[SUOMEN PANKKI]

Urho Lempinen

Optimizing Agents, Exogenous Shocks and Adjustments in the Economy

Real and Nominal Fluctuations in Economies with a Wage Rigidity

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FOREWORD

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Helsinki, August 17, 1984

Urho Lempinen

1 INTRODUCTION

1.1 Purpose

One of the cornerstones of the classical and new classical macroeconomics is Fisher's theory of interest rates, which states that real and nominal interest rates always differ by an amount equivalent to the inflation rate. Moreover, the real interest rate is determined mainly by the long-term rate of time preference, and is therefore considered to remain relatively constant. In an economy in which the Fisher-effect holds, allocation of capital to different uses is unaffected by variations in the price level. Hence, inflation cannot have any real effects in the classical theory, not at least via the capital markets.

The Keynesian view on the consequences of inflation, or variations in inflation, differs vastly from the classical interpretation. The standard Keynesian assumption of rigid or less-than-fully flexible nominal wages provides a simple example of circumstances under which the Fisher-effect does not hold. If, for example, the nominal wage is fixed and the price level drops, the real profits of firms also fall. This reduces real income, and hence the real rate of return, on shares. On the other hand, the nominal terms of government bonds, foreign bonds and other assets without limited liability are typically either regulated or for other reasons partially independent of the price level. The real interest rate on bonds with fully regulated nominal returns rises with deflation. Thus, the real rates of return on shares and such bonds are affected in a different way by variations in the price level.

The reason why the Fisher-effect fails in the Keynesian case is that the rigid nominal wage makes distribution of factor income between capital and labor dependent on the price level. Should all input supply decisions be made by one representative agent, then variations in income distribution need not have any real effects.[1] On the other hand, if capital and labor supply decisions are partly or fully separated, i.e. made by different agents following different decisions rules, a source of distortions exists in the economy. If allocation of capital can be adjusted more flexibly than nominal wages adjust, then optimizing capital suppliers will move capital away from and into firms as exogenous shocks make the price level vary. Through the neoclassical production function, such reallocations of capital have real effects in terms of variations in output and employment.

As a technical construction, the above source of real fluctuations in the economy is similar to what is known as the Tobin effect in the context of growth models (see Tobin (1965), Fischer (1979)). The Tobin effect is due to the fact that the nominal rate of return on money is fixed at zero. Consequently, the real rate of return on money varies one-for-one with the inflation rate. Therefore, an increase in the inflation rate makes investors shift their portfolios towards investments in real capital, which increases output and unemployment.

It is the purpose of this study to analyze some important macro-economic issues in an economy in which the Fisher-effect fails in the sense described above. On one hand, we shall study the familiar question of the neutrality of stabilization policy in the closed economy context. This analysis can be connected to the works of Lucas (1972) and Fischer (1979), and we hope to be able to add some new aspects to this very well known debate. In

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[1]Note that in principle real effects could emerge if the aggregate income of the representative agent is increased by the reallocation of capital. One could imagine such a case to be possible in a small open economy which perceives a very profitable investment opportunity abroad.

particular, however, we will emphasize the view on the policy neutrality issue adopted by Leijonhufvud (1972, 1981). In Leijonhufvud's view, an economy has the corridor property: i.e. the economy absorbs small shocks by means of automatic stabilizers, but large shocks cause real fluctuations unless stabilization policy is conducted.

On the other hand, we shall investigate the adjustment of a small open economy to exogenous shocks. Specifically, we will contribute to the discussion on endogenous exchange rate cycles in a small open economy following a fixed exchange rate regime (for the devaluation cycle, see e.g. Paunio (1969), Korkman (1978), (1980); for exchange rate jumps caused by speculation, see Krugman (1979), Turnovsky (1980), and Obstfeld (1983).

1.2 Stylized facts

Rigidity of the nominal wage and separation of input supply decisions - the two vital assumptions motivating this study - are to a considerable extent empirical facts in the institutional setting of the type of economies found in the Nordic countries (Denmark, Finland, Norway and Sweden). In these economies, a typical empirical observation is that a small number of trade unions, perhaps even only one, represent workers in the labor market. These unions usually negotiate nonstaggered nominal wage contracts with employers. The contracts normally specify a percentage wage increase, which applies throughout the contract period and is less than fully indexed.

Another stylized fact in the Nordic economies is that financial markets are not very well developed and tend to be 'thin' and inefficient (see Kähkönen (1982), Introduction). One manifestation of these features is that a relatively narrow menu of alternative assets is effectively available for average consumers. A large proportion of household savings takes the form of bank deposits, mortgage payments for loans, and life and pension insurance payments. Under these circumstances, actual capital supply decisions are in fact made by highly sophisticated financial agents, such as banks and insurance companies. Given the strong position of trade unions in the labor market and financial intermediaries in the capital market, input supply decisions are indeed strongly, if not completely, separated in the Nordic economies.

In addition to the two empirical observations important for the motivation of the cases to be studied, we will introduce one more commonly observed empirical finding for purely technical reasons. This observation - that substitution possibilities are limited in the production function (i.e. technology is approximately fixed coefficient) in the short run - facilitates the technical analysis in a useful manner.

1.3 Methodology

Given the assumption of nominal wage rigidity and the aim of dynamic analysis, we face a very challenging situation from the analytical point of view. Any rigidity in adjustments of relative prices generates nonlinearities in the standard macroeconomic relationships, which can easily become an obstacle for analytical manipulation. If the variations in income distribution implicitly caused by the wage rigidity are made a potentially important reason for real fluctuations by separating input supply decisions, complexity is further magnified.

Dynamic general equilibrium analysis provides a natural framework for studying effects of distortions when agents follow optimizing behaviour. This is also the approach that we will mostly pursue here. To minimize the complications referred to above, and yet at the same time to insert the desired features into the analysis, we have to make further choices that restrict the general Walrasian model. The first specific assumption is to model capital supply decisions according to the principles of dynamic portfolio theory.

The main advantage of the portfolio theoretic approach is that it is relatively well understood and widely applied in various fields of economics, and that its analytical foundations are firmly established.

Thus, we will, for instance, see later that the approach is very versatile with respect to various types of changes in the environment that set off dynamic adjustment processes. This proves to be particularly useful, since the distortions are transformed into real effects via capital supply decisions in our model. The portfolio approach restricts analysis in two respects. First, the continuous-time representation is much more appropriate for dynamic portfolio analysis than the discrete-time representation, which is more familiar in macroeconomic applications in general. Second, for different assets to be held by a portfolio investor, uncertainty must be introduced into the rates of return on assets. Furthermore, the stochastic processes specifying the rates of return must be of rather specific type.

Uncertainty is introduced into the economy in a very simple manner by assuming that the production process is fundamentally stochastic. Specifically, we propose that the marginal productivities of factors follow stochastic processes, which will be defined later in detail.

The overall analytical approach of the study can then be characterized as stochastic continuous-time general equilibrium analysis. The derivation of the framework as well as the references to the methodological literature will be presented in the second chapter. The only departure from the general approach will be made in the open economy analysis, where the nonlinearity of the key relationships of the model will reach such a high order that strict ad hoc linearizations are needed to carry out even a tentative analysis. These choices are discussed in the fourth chapter and the associated appendix.

1.4 Outline of the study and the main results

As was mentioned above, the main task to be accomplished in the second chapter is to derive analytical frameworks for dynamic macroeconomic analysis in which rational agents operate in a potentially distortionary environment. Some important technical results are derived in the chapter. First, a general competitive equilibrium (with flexible prices) is constructed in the stochastic dynamic economy and the general equilibrium relative prices are solved. Second, the macroeconomic frameworks for closed and open economy analysis are developed from the basic model. The contract wage rigidity, the separation of capital and labor supply decisions, and the short-term fixed-coefficient technology are built into these frameworks. Finally, the speculative behaviour of investors is modelled in open economy conditions in which there are positive probabilities of discrete shifts in exchange rates.

In the third chapter, the familiar policy neutrality question is reconsidered. In the analysis, it will be seen that, similar to the results of Fischer (1979), anticipated shocks cause fluctuations in real variables unless stabilization policy is conducted. On the other hand, it will also be seen that Fischer's distributed lag-lead price level equation only results in our framework if the timing of the shock is known with certainty and if there are quadratic adjustment costs in production. The standard interpretation of an anticipated shock is that both the magnitude and the timing of the shock are exactly known. The results will be very different if, instead, we interpret the notion of an anticipated shock in a nonconventional sense. The interpretation we apply is that a shock is anticipated if its magnitude is known, and if a time-interval within which the shock can randomly occur is known but not the exact timing of the shock. Under this interpretation, gradual price adjustments before the start of the shock will indeed be observed, as Fischer predicts. But also, and differently from Fischer, there is almost certainly overshooting in the price level. Specifically, if the actual shock implies a decrease in the price level, then the expectations on the timing of the shock generate inflation until the shock hits the economy. Thus, under the above circumstances, a positive relationship between the price level and unemployment rate - or a positively sloped Phillips curve - would be empirically observed in the short-run.

The model of the third chapter incorporates Leijonhufvud's corridor property in a certain sense. Thus the economy can adjust to unanticipated shocks through automatic stabilizing mechanisms, in our case through the real balance effect. Stabilization policy is needed for neutralization of the real effects of anticipated shocks. If the timing of the anticipated shocks is known with certainty, then only Keynesian fiscal policy can prevent real effects from emerging. If, on the other hand, the timing of the shocks is uncertain, then a specific monetary policy analogous here to the interest rate targeting policy familiar from e.g. Poole (1970) neutralizes by conducting fiscal policy. Another specific monetary policy, analogous to 'money targeting', can never neutralize anticipated shocks of either kind. The interpretation of small and large is specific and intuitively appealing here. A shock can be large if either its measurable magnitude is large, or if there is little confusion about its timing, i.e. if the distribution for the shock to start within a given short sub-interval is dense. A shock is small if either its magnitude is small or if there is a lot of confusion concerning its timing. If there is no confusion about the timing, then the shock is always large, no matter what the size of the shock is. This extreme case is the standard anticipated shock. If there is no information at all on the timing of the shock, then the shock is always small. This is the case of unanticipated shocks in the standard sense.

The role of speculative behaviour in the macroeconomic adjustment process of a small open economy following the fixed exchange rate regime is analysed in the fourth chapter. In general it will be seen that an economy which has the contract wage rigidity as an institutional feature is likely to experience endogenous devaluations and revaluations. The presence of speculating investors reinforces the tendency for exchange rate jumps to occur. In particular, if the country loses or gains a sufficient amount of reserves for exogenous reasons, then a strong self-fulfilling speculation begins, which only can be stopped by a strong exogenous shock or policy measure operating in the opposite direction. As real fluctuations are associated with the variations in reserves, speculative behaviour can be said to have real costs. Countries with no labor mobility across borders as well as countries with money illusion in wage-setting are likely to experience devaluations more often than revaluations. Countries in which contract wages are set according to an ad hoc policy are more likely to experience cyclical fluctuations and exchange rate changes than countries in which a feedback wage-setting policy is followed. On the other hand, the economies of the former type recover more steadily after a devaluation than economies of the latter type.

2 OPTIMIZING AGENTS AND MACROECONOMICS: TOPICS IN DYNAMIC MODELLING

2.1 Introduction

In the following, some dynamic macroeconomic models of a Cobb-Douglas economy are developed. The purpose of these essentially technical exercises is to derive frameworks for both closed and open economies in which questions of dynamic adjustments of the economies in response to exogenous shocks can be analyzed.

As was pointed out in the first chapter, the economic analysis of this study is going to be carried out under non-Walrasian assumptions, i.e. in a general equilibrium framework in which the presence of distortions is allowed. However, it seems useful to first developed the model under standard Walrasian assumptions of flexible prices, fully rational agents, complete information and perfect competition in all markets. In the Walrasian model, consumers and firms operate in a continuous-time stochastic environment, maximizing intertemporal objective functions. Uncertainty is introduced on a very basic level as a property of the production technology. One of the analytical contributions of this chapter is the solution of the general equilibrium relative prices of the Walrasian model in the stochastic stationary state.

The Walrasian model is then modified by adding the assumptions that introduce the desired distortions into the framework. The consumers are thus divided into two groups: investors, who only supply capital, and workers, who only supply labor. It turns out that the natural behavioural pattern for investors is similar to that of a closed economy investor and quite different, even in open economy conditions, from that of an international consumer-portfolio investor familiar from e.g. some open economy analyses (see Macedo (1981), Meerschwam (1982)). The workers, on the other hand, are assumed to behave so that, in order to hedge against an uncertain income flow, they form a trade union representing all workers in the labor market. The union negotiates wage contracts with employers. The contracts fix the time path of the nominal wage over a finite period of time. Other prices in the economy are assumed to remain fully flexible.

The emphasis of the economic analyses of the third and fourth chapters is on the demand side of the economy, i.e. on the behaviour of investors and workers. Therefore, several simplifying assumptions are made on the supply side of the economy. Because of technical difficulties, no real aggregate capital accumulation in excess of potential replacement investments is allowed. Adjustments of capital stock are assumed costless in each firm. The only exception made in the latter respect is in the applied open economy analysis of the fourth chapter. The technology of the firms is assumed to be fixed-coefficient in the short-run and flexible-coefficient in the long-run. Hence, all shocks that ultimately affect employment and output in the short-run exercise their effects only through responses of investors to these shocks.

The connections between the frameworks to be developed here and the literature are rather diverse. In some sense, one can of course say that the models of this study are related to the entire recent literature on rational expectations. In a specific sense, perhaps only the recent work of Kydland - Prescott (1982) is fairly similar in approach and goals to ours. However, the differences between the studies are vast. The more explicit specifications and the stronger emphasis placed on deriving analytical results force this study to impose considerable simplifications on the supply side of the economy, in particular. For similar reasons, Kydland - Prescott are able to apply a more general consumer utility function. On the other hand, the way in which our models build in 'the persistence of unemployment' may be a more realistic description of the crucial macroeconomic features of small, Scandinavian-type open economies.

In terms of the general approach to macroeconomic issues, our models are very alike in spirit to Tobin's asset accumulation approach (see e.g. Tobin (1980), Tobin-Macedo (1980), Kouri-Macedo (1978)). Asset markets phenomena, including speculative portfolio behaviour, have a central role in our analyses and can have strong and persistent effects on real variables.

Technically, we apply methods developed in stochastic growth theory (see e.g. Bismut (1975), Merton (1975)), in finance (see Merton (1969), (1971), (1973), Malliaris-Brock (1982)), in international finance (see Macedo (1981), Meerschawn (1982), Adler-Dumas (1983)), and in applied mathematics (see Arnold (1974)). Two particularly useful reviews in the technical field are those by Malliaris-Brock (1982) and Chow (1979).

We will proceed as follows. In the second section, the basic nonmonetary closed economy model is derived. In the third section, the public sector and money are added to the basic model. In the fourth section, the basic framework is further extended to include the features of a small open economy. In the last section, the behaviour of an investor speculating on expected devaluation is fully derived. The work in this section constitutes a preliminary analysis for the open economy studies of the fourth chapter. The analytical method itself has, however, a much broader general applicability and we shall use it in the third chapter as well.

2.2 The simple nonmonetary economy with flexible prices

Consider an industry producing one homogeneous good by means of technology, using capital and labor as inputs.

Suppose that the competitive firms are identical and that the representative firm has the following basic Cobb-Douglas technology

(1) $$Y_0 = L^{\beta}K^{1-\beta}$$

where Y_0 = production
L = number of units of labor input
K = number of units of capital
$\beta \in (0,1)$ = constant

In the dynamic context, technology (1) defines the instantaneous rate of production, so that total output over an arbitrary time-interval (t_0,t_1) is given by:

(2) $$Y(t_0,t_1) = \int_{t_0}^{t_1} L^{\beta}K^{1-\beta}ds = (t_1-t_0)L^{\beta}K^{1-\beta}$$

where $t_1 > t_0$.

The firms raise one unit of capital by issuing one share and hire one unit of labour by signing one wage contract. The prices of shares and wage contracts are instantaneously determined in the markets for capital and labor via the competitive mechanism. Let the instantaneous price of a share or the instantaneous price of capital be $Q_B(t)$ and the instantaneous price of a wage contract be $Q_H(t)$. The present value of profits of the representative firm over the time-interval (t_0,t_1), or $V(t_0,t_1)$, is then defined by[2]

$$(3) \qquad V(t_0,t_1) = \int_{t_0}^{t_1} e^{-\rho(s-t_0)} [L^{\beta}K^{1-\beta} - AQ_B(s)K - AQ_H(s)L]\,ds$$

where A is a constant to be determined later in the general equilibrium

ρ is the rate of time preference.

In (3), the two cost terms are dynamic representations of input costs, i.e. stochastic analogues of Jorgensonian user costs. The term A $Q_B(s)$ corresponds to the user cost of capital under certainty in a world with no depreciation, while the term A $Q_H(s)$ is the instantaneous wage rate in general equilibrium. The constant A is determined in the general equilibrium implied by the stochastic stationary state, which will be specified later.

It is useful to apply the Euler decomposition for a function which is homogeneous of degree one, and write (3) as follows:

$$(4) \qquad V(t_0,t_1) = \int_{t_0}^{t_1} e^{-\rho(s-t_0)} ([MP_L - AQ_H(s)]L + [MP_K - AQ_B(s)]K)\,ds$$

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[2]Note that we are analyzing a nonmonetary economy here. Hence, (3) defines real profits, and $Q_B(t)$ and $Q_H(t)$ are the real price of a share and a wage contract, respectively. Note also that no adjustment costs are assumed to be incurred as a result of investments. Thus only two simple forms of investments could be introduced into (3): On one hand, we could view investments as frictionless, instantaneous shifts in the capital stock employed in the production process. On the other hand, we could define output net of replacement investment needed to maintain productive capacity intact.

where $MP_L = \frac{\beta\ L^{\beta}K^{1-\beta}}{L}$ = marginal productivity of labour

$MP_K = \frac{(1-\beta)L^{\beta}K^{1-\beta}}{K}$ = marginal productivity of capital

Under certainty, the firm always maximizes the value of integral (4) by choosing the amounts of inputs so that the marginal productivity of each input is equal to its price. Furthermore, competition among firms drives the value of integral (4) to zero. Formally, the following identities must hold in the dynamic competitive equilibrium:

(5a) $AQ_H(t) = M\ P_L$

(5b) $AQ_B(t) = M\ P_K$

Under uncertainty, matters become more complicated. For one thing, one must be careful about the meaning of uncertainty in this context. If the revenues of the representative firm follow a stochastic process and all prices are perfectly flexible, then the firm can in principle pass the uncertainty in revenues through completely to factor payments and thus make the profits flow riskless, so that maximization of (4) is a valid behavioural assumption for the firm. On the other hand, if there are rigidities in price adjustment - for instance because of wage contracts - then the firm cannot make the flow of profits riskless. In such cases, the assumption about the risk attitudes of the representative firm becomes important. In the relevant parts of this study, i.e. in cases where there are some price rigidities, we will follow the convention that the risk attitudes of firms are the same as those of the shareholders. This assumption implicitly specifies an insurance contract between shareholders and workers. To see this, consider a situation between a firm with stochastic revenues and its infinitely risk averse workers. Such workers are only willing to work at a certain wage rate. Imposing the preferences of the shareholders on the decision making of firms then specifies the competitive risk premium in the general competitive equilibrium which shareholders require for providing complete insurance to the workers. As the shareholders will be assumed to have Cobb-Douglas preferences, the firm facing an uncertain profits flow will, instead of (4), maximize the expected discounted logarithmic utility of the profits flow, i.e.

$$(4') \qquad \tilde{V}(t_0,t_1) = E_{t0} \int_{t_0}^{t_1} e^{-\rho(s-t_0)} \log([M\,P_L - A\cdot Q_H(s)]L + [M\,P_K - AQ_B(s)]K)ds$$

where $\tilde{V}(t_0,t_1)$ is the expected discounted utility over the planning horizon.

Under uncertain profits flow, competitive firm behaviour is defined by the entry condition: If expected instantaneous utility $\tilde{V}(t_0,t_0+dt)$ is larger than a constant log C, then there is costless entry into the industry. The entry condition fixes the level of the competitive risk premium, the determination of which is examined in appendix 1. Note that the actual entry condition is stated in terms of the expected utility from producing over the contract period, but with logarithmic preferences, for which the coefficient of relative risk aversion equals one, the instantaneous time-independent risk premium results (see appendix 1).

To make conditions (5a,b) meaningful in a dynamic environment with uncertainty, we need to specify the dynamic behaviour of the two marginal productivities. This specification also amounts to choosing one basic source of uncertainty in our stylized economy. An analytically simple and empirically somewhat interesting source of variations in real output is the qualities of the labor force. Suppose then that the productivity of the labor force increases over time because of continuing learning-by-doing, but that the increase occurs at an uncertain rate. Hence, when the representative firm hires a fixed amount L of labor input, it actually receives a stochastic sequence of labor input services, denoted by $\tilde{L}(t)$. Furthermore, suppose that this sequence of services can be fully characterized by the following stochastic process of geometric brownian motion type:[3]

$$(6) \qquad d\tilde{L} = \tilde{L}(adt+\sigma_L dz_L)$$

where $Edz_L = 0$

$E(dz_L)^2 = dt.$

$z_L(t) \sim N[0,1]$, with independent increments $z_L(s) - z_L(u)$;

$u > s \geqslant t_0, \forall\, u, s$

This specification of labor input services means intuitively that the effective amount of labour input that can be deployed in production changes in an infinitely persistent manner at a rate at $+ \sigma \int_t dz$. This can be seen clearly when we write explicitly the solution of the stochastic differential equation specified in (6):

$$(7) \qquad \tilde{L}(t) = L \exp\{a(t-t_0)+\sigma_L \int_{t_0}^{t} dz_L\}$$

We interpret the parameter a as the rate of learning-by-doing in production, and the standard deviation σ_L as representing absenteeism and other types of labour disturbances in production.

When the labor force has the property specified in (6), i.e. that the efficiency of each unit of labor changes over time in an uncertain fashion, dynamics with uncertainty are also introduced into the aggregate production. This becomes particularly clearcut

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[3]For a precise definition of the geometric brownian motion process, see Malliaris-Brock (1982), pp. 36 - 38 or Arnold (1974), pp. 45 - 56.

when we study the behaviour of the instantaneous production function with the stochastic process of the efficient labor input included. Denoting the stochastic time path of output by y(t) and directly applying Ito's lemma, we have that

(8) $$dy(t) = [\beta(adt+\sigma_L dz_L) - \frac{1}{2}\beta(1-\beta)\sigma_L^2 dt]y(t).$$

where $y(t) \equiv y_0 \cdot g(t)$, with g(t) as specified below in (9).

