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Investment Behaviour and Market Imperfections with an Application to the Finnish Corporate Sector

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PREFACE

Most of the present doctoral thesis was written in the years 1981 - 1984. The basis of the research was laid down in 1981 - 1982 when I was engaged in full-time research work at the Research Department of the Bank of Finland. While working on the study, I have benefitted greatly from discussions with many people both informally and at various seminars. My greatest debt of gratitude is to Seppo Honkapohja and Erkki Koskela, my official examiners, who have encouraged me during the research work and commented upon several manuscripts. Erkki Koskela has guided the work from the beginning to the end. Matti Virén has given detailed comments and suggestions on several drafts of the empirical part of the study.

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Helsinki, May 1985

Heikki Koskenkylä

CHAPTER I

INTRODUCTION

1.1 Background of the Study

The last decade and a half has seen the rapid growth of the international literature on fixed capital formation by firms, with both theoretical and empirical advances taking place. In particular, the theoretical background of the investment function has been at the centre of modern investment research. In the earlier decades, the investment function was not derived from a rigorously formulated framework, i.e. it did not have a well-formulated basis in intertemporal analysis.

The starting point for the new developments in the investment literature was the reformulation of the neoclassical optimal capital accumulation model by D.W. Jorgenson (1963, 1965, 1967). In this model, factor prices play a key role. The neoclassical model has retained a dominant position in both theoretical and empirical investment studies. However, at about the same time as the seminal work by Jorgenson (1963), the market value approach also gained momentum at both the theoretical and empirical levels (Grundfeld 1960 and Tobin 1961, 1969). Since the beginning of the 1960s, these two approaches have been the main candidates for providing the determinants of investment decisions. However, the simple accelerator model as well as various more 'eclectic' approaches have also existed side by side with these 'new' theories.

It has been argued that Jorgenson's version of the neoclassical investment theory (SNC = standard neoclassical model) is not really a theory of investment as such but merely a theory of alternative levels of the optimal capital stock. This argument is related to the assumptions of this model, which are as follows (see Nickell, 1978):

(1) There exists a perfect capital market.
(2) The environment in which firms operate is one of perfect certainty concerning the future.
(3) Efficient production techniques can be summarized in the form of a twice differentiable production function with two inputs (capital and labour), and there are strictly declining returns to scale.
(4) Capital input has the same productive characteristics regardless of its age and it deteriorates at a constant exponential rate.
(5) The firm acts as a price taker in all markets.
(6) There are no extra costs involved either in the sale and purchase of investment goods or in the installation of new capital goods.

The simplifying assumptions portray a world in which the firm is able to manipulate its stock of fixed capital as if it were operating in 'paper assets'. It should be noted that a seventh assumption is often added to the above list, namely, that there are no taxes of any description. However, the SNC model of Jorgenson incorporates basic corporate tax factors which affect the user cost of capital services (see section 2.1).

The most important simplifying feature of the SNC model is that the demand for factor services depends only upon current parameters and variables. Although the model is formulated as an apparently dynamic problem, it turns out to be essentially static. In this framework, the firm is able to make instantaneous and costless adjustments in the capital stock because of changes in its determinants. Moreover, the model does not provide an investment function with well-defined partial derivatives (see Haavelmo 1960, Nickell 1978). The chief advantage of the present-value maximization rule implied by this model is that it provides an explicit formula for the user cost of capital services (see section 2.1).

The empirical work on neoclassical investment equations was also pioneered by Jorgenson and his collaborators (see section 2.1). Although this approach has many weaknesses, it has flourished over the last two decades. The reason for this is that, since this model is 'structural', it provides a useful frame of reference for policy analysis. However, the model derives only a stock demand for capital and investment is then determined by an ad hoc stock adjustment mechanism (lag structure) in which investment is related to the difference between the optimal (desired) capital stock and the existing capital stock. Adjustment cost literature has subsequently provided a 'theoretical' rationale for the dynamics of the SNC model (e.g. Eisner and Strotz 1963, Lucas 1967, Gould 1968 etc.; see section 3.1.2). In one well-known version of the adjustment cost approach, a determinate rate of investment is derived by appealing to a rising flow supply schedule for capital goods (see section 3.1.2). The rising supply curve is formally obtained by assuming that the firm is faced with a strictly convex adjustment cost function with investment as an argument. In this framework, investment is proportional to the gap between the desired and existing capital stocks. This derivation of the flexible accelerator model from explicit intertemporal optimizing behaviour has provided a partial justification for the stock adjustment model used by Jorgenson and others. Whereas the original SNC model relies on static (stationary) expectations, the adjustment cost model, by contrast, assumes that firms have perfect foresight about the future. Indeed, it is the treatment of expectations which is a critical point in the neoclassical approach as a whole.

The market value approach is based on Tobin's "q"-variable, defined as the ratio of the market value of installed capital to the replacement cost value of capital (Tobin 1961, 1969). The central hypothesis of this model is that investment is an increasing function of "q". The market value of capital is hence directly related to investment behaviour. In contrast to the SNC model, this model is in a sense not structural since it does not look behind the determinants of market value. The expectations hypothesis also differs from that of the SNC model since the market value approach is based on well-functioning asset markets which make forecasts and properly digest expectations. The empirical branch of this approach usually regresses investment directly on some proxy for the "q" variable (see, e.g., Grundfeld 1960, Ciccolo 1975, von Furstenberg 1977, Hayashi 1982).

Although in the earlier investment literature (1960s and 1970s) it was thought that the neoclassical and market value approaches were 'completely' different ways of looking at the determinants of investment behaviour, the recent literature has, however, shown that they are essentially the same and differ only in emphasis (see Abel 1980, 1981 and Hayashi 1982). The similarity follows if the shadow price of installed capital, which is the key variable in the adjustment-cost-based SNC model, is identified with the market price of capital. Thus the "q" approach can be derived from the explicit intertemporal optimization behaviour of firms. In this respect, the above-mentioned 'structural discrepancy' between the SNC and market value approaches vanishes since "q" can now be expressed as a function of the same arguments as investment (or capital) is directly in the former case.

Essentially, these modern theories of investment assume perfect markets and, in particular, highly developed ('perfect') financial markets, which do not, in fact, exist in many countries. More specifically, all prices are usually assumed to be given to the firm (price-taking behaviour) and the firm is almost invariably assumed to be equity-financed (see section 2.1). The only imperfection which is usually allowed is the convex adjustment cost function, i.e. the rising flow supply price of investment goods. The assumption of a perfect capital market implies that at every point of time all firms are in a position to borrow (or lend) as much as they wish at the going rate of interest; that is, there is only one rate of interest (discount rate) and the financial structure of the firm plays no role in its investment policy.

In the real world, the environment in which firms operate is usually characterized by various types of market imperfections. For instance, prices may not be totally exogenous to a firm and taxes may be non-neutral. If the firm is a monopolist in the product market, then it faces a downward-sloping demand curve and the product price is a choice variable. In this case, an exogenous demand variable (parameter) may act as an accelerator-type factor in the investment decision problem (see section 3.3). Firms may also face a demand (sales)-constrained situation, in which case output is exogenous to the firm (excess supply case, see section 3.4) and cost minimization characterizes optimal firm behaviour. In the investment process itself, there may be features which function as market imperfections. For instance, irreversibility and delivery lags are not uncommon features in the world where investment decisions are made and fulfilled. Irreversibility and delivery lags, accompanied by uncertainty about the firm's future prospects, are among the factors which may produce the slow and hesitant reaction often observed in the adjustment of the fixed capital stock by firms (see Nickell 1978).

In most countries, financial markets are characterized by various types of imperfections. The rate of interest on borrowing, or even more generally the cost of capital, may not be given to a firm but may instead depend on, for instance, the debt-equity ratio (leverage). Hence, in this case financial policy is endogenous to the firm (see section 4.1). Firms may also be faced with a binding profit constraint setting an upper bound on investment and implying that self-financing is the marginal source of finance (see section 4.4).

The role of market imperfections has been a rather neglected area in the main strand of the international literature on investment, i.e. in the standard neoclassical and market value approaches. Some aspects of the imperfections referred to above have, however, been analyzed to some extent in the investment literature, notably the effects of demand factors, irreversibility and certain aspects of capital market imperfections on investment behaviour. This literature is not discussed in detail here since the introductory sections of the following chapters of this study give surveys of the relevant literature.

1.2 Aims of the Study

Having outlined the main features of the development of investment models, we can now present the two main aims of the present study. The first aim is theoretically orientated and the second empirically orientated. In the theoretical part our purpose is to examine the investment decision of a single firm in the context of various market imperfections. Our contribution takes the form of both 'new' model specifications and modifications or critical evaluations of some previous models related to our own. Since market imperfections are a crucial phenomenon in this study, it can also be said that our overall aim is devoted to an analysis of the results of relaxing the somewhat unappealing ('unrealistic') assumptions of the standard neoclassical capital accumulation model (see above).

Relaxing the assumptions of the SNC model takes place in three respects. First, we shall examine the effects of imperfections in the financial market on a firm's investment policy. Here, the incorporation of debt finance in the SNC model is crucial. If borrowing is used as a method of finance, then the rate of interest and the discount rate may diverge. The effects of both a price imperfection (a non-linear cost of capital schedule) and quantitative financial constraints will be examined. In the latter circumstances profits (retained earnings) may also be an important determinant of investment behaviour, which is not the case in a perfect capital market.

Second, we shall examine the effects of alternative demand conditions on capital accumulation. Almost all empirical investment studies have found that an accelerator variable (output, sales, capacity utilization etc.) is the major determinant of investment expenditures. If the demand curve is horizontal (i.e. the price of products is given), then there is no role for a scale variable. In order that a scale variable (accelerator) may appear in the investment function, one has to assume that firms face either a downward-sloping demand curve or a fixed output (sales constraint).

Third, we shall also examine in the theoretical part of the study the impact of various tax factors on investment behaviour. Both the role of personal and corporate tax factors will be analyzed. We are also interested in the conditions under which the corporate tax system is non-distortionary with respect to capital formation.

It turns out that market imperfections and taxes play an important role for both the long-run demand for capital (the desired capital stock) and for the dynamics of investment. We have noted above that strictly convex adjustment costs (i.e. an increasing flow supply curve of capital goods) provide one rationale for a determinate rate of investment. There are, however, also other circumstances in which the costless and instantaneous adjustment implied by the SNC model does not occur. A non-myopic investment rule may also occur if the firm is faced with certain types of capital market imperfections. For instance, a high level of investment during the current period will increase earnings in the subsequent periods, thus relaxing a constraint on internal finance in those periods. This implies that a formula for the cost of capital incorporating retained earnings will be relevant to more investment projects than before. If financial constraints are introduced into the firm's optimization problem, the cost of capital will contain the multipliers corresponding to the binding constraints, thereby linking present and future investment (possibly also financial) policies (see also King 1974, 1977 and Nickell 1978). Other forms of capital market imperfections may also have an effect on a firm's investment decisions, very much as if the firm was faced with strictly convex adjustment costs (see also section 4.1).

In the theoretical part of this study we are concerned with examining the dynamic pattern of the investment policy of a firm. We shall use both an adjustment cost framework and various types of capital market imperfections to produce an explanation for the rather sluggish and hesitant behaviour commonly observed when it comes to decisions about investing in fixed capital. In these circumstances, changes in expectations about future levels of demand or other parameters may well affect current investment. Nickell (1974, 1978) has shown that the presence of irreversibility and/or delivery lags jointly with uncertainty about the firm's future profitability may also provide an environment for non-myopic investment behaviour. We do not, however, explore this possibility in the present study.

The second main aim of this study is to carry out an empirical analysis of the investment behaviour of Finnish non-financial firms in the period 1963 - 1980. The neoclassical investment model is used as a theoretical basis for this analysis. We shall make flexible use of the results of our theoretical analysis when it comes to building econometrically testable investment equations. We are especially interested in discovering the relative significance of the variables and parameters implied by our theoretical models as determinants of investment outlays. The critical variables are output (or demand), factor prices, cash flow (profits) and measures for credit rationing. In addition, various proxies will be constructed for each of these variables and their effects tested.

Our main concern is, however, with alternative measures for the user cost of capital services. This variable measures the total cost of capital (depreciation plus cost of financial capital) and it may incorporate various tax factors (personal and corporate), interest rates (rate of interest on debt, discount rate), investment incentives and the expected rate of inflation. In the theoretical part of the study, alternative formulas for the user cost of capital are derived and these will be tested in the empirical analysis.

It was mentioned above that cash flow may be a determinant of investment behaviour in the context of certain capital market imperfections. In such a case, it is of special interest to know whether cash flow affects the long-run demand for capital and/or the timing of investment. These questions, together with the possibly direct effect of credit-rationing proxies, will also be examined empirically using Finnish firm data. Finally, the effects of various types of accelerator variables are a key question attracting attention throughout our empirical investigation.

The empirical analysis uses both annual and quarterly aggregate data. The quarterly data cover only the total aggregate of firms but the annual data are disaggregated into manufacturing firms (the 'open' sector) and firms in the residual sector (the 'closed' sector). Within this data, we shall examine various functional specifications for the investment equation and also carry out a considerable amount of diagnostic checking and testing between competing hypotheses.

Finally, before considering in some more detail the outline of this study, it is worth pointing out some neglected areas in our analysis. This study employs a fairly coherent framework for examining issues in investment behaviour. Although this framework seems to be fruitful, it does, however, suffer from certain shortcomings. The theoretical analysis is restricted to the case of a single firm and hence the results obtained are very partial by nature. However, they do at least illustrate the various adjustment possibilities open to a firm in its investment policy. Another problem is that the theoretical analysis assumes perfect foresight (certainty) on the part of firms. Perfect foresight is a rather convenient approximation in the theoretical analysis, but one must recognize that uncertainty may affect the firm's investment decision in different ways (see, e.g., King 1977, Nickell 1978 and Abel 1983).

In a certainty framework with a perfect capital market, the maximization of the present value of the net income stream produced by a firm is the objective consistent with individuals maximizing the utility of consumption. In an uncertain environment and possibly also one with capital market imperfections, it is not so clear what the firm's objective should be (see Hirshleifer 1958, King 1977 and Nickell 1978). However, while recognizing this crucial question, we have chosen to apply the present value rule throughout our theoretical investigation (see also section 4.1).

One more 'neglected' area in this study is the question of interrelated factor demand functions. We use only a two-factor production function and assume that output price and labour input are more or less freely adjustable. This might be justified on the grounds that this could be true relative to the cost of adjustment in the capital stock and that, for some types of questions, the results provided in our analysis are not too misleading. Finally, section 5.1 presents a brief discussion of some further questions relating to the empirical analysis.

1.3 Outline of the Study

Chapter II examines the impact of inflation, corporate tax factors and a constant debt-capital ratio (or a marginal borrowing ratio) on the long-run demand for capital. This chapter concentrates only on the determinants of the stationary optimal capital stock, i.e. it uses a myopic framework. Various formulas for the user cost of capital are derived. Neutrality of the corporate tax system is considered under different assumptions.

In chapter III we consider the effects of alternative demand conditions on both the long-run demand for capital and on the dynamics of investment. It is assumed that firms face strictly convex adjustment costs. Formulas for the rate of adjustment are derived in the context of alternative demand regimes. In the cases of a downward-sloping demand curve and fixed output, we shall examine the effects of unanticipated and anticipated increases in demand on the time pattern of investment. A critical role in this analysis is played by the price elasticity of the demand for the firm's products and the properties of the production function. Chapter III also assumes a constant debt ratio and the presence of corporate tax parameters. The effects of these factors on investment will be examined subject to alternative demand conditions. More specifically, we shall consider the impact of tax factors both on the long-run demand for capital and on the dynamics of investment. The conditions of the neutrality of the tax system are derived both in respect to the long-run demand for capital (steady state neutrality) and in respect to the rate of adjustment to the equilibrium level of the capital stock (dynamic neutrality); hence we also consider neutrality with respect to investment itself.

Chapter IV is devoted to the analysis of various types of capital market imperfections in relation to capital accumulation. First, it is assumed that the firm faces a rising cost of capital schedule and the effect of this curve is examined with linear and strictly convex adjustment costs. Within this framework, we also consider the impact of new equity issues on the user cost of capital. This implies that personal tax factors will affect the cost of capital through King's tax discrimination variable. Second, the firm is assumed to face quantitative financial constraints in its investment decision (dividend or profit constraint). Because of these assumptions a non-myopic investment rule may follow. The effects of tax factors are examined throughout chapter IV, and in the profit-constrained case somewhat different results follow as compared with the results for our other models.

Chapter V considers the specification of empirical investment equations. Among the issues discussed in this chapter are the dynamic structure of the investment process, the determinants of the desired capital stock and the role of financial factors in the investment equation.

Chapter VI first briefly presents the underlying statistical data used in the econometric analysis and then the empirical results of annual investment equations. Chapter VII presents the results of quarterly investment analysis. Finally, Chapter VIII presents some concluding remarks.

In the Appendices are presented the formulas for the user cost of capital employed in the econometric analysis, a description of the Finnish corporate tax system, a discussion of the measurement problems of some key variables (rate of return, user cost, tax factors etc.) and their respective estimates, a description of the statistical data used and finally some econometric (estimation) results.

PART 1

THEORETICAL ANALYSIS OF INVESTMENT BEHAVIOUR UNDER VARIOUS MARKET IMPERFECTIONS

CHAPTER II

THE EFFECTS OF INFLATION, TAX RULES AND THE DEBT-CAPITAL RATIO ON THE DEMAND FOR CAPITAL

2.1 Basic Issues and the Jorgenson Approach

The aim of this chapter is to consider the fundamental structure of the standard neoclassical capital accumulation model (SNC model). The financial market assumption (perfect market) of the SNC model is, however, relaxed by assuming that firms have an exogenously given and constant debt-capital ratio. The SNC model originally developed by Jorgenson usually assumes that the firm is financed entirely by equity capital so that interest payments do not enter the definition of taxable income. This assumption also implies that the debt-capital (or debt-equity) ratio does not affect the user cost of capital.

The assumption of a constant debt-capital ratio reduces the firm's optimal decision problem to an investment decision and the related decisions concerning output and labour input. The financial policy of the firm is hence exogenously given and the dividend policy, i.e. the allocation of profits between dividends and retained earnings, is of a residual nature.

Although the financial structure of a firm is assumed to be of a simple character, the incorporation of a debt-capital ratio in the SNC model allows a much richer and more realistic analysis of the impact of corporate tax factors and the rate of inflation on the user cost of capital and through it on the demand for capital. Our focus in chapter II is on the long-run effects of taxation, inflation and financial structure (debt-capital ratio) on the demand for capital rather than on the short-run impact on investment, and hence we ignore nonlinear adjustment costs in the models to be developed in this chapter. Effectively, this last-mentioned assumption implies that the flow supply price of capital goods is exogenously given to the firm and hence that adjustment costs are linear. It should, however, be recognized that the inclusion of convex adjustment costs would generally lead to a lower optimal capital stock than in the case with linear adjustment costs, but otherwise the steady-state (long-run) properties of the model would be unaltered (see chapters III and V). We furthermore assume in this chapter that the other basic assumptions of the SNC model hold (assumptions 2 - 6, see section 1.1). Specifically, this means that firms are price takers in the output market and hence face a horizontal demand curve.[1] Since all other assumptions except the financial market one are the same as in the Jorgenson (SNC) model, and, moreover, so as to compare our results with those of the Jorgenson model, we shall next briefly present the basic features of the Jorgenson approach.

In the Jorgenson approach the firm is assumed to maximize the present value of future net cash flows subject to a standard neoclassical production function, $F(K,L)$, and to a capital accumulation equation, $\dot{K} = I - \delta K$, where K is capital input, L is labour input, I is investment, δ is a constant economic rate of depreciation and $\dot{K} = dK/dt$. The net cash flow (Y) is usually defined as

$$(2.1) \quad Y_t = p_t Q_t - w_t L_t - q_t I_t - T_t$$

where p is the price of output, Q is output, w is the wage rate, q is the price of investment goods and T stands for corporate income taxes. Direct corporate income taxes (T) are defined as

$$(2.2) \quad T_t = u(p_t Q_t - w_t L_t - D_t)$$

where u is a (constant) corporate profit tax rate and D represents depreciation for tax purposes. It should be noted that in (2.2) interest cost deductions are not included in the definition of taxable income.

Using the standard calculus of variations method or a more simple 'present value approach', the infinite horizon maximization problem is solved to yield the usual marginal productivity conditions for capital and labour, i.e. $F_K = c/p$ and $F_L = w/p$. A formula for the user cost of capital (c) also follows from this optimization problem and it is given by the following expression

$$(2.3) \quad c_t = \frac{q_t}{(1-u)} (\rho+\delta-g)(1-uz)$$

where ρ is the discount rate, g is the (expected) rate of inflation on capital goods and z is the present value of depreciation charges on one unit of investment (see section 2.2 for the calculation of z).[2] If tax factors are excluded ($u = z = 0$), then $c = q(\rho+\delta-g)$. The implicit demand functions for capital and labour can be solved from the corresponding marginal productivity conditions and they are as follows:

$$(2.4) \quad K^* = K(\frac{c}{p},\frac{w}{p}); \quad L^* = L(\frac{c}{p},\frac{w}{p})$$

The functions in (2.4) determine the long-run (steady-state) equilibrium demand for capital and labour. Jorgenson derives an explicit formula for the desired (optimal) capital stock by assuming a standard Cobb - Douglas production function with diminishing returns to scale, i.e. $Q = AK^aL^b$, $a + b < 1$, and stationary (static) expectations with respect to prices and tax parameters. In this framework, the desired capital stock is given by the equation

$$(2.5) \quad K^* = \frac{ap(1-u)Q}{q(\rho+\delta-g)(1-uz)}$$

It should be noted that, in this profit maximizing framework with a horizontal demand curve (competitive output market), output is also a decision variable (endogenous) and it is determined by a relationship analogous to (2.4), i.e. $Q^* = Q(c/p,w/p)$. Formula (2.5) for K^* is obtained by using only one of the two marginal productivity conditions, i.e. the condition that $F_K = c/p$. This implies that output is treated as exogenous in the investment decision problem (see below).

During the last decade the Jorgenson model of optimal capital accumulation described above was heavily critized by many authors mainly on the grounds of the very restrictive assumptions on which it is built. Here we only briefly list the main points of criticism (for a more detailed discussion, see e.g. Brechling 1975, Helliwell 1976, Nickell 1978). The list is as follows:[3]

i) output is treated as exogenous although the underlying theory would imply that output is also an endogenous decision variable;

ii) the price of the other factor input (labour) does not affect investment decisions;

iii) the firm is assumed to have a very simple financial structure;

iv) only long-run demand for capital is determined within this theory while the optimal rate of investment is not determined;

v) expectations of exogenous factors are very simple (stationary or static);

vi) production technology is assumed to be of an extreme form (putty-putty hypothesis); and

vii) personal taxes are not included in the model.

This chapter is devoted to questions belonging to group (iii) and in later chapters we shall consider the other issues (chapter III is concerned with i) - iv), chapter IV with iii) - v) and vii) and chapter V with all the points in the above list). The outline of this chapter is as follows: Section 2.2 develops the standard neoclassical model with a constant (average) debt-capital ratio and in section 2.3 the SNC model is extended to include a marginal borrowing ratio. These two models are used to derive alternative formulas for the user cost of capital and to examine the long-run effects of taxes, inflation and financial ratios on the demand for capital. Neutrality conditions of the corporate tax system are also discussed in both sections.

2.2 The Standard Neoclassical Investment Model with a Constant Debt-Capital Ratio

2.2.1 Background

In the neoclassical model to be developed in this section it is assumed that the firm has an exogenously given and constant debt-capital ratio. This assumption implies that firms use both debt and equity capital to finance their investment expenditures. Furthermore, it is assumed that the firm uses only internal equity capital (retained earnings) and hence new equity issues as a source of finance are excluded from the analysis (this will, however, be considered in section 4.3). The financial assumption made here means that the rate of interest on debt finance (r) and the discount rate of shareholders (ρ) may diverge. Basically, this financial market assumption implies that a simple form of capital market imperfection is introduced into the SNC model and that the first of the underlying assumptions of the SNC model is modified (see section 1.1, and section 2.2 for the reasons for this 'new' assumption).

The rationale for switching from a perfect capital market approach to an imperfect one is the fact that in the real world firms seem to use many sources of funds at the same time. A second reason for this 'new' assumption is that the inclusion of the financial structure in the SNC model enables a deeper analysis of the complex effects of the rate of inflation on the demand for capital. This is an important issue because the acceleration of inflation in the 1970s caused substantial changes in the real value of debt amortizations (i.e. capital gains to equity holders) in most western countries. A third reason for this 'new' approach is that the neutrality properties of the corporate tax system can be derived more completely within this framework than in the context of the Jorgenson model.

Some previous studies have also recognized the role of the debt-capital ratio and have developed models to tackle the question of the impact of the rate of inflation on investment. Our analysis is related to the work by Sumner (1973), Bergström (1976), Södersten (1977, 1982), Boadway (1978, 1980), Boadway and Bruce (1979), Ylä-Liedenpohja (1976, 1983a, 1983b), Summers (1981) and Poterba and Summers (1983). In the studies by these authors, an average (constant) debt-capital or debt-equity ratio is incorporated in the neoclassical model of the firm. Airaksinen (1979) considers the case of a marginal borrowing ratio, a topic which we shall address in the next section (2.3). The main difference between our analysis and the others is that we make extensive use of a present value formulation in deriving and interpreting the results (see also Boadway, 1980). The basic features of our model are, however, very similar to those of the above authors except that in the work of Summers (1981) and Porterba and Summers (1983) no explicit formula for the user cost is derived since their models are formulated in terms of Tobin's "q"-framework. Instead, these authors derive a hypothetical value for the average (or marginal) Tobin's "q"-variable against which investment (or I/K) is then directly regressed.

This section also seeks to survey and evaluate previous neoclassical models which have incorporated the constancy assumption concerning the debt-capital ratio. We shall also discuss in some detail the neutrality conditions of the corporate income tax system and present some empirical calculations of the real cost of financial capital with parameter values typical of the Finnish manufacturing sector. The chief aim of this section is to present alternative formulas for the user cost of capital which will later be used in our empirical analysis of the investment behaviour of Finnish corporations (chapter V).

2.2.2 Description of the Model and Its Ingredients

The balance sheet of the firm is assumed to be of the following form:[4]

$$(2.6) \quad qK = B + E$$

where qK is total capital at current replacement cost, B is debt (book value) and E is equity capital (internal). The cash flow identity of the firm is defined as

$$(2.7) \quad pQ + \dot{B} = wL + qI + rB + T + Div$$

where Div is dividends and other symbols are the same as in section 2.1. The assumption of a constant debt-capital ratio is given by

$$(2.8) \quad \frac{B}{qK} = s$$

Two arguments can be put forward which may be thought to stand behind (2.8). First, a simple (although not entirely satisfactory) approach used in the finance literature to represent the effect of uncertainty on the financial decision is to assume that the required rate of return on debt increases with leverage (debt-equity ratio), presumably because of increased bankruptcy risks as well as other factors (see, for example, Baumol and Malkiel 1967, Auerbach 1979, Feldstein - Green - Sheshinski 1978). According to this interpretation, s would be the optimal value of the debt-capital ratio which minimizes the cost of capital (see chapter IV where r is assumed to depend on s). Second, firms might be assumed to be constrained in their use of debt (or equity) capital. Whereas the first interpretation would imply that the capital market imperfection is characterized by a non-linear price system in the financial market, the second interpretation effectively means that quantitative credit rationing is present. We do not need to choose between these two interpretations here since our point of departure in this chapter is the question: What, given the firm's (observed) financial behaviour, is the user cost of capital and how do various factors (taxes, inflation etc.) affect it? The important point to note in this connection is that a weighted average cost of capital concept is likely to follow only when firms are constrained in their use of debt or equity, or if uncertainty is present. Otherwise, the appropriate cost of capital is simply the minimum of the costs of debt and equity (see also Auerbach 1983).

Differentiating (2.8) with respect to time gives the time rate of change in debt capital

$$(2.9) \quad \dot{B} = s(q\dot{K}+\dot{q}K)$$

According to (2.9), debt is increased not only in proportion (s) to net investment but also in proportion (s) to the revaluation of the capital stock (capital gains component). Without this second term in (2.9), the debt-capital ratio would not remain constant during periods of a positive rate of inflation on capital goods. In the next section (2.3) we shall discuss the rationale behind the ($s\dot{q}K$) term. Here it suffices to say that if proportion s of gross investment is financed by new loans, then the constancy of the debt ratio requires that the rate of amortization on existing debt must be equal to δ-g (see section 2.3).

Gross retained earnings (gross cash flow) is defined as

$$(2.10) \quad R^g = pQ - wL - rB - T - Div$$

This definition of R^g is the same as that used in, for example, Atkinson and Stiglitz (1980). R^g includes both net profit and current replacement cost value of depreciation ($q\delta K$). Net retained earnings can now be defined as

(2.11) $R^n = pQ - wL - q\delta K - rB - T - Div$

From the balance sheet (2.6) we can obtain the time rate of change in equity

(2.12) $\dot{E} = (1-s)(\dot{q}K+q\dot{K})$

In this model framework gross investment is financed by three types of finance: depreciation, an increase in net borrowing and an increase in internal equity capital. Hence the flow financing identity of investment is

(2.13) $qI = R^n + q\delta K + \dot{B} = R^g + \dot{B}$

Corporate income taxes are defined as[5]

(2.14) $T = u(pQ-wL-rB-D)$

which is equivalent to equation (2.2) except that now the deductibility of nominal interest payments (rB) is also included in the definition of taxable income. Effectively, this deductibility implies that in the following optimization problems the firm is assumed to maximize the wealth of its owners, whereas in the Jorgenson model the maximization is with respect to the total present (market) value of the firm.

Using the cash flow identity (2.7) and the tax expression (2.14), the following formula for dividends results

(2.15) $Div = (1-u)(pQ-wL-rB) - qI + uD + \dot{B}$

The above formulas (2.6 - 2.15) constitute the basic stock and flow identities of the firm which will be used both in this chapter and fairly extensively in the subsequent chapters of this study.

Before turning to consider the optimization problem of the firm, we shall briefly present some useful auxiliary equations and concepts which are necessary and/or useful in the subsequent analysis. Tax depreciation (D) is equal to αK_H, where α is a tax depreciation coefficient and K_H is the historic (original) cost value of the capital stock defined by the equation

$$(2.16)\quad K_{H,t} = \int_{-\infty}^{t} e^{-\alpha(t-v)} q(v)I(v)dv$$

The concept of capital stock stemming from true economic depreciation is defined by

$$(2.17)\quad K_t = \int_{-\infty}^{t} e^{-\delta(t-v)} I(v)dv$$

where I is the volume (at fixed prices) of gross investment. The current replacement cost value of the capital stock is hence qK.

Differentiating (2.16) and (2.17) with respect to time yields the following equations of motion for K_H and K

$$(2.18)\quad \text{i)}\quad \dot{K}_H = qI - \alpha K_H$$

$$\text{ii)}\quad \dot{K} = I - \delta K$$

These differential equations will be used as constraining equations in the subsequent optimization problems. It will, however, be soon seen that the transition equation for K_H is not always necessary if the present value formulation of depreciation deductions is used.

We next consider the treatment of depreciation in the models used below. If a unit of capital is invested at time t, then the undepreciated part of that capital at time v is $e^{-\delta(v-t)}$. Hence depreciation at time v is defined by (assuming g = 0)[6]

$$(2.19)\quad d_v = \delta e^{-\delta(v-t)}$$

The present value of depreciation charges on a unit of investment is then

$$(2.20) \quad z_\delta = \int_t^\infty e^{-\rho(v-t)} d_v dv = \frac{\delta}{\rho+\delta}$$

where ρ is the discount rate (constant). Likewise, the present value of tax depreciation deductions on one unit of gross investment is given by

$$(2.21) \quad z_\alpha = z = \frac{\alpha}{\rho+\alpha}$$

where α is the tax depreciation coefficient.

Finally, we consider the production function basis of our model framework. We assume a standard neoclassical production function, $F(K,L)$, which is twice continuously differentiable and possesses the usual properties that the first partial derivatives (F_K, F_L) are positive and the second partial derivatives (F_{KK}, F_{LL}) are negative. Current operating profits are defined as gross revenue minus the production costs incurred as a result of hiring factors of production other than capital, i.e. $pQ-wL$. The maximized current profits will now be a function of K in the following way: At any instant of time when K is predetermined, the firm will choose the amount of labour so as to satisfy the marginal rule $F_L = w/p$, where p and w are exogenous to the firm. This gives an implicit labour demand function $L = L(K)$. A profit function can now be defined as $f(K) = pF(K,L(K)) - wL(K)$. It is easily shown that $f'(K) > 0$ and

$$(2.22) \quad f''(K) \lesseqgtr 0 \quad \text{as} \quad F_{KK}F_{LL} - F_{KL}^2 \gtreqless 0$$

The concavity of $F(K,L)$ implies that $f'' \leq 0$ and if $f'' < 0$, then F is strictly concave. We shall make use of this profit function below in order to simplify the analysis.

2.2.3 The Optimization Problem and Formulas for the User Cost of Capital

The maximization problem of the firm is now the following:

$$(2.23)\quad \max_{\{I\}} \int_0^\infty e^{-\rho t}[(1-u)f(K_t)-q_tI_t+uzq_tI_t-(1-u)rsq_tK_t+sq_t(I_t-\delta K_t)+s\dot{q}K_t]dt$$

subject to $\dot{K}_t = I_t - \delta K_t$

In (2.23) it is assumed that the usual nonnegativity constraints hold ($K_t \geq 0$, $B_t \geq 0$ and $E_t \geq 0$, which would follow if it is assumed that $0 \leq s \leq 1$) and that there is a given stock of initial capital K_0. It should be noted that in (2.23) only the present value of depreciation allowances on new investment (i.e. zqI) is included. The present value of depreciation allowances on the existing capital stock can be ignored since it is independent of any current or future decisions (see also Abel, 1981 and Summers, 1981 and chapter III). The current value Hamiltonian of this control problem is given by

$$(2.24)\quad H^c = (1-u)f(K) - (1-uz)qI - (1-u)rsqK + sq(I-\delta K) + s\dot{q}K + \lambda(I-\delta K)$$

where λ (= λ_t) is the adjoint variable connected with the equation of motion for K (that is, λ is the shadow price of capital). Assuming an interior solution for an optimum and considering first the case in which prices are fixed (i.e. g = 0), the necessary conditions (Pontryagin's maximum principle) are:

$$(2.25)\quad \text{i)}\quad \dot{\lambda} = (\rho+\delta)\lambda - (1-u)f'(K) + (1-u)rsq + sq\delta$$

$$\text{ii)}\quad \frac{\partial H^c}{\partial I} = -q + uzq + sq + \lambda = 0$$

It should be noted that all variables in (2.25) are evaluated at their 'optimal values' (i.e. at values which satisfy the necessary conditions). It is well-known that in infinite horizon problems there is no general transversality condition and it is necessary to prove such conditions for each case (see Takayama, 1974). We use the sufficiency theorem established by Arrow and Kurz (1970, propositions 8 - 10), which states that, if the maximized Hamiltonian $H^{O}(K,\lambda) = \max_{\{I\}} H^{C}(K,I,\lambda)$ is a concave function of K for a given λ, then any policy is optimal that satisfies the necessary conditions and the following transversality conditions (using our notation)

$$(2.26)\quad \lim_{t\to\infty} \lambda_t e^{-\rho t} > 0, \quad \lim_{t\to\infty} \lambda_t K_t e^{-\rho t} = 0$$

It is easily verified that these conditions hold in our maximization problem (λ_t = constant, $\rho > 0$, $f'' < 0$). It should be noted that the concavity requirement is satisfied since the Hamiltonian is jointly concave in K and I, although the concavity of H^{O} may also hold more generally (see Arrow and Kurz, 1970, chapter V). The sufficiency theorem of Arrow and Kurz will be used throughout this study and it is of more relevance when λ_t is not constant and when I_t has an 'optimal' time path (see chapter III).[7]

From the necessary conditions in (2.25), the marginal condition of capital can be solved as

$$(2.27)\quad f'(K^*) = \frac{q}{(1-u)}[(1-uz)(\rho+\delta)+s((1-u)r-\rho)] = c$$

It should be noted that since q_t is assumed constant, $\dot{\lambda} = 0$ and K* is the equilibrium value of the capital stock. The user cost of capital (c) is now equal to $f'(K^*)$. Using the relationship between f(K) and F(K,L) it then follows that $F_K = \frac{c}{p} = \frac{f'(K^*)}{p}$.

The variable pF_K/q can be defined as the gross rate of return before taxes on investment, and it gives the minimum gross rate of return that a firm can afford to earn on new investment, leaving stockholders no worse off, i.e. pF_K/q is the gross cost of capital (see also Appendix I for definitions of the real user cost).

If the prices of capital goods are assumed to change over time (i.e. $g = \dot{q}/q > 0$) and assuming furthermore that a proportion x ($0 < x < 1$) of 'capital gains' is regarded as taxable income (i.e. xgqK is added to taxable income in (2.14)), then the following formula for the user cost can be solved

$$(2.28)\quad c = \frac{q}{(1-u)}[(1-uz)(\rho+\delta-g)+s((1-u)r-\rho)+uxg]$$

From formulas (2.27) and (2.28) for the user cost, it can be seen that the standard Jorgensonian formula will follow if $s = 0$ (and $x = 0$); see equation (2.3). Hence it can be argued that the Jorgenson approach is based on the assumption that firms use only equity capital.[8]

Next, we consider some transformations of (2.28), and also present some empirical values of the real cost of capital with parameter values typical of Finnish manufacturing. We shall then go on to consider the basic comparative statics properties of our model. In the next subsection (2.2.4), the neutrality issue of the corporate tax system is discussed.

In order to examine the cost of financial capital (i.e. the cost of capital) which is incorporated in the above concepts of user cost, we use the definition of z given by equation (2.21). After some algebraic manipulations the following formula for c is obtained

$$(2.29)\quad c = q[\delta-g+sr+\frac{\rho}{(1-u)}(1-s-\frac{u(\alpha-\delta)}{(\rho+\alpha)})-\frac{ug}{(1-u)}(1-x-\frac{\alpha}{\rho+\alpha})]$$

The term inside the square brackets can be viewed from two different angles. If the Samuelson definition of true economic depreciation is used, it is equivalent to $q(\delta-g)$ (see note 6).[9] In this case all the other terms in the square brackets represent the real cost of capital. If, on the other hand, true economic depreciation is defined to exclude the change in the value of an asset due to inflation, then all the other terms inside the square brackets except δ represent the real cost of capital.[10] Both of these definitions of true economic depreciation will be used in section (2.2.4) when considering the neutrality issue.

The nominal cost of capital in (2.29) is given by

$$(2.30)\quad cc_n = sr + \frac{\rho}{(1-u)}[1-s-\frac{u(\alpha-\delta)}{\rho+\alpha}]$$

where s is the proportion of a firm's investment financed by debt and $(1-s-u(\alpha-\delta)/(\rho+\alpha))$ is the proportion financed by internal funds. When $\alpha > \delta$, the firm is allowed to defer taxes through 'accelerated depreciation'. The weight of internal equity capital implies that a third component of investment, $u(\alpha-\delta)/(\rho+\alpha)$, is financed by deferred taxes, thus ensuring that the weights add up to one. The cost of this third component ('tax credit') is zero and consequently it does not show up in this formula (see also Bergström and Södersten, 1983). In effect, (2.30) gives a formula for the 'weighted average concept' of the nominal cost of capital, which is also used in our empirical analysis (see chapter V and Appendix I).

According to the second definition of true depreciation given above (i.e. $q_t\delta$), the effect of the rate of inflation on the user cost is captured via the following term:

$$(2.31) \quad -\frac{g}{(1-u)}[1-u(z+x)], \quad \text{where } z = \frac{\alpha}{\rho+\alpha}$$

The different ways through which inflation affects the user cost of capital and hence the long-run demand for capital can be deduced from (2.31):

i) Inflation reduces the user cost because nominal interest payments are deductible. This real interest rate effect is captured by the term $-g/(1-u)$. (We shall have more to say about this effect in section 2.3 where the real amortization on loans is taken into account).

ii) When depreciation allowances are based on historical costs under corporate tax laws, inflation reduces their real value and therefore the user cost is increased. This effect is captured by the term $ugz/(1-u)$ in (2.31).

iii) When nominal capital gains are taxed $(0 < x \leq 1)$, inflation increases the user cost and this effect is reflected in the term $ugx/(1-u)$ in (2.31).

The net outcome of these effects on the user cost of capital is ambiguous (i.e. an empirical issue) but it will be seen that for 'reasonable' values of parameters the net effect is to lower capital costs (see below).

Before presenting some empirical estimates of the real cost of capital, it is worth emphasizing here three neglected considerations in the foregoing analysis. First, we have not taken into account the effects of personal taxes which more naturally arise in the context of new equity issues (see Bergström and Södersten, 1983 and chapter IV of this study). Second, it has been assumed that inflation does not influence the costs of equity (ρ) and debt (r), but that it affects the debt-capital ratio (see section 2.2.2). Feldstein and various coauthors (Feldstein, 1976, Feldstein, Green and Sheshinski, 1978) have examined the effects of inflation and tax factors in a general equilibrium framework where interest rates depend on the debt-equity ratio and also react in a Fisherian tradition to inflation. Depreciation is, however, not taken into account in this analysis. Bergström and Södersten (1983), using a more partial model, examine the effects of inflation on capital cost by assuming that real interest rates remain constant during inflationary periods.[11] Third, it has been assumed that only prices of investment goods change ($g > 0$) while p and w stay constant, hence implying that relative prices are changing (see note 2). The same results would follow even if all prices are assumed to change at the same rate (expected relative prices are constant), in which case g would then measure the rate of change in the general price level.[12]

We next present some empirical estimates of the real cost of capital (cc_r) in (2.29) which serve to illustrate the order of magnitude of cc_r with parameter values typical of Finnish manufacturing. Note that cc_r is obtained by adding the inflation component given by (2.31) to cc_n in (2.30). Table 1 shows the values of cc_r at a 'low' and 'high' rate of inflation and assuming alternative values for the tax depreciation rate (α) and for the proportion of taxable capital gains (x).

TABLE 1. Real Cost of Capital (cc_r) with Alternative Values of Parameters

	Rate of Inflation 'Low' ($g = 0.05$)	Rate of Inflation 'High' ($g = 0.12$)
Taxless case	0.070	0.0
$x = 0$, $\alpha = 0.2$	0.051	-0.064
$x = 0.1$, $\alpha = 0.2$	0.058	-0.046
$x = 0$, $\alpha = 0.3$	0.025	-0.080
$x = 0.1$, $\alpha = 0.3$	0.033	-0.062
$x = 0$, $z = 1$	-0.065	-0.135
$x = 0.1$, $z = 1$	-0.058	-0.117

Notes: $s = 0.6$, $r = 0.1$, $\rho = 0.15$, $u = 0.6$ and $\delta = 0.08$; in the taxless case $cc_r = sr + (1-s)\rho - g$
$\alpha = 0.2$ corresponds to $z = 0.571$
$\alpha = 0.3$ corresponds to $z = 0.666$
$z = 1$ implies 'free depreciation'.
See note 13.

It can be seen that in all the cases considered (different assumptions concerning taxation), a 'higher' rate of inflation leads to a considerably lower capital cost. It is also interesting to observe that at a 'low' rate of inflation (about 4 per cent) the real cost of capital is, in the tax case ($x = 0.1, \alpha = 0.2$), equal to the value of cc_r in the taxless case. This implies that at a fairly low rate of inflation the Finnish corporate tax system would be neutral (see also Ylä-Liedenpohja, 1983). It can also be deduced from these results that, given the rate of inflation, an increase in tax depreciation charges (i.e. z increases) will reduce the real cost of capital, although two opposite (in sign) effects are connected with a rise in z. First, since tax depreciation is based on historic cost, the cost of capital is increased through the term $ugz/(1-u)$ in (2.31). Second, the proportion of 'tax credits' which carry a zero cost is increased, hence implying that the weight of internal funds $(1-s-u(\alpha-\delta)/(\rho+\alpha))$ is reduced, and, if it is assumed that $\rho > (1-u)r$, that the capital cost is lowered. With our chosen parameter values, the net effect of a rise in z is hence to reduce the real cost of capital.

In sum, the results presented in table 1 indicate that, with 'reasonable' parameter values and at a 'high' rate of inflation (above 4 per cent), the joint effect of the corporate tax system and the rate of inflation is to reduce capital costs from the level obtained in the taxless case (see also the Appendices of this study).[14] It should be mentioned that a similar result is also obtained by Bergström and Södersten (1983) with rather similar parameter values (the Swedish case) but taking into account personal taxes and, furthermore, assuming that real rates of interest are constant.

Table 2 shows the 'critical' values of the rate of inflation which give a zero value for the real cost of capital (i.e. $cc_r = 0$) with alternative values for the debt ratio (s) and for the tax depreciation coefficient (α). Other parameter values are the same as in table 1.

TABLE 2. The Value of the Rate of Inflation (per cent) Which Gives a Zero Real Cost of Capital ($cc_r = 0$, a Higher Rate of Inflation Gives a Negative cc_r)

Debt-capital Ratio (s)	Tax Depreciation Coefficient $\alpha = 0.2$ ($z = 0.571$)	$\alpha = 0.3$ ($z = 0.666$)	Tax neutral case
s = 0.2	14.8	13.9	
s = 0.4	11.4	10.3	
s = 0.6	8.1	6.7	
s = 0.8	4.7	3.0	
Tax neutral case (s = 0.6)			15.0

Notes: $r = 0.1$, $\rho = 0.15$, $u = 0.6$, $\delta = 0.08$, see also table 1.

From table 2 it can be seen that the 'critical' rate of inflation is lowered as the debt-capital ratio and/or the tax depreciation coefficient is increased. At all levels of the debt ratio, the 'critical' rate of inflation is reduced when tax depreciation charges are accelerated, hence implying that the net effect of accelerated depreciation rules is to reduce the cost of capital despite the two opposing forces.

These empirical observations can be generalized by analyzing more formally the comparative static properties of the model. Table 3 presents the results.

TABLE 3. Long-run Effect of an Increase in Parameters and Variables on the Demand for Capital (i.e. on K*)

parameter/ variable	sign of the effect
q	-
p	+
w	±
g	±
δ	-
r	-
z	+
α	+
ρ	±
s	±
u	±

Note: ± means that the sign of the effect is indeterminate.

Table 3 shows that the signs of the steady-state effects of permanent increases in price levels are of the 'standard' form. The effects of a rise in the interest rate on debt (r), in the economic depreciation coefficient (δ) and in the tax depreciation charges (α) are unambiguous and of expected sign. All the other effects are indeterminate in sign and we shall now consider them in more detail.

The effect of an increase in the discount rate (ρ) will be negative if $uz^2 + s < 1$. With 'reasonable' parameter values (see table 1), this inequality is likely to hold and hence a rise in the discount rate would reduce the long-run demand for capital. It should be emphasized that this model is not very appropriate for studying the effect of discount rate changes, because with the debt-capital ratio fixed, no substitution can take place between debt and equity capital (see section 4.2 where it is assumed that $r = r(e)$, e = debt/equity ratio).

An increase in the debt-ratio (s) will increase the demand for capital if $\rho > (1-u)r$, which is likely to hold under 'normal' conditions. At 'low' values of the discount rate, firms would not invest at all but would accumulate financial capital instead (see e.g. Hochman, Hochman and Razin, 1973).

The effect of inflation on the real cost of financial capital was considered above in the light of some empirical parameter values. More generally, the effect of a rise in the rate of inflation is to increase the demand for capital if $uz + ux < 1$, which is most likely to hold 'in reality' because usually $x \approx 0$ and u,z are between zero and one ($z = 1$ when there is 'free depreciation'). It can hence be concluded that only with a 'high' rate of capital gains taxation is the impact of accelerating inflation such as to reduce the demand for capital.

The last indeterminate sign in table 3 is that relating to the corporate income tax rate (u). The effect of a change in the tax rate will be discussed in the next subsection (2.2.4) where neutrality of the corporate tax system is also discussed.

2.2.4 Neutrality of the Corporate Tax System

Using formula (2.28), the following inequalities can be derived:

$$(2.32) \quad \frac{\partial K^*}{\partial u} \gtrless 0 \text{ if } z \gtrless \frac{\rho + \delta - (1-x)g - s\rho}{\rho + \delta - g}$$

The two 'standard' neutrality conditions in the case of a zero rate of inflation ($g = 0$) are:

(i) $s = 0$, and $z = 1$

These conditions imply that there is 'free depreciation' and no allowance for interest cost deductions in taxation.[15]

(ii) $s = 1$, and $z = \frac{\delta}{\rho + \delta}$

These conditions mean that there is full imputed interest deductibility and true economic depreciation for tax purposes (i.e. $\alpha = \delta$).

Tax neutrality in the case of a nonzero rate of inflation (i.e. $g > 0$) can be analyzed in the following way. The first neutrality result (i.e. $s = 0$, $z = 1$) is directly obtained (assuming $x = 0$, which is natural since $z = 1$). The second neutrality condition is somewhat more complicated and it can be expressed in three alternative ways (note that if $s = 1$, then $\rho = r$).

First, if true economic depreciation is defined as replacement cost depreciation minus the nominal capital gain which accrues on fixed assets, then $\alpha = \delta - g$, and neutrality will follow if all imputed nominal interest payments are tax deductible ($s = 1$). In this case accrued nominal capital gains are not directly included in the tax base, i.e. $x = 0$. In sum, if $s = 1$, $x = 0$, then neutrality follows from (2.32) with $z = (\delta - g)/(\rho + \delta - g)$ implying that $\alpha = \delta - g$.

Second, if all accrued nominal capital gains are directly included in the tax base (i.e. $x = 1$) and nominal interest payments are tax deductible (i.e. $s = 1$), then neutrality follows if z is given by

$$(2.33) \quad z = \int_0^\infty e^{-\rho t} \delta e^{-(\delta - g)t} dt = \frac{\delta}{\rho + \delta - g} = \int_0^\infty e^{-(\rho - g)t} \delta e^{-\delta t} dt$$

These results mean that a nondistortionary tax system follows if there is replacement cost depreciation plus deductibility of real interest payments, or, equivalently, deduction of nominal interest costs and the inclusion in the taxable income of nominal capital gains (i.e. the decrease in the real value of the firm's debt, see also section 2.3, and King 1977).

Third, if prices of output and investment goods change at different rates (i.e. $\bar{g} = \dot{p}/p$, $g = \dot{q}/q$, $\bar{g} \neq g$), then neutrality will follow if real interest payments measured in output prices ($r - \bar{g}$) are tax deductible and the true economic depreciation is defined as $q[\delta - (g - \bar{g})]$, or as $q[\delta - \frac{(\dot{q/p})}{(q/p)}]$, i.e. true economic depreciation is replacement cost depreciation minus the real capital gain accruing on the asset (see note 12).

In sum, it can be noted that full imputed interest deductibility and true economic depreciation alone will not give a neutral tax system. A critical role is also played by nominal accrued capital gains on the stock of capital (see also King, 1977).

The arguments with respect to neutrality can be generalized by applying the present value method more extensively. We shall proceed along similar lines as Boadway (1980), but the formulation is done in terms of our model framework. Assuming for simplicity that $r = \rho$ facilitates the definition of the present value of terms. We shall employ the following present value concepts for a unit of investment: (i) z is as before the present value of tax depreciation charges, (ii) y is the present value of interest deductions and (iii) v is the present value of nominal accrued capital gains on capital included as taxable income. Our basic definitions for these three present value concepts will be:

$$(2.34)\ (i) \qquad z = \int_0^\infty e^{-rt}\alpha e^{-\alpha t}dt = \frac{\alpha}{r+\alpha}$$

$$(ii) \qquad y = \int_0^\infty e^{-rt}sre^{-(\delta-g)t}dt = \frac{sr}{r+\delta-g}$$

$$(iii) \qquad v = \int_0^\infty e^{-rt}xge^{-(\delta-g)t}dt = \frac{xg}{r+\delta-g}$$

It should be noted that in (2.34) z is evaluated at historic (original) cost but y and v are evaluated at replacement cost, i.e. $B = sqK$ and $xgqK$ is taxable capital gains.

If we for simplicity exclude cash-flow effects associated with the debt financing of investment (i.e. $\dot{B}$ and rB), then the following simple maximization problem results[16]

$$\max_{\{I\}} \int_0^\infty e^{-rt}[(1-u)f(K)-(1-uz-uy+uv)qI]dt$$

which is solved subject to $\dot{K} = I - \delta K$ (see section 2.2.3). The following formula for the user cost can be obtained

$$(2.35) \quad c = \frac{q}{(1-u)}[(r+\delta-g)(1-uz-uy+uv)]$$

This new concept of the user cost will be equal to the previous one, given by e.g. (2.28), if $r = \rho$ and $x = v = 0$. The inclusion of $\dot{B}$ and rB with the assumption that $B = sqK$ in the basic cash flow identity of the firm (see eq. 2.7) would change the formula for user cost to $c = q[(\rho+\delta-g)(1-uz-uy+uv)+s((1-u)r-\rho)]/(1-u)$ and lead to corresponding changes in the discount rate in (2.34) but it would not change the general neutrality condition obtained from (2.35), that is

$$(2.36) \quad z + y = 1 + v$$

This result is a generalization of that given in Boadway (1980), where it was found that $y + z = 1$. In (2.36) the present value of taxable capital gains is also formally included in the neutrality condition. It should be emphasized that condition (2.36) is very general in that it is expressed in present value terms of various tax effects and it is hence not necessarily restricted to our basic present value concepts given in (2.34). For example, the tax depreciation system may, instead of the declining balance method in (2.34i), be a straight-line depreciation, the sum-of-the-years'-digit scheme etc., with, however, a corresponding change in the formula for z. It can generally be stated that equation (2.36) defines - for any given rate of interest (r), rate of inflation (g) and depreciation system - a relationship between the parameters s and x which gives a neutral tax system (in this model s is that portion of the full imputed interest cost on total capital which is deductible and x is that portion of the accrued nominal capital gain, $\dot{q}K$, which is included in taxable income).

The two standard neutrality conditions discussed above can easily be obtained from (2.36) using (2.34). The first case with $z = 1$ and $y = 0$ directly follows from (2.36), i.e. free depreciation and no interest deductibility (plus $v = 0$). The case related to true economic depreciation is more complicated, however. The three possibilities for neutrality are:

(i) $z = \frac{\delta - g}{r+\delta-g}$, $y = \frac{r}{r+\delta-g}$ and $v = 0$

In this case true economic depreciation is replacement cost depreciation minus the nominal capital gain which accrues on capital (i.e. $\alpha = \delta - g$), nominal interest payments are deductible and nominal capital gains are not included in the tax base.

(ii) $z = \frac{\delta}{r+\delta-g}$, $y = \frac{r}{r+\delta-g}$ and $v = \frac{g}{r+\delta-g}$

In this case true economic depreciation is at replacement cost, nominal interest payments are deductible and all accrued nominal capital gains are included in taxable income.

(iii) $z = \frac{\delta}{r+\delta-g}$, $y = \frac{r-g}{r+\delta-g}$ and $v = 0$

In this case true economic depreciation is at replacement cost, real interest payments are deductible and nominal capital gains are not included in the tax base.

Finally, using the inequalities in (2.32) or the equivalent present value formulations, it can be deduced that, the greater is z and/or the higher is s (debt ratio), the more likely it is that a rise in the corporate tax rate will increase demand for capital. Using the typical Finnish parameter values (see table 1, $\alpha = 0.2$, $x \approx 0$), it can be observed that at a 'low' rate of inflation the Finnish corporate tax system is neutral. At a 'high' rate of inflation it favours demand for capital (by lowering the capital cost), and a rise in the tax rate (u) will induce more demand for capital (because of almost untaxable capital gains arising from the decrease in the real value of debt).

2.3 User Cost with a Marginal Borrowing Ratio

In order to analyze further how inflation affects capital costs, we shall use a model with a constant marginal debt ratio instead of the average debt ratio. Otherwise this 'new model' is the same as in the previous section. This new financial market assumption also integrates debt repayments (amortizations) into the model and hence the impact of inflation on the real value of debt repayments shows up.[17]

The general equation of motion for debt capital is as follows:

(2.37) $\dot{B} = N - \gamma B$

where N is new loans, and γ is the rate of amortization (i.e. γ is the inverse of the average debt repayment period).

A capital market imperfection is now assumed to be characterized by the following relationship

(2.38) $N = mqI$

which implies that a constant fraction of gross investment is financed by raising new loans (see the discussion in section 2.2.2).[18]

In the following analysis we make use of three stock variables (K_H, K, B) and of the corresponding equations of motion (2.18i, 2.18ii and 2.37). The formulas for K_H and K were given by equations (2.16) and 2.17), and the equivalent equation for B is

$$(2.39) \quad B_t = \int_{-\infty}^{t} e^{-\gamma(t-v)} mq_v I_v dv$$

Taking the time derivative of B_t yields the equation of motion for debt as $\dot{B} = mqI - \gamma B$. It can easily be shown that the constancy relationship ($B = sqK$) of our previous model will follow if $m = s$ and if the rate of amortization (γ) is equal to δ-g. If the rate of amortization is not adjusted for inflation (rate of capital gains), then the average debt-capital ratio would be decreasing over time. If the rate of amortization (δ-g) is negative, then a firm borrows on its appreciated capital stock (in excess of the need for gross borrowing in order to finance investment; see below for the rationality of this assumption).

The dividend expression of the firm can now be presented as (compare (2.15))

$$(2.40) \quad Div = (1-u)f(K) - (1-m)qI - ((1-u)r+\gamma)B + uD - uxgqK$$

where nominal capital gains ($xgqK$) are also included. Total cash flow out of the firm due to debt finance is equal to $((1-u)r+\gamma)B$. In order to simplify the subsequent analysis, we again use the present value method and define the following two present values

$$(2.41) \quad \text{i)} \quad y_1 = \int_0^{\infty} e^{-rt} rme^{-\gamma t} dt = \frac{mr}{r+\gamma}$$

$$\text{ii)} \quad y_2 = \int_0^{\infty} e^{-rt} \gamma me^{-\gamma t} dt = \frac{m\gamma}{r+\gamma}$$

where it is assumed that $\rho = r$ (see the generalization below, eq. (2.45)). One unit of investment leads to m units of new debt, and when the amortization rate is γ the interest payment at t is $rme^{-\gamma t}$ (on investment acquired at zero time). Hence the present value of future interest costs on a unit of new investment is given by y_1. Similarly, the present value of debt amortizations stemming from one unit of investment is determined by y_2.

The maximization problem of the firm can now be formulated as (in the case where $r = \rho$)

$$(2.42) \quad \max_{\{I\}} \int_0^{\infty} e^{-rt}[(1-u)f(K)-(1-m)qI-(1-u)y_1qI-y_2qI+uzqI-uxgqK]dt$$

which is maximized subject to $\dot{K} = I - \delta K$ (for necessary and sufficient conditions, see section 2.2.3).

Solving the above control problem yields the following formula for the user cost of capital

$$(2.43) \quad c = \frac{q}{(1-u)}[(r+\delta-g)(1-uz-uy_1)+uxg]$$

where $z = \alpha/(r+\alpha)$. From (2.43) it can be seen that, in addition to the usual parameters (u, q, r, δ, g, z and x), the user cost of capital now also depends on the marginal borrowing ratio (m) and the amortization coefficient (γ).[19] It is interesting to compare the concept of user cost in (2.43) with that given by the standard Jorgenson formula (2.3), (assuming $x = 0$ and $r = \rho$). It can be seen that the user cost is lower in the former than in the latter case and the difference is due to the present value of tax savings stemming from interest deductions. If debt financing is ignored (i.e. $m = 0$ implying that $y_1 = 0$), then these two concepts of user cost are the same.

The 'general' result for the user cost in the case where the discount rate and the interest rate on debt are unequal (i.e. $\rho \neq r$) is as follows:

$$(2.44) \quad c = \frac{q}{(1-u)}[(\rho+\delta-g)(1-uz+(1-u)y_1-\frac{m\rho}{\rho+\gamma})+uxg]$$

where $y_1 = mr/(\rho+\gamma)$.[20]

Using this formula for c, it can easily be shown that if the rate of debt amortization equalled δ instead of δ-g, then an additional term would be added to the 'standard' concepts of user cost derived in the previous section. This extra term is $gm(\rho-(1-u)r)/(\rho+\delta)$, which implies that if the debt-capital ratio is decreased because of inflation, then the user cost of capital is increased when $\rho > (1-u)r$, which is likely to hold under 'normal' circumstances (see table 1). Because the profit-maximizing firm should aim at a minimum cost of capital, this result means that it is 'optimal' for such a firm to increase debt due specifically to inflation (and not only to finance investment) and thus acquire real capital gains (because of the decline in real value of debt).

The neutrality conditions of taxation can easily be deduced from the two user cost concepts above. From (2.43) it can be seen directly that neutrality is obtained when $y_1 + z = 1$ and $x = 0$. The two 'special' cases, i.e. i) with 'free depreciation' and no interest cost deductions, and ii) with full imputed interest deductions ($m = 1$) and true economic depreciation for tax purposes plus assumptions concerning the taxation of nominal capital gains, also follow from (2.43). The first 'special' neutrality case also follows directly from (2.44), i.e. $z = 1$, $y_1 = 0$, implying that $m = 0$ and that $x = 0$. The second 'special' case is obtained if $z = (\delta-g)/(\rho+\delta-g)$, $m = 1$ and $x = 0$. Notice that this definition of z implies that $\gamma = \delta-g$.

The comparative static properties of this model are broadly similar to those of the previous model (section 2.2) and hence we shall present results only for the 'new' parameters (m and γ) and for the rate of inflation (g), which is of special interest in this connection. Using (2.44) and the definition of y_1 (in the case where $\rho \neq r$), the partial derivative of c with respect to the marginal borrowing ratio (m) is

$$(2.45)\quad \frac{\partial c}{\partial m} = \frac{q}{(1-u)}((1-u)r-\rho)\left(\frac{\rho+\delta-g}{\rho+\gamma}\right)$$

When $\gamma = \delta-g$, the result is the same as in section (2.2.3), i.e. that an increase in the debt-ratio will reduce the user cost if $\rho > (1-u)r$. This result will also hold in the case of (2.45), unless $g > \rho+\delta$. If $(1-u)r > \rho$, then a rise in the borrowing ratio will reduce the user cost and hence increase demand for capital if $g > \rho+\delta$. This last result means that even if the after-tax interest rate is greater than the discount rate, it would be profitable to increase borrowing when the rate of inflation is 'high' (in terms of the parameter values presented in table 1, this 'high' rate of inflation would be 23 per cent). This is due to the 'large' capital gain accruing to the firm in periods of a 'high' rate of inflation in terms of a decrease in the real value of debt.

The partial derivative of the user cost with respect to the rate of amortization is given by

$$(2.46)\quad \frac{\partial c}{\partial \gamma} = \frac{q}{(1-u)}\left[\frac{-m(\rho+\delta-g)((1-u)r-\rho)}{(\rho+\gamma)^2}\right]$$

The sign of the effect of a rise in the rate of amortization (i.e. a shortening in the debt repayment period) on the user cost of capital is generally indeterminate. Various 'special' cases can be delineated. For example, if $\rho > (1-u)r$, then $\partial c/\partial \gamma > 0$ if $g < \rho+\delta$, which might be labelled as a 'normal' case (see again the parameter values in table 1). Hence a shortening in the debt repayment period would 'normally' mean that the demand for capital is reduced.

Finally, we consider the effect of a rise in the rate of inflation on the user cost of capital. The relevant partial derivative is now

$$(2.47)\quad \frac{\partial c}{\partial g} = \frac{q}{(1-u)}\left[u(z+x)-1-(1-u)y_1+\frac{m\rho}{\rho+\gamma}\right]$$

Accelerating inflation (a rise in g) will reduce the user cost of capital if $u(z+x+y_1) - m(r-\rho)/(\rho+\gamma) < 1$. In the special case where $r = \rho$, the requirement is that $u(z+x+y_1) < 1$, which is likely to hold under 'reasonable' parameter values. Using the definition of y_1, the above inequality can also be expressed as $u(z+x) + m((1-u)r-\rho)/(\rho+\gamma) < 1$. If $\rho > (1-u)r$, then this inequality is most likely to hold under 'normal' conditions since the second term on the left-hand side is negative, ensuring that even if $u(z+x)$ is high an increase in the rate of inflation reduces the user cost of capital and induces more demand for capital.

In summary of the discussion in this section (2.3), it can be said that although the model presented here is very similar to that in the previous section (2.2), this 'new' model more clearly highlights the various channels through which inflation may tend to reduce capital costs. The decrease in the real value of debt repayments induced by accelerating inflation, and causing 'sizeable' capital gains to firms under 'normal' conditions, is of special significance in this respect.

2.4 Conclusions of Chapter II

The general aim of this chapter was to incorporate in the standard neoclassical investment model a different assumption concerning the financial capital market than the hypothesis of a perfect market. To achieve this goal it was assumed that firms face a constant debt-capital ratio or a constant marginal borrowing ratio. Other basic assumptions of the standard neoclassical model were kept intact.

The hypothesis that firms use both debt and equity (internal) capital greatly facilitates the analysis of the effects of tax rules, the rate of inflation and the financial structure on the long-run demand for capital. This kind of a model is more suitable than the SNC model for examining the complex joint effects of corporate tax factors and the (expected) rate of inflation on the user cost of capital. It was found that, under rather 'reasonable' parameter value (typical of Finnish manufacturing), an acceleration of inflation tends to reduce the user cost of capital, hence inducing greater demand for capital. Basically, this result is due to the fact that the 'real interest rate effect' or 'the decrease in the real value of debt repayments' is greater than the 'erosion' of the historic cost tax depreciation charges induced by a rising rate of inflation. In this respect it is also significant that nominal capital gains induced by inflation are 'almost' nontaxable.

Various formulas for the user cost of capital were derived in this chapter and they will be used in the empirical analysis of the investment behaviour of Finnish firms (chapter V). The standard neutrality conditions of the corporate tax system were also derived and discussed. Furthermore, we were able to derive a very general neutrality condition with respect to K* (i.e. $y + z - v = 1$), which gives the two standard neutrality results as a special case.

Notes to Chapter II

1. The formulas for the user cost of capital, which are our chief interest in chapter II, are unaffected by the choice of the demand regime for the firm's products. However, in chapter III it will be shown that in a convex adjustment cost framework the dynamics of investment is affected by the choice of the demand regime.

2. In a steady-state analysis it is customary to assume that parameters u, z and δ as well as the discount rate ρ are constant. Prices may be assumed to change over time and if they change at the same constant rate (g), i.e. if $w_t = w_v e^{g(t-v)}$, $p_t = p_v e^{g(t-v)}$ and $q_t = q_v e^{g(t-v)}$, there is an expected rate of inflation of 100 g per cent and all relative prices remain constant over time. In this case, the user cost of capital depends on the real interest rate $(\rho - g)$, which is defined in terms of the general rate of inflation. If, however, p and w are assumed to be constant but q_t changes over time, then the real interest (discount) rate is determined by the rate of change in capital goods prices, and relative prices are changing over time. This issue of constant versus changing relative prices is discussed further in section 2.3. Since the models to be developed in chapter II are basically myopic, the optimization problem of the firm is solved at each instant of time independently of the past or future prospects of the firm's profitability, and hence optimal decision rules depend only upon the current period's values of exogenous parameters and variables. Hence the analysis would be unaltered even if tax parameters u and z were assumed to be time-dependent. If, however, ρ and/or δ were not assumed to be constant, then a completely different and more complex model would follow (see e.g. Auerbach (1983) where an endogenous economic rate of depreciation is assumed).

   It is worth noting that if q_t is also constant, i.e. $g = 0$, then the optimal capital stock is also constant but, if $g \neq 0$, then this capital stock is changing over time on account of changing relative prices (see Takayama, 1974).

3. These criticisms of Jorgenson's model have been evaluated and/or developed further by, for example, the following authors:
   - i) Gould (1969), Gould and Waud (1973), Helliwell (1976), Brechling (1975), Nickell (1978), Abel (1981).
   - ii) King (1972), Brechling (1975), Nickell (1978).
   - iii) Sumner (1973), Bergström (1976), Södersten (1977, 1982), Boadway (1980), Summers (1981), Poterba and Summers (1983) and Ylä-Liedenpohja (1976, 1983); see also section 2.2.
   - iv) Gould (1968), Treadway (1969, 1970, 1974), Abel (1980 etc.) and many other writers in the adjustment cost literature; see Nickell (1978) and chapter III of this study.
   - v) Helliwell and Glorieux (1970), Birch and Siebert (1976), Nickell (1974, 1978), Schiantarelli (1983); see also chapter V.
   - vi) Bischoff (1971), King (1972), Ando-Modigliani-Rasche-Turnovsky (1974), Nickell (1978), Sarantis (1979), Abel (1982), Schiantarelli (1983); see also chapter V.
   - vii) King (1974, 1977), Stiglitz (1973), Auerbach (1979, 1983), Summers (1981), Poterba and Summers (1983), Ylä-Liedenpohja (1982, 1983); see chapter IV.

4. Time indices are usually omitted in the subsequent analysis if no confusion arises. The model is formulated in a continuous time framework and the time derivative of variables is denoted by a 'dot' over the respective variable, e.g. $\dot{B} = \frac{dB}{dt}$ etc.

   Financial assets and inventories are excluded from the balance sheet (2.6), although in the theoretical analysis we shall use qK as a measure of total capital. When calculating empirical values of the debt-capital ratio, it must, however, be assumed that these other assets are also included in the concept of total capital in order that the debt-ratio (s) makes sense, since otherwise it would be more than one (see Appendix V and Poterba and Summers, 1983, p. 154). It can be thought that in the theoretical analysis other assets are implictly assumed to be a fixed proportion of qK and hence that they do not affect the optimality results.

5. In this chapter we assume a 'classical' corporate tax system, implying that only the tax rate on undistributed profits (u) enters. A two-rate system, with different tax rates on distributed and undistributed profits, is considered in section 4.3.

6. If true economic depreciation is defined according to Samuelson (1964), then $d_v = \delta e^{-(\delta-g)(v-t)}$. To see this, consider capital at time t, $q_t K_t$. Taking the time derivative and dividing by $q_t K_t$ yields:

   $(q_t K_t)^{-1} \frac{d(q_t K_t)}{dt} = g + \dot{K}/K = g - \delta$, where $g = \dot{q}/q$.
   Solving this differential equation gives:
   $q_t K_t = q_0 K_0 e^{-(\delta-g)t}$. This simplifies to $e^{-(\delta-g)t}$ for an initial investment of one unit at time zero.

7. Since capital is costlessly and instantaneously adjustable in (2.23), we only get a steady-state (equilibrium) solution and the investment decision depends only on the current values of exogenous variables It suffices to assume here that firms hold subjectively certain expectations about exogenous variables. Optimization is used to derive decision rules which are valid as long as expectations are correct. When they change, the firm instantaneously solves the optimization problem again, using new initial values and new expected values. Effectively, this implies that the firm instantaneously jumps to the new equilibrium position. Hence in this model the investment decision is not really forward looking but rather myopic.

8. The earlier formulas of Jorgenson (1963, 1965) were of the following type:

   (2.28)' $c = q[(\frac{1-uv}{1-u})\delta + (\frac{1-uw}{1-u})r + (\frac{1-ux}{1-u})g]$

   where $v = \frac{D}{\delta qK} = \frac{\alpha qK}{\delta qK} = \frac{\alpha}{\delta}$, and $w = \frac{rB}{rqK}$, or $B = wqK$.

Hence, although a constant debt-capital ratio was actually assumed, only one interest rate enters the formula for the user cost. Furthermore, tax depreciation is assumed to be a constant fraction of economic depreciation at replacement cost, thus implying that tax depreciation is based on current cost, which is usually not the case in the real world. It is interesting to notice that Sandmo (1974) presents the following result:

$$(2.28)'' \quad c = q[r+\delta+\frac{u}{1-u}(\delta-\alpha)]$$

which is a special case of (2.28)' and follows if $v = \alpha/\delta$ and $w = 1$. Hence tax depreciation is based on the current replacement cost value of the capital stock and the firm is assumed to use only debt capital. In Sandmo's case, direct taxes are thus defined as $T = u(f(K)-rqK-\alpha qK)$.

We shall discuss some other formulas for the user cost in chapter IV (section 4.4), where we consider the implications of a binding dividend constraint.

9. According to Samuelson, true economic depreciation is the change in the nominal value of an asset (Samuelson, 1964). The value of one unit of investment at t is equal to $v_t = q_t e^{-\delta t}$, when q_t is the price of investment and investment is made at zero time. The change (decrease) in the value of this asset is hence $-dv/dt = q_t e^{-\delta t}(\delta-g)$ (see also note 6).

10. Formula (2.29) is equivalent to that of Ylä-Liedenpohja (1983) and Bergström and Södersten (1983) except that they both include personal taxes as well. Formula (2.29) is 'restricted' in the sense that tax depreciation is assumed to be of the geometric (exponential) form whereas formulas (2.27) and (2.28) are more general in this respect.

    Formula (2.29) can also be expressed in the following form

$$(2.29)' \quad c = q[\delta-g+sr+(1-s)\frac{\rho}{1-u} - \frac{u}{1-u}(z(\rho+\delta)-\delta)- \frac{ug}{1-u}(1-x-z)]$$

    (see also Appendix I).

11. It might be argued that the assumption of sticky nominal interest rates is a reasonable one in Finnish financial market conditions where the Central Bank controls interest rates on loans and where credit rationing is usually thought to be a permanent phenomenon. Hence real interest rates decline when the rate of inflation accelerates. It should, however, be noted that both the nominal interest rate on loans and the nominal discount rate (measured by the interest rate on government bonds) have increased in a trendwise manner in Finland since the middle of the 1960s (see Appendices). The Finnish financial market system is also undergoing rapid change at present reflecting the influence of market factors, but it still remains a 'mixed system' in which there exist both controlled and noncontrolled interest rates. The effect of inflation on interest rates is difficult to determine in such a system and recent

developments (1982 - 1984) have shown that nominal interest rates have risen even though the rate of inflation is decelerating. This is due to the fact that the Central Bank has pursued a policy which has increased interest rates on debt capital.

12. Neglecting the tax factors ($u = z = x = 0$), the formula for real user cost is $\frac{c}{p} = \frac{q}{p}(r - \dot{q}/q + \delta)$, where it is also assumed that $s = 0$. This can be rewritten as

$$\frac{c}{p} = \frac{q}{p}\left[r - \frac{\dot{p}}{p} + \left(\delta - \left(\frac{\dot{\overline{q/p}}}{q/p}\right)\right)\right]$$

where $\delta - \frac{\dot{\overline{q/p}}}{q/p}$ measures the real change in asset value and $r - \dot{p}/p$ is the real rate of interest. If relative prices are not changing, then the real user cost is $(q/p)(r - \dot{p}/p + \delta)$ and $\dot{p}/p = g$ in our notation. Notice that $(q/p)_t = q_0 e^{gt}/p_0 e^{gt}$, and hence the ratio (q/p) is not affected by the general rate of inflation (see note 2).

13. We have assumed in table 1 that the proportion of capital gains which is taxable is either zero or 0.1. These assumptions imply that the tax rate on capital gains is either zero or 0.06, the latter following since $u = 0.6$. According to the prevailing Finnish tax system, capital gains on fixed capital are tax-free after a ten-year holding period (stamp duty is levied on the value of purchase but we ignore it here) and hence our assumptions may be regarded as 'reasonable'. Shareholders can realize their accrued capital gains tax-free after a five-year holding period. It should be noted that in our model framework the nominal capital gain is basically due to the decline in the real value of debt, which is untaxed, while nominal interest payments on debt are deductible.

14. Another issue is whether firms can in reality use all the available tax deductions. We have assumed that they can, which implies that, even with 'maximum' deductions, the taxable profit would be positive (i.e. $T > 0$), which also guarantees that dividend payments can be made. If this assumption is not satisfied, then firms have unutilized allowances which form an "expense stock". Ylä-Liedenpohja (1983) has shown that in the "expense stock" case, firms behave as if they were operating in a taxless economy and hence the tax system is non-distortionary.

15. It should be noted that condition (i) is written in simplified form since essentially we are dealing here with the neutrality issue with respect to debt finance. In a more complete form taxes can be expressed as $T = u(f(K) - \beta rB - D)$, where $\beta = 0$ if interest payments are not tax deductible and $\beta = 1$ if interest payments are deductible. We now get that

$$\frac{\partial K^*}{\partial u} \gtrless 0 \quad \text{if} \quad z \gtrless \frac{\rho + \delta - (1-x)g - s\rho + sr(1-\beta)}{(\rho + \delta - g)}$$

Condition (i) follows when $\beta = 0$ and $r = \rho$ (plus $g = 0$). If $s = \beta = 1$, then condition (ii) clearly follows (with $g = 0$).

16. The maximization problem can also be solved in the following way. For a profit maximizing equilibrium the price of capital goods must equal the present value of the after-tax returns on investment. Thus

$$q_t = \int_t^{\infty} p_v R_v (1-u) e^{-(\rho+\delta)(v-t)} dv + q_t uy + q_t uz + q_t uv$$

where pR is the marginal before-tax return and the first term on the right-hand side is the present value of the net return to a unit of capital with no deductions taken into account. The other terms are the present values of tax savings due to future interest, depreciation deductions and taxable capital gains on a unit of investment (or q units). To determine the marginal product of capital (R), we differentiate with respect to t, which gives $R = c/p$, where c is given by (2.35).

17. Airaksinen (1979) considers a similar model but in a discrete time framework. The present-value approach used here is technically less cumbersome and the properties of the model can more easily (and more fully) be deduced in our framework. In Airaksinen, the neutrality properties are not discussed whereas our model is suitable for that purpose.

18. It can be argued that the rationale underlying eq. (2.38) is the same as that presented in section 2.2.2 for the assumption of an average debt-capital ratio. The assumption that $N = mqI$ could either stem from credit rationing considerations if banks are willing to finance only a certain (maximum) fraction of investments by granting loans or from an optimal financial policy in an uncertain case. In the latter case, if the amortization rate γ is exogenously given to the firm, then choosing optimal debt-capital ratio implies choosing an optimal value of m (see also Poterba and Summers 1983, p. 141).

19. Assuming that $N = mq\dot{K} = m(qI-\delta qK)$, the following formula for the user cost follows

$$(2.44)' \quad c = \frac{q}{(1-u)}[(1-uz)(r+\delta-g)+m((r+\delta-g)\frac{(-ur)}{(r+\gamma)}+\delta)+uxg]$$

Using this concept of c would not alter the qualitative implications of our model.

20. Alternatively (2.44) can be expressed as

$$c = \frac{q}{(1-u)}[(\rho+\delta-g)(1-uz)+m((1-u)r-\rho)(\frac{\rho+\delta-g}{\rho+\gamma})+uxg]$$

CHAPTER III

THE EFFECTS OF DEMAND CONDITIONS ON THE DYNAMICS OF INVESTMENT BEHAVIOUR

3.1 Introduction

In the preceding chapter we considered the effects of various exogenous factors (taxes, inflation, financial structure etc.) on the long-run (steady state) demand for capital. The model used for this purpose was the same as the standard neoclassical model except that the financial market assumption was more general. The purpose of this chapter is to analyze models which are even more 'general' than the previous ones. We shall retain the financial market assumption made in chapter II but relax assumptions 5 and 6 of the SNC model (see section 1.2).

First, it is assumed that the firm faces strictly convex (external) adjustment costs, i.e. it faces an increasing flow supply price of capital goods. Second, in the previous chapter the firm was assumed to face a horizontal demand curve (perfect output market) but now we shall systematically consider three forms of demand conditions: horizontal, vertical and downward-sloping curves.[1] Taken together, these new assumptions imply that the purpose is to study investment behaviour (and not only demand for capital) in the context of various market imperfections. We believe that these new assumptions might be more 'realistic' than the previous ones and that they will allow the analysis of the effects of some aspects of investment decisions (dynamics, expected demand) which could not be dealt with in the previous models.

The assumption of nonlinear adjustment costs means that the behaviour of firms can no longer be characterized by the myopic rule, according to which optimal decisions depend only upon the current period's values of exogenous variables. In a strictly convex adjustment cost framework, the essential feature of investment decisions is that they are inherently forward-looking since they depend upon expectations of future values of relevant variables. Adjustment costs provide one rationale for the importance of future events (for other rationales, see section 1.1). Since the environment in which the firm operates is assumed to be one of perfect certainty concerning the future, we are effectively dealing with 'perfect foresight' rational expectations models.

