;k_1)$. In the third period, the price of fixprice goods decreases and demand increases. Thus the adjustment process continues indefinately oscillating and converging to the long-run equilibrium values of the variables, which lie between the original full equilibrium values and the first round values.

Comparative static analysis for an increase in the autonomous expenditure G. Short-run multipliers:

$\partial Y^2/\partial G = 1/(1-E_Y^2 2) > 0$, $\partial k/\partial G = E_Y^1 2/(1-E_Y^2 2)(Y_k^1 - E_k^1)) > 0$,

$\partial Y^1/\partial G = \partial E^1/\partial G = Y_k^1 E_Y^1 2/((1-E_y^2 2)(Y_k^1-E_k^1)) > 0$, $\partial p^2/\partial G = 0$.

Long-run multipliers:

$\partial Y^2/\partial G = 1(1-E_Y^2 2) + E_Y^1 2E_p^2 p_k^2/ A(1-E_Y^2 2) > 0$,

$\partial p^2/\partial G = p_k^2 E_Y^1 2/A > 0$, $\partial k/\partial G = E_Y^1 2/A > 0$,

$\partial Y^1/\partial G = \partial E^1/\partial G = Y_k^1 E_Y^1 2/A > 0$,

where $A = (1-E_Y^2 2)(Y_k^1-E_k^1) - E_Y^1 2E_p^2 2p_k^1$.

All the multipliers are positive except the short-run multiplier for the price of fixprice goods p^2, which is zero. The signs of all the multipliers are easily verified except that of the long-run output of fixprice goods Y^2, which includes two terms. A closer examination, however, shows that the second term of $\partial Y^2/\partial G$, which is negative, must be always smaller than the first.

The above model shows that an increase in autonomous government expenditure leads to an increase in output of both fixprice and flexprice goods and to a rise in the prices of both goods. Further, the ratio flexprice good prices to fixprice good prices rises. The diagram suggest that the dynamic model is stable and produces dampening oscillations of the endogenous variables.

One shortcoming of the above model is the assumption of that production of and expenditure on flexprice goods adjust instantaneously to changes in their relative prices. This assumption was needed because of the implicit presupposition that there are no stocks of these goods. When only the market for fixprice goods are considered, this presupposition might be justified because these goods are often produced according to order, i.e. they are purchased before they are produced.

In the case of flexprice goods this assumption is not realistic because such goods are often storeable and well organized markets exist for them. In the following, a stock demand function for flexprice goods is added in the model.

3. A STOCK-FLOW VERSION OF FLEXPRICE-FIXPRICE MODEL

In the stock and flow version of the flexprice-fixprice model, stocks of flexprice goods are added to the analysis. Stocks of flexprice goods are assumed to be held by intermediate traders[1]. Traders are assumed to hold stocks because they expect to earn profits. The expected profits are determined by expected changes in the selling price of stocks of flexprice goods, and on the other hand by expected changes in the cost of holding stocks. Thus the traders' stock demand function may be written.

$$(5) \qquad S^d = S(x^1-x^2) = S(\hat{k}^e),$$

where x^1 is the expected change in the selling price of flexprice goods, x^2 is the expected change in the cost of holdings stocks, which is described by the expected change in fixprices. The term $\hat{k}^e$ is thus the expected change in the ratio of prices of flexprice and fixprice goods.

The traders' stocks are assumed to be at their desired level at every moment of time, i.e. desired stocks are always the same as actual stocks, S^d=S. However, outside supply of and demand for stocks (i.e. flexprice goods) may differ. The response of production and consumption to changes in the price ratio is now assumed to be slow (opposite to the assumption in model I) while stocks are assumed always to be in equilibrium (i.e. the speed of response to a change in the price ratio is infinite). It is further assumed that the traders know in advance the price ratio k, which equilibriates production and consumption of flexprice goods. The traders thus know the response of the supply of and demand for flexprice goods to changes in price ratio k and

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1. Producers' and consumers' stocks are neglected because their effect is assumed to be included in functions E^1 and Y^1.

to changes in the exogenous variables of the model. Traders' speculation is rational in the sense that the equilibrium price ratio is reached via speculation. Thus the traders have perfect foresight concerning k, i.e. they know he model and its parameter values and thus determine via speculation the price ratio, which restores equilibrium after a shock to the exogenous variables of the model.

Next the stock demand function of flexprice goods is added to the flow equilibrium model presented in Fig. 1, and the effects of a permanent increase in autonomous expenditure on the endogenous variables and on their time paths are studied. In the short-run temporary equilibrium, the stock-flow version of the model is as follows, but in full equilibrium $dS = 0$ and $p^1 = p^1_{-1}$.

