Overnight Market Interest Rates and Banks' Demand for Reserves

MARKKU PULLI

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Acknowledgements

Contents

1 Introduction

The objectives of this thesis are twofold. First, the aim is to study the determination of the overnight market interest rate and of banks' demand for reserves in Finland. To gain an insight into and an understanding of these processes, the study reviews relevant empirical and theoretical literature, considers the institutional arrangements in Finland, and finally tries to analyze the working of the overnight market and banks' demand for short-term liquidity both analytically and empirically. One of the reasons for carrying out this investigation is that the Finnish monetary policy system has changed profoundly in recent years as a result of financial market liberalization. The study attempts to shed light on the relations between some key monetary policy variables under this new regime.

The second aim is more general and relates to the problem of describing the effects of uncertainty empirically. A fundamental result of the liquidity management literature, the theoretical framework applied here, is that a bank's demand for reserves depends on the variance of reserves, which is understood to measure the degree of uncertainty. Although few would dispute this result — indeed, it is the way economists have approached the problem of keeping inventories since Edgeworth's time — it has not been treated very thoroughly in empirical research. In traditional regression analysis the variance is treated as a fixed parameter, and therefore it offers very limited scope for modelling the effects of uncertainty empirically. In this study an empirical demand-for-borrowing model is specified under more reasonable assumptions by utilizing recent advances in the econometrics of heteroscedastic processes, namely GARCH methodology. Allowing for changes in the degree of uncertainty provides more scope and more economic content for the empirical application of the traditional reserve model.

From the point of view of this empirical methodology, the aim is to develop a GARCH model with sound theoretical foundations. In much of the work with this technique, the point of departure in analysis has been purely empiricist; the models are designed to account for the fact that in the time series under investigation, large changes tend to cluster. This study attempts to emphasize the implications of the theory for the empirical methodology. Of fundamental interest in empirical work is to test the applicability of the theory of the bank's liquidity management under uncertainty to modelling the overnight market. Because institutional arrangements that are relevant to this study change over time, often rendering

precise estimates from the past obsolete, concentrating on the implications of the theory is, it is to be hoped, a more useful strategy than conducting a purely empirically oriented investigation.

Banks' demand for reserves and the central bank's monetary policy operating procedures are well established research topics. Many of the basic articles on the demand for reserves appeared in the 1960s and early 1970s: for example Orr and Mellon (1961), Whalen (1966), Poole (1968), Frost (1971) and Modigliani, Rashe and Cooper (1972). Comprehensive surveys of this topic are provided by Baltensperger (1980) and Santomero (1984). Changes in the intermediate targets of monetary policy in the USA were followed by a series of articles in which the focus was more on the operating procedures of the central bank, the most notable including Goodfriend (1983) and Poole (1982). One of the more recent contributions is Englund, Hörngren and Viotti (1989). Their study is inspired by the money market conditions and central bank financing system in Sweden, which bear many similarities with circumstances in Finland. Since the end of the credit rationing era in Finland, the determination of short-term interest rates has been studied by Vihriälä (1987). Earlier studies that are related to this subject include Oksanen (1977) and Tarkka (1980).

A general background for the analysis in this study is provided by first presenting a standard model of the reserve market. In this model, the demand for reserves by individual banks is aggregated to form the demand side of the model, the terms applied at the central bank's discount window define the shape of the supply function and monetary policy operations are conducted by horizontally shifting the location of the reserve supply function with respect to banks' total reserves by, for example, buying or selling money market instruments or by adjusting mandatory reserve deposit requirements. The amount of borrowing by banks and the short-term market interest rate are determined at the intersection of the demand and supply curves.

This simple description of the determination of the short-term market interest rate does not seem to fit well with actual observations from the overnight market in Finland. Observed overnight market interest rates are generally not on the supply function of the central bank. Usually, the deviation is several percentage points, which is far too much to be explained by measurement errors. A more plausible explanation is offered in defence of this approach by illustrating how the borrowing schedule changes when liquidity uncertainty is explicitly included in the model of the reserve market. As will be discussed below, under certain conditions that are likely to be met in Finnish circumstances, traditional liquidity management theory is easily

incorporated into the reserve market model without any loss of generality in interpretation.

In order to derive the effects of liquidity uncertainty, a model for a single bank's demand for overnight loans is constructed assuming profits from trade in the overnight market to be a random variable. The demand is solved from an optimization problem based on a balance sheet constraint and expected costs and expected returns from participating in trade. The bank's demand is determined as a function of the market interest rate, the terms for (net) borrowing from the central bank's short-term liquidity facility and the degree of uncertainty the bank is facing. An important feature of the model is that as a result of the nonlinearity introduced into the problem by the shape of the central bank's supply function, the degree of uncertainty changes the optimal behaviour of the bank as compared to the full information case.

In the subsequent analysis of the model, attention is paid to some typically Finnish features of the central bank's facility for liquidity management of the banking system. (Throughout the text this facility is referred to as the 'discount window' or 'call money facility', the first term being more commonly used in the literature while the second accords with the official terminology.) The terms applied to the use of this facility, or the structure of the discount window, have varied in Finland with respect to both administrative regulations and interest rates. In Finland, banks' borrowing has been discouraged by means of a penalizing spread between call money lending and call money deposit rates (the central bank accepts excess funds as demand deposits), penalty rates based on quantitative quotas and additional costs incurred as a result of frequent borrowing. In this study, demand schedules are derived in three cases, which illustrate the effects of these instruments in a simple manner. In all cases it is assumed that there are no restrictions concerning the amount of deposits the bank wishes to make and that deposits earn a constant rate of return. Especially this assumption is closely related to the Finnish system. Of course, other arrangements for deposits of excess funds could be considered in the model.[1]

The first demand schedule is derived assuming that the bank may borrow from the call money facility without limit, but at an interest rate that is higher than the one that is applied to deposits. The result is

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[1] The deposit side of the window could easily be closed in the model by attaching a zero return on deposits. Also, it could be assumed that the bank holds these deposits on its own balance sheet as excess reserves, where they may or may not earn a positive return. This would not necessarily complicate the model, depending, of course, on the exact specification of returns.

an S-shaped schedule, which describes the overnight market interest rate as a weighted average of discount window interest rates. Because of uncertainty, the weights are probabilities of discount window positions. This case is a direct application of a traditional static reserve model under uncertainty, following Poole (1968) and Modigliani, Rasche and Cooper (1970). It is used as a benchmark model in later stages.

The second application of the model concerns the effects of a graduated cost of borrowing function, or a quota system. In this case the demand schedule is a combination of adjoining schedules of the same type as in the basic case. Actually, this follows quite straight-forwardly from the fact that the benchmark model is also a special case of a quota system (a one-step reserve supply function).

In the third application it is assumed that the costs of borrowing are positively related to borrowing in the past. This intertemporal dependency leads to a dynamic model, because the decisions made concerning today's borrowing affect the optimal solution of tomorrow's borrowing.

By aggregating the demand schedules from the single banks case, a market-clearing interest rate can be solved. In the study, aggregation is considered in the benchmark model. The benchmark model is also used to analyze the effects of risk aversion. By assuming that the bank values variability of profits negatively, it is shown that the equilibrium interest rate tends to be higher with risk-averse banks than in the benchmark model.

In all these applications of the model, the differences lie in the specification of the costs of borrowing or in the form of the objective function. A more fundamental extension is made by endogenizing the variance. It is assumed that the bank is able to affect the variance of liquidity shocks through liquidity control measures, which give rise to some cost. This implies that indirectly the variance is also a decision variable to the bank. Combining liquidity control activities with liquidity management decisions yields a two-equation model for the bank's optimal reserves and optimal expenditure on liquidity control; or, effectively, a model with a reserve equation and a variance equation.

In the empirical part of the study, the model is fitted to Finnish daily data from 1987 to 1989. In the estimations, special attention is paid to assumptions concerning the variance of the borrowing function. In this respect, the study is related to a larger body of research.

Empirical literature on discount window borrowing is largely confined to data from the United States. A seminal paper on this subject is Goldfeld and Kane (1966), where borrowing is explained by

the spread between the Federal funds rate and the discount rate using a linear specification. Much of the practical work related to monetary policy operations in the United States has been based on this specification.

In subsequent research, the instability of a simple type of borrowing function has been addressed. A number of studies have employed nonlinear specifications; for example Judd and Scadding (1982) and Peristiani (1991) use polynomials of the spread, and Dutkowsky and Foote (1988) estimate a switching regression model. Instability of the linear model was also noted by Johnson and Spitzer (1981) in the appendix to a report on monetary policy to Congress by the Federal Reserve Board. In addition to nonlinearity, Peristiani (1991) pays attention to heteroscedasticity. He finds that constancy of the variance of the residuals of the borrowing equation is rejected in tests despite nonlinear specification. A theoretical argument explaining this is derived from aggregation considerations assuming that the quantity of borrowing is subject to certain limiting restrictions.

These findings from US data are interesting, as it is argued in this study that observations from Finnish data have similar properties, which, in the Finnish case, can be explained by a simple type of liquidity management model. Thus the theoretical argument that is offered can be derived from maximization of expected profits. The estimations carried out in this study allow for both heteroscedasticity (non-constant conditional variance) and nonlinearity.

To begin with, the model is estimated assuming constant variance. In this specification, the possible presence of liquidity uncertainty is taken into account in the nonlinear functional form, but the degree of uncertainty does not change over time. A theoretically more consistent formulation of the empirical model is obtained when the perceptions of variance by market participants are allowed to change over time. This is achieved by making the variance of liquidity conditional on available information, thereby making it a time-dependent variable. As a result, the empirical model becomes more flexible because the relationship between the market interest rate and liquidity shocks may change over time, depending on volatility. The response of interest rates to reserve shocks may be weaker in the model when the volatility is high than during times when reserves are less volatile. This aspect is certainly embedded in the theoretical analysis, but its implementation in an empirical model is not possible if standard regression analysis techniques are used.

The literature on non-constant conditional variance models, ARCH and GARCH models, has expanded rapidly over the last few years. Seminal papers on the subject were Engle (1982) and Bollerslev

(1986). Hundreds of studies have so far been published (more than two hundred according to Engle and Ng (1991), and the influx of papers has probably been getting stronger since then, if anything). The data used in these applications have usually been on interest rates, stock market prices or exchange rates. However, the liquidity management model offers an appealing framework for applying this method, since a key prediction of the model is that the bank's demand for reserves depends on the variance of reserves. The theory also predicts that the variance enters the demand schedule multiplicatively. This means that it affects the steepness of the schedule only, and not its location. Therefore, the mean equation in the GARCH-M model that is formed is also multiplicative with respect to conditional variance, which constitutes a slight modification to the standard framework. Usually, this "mean" effect is an additive risk premium.

ARCH and GARCH models deal directly with the empirical notion of heteroscedasticity in many financial market time series. In the literature this notion is commonly attributed to Mandelbrot (1963), according to whom in such data "large changes tend to be followed by large changes — of either sign — and small changes tend to be followed by small changes". Mandelbrot (1963) also proposed two other 'stylized facts' associated with such data, namely that the empirical distributions are usually more peaked and that their tails are too long as compared to samples from normally distributed populations. In this study, similar distributional issues are encountered, and to cope with them we apply a less restrictive distribution than the normal distribution (Student's t). As often seems to be the case with financial market data, allowing for heavier tails for the distribution of shocks does improve the model.

Another important feature of the empirical model is the specification of the conditional variance equation. Following normal procedures, standard autoregressive specifications are applied in the first stage. But, this study differs in that it also offers a theoretical explanation for the presence of heteroscedasticity. An expression for optimal variance is derived by endogenizing variance. Utilizing this in specifying the empirical equation for conditional variance proves to be justified. Theoretical considerations lead to a specification of conditional variance that implies asymmetric responses to shocks, which has been one of the empirical issues dealt with in recent work with GARCH and ARCH models.

In the following, a short introduction to the literature on discount window borrowing is provided in Chapter 2. Chapter 3 contains a description of the overnight market and the institutional arrangements for discount window borrowing in Finland. In Chapter 4, a model for

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the bank's demand for reserves is formulated, based on optimizing behaviour by the bank in the presence of liquidity uncertainty and taking into account the specific circumstances in Finland. In Chapter 5, the implications of liquidity control activities are considered and the model is extended to a two-equation framework with equations both for the level of reserves and for their variance. Empirical constant variance applications of the model are discussed in Chapter 6 and this is followed by GARCH estimations in Chapter 7. A brief summary and conclusions are presented in the final chapter.

2 An Overview of the Theory of Banks' Demand for Reserves

The first section of this chapter reviews the aggregate level reserve market model. The aim is to illustrate the relevance for monetary policy of studying the determination of the overnight market interest rate and the discount window borrowing function. Next, some key studies on the borrowing function are reviewed. The third sub-section looks beneath the market level model and presents liquidity management theory as a theoretical justification for the bank's demand for reserves, based on the optimization behaviour of individual banks. The final section discusses the choices made in defining the theoretical framework for the present study.

2.1 The market for reserves

2.1.1 Determination of the short-term interest rate

The reserve market model has been used extensively in analyzing the determination of the short-term interest rate and the effects of different monetary policy procedures and discount window policies. For applications utilizing this framework, see for example Poole (1982), Tabellini (1987), Hardouvelis (1987), Dotsey (1989, 1985), Goodfriend et al. (1986), Cosimano (1987), Roley and Walsh (1985) and Thornton (1988). In the standard form, the model can be summarized in terms of the following set of equations:

E1. $TR^D = cD(i) + ex(i)$

E2. $TR^S = N + B$

E3. $B = B(i, i^{cb})$

E4. $TR^D = TR^S$.

In this model, the demand for reserves by the banking system, TR^D, consists of required reserves, which are a product of the reserve requirement c and deposits D, and excess reserves, ex. Both of these demand components can be thought to be negatively dependent on the

short-term interest rate i, so that the demand curve is downward sloping.

Equation E2 states that the supply of reserves to the banking system is a sum of nonborrowed reserves N and borrowed reserves B. Nonborrowed reserves are exogenous in this simplistic form of the model and the amount of borrowed reserves is determined by the borrowing function in E3. The borrowing function is essentially a supply function in this framework, reflecting supply side factors such as the terms for discount window loans.[1] The last equation is the equilibrium condition.

Combining equations E1—E4 yields

E5. $N - cD(i) = ex(i) - B(i,i^{cb})$,

which is the equation for free reserves. As explained in Poole (1982), this concept is relevant at the level of the banking system and in policy considerations, as it indicates whether the whole banking system has net debt or net holdings with respect to the central bank. A tightening of the banks' position with respect to the central bank is shown as a contraction in free reserves, and vice versa. It should be noted that in this general formulation of the model, the allocation of free reserves between excess reserves (ex) and borrowing from the discount window (B) is not addressed. As regards the liquidity management decision by the banks, this model is ambiguous (Poole, 1982, pp. 581—582).

Solving E5 for the interest rate gives the market-clearing interest rate as a function of the exogenous variables in the model, which in this example are nonborrowed reserves, the reserve requirement, and the central bank's discount window interest rate:

E6. $i = i(N,c,i^{cb})$.

If the level of deposits is also taken as exogenous, the market interest rate can be expressed as a function of free reserves and the central bank's interest rate:

E6'. $i = i(N-cD,i^{cb})$.

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[1] Thornton (1988) discusses the relative merits of "supply" and "demand" views concerning the borrowing function. Formal analysis is not, of course, affected by any labels attached to functions.

Figure 2.1 illustrates the working of the model graphically, assuming a type of discount window that is also relevant to the Finnish system (Kneeshaw and Van den Berg, 1989, pp. 12—14; see also Englund, Hörngren and Viotti, 1989). It is assumed in the figure that the central bank offers reserves to banks through the discount window at an interest rate that gradually rises with the level of borrowing and that the central bank takes excess reserves as deposits, for which it pays a rate of interest equal to rd. The supply curve S is then the sum of the supply of (net) borrowed reserves defined by the above conditions and the amount of nonborrowed reserves N (which is assumed to be controlled by the central bank). Given the demand for reserves, the figure shows the determination of the short-term interest rate as a function of total reserves. It is implicitly assumed that the interbank overnight interest rate must, because of arbitrage, equal the marginal cost of borrowing from the central bank.

Figure 2.1 **The market for reserves**
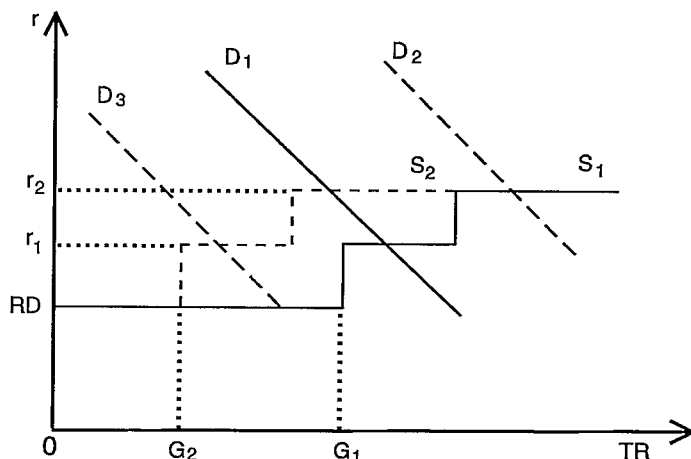
Consider first the effect of an exogenous increase in the demand for reserves by the banking system. A shift of the demand curve from D_1 to D_2 moves banks to the right along the supply curve of the central bank, leading to an increase in the equilibrium interest rate from r_1 to r_2. If the central bank's supply of reserves is totally elastic at the interest rate level r_2, as is assumed in the figure, the equilibrium interest rate is restricted to that level. The market interest rate cannot exceed r_2 because that would imply unlimited profit opportunities for arbitrage between the interbank market and the discount window. Similarly, a downward shift in demand to D_3 lowers the equilibrium

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rate to rd. That level constitutes a floor to the market interest rate if the central bank accepts excess reserves as deposits at the interest rate rd. If it were not willing to accept excess reserves at this rate, the market interest rate would fall further.

The effect of an open market operation, a change in N, can be described as a shift in the supply curve. For example, assuming that the amount of liquidity is cut from OG_1 to OG_2 by reducing N, the supply of reserves curve shifts from S_1 to S_2. Given the demand for reserves, the equilibrium level of the interest rate rises from r_1 to r_2.

This framework can be used to illustrate how different discount window arrangements affect the relationship between market operations and the market interest rate. For example, if the central bank supplies reserves without limit and at a fixed interest rate and pays an equivalent interest rate on deposits, the supply function would be horizontal. In that case open market operations, or changes in nonborrowed reserves in general, would not affect market interest rates. If the level of liquidity in the banking system were changed, the banks would borrow an equal amount from the discount window at the prevailing interest rate and the only change would be in the composition of reserves in favour of borrowing. Market interest rates could be influenced only by changing the discount interest rate.

Continuing the same reasoning, changing the level of nonborrowed reserves will have more impact on interest rates the closer the central bank's supply of reserves is to being a vertical supply function. If the discount window is completely closed, changes in the interest rate are determined solely by the elasticity of the demand for reserves. This elasticity may be considered to be small in the short term, at least as far as required reserves are concerned. Even if excess reserves were assumed to be slightly more elastic, a likely prediction of the model is that interest rates would become considerably more unstable if the supply function were vertical. While this prediction might be reasonable as such, the analysis is obviously incomplete. What one would really expect to happen if the window were closed completely is that the bank would demand more excess reserves in order to compensate for the loss of the central bank's liquidity services. From that point of view it could be argued that it is an unsatisfactory feature of the model that demand for borrowing and demand for excess reserves are considered to be independent of each other.

Including the excess reserves component separately in the demand-for-reserves equation and in the equation for free reserves is probably more appropriate as regards circumstances in the United States than in the case of the Finnish system. In fact, it can be argued that in the Finnish case the right-hand side in E5 should include only one

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variable; or, at least, that the distinction between excess reserves and borrowing is not very informative, because excess reserves can be considered to be negative borrowing. In Finland, any surplus funds are deposited in the central bank through the call money facility. Like borrowing, these 'excess reserves' can also be viewed as being determined as a function of the central bank's interest rates and the market interest rate. In the economic sense, it is hard to discern any dissimilarity between these variables, except for their sign. Moreover, in Finland, excess reserves automatically reduce the amount of a bank's discount window borrowing because these variables are also technically one and the same instrument (same cheque account). In practice, both borrowing and excess reserves are sometimes observed on the same day at the aggregate level, because some banks may have deposits while others have debts. However, that does not make the distinction between excess reserves and borrowing particularly important. One relevant reason for separating these variables could arise from the banks' need to hold cash as excess reserves independently of their borrowing position. But, in Finland, changes in a bank's cash reserves are also automatically reflected in its net borrowing position at the central bank, with cost consequences identical to those that result from direct borrowing.[2] Therefore, not even the cash component need be considered independently.

2.1.2 The overnight market interest rate, the call money facility and monetary policy

From the point of view of monetary policy, the importance of the call money facility derives from the fact that it affects the way changes in liquidity (including open market operations) are related to changes in the overnight market interest rate. Because of this relationship, discount window policies play a key role in monetary policy operating procedures. At the same time it should be emphasized, however, that this study investigates only a strictly limited part of the process of interest rate determination.

The economic relevance of the overnight market interest rate is largely indirect. According to conventional thinking, the overnight interest rate is an important link in the chain from the liquidity of the banking system to longer-term interest rates, although it is not

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[2] This results from a special cash credit arrangement (till-money credit) under which cash reserves are regarded as interest-free loans from the central bank, except for a certain basic amount.

necessarily — at least in Finland — a major target of monetary policy. Of course, the connection between the overnight interest rate and the rest of the yield curve is a research area in itself, and far from being a resolved issue (see for example Shiller, 1990, for a survey of this field). One of the most commonly cited theories on the term structure of interest rates is the rational expectations hypothesis, which states that all interest rates in the yield curve are weighted averages of today's overnight interest rate and expected overnight interest rates in the future plus some premium. This hypothesis implies that a change in today's overnight interest rate feeds into the whole yield curve, *ceteris paribus*, with a declining impact. The condition *'ceteris paribus'* is needed because the effect may be reinforced or restrained by changes in expectations of future overnight rates. However, the empirical evidence concerning the rational expectations hypothesis is mixed. In particular, studies indicate that the premium may not be constant, as the rational expectations hypothesis implies. But that does not imply that overnight market interest rates do not influence the yield curve; in fact, despite the unsettled issues in the theory of the term structure of interest rates, the convention is to view the rest of the yield curve as being influenced by changes in the overnight interest rate.

The determination of the overnight market interest rate in Finland is comprehensibly described in terms of the above reserve market framework, though the openness of the Finnish money market implies that it is reasonable to confine this simple analysis solely to short-term developments. In accordance with the model description, in Finland the central bank influences market interest rates primarily by regulating the total amount of reserves available to banks. The main instruments used for this purpose are the reserve requirement, which is fixed in advance for a month, and money market operations, which can be used more flexibly. In an open economy all fluctuations in liquidity are absorbed via three channels: the public's demand for money, capital movements and banks' reserves. The most immediate is a change in reserves, which under the Finnish system is reflected directly in banks' use of the call money facility. Changes in banks' reserves and in banks' use of the call money facility are, in turn, reflected in short-term interest rates, because for every single bank, the alternative to dealing with the central bank is to buy or sell funds in the interbank market. If this market functions competitively, the interest rate on overnight loans will be such that the demand for and supply of reserves are equal, given the terms for the use of the discount window.

In an open economy changes in interest rates will eventually have an impact on capital flows, which will tend to counteract the changes in liquidity or lead to changes in the exchange rate. Because of the fixed exchange rate regime,[3] absorbing changes in the exchange rate is a limited option. Thus, ultimately, the amount of reserves in the banking system and the corresponding overnight market interest rate are not under the central bank's control in Finland. In terms of the reserve market model, the fixed exchange rate target creates an intervention obligation that effectively endogenizes nonborrowed reserves over a longer horizon.[4]

Consequently, the levels of the overnight market interest rate and banks' reserves are affected by discount window policies only in the short term. By affecting developments in the short term, the form of the call money facility affects the volatility. The above analysis of the reserve model in Figure 2.1 implies that the overnight market interest rate becomes more responsive to all liquidity shocks when the supply function becomes steeper. This suggests that overnight interest rates would, in principle, tend to be more volatile the steeper is the supply-of-reserves function. (This reasoning cannot be generalized to other interest rates if the rational expectations hypothesis is accepted because longer rates are also affected by expectations of future overnight market interest rates.) But, of course, the outcome also depends on the actual policies adopted by the central bank in supplying nonborrowed reserves.

The focus of this study is on the short-term behaviour of reserves and the overnight interest rate and particularly on investigating the properties of the borrowing function. Nonborrowed reserves are taken as exogenous and the central bank's objective function is not included in the analysis. This implies that the positive question of what kind of institutions and other arrangements we should have for conducting

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[3] The exchange rate of the Finnish markka vis-à-vis a specified basket of foreign currencies (since June 1991 vis-à-vis the Ecu) must be kept within a prescribed band, which has been 4.5—6 per cent in recent years.

[4] Effects of procedures used by the central bank in choosing the amount of nonborrowed reserves N have been studied on several occasions. If N is specified as a function of some target variables, the equilibrium interest rate in equation E6 can also be expressed as a function of these targets. The focus in much of the literature has been on the effects of operating procedures and discount window policies on monetary control (money supply), which is not a reasonable point of departure in Finnish circumstances. For example, Dotsey (1989) presents a straightforward modification of this model. Feinman (1988) provides a detailed analysis of the effects of actual practices used in the United States during the period of nonborrowed reserves targeting (see also Goodfriend, 1985). Questions arising from secrecy concerning the target level of N are analyzed in Tabellini (1987).

monetary policy is not addressed. But the analysis of the borrowing function sheds light on the issue of choosing the intermediate target of monetary policy. The analysis that follows focuses on the instability of the borrowing-interest rate relationship: the point of departure is that the observations are not as well behaved as would be expected on the basis of the model of the reserve market. In the theoretical section we will derive explanations for instability, and the empirical description also emphasizes this matter. The existence of instability implies, of course, that it would be difficult to set the targets of monetary policy in terms of borrowing.

2.2   Studies on the borrowing function

At the core of the analysis of the reserve market is a borrowing function, which expresses the amount of banks' discount window borrowing as a function of variables defining the relative attractiveness of obtaining funds from the central bank as compared to other funding. Except for the elasticity of the demand for reserves, other equations in the basic framework are merely identities. If the demand for reserves is also taken as exogenous and assumed to be dictated by the balance sheet constraint, the model collapses to a borrowing function.

In the literature the borrowing function is usually derived from or justified by profit-maximizing behaviour by banks. When faced with reserve needs, the bank is thought to choose between different sources of funding on the basis of relative costs. Therefore, sources can be logically divided into the administratively priced discount window, on the one hand, and market-priced sources, on the other. As the literature is mainly concerned with the system in the United States, these models have had to be modified to take into account the fact that the discount window interest rate in the United States has generally been below the Federal funds rate, suggesting unlimited profit opportunities. But this need not be a problem, of course; it has been recognized, at least since the paper by Goldfeld and Kane (1966), that implicit costs related to borrowing from the discount window may explain this apparent contradiction.[5] In their formulation, borrowing by banks was explained by a spread between the market interest rate and the discount window interest rate. In equilibrium, the effective cost of borrowing from the discount window and from the money market

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[5] An early treatment of this subject can also be found in Polakoff (1960).

should be the same for all banks. The presence of a spread between nominal interest rates has been taken to imply non-price rationing by the authorities.

In the subsequent empirical work this analysis has been extended to take into account nonlinearities in the relationship between borrowing and the interest rate spread. For example, Judd and Scadding (1982) use a zero dummy variable for negative values of spreads and a square root of the spread for positive values. Tinsley et al. (1982) use a hyperbolic function with different weights for negative and positive spreads. Anderson and Rasche (1982) also modify the basic formulation by applying additional dummy variables for some values of the interest rate spread.

Theoretical explanations for the presence of nonlinearities in the static Godfeld-Kane framework have been developed and empirically tested in Dutkowsky (1984), Dutkowsky and Foote (1988) and in a recent study by Peristiani (1991). The explanation offered by Dutkowsky is based on truncation of the borrowing variable from zero and implicit costs attached to borrowing, which together, according to the analysis, introduce an unknown switching point into the borrowing schedule. Switching behaviour results from fact that borrowing is always restricted to zero when the spread is unfavourable, while a normal, continuous borrowing function describes the behaviour when the spread exceeds some threshold value. In Dutkowsky and Foote (1985), this model is extended to a three-regime switching model, in which the middle regime results from aggregation considerations and produces additional nonlinearities.

In Peristiani (1991), it is assumed that the bank's borrowing is subject to collateral limitations. This imposes an upper bound on the amount of borrowing, so that in effect the borrowing variable is truncated from both ends: the lower point of truncation is zero and the upper point of truncation is the collateral limit. Because of this censoring structure, the demand-for-borrowing schedule becomes S-shaped. Again, the explanation is country-specific: neither collateral limitations nor a zero restriction on (net) borrowing have been relevant constraints in Finland. But technically, this solution utilizes exactly the same mechanism that will produce the nonlinearity in the model considered in this study, the difference being that it is the graduated discount window interest rate structure that causes censoring in the Finnish case.

In addition to nonlinearity, the censored structure of the model produces heteroscedasticity when aggregation is taken into account. In an empirical application, Peristiani finds that applying a nonlinear borrowing schedule (eight order polynomial of the spread) will indeed

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result in heteroscedastic errors. The tests presented are supportive of ARCH-type violations (Peristiani, 1991, p. 27), but this finding is not utilized. The model is estimated only under a constant variance assumption. Simulations with the model are presented in Hamdani and Peristiani (1991), where it is shown that the model is capable of producing predictions that imply nonlinearity and heteroscedasticity.

An important distinction in demand-for-borrowing models, at least from the point of view of analytical methods, concerns the time horizon involved. In a seminal article on dynamic discount window policies, Goodfriend (1983) shows how the intertemporal aspects of administrative credit rationing in the United States lead to a dynamic demand schedule for banks' discount window borrowing. In the United States, banks are discouraged by the authorities from borrowing too frequently and/or excessively. Although the intertemporal aspects of the US system are caused by implicit costs related to frequent borrowing, the implicit non-price rationing itself is not an essential part of the model. A comparable demand schedule could be derived from an explicit, dynamic cost of borrowing function.

The banks' optimization problem as formulated by Goodfriend is to choose, at each time period t, a sequence of borrowing w_{t+j} that maximizes the present value of profits from arbitrage between the interbank market and the discount window:

$$\Pi_t = \sum_{j=0}^{\infty} b^j [\delta_{t+j} w_{t+j} - C_{t+j}]$$

$$= \sum_{j=0}^{\infty} b^j [\delta_{t+j} w_{t+j} - c_0 w_{t+j}^2 - 2c_1 w_{t+j} w_{t+j-1} - d_{t+j} w_{t+j}],$$

(2.1)

where $c0$, c_1, $d_t > 0$ and w_t, $w_{t-1} \geq 0$.