The standard adjustment cost literature has almost invariably assumed a perfect output and capital market (see, e.g., Gould 1968, Lucas 1967, Treadway 1969, 1974, Takayama 1974 etc.). Corporate tax factors have usually been excluded from this literature. Abel, however, has recently examined in a series of articles the dynamic effects of corporate tax policy on investment (see Abel 1980, 1981, 1982, also Summers 1981 and Poterba and Summers 1983). Abel assumes that the firm is financed entirely by equity capital so that interest payments do not enter taxable income. Moreover, the firm is usually assumed to operate in a perfect output market.

As regards alternative demand conditions, only a few studies in the adjustment cost field have considered the impact of an imperfect product market (fixed output or downward-sloping demand curve). Grossman (1972) considers basically the same issue as we do here but restricts the analysis to fixed price and fixed output cases. He also assumes a perfect capital market and excludes tax factors. Nickell (1978) considers the cases of horizontal and downward-sloping demand curves but is concerned with a somewhat different issue than in our case (Nickell does not consider explicitly formulas for the rate of adjustment, which are the basic issue dealt with here). Abel (1981b) examines the response of investment to changes in output and price factors when the firm is faced with a fixed output. However, he assumes a perfect capital market and does not consider explicitly formulas for the adjustment speed.

In sum, it can be stated that the basic difference between our approach and that of other authors is that we assume an imperfect capital market whereas they adopt the assumption of a perfect market, implying that the discount rate and the rate of interest on borrowing are equal. It will be seen that, even within a given demand regime, the dynamic properties of investment will change when one moves from a perfect capital market to an imperfect one. We also present a systematic treatment of the three demand regimes and consider the impact of an increase in anticipated future demand on the timing of investment.

The results of this chapter clearly show that the dynamic behaviour of investment is rather sensitive with respect to the following factors: i) the returns to scale assumption of the production function, ii) demand conditions, iii) the capital market assumption and iv) tax parameters. It can generally also be said that the dynamics of investment is endogenous and that Gould's result (constant speed of adjustment) does not generally hold. It is worth mentioning here that an attempt will be made in the empirical part of this study to test the effect of alternative demand regimes on the(endogenous) rate of adjustment.

Before presenting the outline of this chapter we shall briefly consider the implications of nonlinear adjustment costs. As mentioned above it is assumed here that firms face strictly convex adjustment costs when acquiring new investment goods. It is furthermore assumed that adjustment costs are a function of gross investment, i.e. $C(I)$ with $C' > 0$, $C'' > 0$ for $I > 0$, $C(0) = 0$ (see also Takayma 1974, Nickell 1978).

Many forms of adjustment costs have been suggested in the literature (see e.g., Lucas 1967, Treadway 1969, 1971, 1974, Uzawa 1969, Takayama 1974). As far as the general dynamic structure of the investment process is concerned, the implications of different strictly convex formulations are more or less the same since they all produce a slow adjustment path to the equilibrium level of the capital stock (see also Nickell 1978, chapter 3). The long-run (equilibrium) properties of various adjustment cost models can, however, be quite different. The assumption that adjustment costs are a function of gross investment implies that even in the case of a constant-returns-to-scale production function the firm will be of bounded size, which is not the case with net investment adjustment costs.

Although alternative adjustment cost models might produce different equilibrium results and in some cases also different dynamic properties, we have chosen to deal here only with external gross investment adjustment costs.[2] In the real world one might certainly expect that adjustment costs are a mixture of many different forms but since the general features of the adjustment path are the main issue here, it seems acceptable to us to restrict the analysis to one type of adjustment cost (see also Nickell 1978). The choice of gross investment costs can also be defended on the grounds that this type of cost represents a well-defined market phenomenon whereas some other types of adjustment costs are of a more 'unspecified' character.[3]

We can now summarize the advantages of using the adjustment cost approach over the previous SNC approach (chapter II). First, the adjustment cost approach leads directly to a well-defined investment function rather than only to a demand for capital function. Second, a firm that has no determinate factor demand functions in the SNC model when the production function is characterized by constant returns to scale may have determinate functions with adjustment costs. Third, adjustment costs provide a theoretical justification for the distributed lag models which are used extensively in empirical research in this field. Fourth, the myopia that characterizes investment decisions in the SNC model is no longer present and the firm's current investment policy depends on future events.

The outline of the present chapter is as follows: Section 2 develops the basic model and considers it under the assumption of a horizontal demand curve. Section 3 considers the case of a downward-sloping demand curve and section 4 the case of an excess supply (i.e. cost minimization model). In section 4 we also analyze the impact of anticipated and unanticipated increases in demand on investment behaviour. The last section presents the main conclusions of this chapter.

3.2 The Case of a Perfect Output Market

In this section the basic model to be analyzed in this chapter is developed. The model is the same as in the previous chapter except that the firm is now assumed to face strictly convex adjustment costs. Furthermore, it is assumed in this section that the firm faces a horizontal demand curve (i.e. perfect product market with p given). The production function is assumed to possess either decreasing or constant returns to scale and hence the profit function $f(K)$ has the properties $f' > 0$, $f'' \leq 0$. The case $f'' = 0$ corresponds to the assumption of constant returns to scale.

3.2.1 The Model and Necessary Conditions for an Optimum

It is assumed that (external) adjustment costs C(I) possess the properties described in section 3.1 (i.e. $C' > 0$, $C'' > 0$). Furthermore, it is assumed that all costs of investment defined by $q_t[I_t+C(I_t)]$ are capitalized in the (tax) depreciation value of the capital stock (i.e. the book value of the capital stock) and hence they are depreciated over time.[4] Alternatively, it could be assumed that adjustment costs qC(I) are immediately charged for tax purposes rather than amortized over time. This difference in the tax treatment of adjustment costs does not affect the basic character of the results obtained in this chapter (see also note 9).

The net cash flow identity (i.e. dividends) is now given by

$$(3.1)\quad Div_t = (1-u_t)[f(K_t)-rsq_tK_t] - q_t[I_t+C(I_t)] + u_t \int_{-\infty}^{t} D_{t-v}q_v(I_v+C(I_v))dv + sq_t(I_t-\delta K_t) + s\dot{q}K_t$$

where it is assumed that the debt ratio s is constant (the rate of amortization is hence δ-g, see chapter II) and D_{t-v} is the depreciation allowance per unit of investment acquired at time v. Thus, the integral in this formula (multiplied by u) is the tax bill saving at time t due to the tax depreciation allowance on all past investments.

The firm is again assumed to maximize the present value of its net cash flows and hence the objective of the firm is

$$(3.2)\quad \max_{\{I\}} \int_{0}^{\infty} e^{-\rho t}\{(1-u_t)[f(K_t)-rsq_tK_t] - q_t(I_t+C(I_t)] + u_t \int_{-\infty}^{t} D_{t-v}q_v(I_v+C(I_v))dv + sq_t(I_t-\delta K_t) + s\dot{q}K_t\}dt$$

subject to $\dot{K} = I - \delta K$ and a given initial stock of capital.

This maximization problem can be simplified (as in chapter II) by noting that the present value of current and future tax deductions attributable to past investments is independent of current and future decisions of the firm, and hence it can be ignored in the optimization.[5]

The present value of depreciation allowance accruing to a unit of new capital is defined in the usual way (see chapter II) as

$$(3.3) \qquad z = \int_0^\infty e^{-\rho t} D_t dt, \quad \text{and if } D_t = \alpha e^{-\alpha t}, \text{ then } z = \alpha/(\rho+\alpha)$$

The maximization problem in (3.2) can be expressed as

$$(3.4) \qquad \max_{\{I\}} \int_0^\infty e^{-\rho t}\{(1-u_t)[f(K_t)-rsq_tK_t] + sq_tI_t - sq_t(\delta-g)K_t$$
$$- q_t[1-u_t \int_0^\infty D_v e^{-\rho v} dv](I_t+C(I_t))\}dt$$

Solving the necessary conditions from the current value Hamiltonian implied by this maximization problem (with λ as the shadow price of capital) gives

$$(3.5) \qquad \text{i)} \quad \dot{\lambda}_t = (\rho+\delta)\lambda_t - (1-u_t)f'(K_t) + sq_t((1-u_t)r+\delta-g)$$

$$\text{ii)} \quad \lambda_t = q_t(1+C'(I_t))(1-u_t \int_0^\infty D_v e^{-\rho v} dv) - sq_t$$

According to ii), the optimal rate of investment is obtained by equating the net marginal cost of investment with the shadow price of capital (see section 3.1.2).

A solution to the first-order non-linear differential equation (3.5i) is (assuming $g = 0$)

$$(3.6) \qquad \lambda_t = \int_t^\infty [(1-u_v)(f'(K_v)-srq)-s\delta q]e^{-(\rho+\delta)(v-t)}dv$$

which is obtained from the general solution using the transversality condition.[6] The value of λ_t is the present value of the stream of after-tax marginal profits (rentals) accruing to the undepreciated portion of a unit of capital installed at time t minus the net cash flow out of the firm due to debt services. The debt services include both after-tax interest payments (1-u)srq and repayment of debt $q\delta s$, where δ is the rate of amortization (see chapter II). If it is assumed that q_t is time-dependent (i.e. $g = \dot{q}/q \neq 0$), the constancy of the debt-capital ratio s would actually require that the rate of amortization is equal to $(\delta - g)$. If, furthermore, a constant proportion of capital gains is assumed taxable, then (3.6) becomes

$$(3.7) \quad \lambda_t = \int_t^{\infty} [(1-u_v)f'(K_v) - sq_v((1-u)r+\delta-g+uxg)]e^{-(\rho+\delta)(v-t)}dv$$

where $sq((1-u)r+\delta-g+uxg)$ is the cash flow out of the firm due to debt services and capital gains taxes. In the subsequent analysis of this chapter, we shall usually ignore the terms sqg and -suxqg and hence consider the case where q_t is fixed.

Equations (3.5) and (3.7) hold along the optimal path. These equations are quite general in that they allow the profit function f(K) to be either linear or concave and to shift over time. Moreover, very few restrictions are placed on the tax parameters and the tax rate is allowed to change over time. The tax parameters must satisfy the condition $(1 - uz) > 0$ in order that the net cost of investment is positive for $I > 0$ (u_v and D_v now are assumed constant).[7]

Using the relationship between λ and I as given by (3.5ii), equation (3.7) gives an optimal decision rule for investment. However, this condition is not very revealing, because the right-hand side depends upon the complete time path of the capital stock (from t to the infinite future) and the left-hand side is a function of the current rate of investment (see below). Equation (3.7) with $C'(I_t)$ inserted in it also clearly shows the forward-looking character of investment decisions in a strictly convex adjustment cost framework.

3.2.2 Evaluation of the Model

We assume next that the tax rate (u_t), the present value of depreciation charges (z) and prices are fixed (constant). The necessary condition (3.5ii) can then be expressed as

$$(3.8) \quad \lambda_t = q[(1-uz)(1+C'(I_t))-s]$$

The optimal decision rule for investment is now given by

$$(3.9) \quad C'(I_t) = \frac{1}{q(1-uz)} \int_t^{\infty} e^{-(\rho+\delta)(v-t)}[(1-u)f'(K)-sq((1-u)r+\delta)]dv + \frac{s}{(1-uz)} - 1$$

Along the optimal path, the capital stock must satisfy the following relationship:

$$(3.10) \quad f'(K_t) = \frac{q}{(1-u)}[(\rho+\delta)(1-uz)(1+C'(I_t))+s((1-u)r-\rho)-(1-uz)\dot{I}C''(I_t)]$$

The stationary solution ($\dot{K} = 0$, $\dot{I} = 0$ and $\dot{\lambda} = 0$) gives the following formula for the user cost of capital (i.e. $f'(K^*) = c$)

$$(3.11) \quad c = \frac{q}{(1-u)}[(\rho+\delta)(1-uz)(1+C')+s((1-u)r-\rho)]$$

where $C' = C'(I^*)$ and $I^* = \delta K^*$. This formula for the user cost is equivalent to our previous concept (2.26) except that now adjustment costs affect the user cost. Formula (3.11) implies that even at the equilibrium point the firm incurs adjustment costs and hence the steady-state demand for capital is now lower than previously. If it were assumed that $C = C(\dot{K})$, then the marginal costs of adjustment would be zero (if $C(0) = 0$) at the equilibrium point.

Since C(I) is increasing monotonically, I_t can be solved from (3.8) as

$$(3.12)\quad I_t = C'^{-1}\{\frac{\lambda_t}{q(1-uz)} + \frac{s}{(1-uz)} - 1\}$$

This equation determines the optimal time rate of investment as a function of the shadow price of capital. If a perfect capital market were assumed (i.e. $\rho = r$ and $s = 0$), then $\bar{\lambda}_t$ defined as $\lambda_t/q(1-uz)$ would be a marginal Tobin's "q"-variable, since it is the ratio of the marginal shadow price of capital to the tax-adjusted replacement cost of new investment (note that λ is the marginal value of an additional unit of installed capital, see section 3.1.2). We would then have a more common investment function than in (3.12), i.e. $I = C'^{-1}\{\bar{\lambda}_t - 1\}$. It should be noticed that the relationship between investments in perfect and imperfect capital markets cannot be inferred directly from (3.12), since the debt ratio s affects investment in (3.12) both directly through the term $s/(1-uz)$ and indirectly through its effect on λ. However, in the case of a linearly homogeneous production function, a more revealing comparison can be made (see below).

Using (3.5i) and the equation of motion for the capital stock jointly with (3.12), gives the following system of two non-linear differential equations

$$(3.13)\quad \text{i)}\quad \dot{K}_t = I_t - \delta K_t = C'^{-1}(\lambda_t) - \delta K_t$$

$$\text{ii)}\quad \dot{\lambda}_t = (\rho+\delta)\lambda_t - (1-u)f'(K_t) + sq((1-u)r+\delta)$$

where C'^{-1} is written in simplified form (see 3.12).

Since this system is autonomous, its behaviour can conveniently be analyzed in the (λ, K) space. The equation for the $(\dot{\lambda} = 0)$ curve can be solved from (3.13ii)

$$(3.14)\quad \frac{d\lambda}{dK}\bigg|_{\dot{\lambda}=0} = \frac{(1-u)f''(K)}{\rho + \delta} < 0$$

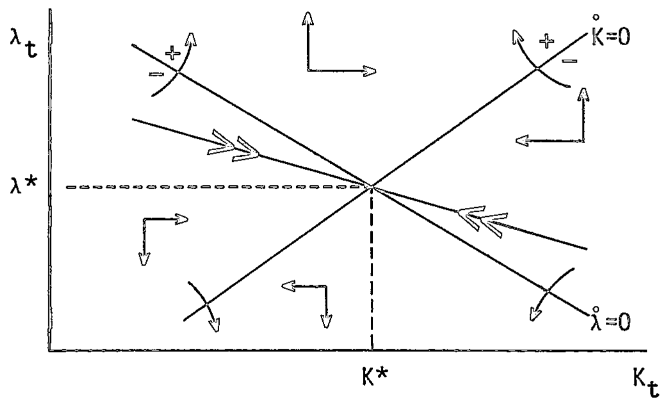
The ($\dot{\lambda}$ = 0) locus slopes downwards since the shadow price of capital along this schedule is equal to the present value of after-tax rentals. An increase in K will reduce the rental value because f"(K) < 0 (decreasing returns to scale assumption).

The equation for the ($\dot{K}$ = 0) curve can be solved from (3.13i). This gives

$$(3.15)\quad \frac{dK}{d\lambda}\bigg|_{\dot{K}=0} = \frac{1}{q\delta(1-uz)C''} > 0$$

and hence the ($\dot{K}$ = 0) locus slopes upward in the (K,λ) space. Figure 1 illustrates the behaviour of this system.

FIGURE 1.



From this figure it is obvious that the steady state is a saddle point. For any value of the capital stock, there exists a unique value of λ which will allow this system (λ,K) to reach a stationary point (λ*,K*). This unique path is described by the two trajectories ">>" and "<<" (all other paths lead to either an infinitely large capital stock or to a zero level of capital stock). At the equilibrium point (K*,λ*) investment is given by $I_t = I^* = \delta K^*$ (i.e. investment is only replacement investment). When $K_0 < K^*$ (the only case considered here), the basic dynamic properties of this system can be summarized as $dK_t/dt > 0$ and $dI_t/dt < 0$, which implies that investment is steadily declining along the optimal path (i.e. monotonic behaviour).

We shall next analyze more formally the dynamics and the saddle-point property of this model. Linearizing the differential equation system in (3.13) around its steady state gives

$$(3.16)\quad \dot{\lambda}_t = (\rho+\delta)(\lambda_t-\lambda^*) - (1-u)f''(K^*)(K_t- K^*)$$
$$\dot{K}_t = \frac{1}{q(1-uz)C''(I^*)}(\lambda_t-\lambda^*) - \delta(K_t-K^*)$$

This linearized system can be expressed in matrix form as

$$(3.17)\quad \begin{bmatrix} \dot{\lambda} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} \rho+\delta & -(1-u)f'' \\ \frac{1}{q(1-uz)C''} & -\delta \end{bmatrix} \begin{bmatrix} \lambda-\lambda^* \\ K-K^* \end{bmatrix}$$

The roots of the characteristic equation of this system are real and opposite in sign. Hence the stationary point is a saddle point as illustrated in figure 1. Since we impose the requirement that the system must approach its steady state, we must discard the positive root and the motion of K_t is governed entirely by the one negative root (say k_1). This gives

$$(3.18)\ (i)\quad K_t = K^* + e^{k_1 t}(K^*-K_0)$$

$$(ii)\quad \dot{K}_t = -k_1(K^* - K_t)$$

Equations (3.18) give the unique perfect foresight path to the equilibrium. In the spirit of rational expectations, it can also be said that, given the current level of the capital stock (and perfect foresight), there is a unique value of the shadow price of capital which will allow the system to reach its steady state.

Recalling that $-k_1$ is positive, we obtain the result that the capital stock moves towards K^* at a rate proportional to the gap $K^* - K_t$. This result is the linear approximation to the 'flexible accelerator' model as suggested in Gould (1968). The speed of adjustment is given as

$$(3.19)\quad -k_1 = -\frac{\rho}{2} + \sqrt{\frac{(\rho+2\delta)^2}{4} - \frac{(1-u)f''}{q(1-uz)C''}}$$

It can be seen that the rate of adjustment is endogenous since it depends on C" and f", and, furthermore, it is larger than the depreciation coefficient δ because the second term in the square root is positive ($C'' > 0$ and $f'' < 0$). The rate of adjustment also depends upon the tax parameters.

Before considering some comparative static and dynamic properties of this model, it is worth analyzing the model in the case of the constant-returns-to-scale production function. In this case $f'' = 0$ and the capital-labour ratio is uniquely determined by the real wage rate. The production function F(K,L) can be written as $L\bar{g}(\ell)$, where $\ell = K/L$. The marginal productivity rule $F_L = w/p$ may be replaced by $\bar{g} - \ell\, \partial\bar{g}/\partial\ell = w/p$ (for all t). Since the marginal product of labour is increasing in the capital-labour ratio this equation may be solved to give $\ell = h(w/p)$, where $h' > 0$. The marginal product of capital can be written as $\partial g(\ell)/\partial\ell$, and, using $\ell = h(w/p)$, the optimal decision rule for investment will be as follows (compare 3.9):

$$(3.20)\quad C'(I_t) = \frac{1}{q_t(1-uz)} \int_t^{\infty} e^{-(\rho+\delta)(v-t)}\Big[(1-u)p_v \frac{\partial\bar{g}}{\partial\ell}\{h(\frac{w_v}{p_v})\} - sq_v((1-u)r+\delta-g)\Big]dv + \frac{s}{(1-uz)} - 1$$

It can be seen that (3.20) gives an optimal investment rule in which the level of investment is determined by the current and future prices (q,p,w). The important point to notice is that current investment decisions depend upon future price developments, implying that the investment decision has a forward-looking character.

Assuming that all prices are expected to remain constant, eq. (3.20) reduces to

$$(3.21) \quad C'(I_t) = \frac{(1-u)p\bar{g}_\ell(w/p) - sq((1-u)r-\rho)}{q(1-uz)(\rho+\delta)} - 1$$

This states that investment should be adjusted so as to make the discounted net marginal benefits from a greater capital stock equal to the marginal cost of investment. It should be noted that the marginal benefits are now larger (if $\rho > (1-u)r$) than in the case of a perfect capital market (i.e. $s = 0$, $r = \rho$) and hence the rate of investment is higher in the imperfect financial market than in a perfect market on account of tax benefits arising from the use of debt finance.

Within this model, however, a very different time path of investment follows as compared to the decreasing returns to scale model (when prices are assumed fixed). We now obtain the following investment function:

$$(3.22) \quad I_t = C'^{-1}\{\frac{(1-u)p\bar{g}_\ell(w/p) - sq((1-u)r-\rho)}{q(1-uz)(\rho+\delta)} - 1\} \equiv I^c$$

where I^c is constant. In the constant returns to scale model, the optimal rate of investment is constant over time (see also Gould, 1968 and Takayma, 1974).

The ($\dot{\lambda} = 0$) locus (in figure 1) is now horizontal since $\frac{d\lambda}{dK}\Big|_{\dot{\lambda}=0} = 0$. The characteristic roots of this linearized system are obtained from (3.17) setting $f'' = 0$, and these roots are $-\delta$ and $\rho+\delta$. The equation of motion of K_t is given by

$$(3.23) \quad \dot{K}_t = \delta(K^*-K_t)$$

It can be seen that the rate of adjustment is equal to the rate of depreciation. Thus it can be concluded that the original result by Gould (1968) also holds in our model including tax factors and a capital market imperfection (the debt ratio $s > 0$, and $r \neq \rho$). It can also be observed that the rate of adjustment is faster in the case of decreasing returns to scale than in the case of constant returns to scale. This issue of relative speeds of adjustment will be discussed further when we analyze the rate of adjustment under alternative demand regimes in the following sections. It should also be noted that, with a linearly homogeneous production function, the rate of adjustment is constant and independent of corporate tax parameters, although these affect the optimal value of gross investment.

3.2.3 Some Comparative Static and Dynamic Results

It should be emphasized that in a perfect foresight model all future shocks are anticipated, since the whole future is known with certainty. The only unexpected changes are surprise shocks that occur in the current (present) time period. Essentially, the distinction between unexpected and expected shocks has as its analogue in the perfect foresight (rational expectations) framework in the distinction between current and future changes. In examining the comparative statics properties, one is basically analyzing the effects of unanticipated (permanent) exogenous changes from an initial position of steady-state equilibrium.

The comparative static properties with respect to K^* and λ^* can be determined by using the following equations

$$(3.24)\quad \text{i)}\quad \dot{\lambda} = 0 \Rightarrow (\rho+\delta)\lambda^* - (1-u)f'(K^*) + sq((1-u)r+\delta) = 0$$

$$\text{ii)}\quad \dot{K} = 0 \Rightarrow \lambda^* - q(1-uz)C'(\delta K^*) - q(1-uz-s) = 0$$

Since I_t is an increasing function of the shadow price of capital, the comparative static properties with respect to I^* can be obtained directly through λ^*. The Jacobian of this system is $J = -(\rho+\delta)q\delta(1-uz)C'' + (1-u)f''$ and hence $J < 0$, implying that the solution is locally unique (since $J \neq 0$).

Using the Jacobian and Cramer's rule, we can examine the comparative static properties, i.e. the partial derivatives of K^* and λ^* with respect to exogenous parameters and variables.

The comparative static results with respect to the optimal stationary level of the capital stock (K^*) are the same in terms of the signs of the effects of permanent increases in parameters and variables as those obtained in chapter II (see table 3) with the decreasing returns to scale assumption ($f'' < 0$). It should, however, be noted that, since the total cost of investment increases with gross investment, the present model leads to a lower optimal capital stock than the corresponding model in chapter II (since $C'(\delta K^*) > 0$, see eq. 3.11). The effect of an increase in the wage rate is generally indeterminate but 'normally' we would expect it to be negative.[8] The effects with respect to the optimal stationary level of investment ($I^* = \delta K^*$) are the same as those with respect to K^* except that the effect of an increase in the rate of depreciation (δ) is ambiguous. When δ increases this must necessarily reduce K^* but the stationary value of investment can either decrease or increase since $I^* = \delta K^*$.

Of special interest in this neoclassical framework is the effect of a (permanent) tax rate change on K^* and on I^*. The analogous result to that in eq. (2.32) is as follows

$$(3.25) \quad \frac{\partial K^*}{\partial u} \gtrless 0 \quad \text{as} \quad z \gtrless \frac{(\rho+\delta)(1+C') - s\rho}{(\rho+\delta)(1+C')}$$

If $C' = 0$, these inequalities reduce to those given in (2.32). The first neutrality condition with $s = 0$ and $z = 1$ (i.e. no interest deductibility and free depreciation) follows directly. The second neutrality condition with tax depreciation equal to economic (true) depreciation is again more complicated. At this point, we should remember that the total cost of investment is assumed to be $q(I+C(I))$ and that this cost is used as a basis for tax depreciation charges (see section 3.2.1). Hence, the marginal cost per unit of investment is $1+C'$. Effectively, we are assuming here that adjustment costs are also tax deductible. Hence the second neutrality condition is equal to $z = 1 - \rho/(\rho+\delta)(1+C')$, where $\rho/(\rho+\delta)(1+C')$ is the present value of imputed interest deductions per unit of capital.

The two standard neutrality conditions can also be directly obtained from (3.11). First, if $z = 1$ and $\rho = r$ with no interest deductibility, neutrality follows. Second, if $z = \delta/(\rho+\delta)$ and $\rho = (1-u)r$, neutrality again follows. The formula of the neutral user cost is given by $c = q(\rho+\delta)(1+C')$.

Comparative dynamic analysis of the effects of (temporary or permanent) changes in exogenous factors on the time path of investment is more cumbersome to carry out. In principle, this kind of analysis can be done assuming either static (stationary) or non-static (variable) expectations about relevant variables and parameters. The most difficult case to analyze is when expectations are not static (an example of comparative dynamic analysis when output price is time-dependent is presented in Gould (1968)).

When prices (and other exogenous factors) are assumed fixed (i.e. static expectations), comparative dynamic analysis can be performed using the relationship between λ and I as given by (3.12) or by (3.22) and noting that I is an increasing function of λ. In the case of constant returns to scale, this analysis is most straightforward since I and λ are constant. Using (3.22) it can easily be shown that the (constant) level of investment is directly (positively) related to the output price and to the depreciation rules (z) and inversely (negatively) related both to the wage rate and to the rate of interest (changes in p, z, w and r must then be understood to be permanent). The effect of an increase in the debt ratio (s) is positive if $\rho > (1-u)r$ and the effect of a change in the tax rate (u) is generally ambiguous, but the two standard neutrality conditions hold in the case of investment equation (3.22). In section 3.4 we shall consider the effects of both anticipated (future) and unanticipated (present) demand changes on the dynamic path of investment.

More generally the neutrality issue of the whole dynamic path of the present investment model can be considered as follows. The equation for the time rate of investment ($\dot{I} = dI/dt$) can be obtained from (3.10) as

$$\text{(3.26)} \qquad \dot{I} = \frac{q[(\rho+\delta)(1-uz)(1+C') + s((1-u)r-\rho)] - (1-u)f'(K)}{q(1-uz)C''}$$

With free depreciation ($z = 1$) and no interest deductibility ($s = 0$ or $r = \rho$, see note 15 in chapter II) neutrality with respect to the rate of investment follows, i.e. $\dot{I}$ is independent of the tax rate u. However, the case of true economic depreciation ($z = s/(\rho + \delta)$) with interest cost deductibility ($\rho = (1 - u)r$) does not directly lead to a nondistortionary situation (see also King 1977, p. 239). Within a simplified version of this model neutrality of $\dot{I}$ can, however, be obtained.

If we use the same approach as in section 2.2.4 (see eq. 2.35) and add the separable external adjustment costs qC(I) to the model (see section 3.2.1), the following equation results

$$(3.27) \quad \dot{I} = \frac{q(r+\delta)(1-uz-uy)(1+C') - (1-u)f'}{q(1-uz-uy)C''}$$

This equation for $\dot{I}$ is derived under the assumptions that $\rho = r$, the firm maximizes the present value of the total company (all assets) and that adjustment costs are depreciated over time as ordinary costs of investment (qI). It can be seen from (3.27) that the time rate of investment is independent of the tax rate u when z + y = 1 and hence neutrality also follows with true depreciation. Our original model incorporating the term $sq((1 - u)r - \rho)$ seems to give neutrality in the case of true depreciation only when the repayment of debt is also tax deductible.[9] Note 9 considers some other approaches used in the literature to achieve neutrality of the dynamic investment path in the case of true depreciation and interest deductibility (see Hartman 1978 and Abel 1983). Finally, it should be noted that the formula for the rate of adjustment (eq. 3.19) is independent of the tax rate u in the case of free depreciation (local neutrality property around K* with z = 1). Furthermore, if we use the same approach as in deriving equation (3.27), then the term q(1-uz)C" in the formula for the rate of adjustment (eq. 3.19) will be replaced by the term q(1-uz-uy)C", and hence the speed of adjustment is neutral when y + z = 1.

3.3 The Case of a Downward-Sloping Demand Curve

The importance of a downward-sloping demand regime for investment has been previously recognized by, among others, Gould and Waud (1973), Chang and Holt (1973), Picou and Waud (1973) and Nickell (1978).[10] Nickell is, however, the only one who considers this demand regime in an adjustment cost framework, and his analysis is theoretically orientated whereas the others estimate an empirical investment equation (see section 5.3). Our model differs from Nickell's model in two respects. First, we include both corporate tax factors and a capital market imperfection (debt-capital ratio). Second, we also consider explicitly the formula for the rate of adjustment and factors affecting it. Otherwise our model is the same as that in Nickell (1978). The main reason for considering the downward-sloping demand model is that this model allows us to examine how current and expected changes in demand influence current investment decisions.

It is assumed that the demand function has the following separable form:[11]

(3.28) $D(p_t)Z_t$, where $\frac{\partial D}{\partial p} = D_p < 0$

Hence the demand function has a constant shape defined by D(p) and it is shifted by an exogenously given parameter Z, which can be time-dependent.

It is furthermore assumed that output is maintained in equality with demand and hence

(3.29) $D(p_t)Z_t = Q_t = F(K_t,L_t)$

This assumption implies that the firm is able and willing to adjust the price of output continuously (instantaneously) and thus has no inventory holdings. The presence of inventories and rigidities in price adjustment would affect the dynamics of investment but this 'simultaneity' problem is beyond the scope of our task here. The basic characteristics of the demand effects on investment can also be derived within this simplified model.

Output price can be solved from (3.29) as

(3.30) $p_t = p\{F(K_t,L_t)/Z_t\}$, where $p\{\ \} = D^{-1}$

The partial derivatives of p with respect to K and L are both negative and they are $\partial p/\partial K = F_K/D_pZ$ and $\partial p/\partial L = F_L/D_pZ$, respectively. Labour input is again assumed to be adjusted instantaneously to its equilibrium level and hence

$$(3.31) \quad F_L(L,K) = \frac{w}{M(p)}$$

where M(p) is the marginal revenue function, which replaces the output price appearing in the perfectly competitive case (see chapter II). (Notice that the time dependency of variables is again suppressed except where it is needed for clarity). Thus, labour is employed up to the point where its marginal revenue product is equal to the wage rate (similarly $F_K = c/M(p)$, see below).

Equations (3.29) and (3.31) can be solved for L and p in terms of K, w and Z (i.e. short-run profit maximization with a given K; see also section 2.2.2 for the case where p is given) and the marginal revenue product of capital ($M(p)F_K$) is given by[12]

$$(3.32) \quad M(p)F_K(K,L) = M[p\{K,w,Z\}]F_K[K,L(K,w,Z)]$$

This can be rewritten in more compact form as

$$(3.33) \quad M(p)F_K = N(K,w,Z)$$

and it can be shown that

$$(3.34) \quad N_K = \frac{M}{F_{LL}}(F_{KK}F_{LL} - F_{KL}^2) + F_K M_p P_K$$

and that $N_K < 0$ (see note 13).

The present value maximization problem of the firm is now as follows:

$$(3.35) \quad \max_{\{I\}} \int_0^\infty e^{-\rho t}\{(1-u)[p\{F(K,L)/Z\}F(K,L)-wL-rsqK] + sq(I-\delta K)+s\dot{q}K$$
$$-q(1-uz)(I+C(I))\}dt$$

subject to $\dot{K} = I - \delta K$ (and an initial value of K, say K_0).

The necessary conditions for an optimum are now

(3.36) i) $\dot{\lambda} = (\rho+\delta)\lambda - (1-u)N + sq((1-u)r+\delta-g)$

ii) $\frac{\partial H^c}{\partial I} = sq - q(1-uz)(1+C') + \lambda = 0$

where λ is the shadow price of capital. Sufficiency follows since the Hamiltonian is jointly concave in K and I and the transversality conditions are satisfied (Arrow and Kurz propositions for sufficiency, see chapter II and section 3.2).

The optimal decision rule for investment can now be obtained from eq. (3.9) by replacing the term f'(K) by N(K,w,Z). It clearly shows that in this adjustment cost framework current investment decisions depend upon current and future prices (p,w and q) and also upon the current and future development of demand (measured by the shift variable Z). Hence current investment decisions are essentially forward-looking.

In the steady state ($\dot{\lambda} = 0$) with constant demand and prices, the equilibrium capital stock must satisfy the marginal condition $F_K = c/M(p)$, where c is given by eq. (3.11). The formula for the user cost is hence the same as in the preceding case (section 3.2.2). Thus the assumption about demand conditions does not affect the user cost of capital except through the $I^* = \delta K^*$ term and the tax neutrality conditions are independent of the demand regime (see also section 3.4). It should be noted that, although the user cost formula is the same, the steady state value of K* depends on the specification of the demand function. The long-run demand function for capital is now given by

(3.37) $K^* = K(w,c,Z)$

In chapter V explicit formulas for K* will be derived assuming a CD or CES production function and a specific form for the demand function (see section 5.3).

From the differential equation system $(\dot{K},\dot{\lambda})$ underlying this model can be determined the slopes of the ($\dot{\lambda} = 0$) and ($\dot{K} = 0$) locus (analogously to the model in section 3.2). The result is that $\frac{d\lambda}{dK}\Big|_{\dot{\lambda}=0} < 0$ and $\frac{dK}{d\lambda}\Big|_{\dot{K}=0} > 0$ and hence the ($\dot{\lambda} = 0$) locus slopes downward and the ($\dot{K} = 0$) locus slopes upward in the (λ, K) space. A diagram analogous to that in section 3.2 shows that the steady state is a saddle point. The interesting point to notice is that $\frac{d\lambda}{dK}|_{\dot{\lambda}=0} = \frac{(1-u)F_K M_p P_K}{(\rho+\delta)} < 0$ if there are constant returns to scale, and hence the ($\dot{\lambda} = 0$) locus is not horizontal as was the case with the perfectly competitive output market (with constant returns to scale). This result also implies that the downward-sloping demand regime does not produce a constant rate of investment as did the horizontal demand curve with constant returns to scale.

The linearized system of the present model is obtained from (3.17) when $f''(K)$ is replaced by N_K given by eq. (3.34). The characteristic roots of the linearized system are both real and opposite in sign (when $N_K < 0$), hence implying that the steady state is a saddle-point solution. The equation of motion for the capital stock is again determined by the negative root (say $\bar{k}_1$) and it is given by

$$(3.38) \quad -\bar{k}_1 = -\frac{\rho}{2} + \sqrt{\frac{(\rho+2\delta)^2}{4} - \frac{(1-u)N_K}{q(1-uz)C''}}$$

The equivalent formula for the rate of adjustment in the case of a competitive output market was given by equation (3.19). It can again be seen that the rate of adjustment is an endogenous decision variable and it depends upon tax parameters, price variables and the demand variable (Z). The interesting point to observe is that the speed of adjustment also depends on the price elasticity of demand for the firm's products (see 3.34).

The formula for the rate of adjustment in the case of constant returns to scale is

$$(3.39) \quad -\bar{k}_1^c = -\frac{\rho}{2} + \sqrt{\frac{(\rho+2\delta)^2}{4} - \frac{(1-u)F_K M_p p_K}{q(1-uz)C''}}$$

where the second term in the squared root is positive since $F_K > 0$, $C'' > 0$, $M_p > 0$ and $p_K < 0$ (see note 13).

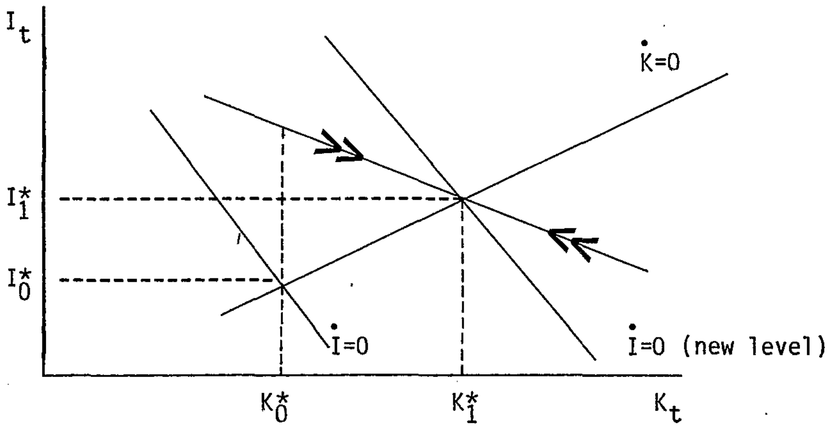
Two observations can be made about the new formulas for the rate of adjustment. First, as in the case of a horizontal demand curve, the rate of adjustment to the equilibrium level of the capital stock is faster in the decreasing returns to scale model than in the case of constant returns to scale. Second, whereas the rate of adjustment was constant in the case of a perfectly competitive output market (and equal to the rate of depreciation δ), it is now variable (endogenous) and greater than the rate of depreciation if there are constant returns to scale. Hence the original result by Gould (1968) does not hold in the case where the firm faces a downward-sloping demand curve.

Using the formula for N_K, it can easily be shown that the partial derivatives of the adjustment rate with respect to the demand parameter (Z) and the elasticity of demand are positive and negative, respectively. Hence a rise in demand will increase the rate of adjustment (and not only the long-run demand for capital). The role of the price elasticity of demand is of special interest in this connection since it causes the main difference between our previous model (section 3.2, perfect output market) and the present one. If the elasticity of demand is infinitely large, then (3.38)reduces to (3.19) because $N_K = f''$ (and $M(p) = p$).[13] If the price elasticity of demand is 'low', then N_K is large and hence the rate of adjustment is 'rapid'.

This last result implies that a firm with 'inelastic' demand for its output will have a higher rate of adjustment than an otherwise similar firm but with 'elastic' demand. Hence the absolute value of the elasticity of demand is indirectly related to the rate of investment. This result can be illustrated by the following example: Consider the effect of an unanticipated increase in demand on investment. In order that the equality between output and demand holds, there must be a jump in the output price since in the adjustment cost framework capacity cannot be instantaneously adjusted to the new equilibrium level (corresponding to the new higher level of demand, which is assumed to be permanent). The rise in demand causes an increase in the shadow price of capital and hence investment also increases (investment first jumps and then the rate of investment declines to the new equilibrium level (δK_1^*) since $dI/dt < 0$). The capital stock moves gradually to its new equilibrium level and the output price also gradually returns to its long-run equilibrium level (see also Nickell, 1978).

Figure 2 illustrates the reaction of investment to an unanticipated jump in demand (i.e. in Z). In its initial position, the firm is assumed to be in equilibrium. Notice that figure 2 is drawn in the (I,K) space and not in the (λ,K) space as is usually the case.

FIGURE 2. The Effect of an Increase in Demand on Investment



Note: K_1^* corresponds to the new level of the equilibrium capital stock.

The important point to notice about the firm's behaviour as depicted in figure 2 is that, the 'less' price elastic the demand is, the 'more' the output price must jump in order that the equality between output and demand is retained. This implies that a firm with 'inelastic' demand experiences a 'large' increase in its marginal revenue and hence its rate of investment must increase more than in the case of a firm which has 'elastic' demand in order that the equality between the marginal cost of investment and the net present value returns on a marginal unit of capital holds.

The unanticipated increase in demand (an exogenous shock) will increase the equilibrium levels of the capital stock and investment as noted above. The other comparative static properties of this model with a downward-sloping demand curve are the same as in the case of a horizontal demand curve.[14]

In sum, the preceding two sections (3.2 and 3.3) have shown that generally the rate of adjustment of investment is rather sensitive with respect to both the demand regime facing the firm (i.e. with respect to the price elasticity of demand) and with respect to the returns to scale hypotheses of the underlying production function. The results hitherto also indicate that generally the rate of adjustment to the equilibrium level of the capital stock depends upon corporate tax parameters (except in the neutrality case in which $z = 1$, see also section 3.2.3).[15]

3.4 The Case of a Fixed Output

It is next assumed that the firm faces a permanent demand (sales) constraint, implying that the demand curve is vertical. This is the polar case to the one where the representative firm faces a horizontal demand curve for its output (the fixed price case, see section 3.2). This new assumption about the demand regime can also be interpreted to mean that excess supply occurs, since the optimal supply of output is greater than the demand for it. Essentially, therefore, this section analyzes the derivation of effective investment demand functions under conditions of excess supply for current output (the previous cases have analyzed 'notional' investment demand functions in the terminology of fix-price or non-Walrasian models). Because the firm is unable to sell the output which it desires to supply, its investment demand will become a function of the level of output which it is able to sell. The current level of output is no longer a choice variable but rather a constraint, whose value is determined by the current level of demand.

The basic structure of the model to be analyzed here is similar to that in Grossman (1972), but we assume that corporate tax factors are present and that the firm has an 'optimal' debt-capital ratio, implying that the discount rate and the rate of interest on debt are unequal. Section 3.4.1 develops the model and examines the dynamics of investment behaviour. Section 3.4.2 analyzes the effect of unanticipated and anticipated demand shocks on the investment path.

3.4.1 The Model and Its Properties

A demand-constrained firm behaves optimally when it equates the marginal rate of substitution $R(K,L)$ with the factor price ratio c/w according to

$$(3.40) \quad R(K,L) = R = -\frac{dL}{dK} = \frac{F_K}{F_L} = \frac{c}{w}$$

This is the standard necessary condition for an optimum in the static case when output is exogenously given (cost-minimization model, see e.g. Brechling, 1975).

We assume that the demand constraint has a constant value ($\bar{Q}$) over the entire future, so that the firm produces $\bar{Q}$ at every point of time. Hence expectations regarding the level of the demand-determined constraint on output are static (stationary). In this set-up, maximization of the present value of expected future net returns involves two stages. First, the firm plans to produce now and in the future the output quantity $\bar{Q}$, and this is done at each point in time with a minimum amount of labour (given the stock of capital at any instant of time). Second, given the 'optimal' amount of labour, the firm chooses an optimal time path of capital accumulation.

The demand for labour is given by

$$(3.41) \quad L_t^d = L(K_t,\bar{Q})$$

such that labour and capital can be used to produce output $\bar{Q} = F(K_t,L_t^d)$. It should be noted that whereas the 'notional' demand for labour is determined by $F_L = w/p$, we now have a case where $F_L > w/p$ since $\bar{Q} < Q_t$ (where $Q_t = F(K_t,L_t)$, and $L_t = L(K_t,w/p)$ such that $F_L = w/p$, see chapter II).

As before, the marginal unit of installed capital should be valued at the present discounted value of the net cash flow which it produces. The net cash flow at time v attributable to the marginal unit of capital is equal to the reduction in the wage bill required to produce the (fixed) output $\bar{Q}$. This wage bill saving is given by

$$(3.42) \quad -w\left.\frac{dL_v}{dK_v}\right|_{Q_v=\text{constant}}$$

which means that the saving in wages resulting from an additional unit of capital is equal to the wage rate multiplied by the marginal rate of substitution of labour for capital (ignoring depreciation and taxes). Increments to the capital stock are profitable as long as the saving in the wage bill is positive (given $\bar{Q}$).

The present value maximization problem of the firm is now as follows (with fixed prices):

$$(3.43)\quad \max_{\{I\}} \int_0^\infty e^{-\rho t}\{(1-u)[\bar{Q}-wL_t^d-rsqK_t] - q(1-uz)(I_t+C(I_t)) + sq(I_t-\delta K_t)\}dt$$

subject to $\dot{K} = I - \delta K$ and $\bar{Q} = F(K_t, L_t^d)$ and with K_0 given (see also sections 2.2.3 and 3.2).

As before, this problem is solved using the relevant Hamiltonian, the necessary conditions for an optimum being

$$(3.44)\quad \text{i)}\quad \dot{\lambda} = (\rho+\delta)\lambda - (1-u)w\frac{F_K}{F_L} + sq((1-u)r+\delta)$$

$$\text{ii)}\quad \frac{\partial H}{\partial I} = sq - q(1-uz)(1+C') + \lambda = 0$$

It should be noted that $-dL/dK = F_K/F_L = R$, where $R = R(K,L;\bar{Q})$ is the marginal rate of substitution of labour for capital in producing the fixed quantity $\bar{Q}$ (see eq. 3.46). The second condition is the same as in our previous models and the first one (i) has the usual interpretation that the realized return on capital (sum of wage savings and capital gains minus debt service charges) is equal to the required return on capital $(\rho + \delta)$.

Sufficiency follows since $L^d(K,\bar{Q})$ is a convex function with respect to K, i.e. $dR/dK < 0$ (see section 3.4.2) and hence the Hamiltonian is jointly concave in K and I ($C' > 0$, $C'' > 0$), (see also sections 2.2.3 and 3.2 where the relevant Arrow and Kurz propositions were discussed and note that we have now assumed that prices w and q are constant). The formula for the shadow price of capital can be solved from the differential equation (3.44i) imposing the transversality condition (see section 3.2 and note 6), and we obtain

$$(3.45)\quad \lambda_t = \int_t^{\infty} e^{-(\rho+\delta)(v-t)}[(1-u)wR(K_v;\bar{Q})-sq((1-u)r+\delta)]dv$$

which also gives the optimal decision rule for investment when the necessary condition (3.44ii) is used. Hence, investment is again an increasing function of the shadow price of capital. In the steady state, the marginal rate of substitution is constant and eq. (3.45) can be solved to give

$$(3.46)\quad \lambda^* = \frac{(1-u)wR(K^*;\bar{Q}) - sq((1-u)r+\delta)}{(\rho+\delta)}$$

where λ^* and K^* are the steady-state values of λ and K, respectively. With $s = 0$ (and $r = \rho$), eq. (3.46) reduces to $\lambda^* = (1-u)wR(K^*,\bar{Q})/(\rho+\delta)$. This equation could be used to compare the response of investment to changes (in the exogenous factors) which lead to equal equilibrium values of λ^* (see Abel, 1981).

The steady-state solution also gives the marginal rule of this cost minimization model, i.e. $F_K/F_L = c/w$, where c is given by eq. (3.11). It can be noted that the user cost of capital is again the same as in the preceding two cases with horizontal and downward-sloping demand curves. Thus it can generally be concluded that the user cost formula is independent of the demand conditions and that the two standard neutrality conditions of the long-run demand for capital do hold (see section 3.2). The steady state value of K^* is, however, again different from the previous demand function cases.

The long-run demand function for capital is now given by

$$(3.47)\quad K^* = K(\frac{c}{w},\bar{Q})$$

In the previous two cases, the analogous results were $K^* = K(w/p,c/p)$ and

$K^* = K(w,c,Z)$ in the horizontal and downward-sloping demand regimes, respectively. Therefore, given r and ρ and the F and C functions, desired capital must now be smaller than in the perfectly competitive output market case (basically, this is due to the fact that F_L exceeds w/p along the optimal path). Relationship (3.47) clearly reflects the fact that, given the demand-imposed constraint on output, the only motivation for increasing the capital stock is to economize in the use of labour.

It can easily be shown that the ($\dot{\lambda} = 0$) locus slopes downwards and the ($\dot{K} = 0$) locus slopes upwards in the (λ,K) space diagram (see section 3.4.2). This kind of phase diagram representation also shows that the steady state is a saddle-point (see also figure 3, section 3.4.2) analogously to our previous cases (sections 3.2 and 3.3). The interesting point to note is that the ($\dot{\lambda} = 0$) locus slopes downwards even in the case of constant returns to scale, hence implying that the rate of investment is not constant along the optimal path (see section 3.4.2). The same result was also obtained in the case of a downward-sloping demand curve (with a linearly homogeneous production function). By contrast, in the case of a perfect output market, the assumption of a linearly homogeneous production function leads to a constant rate of investment. Hence we can again observe that the dynamics of investment is sensitive with respect to both the production and the demand function.

Linearizing the system of two non-linear differential equations behind the above model around the steady state gives

$$(3.48) \qquad \begin{bmatrix} \dot{\lambda} \\ \\ \dot{K} \end{bmatrix} = \begin{bmatrix} \rho+\delta & (1-u)w\,\dfrac{F_{LK}F_K - F_{KK}F_L}{F_L^2} \\ \\ \dfrac{1}{q(1-uz)C''} & -\delta \end{bmatrix} \begin{bmatrix} \lambda-\lambda^* \\ \\ K-K^* \end{bmatrix}$$

The characteristic roots of this linearized system are both real and of opposite sign. Thus the steady state is a saddle-point. The unique path for K_t is given by $\dot{K} = -\bar{\bar{k}}_1(K^*-K)$ where $\bar{\bar{k}}_1$ is the negative root of the linearized system and the rate of adjustment is given by

$$(3.49)\quad -\bar{k}_1 = -\frac{\rho}{2} + \sqrt{\frac{(\rho+2\delta)^2}{4} + \frac{(1-u)w}{q(1-uz)C''} \cdot \frac{F_{LK}F_K - F_{KK}F_L}{F_L^2}}$$

where $|\bar{k}_1|$ may be denoted as the 'effective' rate of adjustment (due to the demand constraint). The second term in the square root is positive since $F_{KK} < 0$.

Using Euler's theorem ($Q = KF_K + LF_L$) and the relationship $F_{LK} = -\frac{K}{L} F_{KK}$ derived from it, the formula for the rate of adjustment in the constant returns to scale model is as follows:

$$(3.50)\quad -\bar{k}_1^c = -\frac{\rho}{2} + \sqrt{\frac{(\rho+2\delta)^2}{4} - \frac{(1-u)w}{q(1-uz)C''} \cdot \frac{F_{KK}\bar{Q}}{LF_L^2}}$$

and the second term in the square root is again positive.

Some interesting observations can be made with respect to both the size and the determinants of the speed of adjustment. First, the rate of adjustment is endogenous (variable) with a linearly homogeneous production function. This result, together with our preceding results with the linearly homogeneous production function, means that the rate of adjustment is constant (and hence exogenous) only in the case of a horizontal demand curve. Thus Gould's original result does not generally hold (i.e. the rate of adjustment is generally different from δ). The second observation which can be made about (3.50) is that the speed of adjustment is greater than the rate of depreciation. Hence, both in the downward-sloping and the vertical demand regimes, the rate of adjustment exceeds the rate of depreciation even though there are constant returns to scale in the underlying production function.

Third, the tax neutrality issue with respect to the dynamic path of the present model can be considered in an analogous manner to the case of a horizontal demand curve (see section 3.2.3). Formulas (3.49) and (3.50) reveal that with free depreciation ($z = 1$) the 'local' neutrality property of the rate of adjustment also holds in the present case (see also section 3.2.3).

Fourth, equation (3.50) shows that a rise in the wage rate or in demand increases the rate of adjustment (as well as the equilibrium level of the capital stock K*). Since $I = \dot{K} + \delta K$ and $\dot{K} = -\bar{\bar{k}}_1(K^*-K)$, the increase in w and $\bar{Q}$ also leads to a higher level of investment (per time unit).[16] This result could be obtained directly from the relationship between λ and I by examining the impact of exogenous shocks (permanent) on the shadow price of capital. Some other comparative static and dynamic properties of the excess supply investment model will be analyzed in the next section.

A direct comparison of the rate of investment can be made between the horizontal (fixed price) and vertical (fixed output)demand regimes (see also Grossman, 1972). We have already observed above that the 'desired' capital stock is lower in the vertical regime than in the horizontal regime (for otherwise similar firms). The relationship between the rate of adjustment is, however, ambiguous. Only in the constant returns to scale model is the speed of adjustment unambiguously higher in the fixed output case than in the horizontal demand regime (see eq. 3.50, which reduces to δ in the latter case). Hence in the case where output is demand-constrained, the rate of investment can be higher than within a perfectly competitive output market. However, this result holds only 'initially', since the steady-state capital stock is smaller in the fixed output case, and thus the rate of investment must eventually decline to a level where $I_1^* < I_0^*$ and $I_1^* = \delta K_1^*$, $I_0^* = \delta K_0^*$ (K_1^* corresponds to the fixed output case and K_0^* to the horizontal case). These results again show that the dynamic path of investment is clearly sensitive with respect to the demand conditions and that this sensitivity is due to the effects of demand conditions on both the long-run demand for capital and the speed of adjustment (see chapter VII, where the effect of demand conditions on the rate of adjustment is tested empirically).

3.4.2 The Effect of an Increase in Demand on Investment

Most of the comparative static properties of our adjustment-cost-based investment models are invariant to the specification of demand conditions (see sections 3.2 and 3.3). The differences occur only with respect to a change in the wage rate and in the demand for the firm's products. The clearest difference is with respect to an increase in the wage rate, since in the demand-constrained model above only a substitution effect occurs and hence $\partial K^*/\partial w > 0$ and $\partial I^*/\partial w > 0$. In the other two demand regimes these partial derivatives are ambiguous in sign, since there is both a substitution and a scale effect (owing to endogenous output).

The effect of a rise in demand (an unanticipated permanent shock) leads to a higher level of the equilibrium capital stock in all demand regimes, but the exact process involved is conditional upon the demand regime which the firm faces. In the horizontal demand curve model, a rise in demand can only be understood in terms of an increase in the output price (see section 3.2). In the case of a downward-sloping curve, an increase in demand is captured through an increase in the exogenous demand parameter (Z, see section 3.3). The quantitative impact of an increase in demand on investment, however, depends in this case on the price elasticity of demand as shown in section 3.3. The 'pure' demand effect can most clearly be examined within the framework of the demand-constrained investment model developed in section 3.4.1.

In this subsection we shall analyze the effects of both unanticipated and anticipated (future) demand shocks on investment behaviour. In order to perform this analysis we shall first develop the phase diagram of the underlying model.

From the two differential equations (eq. 3.44i for $\dot{\lambda}$ and $\dot{K} = I - \delta K$ with $I = C'^{-1}(\lambda)$) of the above excess supply model, the slopes of the ($\dot{\lambda} = 0$) and ($\dot{K} = 0$) curves can be solved as:

$$\text{(3.51)} \quad \text{i)} \quad \left.\frac{d\lambda}{dK}\right|_{\dot{\lambda}=0} = \frac{(1-u)w[F_{KK}F_L - F_K F_{LK}]}{(\rho+\delta)F_L^2} < 0$$

$$\text{ii)} \quad \left.\frac{dK}{d\lambda}\right|_{\dot{K}=0} = \frac{1}{q\delta(1-uz)C''} > 0$$

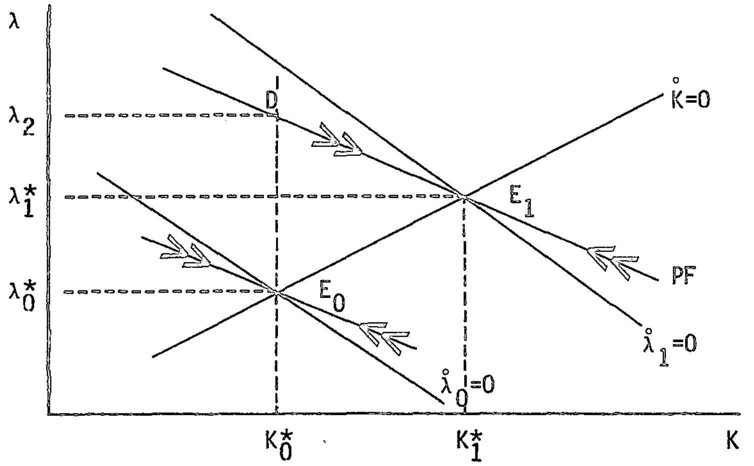
and hence the ($\dot{\lambda} = 0$) locus slopes downward and the ($\dot{K} = 0$) locus slopes upwards. If there are constant returns to scale, then

$$(3.52)\quad \frac{d\lambda}{dK}\Big|_{\dot{\lambda}=0} = \frac{(1-u)wF_{KK}\bar{Q}}{(\rho+\delta)LF_L^2} < 0$$

which means that the ($\dot{\lambda}$ = 0) locus is downward sloping even when the production function is linearly homogeneous.

Figure 3 illustrates the phase diagram apparatus in the (λ,K) space.

FIGURE 3.



We analyze first the effect of an unanticipated demand increase assuming that the firm is initially at the steady-state equilibrium (denoted by E_0 with K_0^* and λ_0^*). The ($\dot{K}$ = 0) locus is not affected by the demand shock but the ($\dot{\lambda}$ = 0) locus shifts upwards. The new steady state is at E_1 with K_1^* and λ_1^* and the level of K_1^* depends upon the elasticity of the marginal rate of substitution with respect to output and, more specifically, upon whether this elasticity is an increasing or decreasing function of K along an isoquant.[17] The shadow price of capital instantaneously jumps to λ_2 (figure 3) and thus induces a higher amount of investment. When the capital stock accumulates towards its new equilibrium value (K_1^*), λ falls smoothly to its new equilibrium level given by λ_1^*.

The impact effect of the unanticipated increase in demand is therefore a vertical jump from E_0 to the point D to place the firm on the unique perfect foresight path (PF) vertically above K_0^*. Thereafter, the firm converges along this path to the point E_1. The implications for investment are now the following: In the original equilibrium, the capital stock is constant and all investment is for replacement purposes, i.e. $I_0^* = \delta K_0^*$. When demand unexpectedly rises, the shadow price of capital λ immediately jumps to λ_2 (and thereafter gradually falls back to λ_1^*). Since investment is, because of adjustment costs, an increasing function of λ, gross investment first jumps to a higher level and then gradually declines to the new steady-state level equal to δK_1^*.

We shall next consider the effect of an anticipated (future) permanent increase in demand on investment. At some initial date t_0, the firm realizes in the light of new information that there will be a permanent rise in demand at some future date t_1. Under rational expectations, the firm will already respond to this new information at t_0, rather than waiting until t_1 when the expected increase in demand actually occurs. This general feature of the rational expectations (perfect foresight) framework can be analyzed in the present situation using figure 4.

FIGURE 4.

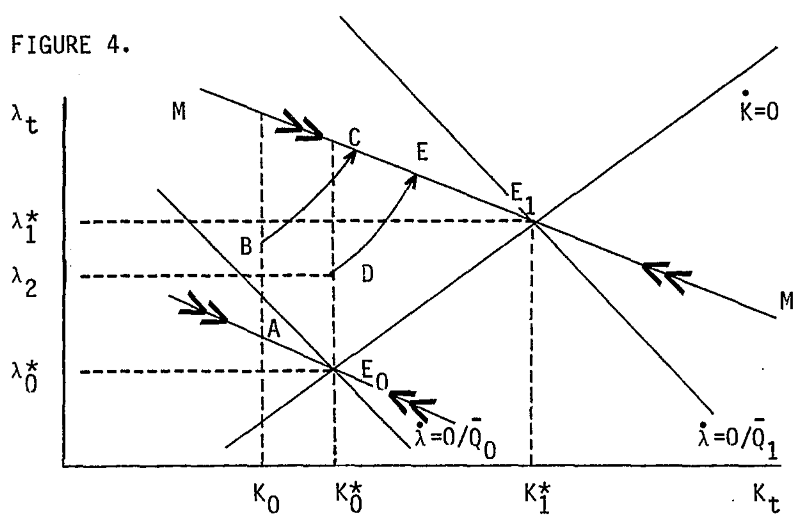


Figure 4 shows two different situations with respect to the initial positon of the firm. First, the firm begins in the original steady state at the point E_0 at time t_0. Second, the firm is initially on the stable path leading to the equilibrium E_0. Since the behaviour of this system is essentially the same in both cases, we shall consider only the first one here.

It is assumed that the firm's expectations about future demand change at t_0 so that it anticipates an increase in demand at t_1 (the original level of demand is $\bar{Q}_0$ and the new level will be, say, $\bar{Q}_1$). Since the arrows of motion depend on the differential equations for λ and K (eq. 3.44i and $\dot{K} = C'^{-1}(\lambda)-\delta K$), the new equations of motion are effective only when demand has actually changed. Between t_0 and t_1 the original equations of motion must be obeyed (see also Begg, 1982, for a general methodological discussion of the properties of perfect foresight systems). Because of the anticipation of a future rise in demand, the system describing the behaviour of the firm must instantaneously jump to the point D (in figure 4). The jump in the shadow price of capital from λ_0^* to λ_2 is caused by the possibility of increased future savings in the wage bill provided by the capital needed to produce the increased amount of output. The unique perfect foresight path requires a jump to (some) point D at time t_0. Following the original equations of motion during the period between t_0 and t_1, the firm reaches the path MM at time t_1. The complete path is hence DEE_1. The precise magnitude of the initial jump in the shadow price of capital will depend upon the structural parameters of the model (see below).

The behaviour of the above system can be analyzed more formally in the following manner. The expected value of demand is $Q_v = \bar{Q}_0$ when $t_0 \leq v \leq t_1$ and $Q_v = \bar{Q}_1$ for $v > t_1$. In the period between t_0 and t_1, the shadow price of capital is given by (3.45) with $\bar{Q} = \bar{Q}_0$ and the 'general' expression for λ is now

$$(3.53)\quad \lambda_t = \int_t^{t_1} e^{-(\rho+\delta)(v-t)}[(1-u)wR(K_v;\bar{Q}_0)-sq((1-u)r+\delta)]dv$$

$$+ \int_{t_1}^{\infty} e^{-(\rho+\delta)(v-t)}[(1-u)wR(K_v;\bar{Q}_1)-sq((1-u)r+\delta)]dv$$

Differentiating (3.53) with respect to time gives the original equation of motion for λ_t, i.e. equation (3.44i), and hence the anticipation of a rise in demand does not influence the equation of motion (before t_1). As described above, the anticipation of a demand increase causes a jump in λ (because of increased possibilities of wage savings) and hence initially investment also jumps so that the time path of the capital stock responds in advance of the increase in demand. Since there are strictly convex costs of adjustment, this response will be spread out over time. After the change in demand has occurred, the new equation of motion for λ is $\dot{\lambda}_t = (\rho+\delta)\lambda_t - (1-u)wR(K_t;\bar{Q}_1) + sq((1-u)r+\delta)$. The critical point in this analysis is the effect of the demand increase on the marginal rate of substitution between labour and capital. If this effect is positive, then following the news of a future rise in demand the ($\dot{\lambda} = 0$) locus shifts upwards and λ_t must jump vertically. It can be shown that the elasticity of the marginal rate of substitution (R) with respect to $\bar{Q}$ is positive (holding K constant).[18] The precise magnitude of this elasticity, however, depends upon the parameters of the production function. The lower is the elasticity of substitution between labour and capital, the greater is the change in R following a given increase in output (see note 18).

The shadow price λ_2, which corresponds to the point D in figure 4, may lie above or below the new steady-state price λ_1^* corresponding to the point E_1. However, at t_1 the shadow price λ reaches its maximum, after which it falls back along MM until the new equilibrium price λ_1^* is attained. Since gross investment is an increasing function of λ, we can infer the time path of investment on the basis of λ_t. If it is assumed that the initial jump to D is such that λ_2 is below λ_1^*, then gross investment first jumps to a higher level and then steadily increases to its maximum level (corresponding to the value of λ at point C in figure 4) after which it starts to decline gradually to its new equilibrium level equal to δK_1^*. Whether investment follows this more complex path or the simple path shown in figure 3 depends upon the parameters of the underlying production function (see also note 18).

In sum, we have again noted that both the production and the demand relationships significantly affect the dynamics of investment behaviour. Moreover, an expected increase in demand will increase both the long-run demand for capital (the equilibrium capital stock) and the rate of investment, and the capital stock responds in advance to the anticipated rise in demand. It can also be shown that the closer to the present the increase in demand is expected to take effect, the larger is the initial rise (jump) in the current rate of investment.

3.5 Conclusions of Chapter III

In chapter III the investment behaviour of the firm was derived from explicit intertemporal optimization in a perfect foresight framework, and it was shown that investment is an increasing function of the shadow price of capital because of strictly convex adjustment costs. Within this framework the dynamics of investment behaviour was analyzed subject to alternative demand conditions (horizontal, vertical and downward-sloping curves). We also assumed that the capital market is imperfect in the sense that the firm has a constant debt ratio and, in addition, we included corporate tax factors in the analysis.

The general outcome of this analysis is that both the long-run demand for capital and the rate of investment depend significantly on the form of demand conditions and on the parameters of the production function (returns to scale and elasticity of substitution). More specifically, the rate of adjustment to the equilibrium level of the capital stock is rather sensitive with respect to the properties of the underlying functions.

The analysis of the downward-sloping demand regime (monopolist case) shows that 'low' price elasticity of demand leads to a more rapid investment response than a 'high' price elasticity case (in response to an expected or unexpected demand increase). If the firm is faced with a binding demand constraint, then the effect of an anticipated rise in demand on the time path of investment depends critically on the elasticity of substitution between labour and capital (in addition to the returns to scale property). In both cases of imperfect product markets, a rise in demand leads to an increase both in the equilibrium level of the capital stock and in the rate of adjustment. The original result by Gould (1968), according to which the rate of adjustment is constant (exogenous) in the case of a linearly homogeneous production function, no longer holds when the product market is imperfect. However, Gould's result holds even when corporate tax factors are present and the capital market is imperfect, if the demand curve is horizontal and there are constant returns to scale.

It was observed that the two standard tax neutrality conditions with respect to the long-run demand for capital (i.e. K* neutrality) are not affected by the choice of the demand regime. We were also able to show that the whole dynamic path of investment (i.e. $\dot{I} = dI/dt$) can be independent of the tax rate in the presence of adjustment costs in these two cases. The neutrality issue with respect to the speed of adjustment is conditional upon the demand regime and upon the returns to scale assumption of the production function. In the case of a horizontal demand curve and constant returns to scale the speed of adjustment is generally independent of the tax parameters. In all other cases considered here the speed of adjustment depends on the tax factors but neutrality follows in the two standard cases (or when $z + y = 1$).

Finally, the capital market imperfection (i.e. $0 < s < 1$, $\rho > (1-u)r$) affects both the long-run demand for capital and the level of investment (dynamics). The deductibility of interest payments increases the level of investment from that obtained in a perfect capital market (see also section 4.2).

Notes to Chapter III

1. The assumptions labelled as a vertical demand curve, an excess supply model, a demand-constrained case or a fixed output can all be used to describe the case of a cost-minimization model subject to a given output.

2. Nonseparable internal adjustment costs are, in certain respects, the most problematic of the standard types of adjustment costs. For example, the effects of output price changes and the slope of the short-run supply curve will be ambiguous, and such well-known results of the static theory as negatively sloped demand curves for a variable factor and the application of the generalized le Chatelier Principle may cease to hold (see e.g. Treadway 1970, 1974, Söderström 1976, Epstein and Denny 1983). The tax neutrality issue concerning the dynamic path of investment is also more complicated in the case of nonseparable adjustment costs than is otherwise the case (see Abel 1983, and note 9).

3. Internal adjustment costs may be thought to represent 'output losses' due to a faster rate of investment, and adjustment costs as a function of (I/K) may stem from the Penrose effect (see e.g. Takayama 1974, Söderström 1976).

4. The same concept of total costs of investment is used, for example, in Sargent (1979). He, however, assumes that $C = C(\dot{K})$ and hence the total investment cost is given by $q(I+C(\dot{K}))$. Using this kind of presentation instead of just $qC(I)$ or $qI+C(I)$ enables us to factor out the term $q(1+C')$ from the user cost concept.

5. The present value of current and future tax deductions attributable to past investments is given by

$$\bar{D}_t = \int_0^{\infty} \{u_t[\int_{-\infty}^{0} D_{t-v}q_v(I_v+C(I_v))dv]\}e^{-\rho t}dt$$

6. The general solution to the differential equation (3.5i) is

$$\lambda_t = \int_t^{\infty} [(1-u_v)(f'(K_v)-srq_v)-s\delta q_v+sq_vg]e^{-(\rho+\delta)(v-t)}dv + C_0e^{(\rho+\delta)t}$$

where C_0 is an arbitrary constant. Imposing the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda_t = 0$ requires that $C_0 = 0$, so that we obtain (3.6). Hence the general solution to the homogeneous equation is eliminated by the transversality condition, leaving the particular solution to the nonhomogeneous equation as the complete solution (see, e.g., Nerlove 1972, Gould 1970).

It should be noted that the necessary conditions are also sufficient since the Hamiltonian is concave in K and I, and the transversality conditions are satisfied (see section 2.2.3, and for the Arrow - Kurz proposition with a stationary problem see Arrow and Kurz (1970), pp. 47 - 51). Hence the optimum (Pontryagin) path is also unique. Basically, the concavity of the integrand in (3.4) and the linearity of the constraining differential equation guarantees that an optimum path exists and that it is unique (see Kamien and Schwarz, 1981).

7. If the term sqI is taken into account, then this condition is $uz + s < 1$ (see also section 4.4.2).

8. If $F_{KL} > 0$, then $\frac{\partial K^*}{\partial w} < 0$ and $\frac{\partial I^*}{\partial w} < 0$. These results are not completely general but simply reflect the strength of the scale (output) effect relative to the substitution effect in this particular model.However, if the wage increase is an industry-wide phenomena, the aggregate (industry) market supply curve is shifted to the left (i.e. output is reduced) and consequently output price (p) increases. Since $\partial K^*/\partial p > 0$, this price increase may lessen the output effect to the extent that in the new equilibrium $\partial K^*/\partial w > 0$ (see also Gould 1968, Gould and Waud 1973, and Virén 1979, p. 36).

9. If adjustment costs qC(I) were directly tax deductible but ordinary costs of investment were depreciated over time, then eq. (3.27) would change to the form $\dot{I} = \frac{q(r+\delta)(1-uz-uy) + q(r+\delta)(1-u)C' - (1-u)f'}{q(1-u)C''}$ and neutrality again follows when $z + y = 1$.

Hartman (1978) shows that true economic depreciation with full deductibility of the cost of capital leads to neutrality even in the case of internal (nonseparable) adjustment costs (see also Honkapohja and Kanniainen 1985). In the models of Hartman and Honkapohja - Kanniainen it is assumed that the objective of the firm is to maximize an intertemporal function of dividends (say $\Phi(Div)$), where Φ':s will affect the optimal debt policy but not the real investment policy. In this model real and financial decisions are completely separable and the discount rate (ρ) is equal to the interest rate on debt (r). Neutrality follows if the total tax allowance on capital D(K) is equal to $(\rho + \delta - g)qK$. This approach corresponds to our case in eq. (3.27), which was derived under the assumption that rB and $\dot{B}$ are excluded from the cash flow equation (except that rB enters through the tax term). See also section 4.2.

Abel (1983) considers the neutrality issue in terms of the following cash flow identity: cash flow = $(1 - u)f(K) - qI + uD(K)$, where D(K) is the depreciation allowance and all the financial flows of debt are excluded. Assuming, however, that $\rho = (1 - u)r$ and $D(K) = (\delta\lambda - \dot{\lambda})K$, neutrality of the dynamic path is obtained (λ is the shadow price of capital). Using Abel's approach, it can be shown that the equations of our necessary conditions in (3.5) are independent of the tax rate if the term $s\delta qK$ is also tax deductible in addition to the fact that $D' = \delta\lambda - \dot{\lambda}$ and $\rho = (1 - u)r$. In sum, it seems that the exact conditions for obtaining neutrality of the whole dynamic path will vary to some extent between different models in the case of true economic depreciation and interest deductibility (see also King 1977, p. 239).

It should be noted that the capital stock equation (3.10) holds along the optimal path and that $\dot{I}$ is derived from that equation. Both $\dot{I}$ and $\dot{K}^*$ must be independent of the tax rate simultaneously because if only $\dot{I}$ is independent then with a change in parameters the initial jump (where $\dot{I}$ is not well-determined) could be non-neutral. However, as shown in section 3.2.3, the same neutrality conditions hold with respect to $\dot{I}$ and K^* (i.e. $z + y = 1$).

10. Gould and Waud (1973), Chang and Holt (1973), and Picou and Waud (1973) all assume the following demand function:

$$p = \alpha_0 S^{\alpha_1} Z^{\alpha_2}, \text{ where } p = \text{price of output,}$$

S = sales or output, and Z = a general shift parameter of demand (income, tastes etc.), see chapter V.

11. The separable form of the demand function has the advantage that the elasticity of demand (ε) and the marginal revenue (M) are both functions of p only (time does not enter as a separate argument, see Nickell 1978). More specifically

$$\varepsilon(p_t) = \frac{p_t}{D(p_t)Z_t} \cdot \frac{\partial D(p_t)Z_t}{\partial p_t} = \frac{p}{D} D_p$$

and

$$M(p_t) = p_t\left[1 + \frac{1}{\varepsilon(p_t)}\right] = p_t\left[1 - \frac{1}{|\varepsilon|}\right], \ \varepsilon < 0$$

A simple form of demand function where the elasticity of demand is a constant is given by $D(p) = Ap^{-\varepsilon}$, or $p(Q) = aQ^{-e}$, $\varepsilon = 1/e$.

12. The short-run profit maximizing equations of L and p are

i) $L_t^d = L(K_t, w_t, Z_t)$

ii) $p_t = p(K_t, w_t, Z_t)$

In the competitive case (chapter II) p is given and $L_t^d = F_L^{-1}[K_t, (\frac{w}{p})_t]$ which reduces to $L^d = L(K)$ when w/p is constant.

13. $$N_K = \frac{M}{F_{LL}}(F_{KK}F_{LL} - F_{KL}^2) + F_K M_p p_K$$

where $p_K = \frac{F_K}{D_p Z} < 0$. Hence $N_K < 0$ if $M_p > 0$ but N_K can be negative even if $M_p < 0$ (notice that $F_{LL} < 0$, $M > 0$, $F_K > 0$ and

$F_{KK}F_{LL} - F_{KL}^2 > 0$ being zero when there are constant returns to scale). Note that $f'' = \frac{p}{F_{LL}}(F_{KK}F_{LL}-F_{KL}^2)$, see chapter II. When demand is perfectly price elastic (infinite case), then the latter term in N_K is zero and hence $N_K = f''$.

14. The effect of a rise in the wage rate on K* and I* is again ambiguous but the sign of the effect now depends on the price elasticity of demand and on the properties of the production function (elasticity of substitution and returns to scale). Increasing wages may yield a positive effect if the substitution effect dominates the output effect (see also note 8, section 5.3 and Nickell 1978, chapters 2-3).

15. Note that, with a horizontal demand curve and constant returns to scale, the rate of adjusment is δ and is hence independent of the tax rate u, but with a downward-sloping demand curve the adjustment rate is endogenous and depends upon u. Thus the impact of tax factors (u, z) on the rate of adjustment is conditional on the choice of the demand regime (see also section 3.4). The general neutrality conditions of the dynamic path of investment presented in section 3.2 also hold in the present case.

16. Assuming a Cobb - Douglas production function $Q = K^a L^{1-a}$, the rate of adjustment is

$$-\bar{k}_1 = -\frac{\rho}{2} + \sqrt{\frac{(\rho+2\delta)^2}{4} + \frac{(1-u)aw}{q(1-uz)(1-a)C''}\left(\frac{\bar{Q}}{K^*}\right)^{\frac{1}{1-a}} \cdot \frac{1}{K^*}}$$

17. Denoting $\omega = \frac{dR/R}{dQ/Q}$ (holding K constant), it can be shown that if $\frac{d\omega}{dK}\Big|_{Q=\text{constant}}$ is negative then a rise in output (demand) causes the $(\dot{\lambda} = 0)$ locus to shift upward proportionately more than in the case when $\frac{d\omega}{dK}\Big|_{Q=\text{constant}}$ is positive (see Abel 1981). Abel (1981) also shows that $\frac{d\omega}{dK}\Big|_{Q=\text{constant}}$ is negative when the elasticity of substitution (σ) between capital and labour is less than one and if $\frac{d\sigma}{dK}\Big|_{Q=\text{constant}}$ is nonnegative. These conditions are clearly satisfied by a CES function with an elasticity of substitution of less than one. Hence, in the case of a CES function with an elasticity of substitution of less than one, the equilibrium value of the capital stock will be higher than in the case of, for example, a CD function (following an increase in demand).

18. See also note 17. The elasticity of the marginal rate of substitution with respect to Q (holding K constant) is given by

(i) $\omega = \frac{dR/R}{dQ/Q} = \frac{Q}{R} \cdot \frac{dR}{dQ} = \frac{Q}{R}\frac{\partial R}{\partial L}\frac{\partial L}{\partial Q} > 0$

because $R = \frac{F_K}{F_L} > 0$, $\frac{\partial L}{\partial Q} = \frac{1}{F_L} > 0$ and

$$\frac{\partial R}{\partial L} = \frac{F_{KL}F_L - F_K F_{LL}}{F_L^2} > 0 \quad (F_{KL} > 0,\ F_{LL} < 0)$$

In the case of constant returns to scale $KF_{KL} + LF_{LL} = 0$. Using the fact that $F_{LL} = (-K/L)F_{KL}$ we obtain for ω

(ii) $\omega = \frac{Q^2 F_{KL}}{LF_K F_L^2}$

and since the elasticity of substitution σ is $F_K F_L/FF_{LK}$ and $v = LF_L/F$ (labour share) we obtain for ω

(iii) $\omega = \frac{1}{\sigma v}$

This result shows that, the smaller is the elasticity of substitution, the larger is the elasticity of the marginal rate of substitution with respect to output (for given K). Hence low elasticity of substitution between capital and labour implies a greater impact effect on investment than a high elasticity (in response to an anticipated increase in demand).

CHAPTER IV

INVESTMENT BEHAVIOUR AND IMPERFECTIONS IN THE CAPITAL MARKET: SOME FURTHER RESULTS

4.1 Some Implications of Imperfect Financial Markets for Investment Decisions

In the preceding two chapters the neoclassical model of the firm's investment behaviour has been analyzed in a framework characterized by a constant (exogenously given) debt ratio. This hypothesis implies that a simple type of financial (capital) market imperfection is assumed, since the borrowing rate of interest and the discount rate are unequal. In this framework, the financial policy of the firm is exogenous and investment is the only decision variable (dividend policy is residual).

The aim of this chapter is to examine some alternative forms of financial market with respect to investment policy. Basically, two types of capital market imperfections will be analysed. First, it is assumed that the firm is faced with a nonlinear cost-of-debt capital schedule. Increasing costs of borrowing at the margin will result when the rate of interest is an increasing function of the debt to equity ratio. In this case, the financial policy of the firm is endogenous (choice variable), although of a simple form. The effect of a rising supply of debt funds on the investment decision is considered in section 4.2. Section 4.3 extends this model to the case where new equity issues are also used to finance investment and the corresponding concept for the user cost of capital is derived.

The second form of capital market imperfection to be examined in this chapter (section 4.4) stems from the possibility of a quantitative constraint on debt finance (an upper bound). The firm is assumed to face permanent or temporary credit rationing when making current investment decisions. Furthermore, two types of constraints will be examined. In section 4.4.1 it is assumed that dividend payments are institutionally (legally) restricted to a maximum amount which is dictated by current profits. This kind of a dividend constraint is shown to imply a borrowing constraint. In section 4.4.2 the firm is assumed to face a given debt to capital ratio which may lead to a permanently (or temporarily) profit-constrained investment function.

The effect of inflation (and taxes) on the demand for capital has already been examined in chapter II. For simplicity it is therefore assumed in this chapter that the rate of inflation is zero. This assumption does not, however, affect the basic qualitative results obtained here. The second restrictive assumption made is that the firm faces a horizontal demand curve since the effects of alternative demand regimes were analyzed in the previous chapter. Occasionally, it will be assumed that the firm faces strictly convex costs of adjustment, but when dealing with effective (binding) constraints it is assumed that the price of new investment goods is exogenously given in order to see the 'pure' effect of constraints on investment behaviour.