The solution of this stochastic differential equation is

(9) $$y(t) = y_0 \exp\{\nu(t-t_0)+\sigma \int_{t_0}^{t} dz_L\} \equiv y_0 g(t)$$

where $\nu = \beta a - \frac{1}{2}\beta(1-\beta)\sigma_L^2$

$\sigma = \beta\sigma_L$

$\exp\{\nu(t-t_0)+\sigma \int_{t_0}^{t} dz_L\} = g(t)$

Assuming that the production process starts exactly at the nonstochastic rate of production,[4] the solution can further be written as

(10) $$y(t) = L^{\beta}K^{1-\beta}\exp\{\nu(t-t_0)+\sigma \int_{t_0}^{t} dz_L\} = Y_0 g(t)$$

By differentiating (10) directly with respect to L and K, the time paths of the marginal products are easily found to be

[4]This is a convention in the case of geometric brownian motion processes, see Malliaris-Brock (1982), p. 34.

$$\text{(11a)} \qquad MP_L(t) = \frac{\beta L^{\beta}K^{1-\beta}}{L} \exp\{\nu(t-t_0)+\sigma \int_{t_0}^{t} dz_L\} \equiv MP_{10}g(t)$$

$$\text{(11b)} \qquad MP_K(t) = \frac{(1-\beta)L^{\beta}K^{1-\beta}}{K} \exp\{\nu(t-t_0)+\sigma \int_{t_0}^{t} dz_L\} \equiv MP_{K0}g(t)$$

Thus the two marginal products follow exactly the same stochastic processes, and there only can be a constant difference between the levels of the marginal products at each point of time. This convenient property is due to the Cobb-Douglas technology. Note that introducing an analogous basic property for capital would only change the parametrization of the g(t) function, and not the general structure at all.[5]

In (11a,b) we have stated the dynamic solutions for the two marginal productivities, given the assumption on the basic dynamics specified in (6). In the general competitive equilibrium

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[5]In conclusion, then, our assumption on the form of the stochastic sequence of labor input services results in a model of production with three possible regimes, which can be interpreted in terms of the parameters of the labor input process.

1. $\nu > 0$, i.e. $a > \frac{1}{2}(1\beta)\sigma_L^2$, or the learning-by-doing effects overcomes the labor disturbances effect; there is stochastic Hicks-neutral technical progress in the economy at an expected rate ν.

2. $\nu < 0$, i.e. $a < \frac{1}{2}(1\beta)\sigma_L^2$, or the labor disturbances effect overcomes the learning-by-doing effect; there is stochastic Hicks-neutral technical regression in the economy at an expected rate ν.

3. $\nu = 0$, i.e. $a = \frac{1}{2}(1\beta)\sigma_L^2$, or the two effects cancel each other out completely; the technology is stationary with stochastic disturbances.

The second regime is somewhat problematic in the general equilibrium and is therefore ruled out from later interpretations.

with flexible prices, conditions (5a,b) must hold. Hence, we find that the real price of a share and a wage contract must have the following general solutions:

(12a) $Q_H(t) = \frac{1}{A} MP_{L0}g(t)$

(12b) $Q_B(t) = \frac{1}{A} MP_{K0}g(t)$

There are an infinite number of solutions (12a,b), because the initial marginal productivities, or the initial general equilibrium price of a share and the corresponding wage rate, are still undetermined. To find the appropriate initial values, we have to model the demand side of the economy.

The consumers in our economy are assumed to be identical in every respect. Thus, they have identical tastes, endowments, and abilities. They plan over an infinite time horizon, which is also the time they can participate in the production process. Each consumer is assumed to have nominal initial endowments $\frac{1}{n}V_{B0}$ and $\frac{1}{n}V_{H0}$ of nonhuman and human wealth, respectively, at the initial moment t_0. Nonhuman wealth consists of the value of a stock of resource, which can be either consumed or used as the capital input in production. The aggregate initial endowment of the resource is $\tilde{K}$ and the initial price Q_{B0}. For one unit of resource given to a firm, the consumer receives one share in that firm.

To define the concept of human wealth, we first state that each consumer has a constant maximum working capacity $\frac{1}{n}H$ in the production process. If the consumer sells the right to use the working capacity to the producer, he will receive a wage contract worth $Q_H(t)$ in return. Hence, one unit of human capital has a real market price $Q_H(t)$ at time t, so that the initial endowment of human wealth owned by agent j is

$$\frac{1}{n} V_{HO} = \frac{1}{n} HV_{HO}.$$

In the whole economy, there are n agents living at any moment, all of whom have the same initial endowments of human capital. Summing up over the n agents we get the following for the economy's aggregate endowments:

$$V_{BO}^{A} = V_{BO}$$

$$V_{HO}^{A} = V_{HO}.$$

Consumers derive utility from consuming the consumption good and from leisure. Thus the consumers can either consume their wealth and the returns on it by buying consumer goods, or alternatively directly consume their human wealth by not working.

We assume that the representative consumer has the following utility function in instantaneous rates of consumption $c_1(t)$ and leisure $c_2(t)$

$$(13) \qquad U(t) = \alpha \log(c_1(t)) + (1-\alpha)\log(c_2(t)$$

$$\alpha \varepsilon (0,1).$$

Thus the instantaneous component of the utility function is of the Cobb-Douglas type. If we now assume that the representative consumer maximizes the expected discounted value over the planning horizon of the instantaneous utility function specified above, we can characterize the consumer's decision making situation by the following dynamic programming problem.

$$(14) \qquad \max_{\{c_1(t),c_2(t)\}} E_{t_0}\{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}[\alpha\log(c_1(t)) + (1-\alpha)\log(c_2(t))]dt\}$$

s.t. appropriate constraints.

where ρ = rate of time preference

Expression (14) is a stochastic dynamic programming problem with two control variables, instantaneous real consumption $c_1(t)$ and instantaneous leisure $c_2(t)$. Since both forms of spending are based on the stocks of and returns on either nonhuman or human wealth, the stock of total wealth must be the state variable of problem (14), so that the solutions for the controls must be functions of the stock of wealth only. We define the total real wealth $W(t)$ as the sum of real human wealth $V_H(t)$ and real nonhuman wealth $V_B(t)$, i.e.

$$(15) \qquad W(t) = V_H(t) + V_B(t) = [\gamma+(1-\gamma)]W(t) = Q_H(t)H + Q_B(t)\tilde{K}$$

where we have defined

$$\gamma \equiv \frac{Q_H(t)H}{Q\ (t)H + Q\ (t)K}$$

Note that in general cases, γ would vary with time. In our case, flexible prices and the homogeneity of the Cobb-Douglas technology guarantee that γ remains constant.

In order to solve problem (14), we have to derive the dynamic equation of motion for the state variable, i.e., the stochastic differential equation governing the behaviour of total real wealth. Differentiating (15), we have

(16) $$dW(t) = \left[\frac{dV_H(t)}{V_H(t)}\gamma + \frac{dV_B(t)}{V_B(t)}(1-\gamma)\right]W(t).$$

The expressions for $\frac{dV_H(t)}{V_H(t)}$ and $\frac{dV_B(t)}{V_B(t)}$ are obtained directly by differentiating stochastically the definitions for $V_H(t)$ and $V_B(t)$ and recalling the general solutions for $Q_H(t)$ and $Q_B(t)$ presented above in (12a,b). Thus we have that

$$\frac{dV_H(t)}{V_H(t)} = \frac{dQ_H(t)}{Q_H(t)} = \frac{dg(t)}{g(t)} = \nu dt + \sigma dz_L$$

$$\frac{dV_B(t)}{V_B(t)} = \frac{dQ_B(t)}{Q_B(t)} = \frac{dg(t)}{g(t)} = \nu dt + \sigma dz_L$$

Substituting this back into (16), we obtain the following form for the stochastic differential equation of real wealth:

(17) $$\begin{aligned} dW(t) &= \left[(\nu dt+\sigma dz_L)\gamma + (\nu dt+\sigma dz_L)(1-\gamma)\right]W(t) \\ &= \left[\nu dt+\sigma dz_L\right]W(t) \end{aligned}$$

Expression (17) is not yet the complete form of the stochastic differential equation for real wealth, because it does not include the effects of the two forms of consumption. Instantaneous real consumption decreases the change in real wealth directly by the amount $c_1(t)$. Instantaneous leisure, on the other hand, decreases the change in real wealth by the flow of real wage income lost by not working, denoted by $f(t)$. In our framework, $f(t)$ is by definition the product of the instantaneous wage rate $w(t)$ and the instantaneous rate of leisure, $c_2(t)$. Formally,

$$f(t) = w(t)c_2(t).$$

Substracting the two consumption value components $c_1(t)$ and $f(t)$ from (17), we finally obtain for the stochastic differential equation governing the time path of real wealth the following:

(18) $\quad dW(t) = [\nu dt + \sigma dz_L]W(t) - c_1(t)dt - w(t)c_2(t)dt.$

Equation (18) is the desired complete form of the equation of motion for problem (14).

Using equation (18), we can rewrite problem (14) in the following revised form:

$$(19) \qquad J(W(t),t) \equiv \max_{\{c_1(t),c_2(t)\}} E_{t_0}\{\int_{t_0}^{t_0+T} e^{-\rho(t-t_0)}[\alpha\log(c_1(t)) + (1-\alpha)\log(c_2(t))]dt\}$$

$$\text{s.t.} \qquad dW(t) = [\nu dt + \sigma dz_L]W(t) - [c_1(t) - w(t)c_2(t)]dt$$

To complete the specification of the problem, we have yet to discuss the transversality condition for the problem. Our economy is a potentially growing or stagnating economy. In both cases it appears reasonable that, because the consumer plans over an infinite horizon, we require that, in order to be optimal, the consumer's decision must be such that the expected value of real wealth neither goes to zero or infinity in finite time. This amounts to requiring that at any point t of time, $t > t_0$, the condition $E_{t_0}[W(t)] = W_0$ holds. With this convention, we choose to study the economy in one specific steady state, the stochastic stationary state, the properties of which will be discussed somewhat later.

By applying Bellman's principle of optimality and adding the transversality condition, we have that the maximizing choices of real consumption and leisure in problem (14) also give the maximizing value of the function $\phi(W(t),t)$ defined by

$$(20)\quad \phi(W(t),t) = 0 \equiv \max_{\{c_1(t),c_2(t)\}} \{\alpha \log(c_1(t)) + (1-\alpha)\log(c_2(t)) - \rho J(W(t)) + \frac{\partial J(W(t),t)}{\partial W(t)}[\nu W(t) - c_1(t) - w(t)c_2(t)] + \frac{1}{2}\frac{\partial^2 J(W(t),t)}{\partial E(t)^2}\sigma^2 W(t)^2\}$$

$$(20a)\quad E_{t_0}[W(t_0+T)] = W_0$$

where (20a) is the transversality condition

As usual, the optimal solutions for the control variables can be found by establishing the first order conditions of (20) and setting them equal to zero. The first order conditions are

$$(21a)\quad \frac{\partial \phi(W(t),t)}{\partial c_1(t)} = \frac{\alpha}{c_1(t)} - \frac{\partial J(W(t),t)}{\partial W(t)} = 0$$

$$(21b)\quad \frac{\partial \phi(W(t),t)}{\partial c_2(t)} = \frac{1-\alpha}{c_2(t)} - w(t)\frac{\partial J(W(t),t)}{\partial W(t)} = 0$$

To obtain explicit solutions for the control rules, we have to find the explicit form of the $J(W(t),t)$ function. Let us start by assuming that the $J(W(t),t)$ function has the general form given by

$$(22)\quad J(W(t),t) = B_0 \log(W(t)) + B_1$$

with undetermined parameters B_0 and B_1. We assume that the control rules take the homogeneous-of-degree-one forms in wealth - $c_1^*(t) = \delta_1(t)\cdot W(t)$, and $c_2^*(t) = \delta_2(t)W(t)$, where $\delta_1(t)$ and $\delta_2(t)$ are the consumption and leisure shares, respectively. The first two partial derivatives of (22) needed for calculations are

$$\frac{\partial J(W(t),t)}{\partial W(t)} = \frac{B_0}{W(t)}$$

$$\frac{\partial^2 J(W(t),t)}{\partial W(t)^2} = - \frac{B_0}{W(t)^2}$$

Substituting the candidate control rules, assumption (22) and the partial derivatives of (22) into equation (20), we obtain the following equation:

$$(23) \qquad \alpha\lfloor \log(\delta_1(t))+\log(W(t))\rfloor + (1-\alpha)\lfloor \log(\delta_2(t))+\log(W(t))\rfloor$$
$$- \rho B_0 \log(W(t)) - \rho B_1 + \frac{B_0}{W(t)} (\nu-\delta_1(t)-\delta_2(t)w(t))W(t)$$
$$- \frac{B_0}{W(t)^2} \sigma^2 W(t)^2 = 0$$

Equation (23) implies the following identities in the coefficients of the terms involving the state variable of the same degree

$$(24a) \qquad \alpha + 1 - \alpha - \rho B_0 = 0 \Rightarrow B_0 = \frac{1}{\rho}$$

$$(24b) \qquad -\rho B_1 = 0 \Rightarrow B_1 = 0$$

Having solved the values of parameters B_0 and B_1, we easily find the optimal control rules c*(t) and c*(t) by substituting the implied value $\frac{\partial J(W(t),t)}{\partial W(t)} = \frac{1}{\rho W(t)}$ into the first order conditions (21a) and (21b). The solutions are:

$$(25a) \qquad c_1^*(t) = \alpha\rho W(t) = \alpha\nu W(t)$$

$$(25b) \qquad c_2^*(t) = \frac{(1-\alpha)\rho W(t)}{w(t)} = \frac{(1-\alpha)\nu W(t)}{w(t)}$$

Note that the transversality condition (20a) requires that $\rho = \nu$ holds in solutions (25a,b). This justifies the second equality signs. If $\rho > \nu$ were to hold, then agents would consume real input resources in the expected value sense and their expected consumption possibilities would end in finite time. On the other hand, if $\rho > \nu$, then there would be net saving in the economy and the expected future consumption possibilities would become infinitely large in finite time. These properties show that the steady state equilibrium that we are going to construct is indeed a stochastic stationary state. Consumers spend all their income and do not accumulate or decumulate capital. All the change in income and consumption is due to variations in productivity change and constant amounts of inputs are employed in production. The consumption rules have a very simple interpretation. Thus, the term $\alpha\nu W(t)$ represents the flow of instantaneous income, measured in real terms relative to consumption of goods. The consumption rules (25a,b) are then exactly analogous to the corresponding rules in a static Cobb-Douglas economy, the only difference being the income factor ν. Consumption of each good is a fixed fraction of the nominal income scaled by the relative price of the type of consumption. The fraction is the preference weight of the good in the consumer's utility function.

Consumption of leisure defines the labor supply behavior of consumers. It will be shown later that the general equilibrium wage rate $w(t)$ equals $\nu Q_{HO} g(t)$. Using this and the definition of real wealth, the instantaneous consumption of leisure can be written as:

$$(26) \qquad c_2^*(t) = \frac{(1-\alpha)\nu}{\nu Q_{HO} g(t)} (Q_{BO}\tilde{K} + Q_{HO}H) g(t)$$

$$= (1-\alpha)\left[\frac{Q_{BO}\tilde{K}}{Q_{HO}} + H\right]$$

The second form of (26) defines the fraction of real income in terms of the wage rate which is consumed in the form of leisure. This fraction remains constant over time, as both the wage income and the capital income grow at the same rates. If (26) is divided by H, then the fraction of potential labor input which remains idle because of the preference for leisure is obtained. Denoting the fraction by $1-\ell$, we have the following expression for it:

$$(27) \qquad 1-\ell = (1-\alpha)\left\lfloor\frac{Q_{BO}\tilde{K}}{Q_{HO}H} + 1\right\rfloor$$

Hence, the labor supply rule of consumers, or the fraction of their working capacity that they want to supply as labor, ℓ, becomes:

$$(28) \qquad \ell = \alpha - (1-\alpha)\,\frac{Q_{BO}\tilde{K}}{Q_{HO}H}$$

Thus, with Cobb-Douglas preferences, the fraction of the potential working capacity supplied as labor equals the weight of consumption in the utility function corrected by a term involving the initial relative capital income in terms of the labor income. Intuitively, the correction term specifies the effect of capital income on the labor supply behaviour of the consumer. The higher is the value of the capital endowment of the consumer, as valued in the general equilibrium via the production function, the less labor the consumer supplies. Empirically, the labor supply rule includes aspects of both intertemporal substitution and wealth distribution. Intertemporal substitution from working to leisure occurs if the capital income rises faster than the labor income. Agents with large capital endowments are less willing to work than those with small endowments.

Having now modelled the behaviour of all the agents in the economy, we have all the ingredients necessary for finding the specific solutions for real prices of shares and the real wage rate. Such relative prices specify the general competitive equilibrium in the economy.[6]

As we stated above on p. 27, finding the specific solutions to prices of shares and wage contracts amounts to finding the initial prices of them. The initial prices are found by analysing the equilibrium conditions in the three markets of the economy.

The equilibrium conditions of the dynamic economy over time interval $[t_0 = 0, h]$ are the following

(29a) $$\int_{t_0=0}^{h} y(s)ds = \int_{t_0=0}^{h} c_1^*(s)ds$$

(29b) $$L = \ell H$$

(29c) $$K = \tilde{K}$$

Condition (29a) defines the goods market equilibrium, (29b) the labor market equilibrium and (29c) the capital market equilibrium.

The last two conditions are time independent, because the respective control rules of the agents are time independent. By letting $h \to 0$, we can write condition (29a) approximately as

(29a') $$y_0 h = \alpha\nu Wh \iff y_0 = \alpha\nu W_0$$

Thus we have a justification for solving the initial equilibrium prices from a rather simple purely static problem.

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[6]In general, the question of the existence of equilibrium in continuous-time stochastic models is a difficult problem of mathematical economics. The commodity space becomes infinite dimensional and, in particular, the compactness of production sets becomes much more difficult to establish than in standard existence proofs. For attempts to prove general results, see e.g. Bewley (1972), Foldes (1978), and Chichilnisky (1981).

A convenient way to find the equilibrium relative real factor prices in the static situation, i.e., for a very short time interval $[t_0=0,h]$, $h \to 0$, is to note first that the first order conditions of a firm are

$$\frac{\beta y_0}{L} - AQ_{H0} = 0$$

$$\frac{(1-\beta)y_0}{K} - AQ_{B0} = 0$$

$$\Rightarrow L = \frac{\beta y_0}{AQ_{H0}}$$

$$K = \frac{(1-\beta)y_0}{AQ_{B0}}$$

Then we get that, if production is to be efficient, the following capital-labor ratio must be observed:

$$(30) \qquad \left(\frac{K}{L}\right)^D = \frac{(1-\beta)Q_{H0}}{\beta Q_{H0}} ,$$

superscript D indicates the demand side (in factor markets

We know, on the other hand, that in the growing economy the factor supplies are given by

$$L^S = \ell H$$

$$K^S = \tilde{K}$$

Thus we have that the capital-labor ratio from the supply side is

$$(31) \qquad \left(\frac{K}{L}\right)^S = \frac{\tilde{K}}{\ell H}$$

In equilibrium, the capital-labor ratios on the supply and demand sides of the factor markets must be equal. Equating (30) and (31) gives us the desired result:

$$(32) \qquad \frac{Q_{HO}}{Q_{BO}} = \frac{\beta \tilde{K}}{(1-\beta)\ell H}$$

The relative factor price (32) must emerge from the input market equilibrium. It depends, as is expected, on endowments, technology parameters and tastes. Equation (32) provides a means of showing that for a flexible price economy the labor supply rule is a constant. Substituting the solution for ℓ in (28) into (32), we obtain the following equation:

$$\frac{Q_{HO}}{Q_{BO}} = \frac{\beta \tilde{K}}{(1-\beta)\left[\alpha-(1-\alpha)\frac{Q_{BO}\tilde{K}}{Q_{HO}H}\right]H}$$

From this equation, the following solution for the ratio of initial nonhuman wealth to initial human wealth can be developed:

$$\frac{Q_{BO}\tilde{K}}{Q_{HO}H} = \frac{\alpha(1-\beta)}{\beta + (1-\beta)(1-\alpha)}$$

If the right hand side of the above identity is substituted into (28) for the left hand side expression, the labor supply rule takes the following form:

$$(28') \qquad \ell = \frac{\alpha\beta}{\beta + (1-\beta)(1-\alpha)}$$

(28') shows that the labor supply rule is indeed a constant, so that ℓ can be treated parametrically in (32).

Hence, we can also solve for the absolute levels of the input prices Q_{H0} and Q_{B0} by using (32) and the goods market equilibrium condition (29a). Note that (32) can be written as

$$Q_{HO} = \frac{\beta\tilde{K}\ Q_{BO}}{(1-\beta)\ell \cdot H}$$

Substituting this solution into (29a') for Q_{H0} and using the appropriate control rules, we can write the equilibrium condition (29a') in the following form:

$$\text{(29a")} \quad \lfloor \ell H \rfloor^{\beta} \tilde{K}^{1-\beta} = y_0 = \alpha\nu \left[\frac{(1-\beta)\ell+\beta}{(1-\beta)\ell}\right] Q_{BO}\tilde{K}$$

This can be solved for Q_{B0} to yield

$$\text{(33)} \qquad Q_{BO} = \frac{(1-\beta)\ell y_0}{\alpha\nu\lfloor (1-\beta)\ell+\beta \rfloor \tilde{K}} = \frac{(1-\beta)y_0}{\nu\tilde{K}}$$

The absolute value of the price of labor input Q_{H0} can then be easily solved by substitution. The solution is

$$\text{(34)} \qquad Q_{HO} = \frac{\beta y_0}{\alpha\nu\lfloor (1-\beta)\ell+\beta \rfloor H} = \frac{\beta y_0}{\nu \ell H}$$

Initial input prices (33) and (34) are prices which - if we assume technology, preferences, endowments and the initial commodity price $p_0 = 1$ as given - will clear factor and goods markets in the initial equilibrium. Note also that (33) and (34) imply that the parameter A used in the profits determination in e.g. (3) must equal ν in the general equilibrium. Due to the time independent control rules of the agents, factor markets will always stay in the same equilibrium state in terms of quantities. The quantities produced and consumed will, on the other hand, change continuously. To see that goods markets also stay continuously in equilibrium, let us write for convenience the complete solution for the price of a share and a wage contract:

$$(35a)\qquad Q_B(t) = \frac{(1-\beta)y_0}{\nu\tilde{K}} \exp\{\nu(t-t_0) + \sigma \int_{t_0}^{t} dz_L\} = Q_{B0}g(t)$$

$$(35b)\qquad Q_H(t) = \frac{\beta y_0}{\nu \ell H} \exp\{\nu(t-t_0) + \sigma \int_{t_0}^{t} dz_L\} = Q_{H0}g(t)$$

Note that the above equations and the fact that $A = \nu$ imply that the user costs of inputs to firms, or the instantaneous capital income and wage rate for consumers, are:

$$AQ_B(t) = \nu Q_{B0}g(t)$$

$$AQ_H(t) = \nu Q_{H0}g(t) = w_0 g(t) = w(t)$$

The latter equalities provide the justification for the substitution applied above in (26).