π = present value of profits, C = cost of borrowing from the discount window, δ = interbank overnight interest rate, d = discount window interest rate, w = discount window borrowing, b = constant rate of time discount.

In specifying the cost function C, Goodfriend (1983) makes the assumption that the cost of borrowing not only increases progressively with the level of borrowing, but also depends on the level of borrowing the previous day. The quadratic form of the cost function is intended to capture the essential elements of the system in the United States. It is defined only for positive levels of borrowing, again in accordance with the system in the United States.

Because lagged borrowing enters into today's costs, borrowing today also affects profits both today and tomorrow. Differentiating today's and tomorrow's (j = 0,1) profits with respect to w_t gives the following Euler equation:

$$\delta_t = 2c_0 w_t + 2c_1 w_{t-1} + 2bc_1 w_{t+1} + d_t.$$ (2.2)

This Euler equation is a second order difference equation in borrowing and it defines the optimal path from date t to infinity. The solution without leads in borrowing is shown by Goodfriend (1983) to be the following:

$$w_t = k_1 w_{t-1} - (k_2)^{-1} h(\delta_t - d_t) - h \sum_{i=2}^{\infty} (k_2)^{-i} (\delta_{t+1-i} - d_{t+1-i}),$$ (2.3)

where $h = 1/(2bc_1)$ and $-1 < k_1 < 0$, $k_2 < -1$.

Equation (2.3) shows that current borrowing depends negatively on past borrowing and positively on current and future spreads between the discount rate and overnight market interest rate. It is a perfect foresight solution, so that in order to derive an operational rule for the bank's borrowing, some process of expectations formation should be specified. This process is probably influenced by the policy of the central bank. In Goodfriend (1983), a case of an autoregressive process is considered. He concludes that the dynamic structure of the demand schedule is a likely cause of the instability in static borrowing functions and that the Fed should make clear its policy intentions towards the spread if it wants to avoid variability in borrowing. The above solution to the Euler equation implies cyclical adjustment of borrowing to liquidity shocks, if parameters c_0, c_1 and b are such that the roots are real valued. Later, Van Hoose (1987) studied policy rules for setting the discount window interest rate on the basis of overnight interest rates, which effectively means the same as setting the spread. He found that borrowing variability is minimized by keeping the spread steady (by adjusting the discount rate accordingly or by pegging the overnight rate to the discount rate using open market operations).

2.3 Liquidity management theory

A long-standing tradition in the research on bank behaviour has been to analyze the bank's decision on liquidity management as a problem of inventory optimization under stochastic demand. Basically, these models consider the optimal amount of reserves that a bank is willing to hold as a consequence of uncertain deposit levels. Holding reserves yields a return to the bank by preventing costs from unexpected deposit drains, but the other side of the bargain is the opportunity cost of reduced lending. The problem the bank faces is to allocate its funds optimally between reserves and lending on the basis of costs and returns related to different assets.

The structure of standard liquidity management models can be presented in a compact form.[6] Assume that a bank has two assets it can choose as investments, loans yielding a net rate of return r and non-interest-bearing reserves (R). It also has a given level of deposits (D), which is subject to withdrawal risk. The net amount of withdrawals is denoted by X, and is distributed with a density function f(X). The bank knows the distribution of X but not its realizations in advance.

If withdrawals exceed reserves, $X > R$, the bank must make up for the deficiency by selling assets or by borrowing. Obtaining additional funds is assumed to be subject to a proportional cost p. The problem is then to maximize the profits from

$$\pi = r(D-R) - \int_R^\infty p(X-R)f(X)d(X).\tag{2.4}$$

Optimization of profits requires that the marginal cost of holding reserves and the marginal return from reserves are equalized, which implies

$$r = p\int_R^\infty f(X)d(X).\tag{2.5}$$

This first order condition, which states that the probability of reserve deficiencies must equal the ratio r/p, characterizes all liquidity

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[6] This presentation of liquidity management theory follows the surveys by Baltensberger (1980) and Santomero (1984).

management models. It has some implications which should be briefly noted.

First, Poole (1968) has shown that the effect of increased uncertainty (mean preserving increase in variance) on the level of reserves is ambiguous, assuming that the distribution of X is symmetric with zero mean. This result is obviously in accordance with intuition, because an increase in variance clearly implies increased probability of large excessive positions on both sides. There is no *a priori* reason for changes on either side to dominate.

Secondly, for the optimal reserves to be positive, it is required that $p > 2r$, given again that the mean of X is zero. This can be seen by setting $R = 0$ in the first order condition, which yields $r/p = 1/2$. Therefore, in order to have an optimal solution in a range $R > 0$, $r/p < 1/2$ must hold. This simple result has potentially important implications. If a well functioning money market exists, it is unlikely that the inequality will hold generally, even if we note that p may include transaction costs and other possible inconveniences. A more likely prediction of the model is that we should observe negative reserves, at least in some time periods.

In the vast literature in this area, the basic liquidity management model has been extended to include other aspects which could be of importance in liquidity management decisions. These additional factors may be institutional circumstances affecting profits, for example legal reserve requirements on deposits. Another direction in which the model has been extended is to consider ways the bank can affect the parameters in equation (2.1) by its own actions, other than choosing the level of reserves.

The existence of legal reserve requirements does not alter the logic of the model, although the details depend on specific rules governing the computation of required reserves and the form in which they are held. A general effect of reserve requirements is a shift in the measure of reserve deficiency, so that excess levels of reserves start from where the legal reserve requirement is met, and not from zero. The optimum condition is still that the cost of holding reserves must equal the cost of obtaining new funds, multiplied by the probability of reserve deficiency (Baltensberger, 1980).

Frost (1971) was the first to consider adjustment costs in a reserve model (see also Baltensberger, 1972a). It is clear that without adjustment costs, the level of reserves in the preceding period is irrelevant in the basic formulation of the model because the bank may rearrange its portfolio continuously. But when adjusting reserves is costly, it is done only to the extent it increases profits; i.e. the cost of adjustment is less than the resulting gain. An interesting consequence

of this is that, in the presence of adjustment costs, no adjustments are made within a certain range around the otherwise optimal level of reserves. Within this range, changes in observed reserves would therefore follow variation in the bank's deposits. Further, any adjustment would always be incomplete as adjustments would never be made to the extent that the limits of this range would be exceeded.

Other extensions of the model take into account the fact that the bank may be able to affect the degree of uncertainty related to its balance sheet or to affect the interest rate it receives on loans (see, for example, Tobin, 1982). These assumptions lead to interesting modifications of the optimality condition (2.5). If it is assumed that the bank faces a declining demand function, the interest rate becomes dependent on the amount of credit extended. As a result, the optimality condition includes a derivative of a revenue function instead of a constant interest rate on loans.

If, on the other hand, the distribution of X is made conditional on available information, which is costly to acquire, the optimality condition requires that the expected marginal return on investment in collecting information be equal to the associated expected marginal costs (Baltensberger, 1972b, Baltensberger and Milde, 1976). Endogenous uncertainty has important implications for reserve demand, as it implies that the burden of adjustment due to changes in interest rates is shared between expenditure on information and precautionary demand for reserves. We shall utilize this literature in the analysis in Chapter 5.

Alternatively, it has been assumed that the bank can influence the distribution of X by diversifying its deposit portfolio (Baltensberger, 1972a). By utilizing the law of large numbers and by concentrating on time deposits, the bank can reduce the optimal level of reserves. This path of modelling is evidently a step towards more complete models of a banking firm, which, however, goes beyond the liquidity management model considered here.

Some reviewers of the liquidity management literature (Santomero, 1984, in particular) have pointed out that there seems to be an apparent contradiction between the sophisticated state of the theory and the modest empirical relevance that banks' total excess reserves have — also in the United States — as a balance sheet item. Indeed, as a model of a banking firm, the liquidity management model is quite restricted and compact. But the interest in this model arises not only from its applicability as a micro-level description of bank behaviour, but also from its usefulness as a framework in which to model the demand for short-term assets in the economy and borrowing from the central bank.

2.4 The theoretical framework of the study

The approach taken in this study is to apply liquidity management theory in modelling banks' demand for borrowing from the central bank in a way that is consistent with the broader framework of the market for reserves. Often the liquidity management model is used to describe the demand for excess reserves, but, as explained above, at least in the Finnish case we can combine both negative and positive reserves in one reserve component only. Therefore, the simultaneity problem of two alternative reserve buffers that confuses the link between the traditional liquidity management model and the reserve market model does not exist. Banks' free reserves consist of net borrowing from the central bank only. The analysis that will be undertaken does not particularly depend on the aggregate level reserve market model of Section 2.1. However, the above discussion (it is to be hoped) helps to relate this analysis to more general descriptions of interest rate determination by showing that it is a special case of the reserve market model and has the same, broader interpretation. The crucial assumptions that are needed in order to make this interpretation are that no independent instrument for excess reserves exists and that the demand for total reserves is exogenous. In the Finnish case, the former is clearly fulfilled and the latter is at least reasonable in short-term analysis.

The liquidity management model has the appealing feature that it explicitly considers the uncertainty banks face. Of course, the existence of uncertainty must underlie all reserve literature, because there would be no reason for a bank to hold reserves if information about future needs were perfect. Even over a horizon of one day there are several factors which can cause the liquidity position of the bank at the end of the day to deviate from what was expected in the morning. A large part of all transfers from and to customers' accounts are not known in advance. The public's demand for money is a random variable for banks even in the short term, though they may try to predict it on the basis of seasonal variation associated with timing of pay-days, weekends, holidays, etc. Also, in extending loans, banks presumably must often meet their customers needs at short notice "and scramble, if necessary, for funds later", simply because it is a business to them, and one of their major economic functions (Poole, 1982, p. 582). The same argument even applies to banks' own operations in the money market. Fixing their position at a given level would prevent them form engaging in potentially profitable transactions.

When the application of the liquidity management model to the Finnish banking environment is considered, heterogeneity among

banks seems to be a feature that should be allowed for in the analysis. It is likely that banks differ with respect to the information they have for liquidity management decisions. A part of the banking sector consists of small banks which are relatively active in the money market and in the foreign exchange market, but which do not have large retail operations. It is possible that a bank without any significant retail business could more easily match transactions and in that way reduce the probability of a reserve deficiency. On the other hand, the law of large numbers works to the benefit of large retail banks, so *a priori* conclusions cannot be made. Also, it is possible that a bank with Treasury funds on its balance sheet faces a completely different degree of uncertainty as compared with other banks because it has one very large customer.

In the subsequent analysis the uncertainty facing banks is treated as exogenous in Chapter 4 and the effects of relaxing this assumption are studied separately in Chapter 5. Assuming exogenous uncertainty does not imply that the analysis does not allow for differences between banks. To the extent that the differences are caused by different volatility in reserves, their existence fits well into the framework of the basic model. The extension of the model to investments in gathering information and other activities affecting the liquidity management decision are of interest in this study mainly because, with endogenous variance, the relationship between reserves and the market interest rate is changed. With endogenous variance, changes in interest rates are reflected both in banks' willingness to hold reserves and in their efforts to control liquidity.

An important and restricting choice is made by assuming that the supply of nonborrowed reserves is exogenous. As was noted above, in several studies nonborrowed reserves have been specified as a function of the central bank's target variables, so that the model includes a policy rule for the central bank's intervention in the money market. Usually, these policy rules are at least to some extent *ad hoc* specifications, but in the Finnish case it would be especially hard to define such a rule explicitly. Certainly, any realistic specification should include the exchange rate, because of the fixed exchange rate regime in force in Finland. But, another important target might be the level of interest rates, because the exchange rate is allowed to fluctuate within a band and, moreover, the weights assigned to the exchange rate and interest rates are not necessarily constant over time. Interesting as it would be to include both the yield curve and the exchange rate block in this analysis, it is certainly beyond our ability to derive any estimable equations in this way. In a theoretical study by Englund, Hörngren and Viotti (1989) one longer-term money market interest

rate is added to the model on the assumption that the exchange rate is fixed, which has strong implications. The course taken here is to focus attention on short-term developments only, which means that even the level of nonborrowed reserves is exogenous.

Before turning to our own application of the liquidity management model, the institutional framework and some characteristics of the relevant data are first discussed in the next chapter.

3 The Market for Overnight Funds and the Call Money Facility in Finland

3.1 Evolution and current structure of the call money facility

3.1.1 Historical background

The evolution of the facilities used in the management of liquidity changes in the banking system in Finland has been closely interrelated with the development of financial markets and the whole central bank financing system. Earlier, when the financial markets were characterized by credit rationing, the central bank's role in providing liquidity management services was considerably more important than it has been since the emergence of interbank markets. Over the last decades, there has been a gradual shift towards a system in which banks rely quantitatively less on the central bank in their short-term liquidity management and more on market-priced interbank trade. In this respect, the changes in Finland follow the same broad pattern that is discernible in many OECD countries. In Finland, the changes occurred somewhat later than in most countries because of the relatively late start in the liberalization process.

Financial markets in Finland were tightly regulated from the end of the Second World War until the second half of the 1980s. Rationing of banks' borrowing from the central bank by means of bank-specific quotas and graduated interest rate schedules was the most important method of monetary control.[1] In addition to the system of quotas for central bank financing, interest rates on bank loans were extensively regulated during most of the period. As a consequence of the lending rate controls, the quantitative quotas applied to banks' central bank financing were reflected in bank lending as credit rationing, and incentives for interbank trade were negligible.

Until the late 1970s, the main instrument of central bank financing was the discounting and rediscounting of bills presented by banks. But, particularly as regards short-term liquidity needs, there were also other

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[1] The history of the system of central bank financing for commercial banks in Finland has been described in detail by Saarinen (1986).

arrangements. Starting from the late 1950s, short-term repurchase agreements in government bonds outside the basic quota were introduced as an instrument for smoothing the liquidity imbalances of individual banks. Another major instrument used for banks' short-term liquidity mangement was based on government cash funds. An important source of liquidity variation in the banking system was, and indeed still is, funds held in government accounts with the state-owned Postipankki. From the late 1960s, Finnish banks had an arrangement among themselves under which commercial banks, which were constantly short of liquidity, could accept deposits from Postipankki, which had a constant surplus of funds. These deposits were included in the central bank financing quotas of commercial banks, which made them a close substitute for direct borrowing from the central bank. In addition, commercial banks could make bilateral interbank deposits. A regular market for overnight funds did not develop at this stage, however.

Starting from the mid-1970s, the Bank of Finland's "call money system" replaced these arrangements, when daily call money credits began to be used as an instrument for the provision of central bank financing to banks. Apart from borrowing overnight, banks were also allowed to make call money deposits. An interesting feature of this arrangement was that it originally imitated 'real' overnight markets to some degree, the central bank being the invisible hand guiding movements in the interest rate and balancing the market. In principle, the central bank only intervened as a borrower in accordance with certain rules, and the call money rate was adjusted to reflect the liquidity needs of banks. But, in a non-market environment, these arrangements for liquidity adjustment required additional balancing regulations. In particular, because of the constantly asymmetric liquidity position of banks, the commercial banks were reimbursed for part of their nominal borrowing costs by the depository for the Treasury's cash funds. Therefore the official call money rate for that period occasionally deviated from the effective marginal cost of borrowing the banks actually faced.

The call money system was originally introduced as a complement to other central bank financing in order to manage short-term variations in banks' liquidity, but its relative significance subsequently increased. In the early 1980s, the rediscounting quotas were reduced significantly, and the graduated interest rate schedule was abolished. Call money credits then became the most important channel of central bank financing and the most important monetary policy tool. An essential feature of the change in the system was that the call money rate was made an explicit policy instrument, decisions on which were made explicitly by the central bank.

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From 1984 to the autumn of 1986 all central bank financing was granted in the form of call money credits. The penalty rate schedule applied to these credits was abolished, so that the banks were allowed to raise overnight loans at a fixed interest rate. This implied that the supply-of-reserves curve of the central bank was horizontal.

The fixed call money interest rate was the key instrument of monetary policy under the horizontal supply curve for call money. In terms of the reserve model, the overnight market rate of interest would have been tightly pegged to the call money rate, had it only been possible to record it at that time. The call money rate effectively represented the short end of the maturity spectrum of interest rates. Also, as there were no other important methods of central bank financing, the fixed call money rate determined the cost of banks' borrowing from the central bank, both at the margin and on average. For that reason, it had a significant direct impact on the profitability of banks, which limited its use as an instrument of monetary policy.

As a part of the liberalization of the financial markets, monetary policy operating procedures in Finland were substantially revised in late 1986 and early 1987. The regulation of bank lending rates was abolished, the previously modest interbank market started to function on a larger scale and the central bank also began to operate in the money market. After a short period of transition, money market instruments began to be used for the major part of central bank financing.

One institutional precondition for this change was that the terms of banks' central bank financing had to be modified in a way that considerably reduced the volume of daily call money credits granted to banks. The new discount window was effectively a penalty rate system. The interest rate on call money deposits was set below the rate on call money credits. This spread between the central bank's borrowing and deposit rates made it profitable for banks to search for other sources of funds or investment opportunities before approaching the window. As a consequence, banks also started to trade in overnight funds in rapidly expanding interbank markets at interest rates determined in the market. The aforementioned perfect peg of the short-term interest rate to the call money interest rate was broken.

When banks' access to the call money facility was restricted and a major part of central bank financing shifted to money market instruments in 1986–1987, one consequence was that the call money credit rate became an instrument affecting merely the marginal costs of borrowing. The call money credit rate did not even directly determine individual banks' marginal costs of borrowing overnight funds any more. The interbank rate became a more relevant measure of marginal cost because buying funds from other banks became the

alternative to using the central bank's call money facility. The call money rate lost much of its status as a policy instrument, but controlling the level of liquidity of the banking system through money market operations became another monetary policy instrument. In all, the guiding of interest rates became indirect by nature.

The importance of the call money facility as a source of funding to banks has declined drastically because of these changes, although it still has a role to play in managing changes in the liquidity of the banking system. The effect of these institutional changes on the use of the call money facility can be seen clearly from the data. There was an obvious regime shift in borrowing behaviour at the time the call money interest rate spread was introduced. Figure 3.1 shows the 22-day moving standard deviation of net borrowing by one of the major banks operating in Finland before and after the penalizing call money interest rate spread was adopted. Mean levels of standard deviation in the respective time periods are indicated by dashed lines. The average level of the moving standard deviation is approximately three times higher in the period of a horizontal discount window supply curve than in the latter period when the penalizing interest rate spread was in effect.

Figure 3.1 **Moving standard deviation of daily net borrowing from the discount window by one of the major banks in Finland, before and after the adoption of the call money interest rate spread**

First period from March 1985 to March 1987, second period from April 1987 to October 1989

1 Moving standard deviation of daily net borrowing
2 Mean of moving standard deviation
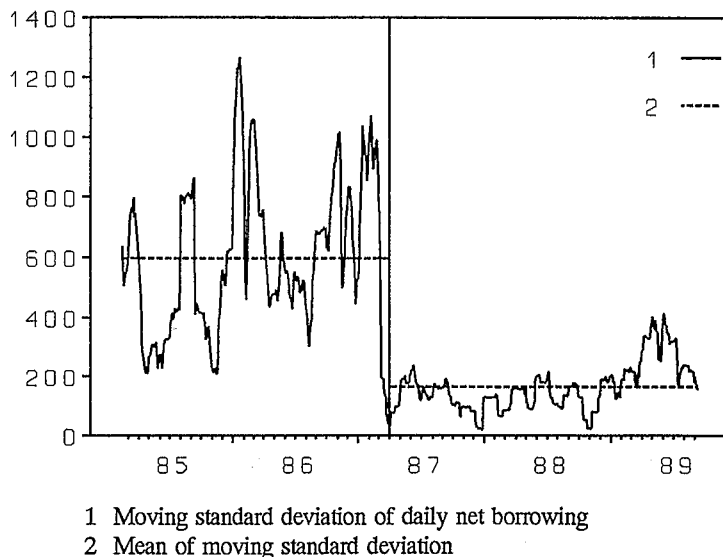
3.1.2 The structure of the call money facility

The current call money facility in Finland consists of elements that are widely used in other countries, although the details vary greatly in different countries. (For a description and analysis of discount windows in several countries, see Kneeshaw and Van den Berg, 1989, and Batten et al., 1989). In most countries there is at least some arrangement that permits banks to borrow from the central bank at short notice and on more or less predetermined terms. Apart from the lending rate, typical restrictions imposed on these loans are quantitative quotas, penalties for frequent borrowing and minimum maturity requirements.

Compared internationally, the distinctive feature of the Finnish call money facility is that it is basically a penalty rate system in that the call money lending rate is normally above market interest rates. The Finnish system also differs in certain other respects from the corresponding arrangements in several other countries. First, banks are allowed to make interest-bearing demand deposits — call money deposits — in their cheque accounts at the Finnish central bank. Secondly, in Finland the penalties for borrowing from the central bank are mostly explicit and predetermined. For example, in the United States, by contrast, more or less discretionary decision making by the authorities is evidently an important element of discount window policies.[2] Further, US banks are allowed to cover required reserves with past excess reserves (carry-over provisions), so that the return on excess reserves depends on the future opportunity cost of reserves. In Finland, on the other hand, required reserves are deposited in the central bank at the end of each month on the basis of the level of deposits in the bank in the previous month. Consequently, the only connection between required reserve deposits and excess reserves is the daily balance sheet constraint.

The exact terms for call money credits and deposits have also been modified since the major overhaul of the system in 1987. Initially, the interest rate on deposits was set at 7.5 per cent and the basic rate on credits at 11 per cent. In addition, borrowing at basic rate was subject to a quota defined separately for each bank according to its size. Borrowing in excess of quota was subject to a penalty rate, which was set at 19 per cent. A further restriction was that average borrowing in any five consecutive banking days should not exceed

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[2] For a discussion see Goodfriend (1983) and Waller (1990). The importance of this feature for the functioning of the discount window in the United States seems to be widely accepted in the literature.

quota. No explicit monetary penalty was stipulated for failing to meet the five day restriction, but the central bank had the option of taking discretionary action.

There have been few changes in interest rates on call money credits and deposits since April 1987; the first change was not until October 6, 1988, when the lending rate was raised to 13 per cent and the deposit rate lowered to 4 per cent. This widened the interest rate spread from 3.5 percentage points to 9 percentage points, and strengthened banks' incentives to undertake transactions in the interbank market for overnight funds. It seems that in practice a relatively wide spread is needed to sustain overnight trade properly in Finnish circumstances. Even though the spread between annual interest rates may seem quite large, it is actually applied to the overnight period only.

The next change in the system was in June 1989, when the borrowing quotas were abolished. At the same time, however, it was stipulated that the five-day moving average of a bank's net borrowing should not be positive. The right to borrow at a fixed lending rate, which was set at 15 per cent, was otherwise unlimited. But the effective cost of borrowing under these arrangements was higher than the loan rate itself. Because of the restriction on the five- day average, a borrowing bank was obliged to make corresponding deposits in the future, and deposits earned interest at a rate of only 4 per cent. In October 1989, this implicit cost was replaced by an explicit penalty rate. Under the new rule, the interest rate charged on borrowing was twice the basic lending rate if a bank's average borrowing over a five-day period was positive.

This multiple structure of the call money facility was probably partly motivated by the authorities' desire to also be able to control borrowing under exceptional circumstances, in particular during times of speculative attacks on currency. Finland has previously experienced a number of attacks during which banks' borrowing tends to increase rapidly as the public's demand for foreign currency increases and markka liquidity is drawn into the central bank in exchange for foreign currency. From the point of view of the central bank, the last limit on borrowing reduces the possibility that the central bank is forced to finance what it regards as speculative currency flows. But it applies equally well to any other cases of severe liquidity drain.

Although the five-day moving average level of borrowing has been restricted since the inception of the system in 1987, the empirical relevance of this condition was much enhanced when the quotas applied to borrowing at the basic call money lending rate were abolished in June 1989. Before that, the aggregate quota for the whole

banking system was about FIM 5 billion. In practice, this was never exceeded, not even for one day. Likewise, borrowing by individual banks seldom exceeded the basic-rate quota. In all cases where it was exceeded, the borrower was a small bank, which had been allotted a very small quota because of its low level of equity capital. There is no indication that any measures was ever taken by the central bank on the basis of the five-day limit during that time. After the quotas were abolished, the five-day restriction became much more binding. Practically all banks exceeded the limit at times. It seems reasonable to conclude that, until June 1989, the call money facility was mostly characterized by the penalizing spread, after which the five-day limit also became important.

3.2   The overnight market: some 'stylized facts'

In Finland, the interbank market for overnight funds started to function on a regular basis in the spring of 1987. The reason for the late start of this trade was primarily the regulation of bank lending rates.

Trading in the overnight market is concentrated at the end of the banking day. The normal practice is that banks start requesting overnight funds shortly after markets for other maturities have closed. Because of this timing, banks can use overnight trade for eliminating deficits or surpluses which could not be cleared during the day, or which were not known earlier in the day. It is logical to presume that banks' information about their final liquidity position becomes more precise as the closing of the books for the day approaches. If a bank knows that it is running a surplus, it has an incentive to offer funds in the overnight market, and conversely, demand funds from other banks if it knows it is on the short side.

The volumes traded in the overnight market have been roughly one-third of the total turnover in interbank markets. This comparison is, of course, affected by the fact that overnight funds are, by definition, sold and bought every day, while this is not necessarily so for claims with longer maturities. Nevertheless, an average turnover of about FIM 4 billion a day (from August 1988 to August 1989) is notable by Finnish standards.

The rate of interest in the overnight market is shown in Figure 3.2, together with the central bank's call money lending and deposit rates. The market interest rate is computed as an average of interest rates applied to all individual deals. Daily variation in the average overnight interest rate seems to have been high; it has, in fact, been much higher than in interest rates on longer maturities. From March

1987 to June 1989, the standard deviation of the overnight interest rate was 1.30 per cent. The corresponding figure for the three-month market interest rate was 0.92 and for the twelve-month interest rate 0.86.

Also, the dispersion of interest rates applied to individual deals in overnight funds between the banks in a given day has been notably large. For example, from January 1989 to June 1989, the mean of the daily standard deviation within the average rate was 2.19 per cent. This somewhat surprisingly large deviation could result from the fact that all deals are not done at the same time, so that recorded interest rates are based on different information about liquidity. Another possible explanation is that all banks are not in equal position in the market. In the thin markets of Finland, it is possible that banks tend to establish relationships which might influence the setting of prices. For example, a large and frequent buyer may be favoured by a frequent seller.

Figure 3.2   **The overnight market interest rate and the central bank's call money interest rates on deposits and credits**
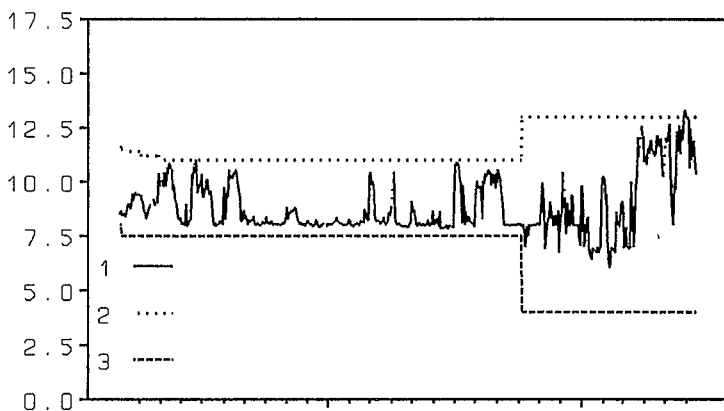
March 1987 — June 1989

1 Overnight market interest rate
2 Call money lending rate
3 Call money deposit rate

From the point of view of the reserve market framework presented above, the most notable feature of Figure 3.2 is that the overnight rate of interest clearly deviates from the interest rates on the central bank's call money. It is usually somewhere between the call money lending and deposit rates. Very seldom is it at the same level as either of these or does it exceed the limits of a band defined by these rates.

This same phenomenon is illustrated in Figures 3.3a and b, which depict the overnight market rate of interest as a function of the net borrowing position of the banks at the central bank. The first figure contains a plot of observations from March 1987 to October 1988 and the second a plot from October 1988 to June 1989. Both plots indicate that there is definitely a positive correlation between these two variables. But it is not so simple as in the basic reserve market framework, which predicts that the market rate will equal the call money deposit rate when reserves are positive and the call money lending rate when reserves are negative. Rather, the plots indicate a continuous, although possibly nonlinear, relationship.

It is interesting to note that the plots reveal very much the same pattern in observations as can be discerned in comparable plots from US data presented by Peristiani (1991) and Tinsley et al. (1982). The relevant y-axis in plotting US data is the spread between the discount rate and the Federal funds rate; in other respects, the plots are the same as depicted here. As noted above, Peristiani found evidence of nonlinearity and heteroscedasticity.

In order to explain the pattern observed in these figures, an asset allocation liquidity management model of the overnight market is studied in the next chapter. The result of this is a demand-for-reserves schedule that allows for both nonlinearity and heteroscedasticity.