Since the interdependence of investment and financing policies is the major issue in this chapter we shall briefly consider some basic approaches to this problem presented in the earlier literature.[1]

The literature on the interdependence of the investment and financing decisions of the firm has basically recognized three different approaches. First, according to the Modigliani - Miller (MM) proposition, financial policy is of no relevance to the value of the firm and hence investment policy is not affected by the financial decision. Second, the 'traditional approach' in the corporate finance literature states that the firm has an optimal financial structure (debt ratio). Overall wealth maximization, in this case, can be viewed as a two-stage strategy in which the firm first chooses a financial policy that minimizes the cost of capital and then utilizes this cost of capital as the discount rate when solving the optimal investment policy (see, e.g., King 1977 and Auerbach 1979). In this case, investment and financing decisions are of a recursive character with respect to each other. Third, investment and financing decisions can be completely interdependent (simultaneous) for two reasons. If the cost of capital depends on the level of investment, then an adjustment-cost-type element is introduced into the behaviour of the firm. The other possibility is that the firm is faced with binding constraints on the use of debt or equity capital and these constraints may link investment and financing policies since investment may affect the exact nature of these constraints (King 1977, Auerbach and King, 1982).

The critical point in the interdependence issue is whether the cost of capital is exogenous to a firm or whether it is dependent upon the firm's own policies. There is no general answer to this question and it still poses a difficult and unresolved problem in the investment and finance literature of the firm.

The Modigliani - Miller proposition and the exact conditions under which it might hold have been widely discussed in the finance literature (see, e.g., Modigliani and Miller 1958, 1963, 1966, Baumol and Malkiel 1967, Stiglitz 1969, 1974, Scott 1976, Haugen and Senbet 1978, Kim 1978, Miller 1977, Hellwig 1981 and Auerbach and King 1982, 1983). Taxes (personal and corporate) and uncertainty (bankruptcy risk) play a central role in this debate. Modigliani and Miller (1963) themselves noted the effect of deductibility of interest payments. The value of a levered firm (V_L) can be found by adding the discounted value of tax savings to the value of an otherwise identical all-equity firm (V_E). The value of a levered firm is thus (in a simplified form) $V_L = V_E + uB$, where u is the corporate tax rate and B is debt.

The problem with this model, however, is that it leads to the extreme use of debt finance if tax savings can increase without limit. The traditional approach to financial policy basically argues that the cost-of-capital schedule is U-shaped (Duesenberry 1958, Lintner 1967, Robichek and Myers 1965). Explicit consideration of the possibility of bankruptcy and bankruptcy costs has later provided the formal basis for a nonlinear cost-of-capital curve (see, e.g. Scott 1976, Kim 1978 etc.). The basic result of this approach can be presented by the following valuation formula for the firm: $V_L = V_E + uB - V(BC)$, where V(BC) stands for the present value of bankruptcy costs. When leverage increases, tax savings (uB) will also increase but this is counterbalanced by rising bankruptcy costs (V(BC)). Hence, this model may yield an optimal financial structure for the firm. Attempts are frequently made to capture the effect of bankruptcy costs (uncertainty) by a nonlinear borrowing cost function, implying that the rate of interest is a rising function of the debt-equity ratio. This assumption will also be used in the next section of this study and it can be noted that essentially it means that the firm may have an 'optimal' financial structure even in the context of certainty (see section (4.2)).[2]

Already long ago, it was argued that investment and financing decisions may be completely interdependent (see, e.g. Kalecki 1937, Lutz and Lutz 1951 and Hirshleifer 1958). As mentioned above, complete interdependence will follow if the cost of capital depends on the level of investment (see Duesenberry 1958 and Nickell 1978). This argument is, however, rather difficult to defend since the cost of capital (in per cent) is measured in completely different units than the level of investment (at current or fixed prices). It would seem more natural to assume that the cost of capital is a function of the ratio of stock variables such as debt and equity. Inselbag (1973) argues that r is a function of both B/K and the rate of change in debt. It should be noted that a cost-of-capital function of the form $c = c(I)$ would be analogous to a strictly convex adjustment cost model when $c' > 0$ and $c'' > 0$ (see also Nickell 1978).

Perhaps a more realistic situation for the interdependence of investment and financing policies occurs if investment is limited by a quantitative constraint on debt finance. Static investment models which assume 'permanent' credit rationing have been analyzed by Hirshleifer (1958), V.L. Smith (1961), Haavelmo (1961) and Vickers (1968). The literature related to credit rationing has subsequently also developed 'rational arguments' for the occurrence of credit rationing (see, e.g., Jaffee 1971, V.L. Smith 1972, Jaffee and Russell 1976, Koskela 1976 and Stiglitz and Weiss 1981). The dynamic effects of credit rationing on investment have been analyzed by Appelbaum and Harris (1978) and Schworm (1980) and our analysis in section 4.4 is related to the work of these authors.

King (1974, 1977) and Boadway (1980) consider neoclassical certainty models of the firm which imply a borrowing constraint because of institutional (legal) restrictions on the maximum amount of dividends. More generally, if in a certainty model there exist differential taxes for dividends and retentions (i.e. capital gains), then one usually has to assume certain constraints on debt or equity in order to prevent an infinite tax arbitrage between the personal and corporate sectors (see also Auerbach and King 1982, 1983). These situations can also give rise to the interdependence between investment and financing decisions because the cost of capital will contain the multipliers corresponding to the binding constraints, thus linking present and future investment and financial policies (see King 1977, and section 4.4.1 of this study).

4.2 The Effect of Increasing Borrowing Costs on Investment

In the preceding two chapters, it was assumed that the firm has an exogenously given (constant) debt-capital ratio. This simple capital market imperfection implies that firms use both debt and equity (internal) capital and that the rate of interest on debt and the discount rate may diverge. In chapter II (section 2.2) we also briefly discussed alternative explanations for this assumption (i.e. constraints on debt or equity, or uncertainty). In this section it is assumed that the firm faces a non-linear cost-of-debt schedule.[3] More specifically, we assume that the rate of interest on the entire debt (r) is an increasing function of the debt-equity ratio ($e = B/E$).[4] Both economic theory and the practice in the real world suggest that the returns demanded by lenders (banks) tend to increase with the leverage of the borrowing firm (see the discussion in section 4.1). This assumption about the financial market is basically linked to the other motivation (uncertainty), given in chapter II, for the existence of an interior solution for the debt-capital ratio (i.e. $0 < s < 1$). The financial policy of the firm is now endogenous, albeit of a simple form.

The present model is similar to the one presented in chapter II. Now, however, it is assumed that the interest rate function is of the following form:

$$(4.1) \quad r = r(e), \; r'(e) > 0$$

where $e = B/E$, or $e = B/(qK-B)$ since $qK = B+E$.

The marginal cost function of borrowed capital is given by

(4.2) i) $h(e) = \frac{\partial r(e)B}{\partial B} = r(e) + er'(e)$

and its derivative is

ii) $h'(e) = 2r'(e) + er''(e)$

It can be noted that $h'(e) > 0$ if $r'(e) > 0$ and hence it is not necessary to assume that $r'' > 0$ in order that the marginal cost of funds is (strictly) increasing. It can easily be shown that the function $r(e)B \equiv y(K,B)$ is convex and linearly homogeneous in K and B.[5]

The maximization problem of the firm can now be expressed as

$$\max_{\{I,N\}} \int_0^\infty e^{-\rho t}[(1-u)f(K)-(1-uz)qI-(1-u)r(e)B+N-\gamma B]dt$$

subject to i) $\dot{K} = I-\delta K$ and ii) $\dot{B} = N - \gamma B$ and the usual nonnegativity constraints on variables (plus initial values for K and B). The equation of motion for debt is the same as used in chapter II. Hence N stands for new loans and γ is the rate of amortization on the existing debt capital. The current value Hamiltonian is now

(4.3) $H^c = (1-u)(f(K)-r(e)B) - (1-uz)qI + N - \gamma B + \lambda_1(I-\delta K) + \lambda_2(N-\gamma B)$

where λ_1 and λ_2 are the costate variables associated with K and B, respectively. The necessary conditions for an optimum are as follows:

(4.4) i) $\dot{\lambda}_1 = (\rho+\delta)\lambda_1 - (1-u)f'(K) - q(1-u)r'e^2$

ii) $\dot{\lambda}_2 = (\rho+\gamma)\lambda_2 + (1-u)h + \gamma$

iii) $\frac{\partial H^c}{\partial I} = -q + uzq + \lambda_1 = 0$

iv) $\frac{\partial H^c}{\partial N} = 1 + \lambda_2 = 0$

Since $r'e = h-r$ and $\partial rB/\partial K = -e(h-r)$, it follows that $q(1-u)r'e^2 = qe(1-u)(h-r)$ where h and r are functions of e.

Sufficiency follows if $f'' < 0$ and since the interest cost function ($r(e)B = y(K,B)$) is convex in K and B. Hence the Hamiltonian is jointly concave in K and B (notice that I enters H only linearly). The usual transversality conditions are also satisfied since λ_1 and λ_2 are constant (for sufficiency propositions, see chapter II).

It should be noted that this model gives only equilibrium values of K and B since both shadow prices (λ_1 and λ_2) are constant. Hence investment and borrowing decisions are myopic in this model (see also chapter II).

The equilibrium condition for the financial market is

$$(4.5)\quad \rho = (1-u)h(e) \text{ or } \rho = (1-u)\frac{\partial r(e)B}{\partial B}$$

According to (4.5), the firm instantaneously adjusts the debt-equity ratio so as to make the after-tax marginal cost of debt equal to the discount rate (constant). Thus, the debt-equity ratio is constant for each t (i.e. $e = e^*$).

The equilibrium condition for the capital stock is

$$(4.6)\quad f'(K^*) = \frac{q}{(1-u)}\left[(\rho+\delta)(1-uz)+e((1-u)r-\rho)\right]$$

which again directly gives the formula for the user cost of capital ($f' = c$). This result for c is equivalent to those obtained previously except that the user cost now depends on e rather than on s. However, since the debt-equity ratio is constant ($e = e^*$), the debt-capital ratio must also be constant and there is a one-to-one correspondence between K and B. The equilibrium debt-capital ratio (s^*) is given by $s^* = e^*/(1+e^*)$.

Using the relationships $\rho = (1-u)h$ and $r = h-r'e$, the formula for the user cost can also be written as $c = q[(\rho+\delta)(1-uz) - (1-u)r'e^2]/(1-u)$. It can be seen directly that if $r' = 0$ (i.e. r is constant), the standard Jorgenson formula for the user cost follows (see section 2.1). The two standard neutrality conditions of the corporate tax system are also satisfied in this model.

The comparative static properties of the present model with respect to K* are the same as in chapter II (with respect to p, q, w, δ, α and u). Some new aspects are also present, however. A positive shift in the marginal return on capital increases the long-run demand for capital but it leaves the equilibrium debt-equity ratio unchanged. A positive shift in the cost-of-funds function decreases the equilibrium values of both the capital stock (K* is lowered) and the debt-equity ratio (e* is lowered). Basically, these results follow because the discount rate is an exogenously given constant. The effect of an increase in the discount rate on K* is ambiguous but it causes a substitution effect between debt and equity finance (i.e. the equilibrium debt-equity ratio increases).

We shall next consider the above model in the presence of strictly convex adjustment costs. The basic characteristics of an adjustment-cost-based investment model were described in chapter III and hence here we only briefly present the underlying model.

The current value Hamiltonian is now given by

$$(4.7) \quad H^c = (1-u)f(K) - (1-uz)q(I+C(I)) - (1-u)r(e)B + N - \gamma B + \lambda_1(I-\delta K) + \lambda_2(N-\gamma B)$$

The equilibrium condition for the financial market is the same as in the case without convex adjustment costs, i.e. $\rho = (1-u)h(e^*)$, see eq. (4.5). The equilibrium condition for the capital stock is given by

$$(4.8) \quad f'(K^*) = \frac{q}{(1-u)}[(\rho+\delta)(1-uz)(1+C')+e((1-u)r-\rho)]$$

which is equivalent to (3.11) except that this formula for the user cost now includes the debt-equity ratio (e) instead of the debt-capital ratio (s).

As usual in an adjustment cost model, investment is an increasing function of the shadow price of capital and the relationship between I and λ_1 is given by

$$(4.9) \quad I = C'^{-1}\{\frac{\lambda_1}{q(1-uz)} - 1\}$$

This relationship (4.9) is, however, different from the corresponding one in chapter III (see eq. (3.12)) because now the marginal cost of funds affects λ_1. Since λ_1 is the marginal value of an additional unit of installed capital and q(1-uz) is the tax-adjusted price of uninstalled investment goods, then the ratio $\lambda_1/q(1-uz)$ is a marginal Tobin's "q"-variable and, according to (4.9), investment is an increasing function of ("q"-1).

The shadow price of capital is given by

$$(4.10) \quad \lambda_{1,t} = \int_0^\infty e^{-(\rho+\delta)(v-t)}[(1-u)f'(K_v)+q(1-u)r'e^2]dv$$

This states that the shadow price of capital is equal to the present value of the marginal benefits of a larger capital stock (net of depreciation). In the present case, these marginal benefits are of two kinds: after-tax profits in production will increase and the rate of interest on debt will go down. It should be noted that r'(e) is a function of K and when we consider changes in one of the state variables (K), holding the other constant (B), then E will always change by the same amount because of the balance sheet identity qK = B+E (notice also that $\partial y(K,B)/\partial K = r'B(\partial e/\partial K) = -r'e^2 < 0$, see also note 5).

In the case of constant returns to scale in the production function, f'(K) can be replaced by $p\bar{g}_\ell(w/p)$, where $\ell = K/L$ (see section 3.2), and, assuming that all prices (q, w and p) are fixed over time, the shadow price of capital becomes $\lambda_1 = (1-u)[p\bar{g}_\ell(w/p) + qr'e^2]/(\rho+\delta)$ and the investment function is

$$(4.11) \quad I = C'^{-1}\{\frac{(1-u)[p\bar{g}_\ell(w/p)+qr'e^2]}{q(1-uz)(\rho+\delta)}\}$$

According to this equation, the rate of investment will be constant along the solution path. The model with a perfect capital market is a special case of this model and is obtained by setting r' = 0, implying that the debt-equity ratio is undetermined. It can be seen from eq. (4.11) that the level of investment is higher in the case of an imperfect financial market than in the case of a perfect market. In section 3.2 the same result was obtained by assuming directly an exogenously given, constant debt ratio. Furthermore, a rise in the product price p will increase investment and a rise in the wage rate will decrease investment provided that $\partial(w/p)/\partial w > 0$. The effect of an increase in the tax rate on investment is ambiguous, and investment is independent of the tax rate u in the two standard neutrality systems (see chapter III).

The model with increasing borrowing costs is a dynamic optimization problem with two state variables (K and B) and two control variables (I and N). The balance sheet identity is $E = qK - B$, which implies that, if we are considering changes in one of the state variables with the other held constant, then E will always be changed by the same amount. However, the financial market equilibrium condition $\rho = (1-u)h$ means that the debt-equity ratio is adjusted so as to equate the after-tax marginal interest rate and the discount rate. Hence the debt-equity ratio is constant along the optimal solution path. Furthermore, since the shadow price of debt λ_2 is a constant ($\lambda_2 = -1$, $\dot{\lambda}_2 = 0$), we can examine the properties of this model in a phase diagram in the (λ_1,K) space. The slope of the ($\dot{\lambda}_1 = 0$) locus is given by

$$(4.12)\quad \left.\frac{d\lambda_1}{dK}\right|_{\dot{\lambda}_1=0} = \frac{(1-u)(f''+qee_K h')}{(\rho + \delta)} < 0$$

The negativity (i.e. downward sloping) in (4.12) follows since $f'' < 0$ and $e_K = \partial e/\partial K = -qe/E < 0$. It should be noted that the ($\dot{\lambda}_1 = 0$) locus can be downward sloping even in the case of a linearly homogeneous production function (i.e. $f'' = 0$), and a horizontal locus follows if either $h' = 0$, implying a perfect capital market ($r' = r'' = 0$), or $e_K = 0$, implying that a change in the capital stock does not affect the debt-equity ratio (i.e. e = constant). The ($\dot{K} = 0$) locus slopes upward as usual (see chapter III).

The phase diagram for this model is similar to that for the constant debt ratio adjustment cost model (see section 3.2). Drawing the arrows of motion in the (λ_1,K) space would show that the steady state is a saddle point. Next, however, we shall consider a bit further the dynamic properties of the present model. The basic question to be answered is: How does the non-linear cost function of borrowed capital affect the speed of adjustment to the equilibrium level of the capital stock?

The linearized system of the differential equations for λ_1 and K is otherwise the same as in eq. (3.17) except that the term $-(1-u)f''$ is replaced by the term $-(1-u)(f''+qee_K h')$. It can be shown that the characteristic roots of this system are real and opposite in sign. Hence the steady state is a saddle point. If this system is to reach its steady state, then the movement of the capital stock must be governed entirely by the negative root, say $\tilde{k}_1$. The speed of adjustment is now given by

$$(4.13) \quad -\tilde{k}_1 = -\frac{\rho}{2} + \sqrt{\frac{(\rho+2\delta)^2}{4} - \frac{(1-u)}{q(1-uz)C''}[f''+qee_K h']}$$

where the term in the squared root is positive since $f'' < 0$, $h' > 0$ and $e_K < 0$.

The following observations can now be made about of the rate of adjustment and factors affecting it. First, the general outcome is that the rate of adjustment is an endogenous choice variable and it depends on the properties of the production function (via f"), the conditions in the financial market (via e, e_K and h') and the tax parameters (u and z). Second, the original result by Gould (1968), according to which the rate of adjustment is equal to the rate of depreciation δ, follows as a special case if either $f'' = h' = 0$, implying constant returns to scale and a perfect capital market, or $f'' = e_K = 0$, implying constant returns to scale and a capital market imperfection such that the cost of debt is non-linear but the debt-equity ratio is constant. Hence linear homogeneity of the production function is not sufficient for Gould's result to hold in an imperfect financial market. However, if the debt-equity ratio is constant, as our model implies, then, with constant returns to scale, the unique optimal path is characterized by a constant λ_1 and consequently a constant investment level. The third feature of the result in (4.13) is that generally the rate of adjustment is greater than the depreciation coefficient δ.

Analogously to the analysis in chapter III, it can be shown that the dynamic path of investment is independent of the tax rate (u) when the two standard neutrality conditions hold, i.e. free depreciation (z = 1) plus non-deductibility of interest ($\rho = h$) or true economic depreciation ($D'(K) = \delta\lambda - \dot{\lambda}$) plus deductibility of interest ($\rho = (1-u)h$) (see also note 15 of chapter III). Equation (4.13) again reveals the 'local' neutrality character of the rate of adjustment when z = 1.

We can now briefly summarize the main results of the present section. The conditions in the capital market will affect the expansion path of the capital stock of the firm. Basically, this is due to the assumption that the discount rate is fixed whereas the rate of interest on debt is a function of the debt-equity ratio. The properties of the non-linear interest cost function also influence the rate of investment up to the equilibrium level of the capital stock. The equilibrium value of the debt-equity ratio is determined by equality between the discount rate and the net-of-tax marginal cost of borrowed capital. Given the constant steady-state value of the debt-equity ratio, the steady-state value of the capital stock is determined by the conditions in the markets for output (demand) and other inputs. In the present model the firm effectively follows a two-stage strategy in its investment behaviour (see also section 4.1). The firm first determines its minimum cost of capital and then uses this to determine its optimal investment policy.

4.3 New Equity Issues and the User Cost of Capital

Hitherto our analysis has dealt with only two sources of funds for financing investment expenditures, i.e. borrowing and retained earnings. In this section, we shall consider a simple way of incorporating the cost of new equity issues (i.e. external equity capital) in the total cost of capital. This extension of the model of the firm implies that personal taxes also enter the formula for the user cost of capital. The chief aim of the present section is to derive a formula for the user cost which is directly comparable to the original Jorgenson formula and to our preceding formulas. The objective of the firm is again to maximize the present value of future dividend payments, i.e. maximization of the wealth of the owners of the firm. It is, furthermore, assumed that adjustment costs are linear, implying that we only consider the long-run demand for capital.

The capital market equilibrium condition is defined in the usual way (see, e.g. King 1977 and Poterba and Summers 1983):

$$(4.14) \quad iV_t = (1-m)Div_t + (1-\tau)(\dot{V}_t-S)$$

where V is the market value of the firm, $\dot{V} = dV/dt$ is the time rate of change of V, i.e. nominal capital gain accruing at t, Div is gross dividend payments and E is new equity issues (paid-in share capital). The constant parameters in (4.14) are as follows: i = nominal required rate of return by stockholders (net of all taxes), m = marginal personal income tax rate (dividend tax rate) and τ = effective tax rate on capital gains (on accrual basis).[6]

It is assumed that the tax rates m and τ correspond to those of a representative (marginal) shareholder. The discount rate i is a risk-free nominal return on some alternative financial investment (after all taxes). If the before-tax return is j, then i = (1-m)j. The cash flow accruing to a shareholder (at t) is given by

$$(4.15) \quad (1-m)Div_t - S_t - \tau(\dot{V}_t-S_t)$$

The value of the firm as seen from the viewpoint of shareholders is the present value of future expected cash flows net of taxes and it is defined by

$$(4.16) \quad V_t = \int_t^\infty e^{-i(v-t)}[(1-m)Div_v-S_v-\tau(\dot{V}_v-S_v)]dv$$

In principle this valuation model shows how the firm is valued by rational investors in efficient markets. As can be seen from (4.16), personal taxes are involved in this valuation principle (i.e. in the pricing of stocks).

In order to simplify (4.16), we take the derivative with respect to the lower limit of integration, t. By integrating and rearranging terms, the value of the capital stock (V_t) can be rewritten as (at time zero)[7]

(4.17) $V = \int_0^{\infty} e^{-\rho t}[\frac{1-m}{1-\tau}Div_t - S_t]dt$

where $\rho = i/(1-\tau)$. If wealth taxes paid by individuals are incorporated in the net cash flow expression (4.15), then $\rho = (i+w)/(1-\tau)$, where w is the marginal wealth tax rate (see Ylä-Liedenpohja 1983). It can be seen from (4.17) that, when S = 0, the firm's maximization problem does not depend on the personal income tax rate m but on the capital gains tax rate τ via the discount factor.

In order to formulate the maximization problem of the firm in the present context, we need to know the cash flow identity of the firm (i.e. dividend expression), which is given by

(4.18) $Div = f(K) - qI - r(e)B - T + \dot{B} + S$

where it is assumed that the rate of interest depends on the debt-equity ratio (see section 4.2). If corporate taxes are defined in the usual way, i.e. $T = u(f(K)-rB-D)$, then T corresponds to the previous cases in the context of a classical corporate tax system. We shall, however, assume in the present section that the tax system corresponds to a two-rate scheme in which a given proportion of distributed profits can be deducted from the tax base.[8] Corporate income taxes are now defined as

(4.19) $T = u(f(K)-rB-D-\beta Div)$

where β is the proportion of paid-out dividends that can be deducted from the taxable income. Eq. (4.19) can be transformed into the following form

(4.20) $T = u(f(K)-rB-D-Div) + u_1 Div$

where $u_1 = u(1-\beta)$ is the tax rate on distributed profits.

Substituting T, given by (4.20), into (4.18), gives the following expression for dividends

$$(4.21)\quad Div = u_2[(1-u)(f(K)-rB)-qI+uD+\dot{B}+S],\ \text{where}\ u_2 = \frac{1}{1-u+u_1}.$$

In order to simplify the subsequent analysis, we shall use the tax discrimination variable defined by King (1974, 1977). This variable (θ) measures the degree of discrimination between retentions and distributions. Variable θ is defined as the opportunity cost of retained earnings in terms of (net) dividends foregone, i.e. the amount of cash flow which shareholders could receive if one unit of retained earnings were distributed. If cash in the hands of the firm and cash in the hands of the shareholder can be interchanged without any additional tax, then there is no discrimination and the value of θ is unity. If θ is less than unity, then dividends are taxed more heavily than retained earnings.

The tax discrimination variable can be defined either in terms of net dividends or in terms of gross dividends foregone. We use the net concept and in our model it is given by

$$(4.22)\quad \theta = \frac{1-m}{(1-\tau)(1-u+u_1)}$$

and if $u = u_1$, then $\theta = (1-m)/(1-\tau)$. If $\tilde{\theta}$ is the 'gross dividend' tax discrimination variable, then $\theta = (1-m)\tilde{\theta}$.

The crucial problem that now remains is the determination of new equity issues (S). We shall use the same assumption as in Södersten (1977) and Ylä-Liedenpohja (1983), namely, that the firm finances an exogenously given proportion of net investment by issuing new ordinary shares. This assumption implies that S is given by the following relationship

$$(4.23)\quad S = nq\dot{K} = nq(I-\delta K)$$

We have already discussed briefly in section 4.1 and in note 1 of this chapter the difficult and as yet unresolved question as to the 'general' optimum of financial and dividend policies of the firm in the neoclassical certainty framework. It should be emphasized that we are, at present, assuming that firms use three types of finance (i.e. borrowing, new issues and retained earnings) and possibly also pay out dividends if retained earnings are in excess of the remaining investment financing need after utilizing borrowing and new issues. We do not try to explain why such a financial policy is actually chosen by the firm. Rather, we pose the question: What, given the firm's observed financial policy, is the concept of the user cost of capital and how do various factors affect it? Effectively, we are assuming that the firm has an interior solution with respect to the three types of finance and that corner solutions (or bounds on investment) are ruled out, i.e. we treat the problem as if there were no bounds, meaning that we examine only free intervals (control problems with bounded investment plans are examined in section 4.4). In note 9, however, we present some simple calculations with typical Finnish parameter values for the 'optimality' of alternative financial methods using the 'pairwise comparison' approach as suggested by King (1977).[9]

The maximization problem of the firm is now given by the following formulation (assuming fixed prices and $f'' < 0$):

$$\max_{\{I,N\}} \int_0^\infty e^{-\rho t}\{\theta[(1-u)f(K)-q(1-uz)I-(1-u)r(e)B+N-\gamma B] - (1-\theta)nq(I-\delta K)\}dt$$

subject to i) $\dot{K} = I - \delta K$ and ii) $\dot{B} = N - \gamma B$. As mentioned above, it should be noticed that the firm is assumed to optimize the debt-equity ratio and since the proportion of new issues is a given constant n then retained earnings are a residual factor in the flow financing identity of gross investment (i.e. $qI = R^g + \dot{B} + S$, see also eq. 2.13).[10]

The necessary conditions for an optimum are now as follows:

$$(4.24)\quad \text{i)}\quad \dot{\lambda}_1 = (\rho+\delta)\lambda_1 - \theta(1-u)f'(K) - \theta(1-u)qe(h-r) - (1-\theta)nq\delta$$

$$\text{ii)}\quad \dot{\lambda}_2 = (\rho+\gamma)\lambda_2 + \theta(1-u)h + \theta\gamma$$

$$\text{iii)}\quad \frac{\partial H^c}{\partial I} = -\theta q + \theta uzq - (1-\theta)nq + \lambda_1 = 0$$

$$\text{iv)}\quad \frac{\partial H^c}{\partial N} = \theta + \lambda_2 = 0$$

where λ_1 and λ_2 are the shadow prices of K and B as in section 4.2.

Condition iv) gives $\lambda_2 = -\theta$ (=constant) and the equilibrium condition for the debt-equity ratio can be solved from ii), i.e. $\rho = (1-u)h$. In this respect the present model is the same as in the previous case (section 4.2). The marginal condition for the capital stock is now

$$(4.25)\quad f'(K^*) = \frac{q}{(1-u)}\left[(\rho+\delta)(1-uz)+e((1-u)r-\rho)+\frac{(1-\theta)}{\theta}n\rho\right]$$

which also gives directly the formula for the user cost of capital (i.e. $f' = c$). If $n = 0$, we obtain the same result as in the previous section (see eq. 4.6). The last term $(1-\theta)n\rho/\theta$ in the brackets is the additional cost of newly raised outside equity in excess of the cost of retained earnings due to the differential taxation of dividends and capital gains. If $\theta = 1$, then this last term is zero and we obtain formula (4.6) for c. Recalling the above definition of θ, we can observe that $(1-\theta)/\theta$ is the ratio of additional tax payments to the additional cash flow for shareholders when one additional unit of retentions is distributed. Hence the additional tax per unit of the shareholder's cash flow is $(1-\theta)/\theta$ and it carries the before-tax opportunity cost $\rho/(1-u)$ (in terms of deferred consumption). The unit cost of new equity finance is equal to $\rho/\theta(1-u)$ (see also King 1977, Södersten 1977).

The presence of personal tax parameters (dividend taxes) in the formula for the cost of capital means that the tax neutrality of the user cost is now somewhat different from the two previous standard cases. We may note that when investment is financed by new issues and retained earnings (i.e. $e = 0$), the tax system will be nondistortionary if (i) $z = 1$ and $\Theta = 1$, implying that $u = u_1$ (classical corporate tax system) and $\tau = m$, or (ii) $z = \delta/(\rho+\delta)$ and $u_1 = 0$ plus $m = \tau$ (for a more thorough discussion of neutrality in the presence of both corporate and personal taxes, see King 1977, p. 237).

We shall next transform the user cost formula (4.25) into a form such that the 'weighted average' concept of the cost of capital shows up. In section 4.2 it was pointed out that the function $y = y(qK,B) \equiv r(e)B$ is linearly homogeneous in qK and B. This function may therefore be rewritten as $y = qKy(1,B/qK) \equiv qK\bar{y}(s)$, where $s = B/qK$ is the debt-capital ratio. Using this transformation of y and $z = \alpha/(\rho+\alpha)$, formula (4.25) for the user cost can be transformed into the following form:

$$(4.26) \quad f'(K^*) = c = q\left[\delta+sr+ \frac{n\rho}{\theta(1-u)} + \frac{(1-s-n)\rho}{(1-u)} - \frac{u\rho(\alpha-\delta)}{(1-u)(\alpha+\rho)}\right]$$

The nominal cost of capital is now:

$$(4.27) \quad cc_n = sr + \frac{n\rho}{\theta(1-u)} + \frac{\rho}{(1-u)}\left[1-s-n- \frac{u(\alpha-\delta)}{(\rho+\alpha)}\right]$$

This is a 'weighted average' form of the cost of capital and the weights are the proportions of investment financed by the three types of funds: s is the portion of investment financed by borrowing, n is the portion financed by new share issues and (1-s-n) is the portion financed by retained earnings (when $\alpha = \delta$). The cost of new issues is $\rho/\theta(1-u)$ and the cost of retentions is $\rho/(1-u)$. If $\theta < 1$, then retained earnings are a less expensive source of funds than new share issues (i.e. capital gains are less heavily taxed than dividend payments). If the rate of tax depreciation is greater than the economic rate of depreciation, i.e. $\alpha > \delta$, then the firm is allowed tax deferrals. This means that it can acquire an interest-free loan ('tax credit') from the government. The weight of this 'tax credit' component is $u(\alpha-\delta)/(\rho+\alpha)$ (see chapter II).

Finally in this section, we shall compare the result given by eq. (4.27) with some other concepts of the cost of capital. The concept of the cost of capital is important since different assumptions about the firm's marginal source of finance have different implications for the investment consequences of dividend taxes. At present, there exist competing views of how dividend taxes affect decisions by firms and shareholders (see e.g. Auerbach 1983, Poterba and Summers 1983). The 'traditional view' argues that, for some poorly understood reason, firms act as if they are required to distribute a substantial proportion of profits as dividends. If p is the dividend-payout ratio and ρ is the post-tax rate of return demanded by investors, then the 'weighted average' cost of capital for the case in which debt and equity finance are used at the margin is given by (see Auerbach, 1983):

$$(4.28) \quad cc_n = sr + (1-s)\frac{\rho}{(1-u)[1-(pm+(1-p)\tau)]}$$

where it is sometimes assumed that $\rho = \rho(p)$, $\rho' < 0$. Since the cost of capital depends on tax rates, changes in either the personal dividend tax rate (m) or in the capital gains tax rate (τ) will affect investment policy. The difficulty with this view of dividend taxes is that it provides no explanation as to why firms pay dividends. Furthermore, the real world parameters are usually such that firms should use only debt finance at the margin and should pay no dividends (see, e.g. Auerbach 1983).[11]

The tax capitalization view ('new view') of dividend taxes was partly developed as a response to the problem of explaining why firms pay dividends (see e.g. King 1977, Auerbach 1983 etc.). The premise of this view is that future taxes are capitalized into share values and hence shareholders are indifferent at the margin between retaining earnings or paying dividends. Raising dividend taxes would result in an immediate decline in the market value of equity but dividend taxes have no impact on a firm's marginal incentive to invest. The cost of equity capital is equal to $\rho/(1-u)(1-\tau)$ and hence it is independent of the dividend tax rate m. It can now be noted that our standard concept of the cost of equity capital is in accordance with this 'new view' (see eq. 4.17 with $S = 0$ or eq. 4.27 with $\theta = 1$, and note that ρ is now the after-tax required rate of return, i.e. $\rho = i$, see eq. 4.14).[12]

According to the 'new view' retained earnings are the marginal source of investment funds. Hence the firm pays a taxable dividend equal to the excess of current net profits over current investment. Dividends are thus determined as a residual (see also note 1). The main problem with this view is that it predicts volatile dividend fluctuations which does not seem to occur in the real world. Another problem is that firms should continue to invest until investors are indifferent between earnings paid out or retained, or when marginal "q" equals: "q" = $\frac{1-m}{1-\tau}$. Empirical evidence, however, suggests that changes in the dividend tax rate (m) do not necessarily influence "q" in a manner consistent with this view (see Poterba and Summers 1983).

The significance of the preceding discussion lies in the fact that in an empirical analysis of investment behaviour one has to choose between different concepts of the equity cost of capital employed. At one extreme there is the 'old view' that this cost is equal to $\rho/(1-u)(1-m)$ if the dividend payout ratio is one, while at the other extreme is the 'new view' that this cost equals $\rho/(1-u)(1-\tau)$. In our empirical analysis of the investment behaviour of Finnish firms (chapter V) we shall apply both of these approaches.[13]

4.4 Financial Constraints and Investment Behaviour

In the previous section it was assumed that firms face a nonlinear price schedule in the credit market. This assumption implies that the firm may obtain an optimal financial structure since the rising marginal cost of funds counterbalances the tax savings arising from interest deductions. This model implies that the rate of interest is perfectly flexible (variable). However, in many cases a hypothesis of more or less rigid interest rates may seem to conform better with the facts of the real world than the perfectly flexible case (see chapter II for a discussion of the Finnish system). Imperfect screening possibilities between different customers (borrowers) or sticky interest rates due to convention or institutional regulations may cause the lenders (banks) to practice credit rationing (see e.g. Jaffee and Russell 1976 and Koskela 1976). Recently, Stiglitz and Weiss (1981) have shown that, even in the case of flexible interest rates, credit rationing may be optimal behaviour for the banks since the expected profits of banks may start to decline at some 'high' level of the interest rate because of 'incentive effects'.

In this section we shall examine the effects of quantitative financial constraints on the firm's investment policy. The financial structure of the firm is now determined solely by constraints on the availability of funds (i.e. borrowing). Basically, we are now dealing with 'optimal' investment behaviour subject to financially bounded investment plans. In the first section (4.4.1) it is assumed that the firm faces a dividend constraint which implies a certain type of borrowing constraint. In the second section (4.4.2) it is assumed that the firm operates under a permanent or temporary profit constraint.

4.4.1 Dividend Constraint and Investment Behaviour

In many countries the corporate tax laws stipulate that dividend payments cannot exceed the current accounting (book) profits. The purpose of such a constraint is to prevent firms from using borrowing merely to increase dividends. It will be shown here that this type of a dividend constraint implies a borrowing constraint on the firm's investment decision. The importance of dividend constraints on the cost of capital has previously been recognized in King (1974) and in Boadway and Bruce (1979).

King (1974) considers various concepts of the cost of capital in a discrete time framework and includes both personal and corporate taxes in his analysis. The main differences between our approach and that of Boadway and Bruce (1979) are as follows: (i) Boadway and Bruce assume that the objective of the firm is to maximize the utility of consumption whereas we assume present value maximization (i.e. wealth maximization), (ii) in Boadway and Bruce personal borrowing is also included and its shadow price affects the user cost of capital; we ignore personal borrowing, and (iii) in Boadway and Bruce the concepts of the two capital stocks (i.e. K and K_H, see section 2.2) differ only with respect to the rate of depreciation, i.e. they assume that $I = \dot{K} + \delta K = \dot{K}_H + \alpha K_H$, whereas we assume that $I = \dot{K} + \delta K$ and $\dot{K}_H = qI - \alpha K_H$.[14] Our alternative assumptions lead to a different formula for the user cost of capital, although the basic idea behind these models is the same. Our assumption concerning the definition of the historic cost capital stock (evaluated at the tax depreciation rate) can be defended on the grounds that it corresponds to the practice in the real world. Boadway and Bruce also restrict their analysis to the case where $r = \rho$ whereas we consider a more general case where $\rho > (1-u)r$ (a special case of which is that $\rho = r$).

In the absence of uncertainty and personal taxes, the ability of firms to deduct interest payments from the tax base makes debt strictly preferable to retentions (equity) as a financing method (see also note 9 of this chapter). Some constraint is clearly needed to prevent firms from engaging in infinite borrowing so as just to be able pay higher dividends. If dividends cannot exceed the current accounting (book) profits, the firm is faced with the following constraint:

(4.29) $(1-u)(f(K)-D-rB) - Div \geq 0$

Using the expression for dividends, i.e. $Div = (1-u)f(K) - qI + uD - (1-u)rB + \dot{B}$, this constraint can be transformed into the following form:[15]

(4.30) $qI - D + \dot{B} \geq 0$ or $\dot{K}_H - \dot{B} \geq 0$

since $\dot{K}_H = qI - D = qI - \alpha K_H$. Noting, furthermore, that $\dot{B} = N - \gamma B$ (see section 4.2), this constraint can also be expressed as

(4.31) $qI - \alpha K_H - N + \gamma B \geq 0$

The constraint $\dot{K}_H - \dot{B} \geq 0$ implies that debt increases at the same rate as the historic cost concept of the capital stock (evaluated at the tax depreciation rate) if the constraint is binding.

Assuming fixed prices, the maximization problem of the firm is now given by

$$\max_{\{I,N\}} \int_0^\infty e^{-\rho t}[(1-u)f(K)-qI+u\alpha K_H-(1-u)rB+\dot{B}]dt$$

subject to i) $\dot{K} = I - \delta K$

ii) $\dot{K}_H = qI - \alpha K_H$

iii) $\dot{B} = N - \gamma B$

iv) $qI - \alpha K_H - N + \gamma B \geq 0$

and the usual non-negativity constraints ($K > 0$, $K_H > 0$, $B > 0$), initial values of variables (K_0, $K_{H,0}$, B_0) and $f'' < 0$.

The current value Lagrangian function (L^c) of this problem is

$$(4.32) \quad L^c = (1-u)f(K) - qI + u\alpha K_H - (1-u)rB + N - \gamma B + \lambda_1(I-\delta K) + \lambda_2(N-\gamma B) + \lambda_3(qI-\alpha K_H) + \mu(qI-\alpha K_H-N+\gamma B)$$

where λ_1, λ_2 and λ_3 are the costate variables (i.e. shadow prices) associated with the state variables K, B and K_H, respectively, and μ is the Lagrangian multiplier associated with the dividend constraint.

The necessary conditions for an optimum are:[16]

(4.33) i) $\dot{\lambda}_1 = (\rho+\delta)\lambda_1 - (1-u)f'(K)$

ii) $\dot{\lambda}_2 = (\rho+\gamma)\lambda_2 + (1-u)r + \gamma - \mu\gamma$

iii) $\dot{\lambda}_3 = (\rho+\alpha)\lambda_3 - u\alpha + \mu\alpha$

iv) $L_I^c = -q + \lambda_1 + q\lambda_3 + q\mu = 0$

v) $L_N^c = 1 + \lambda_2 - \mu = 0$

vi) $\mu \geq 0$ and $\mu(qI-\alpha K_H-N+\gamma B) = 0$

Assuming first that the dividend constraint is not binding (i.e. $\mu = 0$), we can solve the equilibrium value of the capital stock directly from these conditions. It is implicitly given by the following marginal condition:

$$(4.34) \quad f'(K^*) = \frac{q}{(1-u)}(\rho+\delta)(1-uz)$$

which is equivalent to the standard Jorgensonian concept of user cost (see eq. (2.3), when $g = 0$). This concept of user cost does not include the tax savings due to interest deductions. Our model gives the result that $\rho = (1-u)r$ when $\mu = 0$ and thus the firm is indifferent between using debt or equity (retentions) for financing investment.

We assume next that the dividend constraint is binding (i.e. $\mu > 0$). We can solve μ from (4.33v) as $\mu = 1 + \lambda_2$. Inserting this value of μ into (4.33iv) gives $\lambda_1 = -q(\lambda_2+\lambda_3)$ and differentiating this gives $\dot{\lambda}_1 = -q(\dot{\lambda}_2+\dot{\lambda}_3)$. Using now conditions i) - iii) and the values for λ_1 and $\dot{\lambda}_1$, gives the marginal condition for the capital stock as

$$(4.35) \quad f'(K) = q[r+\alpha+(\alpha-\delta)(\frac{\lambda_2+\lambda_3}{1-u})]$$

where K, λ_2 and λ_3 are time-dependent. Equation (4.35) can be transformed into the following forms:

$$(4.36) \quad f'(K) = q[r+\delta+(\alpha-\delta)(1+\frac{\lambda_2+\lambda_3}{1-u})] = q(r+\alpha) + (\delta-\alpha)\frac{\lambda_1}{(1-u)}$$

Equation (4.35) determines the optimal investment policy of the firm. We can observe that the marginal condition of capital will now depend on the time-dependent costate variables and also on the Lagrangian multiplier μ since $\lambda_2 = \mu-1$. Hence the optimal investment policy is nonmyopic even in the absence of strictly convex adjustment costs. The above solution for K_t holds whenever the dividend constraint is binding but we must still examine the effectiveness of the constraint and the nature of the optimal stationary solution for K. However, it can be seen directly from eq. (4.35) that when $\alpha = \delta$, $f'(K)=q(r+\delta)$ and thus the marginal condition is independent of the tax factors (neutral) and the only possible solution is an immediate jump at the initial moment to the equilibrium point K* (if the maximized Hamiltonian is strictly concave in K for a given λ_1, a jump can be optimal only at the initial moment, see Arrow and Kurz 1970).

From (4.33v) we obtain $\mu = 1 + \lambda_2$ and inserting μ into (4.33ii) allows us to integrate λ_2 as

$$(4.37)\quad \lambda_2 = -\frac{(1-u)r}{\rho} + C_0e^{\rho t}$$

where C_0 is the integrating constant. Since $\mu \geq 0$ it follows that $\lambda_2 \geq -1$. Assume that $\rho > (1-u)r$ (this inequality holds even though $\rho = r$). Now $C_0 > 0$ is ruled out by the transversality condition for otherwise the marginal product of capital would increase without bound (when $\alpha > \delta$).

If $C_0 < 0$, sooner or later there occurs a point of time (say $\bar{t}$) at which $C_0e^{\rho\bar{t}} - (1-u)r/\rho = -1$ and from that point onwards $\lambda_2 = -1$. The transversality condition is also satisfied since $\lim_{t\to\infty} e^{-\rho t}(-1) = 0$. However, $\lambda_2 = -1$ implies that $\mu = 0$ and hence $\rho = (1-u)r$ which contradicts our assumption that $\rho > (1-u)r$. Hence it must be so that $C_0 = 0$. Therefore $\lambda_2 > -1$ for $\rho > (1-u)r$ implying that $\mu > 0$.

The above discussion reveals that it is optimal for the firm to operate under the dividend constraint whenever $\rho > (1-u)r$. The reason is simply that in this case borrowing is strictly preferable over equity (retentions) as a method of finance and the optimal solution involves borrowing the maximum possible amount (i.e. $\dot{B} = \dot{K}_H$) until K^* is reached.

We have hitherto established that $\mu = \frac{\rho-(1-u)r}{\rho}$ and $\lambda_2 = \frac{-(1-u)r}{\rho}$ and thus μ and λ_2 are constant. We can now integrate λ_3 from eq. (4.33iii) to get $\lambda_3 = uz + \frac{\alpha((1-u)r-\rho)}{\rho(\rho+\alpha)}$, (the integrating constant must again be assumed zero for otherwise the marginal product of capital would increase or decrease without bound). Inserting the value of λ_3 into (4.33iv) gives the following value for λ_1: $\lambda_1 = q[1-uz + \frac{(1-u)r-\rho}{\rho+\alpha}]$. The last term in the square brackets is the present value of tax savings arising from using debt instead of retentions. If $\rho = r$, then this last term reduces to $-ur/(r+\alpha) = -uy$, i.e. the present value of tax savings arising from interest deductions per unit of investment and hence $\lambda_1 = q(1-uz-uy)$, (see also chapter II). Inserting the above value of λ_1 into eq. (4.33i) and noting than $\dot{\lambda} = 0$, we can now solve the following marginal condition for the steady-state capital stock:

$$(4.38) \quad f'(K^*) = \frac{q}{(1-u)}\left[(\rho+\delta)(1-uz)+\left(\frac{\rho+\delta}{\rho+\alpha}\right)((1-u)r-\rho)\right]$$

and using $z = \alpha/(\rho+\alpha)$ we obtain $f'(K^*) = q(\rho+\delta)\left(\frac{r+\alpha}{\rho+\alpha}\right)$ and hence the investment policy of the firm is independent of the tax parameters when the dividend constraint is binding (see also Boadway and Bruce 1979, p. 98).

The intuition behind this result is as follows. A change (say decrease) in the tax depreciation rate α implies that after-tax profits will increase and that the firm can issue more debt. With the binding borrowing constraint ($\dot{B} = \dot{K}_H$), the reduced tax savings from depreciation are exactly offset by increased tax savings from interest deductions. It should be noted that eq. (4.38) is equivalent to our previous eq. (2.27) for the user cost when $s = (\rho+\delta)/(\rho+\alpha)$ and thus $s < 1$ if $\alpha > \delta$. For example, if $\delta = \rho = 0.1$ and $\alpha = 0.3$, then $s = 0.5$. As noted above, the dividend constraint implies a borrowing constraint of the form $B/qK = s < 1$. If debt were restricted by the undepreciated value of the capital stock evaluated at the true rate δ, then we would obtain $f'(K^*)=q[(\rho+\delta)(1-uz)+((1-u)r-\rho)]/(1-u)$ and neutrality follows when $\delta = \alpha$ (plus full interest deductibility).

The above results imply that the marginal product of capital would increase (or decrease) without limit unless the expression containing the costate variables in (4.35) were constant for all t. Hence the nonstationary cases are not optimal and a profit maximizing firm should use debt finance to the maximum amount dictated by the dividend constraint (when $\rho > (1-u)r$). The basic reason for this 'corner solution' is that we are assuming a certainty environment (r is constant) and thus a minimum value of the cost of capital is achieved with the maximum value of tax savings due to interest deductions. In this context there is no counterbalancing effect to prevent borrowing from increasing to the maximum level implied by the dividend constraint.

4.4.2 Profit Constraint, Retained Earnings and Investment Behaviour

In this section we shall consider the optimal investment behaviour of a firm under the assumption that self-financing is the marginal source of finance. This situation may arise for two different reasons. First, it may be assumed that the firm directly faces a binding constraint on the amount of debt it can obtain at a given interest rate and that the debt-capital ratio is hence an exogenously given constant due to credit rationing considerations, i.e. $B = \bar{s}qK$. Here $\bar{s}$ would be smaller than an optimal ratio (say s^*). Second, the debt-capital ratio could be optimally determined as in section 4.2, where the rate of interest was assumed to be a rising function of leverage.[17] It is, however, assumed that the rate of investment does not affect the debt-capital ratio directly since otherwise the investment and financial decisions would be totally interrelated. Both of these financial assumptions with respect to borrowing imply that there will be an upper bound on the amount of investment the firm can make at any moment of time. With a given debt-ratio, the gross investment of the firm will be limited to the amount given by the volume that absorbs all retained earnings as the equity-financed part. Investment in excess of this amount would call for new equity issues, a possibility which we shall exclude in order to highlight the role of a profit constraint (see also note 17).

The model to be analyzed in this section is basically the same as the one in chapter II, where, however, it was assumed that a profit constraint is not effective. Hence in chapter II the analysis was carried out as if the firm operated in 'free intervals'. The analysis of this section is related to the work of Appelbaum and Harris (1978) and Schworm (1980). Appelbaum and Harris have examined capital accumulation under the hypothesis that investment can be financed only through profits. The financial constraint of their model is a consequence of the following two capital market imperfections: (i) no new borrowing is possible, and (ii) all cash flow which is not invested must be distributed to shareholders. In the Appelbaum and Harris model, a firm that anticipates being financially constrained increases investment over some period in order to increase future profits. Purchasing capital increases future profits and thus reduces the stringency of future financial constraints.

In the model of Schworm (1980), there are two ways of transferring current funds into the future. A firm with current earnings can either purchase capital or accumulate retained earnings. The firm purchases capital if and only if the return on holding capital is greater than or equal to the return on retained earnings. The retained earnings constraint has no anticipatory effect on investment and the firm follows a myopic rule for capital accumulation. At first glance it would seem surprising that the myopic rule is obtained in an imperfect capital market. It would seem plausible that a firm anticipating a binding financial constraint would reduce investment in order to postpone the exhaustion of internal funds. This argument, however, ignores the fact that current investment increases the funds available in the future by increasing future profits.

The model to be developed in this section is a modification and an extension of Appelbaum and Harris's model in the following respects: (1) we assume that the firm is able to borrow up to an exogenously given portion of investment, (2) we assume that basic corporate tax factors are included in the cash flow identity of a firm and hence that we can examine the role of tax factors in a financially-constrained environment, (3) we shall consider the behaviour of the firm under alternative assumptions about the financial constraint and (4) we shall also present some possible extensions of the model as, for example, the incorporation of strictly convex adjustment costs.

The main conclusion of our analysis is that, when the firm is faced with a given borrowing policy, the average profitability (or just cash flow) may become crucial for its investment behaviour. Through profits, taxes, interest payments and depreciation rules can also become a significant factor for the dynamics of investment in addition to the usual 'user cost channel'.

4.4.2.1 The Model and Necessary Conditions for an Optimum

It is assumed that the firm finances a constant portion of its gross investment by raising new loans (N) so that N = sqI (see also chapter II). Furthermore, it is assumed that the debt-capital ratio is s and hence the change in debt is given by $\dot{B} = sq(I-\delta K)$. Since $\dot{B} = N-\gamma B$, where γ is the rate of amortization, we then have $\gamma = \delta$ (see also chapter II).

The flow financing indentity of gross investment is $qI = R^g + N$, where R^g is gross retained earnings: $R^g = f(K)-rB-T-s\delta qK-Div$. Note that $s\delta qK$ is debt repayment. If R^g is defined to equal $f(K)-rB-T-Div$, then $qI = R^g + \dot{B}$ (see also eq. 2.10). If $Div = 0$, then maximum retained earnings, i.e. gross profits (P) after taxes and debt service payments are equal to (D is tax depreciation)

$$(4.39) \quad P = f(K) - rB - sq\delta K - T = (1-u)(f(K)-rsqK) + uD - sq\delta K$$

The firm is assumed to be constrained in its investment policy in such a way that the amount of investment not financed by borrowing must not exceed maximum retained earnings. Thus, the investment policy is constrained to satisfy the following inequalities

$$(4.40) \quad \text{i)} \ (1-s)qI < P \quad \text{or} \quad \text{ii)} \ I < \frac{(1-u)(f(K)-rsqK) - sq\delta K}{q(1-uz-s)}$$

$$\text{or}$$

$$\text{iii)} \ \dot{K} < \frac{(1-u)(f(K) - rsqK) - (1-uz)\delta qK}{q(1-uz-s)}$$

where, as usual, we have used the present value formulation of depreciation deductions.[18,19]

The maximization problem of the firm is now as follows:

$$(4.41) \quad \max_{\{I\}} \int_0^\infty e^{-\rho t}[(1-u)f(K)-(1-uz)qI-(1-u)rsqK+sq(I-\delta K)]dt$$

subject to i) $\dot{K} = I - \delta K$ and

$$\text{ii)} \ I \leqslant \frac{(1-u)(f(K)-rsqK) - s\delta qK}{q(1-uz-s)}$$

and a given initial value of the capital stock, say $K_0 > 0$ (notice that the time dependency of variables is again suppressed in the following except where it is needed for clarity). If the irreversibility constraint were included among the constraints, the admissible values of the control variable I would be given as $0 \leq I \leq P/q(1-s)$. Our analysis will be limited primarily to the effects of an upper bound on investment but at the end of this section we shall briefly comment on the role of a lower constraint.

The Hamiltonian of this model is given by

$$(4.42) \quad H = \omega[(1-u)f(K)-(1-uz-s)qI-(1-u)rsqK-s\delta qK] + \bar{\lambda}(I-\delta K)$$

where $\omega_t = e^{-\rho t}$, and $\bar{\lambda}_t = \omega_t\lambda_t = \lambda_t e^{-\rho t}$ and hence $\bar{\lambda}_t$ is the discounted costate variable associated with the equation of motion for K. Notice also that $\rho = -\dot{\omega}/\omega$ and $\lambda_t = \bar{\lambda}_t e^{\rho t}$.

We next define an auxiliary variable $\bar{p}_t$ such that

$$(4.43) \quad \bar{p}_t = \bar{\lambda}_t - q(1-uz-s)\omega_t \qquad \text{or} \qquad p_t = \lambda_t - q(1-uz-s)$$

and rewrite the Hamiltonian as

$$(4.44) \quad H = \omega[(1-u)f(K)-((1-u)r+\delta)sqK-\lambda\delta K] + \bar{p}I$$

It should be noted that the Hamiltonian is linear in the control variable I and thus the optimal investment policy is given as follows: (i) if $p < 0$, the $I = 0$, (ii) if $p = 0$, then $0 \leq I \leq P/q(1-s)$ and (iii) if $p > 0$, the $I = P/q(1-s)$ (see Takayama 1974, p. 690, also Kamien and Schwarz 1981, Clark 1976). A control that takes on the extreme values is called a bang-bang control and the function p is usually called a switching function. The singular case arises when p vanishes identically over some time interval of positive length, i.e. $p_t = 0$. The case $p \equiv 0$ corresponds to our analysis in chapter II. It should be noted that λ is the shadow price of installed capital and q(1-uz-s) is the tax-adjusted price of new investment goods. When $p = 0$, the capital stock instantaneously jumps to the equilibrium level K^* and K^* is implicitly determined by the Euler equation (see below). The maximum principle implies that the optimal control for our problem must be a combination of bang-bang and singular controls. Further characterization of the optimal solution will be given below. It should also be noted that when $uz + s = 1$, then the rate of investment is momentarily infinite (unbounded) and a jump in K occurs. In the following we shall restrict our analysis to the case where $uz + s < 1$, i.e. the net cost of investment is positive.

We next turn to a more formal treatment of our maximization problem. The Lagrangean function of the above maximization problem is (assuming $\mu > 0$ for all t)

$$(4.45) \quad L = e^{-\rho t}[(1-u)f(K)-(1-uz-s)qI-(1-u)rsqK-s\delta qK] + \bar{\lambda}(I-\delta K) + \bar{\mu}[(1-u)f(K)-(1-uz-s)qI-(1-u)rsqK-s\delta qK]$$

where $\bar{\mu}_t$ is the non-negative Kuhn - Tucker multiplier associated with the profit constraint. Notice that $\bar{\mu}_t = \mu_t e^{-\rho t} = \mu_t \omega_t$ and hence μ_t is the undiscounted multiplier.

The necessary conditions for an optimum are as follows:

$$(4.46) \quad \text{i)} \quad \dot{\bar{\lambda}} = -\omega[(1-u)f'(K)-(1-u)rsq-s\delta q] + \bar{\lambda}\delta - \bar{\mu}[(1-u)f'(K)-(1-u)rsq-s\delta q]$$

or using $\bar{\lambda} \equiv \lambda e^{-\rho t} \equiv \lambda\omega$

$$\dot{\lambda} = (\rho+\delta)\lambda - (1-u)f'(K) + sq((1-u)r+\delta) - \mu[(1-u)f'(K)-(1-u)rsq-sq\delta]$$

ii) $\frac{\partial L}{\partial I} = \bar{\lambda} - \omega q(1-uz-s) - \bar{\mu} q(1-uz-s) = 0$

or

$\lambda - q(1-uz-s) - \mu q(1-uz-s) = 0$

iii) $\bar{\mu} \geqslant 0; \quad \bar{\mu}[P-(1-s)qI] = 0$

The sufficiency results from a theorem by Seierstad and Sydsaeter according to which sufficiency follows if the Hamiltonian is concave in K and I and if the constraint involving state and control variables is quasiconcave in K and I (plus the usual continuity and differentiability assumptions, see Kamien and Schwarz, 1981, section 15). The concavity requirement for H can, however, be relaxed if Arrow's proposition is used to establish sufficiency. This states that, if the maximized Hamiltonian $H^0(K,\lambda) = \max_{\{I\}} H(K,I,\lambda)$ is concave in K for a given λ and provided that the constraint qualification is satisfied, then sufficiency follows (see Kamien and Schwarz, section 15). Since, as usual, we assume that $f''(K) < 0$ and since, moreover, the profit constraint is linear in I, the conditions of the Arrow theorem hold in our model.

It should be noted that, whenever $P - (1-s)qI > 0$, we have $\mu = 0$ and the terms involving partial derivatives of the profit constraint in (4.46) have no impact. However, whenever the profit constraint is tight, choice of the control variable I is restricted to maintain feasibility. The multiplier λ_t gives the marginal valuation of the corresponding state variable K_t at t. Note that condition (4.46i) reflects not only the direct effect of changes in K on the current reward of the objective function and on the state changes through $\dot{K} = I - \delta K$ but also the effect of changes in K on the feasible control region through the profit constraint.

The shadow price of capital can be solved from eq. (4.46i) as

$$(4.47) \qquad \lambda_t = \int_0^\infty e^{-(\rho+\delta)t}(1+\mu_t)[(1-u)f'(K_t)-sq((1-u)r+\delta)]dt$$

Therefore, the value at time t of a marginal unit of capital is again the discounted stream of net marginal profits it generates. Note that the multiplier μ_t affects this valuation principle when the profit constraint is tight. Since the Hamiltonian is linear in the control variable, investment will not be a rising function of the shadow price of capital but it is directly determined through the profit constraint. Equation (4.47) also implies that the cost of capital reflects the anticipated stringency of future financial constraints (see below).

The necessary condition (4.46ii) implies that the multipliers λ and μ are linearly interrelated, i.e.

$$(4.48)\quad \lambda = q(1-uz-s)(1+\mu) \quad \text{and} \quad \mu = \frac{\lambda}{q(1-uz-s)} - 1$$

and hence we can examine the properties of this model either in the (λ,K) space or in the (μ,K) space. Eq. (4.48) implies that $\dot{\lambda} = q(1-uz-s)\dot{\mu}$, and using these values of λ and $\dot{\lambda}$ we can form the following differential equation system underlying the present model:

$$(4.49)\quad \text{i)}\quad \dot{\mu} = (1+\mu)[(\rho+\delta)-(\frac{(1-u)f'(K) - sq(1-u)r - sq\delta}{q(1-uz-s)})]$$

$$\text{ii)}\quad \dot{K} = \frac{(1-u)f(K) - (1-u)rsqK - sq\delta K}{q(1-uz-s)} - \delta K$$

Denoting $\dot{K} \equiv G(K)$, the equation for $\dot{\mu}$ can be rewritten as

$$(4.50)\quad \dot{\mu} = (1+\mu)[\rho - \frac{dG}{dK}]$$

In the special case where $s = u = 0$, we obtain as a steady state solution the well-known myopic rule $f'(K^*) = q(\rho+\delta)$. Formally, the solution has been completely described. We seek four functions of time, K_t, p_t, μ_t and I_t, jointly satisfying the conditions (4.41i), (4.41ii), (4.46i) and (4.46ii) with K_0 given. The initial value of μ has not been explicitly defined; it must be such that all these conditions can jointly be satisfied. However, a good deal more can be said about the structure of the solution.

In the following we wish to compare the optimal investment policies in the financially-unconstrained case (free intervals) and in the financially-constrained case (bounded intervals). It should be noted that in the unconstrained case the firm still has a given constant debt ratio but it can finance all investment without being constrained by the profit constraint, i.e. $\mu = 0$. In the following analysis, the capital stock as well as other relevant variables are occasionally indexed with the superscript c in the constrained case whereas no superscript is used in the unconstrained case.

4.4.2.2 Evaluation of the Model

Using (4.46i) and (4.48), the marginal condition for the constrained capital stock can be solved as

$$(4.51)\quad f'(K_t^c) = \frac{q}{(1-u)}[(\rho+\delta)(1-uz)+s((1-u)r-\rho)-(\frac{(1-uz-s)\dot{\mu}_t}{1+\mu_t})]$$

In the steady state $\dot{\mu} = \dot{\lambda} = 0$, and $f'(K^{*c})$ is given as

$$(4.52)\quad f'(K^{*c}) = \frac{q}{(1-u)}[(\rho+\delta)(1-uz)+s((1-u)r-\rho)]$$

This result implies that $f'(K^{*c}) = f'(K^*)$ and thus even in the profit-constrained case the firm will eventually reach the myopically optimal stationary capital stock (singular solution). Since prices are assumed constant, the financial constraint can be binding only for a finite time period. Therefore, there exists a time point t* such that $\mu_t \equiv 0$ for all $t > t^*$ (see also Clark 1976 and Kamien and Schwarz 1981).

It can now directly be seen from eq. (4.51) that the optimal decision rule for the capital accumulation policy is not myopic since the time dependent multiplier μ_t enter the expression for the user cost of capital. Hence the present model produces a dynamic non-singular capital adjustment path even though adjustment costs are not strictly convex. The reason for the nonmyopic path is the presence of the binding profit constraint, implying that the control variable I is bounded above by a function of the state variable.

The interesting question concerning this model is the relationship between $f'(K_t^c)$ and $f'K_t)$, i.e. whether in the constrained case the user cost of capital is larger than, equal to or smaller than what it is in the unconstrained case. If $\mu_t > 0$, this relationship depends upon the sign of $\dot{\mu}$ in eq. (4.51). If $\lambda > q(1-uz-s)$, then the profit constraing is binding ($p > 0$, $\mu > 0$) the discrepancy $\lambda_t - q(1-uz-s)$, which keeps the system in motion, is eliminated by increasing K. Since the tax-adjusted price of investment goods $q(1-uz-s)$ is constant, this discrepancy is eliminated only when $\dot{\lambda} < 0$. Decreasing returns to scale ensure that $\dot{\lambda} < 0$ and hence also $\dot{\mu} < 0$ (see eq. 4.48). Since $\dot{\mu} < 0$ it follows that $f'(K_t^c) > f'(K^*) = f'(K_t)$, which implies that $K_t^c < K^*$. Therefore, in the profit-constrained case the capital stock is smaller than the myopically optimal capital stock (for a firm identical in all other respects). As noted above, in the steady state it holds that $K^{*c} = K^*$. These results, however, assume that the profit constraint is already binding initially. In section 4.4.2.3 below we shall consider the anticipatory effect of a profit constraint by assuming that this constraint is not initially binding.

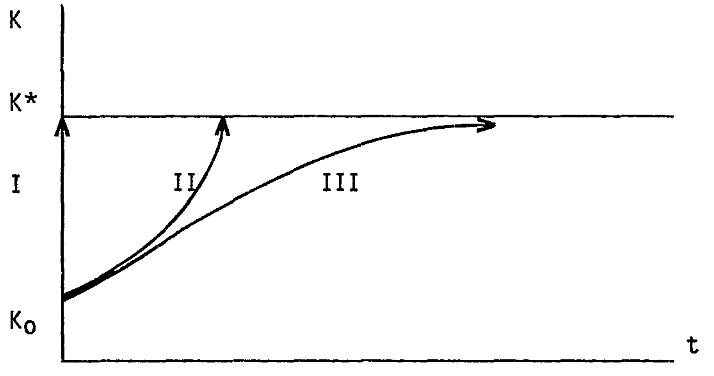
The basic feature of the dynamics of the present model is that if $K_0 < K^*$, then the optimal path represents the fastest possible rate of increase in the capital stock, i.e. $\dot{K} = [(1-u)(f(K)-rsqK)-(1-uz)\delta qK]/q(1-uz-s)$ until K^* is reached after which $\dot{K}$ is chosen so as to maintain K^* (i.e. $\dot{K} = 0$ and $I^* = \delta K^*$), (see Clark 1976, Kamien and Schwarz 1981, section 16). The essential condition for this most rapid approach path (MRAP) is that the objective function of the firm is linear in $\dot{K}$, as in the present model. It should be noted that $\dot{K}$ is maximized for any K when dividends are zero and the maximum amount of debt is used (i.e. $B = sqK$). The values of the controls implied by this model are feasible when it is assumed that dividends must be non-negative, i.e. $Div \geq 0$. Therefore the MRAP is feasible and hence optimal. A firm growing in its market should borrow up to the upper limit (if $\rho > (1-u)r$) and not issue dividends until the stationary optimal capital stock is reached (see also note 18).

The time path of the constrained capital stock can be presented in integral form as follows (see eq. 4.49ii):

$$(4.53) \quad K_t^c = \int_0^t e^{-\delta(t-v)}\left[\frac{(1-u)f(K_v^c) - (1-u)rsqK_v^c - s\delta qK_v^c}{q(1-uz-s)}\right]dv + K_0 e^{-\delta t}$$

At this stage of our study we can summarize the different time paths of the capital stock which may occur. Figure 5 illustrates three different optimal time paths for the capital stock.

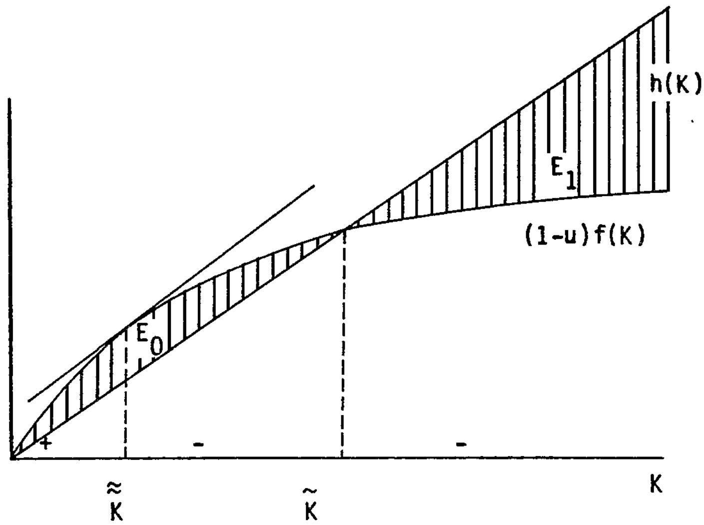
FIGURE 5.



The first time path for K_t describes the case of a singular solution in which the capital stock instantaneously jumps to the equilibrium level K* (path I). Time path III occurs if the model is nonlinear in the control variable, and in this case optimal behaviour is characterized by a gradual (asymptotic) approach to K*. Strictly convex adjustment costs produce the type III path as in chapter III of this study. The most rapid approach path in the case of a binding profit constraint is represented by path II, implying that the equilibrium value of the capital stock is reached gradually but in finite time (if K* is fixed). With respect to investment itself, these three models imply radically different dynamic behaviour. This question will be discussed next on the basis of the present model.

Along the ($\dot{K} = 0$) curve $G(K) = 0$, implying that $(1-u)f(K) - h(K) = 0$, where $h(K) = [(1-u)rs+\delta(1-uz)]qK$. The ($\dot{K} = 0$) curve implicitly defines that value of K, say $\tilde{K}$, which corresponds to $\dot{K} = 0$. Since $dG/dK \neq 0$ at $\tilde{K}$ (see below), we find that K is determined from an implicit function, $K = \phi(u,r,s,z,\delta,q)$, in the neighbourhood of $\tilde{K}$. Figure 6 describes the above situation.

FIGURE 6.



It should be noted that the shaded areas E_0 and E_1 give the cumulative change in K, i.e. $\dot{K}$ is a function of K and the constant parameters of the model. It can be noted that $\dot{K}$ is positive up to point $\tilde{K}$. The maximum rate of $\dot{K}$ can be found as follows:

$$(4.54) \qquad \frac{\partial \dot{K}}{\partial K} = \frac{(1-u)f' - ((1-u)rs + \delta(1-uz))q}{q(1-uz-s)}$$

Furthermore, $\frac{\partial^2 \dot{K}}{\partial K^2} = (1-u)f''/q(1-uz-s) < 0$ and hence the maximum rate of $\dot{K}$ can be found by setting $\partial \dot{K}/\partial K = 0$. Note that the numerator of the partial derivative (4.54) is the derivative of the difference of the two functions (1-u)f(K) and h(K) with respect to K, i.e. the derivative of the shaded area (in figure 6) with respect to K. The maximum value of $\dot{K}$ (say $\overset{\approx}{K}$) is implicitly defined by $(1-u)f'(\overset{\approx}{K}) = q[(1-u)rs+\delta(1-uz)]$. The crucial question is now what is the location of K* as compared to $\overset{\approx}{K}$ and $\tilde{K}$. Note that at point $\tilde{K}$ it holds that $\dot{K} = 0$ and at point $\overset{\approx}{K}$ it holds that $\dot{K}$ takes its maximum value (with respect to K_t). The answer to this question depends on the magnitudes of f'(K*) and $f'(\overset{\approx}{K})$. It can easily be shown that $f'(\overset{\approx}{K}) \gtrless f'(K^*)$ whenever $(1-u)rs+\delta(1-uz) \gtrless (\rho+\delta)(1-uz) + s((1-u)r-\rho$ which reduces to $uz + s \gtrless 1$.

When $uz + s < 1$, then $f'(\overset{\approx}{K}) < f'(K^*)$, implying that $K^* < \overset{\approx}{K}$. If $uz + s = 1$, then $K^* = \overset{\approx}{K}$ and K instantaneously jumps to K* as noted above in section 4.4.2.1. The case $uz + s < 1$ corresponds to our original assumption and implies that the net cost of new investment goods is positive (for $I > 0$). The conclusion now is that in the present model, where the profit constraint is already initially binding, the rate of investment steadily (monotonically) increases until K^* is reached after which it holds that $I^* = \delta K^*$.

We can now compare the investment paths in the three cases illustrated in figure 5. In the unconstrained (myopic) case the rate of investment is momentarily infinite. This simply implies the acquisition of a block of capital goods at some instant of time. In the third case (III), the optimal policy for $K_0 < K^*$ is to invest most heavily in the initial periods and continually decrease the level of investment as K increases towards K^*, i.e. $\dot{I} = dI/dt < 0$. This result, however, follows only when the production function is strictly concave ($f'' < 0$). In the case of constant returns to scale ($f'' = 0$), the level of investment is independent of K and the optimal policy is to maintain a constant rate of investment (see chapter III). In the present model (case II in figure 5) we obtain a completely different adjustment pattern as compared to the strictly convex adjustment cost model (case III) since now it holds that $dI/dt > 0$ along the optimal solution path. It should, however, be emphasized that we have assumed a perfectly competitive output market (i.e. the output price p is given) in the above comparison. If the firm faces a downward-sloping demand curve, then a 'large' increase in the output supply may reduce the output price and the rate of investment need not necessarily increase along the solution path.

Furthermore, in response to anticipated increases in demand or cost conditions, a nonmonotonic reaction pattern of investment may follow with strictly convex adjustment costs as observed in chapter III. A similar situation may also arise in the profit-constrained case, as will be shown below in section 4.4.2.3 in response to an anticipated profit constraint.

The comparative static properties of the present model are the same as in chapter II (see table 3) because of the same singular (myopic) solution. In the profit-constrained case, however, the adjustment path of K is dynamic and hence we can present some comparative dynamic properties of this model. The following partial derivatives can be obtained directly in the case of a strictly binding profit constraint (initially binding):

(4.55) $\frac{\partial I}{\partial p} > 0,\ \frac{\partial I}{\partial q} < 0,\ \frac{\partial I}{\partial w} < 0,\ \frac{\partial I}{\partial r} < 0,\ \frac{\partial I}{\partial \delta} < 0,\ \frac{\partial I}{\partial z} > 0,\ \frac{\partial I}{\partial \rho} < 0,\ \frac{\partial I}{\partial s} = ?,\ \frac{\partial I}{\partial u} = ?.$

Most of these results (signs) are self-explanatory but the ambiguous signs deserve some comments. The effect of a rise in the debt-capital ratio s on investment is ambiguous because of two opposing forces. A rise in s increases debt service payments (interest plus repayment of debt) but on the other hand less investment is left to be financed out of internal funds (1-uz-s decreases) so that investment might also increase (per unit of time). It can be shown that $\partial I/\partial s > 0$ if $f(K)/qK > (1-uz)((1-u)r-s)/(1-u)$ and hence the effect of a change in s on investment depends on the 'average profitability' of the firm as measured by the variable $f(K)/qK$.

The most interesting question in the present context is the effect of the corporate tax rate u on investment. A rise in the tax rate u decreases investment (i.e. $\partial I/\partial u < 0$) if $f(K)/qK > sr - \delta zs/(1-s-z)$ and hence the sign of this effect again depends on the 'average profitability' of the firm. If the firm is sufficiently 'profitable', a rise in the tax rate results in a downward shift of the optimal MRAP path of investment corresponding to each given level of the capital stock. It can therefore be concluded that the usual neutrality conditions do not hold with respect to investment itself, although they still hold with respect to K* (see chapter II). The reason is simply that the firm is now liquidity (profit)-constrained and hence a change in the tax rate changes the maximum amount of retained earnings which directly determines the rate of investment.

It can be argued in the light of these comparative dynamic results that investment incentives (analogous to parameter z, i.e. accelerated depreciation) may generally have a stronger impact on the level of investment in the case where self-financing is the marginal source of finance as opposed to other types of (standard) models in which this effect occurs only through the user cost variable (i.e. via K*). The international literature on the effects of tax factors and investment incentives has largely been concentrated on the equilibrium capital stock through the cost of capital variable (see chapter II). An exception is the work done mainly by Abel, in which the dynamic effects of temporary and permanent changes in taxation and incentive parameters are analyzed in a strictly convex adjustment cost investment model (see references in chapter III). The model developed in this section allows an alternative framework for analyzing the impact of tax and incentive factors on investment behaviour by firms. This model produces the 'usual' type of steady-state effects and some new types of dynamic results since both the cost of capital and cash flow channels are present. In addition, the present model may produce new types of anticipatory effects with respect to temporary and permanent future changes in policy parameters (see section 4.4.2.3 below). It should be mentioned here that the effect of the cash flow channel on the timing of investment has long been recognized in the empirical investment litrature (see, for example, Coen 1971, King 1972, Sarantis 1979 etc., see also chapter V). The present model, however, provides a theoretical explanation for the effects of a cash flow variable on the dynamic investment pattern.

4.4.2.3 The Anticipatory Effect of a Profit Constraint and Some Extensions of the Model

We have hitherto analyzed the optimal investment policy on the assumption that the profit constraint is already binding initially (i.e. $\lambda > q(1\text{-}uz\text{-}s)$. If prices are constant, then, as shown above, the firm will gradually accumulate capital with in the limits delineated by internally-generated funds until the myopically optimal capital stock K^* is reached. A situation in which a financial constraint already exists initially may arise when the initial capital stock is below the unconstrained optimal capital stock and the maximum (feasible) rate of investment does not instantaneously eliminate the gap $(K^* - K_0)$. Another example of financially constrained periods starting at $t = 0$ occurs if the initial capital stock is the unconstrained (myopic) capital stock, but the rate of growth of the myopically optimal capital stock is greater than the feasible rate of growth for the financially constrained firm (over the same intervals).

We shall next consider a situation where the firm thinks that it will have profitable investment opportunities in the future, all of which it will not be able to finance through borrowing (with a given debt ratio). In such a situation the firm might plan to cut its dividend payments in the 'short-run' in order to finance all profitable investment outlays. In an extreme case, the firm is faced with a situation in which investment will be bounded in the future by available maximum retained earnings. It is next assumed that the firm anticipates that it will experience a finite profit-constrained (bounded) time period in the future when pursuing its optimal investment policy in a perfect certainty framework.

By Pontryagin's theorem, λ_t, and hence p_t is a continuous function of time (this also holds for the discounted variables $\bar{\lambda}_t$ and $\bar{p}_t$, see also Appelbaum and Harris 1978). Therefore, in a time interval where $p_t = 0$ it must be so that $\dot{p}_t = 0$. Generally, we have that $\dot{p}_t = \dot{\lambda}_t$ and $\dot{\bar{p}}_t = \dot{\bar{\lambda}} - q(1-uz-s)\dot{\omega}$, (see eq. 4.43). In a financially-constrained time period $\mu > 0$ and $I = P/q(1-s)$. Since $p_t = \lambda_t - q(1-uz-s) = q(1-uz-s)\mu_t$ and assuming that $uz+s < 1$, we hence get that $p_t > 0$ (and $\bar{p}_t > 0$) in a constrained time period. We assume now that the firm faces a finite financially-constrained period $[t_0, t_1]$ when planning its optimal investment policy and that $t_o > 0$, i.e. the bound on investment is not binding initially. Since $\bar{p}_t$ is a continuous function of time (for $t > 0$), it must start and finish in a free (unconstrained) internal and hence $\bar{p}_{t_o} = \bar{p}_{t_1} = 0$. We now obtain

$$(4.56) \qquad \bar{p}_t - \bar{p}_{t_o} = \int_{t_o}^{t} \dot{\bar{p}}_v \, dv \geq 0 \text{ for } t\varepsilon(t_o, t_1)$$

with strict equality when $t = t_1$. The non-negativity in (4.56) follows since $\bar{p}_{t_o} = 0$ and $\bar{p}_t \geq 0$ for $t\varepsilon(t_o, t_1)$, (note that the technique used here is the same as in Nickell 1974 and Appelbaum and Harris 1978). This inequality can be used to examine the effect of an anticipated profit constraint on a firm's capital accumulation.

We noted above that $\dot{\bar{p}} = \dot{\bar{\lambda}} - q(1-uz-s)\dot{\omega}$ and using eqs. (4.46i) and (4.46ii) we get

$$(4.57) \quad \dot{\bar{p}} = \omega[q(\rho+\delta)(1-uz)+sq((1-u)r-\rho)-(1-u)f'(K^c)]$$
$$- \bar{\mu}[(1-u)f'(K^c)-q((1-u)rs+\delta(1-uz))]$$

or

$$\dot{\bar{p}} = \omega\{[(1-u)f'(K) - (1-u)f'(K^c)]$$
$$- \mu[(1-u)f'(K^c)-q((1-u)rs+\delta(1-uz))]\}$$

Substituting $\dot{\bar{p}}$ into eq. (4.56) gives

$$(4.58) \quad \int_{t_0}^{t} e^{-\rho v}[(1-u)f'(K_v)-(1-u)f'(K_v^c)]dv \geqslant$$

$$\int_{t_0}^{t} e^{-\rho v}\mu_v \; [(1-u)f'(K_v^c)-q((1-u)rs+\delta(1-uz))]dv$$

It should be noted that $\int_{t_0}^{t} e^{-\rho v}(1-u)f'(K_v)dv = \int_{t_0}^{t} e^{-\rho v}c_v dv$ is the present discounted value of the after-tax user cost of capital (when $(1-u)f' = c$) and $\int_{t_0}^{t} e^{-\rho v}(1-u)f'(K_v^c)\, dv$ is the present discounted value of the after-tax revenue produced by a marginal unit of capital in the constrained case. When $t = t_1$, these two present values are equal.

The right-hand side (RHS) integral in (4.58) is the numerator of $\partial\dot{K}/\partial K$ (see eq. 4.54), which changes its sign at some $K = \approx{K}$. When the RHS integral in (4.58) is positive, it follows that $f'(K^c) < f'(K)$ for at least some $t\varepsilon(t_0, t_1)$ and hence $K_t^c > K_t$. Therefore during financially-constrained periods it does not generally hold that a firm with constrained investment plans has a lower capital stock relative to a firm which operates without the constraint.