When solutions (35a,b) are inserted into the expression for real wealth and the terms in the goods market equilibrium condition (29a) are explicitly written out in full, condition (29a) becomes:

$$(36)\qquad \int_{t_0=0}^{h} (y_0 \exp\{\nu(s-t_0)+\sigma \int_{t_0}^{s} dz_L\})ds = \int_{t_0=0}^{h} (\alpha\nu[\frac{(1-\beta)y_0}{\nu\tilde{K}}\tilde{K}$$

$$+ \frac{\beta y_0}{\nu \ell H} H] \exp\{\nu(s-t_0)+\sigma \int_{t_0}^{s} dz_L\})ds$$

$$\Leftrightarrow \int_{t_0=0}^{h} y_0 g(s)ds = \int_{t_0=0}^{h} y_0 g(s)ds$$

The last equality proves that there is continuous equilibrium in the goods market. The equality requires that on the right hand side of the first equality the following holds:

$$\alpha\nu\left[\frac{(1-\beta)}{\nu} + \frac{\beta}{\nu\ell}\right] = 1$$

The condition can be checked by carrying out some simple algebraic operations.

Note one particular property of the general equilibrium in the simple model; namely, that the price level is indeterminate. The equilibrium price of a share and the wage rate were defined in real terms with the specific implicit assumption that the price of goods remains constant at $p_0 = 1$. But any other assumption for the time path of the price level can be chosen equally well. Thus, our model demonstrates the general fact that in the competitive equilibrium only relative prices in terms of the numeraire are determined, and the absolute level of all prices remains indeterminate.

2.3 The monetary economy with the public sector included

To include the public sector in the analysis, the framework must be extended on two fronts. On one hand, we need to specify the needs of consumers for public expenditure by changing their preferences. On the other hand, we have to define the forms of participation of the government in the economy.

The role of the public sector in an economy with as much explicit rationality as ours is not obvious. For illustration, it seems useful to first model the public sector as just another competitive sector of the nonmonetary economy, which produces goods to satisfy the needs of consumers. After we have seen the equilibrium characterization of this idealized economy, we will then proceed to develop a model in which the role of the public sector is more conventional.

2.3.1 The public sector and the economy: the idealized case

Consider an industry which has a Cobb-Douglas production technology for instantaneous production at the initial point of time $t = t_0$, defined by

$$(37) \qquad Y_{g0} = L_{g0}^{\beta} K_{g0}^{1-\beta}.$$

The production process defined by (37) uses labor and capital as inputs to produce public goods. It is assumed that when the industry hires an amount L_g of labor it actually receives a stochastic sequence of labor input services defined by the stochastic process:

$$(38) \qquad dL_g = (a_g dt + \sigma_g dz_g) L_g,$$

$$E(dz_g) = 0,$$

$$E\lfloor (dz_t^2) \rfloor = dt.$$

$z_g \sim N(0,1)$, and the increments $z(s) - z(u)$, $u > s \geqslant t_0$, $\forall$ u, s, are independent.

In (38) the parameter a_g describes the continuous process of learning-by-doing at a rate a_g and the parameter σ_g represents the labor disturbances in the production process.

We know from the analysis of section 2.2 that, as a consequence of assumption (38), the rate of production and the instantaneous marginal products of the two inputs are solutions of stochastic differential equations. The solutions take the following forms:

$$(39) \qquad Y_g(t) = Y_{g0} \exp\{\nu_g (t-t_0) + \sigma_g \int_{t_0}^{t} dz_g\},$$

(40) $$MP_{Lg}(t) = \frac{\beta Y_{g0}}{L_g} \exp\{\nu_g(t-t_0)+\sigma_g \int_{t_0}^{t} dz_g\},$$

(41) $$MP_{Kg}(t) = \frac{(1-\beta)Y_{g0}}{K_g} \exp\{\nu_g(t-t_0)+\sigma_g \int_{t_0}^{t} dz_g\},$$

where we have used the notations

$\nu_g = a_g - \frac{1}{2}(1-\beta)\sigma_g^2$, the mean instantaneous rate of change of productivity in the public sector, and

$\sigma_g = \beta\sigma_g$, the instantaneous standard deviation of the rate of change of productivity in the public sector.

From the analysis in section 2.2, we also know that in a perfectly competitive environment with flexible prices such an industry bears no profits risk and has an optimal capital labor ratio of

(42) $$\left(\frac{K_g}{L_g}\right) = \frac{(1-\beta)Q_{H0}^g}{\beta Q_{B0}^g},$$

where Q_{H0}^g and Q_{B0}^g are, as in section 2.2, the initial prices of one unit labor and capital input. The discounted profits of such an industry over an arbitrary time interval (t_0,t_1) are given by:

(43) $$V_g(t_0,t_1) = \int_{t_0}^{t_1} e^{-\rho(s-t\ )}[p_g(s)y_g(s) - A_g Q^g(s)L_g(s) - A_g Q_B^g(s)K_g(s)]ds$$

where A_g is again a constant to be determined in the general equilibrium.

The level of production and the relative prices of the output, capital and labor of the industry are again determined in the general competitive equilibrium via the preferences and initial endowments of consumers in an analogous manner to the determination of corresponding variables in the simple model. The demand-side assumptions differ from those of the basic model only in that the preferences of consumers now include demand for the good of the new industry.[7] The new preferences are:

(44) $$U(t) = \alpha_1 \log c_1(t) + \alpha_2 \log c_2(t) + \alpha_3 \log c_3(t);$$
$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

where α_1, α_2 and α_3 are the preference weights of private consumption, leisure and public expenditure, respectively.

The composition of wealth changes in that consumers now hold a fraction ξ_1 of their nonhuman wealth in the shares of the first industry and a fraction ξ_2 in those of the second industry. Hence, the wealth of the representative consumer becomes

(45) $$W(t) = \xi_1[Q_B(t)\tilde{K}+Q_H(t)H] + \xi_2[Q_B^g(t)\tilde{K}+Q_H(t)H]$$

In (45), we have followed the convention that, with homogeneous labor markets, the wage rates must be the same in the two industries.

[7]Technically, we are analyzing the demand for a publicly produced private good here. By this simplification, the well-known difficulties in representing the aggregate preferences for public goods are avoided. The concept public good is used for convenience in place of the cumbersome concept publicly produced private good.

The dynamic decision making problem of the representative consumer - analogous to problem (19) in section 2.2 - based on preferences (44) and aggregate wealth (45) is then:

$$(46)\qquad J[W(t),t] = \max_{\{c_1(t),c_2(t),c_3(t),\xi_1,\xi_2\}} E_{t_0}\{\int_{t_0}^{t_0+T} e^{-\rho(s-t_0)} [\alpha_1 \log c_1(s)+\alpha_2 \log c_2(s)+\alpha_3 \log c_3(s)]ds\}$$

$$\text{s.t.}\qquad dW(t) = [\xi_1(\nu dt+\sigma dz_L) + \xi_2(\nu_g dt+\sigma_g dz_{Lg})]W(t) - [c_1(t)+w(t)c_2(t)+p_g(t)c_3(t)]dt$$

$$\text{(i)}\qquad \xi_1 + \xi_2 = 1$$

$$\text{(ii)}\qquad E_s[W(t_0+T)] = W_0, \quad s \in [t_0,t_0+T]$$

where the notation of section 2.2 has been used to represent the first industry, or the private sector.

Constraint (ii) in problem (46) can be taken into account as an integral part of the standard solution, but because of constraint (i) the following Lagrangian must be formed for (46):

$$(47)\qquad L = \alpha_1 \log c_1(t) + \alpha_2 \log c_2(t) + \alpha_3 \log c_3(t) - \rho J(W(t),t) + \frac{\partial J(W(t),t)}{\partial W(t)}([\xi_1\nu+\xi_2(\nu_g-\Pi_g)]W(t) - c_1(t) - w(t)c_2(t) - p_g(t)c_3(t)) + \frac{1}{2}\frac{\partial^2 J(W(t),t)}{\partial W(t)^2}[\xi_1^2\sigma^2+2\xi_1\xi_2\sigma_{12}+\xi_2^2\sigma_g^2]W(t)^2 - \lambda(\xi_1+\xi_2-1)$$

where $\sigma_{12} = \text{cov}(dz_L, dz_g)$

Equation (47) implies a set of first order conditions which are straight-forward to determine. To shorten the exposition we only list here the solutions for the control variables implied by such first order conditions:

(48a) $c_1(t) = \alpha_1 \rho W(t)$

(48b) $c_2(t) = \frac{\alpha_2 \rho}{w(t)} W(t)$

(48c) $c_3(t) = \frac{\alpha_3 \rho}{p_g(t)} W(t)$

(48d) $\xi_1 = \phi_M + \phi_s$

(48e) $\xi_2 = 1 - \phi_M - \phi_s$

where $\phi_M = \frac{\sigma_g^2 - \sigma_{12}}{\sigma^2 + \sigma_g^2 - 2\sigma_{12}}$ = minimum variance portfolio fraction

$\phi = \frac{\nu - (\nu_g - \Pi_g)}{\sigma^2 + \sigma_g^2 - 2\sigma_{12}}$ = speculative portfolio fraction

However, the solutions in (48) are not yet completely open to simple interpretations. The role of the preference weights α_1 and α_3, and the income flow of the representative consumer must be analyzed further. First, if we adopt the notation in which $\tilde{W}(t)$ stands for the effective real wealth net of human capital spent in leisure, then α_1 and α_3 must be appropriately adjusted. The consumer consumes all the income earned on net wealth. Hence, it must be assumed that $\alpha_1 + \alpha_3 = 1$ when spending is based on $\tilde{W}(t)$. It is convenient to use the notation $\alpha = \frac{\alpha_1}{\alpha_1 + \alpha_3}$ and $1-\alpha = \frac{\alpha_3}{\alpha_1 + \alpha_3}$ associated with $\tilde{W}(t)$. With the above notation, the two goods markets equilibrium conditions can be written as:

$$(49a)\qquad y_0\exp\{\nu(t-t_0)+\sigma\int_{t_0}^{t} dz_L\}$$

$$=\alpha\rho(\xi_1[Q_{B0}\tilde{K}+Q_{H0}\ell H]\exp\{\nu(t-t_0)+\sigma\int_{t_0}^{t} dz_L\}$$

$$+\xi_2[Q_{B0}\tilde{K}+Q_{H0}\ell H]\exp\{(\nu_g-\Pi_g)(t-t_0)+\sigma_g\int_{t_0}^{t} dz_g-\sigma\int_{t_0}^{t} dz_L\}$$

$$(49b)\qquad y_{g0}\exp\{\nu_g(t-t_0)+\sigma_g\int_{t_0}^{t} dz_g\}=\frac{(1-\alpha)\rho\tilde{W}(t)}{p_g(t)}$$

The equilibrium conditions reveal a very important fact: The goods markets can only be in a dynamic equilibrium if the price of the public good follows the following stochastic process.

$$(50)\qquad \frac{d\,p_g}{p_g}=\Pi_g dt+\sigma_\Pi dz_\Pi=(\nu_g-\nu)dt+\sigma dz_L-\sigma_g dz_g$$

Note that the time path of the price of the public good implied by (50) is

$$(51)\qquad p_g(t)=p_{g0}\exp\{(\nu_g-\nu)(t-t_0)+\sigma\int_{t_0}^{t} dz_L-\sigma_g\int_{t_0}^{t} dz_g\}$$

If (50) and (51) are substituted into the equilibrium conditions (49a,b), the conditions can be written as:

$$(49a')\qquad y_0\exp\{\nu(t-t_0)+\sigma\int_{t_0}^{t} dz_L\}$$

$$=\alpha\rho[Q_{B0}\tilde{K}+Q_{H0}\ell H]\exp\{\nu(t-t_0)+\sigma\int_{t_0}^{t} dz_L\}$$

$$(49b') \quad y_{g0}\exp\{\nu_g(t-t_0)+\sigma_g \int_{t_0}^{t} dz_g\}$$

$$= \frac{(1-\alpha)\rho}{p_g}\lfloor Q_{BO}\tilde{K}+Q_{HO}\ell H\rfloor \exp\{\nu_g(t-t_0)+\sigma_g \int_{t_0}^{t} dz_g\}$$

In (49a',b'), the exponential terms on the supply and demand sides of the markets are identical. Hence, if the initial values of the relevant variables equilibrate the markets in the initial state, then the economy will continuously remain in equilibrium.

The general message of requirement (50) is that in a two good model with Cobb-Douglas preferences the price of the good in the production of which the mean productivity growth rate is higher, must deflate on the average. Also, the required price behaviour of the public good makes the two investment alternatives identical for the investors, so that investors are indifferent between the shares of the two industries. Hence, the portfolio fractions ξ_1 and $\tilde{\xi}_2$ are determined in the goods markets so that $\xi_1 = \tilde{\alpha}_1$ and $\xi_3 = \alpha_3$. The labor supply rule becomes analogous to the one in section 2.2, i.e.:

$$(52) \qquad \ell = 1 - \alpha_2 - \alpha_2 \frac{Q_{BO}\tilde{K}}{Q_{HO}H}$$

The initial state equilibrium conditions of the extended economy which determine the initial relative prices are:

(53a) $y_0 = \alpha\rho[Q_{B0}\tilde{K}+Q_{H0}\ell H]$

(53b) $y_{g0} = \frac{(1-\alpha)\rho}{P_{g0}}[Q_{B0}\tilde{K}+Q_{H0}\ell H]$

(53c) $L + L_g = \ell \cdot H$

(53d) $K + K_g = \tilde{K}$

Conditions (53a-d) imply the relative prices of capital, labor and the public good in terms of the initial price $p_0 = 1$ of the private good. The solutions for such relative prices are obtained in a similar way as those in section 2, and they are:

(54a) $Q_{H0} = \frac{\beta y_0}{\rho\ell H}$

(54b) $Q_{B0} = \frac{(1-\beta)y_0}{\rho\tilde{K}}$

(54c) $P_{g0} = 1$

The above solutions imply that the parameter A_g in the profits function of the public sector must equal ρ. Note again that the absolute prices are not determined in the general equilibrium, as the price of the private good, or the numeraire, can be freely chosen.

2.3.2 The public sector and the economy: the conventional case

In this section, a relatively conventional macroeconomic model is developed in which the idealized two sector model is modified in several important respects. The role of the public sector in the economy will be made technically more conventional by replacing the profits function (43) by the government budget constraint. The

Keynesian features described above in the first chapter, i.e. the separation of capital and labor supply decisions together with nominal wage contracting, are introduced into the analysis. Also, the distinction between the short-run and the long-run technology is stated.

It is clear that when we introduce Keynesian features into the analysis we have to compromise with the notion of rationality to a certain extent. In the first chapter, we proposed the separation of input supply decisions and nominal wage contracting as institutionally plausible assumptions in the economies of the Nordic countries. It seems useless to discuss the rationality of such institutional arrangements from the point of view of welfare economics in this study. However, even the way in which the properties are built into the framework poses some questions. One of the empirical stylized facts stated in the first chapter is that financial intermediaries make the actual capital supply decisions in Nordic economies.

According to the standard neoclassical assumptions, financial intermediaries are owned by consumers, exist to economize the operations of monetary economies and maximize the preferences of the owners. The last characteristic raises the question: why should such intermediaries make decisions which could under some circumstances decrease the wage income of the owners by causing lay-offs? Also, why should owners form a trade union to negotiate wage contracts when they will in any case receive the whole of the national product as factor payments? These questions suggest that, in order to make the effects of variation in income distribution a logically consistent property of the model, financial intermediaries must be consuming agents in technical analysis. Hence, despite the connotations associated with it, we apply the assumption of two types of consumers. Thus there are, on one hand, investors, who supply only capital, and, on the other hand, workers, who supply only labor. We will assume that the relative weights of different forms of consumption (except for leisure) are the same in the preferences of both types of consumers. Note that, with Cobb-Douglas preferences, the above assumption is almost identical to the assumption of homogeneous consumers and one financial intermediary which maximizes the owners' preferences, but only takes the non-human wealth into account in its decision-making. Both technical model constructions will generate the same consumption, income and unemployment consequences if in the latter case we restrict the intertemporal substitution possibilities of consumers in labor supply by requiring that a consumer can only work either a fixed number of hours per day or not at all.

We assume first, then, that there are n consumers in the economy. In the initial state, the investors in the economy owned the total amount $\tilde{K}$ of real capital. There are n_1 investors in the economy, each of whom held a fraction $\frac{1}{n_1}$ of the real capital stock in the initial state. In the initial state general equilibrium, the investors exchanged the units of real capital for shares issued by firms and for government bonds. One unit of capital corresponds to one security, so that the total number of securities equals the number of units of capital. On the average, the investors hold all the shares and bonds, but, at times, some securities may also be held by the workers. The investors do not supply labor at all. The holders of the shares own the firms, and here we take this to mean that, in addition to receiving the general equilibrium factor payments for capital, the shareholders also receive the profits or bear the losses of the firms.

The preferences of all the investors with respect to the real instantaneous consumption of the private good, $c_i(t)$, and the real consumption of the public good, $c_{gi}(t)$, are identical with each other and are defined by the following utility function:

$$(54)\qquad U_i(t) = \alpha_i \log(c_i(t)) + (1-\alpha_i)\log(c_{gi}(t));\ \alpha_i \varepsilon (0,1).$$

The rest, $n-n_1$, of the consumers are workers who supply labor but no capital on the average.[8] Each worker has a maximum instantaneous working capacity $\frac{1}{n-n_1}H$, which he can either consume as leisure or supply as labor. The workers also receive utility from the instantaneous real consumption of the private and the public good. Each worker has identical preferences in the three forms of consumption, which are represented by the following instantaneous utility function:

$$(55) \qquad U_W(t) = \alpha_{1W}\log(c_W(t)) + \alpha_{2W}\log(c_{\ell W}(t)) + \alpha_{3W}\log(c_{gW}(t))$$

$$\alpha_{1W}, \alpha_{3W} \in (0,1); \quad \alpha_{2W} = 1 - \alpha_{1W} - \alpha_{3W}.$$

$c_W(t)$ = worker's consumption of the private good.
$c_{\ell W}(t)$ = worker's consumption of leisure.
$c_{gW}(t)$ = worker's consumption of the public good.

We require also the following to hold:

$$(55a) \qquad \frac{\alpha_{1W}}{\alpha_{3W}} = \frac{\alpha_i}{1-\alpha_i} = \frac{\alpha}{1-\alpha}$$

Condition (55a) indicates that the relative weights of the private and the public good in the preferences of all agents are the same.

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[8]In our analysis, 'trend' saving and capital formation are assumed away. This implies that when the economy is on the trend path, i.e. when the rate of time preference and the rate of instantaneous real income are equal, the workers will not participate in the asset markets at all. However, in short-term analysis, variations in the rate of time preference are, in principle, allowed. This implies that there can be net saving and dissaving in the model in the short run. Consequently, workers can temporarily, at least in principle, participate in the asset markets. Thus the fundamental difference between workers and investors is that the latter do not supply labor at all.

Secondly, we assume that the workers are organized into a trade union.[9] The union negotitates nominal wage contracts with a large number of competitive firms. The contracts with each individual firm start at the same time and their length is fixed at T_c time units. Under the assumed conditions in the labor market, the contract negotiations will be dominated by the union. The union's policy aims at guaranteeing its members the maximum expected real wage rate, given expected full employment. The contract wage, and thus the actual nominal wage rate paid to the workers, is such that the expected rate of change in nominal wages is equal to the expected rate of change in the value of the marginal product of the firm, and the expected utility for the firm from producing during the contract period is at the competitive level. All expectations are formed at the time of contracting. The contract

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[9]The assumption on the presence of the trade union can be theoretically defended by the asymmetric position of the workers and investors with respect to extreme states of nature. If the subsistence rate of consumption is somewhat higher than the lowest possible instantaneous real income then there are some states of nature in which starvation would threaten consumers. Investors could survive such states of nature, if real capital is assumed edible, but the workers do not have any means of survival. This creates a strong incentive for founding a union, which negotiates wage contracts in which the extreme states of nature are eliminated. Note that this is not a decisive argument, since forming a union is just one way of achieving the security objective. Alternative (and possibly superior) ways would be e.g. storage of goods, insurance, or precautionary saving by workers.

wage determination in the economy over an arbitrary contract period $[t_0, t_0+T_C]$ is then given by the following two expressions:[10]

$$(56a) \qquad E_{t0}\left[\frac{dQ_H(t)}{Q_H(t)}\right] = E_{t0}\left[\frac{d(p(t)MP_L(t))}{p(t)MP_L(t)}\right] \equiv \dot{w} = \nu$$

$$(56b) \qquad E_{t0}\left[\frac{Q_H(t)}{p(t)}\right] = e_{t0}\, \frac{1}{\rho}\left[MP_L(t) - \beta d_L y(t)\right],$$

where d_L is the minimum risk premium associated with the use of labor input in production such that the firms are indifferent between producing and not producing.[11] Other notation is adopted from the second section.

$\dot{w}$ = rate of increase in the contract wage

Although simple, our assumption on the nature of wage contracts describes fairly meaningful union behaviour. In addition to equating the expected rate of change in real wages to the expected rate of change in marginal productivity of labor, the specification implies that the expected ratio of wage income to capital income remains constant in the economy. Moreover, even though the trade union introduces an externality into the decision making of

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[10]Expressions (56a) and (56b) can be motivated in a simple way by considering the problem of a profit maximizing firm which operates in a static riskless environment. The first order conditions for such a firm with Cobb-Douglas technology are

$\beta pY - Q_{HL} = 0$

$(1-\beta)pY - Q_{BK} = 0$

If we define that

$\frac{\beta Y}{L} = MP_L$

$\frac{(1-\beta)Y}{K} = MP_K$

then (56b) is a dynamic representation of the first order condition for labor under uncertainty concerning actual productivity. (56a) is obtained from (56b) by direct differentiation.