Figure 3.3a   **Overnight market interest rate and the
                banks' net borrowing from the central bank**

Overnight market interest rate, %

(- = borrowing, + = deposits)   Net borrowing from the
                                     central bank, bill. FIM

March 1987 — October 1988
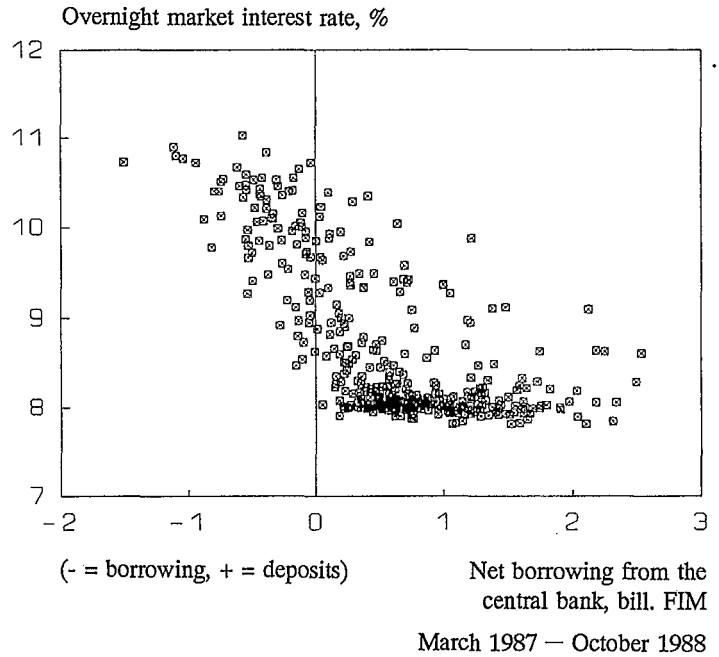
Figure 3.3b   **Overnight market interest rate and the
                banks' net borrowing from the central bank**

Overnight market interest rate, %

(- = borrowing, + = deposits)   Net borrowing from the
                                     central bank, bill. FIM

October 1988 — June 1989

42
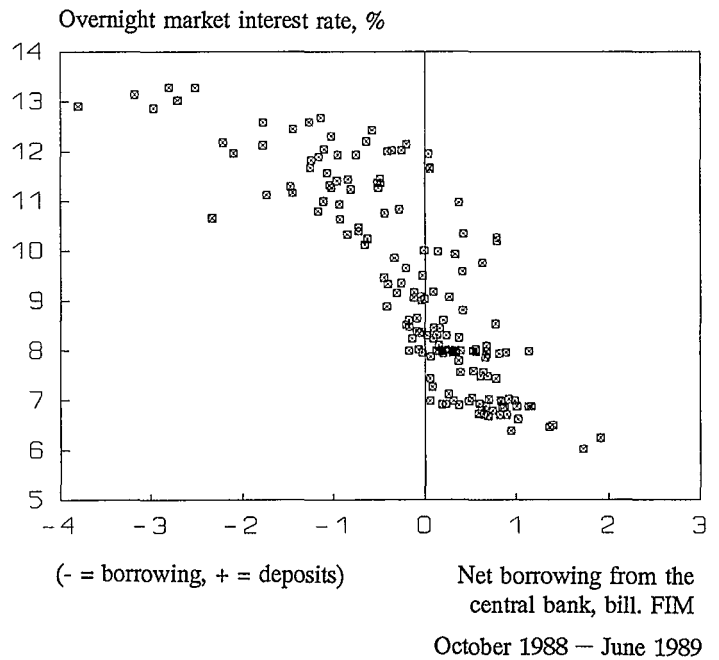
4 A Stochastic Asset Allocation Model of the Overnight Market

In the following, banks' demand for free reserves is derived from an optimization problem in which each bank maximizes its expected profits from trade in the overnight market. The model employed is basically the same as the traditional liquidity management model (Section 2.3) that was first applied to money markets in Modigliani, Rasche and Cooper (1970), in Frost and Sargent (1970) and in Poole (1968).[1] In these pioneering works the approach employed was to describe the bank's portfolio allocation to free reserves and other assets, taking into account the fact that some of the relevant variables affecting reserves are not known for certain. The same framework is applied here but with a specific description of the overnight market and explicit terms concerning discount window borrowing, reflecting the circumstances in Finland. The model will enable us to derive the demand for free reserves solely as a function of the terms for (net) discount window borrowing and the overnight interest rate. After aggregation, the model yields the equilibrium interest rate in the overnight market as a function of the level of borrowing and the borrowing terms.

It is assumed that banks clear their balances at the end of each banking day at the central bank. If a bank has positive free reserves at the time the books are closed, it will make a deposit of an equal amount. When the bank's reserve position is negative, it has the option of borrowing from the central bank. The bank cannot know precisely all transactions that will be carried out on its customers' accounts during the day, so the final reserve position of the bank is uncertain. Without uncertainty the model would coincide with the simple description of the market for reserves that was discussed at the beginning of Chapter 2. The interest rate in the overnight market would always, through arbitrage, approach either the deposit or lending rate at the central bank's discount window. The money market is presumed to function efficiently, so that banks can buy and sell money market instruments without limit, and freely exploit all opportunities for arbitrage that arise if the bank perceives differences between discount window interest rates and money market interest rates.

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[1] A more recent application is found in Dotsey (1991), for example.

Two important assumptions of the model are that trading in the overnight market begins after the normal banking day is over and that the discount window is the last option available to banks after the overnight market is closed. Because of these assumptions, the discussion can be limited to these particular markets and the maximization problem defined without any reference to other markets, i.e. markets for maturities other than overnight loans. This time structure of the availability of different sources of funding to banks was also used by Ho and Saunders (1985) and by Van Hoose (1991) in their analysis of the Federal funds market. What we now have is the following description of a bank's normal business day:

At the beginning of the day the bank is thought to be at its target level of reserves. During the day, the bank's liquidity position may change because of transactions made by the public and also because of its own operations in the money market. The bank also decides in the course of the day whether or not it is willing to trade in the overnight market and at what prices. These decisions must be made under uncertainty, because the bank does not know exactly the liquidity position that it will end up with. After the markets for longer maturities have closed, the deals in overnight loans and deposits between the banks are settled. Finally, at the end of the day, the banks' balances are cleared in the central bank. Negative reserves are then covered with discount window loans and positive reserves are converted into deposits.

Banking Day	O/N trade	Overnight period

O/N market open,   Discount window open,
other markets closed   closing of books
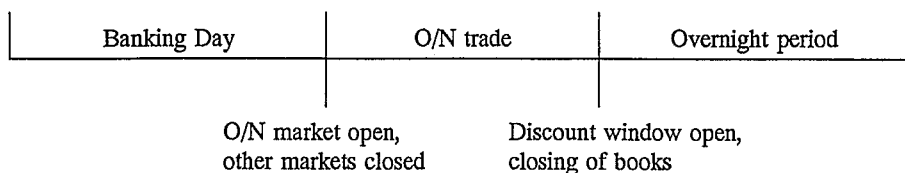
The target level of reserves is taken to be the bank's position at the beginning of the day, so that the final position at the end of the day deviates from the target level exactly by the amount of unforeseen changes in liquidity. Therefore, the adjustment of reserves to the target level is not considered in the model. If a bank's reserve position deviates from the target level during the day, it can always make the necessary correction without any significant costs by buying or selling money market instruments, for example. There is no obvious way to include adjustment costs in the model without abandoning the assumption that funding from the well functioning money market is available to banks.

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To summarize, the following assumptions are made:

— The overnight market is open between normal banking days, i.e. the bank is not able to use any other means to adjust its reserves after overnight trading is over. During the day the bank can freely buy and sell money market instruments.

— When making offers in connection with trade in the overnight market, the bank knows its exact reserve position the previous day. The amount of today's reserves is a random variable with a known distribution.

— After the closing of the overnight market, the bank is allowed to use the central bank's discount window. The balance sheet constraint must be satisfied daily. It is assumed that positive reserves are deposited at the central bank and that negative reserve positions are covered by discount window borrowing.

— The overnight market is competitive.

4.1 The basic model

As a basic model we consider a simple case of a quantitatively unrestricted discount window with a penalizing interest rate spread. The bank may make deposits and borrow without limit, but the interest rate paid on deposits is lower than the interest rate charged on loans. A rationally behaving bank will, of course, use only one of these two options on the same day, so we are interested in the bank's net position on a given day. Under these assumptions concerning the discount window, the rate of return on reserves can be written as follows:

$$\text{Rate of return on reserves} = rd + (rl - rd)\frac{\min(0,w)}{w}, \qquad (4.1)$$

where w = bank's net discount window position
rd = interest rate on discount window deposits
rl = interest rate on discount window loans.

The above equation states that a bank's rate of return on reserves equals the deposit interest rate when its discount window position is positive and the loan interest rate when its position is negative. The

total position is a random variable and its actual level can be written as the sum of the level of reserves at the beginning of the day, the amount of reserves purchased or sold in the overnight market and the net amount of all other transactions carried out during the day. A random variable u is taken to summarize all these other transactions and it is unknown to the bank at the time of trading in the overnight market. All transactions which are known can be thought to be included in the deterministic part of the discount window position.

$$w = W + u = R + Q + u, \tag{4.2}$$

R = target level for reserves
Q = overnight loans
u = random shocks to liquidity, $u \sim N(0, \sigma^2)$.

The bank's profits are defined as the difference between the return on reserves and the cost of obtaining reserves from the overnight interbank market. In this basic model, the maximization of expected profits from overnight trade is taken to be the bank's objective. Thus the bank faces the following maximization problem:

$$\max_{Q} \quad E(\pi) \tag{4.3}$$

$$\pi = rd \cdot w + (rl - rd)\min(0, w) - \delta Q, \tag{4.4}$$

where δ = interest rate in the interbank overnight market.

Because the expected value of the random variable u is zero, equation (4.4) can be written as

$$E(\pi) = rd \cdot W + (rl - rd)E(\min(0, W + u)) - \delta Q. \tag{4.5}$$

To derive the expected value of min(W+u,0), we define a random variable Y that follows a truncated normal distribution. Let

$$Y = \begin{cases} y = W + u, & \text{if } y > 0 \iff u < -W \\ y = 0, & \text{otherwise}. \end{cases} \tag{4.6}$$

The expected value of a truncated variable is a probability-weighted sum of conditional expected values. Using the notation Φ and ϕ to denote the cumulative distribution function and density function for the standard normal, respectively, we obtain

$$E(Y) = Prob(y<0) \cdot E(y \,|\, y<0) + Prob(y=0) \cdot E(y \,|\, y=0)$$

$$= \Phi(-W/\sigma)E(y \,|\, y<0),$$

$$= \Phi(-W/\sigma)(W + E(u \,|\, u<-W))$$

$$= \Phi(-W/\sigma)(W + \int_{-\infty}^{\infty} \frac{u \cdot 1/\sigma \cdot \phi(u/\sigma)}{\Phi(-W/\sigma)} \, du) \tag{4.7}$$

$$= \Phi(-W/\sigma)(W + \frac{-\sigma \cdot \phi(-W/\sigma)}{\Phi(-W/\sigma)}).$$

Using (4.7), the maximization problem in (4.3) can be solved. Taking a derivative with respect to Q gives a first order condition for the maximum and consequently the bank's demand for reserves, which is of very simple form in this case. If the discount window is unrestricted with respect to quantities borrowed or deposited, but involves a penalizing interest rate spread, a bank is willing to trade in the overnight market at an interest rate that is a weighted average of discount window interest rates, the weights being the probabilities of discount window positions. In other words, the optimality condition is that the market interest rate must equal the probability-weighted average of the central bank's interest rates.

$$\delta = rd + (rl - rd)\Phi(-(Q+R)/\sigma)$$

$$= rd + (rl - rd)\Phi(-W/\sigma). \tag{4.8}$$

Equation (4.8) is essentially a modification of the first order condition (2.2) from the general liquidity management model. It is a probit function with the central bank's call money deposit rate as a lower asymptote and the central bank's call money lending rate as an upper asymptote. At the point where reserves switch from positive to negative, the market interest rate equals the arithmetic average of the

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central bank's interest rates. The conditions for positive and negative optimal reserves are

$$\delta < rd + (rl - rd)/2 \Leftrightarrow W > 0$$

$$\delta > rd + (rl - rd)/2 \Leftrightarrow W < 0.$$

The equilibrium interest rate schedule is illustrated graphically in Figure 4.1. It should be noted that in this figure reserves are measured relative to their standard deviation. When equation (4.8) is later solved with respect to the level of reserves (equation 4.9), the standard deviation of reserves will become one of the variables explaining the level of reserves.

Figure 4.1   **Interest rate schedule in the basic model**
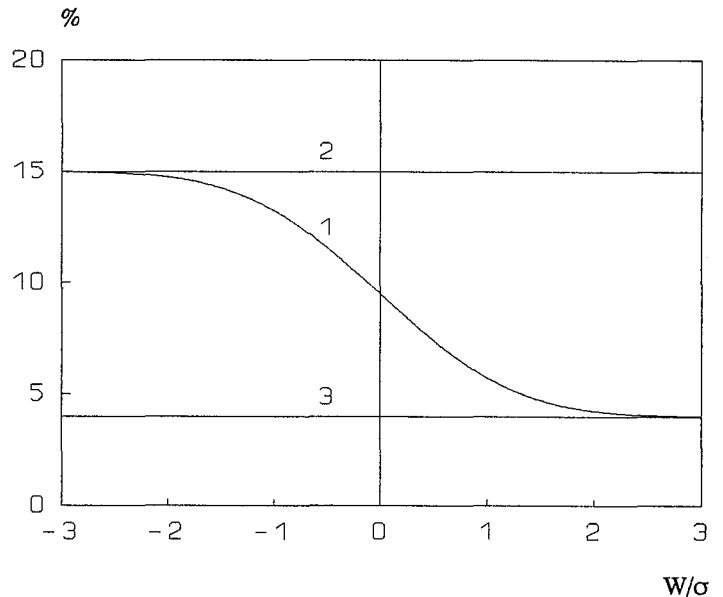
1 Overnight market interest rate
2 Call money lending rate
3 Call money deposit rate

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Aggregation of interest rate schedules

Equation (4.8) was derived for a single bank. The aggregate demand can be derived using the fact that in the interbank market the amount of deposits must equal the amount of loans by definition, so that $\Sigma Q^i = 0$. Aggregating over all banks ($i = 1,...,H$) then gives the market-clearing overnight interest rate as a function of the aggregate target level of reserves, the sum of standard deviations of each bank's reserves and the terms for discount window loans and deposits.

In order to be able to derive the aggregate demand, (4.8) must first be solved with respect to quantities. Because the inverse of the cumulative distribution function of the standard normal has no closed form expression, the only way to solve for W in (4.8) is to use some approximation. A convenient and statistically fitting choice for that purpose is a logistic distribution. It is commonly used in empirical studies instead of the normal distribution and it has a cumulative distribution function that is much easier to manipulate.[2] In the following, those variables that differ across banks are marked with the superscript i and aggregation is done by summing over banks after replacing the normal distribution with the logit distribution.

$$\delta = rd + (rl - rd)\Phi(-(Q+R)^i/\sigma^i)$$

$$\Rightarrow \quad (\delta - rd)/(rl - rd) = \Phi(-(Q+R)^i/\sigma^i)$$

$$\approx 1/(1 + \exp(k(Q+R)^i/\sigma^i))$$

$$\Rightarrow \quad Q^i = \sigma^i/k \cdot \log((rl-\delta)/(\delta-rd)) - R^i$$

$\Sigma Q^i = 0$, which implies

$$\Sigma R^i = \Sigma \sigma^i/k \cdot \log((rl-\delta)/(\delta-rd))$$

$$\Rightarrow \quad \delta = rd + (rl-rd)/(1 + \exp(k\Sigma R^i/\Sigma\sigma^i)) \qquad (4.9)$$

$$\Rightarrow$$

---

[2] A logistic distribution has a cumulative distribution function $L(x) = 1/(1+\exp(-x))$. The transformation $L_k(x) = 1/(1+\exp(-kx))$ closely follows the normal distribution function $\Phi(x)$ when $k = 1.6$ (Amemiya, 1981, p. 1487). According to Amemiya, in empirical work it is difficult to distinguish between these two distributions statistically, unless the observations are heavily concentrated in the tails of the distribution. (The logistic distribution has heavier tails.)

$$\delta \approx rd + (rl - rd)\Phi(-\Sigma R^i/\Sigma\sigma^i).\qquad\qquad (4.10)$$

The aggregated equation for the market interest rate closely resembles the interest rate equation in the case of a single bank. The only differences are that the sum of reserve targets has now replaced the discount window position of individual banks and that reserves are now measured as relative to the sum of standard deviations of individual reserve positions. When empirical applications and policy issues are considered, (4.10) is especially noteworthy because it shows that the aggregate net reserve target of the banking system is the key variable that is linked to the equilibrium interest rate. On the other hand, the equation also illustrates that the relationship between reserve targets and the equilibrium interest rate is affected by the variance of reserves at the disaggregated level. Potentially, this point might be of some importance in empirical applications because the sum of standard deviations of reserves need not behave in a similar fashion in time as the standard deviation of the aggregate position.

Comparative statics

The steepness of the interest rate schedule (4.10) depends on the sum of standard deviations of reserves. This sum of standard deviations can be interpreted as a measure of the degree of uncertainty facing banks, which raises the question as to how increased uncertainty changes the bank's borrowing behaviour. Taking a derivative of the market interest rate with respect to the sum of standard deviations yields

$$\frac{\partial \delta}{\partial \Sigma\sigma} = (rl - rd)\frac{\Sigma R^i}{(\Sigma\sigma)^2}\phi(\Sigma R^i/\Sigma\sigma^i)$$

$$> 0, \quad \text{if } \Sigma R^i > 0$$

$$< 0, \quad \text{if } \Sigma R^i < 0.$$

The effect of an increase in the degree of uncertainty on the market interest rate depends on the sign of the sum of reserve targets. More uncertainty makes the interest rate schedule less steep; it raises the market interest rate if the sum of reserve targets is positive and lowers the market interest rate if the sum of reserve targets is negative (Figure 4.2). Less uncertainty works in the opposite direction, making

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the interest rate schedule steeper. When the sum of standard deviations approaches zero, the schedule becomes a step function that jumps from the central bank's deposit interest rate to the loan interest rate as the sum of reserve targets changes from positive to negative. So, the model does include as a special case the full information case discussed in the previous chapter.

Changes in the central bank's call money deposit and lending rates have straightforward effects on the equilibrium market interest rate. Derivatives of the call money interest rate are positive in both cases, $\Phi(-\Sigma R/\Sigma\sigma)$ with respect to the deposit rate and $\Phi(\Sigma R/\Sigma\sigma)$ with respect to the lending rate. Because of this symmetry, an increase in the width of the interest rate spread between the central bank's interest rates widens the equilibrium interest rate locus evenly with respect to the lending and deposit rates.

Figure 4.2 **The effect of an increase in the degree of uncertainty (the sum of standard deviations of reserves) on the equilibrium market interest rate**
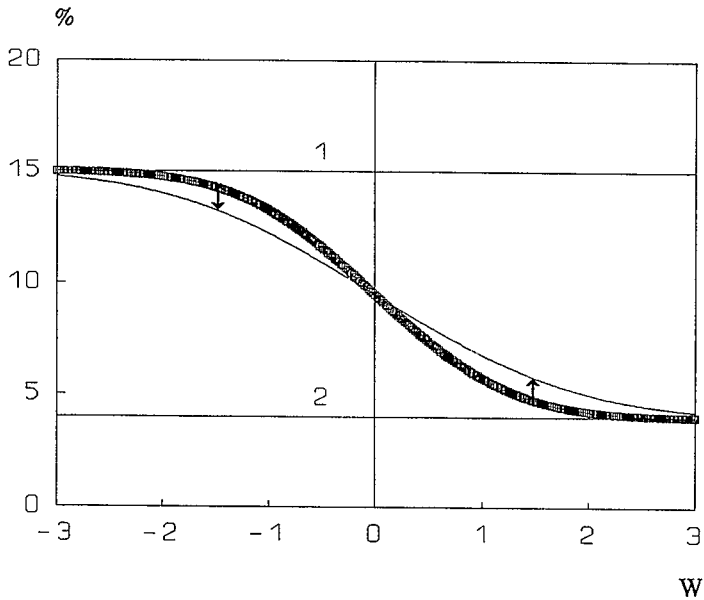
1 Call money lending rate
2 Call money deposit rate

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The impact of changes in the central bank's interest rates on the demand for reserves at a given level of the market interest rate can be obtained from (4.9). An increase in uncertainty obviously increases the reserve target as the sum of deviations of reserves enters the reserve equation multiplicatively. The effects of changes in the deposit rate and lending rate are, of course, symmetric on reserves also, as they were on the market interest rate.

$$\frac{\partial \Sigma R}{\partial rl} = \frac{\Sigma \sigma / k}{rl - \delta} \quad \text{and} \quad \frac{\partial \Sigma R}{\partial rd} = \frac{\Sigma \sigma / k}{\delta - rd}.$$

4.2   Effects of risk aversion

In the basic model, the bank's objective was to maximize expected profits from the allocation of funds to overnight market trade and to net reserves in the central bank. That particular formulation of the objective function implies risk neutrality as the bank is assumed to be interested only in the first moment of the profit distribution. In the following, a mean-variance approach is applied in order to study the effects of risk aversion. For this purpose, the second moment of the distribution, the variance of profits, is included in the objective function of the bank.

Applying a mean-variance approach can be justified with a fairly general, explicit objective function. The following exponential function is commonly used and can be implemented empirically as well:

$$U(\pi) = -\exp(-M \cdot \pi), \tag{4.11}$$

where M = positive constant,
π = profits from interbank trade.

This objective function U is continuous and monotonic and its first derivative with respect to profits is positive and second negative. Because of these properties, the function implies increasing utility and risk aversion. Furthermore, the negative of the second derivative relative to the first derivative equals M, $-U''/U' = M$, which implies that constant M is a measure of absolute risk aversion. A particular advantage of this specific functional form is that it can be shown to lead to the following expression for expected utility, assuming that π is a normally distributed random variable (see Bray, 1985, p. 172):

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$$E(U) = -\exp[-M[E(\pi) - \tfrac{1}{2}M \cdot \text{Var}(\pi)]].$$

The above expression can be further simplified by applying a monotonic transformation. As a result, the objective of a bank's maximization problem can now be expressed as the difference between expected profits and the variance term

$$\max_{Q} \ E(U) = E(\pi) - \tfrac{1}{2}M \cdot \text{Var}(\pi). \tag{4.12}$$

The first part of the right-hand side of (4.12), $E(\pi)$, was already maximized when the basic model was discussed. In addition to expected profits, the solution will now include the derivative of the variance of profits with respect to Q, multiplied by the coefficient of risk aversion. In order to derive the first order condition, the variance will have to be computed and differentiated. These computations are presented in Appendix 1. The result is the following equation for the market interest rate:

$$\delta = rd + (rl - rd)\Phi$$
$$+ (rl - rd)M\Phi(1 - \Phi)\{rd \cdot E[w \,|\, w > 0] - rl \cdot E[w \,|\, w < 0]\}. \tag{4.13}$$

The last part of the interest rate equation is the risk premium. It shows the effect on the interest rate that is due to risk aversion, as compared to risk neutrality, at different levels of (net) discount window borrowing. By assuming a normal distribution, we can write the expected values in explicit form. This yields an estimable expression for the premium.

$$\text{Risk premium} = M(rl - rd)^2\{(b + 1 - \Phi)\sigma\phi - W(1 - \Phi)\Phi\}, \tag{4.14}$$

where $b = rd/(rl-rd)$. The argument in the distribution function Φ and density function ϕ is $-W/\sigma$ (the point of truncation) in both (4.13) and (4.14)

From (4.13) it can be seen that the risk premium approaches zero when W goes to either positive or negative infinity. The term $\Phi(1-\Phi)$, which is the binomial distribution of discount window deposits and loans, approaches zero at both ends of the distribution. This reflects the fact that the uncertainty concerning the interest rate that will be

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applied to a bank's discount window position diminishes when the target level of reserves is either very high or very low. In this framework, the actual rate of return on reserves depends only on the sign of the bank's discount window position, and the sign is unlikely to change at extreme reserve target levels.

The absolute size of the premium depends on the coefficient of risk aversion M, which is unknown. (It is known to be positive, however because of the assumptions made about the objective function U.) It can also be seen that the risk premium is positive at all levels of discount window borrowing because both $\Phi(1-\Phi)$ and $\{rd \cdot E[w \mid w>0] - rl \cdot E[w \mid w<0]\}$ are certain to be positive. On the other hand, the maximum of the premium is generally not at the zero position. In the above expression for the risk premium, conditional expected values of discount window positions are weighted with interest rates applied by the central bank and the rate on loans is presumed to be higher than the rate on deposits. The maximum of the premium is at zero only if the deposit and loan interest rates are equal. Intuitively, the bank's risk is bigger on the loan side of the distribution because the changes in returns that are caused by changes in liquidity are greater on the loan side since the percentage rate applied to loans is higher.

Formally, the maximum of the risk premium can be found from (4.14). Let $\sigma = 1$ for simplicity, and take a derivative of the premium with respect to reserves to obtain (4.15). (The inequality holds because $\phi^2 < \Phi(1-\Phi)$). In the same way it can also be shown that the variance σ^2 and the interest rate spread between loan and deposit rates both have a positive effect on the risk premium.

$$(rl-rd)^2 M \cdot \frac{\phi^2 - (1-\Phi)\Phi}{\phi(b+\Phi)} < 0. \tag{4.15}$$

To summarize the results, we have shown that the equilibrium interest rate is higher if the bank is risk averse than it is when risk neutrality is assumed. The effects of risk aversion are potentially most important when reserves are "close" to zero. The equilibrium locus for a risk averse bank is shown graphically in Figure 4.3 with parameter values M = 0,1 (risk aversion) and M = 0 (risk neutrality). The area between the two loci is the risk premium. It is depicted separately in Figure 4.4.

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Figure 4.3 **Equilibrium locus of a risk averse bank as compared to a case of risk neutrality**

Overnight market
interest rate, %

1 Risk neutrality
2 Risk aversion
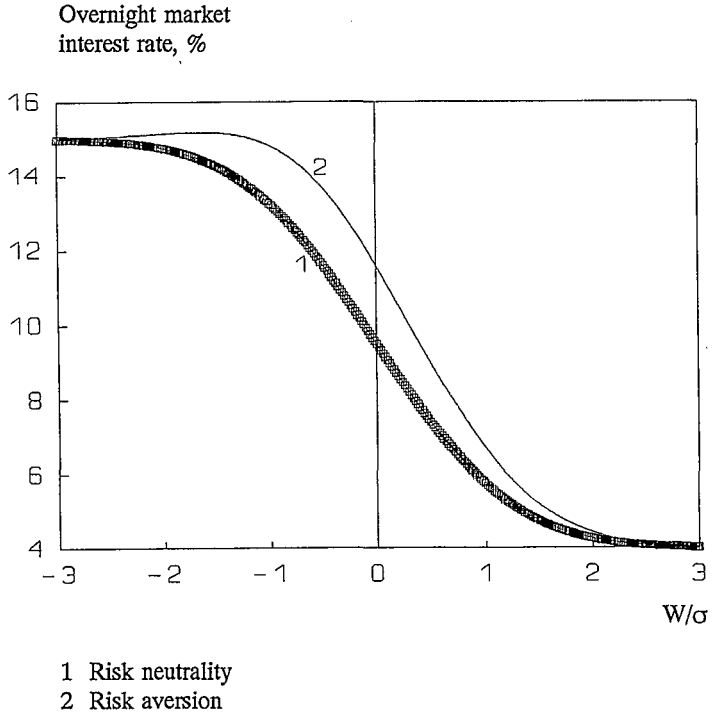
Figure 4.4 **The risk premium**
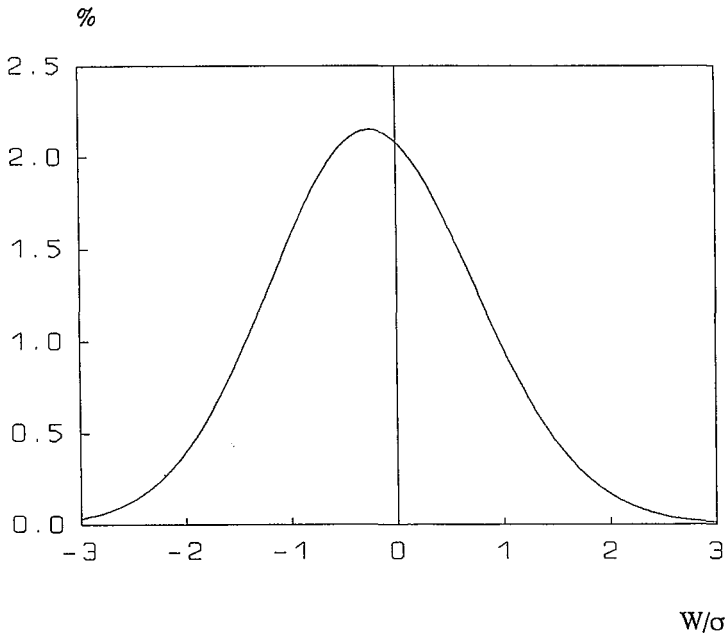
4.3   Quantitative quotas on borrowing

Many central banks restrict the amount of banks' borrowing from the discount window by applying quantitative quotas. In this section, the basic model is modified, in the simplest possible way, so as to allow to the effects of these arrangements to be examined.

It is assumed that, in addition to separate interest rates on deposits and loans, the discount window interest rate schedule has one additional step after some fixed amount of borrowing. As before, it is assumed that the bank may deposit positive reserves without restriction and that an interest rate rd is applied to deposits. The modification made here is that the bank's borrowing from the window at the basic call money lending interest rate rl is limited to an exogenous quota K while the rest of the borrowing is subject to a penalty rate rs. The bank's cost of borrowing therefore jumps discontinuously at K. The bank's profits from overnight trade can then be written as follows:

$$\pi = -\delta Q + \begin{cases} rd \cdot w, & \text{if } w > 0 \\ rl \cdot w, & \text{if } K < w < 0 \\ rl \cdot K + rs(w - K), & \text{if } w < K \end{cases}$$

(4.17)

$$= -\delta Q + \begin{cases} rd(W + u), & \text{if } u > -W \\ rl(W + u), & \text{if } K - W < u < -W \\ rl \cdot K + rs(W + u - K), & \text{if } u < K - W. \end{cases}$$

The expected value of profits is then:

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$$E[\pi] = -\delta Q + \text{Prob}(u > -W)rd \cdot E[W + u \mid u > -W]$$
$$+ \text{Prob}(K - W < u < -W)rl \cdot E[W + u \mid K - W < u < -W]$$
$$+ \text{Prob}(u < K - W)\{(rl - rs)K + rs \cdot E[W + u \mid u < K - W]\}$$

$$\Rightarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4.18)$$

$$E[\pi] = -\delta Q + \{1 - \Phi[-W/\sigma]\}rd \cdot E[W + u \mid u > -W]$$
$$+ \{\Phi[-W/\sigma] - \Phi[(K-W)/\sigma]\}rl \cdot E[W + u \mid K - W < u < -W]$$
$$+ \Phi[(K-W)/\sigma]\{(rl - rs)K + rs \cdot E[W + u \mid u < K - W]\}.$$

Because of the new step in the central bank's discount window interest rate schedule, two points of truncation of the random variable u have to be considered in order to obtain the expected value of profits. Two points of truncation mean that three separate parts of the distribution must be taken into account: one truncated from below, one truncated from above and one truncated from both ends. The expected values of w = W + u with separate upper and lower truncations were already presented when the demand for reserves was derived in the basic model. When the distribution is truncated from both ends, the expected value of w is (see Maddala, 1983, Appendix):

$$E[W + u \mid K - W < u < -W] = \frac{\sigma\phi((K-W)/\sigma) - \sigma\phi(-W/\sigma)}{\Phi(-W/\sigma) - \Phi((K-W)/\sigma)} + W.$$

After substitutions and some manipulation, (4.18) can be written as follows:

$$E[\pi] = -\delta Q - (rl - rd)\sigma\phi(-W/\sigma) - (rl - rs)\sigma\phi((K-W)/\sigma)$$
$$+ W[rd + (rl - rd)\Phi(-W/\sigma)] \qquad\qquad (4.19)$$
$$- (K - W)(rs - rl)\Phi((K-W)/\sigma).$$

The equilibrium locus is now obtained by taking the derivative of the expected profits with respect to Q. The result follows the same pattern as the solution of the basic model. The difference is that the expression for the interest rate now entails an additional term that gives the probability of borrowing in excess of the quota K, weighted with the spread between the central bank's call money lending rate and penalty rate.

$$\delta = rd + (rl - rd)\Phi(-W/\sigma) + (rs - rl)\Phi((K-W)/\sigma). \qquad (4.20)$$

The equilibrium interest rate locus is a probability-weighted combination of two adjoining basic model solutions. The generalization of the model to more interest rate steps in the discount window is not made here, but by the logic of the model it is obvious that adding more steps would only add new probability-weighted interest rate steps to the equilibrium solution. This follows naturally from the fact that the basic model is also a quota system, the quota being zero in that case. The market interest rate is in that case a probability-weighted average of call money interest rates in the neighbourhood of zero reserves, which is the position of the interest rate step in that model. Each new quota and a corresponding interest rate step will yield the same kind of probability term for the equilibrium locus.