This result can be interpreted to mean that if the firm has 'very' profitable investment opportunities (i.e. f' is large), then it might try to 'overinvest' initially in order to overcome the problem of a financial constraint at some later date. If, however, the financial constraint is already binding initially, then, as shown previously, $f'(K_t^c) > f'(K_t)$

and the constrained capital stock is less than the myopically optimal capital stock except at the steady state in which $f'(K^C) = f'(K^*)$. It can be noted that the more accelerated is the tax depreciation system (i.e. z is 'large'), the greater will be the positive value of the RHS. Accelerated depreciation rules increase the likelihood that $f'(K_t^C) < f'(K_t)$. In sum, the present model shows that if the firm has very profitable investment opportunities and if the tax depreciation scheme is 'liberal', the firm can try to mitigate the impending future financial constraint by initially investing more than in the myopic case. By investing more the firm increases internal finance (and interest-free tax credits). In the special case of the Appelbaum and Harris model ($\delta = u = s = 0$), the RHS in (4.58) is always positive since $\mu > 0$ and $f'(K^C) > 0$.

Arrow and Kurz (1971, proposition 12) give some indication of when jumps may or may not occur in a state variable. If the maximized Hamiltonian is strictly concave in K for a given λ, then a jump can never be optimal, except possibly at the initial time point ($t = 0$). The only conditions that must be satisfied across the jump are that p_t is continuous and $p_t = 0$ at the jump point (see also Nickell, 1974). Hence in our model it seems to be the case that the capital stock must jump initially (at $t = 0$) to higher level (investment is momentarily infinite) in order that $K_t^C > K_t$ at some point in the interval (t_0, t_1).

In this model framework, the firm is willing to build up its capital stock in response to an anticipated financial constraint since it can either distribute earnings as dividends or purchase new capital. Purchasing new capital increases current and future profits and thus reduces the stringency of future financial constraints. The expectations of future financial constraints mean a smaller capital stock and therefore higher marginal productivity of capital in the future, which tends to make capital more valuable now.

We shall finally consider in this section two possible extensions of the profit-constrained investment model which are related to the irreversibility assumption and to strictly convex adjustment costs. If our model were made more explicit by specifying the production function, demand conditions and time rates of prices, we could solve explicitly the time paths of λ_t and the cost of investment goods (i.e. $q_t(1-uz-s)$

and hence also consider the conditions under which $\lambda_t \gtrless q_t(1-uz-s)$ or equivalently $p_t \gtrless 0$. In such a case it might also be useful to make the irreversibility assumption, especially if demand for the firm's products may decline over certain time periods (see also Nickell 1974). Investment would then be constrained according to $0 < I_t < P_t/q_t(1-s)$. The critical question would now be when investment follows the lower limit ($I = 0$), the upper bound ($I = P/q(1-s)$), or when there might occur jumps in the capital stock. The answer to this problem would depend on the parameters and variables of the model. In this context the analysis of the 'optimal' timing of switching points is crucial. It is also possible that the switching points between the blocked and free intervals are affected by future financial constraints (see also Schworm 1980).

We have hitherto assumed that adjustment costs are linear (i.e. the price of investment goods is given to a firm). A second possible extension of the financially bounded investment model is to assume that there are strictly convex adjustment costs, i.e. increasing marginal costs of investment. In this case, there is no myopic investment rule that characterizes investment behaviour and current investment decisions are influenced by the anticipation of future financial constraints. In that model there cannot occur jumps in the capital stock and investment accumulation is hence more 'smoothly' determined than in the present model. Very recently, d'Autume and Michel (1985) have shown in an adjustment cost investment model that the effects of an anticipation of a future exogenous investment constraint (i.e. $I < \bar{I}$ for some future t) depend on both the returns to scale assumption and the form of the adjustment cost function. In our profit constrained model, the future financial constraint implies a tendency to make anticipating purchases of capital goods only in the case of decreasing returns to scale. In the case of constant returns to scale and therefore in the absence of an optimal scale of the firm, there is no need for the firm to compensate in advance for the future deficit in investment spending.

4.5 Conclusions of Chapter IV

In this chapter we have analyzed further the effects of various types of capital market imperfections on the firm's investment behaviour. The results of the present chapter clearly indicate that investment behaviour is quite sensitive to the characteristics of financial market imperfections and the resulting financial constraints. More specifically, the conditions of the capital market influence both the long-run demand for capital and the dynamics of investment.

The results of section 4.2 indicate that the properties of the rising interest cost function affect both the demand for capital and the optimal debt-equity ratio. Faced with a rising supply of borrowing curve, the firm is, however, able to choose its optimal financial policy. The firm chooses a minimum cost of capital and uses this to find the optimal investment strategy. In this context, investment and financing decisions are separated (recursive). It was also noted that the rate of adjustment to the equilibrium level of the capital stock is faster in the case of a capital market imperfection (rising interest rate) than in the case of a perfect market. The standard neutrality conditions of the corporate tax system hold in this framework with respect to the steady-state demand for capital and with respect to the dynamic path of investment. In section 4.3 new equity issues as a method of finance were incorporated in a simple way in the preceding model.

In section 4.4 two types of a financial constraints were examined. In the first case the firm is assumed to face a binding dividend constraint, which implies a borrowing constraint. It was shown that the firm will make maximum use of borrowing. In the second case the firm is assumed to face a given (constant) debt ratio and the profit constraint sets an upper bound on investment. In a financially-constrained interval, the firm's borrowing decisions interact with investment decisions. The role of profits is important for investment decisions in this context. The investment decision is non-myopic although adjustment costs are linear. This model gives one theoretical explanation for the existence of the cash flow impact on investment often assumed in empirical investment studies (see chapter V). In the profit-constrained case, the capital stock is usually lower than the myopically optimal capital stock. However, anticipation of a future profit-constrained interval can cause the firm to increase its capital stock above the unconstrained level. By investing more initially, the firm can try to mitigate the impending future constraints, since acquiring capital increases profits (and interest-free tax credits). The standard neutrality results of the corporate tax system hold in the profit-constrained case with respect to the equilibrium capital stock but they do not hold with respect to the dynamic path of investment. It was shown that for 'profitable' firms a rise in the tax rate decreases the level of investment while for 'low profitability' firms the reverse is true. Generally, the impact of tax and incentive parameters on investment depends upon the average profitability of the firm.

Notes to Chapter IV

1. In this study we derive optimal investment policy for a firm which is maximizing its market value (wealth maximization or share price maximization) in a world of perfect certainty. In this framework the dividend decision (payout ratio) is given only implicitly by the outcome of the joint determination of both the investment and financial decisions (exogenous as in B = sqK or endogenous when r = r(B/E)) implied by the maximization of the firm's objective function. Maximization of the market value can yield both an optimal investment rule and an optimal financial structure (when r = r(B/E)), but it leaves only a passive residual role for dividend policy (see also King 1977, chapter 6). Even when taxes (personal and corporate) are included, this model does not produce an interior 'optimal' payout ratio but instead yields only extreme values. For example, if debt were the cheapest source of finance, then this model would predict that the payout ratio is unity (i.e. all retained earnings are distributed), and if retained earnings were the cheapest source of finance, then the payout ratio would be between zero and unity (zero if retentions are inadequate to finance the optimal amount of investment), see also notes 12 and 13.

2. For a discussion of the firm's objective function in a world of uncertainty, see King (1977). If the rate of interest is assumed to be a function of leverage, then the firms and lenders are basically treated in an asymmetrical way. The firms act under subjective certainty about the future, whereas the lenders face a probability distribution of the firm's default risk (see King 1977, Englund 1979).

3. Auerbach and King (1982) derive an equilibrium relationship between the interest rate and the debt-equity ratio from an explicit model of individual optimizing behaviour. This relationship is complex and highly nonlinear. Their model also throws some light on the determinants of the debt-equity ratio in a world of uncertainty.

4. Alternatively, it could be assumed that the rate of interest is a positive constant for 'low' values of the debt-equity ratio (i.e. riskfree interest rate), and that it rises only after some level of e. The impact of a rising interest cost function on the firm's financial and investment policy has previously been considered by, e.g. Hochman, Hochman and Razin (1973), Inselbag (1973), Ylä-Liedenpohja (1976, 1983), Eriksson (1980), Englund (1979) and Steigum (1983). The main difference between our approach and the others is that we explicitly consider the impact of a marginal interest rate on the speed of adjustment in an adjustment cost framework whereas the others do not. The approach of Steigum (1983) is most closely related to ours but he assumes utility maximizing behaviour on the part of the firm whereas we use an adjustment cost model with present value maximization of profits. Steigum also ignores corporate tax factors.

5. Denote $y = y(qK,B) \equiv r(e)B$. The convexity and linear homogeneity of function y with respect to K and B follows since

$$\frac{\partial^2 y}{\partial K^2} > 0, \; \frac{\partial^2 y}{\partial B^2} > 0 \quad \text{and} \quad \frac{\partial^2 y}{\partial K^2}\frac{\partial^2 y}{\partial B^2} - \frac{\partial^2 y}{\partial K \partial B} = 0$$

It should be noted that y is not strictly convex and that convexity follows even though $r''(e) = 0$.

Furthermore, $\frac{\partial y}{\partial K} = r'B\frac{\partial e}{\partial K} = -r'e^2 < 0$ and

$$\frac{\partial y}{\partial B} = r'B\frac{\partial e}{\partial B} + r = r + r'e(1+e) > 0$$

6. In (4.14) it is assumed that new equity issues are deductible in assessing capital gains taxes, i.e. V - S is the taxable capital gain. In practice, accrued capital gains are usually taxed only at the time of realization. However, a deferred capital gains tax imposed only at the time of realization can always be transformed to an equivalent tax on the accruals basis if the holding period is known (see e.g. Bailey 1969, Bergström and Södersten 1982).

7. Taking the derivative of V_t with respect to time gives

$$\frac{dV}{dt} = iV_t - [(1-m)Div_t - S_t - \tau(\dot{V}_t - S_t)]$$

Rearranging gives

$$\frac{dV}{dt} = \frac{i}{1-\tau}V_t - \frac{1-m}{1-\tau}Div_t + S_t$$

From the solution of this differential equation, (4.17) can be obtained at time zero.

8. A two-rate system has existed in Finland since the end of the 1960s (see Appendix III). It should be noted that the basic characteristics of our analysis in chapters II and III would not change if the assumption of a classical one-rate tax system were replaced by the two-rate system.

9. In order to examine the firm's choice of financial policy, King (1977) suggests a pairwise comparison method. This means that we should compare in pairs the three alternative methods of financing open to the firm. Our model in section (4.3) can be thought to correspond to the case where the firm has a given borrowing policy since $e = e^*$. The optimal financing decision is then the following: (i) if $\theta + \tau > 1$, investment is financed only by new equity issues and all net profits are paid out as dividends, (ii) if $\theta + \tau < 1$, investment is financed from retained earnings and surplus retentions are distributed.

   The firm is indifferent with respect to the method of financing only when $\theta + \tau = 1$. In both cases i) and ii) the typical problem of a certainty model with differential taxes arises, i.e. that the firm should go further and use the cheapest financing method in order to finance only higher dividends. Hence constraints are needed to prevent an infinite 'tax arbitrage'. For example, in the first case

i) new share issues whose only purpose is to finance dividends in excess of current net profits might be made illegal. In the second case ii) this problem arises since it is profitable to convert all dividends into capital gains, and hence repurchasing of shares could occur. Thus the constraint in this case would be that the repurchase of own shares is prohibited by law. The typical Finnish parameter values are: θ is between 0.7 and 0.8 (see Appendix III) and τ is close to zero. Hence it can be concluded that $\theta + \tau < 1$, which implies that new equity issues should not be used.

The second comparison is between the two forms of external finance, i.e. borrowing and issuing new shares. For any given level of retentions, the alternative financial policies are: (i) if $1 - m < \theta\varepsilon$, investment is financed by issuing new shares and (ii) if $1 - m > \theta\varepsilon$, investment is financed by borrowing. In these inequalities ε is defined as $\varepsilon = 1-u$ if interest payments are deductible, and $\varepsilon = 1$ if interest payments are not deductible. In the Finnish case, the approximate parameter values are: m is between 0.5 and 0.6, θ is between 0.7 and 0.8 and $1 - u = 0.4$ (see also Appendix III). Hence the second (ii) inequality is likely to hold and borrowing is the 'optimal' financial policy (given the level of retentions).

The final comparison is between financing investment by retentions or by borrowing for a given level of new share issues (which for simplicity we assume to be zero). The alternative financial policies are: (i) if $1 - m > (1-\tau)\varepsilon$, investment is financed by borrowing, and (ii) if $1 - m < (1-\tau)\varepsilon$, investment is financed by retained profits.

In the first case, without any constraints, it would be optimal to borrow not only to finance investment but also to pay out higher dividends. The usual constraint to prevent this is to assume that dividends cannot exceed current net accounting profits (see also section 4.4.1). In the second case, there is an incentive to use retentions to accumulate financial assets (after financing investment and redeeming debt). The restrictions usually assumed are that the level of debt must be non-negative and retentions cannot be increased by paying negative dividends. The typical Finnish parameter values do not in this case give a clear ranking between borrowing and retention policies. If the marginal income tax rate m is 'low', then the borrowing policy would be optimal but, if m is 'high', then the retention policy is optimal. In sum, we can say that in the Finnish case the interest deductibility provision means that debt will always be preferred to new share issues. The preferred order between borrowing and retentions depends critically on the value of the personal income tax rate.

10. Poterba and Summers (1983) in considering the effects of dividend taxes on the cost of capital and investment in a Tobin's "q" framework also assume that the debt-capital ratio is constant.

11. The traditional view treats dividend taxes as additional taxes on corporate profits (double taxation view). This view suggests that the relevant tax burden for firms considering marginal investment is the total tax levied on investment returns at both the firm and the personal level. Dividend tax reductions both raise the share value and provide incentives for investment. Managerial signalling is

usually thought to provide one rationale for dividend payments. If the debt-capital ratio is a given constant, then the marginal source of funds is new equity finance (see e.q. Poterba and Summers, 1983).

12. A basic feature of the tax capitalization view is that the firm should not operate on both the dividend and share issue margins simultaneously (see Auerbach, 1983). Hence the timing of new issues and dividend payments is a critical point in this view. This view is usually assumed to operate so that the marginal source of funds is future retained earnings and the firm uses only new issues initially and pays dividends in the future. Since we assume in section 4.3 that the firm acquires a constant flow of new equity capital, then the cost of equity will depend on the dividend tax rate m unless $\theta = 1$ (i.e. $m = \tau$ which corresponds to the tax irrelevance view). However, the implication of our model (eq. 4.25) is that if current net profits are larger than desired investment then the firm pays dividends at the same time as it uses new equity capital. Our approach may be defended on the grounds that in the real world firms usually behave in the way implied by our model.

13. It should be noted that a weighted average cost of capital is relevant only when firms are constrained in their use of debt or equity or uncertainty is present (see chapter II and section 4.2). However, there need not be a simple marginal source of finance for an aggregate of firms. Different firms may face different financial margins and thus in aggregate investment analysis the weighted average approach may be appropriate (see also Miller 1977 and Auerbach and King 1982).

14. In Boadway and Bruce (1979), K_H and K are defined as

$$\text{(i)}\quad K_{H,t} = K_{H,0}e^{-\alpha t} + \int_{v=0}^{t} I_v e^{-\alpha(t-v)}dv$$

$$\text{(ii)}\quad K_t = K_0 e^{-\delta t} + \int_{v=0}^{t} I_v e^{-\delta(t-v)}dv$$

Differentiating (i) and (ii) gives $I = \dot{K} + \delta K = \dot{K}_H + \alpha K_H$, which gives the following dividend expression:
$Div = (1-u)f(K) - q(\dot{K}_H+(1-u)\alpha K_H) - (1-u)rB + \dot{B}$ when using our notation. In our case the equivalent formula is
$Div = (1-u)f(K) - q\dot{K} - q\delta K + u\alpha K_H - (1-u)rB + \dot{B}$

15. Eq. (4.30) implies that $B_t \leq K_{H,t} + B_0$, where B_0 is some arbitrary constant. The value of B_0 does not affect the necessary conditions and so it can be assumed to be zero (without loss of generality). Hence initially the debt-capital ratio is zero (if $K_{H,0} > 0$ and $K > 0$). If $\alpha > \delta$, then $B \leq K_H$ for all t implies that $B/qK = s < 1$ for all t.

Alworth (1979) suggests a different constraint on the maximum amount of borrowing, i.e. $B \leqslant \hat{K}$, where $\hat{K}$ is the 'tax adjusted' value of the capital stock defined as $\hat{K} = qK - c(qK - K_H)$, where c is the tax rate.

The value of the firm is reduced by the amount of tax which a firm would have to pay if it were to liquidate its assets. The contingent tax liability due to accelerated depreciation (deferred taxes) is treated as a substitute for debt finance. In this model it also holds that $B/qK = s < 1$. Auerbach (1983) assumes that the upper bound on debt financing is the market value of the firm. If "q" = 1 at equilibrium then $B \leqslant qK$. If, however, "q" < 1, then $B < qK$. When personal taxes are taken into account then "q" $= \frac{1-m}{1-\tau} < 1$ and $B < qK$ even at the equilibrium (see also section 4.3).

16. The conditions in (4.33) are also sufficient since the Hamiltonian and the dividend constraint are both concave in the state and control variables. Alternatively, it can be stated that sufficiency results since the maximized Hamiltonian H^{0} is concave in state variables and the requirement of constraint qualification is satisfied (see Kamien and Schwarz 1981, p 209). The transversality conditions are discussed in section 4.4.1.

17. In section 4.3 (see note 9) it was argued that borrowing is the most profitable method of finance, internal finance is next and new equity issues is the least profitable. This ranking of financial methods supports our assumption made in section 4.4.2. If debt financing is given ('optimal'), then the marginal source of finance is self-financing given the usual tax parameters. However, a constant stream of new equity could be assumed in the present model as in section 4.3 without affecting the basic qualitative character of the results in section 4.4.2. It is interesting to notice that the permanent use of new issues implies, according to the 'tax capitalization' view, that dividends should be zero for all t (see note 12). See also note 18.

18. The assumptions made in section 4.4.2 imply the following features of the firm's behaviour. If the firm is using maximum retained earnings to finance investment, its dividend payments are zero. If continuous positive dividend payments were required, the model could be formulated so that investment cannot exceed a given proportion (a) of maximum retained earnings, i.e. $I \leqslant aP/q(1-s)$, where $0 < a < 1$. This assumption would not alter the basic qualitative implications of the model in section 4.4.2. Essentially, it is assumed in the present model that the shareholders are willing to forego dividends if the firm requires all internal funds to finance optimal investment. This kind of a behaviour can follow if the firm can retain all net profits at no cost to the shareholders. Retained earnings provide the firm with a method of 'borrowing' additional funds from shareholders. Rather than issuing new equity, the firm retains earnings and causes existing equity to appreciate. The implication is that the firm's possibility to 'borrow' in this way may mitigate the effect of the inability to issue additional debt or new equity. The assumptions made also imply that the timing of dividend payments does not matter to the shareholders and that the firm would start to pay dividends only after reaching the optimal stationary steady state K* (see also Spence and Starrett, 1975, p. 402). However, if the shareholders can borrow only

in an imperfect capital market, then the timing of dividends might matter. Hence a natural implicit assumption would be that shareholders have access to a perfect capital market so that they are indifferent between income streams with the same present value. Furthermore, if personal tax parameters were incorporated in our model (as in section 4.3), and if the capital gains tax rate (τ) is lower than the dividend tax rate (m), the behaviour implied by our model would be a natural outcome, i.e. shareholders would prefer capital gains over dividend payments (see also Auerbach 1983). In sum it can be stated that there exist various circumstances in which the 'optimal' behaviour from the viewpoint of both the firm and the shareholders might be consistent with the model presented in section 4.4.2.

19. Assuming that the firm finances a constant fraction s of its net investments by borrowing, the upper bound on net investment is then $(1-s)q(I-\delta K) \leqslant f(K) - rB - \delta qK - T$

    which implies that the portion of the firm's net investments not financed by borrowing must not exceed the firm's profits, net of interest payments, depreciation and taxes. This inequality can be transformed into $(1-s)qI \leqslant f(K) - rB - T - sq\delta K$

    which is equal to (4.40) in section 4.4.2.1.

20. It is worth emphasizing here that in a strictly convex adjustment cost model or in a profit-constrained model the basic features of the investment dynamics (i.e. whether $\frac{dI}{dt} > 0$, $\frac{dI}{dt} < 0$, I = constant or I is generally nonlinear) along the optimal solution path do not usually depend on the tax or incentive parameters but on the forms of production and demand conditions the firms are faced with. The tax and incentive parameters only change in a downward (or upward) shift manner the otherwise determined general dynamic path of investment outlays. An exception is when the tax parameters, jointly with the debt-ratio, receive a special value (uz+s = 1, for instance) which alters a slow adjustment pattern to an immediate jump in the capital stock (in the absence of delivery lags, irreversibility etc.).

PART 2

EMPIRICAL ANALYSIS OF THE INVESTMENT BEHAVIOUR OF FINNISH FIRMS

CHAPTER V

SPECIFICATION OF THE EMPIRICAL INVESTMENT EQUATIONS

5.1 From Theory to Testing

Back in chapter II we started with a simple neoclassical model of investment in which the capital stock can be freely and instantaneously adjusted at each point in time. In this framework the optimal capital stock is determined independently of its past history and its future prospects. Even in this abstract environment, however, it was possible to consider the effects of an exogenous debt-capital ratio, basic corporate tax factors and an expected rate of inflation on the user cost of capital services and, through it, on the steady-state demand for capital. Various formulas for the user cost were derived in chapter II.

In chapter III the dynamics of investment decisions was considered in an adjustment cost model. In this framework it is possible to demonstrate that a variant of the conventional Koyck "partial adjustment" specification can be interpreted as a linear approximation to the optimal capital accumulation path of the capital stock. This is a general result of the, nowadays, well-established adjustment cost literature. A particularly important feature of this literature is that the derived partial adjustment coefficient is endogenous and variable, rather than exogenous and fixed, as is the case with the simple Koyck model. In chapter III it was shown that the speed of adjustment is rather sensitive with respect to both the demand conditions and the properties of the production function. A second noticeable feature of the adjustment cost model is that the firm must look to the future when making current investment decisions, a result far more in accord with reality than the myopic investment rules which result from a model without adjustment costs. In chapter III the effects of an expected (temporary or permanent) increase in demand on the dynamic investment path were also formally derived.

In chapter IV we considered various forms of capital market imperfections, especially with respect to the dynamic behaviour of investment.

The main part of the discussion was devoted to two types of imperfections: First, a non-linear cost of capital schedule (marginal interest rate) and, second, temporary or permanent credit rationing (profit constrained model). The general outcome was that the presence of both types of imperfections in the financial market affects not only the long-run demand for capital but also the timing of investment expenditures. In addition, it was noted that the partial adjustment coefficient, and the dynamics of investment in general, is rather sensitive to the specification of the financial market.

These, briefly, were the main issues covered in the foregoing theoretical discussion. It has, of necessity, been somewhat selective and there are aspects of the firm's investment policy which have been omitted (e.g. uncertainty, aggregation problems, interrelated features of factor demands and other assets of the firm).[1] The aim of the present chapter is to analyze empirically the investment behaviour of Finnish non-financial firms on the basis of this theoretical discussion. Both annual and quarterly aggregate data for the period 1963 - 1980 will be used.

There are three rather general questions which we shall attempt to answer by applying econometric methods to this data: First, what are the factors determining the long-run demand for capital? Second, what is the process by which investment expenditures are undertaken to move the actual capital stock towards its desired level? Third, what specifically can be said about the role and impact of different financial variables on capital accumulation?

The empirical analysis devoted to these questions is carried out in the neoclassical framework as described in the theoretical discussion. It can be argued that this frame of reference strongly restricts the analysis to some rather rigorous and well-specified hypothesis testing of the determinants of the investment behaviour of Finnish corporations. Some more eclectic approaches would certainly place the business investment decision in a context of many different motives and reasons for desiring, and constraints on acquiring, new productive capacity, and they would cast doubt on the explicit nature of the neoclassical model. These alternative approaches to the empirical examination of investment behaviour are, however, beyond the scope of this study.[2] It will also be seen from the subsequent analysis that the neoclassical framework is not a uniform model; rather, a researcher has many avenues to choose from in the empirical testing of the general neoclassical hypothesis.

Many important issues are related to the econometric application of an investment equation. These can be classified into three groups: First, the specification of the final reduced-form equation which is to be estimated and which forms the basis for hypothesis testing. Second, the stochastic specification of the equation and the choice of estimation method. Third, hypothesis testing with respect to the underlying maintained hypothesis of the derived model. Furthermore, a general issue in applied econometrics concerns the question as to what to estimate and what to assume. A distinctive feature of recent econometric studies of investment is that explicit theory has, to quite a large extent, been used to place constraints on the structure and functional form of the estimating model. The most well-known examples of this are the studies by Jorgenson and his collaborators. In their work, output and relative prices play a specific pre-ordained role, which is based on a number of simplifying assumptions. It can, however, be argued that the more prior assumptions there are, the less is left to be estimated (see, e.g., Helliwell 1976, Nickell 1978). If the prior information is correct, its use in the form of parameter restrictions will increase the precision of estimation. On the other hand, if the additional restrictions used are incorrect then the resulting equation may be less accurate in terms of accuracy of estimation.

In our empirical investigation we shall use a more flexible approach than in the Jorgensonian-type studies. In the context of the three general questions posed above for this empirical study, we shall analyze the following more specific questions relating to the investment behaviour of Finnish firms:

i) What is the relative significance of accelerator (output or demand) and factor price (wage rate and user cost) variables as determinants of investment?

ii) Do the effects of the wage rate and the user cost on investment behaviour differ from each other?

iii) How do different theoretical measures of the user cost perform in an empirical investment model?

iv) Do cash flow or credit rationing considerations affect investment and, if they do, in what form?

v) How do different kinds of expectations formation hypotheses with respect to exogenous factors perform in investment equations?

The outline of chapter V is as follows: In section 5.2 the general dynamic formulation of an adjustment-cost-based investment equation is considered. In section 5.3 we discuss explicit formulas for the desired capital stock by parametrizing the product and demand functions that were dealt with in implicit function form previously. In section 5.4 the role of financial factors and especially that of cash flow is discussed. Section 5.5 presents the equations to be estimated.

5.2 The Dynamics of Investment Behaviour

Virtually all empirical studies of investment make use of some sort of distributed lag between the changes in the determinants of investment and actual investment. However, there exists such a vast array of possible lag structures for an investment equation that it is sometimes very difficult to see the wood for the trees. The distributed lag structure of an empirical investment model is usually thought to describe many of the underlying reasons for the rather sluggish and hesitant behaviour commonly observed when it comes to decisions concerning investment in fixed capital. At the theoretical level, a fairly common practice is to try to separate lags that arise from the physical time structure of the investment process from other lags which may arise because of, for example, expectations and adjustment costs.

The lag between the time when the firm is faced with a situation in which it requires an increase in its capital stock and the time of the actual investment expenditure may be divided into many different components (see, e.g., Lund 1971, Jorgenson 1971, Rowley and Trivedi, 1975 and Nickell, 1978). The following list is an example of possible components:
(i) Information lags: the time between changes in the determinants of

investment and the firm's knowledge of this. (ii) Decision-making lags: the time taken to draft plans, arrange finance and place orders for new capital goods. (iii) Delivery lag: the time between the placing of orders and the construction of the final product plus bringing it into operation.

In order to analyze how this very complex process affects the structure of investment equations, fairly disaggregated and detailed data would be required (possibly even at company level). The majority of empirical studies do not go into this problem in very explicit detail, although it poses serious questions for empirical work in this area. The studies by Jorgenson (1967) and Chang and Holt (1973) are among the few which try to tackle these questions empirically. In a recent empirical study by Schiantarelli (1983) it is assumed that delivery lags are fixed but that they differ across groups of firms. This assumption leads to an investment specification where an attempt is made to capture the spread of delivery lags by the inclusion of lagged values of the dependent variable on the right-hand side of the estimating equation. There are, however, various other reasons (dynamics, expectations) for the addition of lagged values of the dependent variable and hence we do not explicitly make a similar assumption in our models.

In addition to these 'technical aspects' of the investment process there is always the question of the role of expectations-generated lags. Most investment studies assume, either implicitly or explicitly, the existence of some form of increasing marginal (strictly convex) costs of adjustment. By making an assumption of this type, the actual capital stock which the firm thinks at time t that it would like to possess at some future period is made dependent on existing levels of capital stock (see Nickell, 1978, chapter 11). Futhermore, as our theoretical discussion has shown, a determinate (optimal) rate of investment can be derived in the adjustment cost framework. In this framework, investment is proportional to the gap between the desired (K^*) and the existing (K_t) stock of capital and hence this model describes the familiar flexible accelerator hypothesis:[3]

(5.1) $$I_t^n = \lambda \ (K^*-K_t)$$

which is a continuous time presentation ($I^n = \dot{K}$). The adjustment coefficient λ is the stable root of the characteristic equation of the underlying linear differential equation system (see chapter III and note 3 of this chapter; notice also that λ was used as a symbol for the shadow price of capital in chapters II - IV). The model of the firm from which equation (5.1) is obtained consists of a non-linear objective function incorporating an adjustment cost function as well as production and demand function relationships (see chapters III and IV).

Equation (5.1) will, however, follow only when some rather simplifying hypotheses concerning expectations about the relevant (exogenous) variables are made. The assumption usually made is that expectations are fixed at some level for all future (stationary or static) expectations. Alternatively, it could be assumed that, for example, factor and product prices are expected to increase at a constant exponential rate. This rate can then be factored out of the objective function and combined with the nominal discount rate to transform it into a real discount rate. In this latter case the result is a static expectations assumption concerning relative prices.

These simplistic expectations mechanisms have the convenient property of transforming the problem of the firm into one of adjustment towards a fixed equilibrium position of target capital. In reality, of course, the supposedly fixed target may in fact be moving as prices and demand change over time, but optimization is undertaken by the firm as if it were moving at each point of time towards a fixed target (see also Nerlove, 1972).

Three basic aspects of the adjustment cost rationalization of the flexible accelerator mechanism are: (1) the adjustment coefficient generated by these models is generally dependent on exogenous variables, (2) the local character of the basic model, and (3) forward looking expectations are relevant for current investment decisions. We shall next briefly consider each of these questions.

As the rate of change of the "quasi-fixed" factor (capital) is part of the optimization problem, the theory becomes dynamic in the sense that the time required for adjustment will be endogenously determined. Hence the rate of adjustment to the long-run target level is an endogenous choice variable. In chapters III and IV we derived a number of formulas for the rate of adjustment (see equations 3.19, 3.38, 3.39, 3.49, 3.50, 4.13). The general outcome was that the adjustment coefficient is a function of both the discount rate and the desired capital stock. This coefficient seems to be rather sensitive with respect to the form of the production function, the demand conditions and the assumption made about the financial market.[4] However, if the objective function is quadratic, then its derivatives are constant parameters, implying that the adjustment coefficient varies only with the discount rate. In an extreme case, the implied adjustment coefficient is equal to the constant rate of depreciation (see chapter III).

It was previously noted in our theoretical discussions that the flexible accelerator property will generally hold only locally as a linear approximation around the equilibrium capital stock. Treadway (1974) has shown that the flexible accelerator is a globally optimal solution to the investment problem if and only if the rate of investment is independent of the rate of interest as an a priori restriction. In order to derive a flexible accelerator which is both of the constant coefficient type and globally optimal some even more restrictive assumptions must be imposed on the production and adjustment cost functions.[5]

The general results of the adjustment-cost-based rationalization of the flexible accelerator model imply that the investment function is highly nonlinear and of rather complex form (note that also K^* is generally nonlinear; see section 5.3). The difficulties associated with nonlinear estimation methods have probably been one of the main reasons why the econometric literature on factor demand analysis has not been greatly influenced by the results of the theoretical adjustment cost literature. The studies by Berndt, Fuss and Waverman (1980) and by Epstein and Denny (1983) are among the few in which the endogenous character of the adjustment coefficient is taken into account in empirical applications. In both studies, however, a general quadratic objective function is assumed in order to guarantee econometric tractability of the model.[6]

In empirical investment studies, the arithmetic adjustment process (5.1) is sometimes replaced by the following logarithmic specification (see, e.g., Eisner and Nadiri 1968, King 1972):

$$(5.2) \quad \Delta \log K_t = \lambda \log(\frac{K_t^*}{K_{t-1}})$$

which is a discrete time presentation. This form of the flexible accelerator has the advantage that it gives a log-linear form for the estimating investment equation (see section 5.3). If the logarithmic difference $\Delta \log K$ is approximated by the proportional rate of change ($\Delta K/K$), then the following specification for an investment function results:

$$(5.3) \quad \frac{I_t}{K_{t-1}} = \delta + \lambda \log(\frac{K_t^*}{K_{t-1}})$$

which is derived by using the identity $I = \Delta K + \delta K_{-1}$.[7]

In the empirical analysis of this study, we shall apply the above three specifications of the flexible accelerator model (equations 5.1 - 5.3) in order to check the robustness of the results for alternative functional forms. As was mentioned above the continuous time formulation (5.1) of the flexible accelerator model will result only if very simplistic expectations are assumed with respect to the exogenous variables. In the discrete time version of (5.1) there will inherently always exist a one period expectations lag.[8]

The assumption of the 'fixed target' model in econometric applications has been critized on the grounds that in reality firms are reacting to a moving target, or perhaps to no target at all but rather to changing expectations of the probability distributions of future prices and future demand (see e.g. Taylor 1970, Nerlove 1972, Nickell 1978). If static expectations are not assumed, the optimization problem becomes much more complicated and no simple adjustment pattern of the form (5.1) will follow (see also Begg 1982).[9] In general, the optimal path of investment will depend on the entire future of prices and other exogenous factors. Hence, the presence of adjustment costs has the important implication that firms must consider profit opportunities over their entire planning horizon when making current investment decisions (see chapter III).

The usual way to circumvent the static expectations assumption in (5.1) is to assume that forward expectations are based on the past experience of the firm. This has been the traditional rationale for distributed lag specifications in the vast majority of empirical investment studies. This approach will also be used in our empirical calculations. We shall, however, also perform some experiments with truly forward looking expectations about the main determinants of investment (prices and demand). It should, however, be noted that linear rational expectations modelling allows us to transform the forward looking rule into a finite distributed lag specification (see Begg, 1982).

With non-static expectations there is a different 'desired' capital stock corresponding to each future period and the problem of precisely what firms should aim at is rather complicated. It is clear from our theoretical analysis that optimally behaving firms should aim at some sort of 'moving average' of all the future 'desired' capital stocks (see also Nickell 1978). The analytical solution of this problem depends upon the parametrization of the implicit functions involved, i.e. production, demand and adjustment cost functions. The long-run target is often assumed to be a geometrically weighted average of current and future values of the desired levels of the capital stock, the weights being determined by the adjustment coefficient and by the discount rate. The current target depends on the expected future values of the exogenous variables.[10]

The determination of the 'moving target' capital stock is the basic characteristic of a forward looking investment model. It should be noted that if the adjustment coefficient is endogenous and possibly non-linear, and since K^* is usually non-linear, the resulting estimating equation will be highly non-linear and rather difficult to estimate reliably. Hence it would probably be necessary to make some simplifying assumptions to knock it into some reasonable shape (see chapter VII).

Because of the very complex nature of the reduced form investment equation in the case where there are general assumptions about the underlying hypotheses, we shall proceed in a stepwise manner in our empirical investigation (see also section 5.5). First, both annual and quarterly models will be estimated for the flexible accelerator models of types (5.1) - (5.3) with a constant adjustment rate and assuming either linearized or nonlinear K*. Second, the endogenous nature of the adjustment process is assumed to stem from cash flow considerations of the firm. In this case the adjustment coefficient is assumed to be a nonlinear function of the ratio of cash flow to the desired rate of expansion of the capital stock (see section 5.4). The rate of adjustment will then depend also on the arguments of K* as depicted by our theoretical considerations (see chapter III). Finally, the forward looking expectations hypothesis is examined making flexible use of equation (5.1).

5.3 The Determination of the Desired Capital Stock

In the theoretical discussion a number of formulas for the desired capital stock were derived in an implicit functional form. The main purpose, however, was to consider the formulas for user cost and the dynamics of investment under alternative assumptions with respect to input and output markets. In this section the purpose is to derive explicit functional forms of the desired capital stock by parametrizing the underlying production and demand relationships.

The second issue discussed in this section deals with some other implications of the choice of the production technology for optimal investment policy. Among the issues are the relationship between price and substitution elasticities, the implications of putty-putty and putty-clay technologies for the lag structure of investment, the replacement hypothesis and the rate of technical change. The discussion in this section clearly shows that in the economic theory of investment the form of the optimal investment policy also depends significantly on technological considerations.

5.3.1 Formulas for the Desired Capital Stock

The determination of the desired capital stock is inherently linked with the choice of the production and demand functions. The purpose here is to derive some basic formulas for the desired capital stock by parametrizing these two functions. The previously derived implicit functional forms for the desired capital stock were given by formulas (2.4), (3.37) and (3.47). The results derived here are shown in table 4. Before considering them in more detail, we shall discuss briefly the choices of the underlying functional forms.

We have chosen to deal with two well-known production functions, i.e. the Cobb - Douglas (CD) and constant elasticity of substitution (CES) functions, and with three hypotheses about the product market (demand curves). The demand curves are the same as those used in chapter III, i.e. horizontal (perfect output market), vertical (excess supply model) and downward-sloping curves.

The production functions are given by the following equations:

(5.4) i) $Q = A\,K^{a}\,L^{b}$ (Cobb - Douglas)

ii) $Q = B[vK^{-y}+(1-v)L^{-y}]^{-x/y}$ (CES)

In the case of the CES function the parameters are: B = efficiency parameter, v = distribution parameter, x = degree of homogeneity (scale parameter) and y = substitution parameter. If the elasticity of substitution is σ, then $y = (1-\sigma)/\sigma$ (see Wallis, 1979).

In the case of a monopolist facing a downward-sloping demand curve, the demand function is assumed to be of the form (see also chapter III):

(5.5) $p = \gamma_0 Q^{\gamma_1} Z^{\gamma_2}, \quad \gamma_0 > 0, \quad \gamma_1 < 0, \quad \gamma_2 > 0$

where the exponent γ_1 is the reciprocal of the price elasticity of demand. The variable Z is a general shift term (demand index) and it may represent such variables as consumer income levels, population growth and factors which affect the demand for output. Equation (5.5) has also been used in Gould and Waud (1973) and Chang and Holt (1973). In order to guarantee zero-order homogeneity of demand for capital with respect to nominal prices, the price variable p should be interpreted as a relative price, i.e. the ratio of own product price to other (substitute) product prices (see below).

Technically, the formulas for desired capital are obtained either by static profit maximization or by intertemporal present value maximization using the relevant production and demand function constraints. The concept of user cost is, however, generally not the same in these two cases.[11] We shall next consider the formulas for the desired capital stock presented in table 4.

In the perfect output market case (case 1 in table 4), the formula for the desired capital stock is derived using only one first-order condition ($\frac{\partial Q}{\partial K} = c/p$). This has been the usual practice in the Jorgenson type empirical investment studies. The output variable in this case is, however, the endogenous equilibrium value ($Q = Q^*$). If both first-order conditions are used simultaneously, then $K^* = K(\frac{w}{p},\frac{c}{p})$ (Brechling, 1975).[12]

In the case of a vertical demand curve, output is by assumption exogenous to the firm. It should be noted that a profit maximization model under a demand constraint is equivalent to cost minimization. The desired capital stock now depends on the exogenous output and on the factor price ratio (w/c). The assumption of a vertical demand curve has been preferred by some researchers for three reasons (e.g. Brechling 1975). First, output affects the demand for capital. Second, the relative price variable affects the demand for capital in a way in which the wage rate is also included. Third, the long-run price elasticity of demand for capital is different from unity even in the case of the Cobb - Douglas production function. In the case of a perfect product market, one would have to assume a CES production function in order for price elasticity to be nonunitary (see e.g. Brechling 1975, Helliwell 1975).

Table 4. The Long-Run Steady State Formulas for the Desired Capital Stock under Alternative Assumptions about Production and Demand Functions*

Production function:	Cobb - Douglas (CD)
Demand condition	
1. Horizontal demand curve	(5.6a) $K^* = a \frac{pQ}{c}$
2. Vertical demand curve	(5.6b)[1] $K^* = a' Q^{\frac{1}{a+b}} (\frac{w}{c})^{\frac{b}{a+b}}$
3. Downward-sloping demand curve	(5.6c)[2] $K^* = e_0 Z^{e_1} w^{e_2} c^{e_3}$
Production function:	CES
Demand condition	
1. Horizontal demand curve	(5.6d)[3] $K^* = b' (\frac{p}{c})^{\sigma} Q^{\eta}$
2. Vertical demand curve	(5.6e)[4] $K^* = b'' Q^{\frac{1}{x}} [v + (1-v)^{\sigma} v^{y\sigma} (\frac{w}{c})^{y\sigma}]^{\frac{1}{y}}$
3. Downward-sloping demand curve	(5.6f)[5] $K^* \simeq d_0 Z^{d_1} w^{d_2} c^{d_3}$

---

* w = wage rate, c = user cost, p = price of output, Q = output, Z = demand index (shift parameter)

1) $a' = [\frac{1}{A}(\frac{a}{b})^{b}]^{\frac{1}{(a+b)}}$

2) parameters e_i see note 13

3) $(b')^{(1+y)} = xvB^{(1-\frac{y}{x})}$, $\eta = \sigma + \frac{1-\sigma}{x} = \frac{x+y}{x(1+y)}$, $\sigma = \frac{1}{1+y}$

4) $b'' = (\frac{1}{B})^{\frac{1}{x}}$

5) formula (5.6f) is an approximation, see notes 13 and 14

Somewhat more complicated formulas for the desired capital stock will result when the demand curve is assumed to be downward sloping. In the Cobb - Douglas case the equation for K* is exact but in the CES case it is only an approximation. The derivation of the formulas for K* under this demand regime is discussed in notes 13 and 14. The signs of the parameters of the wage rate and the user cost are ambiguous although one would normally expect K* to depend negatively on c. The sign of the coefficient of the wage rate variable is clearly ambiguous.[13] The problem with this approach is that the effect of a demand variable is rather difficult to quantify. Variable Z, which correctly isolates the effect of demand, is an index which measures the position of the demand curve and this is more or less unobservable. It should also be emphasized that it is the demand curve which is taken as exogenous and actual demand can, of course, be influenced by manipulating the output price. Generally speaking, however, the degree to which firms can control the demand for their products is a rather contentious issue (see, e.g., Nickell 1978).

In the case of a Cobb - Douglas production function and a downward-sloping demand curve, a similar formula to that of (5.6c) can also be derived for an entire competitive industry if it is assumed that each firm acts as a price taker and that the market clearing industry output will be such that the market price is equal to the firm's marginal cost (see Gould and Waud, 1973). An attempt to derive rigorously optimal investment for an industry aggregate in the context of adjustment costs and monopolistic competition is presented in Lucas (1976) and Nerlove (1972).

To summarize the main results of the different formulas for the desired capital stock, the following aspects will be emphasized. First, if both first-order conditions for an optimum are used simultaneously then K* depends in all three cases of demand conditions on both the wage rate and the user cost. Second, an accelerator variable will affect K* only when there exists some form of imperfect competition in the product market. Third, a major problem with the CES function is that, unlike in the Cobb - Douglas case, one cannot transform it into a linear-in-parameters form in vertical or downward-sloping demand curve cases by operations such as taking logarithms. There is no method of linearising that will sort out the parameters and variables to give a directly estimable exact representation; some linear approximation has to be used.

In the empirical applications of investment equations we shall assume that the demand curve is either vertical or downward-sloping. This choice is basically motivated by the fact that we use aggregate data, which implies that, even if a single firm is a price taker, the aggregate of firms usually faces a downward-sloping curve. The second reason for our choice of demand curves is that only when there is some form of imperfection in the output market does it seem legitimate to include an accelerator variable in the investment function (see also Brechling, 1975, Nickell 1978). The third reason for our choice of demand functions is that in these two cases it is indisputable that the wage rate will directly affect investment behaviour. It should, however, be mentioned that especially the problems associated with the measurement of the demand variable (Z) make it rather difficult to distinguish these two demand regimes at an empirical level. But as a general starting-point for the empirical analysis the separation of the demand conditions is useful since it reveals the alternative roles of the accelerator variable and the market conditions.

5.3.2 Some Implications of Technology for Investment

The foregoing discussion has clearly indicated the importance of the underlying production technology for capital accumulation. We shall next briefly discuss four other aspects of the relationship between technology and investment behaviour. These are: (1) the relationship between the elasticity of substitution of factor inputs and the price elasticity of demand for capital, (2) the implications of putty-putty and putty-clay assumptions for the lag structure of the investment process, (3) the replacement hypothesis and (4) the role of technical change in investment models. All these four issues will have direct bearing on the interpretation of the parameter estimates from investment equations to be discussed in chapters VI and VII.

The size of the long-run effect of relative price changes on investment has been among the central issues in empirical investment studies since the seminal work by Jorgenson (1963). It is clear that the effect of a change in the real user cost or in the ratio of the wage rate to the user cost will depend on the elasticity of substitution of the production function (see table 4). If the firm is a price taker in the product market, then the price elasticity is equal to the elasticity of substitution (see equations 5.6a and 5.6d in table 4). Generally, however, it can be said that if there are imperfections in the product market, then the elasticity of the desired capital stock with respect to factor prices is not equal to the elasticity of substitution. The price elasticity will depend on the elasticity of demand for the firm's products and on a number of other parameters as specified in the Marshall-Hicks rules for factor demand elasticities (see, e.g., Nickell 1978, Layard and Walters 1978). Hence the estimated price elasticities of the demand for capital cannot be interpreted directly as estimates of elasticity of substitution between inputs.

The question of the size of the impact of relative prices on investment is also closely linked to another aspect of the underlying technology, i.e. whether the production function is characterized by a putty-putty or putty-clay hypothesis. In our theoretical discussions as well as in the derivation of the formulas for the desired capital stock the putty-putty hypothesis is assumed.

In the case of putty-putty technology, capital is assumed to be as malleable ex post as it is ex ante. In terms of the elasticity of substitution of factor inputs, a putty-putty production function is based on the hypothesis that this elasticity is the same ex post as ex ante. An alternative to the putty-putty technology, putty-clay, has been introduced by Johansen (1959) and applied in empirical work on investment by Bischoff (1971), King (1972), Ando, Modigliani, Rasche and Turnovsky (1974), Sarantis (1979) and Schiantarelli (1983). According to this hypothesis, the elasticity of substitution is smaller ex post than it is ex ante. With an extreme version of putty-clay technology, the elasticity of substitution of capital for labour is zero ex post. The putty-clay hypothesis has two important implications for investment behaviour (see e.g. Hall 1977, Nickell 1978).[15] First, the effect of putty-clay type restrictions is to slow down the response of the aggregate capital-labour ratio to relative price changes since this ratio is (almost) fixed for the existing capital stock. Second, it would not affect the response rate to changes in demand, and hence in a putty-clay model the capital stock responds more quickly to an increase in output than to a decrease in the user cost.

Eisner and Nadiri (1968) and Bischoff (1969), however, noted that, even in a putty-putty investment model, there could be different lag structures with respect to output and relative prices if expectations about these two variables depend only on their own past values and if the weights in the autoregressive expectations processes are different for output and relative prices. Abel (1981) has recently shown that the putty-clay type relative speed of response can be derived from the behaviour of an intertemporally optimizing firm with a putty-putty production function and subject to costs of adjustment. Thus, data consistent with the putty-clay hypothesis could be generated by a model with putty-putty technology and without referring to different expectational lag structures for output and relative prices. In addition, Abel (1981) has shown that, with a putty-putty CES production function and adjustment costs, investment may have different response rates with respect to the wage rate and the user cost.

In summary, it can be said that the putty-putty investment model with adjustment costs is likely to produce lagged response of investment to changes in the exogenous factors similar in nature to that in the putty-clay model. In reality, however, neither the strict putty-putty nor the strict putty-clay hypothesis seems completely plausible, but research has yet to uncover the right compromise between them. At the level of a single firm, it may often be that some form of putty-clay hypothesis is relevant but at an aggregative level there may still exist many ex post substitution possibilities between capital and labour, and thus the putty-putty model might still be a fairly realistic assumption (see Hall, 1977). It should also be noted that it is not possible to specify an investment equation exactly in a vintage (putty-clay) environment since one does not usually have the requisite information on the level and productivity of past investment in each vintage (see King, 1972). This lack of proper data is likely to be the basic reason for the fact that the reduced-form estimating investment equations have been very similar in the putty-putty and putty-clay cases.

Replacement investment is commonly defined to be that part of gross investment which is required in any period to maintain the existing capital stock at a constant level. In empirical investment studies it has almost invariably been assumed a priori that the ratio of replacement investment to the current stock of capital is constant. This assumption has also been used in this study.

Feldstein and Rothschild (1974) and Nickell (1978) have shown that for a general non-exponential depreciation pattern, the replacement-capital stock ratio is constant if and only if the age structure remains constant over time. It has also been argued that replacement investment is subject to optimal decision-making like net investment and hence it should vary in response to economic factors rather than being simply determined mechanistically as would be the case with exponential decay (see e.g. Feldstein and Foot 1971, Feldstein and Rotschild 1974, Eisner 1972).

Although generally a preferred hypotheses might be to assume that the rate of replacement is not simply determined by the mechanical decay of investment goods, but rather is based on economic decisions, this would not necessarily imply that firms do not predict replacement requirements on some fairly mechanistic basis (see Nickell 1978, p. 307). Since it is these predictions which should occur in empirical investment models, it may still be correct to specify replacement investment by making some mechanistic depreciation assumption.[16] The net capital stock series used in our empirical models are based on depreciation rates which change gradually over time but we shall also make some experiments with a more rapidly increasing depreciation coefficients.

Most empirical investment equations typically ignore technical change altogether or deal with it in some way that is analytically convenient. It can also be argued that some unspecified part of technical change has possibly already been taken into account in the existing measures of capital stock and labour input.

In the derivation of the formulas for the desired capital stock, we have ignored the effect of technical change (see table 4). It is possible, however, that this omission causes some bias in the empirical estimates of the effects of output (demand) and relative prices on investment. Hence in the empirical analysis we shall make some simple calculations of investment equations including a proxy variable for technical change. The purpose is not to estimate the rate of technical change per se but to try to discover whether its presence has an effect on the price and output elasticities of the demand for capital.

5.4 The Role of Financial Factors in the Investment Equation

5.4.1 Description of Potential Financial Factors

In the theoretical chapters of this study we discussed various channels through which financial factors affect investment policy by firms. Basically, these can be classified into two forms of transmission mechanisms:

(i) Financial factors which influence investment behaviour via the 'rate of return' effect as captured by the cost of capital variable.

(ii) Financial factors which affect investment through a direct liquidity channel (cash flow effect).

Since the financial variables which influence investment through the cost of capital term in the user cost variable were analyzed fairly extensively in the theoretical chapters, we shall only briefly consider these factors here. Because tax factors are intimately linked with the effects of financial factors, we can summarize the following variables (or parameters): the cost of borrowed capital (average or marginal interest rate), the discount rate, the debt-capital ratio (leverage), tax parameters (corporate and personal) and the amount of retained earnings. The effect of a change in these variables and parameters on the cost of capital and hence on the level of investment is not always unambiguous but this question is not dealt here since it was considered in chapters II-IV.

It could be argued that in an uncertain environment the cost of capital also depends on the market view of the firm's riskiness and on the general level of market risk aversion, both of which are extremely difficult to measure in practice (see Auerbach, 1983). The influences of these uncertainty (risk) aspects should, however, already be incorporated in the two previously mentioned interest rates, i.e. in the rate which the market uses to discount the expected profits generated by the firm (discount rate ρ) and in the rate of interest on borrowed capital (r). Obviously, an increase in ρ and r will raise the cost of capital to firms.

It is clear that in a world of certainty without the possibility of default on loans (bankruptcy) and with a 'typical' corporate tax structure, marginal units of investment are financed solely by debt and all earnings are paid out as dividends (see King 1974, Auerbach 1983). This kind of a world is, however, most likely to be a far cry from reality. What can be observed in the real world is that firms normally use different types of finance and pay out dividends to an amount less than current net profits. On the other hand, if there exists a possibility of bankruptcy, it is not obvious what meaning can be attached to the 'long-run' optimal proportion of debt finance. Moreover, the determination of optimal debt-capital and dividend payout ratios is rather a contentious issue, as recent finance literature has shown (see e.g. Auerbach and King 1983, Auerbach 1983, chapter IV).

Generally, it can be said that, given the firm's level of profits from the previous period and its outstanding debt and equity, it must simultaneously determine its level of real capital expenditure, its total dividend payout and its new levels of debt and equity in the current period. If P_{t-1}, B_{t-1} and N_{t-1} are the profits, debt and equity from period t-1, then the level of dividends (Div) and investment (I) in period t must satisfy the following flow identity: $I_t = \Delta B_t + \Delta N_t + (P_{t-1} - Div_t)$, where I_t is the current value of investment (see also chapter II). If the firm uses different types of finance, then the amount of retained earnings plays a certain role in investment policy through its impact on the weighted average concept of cost of capital (see formulas 2.29, 4.27, 4.28). To the extent that retained earnings are a cheaper form of finance than debt or new equity, an increase in its share of total finance would lower the average cost of capital and hence increase the demand for capital (see also King, 1974).

5.4.2 Profits and Investment

In the investment literature other arguments have been presented in favour of the inclusion of cash flow considerations among the determinants of investment behaviour. It can be noted that cash flow or profit type variables have a long tradition, especially in empirically-orientated investment literature. In the 'residual funds' theory, which was born in the 1950s and which had its greatest impact on investment models in the 1960s, the profit variable played a central role (Duesenberry 1958, Meyer and Kuh 1957, Meyer and Glauber 1964, Kuh 1963, Rowley and Trivedi 1975). The formal theoretical background of this approach, however, remained rather weak. Typically, it was argued that internal funds are the cheapest means of financing investment and hence that they exert a significant impact on investment. A somewhat more elaborate argument was proposed by Lintner (1967), who argued that internally generated funds are an important determinant of investment because of uncertainty and capital market imperfections. Because of these factors, retained earnings affect the slope of the marginal cost of funds curve and hence also the volume of investment.

The earlier literature from the 1950s and 1960s was not, however, very specific with respect to the question of whether internal funds affect the long-run desired capital stock or just the timing of investment. In the 'residual funds' approach it was sometimes argued that in expansionary periods investment is determined by the acceleration principle whereas in cyclical downturns it is confined to a level that can be funded from retained earnings (see Rowley and Trivedi 1975). On the other hand, Eisner (1967) provides some evidence that firms tend to make higher investment expenditures in the periods following higher profits but that over the long run firms earning higher profits do not make markedly greater capital expenditures than firms earning lower profits.

This observation by Eisner indicates that the amount of profits would affect only the timing of investment and not the long-run demand for capital. Nickell (1978) has recently provided some more formal analysis in favour of the impact of profits, chiefly on the timing of investment. Basically, he argues that a direct link between current investment and profits (past and current) can arise only from the essential structure of capital markets under uncertainty and, furthermore, that such capital markets are inherently imperfect (Nickell 1978, p. 167 - 189). In these markets the possibility of bankruptcy for both individuals and firms is taken into account and the marginal cost of capital to the firm will depend upon the riskiness of the firm's future prospects. In this kind of environment, the investment decisions of firms could be affected by the wealth of their owners and, moreover, an important element of this wealth may well consist of the profits earned by the firm. In addition, Nickell argues that, since the cost of capital schedule shifts with the level of current earnings, one would expect the speed at which the firm adjusts to its target level to depend on this same level of earnings, with higher amounts of profits leading to faster adjustment (Nickell 1978, p. 185).

Coen (1971), Prior (1976) and Sarantis (1979) are among the few researchers who have incorporated the flow of internal funds relative to the desired expansion of capital in an adjustment cost mechanism as suggested later by Nickell. Feldstein and Flemming (1971), on the other hand, have integrated the ratio of gross retained earnings to trend output as an additional term in the user cost of capital with its own separate coefficient. As we have noted previously in section 5.2 and in chapters III and IV, the rate of adjustment to the equilibrium capital stock is, in a general case, endogenous and depends, among other things, on retained earnings through the cost of capital variable. Hence, these results imply that cash flow considerations may influence both the long-run demand for capital (K^*) and the timing of investment expenditures. It should, however, be noted that in the profit constrained case considered in section 4.4.2 only the cash flow affects the timing of investment.

These considerations seem to indicate that the flow of internal funds (cash flow) may have an effect on investment both through the long-run demand for capital (user cost effect) and through the endogenous speed of adjustment (timing effect). The effect on timing occurs in the adjustment-cost rationalization of the flexible accelerator model, since the rate of adjustment is dependent upon cash flow, among other factors. What is suggested here is that the speed of adjustment λ in (5.1) and in its transformations (5.2) and (5.3) should be regarded not as a constant but rather as a function of cash flow considerations. Thus the discrete time version of model (5.1) should be written as

$$I_t = \lambda_t(K_t^* - K_{t-1}) + \delta K_{t-1} \tag{5.7}$$

where $\lambda_t = \lambda(CF_t)$, and CF_t is the level of cash flow. Alternatively, the cash flow model could be written as

$$I_t = \lambda_t(K_t^* - (1-\delta)K_{t-1}) \tag{5.8}$$

This equation reflects the hypothesis that a firm can consider the adequacy of internal finance to meet both net expansion and replacement expenditures (see Coen 1971).

The question now remains as to what is the explicit functional form of a cash flow equation. In some applications of the 'residual funds' approach a profit variable was simply added as an additional explanatory variable to a linear investment equation (e.g. Kuh 1963). Greenberg (1964), on the other hand, tested an equation similar to (5.8) and specified λ as an exponential function of after-tax profits (among other factors). A major shortcoming of these kinds of specifications is that they do not take account of the ratio of cash flow to the total amount of funds required in order to accomplish what could be called an investment chore - the expansion from the current to the desired level of the capital stock. Hochman (1966) and Coen (1971) recognize this point in principle but end up through a linearization procedure with cash flow being an additional linear variable in the investment equation.

In addition to the questions of the proper scaling of the CF-variable and the general functional form of the adjustment mechanism, one should also consider the question of whether the rate of adjustment is constrained or unconstrained. A constrained case ($0 < \lambda < 1$) seems preferable on the basis of 'stability' and smooth adjustment to the target level of the capital stock. We have chosen to use a similar specification as in Coen (1971 p. 153) and in Nickell (1978 p. 263). The exact form of the rate of adjustment function is

$$(5.9)\quad \lambda_t = \lambda_0 + (1-\lambda_0)(1-e^{-x_t})$$

where x_t is specified in two alternative forms,

$$(5.10)\quad \text{i)}\quad x_t = \lambda_1 \frac{CF_t^n}{K_t^* - K_{t-1}}$$

$$\text{ii)}\quad x_t = \lambda_1 \frac{CF_t^g}{K_t^* - (1-\delta)K_{t-1}}$$

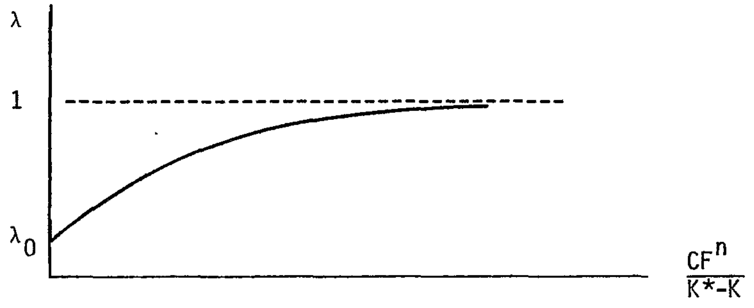
where CF^n and CF^g are net and gross concepts of cash flow, respectively. In the case of equation (5.7), the nonlinear specification of the rate of adjustment with net cash flow gives the following investment model

$$(5.11)\quad I_t = [\lambda_0+(1-\lambda_0)(1-e^{-\lambda_1 \frac{CF_t^n}{K_t^*-K_{t-1}}})](K_t^*-K_{t-1}) + \delta K_{t-1}$$

Inserting (5.10 ii) into (5.9) and the latter into equation (5.7) or (5.8) gives the investment equation in terms of the gross cash flow adjustment mechanism.[17]

We know from the preceding discussion (section 5.2) that an increase in the importance of adjustment costs slows the speed of adjustment ($\frac{\partial\lambda}{\partial b} < 0$ if adjustment costs are bI^2; see note 10). Hence the more internal finance the firm has, the less it is dependent on outside funds and the smaller is the importance of adjustment costs, implying that we would expect λ_1 to have a positive sign. The rather complex structure of this specification ensures that the rate of adjustment is bounded above by unity and has an appropriate shape as a function of available internal funds as illustrated in figure 7.

FIGURE 7.



More formally, the properties of the adjustment mechanism (5.9) can be expressed as

(5.12) i) as $x \to 0$ then $\lambda \to \lambda_0$ (constant speed of adjustment)

ii) as $x \to \infty$ then $\lambda \to 1$ (instantaneous adjustment)

iii) $\frac{\partial \lambda}{\partial x} = (1-\lambda_0)e^{-x} > 0$

iv) $\lim_{x \to \infty} \frac{\partial \lambda}{\partial x} = 0$

The first property implies that, with relatively 'little' cash flow available, the model collapses to the constant speed of adjustment model. The second property ii) means that, when the firm has 'excess' supply of cash flow, its rate of adjustment will be very fast. However, it should be noticed that the extreme limit values of λ in cases i) and ii) are not very likely to occur in the real world since delivery lags and expectational aspects of the investment process also affect investment behaviour. The third property iii) implies that a rise in relative cash flow, ceteris paribus, will increase the rate of adjustment. The fourth property means that, at very high values of relative cash flow, the rate of adjustment is not much affected by a further rise in cash flow.

If one uses the linearized approximation (Taylor series expansion around $x = 0$) of the adjustment mechanism (5.9), the following linear investment equation follows

$$(5.13) \quad I_t = [\lambda_0 + \lambda_1' \frac{CF_t^n}{K_t^* - K_{t-1}}](K_t^* - K_{t-1}) + \delta K_{t-1}$$

where $\lambda_1' = (1-\lambda_0)\lambda_1$. This equation can be transformed into the following form

$$(5.14) \quad I_t = \lambda_0 K_t^* + \lambda_1' CF_t^n + (\delta - \lambda_0) K_{t-1}$$

which is hence a linearized approximation of the non-linear model (5.11). In the case of the 'gross expansion' model the equivalent approximation is

$$(5.15) \quad I_t = \lambda_0 K_t^* + \lambda_1' CF_t^g + \lambda_0 (\delta - 1) K_{t-1}$$

Equations (5.14) and (5.15) are similar to those used by Coen (1971), except that in Coen's model a lagged value of investment is included instead of K_{t-1} and the equation of K* is of linear form, hence implying that a nonspecific production function is assumed (K* depends on w/c and Q in Coen's model).

In summary, it is worth emphasizing that our specification of the endogenous (variable) adjustment rate model according to (5.11) is basically quite close to the 'full' theoretical flexible accelerator models presented in chapter III since the speed of adjustment depends upon both cash flow and the other determinants of the desired capital stock. The exact functional form is, however, chosen here with an eye to the empirical applicability of the model. It should also be noted that the effect of the choice of the demand regime on the rate of adjustment will be reflected in our model through the K* variable in the x-term (equations 5.9 and 5.10). In chapter III it was shown that the rate of adjustment is rather sensitive with respect to the assumption of the demand curve. Using equation (5.11), it is hence possible to test empirically the sensitivity of the rate of adjustment to alternative demand conditions (see chapter VII).

5.4.3. Credit Rationing and Investment

As was mentioned above, it seems likely that cash flow has a role to play in investment policy chiefly in the context of imperfect capital markets. It is this fact which may limit the firm's use of debt finance in an uncertain world. Hence bankruptcy risks can ensure that, beyond a certain level, the costs of borrowed capital will be rising at the margin (see section 4.2). Effective quantity restrictions on debt may even arise if the effective borrowing rate for the firm rises to infinity above a certain level of debt (or leverage), i.e. there exists no interest rate which will give lenders an expected return above the safe rate. In this case the firm will typically make extensive use of internal funds to finance its investment (see also Nickell 1978, p. 211).

In the light of our theoretical discussion in chapter IV, we can distinguish between two basic situations when the firm is faced with an imperfect financial market. First, the firm can finance all of its optimal (desired) investment, although at a rising cost of capital, and, second, the firm cannot finance the full amount of desired investment (constrained case). In the latter case the firm's actual debt-capital ratio (s) is below the optimal ratio (s*) and it cannot make full use of tax savings arising from interest deductions. Hence the realized capital stock is below the unconstrained (optimal) capital stock (see also chapter II, case $\rho > (1-u)r$). Credit rationing will compel the firm to use more expensive methods of finance. In the first case it is assumed that the firm can eventually finance its desired investment outlays using more internally generated funds (cash flow) and/or new equity issues. Since the firm is restricted in using the cheapest form of finance (after-tax), the effect of this capital market imperfection is only to increase the cost of capital and hence also the user cost as compared to a non-rationing situation. In this situation the impact of credit rationing is realized only through price variables (cost of funds).

However, a more complicated situation arises if the anticipations aspects of a credit rationing situation are taken into account, as the analysis of section 4.4.2 demonstrated. There it was shown that if the firm anticipates that at some future date it will face a financial constraint on the quantity of capital goods that it is able to buy, it will invest more during the period preceding the occurrence of this constraint than it would have done in the unconstrained case (under certain conditions, e.g. decreasing returns to scale). Thus a financial constraint may cause anticipatory buying of investment goods.

In the second case mentioned above, where internal funds (and new issues of equity) are insufficient to finance all desired investment (even with a minimum level of dividends), the firm already faces a quantity-constrained situation. In this case investment policy is completely liquidity-constrained and the amount of investment depends only on the available funds (see also section 4.4.2). This situation implies that the timing of investment is also affected by credit rationing factors.

In sum, it can be said that credit rationing (permanent or temporary, anticipated or unanticipated) may affect investment decisions of firms in various ways. Furthermore, different firms may face different situations in the financial market, and for each firm the situation may also change over time (as the analysis of section 4.4.2. reveals). Hence, especially in an aggregate analysis of firms, it is not possible (given the present stage of knowledge) to model the effects of credit rationing in a very precise manner. It might, however, be thought that the main effects of credit rationing (if any) are already captured by the user cost (through various interest rates) and cash flow variables. If the price and other effects (non-price loan terms) are not reflected in these two variables there would be room in an empirical analysis for an additional variable - say, a proxy for the tightness of the credit market - to be included as a determinant of the investment equation. In the empirical investment analysis we shall use different proxy variables for credit rationing as additional explanatory variables in the linear investment equations (see chapters VI and VII).

5.5 Investment Equations to be Estimated

In this section we shall briefly summarize and bring together the discussion of the preceding sections on the major issues in the construction of investment models. In the first section various versions of the equations to be estimated are presented. Since the arguments of the desired capital stock are usually expected values, these 'exogenous' variables are labelled with the superscript 'e' (Q^e, w^e, c^e etc). There are a number of possible approaches to expectations specification and the ones used in our empirical models are briefly discussed in section 5.5.2, which also considers the general characteristics of the stochastic specification of estimating equations.

5.5.1 The Basic Equations

Our main point of departure is to assume a linearized form for the desired capital stock. There is very little empirical work on production functions in the Finnish corporate sector and hence no strong a priori assumptions can be made with respect to the production technology. The linearized version of K* implies that a nonspecific production technology is assumed.

Linearized versions of K* can be thought to stem either from equation (5.6b) or from equations (5.6c,f) and the equations are

$$(5.16)\quad \text{i)}\quad K_t^* = \alpha_0 + \alpha_1 Q_t^e + \alpha_2 \left(\frac{w}{c}\right)_t^e$$

$$\text{ii)}\quad K_t^* = \beta_0 + \beta_1 Z_t^e + \beta_2 w_t^e + \beta_3 c_t^e$$

where the variables Q, w, c and Z are, respectively, production, the wage rate, the user cost and an exogenous shift term for demand. It should be noted that w and c can take either nominal or real values (see section 5.3 and note 13), although we shall use only real values. Note that Q_t^e, $\left(\frac{w}{c}\right)_t^e$ etc. are used as general expressions for the expected values of these exogenous variables.

Inserting the equations for K* into the flexible accelerator model (5.1) gives the following gross investment equations

$$(5.17)\quad \text{i)}\quad I_t = a_0 + a_1 Q_t^e + a_2 \left(\frac{w}{c}\right)_t^e + a_3 K_{t-1}$$

$$\text{ii)}\quad I_t = b_0 + b_1 Z_t^e + b_2 w_t^e + b_3 c_t^e + b_4 K_{t-1}$$

where the parameters are

$a_0 = \lambda\alpha_0$, $a_1 = \lambda\alpha_1$, $a_2 = \lambda\alpha_2$, $a_3 = \delta - \lambda$

$b_0 = \lambda\beta_0$, $b_1 = \lambda\beta_1$, $b_2 = \lambda\beta_2$, $b_3 = \lambda\beta_3$, $b_4 = \delta - \lambda$

It should be noted that our chosen basic adjustment equation (5.1) does not give a linear or log-linear exact presentation for investment when an imperfect product market exists. If, however, the logarithmic specification of the flexible accelerator model (equation 5.2) is used, then a log-linear form for the investment function follows. The equations for K* are now

$$(5.18)\quad \text{i)}\quad K_t^* = \alpha_0[Q_t^e]^{\alpha_1}[(\tfrac{w}{c})_t^e]^{\alpha_2}$$

$$\text{ii)}\quad K_t^* = \beta_0(Z_t^e)^{\beta_1}(w_t^e)^{\beta_2}(c_t^e)^{\beta_3}$$

where equation ii) is derived from either (5.6c) or (5.6f). In the case of (5.6f), the formula for K* is only an approximation based on the CES-function. Inserting these values of K* into equation (5.2) gives the following investment models

$$(5.19)\quad \text{i)}\quad \Delta\log K_t = a_0 + a_1\log Q_t^e + a_2\log(\tfrac{w}{c})_t^e + a_3\log K_{t-1}$$

$$\text{ii)}\quad \Delta\log K_t = b_0 + b_1\log Z_t^e + b_2\log w_t^e + b_3\log c_t^e + b_4\log K_{t-1}$$

where

$a_0 = \lambda\log\alpha_0$, $a_1 = \lambda\alpha_1$, $a_2 = \lambda\alpha_2$, $a_3 = -\lambda$

$b_0 = \lambda\log\beta_0$, $b_1 = \lambda\beta_1$, $b_2 = \lambda\beta_2$, $b_3 = \lambda\beta_3$, $b_4 = -\lambda$

Equations (5.19) are effectively net investment equations since the dependent variable is a transformation of the change in the capital stock.

When the formulas (5.18) for K* are inserted into the adjustment specification (5.3), the following investment equations result

$$(5.20)\quad \text{i)}\quad \frac{I_t}{K_{t-1}} = a_0 + a_1 \log Q_t^e + a_2 \log\left(\frac{w}{c}\right)_t^e + a_3 \log K_{t-1}$$

$$\text{ii)}\quad \frac{I_t}{K_{t-1}} = b_0 + b_1 \log Z_t^e + b_2 \log w_t^e + b_3 \log c_t^e + b_4 \log K_{t-1}$$

where the parameters are

$a_0 = \delta + \lambda \log\alpha_0$, $a_1 = \lambda\alpha_1$, $a_2 = \lambda\alpha_2$, $a_3 = -\lambda$

$b_0 = \delta + \lambda \log\beta_0$, $b_1 = \lambda\beta_1$, $b_2 = \lambda\beta_2$, $b_3 = \lambda\beta_3$, $b_4 = -\lambda$

The six investment equations defined by formulas (5.17), (5.19) and (5.20) are all linear or log-linear in parameters and variables. Hence linear estimation methods are applicable for these models.

If formulas (5.18) for the desired capital stock are directly inserted into our basic equation (5.1) of the flexible accelerator mechanism, a non-linear investment equation results. We shall also try to estimate this kind of a relationship using non-linear estimation methods. The investment equations are now

$$(5.21)\quad \text{i)}\quad I_t = \alpha_0'[Q_t^e]^{\alpha_1}\left[\left(\frac{w}{c}\right)_t^e\right]^{\alpha_2} + a_3 K_{t-1}$$

$$\text{ii)}\quad I_t = \beta_0'(Z_t^e)^{\beta_1}(w_t^e)^{\beta_2}(c_t^e)^{\beta_3} + b_4 K_{t-1}$$

where the parameters are

$\alpha_0' = \lambda\alpha_0$, $a_3 = \delta - \lambda$, $\beta_0' = \lambda\beta_0$, $b_4 = \delta - \lambda$

The case of the flexible accelerator model with a variable (endogenous) speed of adjustment is examined empirically using equation (5.11) or its equivalent in the case of gross cash flow. The estimation problem will also be of a non-linear type when the linearized formulas for K* are inserted into equation (5.11). The linearized specifications of the cash flow investment model follow when formulas (5.16) for K* are inserted into equations (5.14) and (5.15). It can also be argued, however, that this straightforward inclusion of the cash flow variable in the investment regression effectively implies the direct impact of cash flow on the desired capital stock rather than on the timing of investment.

In the above investment models the arguments of the investment decision are derived from the following set of variables: production (or demand), the wage rate, the user cost, cash flow and the lagged capital stock. It was argued at the end of section 5.4 that a direct 'credit rationing' effect might also have a place among the determinants of investment. Although the formal basis for the inclusion of a proxy for credit rationing in the investment regression is rather obscure, we shall nevertheless carry out some experiments using this kind of a variable and the experiments will be conducted within the linearized investment equations (5.17). The linearized equations are also applied in examining the effect of technical progress on investment.

The basic motivation for using a number of functional forms in the empirical analysis stems from the fact that it is very difficult to specify a priori the parametric forms of the underlying functions. We have noted both in the theoretical discussion and in the preceding discussion of this chapter that the form of the reduced-form investment equation is rather sensitive to the parametrization of a number of functions. Using different forms of the investment model makes it possible to check, at least to some extent, the sensitivity of the empirical results with respect to alternative functional specifications. The problems associated with non-linear estimation also support our choice of using the linear equation as a point of departure in the empirical investigation.

5.5.2 Demand Variables, Expectations Hypotheses and Stochastic Specification of the Equations

Almost all empirical investment studies are agreed that some proxy for future output or demand is an important determinant of investment decisions. The proxy variables used to measure demand in this study are discussed below. We also briefly consider the expectations generating mechanisms used in the subsequent analysis. Constructing expectational values for explanatory variables is necessary since the arguments of the desired capital stock are usually expected values in a nonmyopic model.

Finally, we briefly discuss the basic features of the stochastic specification of estimating equations.

The problem with the demand for the firm's output is that it is almost unmeasurable. From equation (5.5) it is obvious that what we are looking for is a measure of the position of the demand curve which is a variable that is truly exogenous to the firm. In the empirical calculations, we have tried different proxies for the Z-variable (demand index). The main alternatives were: (1) output of the respective sector, (2) output of the total Finnish economy, (3) total aggregate demand or its components of the Finnish economy (i.e. investment plus consumption or separately), (4) sales of the respective sector and (5) variables measuring foreign demand as, for example, GDP, industrial production or imports of the OECD countries important for Finnish exports. The own output and sales variables are problematic since they are constrained by the current capital stock and hence are not truly exogenous variables (see also Gould 1969, Nickell 1978, pp. 284-285). These two variables were nevertheless used as the main candidates for the demand variable but we have also carried out a number of experiments with the other possible proxies for the demand effects.

The expected value of output is approximated in the annual models by the current or previous period's value. In addition to these simple expectations assumptions, we have also used in the quarterly models expected values based on the ARIMA-method, Almon polynomial lag distributions and simple moving averages of past values of output. In the 'forward-looking' expectations model the appropriate proxy for expected output is constructed on the basis of the ARIMA-model for output.

Price variables (w/c, w and c) are, by and large, dealt with in an analogous manner to the output and demand variables. However, in both annual and quarterly models we have also tested adaptive and constant expectations models for the rate of inflation, which affects the user cost variable through the real cost of capital term. Tax parameters, the debt-capital ratio and nominal rates of interest are assumed to take either the current or the previous period's value. In summary, the expected values of price variables in the annual investment equations are based on the following expectations hypotheses: perfect foresight, static, constant and adaptive models. In addition to these four hypotheses, we have also tested in the quarterly investment equations price expectations based on the ARIMA-method and Almon polynomial lag distributions.

The other explanatory variables to be tested in the investment equations are cash flow and a proxy for credit rationing. For these variables we use only very simple expectations hypotheses based mainly on the current or the previous period's value and, in the quarterly equations, also on some longer discrete lagged values of these variables.

With respect to the stochastic specification of the estimating equation, two important aspects are worth mentioning here. First, at what stage should the disturbance term be added to the model? Second, the presence of a lagged capital stock in the estimating equations implies that the investment equations are related to stochastic difference equations through the capital stock adjustment equation. We shall only briefly consider these questions here (see also chapters VI and VII).

The purpose of constructing a model is to systematically account for as much of the variation in the observations as possible. The movements not captured by the fitted model are termed residuals, and if the model is reasonably adequate these residuals should be approximately random. Departure from randomness is usually interpreted to mean that the maintained model fails to pick up some systematic component in the data, and that an attempt should therefore be made to find a better model (see also Harvey, 1981, chapter 5). Hence a natural requirement of a good model is that its residuals are random (white noise). This requirement can be interpreted to imply that the disturbance term should be added to the final reduced-form equation to be estimated.

However, alternative approaches do exist. A disturbance term could be added both to the flexible accelerator model (5.1) and to the formula for K^*. If these disturbances were assumed to be nonautocorrelated, then the reduced-form investment equation would contain serially correlated disturbances (see Harvey, 1982). In that case it might be argued that the occurrence of serial correlation in the residuals of the estimated equation would not necessarily imply that the equation is misspecified.

The point of departure here, however, is that the disturbance term is added to the final equation and the randomness of the residuals is tested with different statistics. Residuals play an important role in procedures for detecting structural misspecification. Such procedures usually embody a number of test statistics since lack of randomness may manifest itself in a number of different ways. The behaviour of the OLS residuals under structural misspecification is also not obvious. For these reasons we have employed a number of tests for misspecification in the empirical analysis.

The investment equations considered in this study are not of the usual stochastic difference equation form because the lagged dependent variable (I_{t-1}) is not included directly in the regressions. Past values of investment implicitly affect the equations, however, since the lagged capital stock is defined by the equation $K_{t-1} = I_{t-1} + (1-\delta)K_{t-2}$, and by further substitution of K_{t-2} etc. all the past values of investment would show up in the investment model. The model with K_{t-1} as an explanatory variable seems to us to be more suitable for estimation purposes than a general n^{th} order stochastic difference equation. We shall, however, recognize the implicit stochastic difference equation form of the estimating equations both in the estimation and in the testing procedures.

Details of the stochastic specification and a description of the general strategy of statistical analysis (estimation, diagnostic checking, specification tests etc.) used in our empirical investigation are considered in chapters VI and VII jointly with the estimation results.

Notes to Chapter V

1 Some aspects of investment behaviour neglected in the present analysis are as follows:

(i) Uncertainty: The vast majority of empirical investment studies have assumed a certainty environment. The work of Chang and Holt is a rare example of an empirical study in which the effect of uncertainty is taken into account (see Chang and Holt 1973). For theoretical analysis of the effects of uncertainty on the investment decision, see Nerlove (1972), Nickell (1978) Pindyck (1982), and Abel (1983, 1984).

(ii) Aggregation: If the economic relationships are linear, aggregation is not a serious problem, particularly if the micro units are identical (see Nickell 1978). However, in the non-linear case it is difficult to develop structural restrictions on the aggregate relations corresponding to those which theory imposes on the micro units (see, e.g., Sonnenschein 1972, and Epstein 1983). Epstein (1983) and Blackorby and Schworm (1983) have recently examined conditions for the aggregation of the adjustment-cost-based rationalizations of dynamic factor demand functions. However, any empirical work in this field is bound to be based on approximation and the role of economic theory is chiefly to suggest appropriate independent variables which might also explain the investment behaviour of an aggregate of firms (see, also, Nickell 1978).