[11]The expression for the risk premium d_L is derived in appendix 1.

the firms by forcing them to bear some profits risk, it also allows the firms to adjust to unanticipated distortions in the economy by changing the price of the commodity accordingly within the contract period. In particular, note that, according to the contract setting rule, the contract wage does not respond at all to the potential excess demand and supply situations in the labor market. This feature creates the possibility of persistent unemployment in the analysis.

Note also that we assume the contract wage to be binding on the government as well.

Firms pay the fixed nominal rate of return on shares. Within the wage contract period, exogenous productivity and price shocks may hit the economy. Because of the rigid labor costs, this would affect the profits of the firms. It is assumed that the firms accumulate such profits and losses in each subperiod in a separate fund, which is established by the shareholders as part of the terms of shares. The proceeds of the fund are shared by investors in proportion to their holdings of shares. As a consequence of this assumption, the firms do not substitute between capital and labor within the contract period. Only shocks of a fairly structural nature, such as changes in the contract wage setting rule, will cause variations in the capital-labor ratio. The technology becomes essentially a fixed-coefficient technology in the short run and a Cobb-Douglas technology in the long run. The variations in the balance of the fund may affect the expected returns on shares and thereby the portfolio decisions of the investors. Hence the investors, by their portfolio choices, set the scale of production. The firms adjust unemployment so as to maintain the equilibrium or long-run capital labor ratio.

The last general assumption we state is that the economy is a monetary economy. Thus, the government has established a convention that all prices in the economy are expressed in terms of the currency, which is the only means of payment in the economy. The transactions demand for money is determined by the transactions technology, which is assumed to be such that, for one unit of expenditure, b(t) units of money are needed to carry out the transaction. Thus the quantity theory, or, in a dynamic context, the Cagan money demand function, defines the aggregate transactions demand for money in the economy. Formally, this can be expressed as

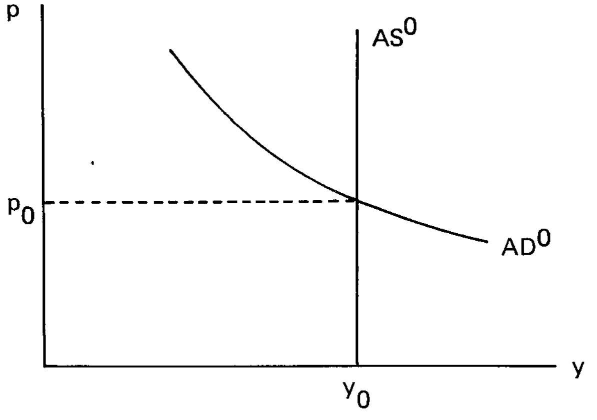
$$(57) \qquad p(t)y(t) = b(t)M(t)$$

where $M(t)$ = transactions demand for money.

The consequences of the above assumptions on the private sector deserve some comments. First, and as usual, the key role of the quantity theory identity (57) is in fixing the price level of the economy. Second, the separation of workers from investors makes labor supply independent of income distribution. Third, the construction of profits funds, which makes the technology fixed-coefficient in the short run, also makes the conventional aggregate supply curve vertical. Thus, all the shocks which affect the profits of firms may affect aggregate supply only by making investors adjust their portfolios; consequently, the aggregate supply curve shifts. In the standard static graphical exposition, the goods markets can be represented by the following figure:

Fig. 1.

GOODS MARKETS EQUILIBRIUM



In the figure, goods markets are in an equilibrium at point (p_0,y_0). A typical demand shock would shift the demand curve to a new position. Whether there will be real effects or not in the economy depends on whether the investors react to the implied change in the price level by adjusting their portfolios. If they do, the supply curve will shift as well and there will be real effects. If they do not, the supply curve remains where it was and the price adjustment equilibrates the goods markets.

The government produces public goods by means of the technology defined in (37), where $\nu_g = \nu$ is assumed for simplicity. To hire the desired amount of labor, the government pays the same contract wage to workers as private firms do. To raise capital and to finance deficits, the government issues two types of bonds. One bond of either type is exchanged for one unit of capital. One of the bonds is the riskless asset of the economy, i.e. its nominal rate of return is riskless. By contrast, the other bond is risky, having a stochastic nominal return. The prices of the two bonds follow the following processes:

$$(58a) \qquad \frac{d\,Q_G^1}{Q_G^1} = r_G dt + \sigma_G dz_G$$

$$(58b) \qquad \frac{d\,Q_G^2}{Q_G^2} = r\,dt$$

where $Q_G^1(t)$ = price of a risky bond at time t

$Q_G^2(t)$ = price of a riskless bond at time t

r = nominal rate of return on the riskless bond

r_G = nominal mean rate of return on the risky bond

We assume that the riskless return r is always positive and fixed. Hence, riskless bonds dominate money holdings as the riskless asset of the economy and money is only demanded for transactions purposes. Note that, with these assumptions, riskless bonds are in effect the speculative money of the economy.

The mean return r_G on the risky bond is determined in the asset market equilibrium in a way to be seen later via the requirement that the total fraction of capital held in public assets is $1-\alpha$. The government raises revenues by levying taxes on income. The aggregate net tax flow is $\tau(t)$, the determination of which is discussed below.

Under the new assumptions on the economy and the public sector, the decision problem of consumers must be split into the problem of a representative investor and the problem of a representative worker. The two problems can be formulated as follows:

$$(59) \qquad J_i(V_B(t),t) = \max_{\{c_i(t)c_{gi}(t),\xi_1,\xi_2,\xi_3\}} E_{t_0}\left\{ \int_{t_0}^{t_0+T} e^{-\rho_i(s-t_0)} \cdot \left[\alpha_i \log c_i(s) + (1-\alpha_i)\log c_{gi}(s)\right] ds \right\}$$

$$\text{s.t.} \quad dV_B(t) = [\xi_1(r_B dt+\sigma_B dz_B) + \xi_2(r_G dt+\sigma_G dz_G) + \xi_3 r dt]V_B(t) - [p(t)c_i(t) + p_g(t)c_{gi}(t)]dt$$

$$\xi_3 = 1 - \xi_1 - \xi_2$$

$$(60) \quad J_W(V_H(t),t) = \max_{\{c_{1w}(t),c_2(t),c_{3w}(t)\}} E_{t_0}\left\{ \int_{t_0}^{t_0+T} e^{-\rho_i(s-t_0)} \cdot [\alpha_{1w}\log c_{1w}(s)+\alpha_{2w}\log c_2(s)+\alpha_{3w}\log c_{3w}(s)]ds\right\}$$

$$\text{s.t.} \quad dV_H(t) = \dot{w}dtV_H(t) - [p(t)c_{1w}(t)-w(t)c_2(t)-p_g(t)c_{3w}(t)]dt$$

where indexes i refer to the representative investor and indexes w to the representative worker. The notation concerning the non-human wealth determination in (59) has been generalized from that used in the idealized case of section 2.3.1. The notation will be defined completely below after equations (61).

The two problems are relatively easy to solve along the lines followed in section 2.2, and therefore we only list here the optimal consumption, labor supply and portfolio rules which are consistent with our assumptions for the extended model:

$$(61a) \quad c(t) = \frac{\alpha\rho}{p(t)} V_B(t) + \frac{\alpha\rho}{p(t)} \tilde{V}_H(t)$$

$$(61b) \quad c_g(t) = \frac{(1-\alpha)\rho}{p_g(t)} V_B(t) + \frac{(1-\alpha)\rho}{p_g(t)} \tilde{V}_H(t)$$

$$(61c) \quad \ell(t) = \frac{(1-\alpha_{2W})\rho}{w(t)} V_H(t) = 1 - \alpha_{2w}$$

$$(61d) \quad \xi_1 = \frac{1}{\sigma_B^2(1-\rho_{BG}^2)}(r_B-r) - \frac{\rho_{BG}}{\sigma_B\sigma_G(1-\rho_{BG}^2)}(r_G-r).$$

(61e) $$\xi_2 = \frac{1}{\sigma_G^2(1-\rho_{BG}^2)}(r_G - r) - \frac{\rho_{BG}}{\sigma_B\sigma_G(1-\rho_{BG}^2)}(r_B - r).$$

(61f) $$\xi_3 = 1 - \xi_1 - \xi_2,$$

where the following notation and assumptions have been used

$c(t) = c_i(t) + c_{1w}(t)$

$c_g(t) = c_{gi}(t) + c_{3w}(t)$

$\alpha = \alpha_i = \dfrac{\alpha_{1w}}{\alpha_{1w} + \alpha_{3w}}$

$1-\alpha = 1-\alpha_i = \dfrac{\alpha_{3w}}{\alpha_{1w} + \alpha_{3w}}$

$\rho_i = \rho_w = \rho$

$\tilde{V}_H(t) = Q_H(t)\ell H$

ξ_1 = portfolio fraction of shares,

ξ_2 = portfolio fraction of risky government bonds,

ξ_3 = portfolio fraction of riskless government bonds

$r_B(t) \equiv r_B dt + \sigma_B dz_B$ = nominal return on shares,
$r_B = \nu$, $\sigma_B dz_B = \sigma dz_L$

$r_G(t) \equiv r_G dt + \sigma_G dz_G$ = nominal return on risky government bonds, $\sigma_G dz_G = \sigma_g dz_g$

r = nominal return on riskless government bonds,

$\ell(t)$ = rate of labor supply,

ρ_{BG} = correlation coefficient between processes (dz_B, dz_G).

Conditions (61d-f) define directly the required mean return r_G on the risky government bond, given the rest of the parameters. To simplify notation and subsequent analysis, we assume that the government bonds are less risky than shares and that the stochastic processes dz_B and dz_G are correlated so that $\sigma_G = \sigma_B \rho_{BG}$. These technical assumptions imply the economic assumption that, when capital moves between the two sectors, the portfolio fraction ξ_3 remains constant and substitution only occurs between shares and risky bonds. Using (61d-f) and the above assumptions, and requiring that $1-\alpha = \xi_1 + \xi_2$, we obtain the following solution for r_G:

$$(62) \qquad r_G = r_B - \alpha\sigma_B^2(1-\rho_{BG}{}^2)$$

By the above assumptions on the public sector asset returns, the prices of government bonds are given by the following functions:

$$(63a) \qquad Q_G^1(t) = Q_{G0}^1 \exp\{(r_B - \alpha\sigma_B^2(1-\rho_{BG}{}^2))(t-t_0) + \sigma_G \int_{t_0}^{t} dz_G\}$$

$$(63b) \qquad Q_G^2(t) = Q_{G0}^2 \exp\{r(t-t_0)\}$$

The analogue of the idealized public sector profit function in the present more realistic case is the government budget constraint. By our assumptions, the government budget constraint is given by:

$$\begin{aligned}(64) \qquad dM(t) &= dQ_G^1(t)B_G^1(t) + dQ_G^2(t)B_G^2(t) + dQ_H(t)L_g(t) \\ &\quad + dB_G^1(t)Q_G^1(t) + dB_G^2(t)Q_G^2(t) + dL_g(t)Q_H(t) - \tau(t) \\ &= (r_G + \sigma_G \frac{dz_G}{dt})Q_G^1(t)B_G^1(t) + rQ_G^2(t)B_G^2(t) \\ &\quad + rB_G^1(t)Q_G^1(t) + dB_G^2(t)Q_G^2(t) + \dot{w}Q_H(t)L_g(t) \\ &\quad + dL_g(t)Q_H(t) - \tau(t) \equiv G(t) - \tau(t)\end{aligned}$$

where $dM(t)$ = instantaneous change in money supply

$$G(t) = dQ_G^1(t)B_G^1(t) + dQ_G^2(t)B_G^2(t) + dQ_H(t)L_g(t)$$
$$+ dB_G^1(t)Q_G^1(t) + dB_G^2(t)Q_G^2(t) + dL_g(t)Q_H(t)$$

According to (64), the instantaneous change in money supply follows a stochastic process determined by several factors. The first three terms on the right hand side of (64) refer to factor payments made by the government to the private sector. Conventionally, these terms can be interpreted as government expenditure. The following two terms are the effects of changes in the stocks of the two types of government bonds. Such changes can be indications of either open market operations, implying that $dB_g^1(t) = -dB_g^2(t)$, or crowding out or crowding in, requiring capital flows between the private and the public sector. The sixth term is the effect of a change in public sector employment on the financial position of the government. The last term is the tax revenue of the government.

All possible government policies can be characterized in terms of the government budget constraint. As to the effects of policies, recall, for instance, that, through the quantity theory equation (57), decisions on the right hand side of (64), which cause money supply to vary, give rise to variations in the price level. The first question of interest is to try to find the policy which would guarantee a full employment general equilibrium. Such a policy is simply found. If the government sets $L_g = (1-\alpha)\ell H$, $B_g^1 + B_g^2 = (1-\alpha)\tilde{K}$ always, and then collects back all the factor payments from consumers in taxes, general equilibrium is reached. This requires different tax flows from workers and investors, however. Specifically, for the general equilibrium the government has to set[12]

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[12]Note that in order for policy (65), (66) not to allow for distortions, the classification of consumers into investors and workers must be exogenous. A distortion would arise if some of the consumers could choose whether to be a worker or not.

(65) $\tau_i(t) = dQ_G^1(t)B_G^1 + dQ_G^2(t)B_G^2 + dM_i(t)$

(66) $\tau_w(t) = dQ_H(t)L_g + dM_w(t)$

where i refers to investors and w to workers.

With the general equilibrium policy, the production and consumption of public goods are at equilibrium levels. Also, all the income created by public activities is taxed so that only the income generated by the private sector is left for purchasing private goods. Hence, as consumption is independent of income distribution between workers and investors, private production and consumption must be equal.

The policy that guarantees full employment equilibrium seems to be very unrealistic, however, if practical situations are considered. Problems with demand revelation mechanisms in the case of public goods are well-known, and we have chosen a very simple way to avoid them by studying publicly produced private goods. Moreover, in reality, lack of information may prevent the government from conducting the perfect policy. For these reasons, we will next briefly describe alternative practical policies and their formulation.

It is clear that there are many alternative policies available to the government which guarantee full employment and government budget balance in the sense of (64) when the economy operates on the long-term trend path. It is only in the distortionary situations that the role and type of policy become critical. To ignore uninteresting cases, let us define that, for any plausible government policy, the budget balance holds in the sense of (64) when there are no distortions present in the economy, i.e.:

(67) $\bar{\tau}(t) = \bar{G}(t) - d\bar{M}(t)$

where $^{-}$ refers to the general equilibrium time paths of variables

The government's role can be simplified in the formal model by using the familiar disposable income notation. The model (61) then becomes:

(61a') $c(t) = \frac{1}{p(t)}\left[\rho(V_B(t)+\tilde{V}_H(t))-\tau(t)\right]$

(61c-f) as above

Note that, if the government is running a balanced budget and no expenditure is financed by money printing, then the following identity must hold in the consumption function (61a'):

(68) $\rho(V_B(t)+\tilde{V}_H(t)) - \tau(t) = p(t)y(t)$

Equation (68) states that consumers spend all national income earned in the private sector on private goods, if the government runs a balanced budget in the restricted sense above. With the notation of the simplified model (61a'), (61c-f), all the characterizations of fiscal policy can be carried out in terms of the disposable income.

Three different policy regimes will be considered in the analysis of the third chapter. A common feature to all the regimes is that the government always employs a fraction $1-\alpha$ of the labor force. The choice of the three specific regimes is based on the type of distortion that the policy maker is to deal with. In our analysis, the potential distortion arises from the fact that, owing to some exogenous events, the price level may fluctuate. This makes the real rate of return on shares fluctuate. As the riskless nominal interest rate is always fixed at r, investors will be willing to reallocate their portfolios, if this is allowed, or if the distortion is not neutralized.

In order to make the analysis interesting, we impose the following general restriction: The government prevents all speculative actions that are taken after the shock has started. This assumption can be motivated by some considerations. First, it is reasonable to assume that the government observes the shock and understands its consequences at the latest at the time when it occurs. If this were not accepted, the government would be believed to have inferior information relative to the investors. Second, the government also knows that any action that the investors take after the onset of the shock aims at samehow circumventing the effects of the contract wage rigidity, which they are already paid for in the form of the risk premium. If the government were to allow the portfolio adjustments after the shock has hit the economy, it would be conducting policy in favor of investors. One example of a policy that would eliminate trading after the onset of such a shock is a specific tax.[13]

What investors can do under the above restriction is to engage in speculative portfolio adjustments before the shock actually occurs. Any government's policy rules must be related to such behaviour in circumstances where the government may have poor information. The first policy regime can be viewed as analogous to the familiar money targeting monetary policy. When the government follows this regime, it fixes a money supply rule and sells as many bonds on the terms defined in (58a,b) as investors want to buy. Since the bonds sell at the general equilibrium terms, the

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[13] An appropriate policy would be to impose a specific capital income tax on investors. If the government announced that all the additional income that investors would receive from portfolio adjustments after the shock would be taxed to finance unemployment benefits indirectly caused by those portfolio adjustments, then investors would have no incentive to reallocate investments.

government knows that, when there are no distortions, it will raise the general equilibrium amount of capital. When a shock that changes the price level hits the economy, there will be redistribution of income between investors and workers. If the investors anticipate the shock in advance, by the substitution assumption stated above after (61) they trade shares for risky bonds, or vice versa, depending on the shock. Hence, the amount of speculative money remains constant and the 'money targeting policy' characterization is justifiable for the regime. The money targeting policy regime can be specified by the following expressions:

$$\text{(69a)} \qquad \frac{dM(t)}{M(t)} = \nu$$

$$\text{(69b)} \qquad \frac{dQ_G^1(t)}{Q_G(t)} = r_G dt + \sigma_G dz_G$$

$$\text{(69c)} \qquad \frac{dQ_G^2(t)}{Q_G(t)} = r dt$$

In (69a), we have determined that the government chooses the specific money supply rule by which the money supply grows at the mean rate of growth of productivity.

The second policy regime can be seen as analogous to the interest rate targeting monetary policy. In this policy, the government tries to adjust the nominal terms on government bonds so that variations in the price level do not cause capital to shift between the two sectors. The rate of return on the riskless bond must be held constant by definition, so that only the mean rate of return or the nominal price of the risky bonds can be adjusted. Later, it will be seen that, under this policy regime, the investors speculate by trading risky bonds for riskless bonds, and vice versa. As riskless bonds are an exact analogue of speculative money in our model, we can let the government fix the transactions money supply and yet have a perfectly flexible supply schedule of riskless claims on the government. This, on the other hand, is the key characteristic of the interest rate targeting policy in traditional analyses (see e.g. Poole (1970)). Formally, we can define the interest rate targeting regime by the following expressions:

(70a) $$\frac{d\,M(t)}{M(t)} = \nu$$

(70b) $$\frac{Q^1_{GR}(t)}{Q_{BR}(t)} = \frac{Q^1_G(t)}{Q_B(t)}$$

(70c) $$\frac{d\,Q^2_G(t)}{Q^2_G(t)} = r\,dt$$

Condition (70b) requires that the ratios between the real price of a risky bond and the real price of a share, indicated by subscript R, and between the respective nominal prices must be equal. This is the requirement that variations in the price level will not cause variations in the relative preferability between risky bonds and shares.

Finally, we will specify the regime of the Keynesian stabilization policy, the objective of which is to minimize real fluctuations in the economy. In our model, this amounts to minimizing the unemployment rate. In the Keynesian regime, the government can freely choose both the net tax flow $\tau(t)$ and the money supply rule. The Keynesian regime can be characterized by the following minimization problem:

(71) $$\min_{\{\tau(t),\, M(t)\}} (un(t))^2$$
s.t. (64)

where $\quad un(t)$ = unemployment rate $= \dfrac{(\tilde{L}(t)+\tilde{L}_g(t)-(L(t)+L_g(t))}{\tilde{L}(t)+\tilde{L}_g(t)}$

$\tilde{}$ = refers to the full employment equilibrium level of the respective variable

In the cases which we will study, the minimization problem (71) turns out to be trivial, as policies guaranteeing a zero rate of unemployment are always available.

In the general framework presented above, various experiments in closed economy macroeconomics can be studied. Particular complexities in the model due to stochastic variations about the trend path can be avoided by studying cases in which there is an exogenous shift in some of the parameters, but during which the stochastic variations, at least in the public sector, are equal to zero. One such case is studied in the third chapter, where the effects of a temporary shift in savings behaviour are analyzed.

2.4 A model of a small open economy

The basic assumptions of the 'conventional' closed economy model are used to a large extent as the underlying assumptions in the open economy model. Thus the assumptions on two different groups of consumers, the trade union, wage contracts and profits funds are retained unchanged in the open economy analysis. Additional assumptions are needed for different blocks of the economy: the preferences of consumers must include the need for imports, the position of the government with respect to the economy has to be simplified slightly, and the links between the home country and the rest of the world must be specified.

It is assumed that the representative investor has a weight α_{4i} in his Cobb-Douglas utility function representing consumption of imported goods, or $c_{mi}(t)$. The corresponding weight of the representative worker is α_{4w}. In particular, we require that

$$(72) \qquad \frac{\alpha_{4i}}{\alpha_{1i} + \alpha_{3i} + \alpha_{4i}} = \frac{\alpha_{4w}}{\alpha_{1w} + \alpha_{3w} + \alpha_{4w}}$$

Condition (72) states again that the relative weights of imports are the same in the preferences of both investors and workers.

The government of the home country, is in principle, similar to the one specified in the closed economy model. Thus the government provides the economy with public services, supplies money and levies taxes to partially finance public expenditure. The main difference is that the government issues only one bond at a riskless rate of return r to raise capital in the home economy. The government fixes the size of the public sector at α_3, i.e. the government always employs a fraction α_3 of the labor and capital of the home economy in public sector activities. At a rate of return r, a fraction ξ_3 of the capital owned by investors is supplied to the government. Hence, if $\xi_3 < \alpha_3$, then the government borrows the implied difference in capital from abroad. If, alternatively, $\xi_3 > \alpha_3$ then the government invests the excess capital abroad. If labor is immobile, the government has no corresponding problem with labor supply, as long as it pays the contract wage to workers. The other difference in the government position is that the autonomy of the government with respect to the money supply rule is limited by the influence of the rest of the world. Our specific assumption in this respect is that the government sets the money supply rule so as to eliminate systematic, anticipated inflation rate differences between the home country and the rest of the world.