For example, in Sweden, banks' borrowing from the discount window has been restricted by means of a large number of quotas and interest rate steps. In 1988, there were 11 such steps, each bank's scale depending on the size of the bank. (Englund, Hörngren, Viotti, 1989). In practice, the penalty rate model that is discussed here could hardly be modified to include so many steps. But it probably would not be necessary either. This analysis suggests that, at least in the case of relatively narrow steps, it might be reasonable to approximate the overnight interest rate with a continuous, linear function. That is, in fact, the assumption made about the supply of reserves from the discount window in the Swedish system by Englund, Hörngren and Viotti (1989). Further, the argument on which they base this assumption is that banks are uncertain about their actual liquidity positions (pp. 531—532). The idea is therefore very similar to the one formally presented here.

Figure 4.5 shows the interest rate schedule that follows from the assumption that a penalty rate is applied to borrowing in excess of a given quota. The horizontal axis measures reserves relative to their standard deviation, as it also did in the basic model. In the figure, the exogenous quota K is set at four times the deviation of the bank's reserves. The exact amount is not important for the interpretation of the figure; using some other quota would only change the scale.

Figure 4.5　　　　　**Interest rate schedule in a model with a penalty rate and a quota on borrowing**
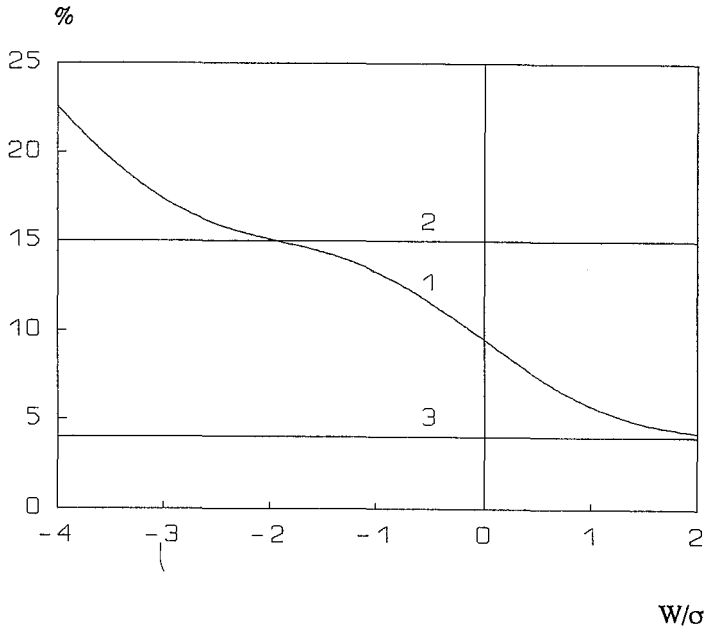
1　Overnight market interest rate
2　Call money lending rate
3　Call money deposit rate

4.4　Time-dependent costs of borrowing

The following section contains a brief discussion on dynamic analysis in the context of this liquidity management framework. The model becomes dynamic if the cost-of-borrowing function is specified as intertemporal. In Section (2.2) above, a study by Goodfriend (1983) was cited as a key contribution to the literature on dynamic borrowing costs.

In order to keep the analysis simple and tractable, the dependency of the bank's cost of borrowing on previous borrowing is included in the model very straightforwardly. It is assumed that an additional penalty rate, r_s, is charged if the bank is a borrower on two consecutive days.

$$\text{Additional penalty costs} = rs_t \cdot \min(w_t, 0) \cdot \frac{\min(w_{t-1}, 0)}{w_{t-1}}. \qquad (4.21)$$

Using a more complicated cost-of-borrowing function would not add any significant new economic insight. Increasing the number of days or formulating more advanced rules for determining the dependency of the cost of borrowing from history would yield basically the same elements in the solution, but in a less transparent form.

As in the previous basic model, the bank is assumed to be concerned about expected profits from arbitrage between the overnight market and the central bank's call money facility. In an intertemporal setting, the time horizon of the objective function becomes important. When considering overnight market transactions, the bank must take into account the consequences that trading today will have on the future. If the cost-of-borrowing function is defined over a two-day time horizon, then borrowing today affects profits both today and tomorrow. Profits in day t and t+1 are written below. Income from later periods is less valuable, and therefore a discount factor c is introduced.

$$\pi_t = rd_t \cdot w_t + (rl - rd)\min(w_t, 0)$$
$$+ rs \frac{\min(w_{t-1}, 0)}{w_{t-1}} \min(w_t, 0) - \delta_t Q_t \qquad (4.22)$$

$$\pi_{t+1} = c\{rd \cdot w_{t+1} + (rl - rd)\min(w_{t+1}, 0)$$
$$+ rs \frac{\min(w_t, 0)}{w_t} \min(w_{t+1}, 0) - \delta_{t+1} Q_{t+1}\}. \qquad (4.23)$$

The expected value of profits on day t are then

$$E(\pi_t) = rd_t W_t + (rl_t - rd_t) E\begin{bmatrix} w_t, & u_t < -W_t \\ 0, \text{ otherwise} \end{bmatrix}$$
$$+ rs_t \cdot E\begin{bmatrix} \dfrac{\min(w_{t-1}, 0)}{w_{t-1}} w_t, & u_t < -W_t \\ 0 \text{ otherwise} \end{bmatrix} - \delta_t Q_t \qquad (4.24)$$

and the derivative with respect to today's trade is

$$\frac{\partial E(\pi_t)}{\partial Q_t} = rd_t + (rl_t - rd_t)\Phi(-W_t/\sigma_t)$$

$$+ rs_t \cdot \frac{\min(w_{t-1}, 0)}{w_{t-1}} \cdot \Phi(-W_t/\sigma_t) - \delta_t. \tag{4.25}$$

Today's borrowing enters tomorrow's cost function only through its effect on expected penalty costs, and therefore the rest of the expression does not affect the solution. The expected penalty costs on day t+1 are

$$E\left[\begin{matrix} c \cdot rs_{t+1} \cdot \min(w_{t+1}, 0), & u_t < -W_t \\ 0 \text{ otherwise} \end{matrix}\right]. \tag{4.26}$$

Taking the derivative with respect to today's trade yields

$$\frac{\partial E(\pi_{t+1})}{\partial Q_t} = -\frac{c}{\sigma_t}\phi(-W_t/\sigma_t)E[rs_{t+1}\min(w_{t+1}, 0)]. \tag{4.27}$$

The total effect of a change in Q_t is the sum of the derivatives of expected profits on days t and t+1. The first order condition for the optimum is that the sum of derivatives equals zero. The solution to this two-day model is therefore defined by the following Euler equation, which expresses the market interest rate as a function of yesterday's, today's and tomorrow's discount window positions. The equation is a second order difference equation with respect to borrowing.

$$\delta_t = rd_t + \left(rl_t - rd_t + rs_t\frac{\min(w_{t-1}, 0)}{w_{t-1}}\right)\Phi(-W_t/\sigma_t)$$

$$- \frac{c}{\sigma_t}\phi(-W_t/\sigma_t)E[rs_{t+1}\min(w_{t+1}, 0)]. \tag{4.28}$$

As in the basic model, the current interest rate is a function of the central bank's interest rates weighted with probabilities of discount window positions. But, in addition to today's probability-weighted deposit and lending rates, this intertemporal solution also includes the penalty rate weighted with a probability that the penalty rate will be applied today. This probability depends on both previous and current borrowing. In this example, it equals zero if previous borrowing is zero because of simplicity of the cost function. With positive values of previous borrowing (negative w_{t-1}), the current penalty rate is weighted with the probability of current borrowing.

The last part of Euler equation takes into account the impact of the current net position on future borrowing costs, which is also a consequence of intertemporal penalty costs. This term is a product of the probability distribution of the current discount window position and expected future values of borrowing and the penalty rate. Because the expected value of future borrowing is non-positive, the effect of this term on the current interest rate is positive in the Euler equation.

To summarize, including intertemporally defined penalty costs in this model yields two additional elements in the solution. These additions can be interpreted as the chance that borrowing today will cause penalty costs today and the chance that borrowing today will cause penalty costs in the future. Intuitively, (4.28) is a probabilistic analogue of the Euler equation in Goodfriend (1983). The next step would be to solve the difference equation for current borrowing so that the dynamics of the solution could be studied. But an inherent difficulty in applying dynamic analysis to this framework is the nonlinearity created by interest rate steps. Even in this simplest case it prevents us from solving the difference equation and reproducing other results presented in Goodfriend (1983).

5 Endogenous Variance: Effects of Liquidity Control

5.1 Introduction

The degree of uncertainty faced by the bank was considered to be exogenous in the preceding discussion. It was assumed that the bank takes the market interest rate, the central bank's interest rates and the variation of reserves as given and chooses the amount of reserves it is willing to hold under these conditions. But, as uncertainty is costly, it seems reasonable to suppose that the bank is willing to make some effort to reduce it, at least over some time horizon. By reducing uncertainty is meant decreasing the value of σ, the standard deviation of net borrowing. That can happen either via a general reduction in the level of liquidity shocks, u, or via a reduction in the frequency of large shocks. In the analysis of this section it is assumed that the bank practices liquidity control with the purpose of affecting the value of σ.

The effects of liquidity control are studied in very general terms here. Clearly, banks are engaged in various activities aimed at controlling their liquidity, though it is difficult to specify the relevant costs and benefits associated with any particular activity. For example, banks are likely to monitor their clients' transactions, synchronize payments, put varying amounts of effort into internal accounting procedures, forecasting reserve needs and collecting information on the general economic environment. These activities produce information about inflows and outflows of funds and they entail costs, as some resources are necessarily devoted to them. So, one could expect that they are pursued to the extent that there is a gain to be achieved.

The following exposition draws especially from studies on information costs in reserve models by Baltensberger (1974) and Baltensberger and Milde (1976). The contribution here is merely the application of their analysis to this particular problem. In this application the central bank's discount window defines the cost of liquidity variation, and therefore the analysis can provide some insight into the effects of the structure of window.

5.2 The model

As in Baltensberger and Milde (1976), efforts to control liquidity are summarized by a variable q measuring resource units spent on relevant activities. The measurement unit of resources can be thought to be man-hours, for example. The cost of one unit of q is denoted by the symbol s and it is assumed to be fixed and independent of the number of the units. The cost of resources devoted to liquidity control is then

$$s \cdot q. \tag{5.1}$$

The relationship between resources used for liquidity control and the degree of uncertainty facing the bank is described by the function a(.). A reasonable assumption is that spending more resources reduces the degree of uncertainty, but at a diminishing rate

$$\sigma = a(q), \quad a'(q) < 0, \quad a''(q) > 0. \tag{5.2}$$

All other assumptions of the previous (basic) model are maintained intact. Allowing for expenditure on liquidity control, expected profits from overnight trade are then

$$E(\pi) = -\delta Q + rd \cdot W + (rl - rd)E(\min(W + u, 0)) - s \cdot q. \tag{5.3}$$

By assumption, the expected value of the random reserve position (W+u) depends on the resources devoted to liquidity control. This is formalized by writing the density function of u in terms of a joint distribution of u and q. The joint distribution is thought to be such that, given the value of q, the distribution of u is symmetric with zero mean. Taking advantage of results presented above, expected profits can be expressed as

$$E(\pi) = -\delta W + rd \cdot W + (rl - rd)F(-W)(W + \int_{-\infty}^{-W} \frac{uf(u,q)}{F(-W)} du) - s \cdot q$$

$$= -\delta Q + rd \cdot W + (rl - rd) \int_{-\infty}^{-W} (W + u)f(u,q)du - s \cdot q. \tag{5.4}$$

Expected profits are then standardized by dividing u by its own standard deviation. The result is that the objective function is again

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expressed in terms of a one dimensional density function. The standardized random variable is independent of q because only its standard deviation depends on q. Define

$$v = u/\sigma, \qquad E(v)=0, \ E(v^2)=1,$$

$$m = W/\sigma. \tag{5.5}$$

Denoting the standardized density function of v by g(v), expected profits in terms of standardized variables are

$$E(\pi) = -(\sigma m - R)\delta + rd \cdot m\sigma + (rl - rd)\sigma \int_{-\infty}^{-m}(v+m)g(v)dv - s \cdot q. \tag{5.6}$$

Taking the derivatives with respect to the decision variables q and m yields the following first order conditions for the optimum. The function G denotes the cumulative distribution function of v such that G'=g

$$\frac{\partial E(\pi)}{\partial m} = -\sigma\delta + rd \cdot \sigma + (rl - rd)\sigma G(-m) = 0 \tag{5.7}$$

$$\Leftrightarrow \quad \delta = rd + (rl - rd)G(-m)$$

$$\frac{\partial E(\pi)}{\partial q} =$$

$$-a'(q)m\delta + a'(q)rd \cdot m + (rl - rd)a'(q) \int_{-\infty}^{-m}(v+m)g(v)dv - s = 0 \tag{5.8}$$

$$\Leftrightarrow \quad a'(q) = \frac{s}{-m\delta + rd \cdot m + (rl - rd)\int_{-\infty}^{-m}(v+m)g(v)dv}.$$

The equilibrium condition for m is of exactly the same form as in the basic model without liquidity control activities. This is so because q enters the interest rate – reserve locus only via its effect on the standard deviation, and therefore it affects m but does not affect w

directly. The latter equation in (5.8) is the equilibrium condition for optimal q. If an explicit expression for a(q) were specified, q could be solved from this equilibrium condition and an explicit equation for the standard deviation would emerge. The result would be a two-equation model for the bank's optimal reserves and optimal degree of liquidity uncertainty, given the exogenous parameters for costs of liquidity control, the central bank's discount window interest rates and the overnight market interest rate.

The economic content in these equations is that the bank is thought to react to uncertainty in two ways: by holding precautionary reserves and by trying to reduce the degree of uncertainty through liquidity control measures. As in the basic model, 'holding reserves' has a slightly different meaning in this context than is usually the case in reserve models. Reserves can be of either sign, the interpretation being that, given the level of interest rates, the bank is not willing to trade away its position completely in the overnight market because of uncertainty. If the bank is on the short side, it will not be prepared to buy the total amount of the shortfall at a price that is slightly less than the central bank's lending rate, as it would be in the perfect information case. The same precautionary behaviour applies equally to long positions as well. The point is that this strategy is costly in comparison to optimal behaviour under perfect information. Therefore, when liquidity control is also included among a bank's options, it will use it as a means to reduce the need to hold costly positions. In that case, the optimal amount of expenditure on controlling liquidity is determined by the above equilibrium condition.

5.3   Comparative statics

Effects on liquidity control

Comparative statics results of the model can be derived directly from the first order conditions 5.7–5.8. These calculations are documented in Appendix 2 and only the results are shown below. First, the effects of changes in exogenous parameters on expenditure on liquidity control are as follows:

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$$\frac{dq}{ds} = \frac{a'}{a''s} < 0$$

$$\frac{dq}{d\delta} = \frac{(a')^2 m}{a''s} \sim Sgn(m)$$

$$\frac{dq}{drl} = -\frac{(a')^2}{a''s} \int_{-\infty}^{-m} (m+v)g(v)dv \geq 0$$

(5.9)

$$\frac{dq}{drd} = \frac{(a')^2}{a''s} \left(-m + \int_{-\infty}^{-m} (m+v)g(v)dv \right) \leq 0.$$

The first result shows the effect on liquidity control of an increase in its own price. It is negative, as it obviously should be, and exactly the same as in Baltensperger and Milde (1976). The next derivative shows that, in this model, the effect of changes in the market interest rate on liquidity control follows the sign of a bank's position in the central bank. This is a direct consequence of allowing reserves to be of either sign in setting up the model; otherwise the derivative is of standard form.

More interesting are the effects of changes in the central bank's interest rates on q because they are distinctive of this particular application. When changes in interest rates are considered separately, we obtain the intuitively reasonable results that raising the discount window lending rate has a positive effect on q while raising the deposit rate has a negative effect. Of particular interest is the combined effect of opposite changes in the central bank's lending and deposits rates, i.e. the effect of widening the central bank's interest rate spread. It can be shown, after some manipulation, that the difference of the derivatives is positive:

$$\frac{dq}{drl} - \frac{dq}{drd} = -\frac{(a')^2}{a''s} \{ m(2G(-m)-1) + 2\int_{-\infty}^{-m} vg(v)dv \} > 0.$$

(5.10)

The result (5.10) above shows that widening the interest rate spread of the discount window will lead to increased investments in liquidity control. A wider discount window interest rate spread will make uncertainty more costly to the banks, and, consequently, the banks are induced to increase expenditure on reducing the degree of uncertainty.

Effects on reserves

Comparative static effects on reserves are calculated by first solving the impact of changes in exogenous parameters on m. The final effects on W are then derived using the above results on the effects of changes on q and, consequently, on σ. The following results are derived in Appendix 2:

$$\frac{dW}{d\delta} = -\frac{a}{(rl-rd)g(-m)} + \frac{m^2(a')^3}{a''s} < 0 \qquad (5.11)$$

$$\frac{dW}{drl} - \frac{dW}{drd} = \frac{2G(-m)-1}{(rl-rd)g(-m)} - \frac{m(a')^3}{a''s}\{m(2G(-m)-1)$$

$$(5.12)$$

$$+ \int_{-\infty}^{-m} vg(v)dv\} \sim -Sgn(m).$$

The first of the above results confirms that an increase in the market interest rate will reduce optimal reserves. The second result concerns the effect of widening the interest rate spread. The signs of the derivatives with respect to discount window deposit and lending rates are ambiguous in both cases when considered separately (see Appendix 2); in this model, raising the discount window lending rate will not necessarily increase optimal reserves. This is because the higher discount window lending rate will, by making reserve deficiencies more costly, increase both m and q, and the latter will reduce σ. In order to obtain an unambiguous effect, some additional restrictions concerning the relationship between σ and q should be introduced.

The combined effect of widening the whole interest rate spread is, however, opposite to the sign of a bank's position. That means that increasing the penalizing interest rate spread will decrease, in absolute terms, both negative and positive reserves. In other words, a wider penalty spread will work to reduce the bank's precautionary positions.

5.4   Solution of the model

Although $a(q)$ was not represented by an explicit function, we can outline the solution of the model and show what elements it would contain. Substituting the equilibrium condition for reserves into the equilibrium condition for optimal q yields

$$a'(q) = \frac{s}{(rl-rd)\int_{-\infty}^{-m} vg(v)dv} \tag{5.13}$$

$$= \frac{s}{(rl-rd)G(-m)E(v|v<-m)}.$$

At this stage an assumption concerning the distribution of v is needed. It will be useful to consider the solution under both normal and logit distributions.

Normal distribution

If G is the cumulative distribution function of standard normal, then $E(v|v<-m) = -\phi(-m)/\Phi(-m)$. The complete model can then be written as follows:

$$\delta = rd + (rl-rd)\Phi(-m)$$

$$a'(q) = \frac{-s}{(rl-rd)\phi(-m)} \tag{5.14}$$

$$\sigma = a(q), \quad m = W/\sigma.$$

In the case of the normal distribution, an explicit solution for m cannot be obtained, but the equilibrium condition for q becomes simple. By taking a Taylor series expansion around m=0, $a'(q)$ can be expressed as

$$a'(q) = c + c \cdot m^2, \qquad c = -\frac{s(2\pi)^{1/2}}{rl - rd}$$
$$= c + c\left(\frac{W}{\sigma}\right)^2.$$

$$(5.15)$$

The above result implies that with a suitable specification of a(q), it is possible to derive an explicit expression for the optimal variance in the model. From (5.15) it follows that the optimal variance is proportional to the squared level of reserves:

$$\sigma^2 = \hat{c}W^2, \quad \text{where} \quad \hat{c} = \frac{c}{(a'(q) - c)}.$$

$$(5.15')$$

This relationship will be utilized later in the study, when empirical equations are specified.

Logit distribution

As was already noted, manipulation of the reserve equation is easier when random shocks are assumed to follow a logit distribution. In that case, the equation for reserves can be solved explicitly, $m = \log(rl-\delta) - \log(\delta-rd)$ and the conditional expected value of v is $E(v \mid v < -m) = -m + \log(1-F(-m))/F(-m)$ (Maddala, 1983, Appendix). Substituting into first order conditions yields

$$m = \log\left(\frac{rl - \delta}{\delta - rd}\right)$$

$$a'(q) = \frac{s}{(rl-\delta)\log\left(\dfrac{rl-\delta}{rl-rd}\right) + (\delta-rd)\log\left(\dfrac{\delta-rd}{rl-rd}\right)}.$$

$$(5.16)$$

The expression for a'(q) states that q depends on s and on a weighted average of (relative) spreads between the market interest rate and discount window interest rates. These spreads are the respective distances from the market interest rate to the central bank's call money lending and call money deposit rates. In order to illustrate the structure of this model more clearly, we denote the upper spread by x and the lower spread by y, implying the following graphical interpretation.

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Figure 5.1   **Partial interest rate spreads**
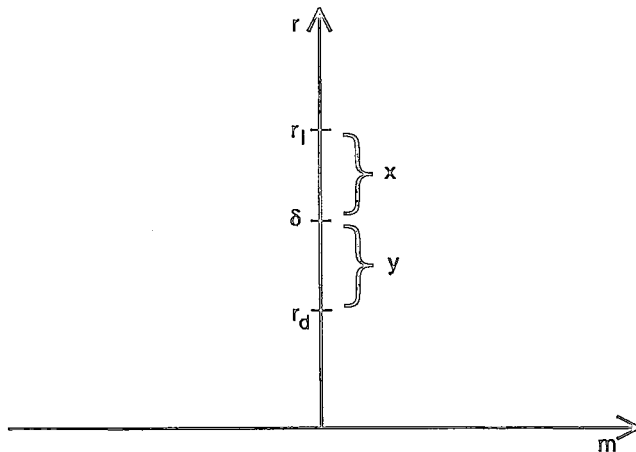
Using these symbols,

$$m = \frac{w}{\sigma} = \log(\frac{x}{y})$$

$$a'(q) = \frac{s}{x\log(\frac{x}{x+y}) + y\log(\frac{y}{x+y})}.$$

(5.17)

According to the above expression, both m and q depend on the relative spreads between the market rate and the upper and lower central bank interest rates. The expression for a'(q) also includes the sum of these partial spreads, measuring the total width of the spread. It can easily be confirmed that an increase in the central bank's interest rate spread will have a positive effect on liquidity control:

$$\frac{\partial a'(q)}{\partial x} - \frac{\partial a'(q)}{\partial y} = -\frac{s}{\psi^2}\left\{\log\left(\frac{x}{x+y}\right) + \log\left(\frac{y}{x+y}\right)\right\} > 0$$

(5.18)

$$\text{where } \psi = x\log\left(\frac{x}{x+y}\right) + y\log\left(\frac{y}{x+y}\right).$$

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5.5   Concluding remarks

The aim of this chapter was to endogenize the variance of liquidity. Holding reserves is the only way the bank is thought to react to uncertainty in a standard liquidity management model. But, because reserves are costly, a logical extension to the model is to consider the implications from including alternative ways to react, and specifically the implications for direct expenditure on liquidity control. Allowing for the possibility of liquidity control measures broadens the interpretation of the model considerably. This study emphasizes the reserve management decision, but one could also view the bank's problem as being primarily a problem of controlling liquidity and holding reserves as a way to economize on costs of liquidity control (Baltensberger and Milde, 1976).

In this two-equation model both the optimal variance and optimal reserves are functions of interest rate spreads between the market interest rate and the central bank's lending and deposit rates. The optimal amount of (standardized) reserves is determined as a function of the upper spread relative to the lower spread just as in the basic model, while optimal variance also depends on the height of the central bank's interest rate step. It was shown that raising this step increases expenditure on liquidity control, reduces optimal variance and reduces both negative and positive reserve positions in absolute terms. Further, it was shown that under certain assumptions optimal variance can be expressed as a function of squared reserves.

These results are consistent with the sudden decline in the variability of borrowing that occurred when the central bank's interest rate step was first introduced in March 1987. Figure 3.1 on page 36 demonstrates that a clear change in borrowing behaviour occurred at that time. This observation cannot be explained in terms of the exogenous variance model presented in the previous chapter. This analysis of endogenous variance model suggests that the banks' were induced by this change to increase their expenditure on liquidity control in order to reduce the variability of borrowing because holding reserves had become costly. Such a regime shift is, of course, an extreme case. The fact that it had noticeable effects on that occasion does not necessarily imply that liquidity control should be an important factor when borrowing behaviour is studied under more normal circumstances. Obviously, some measures to control liquidity are too cumbersome to be flexibly adjusted according to market conditions. For example, the time perspective in decisions concerning the structure of organization cannot be the same as in short-term liquidity management. Probably a major regime shift is needed to

cause changes in organization. But payments synchronization and monitoring clients' transactions, carried out with a varying degree of effort, are examples of activities which might be relevant for short-term considerations as well.

In the empirical section of the study, the variance is specified as time-dependent in GARCH estimations by conditionalizing it on available information. Although the equation for conditional variance is deterministic in the empirical model, it is basically treated as an endogenous variable. The above results are a possible explanation for some of the empirical findings concerning the specification of the conditional variance equation that are presented in Chapter 7. In particular, these results may help to explain the asymmetric response of borrowing to interest rate changes.

6 Empirical Application of the Model with Constant Variance

6.1 Data and organization of the empirical study

The empirical work is organized under two main headings, according to the assumption that is made about the variance of the model. First, constant variance specifications are discussed in this chapter. The aim is to describe the relationship between the key variables of the model using simple econometrics and to carry out conventional analysis of overnight market interest rates. Results from straightforward estimations of a nonlinear overnight market interest rate equation are presented first. After that we estimate theoretically more justified specifications in which the variable to be explained is banks' borrowing from the central bank. The chapter concludes with a description of the observed volatility in borrowing.

In Chapter 7, the empirical model is related more closely to the preceding theoretical discussion by extending the analysis to a GARCH framework, which allows for time-dependent conditional variance. According to the liquidity management model, the steepness of the bank's demand-for-reserves locus depends on the variance of liquidity, and therefore a crucial element of the theory might be missing from constant variance specifications. Autoregressive specifications of conditional variance are considered first. This approach can be viewed as a relatively direct empirical implementation of the one-equation liquidity management model. We then introduce exogenous variables into the conditional variance equation. At this stage, the results from the two-equation model are used as a point of departure, although several other money market variables are also considered. It turns out that the specification corresponding to the theoretical two-equation model is also empirically most promising. Technically, the specification implies asymmetric responses to new innovations. The model exhibits fat-tailed residuals and therefore it is also estimated under the standardized conditional Student's t distribution. When Student's t distribution is used, the thickness of the tails of the distribution varies, depending on the degrees of freedom parameter.

The data used in the empirical applications of the model are from March 1987 to June 1989. During that time, the banks' were discouraged from using the central bank's discount window mainly by

means of a penalizing interest rate spread between the call money deposit and lending rates. After that period, the terms applied to discount window borrowing were modified in a way that made time-dependent costs more relevant than previously. Data from that period are still unsatisfactorily fractured because of a banking strike and major changes in the call money financing system, and are not therefore included in the sample. Moreover, from the point of view of testing the assumptions of the model, the difference between estimations of these two regimes would be in the functional form of the cost-of-borrowing schedule and due only to differences in borrowing terms.

The system also underwent important changes during the period from March 1987 to June 1989. In order to control for the effects of these institutional developments, the model was estimated separately for the whole period and for two shorter periods. During the first six months of the whole estimation period, the central bank did not announce daily figures on the liquidity of the system. Only end-of-week balances were published and with a two-day lag. Starting from August 1987, an estimate of the previous day's central bank deposits and credits was published every morning. Because of these changes, the information available to the banks was significantly poorer in the beginning of the estimation period than in the remaining part. The first six months were therefore excluded from the first estimation period.

Also excluded was a one-month period prior to the widening of the central bank's call money interest rate spread in October 1988, from which time the second period of estimation begins. At that time, the average rate of interest was apparently totally unresponsive to changes in liquidity, indicating that the market was seriously disturbed. Thus, the first sub-period was from August 1987 to August 1988 and the second, during which the interest rate spread was wider, from October 1988 to June 1989. Because of these definitions of the estimation intervals, the two shorter estimation periods do not add up to the whole period under investigation. However, in addition to examining the whole period, the aim was to form sub-samples during which there were no known disturbances of an institutional nature.

Data on banks' discount window deposits and credits are based on daily balance sheet information. All banks with a right to central bank financing have cheque accounts in the Bank of Finland and these accounts are used for the settlement of interbank claims. Technically, the banks do not have to ask for call money credits or to offer call money deposits explicitly. When, after clearing, a bank's overall position is known, its cheque account in the central bank is automatically adjusted so that the balance sheet constraint is satisfied.

Thus the balances on these cheque accounts at the end of the day constitute the net discount window position variable for each bank used in the study. The aggregate position of the whole banking system is the sum of individual banks' positions. The number of banks with access to the call money facility was 10 or 11 during the estimation period. In practice, all commercial banks have had this right. Included in this category are the central banks of savings banks and cooperative banks, through which financing is channelled to local banks.

There is a potential source of error in the balance sheet information when used for the purposes of the model considered here. It has been a practice to accept corrections to banks' clearing up till noon of the following day. It is not possible to know whether all these transactions are actually based on book-keeping errors or if they include new deals made afterwards in order to adjust the discount window position. Another potential source of error is the exceptional liquidity developments of individual banks, i.e. major disturbances in funding. For example, in Peristiani (1991), aggregate borrowing figures are corrected for episodes of this kind. During the period under investigation in this study, this latter problem was probably not serious. At least there were no special financing arrangements for individual banks during that time.