(1) $E^2(p^2, Y^2 - k(E^1 - Y^1)) + G = Y^2$

(2) $Y^2(k) - E^1(Y^2, k) = dS$

(3') $p^2 = p^2(p^1_{-1})$

(4) $k = p^1/p^2$

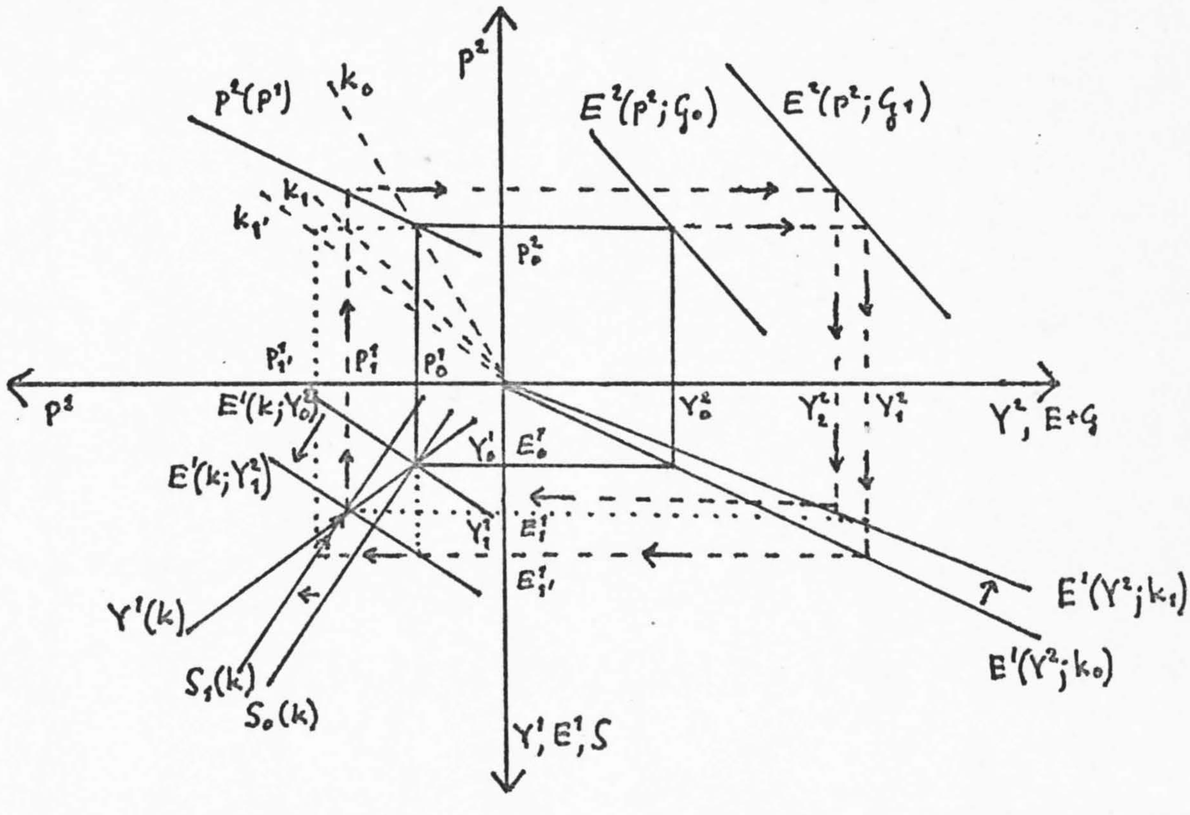
(5) $S(k^e) = S$

(6) $(k^e = k)$

As in version (I) the input cost of flexprice goods in the price equation for fixprice goods (3') is lagged to make graphical analysis of the model easier. In addition to the short-run and long-run equilibria of model (I), a third equilibrium concept is included in model (II). In the very short-run only the market for fixprice goods (eg. 10) and the level of stocks of flexprice goods (eq. 5) are in equilibrium, in the medium-run (the short-run in model I) the demand for and supply of flexprice goods are also in equilibrium i.e. $dS=0$, and in the long-run the prices of fixprice goods are also in equilibrium.

The stock-flow version of the model is analyzed in Figure 2, which is similar to Figure 1 but the stock demand function S(k) of flexprice goods is added. An increase in G shifts the $E^2(p^2;G_0)$-curve to the right to $E^2(p^2;G_1)$. Initially p^2 is given and Y^2 increases to Y_1^2. Although the multiplier process takes time as Hicks (1974, p. 10) has explained, this feature is ignored in the present model.