The home country is a small economy relative to the rest of the world. In particular, the foreign currency prices $p_m^*(t)$ and $p_x^*(t)$ of the imported and exported goods are exogenous to the domestic investors. In the analysis, we ignore variations in the terms of trade and assume that imported and exported goods are physically measured in units such that $p_m^*(t) = p_x^*(t) = p^*(t)$. The world price level is determined by the money supply abroad through the quantity theory. The rate of growth of money supply abroad is given by:

$$\text{(73)} \qquad \frac{dM^*(t)}{M^*(t)} = \dot{m}^*(t) = (\nu^* + \Pi^*)dt + \sigma^* du$$

where $du \sim N(0,1)$ i.i.d.

ν^* = world rate of growth of productivity

Π^* = mean world inflation rate

The time path of the world price level, $p^*(t)$, implied by the quantity theory and the money supply rule (73) is:

$$\text{(74)} \qquad p^*(t) = p_0^* \exp\{\Pi^*(t-t_0) + \sigma^* \int_{t_0}^{t} du\}$$

According to (74), the world price level follows a geometric brownian motion stochastic process, where the random component is due to the presumed inability of the policy authority of the rest of the world to set the money supply precisely. We stated above that the home country government sets the money supply so that no systematic differences occur between domestic and foreign inflation rates. This requires that the domestic rate of growth of money supply under nondistortionary conditions is:

$$\text{(75)} \qquad \frac{dM(t)}{M(t)} \equiv m(t) = (\nu + \Pi^*)dt$$

Money supply rule (75) implies that the time path of the expected home country price level, $E_{t0}p(t)$, becomes

$$\text{(76)} \qquad E_{t0}p(t) = p_0 \exp\{\Pi^*(t-t_0)\}$$

On the other hand, the domestic price level and the world price level must be connected via the exchange rate s(t) as follows:

(77) $p(t) = s(t)p^*(t)$

We assume that the home country tries to keep the exchange rate fixed at the initial level $s_0 = \frac{p_0}{p_0^*}$. Hence, the actual price level in the home country becomes:

(78) $p(t) = s_0 p^*(t) = s_0 p_0^* \exp\{\Pi^*(t-t_0)+\sigma^* \int_{t_0}^{t} du\}$

Thus, the actual price level is the home country varies directly with the world price level variations.

The home country supports the initial exchange rate by intervening continuously in the market for foreign currency. To be able to do so, the central banks of the home country and the rest of the world exchange currencies at the initial exchange rate, so that the expected time path of the foreign exchange reserves of the home country, $E_{t0}R(t)$, is:

(79) $E_{t0}R(t) = R_0 \exp\{\nu(t-t_0)\} \equiv R(t)^P$

When domestic agents respond to exogenous shocks, the time path of actual reserves may deviate from the autonomous path. We assume that there exist a critical lower bound $\underline{R}(t)$ and an upper bound $\bar{R}(t)$ of reserves such that, if the actual level of reserves falls below $\underline{R}(t)$, then the home country devalues, and, if the reserves rise above $\bar{R}(t)$, then the home country revalues. The limits are defined by:

(80a) $\underline{R}(t) = \underline{R}_0 \exp\{\nu(t-t_0)\}$

(80b) $\bar{R}(t) = \bar{R}_0 \exp\{\nu(t-t_0)\}$

The exogeneity of the limits $\underline{R}(t)$ and $\bar{R}(t)$ is assumed for convenience here. Endogenous determination of such limits could possibly be derived along the principles of some inventory theoretic models or models of the demand for transactions balances (models of Miller-Orr, Baumol-Tobin etc.), but such a specification is considered too costly in terms of additional complexities to be worth deriving here.

The percentage of both the devaluation and the revaluation is always assumed to be constant at k. The idea behind the assumption is that such a policy is required by an outside authority, e.g. by the IMF. This type of policy assumption is at first glance very naive. However, it turns out later in the fourth chapter that it is not as restrictive as it may appear. For it will be seen that, if k is arbitrarily set small and e.g. a devaluation must be carried out, there is practically always speculation going on at the time of the devaluation. If a devaluation of k-percent is not large enough, speculation continues and a new devaluation must occur. This process must be repeated until the economy has 'searched' the devaluation which is correct multiple of k-percent.

The interest rate in the world financial market is uncertain. Specifically, the nominal rate of return on bonds denominated in foreign currency is:

(81) $$\frac{dQ^*_B(t)}{Q^*_B(t)} = r^*_B dt + \sigma^*_B dz^*_B$$

where r^*_B = mean foreign interest rate

σ^*_B = standard deviation of the process dz^*_B

$dz^*_B \sim N(0,1)$ i.i.d

At this rate of return, domestic investors and the government can invest in foreign bonds or borrow from the foreign financial market. To simplify the analysis, we adopt the assumption that substitutions in the porfolios of domestic investors occur only between shares and foreign bonds, and thus the portfolio share of the government bonds remains constant. Technically, we assume that the risks attached to foreign bonds are smaller than those attached to shares. In that case $\sigma_B^* = \rho_{B*}\sigma_B$, $\rho_{B*} = \text{corr}(dz_B dz_B^*)$, guarantees the proposed portfolio adjustments.

Given the above assumptions, we could reformulate decision problems of the representative investor and worker. Since, however, such formulations would be exactly analogous to those presented in section 2.3, we do not present them here. In conclusion, we list the aggregate consumption and portfolio rules of the economy.[14]

(82a) $$c(t) = \frac{\alpha_1 \rho}{p(t)}\left[V_B(t)+\tilde{V}_H(t)\right]$$

(82b) $$c_g(t) = \frac{\alpha_3 \rho}{p_g(t)}\left[V_B(t)+\tilde{V}_H(t)\right]$$

(82c) $$c_m(t) = \frac{\alpha_4 \rho}{s_0 p^*(t)}\left[V_B(t)+\tilde{V}_H(t)\right]$$

[14]Note the interesting difference between portfolio rules (82e,f) and the conventional portfolio rules of an international consumer-investor found in e.g. Macedo (1981) and Meerschwam (1982). Rules (82e,f), derived under the fixed exchange rate regime, are similar to closed economy portfolio rules, while the conventional rules of the international investor, obtained under flexible exchange rates, include hedging against variations in the international purchasing power of the portfolio. The latter type of hedging becomes effective under fixed exchange rates only through speculation against discrete jumps in the exchange rate. This effect will be derived in section 2.5.

(82d) $\ell = 1-\alpha_{2w}$

(82e) $\xi_1 = \frac{1}{\sigma_B^2(1-\rho_{B*}^2)}(r_B - r_B^*)$

(82f) $\xi_2 = \frac{1}{\sigma_B^2\rho_{B*}^2(1-\rho_{B*}^2)}(r_B^* - r) - \frac{1}{\sigma_B^2(1-\rho_{B*}^2)}(r_B - r)$

(82g) $\xi_3 = 1 - \xi_1 - \xi_2$

where $\alpha_1 = \alpha_{1i}$, $\alpha_3 = \alpha_{3i}$, $\alpha_4 = \alpha_{4i}$
$c_m(t)$ = demand for imports
ξ_1 = portfolio fraction of shares
ξ_2 = portfolio fraction of foreign bonds
ξ_3 = portfolio fraction of government bonds

We can again abstract from the public sector and leisure consumption in the consumption rules above, and state the rules in terms of disposable income. Let, then, α be the preference weight of consumption of the domestic good and $1-\alpha$ that of imports. Then (82a) and (82c) can be replaced by

(82a') $c(t) = \frac{\alpha}{p(t)}(\rho\lfloor V_B(t)+\tilde{V}_H(t)\rfloor - \tau(t))$

(82c') $c_m(t) = \frac{(1-\alpha)}{s_0 p^*(t)}(\rho\lfloor V_B(t)+\tilde{V}_H(t)\rfloor - \tau(t))$

On the basis of (82a'), (82c') and (82e-g) we can state the important open economy variables relevant for further analysis. Since production in the domestic private sector is determined in the same way as in the closed economy model, exports are given by the following residual

(83) $x(t) = y(t) - c(t)$

The current account surplus in home currency then becomes:

(84) $S(t) = s_0 p^*(t)[c(t)-c_m(t)]$

The stock of capital exports is the value of domestic capital held in foreign bonds. The stock of capital imports, on the other hand, is the residual capital imports by the government needed to satisfy capital needs in the public and, possibly, the private sector.[15] Denoting the home currency values of the stock of exports by $X_c(t)$ and the stock of imports by $M_c(t)$, the expressions for them are:

(85a) $X_c(t) = s_0 \xi_2(t) Q_B^*(t) \tilde{K}$

(85b) $M_c(t) = s_0(1-\xi_1(t)-\xi_2(t)) Q_B(t) \tilde{K}$

The net inflow of capital, C(t), measured in home currency, follows from instantaneous changes in the stocks in (85a,b), and is given by:

(86) $C(t) = -s_0 d\xi_2(t) Q_B^*(t) \tilde{K}$

Note that valuation changes in (85a,b) represent interest receipts and payments, and are included in the disposable income and hence in the current account surplus.

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[15]The needs in the public sector can include the equilibrium gap in finance, or $\alpha_3 - \xi_3$, and the possible finance for government budget deficit. The latter will be ignored here. The needs in the private sector can be due to the equilibrium gap $1-\alpha_3-\xi_1$, as investors could in principle be net exporters or importers of capital in the equilibrium. Full employment equilibrium in the home country requires capital imports by the government, if investors are net exporters of capital in the equilibrium. This could happen in the general equilibrium, if the government lets investors hedge optimally against all risks and bears the aggregate risk itself. In the open economy model, the government would then be financing firms by loans.

The current account surplus and the net capital inflow together determine the instantaneous change in reserves in excess of the exogenous policy-generated change that was implicitly defined in (79). Measured in home currency, the non-policy-generated instantaneous change in reserves is simply:

(87) $$dR(t)^n = S(t) + C(t)$$

Expression (87) defines a complicated stochastic differential equation. This can be seen by writing out its complete explicit form as follows:

(88) $$\begin{aligned} dR(t)^n = {} & s_0 p_0^* \exp\{\Pi^*(t-t_0)+\sigma^* \int_{t_0}^{t} du\}[y_0 \exp\{\nu(t-t_0) \\ & + \sigma \int_{t_0}^{t} dz\} - \{\nu([\xi_1(t)Q_{B0}\exp\{r_B(t-t_0)+\sigma_B \int_{t_0}^{t} dz_B\} \\ & +\xi_2(t)Q_{B0}^*\exp\{r_B(t-t_0)+\sigma_B^* \int_{t_0}^{t} dz_B^*\} \\ & +\xi_3 Q_{G0}\exp\{r(t-t_0)\}]\tilde{K} \\ & + (\xi_1(t)+\alpha_3)Q_{H0}\exp\{\nu(t-t_0)\}\ell H) - \tau(t)\}] \\ & - s_0 d\xi_2(t)Q_{B0}^*\exp\{r_B^*(t-t_0)+\sigma_B^* \int_{t_0}^{t} dz_B^*\} \end{aligned}$$

It is clear that the global general equilibrium mean of the non-policy-generated change in reserves is zero, even though the expected change for other than policy reasons can temporarily differ from zero. Such temporary deviations are due to speculative responses of agents to exogenous events. The stochastic parts in (88) will not in general sum to zero, except under very specific assumptions concerning government policy analogous to assumptions (65) and (66) in the closed economy case.

The change in total reserves consists of the policy-generated change defined in (79) and the non-policy-generated change specified in (88), and is

$$(89) \qquad dR(t) = dR(t)^p + dR(t)^n$$

The above remarks concerning the non-policy-generated change indicate that the probability distribution of the change in total reserves is likely to have time dependent parameters. In addition, the time dependence of the moments of (89) may be highly nonlinear.

The model developed so far provides a sufficient framework for open economy analysis except in one important aspect. The policy of the home country government to try to follow the fixed exchange rate regime can, with appropriate sequences of shocks, to which agents respond, create devaluation or revaluation expectations. These expectations can generate speculative behaviour, which may have strong feedback effects on the economy and can lead to temporary deviations from zero of the mean of (88) as mentioned above. Therefore, we have to examine in what ways exchange rate expectations and the implied speculation may affect decision making.

2.5 Speculation and fixed exchange rates

In principle, expectations of discrete jumps in the exchange rate can affect the economy through both the goods and the financial markets. If goods are storable, devaluation expectations would, for instance, most likely lead to a speculative build-up of inventories of the imported goods before devaluation. Furthermore, such expectations would cause exports of capital, since the expected rate of return on foreign asset would increase. In our analysis, we have assumed goods to be non-storable, so that only the asset market effects of jump expectations are relevant for our purposes. Consider then the representative investor in the open economy environment. He can, as was assumed above, invest his wealth in shares, government bonds and foreign bonds. With the assumed preferences, his consumption and portfolio problems are separable, and hence we will suppress the consumption decisions for simplicity at this stage. The portfolio problem of the investor can be completely characterized by the following Bellman-equation:

$$(90) \qquad 0 = \max_{\{\xi_1,\xi_2,\xi_3\}} E_{t0}\Big\{u[c(t)] - \rho J(V_B(t,s),s,t) + \frac{\partial J(V_B(t,s),s,t)}{\partial V_B(t,s)}\, dV_B(t,s) + \frac{1}{2}\frac{\partial^2 J(V_B(t,s),s,t)}{\partial V_B(t,s)}[dV_B(t,s)]\Big\}$$

where $dV_B(t,s) = [\xi_1(r_B dt+\sigma_B dz) + \xi_2(r_B^* dt+\sigma_B^* dz^*) + \xi_3 r dt]V_B(t,s)$

$\xi_3 = 1 - \xi_1 - \xi_2$

$u[c(t)]$ = generalized utility index

In order to find out how the behaviour of the investor changes with exchange rate jump expectations, we have to determined in what ways equation (90) must be adjusted so that it takes discrete jumps in the exchange rate into account.

First, we construct a purely technical derivation and give an interpretation to it at a later stage. Suppose, then, that there is a small probability $\tilde{\lambda}$ that, at any instant, the exchange rate can discretely jump from the initial level s_0 to the level $s_1 = (1+k)s_0$. In economic terms, this means that a k-percent devaluation is likely to happen in the home country, but that the timing of the event is unknown within a finite time interval and can occur with equal likelihood at any moment. With these assumptions, the exchange rate follows a Poisson process defined by[16]:

(91) $ds = k\,dv$

where $dv = 0$ with probability $1 - \tilde{\lambda}$

$dv = 1$ with probability $\tilde{\lambda}$

$\tilde{\lambda}$ is defined by the following statements:

Pr{devaluation occurs once in the time interval $(t,t+h)$} $= \tilde{\lambda}h + O(h)$

Pr{devaluation does not occur in the time interval $(t,t+h)$} $= 1 - \tilde{\lambda}h + O(h)$

Pr{devaluation occurs more than once in the time interval $(t,t+h)$} $= O(h)$

and $O(h)$ is of the asymptotic order zero in the following sense: $\psi(h)$ is $O(h)$ if $\lim_{h\to 0} (\psi(h)/h) = 0$

It can be shown that, with assumption (91), the Bellman equation representing the decision making situation of the investor changes into the following form:[17]

(92)
$$0 = \max_{\{\xi_1,\xi_2,\xi_3\}} E_{t0}\Big\{u[c(t)] - \rho J(V_B(t,s),s,t) + \frac{\partial J(V_B(t,s),s,t)}{\partial V_B(t,s)}\, dV_B(t,s) + \frac{1}{2}\frac{\partial^2 J(V_B(t,s),s,t)}{\partial V_B(t,s)}[dV_B(t,s)]\Big\} + \tilde{\lambda}[J(V_B(s_1,t),s_1,t) - J(V_B(s_0,t),s_0,t)]\Big\}$$

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[16]See Merton (1971), pp. 395 - 396.

[17]See Malliaris- Brock (1982), pp. 121 - 124.

Intuitively, the change in the Bellman equation due to the possible jump in the exchange rate is easy to interpret. The last term in (92) represents the expected value of the change in the utility of the investor, given that the wealth level of the investor may jump with the probability $\tilde{\lambda}$.

Equation (92) can be developed into an operational form by studying the change in real wealth due to the jump in the exchange rate. The total wealth levels of the investor measured in home currency before and after the devaluation are the following:

$$(93a) \qquad V_B(t,s_0) = [\xi_1(t)Q_B(t)+\xi_2(t)s_0Q_B^*(t)+\xi_3(t)Q_G(t)]\tilde{K}$$

$$(93b) \qquad V_B(t,s_1) = [\xi_1(t)Q_B(t)+\xi_2(t)s_1Q_B^*(t)+\xi_3(t)Q_G(t)]\tilde{K}$$

Thus, as is to be expected, the change in real wealth is a fraction k of the home currency value of the foreign currency denominated assets in the portfolio, where k is the devaluation percentage. By using (94), we can now develop an operational representation for the last term of equation (92). Applying the Taylor approximation, the term can be written as follows:

$$(95) \qquad \tilde{\lambda}[J(V_B(t,s_1),t,s_1) - J(V_B(t,s_0),t,s_0)]$$
$$= \tilde{\lambda}\left(\frac{\partial J(V_B[t,s_0),t,s_0])}{\partial V_B[(t,s_0),t,s_0]}\xi_2 k\, V_B(t,s_0)\right.$$
$$\left. + \frac{\partial^2 J(V_B[t,s_0),t,s_0])}{\partial V_B[t,s_0),t,s_0]^2}\xi_2^2(s_1-Es_1)^2 V_B(t,s_0)^2\right)$$

The approximation in (95) becomes useful when one observes that the other partials of the J-function in (92) are also evaluated at $s = s_0$. If we assume first that the devaluation percentage is known with certainty to be equal to k, then the second order term in (95) disappears from the point of view of equation (92), in which expectations are taken over the entire right hand side of the equation. Substituting (95) now into (92), taking into account the constraints of (90) and reordering terms, we can write equation (92) in the following form:

$$(96)\qquad 0 = \max_{\{\xi_1,\xi_2\}} \{u[c(t)] - \rho J(V_B(t,s_0),s_0,t) + \frac{\partial J(V_B(t,s_0),s_0,t)}{\partial V_B(t,s_0)}\{[\xi_1(r_B-r)+\xi_2(r_B^*+\tilde{\lambda}k-r)-r]V_B(t,s_0) - c(t)\} + \frac{1}{2}\frac{\partial^2 J(V_B(t,s_0),s_0,t)}{\partial V_B(t,s_0)^2}[\xi_1^2\sigma_B^2+2\xi_1\xi_2\rho_{B*}\sigma_B\sigma_b^* + \xi_2^2\sigma_B^{*2}]V_B(t,s_0)^2\}$$

$$\xi_3 = 1 - \xi_1 - \xi_2$$

With the Cobb-Douglas preferences, the above Bellman-equation implies the following optimal portfolio rules:

$$(97a)\qquad \xi_1 = \frac{1}{\sigma_B^2(1-\rho_{B*}^2)}(r_B-r) - \frac{\rho_{B*}}{\sigma_B\sigma_B^*(1-\rho_{B*}^2)}(r_B^*+\tilde{\lambda}k-r)$$

$$(97b)\qquad \xi_2 = \frac{1}{\sigma_B^{*2}(1-\rho_{B*}^2)}(r_B^*-\tilde{\lambda}k-r) - \frac{\rho_{B*}}{\sigma_B\sigma_B^*(1-\rho_{B*}^2)}(r_B-r)$$

$$(97c)\qquad \xi_3 = 1 - \xi_1 - \xi_2$$

Thus the effect of devaluation speculation on investment behaviour is startlingly simple. If the devaluation percentage k is known with certainty, the effect is simply that the mean return on foreign assets is increased from the investor's point of view by $\tilde{\lambda}k$ percent. Otherwise, the portfolio rules remain exactly the same as they were without devaluation expectations.

If, alternatively, it is assumed that k is perceived to be an independent random variable with distribution $k \sim F(k,\sigma_k^2)$ and that the higher-than-the-second moments of the distribution are not critical for the validity of approximation (95), then matters become only slightly more complicated. With such assumptions, equation (96) becomes:

$$(98) \qquad 0 = \max_{\{\xi_1,\ \xi_2\}} \{u[c(t)] - \rho J(V_B(t,s_0),s_0,t) + \frac{\partial J(V_B(t,s_0),s_0,t)}{\partial V_B(t,s_0)} \{[\xi_1(r_B-r)+\xi_2(r_B^*+\tilde{\lambda}k-r)]V_B(t,s_0) - c(t)\} + \frac{1}{2}\frac{\partial^2 J(V_B(t,s_0),s_0,(t))}{\partial V_B(t,s_0)^2}[\xi_1^2\sigma_B^2+2\xi_1\xi_2\rho_{B*}\sigma_B\sigma_B^* + \xi_2^2(\sigma_B^{*2}+\tilde{\lambda}\sigma_k^2)]V_B(t,s_0)^2$$

Equation (98) leads to the following optimal portfolio rules:

$$(99a) \qquad \xi_1 = \frac{(\sigma_B^{*2}+\tilde{\lambda}\sigma_k^2)}{(\sigma_B^2\sigma_B^{*2}(1-\rho_{B*}^2)+\sigma_B^2\tilde{\lambda}\sigma_k^2)}(r_B-r) - \frac{\rho_{B*}\sigma_B\sigma_B^*}{(\sigma_B^2\sigma_B^{*2}(1-\rho_{B*})^2+\sigma_B^2\tilde{\lambda}\sigma_k^2)}(r_B^*+\tilde{\lambda}k-r)$$

$$(99b) \qquad \xi_2 = \frac{\sigma_B^2}{(\sigma_B^2\sigma_B^{*2}(1-\rho_{B*}^2)+\sigma_B^2\tilde{\lambda}\sigma_k^2)}(r_B^*+\tilde{\lambda}k-r) - \frac{\sigma_{B*}\sigma_B\sigma_B^*}{(\sigma_B^2\sigma_B^{*2}(1-\rho_{B*})^2+\sigma_B^2\lambda\sigma_k^2)}(r_B-r)$$

$$(99c) \qquad \xi_3 = 1 - \xi_1 - \xi_2$$

The portfolio rules in (99) with an uncertain devaluation percentage differ from (97) in only two respects. First, the uncertainty of the devaluation percentage increases the total uncertainty of the devaluation percentage increases the total variance of the portfolio, thus increasing the common denominators in ξ_1 and ξ_2. This means an adjustment of portfolios towards the riskless asset. Secondly, the risky devaluation speculation adds a discount to holdings of shares, because shares are free from this type of risk. Consequently, the demand for shares becomes larger with a risky than a riskless devaluation percentage.

The technical derivation of the effects of exchange rate speculation on the behaviour of an investor is now complete. In the analysis, the probability $\tilde{\lambda}$ was treated purely parametrically and here we seek to present an interpretation for the endogenous determination of $\tilde{\lambda}$.