The interest rate variable used is the overnight market rate of interest computed and published by the Bank of Finland. It is calculated as a weighted average of rates of interest applied in, all interbank overnight transactions. This might introduce some problems, which will be discussed later. It would be more appropriate to use interest rates based on actual quotations, and preferably from the same point in time. Unfortunately, such data are not available for the overnight market; banks do not quote prices of overnight loans systematically, nor do they act as market makers as they do in the markets for longer maturities.

Interest rates for discount window deposits and credits, and all exogenous variables used in explaining volatility, are taken from the Bank of Finland's database. Throughout the empirical work, all data used are daily. In Finland, the interval between settlements is one day, which makes this data frequency a logical choice.

6.2 A model with constant variance

The interest rate schedule that was derived from the basic liquidity management model expressed the overnight market rate of interest as an average of the central bank's deposit and lending rates, weighted with probabilities of target levels of discount window positions. Assuming normally distributed deviations of the bank's actual liquidity position from the predicted position, this schedule was shown to be a probit function with the central bank's call money interest rates as limiting asymptotes.

We begin the empirical analysis with a simple regression of discount window positions on the overnight market interest rate, using a cumulative normal distribution function as a functional form of specification. Although the theoretical model is a demand-for-reserves equation and not an interest rate equation, explaining market interest rates with the discount window position of the banking system as an explanatory variable is of interest especially from the point of view of the aggregate level reserve market model. The interest rate equation corresponds to the reduced form of that model (eq. E6' in Chapter 2).

The derived interest rate schedule was

$$\delta = rd + (rl - rd)\Phi(-\Sigma Ri/\Sigma\sigma_i)$$
$$= rd \cdot \Phi(\Sigma Ri/\Sigma\sigma_i) + rl \cdot \Phi(-\Sigma Ri/\Sigma\sigma_i). \tag{6.1}$$

In order to implement it empirically, the following equation was specified:

$$\delta_t = a_2 rd_t \Phi(a_1 w_t) + a_3 rl_t \Phi(-a_1 w_t) + \varepsilon_t, \tag{6.2}$$

where w_t = banks' aggregate net discount window position in the central bank, and

ε_t = residual (a measurement error).

In the parameterization of the interest rate equation, coefficient a_1 was included in the cumulative distribution function. It can be interpreted as an inverse of the standard deviation variable in the interest rate schedule, the assumption being that the degree of uncertainty is constant over time, $\sigma_t^2 = \sigma_{t+k}^2$. All time-dependent variables of the model were specified to be from the same day.

The results from the estimation of (6.2) are reported in Table 6.1. Estimations were carried out separately for the whole period and for the two sub-samples described above. The equation for the whole period performed especially poorly without any adjustment for the change in the width of the penalizing interest rate spread, and therefore a dummy variable that permitted coefficient a_1 to change in the latter part of the period was added to the equation.

The parameter estimates for the coefficients of asymptotes are in most cases reasonably close to one, which is the value implied by the model (strict parameter restrictions are statistically rejected in several cases, however). The estimate for the parameter defining the steepness of the schedule varies in different estimation periods. R-squared in these estimation runs from .62 to .74.

Table 6.1   **Interest rate model, OLS**

Sample:	All obs.		First period		Second period	
Coefficient						
a1	1.89	(.324)	2.75	(.375)	.801	(.145)
a2	1.06	(.014)	1.08	(.006)	1.36	(.170)
a3	1.01	(.028)	.957	(.009)	.976	(.015)
a4	-.68	(.058)				
R²:	.628		.745		.687	
SSR:	373.8		49.2		207.6	
SEE:	.803		.418		1.098	
Durbin—Watson	.885		1.66		.912	
Observations	584		295		175	
Wald test statistics for parameter restrictions:						
a2 = a3 = 1	47.8**		193.4**		4.6	
a2 = 1	21.9**		189.0**		3.5	
a3 = 1	.16		21.2**		.57	

The estimated equation was for the whole period $\delta_t = a_2 rd_t \Phi((1 + a_4 D)a_1 w_t) + a_3 rl_t \Phi(-(1 + a_4 D)a_1 w_t) + \varepsilon_t$, where coefficient a_4 is for a slope dummy D that effectively allows for a shift in parameter a_1. The dummy equals one in the latter period and zero elsewhere. In estimations of sub-periods, the dummy is not included.

SEE = Standard error of estimate
SSR = Sum of squared residuals
Standard deviations of estimated parameters are in parentheses (computed from the covariance matrix modified using a procedure developed by Hansen, 1982).
The Wald test statistics are distributed under the null hypothesis as chi-squared with degrees of freedom equal to the number of restrictions.
Asterisks indicate significance levels (** = 1 %, * = 5 %) for tests of parameter restrictions.

Figure 6.1   **Overnight rates of interest and fitted values from the constant variance interest rate equation**

%

March 1987 — June 1989
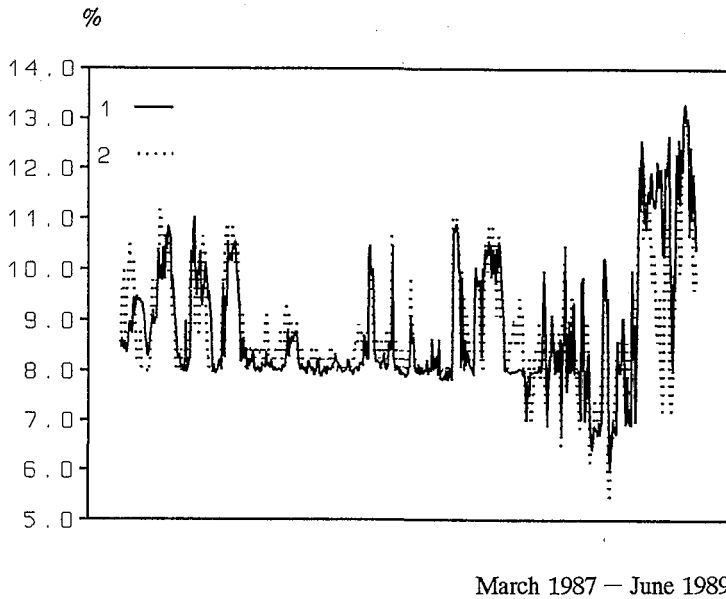
1 Overnight market interest rate
2 Fitted values

The residuals of the model are serially correlated, the values of Durbin—Watson statistics ranging from .756 to 1.66. Because of autocorrelation, a direct estimate for the variance-covariance matrix of parameters is not consistent. Therefore, a procedure developed by Hansen (1982) was used to modify the estimate for the covariance matrix. (A lag length of 12 periods was applied. The effect was that the original standard deviations of the parameters more than doubled.) All tests for parameter restrictions were calculated from the modified covariance matrix.

Fitted values from the interest rate model (6.2) are shown in Figures 6.1 and 6.2. In the first of these figures, the values generated from the estimated model are compared to actual values of overnight interest rates. The fit does follow the main developments in the data, but the problem of serial correlation is also clearly evident from the graph. Periods during which the model systematically under- or overestimated observed interest rates can be visually detected. The model fails most notably in the spring of 1988, for a period of approximately one month. This is probably connected with the revaluation of the currency at that time. A change in the fluctuation

limits of the markka was followed by a tightening of conditions in the money market, increased discount window borrowing by the banks and a steep rise in short-term interest rates. The model does not adequately capture these large changes.

Figure 6.2 illustrates fitted values from the model as a function of the discount window position of the banking system. These estimated interest rate schedules, or inverted demand-for-borrowing schedules, are graphed for two discount window interest rate regimes. The narrow schedule relates to discount window interest rates from March 1987 to October 1988 (7.5 per cent and 11 per cent), and the other to interest rates from October 1988 to June 1989 (4 per cent and 13 per cent).

Figure 6.2   **Fitted values from the constant variance interest rate model as a function of the discount window position of the banking system**

%

– FIM 4 bill.   + FIM 4 bill.

1 March 1987 – October 1988
2 October 1988 – June 1989

80
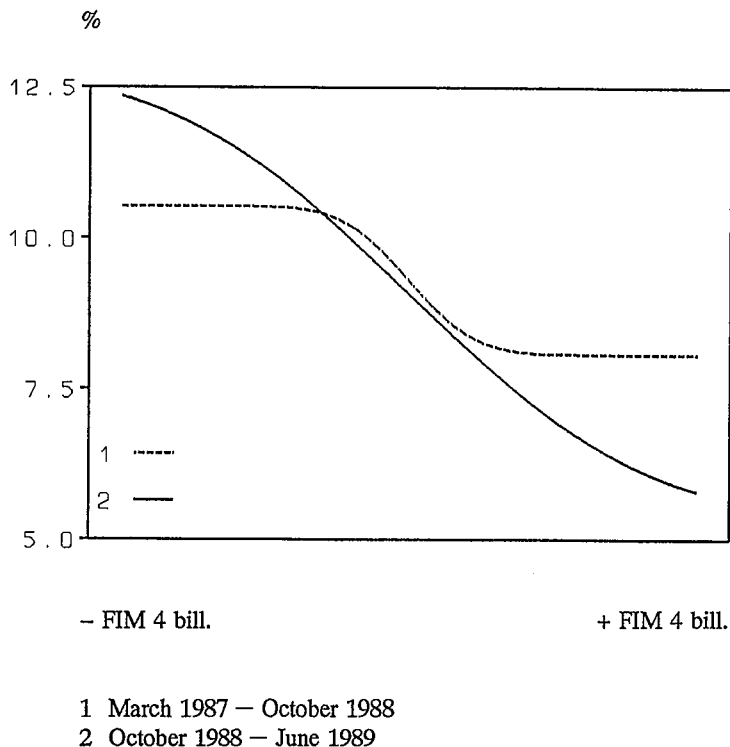
The analysis of the theoretical model in the preceding section suggests that the empirical interest rate equation (6.2) might suffer from a problem of missing variables. The actual terms for discount window borrowing were, in fact, more complicated during the period under investigation than is assumed in (6.2). Most notably, there was an upper bound for borrowing at the interest rate rl, and borrowing in excess of quota was subject to a penalty rate rs. It was shown that the interest rate schedule should include the effect of the probability of exceeding this limit. Another point made in the analysis was that risk averse behaviour by banks would add a positive risk premium to interest rates.

In order to take these considerations into account, variables arising from the quota system, $(rs-rl)\Phi((K-w)/\sigma)$, and risk aversion, $(rl-rd)^2\{(b+1-\Phi)\sigma\phi-w(1-\Phi)\Phi\}$, were added to (6.2) (see pp. 53 and 58). In both cases the variance term was replaced with coefficient a_1 as in (6.2) and the model was estimated using OLS. Neither of additional variables proved to be significant. It should be noted that there were no instances where aggregate borrowing was observed to be in excess of the aggregate quota. Although the existence of a quota should, according to the theory, affect borrowing behaviour, it is not detected empirically. The results are reported in Appendix 3.

But a major problem in the above specification of the empirical interest rate equation is that it is based on a significant simplification of the error structure implied by the theoretical model. In the theoretical model, ΣR_i was the aggregate target level of liquidity, which is an unobservable variable. The observed variable is the net daily position of the banks, $w_t = \Sigma R_{it} + \Sigma u_{it}$. In the interest rate model the observed net position was used instead of the target level, the implicit assumption being that $\Sigma u_{it} = 0$. If this is not the case, as might be expected, then it is not reasonable to assume that the residuals in the empirical model follow a known distribution.

Therefore, in order to maintain the interpretation of the theoretical model, the interest rate schedule has to be inverted for estimation. The model is then a normal demand-for-reserves equation, in which the level of reserves is explained by the market interest rate and the cost of net discount window borrowing. As noted above on p. 49, the interest rate equation can be inverted to form a net borrowing equation by applying the logistic distribution as an approximation for the normal distribution. The cumulative distribution functions of the standard normal are then replaced by exponential functions and the unobservable sum of reserve targets, ΣR_{it}, can be solved out. Writing this as $w_t - \Sigma u_{it}$ gives the reserve model directly in a form that can be estimated

$$\delta_t = rd_t + (rl_t - rd_t)\Phi(-\Sigma R_{it}/\Sigma\sigma_{it})$$

$$\Rightarrow$$

$$\Sigma R_{it} = \Sigma\sigma_{it}/k[\log(rl_t - \delta_t) - \log(\delta_t - rd_t)] \tag{6.3}$$

$$\Rightarrow$$

$$w_t = \Sigma\sigma_{it}/k[\log(rl_t - \delta_t) - \log(\delta_t - rd_t)] + \Sigma u_{it}.$$

In (6.3), the level of net borrowing is given as a product of the standard deviation of reserves and a logarithmic expression for spreads between the market interest rate and the central bank's lending and deposit rates, plus random shocks to liquidity. These shocks, u_i, are normally distributed with mean zero and variance σ_i^2, which implies that the sum of shocks, Σu_i, also follows a normal distribution with $\Sigma u_i \sim N(0, \Sigma\sigma_i^2)$. In order to simplify the notation, indexing over banks is not used from here on, so that $\sigma^2 = \Sigma\sigma_i^2$, and $\Sigma u_i = u$, $u \sim N(0, \sigma^2)$. It should be noted, however, that the volatility term in the reserve equation is a function of the standard deviations of individual banks' borrowing, $\sigma = (\Sigma\sigma_i^2)^{1/2}$. Thus there might be an aggregation problem in the specification of the empirical model. For example, Peristiani (1991) and Dutkowky and Foote (1985) emphazise aggregation considerations.

In the empirical applications aggregate data on borrowing were used, which means that it was implicitly assumed that the sum of the standard deviations of borrowing is proportional to the standard deviation of the sum of borrowing. In the light of historical variances from both aggregated and disaggregated data shown in Figure 6.3 on page 86, this is not a particularly controversial assumption. Of course, a coefficient was included in the specification in order to allow for a constant difference between the levels of aggregate and disaggregated variables.

The equation for banks' net borrowing was first estimated with the same constant variance assumption that was made in the estimation of the interest rate model. In order to ensure compatibility with later specifications, the model was parametrized to include the variance parameter explicitly and estimated using the method of maximum likelihood. The specification of the empirical constant variance net borrowing equation was

$$w_t = b_1\sqrt{c_0}/k(\log(b_2rl_t - \delta_t) - \log(\delta_t - rd_t)) + u_t \tag{6.4}$$

$$u_t \sim N(0, c_0).$$

Table 6.2 **Equation for net borrowing, constant variance (ML)**

Sample:	All obs.		First period		Second period	
Coefficients						
b1	1.57	(.074)	1.80	(.104)	2.21	(.175)
b2	1.02	(.003)	1.06	(.014)	1.03	(.008)
c0	.280	(.013)	.178	(.012)	.256	(.025)
LogL	-455.55		-158.09		-128.44	
R^2:	.588		.551		.705	
SSR:	162.89		50.62		44.65	
Ljung-Box						
LB(5)	552.7**		144.4**		121.8**	
LB^2(5)	313.8**		43.1**		78.27**	
Durbin—Watson	0.72		0.96		0.95	
TR^2	172.4**		20.52**		33.41**	

Estimated model:

$$w_t = b_1 \sqrt{c_0} / k(\log(b_2 rl_t - \delta_t) - \log(\delta_t - rd_t)) + u_t$$

$$u_t \sim N(0, c_0)$$

Standard errors of the estimates are in parentheses.
The Ljung-Box(P) statistic tests the randomness in standardized residuals (LB) and in standardized squared residuals (LB^2) indicated by first P autocorrelations. Under the null hypothesis, it follows a chi^2 distribution with P degrees of freedom.
TR^2 is the Lagrange multiplier test statistic for the presence of first order ARCH as proposed in Engle (1982a). It is computed as TR^2 from a regression of squared standardized residuals on its own first lag. Under the null hypothesis of no ARCH, TR^2 is distributed as chi^2 with one degree of freedom.

The main reason for including coefficient b_2 in the specification is that the data contain five observations of the market interest rate that are a few basis points higher than the prevailing discount window loan rate. Coefficient b_2 is needed to ensure that the equation is also defined in these cases. The scale parameter k=1.6 is included merely to preserve the original interpretation of the parameters. The results are presented in Table 6.2. The important thing to note is that the constant variance reserve model also performs inadequately in diagnostic tests; the residuals are serially interdependent and exhibit heteroscedasticity. This is clearly illustrated by the high values of the Ljung-Box test statistics computed from standardized residuals and squared standardized residuals.

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The presence of serial correlation in the residuals of the model is, of course, a sign of misspecification. At worst, it is possible that some of the key assumptions underlying the model are violated, so that the theoretical model should be rejected and derived again from the beginning under new assumptions. The framework of the analysis could be modified, for example, to allow for imperfect competition between the banks. But the problem may also be due to less serious problems connected with empirical implementation of the model.

It is evident that the severeness of the problem of serial correlation, as detected by the tests, is partly due to the high frequency of the data. Using daily data provides a large number of observations, so the chances of rejecting a null hypothesis are statistically better than with lower frequency data from the same time span. But one might suspect that the adjustment of interest rates is completely instantaneous, irrespective of the time frame of sampling.

The model was also estimated using only one observation from each week. The estimations were carried out separately for each day of the week, so that any bias that could have resulted from averaging over weeks could be avoided. One data set contained the observations from Mondays, one from Tuesdays, and so on. The results of these estimations are reported in Appendix 3. The main point is illustrated in Table 6.3, which shows test statistics on serial correlation from week-day estimations compared with the statistics on the whole data set. The problem of serial correlation is substantially reduced in week-day estimations; but at a cost of using less information. The estimations of daily models do not reveal any obvious differences between different days. Friday might be expected to be an exceptional day, because Friday's position determines the cost for the weekend as well. However, there is only slight evidence of this. Mostly, it is the problem of serial correlation which is the worst in data for Fridays. The data also show that, on average, the banks' position has been somewhat smaller at weekends than during the week, although no systematic differences are found between interest rates on Fridays and those on other days. The mean values of variables and tests for differences between means are also reported in Appendix 3.

| Table 6.3 | **Statistics on serial correlation in the residuals of week-day models** | |
	Q(5)	D–W
Monday	8.77	1.56
Tuesday	7.38	1.60
Wednesday	8.28	1.42
Thursday	18.92**	1.42
Friday	21.53**	1.31
All days	527.78**	0.72

The estimates of the model might also be affected by a simultaneity bias, because the estimated equation is a demand equation and the observations may reflect changes in both demand and supply, as was discussed in Chapter 2 when the aggregate level reserve market model was presented. Estimated parameters are unbiased only if all variation is caused by supply side factors and the demand relationship is stable. The supply side in this context is the fixed amount of liquidity that is available in the market at the time the overnight market is open. Therefore, observations should reflect daily changes in liquidity only, otherwise the estimates are biased. If the theoretical model is taken extremely seriously, then simultaneity should not be a problem in estimations of the borrowing equation. By assumption, the only stochastic component in the model is the amount of available liquidity, and the specification of the model was derived from that assumption. But of course, one cannot rule out the possibility that the demand relationship is subject to errors. Our best defence is that it seems unlikely that this component could be important as compared to changes in daily liquidity. If it were important, then instrumental variables techniques should be used in estimation and perhaps a less exact functional specification should be preferred. However, the strategy adopted here was to maintain the assumptions of the theoretical model, also those concerning the stochastic properties, and to proceed with the specification that was derived.

6.3   Changes in the standard deviation of borrowing

In the light of liquidity management theory, an obvious question to ask is whether the assumption of constant variance of discount window borrowing is reasonable; or is it the cause of the symptoms of misspecification that can be detected? Because the theory predicts that

the variance of borrowing explains the level of borrowing, the constant variance assumption is a particularly interesting issue in the empirical implementation of the model. If changes in the variance are a source of variation in borrowing, then constant variance specifications suffer from the missing variables problem, in addition to the usual problem of inefficiency of estimates.

One way to approach this question is to examine the variance in the data at different time periods. Figure 6.3 contains two moving standard deviation series computed from net discount window borrowing. One series is computed from the aggregate net discount window position of the banking system, and the other is a sum of deviations of individual banks' borrowing. The latter is probably closer to the concept that was used in the derivation of the model. On the other hand, the former is much easier to obtain, as well as being based on publicly available data. The time span in these calculations was approximately one month (22 banking days), so that each observation in the series is a standard deviation of the previous month's borrowing.

Figure 6.3 **Moving standard deviation of the banks' net discount window position**

Mill. FIM
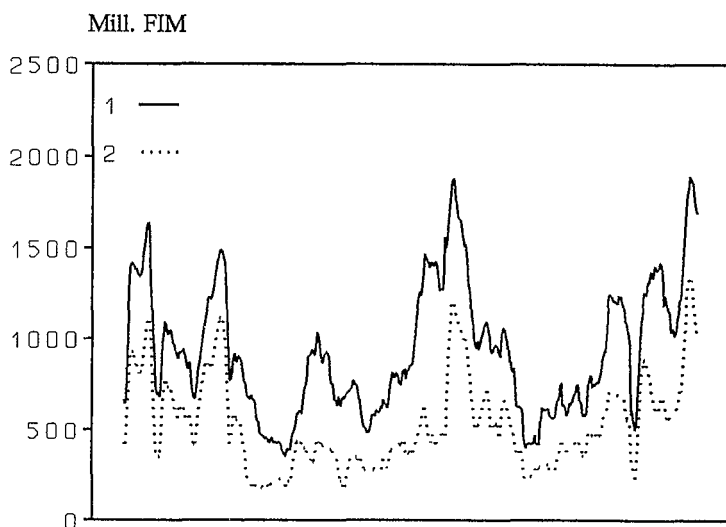
March 1987 – June 1989

1 Sum of standard deviations of individual banks' positions
2 Standard deviation of the aggregate net position

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The average level of the moving standard deviation of aggregate borrowing is close to FIM 500 million and the corresponding figure for disaggregated data is about twice as high. The correlation between these two series is high, over .90, which indicates that the standard deviation from the aggregate data might be an appropriate proxy for the sum of deviations.

Judging from graphs of both series of moving standard deviations, the constancy of variance is not supported by the data. This suggests that it should be included in the equation for borrowing as an explanatory variable. Related evidence is provided by the Ljung-Box tests for non-linear dependence and the Lagrange multiplier test for ARCH that were presented in Table 6.2, and which both indicate heteroscedasticity. Because the model is unsatisfactorily specified, these tests cannot be considered to be reliable. When serial correlation is eliminated by means of the AR(1) correction, evidence of heteroscedasticity is still found, although it is much weaker.[1]

A crude approach would be to estimate the model using the above computed moving standard deviation series as explanatory variables. But simply adding such exogenously computed proxies to the model must be considered an inadequate solution to the problem for at least two reasons. First, the time span and the weights used in the computation of these variables are completely arbitrary. Using some other formula would yield a different variable. Therefore, some criterion would be needed for choosing between alternatives. Second, any method of computation that treats the variance as an exogenous variable would result in inefficient estimation of the borrowing equation. Because the amount of discount window borrowing in different time periods and the variance of borrowing are determined by the same process, it would be inappropriate to estimate either of these assuming the other to be exogenous. But, especially because variance enters the demand schedule and affects the estimates of associated parameters, the variance of residuals from estimation should be linked to the variance in the demand schedule and the system should be estimated as a whole.

These considerations can be taken into account in ARCH and GARCH models, which are the subject of the next chapter. They provide a framework in which a liquidity management model can be empirically specified and estimated in a way that encompasses the essential feature of the theoretical model, i.e. that the steepness of the demand-for-reserves schedule depends on the variance of liquidity shocks.

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[1] The presence of heteroscedasticity could be further studied by applying the test presented in White (1980) to equation (6.4). This was not considered necessary here, however, because the heteroscedasticity properties of the model are investigated using tests for parameter restrictions in a more general model in the next chapter.

7 Estimation of the Reserve Model with Time-Dependent Conditional Variance

7.1 A nonlinear GARCH-in mean model

A standard, albeit implicit, assumption made about the variance in econometric models is that it is in the information set of the agents whose behaviour is being described. Of course, there is usually no particular reason why this should be the case. There are very strong grounds for questioning the validity of this assumption in reserve models, because the whole need for holding reserves in the theoretical model derives from the expected variation of reserves in the future. Fixing the variance by presuming that it is known to agents, *ex ante* and *ex post*, is, in principle, a very restrictive assumption in these models.

ARCH and GARCH models constitute a class of econometric models that explicitly allow for non-constant variance, or conditional dynamic heteroscedasticity. In these models, the conditional variance may vary over time, although the unconditional, or theoretical, variance is constant. More precisely, the expected value of the second moment of the distribution of a random variable is specified to be conditional on the set of information that is available at the time expectations are formed. In other words, if we consider the distribution of a random variable ε and the information set I, the idea in these models is to make a distinction between time-varying expectation $E_t(\varepsilon^2 \mid I_{t-1})$ and constant, unconditional variance $E(\varepsilon^2)$. Important contributions to the literature on these models include Engle (1982), Bollerslev (1986), Engle, Lilien and Robins (1987) and Bollerslev, Engle and Wooldridge (1988).

Because of this focus on information in the specification of the variance of the model, ARCH and GARCH models are potentially applicable, especially when the problem to be modelled econometrically involves some aspects of uncertainty. In the model considered here the link is obvious. The point of departure in discussing banks' demand for reserves above was that their liquidity position at the end of the day was unknown at the time the decisions on trade in the overnight market were made. Without further assumptions, it is logical to take account of the fact that the variance of liquidity is also unknown to banks. The notion that banks act on the

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basis of the expected variance of liquidity, conditional on available information, fits very well into this framework.

The information set in ARCH and GARCH models normally consists of past observations of the variables in the model and the equation for the conditional variance is autoregressive. In principle, there is no reason why an autoregressive equation should always be applied. If the conditional variance has the interpretation in the model that it is an expectation formed by some agent, then it might be possible to model the process using truly exogenous variables. Without any knowledge of the true process, the autoregressive equation is, of course, a natural choice.

In the following a GARCH-in mean model is used to explain banks' demand for reserves. The model is formed by adding an equation for the conditional variance of liquidity, on the assumption that it is the conditional variance that enters into the banks' demand schedule. In this setting, the conditional variance can be regarded as the banks' expectation concerning the variance of liquidity in the same day.

From here on, the symbol h^2 is used to denote the conditional variance and the symbol σ^2 is reserved for unconditional variance. The banks' information set is denoted by the symbol I. In order to keep the notation simple, the variable X_t with a parameter vector ß is used to denote the logarithmic expression for exogenous variables in the reserve equation (6.4). The explanatory variables in the equation for the conditional variance are expressed as $Z_t\tau$, where Z_t is a matrix consisting of past errors and past conditional variances and τ the corresponding vector of parameters. Using these symbols, the model can now be written in the following form:

$$w_t = h_t X_t \beta + u_t; \qquad u_t | I_{t-1} \sim N(0, h_t^2)$$

$$h_t^2 = Z_t'\tau,$$

(7.1)

where $\quad Z_t' = (1, h_{t-1}^2, h_{t-2}^2, ..., h_{t-q}^2, u_{t-1}^2, u_{t-2}^2, ..., u_{t-p}^2)$

and $\quad X_t = \log((rl_t - \delta_t)/(\delta_t - rd_t)).$

The conditional variance equation in (7.1) is of the form GARCH(p,q), where h_t^2 is a function of past residuals, u_{t-i}^2, $i = 1, ..., q$, and lagged values of conditional variance, h_{t-i}^2, $i = 1, ..., p$. If h_t^2 is specified as a function of past residuals only, the model is said to be ARCH(q)

(autoregressive conditional heteroscedasticity). The ARCH model, first introduced in Engle (1982), is a special case of the GARCH model, as the difference between these models can be expressed as linear restrictions on parameters. Moreover, it can be shown that the GARCH(q,p) process can always be expressed as the ARCH(∞) process, i.e. with an infinite number of lagged residuals (Bollerslev, 1986, p. 309).

Important extensions of simple ARCH and GARCH models are ARCH- and GARCH-in mean models, where the conditional variance is included as an explanatory variable in the mean equation (equation for w_t here). In mean models clearly have more economic content than simple models because the conditional variance is also allowed to affect the predictions of the model in addition to a rather technical correction for heteroscedasticity. Mean-effect implies that uncertainty has a genuine effect on behaviour. In the first GARCH-in mean model presented in the literature, the conditional variance was used to account for a risk premium in the interest rate equation (Engle, Lilien, Robins, 1987).

In model (7.1) above, the square root of the conditional variance, h_t, enters the reserve equation as a result of banks' maximizing behaviour. So, the model belongs to the class of in mean models. It differs somewhat, however, from the original formulation of the model by Engle et al. (1987). In their presentation the effect of the conditional variance term in the mean equation was linear while in (7.1) it is nonlinear or multiplicative.

It should be emphasized that the nonlinearity of the mean equation with respect to conditional variance has a meaningful economic interpretation in this context. Because of this nonlinearity, uncertainty affects the steepness of the demand schedule and the sign of the effect of uncertainty on the market interest rate depends on the level of reserves. An increase in uncertainty raises the market interest rate if banks are net borrowers and lowers it if they are net lenders. Linearization of the equation would substantially alter the logic of the model. In a linearized model the changes in variance would shift the interest rate schedule independently of the level of reserves. This would change the interpretation of the effect of uncertainty, so that it would be the same as in the case of a risk premium, which is not consistent with the theoretical model discussed above.

As is explained in Engle and Bollerslev (1986) and in Chou (1988), the parameters of the variance equation of a GARCH(1,1) model are particularly informative as regards the persistence of shocks. This can be seen by considering the time path of conditional variance in a model where $h_t^2 = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1}^2$. The conditional variance at

period t+s and its expected value conditional on information available at period t are then

$$h_{t+s}^2 = c_0 + c_1 E_t(u_{t+s-1}^2) + c_2 E_t(h_{t+s-1}^2)$$

$$E_t(h_{t+s}^2) = c_0 + c_1 E_t(u_{t+s-1}^2) + c_2 E_t(h_{t+s-1}^2) \qquad (7.2)$$

$$= c_0 + (c_1 + c_2) E_t(h_{t+s-1}^2).$$

On the other hand, the unconditional variance of the model is[1]

$$E(u_t^2) = \sigma^2 = c_0/(1 - (c_1 + c_2))$$

$$\Rightarrow \quad c_0 = \sigma^2 - (c_1 + c_2)\sigma^2. \qquad (7.3)$$

Solving (7.3) for c_0 and substituting the result into the equation for conditional variance yields

$$E_t(h_{t+s}^2) = \sigma^2 + (c_1 + c_2)(E_t(h_{t+s-1}^2) - \sigma^2)$$

$$\Leftrightarrow \qquad (7.4)$$

$$E_t(h_{t+s}^2 - \sigma^2) = (c_1 + c_2)^s(h_{t+1}^2 - \sigma^2).$$

The formulation above reveals directly that for all parameter values $c_1 + c_2 < 1$ (stationary variance equation) the conditional variance $E_t(h_{t+s}^2)$ approaches the unconditional variance σ^2 when s goes to infinity. Further, the impact of shocks on volatility decays at a constant rate and the speed of the decay is measured by the sum of the parameters c_1 and c_2 (Chou, 1988, p. 282; see also Engle and Bollerslev, 1986).