FIGURE 2



Increased fixprice production Y_1^2 requires increased flexprice input E_1^1, and the $E^1(k;Y_0^2)$-curve shifts to $E^1(k;Y_1^2)$ to the point $(E_{1'}^1, p_0^1)$ price ratio k could instantly reach the new equilibrium k_1 (or p_1^1) at the intersection of $Y^1(k)$ and $E^1(k;Y_1^2)$ curves, the process of adjusting actual flows would, as earlier was assumed,take time. Thus the increased raw material needs must be satisfied from existing stocks, but because traders have perfect foresight of the new price after the shock nobody would sell at the existing price. Because of the perfect foresight of the traders, the $S_0(k)$-curve must shift upwards in the figure 2 to $S_1(k)$, where it goes through the new equilibrium point (E_1^1,p_1^1) of $Y^1(k)$ and $E^1(k;Y_1^2)$-curves. Thus the price or the price ratio of flexprice goods must rise exactly to p_1^1, or k_1,, which is determined by the intersection of a horizontal line at E_1^1, and the $S_1(k)$-curve. At this price the traders start to expect flexprices to decline and start to sell flexprice goods instead of buying them. This is the short-run or momentary equilibrium of the model; fixprice production can satisfy its needs for flexprice goods for a while, but the prices of flexprice goods have risen to p_1^{1}'.

In the medium-run (the short-run of model I), the supply of flexprice goods exceeds demand at price p_1^1,. The traders sell their stocks and the price decreases, but at the same time the supply of flexprice goods increases and the demand decreases until the medium-run equilibrium price p_1^1 is reached. Simultaneously, the $E^1(Y^2;k_0)$-curve rotates to $E^1(Y^2;k_1)$. Thus in the medium-run equilibrium, the price level of fixprice goods remains at the initial level p_0^2, the production of fixprice goods is Y_1^2, expenditure on and production of flexprice goods Y_1^1 and E_1^1, the price of flexprice goods p_1^1 and the price ratio is k_1.

In the long-run, the increased price of flexprice goods raises the price of fixprice goods as explained in model I. This reduces demand for and production of fixprice goods and, further, the need for flexprice good inputs in the production of fixprice goods. This in turn shifts the stock demand curve $S_k(k\ )$ downwards in figure 2 and causes a new adjustment process of stocks and flows of flexprice goods. The resulting change in the price level of flexprice goods affects fixprices, the production of fixprice goods, the consumption of flexprice goods, etc. Finally, the long-run equilibrium is reached at p^{1*}, p^{2*}, Y^{2*}, E^{1*} and Y^{1*}, but the stocks of flexprice goods have remained at the equilibrium corresponding to the current price ratio k, and when this is reached at k^*, the stocks are also at their long-run equilibrium S^*. The time path of the variables is cyclical, and the multipliers are the same as before.

An interesting alternative specification of the model would be the case where the production of flexprice goods is exogenous ($Y^1_k = 0$) and spending on flexprice goods does not respond to changes in price ratio k ($E^1_k = 0$). The model would then describe the case where the supply of raw materials, e.g. energy, is fixed and no substitutes exist. The model is