(iii) Interrelated aspects of factor demand analysis: Firms usually possess various adjustment mechanisms which can be used to respond to changes in exogenous factors, i.e. prices of products, inventories, financial assets, the rate of capacity utilization and all factor inputs (capital, labour, energy etc.). The pioneering work in interrelated factor demand analysis is that by Nadiri and Rosen (1969) and Brechling (1975). Dynamic analysis of interrelated factor demands has recently been presented by Berndt, Fuss and Waverman (1979), Meese (1980) and Epstein and Denny (1983). The interrelated nature of various assets (i.e. capital, inventories, financial assets) has been examined by Dhrymes and Kurz (1967) and Schramm (1970).

2 The following authors have provided surveys of econometric and theoretical investment studies: Eisner and Strotz (1963), Jorgenson (1971), Rowley and Trivedi (1975), Helliwell (1976) and Nickell (1978).

3 The discrete time equivalent of (5.1) is $\Delta K_t = (1-e^{-\lambda})(K_t^*-K_{t-1})$

(see Berndt, Fuss and Waverman 1979). The discrete time approximation used in our empirical investigation is $\Delta K_t = \lambda(K_t^*-K_{t-1})$

This can be justified on two grounds. First, it can be shown that λ is a linear approximation to $(1-e^{-\lambda})$. The use of this approximation will reduce the non-linearity of the dynamic model. Second, λ would be exact if the model were originally formulated in discrete time.

4 Our theoretical analysis has been restricted to adjustment costs only because of the existence of a monopsony in the capital goods market, i.e. to the case of a rising flow supply price of new investment goods (external adjustment costs). Furthermore, it has been assumed that adjustment costs are a function of gross investment only. A large number of other possibilities have been investigated in the adjustment cost literature (see chapter III).

5 The endogenous form of the adjustment path was noted earlier by Mundlak (1966, 1967). He also emphasized that some important implications can be drawn from comparative statics theory with respect to the dynamics. That is to say, the correspondence principle is applied in the other direction. The comparative statics results of an adjustment cost model are crucially linked to the determination of the short-run and long-run elasticities of the demand for capital with respect to exogenous factors and, in addition, to the lower and upper bounds of the adjustment speed. Usually, restrictions are made which guarantee that $0 < \lambda \leq 1$.

6 The quadratic specification implies that the optimal solution is globally as well as locally valid. The problem with the quadratic objective function is that one cannot make use of some well-known parametrizations of the production function (CD or CES). It might also be argued that the assumption of a quadratic functional form to guarantee the global nature of the result is not necessary since linearization could be a realistic assumption in the case of capital accumulation models. The capital stock usually changes relatively little from period to period (that is $\Delta K/K$ is 'small') and it might be thought that the actual capital stock is always fairly close to the next period's desired level.

7 Investment models with (I/K) as the dependent variable have been estimated by Chenery (1952), Hayashi (1982), Abel (1980) and Tarkka (1983). The (I/K) -transformation would also follow if adjustment costs are a function of I/K rather than a function of investment itself, see Poterba and Summers (1983).

8 As Abel (1980) has emphasized, the distinction between a continuous and a discrete time model is not a trivial matter (see also Nickell 1978). In a discrete time formulation, one period expectation is always inherently assumed: For example, the simple expression for the user cost ($c = q(r+\delta-g)$) is equivalent in discrete time to $q_t(r_t+\delta)-\Delta q_t$, where $\Delta q_t = q_{t+1}-q_t$. This implies that the user cost is a function of, among other things, the price of investment goods one period ahead.

9 If future values of exogenous variables are not assumed to be known with certainty, the problem of deriving an empirically tractable investment model under dynamic conditions becomes even more difficult. Some attempts in this direction have been made by Taylor (1970) and Nerlove (1972).

10 With non-static expectations there is a different 'desired' capital stock corresponding to each future period and the problem of precisely what firms should aim at is rather complicated (see Nickell 1978). Nickell (chapter 11) assumes an imperfectly competitive firm which

faces a downward-sloping demand curve and adjustment costs of the quadratic form, $C(I) = bI^2$. In this framework, the optimal level of investment is approximately given by the following equation:

$$I_t \simeq \lambda \left[K_t^* + \sum_{i=t+1}^{\infty} w_i (K_i^* - K_t^*) - K_{t-1}\right] + \delta K_{t-1}$$

where the 'moving average' weights w_i are equal to $\frac{\lambda+\rho}{1+\rho}\left(\frac{1-\lambda}{1+\rho}\right)^{i-t}$. The adjustment coefficient is λ and K_i^* is the desired capital stock defined for prices and demand ruling at time i ($i > t+1$). The target capital stock is now a power series in the forward differences of K^*. With stationary expectations, all these future differences are zero and the standard flexible accelerator model follows. In this model, the firm does not aim at a simple (static) target level of capital stock but instead at a level which is the sum of the next period's target level and of the exponential weighted sum of the difference of all future targets. The determination of the 'moving target' capital stock is the basic characteristic of a forward looking investment model. For a recent empirical application of this approach, see Schiantarelli (1983).

11 In the static case, the profit function to be maximized is $\Pi(Q,K,L) = pQ - wL - cK$, where c is simply a general expression for the cost of capital. In the intertemporal case, the profit function is equivalent to that used in our theoretical analysis in chapters III and IV. The intertemporal model provides an explicit formula for the user cost of capital. In addition, the user cost of capital services will depend upon the adjustment cost term (see, e.g., section 3.2). If adjustment costs are of the external form, i.e. they represent the rising flow supply curve of investment goods, it is difficult to separate an adjustment cost term from the actual price of new investment goods. Hence, in our empirical analysis with the aggregate data it is assumed that the market price of new capital goods also includes the effects of adjustment costs (see also Eisner and Strotz, 1963).

12 Brechling (1975) and Gould (1969) have critized the Jorgensonian approach since in this model output is by definition endogenous. In the Cobb - Douglas case, the 'correct' specification of K^* would be

$$\log K^* = \text{constant} + \frac{\omega}{1-a-b}\, t - \frac{b}{1-a-b}\log\left(\frac{w}{p}\right) - \frac{1-b}{1-a-b}\log\left(\frac{c}{p}\right)$$

which is based on the assumption that $Q = Ae^{\omega t}K^aL^b$, where ω is the rate of technical change (see Brechling 1975).

13 Equation (5.6c) for K^* follows from the following maximization problem:

$$\max_{\{K,L\}} \Pi = pQ - wL - cK \text{ subject to (i) } Q = AK^aL^b,\ p = \gamma_0 Q^{\gamma_1} Z^{\gamma_2}$$

Solving this maximization problem gives the following expressions for the coefficients e_i in (5.6c):

$$e_1 = -\frac{\gamma_2}{(a+b)(1+\gamma_1)-1}, \quad e_2 = \frac{b(1+\gamma_1)}{(a+b)(1+\gamma_1)-1}$$

$$e_3 = \frac{1-b(1+\gamma_1)}{(a+b)(1+\gamma_1)-1}$$

What can be said on a priori grounds about the signs of the e_i coefficients? The sign of the coefficient of the demand index Z is positive provided that $\gamma_2 > 0$, $\gamma_1 < 0$ and $a + b \leq 1$, but it may also be positive with increasing returns to scale provided that γ_1 is sufficiently small. Note that γ_1 is the inverse of the price elasticity of demand and $a + b \leq 1$ means that the production function has constant or decreasing returns to scale. The sign of the coefficient of the own price of capital (i.e. user cost) is always negative ($e_3 < 0$) if $a + b \leq 1$. In the case $-1 < \gamma_1 < 0$, the sign of e_3 may be positive if $a + b > 1$. If it is assumed that a monopolist sets the product price in the elastic range of the demand curve, i.e. $-1 < \gamma_1 < 0$, the sign of the coefficient of the wage rate is negative if $a + b \leq 1$. A positive sign for the coefficient of the wage rate follows with increasing returns to scale.

Since in the empirical analysis we use aggregate data of firms it is worth considering here also the case of a perfectly competitive industry. In a competitive industry, equilibrium occurs where the industry supply curve cuts the demand curve. The elasticity of the market demand curve at the equilibrium price may be anything from zero to infinity (see Layard and Walters 1978). Thus it is quite possible for a competitive industry to be in equilibrium at a point where the demand is inelastic; such a situation is not possible for a profit-maximizing monopolist. Hence in a competitive industry operating in the region where demand is inelastic (i.e. $\gamma_1 < -1$), wage increases will lead to an increased use of capital, i.e. the coefficient of the wage rate is positive (with decreasing returns to scale). In the case of a monopolistic or competitive industry such that $-1 < \gamma_1 < 0$, increases in the price of labour will decrease the amount of capital demanded. This difference arises because of the interaction of the scale and substitution effects (see, also, Gould, 1973). For industries facing an elastic demand curve the scale effect will swamp the substitution effect; that is, although capital will be substituted for labour (as w increases), the rise in costs will reduce output enough to overcome this increase and causes a net decline in the demand for capital. The coefficient of a technological change term (Hicks neutral case) is $-(1 + \gamma_1)/\gamma_1$ with constant returns to scale and hence it has the opposite sign of the coefficient for the wage rate. This means that an improvement in technology in a monopolistic industry or a competitive industry facing an elastic demand function will lead to increases in capital stock (and in labour input). However, for a competitive industry facing an inelastic demand function, technical change will reduce the demand for capital (and for labour).

In the case of the CES production function (eq. 5.4ii) with constant returns to scale ($x = 1$), the exact formula for K* can be shown to be

$$K^* = \left[\frac{c}{v\gamma_0(1+\gamma_1)Z^2}\right]^{\frac{1}{\gamma_1}} \left[B^{-y}\{v+(1-v)^{\sigma} \left(\frac{c}{vw}\right)^{-y\sigma}\}\right]^{\frac{\gamma_1+y+1}{\gamma_1 y}}$$

The effect of increasing capital costs is unambiguously negative, whereas the effect of an increasing wage rate depends on the sign of $(\sigma+\eta)$, where $\eta = 1/\gamma_1$. If the substitution effect dominates the scale effect (output effect), $\sigma + \eta > 0$ and an increase in the wage costs will increase the demand for capital (see, also, Nickell 1978, pp. 18-19). In general in the case of non-constant returns to scale, the cross price elasticity of demand for capital is equal to $k_w (\sigma + x\eta)$ where k_w is the cost share of labour input, σ = elasticity of substitution between K and L, η = elasticity of demand for output and x = returns to scale (see Layard and Walters, 1978).

In addition to the above considerations it should he noted that the net effect of wage rate changes on investment also depends upon whether the investment equation includes a cash flow variable through which $(\partial I/\partial w) < 0$, and hence the total effect is clearly an empirical issue (see, also, section 4.4.2 where the theoretical discussion on the cash flow channel is presented).

It should finally be noted that in equation (5.6c) for K* both the wage rate and the user cost are nominal variables. If the underlying demand curve is interpreted so that p is the ratio of the own product price ($\bar{p}$) and the prices of substitute products ($\bar{\bar{p}}$), i.e. $p = \bar{p}/\bar{\bar{p}}$, then K* depends on the real wage rate ($w/\bar{p}$) and the real user cost ($c/\bar{p}$). In our empirical analysis, we shall measure real factor costs in terms of the general (aggregate) output price index, i.e. w and c are deflated by p. This choice can be defended on the grounds that we use only data from large aggregates of firms in the empirical analysis.

14 The derivation of the equation (5.6f) for K* in the case of the CES production function and the downward-sloping demand curve is rather lengthy and it is not shown here. The approximate formula for K* is obtained by using the Taylor series approximation of the CES function as

$$\log Q \approx \log B + xv \log K + c(1-v)\log L - \frac{1}{2} xyv(1-v)\left[\log \frac{K}{L}\right]^2$$

(see, e.g., Wallis 1979). The squared logarithm of the capital-labour ratio is added to the CD log-linear version and this last term indicates the departure from the unit elasticity of substitution.The coefficients of equation (5.8f) are functions of the underlying parameters B, v, x, y (production function) and γ_0, γ_1, γ_2 (demand function).

15 In the earlier literature, the distinction between the putty-putty and putty-clay investment models was made on the basis of the form of the explanatory variables (see Bischoff, 1971). In the putty-clay model, investment was assumed to depend on the level of relative prices and in the putty-putty model on the change of the relative prices. This distinction is not valid, however, when the putty-putty model is derived in the context of convex adjustment costs since in

this case investment depends on the level of factor prices. A typical feature of the putty-clay investment model is that the real rate of interest (discount rate) is defined in terms of wage rate inflation (see Ando - Modigliani - Rasche - Turnovsky 1974 and Nickell 1978).

16 Nickell (1978) has suggested that in order to capture the possible 'echo' effects of past capital stocks, one should include past values of investment as additional explanatory variables in an investment regression.

17 It might also be argued that it would be reasonable to assume that one of the effects of a capital cost structure in which internal funds are included is akin to that of strictly convex adjustment costs in which the importance of adjustment costs depends on the flow of funds relative to the desired quantity of investment (see Nickell 1978, pp. 262 - 263).

CHAPTER VI

DESCRIPTION OF DATA AND EMPIRICAL RESULTS OF ANNUAL INVESTMENT EQUATIONS

6.1 Introduction

The general aim of the econometric analysis is to examine whether the investment behaviour of Finnish firms can be explained by the neoclassical investment theory. The more specific aims were described at the end of section 5.1. Various versions of the neoclassical model will be tested empirically as described in the previous section (5.5). Special attention is paid to the role of alternative measures of the user cost of capital (see Appendix I).

Since the statistical data used in this analysis is described in some detail in the Appendices, we shall only briefly consider certain general features here. The firms included in this analysis are those operating in sectors 2 - 7 according to the standard SIC-classification. These sectors cover about 85 per cent of investment in fixed capital by the total corporate sector. The remaining part of the enterprise sector has been excluded mainly because of data limitations.

The aggregate sector (A) is disaggregated into two subsectors, namely the manufacturing (M) and residual (R) sectors. The residual sector is defined as the difference of the aggregate and manufacturing sectors, and hence it includes trade and commerce, construction, transportation and other service industries. The basic reason for this disaggregation is that a priori it can be thought that there exist some important differences in the investment behaviour of firms in the M- and R-sectors.

First, manufacturing can be characterized as an 'open sector', whereas the residual sector is mainly a 'closed sector'. Developments in foreign demand (exports) are therefore of importance for manufacturing firms. By constrast, the residual sector is mainly influenced by developments in domestic demand. The second difference between these two sectors is related to the capital intensity of production and investment. Manufacturing is more capital-intensive than the residual sector, and hence the capital-labour ratio is higher in manufacturing than in the residual sector. The third difference stems from the fact that the average real rate of return on capital has tended to be clearly higher in manufacturing than in the residual sector. Moreover, the volatility of the rate of return has generally been much greater in the former than in the latter sector (see Appendix II).

These three aspects - the degree of openness, the capital-labour ratio and developments in the rate of return - may have significant implications for the investment behaviour of the respective sectors. By disaggregating the total data into these two subgroups, we aim to take into account in the econometric analysis some important aspects of heterogeneity among firms in the corporate sector.

Detailed descriptions of sources of data and of the construction of variables and parameters are presented in Appendices I - V. Investment, capital stock, output and cash flow variables are measured at 1975-prices (million FIM). The wage rate variable is measured in markkaa per manhour. The units in which the user cost is measured are defined as follows: Consider, for example, the 1975 figure of 12.30 for manufacturing in the case of the c_1-variable (Appendix II). This figure indicates that the implicit rental cost of one unit of new capital selling for FIM 100 is 12.30 per year. The unit of measurement of the user cost is not indicated in the tables of Appendices II and V, because the user cost is only an index number. This implies that the relative price variable w/c is measured in markkaa per unit of time. The marginal interest rate variables (RM1 and RM2), which are used as proxies for credit rationing effects, are measured in percentage points and hence have a different unit of measurement than the other variables. To check the effect of the RM-variables when they are expressed in the same unit of measurement as other variables, we have also estimated the regression equations in a form such that the RM-variables are multiplied by K_{-1}. This multiplication does not, however, significantly affect the estimation results.

The effect of factor cost variables on investment is of special interest in the neoclassical approach. We have chosen to use two forms of the factor price variables in the empirical analysis (see sections 5.3 and 5.5). They appear in the investment equations either in ratio form (w/c) or separately (w/p, c/p). The former case arises from the assumption of a vertical demand curve (i.e. cost minimization) and the latter case from the assumption of a downward-sloping demand curve (plus profit maximization, see table 4). For the user cost variable we have systematically used two basic variants (c_1 and c_2, see Appendix V) for all sectors. Since the user cost variable is rather difficult to construct and many formulas for it have been proposed both in this study and in the investment literature, it did not seem reasonable to rule out a priori any basic variant of this variable from the analysis.

The general formula for the user cost variable is $q(cc_r+\delta)$, where cc_r is the real cost of financial capital, δ is the economic rate of depreciation and q is the price index of investment goods (see Appendix V). The c_1-variable is based on a "rate of return" approach in which the basic formula for the user cost is defined by $c_1 = q(RR/q_T+\delta)$, where RR is the real rate of return and q_T is Tobin's "q"-variable. In this approach, the real cost of financial capital is measured by the ratio of rate of return to Tobin's "q". Alternatively, the cost of capital can be defined as cc = (corporate profits after tax + net monetary interest) / (value of securities). In Tobin's "q" approach the value of securities is expressed at market prices. In empirical investment studies the value of securities is sometimes expressed at current replacement cost. If the numerator and denominator in cc are both expressed at current cost, then cc is the real cost of capital (see Appendix II; also Coen 1971, Picou and Waud 1973 and Holland (ed.) 1984). Basically, the "q" variable should reflect all relevant current and future information. Hence it incorporates expectations about the firm's future prospects and no explicit modelling with respect to the rate of inflation, for example, is needed in this approach (see, also, Abel 1980).

The c_2-variable is based on a "weighted average" approach where the real cost of capital (cc_r) is calculated as a weighted average of the costs of different forms of finance, the weights being the corresponding portions of finance in total capital (see Appendix V). In this method of constructing user cost, an explicit assumption is needed with respect to the expectations formation of the rate of inflation as well as with respect to other arguments of the cost of capital. It should, however, be emphasized that if the weights are based on market values of debt and equity and if the expectations are the same as those implied by the "q" -variable, then the two approaches (c_1, c_2) are essentially the same (see Appendices II and V).

The empirical analysis covers the years 1963 - 1980. Both annual and quarterly data are used in the analysis. For all three sectors (M, R and A) annual data is used systematically. For the aggregate sector (A), we also use quarterly data, which is partly official and partly self-constructed (see Appendix IV). Especially the data used to calculate the rental value of capital are available in part only annually. The within-year variations in the quarterly user cost series are therefore interpolations from the annual observations. It could also be argued that the annual observations provide a more natural basis for analyzing the investment behaviour of firms since investment decisions are usually made with respect to longer time periods, especially in the case of fixed capital outlays. Quarterly investment observations can be regarded as being mainly approximations of the quarterly financial (cash flow) outlays arising from annual or even longer period investment decisions.

The quarterly data is seasonally adjusted. The seasonal adjustment has been carried out in the construction of the quarterly data for the BOF3 model (see Suomen kansantalouden neljännesvuosimalli, Bank of Finland, Research Department, Research Papers No. 2, 1983). Quarterly data is expressed at quarterly rates except in the case of the user cost, which has been calculated by using annual rates of cost of capital, depreciation rules and the rate of inflation. Hence, the method used to calculate quarterly values of the user cost means that ($cc_r+\delta$) is expressed at annual rate but that q is the quarterly price index of investment goods. Although there are problems with the quarterly data, it was nevertheless felt that extending the analysis on a quarterly basis at the aggregate level, where the most reliable quarterly data are available, might provide some more information on the time pattern of investment behaviour as compared to the annual data.

6.2 Annual Investment Equations

Sections 6.2 and 6.3 present the estimation results of the econometric analysis based on annual time series observations. The first subsection (6.2.1) considers the 'encompassing' test results with respect to alternative expectations hypotheses of the rate of inflation in the user cost. The next three subsections present the estimated linear investment equations for the three sectors (M, R and A) under consideration. Section 6.3 presents the estimation results of non-linear investment equations for all sectors. In sections 6.4 and 6.5 are presented estimates of some structural parameters (rate of adjustment, long-run elasticities). Section 6.6 considers the effect of some additional variables and hypotheses on investment in all sectors (credit rationing, technical change and replacement hypothesis). Finally, section 6.7 summarizes the results of annual investment equations.

6.2.1 Tests of the Alternative Expectations Hypotheses for the Rate of Inflation and User Cost Variables in Annual Equations

The expectations hypotheses with respect to the explanatory variables (Q, Z, w/c, w, c, CF and RM) used in the investment models were described in section 5.5.2. For all of these variables, the estimation results are shown either as current values or one-period lagged values (or as both in some cases). In the preliminary analysis (not reported here), the adaptive expectations hypothesis was also tried for output and relative prices variables, but since this hypothesis seemed to point either to static or perfect foresight (current period) values of variables (in terms of R^2 and t-statistics of the estimated equations) we have chosen to use only these two simple expectations models for further analysis in the case of the annual data. For the rate of inflation, however, some other expectations hypotheses were also tried in the case of the c_2-variant of user cost. We shall next consider the role of various expectation hypotheses of the rate of inflation in investment equations through a specification test based on the 'encompassing' principle as suggested by Mizon and Richard (1982). The alternative c_2-variables are also compared to the c_1-variant of the user cost.

The availability of alternative user cost variables as candidates for an explanatory variable in the investment equation effectively creates a situation of non-nested hypotheses testing. On the basis of the original results by Cox (1961), Pesaran (1974) has shown how the problem of comparing two linear regression specifications can be solved practically. Research since then is summarized by Mizon and Richard (1982). In the Mizon - Richard terminology, our problem of comparing two user cost variables reduces to the following two hypotheses:

$$H_0 : E(I|c_1,c_2) = a_0 + a_1Q_{-1} + a_2(\frac{w}{c_1}) + a_3K_{-1} + v$$

$$H_1 : E(I|c_1,c_2) = a_0' + a_1'Q_{-1} + a_2'(\frac{w}{c_2}) + a_3'K_{-1} + u$$

where H_0 corresponds to the c_1-variant of the user cost, since it proposes that, conditional on the two variants of the user cost, investment depends only on w/c_1. Similarly, H_1 corresponds to the c_2-variant of the user cost variable. Alternative variants of c_2 in the case of different expectations hypotheses for the rate of inflation can also be compared with each other or with c_1.

The above formulation of the problem is based on the investment equation (5.17i), where the price ratio (w/c) is an explanatory variable. Alternatively, this formulation can be based on investment equation (5.17ii), where the wage rate (w/p) and the user cost (c/p) appear as separate regressors. Both of these investment equations will be applied here.

Cox's original encompassing principle states that one can test H_0 by analyzing its ability to predict what is observed when estimating the misspecified (under H_0) equation described by H_1. Mizon and Richard (1982) have shown that the Wald encompassing test of H_1 when H_0 is the null hypothesis is the 'orthodox' F-test of $d_3 = 0$ in the equation:

$$I = d_0 + d_1Q_{-1} + d_2(\frac{w}{c_1}) + d_3(\frac{w}{c_2}) + d_4K_{-1} + \varepsilon$$

Encompassing tests are fairly easy to compute in the models under consideration here. Tables 5 and 6 show several such calculations. In both tables, the null hypothesis in case I is the c_1-variant of the user cost and the competing hypotheses are based on c_2 with different expectations models for the rate of inflation in the cost of capital variable (see Appendix I and IV). Case II in tables 5 and 6 is based on the null hypothesis that c_2 incorporates adaptive expectations of the rate of inflation. This expectations hypothesis is then tested against other price expectations in c_2 and also against c_1.

In a preliminary analysis we have tested the adaptive expectations hypothesis with different values for the rate of adaptation (0, 0.1, 0.2,..., 0.9, 1.0). That value of the coefficient was chosen which gave the minimum residual sum of squares for the investment equation. The values of the adaptation coefficients were 0.5, 0.2 and 0.1 in the M-, R- and A-sectors, respectively, (the search method was used to find the 'optimum' value of the coefficient, see Maddala, 1981, p. 146).

In table 5 the encompassing tests are carried out in the framework of investment equation (5.17i), where w/c is an explanatory variable, while in table 6 the tests are based on equation (5.17ii), where w/p and c/p are independent variables. The accelerator variable (Q_{-1}) is the same in both equations (see sections 6.2.2 - 6.2.4).

The test results with respect to alternative expectations hypotheses about the rate of inflation are quite clear as to which model is more data-compatible (see especially case II). When the adaptive expectations model is the null hypothesis, in all cases except one the null cannot be rejected. When the roles of the null and alternative hypotheses are reversed, the encompassing tests reject all the null hypotheses (non-adaptive expectations). These reversal tests are not shown here.

The test calculations in the case where c_1 and adaptive-c_2 variables are the competing hypotheses are not so clear-cut. When c_1 is the null, then only in the manufacturing sector (table 5) is the null rejected in the case of w/c-models, but in the case of the (w/p,c/p)-models of table 6 the c_1-null is rejected in all sectors. When the roles of the null and alternative hypotheses are reversed, the c_2-adaptive is rejected in all cases except one (table 6).

TABLE 5. Encompassing Tests of Competing Expectations Hypotheses for the Rate of Inflation in the User Cost (investment equation 5.17i)

I. Null hypothesis (H_0): c_1	F-statistics		
Alternative hypothesis:	M	R	A
c_2/constant price expectations	0.47	0.01	0.12
c_2/static price expectations	0.61	0.46	3.54
c_2/perfect foresight price expectations	0.52	0.33	2.11
c_2/adaptive price expectations	6.12	2.40	2.34
II. Null hypothesis (H_0): c_2/adaptive case	**F-statistics**		
Alternative hypothesis:	M	R	A
c_2/constant price expectations	1.73	0.02	0.03
c_2/static price expectations	2.32	0.11	2.53
c_2/perfect foresight price expectations	1.99	0.21	2.11
c_1	5.85	11.21	10.68

Notes: The 95 % value for F(1,13) is 4.67 and the 99 % value is 9.07 and the respective F(1,12) are 4.75 and 9.33 (table 6). If the calculated F-statistics are smaller than these critical values, then the null hypothesis is not rejected; otherwise it is rejected. The reported test statistics are Wald tests of the restriction that the coefficients on the alternative hypotheses of the c-variables are zero.

TABLE 6. Encompassing Tests of Competing Expectations Hypotheses for the Rate of Inflation in the User Cost (investment equation 5.17ii, $Z^e = Q_{-1}$)

I. Null hypothesis (H_0): c_1	F-statistics		
Alternative hypothesis:	M	R	A
c_2/constant price expectations	0.12	0.25	0.11
c_2/static price expectations	2.56	0.41	3.06
c_2/perfect foresight price expectations	2.11	0.62	2.86
c_2/adaptive price expectations	9.76	6.88	17.58
II. Null hypothesis (H_0): c_2/adaptive case	**F-statistics**		
Alternative hypothesis:	M	R	A
c_2/constant price expectations	0.66	4.03	1.74
c_2/static price expectations	1.49	1.41	1.09
c_2/perfect foresight price expectations	1.12	2.01	0.86
c_1	1.16	3.35	4.35

Notes: see notes to table 5.

In the light of these encompassing test results, it seems quite legitimate to use both basic variants of the user cost (c_1 and c_2) in the econometric analysis. From the alternatives of the c_2-variant with respect to competing expectations hypotheses about the rate of inflation, we have chosen the adaptive expectations model with the above-mentioned coefficient values.

6.2.2 Estimated Equations for the Manufacturing Sector

The main results of the annual linear investment equations for the manufacturing sector are presented in tables 7 and 8. Table 9 presents encompassing tests of alternative c_2-variables and table 10 stability tests of some basic investment equations. Additional empirical results for manufacturing are presented in the Appendices; see tables A18 - A25.

In tables 7 and 8 the estimated investment equations are of the linearized form (see section 5.5.1). The equations of table 7 are based on the assumption of a vertical demand curve (cost-minimization approach) whereas the equations of table 8 are derived in the context of a downward-sloping demand curve (output is used as proxy for the demand index). The equations have been estimated systematically with both variants of the user cost (c_1 and c_2 with adaptive price expectations).

In order to evaluate the performance of the estimated equations, tables 7 and 8 incorporate the following statistics: estimates of coefficients and their t-statistics, goodness of fit (R^2), standard errors of the regressions (SEE), test statistics for serial correlation (DW and Durbin-m) and, when needed, estimates of the autocorrelation coefficient (RHO). The estimation method used in each equation is indicated in the column labelled EST.

The calculated test statistics seem to indicate that the DW-test alone gives very precise evidence of the presence of serial correlation. The generalized Durbin m-test was also used to try to detect whether the annual equations contain second-order autocorrelation in the residuals of equations where the DW-test alerts, but these calculations clearly indicated the presence of first-order autocorrelation only.

It is well known that autocorrelation in the residuals results in the OLS estimators being unbiased but less efficient than e.g. the GLS estimates that take account of the autocorrelation (see Maddala, 1981). Also, the variances of the estimators are themselves biased. Thus, by using proper estimation methods we gain higher efficiency and correct estimated variances. We have applied the rule of thumb suggested by Rao and Griliches (1969) according to which it is advisable to use an estimation method (e.g. Cochrane - Orcutt, Durbin, ML etc.) that takes account of autocorrelation if the autocorrelation coefficient (absolute value) is greater than 0.3 (see Maddala, 1981, p. 283).

Two estimation methods are used to tackle the autocorrelation problem: First, the standard Cochrane - Orcutt (CO) procedure and second, the Hatanaka two-step procedure (HAT). The HAT-method is used since the CO-method is not usually regarded as applicable in models where the lagged endogenous variable and the autocorrelated disturbance coexist (see Hatanaka, 1974). The Hatanaka two-step procedure consists of the instrumental variable method in the first step and the OLS-method in the second step. In all sectors our calculations showed that the HAT-method yields results which are very close to those obtained with the Cochrane - Orcutt procedure. Moreover, it was found that, if OLS is used in the first step instead of the instrumental method, the results are practically unchanged.

In addition to the OLS, CO and HAT estimation methods, we have also used the two-stage least squares (TSLS) method. The reason for this is that the output variable can be regarded as endogenous since it depends upon the current capital stock (see, e.g., Gould, 1969 and Hall, 1977). The instruments which were used in the TSLS procedure are listed in table A18. Our results show that the TSLS-method produces estimates which are very close to the OLS-estimates and hence the endogeneity problem of the output variable does not seem to be severe.

We shall next consider the general features of the estimation results in the manufacturing sector. Encompassing tests are then presented for alternative c_2-variables and finally some stability tests are calculated for the basic equations of tables 7 and 8.

General features of the estimation results for the manufacturing sector

We consider first the estimation results of table 7, where the explanatory price variable is in the ratio form (w/c). In terms of goodness of fit (R^2 and SEE) and plausibility of the coefficient estimates (signs and t-statistics), these equations seem to perform rather well. The most noticeable feature of the w/c-models is that the coefficient estimates of the two user cost variables differ very significantly. The c_1-variable gives a much higher value for the coefficient of the factor price ratio than does the c_2-variable. Estimates of coefficients of other explanatory variables seem to be practically unaffected by the choice of the relative price variable.

The basic difference in the development of the w/c_1 and w/c_2 variables stems from the fact that the c_1-variable grows very 'smoothly' in time and hence the relative price w/c_1 is also a fairly stably rising series (in 1963 - 1980). By contrast, the c_2 variable with adaptive price expectations declines to a very low level in the years after 1974 (it even takes some negative values) because of the acceleration of the rate of inflation, which reduces considerably the real cost of financial capital (see Appendix V). This decline in the c_2-variable is mainly due to the decrease in the real value of debt amortizations and interest expenses caused by the rise in the rate of inflation and to the almost free depreciation allowances for tax purposes granted to Finnish manufacturing firms during 1976 - 1979 (see chapter II for theoretical discussion and Appendix III for tax parameters). Since the price variable is in ratio form (w/c_2), the time pattern of c_2 causes large fluctuations in the w/c_2-variable and thus the coefficient of this variable takes a rather low, albeit significant, value.

The w/c_2-variable behaves similarly in all three sectors (M, R and A), although the fluctuations are less pronounced in the R- and A-sectors than in manufacturing, and it undoubtedly raises the question of the stability of the investment equations, especially after the 'oil crisis' in 1974 when the acceleration of the rate of inflation started.

Table 7. Annual Estimation Results of Linear Investment Equations for Manufacturing (eqs. 5.14, 5.15, 5.17i)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
constant	1.281	1.078	2.296	2.275	1.136	0.871	2.090	2.113	2.615	2.588	4.089	1.500	3.628
	(1.55)	(1.23)	(3.97)	(3.91)	(1.24)	(0.96)	(2.50)	(2.52)	(4.98)	(4.89)	(2.29)	(2.90)	(1.85)
Q	0.450	0.605					0.304	0.346					
	(3.54)	(3.91)					(2.50)	(2.60)					
Q_{-1}			0.626	0.595	0.352	0.277			0.634	0.608	0.512	0.547	0.415
			(6.20)	(5.38)	(1.78)	(1.37)			(6.26)	(5.47)	(5.86)	(5.01)	(2.37)
(w/c_1)	4.113	5.434	1.691	1.620	1.827	1.567							
	(2.11)	(2.53)	(1.45)	(1.37)	(1.64)	(1.46)							
(w/c_2)							0.005	0.006	0.018	0.017	0.027	0.028	0.025
							(0.27)	(0.30)	(1.52)	(1.45)	(4.01)	(3.13)	(3.54)
CF^g_{-1}					0.277								
					(1.59)								
CF^n_{-1}						0.334							0.087
						(1.94)							(0.65)
K_{-1}	-0.281	-0.400	-0.319	-0.300	-0.184	-0.114	-0.103	-0.128	-0.283	-0.269	-0.251	-0.260	-0.193
	(2.66)	(3.17)	(4.67)	(4.07)	(1.71)	(0.93)	(1.42)	(1.61)	(4.89)	(4.24)	(4.18)	(3.59)	(1.78)
R^2	0.595	0.552	0.795	0.794	0.815	0.829	0.469	0.464	0.798	0.797	0.870	0.700	0.863
SEE	727.3	764.9	517.3	519.1	491.8	472.7	833.1	836.9	513.8	515.0	382.1	407.8	390.9
DW	1.25	1.34	1.85	1.78	2.24	2.37	0.76	0.76	0.82	0.80	1.32	1.26	1.48
D-m	0.47	0.39	0.02	0.04	-0.38	-0.40	0.90	0.90	0.85	0.86	0.23	0.35	0.20
	(2.11)	(1.11)	(0.06)	(0.11)	(1.01)	(1.21)	(3.36)	(3.37)	(2.83)	(3.09)	(0.90)	(1.17)	(0.78)
RHO	-	-	-	-	-	-	-	-	-	-	0.74	0.68	0.74
											(5.69)		(5.60)
EST	OLS	TSLS	OLS	TSLS	OLS	OLS	OLS	TSLS	OLS	TSLS	CO	HAT	CO

Note: Equations 1, 3, 5, 6, 7, 9 are estimated by OLS, equations 2, 4, 8, 10 by TSLS, equations 11 and 13 by CO (Cochran - Orcutt) and equation 12 by HAT (Hatanaka two-step method). The list of instruments used in TSLS is presented in table A18 (see Appendices). CF^g is gross cash flow and CF^n is net cash flow. The dependent variable I and independent variables Q, CF and K are expressed in million FIM (at 1975 prices). Constant and (w/c) variables are multiplied by 10^3. D-m indicates Durbin's autocorrelation statistic (m-statistic), RHO is the first-order autocorrelation coefficient, t-ratios are shown in parentheses and SEE is the standard error of the regression. In the case of CO-method R^2 is at the original level of variables, in the case of HAT-method it is at the transformed level.

Table 8. Annual Estimation Results of Linear Investment Equations for Manufacturing (eqs. 5.14, 5.15, 5.17ii)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
constant	6.098	4.006	3.025	6.445	6.667	4.846	4.843	5.224	6.867	3.798	4.073	3.685
	(2.81)	(3.09)	(2.30)	(6.62)	(6.61)	(5.95)	(5.94)	(8.53)	(7.41)	(3.75)	(8.77)	(6.86)
Q	0.478			0.401	0.467							
	(3.39)			(5.39)	(5.55)							
Q_{-1}		0.630	0.309			0.549	0.550	0.559	0.556	0.319	0.283	0.266
		(5.89)	(1.52)			(6.55)	(6.18)	(8.88)	(8.01)	(1.95)	(3.55)	(3.14)
(w/p)	-0.261	-0.155	-0.230	0.559	0.559	0.277	0.276	0.422	0.488	0.202	0.492	0.443
	(0.69)	(0.58)	(0.93)	(1.95)	(1.89)	(1.10)	(1.09)	(1.92)	(1.40)	(0.83)	(3.43)	(3.25)
(c_1/p)	-0.365	-0.123	-0.156									
	(1.93)	(1.13)	(1.52)									
(c_2/p)				-0.162	-0.170	-0.086	-0.086	-0.097	-0.101	-0.082	-0.101	-0.091
				(5.12)	(5.15)	(3.15)	(3.14)	(4.56)	(3.70)	(3.15)	(8.02)	(7.01)
CF^g_{-1}			0.331							0.234	0.291	
			(1.80)							(1.60)	(4.03)	
CF^n_{-1}												0.303
												(3.98)
K_{-1}	-0.083	-0.218	-0.027	-0.465	-0.507	-0.392	-0.393	-0.460	-0.485	-0.246	-0.336	-0.292
	(0.57)	(1.97)	(0.28)	(3.69)	(3.86)	(3.75)	(3.73)	(5.13)	(3.50)	(1.82)	(4.90)	(3.80)
R^2	0.561	0.775	0.808	0.812	0.801	0.860	0.860	0.869	0.910	0.875	0.928	0.927
SEE	757.3	542.5	500.9	495.0	509.6	427.8	428.0	396.7	412.8	404.4	281.6	284.2
DW	1.17	1.65	2.07	2.29	2.20	2.49	2.50	2.00	1.93	2.98	2.47	2.44
D-m	0.51	0.21	-0.11	-0.27	-0.49	-0.56	-0.57	-0.08	0.02	-0.77	-0.31	-0.29
	(2.67)	(0.42)	(0.23)	(0.49)	(1.13)	(1.38)	(1.39)	(0.10)	(0.09)	(2.68)	(1.47)	(1.30)
RHO	-	-	-	-	-	-	-	-0.40	-0.47	-	-0.78	-0.77
								(1.83)			(4.92)	(4.90)
EST	OLS	OLS	OLS	OLS	TSLS	OLS	TSLS	CO	HAT	OLS	CO	CO

Note: In the equations of table 8, $Z^e = Q_t$ or Q_{t-1}. Equations with other proxies for Z^e are presented in the Appendices, see table A19.

Cash flow variables perform quite well in the models of the M-sector although they are only 'moderately' significant. Several versions of cash flow - net and gross measures, current and lagged values - were tried and it turned out that the estimation results are not very sensitive to the choice of the CF-variable, except that gross cash flow seems to lower the coefficient on the output variable (this phenomenon is observed in all three sectors). In general, the net concept of cash flow, i.e. retained earnings, might be regarded as preferable since it causes less multi-collinearity problems than the gross cash flow concept (the correlation between CF^g and Q is about 0.80 but only 0.10 between CF^n and Q).

Table 8 presents estimation results in the case where the real wage rate (w/p) and the real user cost (c/p) appear as separate explanatory variables. Lagged output (Q_{-1}) is used as a proxy for the demand variable in these models. In terms of goodness of fit and significance of the coefficient estimates, these equations seem to perform somewhat better than the w/c-models. It can also be noted that the c_2-variant of user cost performs clearly better than the c_1-variant (in terms of R^2 and t-statistics). In these models, the efficiency of estimates seems to increase significantly when the autocorrelation of the residuals is corrected by either the Cochrane - Orcutt or Hatanaka procedures. Cash flow variables are also highly significant in these equations. The sign of the coefficient of the wage rate variable is positive. The various possible explanations for a positive sign were discussed in note 13 to chapter V (see also section 6.5). It should be noted that the net effect of a wage increase on investment is ambiguous since in addition to a positive effect (substitution effect) the wage increase also reduces cash flow (negative liquidity effect).

The use of TSLS, on the other hand, has very little effect on the coefficient estimates. Since the equations of table 8 are based on the assumption that firms face a downward-sloping demand curve, the acceleration variable should be a truly exogenous demand index (see sections 5.3 and 5.5.2). As mentioned in section 5.5.2 we have also used other proxies for the exogenous demand index. Two variables are used to try to capture the effects of foreign demand (see table A19), namely the volumes of industrial production and imports in five OECD countries important for Finnish exports. The other demand proxies are manufacturing sales, total

Finnish GDP and measures of aggregate demand in Finland (consumption plus investment (plus exports)).

The estimation results (table A19) clearly indicate that the choice of the demand proxy does not have much effect on the coefficient values of other variables (wage rate, user cost, capital stock). In addition, all these demand proxies have a positive sign and they imply a long-run elasticity of somewhat less than one for the demand for capital with respect to expected output demand. Proxies measuring 'foreign demand' do not, however, perform as well as proxies for 'domestic demand' pressures. In terms of R^2, the best results are obtained by using GDP or aggregate demand of the total Finnish economy, but the results with these proxies are very similar to those obtained with lagged output. We may hence conclude that, despite the a priori suspicions concerning the output variable, it might nevertheless be a fairly good proxy for expected demand pressure in the manufacturing sector.

In table A20 the estimation results of the manufacturing investment models are presented for alternative functional forms in which $\Delta \log K$ and I/K_{-1} are dependent variables. These functional forms can be applied only in the case of the c_1-variable since the c_2-variable can take negative values. These alternative equations perform very similarly to the basic linear models and hence the linearization procedure seems to give a reliable approximation of the underlying log-linear or non-linear investment model (see also section 6.3 where truly nonlinear models are estimated). The sensitivity of the long-run elasticities of desired capital with respect to price and output variables within different functional specifications is discussed in section 6.5.

In sum, it seems that in the manufacturing sector the best performing equations are of the form where the real wage rate and the real user cost enter as separate arguments and the user cost is of the second type (c_2).

Encompassing tests of competing c_2-formulas in the manufacturing sector

In the investment equations of tables 7 and 8 we have used the standard formula of the c_2-variant of the user cost (see Appendix V). The tax depreciation coefficient is of the 'average' form, the discount rate is the 'corrected' interest rate on state bonds and new equity issues are excluded as a source of finance in the standard c_2-measure. Table 9 presents encompassing test results for competing c_2-variables when the null hypothesis is the standard c_2. The first case (I) is based on the (w/c)-investment models and the second case (II) on the (w/p,c/p)-investment models. The methodology underlying the encompassing principle was described above in section 6.2.1. All the c_2 variables used in table 9 are based on adaptive price expectations with a coefficient 0.5 (see section 6.2.1).

The alternative c_2-measures are as follows:

- c_{21}: standard c_2 with a tax depreciation coefficient equal to the book value of that coefficient (see Appendix III, table A10)
- c_{22}: standard c_2 with the present value of tax depreciation charges (z) equalling the marginal value; in the years 1963 - 1975 $c_{22} = c_2$, but since 1976 the z-variable used in c_{22} has been almost 1 and hence $c_{22} \neq c_2$ (see Appendix III, table A11)
- c_{23}: c_2 also includes new equity issues as a source of finance and hence personal tax factors are integrated in the standard c_2 (see Appendix I equation A8 and Appendix III for the values of the tax discrimination variable θ and the proportion (n) of new issues in total finance)
- c_{24}: $q(r+\delta-g^e)$, which corresponds to a tax-neutral case where g^e is the expected rate of inflation on an adaptive basis
- c_{25}: $q(r+\delta)$, which is based on the assumptions of a zero rate of inflation and tax neutrality
- c_{26}: standard c_2 with the interest rate on state bonds used directly as the discount rate (see Appendix II)
- c_{27}: standard c_2 with the average (constant) earnings price ratio used as a proxy for the discount rate (see Appendix II)

TABLE 9. Encompassing Tests of Competing Measures of the User Cost Variable in the Manufacturing Sector (investment equations 5.17i and 5.17ii)

I. Equation 5.17i; null hypothesis: c_2 = standard formula

Alternative hypothesis:	F-statistic
c_{21}/book value of depreciation coefficient	0.44
c_{22}/marginal value of depreciation coefficient	18.13
c_{23}/c_2 includes personal taxes (cost of new equity)	0.59
c_{24}/tax-neutral c_2 with adaptive price expectations	3.07
c_{25}/tax-neutral c_2 with zero rate of inflation	0.68
c_{26}/interest rate on state bonds as discount rate	0.98
c_{27}/average (constant) earnings-price ratio as discount rate	1.03

II. Equation 5.17ii; null hypothesis: c_2 = standard formula

Alternative hypothesis:	F-statistic
c_{21}/book value of depreciation coefficient	0.24
c_{22}/marginal value of depreciation coefficient	0.26
c_{23}/c_2 includes personal taxes (cost of new equity)	0.34
c_{24}/tax-neutral c_2 with adaptive price expectations	0.06
c_{25}/tax-neutral c_2 with zero rate of inflation	0.98
c_{26}/interest rate on state bonds as discount rate	0.46
c_{27}/average (constant) earnings-price ratio as discount rate	1.32

Notes: See tables 5 and 6. The 95 % value of $F(1,13)$ is 4.67 and the 99 % value 9.07.

The results of the encompassing tests indicate that the null (standard c_2) hypothesis cannot be rejected, except in one case. When the alternative hypothesis is the c_{22}-variable in the case of the w/c-investment model, the null is rejected. These encompassing test results have some rather interesting implications with respect to the effects of tax factors on the investment behaviour of manufacturing firms.

First, firms seem to base investment decisions on statutory (maximum) depreciation rules, either in average or marginal form, and the book value of the depreciation coefficient is not the appropriate basis, although in the 1970s it was almost continuously below the statutory values (see Appendix III). Second, personal taxes do not seem to significantly affect the investment policy of manufacturing firms, implying that only taxes levied at the corporate level are important in assessing the tax system's impact on capital formation (Poterba and

Summers (1983) have presented evidence to the contrary using U.K. investment data). Third, tax-neutral measures of the user cost are inferior to those including corporate tax factors, thus confirming the preceding conclusion. Fourth, investment policy is not very sensitive to the choice of the discount rate (see c_{26} and c_{27}).

Summarizing these encompassing tests, it can be concluded that our basic choice of the second variant of the user cost (standard c_2) seems to represent a reasonably good measure for use in further analysis of the manufacturing sector's investment behaviour.

Stability tests of investment equations in the manufacturing sector

It was already noted in the preceding discussion of the manufacturing investment equations that the problems arising with the c_2-variant of the user cost, especially in the models where the factor price ratio (w/c) is an explanatory variable, might be connected with the rapid increase in the rate of inflation after the 'oil crisis' in 1974. Since our estimation period 1963 - 1980 is rather long and a number of significant institutional changes took place during this time both in the international and domestic markets, it is of special importance to try to check whether the coefficients of the investment equations should be regarded as invariant over time or not. For this purpose we have conducted a number of stability tests which are presented in table 10.

For the most part, the stability tests were conducted for the estimated equations without the correction for first-order serial correlation. The term stability is defined here in the statistical sense of the estimated coefficients of the explanatory variables remaining constant over time. The tests used in table 10 are thoroughly described in Brown, Durbin and Evans (1975), Cameron (1979), Uri (1982), Koskela and Virén (1982). All the tests employed here are included in the RAL-program used at the Bank of Finland.

TABLE 10. Stability Tests of Annual Linear Investment Equations (Manufacturing Sector)

Table No./ Equation No.	F- H	C_B	C_F	C_B^2	C_F^2	Chow	DW
7/3	F(4,10)=1.053	0.674	0.496	0.297	0.302	F(4,10)=0.936	1.85
7/15	F(5,8) =1.844	0.712	0.343	0.414*	0.344*	F(5,8) =0.618	2.24
7/9	F(4,10)=7.049*	1.183*	0.728	0.200	0.544*	F(4,10)=1.402	0.82
A20/1	F(4,10)=0.349	0.549	0.315	0.236	0.180	F(4,10)=0.493	1.98
A20/2	F(4,10)=0.590	0.577	0.334	0.229	0.186	F(4,10)=0.676	1.85
8/2	F(5,8) =5.432*	1.123*	0.867	0.581*	0.622*	F(5,8) =6.233*	1.65
8/6	F(5,8) =0.344	0.464	0.322	0.314	0.257	F(5,8) =0.807	2.49
8/10	F(6,6) =0.493	0.298	0.269	0.421*	0.324	F(6,6) =0.390	2.98
8/11	F(6,6) =1.232	0.489	0.303	0.201	0.352*	F(6,6) =1.083	2.23

Notes: Critical values of test statistics

	F(4,10)	F(5,8)	F(6,6)	$C_{B,F}$	C_7^2	C_6^2
5 %	3.48	3.69	4.28	0.948	0.339	0.355
1 %	5.99	6.63	8.47	1.143	0.433	0.454

Equations with F(4,10) correspond to C_7^2 and those with F(5,8) to C_6^2.
*) implies that the test statistic exceeds the critical value at the 5 per cent significance level and hence the null hypothesis of stability is rejected. A20/1 refers to equation 1 in table A20 in the Appendices etc.

In table 10 the test statistics are as follows: (1) F-H is the Farley-Hinich test with first degree time-trending regressions and, if the basic model is $y_t = bx_t + u_t$, then the time-trending model is $y_t = (b_0+b_1t)x_t + u_t$; (2) in the CUSUM- and CUSUM - SQUARES -tests the letters B and F indicate whether recursive residuals have been calculated on the basis of background or forward forecast errors, respectively; (3) in the CHOW-test the time period is split into two segments (1963-72) and (1972-80) and the test statistic is the standard F-test.

These tests differ in terms of their power and alternative hypothesis (see, e.g., Harvey, 1982 and Cameron, 1979). However, they can also be regarded as a test for general misspecification without reference to any particular type of specification error (see Harvey, 1982). It is usually also rather difficult to specify exactly the alternative hypothesis in aggregated data (see Cameron, 1979). These stability tests are typically also tests for the stability of the whole regression relationship, and hence the source of variability is not easily identified. For these reasons we have not tried to detect the possible alternative hypothesis but instead have used these stability tests as general tests against misspecification.

Before considering the formal stability tests, we have conducted some informal comparisons of graphs of the parameters and Quandt's log-likelihood ratios over time. Such informal comparisons can give useful information about the stability itself and about the point in time when any shift in the regression relation might have occurred. However, the informal inspection is not very revealing in the case of annual data with a rather small sample size and hence no graphs are presented here for the annual data.

The formal stability tests for the most important manufacturing investment equations are presented in table 10. The general conclusion of these tests is clearly that the null hypothesis of parameter stability is not rejected. Only in a few cases do the test statistics exceed their critical values, implying the rejection of the null. In these few cases it is also evident that the coefficient estimates are not precise (low t-values) and/or that the autocorrelation correction has not succeeded. Hence, the stability tests are not very helpful for discriminating between competing hypotheses (measures of c, functional forms and demand regimes). In the cases where DW and Durbin's m-statistics suggest that there is first-order autocorrelation in the residuals, the significance levels for the test statistics should be considered with due care (see Cameron, 1979).

The stability hypothesis is rejected in more than one test mainly in the case of the c_2-variant of the user cost in the w/c-type investment equation. As was already noted above, it is likely that there are problems in correctly modelling the change in price expectations after the 'oil crisis' in 1974 and this might be reflected in the 'instability' of the investment equations based on the price variable w/c_2. Investment equations which include the real wage rate (w/p) and the real user cost (c/p) as separate explanatory variables do not suffer significantly from this problem. Hence it can be argued that the stability tests conducted here support the downward-sloping demand curve version of the investment equation in the manufacturing sector (i.e. w/p and c_2/p as separate arguments).

6.2.3 Estimated Equations for the Residual Sector

The estimation results of the linear investment equations for the residual sector are presented in tables 11 and 12. Table 13 shows encompassing tests of alternative user cost (c_2) variables and table 14 stability tests. Additional empirical results for the residual sector are presented in the Appendices; see tables A18 - A25.

Table 11 shows investment equations which incorporate the price ratio (w/c) as an explanatory variable. It can be noted that in terms of goodness of fit (R^2 and SEE) and significance of coefficient estimates (t-statistics) these equations perform quite well. The results are reported with both current and lagged output as an accelerator variable since the models with a lagged value have a rather high DW statistic. As in the case of the manufacturing sector, the equations incorporating the c_1-variant of the user cost perform better than those with the c_2-variable but the difference between the coefficient estimates of (w/c_1) and (w/c_2) is smaller in the R-sector than in the M-sector. The problems connected with the w/c_2-variable are similar to those observed in the M-sector except that the c_2-variable behaves much more 'smoothly' in the R-sector than in the M-sector. Furthermore, the c_2-variable of the residual sector does not decrease after 1974 as in the case of the M-sector, partly due to the fact that firms in the residual sector were not entitled to 'free depreciation' after 1976.

From the results presented in table 11 it can be seen that the relative price variable (w/c) is more significant in the residual than in the manufacturing sector (see table 7) and that the coefficient of the (w/c)-variable is much higher in the former than in the latter sector (a comparison of the long-run price elasticities of desired capital is presented in section 6.5). Alternative estimation methods indicate that the results are not very sensitive to the choice of the estimation procedure (OLS, CO or TSLS). Contrary to the results for the manufacturing sector, the cash flow variable is not usually significant in the residual sector and it can even take a negative coefficient value.

Table 11. Annual Estimation Results of Linear Investment Equations for the Residual Sector (eqs. 5.14, 5.15, 5.17i)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
constant	-1.848	-1.654	-1.813	-1.721	-1.834	-2.521	-1.815	-0.215	0.262	-0.845	-0.940	-1.288	-1.061
	(3.07)	(2.61)	(4.11)	(3.73)	(5.75)	(5.34)	(2.48)	(0.26)	(0.30)	(1.28)	(1.65)	(1.52)	(1.17)
Q	0.260	0.155						0.526	0.443				
	(2.75)	(1.42)						(6.14)	(4.66)				
Q_{-1}			0.478	0.380	0.515	0.519	0.478			0.709	0.725	0.725	0.702
			(4.94)	(3.10)	(6.26)	(5.56)	(4.68)			(8.59)	(11.34)	(10.60)	(7.98)
(w/c_1)	7.722	9.049	5.394	6.541	4.762	4.691	5.397						
	(4.47)	(4.77)	(3.66)	(3.77)	(3.70)	(3.14)	(3.50)						
(w/c_2)								0.836	0.868	0.402	0.390	0.388	0.444
								(2.80)	(2.81)	(1.67)	(1.76)	(1.62)	(1.63)
CF^g_{-1}							0.083						
							(0.48)						
CF^g													0.087
													(0.36)
K_{-1}	-0.138	-0.121	-0.182	-0.164	-0.184	-0.184	-0.182	-0.149	-0.120	-0.198	-0.203	-0.203	-0.200
	(5.14)	(4.20)	(7.58)	(5.92)	(10.59)	(9.86)	(7.26)	(4.27)	(3.16)	(6.53)	(9.09)	(8.64)	(6.29)
R^2	0.913	0.905	0.951	0.948	0.947	0.972	0.947	0.864	0.855	0.920	0.914	0.953	0.915
SEE	470.0	490.1	352.3	364.7	336.9	350.3	365.6	586.4	605.5	450.1	433.8	451.5	464.7
DW	1.53	1.74	2.66	2.49	1.96	1.93	2.66	1.62	1.57	2.62	2.11	2.11	2.59
D-m	0.43	0.38	-0.49	-0.41	-0.04	0.04	-0.47	0.24	0.32	-0.40	-0.09	-0.63	-0.39
	(1.13)	(1.23)	(1.66)	(1.48)	(0.11)	(0.05)	(1.64)	(0.71)	(1.05)	(1.36)	(0.28)	(0.72)	(1.37)
RHO	-	-	-	-	-0.42	-0.44	-	-	-	-	-0.38	-0.38	-
					(1.76)						(1.59)		
EST	OLS	TSLS	OLS	TSLS	CO	HAT	OLS	OLS	TSLS	OLS	CO	HAT	OLS

Note: See table 7.

TABLE 12. Annual Estimation Results of Linear Investment Equations for the Residual Sector (eqs. 5.14, 5.15, 5.17ii)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
constant	5.594	3.564	3.965	4.341	2.462	1.538	1.655	1.442	1.595
	(1.89)	(0.91)	(1.83)	(2.56)	(1.85)	(1.47)	(1.53)	(1.36)	(1.50)
Q	0.151	0.397			0.085				
	(0.78)	(1.08)			(0.50)				
Q_{-1}			0.475	0.595		0.429	0.367	0.483	0.502
			(3.41)	(5.80)		(3.29)	(2.11)	(3.28)	(3.13)
(w/p)	1.023	2.239	0.541	0.272	1.066	0.537	0.643	0.570	0.546
	(2.22)	(2.64)	(1.98)	(1.41)	(2.61)	(2.14)	(2.03)	(2.22)	(2.15)
(c_1/p)	-0.668	-0.420	-0.528	-0.557					
	(2.35)	(1.09)	(2.60)	(3.54)					
(c_2/p)					-0.174	-0.141	-0.144	-0.113	-0.123
					(3.05)	(3.23)	(3.25)	(2.07)	(2.48)
CF^g_{-1}								-0.220	
								(0.83)	
CF^n_{-1}									0.228
									(0.80)
K_{-1}	-0.168	-0.222	-0.193	-0.181	-0.196	-0.209	-0.207	-0.224	-0.242
	(3.88)	(3.60)	(6.59)	(9.57)	(5.35)	(8.16)	(8.08)	(7.09)	(4.97)
R^2	0.898	0.834	0.943	0.978	0.915	0.952	0.951	0.950	0.950
SEE	509.0	649.5	381.1	341.8	463.7	349.9	351.1	354.3	354.9
DW	1.33	1.06	2.56	2.36	1.30	2.29	2.10	2.39	2.36
D-m	0.40	0.77	-0.41	-0.30	0.39	-0.25	-0.21	-0.40	-0.38
	(1.21)	(2.61)	(1.52)	(0.99)	(1.20)	(0.63)	(0.50)	(1.12)	(1.10)
RHO	-	-	-	-0.47	-	-	-	-	-
				(1.60)					
EST	OLS	TSLS	OLS	CO	OLS	OLS	TSLS	OLS	OLS

Note: See tables 7 and 8. In the equations of table 12, $Z^e = Q_t$ or Q_{t-1}. Equations with other proxies for Z^e are presented in the Appendices, see table A19.

Table 12 presents the residual sector's investment equations in the case of the downward-sloping demand curve with the real wage rate (w/p) and the real user cost (c/p) as separate explanatory variables. These equations seem to perform slightly better than the w/c-models but the difference between these two types of investment model is now much smaller than in the M-sector. Both variants of the user cost (c_1 and c_2) seem to do almost equally well in the residual sector although there is some difference in the coefficient values of the wage rate and the user cost.

TABLE 13. Encompassing Tests of Competing Measures of the User Cost Variable in the Residual Sector (investment equations 5.17i and 5.17ii)

I. Equation 5.17i; null hypothesis: c_2 = standard formula

Alternative hypothesis:	F-statistic
c_{21}/book value of depreciation coefficient	0.25
c_{24}/tax-neutral c_2 with adaptive price expectations	2.14
c_{25}/tax-neutral c_2 with zero rate of inflation	1.93
c_{26}/interest rate on state bonds as discount rate	1.82

II. Equation 5.17ii; null hypothesis: c_2 = standard formula

Alternative hypothesis:	F-statistic
c_{21}/book value of depreciation coefficient	0.49
c_{24}/tax-neutral c_2 with adaptive price expectations	2.67
c_{25}/tax-neutral c_2 with zero rate of inflation	0.83
c_{26}/interest rate on state bonds as discount rate	1.94

Notes: See table 5. The 95 % and 99 % values of F(1,13) are 4.67 and 9.07, respectively (case I). The 95 % and 99 % values of F(1,12) are 4.75 and 9.33, respectively (case II).

As in the manufacturing sector, we have also used other proxies for foreign and domestic demand as an accelerator variable in the R-sector (see table A19). The foreign demand proxies do not perform well in this sector (the sign of the coefficient of foreign demand may even be negative) but the domestic demand proxies all perform very well (eqs. 7-9 in table A19). As in the M-sector, the coefficient estimates of the price variables are not, however, much affected by the choice of the demand variable. In sum, it seems that the 'closed' residual sector is more dependent upon domestic demand, which seems to be reasonably well approximated by the output variable.

Table 13 shows encompassing tests of alternative measures of the c_2-variant of the user cost in the residual sector. Because of data limitations, not all the alternative c_2-variables employed in the M-sector are presented in table 13. The test results of table 13 clearly indicate that the null hypothesis of the standard c_2 is not rejected. Hence corporate tax factors seem to affect the investment behaviour of firms in the residual sector.

TABLE 14. Stability Tests of Annual Linear Investment Equations (Residual Sector)

Table No./ equation No.	F-H	C_B	C_F	C_B^2	C_F^2	Chow	DW
11/1	F(4,10)=7.439*	0.366	0.430	0.203	0.380*	F(4,10)=2.317	1.53
11/3	F(4,10)=1.080	0.252	0.417	0.217	0.234	F(4,10)=1.054	2.66
11/8	F(4,10)=6.521*	0.985*	0.551	0.438*	0.323	F(4,10)=2.330	1.62
11/10	F(4,10)=2.113	0.772	0.582	0.311	0.415*	F(4,10)=1.029	2.62
A20/5	F(4,10)=1.003	0.258	0.553	0.376*	0.339	F(4,10)=0.895	2.41
A20/6	F(4,10)=0.826	0.325	0.524	0.359*	0.386*	F(4,10)=0.579	2.64
12/3	F(5,8) =3.335	0.316	0.535	0.191	0.167	F(5,8) =2.515	2.56
12/6	F(5,8) =4.479*	0.623	0.233	0.148	0.237	F(5,8) =2.038	2.29

See notes to table 10.
*) implies that the test statistic exceeds the critical value at the 5 per cent significance level and hence the null hypothesis of stability is rejected.

Table 14 presents stability tests for the residual sector's investment equations. The methodological aspects of these tests were discussed above in connection with the manufacturing sector. The null hypothesis of constant coefficients over time is not rejected in the majority of the tests. As in the M-sector, the w/c_2-variable also seems to cause instability problems in the R-sector and probably for similar reasons (see section 6.2.2). Otherwise, the stability tests do not provide much new information for facilitating discrimination between alternative hypotheses (competing c-formulas, functional forms and demand regimes).

In sum, it seems that the cost-minimization approach (based on w/c_1) and the profit-maximization approach (based on w/p and c_2/p) yield practically equally satisfactory results. The basic difference in the results of these two model groups in that the latter implies an unequal coefficient value for the wage rate and user cost variables whereas in the former case the coefficients are equal (see also section 6.5, where long-run elasticities are discussed.

6.2.4 Estimated Equations for the Aggregate Sector and the Results of SURE-Estimation

The estimation results of the linear investment equations for the aggregate sector are presented in tables 15 and 16. Table 17 shows encompassing tests of alternative user cost variables and table 18 gives stability tests for the aggregate sector. Additional empirical results for the aggregate sector are presented in the Appendices; see tables A18-A25.

Table 15 includes investment equations that are based on the assumption of cost-minimization, implying that the price ratio (w/c) is an explanatory variable. The estimated investment equations perform quite well in terms of goodness of fit and significance of coefficient estimates. The equations with (w/c_1) as an explanatory variable seem to do somewhat better than those with (w/c_2), as was also the case in the M- and R-sectors. The size of the coefficient estimate of the (w/c_1)-variable is again considerably larger than that of the (w/c_2)-variable, and basically for the same reasons as in the M- and R-sectors.

Table 16 shows investment equations based on the assumption of a downward-sloping demand curve and lagged output is again used as a first proxy for the exogenous demand index (Z). Both variants of the user cost produce rather similar results, although there are some differences in the magnitudes of the coefficients. The coefficient of the real wage rate is positive and that of the real user cost is negative, as was also the case in the investment models of the M- and R-sectors. The investment equations with the real wage rate and the real user cost as separate regressors seem to give the lowest residual sum of squares when the second variant (c_2) of the user cost is applied. From the results of tables 15 and 16, it can be seen that the choice of the estimation method has very little effect on the estimates. This is due to the fact that there is almost no autocorrelation in the residuals of the aggregate investment equations, as evidenced by the DW- and Durbin's m-statistics.

TABLE 15. Annual Estimation Results of Linear Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15, 5.17i)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
constant	-1.098	-1.084	0.223	0.229	-0.498	3.772	3.507	4.317
	(0.84)	(0.82)	(0.30)	(0.30)	(0.41)	(2.33)	(2.01)	(2.44)
Q	0.404	0.401						
	(5.00)	(4.91)						
Q_{-1}			0.619	0.609	0.525	0.834	0.802	0.913
			(10.01)	(9.26)	(3.78)	(7.20)	(5.79)	(5.98)
(w/c_1)	12.093	12.099	6.449	6.548	7.123			
	(3.86)	(3.87)	(3.33)	(3.35)	(3.30)			
(w/c_2)						1.382	1.199	1.318
						(1.61)	(1.24)	(1.51)
CF^g_{-1}								0.144
								(0.81)
CF^n_{-1}					0.133			
					(0.85)			
K_{-1}	-0.222	-0.221	-0.263	-0.260	-0.224	-0.354	-0.333	-0.382
	(4.65)	(4.95)	(9.14)	(8.71)	(3.74)	(4.28)	(3.40)	(4.22)
R^2	0.840	0.840	0.945	0.945	0.944	0.920	0.917	0.915
SEE	1058.0	1057.9	619.3	619.7	629.0	761.1	763.1	770.6
DW	1.42	1.42	2.01	2.00	2.12	1.67	1.59	1.79
D-m	0.62	0.63	-0.12	-0.11	-0.26	0.10	0.13	0.09
	(1.54)	(1.56)	(0.37)	(0.34)	(0.50)	(0.32)	(0.42)	(0.21)
EST	OLS	TSLS	OLS	TSLS	OLS	OLS	TSLS	OLS

Note: See table 7.

The results in tables 15 and 16 show that the cash flow variables are not very significant determinants of firms investment policy in the aggregate sector. However, the results for the manufacturing sector indicated that cash flow is an important determinant of the investment behaviour of manufacturing firms. As in the M- and R-sectors, we also tested investment equations based on other proxies for the demand index (see table A19). The domestic demand variables perform clearly better than proxies for foreign demand pressures. The own output variable (Q) again gives the lowest SEE, however.

TABLE 16. Annual Estimation Results of Linear Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15, 5.17ii)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
constant	9.923	9.589	6.499	6.408	6.628	15.309	15.025	16.095
	(1.95)	(1.86)	(2.55)	(2.49)	(2.48)	(4.59)	(4.43)	(4.78)
Q	0.346	0.327						
	(2.15)	(1.95)						
Q_{-1}			0.618	0.594	0.579	0.646	0.609	0.739
			(6.14)	(5.12)	(3.63)	(8.46)	(6.72)	(6.65)
(w/p)	1.454	1.547	0.611	0.709	0.586	1.144	1.287	1.193
	(1.47)	(1.52)	(1.13)	(1.19)	(1.03)	(2.71)	(2.76)	(2.84)
(c_1/p)	-0.919	-0.888	-0.524	-0.518	-0.564			
	(1.98)	(1.89)	(2.27)	(2.24)	(2.10)			
(c_2/p)						-0.742	-0.730	-0.750
						(4.36)	(4.23)	(4.45)
CF^n_{-1}					0.068			0.151
					(1.33)			(1.14)
K_{-1}	-0.260	-0.264	-0.275	-0.277	-0.251	-0.456	-0.457	-0.510
	(2.84)	(2.86)	(5.47)	(5.44)	(2.80)	(8.29)	(8.21)	(7.08)
R^2	0.800	0.799	0.930	0.929	0.925	0.960	0.960	0.961
SEE	1183.0	1184.0	698.5	700.3	723.7	525.9	531.0	519.9
DW	1.19	1.19	1.78	1.75	1.81	1.86	1.77	1.95
D-m	0.96	0.97	0.03	0.06	0.02	0.03	0.14	-0.02
	(3.10)	(3.11)	(0.16)	(0.15)	(0.07)	(0.10)	(0.40)	(0.02)
EST	OLS	TSLS	OLS	TSLS	OLS	OLS	TSLS	OLS

Note: See tables 7 and 8. In the equation of table 16, $Z^e = Q_t$ or Q_{t-1}. Equations with other proxies for Z^e are presented in the Appendices see table A19.

Table 17 gives encompassing tests for alternative user cost (c_2) variables in the aggregate sector. Because of data limitations it was not possible to calculate all the choices of c_2 applied in the M-sector. In the first case (I), tests are based on the investment equation in which the price ratio (w/c) is an explanatory variable. The null hypothesis of the standard c_2-variable is rejected according to both the marginal depreciation rule (as, too, in the M-sector) and the tax-neutral user cost measures. In case II, which is based on the investment equation with real factor prices (w/p,c/p) as separate regressors, the null is not rejected.

TABLE 17. Encompassing Tests of Competing Measures of the User Cost Variable in the Aggregate Sector (investment equations 5.17i and 5.17ii)

I. Equation 5.17i; null hypothesis: c_2 = standard formula

Alternative hypothesis:	F-statistic
c_{21}/book value of depreciation coefficient	0.63
c_{22}/marginal value of depreciation coefficient	10.46
c_{24}/tax-neutral c_2 with adaptive price expectations	4.11
c_{25}/tax-neutral c_2 with zero rate of inflation	4.53
c_{26}/interest rate on state bonds as discount rate	1.62

II. Equation 5.17ii; null hypothesis: c_2 = standard formula

Alternative hypothesis:	F-statistic
c_{21}/book value of depreciation coefficient	1.75
c_{22}/marginal value of depreciation coefficient	0.03
c_{24}/tax-neutral c_2 with adaptive price expectations	0.05
c_{25}/tax-neutral c_2 with zero rate of inflation	0.20
c_{26}/interest rate on state bonds as discount rate	0.72

Note: See table 5 for critical values of F.

Although the encompassing tests for the aggregate sector give some support for the view that corporate tax factors are not important determinants of investment (case I), we are inclined to prefer the results of the second test (II) since the aggregate investment equations perform better in this case than with the w/c-investment models (in terms of goodness of fit and significance of coefficient estimates).

Table 18 shows stability tests for the investment equations of the aggregate sector. These tests tell a consistent story about the stability of the coefficients in all cases except where the explanatory price variable is w/c_2. As in the M- and R-sectors, this second variant of the user cost seems to cause instability in the w/c-investment models. The reasons for instability are likely to be the same as in the M- and R-sectors.

TABLE 18. Stability Tests of Annual Linear Investment Equations (Aggregate Sector)

Table No./ equation No.	F-H	c_B	c_F	c_B^2	c_F^2	Chow	DW
15/3	F(4,10)=0.791	0.614	0.297	0.287	0.157	F(4,10)=0.683	2.00
A20/9	F(4,10)=0.220	0.494	0.337	0.203	0.255	F(4,10)=0.466	1.98
A20/10	F(4,10)=0.407	0.530	0.364	0.228	0.235	F(4,10)=0.537	1.93
15/6	F(4,10)=4.518*	0.642	0.540	0.360*	0.400*	F(4,10)=0.720	1.67
16/3	F(5,8) =1.114	0.323	0.697	0.291	0.279	F(5,8) =1.813	1.78
A20/11	F(5,8) =0.936	0.254	0.917	0.221	0.160	F(5,8) =1.487	1.95
A20/12	F(5,8) =1.032	0.287	0.856	0.263	0.145	F(5,8) =1.515	1.89
16/6	F(5,8) =1.502	0.541	0.483	0.315	0.345	F(5,8) =0.506	1.86

See notes to table 10.
*) implies that the test statistic exceeds the critical value at the 5 per cent significance level and hence the null hypothesis of stability is rejected. A20/9 refers to equation 9 in table A20 in the Appendices etc.

In the aggregate sector it seems clear that the best performing investment equations are those which have the real wage rate and the real user cost as separate arguments and where the second variant of user cost (c_2) is used.

The investment equations for the manufacturing and residual sectors have hitherto been estimated separately. Since these two sectors jointly form the aggregate of all firms (i.e. aggregate sector), it is natural to ask whether there might exist some common factors which are not taken into account in a separate analysis and which could be reflected in the residuals of both sector's equations. Tentatively, it might be thought that, for example, certain financial factors (e.g. credit rationing) and/or variables reflecting the rapidly changed energy situation might affect investment behaviour in both sectors. In such a case it is sometimes possible to obtain better estimators by viewing the separate equations as part of a system (see Harvey, 1982).

When the disturbances in a particular equation are contemporaneously correlated with the disturbances in other equations (e.g. because of a common neglected variable), the relevant equation systems are known as seemingly unrelated regression equations (SURE, see Harvey, 1982). The seemingly unrelated regressions case is also known as Zellner estimation. We applied the SURE-(Zellner)-estimation method included in the

SHAZAM-program to the investment equations of the M- and R-sectors. The SHAZAM-program estimates a set of equations and performs a joint generalized least squares procedure of using the covariance matrix of residuals across equations. SHAZAM can also impose restrictions on the coefficients within or across equations. We have used it to impose the equality of the coefficients of the explanatory variables across the equations for the M-and R-sectors.

The SURE-estimation results are presented in the Appendices (see table A21). It can be seen that SURE leads to some gain in the efficiency of estimates, implying that the t-statistics of the coefficient estimates are somewhat higher than in the OLS-estimation. The SEE of regressions is about 15 per cent lower in SURE than in the OLS-estimation. However, the SURE estimation seems to have very little effect on the magnitude of the coefficient estimates.

Table A22 (see Appendices) presents tests for the equality of coefficients of the output (Q_{-1}), prices (w/c or w/p and c/p) and lagged capital stock (K_{-1}) variables in the M- and R-sectors. The results show that the null hypothesis of simultaneous equality of all coefficients is rejected in all cases. The evidence with respect to the separate equality hypothesis is rather mixed, however. The coefficient of the output variable seems to be equal in all cases as the estimates of the long-run output elasticities also indicate (see table 22). By contrast, the coefficients of the lagged capital stock seem to be unequal in the M- and R-sectors. Since the depreciation coefficients in the M- and R-sectors are almost equal (0.078 and 0.075, respectively), the divergence of the coefficients of K_{-1} implies that the rates of adjustment are not equal in these two sectors (see also table 21).

The null hypothesis of the equality of the coefficients for the price variables in manufacturing and residual sectors is rejected in equations 1 and 3 (at 5 per cent significance level) but this null is not rejected in equations 2 and 4 (see table A22). Hence the evidence with respect to the equality of the parameters of the price variables is rather mixed. However, even though the null hypothesis for the price variables is not rejected in the case of equation 4, which is the 'best' equation (lowest SEE) in both the M- and R-sectors, the price elasticities of the desired capital stock can be quite different in these two sectors. The reason for this is that the price elasticities depend both on the coefficient of the lagged capital stock (being unequal in the light of our test) and on the mean values of the price and capital stock variables. Hence, it is possible that the effects of factor price variables on investment diverge significantly in the manufacturing and residual sectors (see table 22).

6.3 Non-Linear Estimation Results for All Sectors

Table 19 reports estimates of the nonlinear regression coefficients for all three sectors. The nonlinear equations have been estimated only for the c_1-variant of the user cost because the c_2-variables contain negative values, thus prohibiting the use of nonlinear estimation.

In the nonlinear investment equations, a minimum distance estimation (Mindis) method is used. The algorithm employed is due to Amemiya as extended and implemented by Berndt, Hall, Hall and Hausman (1974). Because of nonlinearities, the likelihood function is not necessarily "well-behaved". This means that there may exist different estimates of the parameters that result in a good 'fit' of the regression equation on the data.

In each sector, the initial values of the coefficients chosen for starting the iteration process of the nonlinear regression programme were 1.0, 1.0, 0.2 and -0.05 for the constant term, output, relative prices (w/c) and capital stock variables, respectively (in equations 1, 3 and 5) and 1.0, 1.0, 0.2, -0.2 and -0.05 for the constant term, output, the real wage rate, the real user cost and capital stock variables, respectively (in equations 2, 4 and 6). However, we also tried a large number of other initial values in order to check the sensitivity of estimates to the choice of initial values. The final coefficient estimates chosen were those which gave the lowest residual sum of squared errors about the regression. Our calculations indicate that the estimates are not very sensitive to the initial values of the coefficients. In addition, we also estimated the coefficients by another nonlinear method - the Marquardt method, also included in the RAL program - and the results were very similar to those obtained by the Mindis method. Thus we feel that the parameter estimates are not unreasonable.

TABLE 19. Annual Estimation Results of Non-Linear Investment Equations in All Sectors (eqs. 5.21i, 5.21ii)

(5.21i) $$I_t = \alpha_0' [Q_t^e]^{\alpha_1} [(\frac{w}{c})_t^e]^{\alpha_2} + a_3 K_{t-1}$$

(5.21ii) $$I_t = \beta_0' (Z_t^e)^{\beta_1} (w_t^e)^{\beta_2} (c_t^e)^{\beta_3} + b_4 K_{t-1}$$

Eq. No./ Sector	(1/M)	(2/M)	(3/R)	(4/R)	(5/A)	(6/A)
constant	9.281	2.580	10.294	14.950	7.011	5.846
	(1.81)	(0.60)	(0.95)	(0.65)	(1.85)	(0.73)
Q_{-1}	0.756	0.933	0.723	0.667	0.793	0.817
	(15.30)	(4.21)	(6.65)	(3.37)	(15.70)	(4.83)
$(\frac{w}{c_1})$	0.097		0.403		0.197	
	(1.53)		(3.56)		(3.44)	
$(\frac{w}{p})$		0.112		0.457		0.172
		(0.42)		(2.33)		(0.96)
$(\frac{c_1}{p})$		-0.144		-0.373		-0.205
		(1.53)		(2.61)		(2.65)
K_{-1}	-0.316	-0.241	-0.184	-0.189	-0.263	-0.258
	(4.84)	(2.42)	(7.77)	(6.55)	(9.05)	(5.43)
R^2	0.85	0.85	0.96	0.96	0.95	0.95
SEE	495.5	494.6	345.3	356.7	619.6	642.5
DW	1.98	2.04	2.75	2.60	2.03	2.05
LLF	-135.0	-134.3	-128.5	-128.4	-139.0	-139.0

Note: The coefficient estimates of Q, w/c, w/p and c/p are the long-run elasticities of the desired capital stock with respect to these variables. LLF is the value of the log likelihood function. The estimation is based on the Mindis-method (minimum distance estimation) in the RAL-program. The algorithm employed is due to Amemiya (1974) as extended and implemented by Berndt, Hall, Hall and Hausman (1974). t-ratios are shown in parentheses. In the equations of table 19, $Q_t^e = Q_{-1}$ and $Z_t^e = Q_{-1}$

In terms of goodness of fit (R^2 and SEE) and significance of the parameter estimates (t-statistics), the nonlinear models behave very much the same as the linear models. The relative magnitudes of the coefficient estimates of the output and price variables are quite close to those in the linear equations. It should be noted that the estimated values of the parameters of output and price variables may be interpreted as the estimated long-run elasticities of the desired capital stock with respect to these variables, and hence a direct comparison with the coefficient estimates of linear models is not meaningful. In section 6.5, the long-run elasticities obtained with different functional specifications are compared.

Table 19 also contains the values of the log-likelihood function for each equation. These can be used to obtain a LR-test for the restriction that the real wage rate and the real user cost have equal coefficients in each sector. Table 20 gives the results of this LR-test.