If the sum of the coefficients c_1 and c_2 is one, the unconditional variance of the model is no longer defined. Engle and Bollerslev (1986) call this case the integrated GARCH process. It has the

---

[1] The unconditional variance $E(u_t^2)$ can be derived using the law of iterative expectations, according to which the unconditional expected value of a variable is found by repeatedly taking conditional expectations starting at time t–1. Thus $E(x_t) = E_0 \cdots E_{t-2} E_{t-1}(x_t)$. (Harvey, 1990, p. 212).

property that all shocks have a permanent impact on conditional variance. The conditional variance at t+s is simply the next period's conditional variance plus a constant trend, i.e.

$$E_t(h_{t+s}^2) = sc_0 + h_{t+1}^2. \qquad (7.5)$$

In Engle and Ng (1991), the concept of News Impact Curve is introduced as a means to analyze and compare different formulations of conditional variance. This curve is the relationship between lagged innovations, u_{t-1}, and conditional variance, h_t^2, assuming all previous information to be constant. It shows the effect that new innovations have on volatility. In the basic GARCH(1,1) model this curve is of the form $h_t^2 = (c_0 + c_2 h^2) + c_1 u_{t-1}^2$, implying a symmetric, parabolic relationship between innovations and conditional variance. Thus the effect of an innovation is stronger the larger is its size, and the effect does not depend on the sign. But, for example in the exponential GARCH developed by Nelson (1990), the slopes of the News Impact Curve are different on the positive and negative sides of the distribution of innovations, allowing for shocks of different sign to have different impacts on volatility. (The argument Nelson makes is that negative shocks to asset markets increase conditional variance more than positive shocks.)

Maximum likelihood estimation of the model is considered in some detail in Appendix 4. Because of the nonlinear structure of the mean equation with respect to effects of conditional variance, expressions for derivatives are also derived there. (When numerical derivatives are applied, the estimation procedures do not differ from estimating a linear ARCH-in mean or GARCH-in mean model.) If the random variable u follows a normal distribution, the log-likelihood function of the model, $L(\Theta)$, where $\Theta = (\beta', \tau')$ is the vector of all exogenous parameters, can we written as follows:

$$L(\Theta) = \Sigma L_t(\Theta);$$

$$L_t(\Theta) = -\tfrac{1}{2} \log(2\pi) - \tfrac{1}{2} \log(h_t^2) - \tfrac{1}{2} u_t^2 / h_t^2 \qquad (7.6)$$

where $h_t^2 = c_0 + c_1 u_{t-1}^2 + \ldots + c_q u_{t-q}^2 + a_1 h_{t-1}^2 + \ldots + a_q h_{t-p}^2$

$\qquad u_t^2 = (w_t - h_t X_t' \beta)^2.$

Under sufficient regularity conditions, the maximum likelihood estimator is consistent and asymptotically normal. Weiss (1986) presents a set of sufficient conditions in the case of the ARCH model, but for models with a conditional variance in mean equation such conditions have not been derived (Nelson, 1991). Following others, we also make the standard assumption that the maximum likelihood estimator is consistent and asymptotically normal.

7.2 Estimation results from GARCH(1,1)-in mean

The empirical borrowing equation with time-dependent conditional variance was first specified as GARCH(1,1)-in mean in the following nonlinear form:

$$w_t = b_1 h_t (\log(b_2 r l_t - \delta_t) - \log(\delta_t - r d_t)) + u_t, \qquad (7.7)$$

where $u_t | I_{t-1} \sim N(0, h_t^2)$

and $\quad h_t^2 = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1}^2.$

The results from the GARCH(1,1)-in mean model are reported in Table 3. Using the whole sample, the estimated coefficients for the lagged residual and the lagged variance term in the conditional variance equation were .35 and .48, respectively. Dividing the sample into two sub-periods reduced the estimated coefficients for lagged residuals and increased the coefficients for lagged variance terms in both sub-samples, the coefficient for lagged variance being a little higher in the latter period. The higher is the value of the coefficient for lagged variance, the smoother the time series of conditional variance becomes.

As was shown earlier, the sum of the coefficients c_1 and c_2 gives an estimate of the speed of decay of shocks in the model because of the properties of GARCH(1,1) process. For the whole period, the sum of c_1 and c_2 is .833, and in the two subperiods .778 and .925, respectively. In the light of these estimates, the effects of shocks are less permanent in the former period; after a week, the effect is reduced to less than 30 per cent in the first period and to 68 per cent in the second period. For the whole period, the estimates imply that about 40 per cent of the effect of a shock remains after a week (see Engle and

Bollerslev, 1986, and Nelson, 1990) on issues related to defining persistence in the GARCH-model).

The GARCH(1,1)-in mean model was tested against both one step higher and lower order specifications of the variance equation. Based on the likelihood ratios computed, the model passed tests in both cases with clear margins. Consequently, the constant conditional variance hypothesis with one more degree of freedom was also rejected.

The model explains a notable part of the variation in reserves, R^2 being .62 and .74 in the respective subperiods and .67 in the whole period. These values indicate a considerable improvement as compared to the constant conditional variance model (6.4). Visual inspection also confirms that the fitted values of the model do follow the main developments in the data. Figure 7.1 contains fitted values of the model evaluated on the parameters estimated from the whole sample, together with actual values of the bank's net discount window borrowing.

Figure 7.1   **Banks' net discount window positon and fitted values from the GARCH(1,1)-in mean model**

Bill. FIM
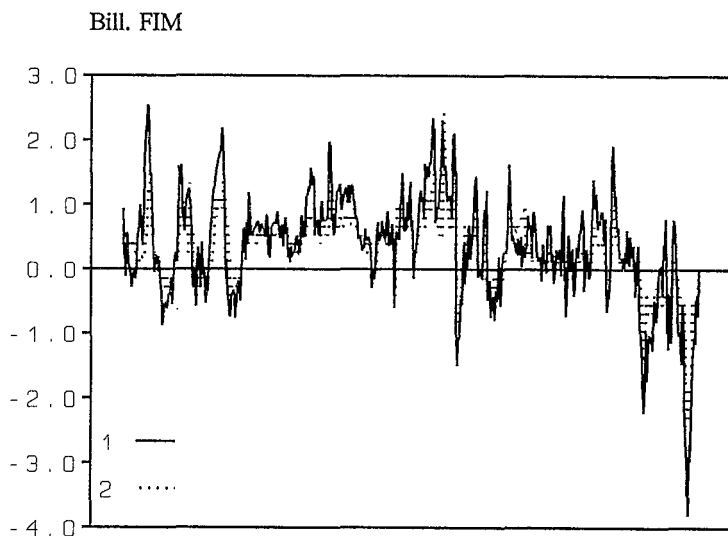
March 1987 − June 1989

1 Banks' discount window position
2 Fitted values from GARCH(1,1)-in mean

Table 7.1 **GARCH(1,1)-in mean**

Sample	All obs.		First period		Second period	
Mean equation						
b1	1.40	(.055)	1.75	(.086)	2.35	(.238)
b2	1.07	(.014)	1.06	(.017)	1.15	(.231)
Variance equation						
c0	.041	(.006)	.032	(.007)	.025	(.009)
c1	.354	(.050)	.228	(.040)	.281	(.091)
c2	.479	(.039)	.550	(.070)	.644	(.055)
LogL	-333.78		-111.94		-105.61	
R^2:	.671		.621		.738	
SSR:	129.985		42.73		39.67	

Likelihood ratio tests for model specification

LR for:

$c_3 u_{t-2}^2 = 0$	0.01	1.24	0.57
$c_2 h_{t-1}^2 = 0$	32.82**	19.35**	68.64**
$h_t^2 = constant$	92.30**	45.66**	243.55**

The estimated GARCH(1,1)-in mean model was

$$w_t = b_1 h_t / k (\log(b_2 rl_t - \delta_t) - \log(\delta_t - rd_t)) + u_t$$
$$h_t^2 = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1}^2$$
$$u_t \sim N(0, h_t^2).$$

Standard errors of estimates are in the parentheses.
LR = likelihood ratio test statistics = -2(LogL(H0)-LogL(H1)).
Under the null hypothesis, LR is distributed as chi^2 with degrees of freedom equalling the number of restrictions.
Asterisks indicate significance levels (* = 5 %, ** = 1 %) for LR statistics.

The values of R^2 are about the same size in the GARCH(1,1)-M model for reserves and in the constant variance interest rate model. Because these models are non-nested, any comparisons might be misleading. However, there is an obvious reason why R^2 values tend to be higher in the interest rate model than in the reserve model. In the interest rate model, the functional form used makes it possible for the observations to fall outside the estimated asymptotes. This is in contrast with the theoretical model, because no-one would buy at a higher price than the central bank's lending rate or sell at a lower price than the central bank's deposit rate. Allowing for that possibility in estimation understandably reduces the sum of squared residuals, however. In the reserve model, the estimated logarithmic function is

not defined outside the asymptotes, which means that in that specification both asymptotes must lie outside the extremum values of the observed distribution.

The interest rate model failed especially during the spring of 1989, when money and currency markets were in turmoil because of the change in the fluctuation range of the currency index. This period is explained somewhat better by the reserve model, in which the conditional variance responds to the change in the level of discount window borrowing that followed the revaluation of the markka. The model does not, however, adequately explain the reduction in borrowing that began a few weeks after the revaluation.

The statistics on the residuals of the model reveal serious defects in the specification. Table 7.2 contains some of the diagnostic statistics that were computed. The Ljung-Box test statistics on the standardized conditional residuals are statistically very significant, indicating the presence of serial correlation in the residuals.

The same Ljung-Box statistics based on squared standardized conditional residuals are not statistically significant, which means that no evidence of heteroscedasticity is found. This is a clear improvement in comparison to the model with constant conditional variance. The results from the constant variance reserve model reported above in Table 6.2 indicate heteroscedastic residuals even after correction for serial correlation. In addition, a standard Lagrange multiplier test for the presence of ARCH, based on autocorrelations between the squares of residuals, was applied to the constant variance model (see Engle, 1982). This test also showed that there are strong grounds for doubting the constant variance assumption. Of course, the likelihood ratio tests of the specification of the conditional variance, which were presented in Table 7.1, support the same conclusion. All this evidence suggests that applying GARCH methodology to this problem is a reasonable approach, even though the specification is still not adequate for conclusions to be made.

Apart from serial correlation, the tests indicate other problems in the residuals of the GARCH(1,1)-M model. First of all, the mean of the residuals deviates significantly from zero in the whole sample and in the first sub-sample. Unlike OLS with a constant term, the maximum likelihood method does not force the mean of the residuals to be zero. The mean of the residuals is an assumption to be tested, and its violation must be taken as a sign of a possible misspecification. Adding a constant term to the mean equation did not change the properties of the residuals in this respect. One is tempted to conclude that the misspecification is of a more complicated nature and has something to do with the asymmetry of the distribution, and not with its location.

Table 7.2   **GARCH(1,1)-in mean: statistics on residuals**

Sample	All obs.	First period	Second period
Ljung-Box			
LB(5)	233.0**	43.5**	63.32**
$LB^2(5)$	1.59	5.02	9.00
LB(25)	305.9**	76.6**	107.8**
$LB^2(25)$	15.1	38.4	22.9
Durbin—Watson	1.10	1.30	1.22
t-test for mean (u) = 0	4.68**	3.35**	1.10
Skewness	-.09	0.39	-.05
Kurtosis	3.85**	3.76**	3.60
Bera-Jarque	18.43**	14.18**	2.75

Asterisks indicate significance levels (* = 5 %, ** = 1 %) of the reported test statistics for the Ljung-Box tests, t-test for mean and Bera-Jarque. For skewness and kurtosis, they indicate significance of deviation from zero and three, respectively.

In the maximum likelihood estimation of the model it was also presumed that the residuals are normally distributed. This implies restrictions for the third and fourth moments of the distribution. Table 7.2 also contains coefficients of skewness and kurtosis computed from the actual distribution. Under the normality assumption, these should be close to zero and three, respectively. A joint test for the above restrictions, often called the Bera-Jarque statistic, was carried out to test for normality.[2] Except for the latter sub-sample, the resulting test statistics led to rejection of the null hypothesis. This resulted mainly from the high values of the coefficient of kurtosis.

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[2] Bera and Jarque (1980). The test statistic is computed as $T(\frac{m_3}{6} + \frac{(m_4-3)^2}{24})$, where $(m_3)^{1/2} = E(\frac{u_t^3}{h_t^3})$ is the coefficient of kurtosis and $m_4 = E(\frac{u_t^4}{h_t^4})$ is the coefficient of skewness. Under the null hypothesis of normality, the test statistic is distributed asymptotically as chi^2 with two degrees of freedom.

Figure 7.2 Cumulative probability distributions of
conditional residuals from the GARCH(1,1)-in
mean model, as compared to the standard
normal

1 Theoretical distribution
2 Actual distribution
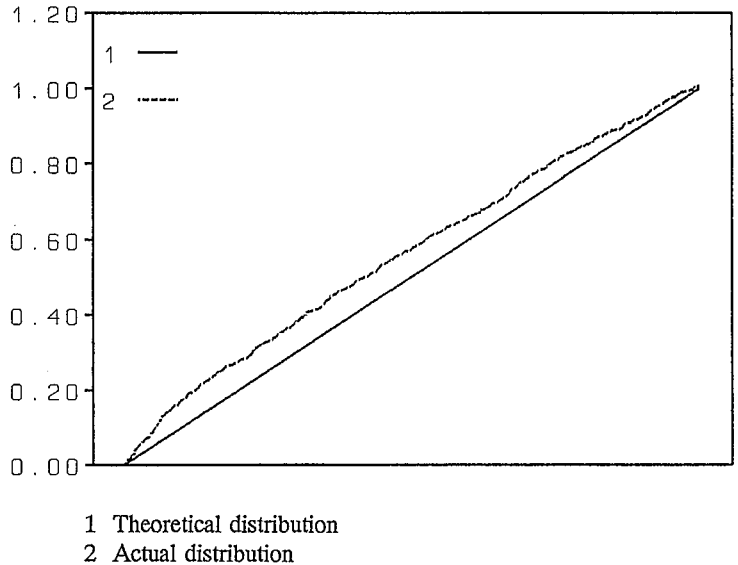
Normality plots of residuals from the GARCH(1,1)-in mean model are illustrated in Figure 7.2. Cumulative probability distributions of standardized conditional residuals were calculated for the whole period and separately for both sub-periods. The plot shows the results from the whole period. The scale of the plot is such that the straight forty-five degree line in the figure corresponds to the cumulative distribution of the standard normal, which was assumed to be the underlying theoretical distribution. Basically, if the empirical distribution is leptokurtic, then the observed cumulative distribution should run above the forty-five degree line, and vice versa if the empirical distribution has thinner tails than the standard normal. It seems that the empirical distribution is thinner than the standard normal near the mean value and has excessive probability mass in the tails of the distribution. This is the same pattern that was also indicated by the statistically significant values for excess kurtosis in Table 7.2.

A standard solution to the problem of serial correlation is to postulate that the residuals follow a AR or MA process. The plot of the autocorrelation functions and partial autocorrelation functions indicates that here the serial dependence between residuals is of the

type AR(1). These plots are shown in Appendix 6.[3] But, the difficulty with this approach is, of course, that there is no reliable justification for the presence of AR(1) in this context. There were no dynamic elements in the theoretical model that could be argued to be a cause of serial correlation, and even if some potential explanation for it could be found, the solution would be somewhat *ad hoc*.[4] However, because of the assumptions on which the maximum likelihood approach is based, there are strong arguments for correcting for AR(1), as the fact is that serial correlation is present in the data. A proper likelihood function under the assumption that the residuals follow the AR(1) process is also derived in Appendix 6.

The results from the estimation of the GARCH(1,1)-in mean model with the AR(1) correction are reported in Table 7.3. Applying the AR(1) assumption corrects, to a large extent, for the problem of serial correlation. This can be seen from the Ljung-Box test statistics computed from the conditional residuals. In both of the shorter estimation periods, these values are statistically insignificant if five sample autocorrelations are used. For the whole period, the value of the test statistic is 11.56, which slightly exceeds the 5 per cent level critical value (11.1), but is clearly less than 1 per cent level critical value (15.1). Increasing the number of lags to 25 still reveals some problems in the first estimation period as well. The value of the AR(1) coefficient p is notably high, ranging from 0.664 to 0.777.

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[3] A partial autocorrelation function of an AR(p) process has a cutoff after a lag p and its autocorrelation function tails off, while the opposite is true for a MA process (Box and Jenkins, p. 175). The plots in Appendix 2 show that the residuals are more likely to follow the former pattern than the latter.

[4] When considering the dynamic properties of the specification of model (7.1), one interesting point to note is that the interest rate variable is computed as a weighted average of interest rates applied to all overnight transactions during the day. It is possible, and indeed quite likely, that all deals are not done at exactly the same time. Consequently, one might suspect something akin to the problem of non-synchronization that is frequently encountered in stock market data (see, for example Baillie and DeGennaro, 1990). According to well-known argumentation, non-synchronization induces spurious autocorrelation in returns (see, for example, Scholes and Williams, 1977, Lo and MacKinlay, 1988 and Conrad et al., 1991).

Table 7.3   GARCH(1,1)-in mean, AR(1)

Sample	All obs.		First period		Second period	
Mean equation						
b1	1.04	(.120)	1.30	(.149)	2.13	(.436)
b2	1.35	(.111)	1.46	(.134)	1.11	(.063)
Variance equation						
c0	.027	(.006)	.007	(.002)	.026	(.015)
c1	.189	(.041)	.095	(.024)	.110	(.058)
c2	.623	(.065)	.852	(.033)	.737	(.113)
p	.777	(.026)	.688	(.042)	.664	(.079)
LogL	-227.75		-84.20		-86.63	
R^2:	.796		.695		.810	
Ljung-Box						
LB(5)	11.56*		7.85		8.31	
$LB^2(5)$	4.84		5.37		6.75	
LB(25)	73.6*		52.5**		32.7	
$LB^2(25)$	31.2		28.7		23.0	
Durbin—Watson	2.00		2.04		2.17	
t-statistics						
for mean(u) = 0	0.35		.37		.79	
Skewness	-.20*		-.04		-.26	
Kurtosis	4.46**		4.71**		4.17**	
Bera-Jarque	55.67**		34.69**		11.97**	
<u>Tests for model specification</u>						
LR: $c_2 h_{t-1}^2 = 0$	23.5**		20.0**		6.8**	
h_t^2 = constant	43.1**		29.1**		10.6**	

The estimated model was

$$w_t = b_1 h_t / k(\log(b_2 rl_t - \delta_t) - \log(\delta_t - rd_t)) + u_t$$
$$\varepsilon_t = u_t - p\, u_{t-1}$$
$$h_t^2 = c_0 + c_1 \varepsilon_{t-1}^2 + c_2 h_{t-1}^2$$
$$\varepsilon_t \sim N(0, h_t^2).$$

Standard errors of estimates are in parentheses. All statistics on residuals were computed using the conditional distribution. LR = likelihood ratio test statistic = -2(LogL(H0)-LogL(H1)). LR is distributed under the null hypothesis as chi^2 with degrees of freedom equalling the number of restrictions. Asterisks after LR, Ljung-Box, t-statistics for mean, and Bera-Jarque indicate the significance levels of reported statistics (* = 5 %, ** = 1 %). For skewness and kurtosis they indicate the significance of deviation from zero and three, respectively.

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An important thing to note is that the presence of GARCH is still detected in the residuals despite the AR(1) correction. This can be seen from the likelihood ratio tests, in which GARCH(1,1) was set against ARCH(1) and constant variance models. These tests rejected lower order specifications of conditional variance in favour of GARCH(1,1) by clear margins. Nor is there any need to further increase the order of the conditional variance equation. The Ljung-Box test statistics from the squared conditional residuals are insignificant, indicating that no heteroscedasticity remains.

Other diagnostic checks on the residuals were also carried out. Bera-Jarque statistics indicate that the normality assumption of the conditional residuals is violated in all cases. The cause of this is excess kurtosis, which means that the empirical distribution has heavier tails than the normal distribution. On the other hand, the first and third moments of the empirical distribution are not found to be in conflict with the assumptions.

Even though one cannot be completely satisfied with the AR(1) model because of its weak theoretical background, in purely empirical terms it is a fairly good description of the data, except for the above-mentioned problem of excess kurtosis in residuals. In the econometric literature, it is frequently the case that researchers do not hesitate to use the AR or MA correction (examples that are closely related to this study include Baillie and DeGennaro, 1990, Nelson, 1991, and Schwert and Sequin, 1990). The rejection of the constant conditional variance model in favour of the GARCH(1,1)-M with AR(1) is empirically a major result of the study.

7.3   Specification of the variance equation

In order to maintain a close link between the theoretical model of the previous sections and the empirical model, the strategy adopted here is to consider alternative distributional assumptions (in Section 7.4) and to improve the specification of the variance equation, rather than to close the discussion with the AR(1) correction. From the point of view of the theoretical model, both the distribution and the variance equation are open issues in the above specification of a GARCH(1,1)-M model for the conditional variance equation. Functional forms other than simple linear specifications could be tried, but some of the exogenous variables included in the information set of agents could also be added to the equation. An extensive collection of specifications used in the literature is provided in Engle and Ng (1991). In fact, a number of different functional forms for conditional

variance were applied. These experiments did not produce any important results and are not reported here. Instead, specifications of the conditional variance equation with exogenous variables are considered next. Examples of this approach are found in Baillie and DeGennaro (1990) and Lastrapes (1989).

When the model is extended from autoregressive specification of the conditional variance equation to specifications with additional information, a logical way to proceed is to consider the implications of a theoretical model with endogenous variance. In Chapter 5, it was shown that if the bank is assumed to be able to affect the variance of liquidity shocks by devoting resources to liquidity control, then the optimal variance depends on essentially the same variables that affect the demand for reserves, along with the direct cost of liquidity control. It was shown that, with a logistic distribution, both optimal reserves and optimal variance were functions of spreads between the overnight interest rate and the central bank's call money interest rate, although the latter in a more complicated way. On the other hand, a very convenient representation of the equilibrium condition for optimal variance was derived by assuming normally distributed errors and by applying Taylor series linearization around W=0. It was shown that the optimal variance is in that case proportional to the squared level of reserves, given the level of expenditure on liquidity control, i.e.

$$\sigma^2 = \hat{c} W^2, \quad \text{where} \quad \hat{c} = \frac{c}{(a'(q) - c)}. \tag{7.8}$$

Based on the above relationship, an empirical equation for conditional variance was formed by combining the squared level of liquidity with autoregressive terms. A proper interpretation of this equation is that it describes the banks' expectations concerning variance. Even though it is not the conditional expectation that is determined in the theoretical model, it is assumed that the same factors that affect the optimality condition also affect the expectation of variance that is formed at any given time. To put it in another way, if one specifies an equation for some agents' expectations about some variable, then the optimality condition of the variable in question is a reasonable point of departure.

Since, however, the actual process generating the expectations of variance is really not known, restricting the empirical analysis exclusively to the specification in (7.8) might be unwarranted. Looking at the problem from a more empirical viewpoint, there are a large number of other variables that could be useful in predicting the conditional variance of liquidity, other than the past levels of liquidity.

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Clearly, one would expect that the more closely a variable is related to the money market, the more information it carries on the expected volatility. On these grounds, among the variables chosen for the empirical tests, along with the level of liquidity, were the spread between the overnight interest rate and the 30-day interest rate (yield curve spread) and the lagged change in the stock market index. Further, as the money market in Finland is closely integrated into the foreign exchange market, the changes in the exchange rate and in the foreign reserves of the central bank were included in the list of variables that were tested empirically. For all these variables, lagged values were used because of the definition of the information set on which the variance is conditionalized.

One way to investigate the usefulness of these variables, including the lagged level of liquidity as implied by the liquidity control model, would be to add each in turn to the conditional variance equation and then use the resulting values of the likelihood function for constructing likelihood ratio tests for restricting the added coefficients of the general model to zero. Yet another, and equally powerful, test is the Lagrange multiplier test. This requires the estimation of the restricted model only. In this case, the restricted model is GARCH(1,1), so that, in fact, no new estimations are needed.

A general approach to LM tests, also applicable to GARCH models, is presented in Engle (1982b) (for an application, see also Engle, Lilien and Robins, 1987). Intuitively, the idea in this test is to view the restricted model as a problem of maximizing a function under constraints. In such a maximization problem, Lagrange multipliers give the shadow prices of constraints. When applied to testing restrictions in an econometric model, these shadow prices must be large if the null hypothesis is false and small if the null is true. Because shadow prices are associated with the constrained optimum, parameters maximizing the constrained model are used in the test, rather than parameters that maximize the general model. Thus the test is based on the matrix of first derivatives of the likelihood function of the explicitly constrained model, which is, in other words, the matrix of derivatives from the general model evaluated at the parameter estimates under the null.

In practice, this matrix can be formed by taking the matrix of scores, S, from the converged GARCH(1,1) estimation (the matrix of scores consists of vectors of derivatives of the log-likelihood function with respect to the parameters; see Appendix 4). The procedure is to concatenate this with a matrix of derivatives with respect to the constrained parameters, evaluated at zero in this case. The resulting

matrix, S^0, then has the dimensions of the general model and the last columns of it consist of Lagrange multipliers. The LM test statistics is

$$LM = I'S^0(S^{0'}S^0)^{-1}S^{0'}I,$$

which is easily computed from the first BHHH iteration[5] of the general model starting from the parameters under the null. In practice, this test is considerably more convenient to apply than the likelihood ratio test if the model to be estimated is complicated and particularly if the test is to be repeated for several variables. Note that the expression for LM is the uncentred correlation coefficient between the unit matrix I and the matrix of scores, S^0. Therefore, it can also be calculated as $T*R^2$ from an OLS regression of S^0 on I.

Values of the computed Lagrange multiplier test statistics are presented in Table 7.4. The restricted model was the GARCH(1,1)-in mean (model 7.1 above) and the results refer to estimates from the whole period from March 1987 to June 1989.

Table 7.4   **Lagrange multiplier statistics for variables omitted from the conditional variance equation**

	LM
w_{t-1}^2	31.67**
$spread_{t-1}^2$	10.06**
HEX_{t-1}^2	0.74
e_{t-1}^2	1.13
FR_{t-2}^2	2.20

Explanations: w = banks' position in the central bank, spread = o/n interest rate - 30-day interest rate, HEX = change in the stock market index, e = change in the exchange rate, FR = change in the foreign exchange reserves of the central bank.

All variables were entered in squared form. Under the null the test statistic is distributed as chi-squared with degrees of freedom equal to the number of restrictions. ** = test statistic significant at 1 % level.

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[5] An algorithm by Berndt, Hall, Hall and Hausman (1974) is used in all maximum likelihood estimations in this study. See Appendix 4 for details.

The values of the LM test statistics indicate that the variables that significantly improve the model are the lagged level of liquidity and the interest rate spread. By experimentation, it was also found that it is indeed the squared forms of these variables that should be used. Thus, the effect depends on the magnitude of these variables and not on their sign. This is of importance for the consistency of the model, as these variables are entered into the variance equation, in which negative values are generally not allowed. In principle, it is not necessary to restrict the variables in the equation to be non-negative as long as the implied conditional variance is positive. For example, in Schwert (1989), volatility in the stock market is modelled without this restriction. But, if the model is expected to be valid in out of sample predictions, allowing for the possibility that predictions are negative with some values of variables might be problematic. And in any case, one avoids a lot of checking if all variables are positive.

For the same reason it is usually also required that all parameters in the conditional variance equation be positive. The signs of parameters could clearly be a problem in a model in which the conditional variance equation includes additional variables. This problem is often also encountered in higher order ARCH models, for which the usual practice is to include some explicit parameter restrictions.

Based on the values of the LM statistics presented above in Table 7.4, the basic GARCH(1,1)-in mean model was modified to include w_{t-1}^2 in the conditional variance equation, as was suggested by theoretical analysis of the liquidity control model. It was also checked, using LM statistics, that after adding w_{t-1}^2 to the model, the effect of the yield curve spread becomes insignificant (LM = 0.33). The specification of the conditional variance of the model was then

$$h_t^2 = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1}^2 + c_3 w_{t-1}^2. \tag{7.9}$$

Technically, the effect of this formulation, as compared to the basic GARCH(1,1)-M model, is that is permits the conditional variance to increase when the level of liquidity is either low or high, given that c_3 is positive. This adds to the flexibility of the model in explaining the peak values of observed discount window borrowing. Figure 7.1, which presented fitted values from the basic GARCH(1,1)-M model, suggests that a systematic underestimation of large values of liquidity might be the source of serial correlation in the residuals of that model.

The economic interpretation of the formulation is that the banks tend to expect high volatility when the level of liquidity is especially

high or low. Therefore, the market interest rate does not fall with large positive levels of liquidity as much as it would otherwise do, and, consequently, we observe higher levels of discount window deposits being associated with higher interest rates than in the absence of this effect. When the liquidity of the banking system is very tight, on the other hand, a rise in the equilibrium interest rate is damped by the increase in expected volatility.

The News Impact Curve (see p. 92) can be used to illustrate the effect of including w_{t-1}^2 in the conditional variance equation. The dynamic structure of this particular GARCH process is affected by the fact that w_{t-1}^2 is not "truly" exogenous to the model, because it consists of a lagged predicted mean and innovations. Substituting the mean equation into conditional variance, holding previous information constant, gives the following relationship

$$h_t^2 = (c_0 + c_2 h^2) + c_1 u_{t-1}^2 + c_3 (h_{t-1} X_{t-1} + u_{t-1})^2. \qquad (7.9')$$

The predictable volatility in this model thus depends on the exogenous variable $h_{t-1} X_{t-1}$ as well as new innovations. The News Impact Curve is consequently somewhat more complicated than in the basic GARCH model. The slope of the curve depends on the sign of X_{t-1}, which means that the effect of a new innovation is different on different levels of (lagged) market interest rates. In addition, the location of the minimum of the curve depends on the mean, $h_{t-1} X_{t-1}$. In this respect the formulation resembles the NGARCH model of Engle and Ng (1991). The News Impact Curve of the model is depicted in Figure 7.3 below.

The News Impact Curves in Figure 7.3 are depicted separately for positive and negative values of the mean of the model. When the mean is negative, meaning that the model predicts that banks will be net borrowers, "further" negative innovations increase the predictable variance of the model more than positive innovations. Likewise, on the deposit side, positive deviations from the predicted mean have a stronger impact on predicted volatility than negative deviations. This may be taken as an empirical finding concerning expectations, but the explanation suggested by the theory is that the variance increases because expenditure on liquidity control is reduced when there are large positions of either sign. These volatility effects are then reflected in the steepness of the demand-for-borrowing schedule via the in mean term in the complete demand model.