$$E^2(p^2, Y^2 - k(E^1 - Y^1)) + G = Y^2$$

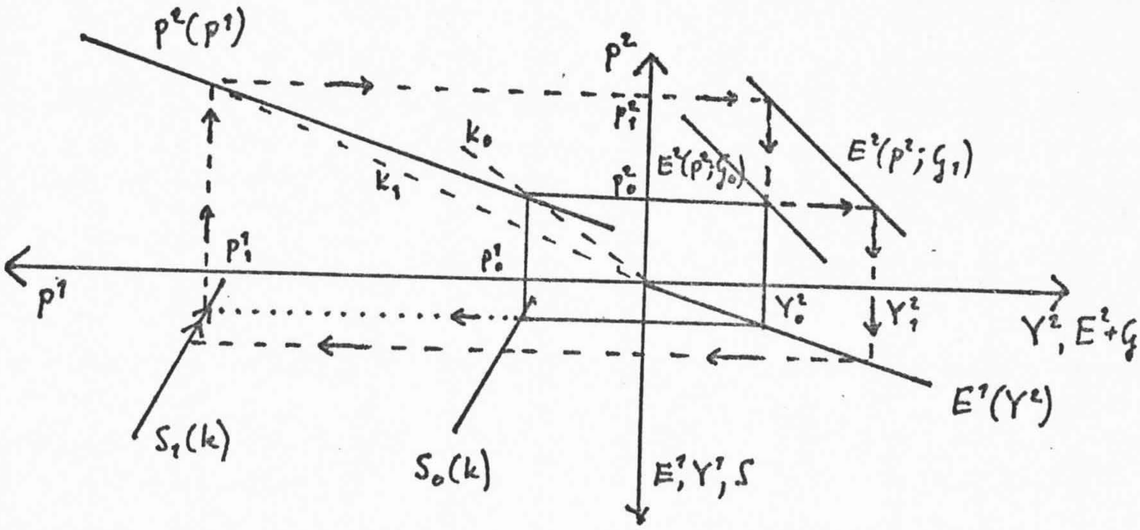
$$E^1(Y^2) - Y^1 = dS$$

$$p^2 = p^2(p^1)$$

$$S(k) = S$$

The model is analyzed in Fig. 3.

FIGURE 3



An increase in G would increase spending on Y^1. Since their production cannot be increased, the increase in demand must be satisfied from existing stocks, but because the stock-holders have rational expectations, the stock demand curve shifts from $S_0(k)$ to $S_1(k)$, passing through the new full equilibrium point (E_0^1, p^{1*}). The instantaneous equilibrium price of flexprice goods may exceed the new full equilibrium price p^{1*} depending on the shape of the stock demand curve. However, the price level of flexprice goods falls to the new equilibrium, where the output of fixprice goods is at the initial level, but the price level of fixprice goods is p^{2*}.

It is clear from the above model that if the full employment level of output of fixprice goods is higher than the initial level, persistent unemployment would result. Hicks has considered this case in his recent book (1977), and he suggests a way to get rid of this problem: change in the consumption and production patterns which would cause substitution of labour for raw material inputs, i.e. the economic model would change.

The stock demand function brings nothing essentially new compred to model (I) except the short-run overshooting of the real value of stocks and prices of flexprice goods. This, however, explains the so-called "overshooting" phenomenon, the volatility of raw material prices (and exchange rates) in the 1970s.

In the following, a mathematical treatment of model (II) is developed. The price equation for fixprice goods is altered adding wages, which, in turn are affected by the Phillips-curve.

4. THE DYNAMIC ADJUSTMENT EQUATIONS OF THE MODEL

In the following, the model is expanded by taking into account price expectations x^2 concerning fixprice goods. These expectations affect positively demand for these goods because a decrease in the expected inflation rate x^2 raises the real rate of interest and discourages expenditure (see Tobin, 1975, p. 197). Thus in full equilibrium the following conditions hold:

$$E^2(p^2,x^2Y^2-k(E^1-Y^1)) + G - Y^2 = 0$$
$$Y^1(k) - E^1(Y^2,k) = 0$$
$$k - k^* = 0$$
$$Y^2-Y^{2*} = 0$$

where k^* is the full equilibrium value of the terms of trade and Y^{2*} is the full employment or "natural" unemployment rate of output of fixprice goods.

The intermediate traders of flexprice goods were assumed to hold stocks because they expected to earn profits or a receive real capital gains. Thus the stock demand function for flexprice goods was written

(5) $S = S\ (x^1-x^2) = S\ (\hat{k}^e)$

where x^1 is the expected change in prices of flexprice goods, x^2 is the expected change in prices of fixprice goods, so that the expected change in the price ratio is $\hat{k}^e = x^1-x^2$.

The traders are assumed to expect that the relative price of flexprice goods k will return to the equilibrium price ratio k^* at speed λ. Thus the traders' expectations adjustment function may be written

(6) $\hat{k}^e = \lambda(k^*-k).$

Substituting for $\hat{k}^e$ in (5) and differentiating S gives

$$(7) \quad \dot{S} = S_k \lambda k \pi^2 - S_k \lambda k \pi^1,$$

where $\pi = \dot{p}/p$. Because the traders have perfect foresight, the chance in stocks due to expectations must be equal to the actual change in stocks, i.e. $\dot{S} = Y^1 - E^1$. Substituting $\dot{S} = Y^1 - E^1$ in (7), the price equation for flexprice goods is

$$(8) \quad \pi^1 \doteq (B_p/k)(E^1 - Y^1) + \pi^2,$$

where $B_p = 1/S_k\lambda$.

The parameter S_k describes the slope of the stock demand curve S(k) at the intersection of the flow demand and supply curves $E^1(k;Y^2)$ and $Y^1(k)$ for flexprice goods. Parameter λ describes the speed of adjustment of the price of flexprice goods (or the price ratio) along the stock demand curve consistent with the speeds of adjustment along the flow demand and supply curves of flexprice goods and also the shifts in these curves arising from the endogenous adjustment of the output of fixprice goods to the changing price p^2 of these goods.