TABLE 20. Likelihood ratio (LR) tests for the equality of the coefficients of the real wage rate (w/p) and the real user cost (c_1/p) within each sector

	χ_1^2	critical values of χ_1^2	
		5 %	1 %
Manufacturing	1.2	3.84	6.63
Residual sector	0.2		
Aggregate sector	0.0		

The results of table 20 clearly show that the null hypothesis of coefficient equality is not rejected, and hence the real wage rate and the real user cost have equal coefficients in each sector. It should be noted that these tests were not performed with respect to parameter equality across sectors. Such tests were carried out in the previous section in the context of SURE-estimation. It should also be emphasized that the LR-tests were performed only with the c_1-variant of the user cost. In section 6.5, however, we also present the average long-run elasticities of the desired capital stock in the case of linear equations with c_2-variables. It should be noted that the LR-tests indicate that the cost-minimization model is not rejected in any sector when c_1 is the user cost variable. By contrast, as noted previously, the linear equations indicate that the cost-minimization approach is inferior to the profit-maximization models with a donward-sloping demand curve (in M- and A-sectors) when c_2 is used.

6.4 Estimates of the Rate of Adjustment in All Sectors

In addition to examining the signs and significance of the coefficients and the closeness of fit, one can evaluate the investment relations by inspecting the structural parameters estimated from them. Table 21 presents estimates of the rate of adjustment based on annual data. In section 6.5, we consider estimates of the long-run elasticities of the demand for capital.

The estimates of the adjustment rate indicate the annual rate by which the gap between the desired and actual capital stock is eliminated. Table 21 gives the estimation results of the main investment equations for each sector. Both constant and variable adjustment rate equations are considered. A cash flow variable is not included among the explanatory variables in the constant speed of adjustment models, but is included in the variable adjustment rate models (see the discussion in section 5.4.4).

The estimates of the constant rate of adjustment (λ) are obtained through the relationships between the structural and reduced-form coefficients of the respective investment equations (see section 5.5.1). The estimated coefficient of the lagged capital stock gives, either directly or indirectly, an estimate of the adjustment speed. In the indirect case, the estimates are obtained by using an a priori value of the depreciation coefficient ($\delta_M = 0.078$, $\delta_R = 0.075$, $\delta_A = 0.076$).

Estimates of the variable rate of adjustment are calculated from investment equations (5.14) and (5.15) when either of the two alternative linearized K*-equations (5.16i and ii) is transformed into these investment models. In the variable case, the 'total' rate of adjustment is the sum of the constant (initial) rate of adjustment (λ_0) and the variable component, which is the product of λ_1 and the cash flow gap variable. If the cash flow is a net concept, then this gap variable is $CF^n/(K^*-K_{-1})$, and if it is a gross concept, then the gap is $CF^g/[K^*-(1-\delta)K_{-1}]$.

TABLE 21. Estimates of the Rate of Adjustment for All Sectors

Table No./ Eq. No.	Relative prices	Cash Flow	λ	Rate of Adjustment λ_0	λ_1	$\lambda_0+0.5\lambda_1$	$\lambda_0+\lambda_1$
Manufacturing							
7/3	w/c_1	-	0.40				
7/5	w/c_1	CF^g	-	0.20	0.35	0.38	0.55
7/9	w/c_2	-	0.36				
8/6	$w/p, c_2/p$	-	0.47				
8/11	$w/p, c_2/p$	CF^g	-	0.36	0.45	0.58	0.81
8/12	$w/p, c_2/p$	CF^n	-	0.37	0.48	0.61	0.85
19/1	w/c_1	-	0.39				
19/2	$w/p, c_1/p$	-	0.32				
A20/1	w/c_1	-	0.40				
A20/2	w/c_1	-	0.43				
A20/3	$w/p, c_1/p$	-	0.37				
A20/4	$w/p, c_1/p$	-	0.38				
		average	0.39				0.74
Residual sector							
11/3	w/c_1	-	0.26				
11/7	w/c_1	CF^g	-	0.20	0.10	0.25	0.30
11/10	w/c_2	-	0.27				
12/3	$w/p, c_1/p$	-	0.27				
12/6	$w/p, c_2/p$	-	0.28				
12/9	$w/p, c_2/p$	CF^n	-	0.32	0.34	0.49	0.66
19/3	w/c_1	-	0.26				
19/4	$w/p, c_1/p$	-	0.27				
A20/5	w/c_1	-	0.28				
A20/6	w/c_1	-	0.31				
A20/7	$w/p, c_1/p$	-	0.28				
A20/8	$w/p, c_1/p$	-	0.31				
		average	0.28				0.48
Aggregate sector							
15/3	w/c_1	-	0.34				
15/5	w/c_1	CF^n	-	0.30	0.19	0.40	0.49
15/6	w/c_2	-	0.43				
16/3	$w/p, c_1/p$	-	0.36				
16/6	$w/p, c_2/p$	-	0.53				
16/8	$w/p, c_2/p$	CF^n	-	0.59	0.37	0.77	0.96
19/5	w/c_1	-	0.35				
19/6	$w/p, c_1/p$	-	0.33				
A20/9	w/c_1	-	0.35				
A20/10	w/c_1	-	0.38				
A20/11	$w/p, c_1/p$	-	0.36				
A20/12	$w/p, c_1/p$	-	0.40				
		average	0.38				0.73

Notes: Table A20 is presented in the Appendices. Variables in the "Relative prices" and "Cash flow" columns indicate the explanatory variables used in the corresponding investment equations. Estimates of λ are from the constant speed of adjustment equations (without the CF-variable) and estimates of λ_0 and λ_1 are connected to the variable speed of adjustment equations (including the CF-variable). Hence, if the cash flow gap is 0.5, the rate of adjustment is given as $\lambda_0 + 0.5\lambda_1$.

Table 21 contains the estimates of λ_0 and λ_1 together with the estimates of the 'total' rate of adjustment with two values (0.5 and 1.0) for the cash flow gap variable. Hence, the estimate of $(\lambda_0+\lambda_1)$ gives the rate of adjustment when the firm can finance all net (CF^n-case) or gross (CF^g-case) investment by using only internal finance. In the other extreme case, where no internal finance is available, the rate of adjustment is simply λ_0.

When evaluating the estimates of the rate of adjustment, it should be recalled from the estimation results of the underlying investment equations that the coefficient estimates of the cash flow variables are only 'mildly' significant, especially in the residual and aggregate sectors. The significance of the coefficient estimates of the cash flow variables also varies from equation to equation. Hence the estimates of the adjustment rate must be regarded with some caution and they are reported here mainly to illustrate the order of magnitude of the adjustment speed in the annual data.

Table 21 also contains the averages of various estimates of the adjustment rate for both the constant and variable cases in each sector. The estimates of the constant rate of adjustment are roughly of the order 0.3 - 0.4 in all sectors, implying that about 30 to 40 per cent of the gap between the desired and actual capital stocks is eliminated in each year. The variable adjustment rate models imply a much higher speed, especially if firms have ample cash flow for financing investment outlays (notably in the M-sector).

The estimates of the rate of adjustment seem rather high but one should note that substantial expectations lags have already been imposed on the variables (especially c_2) determining the optimal capital stock. Generally, it would seem that the variable speed of adjustment model is 'better' than the constant speed model only in the case of the manufacturing sector (see sections 6.2.2 - 6.2.4 above). The results obtained here accord very well with the results given in Coen (1971). The annual estimates by Coen (obtained from quarterly equations) are $\lambda \approx 0.34$ and $\lambda_0 + \lambda_1 \approx 0.8$.

6.5 Estimates of the Long-Run Elasticities of the Desired Capital Stock with Respect to Output and Price Variables in All Sectors

Table 22 gives estimates of the long-run elasticities of the optimal capital stock with respect to the accelerator (output, demand) and price (w/c or w/p and c/p) variables. In the log-linear (ΔlogK and I/K as dependent variables) and nonlinear investment equations the estimates of elasticities are constant and are obtained from the coefficient estimates of the corresponding variables. In the case of the linear equations, an effort has been made to convert the slope coefficients of the demand for capital into elasticities using the mean values of the explanatory variables. Table 22 also contains averages of various elasticity estimates for each sector.

The investment equations from which the elasticity estimates are derived are the main versions of each form of model and they do not include cash flow variables. It should be noted that in the case of the c_2-variant of the user cost only elasticities for the linear models are obtained, since it was not possible to estimate the logarithmic transformations of equations with this variable.

From table 22 it can be seen that the estimates of output elasticities are within a rather narrow range. The long-run output elasticity of the desired capital stock seems to be somewhat below one (0.75 - 0.85), hence implying slightly increasing returns to scale in the production function (for the case of a cost-minimization model, see table 4). The price elasticities display a much wider range, mainly because of the difference in the estimates of coefficients arising from c_1 and c_2 variables (see the discussion in section 6.2.2).

Table 22. Estimates of the Long-Run Elasticities of the Desired Capital Stock with Respect to Output and Price Variables for All Sectors

Sector	Price Variable	Functional Form of the Investment Equation (dependent variable): linear - I				ΔlogK				I/K_{-1}				non-linear I			
		ε_Q	$\varepsilon_{w/c}$	$\varepsilon_{w/p}$	$\varepsilon_{c/p}$	ε_Q	$\varepsilon_{w/c}$	$\varepsilon_{w/p}$	$\varepsilon_{c/p}$	ε_Q	$\varepsilon_{w/c}$	$\varepsilon_{w/p}$	$\varepsilon_{c/p}$	ε_Q	$\varepsilon_{w/c}$	$\varepsilon_{w/p}$	$\varepsilon_{c/p}$
M	w/c_1	0.85	0.12			0.77	0.072			0.79	0.065			0.76	0.097		
M	w/c_2	0.94	0.0014														
M	w/p, c_1/p	1.14		-0.19	-0.13	0.84		0.013	-0.09	0.89		0.05	-0.09	0.93		0.11	-0.14
M	w/p, c_2/p	0.63		0.21	-0.013												
R	w/c_1	0.84	0.44			0.78	0.33			0.79	0.34			0.72	0.40		
R	w/c_2	1.17	0.038														
R	w/p, c_1/p	1.05		0.25	-0.45	0.77		0.34	-0.33	0.70		0.41	-0.29	0.67		0.45	-0.37
R	w/p, c_2/p	0.68		0.44	-0.097												
A	w/c_1	0.89	0.22			0.80	0.18			0.83	0.17			0.79	0.20		
A	w/c_2	0.95	0.057														
A	w/p, c_1/p	0.86		0.25	-0.19	0.76		0.22	-0.17	0.75		0.25	-0.15	0.82		0.17	-0.21
A	w/p, c_2/p	0.60		0.31	-0.14												

Average of output elasticities (ε_Q) in the (w/c)-equations: M = 0.82, R = 0.86, A = 0.85

Average of price elasticities ($\varepsilon_{w/c}$) in the (w/c)-equations: M = 0.072, R = 0.31, A = 0.16

Average of output elasticities in the (w/p, c/p)-equations: M = 0.87, R = 0.77, A = 0.76

Average of real wage rate elasticities in the (w/p, c/p)-equations: M = 0.13, R = 0.38, A = 0.24

Average of real user cost elasticities in the (w/p, c/p)-equations: M = -0.092, R = -0.31, A = -0.17

M = manufacturing

R = residual sector

A = aggregate sector

However, some systematic picture can be observed between the price elasticities of different sectors. On average, price elasticities are lower in manufacturing than in the residual sector, while elasticities in the aggregate sector fall in between. The price elasticities of the desired capital stock are rather low in the M-sector (somewhat below 0.10), hence implying indirectly a seemingly low elasticity of substitution between capital and labour (see section 5.3.2). In the residual sector, the average estimate of price elasticities is roughly 0.3 - 0.4, which can be thought to be of a reasonable order of magnitude since in the labour-intensive residual sector there should be more substitution possibilities between factor inputs. Alternatively, it could be argued, on a somewhat tentative basis, that the production technology of the manufacturing sector is more likely to be of a putty-clay type than that of the residual sector, which is akin to a putty-putty technology (see the discussion of putty-putty and putty-clay models in section 5.3.2).

As mentioned above, the cost-minimization equations with w/c as the price variable indicate slightly increasing returns to scale (about 1.1 - 1.3) in the production function. We have noted previously (see sections 6.2.2. - 6.2.4.) that the sign of the coefficient of the wage rate variable is positive in all sectors (this result holds for all functional specifications and demand proxies used) in the profit-maximization equations based on the assumption of a downward-sloping demand curve (w/p and c/p as separate variables). In note 13 of chapter V, we discussed at some length the various circumstances which may yield a positive coefficient for the w-variable. In the monopolistic industry case with an elastic demand function, the requirement is that there are increasing returns to scale in the production function (CD). A positive sign for the coefficient of the wage rate may, however, also follow if a competitive industry faces an inelastic demand for its output (with decreasing returns to scale). Furthermore, if the equations with the wage rate and the user cost as separate arguments may be interpreted as approximations to the CES production function, then other possibilities for a positive sign of the coefficient of w-variable arise (note that in the linear investment equations the production function is basically nonspecific, see section 5.3). Hence, generally the sign of the coefficient of the wage rate depends on the market structure the firms are faced with, the parameters of the production function (elasticity of substitution and returns to scale) and the price elasticity of demand for the products. Because of the multitude of hypotheses involved and the non-existence of reliable information about the market structure, we do not consider it possible to distinguish between different explanations for the positive wage rate coefficient. This result is hence regarded here as an empirical fact. However, there is some support for the increasing returns to scale argument since a recent study by H. Tarkka (1984) on factor demand in Finnish manufacturing reports increasing returns to scale (1.2-1.3).

6.6 The Effect of Credit Rationing, Technical Change and the Replacement Hypothesis on Investment in All Sectors

In this section we present some preliminary empirical results for two additional factors that might affect the investment behaviour of Finnish firms. First, two different variables are used as a proxy for credit rationing and these variables are added to the linear investment equations for each sector in order to see whether a direct credit rationing effect exists. Second, a simple trend variable is added to the linear investment equations of each sector in order to take account of the possible impact of technical change. The estimation results with respect to credit rationing and technical change effects are presented in tables A23 and A24 (Appendices). Table A25 presents some experiments with higher depreciation rates than in our basic equations.

In section 5.4.3 we discussed, rather tentatively, the role of credit rationing in investment policy. The discussion was based to some extent on the theoretical results presented in chapter IV. It was argued that credit rationing might have either permanent or temporary effects on the investment of a single firm. If firms are permanently rationed in the loan market, then their investment function is liquidity-constrained all the time and only financial variables (cash flow and the rationed amount of credit) affect investment behaviour. The situation is quite different if a firm anticipates being financially constrained in the future (permanently or temporarily). In this case the anticipation of a future binding credit constraint might even increase investment in the periods before this constraint becomes effective (see section 4.4).

In the light of the above discussion, it is not very clear how the possible effect of credit rationing should be modelled in an empirical investment equation, i.e. whether it should affect the desired capital stock or the timing of investment or both. This question is even more difficult if aggregate data are used in the empirical analysis. It can be argued that not all firms face credit rationing in a given time period and, furthermore, that single firms may face changing situations with respect to the availability of credit. Hence in an aggregate analysis of firms, the credit rationing effect is likely to occur only in some 'average' form which is probably not time invariant.

We shall try to capture the possible effect of credit rationing by applying two different variables as proxies for the expected credit rationing. The first proxy is based on the assumption that the supply of credit by banks at the margin is chiefly affected by the marginal cost of central bank borrowing (for a discussion of the banking system and financial markets in Finland, see Oksanen, 1977 and Tarkka, 1983). The second proxy for the credit rationing variable is constructed on the basis that the supply of loans depends upon the difference between the marginal rate and the average lending rate (see Tarkka, 1983).

For the marginal interest rate on central bank borrowing (RM), two alternative series were used (RM1, RM2, see Appendix V). Hence two variants were also available for the interest rate differential variable (RM-r, r = average lending rate). The expectations hypotheses with respect to the credit rationing proxies are of a simple form: only the current period and one or two period lagged values were tested empirically. The estimation results obtained for the credit rationing variables are shown in the Appendices (table A23) for two functional forms of the investment equations (w/c or w/p and c/p as price terms) and for two variants of the user cost variable (c_1 and c_2/adaptive).

The estimation results are rather clear-cut. Credit rationing does not seem to have a significant effect on the investment behaviour of Finnish firms. With the c_1-variant of the user cost, all the t-statistics of the coefficient estimates of credit rationing variables are clearly insignificant (only selected results are presented in the Appendices, but many other estimation results confirmed this conclusion). With the c_2-variant of the user cost, the coefficient estimate is mildly significant in the residual sector. It can also be noted that the coefficient estimates take either a positive or a negative sign. In the residual sector, the results obtained with the c_2-equations indicate that the anticipation of credit rationing has a positive impact on investment. The experiments with credit rationing factors must be regarded as rather preliminary, and it is possible the impact of credit rationing occurs indirectly through the accelerator (output, demand) variables. The regression models for output in each sector indicate that an increase in credit affects output positively (see table A18) and if lending is a negative function of the marginal interest rate (or interest rate differential) then the effect of credit rationing might indeed be realized through the output variable. An economy-wide empirical macro-model would, however, be needed to simulate the impact of 'credit rationing' proxies on investment behaviour.

In section 5.3.2 we discussed some basic aspects of the role of technical progress in investment models. Table A24 shows some simple calculations carried out with linear investment equations including a time trend variable (T) as a proxy for technical change. In the cost-minimization models (w/c as the price variable), the sign of the T-variable should be negative while in the profit maximizing models (w/p and c/p as the price variables), it is generally ambiguous. However, in the CD-function the sign should be the opposite of that of the wage rate coefficient and, since the wage rate has a positive coefficient, the sign of technical progress (Hicks-neutral) should be negative (see note 13 of chapter V). From the estimation results it can be concluded that, in the w/c-equations, the sign of the coefficient of the trend variable is usually negative but clearly insignificant. In the (w/p,c/p)-equation, the sign of the T-variable varies and its coefficient estimate is significant only in the residual sector (eq. 4) and in the second equation of the aggregate sector (eq. 6 with c_2). In the residual sector the inclusion of the T-variable seems to lower the coefficient of output and to increase the coefficients of price variables.

Table A25 reports some experiments with higher depreciation rates than in our basic equations. It has often been argued that the economic rate of depreciation has increased rapidly since the 'oil-price shock' in 1974. Our estimation results indicate that the coefficient estimates of linear investment equations are not very sensitive to at least modest changes in the depreciation coefficient.

6.7 Summary of the Empirical Results of Annual Investment Equations

In this summary we briefly point out the most important features of the preceding empirical results obtained by using annual data on the Finnish corporate sector. They are as follows:

(i) The accelerator (output or demand) is the most important single determinant of investment, although relative factor prices are also a significant factor. On average, accelerator variables affect investment with a shorter time lag than price variables.

(ii) Price variables (the real wage rate and real user cost) seem to affect investment behaviour separately (with unequal coefficients) and the elasticity of demand for capital with respect to the wage rate is higher than with respect to the user cost (see also below).

(iii) Different measures of capital cost (user cost) yield rather different results with respect to, for example, the long-run price elasticities of the desired capital stock. The second variant of the user cost (c_2) seems to perform better than the c_1-variant. Long-run output elasticity (slightly less than one) is not greatly affected by the choice of the capital cost variable. The modelling of expectations for the rate of inflation is crucial to the measurement of the capital cost variable in the case of the c_2-variable.

(iv) Corporate tax factors are quite important for the investment policy of firms, but personal tax factors do not seem to have a significant effect on investment decisions.

(v) Cash flow considerations have a significant influence on investment (its timing) by manufacturing firms, but they are not important for firms in the 'closed' residual sector.

(vi) Credit rationing does not seem to affect investment policy. It may, however, have an indirect negative impact through the accelerator variable.

(vii) Generally, investment behaviour does not seem to be sensitive to 'modest' changes in the replacement hypothesis (i.e. depreciation rates) or to the inclusion of technical change as a separate argument (except perhaps in the residual sectors).

Sectoral Comparisons:

(viii) Accelerator variables are almost equally important in all sectors. Foreign demand seems to affect only investment by manufacturing firms whereas investment decisions of firms in the 'closed' residual sector are mainly affected by developments in domestic demand.

(ix) Price variables are more important in the residual sector than in the manufacturing sector. The long-run price elasticity of the optimal capital stock is about 0.10 in manufacturing and about 0.35 in the residual sector (with the price variable being in the ratio form w/c). The long-run elasticities with respect to the real wage rate and the real user cost also seem to diverge in the two sectors, being roughly of the same order of magnitude as the elasticity with respect to the price ratio (the wage rate has a positive and the user cost a negative sign). Indirectly, the elasticity estimates imply that there are more substitution possibilities between capital and labour in the residual sector (labour-intensive) than in the manufacturing (capital-intensive) sector.

On the whole, the results with annual data indicate that the neoclassical models of investment provide a fairly adequate explanation of the investment behaviour of Finnish corporations in the period under consideration (1963 - 1980).

CHAPTER VII

EMPIRICAL RESULTS OF QUARTERLY INVESTMENT EQUATIONS

7.1 Tests of Alternative Expectations Hypotheses for the Rate of Inflation and a General Outline of the Quarterly Analysis

The statistical data used in the empirical analysis were described in section 6.1. Before considering the general outline of the quarterly analysis, we shall present some encompassing tests of competing expectations hypotheses for the rate of inflation, which is a crucial determinant of the user cost variable.

The tests were carried out for the two variants of the user cost (c_1 and c_2) and for two forms of linear investment equations (eqs. 5.17i and 5.17ii). The expectations assumptions were also the same as those used with annual data. With adaptive price expectations, the adaptation coefficient is 0.1 and it was selected on the basis of the minimum sum of residual squares. It should, however, be emphasized that coefficient values between 0.1 and 0.5 gave very similar results to the coefficient (0.1) used in the subsequent analysis (especially in the case of eq. 5.17ii).

The encompassing test results presented in table 23 are fairly clear-cut. In the first case (I), with the price ratio (w/c) as an explanatory variable (other variables included in the regression were a constant term, Q_{-1} and K_{-1} for all equations), the null hypothesis c_1 is not rejected with any of the alternatives. When the test is reversed, all the null hypotheses with different c_2-variables are rejected. These results clearly indicate that, for investment equations containing the factor price ratio as an explanatory price variable, the first variant (c_1) of the user cost performs best. Similar results were obtained using annual data. The investment equations with the (w/c_2)-variable perform rather poorly for quarterly data and the coefficient estimate of the w/c_2 is insignificant in most cases, often receiving the wrong sign (negative). Hence, the estimation results below are reported for equations including c_1. In the Appendices are shown some estimation results with w/c_2. The reasons for the poor performance of the c_2-variables are the same as in the case of annual data (w/c_2 behaves in a very erratic manner) but the problems with c_2 seem to be aggravated when quarterly data are used. The encompassing tests were also made with longer lags of the output variable but the results were essentially the same as with Q_{-1}.

Table 23. Encompassing Tests of Competing Expectations Hypotheses for the Rate of Inflation in the User Cost Variable, (quarterly data of the aggregate sector)

I. Investment equation 5.17i (w/c as price variable)

			F-statistics		
Null hypothesis H_0:	c_1	c_2/adaptive	c_2/static	c_2/perfect	c_2/constant
Alternative hypothesis:					
c_2/constant price expectations	0.09	1.71	1.30	0.86	
c_2/static price expectations	0.01	0.01		2.11	0.01
c_2/'perfect foresight' price expectations	0.17	0.24	1.09		0.92
c_2/adaptive price expectations	0.30		1.43	4.98	0.15
c_1		22.45	24.28	18.67	22.56

II. Investment equation 5.17ii (w/p,c/p as price variable)

			F-statistics		
Null hypothesis H_0:	c_1	c_2/adaptive	c_2/static	c_2/perfect	c_2/constant
Alternative hypothesis:					
c_2/constant price expectations	0.20	4.09	5.67	2.69	
c_2/static price expectations	0.14	9.82		3.11	1.32
c_2/'perfect foresight' price expectations	0.29	3.63	2.66		2.01
c_2/adaptive price expectations	33.15		69.15	53.06	51.45
c_1		0.18	12.81	13.17	8.10

Notes: The 95 and 99 per cent values of $F(1,68)$ are approximately 3.99 and 7.06, respectively. If the calculated F-statistics are smaller than the critical values, the null hypothesis is not rejected; (see also table 5).

Encompassing tests in case (II), where the real wage rate (w/p) and the real user cost (c/p) are price variables, leave some room for interpretation. The null hypotheses with different c-variables are rejected when the alternative is the adaptive version of c_2. The null hypotheses with static, perfect and constant price expectations are also rejected when the alternative is the c_1-variable. The adaptive-c_2 seems to perform best in the light of these tests. However, in order to carry out a comparable analysis with annual models and the (w/c_1)-models, we have selected for further analysis the two variants of the user cost (c_1 and c_2), where c_2 is calculated under the assumption of adaptive price expectations with a coefficient 0.1.[1]

After having made our choices with respect to the price variables, we shall briefly consider the general strategy of the quarterly analysis. The main features of the analysis are the same as with the annual data but some differences do, however, arise because of the quarterly basis of the time series observations. It should be emphasized that before performing any estimation, diagnostic checking or specification tests, we have a priori selected some possible specifications of investment equations which were found to be reasonable and interesting for a detailed analysis (e.g. functional form: linear, log-linear, non-linear without cash flow and non-linear with cash flow, and the different measures of the user cost etc; see the discussion in sections 5.2 - 5.5). Hence, various alternative formulations of the investment equation suggested by the neoclassical theory have been delineated and the next step is to estimate the equations and to carry out specification and misspecification testing with competing hypotheses.

Specification and misspecification tests will be carried out mainly within the context of linear (or log-linear) models since statistical inference and hypothesis testing are not yet well developed for non-linear regression models (see, e.g., Harvey, 1982). Thus, the estimation of non-linear equations is chiefly motivated by theoretical considerations, although some comparisons with linear models can be made in the light of the standard errors of regression and the estimates of structural parameters (rate of adjustment, elasticities). Some tests of the nested hypotheses can also be made within different types of non-linear models.

The quarterly analysis can be classified under the following broad categories:

i) Estimation of basic linear investment equations:
Estimation results are reported in tables 25, 27 and 28 and in the Appendices. The estimation methods used are OLS, TSLS, Cochran - Orcutt (CO) and two-stage Hatanaka (HAT); see the discussion in section 6.2.2.

ii) Tests against misspecification:
Unlike a test of specification (e.g. encompassing tests of c-measures), a test of misspecification is constructed with no clear alternative hypothesis in mind. Hence, such tests are designed for assessing the goodness of fit of the model implied by a particular maintained hypothesis (see Harvey, 1982). However, although the alternative hypothesis is usually rather vague, the choice of a particular misspecification test is generally based on some possible departure from the basic hypothesis in some suspected direction.

However, it should be emphasized that the appropriate conclusion to be drawn from the tests depends upon whether one is willing to maintain the correctness of the model's specification. Thus, when the model is thought to be correct, rejection may reasonably be attributed to either heteroscedasticity or serial correlation, so that there is a potential efficiency gain to be realized from a more careful modelling of the variance structure (i.e. disturbance term). If the researcher is less confident about the correctness of the linear model, the tests indicate only that something is wrong, but not what. A more thorough investigation of the model's specification is hence indicated.

In the quarterly models, serial correlation of residuals is usually a more serious problem than in annual models and hence more attention is paid here to testing for the presence and form of autocorrelation. The following test statistics are employed: DW, Durbin-m, the Breusch LM(4) for fourth-order (AR(4)) autocorrelation and the Wallis d_4-test for seasonal aspects of autocorrelation (see, also, table 24).

iii) Specification tests with respect to the dynamics:
Our basic point of departure with both annual and quarterly equations is the form in which the lagged capital stock is an explanatory variable. Since the tests for serial correlation reveal that autocorrelation is present in the residuals of quarterly equations, we have also considered some more general dynamic specifications where an additional lagged capital stock (K_{t-2}) and/or lagged investment itself (I_{t-1}) enter as explanatory variables. Formal likelihood ratio tests are carried out between these more general dynamic specifications and the original or the constrained (first-order autocorrelation) specifications mainly in order to check whether a first-order autocorrelation correction is an appropriate way of modelling the dynamics of investment equations. LR-test results are reported in tables 26 and 29.

iv) Analysis of other functional specifications of the investment models:
As in the case of the annual data, we have also used quarterly data to examine some alternative functional specifications of the investment equation. This procedure can also be seen as a way of trying to tackle the autocorrelation problems of the residuals in linear equations since autocorrelation of residuals is not full evidence of serially correlated disturbances (see, e.g., Harvey, 1982). Functional misspecification, omitted variables or errors in variables may also cause serial correlation in the residuals. The main motivation, however, with the endogenous rate of adjustment models (non-linear cash flow models) is that theoretical considerations imply that the constant rate of adjustment is obtained only under rather restrictive assumptions (see chapters III and IV).

In log-linear models, the dependent variable is either $\Delta \log K$ or I/K_{-1} (see Table A28). The estimation results of the standard non-linear equations (without a cash flow variable) are presented in table 31. Tables 32 and 33 show the results of the non-linear equations with the endogenous (variable) rate of adjustment due to cash flow considerations. In order to check the sensitivity of the non-linear estimation results, three different estimation methods (programmes) have been employed (Mindis, Marquardt and Shazam).

v) Specification tests of alternative measures for the user cost: Analogously to the annual models, we have performed some encompassing tests of alternative concepts of the user cost. The alternatives deal with different specifications of the tax factors (tax neutral case etc.) in the user cost variable and they are formulated within the second variant of the user cost (c_2). These tests were carried out for the linear equations and they are reported in table 34.

vi) Estimates of the structural parameters:
Estimates of the rate of adjustment are presented in table 35. They cover both the constant and variable speed of adjustment models. Table 36 shows some estimates of long-run elasticities of the demand for capital with respect to the accelerator (output) and price variables.

As was mentioned in section 5.5.2, in the quarterly analysis we shall examine some proxies for the independent variables that are more expectational than was the case in the annual analysis. ARIMA-models of output and price variables are considered in Appendix VI. ARIMA-models are used to generate short-term (one-period) expected values for variables. Almon polynomial lag structures are also used to generate expectational values. In all these cases as well as in the cases of static and adaptive expectations hypotheses, the forecast horizon is rather short and the expected values of variables are based on their own past values.

The essential characteristics of the investment decision are, however, that they are inherently forward-looking since they depend on the expectations of future values of the relevant variables (see section 5.2). At the same time, however, the usual practice in empirical work has been to apply the 'short-sighted' approach mentioned above. Because of the importance of the forward-looking character of the investment process, we shall also perform some preliminary analysis with truly forward-looking expectational values of the output and price variables.[2]

Table 24 shows the definitions (abbreviations) of the variables and test statics which are used in the subsequent quarterly analysis.

7.2 Estimation Results of Linear Quarterly Investment Equations for the Aggregate Sector

Since the c_2-variable with adaptive price expectations receives negative values in the mid-1970s when the rate of inflation accelerated rapidly, we have also constructed a new series for the c_2-variable which is obtained from the original c_2-variant by adding a constant risk premium to the cost of capital component (see Asteraki 1984). This premium might be thought to reflect the effects of exchange rate risks inherent in foreign borrowing but which are not reflected in the nominal interest rate on debt capital. The 'corrected' c_2 is positive in the period 1963.1 - 1980.4. The correlation between the original c_2 and the 'corrected' c_2 is 0.97 and the estimation results with these two series are almost similar with both the (w/c_2) - and $(w/p, c_2/p)$ - type models (see below). The advantage of the positive c_2-variable is that it can also be used in log-linear and non-linear equations in order to make a full scale comparison between investment equations based on c_1 and c_2.

The estimation results of the linear quarterly investment equations with the factor price ratio (w/c) as an explanatory price variable are presented in table 25 and in the Appendices, tables A26 and A27. In terms of standard statistical criteria (goodness of fit, SEE, autocorrelation tests etc.), it seems rather clear that the estimated equations perform more satisfactorily with w/c_1 than with w/c_2 as the price variable, a result which is in accordance with the annual data. In the quarterly data, the w/c_2 variable seems to perform even more poorly than in the annual data because the sign of the coefficient estimate of w/c_2 is uncertain (see table A27). The reasons for the problematic behaviour of the w/c_2-variable are likely to be the same as in the case of the annual data (see sections 6.2.2 and 6.2.4).

TABLE 24. Definitions of Variables and Statistics Used in the Quarterly Analysis

1. Output (Q^e):

   $Q_1 = Q_{t-1}$, $Q_4 = Q_{t-4}$, Q_A = Almon polynomial lag structure for Q
   Q_{AR} = ARIMA-model for Q, Q_F = Forward-looking expectations of Q (see note 2)

2. Price Variables ($(\frac{w}{c})^e$, $(\frac{w}{p})^e$, $(\frac{c}{p})^e$):

   $wc_0 = (\frac{w}{c_1})_t$, $wc_1 = (\frac{w}{c_1})_{t-1}$, $wc_{20} = (\frac{w}{c_2})_t$ etc.

   wc_A = Almon-model for (w/c_1), wc_{AR} = ARIMA-model for (w/c_1)

   wc_F = Forward-looking model for (w/c_1)

   $\bar{w}_0 = (\frac{w}{p})_t$, $\bar{w}_1 = (\frac{w}{p})_{t-1}$, $\bar{w}_{AR}$ = ARIMA-model for (w/p)

   $\bar{w}_F$ = Forward-looking model for (w/p)

   $\bar{c}_1 = (\frac{c_1}{p})_t$, $\bar{c}_{1,A}$ = Almon-model for (c_1/p)

   $\bar{c}_{1,AR}$ = ARIMA-model for (c_1/p), $\bar{c}_{1,F}$ = Forward-looking model for (c_1/p)

   $\bar{c}_2 = (\frac{c_2}{p})_t$, $\bar{c}_{2i} = (\frac{c_{2i}}{p})_t$, i = 1, 2, 4, 5, 6 (see table 17)

   $\bar{c}_{2,F}$ = Forward-looking model for (c_2/p)

3. Cash-Flow (CF^e):

   $CF_1^n = CF_{t-1}^{net}$, $CF_4^n = CF_{t-4}^{net}$, $CF_1^g = CF_{t-1}^{gross}$,

   $CF_4^g = CF_{t-4}^{gross}$ etc., CF_A = Almon-model for CF

4. Other variables T, RM, see Appendices, tables A35, A36.

5. Test Statistics:
   - R^2, SEE, DW, D-m, RHO: see tables 7 - 8
   - LM(4) is the Breusch (1978) LM-autocorrelation statistic for the AR(4) process and the critical values of χ_4^2 are 1 %/13.28 and 5 %/9.49; $e_1,\ldots,e_4$ are lagged residuals from OLS-estimation used to calculate LM(4).
   - d_4 is the Wallis (1972) test statistic analogous to DW and it is derived from the residual relationship $e_t = \rho_4 e_{t-4} + u_t$. The 5 per cent significant points for (72,4) degrees of freedom are $d_{4.L} = 1.418$, $d_{4,U} = 1.648$, and the null hypothesis $\rho_4 = 0$ is rejected if $d_4 < d_{4,L}$, the null is accepted if $d_4 > d_{4,U}$ and the inconclusive region is $d_{4,L} < d_4 < d_{4,U}$ (see Wallis, 1972).

TABLE 25. Quarterly Estimation Results of Linear Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15 and 5.17i; w/c_1 as the price variable)

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	-0.160	-0.477	0.107	0.029	0.582	0.181	-0.068
	(0.91)	(2.60)	(0.82)	(0.16)	(1.85)	(1.47)	(1.60)
Q^e	0.451	0.298	0.649	0.613	0.537	0.620	0.309
	(9.47)	(4.91)	(14.25)	(8.31)	(6.01)	(9.73)	(4.23)
$(w/c_1)^e$	2.745	2.668	1.665	1.744	0.478	1.165	1.890
	(6.32)	(6.68)	(4.93)	(4.82)	(0.97)	(2.65)	(3.64)
CF^e		0.305		0.054			0.204
		(3.68)		(0.83)			(2.42)
K_{-1}	-0.058	-0.038	-0.070	-0.066	-0.047	-0.062	-0.033
	(8.12)	(4.41)	(12.31)	(9.26)	(4.14)	(7.68)	(3.20)
R^2	0.810	0.839	0.889	0.886	0.904	0.766	0.683
SEE	288.7	265.2	220.1	221.1	201.5	204.7	245.7
DW	1.03	1.25	1.21	1.24	2.22	1.95	1.98
D-m	0.51	0.38	0.38	0.37	-0.25	-0.19	-0.26
	(4.35)	(2.93)	(3.07)	(2.95)	(1.95)	(0.61)	(0.78)
d_4	1.91	1.96	1.95	1.93			
RHO					0.63	0.42	0.44
					(6.59)		
LM(4)	27.12	15.91	12.96	12.74	11.26	15.12	28.08
e_1	0.254	0.228	0.340	0.330	-0.242		
	(1.99)	(1.73)	(2.53)	(2.46)	(1.75)		
e_2	0.385	0.324	0.120	0.128	0.057		
	(2.92)	(2.36)	(0.85)	(0.90)	(0.41)		
e_3	0.215	0.181	0.170	0.192	0.348		
	(1.48)	(1.23)	(1.17)	(1.27)	(2.32)		
e_4	0.083	0.076	-0.163	-0.149	0.095		
	(0.59)	(0.46)	(1.19)	(1.07)	(0.64)		
EST	OLS	OLS	OLS	OLS	CO	HAT	HAT
Definition of	Q_1	Q_1	Q_4	Q_4	Q_4	Q_4	Q_1
Variables	wc_0	wc_0	wc_0	wc_0	wc_0	wc_0	wc_0
		CF_4^n		CF_4^g			CF_4^n

Notes: See table 24. In the HAT-estimation, R^2 is expressed at the transformed level of variables; in all other cases R^2 is at the original level of variables.

Results of sensitivity analysis carried out with respect to different lag specifications of the output variable are presented in Table A26. The highest t-value for the coefficient estimate of output is obtained with a one year lag, and the output variable with this lag (Q_{t-4}) and with a one quarter lag (Q_{t-1}) were chosen as the basic 'discrete' lag specifications of this variable. In the case of the price variable (w/c_1), the current period's value seems to perform best. Other expectational values for the two independent variables (Q and w/c) are: Almon polynomial lag structure, ARIMA-models and forward-looking expectations. The construction of the ARIMA-models is presented in Appendix VI and the construction of forward-looking models in note 2 of this chapter.

An attempt is made here to capture the basic idea of a forward-looking model, where the target capital stock depends upon the power series of the forward differences of the desired capital stock (see chapter V), by a linear specification for K* and by constructing directly forward-looking expectational values for the levels of both the output (Q) and the price variables. For example, Q_F is effectively a weighted average future output and the future values have been generated by an ARIMA-model for Q in which the weights are assumed to decline geometrically (see note 2 and Appendix VI).

In the light of the estimation results and test statistics (DW, D-m and LM(4)), it seems fairly evident that the residuals of the w/c_1-type investment equations are serially correlated and that the autocorrelation might even be of a higher order than one. It should, however, be emphasized that, although a low value of DW (or the values of other autocorrelation tests) might indicate a serially correlated disturbance term, it could also be symptomatic of some other type of misspecification (see, also, section (7.1). In particular, it is possible that positive serial correlation can be expected when the functional form is inappropriate. A similar pattern may also emerge when a variable has been omitted (see Harvey 1982, p. 155).

We have also made an attempt to model the OLS-residuals by the ARMA-procedure (see table A32). The estimation results of table A32 indicate that there is not much to choose between the pure AR-processes and the ARMA-processes. The pure MA-processes are clearly inferior to the

AR- and ARMA-processes (the MA-results are not reported in table A32). Table A27 reports estimated investment equations based on the AR(2) -assumption (eqs. 2, 11 and 12). The results indicate that the coefficient estimates of the relative price variable (w/c) become insignificant.

The different expectational values of independent variables can be viewed as an effort to try to check the effect of alternative independent variables on the performance of w/c-type equations. Alternative functional specifications (log-linear) are presented in Table A28 with $\Delta \log K$ and I/K_{-1} as dependent variables. An attempt has been made to deal with the case of omitted variables as a cause of serial correlation by adding a cash flow variable to the equations. Estimation results with proxy variables for credit rationing are presented in table A35 and the results with the technical change included are shown in table A36 (see, also, the discussion in section 7.5).

The results of the alternative functional specifications, as well as the expectational values of independent variables and omitted variables, clearly indicate the rather persistent presence of serial correlation in the equations including w/c_1 as a price variable. In some cases, however, it is of a somewhat 'milder' type than in the original equations.

In the light of these results, it seems quite legitimate to ask whether the dynamic specification of the investment equations could be improved by accepting first-order serial correlation as an integral part of the model. Table 26 contains the results of some likelihood-ratio tests of alternative dynamic specifications. The alternative hypothesis is an equation which includes additional lagged values of variables. The null hypothesis is either the original equation or a constrained form of the alternative hypothesis corresponding to the case of first-order serial correlation in the disturbance term. The constrained form is estimated by a non-linear estimation method (Mindis).

The test results of table 26 indicate that the null hypotheses are rejected. Hence, the first-order autocorrelation correction does not seem to be an appropriate way of modelling the dynamic structure of the w/c-type investment equations; however, the estimation results obtained using the CO- and HAT-methods are reported in table 25.

TABLE 26. Likelihood-Ratio (LR-) Tests of Alternative Hypotheses with Respect to the Dynamic Specification of Quarterly Investment Equations for the Aggregate Sector (w/c_1, w/c_2, w/p and c_1/p as price variables)

Alternative hypothesis (H_1):	'free lag specification'			
Null hypothesis (H_0):	LR	χ^2_p (p=)	Critical values of χ^2_p 1 %	5 %
1. Equation 3/Table 25				
- original form	30.00	4	13.27	9.49
- constrained form (CO)	16.41	3	11.35	7.82
2. Equation 2/Table 25				
- original form	30.80	5	15.09	11.07
- constrained form (CO)	20.40	4	13.27	9.49
3. Equation 10/Table A27				
- original form	50.00	4	13.27	9.49
- constrained form (CO)	14.80	3	11.35	7.82
4. Equation 2/Table 27				
- original form	46.60	5	15.09	11.07
- constrained form (CO)	16.60	4	13.27	9.49
5. Equation 3/Table 27				
- original form	48.00	6	16.81	12.59
- constrained form (CO)	18.00	5	15.09	11.07

Notes: $LR = -2(LLF_0 - LLF_1)$, where LLF_0 and LLF_1 are the logs of the likelihood function obtained under the null and alternative hypothesis, respectively. In case 1 (eq. 3/Table 25) the 'free lag specification is:

$$I_t = a_0 + a_1 Q^e_t + a_2 Q^e_{t-1} + a_3 \left(\frac{w}{c_1}\right)^e + a_4 \left(\frac{w}{c_1}\right)^e_{t-1} + a_5 K_{t-1} + a_6 K_{t-2} + a_7 I_{t-1}$$

The original form is obtained when $a_2 = a_4 = a_6 = a_7 = 0$ and the constrained form is obtained by using parameter restrictions: $a_2 = -a_1 a_7$, $a_4 = -a_3 a_7$, $a_6 = -a_5 a_7$. The constrained form is equivalent to the Cochran - Orcutt (CO) correction.

The estimation results of the linear equations with the real wage rate (w/p) and the real user cost (c_1/p) as separate price variables are reported in table 27 and in the Appendices, tables A29 - A30. Likelihood-ratio tests of alternative dynamic specifications are presented in table 26 (cases 4 and 5). The estimation and test results obtained for equations of the (w/p,c_1/p)-type display fairly similar features to those of the (w/c_1)-type equations with serially correlated residuals, but without it being possible to accept the constrained form as an appropriate dynamic specification. However, the estimation results obtained by applying the CO- and HAT-methods seem to perform better than in the case of the w/c-models (see table 27).

On the basis of the above results, it can be concluded that the linear equations which are based on the assumption of the c_1-variant of the user cost are not adequate to explain the behaviour of investment in the aggregate sector in the period under consideration (the stability tests also support this conclusion, see table 30). However, some support has been found for the importance of both the expected output and expected factor prices for the investment decisions, but the general structure of the equation needs to be improved.

There are different ways in which an improvement might be achieved. First, by accepting the hypothesis that the disturbances are truly correlated, one could try to model higher-order autocorrelation and use the generalized Durbin-method for estimation purposes. Second, COMFAC-analysis could be used to model the relationship between systematic dynamics and autocorrelated disturbances. Third, we could try to change the model specification with respect to, for example, the form of the c-variable. The first and third avenues are chosen here. Table A32 reports some ARMA-estimation results for the OLS residuals (w/p, c_1/p equations). As in the case of the (w/c) -equations the pure MA-processes were inferior to other specifications. An AR(2) process or even some mixed process (ARMA) seems to perform better than the pure AR(1) process. Table A29 reports an estimated investment equation based on the AR(2) -assumption (eq. 4). It can be seen that the model performs quite well except that the coefficient of the user cost variable is not significant. Hence some problems seem to remain with the c_1-variant of the user cost.

TABLE 27. Quarterly Estimation Results of Linear Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15 and 5.17ii; w/p and c_1/p as price variables)

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	2.509	1.681	2.201	1.676	1.867	0.457	0.338	0.476
	(4.54)	(4.29)	(4.22)	(4.26)	(0.21)	(1.67)	(1.02)	(1.70)
Q^e	0.434	0.646	0.292	0.620	0.283	0.555	0.220	0.560
	(6.30)	(11.12)	(3.83)	(7.52)	(2.58)	(7.20)	(2.64)	(5.84)
$(w/p)^e$	0.259	0.162	0.224	0.162	0.159	0.182	0.282	0.182
	(2.57)	(2.18)	(2.38)	(2.16)	(2.33)	(2.34)	(3.18)	(2.32)
$(c_1/p)^e$	-0.221	-0.131	-0.222	-0.134	0.091	-0.040	-0.038	-0.042
	(4.29)	(3.62)	(4.64)	(3.60)	(1.94)	(0.86)	(0.67)	(0.90)
CF^e			0.310	0.038			0.159	0.130
			(3.43)	(1.44)			(1.94)	(0.78)
K_{-1}	-0.061	-0.073	-0.038	-0.069	-0.041	-0.068	-0.043	-0.068
	(5.48)	(8.78)	(3.08)	(6.20)	(2.58)	(6.33)	(3.25)	(5.50)
R^2	0.791	0.878	0.812	0.876	0.916	0.711	0.603	0.710
SEE	309.3	231.5	286.9	232.8	188.8	203.3	236.7	205.2
DW	0.86	1.06	1.06	1.08	2.41	1.88	1.88	1.87
D-m	0.61	0.48	0.51	0.46	-0.36	0.02	0.04	0.05
	(5.43)	(4.01)	(3.93)	(3.62)	(2.99)	(0.08)	(0.21)	(0.26)
d_4	1.29	1.72	1.35					
RHO					0.86	0.53	0.63	0.52
					(14.54)			
LM(4)	35.50	17.64	26.50	17.21	16.21	20.95	43.08	20.26
e_1	0.244	0.389	0.257	0.382	-0.400			
	(1.87)	(2.88)	(1.91)	(2.79)	(2.96)			
e_2	0.391	0.129	0.323	0.136	-0.125			
	(3.00)	(0.90)	(2.37)	(0.94)	(0.88)			
e_3	0.285	0.223	0.312	0.232	0.215			
	(1.96)	(1.45)	(2.14)	(1.48)	(1.49)			
e_4	0.132	-0.145	0.173	-0.140	0.025			
	(0.96)	(1.04)	(1.23)	(0.59)	(0.18)			
EST	OLS	OLS	OLS	OLS	CO	HAT	HAT	HAT
Definitions of Variables	Q_1	Q_4	Q_1	Q_4	Q_4	Q_4	Q_1	Q_4
	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$
	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$
			CF_4^n	CF_4^n			CF_4^n	CF_4^n

Notes: See tables 24 and 25.

Table A29 presents estimation results based on more complicated expectations hypotheses for the exogenous variables (eqs. 5-10). In the light of the general performance (R^2, SEE, DW etc.) of these equations, only the Almon lag specification (eq. 5) seems to do slightly better than the previous models (table 27). However, the coefficient of the user cost is also insignificant in the Almon-case. Table A30 reports some experiments with other proxies for the demand variable (Z). These proxies are the same as those used in the annual equations (see table A19). The equations which are based on the domestic demand variables (eqs. 7 and 8) perform equally well as the equations based on the own output variables (table 27). These equations are also free of serial correlation. A similar result was also obtained with annual data (see table A19). The coefficient estimate of the wage rate variable is not, however, significant in these equations.

The basic advantage of the c_1-variant of the user cost over the c_2-variant is that the negative value problem is not present (see section 6.2). Some other problems exist with the c_1-variable, however. Although in principle the effects of all lagged variables on the expectations of relevant future factors affecting the cost of capital should be captured by Tobin's "q"-variable, this may not happen in practice because of, for example, capital market imperfections (see, also, Appendix II). A second reason for the problems with c_1 may stem from the fact that the calculation of the real cost of financial capital is based on the estimates of the "q"-variable, which are probably very crude measures in the Finnish case, where the stock market plays a rather minor role and various approximations have to be made in order to obtain the "q"-values. A third possible reason for the problems with c_1 is that its calculation is based on average "q"-values (the market value of the firm divided by the replacement cost of its total capital) rather than marginal "q"-values (the valuation of an additional unit of capital relative to the cost of this capital). It is the marginal "q" which is the relevant determinant of investment (see e.g. Abel 1981). In contrast, the c_2-variable avoids some of these problems since its calculation is based on explicit modelling of expectations with respect to its determinants, such as the rate of inflation, and because marginal (or average) investment incentives are used (see Appendices I and IV).

TABLE 28. Quarterly Estimation Results of Linear Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15 and 5.17ii; w/p and c_2/p as price variables)

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	1.899	1.612	1.477	1.455	1.872	1.508	1.677	0.928	0.940
	(9.59)	(7.89)	(7.82)	(7.77)	(6.20)	(5.67)	(6.03)	(5.65)	(5.70)
Q^e	0.240	0.147	0.417	0.325	0.213	0.405	0.151	0.411	0.334
	(5.27)	(2.87)	(7.44)	(4.05)	(3.52)	(5.81)	(2.44)	(5.86)	(3.64)
$(w/p)^e$	0.319	0.296	0.253	0.255	0.309	0.240	0.301	0.231	0.238
	(4.55)	(4.48)	(4.04)	(4.11)	(4.23)	(3.55)	(4.29)	(3.28)	(3.40)
$(c_2/p)^e$	-0.070	-0.067	-0.050	-0.054	-0.067	-0.049	-0.066	-0.049	-0.053
	(10.43)	(10.59)	(7.23)	(7.34)	(6.47)	(5.16)	(7.29)	(5.16)	(5.41)
CF^e		0.211		0.120			0.160		0.099
		(3.30)		(1.60)			(2.47)		(1.30)
K_{-1}	-0.063	-0.047	-0.070	-0.062	-0.058	-0.067	-0.050	-0.067	-0.061
	(8.16)	(5.46)	(10.39)	(7.39)	(5.92)	(7.80)	(4.91)	(7.57)	(6.14)
R^2	0.893	0.911	0.918	0.920	0.910	0.927	0.916	0.820	0.831
SEE	215.6	201.2	189.6	187.5	195.5	176.2	188.8	177.8	177.0
DW	1.14	1.29	1.22	1.25	2.18	2.05	2.11	2.05	2.03
D-m	0.43	0.36	0.39	0.37	-0.12	-0.05	-0.087	-0.34	-0.33
	(3.76)	(2.93)	(3.30)	(3.08)	(0.93)	(0.43)	(0.67)	(1.01)	(0.93)
d_4	2.04	1.87	2.00	2.06					
RHO					0.45	0.38	0.38	0.43	0.37
					(4.23)	(3.54)	(3.46)		
LM(4)	17.28	11.30	13.82	12.88	5.70	3.24	4.02	5.83	7.20
e_1	0.311	0.281	0.344	0.323	-0.146	-0.060	-0.108		
	(2.35)	(2.08)	(2.63)	(2.43)	(1.10)	(0.42)	(0.79)		
e_2	0.246	0.185	0.120	0.137	0.194	0.104	0.173		
	(1.75)	(1.30)	(0.86)	(0.97)	(1.46)	(0.77)	(1.27)		
e_3	0.096	0.077	0.146	0.123	0.201	0.184	0.159		
	(0.66)	(0.52)	(1.01)	(0.84)	(1.43)	(1.31)	(1.11)		
e_4	-0.043	-0.073	-0.174	-0.186	0.064	-0.051	0.015		
	(0.31)	(0.53)	(1.31)	(1.38)	(0.46)	(0.37)	(0.11)		
EST	OLS	OLS	OLS	OLS	CO	CO	CO	HAT	HAT
Definitions of Variables	Q_1	Q_1	Q_4	Q_4	Q_1	Q_4	Q_1	Q_4	Q_4
	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$
	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$
		CF_4^n		CF_4^g					CF_4^g

Notes: See table 24.

The last form of the price variables used includes the real wage rate and the second variant of the real user cost (c_2/p). The estimation results of linear equations with c_2/p as the user cost variable are presented in table 28 and in the Appendices, see tables A28, A31, A33, A34. The estimation results and test statistics clearly indicate that this form of investment equation is superior to the ones presented above. Standard errors of regression are about 10 - 20 per cent lower and t-statistics of coefficient estimates clearly higher than in the previous models with the quarterly data. Tests of serial correlation in the residuals indicate that autocorrelation is of a rather 'mild' form, usually of first-order, and that no seasonal element seems to be present in the residuals. There seems to be some sensitivity in the coefficient estimates of the output variable with respect to alternative expectations hypotheses but the coefficient estimates of the wage rate and the user cost are rather insensitive with respect to the competing hypotheses. The cash flow variable is also significant in most regressions. Its coefficient obtains its highest t-value with a one year lag, although it also has a positive sign with shorter lags (not reported here).

The likelihood-ratio tests reported in table 29 indicate that the null hypothesis of a constrained dynamic specification stemming from the first-order autocorrelation in the residuals is not rejected. Estimation results also show that the CO- or HAT-methods increase the efficiency of the coefficient estimates (t-statistics increase). The standard errors of regression (SEE) are about 6 per cent of the mean value of the dependent variable in the best equations, thus indicating that these equations have a good tracking ability. Hence, the investment equation containing the real wage rate and the second variant of the real user cost as price variables would seem to be our preferred equation among the linear specifications in the quarterly data and when output is used as a proxy for the demand index.

Table A28 reports the estimation results of log-linear investment equations with the 'corrected' c_2-variant of the user cost. In addition to using output as a proxy for expected demand (eqs. 9-11), we also experimented with aggregate domestic demand variables (eqs. 13-15) as in the case of the (w/p, c_1/p) -equations. In terms of general performance (R^2, SEE), these equations do very well and in the case of the domestic demand variables the OLS-residuals are clearly white noise (in the light of DW, D-m and LM(4) test statistics). All the coefficient estimates are also highly significant. The (w/p, c_2/p) -investment equations are also somewhat better than the corresponding (w/p, c_1/p) -equations (in terms of R^2, SEE), which contain serially correlated residuals.

Alternative demand proxies were also tried for the basic linear equations of the (w/p, c_2/p) -form (see table A31). Both foreign and domestic demand proxies were used. The estimation results are very similar to those based on the lagged output variables, and the coefficient estimates of the price and cash flow variables are not sensitive to the choice of the demand proxy. The proxies measuring domestic demand pressures again perform best, but measured in terms of R^2 the difference is very small as compared to the equations with output variables.

Table A34 reports estimation results of linear (w/p, c_2/p) -equations based on various expectations hypotheses with respect to the 'exogenous' variables (ARIMA-models, Almon-lags and forward-looking expectations). The coefficient estimates of the price and cash flow variables again seem to be very insensitive to the choice of the expectations hypotheses but the parameter of the accelerator variable varies considerably and is even insignificant in some cases. The best fit is again obtained with Almon-lags for the output variable. Table A33 presents estimation results with different measures of the c_2-variable (see, also, table A34 where encompassing tests are presented for alternative c_2-variables). The estimation results in table A33 are generally very insensitive to the choice of the c_2-variable. In sum, the foregoing experiments with the (w/p, c_2/p) -equations indicate a rather surprising amount of invariance with respect to various specifications (functional form, demand proxies and c_2 -formulas). The results also indicate that the investment equations based on the c_2-variable perform generally somewhat better than equations based on the c_1-variable.

TABLE 29. Likelihood-Ratio (LR-) Tests of Alternative Hypotheses with Respect to the Dynamic Specification of Quarterly Investment Equations for the Aggregate Sector (w/p, c_2/p as price variables)

Alternative hypothesis (H_1):	'free lag specification'			
Null hypothesis (H_0):	LR	χ^2_p (p=)	Critical values of χ^2_p 1 %	5 %
1. Equation 1/Table 28				
- original form	33.60	5	15.09	11.07
- constrained form (CO)	15.50	4	13.27	9.49
2. Equation 3/Table 28				
- original form	20.10	5	15.09	11.07
- constrained form (CO)	9.00	4	13.27	9.49
3. Equation 2/Table 28				
- original form	30.80	6	16.81	12.59
- constrained form (CO)	8.20	5	15.09	11.07
4. Equation 4 with CF^n_4/Table 28				
- original form	19.00	6	16.81	12.59
- constrained form (CO)	6.10	5	15.09	11.07
5. Equation 10/Table 33				
- original form	16.00	5	15.09	11.07
- constrained form (CO)	8.60	4	13.27	9.49

Notes: See table 26. If the LR-statistic exceeds the corresponding critical value of χ^2_p, then the null hypothesis is rejected; otherwise the null is not rejected. The constrained form corresponds to the Cochran - Orcutt first-order autocorrelation estimation of the original equation (non-linear estimation is performed by applying the Mindis-method as in table 32).

The formal stability test results reported in table 30 also support the above conclusion (see the discussion on stability tests in section 6.2.2). The investment equations with w/p and c_2/p as the price variables almost universally pass the stability tests and hence the null hypothesis of parameter constancy is not rejected. In investment models with other price variables, there seem to be instability problems, although some of the equations with w/p and c_1/p as the price variables also perform quite well.

The reliability (power) of stability tests usually requires that the residuals of the regressions should be serially uncorrelated and homoscedastic (especially the Chow test and other homogeneity tests). Corsi, Pollock and Prakken (1982) have recently presented some evidence concerning the appropriateness of the Chow test in the presence of first-order serially correlated residuals. Their results suggest that the Chow test is biased towards rejecting the hypothesis of stability in the regression coefficients.

Toyoda (1974), Jaytissa (1977) and Schmidt - Sickles (1977) have investigated the accuracy of the Chow test under conditions of heteroscedasticity. They have found that heteroscedasticity in residuals significantly affects the 'true' level of significance of the Chow test. Because of this, we have calculated White-tests for the equations presented in table 28. These tests show that heteroscedasticity does not seem to be a significant cause of bias in the stability tests. On the other hand, the autocorrelation tests (DW, D-m and LM(4)) show that serial correlation of residuals is fairly mild in the $(w/p,c_2/p)$-investment models. QUSUMSQ-tests and plots are also regarded as general tests against heteroscedasticity of residuals and they also support the conclusion made here (graphical plots are not shown here).

We also carried out informal comparisons of the graphs of the parameters obtained in the F-H and homogeneity tests and of the plots of the CUSUM and CUSUMSQ residuals and these confirmed our conclusions fairly well. Quandt's log-likelihood ratio statistic was also examined. In the case of equations where the price variable is either w/c_1 or $(w/p,c_1/p)$, there seems to be a clear turning point around the years 1973 - 75, the time of the first 'oil crisis'. In the equations with w/p and c_2/p as the price variables, a turning point is also present, but it is not so evident as in the preceding cases. Hence, it can be concluded that the mid-1970s seem to be a period in which a change in the coefficients or in the structure of the whole investment regression might have taken place. This result confirms the problem encountered above in connection with modelling expectational values for the rate of inflation. It might also be argued that the 'oil crisis' caused an abrupt increase in the rate of deterioration and obsolescence of the existing capital stock. This possibility is examined in section 7.5.

Table 30. Stability Tests of Quarterly Linear Investment Equations for the Aggregate Sector

Table No./ Eq. No.	Price variable	F-H-1°	F-H-2°	C_B	C_F	C_B^2	C_F^2	H_{24}	H_{36}	DW	D-m	W/χ_p^2
25/1	w/c_1	F(4,64)=10.06	F(4.60)=8.90	1.279	1.056	0.276	0.308	F(8,60) =14.53	F(4,64)=1.26	1.03	0.51	8.90/12.59
25/3	w/c_1	F(4,64)= 3.10	F(4,60)=1.49	1.153	0.766	0.216	0.190	F(8,60) = 5.94	F(4,64)=1.73	1.21	0.38	4.52/12.59
25/4	w/c_1	F(5,62)= 3.47	F(5,57)=1.23	1.105	0.786	0.214	0.201	F(10,57)= 4.83	F(5,62)=1.52	1.24	0.37	4.11/18.31
A27/9	w/c_2	F(4,64)=23.55	F(4,60)=6.74	1.923	0.951	0.273	0.468	F(8,60) =21.35	F(4,64)=4.87	0.73	0.49	22.92/12.59
A28/1	w/c_1	F(4,64)= 1.11	F(4,60)=1.43	1.025	0.900	0.210	0.166	F(8,60) = 4.30	F(4,64)=0.95	1.15	0.42	4.23/12.59
A28/3	w/c_1	F(4,64)= 1.79	F(4,60)=1.58	1.115	0.858	0.221	0.172	F(8,60) = 5.16	F(4,64)=1.38	1.11	0.42	3.55/12.59
27/1	w/p,c_1/p	F(5,62)=14.25	F(5,57)=8.41	1.197	0.793	0.252	0.407	F(10,57)=21.40	F(5,62)=3.83	0.86	0.61	8.32/18.31
27/3	w/p,c_1/p	F(6,60)=14.43	F(6,54)=5.44	1.408	0.778	0.210	0.424	F(12,54)=15.50	F(6,60)=3.78	1.06	0.51	10.06/25.00
A28/6	w/p,c_1/p	F(5,62)= 2.26	F(5,57)=3.23	0.864	0.840	0.198	0.159	F(10,57)= 6.38	F(5,62)=2.17	1.08	0.46	7.63/18.31
A28/7	w/p,c_1/p	F(5,62)= 2.50	F(5,57)=3.49	0.953	0.811	0.209	0.164	F(10,57)= 6.99	F(5,62)=2.35	1.05	0.47	7.00/18.31
28/1	w/p,c_2/p	F(5,62)= 2.59	F(5,57)=3.06	0.840	0.502	0.125	0.249	F(10,57)= 3.22	F(5,62)=4.93	1.14	0.43	16.98/18.31
28/3	w/p,c_2/p	F(5,62)= 1.30	F(5,57)=1.29	1.107	0.696	0.152	0.176	F(10,57)= 1.86	F(5,62)=1.93	1.22	0.39	14.61/18.31
28/2	w/p,c_2/p	F(6,60)= 2.61	F(6,54)=2.53	0.966	0.508	0.134	0.221	F(12,54)= 2.70	F(6,60)=3.05	1.29	0.36	19.86/25.00
A33/4	w/p,c_2/p	F(5,62)= 1.33	F(5,57)=2.48	1.140	0.711	0.183	0.189	F(10,57)= 1.35	F(5,62)=1.42	1.36	0.32	14.12/18.31
A33/8	w/p,c_2/p	F(6,60)= 1.86	F(6,54)=2.51	0.956	0.445	0.164	0.198	F(12,54)= 2.01	F(6,60)=2.15	1.43	0.29	17.81/25.00
A34/6	w/p,c_2/p	F(6,60)= 2.60	F(6,54)=3.02	0.568	0.651	0.151	0.230	F(12,54)= 1.55	F(6,60)=2.23	1.33		

Notes: Critical values of test statistics:

	F(4,64)	F(5,62)	F(6,60)	$C_{B,F}$	C_{32}^2	C_{33}^2	C_{34}^2
5 %	2.53	2.37	2.25	0.948	0.192	0.189	0.187
1 %	3.65	3.34	3.12	1.143	0.242	0.238	0.235

F(4,60), F(5,57) and F(6,54) are approximately equal to F(4,64), F(5,62) and F(6,60), respectively. Equations with (4,64) degrees of freedom for F-H-1° correspond to C_{34}^2 etc.

F-H-1°: Farley - Hinich test with first degree time-trending regressions (see table 10)
F-H-2°: Farley - Hinich test with second degree time-trending regressions
C_B, C_F: CUSUM tests (backward, forward)

C_B^2, C_F^2: CUSUM SQUARES tests (backward, forward)

H : homogeneity statistics for moving regressions (the lengths of the time segments are 24 and 36 quarters and hence H_{36} is the Chow-test)

W : White test for heteroscedasticity (see White, 1980), χ_p^2 gives the 5 per cent critical value of this test statistic with p ranging from 6 to 21 ($p = \frac{k(k+1)}{2}$, k = number of independent variable)

Summarizing the estimation and misspecification (diagnostic checking) test results for the linear quarterly investment equations, it can be said that, although we are tempted to draw the conclusion that the serial correlation of residuals is not a serious problem in the $(w/p, c_2/p)$-type models and that the first-order autocorrelation correction seems to be a legitimate method to apply when it is needed, it should nevertheless be emphasized that there may exist many different reasons for the 'mild' autocorrelation in some specifications. The LR-test results for alternative dynamic specifications indicate that even the 'best' equations might require the presence of lagged investment and/or an additional lagged capital stock in order to be 'completely' satisfactory. The following theoretical, methodological and data considerations might provide explanations for the inclusion of the additional lagged variables in the investment regression.

i) Linear approximation may not perhaps be the 'best' functional form and hence an attempt should be made to fit non-linear models to the data.

ii) The rate of adjustment is not constant and hence models with the endogenous (variable) speed of adjustment might work better, as theoretical considerations also suggest (see chapters III and IV).

iii) The proxies used for the exogenous demand variable might be inappropriate, although the endogeneity problem with the output variable does not seem to be severe in the light of the TSLS-estimation results.

iv) The replacement ratio might not be constant as some of the stability test and plot analyses could be interpreted as implying. If the replacement ratio (depreciation coefficient) is variable over time and possibly subject to rational economic decision-making, then lagged values of investment or the capital stock would show up in the investment regression in order to capture 'echo' effects of past capital stocks (see Nickell, 1978, p. 274, Feldstein and Rotschild, 1974 and Schiantarelli, 1983).

v) The putty-putty assumption of the production technology could be inappropriate. The putty-clay hypothesis, on the other hand, usually leads to a specification in which the reduced-form investment equation contains lagged values of investment among the independent variables (see, e.g., King 1972, Sarantis 1979;see also section 5.3.2).

vi) The dynamic structure of the model could be misspecified. King (1972) and Sarantis (1979), for example, specify a partial adjustment mechanism in terms of investment, i.e. $\log I_t = \lambda \log I_t^* + (1-\lambda)\log I_{t-1}$, whereas we have used a similar model in terms of the capital stock.

vii) The simultaneity of investment and financing decisions, the endogeneity of the cost of capital and the specification of credit rationing effects might all be causes of the serial correlation of residuals (see section 5.4).

viii) It has been assumed that the output price and labour input are more or less freely adjustable. This can be justified only on the grounds that this is true relative to the costs of capital adjustment. Nevertheless, it is possible that the 'true' relationship is between the demands for various factors of production (capital, labour, energy etc.) and hence an interrelated factor demand approach should be used.

ix) The question of the aggregation of variables (output, capital and labour input) and micro relationships is always a severe problem in empirical analysis at industry or higher levels of aggregation.

x) Data inaccuracies can always be a cause of serial correlation (errors in variables). In the case of quarterly data, which is based partly on interpolation from annual observations and where seasonal adjustment might be inaccurate, autocorrelation is more common than expected.

It is not possible here to discuss at a greater length all the points presented above nor to carry out further analysis with respect to all the questions raised and we shall therefore confine our analysis mainly to the first four issues listed above. However, when evaluating the results of the quarterly analysis, it is worth keeping all these points in mind.

7.3 Estimation Results of Non-Linear Quarterly Investment Equations for the Aggregate Sector

Estimation of equations which are inherently non-linear in their coefficients is carried out in two steps. Table 31 gives estimation results of equations excluding the cash flow variable. Hence the rate of adjustment is constant in these models. Tables 32 and 33 report some experiments with equations including an endogenous (variable) rate of adjustment to take account of cash flow considerations (see section 5.4).

The general methodology of non-linear estimation was discussed in section 6.3. One problem which bedevils econometric work with non-linear equations is that the objective function is often relatively 'flat' around the optimum. Such problems are normally encountered when the objective function depends on more than two parameters (see Harvey 1982, pp. 124-125). Given the multicollinearity of economic time series and the small samples available, flat objective functions are quite common. In order to try to guarantee that a global solution has actually been obtained we have used three methods (Mindis, Marquardt and Shazam). Mindis is a minimum distance estimation method (see Berndt, Hall, Hall and Hausman 1974/RAL-program), Marquardt is based on the same principle as the quadratic hill-climbing method (see Marquardt 1963, Harvey 1982/RAL-program) and the Shazam program includes a quasi-Newton estimation method. Estimation results are reported here only for the Mindis-method since, after a large amount of computational work, we were able to derive very similar results with the other two methods. A large set of initial values was used in order to guarantee a global estimation result (see below).

Both constant and variable speed of adjustment equations were estimated for the two basic forms of investment models (eqs. 5.21i and 5.21ii) and for some alternative expectational values. In the case of equation (5.21ii), both output and other proxies are used for the exogenous demand index (see section 5.5.2). All non-linear equations are applied for both variants of the user cost.

TABLE 31. Quarterly Estimation Results of Non-Linear Investment Equations for the Aggregate Sector (eqs. 5.21i, and 5.21ii)

(5.21i) $$I_t = \alpha_0' \, [Q_t^e]^{\alpha_1} [(\tfrac{w}{c})_t^e]^{\alpha_2} + a_3 \, K_{t-1}$$

(5.21ii) $$I_t = \beta_0' \, (Z_t^e)^{\beta_1} (w_t^e)^{\beta_2} (c_t^e)^{\beta_3} + b_4 \, K_{t-1}$$

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	5.641	2.642	6.167	28.567	0.738	8.455	45.630	108.727
	(3.25)	(4.62)	(1.67)	(1.90)	(0.96)	(1.26)	(1.46)	(2.43)
Q^e	0.791	0.856	0.778	0.522				
	(24.21)	(33.06)	(9.03)	(7.04)				
Z_1^e					0.926	0.601		
					(7.23)	(6.21)		
Z_2^e							0.473	0.326
							(4.84)	(5.82)
(w/c_1)	0.193							
	(5.18)							
(w/c_2)		0.020						
		(4.40)						
(w/p)			0.207	0.387	0.209	0.400	0.554	0.581
			(2.26)	(5.04)	(1.83)	(4.72)	(5.72)	(9.77)
(c_1/p)			-0.189		-0.173		-0.245	
			(4.23)		(3.27)		(3.51)	
(c_2/p)				-0.021		-0.023		-0.031
				(6.20)		(9.42)		(8.81)
K_{-1}	-0.070	-0.054	-0.071	-0.075	-0.064	-0.071	-0.068	-0.077
	(12.19)	(10.19)	(8.84)	(10.34)	(7.20)	(9.42)	(5.90)	(9.62)
R^2	0.891	0.895	0.890	0.915	0.868	0.908	0.782	0.896
SEE	220.0	218.7	221.6	198.9	247.3	206.6	317.8	219.6
DW	1.20	1.05	1.19	1.17	1.66	1.76	0.85	1.46
LLF	-488.5	-488.0	-488.4	-480.6	-496.3	-483.4	-514.4	-487.8

Notes: See table 19. The coefficient estimates of Q, Z, w/c, w/p and c/p are the long-run elasticities of the desired capital stock with respect to these variables. LLF is the value of the log likelihood function. Estimation is based on the Mindis-method (see text). $Q^e = Q_{t-4}$, $Z_1^e = CI_{-1}$ and $Z_2^e = S_{-4}$ (see also table A30), c_2 is the variant which is positive for all t.

LR-test: χ_1^2

eqs. (1) & (3) 0.2
eqs. (2) & (4) 14.8

The estimation results presented in table 31 for the constant speed of adjustment equations with c_1 as the user cost show great similarities to the corresponding annual models of the aggregate sector (see table 19). In terms of goodness of fit of the regression, the non-linear equations are very similar to the linear quarterly equations, thus indicating that the linear approximation works quite well with both user cost variables. The coefficient estimate of the relative price variable (w/c) is much higher with c_1 than with c_2. A similar result was also obtained with linear equations. The residuals of the non-linear (w/c)-equations are also serially correlated. The equations with w/p and c_2/p as separate price variables are clearly better than the (w/c) - or (w/p, c_1/p)-equations. Hence, this result is in accordance with the linear equations. The lowest SEE is obtained with lagged output (Q_{-4}) as an accelerator variable, although the coefficient estimates of the wage rate and the user cost are not very sensitive to the choice of the demand proxy. LR-tests indicate that the null hypothesis of the equality of the wage rate and user cost coefficients is not rejected in the case of the c_1-variable but the null is clearly rejected in the case of the second variant of the user cost (see table 31, notes). The estimation results with linear investment equations also suport this conclusion. It should also be noted that a random residual is obtained when the domestic demand variable (CI) is used as a proxy for the demand effects. The estimation results with Q_{-4} and CI_{-1} variables are, however, very similar.

Tables 32 and 33 report our experiments with non-linear equations in which the rate of adjustment is endogenous because of cash flow and K* effects. Because of the rather complex nonlinearity, there were more estimation problems in this case than in the preceding case with the constant speed of adjustment. Hence, a large number of iterations with respect not only to functional forms (equations of K*), measures of cash flow (net, gross), and lags in all variables (Q, w/c, w/p, c/p and CF) but also to methods (initial values and algorithms) were employed before the results shown in tables 32 and 33 were selected.

TABLE 32. Quarterly Estimation Results of Non-Linear Cash Flow Investment Equations for the Aggregate Sector (eq. 5.11, variable speed of adjustment model)

(5.11) $$I_t = [\lambda_0 + (1-\lambda_0)(1-e^{-\lambda_1 CF_t^{gap}})](K_t^* - K_{t-1}) + \delta K_{t-1}$$

$$CF^{gap} = \frac{CF^{net}}{K^* - K_{-1}}, \quad \text{or} \quad \frac{CF^{gross}}{K^* - (1-\delta)K_{-1}}$$

(5.18) $$K_t^* = \alpha_0 \, [Q_t^e]^{\alpha_1} [(\tfrac{w}{c})_t^e]^{\alpha_2}$$

Parameters and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)
λ						0.069	0.058
						(9.85)	(7.41)
λ_0	0.049	0.044	0.011	0.074	0.015		
	(5.65)	(4.17)	(1.90)	(6.94)	(2.59)		
λ_1	0.319	0.265	0.330	0.075	0.362		
	(3.47)	(3.89)	(6.14)	(6.78)	(6.01)		
λ_0	555.24	519.50	44.79	83.92	30.34	284.42	444.83
	(1.61)	(1.91)	(1.47)	(2.72)	(1.27)	(2.37)	(1.95)
λ_1	0.553	0.560	0.800	0.761	0.844	0.629	0.504
	(8.18)	(9.87)	(11.17)	(18.96)	(10.10)	(13.84)	(6.70)
λ_2	0.513	0.479	0.031	0.217	0.021	0.367	0.445
	(5.86)	(7.00)	(5.16)	(4.28)	(2.22)	(7.23)	(6.84)
λ_3							0.095
							(2.28)
R^2	0.672	0.669	0.569	0.766	0.578	0.610	0.648
SEE	260.5	263.2	301.3	220.0	298.9	282.0	269.7
DW	1.19	1.20	0.79	1.23	0.84	1.03	1.12
LLF	-500.1	-500.8	-510.6	-487.9	-510.0	-506.3	-502.6
ε_Q	0.553	0.560	0.800	0.761	0.844	0.629	0.504
$\varepsilon_{w/c}$	0.513	0.479	0.031	0.217	0.021	0.367	0.445
Definitions of Variables	Q_1	Q_1	Q_1	Q_4	Q_4	Q_1	Q_1
	w/c_1	w/c_1	w/c_2	w/c_1	w/c_2	w/c_1	w/c_1
	CF_4^n	CF_4^g	CF_4^g	CF_4^g	CF_4^g		CF_4^g

Notes: See table 24. LLF = log-likelihood function, SEE = standard error of the regression, ε_Q and $\varepsilon_{w/c}$ are elasticities of demand for capital with respect to output and relative prices, respectively. Estimation is based on the Mindis-method but other methods (programmes) have also been used (Marquardt, Shazam) and δ = 0.019 in all equations.

Eq. (6): $$I^{net} = \lambda \, [\alpha_0 Q^{\alpha_1} (w/c)^{\alpha_2} - K_{-1}]$$

Eq. (7): $$I^{net} = \lambda \, [\alpha_0 Q^{\alpha_1} (w/c)^{\alpha_2} CF^{\alpha_3} - K_{-1}]$$

TABLE 33. Quarterly Estimation Results of Non-Linear Cash Flow Investment Equations for the Aggregate Sector (see table 32, investment equation 5.11)

(5.18ii) $K_t^* = \beta_0 (Z_t^e)^{\beta_1} [(\frac{w}{p})_t^e]^{\beta_2} [(\frac{c}{p})_t^e]^{\beta_3}$

Parameters and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
λ							0.069	0.058
							(6.79)	(4.96)
λ_0	0.022	0.100	0.070	0.081	0.072	0.072		
	(2.45)	(10.16)	(5.06)	(8.11)	(5.57)	(6.28)		
λ_1	0.382	0.061	0.094	0.033	0.120	0.095		
	(6.38)	(0.64)	(0.94)	(1.44)	(1.40)	(1.12)		
β_0	249.42	2048.13	68.41	367.93	1205.24	115.72	276.56	322.43
	(1.72)	(2.68)	(1.53)	(1.75)	(2.15)	(1.17)	(1.29)	(1.12)
β_1	0.754	0.239	0.799	0.503	0.326	0.586	0.635	0.562
	(8.64)	(4.25)	(7.61)	(5.88)	(4.81)	(5.25)	(4.74)	(3.70)
β_2	0.311	0.661	0.180	0.386	0.560	0.397	0.362	0.381
	(3.36)	(10.73)	(1.63)	(4.33)	(7.92)	(4.08)	(2.63)	(2.41)
β_3	-0.736	-0.028	-0.236	-0.023	-0.035	-0.026	-0.370	-0.489
	(7.79)	(9.56)	(3.24)	(4.57)	(7.18)	(5.45)	(4.21)	(3.92)
β_4								0.103
								(2.12)
R^2	0.701	0.764	0.767	0.807	0.780	0.799	0.610	0.650
SEE	251.5	228.2	221.5	201.4	215.3	205.7	284.1	271.3
DW	1.60	1.13	1.22	1.17	1.40	1.67	1.03	1.18
LLF	-497.0	-490.0	-487.8	-481.4	-485.8	-482.5	-506.3	-502.5
ε_Q	0.754	0.239	0.799	0.503	0.326	0.586	0.635	0.562
$\varepsilon_{w/p}$	0.311	0.661	0.180	0.386	0.560	0.397	0.362	0.381
$\varepsilon_{c/p}$	-0.736	-0.028	-0.236	-0.023	-0.025	-0.026	-0.370	-0.489
Definitions of Variables	Q_1	Q_1	Q_4	Q_4	S_4	CI_1	Q_1	Q_1
	c_1/p	c_2/p	c_1/p	c_2/p	c_2/p	c_2/p	c_1/p	c_1/p
	CF_4^g	CF_4^g	CF_4^g	CF_4^g	CF_4^g	CF_4^g		CF_4^g

Notes: See tables 19, 24, 31, 32 and A30; ε_Q, $\varepsilon_{w/p}$ and $\varepsilon_{c/p}$ are elasticities of demand for capital with respcet to output, the real wage rate and the real user cost, respectively, $\delta = 0.019$ in all equations. Estimation is based on the Mindis-method.

Eq. (7): $I^{net} = \lambda\ [\beta_0 Q^{\beta_1} (w/p)^{\beta_2} (c/p)^{\beta_3} - K_{-1}]$

Eq. (8): $I^{net} = \lambda\ [\beta_0 Q^{\beta_1} (w/p)^{\beta_2} (c/p)^{\beta_3}\ CF^{\beta_4} - K_{-1}]$

δ = sales and CI = investment plus consumption (see table A30).

Our experiments clearly showed that there may be several different estimates of the parameters which result in a 'good fit' to the data. Our reported parameters must, therefore, be treated with more caution than in the preceding case (table 31) since somewhat different values may be found by using different techniques. We feel, however, that the parameter estimates are not unreasonable since different algorithms and initial values gave essentially similar results.