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Figure 7.3   **News Impact Curve for the GARCH-M model with w_{t-1}^2 in the conditional variance equation**

h_t^2

0

u_{t-1}

1   Previous mean negative, $x_{t-1} < 0$
2   Previous mean positive, $x_{t-1} > 0$
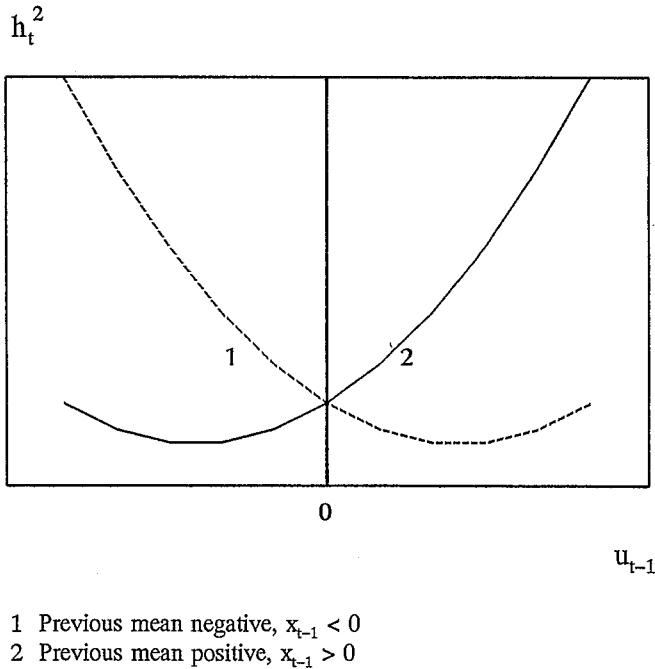
It is interesting to note that evidence of such asymmetrical responses to shocks is fairly common in studies on financial market data employing conditional heteroscedasticity techniques. For example, Spanos (1991) cites this as one of the empirical issues to which the first applications of ARCH specifications gave rise. For example, the exponential GARCH of Nelson (1991), the NGARCH of Engle and Ng (1991), the QARCH of Sentana (1991) and the volatility feedback model of Campbell and Hentschell (1991) have, in fact, been developed to deal with this issue.

Before turning to the estimation results from this enhanced model, ways to account for the thick tails of the distribution of conditional residuals are considered in the next section.

7.4 Alternative distributional assumptions

One commonly accepted 'stylized fact' associated with financial market time series is that empirical distributions are often too thick-tailed and too peaked as compared to the normal distribution. This property, leptokurtosis, was also exhibited by the residuals of the GARCH(1,1) model above. Our concern is that it invalidates the maximum likelihood estimation procedures applied to that model. Ways to overcome this obstacle are considered in this section.

The distributional assumption about residuals could be relaxed altogether, at least in principle. Rich, Raymond and Butler (1991) show how generalized methods of moments estimation (GMM) can be used to estimate ARCH models in the presence of non-normal residuals. However, the model considered here is more complicated because of the mean effect of the variance term. That would make generalized instrumental variables estimation considerably more labourious.

The restrictions imposed on the distribution of residuals can be made somewhat more flexible by applying the conditional Student's t distribution instead of the conditional normal. The t distribution is also symmetric around the mean, but it may have heavier tails than the normal distribution, depending on the degrees of freedom. At the extreme, when the degrees of freedom parameter d goes to infinity, the t distribution $f_t(0, \sigma^2, d)$ approaches the normal distribution $N(0, \sigma^2)$. The parameter values $d < \infty$, in turn, imply that more probability mass is concentrated in the tails of the distribution than in a normal distribution with equal variance. Therefore, the normal distribution can be derived from Student's t with a simple parameter restriction and these distributional assumptions can be tested against each other. Examples of studies using the t-distribution in modelling financial market data with the GARCH include Bollerslev (1987), Baillie and DeGennaro (1990) and Booth, Hatem, Virtanen and Yli-Olli (1990).

Adopting the standardized[6] Student's t implies the following distribution for conditional residuals:

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[6] The standardization scheme used implies Var(u) = v/(v-2) (i.e. unequal to one). For details concerning the standardization of a Student random variable, see, for example, Blattberg and Gonedes (1974).

$$u_t|I_{t-1} \sim f_t(u_t|I_{t-1}) = f_t(0, h_t^2, d)$$

$$= \Gamma(\frac{d+1}{2})\Gamma(\frac{d}{2})^{-1}\frac{1}{\sqrt{(d-2)h_t^2\pi}}(1+\frac{u_t^2}{h_t^2(d-2)})^{\frac{-(d+1)}{2}}, \quad d>2,$$

where $\Gamma(.)$ = gamma function.

The log-likelihood function of the model with a standardized Student's t distribution is then

$$L = \sum_t \log(f_t(0, h_t^2, d))$$

$$= \sum_t \{\log(\Gamma(\frac{d+1}{2})\Gamma(\frac{d}{2})^{-1}) - 1/2\log((d-2)\pi) - 1/2\log(h_t^2)$$

$$- \frac{d+1}{2}\log(1+\frac{u_t^2}{h_t^2(d-2)})\}.$$

In the case of leptokurtic, or thick-tailed, conditional residuals, the above likelihood function should provide a more accurate description of the data than the one under the conditional normal. It should be noted that the unconditional distribution corresponding to conditionally normal residuals is also leptokurtic. Applying Student's t distribution to the conditional residuals is another way to deal with fat tails in the distribution of underlying time series (Bollerslev, 1987).

In the estimations, the degrees for freedom parameter d in the above likelihood function was replaced by $1/d$, so that in comparisons with the normal distribution the null hypothesis could be written in a testable form as $1/d = 0$. For computational reasons the values of the gamma function were evaluated using Stirling's formula (see, for example, Cramer, 1971, p. 130). The derivation of the estimated model is documented in Appendix 5.

As in the case of the conditional normal distribution, the appropriateness of the model can be assessed by comparing the resulting empirical distribution to its theoretical counterpart. The second and third moments of the conditional Student's t distribution are the same as in the conditional normal (h_t^2 and 0, respectively), but the fourth moment is defined as follows (see Bollerslev, 1987):

$$E(u_t^4|I_{t-1}) = 3(d-2)(d-4)^{-1}h_t^4, \quad d>4.$$

7.5 Estimation results from the modified GARCH-in mean model

The modified GARCH-in mean model was estimated for all estimation periods using both the conditional normal and conditional Student's t to describe the distribution of residuals. Table 7.4 reports the results from the model with the lagged squared level of liquidity in the variance equation, assuming a conditional normal distribution of errors.

According to the likelihood ratio tests, the values of the log-likelihood function of the normal distribution model are significantly higher in all estimation periods than in the results from the basic GARCH(1,1)-in mean formulation of the conditional variance. The results also indicate that adding the squared lagged level of net borrowing to the conditional variance equation clearly improved the quality of residuals. Serial correlation was reduced in all estimation periods, particulary in the first period where only weak evidence of autocorrelation is found. Further, the mean of the residuals is zero in both sub-periods, as is presumed in the maximum likelihood estimation.

A joint test for higher moments still indicates a departure from the normality assumption, except for the second estimation period. The value of the Bera-Jarque statistic is 57.1 in the first period and 2.7 in the second period, while the critical value at the 1 per cent level is 9.21.

The parameter estimates for c_3, which is the coefficient of lagged squared borrowing in the conditional variance equation, are statistically significant and positive. As all variables of the conditional variance equation are in squared form and all parameter estimates positive, the estimate of the conditional variance is certain to be always positive. Estimates for c_1 and c_2 are reduced in all cases as compared to the basic GARCH-M, and in the first period the coefficient for lagged variance is not statistically significant. Including additional lags of residuals in the equation for conditional variance is not supported by the LM test.

The coefficient of the slope of the demand schedule, b_1, also differs from the basic formulation of the conditional variance. The results from the enhanced model imply a steeper demand-for-borrowing schedule, or equivalently, a less steep interest rate locus.

Table 7.5 **GARCH(1,1)-in mean, w_{t-1}^2 in the conditional variance equation**

Sample	All obs.		First period		Second period	
Mean equation						
b1	1.60	(.071)	2.04	(.105)	2.62	(.249)
b2	1.05	(.009)	1.06	(.011)	1.12	(.018)
Variance equation						
c0	.059	(.009)	.047	(.010)	.046	(.020)
c1	.201	(.051)	.100	(.054)	.179	(.082)
c2	.176	(.083)	.036	(.107)	.428	(.191)
c3	.094	(.018)	.079	(.016)	.054	(.031)
LogL	-304.06		-84.49		-97.51	
R^2:	.726		.701		.782	
SSR:	108.05		33.74		32.97	
Ljung-Box						
LB(5)	157.7**		11.49*		35.04**	
LB^2(5)	4.95		5.65		4.58	
LB(25)	246.1**		40.5*		74.5**	
LB^2(25)	36.3		17.0		27.3	
t-statistics for mean(u)=0	3.0**		0.92		0.06	
Skewness	.09		.53**		0.05	
Kurtosis	4.05**		4.94**		3.53	
Bera-Jarque	27.4**		57.1**		2.74	
LR for $c_3 w_{t-1}^2 = 0$	59.4**		54.9**		16.20**	
LM for $u_{t-2}^2 = 0$	2.53		.77		.02	

The estimated GARCH(1,1)-in mean model was

$$w_t = b_1 h_t / k (\log(b_2 r l_t - \delta_t) - \log(\delta_t - r d_t)) + u_t$$
$$h_t^2 = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1}^2 + c_3 w_{t-1}^2.$$
$$u_t \sim N(0, h_t^2).$$

Standard errors of the estimates are in parentheses.
LR = likelihood ratio test statistics
LM = Lagrange multiplier test statistics
LR and LM are both distributed as chi^2 with degrees of freedom equalling the number of restrictions under the null hypothesis.
Asterisks indicate the significance levels (* = 5 %, ** = 1 %) of reported test statistics for Ljung-Box, LR, LM, t-test for mean and Bera-Jarque. For skewness and kurtosis, they indicate significance of deviation from zero and three, respectively.

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Finally, the estimation results from the model with the conditional Student's t distribution of residuals are reported in Table 7.6. As noted before, the difference between the normal and Student's t distributions is that applying the latter allows for thicker tails.

According to the estimates, the inverse of the degrees of freedom parameter, 1/d, deviates significantly from zero in the whole sample and in the first period, indicating that in these cases a statistically thicker-tailed distribution fits better than a normal distribution. The likelihood ratio tests also show the same result. The estimate for the number of degrees of freedom is 4.79 in the first period, in which the model passed the likelihood ratio test against the normal distribution model, and 7.85 in the second, in which it did not pass.

In order to compare the observed distribution of the conditional residuals with the standardized Student's t distribution, Kolmogorov-Smirnov[7] test statistics were computed. These statistics were statistically insignificant in both separate estimation periods. However, because the inverse of the degrees of freedom parameter did not deviate from zero in the latter period, a reasonable conclusion is that the sample size is in that case so small that we cannot discriminate between these alternative distributions with these tests. The values of the observed coefficients of kurtosis are somewhat smaller than the theoretical values implied by degrees of freedom estimates, but not alarmingly. If the implied theoretical values, $3(d-2)/(d-4)$, are computed using a $1/d \pm$ one standard deviation, the observed values are well within that range.

The residuals are clean from serial correlation in the first estimation period, but not in the second period or in the whole sample. There are no signs of further heteroscedasticity. As in previous models, restricting the conditional variance to a constant yields highly significant likelihood ratio test statistics. Also, the LR tests are supportive of the enhanced version of the conditional variance equation, in which the squared lagged level of liquidity is present.

The overall picture that emerges from these experiments with Student's t distribution is that permitting thicker tails does not change the estimates very much. This finding is not uncommon (see, for example, Spanos, 1991). The model is, of course, statistically better, because it is in better accordance with the assumptions. But especially the parameter estimates of the demand-for-reserves equations are practically unchanged.

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[7] The Kolmogorov-Smirnov test statistic is $\max|F_n(x)-F(x)|$, where F_n is the cumulative sample distribution function and F the assumed cumulative distribution function. Asymptotic formulas for the critical values of the 5 % and 1 % levels are $1.36/\sqrt{N}$ and $1.63/\sqrt{N}$, respectively. (Lindgren, 1969, p. 486).

Table 7.6   **GARCH(1,1)-in mean, Student's t, w_{t-1}^2 in the conditional variance equation**

Sample	All obs.		First period		Second period	
Mean equation						
b1	1.55	(.095)	2.02	(.169)	2.52	(.325)
b2	1.06	(.009)	1.04	(.009)	1.12	(.018)
Variance equation						
c0	.061	(.010)	.048	(.009)	.045	(.026)
c1	.222	(.068)	.070	(.047)	.236	(.126)
c2	.096	(.080)			.372	(.204)
c3	.127	(.027)	.098	(.020)	.069	(.041)
1/d	.161	(.050)	.201	(.065)	.127	(.108)
LogL	-294.62		-74.99		-96.29	
R^2:	.733		.705		.785	
SSR:	105.80		33.27		32.55	
Ljung-Box(5)						
LB(5)	131.6**		6.86		27.91**	
$LB^2(5)$	5.11		6.77		3.30	
LB(25	209.8**		36.5		61.2**	
$LB^2(25)$	38.4*		20.3		23.1	
t-statistics						
for mean(u)=0	3.3**		1.63		0.11	
Skewness	.11		0.61		-0.01	
Kurtosis	4.31		5.37		3.82	
v	6.20		4.79		7.85	
3(v-2)/(v-4)	5.72		9.17		4.56	
Kolmogorov-Smirnov	.075**		.063		.041	
LR:						
h_t^2 =const.	296.0**		163.4**		62.8**	
$c_3 w_{t-1}^2$ =0	70.8**		66.3**		18.3**	
1/d =0	18.9**		19.0**		2.4	

Estimated model

$$w_t = b_1 h_t/k(\log(b_2 r l_t - \delta_t) - \log(\delta_t - rd_t)) + u_t$$

$$h_t^2 = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1}^2 + c_3 w_{t-1}^2.$$

$$u_t \sim \text{Student's } t(0, h_t^2, v)$$

Standard errors of the estimates are in parentheses.
LR = likelihood ratio test statistics.
LM = Lagrange multiplier test statistics.
LR and LM are both distributed as chi^2 with degrees of freedom equalling the number of restrictions under the null hypothesis.
Asterisks indicate the significance levels (* = 5 %, ** = 1 %) of reported test statistics for Ljung-Box, LR, LM, the t-test for mean and Kolmogorov-Smirnov.

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Figure 7.4   **Actual and fitted values from the enhanced GARCH(1,1)-in mean model**

Bill. FIM
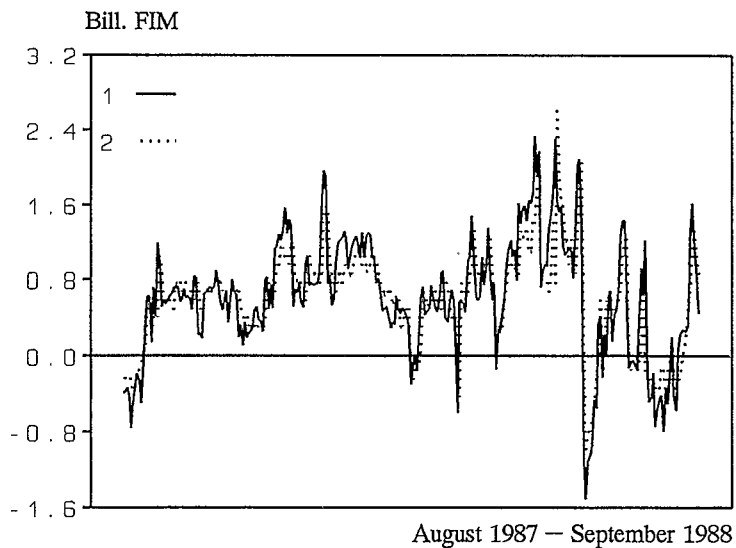
1 Actual values
2 Fitted values

Figure 7.5   **Actual and fitted values from the enhanced GARCH(1,1) -in mean model**

Bill. FIM

1 Actual values
2 Fitted values

Figures 7.4 and 7.5 contain fitted values from the enhanced GARCH-in mean model with the standardized Student's t distribution in the two sub-samples. In the first period, the fit seems to be good; the model now explains peak values as well. In the second period, the model still fails systematically during some episodes. In particular, after the revaluation of the currency in March 1989, overnight interest rates were for a month at a higher level than the model predicts as being consistent with observed levels of liquidity.
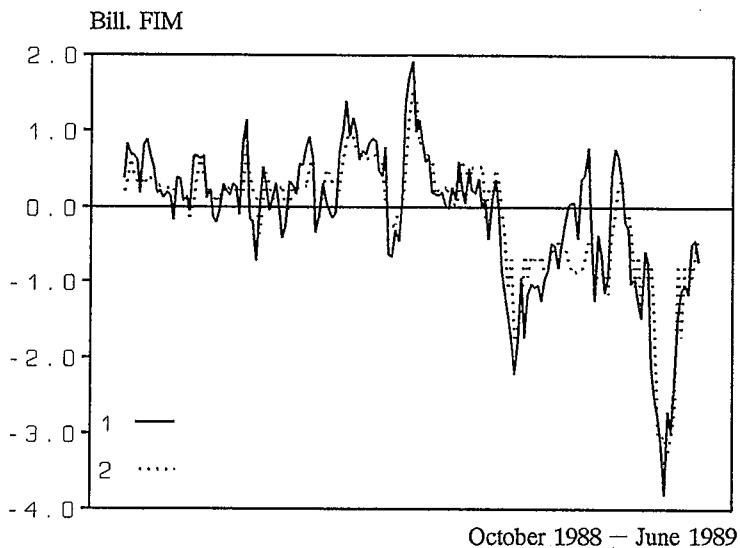
7.6 Concluding remarks

This chapter investigated an empirical demand-for-reserves model with time-dependent conditional variance. Because the discussion was mainly concerned with econometric difficulties, the economics involved deserves some additional comments.

The choice of the empirical approach was based on the notion that liquidity management theory emphasizes uncertainty, which in empirical work can be duly accounted for in dynamic heteroscedasticity models that allow for a changing conditional variance. Particularly in the short-term money market, a constant degree of uncertainty must be considered an extremely special case. In ARCH and GARCH models, the probabilistic information on new innovations is conditional on information that is available at any given time. When new innovations emerge, the information set and conditional expectations are updated accordingly.

In the empirical model that was constructed, changes in conditional variance affect the slope of the demand-for-reserves schedule. At times, when the volatility is low and there is little uncertainty about the actual value of liquidity, the elasticity of the demand-for-reserves with respect to market interest rates is higher than in more uncertain periods. Therefore, the main problem with heteroscedasticity in this model is not that it makes parameter estimates inefficient or the variance estimate biased. Rather, a far more serious problem is that omitting the effects of changing variance gives the wrong picture of the effects of liquidity shocks on borrowing and on equilibrium interest rates. In particular, because intervention by the central bank is actually a liquidity shock, the heteroscedasticity model implies that the impact of intervention diminishes when volatility in the market increases.

When implementing the empirical model, it was found that constant variance applications do not adequately describe the data. Models with time-dependent conditional variance performed much better, although not completely satisfactorily. It seems evident that this model is also

rejected if the estimation period is chosen arbitrarily from the period during which the overnight market has existed in Finland. But it is likely that the functioning of the market has been influenced by developments that are beyond the scope of this simple model. The point emphasized here is that if some additional considerations are taken into account in choosing the estimation period, then the conditional heteroscedasticity model survives.

It was also found that dealing with some of the empirical issues that are commonly encountered in GARCH applications, such as asymmetric effects of new innovations on volatility and the thick-tailed distribution of conditional residuals, did improve the statistical properties of the model. From the point of view of economics, there is not very much that can be said about the choice of the distribution. The normal distribution would probably be the first choice for most because of the central limit theorem. However, there are no grounds to discriminate *a priori* against alternatives in this application. Thick-tailed distributions have often been found to be appropriate in the context of financial markets time series. It is not clear why that is so in this case, but it is well known that the central limit theorem is not valid if the number of underlying random variables is too small, or if, loosely speaking, the contribution of any single random variable is very large relative to the whole. Only a few banks have access to the call money facility in Finland, and variation in liquidity is likely to be dominated by government finances and the central bank's intervention. In these circumstances it is not surprising that liquidity shocks are found to be non-normally distributed.

The model was made to include asymmetric responses to shocks by adding the squared lagged level of liquidity to the specification of the conditional variance. This formulation implies that conditional variance increases if the previous day's liquidity is either very tight or very plentiful. It was also shown that this effect can be theoretically justified with liquidity control activities. In the theoretical model with liquidity control expenditure, variance increases with the size of the bank's net position, because less resources are used for liquidity control when uncertainty concerning the call money interest rate diminishes. If a bank knows for certain on which side it will be, it is not willing to pay for liquidity control, but instead utilizes the overnight market. If the bank is uncertain about the result, then it will use some resources in order to reduce variance. Because uncertainty concerning the result diminishes as the size of the position increases, large positions are associated with low expenditure on liquidity control and high values of variance. The same property was incorporated in the empirical model by adding the squared lagged level of liquidity to the equation for conditional variance.

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8 Summary and Conclusions

This study has investigated the determination of overnight interest rates and of banks' overnight borrowing from the central bank in Finland both theoretically and empirically. The approach employed was to study this subject in a liquidity management theory framework and to use econometric models of conditional heteroscedasticity to implement the theoretical model empirically. Thus, the theoretical framework emphasizes the effects of uncertainty and the empirical method is such that it allows for a changing degree of uncertainty.

In our application of liquidity management theory, banks' demand for short-term liquidity in the interbank overnight market was explained by the opportunity costs of liquidity, defined by the central bank's call money facility. In terms of the standard reserve market model, the call money facility in Finland can be described as a continuous, stepwise reserve supply function, which is defined for both negative and positive levels of reserves. Based on reasoning that, during the banking day, the banks have only a forecast available on their final liquidity position, the costs and returns related to overnight trade were specified as random variables. Because of liquidity uncertainty, the resulting equilibrium market interest rate locus becomes continuous and smooth, even though the costs of call money from the central bank are defined by a less well-behaved interest rate schedule. Since market interest rates are known to deviate from the central bank's stepwise interest rate schedule in Finland, a smooth relationship is obviously more realistic. The analysis illustrated that this evidence on interest rates does not need to be in contradiction with the standard supply — demand framework of reserve markets.

The benchmark case in the analysis was a model in which the central bank's supply mechanism of (net) reserves was characterized by separate interest rates on call money credits and call money deposits. The equilibrium overnight market interest rate schedule was shown to be a declining function of banks' free reserves, asymptotically approaching official interest rates. The steepness of the schedule depended on the degree of uncertainty. When the variance of reserves increases, the schedule becomes flatter, which implies weaker responses of interest rates to changes in liquidity. On the other hand, with perfect information (zero variance), the interest rate schedule collapses to the central bank's supply function. When the interest rate schedule is inverted, the equation defines banks' demand for reserves, i.e. it shows the level of reserves that is optimal for banks at given levels of the market interest rate and the central bank's interest rates.

In this model, the demand for reserves is more appropriately understood as willingness to hold a certain liquidity position, positive or negative, rather than eliminate it by buying or selling at interest rates that are the same as the central bank's interest rates. Without uncertainty, a bank with surplus funds would always be ready to sell if the market interest rate is just higher than the central bank's call money deposit rate and a bank with a deficit would always offer to purchase funds if the market interest rate is less that the central bank's call money lending rate.

Typically, the facilities used by central banks for providing short-term liquidity to banks involve more complicated borrowing terms than those described in the benchmark model. Two additional types of instrument were considered as applications. First, the effects of a quota system were illustrated in an example where an additional penalty rate was applied to borrowing in excess of certain fixed amount. Because penalty rates based on quantitative quotas are simply additional interest rate steps in the supply schedule, the result was an equilibrium interest rate schedule consisting of two adjoining basic model solutions. In the basic model, the expression for the market interest rate was a probability-weighted average of the central bank's interest rates, and adding one more step to the supply schedule yielded another probability-weighted interest rate term in the solution. Obviously, any number of interest rate steps can be introduced into the interest rate equation because of this simple additive functional form. But, at the same time, approximating the supply schedule with a linear function becomes more and more tempting as the distances between the steps get shorter (relative to variance). The second extension of the model towards more complicated supply schedules concerned time-dependent borrowing costs. The model becomes dynamic if previous borrowing leads to a penalty cost being charged on present borrowing. The result from a very simple example was that two additional probability terms appeared in the solution of the market interest rate. One was the probability that borrowing today is penalized because of past borrowing and the other the effect of today's borrowing on the probability that penalties will be encountered in the future.

As a further modification, the variance of profits was included in the objectives of the maximizing bank. This implies risk aversion, and was shown to lead to a positive risk premium in the interest rate. The expected variance of profits diminishes as the absolute size of the reserve position increases, and consequently the risk premium also approaches zero when positions are large. The risk premium was, of course, a function of the variance of reserves.

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In these applications of the model it was assumed that the bank takes the degree of uncertainty as given and adjusts the demand for net borrowing as a response to changes in variance. It was argued that this is unlikely to be an exhaustive description of borrowing behaviour; particularly if one looks at the changes in borrowing behaviour that occurred when the interest rate step was first introduced, it seems obvious that banks are able to affect the variance as well. Even though the study is concerned mainly with very short-term behaviour, it is not evident that liquidity control activities should be by-passed in the analysis. Some liquidity control measures might be related to long-term developments, but others might be relevant in the short term, too. In our static theoretical analysis, liquidity control was defined as resources devoted to activities that reduce the variance of liquidity shocks. After making that assumption, the literature on liquidity management with information costs could be applied to our model.

Including liquidity control activities in the analysis endogenizes the variance and the model becomes a two-equation system with equations for the level of reserves and for the variance of reserves. This simultaneous model presumes that the bank adjusts both its demand for liquidity and expenditure on liquidity control when faced with uncertainty concerning the interest rate that is applied to its final liquidity position. As with exogenous variance, the solution collapses to the central bank's supply schedule if there is no uncertainty. The model separates conveniently, so that the reserve equation turns out to be exactly the same as with exogenous variance. The other equation is the equilibrium condition for expenditure on liquidity control, from which variance is determined. Because it is not specified explicitly how spending resource on liquidity control affects variance, the resulting variance equation is also implicit. But it is shown that the optimal variance depends on the same variables as those that determine the demand for reserves; i.e. the central bank's interest rates and the market interest rate. Under certain conditions, optimal standardized reserves can be expressed as a function of the 'upper spread' relative to the 'lower spread', where the upper and lower spreads are the distances between the market interest rate and the discount window interest rate. The corresponding variance then also depends on the upper and lower spreads, though the absolute size of the total spread matters, too. Thus the optimal variance is affected by the size of the interest rate step in the supply schedule, while optimal standardized reserves depend only on the location of the market interest rate relative to this step. It was also shown that, under some

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specific assumptions, optimal variance is proportional to the square of optimal reserves.

The analysis of the theoretical model suggests some implications for the choice of the intermediate target of monetary policy. An essential feature of the analysis as regards the methods of monetary control is that it focuses on the instability of the borrowing — interest rate relationship. In practice, borrowing is the operational target of monetary policy if the intermediate target is set in terms of quantities, because it is the only indicator of the changes in the level of banks' reserves the central bank can constantly observe. This analysis implies that targeting the level of borrowing would probably result in undesired volatility in interest rates because of the instability of the borrowing function. Interest rate changes would be needed merely to compensate for the changes in the demand for borrowing that are caused by the changes in volatility resulting from liquidity shocks.

In addition to liquidity shocks and changes in interest rates, a third source of borrowing variability is identified in the analysis: changes in the costs of liquidity control, or in the effectiveness of liquidity control measures. This source of variability is clearly beyond the control of the monetary authority as far as fine-tuning is concerned. The argument we wish to make is that the relationship between borrowing and interest rates might be unstable because of this factor as well, and consequently, following either an interest rate target or a reserve target in the conduct of monetary policy operations would yield different outcomes. This, too, might be seen as an argument in favour of targeting interest rates rather than fixing the level of borrowing. Given that the borrowing schedule is subject to noise because of liquidity control activities, targeting the level of borrowing would result in volatility in interest rates.

Another, closely related, point highlighted in the analysis is that the policies of the central bank affect the borrowing-interest rate relationship. Clearly, if the policy of the central bank is to smooth liquidity shocks, then the borrowing function becomes steeper because the variance of borrowing is reduced. The smaller the variance, the less reserves (in absolute terms) the banks would hold at a given level of interest rates and the larger the interest rate changes that would follow from shocks. The model suggests that it is possible that a policy of actively smoothing borrowing might actually turn out to be counterproductive as the banks' demand for borrowing becomes more responsive to changes in interest rates.

In the empirical section of the study, the model was fitted to data from the overnight market and call money facility from March 1987 to June 1989. Theoretically, it is clear that borrowing is the endogenous

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variable in the model, so that the equation should be estimated with borrowing variable on the left-hand side. But applications of the model with borrowing quotas and risk aversion yielded expressions for the interest rate that could not be inverted to form borrowing equations. These are both variables that potentially might have had influence on equilibrium interest rates during the period under investigation. It was therefore decided to start the empirical investigation with an interest rate specification in which these additional features could be included without complications. However, neither the risk premium nor quotas had any explanatory power with respect to interest rates. Although this model was obviously incorrectly specified, it was determined that these variables can be excluded from further empirical investigation.

It was possible to explore the implications of liquidity management theory much more fully in estimations of the borrowing equation. Because the variance of reserves determines the steepness of the borrowing function, conditional heteroscedasticity techiques were applied. Adopting time-dependent conditional variance means that the relationship between the demand for borrowing and interest rates is made dependent on the perceived volatility in liquidity. The demand schedule becomes flatter with respect to interest rates in times of large changes in liquidity and steeper when changes are small. This means that the more volatile the reserves are, the more sizeable is the liquidity shock needed to cause a given reaction in interest rates. This feature of the empirical model follows directly from liquidity management theory. The empirical section of the study was largely devoted to investigating — applying GARCH methodology — how this process should actually be described in estimations. Of course, the most important conclusion is that the evidence does not support the assumption that it should be described as being constant over time.

The implications of the theory concerning the specification of the demand-for-borrowing equation, or mean equation, are considerably stronger than those concerning the conditional variance equation. The conditional variance equation describes expectations formation, about which we actually did not have any prior knowledge. The usual solution is to apply autoregressive equations using various functional forms of specification and to choose the one that is statistically most promising. In a number of studies, autoregressive specifications have been supplemented with some additional exogenous variables which can be thought to in the information set. Following that approach, we included the square of (lagged) reserves in the conditional variance equation. The choice was also justified by theoretical analysis of the determinants of optimal variance in an endogenous variance version of the liquidity management model. A set of other variables that are

closely related to developments in the money market was also considered, but each was found to be unsatisfactory. Another loose end in the specification of our GARCH model was the distribution of conditional residuals. In order to account for thick tails, Student's t distribution was employed instead of the normal distribution in an additional application.