The change in the price of fixprice goods π^2 is assumed to depend on wage and raw material costs[1], where the latter is described by the change in the price of flexprice goods. Wages are assumed to depend on excess demand for labour and on the expected change in prices of fixprice goods. Thus

$$(9) \quad \pi^2 = \beta(A_p(Y^2 - Y^{2*}) + x^2) + (1-\beta)(B_p(E^1 - Y^1)/k + \pi^2)$$

where β is the share of wages in the pricing of fixprice goods. Solving for π^2 yields

---

1. In a study of the price-wage mechanism in the United States, Eckstein and Girola (1978, p. 332) found that independent movements of material prices affect finished goods' prices directly and wages indirectly. Note also the equations in the appendix.

$$(10) \qquad \pi^2 = A_p(Y^2-Y^{2*}) + \beta' B_p(E^1-Y^1)/k + x^2$$

where $\beta' = (1-\beta)/\beta$.

The price expectations relating to fixprice goods are assumed to adjust adaptively. This is in line with the empirical observation that the coefficient of expected prices in the wage equation is smaller than one[1] (Tobin 1972, p. 11). Thus

$$(11) \qquad \dot{x}^2 = A_x(\pi^2-x^2)$$

Output of fixprice goods is assumed to respond in a Keynesian way to the difference between very short-run demand for and supply of these commodities (see Tobin 1975, p. 198). Thus

$$(12) \qquad \dot{Y}^2 = A_y(E^2-Y^2).$$

Equations 10-12 form a dynamic model, where k is exogenous and, consequently, so is Y^1. However, k may be such that full employment output Y^{2*} is never achieved since the output of fixprice goods depends on both labour and raw material inputs. If k is fixed, Y^2 and employment are also fixed and not necessarily at the full employment level.

The model may be expanded by making k endogenous. The dynamic adjustments equation for k is derived by differentiating k, which yields

$$\frac{\dot{k}}{k} = \pi^1 - \pi^2.$$

Using equation (8) to substitute for π gives

$$(13) \qquad \frac{\dot{k}}{k} = B_p(E^1-Y^1)/k.$$

Equations 10-13 thus form a four equation model, in which the ratio of the prices of flexprice and fixprice goods is allowed to change. The output of flexprice goods may also respond to a change in this price ratio at a given speed. This will give an additional equation, but the process is independent of the model given by 10-13 and is not explicitly considered here.

5. THE LOCAL STABILITY OF THE MODELS

The dynamic adjustment equations, for the model are rewritten below. The first three equations of the complete system form a sub-model in which the price ratio k is exogenous.

$$
\begin{aligned}
\dot{Y}^2 &= A_Y(E^2-Y^2) \\
\dot{p}^2 &= A_p(Y^2-Y^{2*})p^2 + \beta' B_p(E^1-Y^1)p^2/k + x^2p^2 \\
\dot{x}^2 &= A_xA_p(Y^2-Y^{2*}) + \beta' A_xB_p(E^1-Y^1) \\
\dot{k} &= B_p(E^1-Y^1)
\end{aligned}
\tag{14}
$$

The following matrix shows the equations of the model after they have been linearized around their equilibrium values $Y^2 = Y^{2*}$, $p^2 = p^{2*}$, $x^2 = 0$, and $Y^1 = Y^{1*}$.

$$
\begin{bmatrix} \dot{Y}^2 \\ \dot{p}^2 \\ \dot{x}^2 \\ \hline \dot{k} \end{bmatrix}
=
\left[\begin{array}{ccc:c}
A_Y[E^2_Y(1-k^*E^1_Y2)-1] & A_YE^2_p2 & A_YE^2_x2 & -A_YE^2_Y(E^1_k-Y^1_k)k^* \\
p^{2*}(A_p+\beta' B_pE^1_Y2/k) & 0 & p^{2*} & \beta' B_p(E^1_k-Y^1_k)p^{2*}/k^* \\
A_x(A_p+\beta' B_pE^1_Y2/k) & 0 & 0 & \beta' B_p(E^1_k-Y^1_k)A_x/k^* \\
\hdashline
B_pE^1_Y2\ k^* & 0 & 0 & B_p(E^1_k-Y^1_k)\ k^*
\end{array}\right]
\cdot
\begin{bmatrix} Y^2-Y^{2*} \\ p^2-p^2 \\ x^2 \\ k-k^* \end{bmatrix}
\tag{15}
$$

The sub-model given by the first three equations (shown by the dotted lines) is much the same as Tobin's WKP-(Walras-Keynes-Phillips) -model. The only difference is the addition of the Marshallian price adjustment mechanism to the price equation. Consequently, the necessary critical condition for the local stability of the sub-model is

(16) $$p^{2*}E^2_p2 + A_x E^2_x 2 < 0,$$

Which is the same condition as in Tobin's model (Tobin 1975, p. 199). The first term of (16) is negative and the second term positive. Thus the effect of the price level on aggregate demand (on demand for fixprice commodities) must be strongly negative and the price expectation effect must be weak and slow to react to experience if the sub-model is to be stable.