In the preliminary calculations, it turned out that the variable speed of adjustment model is more easily estimated when the depreciation coefficient is fixed a priori (δ = 0.019/per quarter), even though the results of the 'full' model accord quite well with those of the restricted one. To avoid computational difficulties, we have therefore selected the 'net investment approach' for these types of models. A typical example of initial values used where w/c is the price variable (table 32) is as follows: $\lambda_0 = 0.05$, $\lambda_1 = 0.5$, $\alpha_0 = 10$, 100 or 500, $\alpha_1 = 1.0$ and $\alpha_2 = 0.5$.

As the results of tables 32 and 33 reveal, the coefficient estimates show some sensitivity with respect to different expectational values of independent variables. Although some sensitivity does seem to exist in the coefficient estimates, we nevertheless feel that some important common features can also be observed among the different specifications. These are as follows:

i) Estimates of the 'initial' rate of adjustment (λ_0) are within a rather narrow range (about 0.02 - 0.10) in all experiments. Estimates of the speed in the previous specifications (linear or nonlinear) with a constancy assumption were a little higher when comparison is made on an equation-to-equation basis (see also section 7.4).

ii) There is some more variability in the estimates for the variable component (λ_1) of the 'total' speed of adjustment, especially in the ($\frac{w}{c}$)-equations. The value of λ_1 seems to be significantly different from zero and to lie below 0.2.

iii) Estimates of the long-run output (demand) elasticity of the demand for capital are below one universally and, on the average, higher values are obtained in the case where the wage rate and the user cost do not appear separately in the regression (table 33).

iv) Estimates of the long-run price elasticities of the demand for capital show more variability than the output (demand) elasticities. The elasticity with respect to (w/c_2) is again much lower than with respect to (w/c_1), table 32. The long-run price elasticities of the variable speed of adjustment equations are approximately the same as those obtained from the constant speed of adjustment models, as our theory would suggest (a comparison should be made between eq. 4/table 32 versus eq. 1/table 31 and between eq. 5/table 32 versus eq. 2/table 31).

v) The estimation results with (w/p, c/p) -equations in table 33 again clearly indicate that the models based on the c_2-variable perform better than those based on c_1. Furthermore, the $(w/p, c_2/p)$ -equations are clearly better than the best (w/c_1) -equations in table 32. The coefficient estimates of the wage rate and the c_1 - user cost are again very close to each other but in the case of c_2 these coefficients differ significantly (c_2/p again receives a much lower coefficient value than w/p). The coefficient estimates of price variables are not very sensitive to the choice of the demand proxy (see table 33). Various cash flow measures (gross or net and current or lagged values) perform very well but results are reported mainly for the gross cash flow variable with a one year lag.

An interesting comparison among different net investment equations can be made with respect to the role of cash flow by using the LR-test procedure. Equation (7) in table 32 is estimated on the assumption that cash flow directly affects the desired capital stock (see footnote to table 32) and in equation (6) the coefficient of cash flow is restricted to zero ($\alpha_3 = 0$). The likelihood-ratio statistic ($LR = 2(LLF_1-LLF_0)$) takes a value of 7.4, which exceeds the 1 and 5 per cent critical values of χ_1^2 (one restriction is needed to define the null hypothesis). The same result is also obtained with the Q_4 variable although not so clearly. These results may be interpreted as corresponding to that of the linear equations where the cash flow variable is added linearly to the regression and its coefficient receives a significant t-value. Hence, cash flow considerations seem to be an important determinant of the investment decisions.

A second LR-test with respect to the role of cash flow in the investment equation can be obtained by comparing equation (6) and equation (2) in table 32. The value of the LR statistic is 11.0, which clearly exceeds the 5 and 1 per cent significance values of χ_1^2. Hence, in the (w/c_1) -investment equations cash flow seems to significantly affect the rate of adjustment. This result, however, does not follow with the (w/c_2) -variable but it should also be noted that the (w/c_2) -equations perform more poorly than the (w/c_1) -equations.

Equivalent LR-tests can be performed for the equations in table 33 (eq. 7 vs. eq. 8 and eq. 7 vs. eq. 3). These tests indicate that the proper way to model cash flow effects in an investment model is through the rate of adjustment mechanism. It should, however, be emphasized that the cash flow variable affects investment decisions in our nonlinear specifications both through the cost of capital variable (via the desired capital stock) and through the timing effect. The former effect follows since in the context of imperfect capital markets firms use various sources of finance, among them internally generated funds (cash flow), which is hence one component in the weighted average concept of the cost of capital (see the discussion in section 5.4). It should also be emphasized that in the endogenous rate of adjustment models all the arguments of K* also affect the rate of adjustment (i.e. output or demand, the wage rate, the user cost and through the user cost also interest rates, tax factors, the debt ratio and investment incentives).

In chapter III of this study it was found that the endogenous rate of adjustment is sensitive with respect to both the form of the production function and the demand conditions of the firm's products. The nonlinear cash flow investment equations give some empirical evidence of this sensitivity since we have employed two different forms for the desired capital stock. These are as follows: i) nonlinear K* with w/c as the price variable, implying a Cobb - Douglas production function and a vertical demand curve (table 32), and ii) nonlinear K* with w/p and c/p as the price variables, implying a downward-sloping demand curve and either a Cobb - Douglas (exact representation) or CES (approximation, see table 4, section 5.3) production function (table 33).

The estimation results seem to indicate that the rate of adjustment is not necessarily very sensitive with respect to alternative production and demand function specifications. On the contrary, the results seem to indicate that the rate of adjustment is more sensitive with respect to different expectational values of explanatory variables and the choice of the demand proxy than with respect to different assumptions about the underlying functional relationships. In the theoretical part we were able to consider only fixed (stationary) expectations.

In summary of the empirical results with nonlinear investment equations, the following general points can be stated: First, both forms of the nonlinear models (constant and variable speed) support the conclusion that both output and price variables are important determinants of the investment decisions. The long-run elasticity of the demand for capital with respect to the accelerator (output) seems to be between 0.5 and 1.0 and the corresponding price elasticities are between zero and 0.5, although significantly different from zero (see also section 5.6.3.4.). The second variant of the user cost (c_2) again seems to perform better than the first variant (c_1). In the (w/p, c_2/p) -equations, the coefficient of the user cost is again much lower than the coefficient of the wage rate (see also section 7.4.). Second, the assumption of an endogenous rate of adjustment seems to be a somewhat more acceptable hypothesis than the assumption of a constant rate of adjustment. This result is also supported by theoretical considerations. Third, the rate of adjustment might be more sensitive with respect to different expectations hypotheses about the independent variables than with respect to different underlying functional specifications (production and demand functions). This last conclusion is, however, rather preliminary and more work should be done to obtain a firmer conclusion.

7.4 Encompassing Tests of Alternative User Cost Measures and Estimates of Structural Parameters

In table 34 are reported encompassing test results for competing formulas of the second variant of the user cost (c_2). All the alternative measures of c_2 are based on adaptive expectations for the rate of inflation with coefficient 0.1 and output as an accelerator variables (see section 7.1). The tests are analogous to those calculated with the annual data (see sections 6.2.1 and 6.2.4, see also note 1).

As with the annual models, the alternative hypotheses of the c_2 user cost are calculated with respect to the depreciation coefficient, tax parameters and the discount rate. In case I, tests are based on equations in which the price ratio (w/c) is an independent variable. Since these equations perform rather poorly, the tests should be treated rather cautiously. The interesting feature of the results is that the null hypotheses are rejected when the alternative hypothesis is the tax neutral c_2 excluding the rate of inflation, i.e. $c = q(r+\delta)$. The reason for this result is clearly that this measure of the user cost performs quite smoothly and is always positive. This implies that the price ratio (w/c) calculated with this c-variable avoids the 'abnormalities' of other w/c-variables. This result also emphasizes the importance of the proper modelling of expectations for the rate of inflation.

In case II, encompassing tests are based on an equation in which the wage rate and the user cost are separate variables. Different model specifications were also tried. The test results are no longer so clear-cut. The standard c_2 measure is rejected in three cases. It should be recalled that the standard c_2 is based on the statutory (maximum) rate of tax depreciation and the 'corrected' discount rate (see Appendix IV). The null hypothesis based on the book value of the depreciation coefficient seems to pass these tests most clearly. This may be interpreted as implying that firms base their investment decisions on actual book values rather than on the higher statutory rates, which they have not been able to realize, especially in the 1970s (see Appendix IV). This result is at variance with the annual equations (see table 17) so that no firm conclusion can be drawn with respect to the statutory or book value

TABLE 34. Encompassing Tests of Competing Measures of the User Cost Variable in the Aggregate Sector (quarterly data)

I. Investment equation 5.17i (w/c as price variable)

Null hypothesis H_0:	F-statistics					
	c_2	c_{21}	c_{22}	c_{24}	c_{25}	c_{26}
Alternative hypothesis:						
c_2/standard formula		0.26	0.42	0.06	0.94	0.06
c_{21}/book value of depreciation	0.50		3.20	0.31	0.11	0.25
c_{22}/marginal value of depreciation	2.01	4.42		1.61	3.38	1.34
c_{24}/tax-neutral c_2 with adaptive price expectations	0.66	0.54	0.67		0.59	0.44
c_{25}/tax-neutral c_2 with zero rate of inflation	13.37	13.72	16.21	14.00		13.77
c_{26}/interest rate on government bonds as discount rate	0.22	0.11	0.03	0.07	1.16	

II. Investment equation 5.17ii (w/p, c/p as price variables)

Null hypothesis H_0:	F-statistics					
	c_2	c_{21}	c_{22}	c_{24}	c_{25}	c_{26}
Alternative hypothesis:						
c_2/standard formula		2.45	1.43	5.22	28.02	4.46
c_{21}/book value of depreciation	11.14		5.16	2.73	53.41	10.13
c_{22}/marginal value of depreciation	5.15	0.61		0.01	45.82	5.29
c_{24}/tax-neutral c_2 with adaptive price expectations	6.67	0.20	1.91		47.61	10.70
c_{25}/tax-neutral c_2 with zero rate of inflation	2.76	1.88	1.88	1.04		2.98
c_{26}/interest rate on government bonds as discount rate	0.12	0.71	0.72	3.77	40.38	

Notes: See table 17. The 95 and 99 per cent critical values of F(1,68) are 3.99 and 7.06, respectively.

depreciation rates. They seem to perform almost equally well (see table A33) where estimation results are presented with alternative c_2 -measures.

The null hypothesis of the standard c_2 is rejected when the alternative is the tax-neutral c_{24}, i.e. $c = q(r+\delta-g^e)$. On the other hand, when the null is the c_{24} variable, it is also rejected by the standard c_2. Thus the results indicate that there is some uncertainty about the effects of corporate tax factors on the investment decisions in the aggregate sector. In the annual analysis, the results were more clear-cut (see table 17, case II). Ylä-Liedenpohja (1983) has argued that Finnish firms might operate in a situation which can be described by an "expense stock doctrine". This means that companies effectively operate in a taxless economy since they are not able to make full use of the available deductions for tax purposes because of a nonnegativity constraint on the taxable income (positive dividend payments are usually required) and because of low average profitability (see also Bergström and Södersten, 1984).

Table A33 shows estimation results for different user cost variables but they behave in such a similar manner that it does not seem possible to discriminate strongly between them. In conclusion of this examination of the encompassing tests, it can be said that there seems to be some ambiguity about the effects of corporate tax factors on firms' investment policy in the aggregate sector on the basis of the quarterly analysis. It is possible that a more reliable picture could only be obtained by disaggregating firms into, for example, two groups, one consisting of firms that can make full use of tax deductions and the other of firms which operate under the assumption of a taxless economy. Available statistical data does not render this kind of disaggregated analysis possible, however. The reliability of the encompassing tests may also have been reduced by the interpolative nature of the quarterly tax parameters.

Table 35 presents some selected estimates of the rate of adjustment calculated from various types of quarterly investment equations. Some of the estimates have already been considered in the previous section in connection with non-linear models. A rough idea of the overall magnitude of estimates is obtained by looking at the average values of all estimates. The estimates of the constant rate of adjustment imply that about 8.6 per cent of the gap between the desired and existing capital stock is eliminated in each quarter. This result accords well with the annual estimates (see table 22). Our result is well in accordance with that by Coen (1971), who reports 0.1 for the constant rate of adjustment also using quarterly investment models.

In the case of the variable speed of adjustment models, the estimated speed is much higher when firms have ample cash flow. Similar results were also obtained with annual data. The results by Coen were $\lambda_0 \approx 0.1$

and $\lambda_1 \approx 0.2$ and hence the maximum rate of adjustment is 0.3 (per quarter), and our results are $\lambda_0 = 0.06$, $\lambda_1 = 0.19$ and $\lambda_0 + \lambda_1 = 0.25$. It should, however, be noted that especially with respect to the estimate of λ_1, there is quite considerable variability among different equation specifications. The results of both quarterly and annual data indicate that even in the case where firms can finance all desired investment by internal funds there are some other factors (delivery lags, irreversibility etc.) which prevent an instantaneous adjustment. Generally, these results seem to be fairly reasonable, since it can be thought that when companies have an 'excess supply' of cash flow they can avoid the frequently time-consuming negotiations and arrangements associated with raising outside finance (especially if the credit market is tight).

Table 36 presents estimates of the long-run elasticities of the demand for capital with respect to the accelerator (output etc.) and price variables. In the case of linear equations the estimates are calculated at the mean values of the variables as also in the annual equations. The output (demand) elasticity is again somewhat below one. Hence, this result is well in accordance with that of the annual equations (see table 22) where, however, it was slightly higher (on average). In both the annual and quarterly models output (demand) elasticities are somewhat lower in equations where the cash flow variable is included than in equations which exclude cash flow. Elasticities with respect to the price variables again show more variability but the general features are the same as in the annual models. In the cost-minimization models (w/c as the price variable) the average price elasticity is about 0.3, but clearly higher in the case of (w/c_1) than in the case of (w/c_2). It should be recalled that the (w/c_1) -equations perform substantially better than those based on (w/c_2).

The estimates of the long-run elasticity of the desired capital stock with respect to the real wage rate are within a rather narrow range (between about 0.2 and 0.5). The wage rate elasticity generally seems to be rather insensitive with respect to alternative specifications of the investment function. The user cost elasticity varies considerably more between different equations. Generally, the elasticity estimates (in absolute value) with respect to c_1 are clearly higher than with respect to c_2 (as also in the annual models). Furthermore, in the case of the c_2 -variable the elasticity estimate (absolute value) is much lower than the wage rate elasticity.

TABLE 35. Estimates of the Rate of Adjustment in the Aggregate Sector (quarterly data)

Table No. /Eq. No.	Price variables	Cash Flow	λ	Rate of Adjustment λ_0	λ_1	$\lambda_0+0.5\lambda_1$	$\lambda_0+\lambda_1$	Form of equation
25/1	w/c_1	-	0.077					linear
25/2	w/c_1	CF^n		0.057	0.323	0.219	0.380	-"-
25/3	w/c_1	-	0.089					-"-
25/4	w/c_1	CF^g		0.067	0.058	0.096	0.125	-"-
A27/9	w/c_2	-	0.077					-"-
31/1	w/c_1	-	0.089					non-linear
31/2	w/c_2	-	0.073					-"-
32/1	w/c_1	CF^n		0.049	0.319	0.208	0.368	-"-
32/4	w/c_1	CF^g		0.074	0.075	0.112	0.149	-"-
32/5	w/c_2	CF^g		0.015	0.362	0.195	0.377	-"-
A28/1	w/c_1	-	0.095					log-linear
A28/4	w/c_2	-	0.082					-"-
27/1	$w/p, c_1/p$	-	0.080					linear
27/3	$w/p, c_1/p$	CF^n		0.057	0.328	0.221	0.385	-"-
31/3	$w/p, c_1/p$	-	0.090					non-linear
33/3	$w/p, c_1/p$	CF^g		0.070	0.094	0.117	0.164	-"-
33/7	$w/p, c_1/p$	-	0.069					-"-
A28/6	$w/p, c_1/p$	-	0.099					log-linear
28/1	$w/p, c_2/p$	-	0.082					linear
28/2	$w/p, c_2/p$	CF^n		0.066	0.226	0.179	0.292	-"-
28/4	$w/p, c_2/p$	CF^g		0.063	0.128	0.127	0.191	-"-
31/4	$w/p, c_2/p$	-	0.094					non-linear
33/4	$w/p, c_2/p$	CF^g		0.081	0.033	0.097	0.114	-"-
33/5	$w/p, c_2/p$	CF^g		0.072	0.120	0.130	0.192	-"-
A28/9	$w/p, c_2/p$	-	0.103					log-linear
average of estimates:			0.086	0.061	0.188	0.155	0.249	

Notes: See table 21.
λ = constant rate of adjustment (equations without cash flow)
λ_0 = initial rate of adjustment (CF = 0)
λ_1 = variable rate of adjustment (CF^{gap})
$\lambda_0+\lambda_1$ = maximum rate of adjustment (CF^{gap} = 1.0)
$\lambda_0+0.5\lambda_1$ = rate of adjustment when CF^{gap} = 0.5

$$CF^{gap} = \frac{CF^{net}}{K^* - K_{-1}}, \text{ or } \frac{CF^{gross}}{K^* - (1-\delta)K_{-1}}$$

The estimated long-run elasticities with respect to the user cost variables could be used to calculate the long-run effects of changes in the interest rates (r and ρ), the tax parameters (u, z, m etc.), the debt-equity ratio (e) and various investment incentives on the long-run demand for capital. We only present some experiments with respect to the interest rate on borrowing. Here one has to ask first what is the elasticity of the user cost itself in relation to a (permanent) change in the real interest rate. We carried out calculations with various user cost formulas and various values of parameters (variables) which affect this result (i.e. the rate of inflation, the depreciation coefficient, the present value of tax depreciation deductions etc.). The details of the calculations are not shown here but the results can be summarized as follows: a one percentage point permanent increase in the real interest rate will increase the user cost by about 4.7 - 12.5 per cent. In these experiments the rate of inflation (g) varied between 0 and 10 per cent (at annual rate), the rate of economic depreciation varied between 8 and 11 per cent and the initial level of the nominal interest rate (r) between 10 and 13 per cent. At the present level of the parameters ($r = 0.11$, $\delta = 0.08$, and $g = 0.06$, February 1985) a one percentage point rise in the real (expected) interest rate will increase the user cost by about 7.1 per cent. Assuming, for convenience, that this effect is 10 per cent, the average user cost elasticities presented in table 36 have the following implications (the (w/p, c/p) -equations are used): In the case of c_1 -user cost, a one percentage point increase in the real interest rate will, in the long-run, reduce the demand for capital by about 2.6 per cent and in the case of c_2 -variable the equivalent impact is about 0.3 per cent. By way of a comparison it can be mentioned that simulation results with a total-economy macromodel (BOF3) yield about 0.9 per cent. If investment equations based on the c_2 -variable are regarded as the preferred ones, then the effects of a (permanent) interest rate increase is rather modest on capital formation (in the long-run). In the short-run, the impact on investment may, however, be much larger because of liquidity (cash flow) effects.

Table 36. Estimates of the Long-Run Elasticities of the Desired Capital Stock with Respect to Output and Price Variables in the Aggregate Sector (quarterly data)

Table No. /Eq. No.	Form of equation	User cost	Cash Flow	Estimates of Elasticities ε_Q	$\varepsilon_{w/c}$	$\varepsilon_{w/p}$	$\varepsilon_{c/p}$
25/1	linear	c_1	-	0.727	0.426		
25/2	-"-	c_1	CF^n	0.648	0.559		
25/3	-"-	c_1	-	0.903	0.223		
A27/9	-"-	c_2	-	1.117	0.142		
31/1	non-linear	c_1	-	0.791	0.193		
31/2	-"-	c_2	-	0.856	0.020		
32/4	-"-	c_1	CF^g	0.761	0.217		
32/5	-"-	c_2	CF^g	0.844	0.021		
A28/1	log-linear	c_1	-	0.789	0.168		
A28/4	-"-	c_2	-	0.866	0.020		
27/2	linear	c_1	-	0.870		0.255	-0.186
27/3	-"-	c_1	CF^n	0.635		0.568	-0.508
31/3	non-linear	c_1	-	0.778		0.207	-0.189
31/5	-"-	c_1	-	0.926		0.209	-0.173
33/3	-"-	c_1	CF^g	0.799		0.180	-0.236
A28/6	log-linear	c_1	-	0.727		0.262	-0.152
A28/8	-"-	c_1	CF^n	0.592		0.434	-0.342
28/3	linear	c_2	-	0.582		0.411	-0.040
28/2	-"-	c_2	CF^n	0.276		0.648	-0.070
31/4	non-linear	c_2	-	0.522		0.387	-0.021
31/6	-"-	c_2	-	0.601		0.400	-0.023
33/4	-"-	c_2	CF^g	0.503		0.386	-0.023
A28/9	log-linear	c_2	-	0.505		0.398	-0.021
A28/14	-"-	c_2	-	0.590		0.397	-0.022
average of estimates:				0.717	0.199	0.367	-0.143
average of estimates: (c_1-equations)				0.765	0.298	0.302	-0.255
average of estimates: (c_2-equations)				0.661	0.051	0.432	-0.031

Notes: See table 22.
ε_Q, $\varepsilon_{w/c}$, $\varepsilon_{w/p}$, $\varepsilon_{c/p}$: elasticities with respect to output the price ratio, the real wage rate and the real user cost, respectively.

Estimates for other structural parameters of the investment equations are fairly difficult to obtain and since the results are very similar to those of the annual aggregate equations we shall only briefly consider this issue here. Parameters of interest are the returns to scale and the elasticity of substitution of the underlying production function and the price elasticity of demand for the firms' products. Like the annual models, the quarterly equations also imply that there are slightly increasing returns to scale since the output elasticity in the cost-minimization models (w/c -equations) is below one (see, also, table 4). Since these equations (log-linear and nonlinear versions) are based on the assumption of a Cobb - Douglas production function, the elasticity of substitution between capital and labour is by definition unity and, furthermore, output is demand constrained.

When considering the structural parameters associated with the investment equations based on the hypothesis of a downward-sloping demand curve, it should be noted that an exact specification is obtained only in the case of the CD-function but only an approximation in the case of the CES production function (see, also, table 4 and notice that the underlying production function is nonspecific in the linear investment equations). The critical point in the (w/p, c/p) -equations is the positive sign of the coefficient of the wage rate variable. The three circumstances (parameter combinations) which are consistent with the positive wage rate coefficient were already considered in connection with the annual equations (see section 6.5). The aggregate quarterly equations do not offer any new information in this respect over the annual models. Our cautiously preferred interpretation was that the positive wage rate coefficient might be consistent with monopolistic competition in the product market and increasing returns to scale in the production function (in the case of the CD-function) and hence the price elasticity of demand would be over one. If, however, the CES approximation is used as a basis for parameter evaluation, then the positive wage rate coefficient can also follow if the elasticity of substitution is greater than the price elasticity (when there exist constant returns to scale). It should be emphasized that we are discussing here long-run parameter values and, as Sato (1975), for example, has forcefully stated, there are reasons to believe that in a large aggregate of firms the long-run elasticity of substitution may be much higher than conventionally believed and presumably much above unity (Sato, chapter 10).

We feel that no firm conclusions can be drawn with respect to the above-mentioned three structural parameters because of the multitude of hypotheses involved and since no reliable a priori information exists on the market structure (demand conditions) of Finnish firms. Furthermore, it is most likely that a large aggregate of firms includes both companies with price-elastic (e.g. open sector) and price-inelastic (closed sector) demand conditions. One additional problem is the concept of output price (p) used to define the real concept of the wage rate and the user cost (see, also, note 13 to chapter V). Our basic choice was to use the price index of own output, although experiments were also carried out with the price index of the total Finnish GDP. The estimation results seem, however, to be very insensitive to the choice of the price index (p). Finally, it is worth emphasizing that it is generally very hard to get robust estimates of the elasticity of substitution and the bias of technical change simultaneously (see e.g. Sato 1975). It is possible that our measures of K and L already incorporate some amount of technical progress, i.e. K and L are not efficiently constant. This kind of a bias is usually thought to lead to overestimates of the returns to scale parameter (see Sato 1975, Wallis 1979). Hence, if capital and labour inputs were really measured in constant (base year) efficiency units, there might well be decreasing returns to scale and this would significantly affect our tentative conclusions.

7.5 The Effects of Credit Rationing, Technical Change and the Replacement Hypothesis in the Quarterly Data

In section 6.6 we considered annual estimation results with respect to the effect of credit rationing on investment behaviour. Two proxy variables were used in order to capture this effect. Table A35 presents some analogous experiments with the quarterly data for the aggregate sector.

The 'theoretical' arguments for including a direct proxy for credit rationing in the investment regression were discussed in sections 5.4.3 and 6.6. In the quarterly data, we use somewhat different expectational values for the proxy variables than in the annual data (i.e. longer lags). The quarterly analysis is based only on the first variant of the RM-variable (see Table A35). The estimation results indicate that credit rationing has only a negligible direct effect on investment decisions if any. The coefficient estimates of the RM-variables are usually insignificant and of varied sign. Many other experiments with different lagged values, for example, confirm this result (they are not reported here). These results should, however, be interpreted rather cautiously because of the somewhat shaky theoretical foundations and probably rather rough approximations of the relevant proxy variables.

In section 6.6 we also considered the impact of technical change on investment using the annual data. We tried to capture the effect of technical change by a trend variable (an approximation to Hicks neutral technical progress). In the quarterly analysis we attempt to capture the effect of technical change in two ways. First, equations 1 and 2 in table A36 report the estimation results of equations in which the price ratio is a 'corrected' form of the original variable (w/c). In this 'corrected' form, the wage rate is expressed in 'efficiency' units to take account of Harrod-neutral technical change. This correction is calculated by using an estimate of labour quality. The inclusion of technical change does not seem to have much effect on the general goodness of fit of the equations. When the impact of technical progress is explicitly taken into account, the coefficient of the output variable is slightly increased and the coefficient of the factor price ratio is somewhat decreased.

The second way to try to check the effect of technical change on investment consists of adding a trend variable to the regressions. Table A36 also reports these experiments, which can be interpreted to be approximations of Hicks-neutral technical progress. The coefficient of the trend term is significant in most regressions and of negative sign, but the presence of this term seems to lower the significance of the coefficient of the lagged capital stock, probably because of the multicollinearity between these variables. The coefficient estimate of the output variable is slightly higher in the presence of technical change than without it. The effect of the inclusion of the trend variable on the coefficient value of the price variables is small and ambiguous.

Table A37 reports some experiments with the replacement hypothesis of the investment equation. In these experiments the economic rate of depreciation was increased after the first 'oil crisis' in 1973 and the corresponding new net capital stock series were constructed. As with the annual equations, the estimation results are not sensitive, at least to slight changes in the depreciation rates.

To sum up the results of the estimation experiments considered above, it can be said that the addition of credit rationing and/or technical change proxies to our basic linear investment equations does not seem to result in any significant improvement or change in relation to the basic equations. Modest changes in the depreciation coefficient do not have a significant effect on the estimation results either. These results should, however, be regarded as rather preliminary and tentative because of the approximations used to capture the effects of these three factors.

7.6 Summary of the Empirical Results of Quarterly Investment Equations

The results of the quarterly analysis of the determinants of investment decisions of Finnish firms accord quite well with those obtained with the annual analysis (see section 6.7). The main results of the quarterly models are as follows:

i) The accelerator variable is the most important factor determining investment decisions and it seems to influence investment policy with a shorter time lag than the price variables measuring factor costs. The long-run elasticity of the demand for capital with respect to output (or demand) seems to be somewhat below one, indicating that there are increasing returns to scale in the production function. Various proxy variables for the expectational values of output and demand support these conclusions.

ii) The effect of the price variables (factor costs) on investment behaviour is also significant but rather sensitive with respect to the specification of the user cost variable. Estimates of the long-run elasticity of the demand for capital with respect to the wage rate are within a rather narrow range (0.2 - 0.5) but the corresponding elasticities with respect to the user cost differ significantly between c_1 and

c_2 variables. In our preferred specifications (based on c_2) the user cost elasticity is much lower (about 0.02 - 0.04 in absolute value) than the wage rate elasticity. Our results indicate that a one percentage point permanent increase in the real interest rate would reduce (on average) the demand for capital by 0.3 per cent in the long-run. The modelling of expectational values for the rate of inflation is crucial in the case of the c_2 -variant of user cost. As regards both the magnitude of long-run elasticities and the importance of the price expectations, the results of annual and quarterly analysis are very similar.

iii) There is some ambiguity about the effect of corporate tax factors on investment policy. Although most of the results seem to support the conclusion that tax factors are relevant, some at least point to the possibility of neutrality of the corporate tax system (see section 7.4). In the annual analysis of the aggregate sector, the results were more unambiguous, supporting the importance of tax factors.

iv) Internally generated funds (cash flow) seem to affect investment decisions both through the cost of capital term (desired capital) and through the liquidity channel (timing effect). The former effect follows since internal funds influence the weighted average cost of capital variable and hence the profitability (rate of return) of investment projects. In addition, firms seem to be able to react faster to changes in the determinants of investment (demand, prices) when cash flow is ample than when it is scarce. Credit rationing does not seem to exert a significant impact on investment behaviour.

The results of both the annual and quarterly analysis can together be interpreted to mean that the investment behaviour of Finnish firms is explained quite well by the general neoclassical approach in which both demand and factor cost variables play a significant role. However, different versions of the neoclassical model yield somewhat different results with respect to the exact quantitative magnitudes of the effects of the relevant factors.

Notes to Chapter VII

1 We have also calculated analogous tests to those presented in table 23 (also table 34) using the Davidson - Mackinnon J-test in which the fitted value of one specification is introduced as an extra explanatory variable in the other specification. Since the J-tests give essentially the same results as the encompassing tests, these tests are not reported here.

2 The construction of forward-looking expectations values for the output and relative price variables (aggregate data, quarterly analysis) is as follows: In the case of the forward-looking expectations hypotheses for the investment equation, the expected future values of output and relative prices were constructed by using the ARIMA models of respective variables (see Appendix VI). In producing the forecasts, the ARIMA-models were used to generate a twelve period (t+1, t+2,..., t+12) forecast for each past period starting from the first quarter of 1963 (1963.1, 1963.2,...). The forward-looking expectations time series is then obtained by weighting the forecasts of the twelve periods. The method gives a single-valued expectations variable corresponding to each past period. The weights are normalized to sum to one. Various decay parameters were tried (0.1, 0.5, 0.95), and they were assumed to stem from the geometric series. Experimentation with different rates of decay made fairly little difference. All the series of output and price variables are so trend-dominated and the innovations in these series are so persistent that the forecasts for succeeding years are highly collinear. In summary, it seems that the particular forecast weights chosen should have rather little effect on the results (for the methodology of constructing future forecast values, see e.g. Altonji 1982, Virén 1983).

The forward-looking expectations approach in modelling investment behaviour seems to produce very similar results to those of the more traditional methods where expected values of variables are based on past experience (static, adaptive). Comparison of various approaches was made in terms of the parameter and elasticity estimates, significance of coefficients (t-statistics), the goodness of fit (R^2 and SEE) and autocorrelation properties of the residuals. Especially in the case of price variables, the forward-looking method produced results which were so similar to those of the other approaches, that only few estimation results are shown here. In the case of output variables, there is less similarity in the estimation results of different methods and hence some more calculations are presented. In the study by Schiantarelli (1983) in which the forward-looking approach was applied in investment models for the Italian industrial sector, the future expectational values were constructed only for the output variable.

CHAPTER VIII

CONCLUDING REMARKS

The present study has examined the investment behaviour of firms both at the theoretical and empirical levels. In the theoretically-orientated chapters various models of investment and the demand for capital were developed subject to alternative assumptions about the output and input markets.

In chapter II we examined the effects of inflation, tax rules and a constant debt-capital ratio on the long-run demand for capital. It was found that in a partial equilibrium framework an acceleration of the rate of inflation tends to decrease the user cost of capital, thereby increasing the demand for capital. The standard neutrality results of the corporate income tax system were found to hold in this model.

In chapter III the basic model developed in the previous chapter was enlarged by incorporating strictly convex adjustment costs in the model. Alternative assumptions concerning the demand conditions of the firm were made in this chapter. The adjustment cost framework allowed the analysis of some dynamic effects on investment decisions. It was found that the rate of adjustment to the equilibrium level of the capital stock significantly depends on both the demand conditions and the returns to scale assumption of the production function. Generally, it can be said that the speed of adjustment is an endogenous choice variable to the firm and is hence variable rather than an exogenously given constant. The original result of Gould (1968), according to which the rate of adjustment is equal to the depreciation coefficient in the case of a linearly homogeneous production function, does not hold if the product market is imperfect, i.e. the firm faces a vertical or a downward-sloping demand curve. In the case of diminishing returns to scale the rate of adjustment is always endogenous. In chapter III it was shown that the dynamic path of investment itself also depends on the structural parameters of the underlying model. Of crucial importance in this respect are the price elasticity of the demand for the firm's output and the elasticity of substitution between factor inputs. The standard neutrality results of the corporate tax system hold in the present adjustment-cost-based investment model with respect to the long-run demand for capital, i.e. with respect to the user cost of capital services. In chapter III it was shown that the speed of adjustment also depends on corporate tax factors. However, the neutrality conditions of the corporate tax system with respect to the speed of adjustment, and even more generally, to investment itself may coincide with the user cost neutrality.

Chapter IV examined the effects of various forms of capital market imperfections on investment decisions. First, a rising cost of capital schedule was assumed. Second, two forms of quantitative constraints on debt finance were examined. As a general conclusion it can be said that alternative forms of capital market imperfections have quite different effects both on the long-run demand for capital and on the time rate of investment. In addition, investment behaviour under capital market imperfections is usually different from the behaviour in a perfect capital market. The rate of adjustment is quite sensitive with respect to the form of the capital market and in certain situations (expected future profit constraint) retained earnings (cash flow) may be an important determinant of investment outlays. In a liquidity-constrained investment model, the standard neutrality conditions of the tax system do not hold and the effects of tax factors on capital accumulation depend on the average profitability of firms.

In chapters V - VII an empirical analysis of the determinants of the investment behaviour of Finnish firms was carried out. The analysis, which covered the years 1963 - 1980, was based on both annual and quarterly observations. The data was disaggregated into three sectors - manufacturing, residual and aggregate - and it was found that investment behaviour in the three sectors can be explained fairly satisfactorily by the neoclassical approach. However, the accelerator (demand) variable seems to be the main determinant of investment decisions in each sector. The relative price variables have the strongest effect in the residual sector. In the manufacturing and aggregate sectors cash flow considerations add significantly to the explanation of investment expenditures. There do not seem to be any significant credit rationing effects on the investment outlays of Finnish firms.

The empirical results of this study imply that monetary and fiscal policies affect investment behaviour both via the user cost variable ('a rate of return effect') and via the cash flow variable ('a liquidity effect'). The effects through the user cost channel are felt with a considerable lag but the impact of cash flow has a much shorter lag. In general, the empirical results indicate that monetary and fiscal policy measures may have a considerable impact both on the long-run demand for capital and on the timing of investment expenditures.

Through the user cost and cash flow variables, corporate tax rules and the rate of inflation also have a significant effect on the capital formation of Finnish firms, on average. However, at a low rate of inflation the Finnish tax system is effectively neutral with respect to capital formation. There might also be differences in the effects of tax factors on investment behaviour between different kinds of firms, as, for example, with respect to firms' profitability. The empirical calculations with alternative user cost measures indicate that the decrease in the real value of tax depreciation deductions is more than offset by the decrease in the real value of debt caused by an acceleration in the rate of inflation. However, it should be emphasized that our models do not take into account the fact that accelerating inflation may reduce the price competitiveness of output in an open economy and may therefore cause a reduction in revenues, i.e. a decrease in cash flow.

Generally, the results of this study can be interpreted as indicating that in the financial market conditions prevailing in Finland firms are better sheltered against the 'erosion' effects of inflation than in conditions where nominal rates of interest rise in line with the rate of inflation.

APPENDICES p.

APPENDIX I

Formulas for the User Cost of Capital (c_2-approach)

In the empirical analysis of investment equations two basic variants of the user cost variable are used. The first variant is based on a 'rate of return' approach which does not require the explicit modelling of expectations with respect to the rate of inflation. The construction of the c_1-variable is discussed in Appendices II and IV (see also section 5.6.1). The second variant of the user cost is based on the 'theoretical' formulas which were derived in chapters II and IV. The c_2-variable is based on the 'weighted average' concept of the cost of capital. Its empirical implementation requires explicit modelling of expectations with respect to the rate of inflation, taxes etc. This Appendix summarizes and extends the formulas for the c_2-variables. Empirical values of the c_2-variables are presented in Appendix V (c is used here as a general symbol for the user cost). A list of symbols for parameters and variables used in the c_2-formulas is presented at the end of this Appendix.

The marginal productivity rule of capital is in a general form (perfect competition in the output market) given by

(A1) $F_K = \frac{qA}{p}$, where $c = qA$, and $A = cc_r + \delta$

Variable qA is often referred to as the nominal user cost and qA/p as the real user cost. However, since variable A also incorporates the real cost of financial capital (cc_r) in addition to the depreciation coefficient, variable qA can itself be referred to as the real user cost and variable $q\bar{A}$, where $\bar{A} = cc_n + \delta$, as the nominal user cost, and hence c/p is simply a relative price variable.

Assuming that the rate of inflation is zero (g = 0), A is given by (see equation 2.29)

(A2) $A = \delta + sr + \frac{(1-s)\rho}{(1-u)} - \frac{u}{(1-u)}[z(\rho+\delta)-\delta]$

where the parameters (variables) have the same context as in the theoretical analysis (chapter II).

The nominal cost of capital (cc_n) is given by $cc_n = A-\delta$. Using the definition of z, i.e. $z = \alpha/(\rho+\alpha)$, cc_n can be transformed into the following form

(A3) $cc_n = sr + \frac{\rho}{(1-u)}[1-s-\frac{u(\alpha-\delta)}{(\rho+\alpha)}]$

Assuming next that the rate of inflation is nonzero ($g \neq 0$), variable A can be expressed as

(A4) $A = [(1-uz)(\rho+\delta-g)+s((1-u)r-\rho)]/(1-u)$

This can be transformed into the following form

(A5) $$A = \delta + sr + \frac{\rho}{(1-u)}\left[1-s-\frac{u(\alpha-\delta)}{(\rho+\alpha)}\right] - \frac{g}{(1-u)}\left[1-ux-\frac{u\alpha}{(\rho+\alpha)}\right]$$

where it is assumed that proportion x of capital gains is taxable.

The real cost of capital is now given by

(A6) $$cc_r = sr + \frac{\rho}{(1-u)}\left[1-s-\frac{u(\alpha-\delta)}{(\rho+\alpha)}\right] - \frac{g}{(1-u)}\left[1-ux-\frac{u\alpha}{(\rho+\alpha)}\right]$$

and using the definition of z, cc_r is

(A6)' $$cc_r = sr + \frac{\rho(1-s)}{(1-u)} - \frac{u}{(1-u)}[z(\rho+\delta)-\delta] - \frac{g}{(1-u)}[1-u(x+z)]$$

In a 'compact' form the formula for the real user cost is given by

(A7) $$c = \frac{q}{(1-u)}[(1-uz)(\rho+\delta-g)+s((1-u)r-\rho)+uxg]$$

Next the effect of the cost of new equity issues is incorporated in the user cost (see section 4.2.2). The basic formula for the user cost is given by (see eq. 4.25).

(A8) $$c = \frac{q}{(1-u)}\left[(1-uz)(\rho+\delta-g)+s((1-u)r-\rho)+\frac{(1-\theta)}{\theta}n\rho+uxg\right]$$

Within this formula the nominal cost of capital cc_n given by

(A9) $$cc_n = sr + \frac{n\rho}{\theta(1-u)} + \frac{\rho}{(1-u)}\left[1-s-n-\frac{u(\alpha-\delta)}{(\rho+\alpha)}\right]$$

and the real cost of capital cc_r can be derived analogously to equation (A6).

The effect of a corporate wealth tax rate (w) can easily be incorporated in the formulas for the user cost. If wealth taxes are based on net wealth (qK-B), then the sum of income and wealth taxes is

(A10) $$T = u(pQ-wL-rB-D) + w(qK-B)$$

Assuming that B = sqK, the following formula for the user cost can be derived (excluding new equity issues)

(A11) $$c = \frac{q}{(1-u)}[(1-uz)(\rho+\delta-g)+s((1-u)r-\rho)+uxg+w(1-s)]$$

In the years 1976 - 79 an extra deduction worth 3 per cent of part of the acquisition costs of new investment was allowed in taxation (see Appendix III). In a general form, if the extra deduction (in per cent) is 100η and the portion of gross investment that is subject to this deduction is k, then income taxes are given by $T = u(pQ-wL-rB-D-\eta kqI)$ and the equivalent user cost is

(A12) $c = \frac{q}{(1-u)}[(1-u(z+\eta k))(\rho+\delta-g)+s((1-u)r-\rho)]$

The effect of extra accelerated depreciation deductions (applied in Finland in 1976 - 79) can be incorporated in the user cost through the present value of tax depreciation deductions on one unit of new capital, i.e. via the z-variable. If free depreciation on new investment goods is allowed, then z = 1. The corresponding values of z and α can always be solved through the equation

(A13) $\alpha = \frac{z\rho}{(1-z)}$

<u>List of parameters and variables used to define c-variables:</u>

s = ratio of debt to total capital (B/qK)
r = interest rate on debt
ρ = discount rate
u = corporate profit tax rate
α = tax depreciation coefficient
δ = economic rate of depreciation
z = present value of depreciation deductions on one unit of investment
x = proportion of taxable capital gains
θ = tax discrimination variable (see section 4.3)
n = proportion of new equity issues in total investment
w = corporate wealth tax rate
η = proportion of extra investment allowance
k = proportion of investment on which extra investment allowance is applied
q = price index of investment goods
p = price index of output
g = rate of inflation on investment goods, i.e. $g = \dot{q}/q$

APPENDIX II

Measurement of the Real Rate of Return,
Tobin's "q"-Variable and the Discount Rate

This appendix presents the formulas for and estimates of the real rate of return, the discount rate and Tobin's "q"-variable. Estimates of the real rate of return and the "q"-variable are used to calculate the first variant of the user cost c_1.

The estimates of the real rate of return are calculated by means of a method that implies that both the numerator and the denominator are "inflated" to current (replacement cost) values. This approach has been used by, e.g., Feldstein and Summers (1977), Hill (1979), Holland and Myers (1980) and Feldstein (1982). Our method is similar to that of Holland and Myers (1980, 1984), who have directed an international rate-of-return project in which the author of this study has also participated. Detailed descriptions of the calculations for the manufacturing sector are presented in Koskenkylä (1983) and for other sectors in an unpublished paper by Koskenkylä (1982), see also Holland (ed., 1984).

The basic formula for calculating the real rate of return on total capital is given as

$$\text{(A14)} \quad RR^a = \frac{\Pi^a + IR}{TC}$$

where

$\Pi^a = pQ - wL - \delta qK - rB - T$
$IR = rB$
TC = replacement cost value of total capital (fixed assets plus inventories)

RR^a is an after-tax rate of return. The before-tax rate of return is calculated by replacing Π^a with $\Pi^b = \Pi^a + T$. The Π-variable is a measure of the real operating income. Strictly speaking the operating income equals the real total income only if real holding gains on total capital are zero. This would mean that the general price level, say output prices (p), increases at the same rate as the implicit price index of total capital (g_T). Taking into account real holding gains, the real rate of return is defined as

$$\text{(A15)} \quad \overline{RR}^a = \frac{\Pi^a + IR + RHG}{TC}$$

where $RHG = (g_T' - p')TC$, $g_T' = \dot{g}_T/g_T$ and $p' = \dot{p}/p$. The series of RHG is, however, quite volatile (erratic) and we have chosen to use the first measure of RR (eq. A14), see also Holland (1984).

The estimates of RR both after- and before-taxes are presented in Table A1.

TABLE A1.

Estimates of the Real Rate of Return on Total Capital in the Manufacturing, Residual and Aggregate Sectors in 1961 - 80, per cent

	RR_M^a	RR_M^b	RR_R^a	RR_R^b	RR_A^a	RR_A^b
1961	10.80	12.61	7.79	8.76	9.09	10.72
1962	8.17	9.77	6.93	8.50	7.46	9.41
1963	8.63	10.01	7.16	8.17	7.79	9.29
1964	8.46	9.74	6.80	8.43	7.53	9.32
1965	7.24	8.30	6.78	8.46	6.98	8.75
1966	6.07	7.44	6.48	8.08	6.29	8.02
1967	6.16	7.22	6.05	7.37	6.10	7.53
1968	8.60	9.27	5.77	7.17	7.03	8.45
1969	12.29	13.36	6.19	7.37	8.92	10.19
1970	12.12	12.53	6.30	7.46	8.92	10.09
1971	7.90	8.18	5.71	6.81	6.72	7.79
1972	8.24	8.35	6.14	7.12	7.12	8.05
1973	9.35	9.39	6.37	7.39	7.76	8.72
1974	12.29	12.01	6.53	7.56	9.26	10.20
1975	6.18	6.24	4.57	5.72	5.34	6.34
1976	4.78	5.14	3.53	4.83	4.12	5.24
1977	4.37	4.56	3.79	4.78	4.06	4.95
1978	6.59	7.25	4.02	5.01	5.18	6.04
1979	9.35	10.17	5.08	6.02	6.98	7.87
1980	9.02	9.83	5.24	5.91	6.95	7.69
mean	8.33	9.07	5.86	7.05	6.98	8.23
st. deviation	2.31	2.41	1.16	1.26	1.50	1.64

Note: RR_M, RR_R and RR_A refer to the rate of return in the manufacturing, residual and aggregate sectors, respectively. RR^a is an after-tax concept and RR^b is a before-tax concept.

It can be seen that the rate of return has, on average, been much higher in manufacturing than in the residual sector. The fluctuations of RR are larger in manufacturing than in the residual sector. The aggregate sector falls between the disaggregated sectors in both respects.

We next turn to the calculation of Tobin's "q"-variable. The average "q"-variable is defined as

$$(A16) \quad q_T = \frac{MV}{TC}$$

where MV is the market value and TC is the replacement cost value of total capital (fixed assets + inventories + net financial assets). The estimates of market values have been calculated on the basis of the companies quoted on the Helsinki Stock Exchange. In manufacturing, listed companies represent about 15 per cent of total manufacturing as measured by the share of sales. Since no other data is available in Finland, we have had to base these calculations on rather 'rough' measures. In the calculations it is assumed that developments in companies which are not quoted on the stock market follow those of the quoted companies.

In order to calculate the "q"-variable, an auxiliary variable was first measured from the stock market data. This variable (F) is the ratio of the market value of equity to the nominal value of the equity of listed companies. The variable F is then used to calculate the market value of the equity capital of the total sector, i.e. $E^M = F \cdot E$, where E is the book value of equity capital. Since very few companies are quoted in the residual sector, we have not used an estimate of F for this sector, which turned out to be a very 'erratic' series. For the aggregate sector, the F-variable has been calculated on the basis of all listed non-financial companies on the Stock Exchange.

The total market value is estimated by using the nominal (book) value of debt capital since only a negligible portion of debt is quoted in Finland.

Table A2 presents estimates of the F-variable and Table A3 estimates of the "q"-variable. Two estimates have been calculated for the manufacturing sector. The first is based on the assumption that only equity capital, which is used as a basis for dividend payments, has a market value ("q"$_1$). The second estimate ("q"$_2$) is based on the assumption that total own capital has an implicit market value. Total own capital (OC) is estimated by the formula: $OC = TC - (B+B_C)$, where B_C is a measure of deferred taxes (tax credits). For the residual and aggregate sectors, the data allowed only the calculation of the second variant of "q".

TABLE A2

Ratio of the Market Value to the Nominal Value of the Equity (Own) Capital of Listed Companies in the Manufacturing and Aggregate Sectors

	F_M	F_A
1961	2.04	2.12
1962	2.12	2.09
1963	2.01	1.97
1964	2.02	1.91
1965	1.70	1.82
1966	1.35	1.51
1967	1.14	1.25
1968	1.46	1.61
1969	1.65	1.81
1970	1.93	1.76
1971	2.00	1.91
1972	2.69	2.88
1973	3.40	3.21
1974	2.48	2.44
1975	2.06	2.15
1976	1.56	1.67
1977	1.35	1.46
1978	1.47	1.51
1979	1.61	1.65
1980	1.44	1.50
mean	1.90	1.97
st. deviation	0.53	0.47

The ratio used for the residual sector is the same as for the aggregate.

TABLE A3

Estimates of Tobin's "q"-Variable in the Manufacturing, Residual and Aggregate Sectors

	M "q_1"	M "q_2"	R "q_2"	A "q_2"
1961	1.14	1.72	1.83	1.81
1962	1.14	1.75	1.78	1.76
1963	1.14	1.67	1.71	1.68
1964	1.14	1.66	1.64	1.61
1965	1.07	1.44	1.58	1.55
1966	1.07	1.21	1.36	1.33
1967	1.07	1.08	1.17	1.16
1968	1.09	1.27	1.44	1.40
1969	1.10	1.36	1.56	1.51
1970	1.13	1.51	1.53	1.48
1971	1.13	1.50	1.63	1.55
1972	1.19	1.80	2.32	2.12
1973	1.24	2.20	2.55	2.34
1974	1.15	1.74	2.01	1.87
1975	1.14	1.48	1.78	1.66
1976	1.10	1.24	1.47	1.38
1977	1.09	1.15	1.32	1.26
1978	1.10	1.20	1.35	1.29
1979	1.10	1.27	1.44	1.37
1980	1.07	1.19	1.34	1.29
mean	1.12	1.47	1.64	1.57
st. deviation	0.04	0.30	0.34	0.30

An estimate of the real cost of capital used to measure the first variant of the user cost (c_1) can now be calculated as follows: The market value of a firm equals the capitalized value of the long-run average earnings from assets now in place plus the present value of growth opportunities (PVGO), (see, e.g., Holland and Myers, 1980). Hence

$$(A17) \quad MV = \frac{Y}{\rho_c} + PVGO$$

where ρ_c is the capitalization rate and Y equals average long-run earnings. Variable Y can be defined as: $Y = RR \cdot TC$ and hence the market value is given as

$$(A18) \quad MV = \left(\frac{RR}{\rho_c}\right)TC + PVGO$$

It can now be seen that "q" depends on the ratio of the rate of return to the capitalization rate (assuming for simplicity PVGO = 0). The capitalization factor can be solved as

$$(A19) \quad \rho_c = \frac{RR}{\text{"}q\text{"}}$$

This ratio can be thought to represent a generalized earnings-price ratio, where the "price" equals the market value of total capital and the "earnings" equal the real operating income, i.e. Π + IR (see eq. A14). Since it is difficult to obtain a measure for PVGO, it is assumed to be zero. This growth factor has also been neglected in some other studies (see, e.g. Nickell, 1978, p. 267). It should be noted that since we are dealing with total capital and total operating income (i.e. interest costs are included in income), the ρ_c variable is a measure for the real cost of capital.

The before-taxes estimates for ρ_c are presented in Table A4.

TABLE A4

Estimates of the Real Cost of Capital Based on the "q" Approach in the Manufacturing, Residual and Aggregate Sectors

	M ρ_{c1}	M ρ_{c2}	R ρ_{c2}	A ρ_{c2}
1961	11.08	7.34	4.78	5.94
1962	8.54	5.57	4.76	5.34
1963	8.75	5.98	4.79	5.53
1964	8.58	5.88	5.14	5.77
1965	7.76	5.77	5.35	5.64
1966	6.93	6.13	5.95	6.01
1967	6.76	6.70	6.28	6.50
1968	8.48	7.30	4.98	6.03
1969	12.11	9.82	4.72	6.74
1970	11.11	8.30	4.89	6.84
1971	7.25	5.44	4.19	5.04
1972	6.99	4.64	3.07	3.79
1973	7.57	4.26	2.90	3.72
1974	10.42	6.90	3.77	5.46
1975	5.50	4.23	3.21	3.83
1976	4.67	4.14	3.30	3.79
1977	4.20	3.98	3.62	3.92
1978	6.59	6.05	3.82	4.68
1979	9.24	8.02	4.17	5.74
1980	9.16	8.24	4.40	5.97
mean	8.08	6.23	4.40	5.31
st. deviation	2.11	1.60	0.94	1.02

Note:
- for the residual and aggregate sectors only the "q_2"-variable is available for estimate ρ_c;
- the estimates of ρ_c are before-tax values since they are based on the RR^b variable.

Finally in this Appendix we present estimates for the discount rate of shareholders (ρ). Three methods are used: (i) earnings-price ratio, (ii) interest rate on government bonds and (iii) the rate of return on own capital (see section 4.3 where the question of the after- or before-personal income tax rate is considered, i.e. ρ or $\rho/(1-m)$ etc.). The earnings-price ratio is defined as

(A20) $$\rho_{ep} = \frac{Div + \Delta E^M}{E^M}$$

where Div = dividends and E^M = market value of equity (own capital). Because of the very high volatility of the ρ_{ep} series (as also observed in other countries, see, e.g., Holland and Myers, 1984), we present only the average values of ρ_{ep} in the period 1961 - 1980.

TABLE A5

Average Earnings-Price Ratio for the Manufacturing, Residual and Aggregate Sectors, per cent

	M	R	A
mean of ρ_{ep}	13.77	13.72	12.96
st. deviation	21.66	23.28	23.07

It can be seen that the average values of the ρ_{ep} estimates are quite close to each other and that the ρ_{ep} variables are extremely volatile.

The second method of estimating the discount rate is based on the interest rate on government bonds (completely tax free). Table A6 shows this interest rate series (ρ) and two transformations. The first is calculated by dividing ρ by (1-m), where m is the marginal personal income tax rate. This series was constructed in order to compare the ρ-variable with the ρ_{ep}-variable. The ρ_{ep}-variable is a before-personal-tax concept and the transformation of ρ is made since interest earnings from government bonds have usually been tax-free in Finland. The value of m used to calculate ρ_m is 0.5. The second transformation corresponds to the formula presented in section 4.3 where the discount rate was found to equal $(i+w)/(1-\tau)$, and $i = \rho$, w = marginal wealth tax rate, and τ = tax rate on capital gains. It was assumed that $w = 0.02$ and $\tau = 0.15$.

The average value of $\rho/(1-m)$ is about 2.5 percentage points higher than the average value of the earnings-price ratio. It is likely that the estimated earnings-price ratios are underestimates since in Finland shareholders receive a considerable portion of their total capital income (from holding stocks) in the form of scrip issues. If these were taken into account, then the 'capital gain' component in formula (A20) would be larger than just the change in the market value of equity.

TABLE A6

Interest Rate on Government Bonds (After and Before Personal Income Taxes and Corrected for Other Personal Taxes) and the Rate of Return on Own Capital

	ρ	$\rho/(1-m)$	$\frac{\rho+w}{1-\tau}$	RRO
1960	7.06	14.12	10.66	14.88
1961	7.06	14.12	10.66	14.44
1962	7.06	14.12	10.66	12.46
1963	7.06	14.12	10.66	13.62
1964	7.06	14.12	10.66	17.18
1965	7.06	14.12	10.66	11.39
1966	7.06	14.12	10.66	9.33
1967	7.06	14.12	10.66	10.52
1968	8.12	16.24	11.91	18.07
1969	7.81	15.62	11.54	18.29
1970	7.69	15.38	11.40	18.78
1971	7.96	15.92	11.72	16.31
1972	7.86	15.72	11.60	17.54
1973	8.28	16.56	13.09	24.15
1974	8.52	17.04	12.38	34.47
1975	9.70	19.40	13.76	27.22
1976	10.31	20.62	14.48	21.91
1977	10.32	20.64	14.49	18.36
1978	9.26	18.52	13.35	16.50
1979	9.45	18.90	13.47	20.60
1980	10.47	20.94	14.67	24.18
mean	8.20	16.40	12.00	18.11
st. deviation	1.22	2.45	1.44	5.95

Column RRO in table A6 stands for an estimate of the real rate of return on own capital in the manufacturing sector. The estimate is taken from Koskenkylä (1984). It is calculated by the formula

$$\text{(A20)'} \quad RRO = \frac{P^a + INF_1 - INF_2}{TC - B}$$

where
TC = total capital (replacement cost value)
B = debt
P^a = profit after corporate income taxes
INF_1 = gB (gearing adjustment for debt)
INF_2 = gM (gearing adjustment for financial assets)
M = monetary assets
g = rate of change in the consumer price index

This third way of estimating the discount rate gives the highest value because the decline in the real value of the indebtedness of firms results in capital gains for equity owners and, conversely, losses to creditors. However, changes in the real value of debt have no net effect on the real rate of return on total capital since capital gains (owners) and losses (creditors) cancel out.

APPENDIX III

Description of the Finnish Corporate Tax System and Estimates of Tax Parameters

Finnish corporations are obliged to pay income taxes to both the central and local governments as well as church taxes to the local parish. Up to 1968, and again temporarily in 1976, corporations were also liable to pay a wealth tax. The present income taxation is governed by the Act on the Taxation of Business and Professional Income, which came into force in 1968. This act is applied to all business activities, regardless of the type of business unit (see L. Aarnio, Bank of Finland Monthly Bulletin, July 1978). This new legislation represented an important reform by which the regulations concerning the determination and apportioning of taxable net income were changed. At the same time, a tax system was introduced which allowed losses to be offset against profits in the five subsequent years. Prior to 1968, the corporate tax system was, in general, also based on a concept of net income. Since 1969 the corporate tax system has been of a dual-rate type (split-rate system). The income of a firm has been subject to double taxation (at company and personal levels). This was alleviated in central government income taxation by allowing a company to deduct 40 per cent of its dividend payments from its taxable income (under certain conditions). Later (1977) this deduction was increased to 60 per cent

In 1968 corporations were exempted from wealth tax, which had been 1 per cent of net wealth prior to that year. The fiscal effects of this removal of wealth tax were offset by a temporary increase in corporate income taxes in 1968 - 70 (see Table A7). The income tax rate was raised by 9 percentage points in 1968. The increment was gradually reduced and was cancelled altogether in 1971. Since 1971 the central government tax rate has been 43 per cent (see Table A7).

In the determination of annual taxable income there are many regulations on the allocation of expenses. Only the main ones will be considered here. Depreciation of fixed capital is determined according to a degressive (geometric) depreciation system. The maximum allowable depreciation is 30 per cent (of the undepreciated value) for machinery and equipment. This rate is 9 - 10 per cent for buildings. In the case of certain special structures, even higher depreciation rates can be applied.

Inventories are valued according to the FIFO principle. Inventories can, however, be undervalued to a maximum amount of 50 per cent of the original cost of inventories. Up to 1968, there was completely free undervaluation (i.e. 100 per cent).

Depreciation charges and undervaluation of inventories are the most important methods which firms can use to manipulate their taxable income. Some minor items also exist, as, for example, provisions for trade credit risks, provisions for the guaranteeing of a product and some regional arrangements allowing extra depreciation and interest rate subsidies etc. In 1978 a new system was introduced mainly for firms which do not have 'eligible' assets for depreciation and inventory reserves. The new operating reserve system allows an enterprise to deduct up to a maximum of 50 per cent of its net earnings for transfer to an operating reserve. However, the total amount of this reserve cannot exceed a certain percentage of the firm's total wage bill for the previous year. Since this system came into force only in 1978 - 1979 and our empirical analysis ends in 1980, we have not taken this system into account in the measurement of the user cost.

Joint-stock companies (and some other firms) are allowed to make investment reserves for central government income tax purposes. A certain percentage of net income can be deducted provided that the company deposits in the central bank an amount equal to the income tax payable on the deduction. The central bank pays a certain interest rate on this deposit. Prior to 1979 this investment reserve system was very seldom used and hence we have not incorporated it in our measures of the cost of capital. In 1979, however, the system was made more attractive and it has subsequently been more widely used.

Because of the poor performance of the Finnish economy in the years 1976 - 1978, the government took some stimulative fiscal policy action in order to accelerate investment by manufacturing firms. The temporary right to almost free depreciation (with some exceptions, however) was introduced in 1976. Later an extra deduction equivalent to 3 per cent of the acquisition costs of buildings was permitted. These concessions remained in force up to 1980 with some modifications in 1977 - 1978. The tax concession policies were supplemented in 1977 by a reduction in the sales tax on investment goods. This reduction has been partly maintained up to the present time.

We shall next present the estimates of tax parameters which are needed in the measurement of the user cost variable (in the c_2-case). Table A7 shows the corporate income tax parameters.

TABLE A7

Estimates of Corporate Income Tax Parameters in 1960 - 1980

	u_g	u_1	u_0	u_L	u
1960	0.38	0.38	0.45	0.12	0.50
1961	0.38	0.38	0.43	0.12	0.50
1962	0.43	0.43	0.50	0.12	0.55
1963	0.38	0.38	0.47	0.12	0.50
1964	0.45	0.45	0.53	0.12	0.57
1965	0.58	0.42	0.56	0.13	0.61
1966	0.48	0.42	0.57	0.13	0.61
1967	0.49	0.47	0.58	0.13	0.62
1968	0.58	0.56	0.59	0.13	0.72
1969	0.49	0.25	0.50	0.14	0.63
1970	0.47	0.24	0.48	0.14	0.61
1971	0.43	0.22	0.44	0.15	0.58
1972	0.43	0.22	0.44	0.15	0.58
1973	0.43	0.23	0.44	0.15	0.58
1974	0.43	0.22	0.44	0.15	0.58
1975	0.43	0.23	0.44	0.16	0.59
1976	0.43	0.22	0.58	0.16	0.59
1977	0.43	0.16	0.44	0.16	0.59
1978	0.43	0.15	0.44	0.16	0.59
1979	0.43	0.15	0.44	0.16	0.59
1980	0.43	0.15	0.44	0.16	0.59

u_g = central government (state) income tax rate on undistributed profits
u_L = local government income tax rate

u_1 = central government income tax rate on distributed profits
u = $u_g + u_L$
u_0 = central government "income tax rate" corrected for wealth taxes, i.e.

$$u_0 = \frac{\text{income taxes + wealth taxes}}{\text{taxable income}}$$

The total corporate tax rate (u) has been calculated on the assumption that the taxable income is the same in central and local government taxation. In reality, there is some difference in the assessment of the two taxable incomes but we have neglected it here. However, a rough calculation of the effect of this difference on the estimates of the user cost showed that it was negligible.

Next we shall consider estimates for the tax discrimination variable (θ) (see section 4.3. for a definition of θ, also King 1977). In a classical tax system θ is equal to (1-m). In a two-rate system θ is given by

$$\text{(A21)} \quad \theta_1 = \frac{1 - m}{1 - u_g + u}$$

using our notation for tax parameters. In place of u_g and u_1 we could use u (= u_g+u_L) and (u_1+u_L) but u_L cancels out. King has presented an estimate of θ for Finland; his estimate is 0.83 (see, King 1977, p. 55). The effect of capital gains taxation could also be incorporated in θ. In this case θ is equal to $\theta_1/(1-\tau)$, where τ is the effective tax rate on capital gains (accrual basis). Table A8 shows our estimates for the tax discrimination variable assuming that m = 0.5 and τ = 0.15.

TABLE A8

Estimates of the Tax Discrimination Variable (θ) in 1960 - 1980

	θ_1	$\theta_2=\theta_1/(1-\tau)$		θ_1	$\theta_2=\theta_1/(1-\tau)$
1960	0.50	0.59	1971	0.63	0.74
1961	0.50	0.59	1972	0.63	0.74
1962	0.50	0.59	1973	0.63	0.74
1963	0.50	0.59	1974	0.63	0.74
1964	0.50	0.59	1975	0.63	0.74
1965	0.53	0.62	1976	0.63	0.74
1966	0.53	0.62	1977	0.68	0.80
1967	0.51	0.60	1978	0.69	0.81
1968	0.51	0.60	1979	0.69	0.81
1969	0.66	0.78	1980	0.69	0.81
1970	0.65	0.76			

An important parameter in the measurement of the user cost is the tax depreciation coefficient. Estimates for the statutory (maximum) tax depreciation rates - parameter α in the theoretical analysis - are presented in Table A9.

The coefficients α have been calculated for each sector as a weighted average of the depreciation rates for machinery and equipment and for the structures, the weights being the corresponding shares of machinery and equipment and structures in total fixed capital (the weights change over time). The effect of the extra depreciation deductions in the manufacturing sector in 1976 - 1979 can clearly be seen from this table. Without these extra accelerated rules, the value of α in manufacturing in 1976-1980 would be 0.21.

Table (A10) presents estimates of the 'book value' tax depreciation coefficient, i.e. the value of the depreciation coefficient which has been used to calculate taxable income.

TABLE A9

Estimates of the Statutory Tax Depreciation Coefficients in the Manufacturing, Residual and Aggregate Sectors in 1961 - 1980

	α_M	α_R	α_A
1961	0.20	0.15	0.17
1962	0.20	0.15	0.17
1963	0.20	0.15	0.17
1964	0.20	0.15	0.18
1965	0.20	0.16	0.18
1966	0.20	0.16	0.18
1967	0.20	0.16	0.18
1968	0.20	0.16	0.18
1969	0.21	0.19	0.20
1970	0.21	0.19	0.20
1971	0.21	0.19	0.20
1972	0.21	0.19	0.20
1973	0.21	0.19	0.20
1974	0.21	0.19	0.20
1975	0.21	0.19	0.20
1976	0.32	0.19	0.25
1977	0.33	0.19	0.25
1978	0.29	0.19	0.23
1979	0.35	0.19	0.26
1980	0.32	0.19	0.25

TABLE A10

Estimates of the 'Book Value' Tax Depreciation Coefficient in the Manufacturing, Residual and Aggregate Sectors in 1961 - 1980

	α_M^B	α_R^B	α_A^B
1961	0.20	0.09	0.17
1962	0.17	0.09	0.15
1963	0.17	0.09	0.14
1964	0.17	0.10	0.15
1965	0.16	0.08	0.13
1966	0.15	0.08	0.12
1967	0.13	0.08	0.11
1968	0.14	0.10	0.13
1969	0.19	0.12	0.17
1970	0.19	0.13	0.17
1971	0.14	0.12	0.14
1972	0.13	0.11	0.12
1973	0.14	0.11	0.13
1974	0.18	0.17	0.18
1975	0.13	0.17	0.15
1976	0.12	0.15	0.13
1977	0.11	0.14	0.12
1978	0.11	0.15	0.13
1979	0.16	0.15	0.15
1980	0.16	0.17	0.16

The α^B parameters have been calculated from balance sheet data (see Appendix IV). The balance sheet data for manufacturing is available in 'official' form for the period under consideration. For the residual sector we have formed a sample from both published and unpublished data sources.

Comparing the α and α^B parameters, it can be noted that, especially in the 1970s, firms were not able to make full use of the statutory depreciation rates mainly because of the low level of corporate profitability.

Table A11 presents estimates of the present value of tax depreciation charges (per unit of investment) calculated by the formula $z = \alpha/(\rho+\alpha)$, where ρ is an estimated discount factor (see Table A6) and α:s are taken from Table A9.

TABLE A11

Estimates of the Present Value of Tax Depreciation Deductions (per unit of investment) in the Manufacturing, Residual and Aggregate Sectors in 1961 - 1980

	z_M	$z_{M,m}$	z_R	z_A
1961	0.74	0.74	0.68	0.71
1962	0.74	0.74	0.68	0.71
1963	0.74	0.74	0.69	0.71
1964	0.74	0.74	0.69	0.71
1965	0.74	0.74	0.69	0.71
1966	0.74	0.74	0.69	0.72
1967	0.74	0.74	0.69	0.72
1968	0.71	0.71	0.66	0.69
1969	0.73	0.73	0.70	0.72
1970	0.73	0.73	0.71	0.72
1971	0.73	0.73	0.70	0.71
1972	0.73	0.73	0.71	0.72
1973	0.72	0.72	0.70	0.71
1974	0.71	0.71	0.69	0.70
1975	0.68	0.68	0.66	0.67
1976	0.76	0.79	0.65	0.71
1977	0.76	0.82	0.65	0.71
1978	0.76	0.89	0.67	0.71
1979	0.79	0.93	0.67	0.73
1980	0.75	0.78	0.64	0.70

The estimates z_M, z_R and z_A are the 'average' values since they are based on the average values of the tax depreciation coefficient (table A9). In manufacturing, this method implies that z_M is a weighted average of the 'normal' statutory depreciation rate (0.21 in 1976 - 1980) and of the marginal accelerated depreciation rules applying to new investment goods. The estimate of the present value of marginal depreciation rules is given by the variable $z_{M,m}$. It can be seen that $z_{M,m} = z_M$ in 1961 - 1975 when no extra depreciation allowances were permitted. In the residual sector the average and marginal present values are equal since no extra accelerated rules have been permitted. For the aggregate sector, only average z_A is presented: the marginal z_A could be calculated approximately as an average of $z_{M,m}$ and z_R. Present values based on the 'book depreciation' deductions could also be calculated by the formula $z = \alpha^B/(\rho+\alpha^B)$ and would be somewhat lower than the figures in table A11.

The final parameter to be presented here measures the portion of new equity issues in total investment as an additional source of finance. In the theoretical analysis (see section 4.2.2), this share-parameter was denoted by n. Because of data deficiencies, n could be estimated only for manufacturing and this estimate should be regarded as a rough measure. The estimate of n is probably biased upwards since it was calculated as a ratio of the change in total own (equity) capital to the current value of investment.

TABLE A12

Estimate of the Proportion of New Equity Issues in Total Investment in the Manufacturing Sector in 1961 - 1980, parameter n

1961	0.06	1971	0.12
1962	0.08	1972	0.17
1963	0.23	1973	0.12
1964	0.02	1974	0.13
1965	0.15	1975	0.13
1966	0.09	1976	0.14
1967	0.20	1977	0.09
1968	0.19	1978	0.16
1969	0.20	1979	0.14
1970	0.14	1980	0.10

APPENDIX IV

Estimates of the User Cost of Capital

As was mentioned in Appendix I, two basic variants of the user cost have been calculated for the econometric analysis. The first approach is based on the real rate of return and Tobin's "q" variable (see Appendix II). User Cost is then defined as

(A24) $c = q(\rho_c+\delta)$, where $c = c_1$

where ρ_c = RR/"q" is a measure of the real cost of capital (see Table A4 and eq. A19). Table A13 presents estimates of the user cost of capital with the "Rate of Return" approach and using the ρ_{c_2} estimates.

TABLE A13

Estimates of the Real User Cost in the Manufacturing, Residual and Aggregate Sectors in 1961 - 1980 using the 'Rate of Return' Approach (c_1-approach)

	c_M	c_R	c_A
1961	4.09	3.63	3.84
1962	3.76	3.84	3.87
1963	4.04	3.95	4.06
1964	4.21	4.19	4.30
1965	4.51	4.43	4.50
1966	4.78	4.88	4.82
1967	5.26	5.31	5.31
1968	6.26	5.39	5.79
1969	7.66	5.73	6.51
1970	7.73	6.27	7.14
1971	7.19	6.56	6.99
1972	7.60	6.63	7.08
1973	8.64	7.43	8.11
1974	13.11	9.83	11.52
1975	12.30	10.97	11.72
1976	13.39	12.06	12.78
1977	14.75	13.95	14.47
1978	17.68	15.37	16.42
1979	21.38	17.08	18.95
1980	23.98	19.59	21.38

The second method of calculating the user cost is based on the formulas presented in Appendix I. This c_2-method is called a 'weighted average' approach since the cost of capital is a weighted average of the costs of different financial sources. A multitude of different empirical values for the user cost can be constructed using both of the theoretical formulas (see Appendix I) and alternative estimates of the parameters in these formulas (see Appendices III and IV). We present here only some basic estimates of the user cost variant c_2 which are applied in the econometric analysis.

The standard estimates of the c_2-variables are based on the following assumptions: (i) the depreciation coefficient is the statutory (maximum) rate; (ii) the discount rate is measured by the 'corrected' interest rate on government bonds; (iii) wealth taxes are excluded; (iv) the cost of new equity issues is excluded; (V) extra depreciation deductions are taken into account at 'average values'; and (vi) the real cost of capital is estimated assuming that price expectations are adaptive.

In the econometric analysis a number of alternative assumptions are made with respect to expected values of the rate of inflation, resulting in alternative estimates of the real cost of capital (see section 5.6).

Quarterly estimates of the nominal user cost (c^Q) are calculated by the following formula:

$$\text{(A25)} \quad c^Q = q^Q(cc_n+\delta)$$

where q^Q is the quarterly price index of investment goods, cc_n and δ are annual rates of the nominal cost of capital and the depreciation coefficient, respectively. In the empirical analysis, a method is often used in which ($cc_n+\delta$) is divided by four (see e.g. Bischoff 1969). A more exact formula would be to transform the annual values of cc_n and δ to quarterly values using the following formula (for δ):

$$\text{(A26)} \quad \delta = 1 - (1-\delta^Q)^4$$

where δ^Q is the quarterly value of δ. This formula gives values of δ^Q which are, however, very close to those obtained by dividing δ by four. Dividing by four does not affect the basic results since the divisor is constant.

The quarterly estimate of the real user cost is obtained by replacing cc_n with cc_r (real cost of capital). In measuring cc_r, the quarterly rate of inflation has, however, been transformed to annual rate.

Table A14 presents various estimates of the nominal cost of capital which are used to measure the corresponding values of the user cost. Estimates of the real cost of capital are obtained by substracting an inflation factor from cc_n (see eq. A6 Appendix I). The values of cc_n variables in Table A14 are all before-tax concepts, and hence multiplying by (1-u) gives an illustration of the after-tax values (say, u = 0.6).

TABLE A14

Estimates of the Nominal Cost of Capital (cc_n) in the Manufacturing, Residual and Aggregate Sectors in 1961 - 1980, per cent (before-tax values)

	M	M	M	M	R	R	R	A	A	A	A
	$cc_{n,1}$	$cc_{n,2}$	$cc_{n,3}$	$cc_{n,4}$	$cc_{n,1}$	$cc_{n,2}$	$cc_{n,3}$	$cc_{n,1}$	$cc_{n,2}$	$cc_{n,3}$	$cc_{n,4}$
1961	12.91	12.92	7.44	12.91	14.49	16.98	8.82	13.75	13.84	8.17	13.80
1962	12.73	13.72	7.35	12.73	15.70	18.72	9.32	14.33	15.31	8.40	14.40
1963	11.92	12.91	7.23	11.92	14.04	16.63	8.70	13.06	14.07	8.01	13.10
1964	12.37	13.76	7.27	12.37	15.10	18.01	9.09	13.81	15.09	8.22	13.88
1965	12.29	14.31	7.15	12.29	14.46	19.69	8.76	13.41	15.83	7.96	13.48
1966	11.70	14.45	6.85	11.70	15.32	21.12	9.14	13.61	16.75	8.03	13.68
1967	10.82	14.73	6.57	10.82	14.54	20.71	8.70	12.81	16.84	7.68	12.88
1968	14.18	19.83	7.31	14.18	20.82	27.94	11.06	17.73	23.07	9.27	17.84
1969	11.72	12.92	6.98	11.72	15.70	20.26	9.07	13.87	15.48	8.09	13.89
1970	10.68	11.39	6.48	10.68	15.09	18.67	8.71	13.05	14.55	7.66	13.07
1971	10.25	13.36	6.53	10.25	15.21	18.69	8.91	12.85	15.91	7.75	12.87
1972	9.74	13.76	6.23	9.74	15.29	19.27	8.85	12.65	16.50	7.58	12.67
1973	10.05	13.48	6.35	10.05	16.43	20.69	9.81	13.34	16.85	8.07	13.35
1974	11.56	13.08	7.18	11.56	17.53	18.48	10.33	14.59	15.50	8.73	14.60
1975	11.88	16.28	7.04	11.88	19.62	20.96	10.90	15.89	18.90	9.00	15.90
1976	7.35	17.40	4.47	4.03	21.10	23.46	11.60	14.28	20.98	8.03	13.33
1977	7.17	18.47	4.28	-1.15	22.05	25.28	12.20	14.96	22.51	8.48	11.79
1978	7.52	16.13	5.12	-3.01	18.83	21.12	10.73	13.68	18.96	8.09	9.33
1979	6.26	13.53	4.35	-4.44	18.94	21.24	10.71	12.80	17.97	7.51	8.82
1980	8.15	15.19	5.35	0.98	22.05	23.11	12.56	15.19	19.47	8.87	12.52

Notes: $cc_{n,1}$ = standard cc_n based on the statutory depreciation coefficient and on the 'corrected' discount rate ($\rho/(1-m)$)
$cc_{n,2}$ = based on the 'book value' depreciation coefficient, otherwise as $cc_{n,1}$
$cc_{n,3}$ = based on the discount rate ρ, otherwise as $cc_{n,1}$
$cc_{n,4}$ = based on the marginal depreciation coefficient, otherwise as $cc_{n,1}$

It can be noted that cc_n is much lower (especially in the years 1976 - 1979) in manufacturing than in the residual or aggregate sectors mainly because of the rising debt-equity ratio ($r < \rho$) and the investment incentives applied in manufacturing.

Table A15 presents some estimates of the nominal and real user cost calculated by formulas $c_n = q(cc_n+\delta)$ and $c_r = q(cc_r+\delta)$. Estimates of the real user cost are based on either constant or adaptive expectations of the rate of inflation (g-variable, see eq. A6 in Appendix I). In the empirical analysis of investment equations some other proxies for the rate of inflation are also tried (see sections 5.5.2 and 5.6). It can be seen from table A15 that the real user cost is much lower in the manufacturing sector than in the residual and aggregate sectors. In the R- and A-sectors the real user cost is a steadily and fairly smoothly rising series whereas it is a rather volatile series in manufacturing. The low level of the user cost in the M-sector is attributable to the low level of the real cost of capital caused by an increase in the rate of inflation in the latter half of 1970s (a rise in capital gains owing to a high level of the debt-ratio) and the accelerated depreciation rules and other investment incentives applied in the years 1976 - 1979. The estimates of the user cost presented in Table A15 are rather similar to those presented in Table A13 in the case of the R- and A-sectors. By contrast in the case of the M-sector the two variants (c_1 and c_2) of the user cost differ significantly. We do not, however, feel it is possible to rule out either of these measures even though the negative values of the user cost are 'unsatisfactory'.

TABLE A15

Estimates of the Nominal and Real User Cost in the Manufacturing, Residual and Aggregate Sectors in 1961 - 1980 using the 'Weighted Average' Approach (C_2-approach)

	M c_n	M $c_{r,1}$	M $c_{r,2}$	M $c_{r,2}$	R c_n	R $c_{r,1}$	R $c_{r,2}$	A c_n	A $c_{r,1}$	A $c_{r,2}$
1961	5.89	2.38	4.17	4.19	6.82	2.80	5.01	6.37	2.59	4.52
1962	6.14	2.20	4.27	4.49	7.71	3.09	5.72	6.96	2.66	4.92
1963	6.13	2.23	4.21	4.08	7.31	2.89	5.23	6.76	2.57	4.70
1964	6.61	2.15	4.39	4.17	7.93	2.90	5.46	7.30	2.54	4.92
1965	7.12	2.08	4.62	4.57	8.03	2.50	5.35	7.58	2.29	4.96
1966	7.10	1.90	4.35	3.62	8.71	2.97	5.52	7.94	2.45	5.03
1967	7.16	1.54	4.34	4.56	8.93	2.71	5.85	8.11	2.16	5.12
1968	9.12	1.08	5.01	4.85	12.27	3.28	7.63	10.77	2.23	6.40
1969	8.49	1.72	4.39	1.78	10.79	3.35	5.56	9.71	2.57	5.39
1970	8.87	1.61	4.60	3.69	11.36	3.56	6.30	10.18	2.63	5.74
1971	9.76	2.09	4.81	2.59	12.63	4.50	6.53	11.24	3.32	6.13
1972	10.67	2.02	4.54	1.07	14.15	5.10	6.39	12.48	3.62	6.20
1973	12.72	2.45	4.71	-0.26	16.87	6.49	6.81	14.85	4.53	6.80
1974	17.17	4.28	5.67	-3.22	21.59	8.71	6.71	19.40	6.52	7.91
1975	19.95	4.68	3.75	-11.82	27.38	11.82	4.96	23.78	8.35	7.41
1976	17.85	2.35	0.43	-10.98	33.71	16.29	8.01	25.91	9.44	7.52
1977	17.67	0.58	-1.83	-8.40	36.96	17.25	9.17	27.60	9.01	6.79
1978	19.55	1.63	-0.90	-5.84	36.07	14.91	6.56	28.38	8.62	5.83
1979	19.44	0.57	-0.41	3.54	38.89	15.96	12.13	29.23	8.32	7.22
1980	27.32	2.62	2.19	7.22	48.64	22.53	20.44	35.89	12.30	11.82

Notes: c_n = nominal user cost based on $cc_{n,1}$ (table A14)
$c_{r,1}$ = real user cost based on constant price expectations
$c_{r,2}$ = real user cost based on adaptive price expectations (values of adaptation coefficient: M = 0.1 and 0.5, R = 0.2, A = 0.1

APPENDIX V

Data Sources and Construction of Variables

The main sources of the statistical data used in this study are

- national income account statistics (NIA)
- balance sheet data (enterprise statistics), (BSD)
- financial market statistics (FMS)
- statistics on income and wealth taxes (IWT)
- industrial statistics (IS)

The sectoral classification is as follows:

- manufacturing covers SIC 2 - 3 industries
- residual sector covers SIC 4 - 7 industries
- aggregate sector covers SIC 2 - 7 industries

The construction and content of the main economic variables used in the empirical analysis is next described:

1 Investment (I)

Annual data is derived from NIA (at 1975 prices). Quarterly data for the aggregate sector is constructed by using the annual data as a base series and taking the quarterly fluctuations from the quarterly investment series of the BOF-model of the Bank of Finland.