The determination of reserves was specified above in (89), where exchange rate speculation was not explicitly included. An equation of type (89) determines the instantaneous probability distribution of the change in reserves. Let the density function and the distribution function of $dR(t)$ at time t be $f_t(dR(t))$ and $F_t(dR(t))$, respectively. An evaluation point of $F_t(dR(t))$ defines the probability for the event that reserves change by the specified amount during the following time-interval dt. The two evaluation points that are analytically of special interest to us are the devaluation and revaluation limits $\underline{R}(t)$ and $\bar{R}(t)$ defined above in (80a,b). Two probabilities can be defined in terms of $\underline{R}(t)$ and $\bar{R}(t)$ as follows:

(100a) $\quad Pr\{\text{devaluation occurs within the time interval } (t,t+dt)\}$

$$= F_t(\underline{R}(t)) \equiv \lambda_t^d$$

(100b) $\quad Pr\{\text{revaluation occurs within the time interval } (t,t+dt)\}$

$$= 1-F_t(\bar{R}(t)) \equiv \lambda_t^r$$

The probabilities λ_t^d and λ_t^r are the gross probabilities for a devaluation or a revaluation to occur within a short interval. Note that, in the technical portfolio rules, the gross probabilities would enter symmetrically but with opposite signs, since the sign of the rate of return effect depends on the sign of the change in the exchange rate. It is then useful to define two alternative probabilities which combine the effects of the gross probabilities. Specifically, we state that

(101a) $\lambda_t = \Pr\{\text{devaluation occurs and revaluation does not occur within the time interval } (t,t+dt)\}$

$= \lambda_t^d - \lambda_t^r, \text{ if } \lambda_t^d > \lambda_t^r$

(101b) $\lambda_t = \Pr\{\text{revaluation occurs and devaluation does not occur within the time interval } (t,t+dt)\}$

$= \lambda_t^r - \lambda_t^d, \text{ if } \lambda_t^r > \lambda_t^d$

The probabilities λ_t defined by (101a,b) are the operational counterparts of the probability $\tilde{\lambda}$ used in the technical analysis. Note in particular that, if $\lambda_t^d > \lambda_t^r$ and there are 'net' expectations of devaluation, then the $\tilde{\lambda}$'s in the portfolio rules (97) and (99) can be replaced directly by the λ_t defined in (101a). If, alternatively, $\lambda_t^r > \lambda_t^d$, then the $\tilde{\lambda}$'s can again be replaced by the λ_t defined in (101b), but the signs of all the terms involving revaluation speculation must be changed. How the probabilities λ_t are actually obtained is an entirely different issue, the complexity of which is easily hidden by technical treatments such as the one above. In principle, agents form estimates of λ_t by interpreting economic events in light of the open economy model that generates the instantaneous distribution $F_t(dR(t))$ of the change in reserves. It would be natural to assume that such estimates were rational expectations on the probabilities λ_t, However, the information processing problem of agents in forming such estimates is a formidable challenge. The dynamics in the change in reserves is likely to be of a high order and the problems in determining the time path of the probabilities λ_t in the fourth chapter where the macroeconomic consequences of speculation are analyzed in a linearized version of the open economy model developed above.

It should be clear that the procedure developed above is applicable to a variety of situations in which an agent is speculating on an event the timing of which he does not know precisely. We will also apply the procedure in the closed economy analysis of the third chapter, where one case to be analyzed includes a demand shock, the magnitude of which agents know with certainty, but the timing of which they are unsure of.

2.6 Concluding remarks

In the above analysis, three different frameworks were developed. First, a simple closed economy nonmonetary model was derived. The computation of the general equilibrium relative prices of this economy was presented. The basic framework was then extended to include the public sector, demand for money and the input market mechanism generating Keynesian rigidities in the economy. Finally, the model was extended further to include exports and imports of goods and capital under the small open economy assumptions.

As a somewhat separate piece of analysis, the behaviour of an international investor making decisions under devaluation expectations was examined. The portfolio rules of the investor were derived, given both a certain and an uncertain devaluation percentage. It was shown that a positive probability of a k-percent devaluation, where k is certain, makes the investor favour assets denominated in foreign currency. It was also shown that if k is a random variable, then demand for the riskless asset increases relative to the case of a certain k and more at the cost of the foreign assets.

3 SPECULATION AND THE REAL BALANCE EFFECT: THE NEUTRALITY OF STABILIZATION POLICY RECONSIDERED

3.1 Introduction

In the historical debate on the policy-neutrality question, three phases can be distinguished. The basic classical position was that, with flexible prices, the Walrasian general equilibrium world operates continuously in full employment and there is no room for welfare improvements by means of economic policy. The opposing claim was that, by introducing Keynesian rigidities into price adjustments, it became possible to model situations in which the full use of real resources required policy actions from the public authority. The classical school found a counterargument in the real balance effect (see Pigou (1943) and Patinkin (1948, 1956) for the traditional position, Tobin (1980) and Fischer (1981) for a reconsideration, and McCallum (1983) for a recent application in a dynamic rational expectations model), being again able to claim that no policy was necessary. The essence of the real balance effect was that variations in the price level create capital gains or losses for those agents who are holding money. These capital gains or losses have a balancing effect on aggregate demand, thus allowing the economy to adjust in full employment.

The same kind of policy effectiveness debate has been going on again since the early 1970's. The emphasis in the early phases of the controversy (e.g. in the early works of Lucas) was on the rational expectations hypothesis. In short, it was claimed that, if agents form their expectations rationally, then systematic policy would, once again, have no effects on real economic variables. Later on, it became clear through the counter examples of e.g. Phelps - Taylor (1977) and Fischer (1977) that the crucial assumption for policy neutrality was, in fact, the price flexibility assumption, and not so much the rational expectations hypothesis. This point was especially emphasized in Tobin (1980, essay 2).

In the following, we try to carry out a dynamic policy effectiveness analysis in which the separation of capital and labor supply decisions and a Keynesian rigidity in the form of nominal wage contracts are allowed to create potentially feasible conditions for nonneutrality results. However, we also allow for the presence of the real balance effect, which provides the economy with a potential automatic stabilizing mechanism.

Specifically, we will study effects of demand disturbances in the closed economy model developed above in section 2.3. The money targeting regime, as defined in section 2.3, is the neutral government policy in our model, because it does not eliminate in any way the fundamental distortion in the economy. Hence, we say that the automatic stabilizing mechanism, i.e. the real balance effect, neutralizes the demand shock if the shock causes no real effects under the money targeting policy. The consequences of the same shocks are then compared under the alternative monetary policy, or the interest rate targeting, regime and under the Keynesian stabilization policy. If the shock causes no real effects under the policy regime, we say that the policy is nonneutral.

The basic demand disturbance - a temporary shift in the rate of time preference of consumers, or a savings shock - is analyzed under three different informational assumptions. First, it is assumed that the shock is unanticipated. Second, the shock is assumed to be anticipated in the sense that agents know precisely the timing of the demand shift. Finally, the shock is assumed to be anticipated, but now in the sense that agents know exactly the time-interval within which the shock hits, but do not know the precise timing of the shock within the time-interval. A specific feature and novelty of the analysis is that, while the analysis itself is carried out in continuous time, the shocks are discrete events in all the three cases. This choice reflects the view that, in many or even most cases, events are fundamentally discrete in nature but occur in a timewise continuous environment.

The two types of anticipated shocks cause very different responses in the economy. If the timing of the shock is known precisely, then both the price level and output adjust an instant before the shock occurs, and never start adjusting gradually at the time when the shock is announced. Gradual adjustment patterns would only be obtained if costs of adjustment were assumed. If the timing of the shock is uncertain, then both the price level and output start adjusting gradually at the time when investors get to know about the shock. The price path always includes overshooting. Thus, during the speculation preceding the actual onset of the shock, the price level changes in the opposite direction to that implied by the shock itself. Consequently, during the speculative periods before the shock, the price level and the unemployment rate are positively associated, i.e., the observed Phillips-curve relationship is positively sloped during these periods.

The effects of anticipated shocks on the price level and output are similar to those obtained in Fischer (1979) during the period when the shock is in effect. However, there are marked differences in the dynamic responses of the variables before and after the shocks. Contrary to what was stated out above about the responses in our model, in Fischer's model the price level always adjusts gradually to anticipated temporary shocks, with geometric lead and lag weights assigned to the shocks. Moreover, overshooting in the price level never results in Fischer's model, given the parameter assumptions that he uses.

Our findings can also be connected to a small branch of the literature initiated by Leijonhufvud (1973, 1981). The main point of Leijonhufvud's analysis was that an economy has the corridor property: the economy can adjust to small shocks without policy intervention, but needs policy measures to absorb large shocks in full employment. Furthermore, Keynesian fiscal policy rather than monetary policy is needed to neutralize the real effects of very large shocks. We will reach analogous conclusions in terms of the unanticipated and anticipated shocks, and in terms of the money targeting, interest rate targeting and Keynesian policy regimes. In particular, we will see that the real balance effect combined with the money targeting policy regime will only neutralize shocks if they are unanticipated. The interest rate targeting policy will neutralize anticipated shocks which are of moderate size, while Keynesian fiscal policy is needed to eliminate the real effects of very large anticipated shocks.

The fundamental reason for the results obtained is variations in income distribution implied by the fixed contract wage. Investors react to such variations within the contract period, and their responses make the vertical supply curve shift. Viewed from a slightly different angle, variations in the price level make the relative preferabilities of different investment opportunities vary, and hence make investors adjust their portfolios. This mechanism, generally called the Tobin effect and first introduced in Tobin (1965) and Mundell (1963), is widely used as a way of introducing the non-neutrality of money into a neoclassical growth model and is the driving force of e.g. the analysis by Fischer (1979).

3.2 Description of the experiments and the basic argument

The model of the monetary economy with the conventional public sector developed in section 2.3.2 will be used in the following analysis. It was seen that the model is a general equilibrium model, in which agents behave optimally and form their expectations rationally. A Keynesian rigidity is built into the economy by the assumption of nominal wage contracts, which are negotiated between individual firms and the trade union representing the workers. Both the technology and preferences in the economy are of the Cobb-Douglas type, but the simplifying assumptions on profits funds described in section 2.3.2 and the nonresponsiveness of union behaviour to disequilibria in labor markets make technology fixed-coefficient in the short run.

The equations of the model and the notation used are listed here for convenience:

(102a) $$c(t) = \frac{1}{p(t)}[\rho(V_B(t)+\tilde{V}_H(t))-\tau(t)]$$

(102b) $$\ell = 1 - \alpha_{2w}$$

(102c) $$\xi_1 = \frac{1}{\sigma_B^2(1-\rho_{BG}^2)}(r_B-r) - \frac{\rho_{BG}}{\sigma_B\sigma_G(1-\rho_{BG}^2)}(r_G-r)$$

(102d) $$\xi_2 = \frac{1}{\sigma_G^2(1-\rho_{BG}^2)}(r_G-r) - \frac{\rho_{BG}}{\sigma_B\sigma_G(1-\rho_{BG}^2)}(r_B-r)$$

(102e) $$\xi_3 = 1 - \xi_1 - \xi_2$$

(102f) $$y(t) = \xi_1(\ell H)^{\beta}(\tilde{K})^{1-\beta}\exp\{\nu(t-t_0)+\sigma\int_{t_0}^{t} dz_L\}$$

(102g) $$p(t)y(t) = b(t)M(t)$$

where $c(t)$ = aggregate demand, or consumption

$y(t)$ = aggregate supply

$p(t)$ = price level

$V_B(t)$ = nonhuman wealth $[\xi_1 Q_B(t)+\xi_2 Q_G^1(t)+\xi_2 Q_G^2(t)]\tilde{K}$; $\tilde{K}$ = stock of real capital

$\tilde{V}_H(t)$ = actual human wealth = $\ell Q_H(t)H$

ℓ = labor supply; H = stock of working capacity

$\tau(t)$ = tax flow

ξ_1 = portfolio fraction of shares

ξ_2 = portfolio fraction of risky government bonds

ξ_3 = portfolio fraction of riskless government bonds

$r_B(t) \equiv r_B dt + \sigma_B dz_B = \nu dt + \sigma dz_L$ = nominal rate of return on shares

$r_G(t) \equiv r_G dt + \sigma_G dz_G$ = nominal rate of return on risky government bonds

r = nominal rate of return on riskless government bonds

ρ_{BG} = correlation coefficient between processes (dz_B, dz_G)

$M(t)$ = transactions demand for money

$b(t)$ = velocity of money

The contract wage, which is also binding on the public sector, is always set in the way specified above in section 2.3.2 i.e.:

$$(103a) \quad \dot{w} = \nu$$

$$(103b) \quad E_{t0}\left[\frac{Q_H(t)}{P(t)}\right] = E_{t0} \cdot \frac{1}{\rho}\left[MP_L(t) - \beta d_L y(t)\right]$$

To simplify the exposition - without making the analysis any less general - we assume that the mean velocity of money equals one and that short-term variations in the velocity completely absorb random variation in the instantaneous output. These assumptions can be formalized as follows:

$$(104b) \quad b(t) = \exp\left\{\sigma \int_{t_0}^{t} dz_L\right\}$$

In our case, this assumption guarantees that the time path of the price level will not be affected by stochastic variations in output about the mean.

The effects on the economy of very simple demand shocks are analyzed in the following. In principle, analysis of two types of shifts in demand behaviour is possible in our framework. From the point of view of economic motivation, a shift in the preference weights of consumers is the simpler case. If the preference weight of public services increases in the utility functions of consumers, there is a decrease in the equilibrium size of the private sector. If the shift of inputs between sectors is frictionless, then this type of shock has no analytical interest, as full employment general equilibrium is always guaranteed. If, on the other hand, moving resources between sectors is costly, then the case becomes interesting, but technically complicated even with the simplest adjustment cost assumptions.

The second type of shocks are shifts in saving behaviour. The transversality condition (46 iii) set above in section 2.3.1 for converting the infinite horizon problem into a finite horizon one implies that $\rho = \nu$ always holds. This is the same as requiring that no capital formation, or no net saving, ever occurs in the economy. To allow for the possibility of saving shocks, we have to weaken the transversality condition.

Interpret then the representative investor and worker, who are optimizing over infinite time horizons, as a sequence of overlapping generations of agents with constant life-times of T_L time units and with some weight for a bequest motive in preferences. Consider the agents whose life-times end at t_0+NT_L, where N is a very large integer. In discounting problems, the decisions of the N^{th} generation must have a positive amount of capital at the end of its life-time, because the bequest motive has some weight in the utility function of the generation. We can then assume - without significantly affecting the decision making of the generations in the near future - that the representative investor of the N^{th} generation has a stock $\tilde{K}$ of capital at the end of his life-time. With equally minimal effects, we can assume that at the same time the representative worker has no capital. At time

t_0+NT_L, then, the economy has the same endowments as at time t_0. If we assume further that later generations have identical preferences to the N^{th} generation, we no longer restrict the choices of the generations in the near future. On these assumptions, the transversality condition (46 ii) in section 2.3.1 must be replaced by the following condition:

$$(105) \qquad E_{t0}\tilde{K}(t_0+T) = \tilde{K} \qquad\qquad \forall\, T \geqslant t_0+ NT_L$$

Condition (105) differs from (46 ii) in that, in the former, the equality $\rho = \nu$ must only hold from time t_0+NT_L on, whereas, in the latter, it must hold throughout the time horizon.

With condition (105), we have a theoretical justification for allowing for differences between the rate of time preference ρ and the mean rate of growth of productivity ν in the near future. Such differences imply temporary accumulation or decumulation of capital, i.e. periods of net saving or dissaving. A complete theory of saving and investment could be constructed in our model by specifying a relationship between the rate of time preference ρ, the rates of return on different assets, the contract wage rate increase and the bequest targets of each generation.[18] To keep matters simple, we are much less ambitious and will only consider temporary exogenous shifts in the rate of time preference ρ. Specifically, we will study the adjustments in the economy if the rate of time preference drops from the zero saving level $\rho = \nu$ to the level $(1-k)\rho$, where $k \in (0,1)$. After a while, a shift back to the initial level ρ occurs. A shock of this type can be thought to represent an exogenous increase in the preference weight of the bequest motive. Note that in order for condition (105) to hold, at

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[18]This type of theory development would imply endogenization of the rate of time preference e.g. in the sense developed by Uzawa (1968) and recently applied by, for example, Obstfeld (1981).

least one generation before t_0+NT_L must dissave to an appropriate extent.

The above basic savings shock will be studied under three different informational assumptions and the assumption that agents hold rational expectations. In the first case, all consumer start saving at the same time, but they are only aware of their own plan in advance. A similar asymmetry concerning information on individual and aggregate decisions is assumed to hold when agents return to the trend behaviour. Thus, the temporary demand disturbance is an unanticipated event in the first experiment. In the second case, all consumers both start and stop saving at the same time, and are fully aware of this fact in advance. The demand shock is then anticipated and agents know the timing of the shock with certainty.

In the third case, a more general shock is analyzed. The basic structure of the case is that all agents are known to make the same savings decision within a certain time-interval. The decisions of a fraction of agents are smoothly distributed over the time-interval, while the decisions of other groups of agents are truncated about different time points within that interval. None of the agents knows which segment he belongs and no one can infer better than probabilistic information on the timing of the truncated decisions. Thus the agents have subjective probability distributions for both the decisions of the fraction of agents in the first cohort and for the timing of the truncated decisions within the interval. Note that the decisions of agents in the first cohort accumulate the aggregate shock continuously, while the effects of the truncated decisions are discrete.

The idea behind the above general shock structure is that while, in general, the decisions of 'atomic' agents can be thought to be spread over the whole time interval according to some smooth distribution, there could be factors unspecified in the model which would implicitly 'group' the decisions of some fractions of agents about the same time point. Empirically, such unspecified factors could, for example, be exogenous changes in pension schemes, which would probably make agents close to retirement and young agents respond very differently. One could imagine that the former group would react quickly and at about the same time by changing their saving behaviour, while the reaction of the young would be more diffuse.

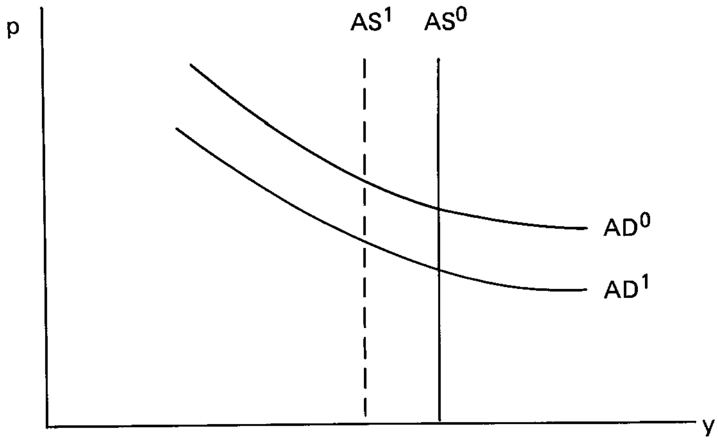
We abstract from the complicated general structure by studying the simple case in which only the workers start and stop saving at the same times, but in which investors know only the scale of the decision and the time-intervals within which the commencement and termination of saving occur with probability one. To make the magnitude of the aggregate shock comparable to the other cases, we - knowing that the income share of workers is $\beta(1-d_L)$ in the stationary state - assume that the workers' rate of time preference falls to the level $(1 - \frac{k}{\beta(1-d)_L})\rho$, which investors know.

The saving increases the potential supply of capital in the economy. If the consumers actually supply the accumulated capital to firms and the public sector, then the economy will move to a new equilibrium. In the new steady state, both production and consumption would be greater than in the initial state. The relative price of capital would be lower, since the economy would move along the aggregate production function. For simplicity, we want to abstract from shifts in stationary states and merely concentrate on analyzing pure demand disturbances. Hence, we make the final simplifying assumption that the net saving is pure hoarding, i.e. that the aggregate supply of capital in the input markets remains constant. With this assumption, the only direct consequence of the savings shock is a shift in aggregate demand.

The three experiments are analyzed under the three policy regimes defined above in section 2.3.2. The basic argument concerning the policy neutrality issue can be illustrated in terms of Figure 2.

Fig. 2.

DEMAND DISTURBANCE AND REAL EFFECTS



In terms of the familiar static demand-supply diagram, the savings shock implies, in all three experiments, a temporary shift of the aggregate demand curve form the initial position AD^0 to position AD^{-1}. With a vertical aggregate supply curve, there will only be real effects as a result of the shock if the supply curve moves from the initial position AS^0 to an arbitrary new position AS^{-1}. The shift in the supply curve will take place if investors take speculative actions against the demand shock. Investors are likely to do so, as the decrease in the price level implied by the demand shift will change their investment opportunities. The latter is caused by the wage contracting, which makes the Fisher effect fail. If any of the policy regimes is such that under it the shift in the supply curve will not occur, then we can say that this policy is not neutral relative to the disturbance.

3.3 Adjustment of the economy to unanticipated shocks

Let us consider first the case of unanticipated shocks. Suppose then that at time t_s each agent starts to hoard a fraction k of his income flow. Furthermore, let all agents be aware only of their own saving plans in advance so that the aggregate shock emerges as a surprise to all agents. The aggregate shock implies that the nominal money balances available for transactions uses drop to a new time path (1-k)M(t). The quantity theory equation determines the price level of the economy continuously, i.e. the following identity must hold:

$$(106) \qquad y(t) = b(t)\,\frac{M(t)}{p(t)} \iff E_0\lfloor y(t) \rfloor = \frac{M(t)}{p(t)}$$

where the latter identity is based on (104).

In the pre-shock equilibrium, money supply grows at a constant rate ν in all policy regimes so that the price level stays constant at p_0. Let us therefore denote the general price p(t) by the multiplicative form $h(t)p_0$. Using this notation, the quantity theory equation becomes:

$$(107) \qquad E_0\lfloor y(t) \rfloor = \frac{M(t)}{h(t)p_0}$$

Because the shock is unanticipated, investors cannot take speculative actions in advance and at time t_s production y(t) remains unchanged. Consequently, the price must drop to the level $(1-k)p_0$ in order to make the remaining money balances sufficient for carrying out all the transactions in the goods market.

Investors observe at time t_s that the profits of firms fall as the price level drops. This makes shares a less favourable investment for them relative to government bonds and investors would, ceteris paribus, be willing to adjust their portfolios towards bonds. But at the same time the government observes the shock as well, and, by our assumption stated in section 2.3.2 above, prevents such profitable trading. Hence, investors are stuck with their portfolios and income distribution changes in favor of workers. Because of the Cobb-Douglas preferences, this does not have any effects on aggregate consumption, as it is neutral to variations in income distribution.