The conditions for the local stability of the complete four equation model are difficult to evaluate.However, all the parameters a_n of the characteristic equation of the complete model

(17) $$/A-\lambda I/ = a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

are positive, if

(18) $$p^{2*}E^2_p2 + A_x E^2_x 2 + A_x E^2_x 2[A_p(1-\beta)/k^* \; B_p E^1_Y 2 \;] < 0.$$

Thus the necessary condition for local stability is stronger than in the sub-model, i.e., the price level effect must be stronger than the price expectations effect multiplied by the expectations adjustment speed, by the positive factor given in parentheses in (18).

In summary, even if the necessary critical condition for stability holds, full employment may never be reached in the sub-model if the ratio between the prices of flexprice and fixprice goods (raw materials and manufactured goods) is not allowed to adjust or if $Y_k^1 = E_k^1 = 0$. Even if the price ratio can adjust to the value corresponding to full employment equilibrium, the process will be slower the smaller the response of the supply of and expenditure on flexprice goods to a change of the price ratio. Full employment is also more difficult to reach than in Tobin's WKP-model.

To complete the dynamic analysis of the flexprice-fixprice model, a partial model of two equations is studied graphically.

6. A GRAPHICAL ANALYSIS OF THE DYNAMICS OF A PARTIAL MODEL

In the following, the dynamics and the stability of the two equation model consisting of the price equations for flexprice and fixprice goods (8) and (9) are studied using phase diagrams. To build the equations, it is assumed that expenditure on and output of fixprice commodities are always equal, $Y^2 = E^2$, and that expectations about changes in prices of fixprice goods are exogenous. It is also assumed for simplicity that the price ratio k only affect the sypply of flexprice goods and not expenditure on them. The price equations are

$$\pi^1 = (B_p/k)\{E^1[E^2(p^2,\bar{x}^2,Y^2-k\{E^1(Y^2)-Y^1(k)\})+G] - Y^1(k)\} + \pi^2 \tag{19}$$

$$\pi^2 = \beta A_p\{[E^2(p^2,\bar{x}^2,Y^2-k\{E^1(Y^2)-Y^1(k)\})+G] - Y^{2*}\} + \bar{x}^2 + (1-\beta)\pi^1$$

The model can be linearized around its equilibrium values $k = k^*$ or $p^1 = p^{1*}$ and $p^2 = p^{2*}$:

$$\begin{bmatrix} \dot{p}^1 \\ \dot{p}^2 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \cdot \begin{bmatrix} p^1-p^{1*} \\ p^2-p^{2*} \end{bmatrix} + \begin{bmatrix} F_{13} & F_{14} \\ F_{23} & F_{24} \end{bmatrix} \begin{bmatrix} \bar{x}^2 \\ G \end{bmatrix} \tag{20}$$

where

$$F_{11} = [E^2_Y(\alpha_1+1) - 1]Y^1_p/\Delta$$

$$F_{12} = \{E^2_p 2(\alpha_1+E^1_Y 2) - [E^2_Y(\alpha_1 + 1) - 1]Y^1_p\}/\Delta$$

$$F_{21} = [E^2_Y(\alpha_2 + 1) - 1]Y^1_p/\Delta$$

$$F_{22} = \{E^2_p 2(\alpha_2 + E^1_Y 2) - [E^2_Y(\alpha_2 + 1) - 1]Y^1_p\}/\Delta$$

$$F_{13} = 1+(1+B_p/\beta k^*)E_x^2 2$$

$$F_{14} = (1+B_p/\beta k^*)\Delta$$

$$F_{23} = 1+[1+(1-\beta)B_p/\beta k^*]E_x^2 2/\Delta$$

$$F_{24} = [1+(1-\beta)B_p/\beta k^*]/\Delta$$

and

$$\alpha_1 = \beta k^{*2}A_p/B_p$$

$$\alpha_2 = \beta k^{*2}A_p/(1-\beta)B_p$$

$$\Delta = 1 - E_Y^2(1 - k^*E_Y^1 2).$$

The model is stable, since the stability conditions $F_{11} + F_{22} < 0$ and $F_{11}F_{22} - F_{21}F_{12} > 0$ hold.