2 Capital Stock (K)

The annual capital stock series is derived from the data of the Central Statistical Office (at 1975 prices). The average economic depreciation rates (δ) implied by these capital stock series are

- manufacturing 0.078
- residual sector 0.075
- aggregate sector 0.076

In the annual models for manufacturing, we have also used a capital stock series constructed by Koskenkylä (1978) which is based on a 5.4 per cent rate of depreciation (to check the sensitivity of results with respect to K-variable). The quarterly capital stock series was constructed using the identity $K_t = I_t + (1-\delta)K_{t-1}$ and the quarterly investment series.

3 Production (Q)

Both annual and quarterly data are taken directly from NIA (at 1975 prices).

4 Labour Costs (w)

The underlying data consist of the total compensation of employees (production and non-production workers) and total man-hours worked at annual rates. The wage rate (w) is simply the total compensation per man-hour. Compensation is measured in billions of FIM and man-hours in billions and hence the wage rate is measured in markkaa per man-hour. Quarterly data is constructed using the series of the BOF-model and the NIA-statistics.

5 Interest Rate on Debt (r)

The reference series is the average lending rate of commercial banks. We have, however, also calculated 'own' average interest rates for each sector using the balance sheet data gathered for this project. The formula used is

$$r = \frac{IR_t}{\frac{1}{2}(B_t + B_{t-1})}$$

where IR = interest expenses (net) and B_t is the end of period value of debt. Some attempts to deduct the 'interest-free' portion of debt were also made.

6 Marginal Interest Rate (RM)

At present, two series at annual level exist for the marginal interest rate. The first measure (RM1) has been constructed by Tarkka (1981) and the second variant (RM2) by Huomo and Korkman (1980). Both series were used in the annual equations although their correlation coefficient is only 0.21. The quarterly series was constructed by Tarkka (for the BOF-model).

7 Stock of Debt (B)

For the manufacturing sector two annual series can be obtained: (i) from the balance sheet data and (ii) from the statistics on credit outstanding (FMS). There is some difference between these two series. The balance sheet data is used in this study.

For the residual sector a debt series was constructed from the balance sheet data gathered for this study. For the years 1960 - 1967 the estimate of B is based on the growth rate of debt in the sample of the residual sector firms. For the aggregate sector two sources are available for constructing the B-variable. First, B can be calculated directly as a sum of the M- and R-sector debt series. The second estimate can be obtained from the statistics on outstanding credit (FMS). Although we have tried to limit the difference between these two series, some difference nevertheless remains probably because of the different coverage of trade credits.

8 Total Capital (TC)

In the theoretical analysis total capital at replacement cost is the qK-variable. In the empirical calculations of the rate of return, however, it is natural to use a total measure of capital (fixed assets plus inventories plus net financial assets). If a TC-measure is not used, then the debt-capital series (s) do not make sense because s can be greater than one. This discrepancy between theoretical and empirical analysis may, however, be explained by assuming that in the theoretical analysis other assets (inventories plus financial assets) are a constant proportion of total capital (see also Poterba and Summers (1983)). The method for calculating the replacement cost value of total capital is described in Koskenkylä (1984).

9 Price Indices of Investment Goods (q) and Output (p)

These price indices are taken directly from NIA statistics at annual rates and the quarterly series are obtained from the BOF-model data. The price indices have 1975 as a base year (q_{1975} = 1.00 etc.)

10 Cash Flow (CF) and Dividends (Div)

Net cash flow (retained earnings) is calculated as

$$\text{(A22)} \quad CF^n = pQ - wL - \delta qK - rB - T - Div$$

$$\text{(A23)} \quad CF^g = CF^n + \delta qK$$

which implies that CF^g is the sum of depreciation charges and profits after taxes. Quarterly CF series were obtained using the quarterly values of the respective variables and partly interpolating some of the variables (rB, T and Div).

The annual series of taxes and dividends were obtained from BSD, IWT and some unpublished data of the Central Statistical Office. There are differences between the series for dividends and taxes thus obtained but they do not have much effect on the estimates of CF. Two other series for CF can also be constructed using a) tax depreciation instead of true economic depreciation and b) labour costs measured on the basis of the balance sheet data. Using these variables would affect the CF-estimates somewhat. We have, however, assumed that it is the 'true' economic cash flow which affects investment behaviour.

Cash flow variables used in the econometric analysis are deflated values of the nominal CF since it is the purchasing power of cash flow relative to the cost of new investment goods which is thought to affect investment decisions.

Tables A16 and A17 present the basic annual and quarterly data used in the econometric analysis.

TABLE A16. Annual Data for All Sectors

Manufacturing

	I	K	Q	B	TC	CF^g	CF^n	T	Div	w	p	q	s	e
1962	3682	24881	12590	6037	11921	3746	2134	278	58	3.01	0.35	0.29	0.51	1.03
1963	3083	26142	13076	6813	12855	4062	2330	264	59	3.26	0.36	0.30	0.53	1.13
1964	3489	27691	13963	7627	14324	4172	2310	269	66	3.69	0.38	0.32	0.53	1.14
1965	3718	29340	14808	8779	16106	3736	1813	285	69	4.09	0.39	0.34	0.55	1.20
1966	3935	31052	15520	9801	17333	3531	1359	315	70	4.47	0.39	0.35	0.57	1.30
1967	3306	31997	15971	11471	19014	3544	1305	286	63	4.98	0.41	0.37	0.60	1.52
1968	3406	32925	16846	12744	21801	4604	2414	294	80	5.55	0.46	0.42	0.58	1.41
1969	4051	34373	19012	14894	24387	6994	4490	323	118	6.05	0.51	0.43	0.61	1.57
1970	5317	36979	21097	18350	29267	7094	4641	315	143	6.79	0.53	0.48	0.63	1.68
1971	5808	39815	21361	22739	34839	5025	2364	329	123	7.89	0.55	0.53	0.65	1.88
1972	5505	42143	23930	26475	40119	5376	2562	326	151	9.10	0.59	0.60	0.66	1.94
1973	4905	43690	25477	32977	49575	6090	3227	401	231	10.88	0.69	0.71	0.67	1.99
1974	6508	46649	26739	44086	68344	8193	5319	498	260	13.69	0.91	0.87	0.65	1.82
1975	6652	49537	25647	53714	80776	4819	1537	625	125	17.12	1.00	1.00	0.66	1.98
1976	5833	51415	26064	61417	89271	3715	133	782	361	20.33	1.11	1.10	0.69	2.20
1977	4837	52177	25855	67857	98170	3247	-387	725	315	22.77	1.19	1.24	0.69	2.24
1978	3919	51975	26922	72883	104808	5337	1310	703	337	24.37	1.29	1.27	0.70	2.28
1979	4465	52385	29874	78856	115429	7449	3647	900	460	27.05	1.41	1.35	0.68	2.16
1980	5788	54088	32368	93986	136772	6821	3128	1023	620	30.91	1.50	1.50	0.69	2.20

TABLE A16 cont.

Residual Sector

	I	K	Q	B	TC	CF^g	CF^n	T	Div	w	p	q	s	e
1962	4288	33840	15881	7745	15774	3956	1823	236	93	2.76	0.35	0.32	0.49	0.96
1963	3829	35225	16375	8706	16549	4391	2028	164	99	3.08	0.37	0.33	0.53	1.11
1964	4136	36794	17130	9283	17599	4346	1848	278	102	3.51	0.40	0.34	0.53	1.12
1965	4580	38717	18339	11014	19566	4403	1865	313	118	3.77	0.41	0.35	0.56	1.29
1966	4663	40491	18904	11676	21625	4602	1807	330	126	4.13	0.44	0.36	0.54	1.17
1967	4880	42329	19243	13344	23525	4505	1638	298	138	4.58	0.46	0.38	0.57	1.31
1968	3906	43091	19218	14289	26793	4276	1484	352	104	5.12	0.50	0.43	0.53	1.14
1969	4825	44595	21153	16508	30311	5093	1972	335	90	5.64	0.51	0.46	0.54	1.20
1970	5849	47010	22882	18972	35071	5259	2089	378	156	6.35	0.54	0.50	0.54	1.18
1971	6306	49693	23346	21033	39303	4918	1650	409	209	7.20	0.58	0.55	0.54	1.15
1972	6876	52735	25552	23968	45985	5576	2147	415	188	8.27	0.63	0.62	0.52	1.09
1973	8003	56645	27908	30273	56541	5871	2262	522	154	9.79	0.70	0.70	0.54	1.15
1974	7779	60043	28807	38702	74432	5897	2327	677	221	12.25	0.85	0.85	0.52	1.08
1975	8962	64350	29343	45866	89505	4769	788	938	504	15.30	1.00	1.00	0.51	1.05
1976	8228	67681	28448	51455	101301	4572	143	1244	229	17.66	1.14	1.11	0.51	1.03
1977	7398	69968	28052	58507	118404	4773	247	1088	249	19.47	1.26	1.25	0.49	0.98
1978	5913	70632	28414	66532	128937	5393	607	1231	218	20.49	1.34	1.37	0.52	1.07
1979	6393	71762	30184	74792	143824	6480	1556	1284	297	22.82	1.46	1.47	0.52	1.08
1980	6440	72853	31731	84805	164164	6077	1265	1044	230	26.02	1.58	1.65	0.52	1.07

Aggregate Sector

	I	K	Q	B	TC	CF^g	CF^n	T	Div	w	p	q	s	e
1962	7970	58720	28471	13782	27695	7683	3923	514	151	2.87	0.35	0.31	0.50	0.99
1963	6912	61367	29450	15519	29403	8430	4327	428	158	3.15	0.37	0.32	0.53	1.12
1964	7625	64484	31093	16910	31923	8501	4131	547	168	3.58	0.39	0.33	0.53	1.13
1965	8298	68056	33147	19793	35672	8136	3670	599	187	3.90	0.40	0.35	0.55	1.25
1966	8598	71543	34423	21477	38958	8141	3168	645	196	4.27	0.42	0.36	0.55	1.23
1967	8186	74326	35214	24814	42539	8035	2937	583	201	4.74	0.44	0.38	0.58	1.40
1968	7312	76016	36064	27033	48594	8860	3869	646	184	5.30	0.48	0.43	0.56	1.25
1969	8876	78968	40165	31403	54698	11991	6360	657	209	5.81	0.51	0.45	0.57	1.35
1970	11166	83989	43980	37322	64338	12302	6667	693	299	6.54	0.54	0.49	0.58	1.38
1971	12115	89508	44707	43772	74141	9934	3996	738	332	7.49	0.57	0.54	0.59	1.44
1972	12381	94879	49482	50443	86104	10942	4698	742	339	8.63	0.61	0.61	0.59	1.41
1973	12907	100335	53385	63250	106116	11981	5503	923	385	10.26	0.70	0.70	0.60	1.48
1974	14287	106693	55546	82788	142776	14127	7684	1175	481	12.87	0.88	0.86	0.58	1.38
1975	15614	113887	54990	99580	170281	9589	2326	1563	628	16.07	1.00	1.00	0.58	1.41
1976	14060	119096	54511	112872	190571	8287	276	2026	590	18.82	1.12	1.11	0.59	1.45
1977	12235	122146	53907	126364	216574	8019	-137	1813	564	20.89	1.23	1.24	0.58	1.40
1978	9832	122607	55335	139415	233745	10648	1874	1933	555	22.14	1.32	1.33	0.60	1.48
1979	10858	124147	60058	153648	259253	13609	4893	2184	757	24.76	1.43	1.42	0.59	1.45
1980	12229	126941	64099	178791	300936	12829	4316	2067	850	28.17	1.54	1.57	0.59	1.46

TABLE A16 cont.

Interest Rates

	r_M	r_R	r_A	r_B	RM1	RM2
1962	7.16	8.99	8.12	7.03	24.88	7.18
1963	7.37	8.74	8.09	7.14	20.40	12.99
1964	7.50	8.94	8.23	7.25	18.58	12.33
1965	7.50	8.84	8.16	7.47	7.00	15.48
1966	7.29	9.03	8.13	7.53	19.52	18.57
1967	7.54	8.59	8.04	7.56	10.61	23.50
1968	7.68	9.33	8.44	7.70	7.65	7.00
1969	7.87	9.05	8.42	7.71	7.12	7.00
1970	7.41	8.56	7.94	7.76	17.83	19.18
1971	7.50	8.71	8.02	8.74	9.75	9.00
1972	7.27	8.55	7.79	8.17	7.75	7.88
1973	7.34	10.03	8.41	8.96	12.88	11.64
1974	7.96	10.48	8.97	9.84	14.62	24.26
1975	7.50	9.60	8.33	10.02	19.64	24.80
1976	7.46	9.99	8.43	10.14	18.63	20.50
1977	7.89	10.80	9.01	9.99	17.36	17.39
1978	8.00	10.25	8.88	8.51	11.75	10.49
1979	7.81	10.06	8.73	8.51	9.26	13.00
1980	8.58	11.83	9.90	10.00	14.58	13.00

TABLE A17. Quarterly Data for the Aggregate Sector

	I	K	Q	CF^g	CF^n	T	Div
1963.1	1687	59356	7176	1930	894	107	39
1963.2	1674	59968	7353	2387	1354	107	39
1963.3	1758	60653	7431	2181	1155	107	39
1963.4	1793	61367	7490	1936	926	107	39
1964.1	1746	62008	7637	2502	1384	137	42
1964.2	1887	62779	7611	1958	850	137	42
1964.3	1961	63610	7858	1807	720	137	42
1964.4	2031	64484	7987	2238	1180	137	42
1965.1	2029	65359	8149	1969	843	150	47
1965.2	2049	66239	8210	1783	665	150	47
1965.3	2092	67145	8346	2093	981	150	47
1965.4	2127	68056	8441	2287	1178	150	47
1966.1	2159	68963	8395	1749	493	161	49
1966.2	2077	69771	8499	2071	825	161	49
1966.3	2155	70642	8626	1965	726	161	49
1966.4	2207	71543	8903	2351	1119	161	49
1967.1	2197	72409	8803	2026	709	146	50
1967.2	2207	73269	8865	1959	658	146	50
1967.3	1811	73717	8800	2048	750	146	50
1967.4	1972	74326	8746	2003	813	146	50
1968.1	1903	74839	8856	1949	688	161	46
1968.2	1932	75372	8986	2056	809	161	46
1968.3	1773	75736	9081	2470	1224	161	46
1968.4	1704	76016	9141	2379	1143	161	46
1969.1	2246	76802	9692	2732	1289	164	52
1969.2	2023	77351	9822	2943	1513	164	52
1969.3	2199	78065	10246	3234	1833	164	52
1969.4	2407	78968	10405	3071	1711	164	52

TABLE A17 cont.

1970.1	2889	80356	10695	2950	1518	173	75
1970.2	2674	81503	10953	3369	1920	173	75
1970.3	2773	82728	11147	3163	1757	173	75
1970.4	2831	83989	11185	2835	1483	173	75
1971.1	2667	85044	10593	2119	534	184	83
1971.2	3144	86555	11268	2571	1056	184	83
1971.3	3100	87993	11409	2677	1220	184	83
1971.4	3204	89508	11437	2537	1144	184	83
1972.1	3146	90945	11962	2478	867	185	85
1972.2	3060	92268	12271	2802	1193	185	85
1972.3	3037	93543	12429	2926	1386	185	85
1972.4	3138	94879	12821	2726	1237	185	85
1973.1	3332	96390	13131	2755	1048	231	96
1973.2	2859	97398	13008	2857	1133	231	96
1973.3	3365	98892	13479	2851	1289	231	96
1973.4	3352	100335	13767	3466	1959	231	96
1974.1	3267	101666	13845	3362	1628	294	120
1974.2	3422	103125	13919	3479	1828	294	120
1974.3	3842	104977	13868	3407	1844	294	120
1974.4	3756	106693	13913	3847	2335	294	120
1975.1	4030	108674	14058	2596	674	391	157
1975.2	4080	110668	13853	2067	255	391	157
1975.3	3778	112321	13604	2382	603	391	157
1975.4	3726	113887	13475	2549	792	391	157
1976.1	3799	115511	13468	1654	-412	506	148
1976.2	3330	116635	13628	2072	-4	506	148
1976.3	3554	117961	13607	2358	402	506	148
1976.4	3378	119096	13808	2178	256	506	148
1977.1	3142	119963	13424	2078	-60	453	141
1977.2	3257	120929	13332	2039	-6	453	141
1977.3	2848	121467	13584	1952	-73	453	141
1977.4	2988	122146	13567	1954	-2	453	141
1978.1	2572	122434	13629	2735	467	483	139
1978.2	2557	122638	13773	2928	713	483	139
1978.3	2365	122642	13815	2589	396	483	139
1978.4	2338	122607	14119	2410	306	483	139
1979.1	2662	122940	14614	3356	1085	546	189
1979.2	2738	123343	14967	3281	1081	546	189
1979.3	2634	123633	15059	3378	1222	546	189
1979.4	2823	124147	15419	3755	1659	546	189
1980.1	2966	124754	15712	3056	845	517	212
1980.2	2991	125375	15851	2982	826	517	212
1980.3	3307	126300	16280	3284	1198	517	212
1980.4	2965	126941	16256	3490	1423	517	212

	s	e	w	p	q	r	c_1	c_{2c}	c_{2a}
1963.1	0.52	1.08	3.01	0.36	0.31	8.13	4.05	2.55	4.97
1963.2	0.53	1.12	3.12	0.38	0.32	8.07	4.04	2.56	4.90
1963.3	0.53	1.14	3.21	0.37	0.32	8.06	4.06	2.57	4.87
1963.4	0.53	1.13	3.28	0.36	0.32	8.08	4.11	2.62	4.92
1964.1	0.53	1.11	3.43	0.40	0.32	8.24	4.19	2.48	4.95
1964.2	0.53	1.11	3.55	0.39	0.32	8.27	4.23	2.50	5.06
1964.3	0.53	1.12	3.63	0.38	0.33	8.26	4.32	2.55	5.21
1964.4	0.54	1.16	3.72	0.40	0.34	8.17	4.46	2.62	5.32
1965.1	0.55	1.22	3.79	0.40	0.34	8.27	4.46	2.27	5.05
1965.2	0.56	1.26	3.87	0.39	0.35	8.17	4.50	2.28	4.80
1965.3	0.56	1.27	3.91	0.40	0.35	8.12	4.52	2.29	4.56
1965.4	0.55	1.24	4.03	0.41	0.35	8.09	4.52	2.29	4.44

1966.1	0.55	1.21	4.02	0.40	0.35	8.15	4.80	2.43	4.67
1966.2	0.55	1.20	4.15	0.41	0.36	8.16	4.82	2.45	4.84
1966.3	0.55	1.22	4.37	0.42	0.36	8.14	4.83	2.46	5.02
1966.4	0.56	1.29	4.53	0.43	0.36	8.08	4.85	2.47	5.24
1967.1	0.58	1.37	4.63	0.43	0.37	8.03	5.10	2.08	5.14
1967.2	0.59	1.42	4.70	0.43	0.37	8.00	5.19	2.11	5.32
1967.3	0.59	1.43	4.77	0.44	0.37	8.03	5.22	2.12	5.39
1967.4	0.58	1.38	4.85	0.45	0.40	8.10	5.73	2.32	5.93
1968.1	0.57	1.31	5.11	0.46	0.42	8.33	5.75	2.20	6.15
1968.2	0.55	1.25	5.26	0.47	0.43	8.43	5.81	2.23	5.16
1968.3	0.55	1.22	5.35	0.50	0.43	8.50	5.79	2.24	4.22
1968.4	0.55	1.24	5.47	0.49	0.43	8.52	5.79	2.25	3.45
1969.1	0.56	1.29	5.64	0.50	0.44	8.48	6.39	2.52	3.64
1969.2	0.57	1.34	5.70	0.51	0.44	8.45	6.40	2.53	4.05
1969.3	0.58	1.38	5.89	0.52	0.45	8.41	6.51	2.58	4.54
1969.4	0.58	1.39	6.02	0.52	0.46	8.33	6.73	2.67	4.89
1970.1	0.58	1.38	6.35	0.53	0.48	8.25	6.96	2.59	4.76
1970.2	0.58	1.37	6.39	0.54	0.47	8.09	6.93	2.56	4.39
1970.3	0.58	1.38	6.58	0.54	0.49	7.85	7.18	2.64	4.33
1970.4	0.58	1.40	6.84	0.54	0.51	7.55	7.49	2.74	4.18
1971.1	0.59	1.43	7.16	0.56	0.51	8.17	6.56	3.12	4.14
1971.2	0.59	1.45	7.31	0.56	0.53	7.97	6.86	3.26	4.39
1971.3	0.59	1.45	7.50	0.57	0.55	7.92	7.12	3.38	4.12
1971.4	0.59	1.44	8.01	0.59	0.58	8.01	7.44	3.54	3.86
1972.1	0.59	1.41	8.23	0.59	0.59	7.63	6.88	3.49	3.37
1972.2	0.58	1.40	8.49	0.60	0.59	7.78	6.88	3.51	2.72
1972.3	0.58	1.41	8.75	0.62	0.62	7.87	7.17	3.67	2.70
1972.4	0.59	1.43	9.03	0.62	0.64	7.86	7.41	3.80	2.55
1973.1	0.59	1.47	9.48	0.65	0.66	7.80	7.67	4.31	2.82
1973.2	0.60	1.49	9.89	0.68	0.66	7.75	7.60	4.26	2.34
1973.3	0.60	1.49	10.58	0.70	0.73	9.03	8.42	4.69	2.25
1973.4	0.59	1.46	11.07	0.75	0.75	9.08	8.76	4.85	1.23
1974.1	0.59	1.42	11.60	0.81	0.80	9.07	10.64	6.05	1.10
1974.2	0.58	1.38	12.70	0.87	0.84	9.05	11.23	6.35	-0.11
1974.3	0.58	1.36	13.18	0.88	0.89	8.98	11.90	6.71	-2.28
1974.4	0.58	1.36	14.01	0.96	0.92	8.77	12.33	6.95	-3.42
1975.1	0.58	1.37	14.84	0.94	0.94	8.53	11.11	7.87	-4.06
1975.2	0.58	1.39	16.00	0.97	1.00	8.33	11.77	8.37	-4.36
1975.3	0.59	1.42	16.46	1.02	1.02	8.22	11.95	8.53	-4.65
1975.4	0.59	1.45	17.00	1.07	1.03	8.21	12.06	8.64	-4.13
1976.1	0.59	1.46	17.97	1.07	1.07	8.28	12.38	9.18	-2.68
1976.2	0.59	1.47	18.26	1.10	1.07	8.38	12.31	9.12	-2.09
1976.3	0.59	1.45	19.21	1.16	1.13	8.47	13.08	9.66	-0.45
1976.4	0.59	1.43	19.90	1.16	1.15	8.59	13.34	9.80	0.20
1977.1	0.58	1.40	19.93	1.20	1.19	8.74	13.79	8.58	-0.91
1977.2	0.58	1.39	20.75	1.23	1.24	9.00	14.44	8.97	-0.48
1977.3	0.58	1.39	21.38	1.23	1.25	9.38	14.59	9.08	-1.11
1977.4	0.59	1.42	21.54	1.24	1.30	8.91	15.08	9.43	-0.30
1978.1	0.59	1.45	21.46	1.30	1.29	9.29	15.91	8.31	-0.66
1978.2	0.60	1.48	21.23	1.32	1.32	8.65	16.26	8.54	1.27
1978.3	0.60	1.49	22.71	1.33	1.33	8.78	16.40	8.63	3.72
1978.4	0.60	1.49	23.20	1.32	1.39	8.82	17.12	8.98	6.19
1979.1	0.60	1.47	24.20	1.41	1.36	8.47	18.06	8.01	8.62
1979.2	0.59	1.45	24.10	1.40	1.41	8.47	18.67	8.24	9.68
1979.3	0.59	1.41	25.00	1.44	1.44	8.41	19.22	8.39	10.45
1979.4	0.58	1.38	25.62	1.48	1.48	9.55	19.84	8.63	11.02
1980.1	0.58	1.40	26.72	1.48	1.52	10.16	20.49	11.80	15.07
1980.2	0.59	1.45	27.67	1.52	1.55	10.00	21.08	12.13	14.96
1980.3	0.59	1.45	28.57	1.54	1.61	9.82	21.66	12.57	15.25
1980.4	0.60	1.48	29.68	1.62	1.62	9.64	22.25	12.72	14.96

c_{2c} = user cost c_2 with constant price expectations
c_{2a} = user cost c_2 with adaptive price expectations

APPENDIX VI

ARIMA-models of the Explanatory Variables (Output and prices) Used in the Quarterly Investment Equations of the Aggregate Sector

In order to construct the ARIMA-models of explanatory variables we have used the three-stage approach suggested by Box and Jenkins. The steps are: identification, estimation and diagnostic checking.

Identification is the procedure for obtaining an approximate idea of the structure of the model, i.e. the degree of (p,d,q) of an ARIMA(p,d,q) model. This is done by studying the autocorrelation and partial autocorrelation functions of the variables. Since most time series also contain a trend and are thus nonstationary, we have first eliminated the trend by successive first differences of the logs of the variables. The autocorrelograms and partial autocorrelograms of the log differences, together with the respective correlation coefficients, are then used to judge the degree of ARIMA (p,q)-models.

The results suggest that the log-differences of the following variables are (almost) white noise:

- c_1 = first variant of the nominal user cost
- c_{2c} = second variant of the nominal user cost with constant price expectations
- w/c_{2c}
- w/c_1

The 'white noise' property of the first log-differences, in turn, implies that the expected future changes of the corresponding variables are almost constant (or zero in the case where there is no trend). This 'white noise' property is often used to justify the 'static' expectations hypothesis, i.e. that the one-period forecast of a variable (X^e_{t+1}) is equal to the current value (X_t) of that variable, i.e. $X^e_{t+1} = X_t$ or $X^e_t = X_{t-1}$. Hence it can be argued that our estimated models which include $(w/c_1)_t$ or $(w/c_1)_{t-1}$ as a price variable correspond to the ARIMA-model of (w/c_1).

The ML-estimation results of other explanatory variables are:*)

i) output (Q): ARIMA (2,1,0)

$$\underset{(0.110)}{(1 + 0.152\,B} + \underset{(0.112)}{0.362\,B^2)}\bar{Q}_t = u_t, \quad Q_1(10) = 4.13$$

Q_1 indicates the Box-Pierce statistics for 17 lags (in parentheses the respective degrees of freedom, $\bar{Q}_t = (1-B)\log Q_t$).

* B is a backward-shift operator ($BX_t = X_{t-1}$); standard errors are in parentheses.

ii) rate of inflation (g): ARIMA (0,1,4)

$$\bar{g}_t = (1 + \underset{(0.065)}{0.146}\, B^2 - \underset{(0.063)}{0.789}\, B^4)u_t, \quad Q_1(10) = 11.47$$

where $\bar{g}_t = (1-B)\log g_t$

iii) nominal wage rate (w): ARIMA (0,2,1)

$$\log w_t - 2\log w_{t-1} + \log w_{t-2} = (1 - \underset{(0.072)}{0.785}\, B)u_t, \quad Q_1(11) = 11.36$$

iv) real wage rate (w/p): ARIMA (2,1,0)

$$(1 + \underset{(0.100)}{0.264}\, B + \underset{(0.101)}{0.453}\, B^2)(\frac{\bar{w}}{p}) = \underset{(0.003)}{0.02} + u_t, \quad Q_1(9) = 6.53$$

where $(\frac{\bar{w}}{p}) = (1-B)\log(\frac{w}{p})_t$

v) real user cost (c_1/p): ARIMA (0,1,1)

$$(\frac{\bar{c}_1}{p})_t = (1 - \underset{(0.107)}{0.324}\, B)u_t, \quad Q_1(14) = 8.21$$

where $(\frac{\bar{c}_1}{p}) = (1-B)\log(\frac{c_1}{p})_t$

The simplest forecast values of the variables are obtained by using either the fitted values of the ARIMA-models or generating one-period (t+1) forecasts with these models.

Table A18. Regression Models for Output in All Sectors (annual and quarterly data)

Eq. No./ Sector	(1/M)	(2/M)	(3/R)	(4/R)	(5/A)	(6/A)	(7/A)	(8/A)
constant	-6122.12	-	-7396.46	-7036.17	-8483.10	-	-1955.11	-3214.48
			(2.61)	(2.51)	(2.30)		(4.73)	(5.26)
X_t	257.33	347.80			145.22	268.52		
	(10.98)	(6.31)			(1.77)	(1.90)		
X_{t-1}			164.48	158.87				
			(2.31)	(2.03)				
X_{t-4}							81.82	105.68
							(7.00)	(6.82)
G_t			2.34	2.35	4.22	5.64	2.06	2.51
			(2.48)	(2.40)	(4.33)	(3.54)	(4.82)	(4.07)
P_t	26.19	33.44	29.82	29.45	52.41	89.80	5.92	9.73
	(7.94)	(2.58)	(2.26)	(2.13)	(3.29)	(4.04)	(3.57)	(3.68)
L_t	54.06				106.96		70.64	
	(4.00)				(4.64)		(6.84)	
RM1*		-21.74		-8.74		-54.95		-15.50
		(0.42)		(0.22)		(0.71)		(1.96)
W_t		-27.74	-258.15	-257.09	-384.86	-631.03	-39.26	-59.67
		(0.69)	(2.40)	(2.30)	(3.22)	(3.56)	(3.29)	(3.44)
R^2	0.989	0.977	0.978	0.976	0.994	0.985	0.990	0.977
SEE	619.9	899.6	757.1	786.5	825.1	1350.2	300.8	407.6
DW	2.15	1.45	1.50	1.44	1.98	1.95	1.44	0.71

Notes: Equations 1-6 are estimated from annual data and equations 7-8 from quarterly data. Variables X, G, P, L (or RM1) and W are used as instruments in the TSLS estimation of linear investment equations.

X = industrial production in five important countries for Finnish exports (volume index)
G = public consumption in Finland (at 1975 prices)
P = price index of imports of fuel products
L = absolute change in the volume of credit granted by commercial banks
W = index of the negotiated wage rate
RM1* = proxy variable for credit rationing (marginal interest rate on central bank debt, see table A 23); in equations 1-6 RM1 is lagged by one year and in equations 7-8 it is lagged by four quarters.

Table A19. Annual Estimation Results of Linear Investment Equations for All Sectors with Alternative Proxies for the Demand Variable (eq. 5.17ii, Z = proxy for demand index)

Eq. No./ Sector	(1/M)	(2/M)	(3/M)	(4/M)	(5/M)	(6/R)	(7/R)	(8/R)	(9/R)	(10/A)	(11/A)	(12/A)	(13/A)	(14/A)	(15/A)
constant	-1.88	7.92	5.50	1.12	2.65	4.95	2.50	0.55	2.02	2.42	14.15	11.97	12.48	5.23	6.52
	(1.18)	(7.51)	(6.29)	(1.32)	(3.13)	(3.02)	(2.00)	(0.48)	(0.83)	(0.28)	(1.60)	(1.83)	(2.68)	(1.52)	(2.09)
X_{-1}	0.18					-0.11				0.20					
	(5.43)					(2.15)				(1.82)					
M_{-1}		0.14									0.09				
		(5.53)									(1.13)				
S_{-1}			0.35									0.34			
			(5.98)									(2.89)			
GDP_{-1}				0.24			0.11						0.46		
				(8.08)			(1.53)						(5.40)		
CIX_{-1}					0.18										
					(7.01)										
CI_{-1}								0.16	0.17					0.43	0.32
								(3.37)	(3.15)					(8.10)	(4.60)
(w/p)	0.31	0.52	0.32	0.26	0.45	0.19	0.65	0.46	0.49	0.20	2.99	2.36	1.17	1.27	0.74
	(1.08)	(1.84)	(1.18)	(1.33)	(1.94)	(5.59)	(1.55)	(1.72)	(1.60)	(1.83)	(3.30)	(3.17)	(1.87)	(2.95)	(1.20)
(c_1/p)									-0.40						-0.73
									(1.91)						(2.52)
(c_2/p)	-0.08	-0.15	-0.09	-0.09	-0.10	-0.17	-0.20	-0.13		-0.40	-0.63	-0.65	-0.84	-0.50	-
	(2.57)	(4.98)	(3.26)	(3.97)	(3.93)	(3.43)	(3.54)	(2.93)		(1.15)	(1.50)	(3.26)	(3.42)	(2.85)	
K_{-1}	-0.35	-0.56	-0.39	-0.51	-0.60	-0.24	-0.22	-0.23	-0.22	-0.37	-0.50	-0.48	-0.53	-0.48	-0.35
	(2.93)	(4.28)	(3.44)	(5.46)	(5.44)	(7.20)	(6.37)	(8.64)	(6.78)	(2.61)	(3.65)	(4.53)	(6.71)	(8.34)	(5.81)
R^2	0.82	0.82	0.84	0.90	0.87	0.94	0.93	0.95	0.94	0.80	0.77	0.84	0.92	0.96	0.90
SEE	490.1	484.8	457.9	361.6	405.8	402.2	436.5	346.4	394.9	1194.1	1281.0	1046.9	745.1	545.6	851.4
DW	1.96	2.82	2.29	2.12	2.20	1.56	1.66	2.29	2.47	0.78	0.87	0.94	1.12	1.84	1.58

Notes: The proxy variables used for the expected demand variable (Z^e) are as follows:

X = volume index of industrial production in five countries (Sweden, W-Germany, United Kingdom, the USA, France) important for Finnish exports (see table A18).

M = volume index of imports of the five countries as in X.

S = volume of sales (for the aggregate sector S has been constructed by J. Pesola)

GDP = volume of gross domestic production (Q) of total Finnish economy

CIX = volume of total aggregate demand in Finland (i.e. consumption + investment + exports)

CI = consumption + investment (compare CIX)

Table A20. Annual Estimation Results of Investment Equations for All Sectors (eqs. 5.19, and 5.20)

Eq. No./ Sector	(1/M)	(2/M)	(3/M)	(4/M)	(5/R)	(6/R)	(7/R)	(8/R)	(9/A)	(10/A)	(11/A)	(12/A)
Dependent Variable	$\Delta\log K$	I/K_{-1}	$\Delta\log K$	I/K_{-1}	$\Delta\log K$	I/K_{-1}	$\Delta\log K$	I/K_{-1}	$\Delta\log K$	I/K_{-1}	$\Delta\log K$	I/K_{-1}
constant	1.226	1.281	0.962	0.933	0.878	1.007	0.903	1.192	1.030	1.094	1.151	1.367
	(4.44)	(4.19)	(1.44)	(1.26)	(2.61)	(2.87)	(1.82)	(2.33)	(4.44)	(4.30)	(1.96)	(2.13)
$\log Q_{-1}$	0.310	0.338	0.311	0.339	0.216	0.241	0.214	0.220	0.280	0.314	0.272	0.296
	(6.11)	(6.02)	(5.95)	(5.88)	(4.42)	(4.73)	(3.34)	(3.34)	(8.77)	(8.97)	(5.78)	(5.79)
$\log (w/c_1)$	0.029	0.028			0.093	0.104			0.062	0.064		
	(1.55)	(1.31)			(2.63)	(2.84)			(2.50)	(2.35)		
$\log (w/p)$			0.005	0.018			0.096	0.131			0.077	0.099
			(0.07)	(0.19)			(1.53)	(2.03)			(1.05)	(1.24)
$\log (c_1/p)$			-0.034	-0.035			-0.091	-0.091			-0.059	-0.058
			(1.12)	(1.01)			(2.05)	(2.00)			(2.15)	(1.94)
$\log K_{-1}$	-0.404	-0.427	-0.370	-0.383	-0.278	-0.306	-0.279	-0.314	-0.349	-0.379	-0.356	-0.396
	(6.31)	(6.03)	(3.63)	(3.44)	(9.39)	(9.92)	(8.28)	(9.02)	(10.83)	(10.74)	(7.53)	(7.69)
R^2	0.75	0.72	0.74	0.71	0.85	0.86	0.84	0.86	0.88	0.87	0.87	0.87
SEE	0.011	0.012	0.011	0.012	0.007	0.007	0.007	0.006	0.006	0.007	0.006	0.007
DW	1.98	1.85	1.98	1.86	2.41	2.64	2.39	2.41	1.98	1.93	1.95	1.89

Notes: In all equations Q_{-1} is used as a proxy for Z_t^e and Q_t^e (see eqs. 5.19 and 5.20)

Table A21. SURE-Estimation Results for the Manufacturing and Residual Sectors

Eq. No./ Sector	(1/M)	(2/M)	(3/M)	(4/M)	(5/R)	(6/R)	(7/R)	(8/R)
constant	2.296	2.616	4.007	4.844	-1.812	-0.836	3.929	1.532
	(4.49)	(5.64)	(3.63)	(6.99)	(4.66)	(1.31)	(2.12)	(1.73)
Q_{-1}	0.626	0.634	0.630	0.548	0.478	0.700	0.473	0.423
	(7.03)	(7.10)	(6.92)	(7.69)	(5.59)	(8.91)	(3.95)	(3.81)
(w/c_1)	1.691				5.396			
	(1.65)				(4.16)			
(w/c_2)		0.018				0.395		
		(1.21)				(1.56)		
(w/p)			-0.155	0.277			0.547	0.550
			(0.68)	(1.30)			(2.33)	(2.56)
(c_1/p)			-0.123				-0.524	
			(1.33)				(3.03)	
(c_2/p)				-0.086				-0.140
				(3.71)				(3.80)
K_{-1}	-0.319	-0.283	-0.218	-0.392	-0.182	-0.193	-0.193	-0.210
	(5.29)	(5.54)	(2.31)	(4.41)	(8.60)	(7.01)	(7.75)	(9.64)
R^2	0.80	0.81	0.79	0.87	0.95	0.92	0.95	0.96
(1) SEE/SURE	456.2	453.1	461.1	363.7	310.6	408.2	323.8	296.3
(2) SEE/OLS	517.2	513.8	542.5	427.8	352.3	450.1	381.1	349.9
(1)/(2)	0.88	0.88	0.85	0.85	0.88	0.91	0.85	0.85

Table A22. Tests for the Equality of Coefficients in the Manufacturing and Residual Sectors (annual data, SURE-estimation)

1. Equation $I = a_0 + a_1Q_{-1} + a_2(w/c_1) + a_3K_{-1}$

	F	F-critical: 5 %	1 %
Equality of a_1	1.41		
Equality of a_2	4.91	4.20	7.64
Equality of a_3	4.59		
All a_i equal	32.92	2.95	4.57

2. Equation $I = a_0 + a_1Q_{-1} + a_2(w/c_2) + a_3K_{-1}$

	F	F-critical: 5 %	1 %
Equality of a_1	0.051		
Equality of a_2	0.147	4.20	7.64
Equality of a_3	10.05		
All a_i equal	26.33	2.95	4.57

3. Equation $I = b_0 + b_1Q_{-1} + b_2(w/p) + b_3(c_1/p) + b_4K_{-1}$

	F	F-critical: 5 %	1 %
Equality of b_1	1.11		
Equality of b_2	4.56		
Equality of b_3	4.37	4.22	7.72
Equality of b_4	0.082		
All b_i equal	16.03	2.74	4.14

4. Equation $I = b_0 + b_1Q_{-1} + b_2(w/p) + b_3(c_2/p) + b_4K_{-1}$

	F	F-critical: 5 %	1 %
Equality of b_1	0.59		
Equality of b_2	0.30		
Equality of b_3	0.95	4.22	7.72
Equality of b_4	5.44		
All b_i equal	32.50	2.74	4.14

Table A23. The Effect of Credit Rationing in the Annual Investment Equations for All Sectors

Eq. No./ Sector	(1/M)	(2/M)	(3/M)	(4/R)	(5/R)	(6/R)	(7/A)	(8/A)	(9/A)
constant	2.242	4.905	4.780	-1.764	3.117	2.142	0.247	6.846	14.795
	(3.36)	(2.78)	(5.14)	(3.85)	(1.25)	(2.05)	(0.32)	(2.36)	(4.06)
Q						0.256			
						(1.61)			
Q_{-1}	0.621	0.611	0.554	0.485	0.437		0.614	0.620	0.643
	(5.80)	(5.48)	(5.49)	(4.86)	(2.88)		(9.50)	(5.93)	(8.12)
(w/c_1)	1.600			5.368			6.847		
	(1.22)			(3.56)			(3.14)		
(w/p)		0.180	0.257		0.663	0.758		0.598	1.199
		(0.66)	(0.90)		(2.04)	(2.10)		(1.06)	(2.64)
(c_1/p)		-0.181			-0.472			-0.553	
		(1.35)			(2.14)			(2.13)	
(c_2/p)			-0.083			-0.146			-0.727
			(2.63)			(4.14)			(4.05)
RM1	5.29								
	(0.18)								
$(RM1-r)_{-1}$		-24.68			15.97				12.05
		(0.77)			(0.72)				(0.44)
$RM2_{-1}$				-9.23			-13.02		
				(0.61)			(0.45)		
$(RM2-r)_{-1}$			3.81			44.94		-9.83	
			(0.17)			(2.31)		(0.23)	
K_{-1}	-0.315	-0.199	-0.386	-0.182	-0.203	-0.200	-0.264	-0.273	-0.459
	(4.20)	(1.72)	(3.40)	(7.44)	(6.15)	(6.87)	(8.89)	(5.21)	(8.02)
R^2	0.78	0.77	0.85	0.95	0.94	0.95	0.94	0.93	0.96
SEE	536.1	551.2	444.7	360.5	388.3	373.5	637.6	724.5	543.2
DW	1.83	1.78	2.49	2.52	2.44	2.28	1.89	1.69	1.97

Notes: See Appendix V; RM1 and RM2 are marginal interest rates on the central bank borrowing of commercial banks and r is the average lending rate of these banks.

Table A24. The Effect of Technical Change in the Annual Investment Equations for All Sectors

Eq. No./ Sector	(1/M)	(2/M)	(3/M)	(4/M)	(5/R)	(6/R)	(7/R)	(8/R)	(9/A)	(10/A)	(11/A)	(12/A)
constant	2.234	-1.706	9.234	8.764	-2.700	-0.297	-7.146	-6.171	0.556	1.492	3.236	8.529
	(0.93)	(0.78)	(0.85)	(2.70)	(2.31)	(0.12)	(1.53)	(2.41)	(0.25)	(0.31)	(0.24)	(1.86)
Q_{-1}	0.627	0.745	0.584	0.465	0.502	0.694	0.202	0.209	0.611	0.833	0.609	0.634
	(5.35)	(6.98)	(4.01)	(4.39)	(4.91)	(6.60)	(1.29)	(1.72)	(7.66)	(7.01)	(5.50)	(9.13)
(w/c_1)	1.653				5.481				6.617			
	(0.87)				(3.67)				(2.91)			
(w/c_2)		0.026				0.429				1.172		
		(2.31)				(1.57)				(1.21)		
(w/p)			-0.302	0.319			1.658	1.452			0.832	1.535
			(0.74)	(1.38)			(3.39)	(4.22)			(0.79)	(3.55)
(c_1/p)			-0.311				-0.053				-0.387	
			(0.77)				(0.21)				(0.64)	
(c_2/p)				-0.116				-0.819				-0.659
				(3.23)				(2.15)				(4.12)
T	-8.29	-15.22	381.06	110.67	-94.49	-16.39	-641.03	-203.90	41.71	-65.66	-191.65	-165.18
	(0.03)	(2.03)	(0.48)	(1.25)	(0.82)	(0.24)	(2.58)	(3.17)	(0.18)	(0.52)	(0.25)	(1.97)
K_{-1}	-0.315	-0.139	-0.331	-0.528	-0.155	-0.210	-0.072	-0.101	-0.272	-0.308	-0.259	-0.399
	(1.67)	(1.58)	(1.28)	(3.53)	(3.74)	(3.64)	(1.41)	(2.57)	(4.64)	(2.50)	(3.16)	(6.91)
R^2	0.78	0.83	0.76	0.87	0.95	0.91	0.96	0.97	0.94	0.91	0.93	0.97
SEE	536.7	464.7	559.2	419.0	356.5	466.0	318.1	267.3	642.0	781.7	725.2	475.9
DW	1.85	0.95	1.79	2.76	2.90	2.59	2.06	2.35	2.02	1.69	1.74	2.28

Notes: T = linear trend variable (1963 = 1, 1964 = 2, ...)

Table A25. Tests for the Replacement Hypothesis in the Annual Investment Equations for All Sectors

Eq. No./ Sector	(1/M)	(2/M)	(3/R)	(4/R)	(5/A)	(6/A)
constant	4.287	5.556	5.087	2.365	8.353	17.781
	(3.07)	(5.67)	(2.28)	(2.26)	(2.91)	(3.48)
Q_{-1}	0.624	0.550	0.506	0.452	0.666	0.719
	(5.43)	(6.29)	(3.40)	(3.39)	(5.84)	(6.81)
(w/p)	-0.269	0.242	0.552	0.568	0.390	0.961
	(1.05)	(0.93)	(1.89)	(2.21)	(0.62)	(1.63)
(c_1/p)	-0.137		-0.616		-0.641	
	(1.19)		(2.92)		(2.43)	
(c_2/p)		-0.101		-0.169		-0.819
		(3.22)		(3.84)		(3.19)
K^1_{-1}	-0.175	-0.399	-0.213	-0.239	-0.271	-0.486
	(1.57)	(3.46)	(6.14)	(8.05)	(4.42)	(5.49)
R^2	0.75	0.85	0.94	0.95	0.91	0.93
SEE	566.4	444.7	401.8	353.9	800.2	722.7
SEE*	542.5	427.8	381.1	349.9	698.5	525.9
DW	1.53	2.39	2.47	2.31	1.56	1.34

Notes: The net capital stock K^1 is calculated by increasing the rate of depreciation (δ) by 0.5 percentage point from 1974 onwards as compared to the original value of δ. The average original values of δ are: δ_M = 7.8 %, δ_R = 7.5 %, δ_A = 7.6 % (see Appendix V). SEE* is the standard error of the regression with the original K-variables.

Table A26. Quarterly Investment Equations for the Aggregate Sector with Alternative Lags for the Output Variable

Variables and statistics:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					Equation No.								
Constant	-0.199	-0.160	-0.109	0.098	0.108	0.252	0.084	1.989	1.903	1.774	1.647	1.480	1.359
	(1.06)	(0.90)	(0.70)	(0.68)	(0.82)	(1.82)	(0.60)	(10.02)	(9.61)	(9.30)	(8.73)	(7.83)	(6.42)
Q_t	0.411						0.154	0.223					
	(8.57)						(1.16)	(5.02)					
Q_{t-1}		0.452					-0.314		0.240				
		(9.46)					(1.71)		(5.25)				
Q_{t-2}			0.517				0.209			0.295			
			(11.39)				(1.14)			(6.21)			
Q_{t-3}				0.573			0.025				0.348		
				(12.53)			(0.14)				(6.80)		
Q_{t-4}					0.649		0.579					0.417	
					(14.25)		(3.91)					(7.43)	
Q_{t-5}						0.708							0.439
						(3.11)							(6.50)
(w/c_1)	2.895	2.744	2.467	2.032	1.665	1.208	1.696						
	(6.30)	(6.32)	(6.36)	(5.54)	(4.92)	(3.27)	(4.62)						
(w/p)								0.312	0.319	0.289	0.269	0.253	0.294
								(4.29)	(4.55)	(4.33)	(4.14)	(4.05)	(4.55)
(c_2/p)								-0.074	-0.070	-0.064	-0.058	-0.050	-0.044
								(11.03)	(10.43)	(9.75)	(8.66)	(7.23)	(5.59)
K_{-1}	-0.055	-0.058	-0.063	-0.064	-0.069	-0.072	-0.070	-0.061	-0.063	-0.065	-0.066	-0.070	-0.077
	(7.35)	(8.11)	(9.74)	(10.74)	(12.31)	(11.55)	(12.20)	(7.73)	(8.18)	(8.80)	(9.45)	(10.42)	(10.59)
R^2	0.80	0.82	0.85	0.87	0.89	0.88	0.89	0.90	0.90	0.91	0.92	0.93	0.91
SEE	304.7	288.7	257.6	241.7	220.1	234.0	220.3	218.3	215.5	204.1	197.0	189.6	200.6
DW	0.69	1.03	1.05	1.28	1.21	1.44	1.03	0.90	1.14	1.13	1.27	1.22	1.41

Table A27. Quarterly Estimation Results of Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15 and 5.17i with w/c as the price variable)

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	-0.240	1.383	0.177	0.235	-0.072	0.097	-0.164	-0.166	0.437	0.543	1.417	2.446
	(1.26)	(1.67)	(1.29)	(1.67)	(0.39)	(0.57)	(0.76)	(0.77)	(3.91)	(4.14)	(1.74)	(1.54)
Q^e	0.447	0.366	0.735	0.792	0.483	0.756	0.309	0.310	0.694	0.647	0.364	0.119
	(8.58)	(3.68)	(14.54)	(16.18)	(9.58)	(9.94)	(6.49)	(6.51)	(7.17)	(12.34)	(3.68)	(1.21)
$(w/c)^e$	2.918	0.870	1.353	1.288	2.441	1.281	2.973	2.954	0.331	0.131	0.021	0.014
	(6.32)	(0.19)	(3.69)	(2.88)	(5.15)	(3.33)	(5.70)	(5.71)	(0.24)	(2.69)	(0.44)	(0.30)
CF^e						0.075						
						(1.52)						
K_{-1}	-0.058	-0.030	-0.077	-0.082	-0.059	-0.077	-0.042	-0.044	-0.058	-0.055	-0.029	-0.009
	(7.52)	(2.08)	(13.45)	(11.83)	(7.44)	(8.96)	(5.51)	(5.51)	(9.68)	(9.18)	(2.18)	(0.55)
R^2	0.786	0.913	0.900	0.887	0.803	0.904	0.745	0.746	0.851	0.861	0.913	0.900
SEE	305.9	191.6	209.5	216.7	302.0	207.0	345.2	344.9	256.3	243.8	191.7	205.8
DW	0.68	2.06	1.23	1.21	0.66	1.35	0.65	0.65	0.77	0.91	2.08	2.00
D-m	0.70	-	-	-	0.71	-	0.69	0.70	0.67	0.51	-	-
	(6.89)				(7.27)		(7.00)	(7.17)	(6.11)	(4.96)		
RHO_1	-	0.52									0.52	0.55
		(4.44)									(4.41)	(4.80)
RHO_2	-	0.35									0.35	0.37
		(2.98)									(3.01)	(3.33)
EST	TSLS	CO	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	CO	CO
Definitions of variables	Q_0	Q_4	Q_A	Q_A	Q_{AR}	Q_A	Q_F	Q_F	Q_4	Q_4	Q_4	Q_1
	wc_0	wc_0	wc_0	wc_A	wc_{AR}	wc_0	wc_0	wc_F	wc_2	wc_2	wc_2	wc_2
						CF_4^n						

Notes: See table 24 for abbreviations of variables. In equations 11 and 12 c_2 is the value of c_2 which is positive for all t. In equations 2, 11 and 12, an AR(2) process is assumed for the disturbance.

Table A28. Quarterly Estimation Results of Log-Linear Investment Equations for the Aggregate Sector ($\Delta \log K$ and I/K - equations)

Variables and statistics:	(1) $\Delta\log K$	(2) $\Delta\log K$	(3) I/K_{-1}	(4) $\Delta\log K$	(5) I/K_{-1}	(6) $\Delta\log K$	(7) I/K_{-1}	(8) I/K_{-1}	(9) $\Delta\log K$	(10) I/K_{-1}	(11) I/K_{-1}	(12) I/K_{-1}	(13) I/K_{-1}	(14) I/K_{-1}	(15) I/K_{-1}
constant	0.385	0.381	0.395	0.291	0.302	0.454	0.488	0.441	0.604	0.645	0.700	0.234	0.482	0.763	0.784
	(9.27)	(7.09)	(9.24)	(13.30)	(13.51)	(5.59)	(5.86)	(4.04)	(7.27)	(7.70)	(8.39)	(2.36)	(5.02)	(9.59)	(9.42)
$\log Q^e$	0.075	0.044	0.079	0.071	0.075	0.072	0.075	0.045	0.052	0.055	0.039	0.078	0.062	0.092	0.032
	(12.94)	(5.64)	(12.93)	(10.99)	(11.34)	(9.97)	(10.11)	(4.98)	(6.95)	(7.19)	(6.65)	(9.13)	(7.44)	(6.56)	(5.73)
$\log(w/c_1)^e$	0.016	0.027	0.016												
	(3.81)	(5.23)	(3.65)												
$\log(w/c_2)^e$				0.0016	0.0016										
				(3.66)	(3.59)										
$\log(w/p)^e$						0.026	0.029	0.033	0.041	0.045	0.052	0.022	0.042	0.062	0.065
						(2.47)	(2.70)	(2.48)	(4.03)	(4.35)	(5.04)	(1.96)	(4.06)	(6.52)	(6.55)
$\log(c_1/p)^e$						-0.015	-0.014	-0.026				-0.012			
						(3.32)	(3.10)	(4.33)				(2.62)			
$\log(c_2/p)^e$									-0.0021	-0.0022	-0.0033		-0.0023	-0.0035	-0.0035
									(5.09)	(5.19)	(8.83)		(5.87)	(9.15)	(8.45)
$\log CF^e$		0.0018						0.0017							
		(2.10)						(1.86)							
$\log K_{-1}$	-0.095	-0.069	-0.097	-0.082	-0.085	-0.099	-0.104	-0.076	-0.103	-0.109	-0.109	-0.091	-0.103	-0.156	-0.107
	(14.60)	(7.78)	(14.57)	(13.67)	(13.77)	(11.89)	(12.12)	(6.19)	(13.33)	(13.81)	(13.80)	(10.41)	(13.32)	(13.75)	(12.49)
R^2	0.76	0.67	0.76	0.76	0.76	0.76	0.76	0.66	0.80	0.80	0.79	0.76	0.81	0.79	0.77
SEE	0.0023	0.0027	0.0024	0.0024	0.0024	0.0023	0.024	0.0028	0.0021	0.0021	0.0022	0.0024	0.0021	0.0022	0.0023
DW	1.15	1.16	1.11	1.00	1.01	1.08	1.05	1.08	1.06	1.10	1.32	1.72	1.83	1.41	1.40
D-m	0.42	0.41	0.45	0.51	0.51	0.46	0.47	0.49	0.48	0.46	0.34	0.14	0.07	0.30	0.32
	(3.61)	(3.52)	(3.88)	(4.75)	(4.73)	(3.93)	(3.99)	(3.97)	(4.29)	(4.04)	(2.85)	(1.10)	(0.57)	(2.43)	(2.58)
LM(4)	16.42	15.14	16.56	23.98	23.47	16.89	17.12	21.31	19.51	18.07	10.51	10.96	4.25	6.98	10.87
Definitions	Q_4	Q_1	Q_4	Q_4	Q_4	Q_4	Q_4	Q_1	Q_4	Q_4	Q_1	CI_1	CI_1	C_1	S_4
of						$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$
variables	wc_0	wc_0	wc_0	wc_2	wc_2	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$
		CF_4^n						CF_4^n							

Notes: See table 24 for abbreviations of variables. Variables CI, C and S are used as proxies for the demand variable Z^e and the coefficient estimates of these variables are prsented in the $\log Q^e$ row in table A28. CI = consumption + investment of the Finnish economy, C = consumption and S = sales of the aggregate sector (constructed by J. Pesola), see also tables A30 and A31.

Table A29. Quarterly Estimation Results of Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15 and 5.17i; w/p and c_1/p as price variables)

Variables and statistics:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
constant	1.621	2.730	2.764	0.766	0.650	3.003	3.103	2.259	2.198	2.110
	(4.10)	(4.77)	(4.43)	(0.59)	(1.55)	(4.51)	(5.02)	(3.37)	(3.71)	(2.76)
Q^e	0.651	0.516	0.436	0.271	0.824	0.583	0.389	0.233	0.128	0.235
	(11.10)	(6.16)	(4.89)	(2.70)	(9.55)	(7.61)	(4.26)	(3.31)	(1.93)	(3.31)
$(w/p)^e$	0.159	0.188	0.225	0.153	0.174	0.027	0.058	0.394	0.293	0.352
	(2.21)	(1.71)	(1.87)	(2.41)	(2.71)	(0.20)	(0.45)	(3.29)	(2.70)	(3.11)
$(c_1/p)^e$	-0.138	-0.243	-0.247	0.070	-0.050	-0.254	-0.308	-0.198	-0.253	-0.210
	(3.60)	(4.54)	(4.20)	(1.56)	(1.09)	(4.07)	(5.13)	(3.14)	(4.18)	(2.96)
CF^e					0.242		0.381		0.441	
					(1.62)		(3.42)		(4.43)	
K_{-1}	-0.069	-0.060	-0.056	-0.043	-0.099	-0.038	-0.031	-0.057	-0.035	-0.060
	(8.70)	(5.36)	(4.71)	(2.59)	(9.63)	(2.50)	(2.17)	(4.31)	(2.76)	(4.50)
R^2	0.88	0.78	0.75	0.92	0.91	0.78	0.82	0.72	0.78	0.74
SEE	232.9	312.5	328.1	181.9	195.1	320.9	297.9	361.9	320.1	350.7
DW	1.07	0.94	0.60	2.03	1.15	0.54	0.76	0.52	0.85	0.66
D-m	0.46	0.56	0.77	-	-	0.76	0.66	-	-	-
	(4.11)	(5.00)	(8.42)			(8.41)	(6.11)			
RHO_1				0.61						
				(5.10)						
RHO_2				0.31						
				(2.61)						
EST	OLS	TSLS	TSLS	CO	OLS	OLS	OLS	OLS	OLS	OLS
Definitions of variables	Q_4	Q_1	Q_1	Q_4	Q_A	Q_{AR}	Q_{AR}	Q_F	Q_F	Q_F
	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_{AR}$	$\bar{w}_{AR}$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_F$
	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_{1,AR}$	$\bar{c}_{1,AR}$	$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_{1,F}$
					CF_4^g		CF_4^g		CF_4^g	

Notes: See table 24 for the abbreviations of variables. In equation 1 the price variable p is the price index of the total GDP of the Finnish economy, in all other equations p is the price index of the output of the aggregate firm sector (A).

Table A30. Quarterly Investment Equations for the Aggregate Sector with Alternative Proxies for the Demand Variable (eq. 5.17ii, c_1/p as the user cost, Z = proxy for demand index)

Variables and statistics:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Equation No.				
constant	0.258	0.397	0.174	0.111	0.119	0.117	0.027	0.470
	(0.54)	(4.55)	(3.13)	(2.36)	(2.06)	(2.27)	(0.60)	(1.07)
X_{-4}	0.986							
	(8.57)							
M_{-1}		0.485						
		(4.10)						
S_{-4}			0.347					
			(5.43)					
GDP_{-4}				0.419				
				(8.10)				
Y_{-1}					0.467			
					(4.46)			
C_{-1}						0.771		
						(6.16)		
CI_{-1}							0.366	0.308
							(9.80)	(6.73)
CF^{n}_{-4}								0.167
								(2.06)
(w/p)	0.100	0.408	0.389	0.189	0.301	0.325	0.124	0.119
	(1.06)	(3.79)	(4.00)	(2.17)	(2.56)	(3.36)	(1.47)	(1.45)
(c_1/p)	-0.275	-0.270	-0.166	-0.180	-0.157	-0.210	-0.114	-0.134
	(5.93)	(4.00)	(3.20)	(4.07)	(2.88)	(4.08)	(2.93)	(3.41)
K_{-1}	-0.049	-0.082	-0.070	-0.084	-0.070	-0.132	-0.063	-0.050
	(5.02)	(6.23)	(5.98)	(8.24)	(5.66)	(8.46)	(7.05)	(4.69)
R^2	0.84	0.72	0.76	0.82	0.73	0.78	0.86	0.86
SEE	269.5	349.2	325.4	277.5	342.9	312.1	250.3	244.5
DW	1.16	0.66	0.76	1.02	0.89	0.85	1.61	1.62
D-m	0.32	0.74	0.64	0.51	0.74	0.62	0.17	0.19
	(2.92)	(7.56)	(6.18)	(4.31)	(6.28)	(5.63)	(1.30)	(1.32)
LM(4)	24.33	38.66	32.69	18.65	34.70	29.81	11.51	11.38

Notes: See table A19. The proxy variables used for the expected demand variable (Z^e) are as follows:

X = volume index of industrial production in five countries important for the Finnish exports (see table A19)
M = volume index of the imports of the five countries as in X (see table A19)
S = volume of sales (see table A19)
GDP = volume of output (see table A19)
Y = volume of disposable income in the Finnish economy
C = volume of consumption (see table A19)
CI = volume of consumption plus investment (see table A19)

Table A31. Quarterly Investment Equations for the Aggregate Sector with Alternative Proxies for the Demand Variable (c_2/p as the user cost)

Variables and statistics:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Equation No.						
constant	0.780	2.521	1.751	1.135	1.159	1.335	0.737	0.838	1.819	1.945
	(1.85)	(10.03)	(7.74)	(4.92)	(5.13)	(1.24)	(2.68)	(3.16)	(4.34)	(4.78)
X_{-4}	0.352									
	(3.52)									
M_{-1}		0.226								
		(3.53)								
S_{-4}			0.176							
			(3.78)							
GDP_{-4}				0.252	0.180					
				(6.50)	(3.48)					
Y_{-1}						0.199				
						(2.92)				
CI_{-1}							0.229	0.162	0.221	0.160
							(6.43)	(3.88)	(6.04)	(3.74)
CF^n_{-4}					0.145	0.229		0.171		0.158
					(2.05)	(3.82)		(2.73)		(2.49)
(w/p)	0.341	0.412	0.414	0.276	0.277	0.273	0.241	0.239	0.245	0.242
	(4.11)	(5.76)	(5.95)	(4.17)	(4.26)	(3.94)	(3.41)	(3.54)	(3.44)	(5.53)
(c_2/p)	-0.065	-0.078	-0.068	-0.062	-0.063	-0.068	-0.052	-0.054	-0.049	-0.051
	(8.19)	(10.75)	(9.16)	(9.63)	(9.88)	(10.83)	(7.04)	(7.69)	(6.94)	(7.45)
K_{-1}	-0.059	-0.074	-0.069	-0.077	-0.062	-0.050	-0.064	-0.051	-0.071	-0.059
	(6.88)	(8.57)	(8.24)	(10.51)	(6.13)	(5.61)	(8.95)	(6.07)	(9.61)	(6.48)
R^2	0.88	0.87	0.88	0.91	0.91	0.91	0.91	0.92	0.91	0.91
SEE	235.2	235.5	232.8	200.7	196.1	200.8	201.6	192.6	202.8	195.4
DW	0.88	0.91	1.09	1.29	1.32	1.44	1.66	1.64	1.59	1.56
D-m	0.40	0.56	0.47	0.36	0.34	0.30	0.18	0.19	0.21	0.23
	(3.11)	(5.35)	(4.21)	(3.03)	(2.86)	(2.40)	(1.40)	(1.45)	(1.69)	(1.82)
LM(4)	21.76	23.83	18.86	10.73	10.15	8.21	7.41	6.26	8.86	7.99

Notes: See table A30 for the definitions of variables. In equations 9 and 10 c_2 is the variant which is positive for all t (see section 5.6.3.1)

Table A32. ARMA-Estimation Results for Selected OLS-Residuals in the Aggregate Sector

Form of the Residual Process	AR(1)	ARMA(1,1)	ARMA(1,2)	AR(2)	ARMA(2,1)
Type of Equation:					
$(Q_{-4}, w/c_1)$	SEE 2.86	SEE 2.81	SEE 2.79	SEE 2.81	SEE 2.81
$\chi^2_{14} = 38.27$	A1 0.38	A1 0.62	A1 0.56	A1 0.33	A1 0.47
	(3.32)	(2.63)	(1.71)	(2.67)	(0.52)
	χ^2_{13} 18.19	M1 0.27	M1 0.26	A2 0.13	A2 0.09
		(0.95)	(0.78)	(1.08)	(0.23)
		χ^2_{12} 18.10	M2 -0.11	χ^2_{12} 18.10	M1 0.14
			(0.68)		(0.16)
			χ^2_{11} 17.82		χ^2_{11} 18.14
$(Q_{-4}, w/c_2)$	SEE 2.96	SEE 2.83	SEE 2.73	SEE 2.86	SEE 2.83
$\chi^2_{14} = 59.71$	A1 0.55	A1 0.78	A1 0.69	A1 0.45	A1 0.66
	(5.19)	(5.94)	(4.71)	(3.61)	(1.21)
	χ^2_{13} 34.89	M1 0.33	M1 0.45	A2 0.20	A2 0.10
		(1.71)	(2.97)	(1.62)	(0.30)
		χ^2_{12} 32.54	M2 -0.40	χ^2_{12} 35.90	M1 0.23
			(3.32)		(0.43)
			χ^2_{11} 26.18		χ^2_{11} 32.92
$(Q_{-1}, w/p, c_1/p)$	SEE 4.44	SEE 3.99	SEE 3.82	SEE 3.88	SEE 3.86
$\chi^2_{14} = 87.40$	A1 0.57	A1 0.86	A1 0.82	A1 0.37	A1 0.48
	(5.63)	(8.69)	(7.08)	(3.21)	(1.51)
	χ^2_{13} 26.61	M1 0.42	M1 0.50	A2 0.36	A2 0.31
		(2.59)	(3.23)	(3.17)	(0.31)
		χ^2_{12} 15.63	M2 -0.21	χ^2_{12} 9.56	M1 0.13
			(1.60)		(0.39)
			χ^2_{11} 8.77		χ^2_{11} 9.62
$(Q_{-4}, w/p, c_1/p)$	SEE 2.91	SEE 2.84	SEE 2.83	SEE 2.84	SEE 2.84
$\chi^2_{14} = 43.49$	A1 0.46	A1 0.70	A1 0.65	A1 0.40	A1 0.57
	(4.11)	(3.83)	(2.60)	(3.15)	(0.77)
	χ^2_{13} 17.49	M1 0.30	M1 0.30	A2 0.16	A2 0.08
		(1.24)	(1.11)	(1.27)	(0.21)
		χ^2_{12} 16.88	M2 -0.10	χ^2_{12} 16.87	M1 0.18
			(0.65)		(0.25)
			χ^2_{11} 16.48		χ^2_{11} 16.48
$(Q_{-4}, w/p, c_2/p)$	SEE 2.06	SEE 2.03	SEE 2.02	SEE 2.02	SEE 2.02
$\chi^2_{14} = 41.82$	A1 0.38	A1 0.60	A1 0.55	A1 0.33	A1 0.45
	(3.49)	(2.57)	(1.58)	(2.76)	(0.47)
	χ^2_{13} 13.72	M1 0.25	M1 0.23	A2 0.13	A2 0.08
		(0.89)	(0.65)	(1.03)	(0.21)
		χ^2_{12} 11.66	M2 -0.09	χ^2_{12} 11.37	M1 0.11
			(0.54)		(0.12)
			χ^2_{11} 10.65		χ^2_{11} 11.26

Notes: In the column 'type of equation' are presented the independent output and price variables of the relevant equations. SEE is the sum of residual squares; A1, A2, M1, M2 refer to the maximum likelihood estimates of the AR- and MA-parameters and t-values are in parentheses. χ^2_p is a test for the hypothesis that the residual series is white noise. SEE should be multiplied by 10^6 to obtain the correct absolute value.

Table A33. Quarterly Estimation Results of Linear Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15 and 5.17ii with w/p and c_2/p as price variables, alternative measures for c_2-variable)

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
constant	2.484	1.818	1.945	1.397	1.357	0.815	1.647	1.846	1.179	1.398	1.275
	(7.73)	(4.34)	(4.78)	(8.60)	(8.29)	(3.16)	(8.29)	(8.80)	(8.07)	(3.87)	(7.57)
Q^e	0.405			0.351	0.272		0.435	0.171	0.414	0.676	0.425
	(7.00)			(6.22)	(3.49)		(8.21)	(3.48)	(7.77)	(11.12)	(7.59)
CI^e		0.221	0.160			0.184					
		(6.04)	(3.74)			(5.05)					
$(w/p)^e$	0.257	0.245	0.242	0.309	0.312	0.310	0.283	0.326	0.245	0.169	0.259
	(4.03)	(3.44)	(3.53)	(5.07)	(5.16)	(4.47)	(4.55)	(5.14)	(4.10)	(2.22)	(4.08)
$(c_2/p)^e$	-0.047	-0.049	-0.051	-0.050	-0.053	-0.052	-0.056	-0.076	-0.075	-0.195	-0.055
	(7.06)	(6.94)	(7.45)	(8.20)	(8.24)	(7.91)	(7.66)	(11.28)	(7.90)	(3.13)	(7.12)
CF^e			0.158		0.101			0.219			
			(2.49)		(1.45)			(3.59)			
K_{-1}	-0.076	-0.071	-0.059	-0.069	-0.062	-0.064	-0.079	-0.057	-0.068	-0.071	-0.072
	(11.05)	(9.61)	(6.84)	(10.75)	(7.76)	(9.35)	(11.75)	(6.77)	(10.47)	(8.44)	(10.52)
R^2	0.92	0.91	0.91	0.93	0.93	0.92	0.92	0.91	0.92	0.87	0.92
SEE	191.7	202.9	195.4	178.8	177.4	191.1	184.7	193.1	183.5	236.3	190.9
DW	1.17	1.59	1.56	1.36	1.40	1.68	1.32	1.43	1.30	1.05	1.23
LM(4)	14.11	7.49	7.86	9.65	8.92	6.91	10.22	8.11	9.96	21.72	18.17
Definitions	Q_4	CI_1	CI_1	Q_4	Q_4	CI_1	Q_4	Q_1	Q_4	Q_4	Q_4
of	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$
variables	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_{21}$	$\bar{c}_{21}$	$\bar{c}_{21}$	$\bar{c}_{22}$	$\bar{c}_{22}$	$\bar{c}_{24}$	$\bar{c}_{25}$	$\bar{c}_{26}$
			CF_4^n		CF_4^g			CF_4^n			

Notes: See tables 24, 28 and A31. $CI^e = CI_{-1}$ (lagged value of consumption plus investment). The definitions of the c_2-variables are presented in table 17. In equations 1-3, c_2 is that variant which is positive for all t (1963.1 - 1980.4)

Table A34. Quarterly Estimation Results of Linear Investment Equations for the Aggregate Sector (eqs. 5.14, 5.15, 5.17ii; w/p and c_2/p as price variables)

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)
constant	1.989	1.993	1.510	1.237	1.832	1.514	1.613
	(10.02)	(10.03)	(5.08)	(5.55)	(10.77)	(8.68)	(9.16)
Q^e	0.223	0.212	0.398	0.442	0.115	0.054	0.127
	(5.03)	(4.10)	(5.43)	(4.62)	(3.13)	(1.36)	(3.60)
$(w/p)^e$	0.312	0.323	0.223	0.237	0.421	0.375	0.411
	(4.29)	(4.15)	(3.33)	(3.77)	(6.13)	(5.91)	(6.70)
$(c_2/p)^e$	-0.074	-0.074	-0.048	-0.044	-0.072	-0.068	-0.010
	(11.04)	(11.03)	(4.61)	(4.77)	(12.73)	(13.05)	(6.16)
CF^e				0.099		0.234	
				(1.50)		(3.92)	
K_{-1}	-0.061	-0.061	-0.064	-0.069	-0.060	-0.049	-0.042
	(7.73)	(7.72)	(6.88)	(6.73)	(7.94)	(6.47)	(6.17)
R^2	0.89	0.89	0.93	0.92	0.91	0.92	0.89
SEE	218.3	218.4	175.8	184.8	209.7	190.2	231.6
DW	0.90	0.91	1.98	1.30	1.10	1.33	1.07
RHO_1			0.34				
			(2.78)				
RHO_2			0.15				
			(1.25)				
EST	OLS	TSLS	CO	OLS	OLS	OLS	OLS
Definitions	Q_0	Q_0	Q_4	Q_A	Q_F	Q_F	Q_F
of	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_F$
variables	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_{2,F}$
				CF_4^n		CF_4^g	

Table A35. The Effect of Credit Rationing in the Quarterly Investment Equations for the Aggregate Sector

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
constant	-0.231	-0.536	0.020	2.033	1.046	1.513	1.607	1.451	1.292
	(1.27)	(2.70)	(0.12)	(3.70)	(2.25)	(7.35)	(7.18)	(7.19)	(6.71)
Q^e	0.462	0.324	0.602	0.294	0.685	0.143	0.346	0.418	0.346
	(9.63)	(4.65)	(7.95)	(3.87)	(6.76)	(2.79)	(4.55)	(7.41)	(6.12)
$(w/c_1)^e$	2.715	2.624	1.726						
	(6.29)	(6.48)	(4.87)						
$(w/p)^e$				0.248	0.190	0.319	0.286	0.260	0.329
				(2.57)	(2.60)	(4.91)	(4.51)	(3.98)	(5.14)
$(c_1/p)^e$				-0.212	-0.083				
				(4.37)	(1.66)				
$(c_2/p)^e$						-0.069	-0.059	-0.049	-0.049
						(11.12)	(7.74)	(7.16)	(7.93)
CF^e		0.288	0.065	0.298	0.167	0.221	0.113		
		(3.34)	(0.77)	(3.29)	(1.01)	(3.28)	(1.62)		
RM^e	7.604	4.234	-0.091	6.571	-1.165	7.629	-0.367	1.431	4.774
	(1.39)	(0.78)	(0.20)	(1.16)	(0.36)	(2.04)	(0.11)	(0.40)	(1.03)
K_{-1}	-0.058	-0.040	-0.064	-0.041	-0.083	-0.054	-0.069	-0.071	-0.071
	(8.24)	(4.38)	(7.42)	(3.27)	(7.72)	(6.66)	(7.49)	(9.98)	(10.59)
R^2	0.81	0.84	0.89	0.81	0.89	0.91	0.92	0.92	0.93
SEE	286.7	266.0	222.4	286.3	212.1	195.3	183.8	190.8	178.8
DW	1.10	1.28	1.24	1.04	1.20	1.37	1.38	1.24	1.39
Definitions	Q_1	Q_1	Q_4	Q_1	Q_A	Q_1	Q_4	Q_4	Q_4
of	wc_0	wc_0	wc_0	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$
variables				$\bar{c}_1$	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_{22}$	$\bar{c}_2$	$\bar{c}_{21}$
		CF_4^n	CF_4^n	CF_4^n	CF_A^g	CF_4^g	CF_4^n		
	RML_1	RM_4	RM_1	RML_1	RM_4	RM_1	RM_4	RML_4	RMM

Notes: See table A23. RM is the marginal interest rate on central bank borrowing of commercial banks (RM1). $RM_1 = RM_{t-1}$, etc. RML = RM-r, r = average interest rate on bank lending.

$$RMM = \frac{1}{4}\sum_{i=0}^{3} RM_{t-i} \quad .$$

Table A36. The Effect of Technical Change in the Quarterly Investment Equations for the Aggregate Sector

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
constant	-2.313	-2.753	-1.447	-1.791	-1.723	0.131	0.697	0.869
	(4.50)	(5.44)	(2.62)	(3.43)	(1.30)	(0.21)	(1.36)	(1.78)
Q^e	0.508	0.366	0.538	0.389	0.373	0.233	0.443	0.374
	(9.95)	(5.47)	(9.39)	(5.77)	(5.14)	(4.37)	(7.69)	(6.26)
$(w^*/c_1)^e$	2.023	2.110						
	(5.60)	(6.16)						
$(w/c_1)^e$			2.244	2.369				
			(4.99)	(5.66)				
$(w/p)^e$					0.367	0.349	0.294	0.333
					(3.76)	(5.40)	(4.40)	(5.18)
$(c_1/p)^e$					-0.086			
					(1.34)			
$(c_2/p)^e$						-0.058	-0.044	-0.046
						(8.38)	(5.78)	(6.61)
CF^e		0.297		0.304	0.253	0.216		
		(3.06)		(3.55)	(2.77)	(3.22)		
RM^e						5.346		
						(0.83)		
T			-1.468	-1.425	-2.592	-1.176	-0.664	-0.455
			(2.59)	(2.71)	(3.39)	(1.99)	(1.63)	(1.15)
K_{-1}	-0.036	-0.023	-0.029	-0.017	-0.013	-0.036	-0.063	-0.064
	(6.07)	(3.28)	(2.26)	(1.41)	(0.96)	(2.63)	(7.59)	(8.10)
R^2	0.79	0.82	0.83	0.85	0.84	0.92	0.92	0.93
SEE	301.0	284.1	274.5	254.4	267.6	188.8	187.4	178.4
DW	0.92	1.03	1.08	1.24	0.87	1.56	1.30	1.41
Definitions	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_4	Q_4
of	w^*c_0	w^*c_0	wc_0	wc_0	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$
variables					$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_{21}$
		CF_4^g		CF_4^g	CF_4^g	CF_4^n		
						RMM		

Notes: See table A24 and A35.
$w^* = w/y$, where y = exponential trend of average labour productivity (index of labour quality) and T = linear time trend (1963.1 = 0.25, 1963.2 = 0.50 etc., the coefficient of T should be multiplied by 10^2 to get the correct scale.

Table A37. Tests for the Replacement Hypothesis in the Quarterly Investment Equations for the Aggregate Sector

Variables and statistics:	Equation No. (1)	(2)	(3)	(4)	(5)	(6)
constant	2.193	2.053	3.440	2.147	1.952	3.302
	(4.92)	(10.64)	(9.97)	(4.86)	(10.10)	(9.57)
Q^e	0.705	0.424	0.408	0.685	0.404	0.386
	(10.54)	(7.38)	(6.85)	(10.41)	(6.99)	(6.43)
$(w/p)^e$	0.102	0.278	0.285	0.100	0.267	0.271
	(1.22)	(4.18)	(4.19)	(1.22)	(4.02)	(4.00)
$(c_1/p)^e$	-0.154			-0.158		
	(3.74)			(3.84)		
$(c_2/p)^e$		-0.064	-0.062		-0.063	-0.061
		(8.86)	(8.61)		(8.72)	(8.43)
K^1_{-1}	-0.074	-0.081	-0.090			
	(6.87)	(10.01)	(10.59)			
K^2_{-1}				-0.071	-0.075	-0.083
				(6.98)	(9.89)	(10.43)
R^2	0.85	0.91	0.91	0.85	0.91	0.91
SEE	260.4	194.2	197.2	258.7	195.7	199.1
DW	0.95	1.24	1.19	0.95	1.19	1.13
Definitions	Q_4	Q_4	Q_4	Q_4	Q_4	Q_4
of	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$	$\bar{w}_0$
variables	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_2$	$\bar{c}_1$	$\bar{c}_2$	$\bar{c}_2$

Notes: See table A25. The net capital stock K^1 is calculated by increasing the rate of depreciation (δ) by 0.5 percentage points (at annual level) from 1974 onwards and K^2 is calculated by increasing the rate of depreciation gradually from 1974 onwards so that at the end of 1980 δ is 0.5 percentage point higher than its original value. In equations 3 and 6, c_2 is that variant which is positive for all t.

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