At time t_s+T_s, again as a surprise, agents stop accumulating hoarding balances. Nominal transactions balances return to the path M(t) and, again, there is no change in output since investors cannot speculate on the unanticipated event. With the unanticipated shock, the function h(t) governing the time path of the price level takes the following form:

$$(108) \qquad h(t) = \begin{cases} 1 - k & \text{for } t \in (t_s, t_s+T_s) \\ 1 & \text{for } t \notin (t_s, t_s+T_s) \end{cases}$$

Graphically, the time path of the price level in the case of the unanticipated shock is shown in Figure 3.

In the aggregate demand-aggregate supply framework, the effects of the unanticipated shock can be represented as a temporary shift in the demand curve. This is done in Figure 4.

The demand curve shifts at time t_s from position AD^0 to position AD^1, and moves back at time t_s+T_s. Because investors cannot make speculative arrangements to eliminate the effects of the shock on their wealth position, the aggregate supply curve stays in its initial position AS. The economy adjusts to the shock purely in terms of price adjustment. The temporary price decrease raises the real value of the residual money balances so that the goods market equilibrium is guaranteed. Thus, we can say that the real balance

Fig. 3.

THE TIME PATH OF THE PRICE LEVEL ASSOCIATED WITH AN UNANTICIPATED TEMPORARY INCREASE IN SAVING

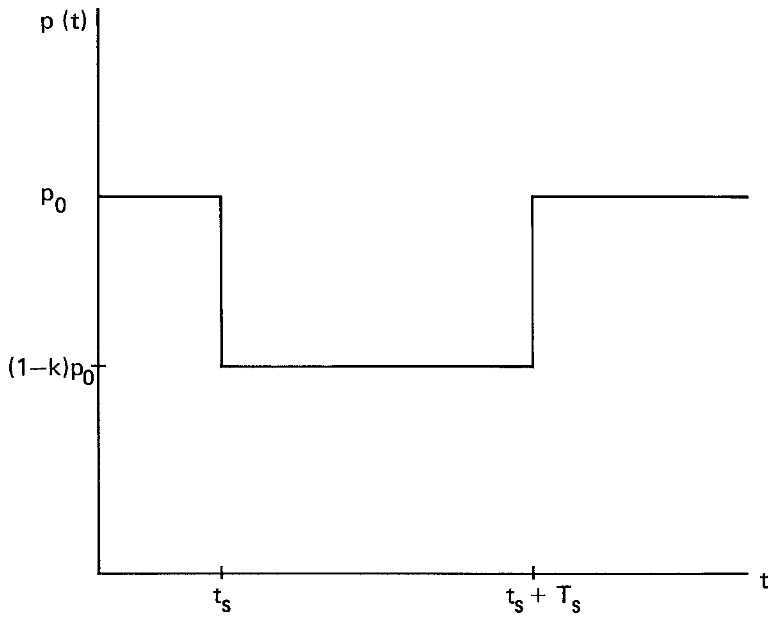
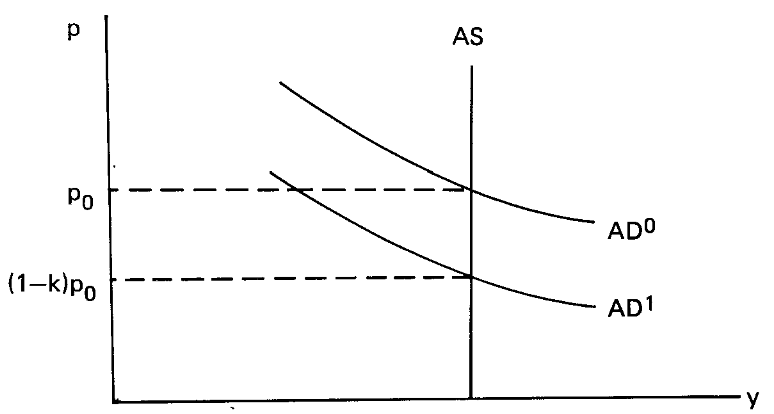


Fig. 4.

SHIFT IN AGGREGATE DEMAND ASSOCIATED WITH AN UNANTICIPATED TEMPORARY INCREASE IN SAVING



effect fully neutralizes the unanticipated disturbance, and none of the policy regimes can do any better in stabilizing real fluctuations.

3.4 Adjustment of the economy to anticipated shocks with certain timing

Alternatively, let us consider the same saving shock with the only difference being that private agents know at time $t_0 < t_s$ that all agents start hoarding a fraction k of their income at time t_s and continue net saving for T_s time units. Both workers and investors then know that at time t_s the nominal transactions balances drop discretely to the new time path $(1-k)M(t)$, returning back to the original path $M(t)$ at time t_s+T_s.[19]

Even with perfect information on the event, workers cannot hedge in any way against price level variations, as they have no alternative ways of earning their income and have already fixed the contract wage before t_s. In this case they obviously would have no motivation for such behaviour, as the price level is likely to fall if there were no employment effects. Investors, on the other hand, can hedge against price level variations, as they can in principle adjust the composition of their portfolios.

To see the hedging alternatives of the representative investor, let us study the change in the value of one share relative to the value of other assets. In the second chapter, we adopted the convention that the riskless government bonds are identical with money except for having a positive nominal interest rate. Hence the real price of a riskless bond varies in exactly the same way as does the real price of money, or follows the time path given by:

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[19]Note that, for simplicity, the shock is assumed to start and stop within one contract period in all the three cases. This restriction is only relevant in the anticipated cases.

$$(109) \qquad Q^2_{GR}(t) = \frac{1}{h(t)} Q^2_G(t)$$

where $h(t): p(t) = h(t)p_0$

Equation (109) implies that if the price level falls, i.e. if $h(t) < 1$, then the real price of the riskless bond rises. Because $h(t) = 1$ before the shock, the investor can trade one share for $Q_B(t)/Q^2_G(t)$ riskless bonds any time before the shock. When the shock occurs and the price level changes, this exchange ratio changes as well. Note that the price level change is deterministic and does not change the stochastic properties of the time path of the price of shares. Consequently, we can simplify the analysis by studying the change in the mean price path of shares.

With the assumed contract wage determination and assuming a discrete shock, the mean real price of one share is determined by the profits function of the firm as follows:

$$(110) \qquad \bar{Q}_{BR}(t) = \frac{1}{p_0 h(t)}[h(t)-B]\frac{p_0\bar{y}(t)}{\nu K}$$

$$= \left[\frac{h(t)-B}{h(t)}\right] \frac{\bar{y}(t)}{\nu K}$$

where $Q_{BR}(t)$ = mean real price of one share
B = $\beta(1-d_L)$ = risk-premium-corrected income share of labour
$\bar{y}(t)$ = mean output

The mean real price of a share in no-shock conditions is obtained from (110) simply by setting $h(t) = 1$. Equation (110) implies that the real price of a share decreases with the saving shock. This is due to the fact that the nominal wage is independent of the price level, and therefore the real labor costs rise as the price level falls. As the owners of firms, investors must bear all the losses of sales revenues. By comparison, recall that the real price of one riskless bond rises with the saving shock. The difference in the asymmetric changes in real prices of different assets is an indication of the fact that the Fisher-effect does not hold in the rigid contract wage economy.

The shock-adjusted mean real price of a share can also be expressed in terms of the no-shock real price as follows:

$$(111)\qquad \bar{Q}_{BR}(t) = \frac{(h(t)-B)}{h(t)(1-B)}\,(1-B)\,\frac{\bar{y}(t)}{\nu K} = \frac{(h(t)-B)}{h(t)(1-B)}\,E_0\lfloor Q_{BR}(t)\rfloor$$

Presuming that $h(t) < 1$ during the saving period, it is clear that the investor has two different mean relative prices at which he can trade shares for riskless bonds: one price at time t_s-dt and another - a less favourable one - at time t_s+dt. By using (111), the two relative prices can be expressed in easily comparable forms

$$(112a)\qquad E_0\left\lfloor\frac{Q_{BR}(t)}{Q^2_{GR}(t)}\right\rfloor^{-} = \frac{E_0 Q_{BR}(t)}{E_0 Q^2_{GR}(t)}$$

$$(112b)\qquad E_0\left\lfloor\frac{Q_{BR}(t)}{Q^2_{GR}(t)}\right\rfloor^{+} = \frac{\frac{(h(t)-B)}{h(t)(1-B)}\,E_0 Q_{BR}(t)}{\frac{1}{h(t)}\,E_0 Q^2_{GR}(t)} = \frac{(h(t)-B)}{(1-B)}\,\frac{E_0 Q_{BR}(t)}{E_0 Q^2_{GR}(t)}$$

where - refers to time t_s - dt and + to time t_s + dt, and E_0 to the expectation formed at time t_0.

Equation (112b) reveals that the real value of one share in terms of the price level changes by a factor $\frac{(h(t)-B)}{(1-B)}$ at time t_s. This observation gives us a foundation for deriving the trading policy for an optimizing investor who knows the timing and the magnitude of the shock in advance.

In his trading strategy, the investor has to consider two factors. First, the saving shock tends to decrease the price level in a similar way as in the case of the unanticipated shock. Second, the investor reduces aggregate supply in the market as he moves capital from firms to the public sector. Through the quantity theory equation, this tends to eliminate the price level effect of the saving shock. The two contributions to the price level can be seen by writing out the quantity theory equation which takes the speculative action of the investor into account:[20]

$$(113) \qquad (1-v(t))y(t) = \frac{(1-k)M(t)}{h(t)p_0}$$

where $v(t)$ = fraction of output lost by the actions of investors.

Identity (113) can only hold if

$$(114) \qquad h(t) = \frac{(1-k)}{(1-v(t))}$$

The investor's problem of optimal speculation (or arbitrage in the riskless case) can now readily be specified. As the production technology of the firms has the fixed-coefficient property in the short run, then the variable $v(t)$ can be interpreted as the fraction of the share of capital allocated to firms in the investor's no-shock equilibrium portfolio that he trades for bonds. Formally, the variable $v(t)$ can be defined as:

[20] Note that in (113) we have used assumption (104) on the specific form of $b(t)$.

$$(115) \qquad v(t) = \frac{\bar{\xi}_1 - \xi_1(t)}{\bar{\xi}_1} = 1 - \frac{\xi_1(t)}{\bar{\xi}_1}$$

where $\bar{\xi}_1$ = equilibrium value of the fraction of shares

The gain for the investor from trading a fraction $v(t)$ of shares for bonds at time t_s-dt rather than at t_s+dt, denoted by G^S, is given by the following expression:

$$(116) \qquad G^S = v(t)\left[1 - \frac{(h(t)-B)}{1 - B}\right]$$

$$= v(t)\left[\frac{1 - \frac{1 - k}{1 - v(t)}}{1 - B}\right]$$

The motivation contained in expression (116) can most clearly be seen from the first form. The first form states, namely, that, by trading at time t_s-dt, the agent avoids a loss $\left[1 - \frac{(h(t)-B)}{1 - B}\right]$ per unit, or the drop in the real value of a share. The optimal policy of the investor is obtained by maximizing (116). Specifically, the investor's trading problem is:

$$(117) \qquad \max_{\{v(t)\}} \; v(t) \left[\frac{1 - \frac{1 - k}{1 - v(t)}}{1 - B}\right]$$

The feasible solution for (117) is given by:

$$(118) \qquad v(t) = v = 1 - \sqrt{(1-k)}$$

Thus the investor moves a fraction $1 - \sqrt{(1-k)}$ of his capital in firms into government bonds at time t_s-dt. Private production then drops by a factor $\sqrt{(1-k)}$. Using (114) the price level equation becomes

(119) $p(t) = h(t)p_0 = \sqrt{(1-k)}\ p_0$

Therefore, the price level drops at time t_s by a factor $\sqrt{(1-k)}$, or equiproportionately with production.

An exactly analogous sequence of events occurs at time t_s+T_s. Investors move capital from the public sector to the private sector and the equilibrium savings behavior resumes. The time path of the price level is drawn in Figure 5.

The price level drops to the level $\sqrt{(1-k)}\ p_0$ at time t_s and then rises back to p_0 at time t_s+T_s. Note the difference between the standard results for price level behaviour in the case of anticipated shocks, presented e.g. in Fischer (1979), and the path drawn in Figure 5. We shall discuss the difference in detail in section 3.7.[21]

In the static supply-demand diagram in Figure 6, the effects of the shock can be demonstrated very clearly. At time t_s, both the demand and supply schedules shift to the left so that both the price level and output drop by a fraction $\sqrt{(1-k)}$ on the new equilibrium paths. At time t_s+T_s, both curves move back to their initial positions.

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[21]Note that we have ignored the fact that the trading must occur dt time units before the shock. This would imply another discontinuity in the price level at time t_s-dt. However, when we let dt approach zero, then at the limit we will approximately have the price path of the form obtained in the analysis.

Fig. 5.

THE TIME PATH OF THE PRICE LEVEL ASSOCIATED WITH AN ANTICIPATED TEMPORARY INCREASE IN SAVING

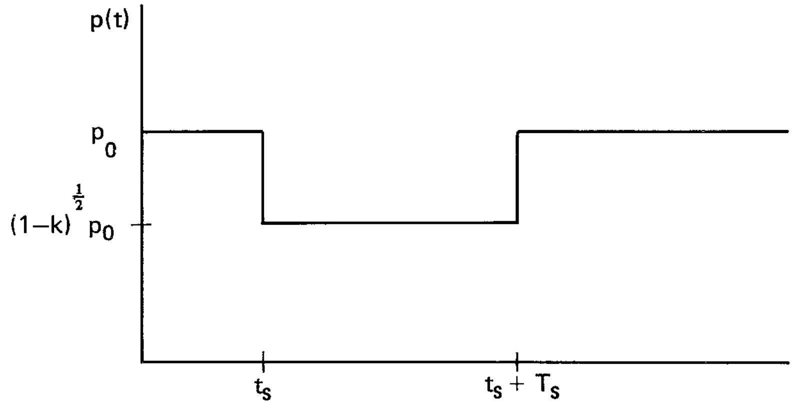
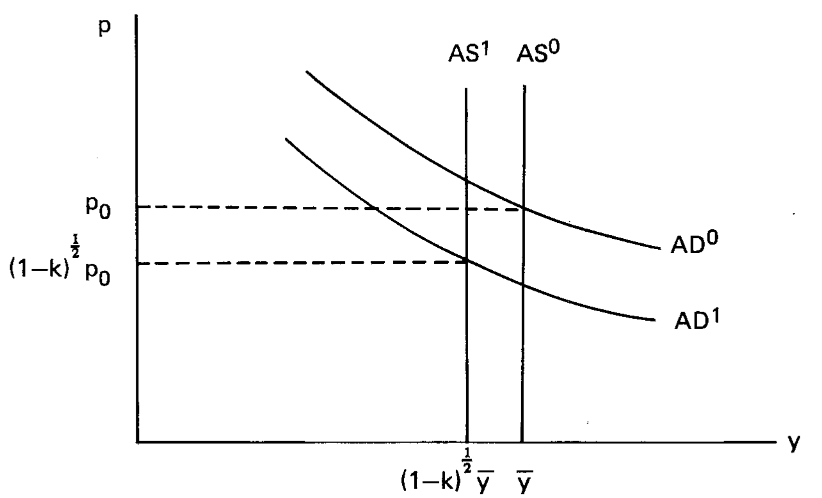


Fig. 6.

SHIFTS IN AGGREGATE DEMAND AND SUPPLY ASSOCIATED WITH AN ANTICIPATED TEMPORARY INCREASE IN SAVING



So far, we have not made any reference to the policy regimes in effect. This is because the above analysis is valid for both the money targeting and the interest rate targeting policies, and, because under the Keynesian regime, the disturbance can be completely neutralized. In the money targeting regime, the government is always ready to trade any amounts of risky bonds at the prevailing general equilibrium terms. Thus investors will effectively trade shares for risky bonds before the start of the saving period and risky bonds for shares before the return to the trend saving behaviour. On the other hand, in the interest rate targeting regime, the government ties the price of the risky bonds to the price level so that the ratios between the real prices of shares and risky bonds and their nominal prices remain the same. Hence, investors will perceive shares and risky bonds as neutral investment opportunities relative to price level variations. But, by assumption, the nominal price of riskless bonds (speculative money) is fixed. Therefore, investors will shift capital between the private and the public sector by exchanging shares and riskless bonds in the same way as before. In conclusion, both types of monetary policy regimes will lead to the same real effects described above. In this sense, both policy regimes are neutral relative to the demand disturbance.

It is easy to construct an example in which a Keynesian stabilization policy fully neutralizes the real and the price effects. Let the government run a budget deficit during the saving period by, e.g., decreasing the tax flow and assume that the deficit is financed by money printing. Specifically, let the deficit be of such a magnitude that at time t_s, money supply increases by a fraction kM(t) and continues to follow the escalated time path until time T_s. Then the nominal transactions balances in the economy remain on the equilibrium time path regardless of the hoarding decision of consumers. Hence, the price level stays at p_0 throughout the time axis. But then there is no motivation for investors to speculate and there will be no real effects either. The Keynesian stabilization policy is nonneutral in that it completely eliminates the real effects of the demand shock.

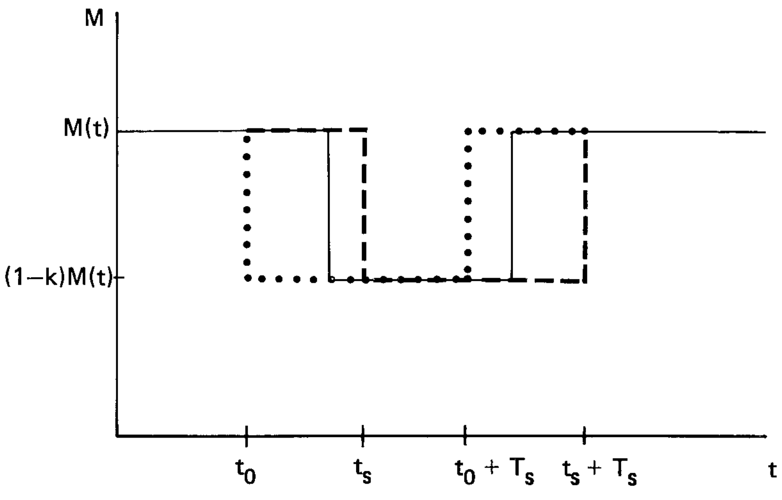
The Keynesian stabilization policy is nonneutral in that it completely eliminates the real effects of the demand shock.

3.5 Adjustment of the economy to anticipated shocks with uncertain timing

As the final case, we will now consider a more general anticipated shock. Investors know that a fraction k of the aggregate transactions balances is temporarily hoarded by workers, but they do not know precisely the timing and the duration of the shock. Specifically, at time t_0 investors form expectations that, by time t_s, the described aggregate saving starts discretely with probability one. Once the shock has started, investors find out that, by time $t_s+T_s > t_s$, the trend behaviour resumes, again discretely and with probability one. The time path of the transactions balances net of hoarding are depicted in Figure 7.

Fig. 7.

THE SAVING SHOCK WITH UNCERTAIN TIMING



The figure indicates that the shock can start at any moment within the time interval (t_0, t_s). The shock can stop at any time within the whole interval (t_0, t_s+T_s). If the shock startes at time t_0, then it will stop by t_0+T_s at the latest, but can stop at any time before t_0+T_s. If the shock starts at the last feasible moment t_s, then the last possible time by which the shock will end is at t_s+T_s.

For simplification, we assume that the timing of both the start and the end of the shock is expected to be uniformly distributed over the respective time interval. Consider the start of the shock. If the probability that the shock will start within the time interval (t_0, t_s) is one and the event has a uniform distribution, then the following holds:

(120a) $\Pr\{\text{shock starts within an arbitrary subinterval } h,\ h \to dt\}$

$$= \frac{1}{t_s - t_0} \equiv \lambda h$$

(120b) $\Pr\{\text{shock does not start within interval } h\}$

$$= 1 - \frac{1}{t_s - t_0} \equiv (1-\lambda)h$$

(120c) $\Pr\{\text{shock starts more than once within interval } h\} = 0$

Thus the instantaneous probability for the start of the shock satisfies the conditions of the Poisson process and we can invoke the approach developed for devaluation speculation in section 2.5 for modelling the behaviour of investors here.

Note the important fact that the instantaneous probability λ depends on time. To see this, assume that the shock has not started by time $t_1 \in (t_0, t_s)$. The probability that the shock starts within interval (t_1, t_s) is one and the occurrence of the event within small subintervals of equal length is still uniformly distributed. Then the probability, denoted by λ_1, that the shock starts within an arbitrary small subinterval $h \to dt \in (t_1, t_s)$, given that it did not start by t_1, is given by the following expression:

$$\lambda_1 = \frac{1}{t_s - t_1} \tag{121}$$

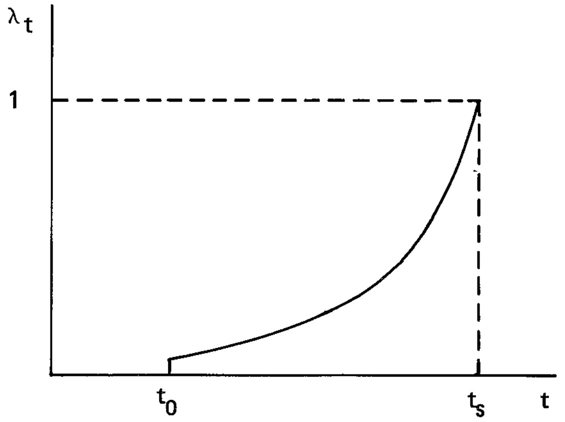
The general instantaneous probability λ_t corresponding to λ_1 in (121) is obtained as

(122) $\lambda_t dt \equiv$ Pr{shock starts within an arbitrary subinterval $h \to dt$, $h \in (t_1\ t_s)$, when it is known that it did not start by time t} $= \dfrac{1}{t_s - t} = \dfrac{t_s - t_0}{t_s - t}\lambda dt$

The time path of the instantaneous probability λ_t as defined in (122) is drawn in Figure 8.

Fig. 8.

TIME PATH OF INSTANTANEOUS PROBABILITY λ_t



Note that if the shock did not start by time t_s-h, where h is the unit of measurement, then the probability that the event will occur within the last interval h equals one.