The condition $F_{11}F_{22} - F_{12}F_{21} > 0$ implies that the difference between the slopes of the curves for $\dot{p}^1 = 0$ and $\dot{p}^2 = 0$ must be positive in the stable case. Parameters F_{12} and F_{22} are presumably negative (if $E_Y^1 2E_p^2 2 - E_Y^2 + 1 < 0$), but the signs of parameters F_{11} and F_{21} are sensitive to the absolute values of E_Y^2, β, A_p and B_p. When $E_Y^2 = 1$, F_{11} and F_{21} are positive. If $E_Y^2 < 1$ and β is fixed, a high value of B_p relative to A_p results in negative values for F_{11} and F_{21}. If β grows, F_{21} tends to become positive quicker than F_{11}. These three cases are shown in Figures 3-5, of which the first case, when E_Y^2 is close to 1, is the most probable.

In Figure 4, $F_{11}, F_{21} > 0$, and both singular curves for $\dot{p}^1 = 0$ and $\dot{p}^2 = 0$ are upwards sloping. The movement from disequilibrium towards equilibrium is described by the arrows. The phase path (the dotted curve from D to E) describes the dynamic process of the model towards equilibrium. At point D the prices of flexprice goods are at their full equilibrium

FIGURE 4

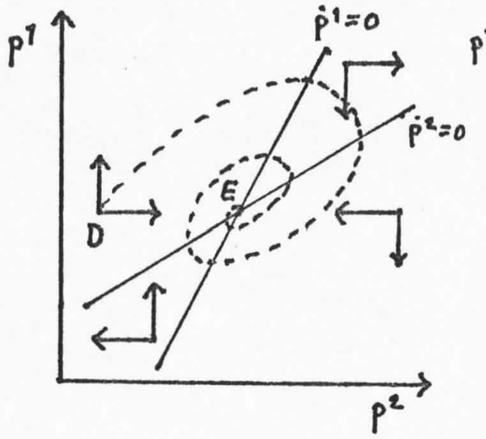


FIGURE 5

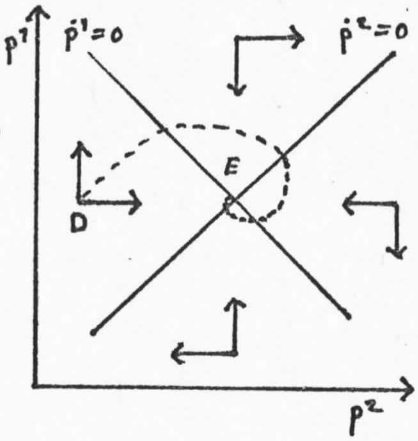
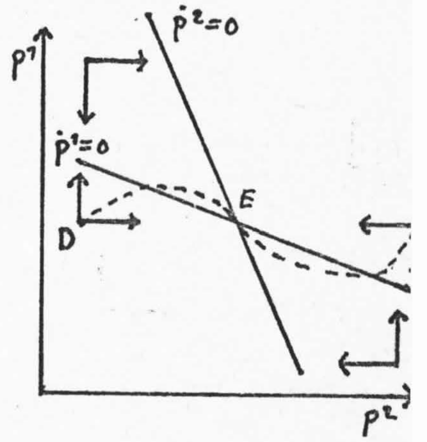


FIGURE 6



level E, but the prices of fixprice commodities are below that level. Spending on and production of fixprice goods tend to increase because of the price level effect $E_p^2 2$, and the price of fixprice goods rises. The supply of flexprice goods also tends to decrease through the price ratio effect Y_k^1, but the increase in spending on flexprice goods generated by the increased output of fixprice and flexprice goods is greater. Thus the prices of flexprice goods also initially increase. However, when they reach their temporary equilibrium value on curve $\dot{p}^1 = 0$ and the price of fixprice goods continues to rise, the supply of flexprice goods also continues to increase but by more than spending on them because of the dampening effect of the higher price of fixprice goods on their output. Thus the price of flexprice goods decreases. When the price of fixprice goods reaches its temporary equilibrium value on the $\dot{p}^2 = 0$-curve, which is greater than E, it starts to decrease, and the price of flexprice goods continues to decrease. At the $\dot{p}^1 = 0$-curve flexprices again start to increase, while fixprices decrease until $\dot{p}^2 = 0$-curve is reached. Thus the process converges, oscillating to the full equilibrium value of both prices at E.