These modifications were found to be reasonable on statistical grounds, but they also carry a certain economic interpretation. The specification of the conditional variance equation implies asymmetric responses to shocks, because the effect of a new innovation depends on the level of liquidity. Asymmetricity has been found in several other studies, too. In this model the interpretation is that banks expect large changes in liquidity when the observed level of liquidity is especially high or especially low. But our analysis of the theoretical two-equation model suggests that the underlying reason might be that the variance and the mean are, in fact, determined simultaneously. The interpretation that can be given to empirical results concerning the distribution is more vague; it merely implies that the liquidity shocks do not consist of a large number of independent random variables. Further conclusions cannot be drawn, but we note that the evidence does not, at least, contradict the interpretation that liquidity shocks are caused mainly by the central bank's intervention and by changes in government cash funds.

It turns out that unless the relatively inflexible specification of the demand equation is somewhat relaxed with an empirically oriented AR(1) correction, the model performs statistically adequately only in a part of the whole estimation period. The reluctance to accept the AR(1) model derives from the suspicion that a technical correction for dynamic misspecification might conceal genuine problems concerning the applicability of the theory to this particular data and distract attention from the true limitations of the model. The adopted model describes arbitrage behaviour in competitive markets, and it was noted that the overnight market in Finland has even collapsed during some episodes. Against this background, it is not unexpected that the model is found satisfactory in only a limited sample.

Finally, some general remarks concerning the analysis should be pointed out. Our experience was that, by using conditional heteroscedasticity techniques, a straightforward application of a long-standing theory of inventory optimization under stochastic demand could be employed empirically in a much richer way in modelling the demand for reserves. By adopting this strategy, we were able to implement empirically the key explanation that this theory provides for optimal demand, i.e. the variability of the underlying variable. Further,

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this theory provides a good subject for applying this methodology, because the effects of non-constant variance are derived from the theory.

Another remark relates to the empirical description of the effects of the central bank's intervention and of the effects of liquidity changes on short-term interest rates in general. It is probably a common consensus that short-term interest rates and banks' borrowing are closely related with each other and that the central bank's intervention, being liquidity shocks, affect both of these. At the same time, most would agree that precise statistical estimates concerning the dependencies are hard to present, and, in particular, that the effect of intervention on interest rates varies. In this study we have analyzed the reasons for the observed instability of the borrowing-interest rate relationship and found that it may very well be explained by the implications of liquidity uncertainty for banks' borrowing behaviour. According to this study, conditional heteroscedasticity models do seem to provide a framework in which these processes can be consistently described, maybe not for actual forecasting purposes, but anyway for understanding the behaviour of the market.

Appendix 1.   Risk premium

In the case of a risk averse bank, the maximization problem is defined as

$$\max_{Q} \; E(U) = E(\pi) - \tfrac{1}{2}M \cdot Var(\pi), \qquad\qquad (A1.1)$$

where

$$Var(\pi) = E(\pi - E(\pi))^2$$

$$= E\!\left[w \cdot rd + (rl - rd)Y - \delta Q - W \cdot rd - (rl - rd)E(Y) + \delta Q\right]^2$$

$$= E\{u \cdot rd + (rl - rd)[EY - E(Y)]\}^2 \qquad\qquad (A1.2)$$

$$= E\{u^2 rd^2 + 2rd(rl - rd)u[Y - EY] + (rl - rd)^2[Y - EY]^2\}$$

$$= \sigma^2 rd^2 + 2rd(rl - rd)E(u \cdot Y) + (rl - rd)^2 Var(Y).$$

Next, we define a random variable Z such that Z = Y-W = min(u,-W). Subtracting a constant will not affect variance or covariance, so A1.2 can be written as

$$Var(\pi) = \sigma^2 rd^2 + 2(rl - rd)[rd \cdot E(Z \cdot u) + (rl - rd)Var(Z)]$$

$$= \sigma^2 rd^2 + 2(rl - rd)\{rd \cdot E(Z \cdot u) + (rl - rd)[E(Z^2) - (EZ)^2]\}, \qquad (A1.3)$$

$$\text{where} \quad Z = \min(u,-W) = \begin{cases} u, & \text{if } u < -W \\ -W, & \text{if } u \geq -W \end{cases}.$$

In order to maximize the objective function, it is therefore necessary to differentiate $E(Z \cdot u)$, $E(Z^2)$ and $(EZ)^2$ with respect to Q. These expressions are derived below, assuming that u's distribution is defined by a general symmetric cumulative distribution function F

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$$E(u \cdot Z) = F \cdot E(u^2 | u < -W) - (1-F)W \cdot E(u \geq -W)$$

$$= F \int_{-\infty}^{-W} \frac{u^2 f(u)}{F} du - (1-F)W \int_{-W}^{\infty} \frac{u f(u)}{1-F} du$$

$$\frac{\partial E(u \cdot Z)}{\partial Q} = -W^2 f(w) + \int_{-\infty}^{-W} u f(u) du + W^2 f(w)$$

$$= -(1-F)E(u | u \geq -w)$$

$$E(Z^2) = F \cdot E(u^2 | u < -W) + (1-F)W^2$$

$$\frac{\partial E(Z^2)}{\partial Q} = 2W(1-F)$$

$$E(Z) = F \cdot E(u | u < -W) - (1-F)W$$

$$\frac{\partial (EZ)^2}{\partial Q} = -2E(Z)(1-F).$$

The derivative of the variance term in the expected profits equation is then found by substitution

$$\frac{\partial(-\tfrac{1}{2}M \operatorname{Var}(\pi))}{\partial Q} = -M(rl-rd)(1-F)\{-rd E(u | u \geq -W) + (rl-rd)(W+EZ)\}$$

$$= M(rl-rd)(1-F)\{rd[E(u+W | u \geq -W) + EZ] - rl \cdot (W+EZ)\}$$

$$= M(rl-rd)(1-F)\{rd[E(u+W | u \geq -W) - (1-F)E(u+W | u \geq -W)]$$

$$-rl \cdot F \cdot E(u+W | u < -W)\}$$

$$= M(rl-rd)(1-F)F\{rd \cdot E(u+W | u \geq -W) - rl \cdot E(u+W | u < -W)\}.$$

If u is normally distributed, the premia can also be written in explicit form

$$M(rl-rd)(1-\Phi)\Phi\{rd \cdot [W+\frac{\sigma\phi}{1-\Phi}] - rl \cdot [W-\frac{\sigma\phi}{\Phi}]\}. \qquad (A1.4)$$

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Appendix 2. Comparative statics of the liquidity control model

Following Baltensberger and Milde (1976), comparative statics results are derived from the differentiated form of the system of first order conditions

$$
\begin{bmatrix} -(r_L-r_D)g(-m) & 0 \\ \\ 0 & a''s/a' \end{bmatrix} \begin{bmatrix} dm \\ \\ dq \end{bmatrix} =
$$

$$
\begin{bmatrix} -dr_D + dr_D \cdot G(-m) + d\delta \\ \\ ds + d\delta \cdot a'm - dr_D \cdot a'm + dr_D \cdot a' \int_{-\infty}^{-m}(v+m)g(v)dv \\ \\ -dr_L \cdot a' \int_{-\infty}^{-m}(v+m)g(v)dv \end{bmatrix}. \qquad (A2.1)
$$

Applying Cramer's rule yields the required expressions. In the following, J is the matrix of second derivatives. The notation J_{yx} is used for the appropriate modification of J needed to solve the effect of x on y. First, the effects on q are:

$$
|J| = -(r_L-r_D)g(-m)a''s/a' > 0 \qquad (A2.2)
$$

$$
|J_{qs}| = -(r_L-r_D)g(-m) \implies \frac{|J_{qs}|}{|J|} = \frac{dq}{ds} = \frac{a'}{a''s} < 0 \qquad (A2.3)
$$

$$
|J_{q\delta}| = -a'm(r_L-r_D)g(-m) \implies \frac{dq}{d\delta} = \frac{(a')m}{a''s} \sim Sgn(m) \qquad (A2.4)
$$

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$$\left|J_{qr_L}\right| = (r_L - r_D)g(-m)a' \int_{-\infty}^{-m} (m+v)g(v)dv$$

$$\text{(A2.5)}$$

$$\Rightarrow \frac{dq}{dr_L} = \frac{-(a')^2 \int_{-\infty}^{-m} (m+v)g(v)dv}{a'' s} \geq 0$$

$$\left|J_{qr_D}\right| = (r_L - r_D)g(-m)\{a'm - a' \int_{-\infty}^{-m} (m+v)g(v)dv\}$$

$$\text{(A2.6)}$$

$$\Rightarrow \frac{dq}{dr_D} = \frac{(a')^2(-m + \int_{-\infty}^{-m} (m+v)g(v)dv)}{a'' s} \leq 0.$$

The impact of a wider interest rate spread on liquidity control expenditure is:

$$\frac{dq}{dr_L} - \frac{dq}{dr_D} = \frac{-(a')^2}{a'' s} \{\int_{-\infty}^{-m} (m+v)g(v+m)dv - m$$

$$+ \int_{-\infty}^{-m} (m+v)g(v)dv\} \qquad \text{(A2.7)}$$

$$= \frac{-(a')^2}{a'' s} \{m(2G(-m)-1) + 2\int_{-\infty}^{-m} vg(v)dv\} > 0.$$

The effects on m are:

$$\left|J_{ms}\right| = 0 \Rightarrow \frac{dm}{ds} = 0 \qquad \text{(A2.8)}$$

$$\left|J_{m\delta}\right| = a'' s/a \Rightarrow \frac{dm}{d\delta} = -\frac{1}{(r_L - r_D)g(-m)} < 0 \qquad \text{(A2.9)}$$

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$$\left|J_{mr_L}\right| = -a'' s/aG(-m) \Rightarrow \frac{dm}{dr_L} = \frac{G(-m)}{(r_L-r_D)g(-m)} > 0 \qquad (A2.10)$$

$$\left|J_{mr_D}\right| = a'' s/a(G(-m)-1) \Rightarrow \frac{dm}{dr_D} = \frac{1-G(-m)}{(r_L-r_D)g(-m)} > 0 \qquad (A2.11)$$

Finally, effects on W:

$$\frac{dW}{d\delta} = \frac{dm}{d\delta} \cdot a + ma' \frac{dq}{d\delta} = \frac{-a}{(r_L-r_D)g(-m)} + \frac{m^2(a')^3}{a'' s} < 0 \quad (A2.12)$$

$$\frac{dW}{dr_L} = \frac{G(-m)}{(r_L-r_D)g(-m)} - \frac{m(a')^3}{a'' s} \int_{-\infty}^{-m}(m+v)g(v)dv \qquad (A2.13)$$

$$\frac{dW}{dr_D} = \frac{1-G(-m)}{(r_L-r_D)g(-m)} \cdot a + \frac{m(a')^3}{a'' s} \{-m$$

$$+ \int_{-\infty}^{-m}(m+v)g(v)dv\}. \qquad (A2.14)$$

The effect of a wider spread on reserves is:

$$\frac{dW}{dr_L} - \frac{dW}{dr_D} = \frac{2G(-m)-1}{(r_L-r_D)g(-m)} - \frac{m(a')^3}{a'' s} \{m(2G(-m)-1)$$

$$+ 2\int_{-\infty}^{-m} vg(v)dv\} \qquad (A2.15)$$

$$< 0, \text{ if } m > 0$$

$$> 0, \text{ if } m < 0.$$

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Appendix 3. Additional estimation results from applications with constant conditional variance

This appendix contains the additional estimation results from the constant variance application of the model which were referred to in the text. Tables A3.1, A3.2 and A3.3 report the results from the basic interest rate model, the model with a risk premium and the model with a borrowing quota, respectively. The conclusion is that the estimates of the basic equation are not changed by these two modifications and that the coefficients of the additional variables do not deviate significantly from zero.

Table A3.4. contains the results from week-day estimations of the borrowing function under the assumption that the conditional variance is constant. As was noted in the text, the main result of these estimations is that serial correlation largely disappears when only one observation from each week is used.

Table A3.1. **Interest rate model, OLS**

Sample	All obs.		First period		Second period	
Coefficient						
a1	0.44	(.048)	2.76	(.376)	0.76	(.136)
a2	0.84	(.066)	1.08	(.006)	1.33	(.177)
a3	1.12	(.051)	0.96	(.009)	0.99	(.017)
R^2	.619		.745		.687	
SSR	380.7		49.2		208.3	
SEE	.809		.418		1.10	
Durbin-Watson	.761		1.66		0.91	
Observations	584		284		176	
Wald test statistics for parameter restrictions:						
a2=a3=1	6.42*		193.4**		4.56	
a2=1	6.31*		189.0**		3.49	
a3=1	5.09*		21.2**		0.57	

Estimated equation was:

$$\delta_t = a_2 rd_t \Phi(a_1 w_t) + a_3 rl_t \Phi(-a_1 w_t) + \varepsilon_t.$$

Table A3.2.　　　**Interest rate model with a risk premium, OLS**

Sample	All obs.		First period		Second period	
Coefficient						
a1	0.46	(.040)	3.42	(.787)	0.77	(.146)
a2	0.89	(.048)	1.08	(.006)	1.69	(.193)
a3	1.03	(.058)	1.21	(.371)	0.41	(.533)
a4	0.01	(.005)	-0.06	(.082)	0.04	(.035)
R^2	.624		.784		.692	
SSR	375.2		48.7		204.9	
SEE	.804		.417		1.09	
Durbin-Watson	.763		1.69		.931	

The estimated equation was:

$$\delta_t = a_2 rd_t \Phi(a_1 w_t) + a_3 rl_t \Phi(-a_1 w_t)$$

$$+ a_4 (rl_t - rd_t)^2 \{ \varnothing(a_1 w_t) \left(\frac{rl_t}{rl_t - rd_t} + \Phi(-a_1 w_t) \right)$$

$$- a_1 w_t \Phi(-a_1 w_t) \Phi(a_1 w_t) \} + \varepsilon_t.$$

Table A3.3.　　　**Interest rate model with a quota, OLS**

Sample	All obs.		First period		Second period	
Coefficient						
a1	0.45	(.038)	2.76	(.375)	0.85	(.154)
a2	0.84	(.055)	1.08	(.006)	1.41	(.160)
a3	1.11	(.052)	0.96	(.010)	0.96	(.019)
a4	-.80	(.200)	-573.0	(1367)	1.81	(1.39)
R^2	.620		.745		.687	
SSR	380.0		49.2		208.0	
SEE	.809		.419		1.10	
Durbin-Watson	.759		1.66		.920	

The estimated equation was:

$$\delta_t = a_2 rd_t \Phi(a_1 w_t) + a_3 rl_t \Phi(-a_1 w_t)$$

$$+ a_4 (rs_t - rl_t) \Phi(a_1 (K_t - w_t)) + \varepsilon_t.$$

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Table A3.4.   Week-day estimations of the borrowing function, ML

	Monday		Tuesday		Wednesday		Thursday		Friday	
b1	1.79	(.167)	1.62	(.168)	1.54	(.158)	1.50	(.184)	1.58	(.168)
b2	1.06	(.011)	1.05	(.010)	1.01	(.003)	1.04	(.015)	1.00	(.012)
c_0	.269	(.030)	.297	(.035)	.259	(.028)	.306	(.039)	.231	(.022)
LogL	-89.3		-94.9		-86.6		-96.7		-80.1	
R^2	.621		.566		.623		.544		.669	
SSR	31.5		34.7		30.1		35.8		26.9	
Ljung-Box(5)										
u_t	8.77		7.38		8.28		18.92		21.53	
u_t^2	1.47		1.03		1.74		1.81		2.49	
D-W	1.56		1.60		1.42		1.43		1.32	
NOBS	117		117		117		117		117	
$\frac{1}{T}\sum_t \delta_t$	8.96		8.83		8.79		8.74		8.73	
$\frac{1}{T}\sum_t w_t$	442		443		391		365		204**	

Asterisks indicate that the mean deviates significally from the mean of the observations from the rest of the week (assuming normality and equal variances).

Appendix 4. Maximum likelihood estimation of a nonlinear GARCH-in mean model

Maximum likelihood estimation of a GARCH(p,q) model is considered here under the assumption that the random variable u follows a normal distribution. The symbol $\Theta = (\beta',\tau')$ is used to denote the vector of all exogenous parameters, consisting of a vector of mean equation parameters β and a vector of variance equation parameters τ. The log-likelihood function of the model, $L(\Theta)$, can then be written as follows:

$$L(\Theta) = \Sigma L_t(\Theta);$$

$$L_t(\Theta) = -\tfrac{1}{2}\log(h_t^2) - \tfrac{1}{2}\log(h_t^2) - \tfrac{1}{2}u_t^2/h_t^2,$$

(A3.1)

where
$$h_t^2 = Z_t'\tau = c_0 + c_1 u_{t-1}^2 + ... + c_q u_{t-q}^2 + a_1 h_{t-1}^2 + ... + a_q h_{t-p}^2$$

$$u_t^2 = (w_t - h_t X_t'\beta)^2.$$

The task is to find the vector of parameters that maximizes the value of the log-likelihood function. A necessary condition for maximum is that the derivatives with respect to the parameters equal zero. Because of the nonlinear structure of the mean equation, analytical expressions for the derivatives are also derived. These derivatives are of following form:

$$\frac{\partial L_t}{\partial\Theta} = -\frac{1}{2}h^{-4}\frac{\partial h_t}{\partial\Theta}\left\{h_t^2 - u_t^2 - h_t u_t X_t'\beta\right\} + \frac{u_t}{h_t}\frac{\partial X_t'\beta}{\partial\Theta}.$$

(A3.2)

These expressions for the derivatives differ only slightly from those of a linear ARCH-M model (Engle, Lilien and Robins, 1987, p. 398). As always in ARCH and GARCH models, the derivatives are recursive. This is seen more clearly after these expressions are elaborated a little further. Each derivative includes the derivative of the conditional variance with respect to the parameters. First, with respect to the parameters of the variance equation, we have

$$\frac{\partial h_t^2}{\partial\tau} = Z_t + \sum_{i=1}^{p} a_i \frac{\partial h_{t-i}^2}{\partial\tau} - \sum_{i=1}^{q} c_i \frac{\partial u_{t-i}^1}{\partial\tau}.$$

(A3.3)

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Accordingly, the derivatives with respect to the parameters of the mean equation are

$$\frac{\partial h_t^1}{\partial \beta} = \sum_{i=1}^{p} a_i \frac{\partial h_{t-i}^2}{\partial \beta} - 2\sum_{i=1}^{q} c_i u_{t-i} h_{t-i} X_{t-i}$$

$$- \sum_{i=1}^{q} c_i \frac{u_{t-i}}{h_{t-i}} X_{t-i} \beta \frac{\partial h_{t-i}^2}{\partial \beta}.$$

(A3.4)

Except for the last term in (A3.4), the above expressions are the same as in a GARCH model without a mean effect (see Bollerslev, 1986, p. 316). They are highly recursive since the derivatives of the variance equation depend on the derivatives of the lagged variances and lagged residuals, which, in turn, both depend on the derivatives of the lagged variances. Engle et al. have applied both numerical and analytical derivatives, and they recommend the use of the former because they allow the specification of the model to be changed flexibly. (Engle, Lilien and Robins, 1987, p. 396).

The estimation method that is usually applied to complicated maximum likelihood problems is the BHHH algorithm (Berndt, Hall, Hall and Hausman, 1974). It has the important advantage that it only uses the first derivatives of the log-likelihood function. The BHHH algorithm was also used for iterations in the estimations carried out here and the derivatives were evaluated numerically. Advanced statistical program packages nowadays include maximization algorithms for functions of general form and also the possibility to evaluate derivatives recursively. Much of the estimation work in this study was carried out using standard procedures of the RATS program. However, relying solely on standard packages would limit the possibilites for diagnostic testing, or in some cases even the specification of the model. Performing tests is much easier if all the intermediate results from the estimation procedure are available, which is generally not the case with standard procedures. In particular, LM tests for specification of the model can be computed from the matrix of first derivatives (matrix S below).

Because of the lag structure of the model, an assumption has to be made about the values of residuals prior to the estimation period. Here they were set according to the expected values of residuals in the model, i.e. to zero. Strictly speaking, the estimation results are conditional on the values chosen for pre-sample residuals.

The steps of the estimation procedure can be summarized as follows (Engle, Lilien and Robins, 1987):

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1. Starting from a given vector of parameters, Θ^1, compute the value of the log-likelihood function $L(\Theta^1)$. This involves the evaluation of the residual term and the conditional variance term. These can be computed recursively from their equations, assuming that pre-sample residuals are zero.

2. Compute the derivative of the log-likelihood function with respect to all parameters at each time period and from these form a (T x m) matrix S, where T = number of periods and m = number of parameters

$$S = \begin{bmatrix} \dfrac{dL_1(\Theta)}{d\Theta_1} & \cdots & \dfrac{dL_1(\Theta)}{d\Theta_m} \\ & & \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \dfrac{dL_T(\Theta)}{d\Theta_1} & \cdots & \dfrac{dL_T(\Theta)}{d\Theta_m} \end{bmatrix}.$$

3. The iterations of the Berndt, Hall, Hall, Hausman algorithm (1974) are computed using this matrix S. They have shown that there always exists constant c^i such that the iteration

$$\beta^{i+1} = \beta^i + c^i(S'S)^{-1}S'I,$$

where I = unit matrix (Tx1),

converges to a stationary point of the log-likelihood function (a local maximum). Moreover, S'S converges to the Hessian of the function when T goes to infinity. The inverse of this matrix, $(S'S)^{-1}$, is a consistent estimate of the variance-covariance matrix of the parameters (Berndt, Hall, Hall and Hausman, 1974, p. 658).

Appendix 5. Estimating a GARCH-in mean model with Student's t distribution

The Student density function is

$$f(x \mid m, H, d) = \frac{\Gamma\left(\frac{1+d}{2}\right) d^{\frac{d}{2}} \sqrt{H}}{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{d}{2}\right)} \left[d + H(x-m)^2\right]^{-\frac{d+1}{2}}, \qquad (A5.1)$$

where m is the location parameter, H is the scale parameter, d is the degrees of freedom parameter, $d > 0$ and $\Gamma(.)$ is the gamma function. The first two moments of the Student distribution are $E(x) = m$, for $d > 1$, and $E(x^2) = H^{-1}d/(d-2)$, for $d > 2$.

In this particular application, the moments of the conditional distribution are $E(u_t^2 \mid I_{t-1}) = h_t^2$ and $E(u_t \mid I_{t-1}) = 0$. Using these, the conditional density function of the model can be written in standardized form as

$$g(u_t \mid I_{t-1}) = \frac{\Gamma\left(\frac{1+d}{2}\right)}{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{d}{2}\right)} d^{\frac{d}{2}} \sqrt{\frac{d}{(d-2)h_t^2}} \left[d + \frac{du_t^2}{h_t^2(d-2)}\right]^{-\frac{d+1}{2}}$$

$$ \qquad (A5.2)$$

$$= \frac{\Gamma\left(\frac{1+d}{2}\right)}{\Gamma\left(\frac{d}{2}\right)} \frac{1}{\sqrt{(d-2)h_t^2 \pi}} \left(1 + \frac{u_t^2}{h_t^2(d-2)}\right)^{-\frac{d+1}{2}}, \quad d > 2.$$

The expression in (A5.2) is the same as in Bollerslev (1987). The log-likelihood function of the model is

$$L = \sum_t L_t;$$

$$L_t = \log(g(u_t | I_{t-1}))$$

$$= \log\left(\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)}\right) - \frac{1}{2}\log((d-2)\pi) - \frac{1}{2}\log h_t^2 + \frac{d+1}{2}\log\left(1 + \frac{u_t^2}{h_t^2(d-2)}\right).$$

This expression for the log-likelihood function is still computationally inconvenient because its use would require evaluating values of the gamma function. In order to circumvent this, we use Stirling's formula (see Kendall and Stuart, 1958, p. 81), which states that

$$\log(\Gamma(p)) = (p - \frac{1}{2})\log p - p - \frac{1}{2}\log(2\pi) + R(p),$$

where $R(p)$ is a certain polynomial of p of order $-(2k-1)$, $k = 1,2,...$ Reasonable adequacy for our purposes is achieved by taking the first three terms of this polynomial[1]

$$R(p) = \frac{1}{12p} - \frac{1}{360p^3} + \frac{1}{1260p^5}.$$

Using these formulas, the gamma functions in the log-likelihood function are expanded into polynomials. The resulting expression can be further simplified, so that finally the log-likelihood function is not computationally much more demanding than the corresponding function under normally distributed errors

$$L_t = \frac{d}{2}\log\left(\frac{d+1}{d}\right) + \frac{1}{2}\log\left(\frac{d}{2(d-2)\pi}\right) - \frac{1}{2} - \frac{1}{2}\log h_t^2 - \frac{d+1}{2}\log\left(1 + \frac{u_t^2}{h_t^2(d-2)}\right)$$

$$+ \frac{2}{12}((d+1)^{-1} - d^{-1}) - \frac{2^3}{360}((d+1)^{-3} - d^{-3}) + \frac{2^5}{1260}((d+1)^{-5} - d^{-5}).$$

---

[1] The adequacy of approximation can be checked against theoretically known properties of the gamma function (see Cramer 1971).

Appendix 6. Modelling the residuals as an AR(1) process

Test statistics from conditional residuals of the basic GARCH(1,1)-in mean model indicate the presence of serial correlation. To investigate the type of serial dependence, autocorrelation and partial autocorrelation functions of conditional residuals were calculated. These functions are plotted in Figures A6.1 and A6.2.

The plot in Figure A6.1 shows that the partial autocorrelation function decays rapidly. By contrast, the decay of the autocorrelation function depicted in Figure A6.2 is rather slow and appears to be characterized by an exponential process. The pattern of these functions implies that the type of serial correlation is indeed AR and not MA or a combination of these two processes, ARMA. It also seems that the order of the AR process is close to one, indicating that applying the first order correction is justified.

Figure A6.1.   **Partial autocorrelation function of conditional residuals from the GARCH(1,1)-in mean model: the whole estimation period**
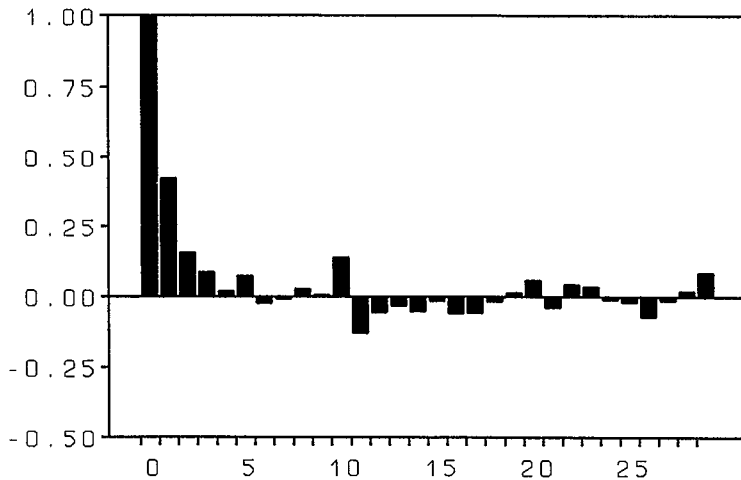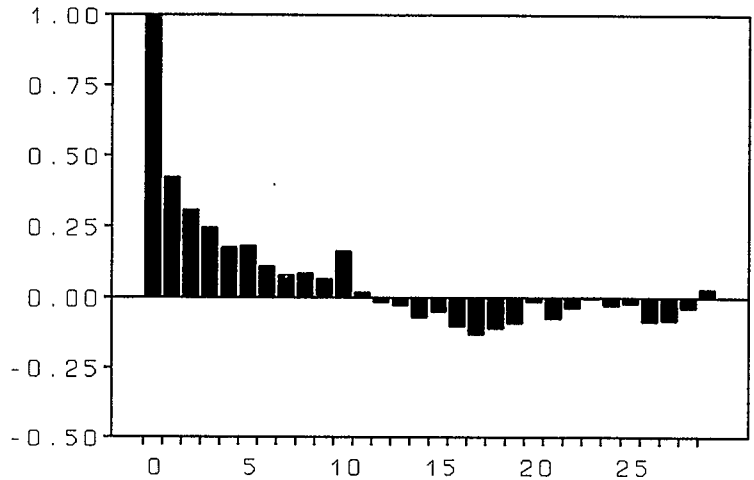
Figure A6.2.   Autocorrelation function of conditional
residuals form the GARCH(1,1)-in mean model:
the whole estimation period

In the case of AR(1), the residuals are thought to be governed by the following process:

$$u_t = p\ u_{t-1} + \varepsilon_t. \tag{A6.1}$$

It is still assumed that the innovations of the model, now denoted by ε, are conditionally normally distributed. Under these assumptions, the complete model is

$$w_t = b_1 h_t / k (\log(b_2 r l_t - \delta_t) - \log(\delta_t - rd_t)) + u_t$$

$$\varepsilon_t = u_t - p\ u_{t-1}$$

$$\varepsilon_t \sim N(0, h_t^2) \tag{A6.2}$$

$$h_t^2 = c_0 + c_1 \varepsilon_{t-1}^2 + c_2 h_{t-1}^2.$$

In order to implement these changes in estimation, the likelihood function of the model must be modified accordingly. The probabilities of the observations w_t must be expressed as being conditional on the previous observation, w_{t-1}, because the previous day's observation affects the current day's value. Noting, however, that the probability of the first observation is not changed, the probabilities of all observations can be written as (see Judge, Griffiths, Lutkepohl and Lee, 1982, p. 289)

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$$f(w_1) = (2\pi h^2 u_1)^{-\frac{1}{2}} \exp(-\frac{1}{2} h_1^{-2} u_1^2)$$

$$f(w_t | w_{t-1}) = (2\pi h_t^2)^{-\frac{1}{2}} \exp(-\frac{1}{2} h_t^{-2} \varepsilon_t^2), \tag{A6.3}$$

when $t = 2, ..., T$.

The log-likelihood function is

$$L = \Sigma L_t, \tag{A6.4}$$

where

$$L_1 = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(h_{u_1}^2) - \frac{1}{2} u_1^2 / h_{u_1}^2, \text{ and}$$

$$L_t = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(h_t^2) - \frac{1}{2} (u_t - p \cdot u_{t-1})^2 / h_t^2,$$

when $t = 2, ..., T$.

Results from the estimation of the above model are reported in the text in Chapter 7.

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