In Figure 5, $F_{11} < 0$ and $F_{12} > 0$, and the process is about the same as in Figure 1. In Figure 6, F_{11}, $F_{12} < 0$, and both singular curves are downward sloping. In this case either flexprices or fixprices can move up and down once before full equilibrium, but the other price must converge towards equilibrium monotonically.

7. AN OPEN ECONOMY "FLEXPRICE-FIXPRICE" MODEL

Following Hicks (1974, p. 29), the models given above may be also used to analyze open economy problems. The flexprice goods are then internationally traded goods (or foreign exchange), their production Y^1 is exports and consumption E^1 represents imports and $\dot{S}=Y^1-E^1$ stands for the change in foreign exchange reserves. The prices of traded goods measured in foreign currency are p^1 and those of domestically produced and consumed goods p^2. The terms of trade are $k = ep^1/p^2$, where e is the exchange rate (domestic currency/foreign currency).

If the exchange rate is fixed and p^1 is determined outside the model (as in the sub-model of section 5) and e is such that k is below its equilibrium value, imports tend to increase and exports and exchange reserves to decrease. Output of traded goods decreases because of the price ratio effect on their export demand Y^1_k and the substitution effect of imports E^1_k used as input in domestic production. The multiplier effects and restrictive economic policy also reduce output of nontraded goods. If e is raised, exports increase, which, however, is not self-evident in the Finnish case because so many of her export prices are determined in the foreign flexprice market (as is shown in the appendix) and imports decrease if $E^1_k < 0$. However, output of non-traded goods increases. If e and k are above their equilibrium value, domestic output increases to full capacity and inflation accelerates.

The complete model of section 5 and the partial model of sections 2,3,6 describe flexible exchange rate cases. The foreign exchange reserves are then always in temporary equilibrium as a result of exchange rate adjustment, and output and prices may reach their stable full equilibrium values. However, cyclical oscillations in output and inflation may result.

Appendix

The estimation results for Finnish foreign trade price equations.

In this appendix the main results of empirical research (Hämäläinen, 1977) on the behaviour of Finnish foreign trade prices are given (t-values of the coefficients are in brackets and ρ is the first order autocorrelation coefficient:

Period (quarterly data)

1) $\dot{PMR\$} = \sum_{j=0}^{6} b_j^{(2)} \dot{WCP}_{t-j}$ (1950.4-1976.2)

$b_{0-6} = .07, .12, .15, .16, .15, .12, .07$ $\quad \sum_j b_j = .82$ (12.5) $\quad R^2 = .90$, $DW = 1.8$, $\rho = .7$

2a) $\dot{WCP} = -.014 + \sum_{j=0}^{2} b_j^{(2)} \dot{IP}_{t-j}$ (1956.4-1976.2)

$b_o = .52, b_1 = .70, b_2 = .52,$ $\quad \sum b_j = 1.74$ (4.9) $\quad R^2 = .84$, $DW = 1.7$, $\rho = .9$

2b) $\dot{WCP} = 4.06 + \sum_{j=0}^{2} b_j^{(2)} \dot{IP}_{t-j}$ (1970.1-1976.2)

$b_o = .95, b_1 = 1.26, b_2 = .95$ $\quad \sum b_j = 3.16$ (5.2) $\quad R^2 = .88$, $DW = 2.1$, $\rho = .8$

3) $\dot{PMV} = \underset{(1.3)}{.008} + \underset{(4.3)}{.43}\, \dot{ULC}_f + \underset{(2.0)}{.18}\, \dot{ULC}_d + \underset{(19.5)}{.93}\, \dot{ER} + \underset{(2.1)}{.26}\, \dot{PMR\$}$ (1956.1-1976.2)

$R^2 = .89$, $DW = 2.0$, $\rho = .2$

4) $\dot{PX} = \underset{(2.4)}{.23}\, \dot{ULC}_d + \underset{(1.7)}{.19}\, \dot{ULC}_f + \underset{(14.5)}{.85}\, \dot{ER} + \underset{(8.5)}{.41}\, \dot{WCP}_{t-3}$ (1956.1-1976.2)

$R^2 = .93$, $DW = 1.9$, $\rho = .7$

PMR$ = dollar price of raw material imports
WCP = The Economist world commodity price (dollar) index
IP = OECD industrial production
PMV = Unit value of finished goods imports in domestic currency
PX = Unit value of total exports in domestic currency
ULC_f, ULC_d = foreign and domestic unit labour cost
ER = Exchange rate

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