We saw in the previous section that the start of the saving shock makes shares less desirable to investors, so that they should adjust their portfolios appropriately. On the other hand, in section 2.5 we developed an approach by means of which the effect of a devaluation with uncertain timing could be taken into account in the decision making of an investor. The saving shock as modelled here is an exact analogue of a devaluation with uncertain timing, as to the analytical technique. Thus similar portfolio rules to the rules in (97) specify precisely the time dependent substitution in the representative investor's portfolio. Applying directly portfolio rule (97a), we obtain the following for the fraction of shares which takes into account the uncertain timing of the saving shock:

$$(123) \qquad \xi_1(t) = \frac{1}{\sigma_B^2(1-\rho_{BG}^2)} \lfloor r_B - r_G - \lambda_t \tilde{k} \rfloor$$

where $\tilde{k} = \frac{k}{1-B}$

In (123), the last term in the square brackets captures the effect on portfolio composition of the event that a drop of k per cent occurs in the price level with a probability λ_t.[22] We saw above that λ_t is an increasing function, so that (123) implies a gradual shift of capital from the private sector to government bonds. But this decreases private sector output by exactly the same percentage as the portfolio composition changes. Through the quantity theory equation, the price level must inflate as the shock has not

[22]Recall from section 3.4 that a decrease of k-percent in the price level does not affect the real price of a share equiproportionately. The actual effect k is obtained by substituting 1-k for h(t) into (112b) and subtracting the result from one.

yet occurred, and therefore rule (123) must be corrected for the possible effects of inflation.

How the rule is affected by inflation rate depends on how the mean real rates of return on various assets are related to a non-stochastic inflation rate. The mean real rates of return on government bonds and shares are obtained by logarithmically differentiating the mean real price of the respective securities and are given by:

$$(124a)\qquad \frac{d\bar{Q}^{1}_{GR}(t)/dt}{\bar{Q}^{1}_{GR}(t)} = r_G - \frac{dh(t)}{h(t)}$$

$$(124b)\qquad \frac{d\bar{Q}_{BR}(t)/dt}{\bar{Q}_{BR}(t)} = r_B + \frac{B\,\frac{dh(t)}{h(t)}}{h(t)-B}$$

From (124a,b) it is clear that the Fisher effect does not hold in our model. As was pointed out in the previous section for a discrete jump in the price level, it is the fixed contract wage that makes the real rate of return on shares respond nonneutrally to inflation. While a positive inflation rate decreases by its absolute magnitude the mean real interest on government bonds, it actually increases the mean real rate of return on shares.

The inflation-adjusted portfolio rule for shares is obtained by substituting the right hand sides of (124a,b) for r_G and r_B respectively, in equation (123). The inflation-adjusted rule then becomes:

$$(125) \qquad \xi_1(t) = \frac{1}{\sigma_B^2(1-\rho_{BG}^2)}\left[r_B - r_G + \frac{\frac{dh(t)/dt}{h(t)}}{[1 - \frac{B}{h(t)}]} - \lambda_t\tilde{k}\right]$$

$$= \bar{\xi}_1 + \frac{1}{\sigma_B^2(1-\rho_{BG}^2)}\left[\frac{\frac{dh(t)/dt}{h(t)}}{[1 - \frac{B}{h(t)}]} - \lambda_t\tilde{k}\right]$$

Equation (125) is a complete expression for describing the gradual shift of resources from the private to the public sector. Note that the two terms in the square brackets have different signs but that the second terms must dominate in magnitude. The second term specifies the effect on the investor's behaviour of speculation on the event that the shock hits the economy. The first term specifies the counter effect due to the fact that investors speculate on the possible shock before it occurs. This implies a movement of capital between the two sectors and makes the actual price level shift in the opposite direction to that implied by the actual shock.

Equation (125) is useless, however, unless we are able to determine the implied time path of the price level prior to the occurrence of the actual shock, or the form of the function h(t). In order to do so, we form an expression for the variable v(t) - the fraction by which the private sector output changes because of speculation - and then use the quantity theory equation to determine the function h(t).

Equation (125) implies a representation for the variable v(t). Applying directly the definition of v(t) stated in (115) on (125), we obtain

$$(126) \qquad v(t)^s = -\frac{1}{\bar{\xi}\,(r_B-r_G)}\left[\frac{\frac{dh(t)/dt}{h(t)}}{(1-\frac{B}{h(t)})} - \lambda_t\tilde{k}\right]$$

$$= -f_0\left[\frac{\Pi(t)}{(1-\frac{B}{h(t)})} - \lambda_t\tilde{k}\right]$$

where $v(t)^s$ = percentage deviation of capital in the private sector from the trend value $\bar{\xi}_1$, as implied by (124)

$$f_0 = \frac{1}{\bar{\xi}_1(r_B-r_G)}$$

$$\Pi(t) = \frac{dh(t)/dt}{h(t)}$$

As was seen in equations (113) and (114), the quantity theory equation implies directly that prior to the occurrence of the shock

$$(127) \qquad h(t) = \frac{1}{1 - v(t)^s}$$

With the right hand side of (126) substituted into it for $v(t)^s$, equation (127) determines implicitly the time path of the actual price level before the shock occurs. It is clear that equation (127) contains so much nonlinearity that it cannot be analytically solved. For small shocks, we can simplify equation (127) by using the evaluation $h(t) = 1$ for all t. Then (127) can be solved for the inflation rate as follows:

$$(128) \qquad \Pi(t) = (1-B)\lambda_t\tilde{k} = \lambda_t k = \frac{1}{(t_s-t)}\,k$$

According to (128), the price level rises at an accelerating rate $(\frac{d\Pi(t)}{dt} > 0)$ before the shock enters.

The solution for the function h(t) implied by (128) is of the general from:

$$(129) \qquad h(t) = h_0 \exp\{-k \log(t_s - t)\}$$

The value of the constant h_0 can be found by considering the situation just before time t_s, the last possible moment for the shock to enter. If the shock has not occurred and $t_s - t \to dt$, then the probability λ_t grows large. At time t_s-dt, or one unit before t_s; the probability that the shock hits in the following interval dt is one. Therefore, at time t_s-dt, the investor is exactly in the same position as he is if he knows the timing of the shock with certainty. It was shown above that in that case the optimal price level for the investor at time t_s is $\sqrt{(1-k)}\ p_0$, or that $h(t_s) = \sqrt{(1-k)}$. If the shock has not hit by time t_s-dt, then, by the principle of optimality, the following must hold:

$$(130) \qquad h(t_s - dt) = \sqrt{(1-k)} + k$$

Identity (130) is simply due to the fact that, within the subinterval (t_s-dt, t_s), the price level must fall by a fraction k and it must end up at the level $\sqrt{(1-k)}\ p_0$. If the right hand side of (130) is substituted into (129) for the left hand side and the right hand side of (128) is evaluated at $t = t_s$-dt, the following is obtained:

$$(131) \qquad k + \sqrt{(1-k)} = h_0 \exp\{-k \log (1)\}$$

(131) implies that

$$h_0 = k + \sqrt{(1-k)}$$

The particular solution of (129) which is feasible from the point of view of the principle of optimality is then

$$\tilde{h}(t) = \lfloor k + \sqrt{(1-k)} \rfloor \exp\{-k \log (t_s - t)\} \tag{132}$$

where $\tilde{h}(t)$ is the approximate solution for $h(t)$

Recall here that solution (132) is only an approximation for the actual function $h(t)$. Solution (132) approximates by first order dynamics the actual higher order nonlinear dynamics. The result seems plausible, however. To see this, substitute (132) and (128) into (126) for $h(t)$ and $\Pi(t)$, respectively. At $h(t) = 1$, $v(t)^S$ equals zero. At $h(t) > 1$, the second term of (126) in square brackets becomes larger than the first term. Hence, $v(t)^S$ becomes strictly positive. Furthermore, $v(t)^S$ increases at an accelerating speed as $h(t)$ increases. All these properties implied by the approximate solution (132) are in harmony with the fact that the driving force in the price dynamics is speculation on an event, the probability of which increases with time at an accelerating speed.

Because of the solution for $h(t)$ is only approximate, substitutions of $h(t)$ and $\Pi(t)$ into (126) will not give a solution for $v(t)^S$ which will satisfy the quantity theory equation (127). For the correct $h(t)$ with higher than first order dynamics, the terminal condition (130) due to the principle of optimality would be just one boundary condition fixing one of the unknown constants of integration. Other conditions would be needed to guarantee that $v(t)^S$ generated by (126) would satisfy the quantity theory equation. Therefore, we have to solve the approximate $v(t)^S$ directly from (127) by substituting the approximate solution (132) into (127) for $h(t)$. Such an approximate solution for $v(t)^S$, denoted by $\tilde{v}(t)^S$, then becomes

$$(133) \qquad \tilde{v}(t)^s = 1 - [\tilde{h}(t)]^{-1} = 1 - [k + \sqrt{(1-k)}]^{-1}\exp\{k \log(t_s - t)\}$$

The approximate time paths of the price level and the portfolio fraction $\xi_1(t)$ are drawn in Figure 9. Note that, with the short-term fixed-coefficient technology, the portfolio fraction $\xi_1(t)$ is a representation of output that abstracts away from the potentially confusing stochastic trend in output itself.

The figure is drawn assuming that the shock actually starts at time t_s. Were this not the case, the only difference would be that price level and output would follow the same time paths for a shorter period of time.

At time t_0, when agents first get to know about the shock, both the price level and output jump, the former upwards and the latter downwards. After that, the price level inflates and output decreases gradually until the shock enters.

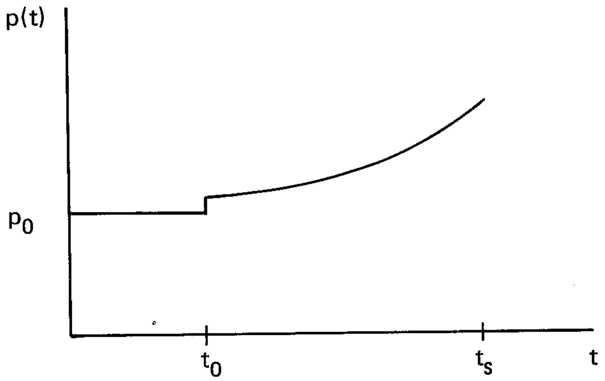
At the time when the saving period actually begins, the price level jumps discretely downwards by k-percent. At the same time, investors start forming expectations and speculate on the end of the saving period, as well as start increasing investments in the private sector. This begins to flood the goods market with excess supply, so that via the quantity theory equation the price level begins to deflate. The price level and the portfolio fraction $\xi_1(t)$ follow time paths analogous to (132) and (133) until again the shock enters, i.e. the saving period ends.

In conclusion, we present in Figure 10 the time paths of the price level and the portfolio fraction $\xi_1(t)$ throughout the entire period when speculative behaviour is in effect.

Fig. 9.

PRICE LEVEL AND PORTFOLIO FRACTION $\xi_1(t)$ WHEN INVESTORS SPECULATE ON A SAVING SHOCK WITH UNCERTAIN TIMING: SITUATION BEFORE THE SHOCK

a) Price level



b) Portfolio fraction $\xi_1(t)$

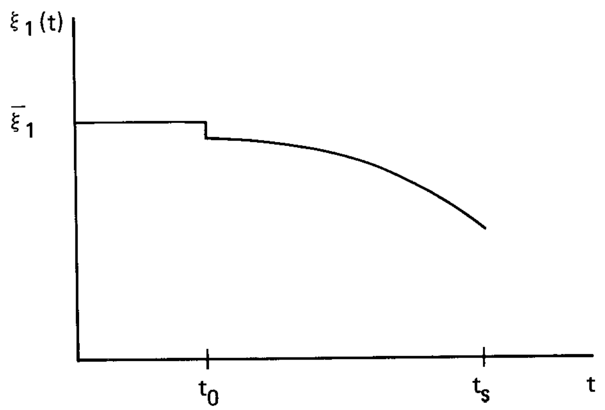
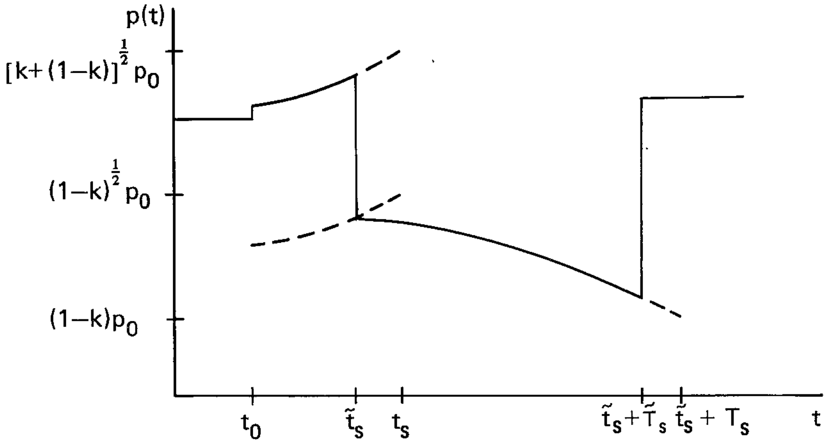


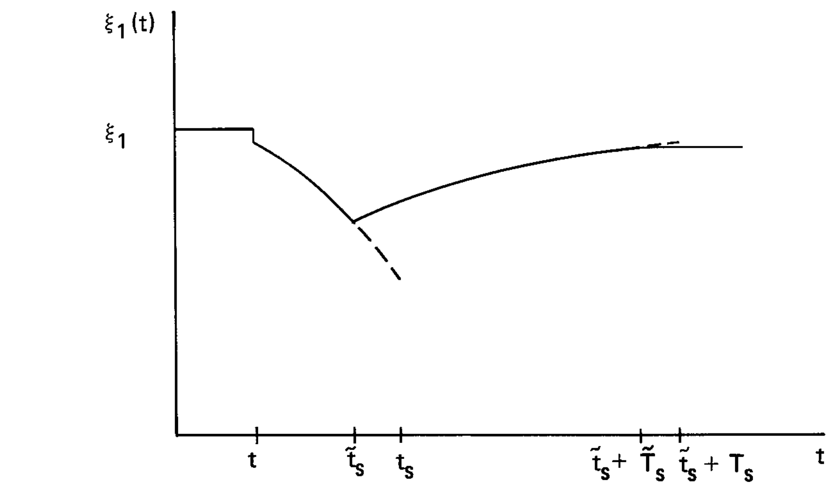
Fig. 10.

PRICE LEVEL AND PORTFOLIO FRACTION $\xi_1(t)$ WHEN INVESTORS SPECULATE ON A SAVING SHOCK WITH UNCERTAIN TIMING: THE WHOLE PERIOD

a) Price level



b) Portfolio fraction $\xi_1(t)$



At time t_0, investors get to know about the aggregate saving shock within the next $t_s - t_0$ time units. They begin to speculate on the start of the shock so as to derive capital gains from the expected change in the real relative prices of various investment opportunities. It is optimal for the investors to choose a speculative path, which implies a drop in the price level to $\sqrt{(1-k)}\ p_0$ if the shock occurs at time t_s, the last possible time. The optimal speculative behaviour of investors leads to gradual decreases in capital - hence in output - in the private sector. Through the quantity theory equation, the gradual decline in aggregate supply causes a gradual increase in the price level. The implied price path has the property that, if the shock hits at time t_s, then the price level does indeed jump to the level $\sqrt{(1-k)}\ p_0$.

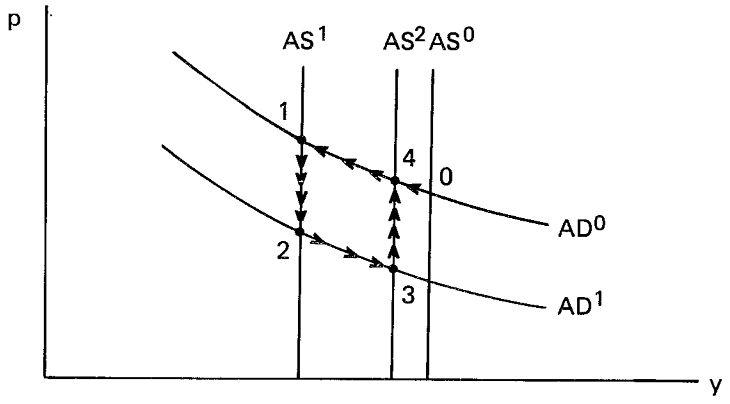
If the shock actually starts at time $\tilde{t}_s < t_s$, then the price level jumps down immediately by k percent to the path shown by the broken line in the figure, which is parallel to the original path valid before the shock. Note that the actual price level after the start of the shock is now lower than it would have been if the timing of the shock had been known with certainty. This is an indication of the fact that investors take a more cautious speculative position if there is uncertainty concerning the timing of the shock.

At time $\tilde{t}_s$, or when the aggregate saving period starts, investors form the view that, within the next T_s time units, the stationary state consumption-saving behaviour resumes. They gradually start investing more and more in the private sector. The optimal speculative path is again, in principle, determined by the terminal condition that the price level must end up being $\sqrt{(1-k)} \cdot p_0^1$ at the last possible time for the stationary state behaviour to resume, or at $\tilde{t}_s + T_s$. This cannot be right, however. The initial price level p_0^1 for the new speculation period, or $p(\tilde{t}_s)$, is lower than the corresponding price level $\sqrt{(1-k)}\ p_0$ in the case of the shock with certain timing. This would imply that the new stationary state price level would be lower than the corresponding price level before the shock if the drop in saving occurred at the latest possible time. This means, on the other hand, that output would be on a higher path in such a stationary state equilibrium. Because of the labour constraint, this cannot be possible and therefore optimizing investors cannot plan such an investment policy. The relevant terminal condition for the investors must then be that, if the shock hits at the latest possible time, then the price level must jump to the initial stationary state level, i.e. to p_0. This is the same thing as requiring that the fraction $\bar{\xi}_1(t)$ must be at the stationary state level at time $t_s+\tilde{T}_s$, or at the last possible time for the shock to occur.

With this terminal condition, the time paths of p(t) and $\xi_1(t)$ shown by the solid lines result. If the stationary state consumption-saving behaviour resumes at time $\tilde{t}_s+\tilde{T}_s$, or before the last possible time, then the stock of capital in the private sector has not returned to the stationary state level, but is below that level. Consequently, aggregate supply ends up on a lower and the price level on a higher time path than in the general equilibrium stationary state before the shock. There is no mechanism in this model which would move the economy back to the stationary state in the short run. However, in the ensuing wage contracting the implied permanent shift in income distribution in favour of capital will be corrected. According to the wage contracting condition (103b), the expected real wage over the contract period must be equal to the value of the marginal product of labor corrected for the risk premium. Enforcement of this condition decreases the real return on shares so that investors move back to their initial stationary state portfolio composition. With the fixed money supply rule, this implies a drop in the price level back to p_0 so that the initial stationary state resumes. The adjustments in the economy can again be illustrated in the static demand-supply diagram. This is done in Figure 11.

Fig. 11.

AGGREGATE DEMAND AND SUPPLY IN THE CASE OF AN ANTICIPATED SAVING SHOCK WITH UNCERTAIN TIMING



When speculation on the falling price level begins, the supply curve starts moving to the left. The shock hits when the supply curve is in position AS^1 and moves the demand curve to position AD^1. Speculation on the upward jump in the price level then starts, shifting the supply curve to the right. When the supply curve is in position AS^2, the saving period ends, shifting the demand curve back to position AD^0. The goods market equilibrium moves first along the demand curve AD^0 from point 0 to point 1, then jumps along the supply curve AS^1 from point 1 to point 2, again moves gradually along the demand curve AD^1 from point 2 to point 3, and finally jumps along the supply curve AS^1 from point 3 to point 4. At the following contracting time, the supply curve shifts back to the initial position AS^0 and the initial stationary state general equilibrium resumes. This must happen because the long-term determinants of the equilibrium - technology, preferences, and endowments - remain unchanged.

Finally, we note that the dynamic paths in the present case contain an interesting prediction. During periods when investors speculate on an event but the event has not occurred, the price level and output are positively related. Hence, during such periods, positively sloped short-term Phillips curves would be empirically observed. The average medium-term implicit Phillips-curve relationship would have the standard negative slope, while the long-term implicit Phillips-curve would be vertical. In the example, medium-term would mean e.g. the period of speculation on the start of the shock and the actual start, while the long-term would include the whole adjustment process.

3.6 Anticipated shocks with uncertain timing and the neutrality of policy

In the analysis of the previous section, we assumed implicitly that the government is following the money targeting policy regime. The analysis showed that real effects result under this policy regime. Hence, we say that the money targeting monetary policy is neutral in eliminating the real effects of demand disturbances.

Consider, alternatively, the economy in which agents are anticipating the same shock but know that the government is following the interest rate targeting policy. Under this policy, the government ties the nominal rate of return on risky bonds to the inflation rate in such a way that the real rates of return on both shares and risky bonds vary identically as the inflation rate varies. In (124a,b), we stated the 'non-administered' real rates of return on the risky government bonds and shares. Let us denote by $r_G(t)$ the inflation dependent nominal rate of return on the risky bonds. We can then solve for this rate of return by establishing the following condition:

$$(134) \qquad r_B + \frac{B\,\frac{dh(t)}{h(t)}}{h(t) - B} - \left[r_G(t) - \frac{dh(t)}{h(t)}\right] = r_B - r_G$$

The inflation dependent nominal rate of return implied by (33) is:

$$r_G(t) = r_G + \frac{dh(t)}{h(t) - B} \tag{135}$$

Recall that the interest rate targeting policy makes the real price of a risky bond remain in constant proportion to the real price of a share. Hence capital gains and losses due to discrete price level changes are the same for holding either risky bonds or shares in numbers worth the same amounts of money. Using the latter fact and (135) in equation (125), which gives the time-dependent equation for the portfolio fraction $\xi_1(t)$, one observes that, under the interest rate targeting policy, the portfolio fraction of shares becomes independent of the inflation rate. Mathematically, this can be seen by noting that, if $r_G(t) - \frac{dh(t)}{h(t)} - \lambda_t\tilde{k}$ is substituted into (125) for r_G, then the second term in the latter form of the solution becomes equal to zero.

The above conclusion clarifies the motivation for the interest rate targeting policy. By following such a policy, the government tries to neutralize the consequences of the failure of the Fisher-effect or the basic distortion. What the interest rate targeting policy actually does is to transform the movements of capital between the private and the public sector caused by anticipated price level variations into portfolio adjustments among the public sector assets. Thus, in effect, the interest rate targeting policy eliminates crowding out and crowding in phenomena that are the manifestation of the real effects under the money targeting policy. The portfolio adjustment among the public sector assets implied by the interest rate targeting policy can be seen by substituting $r_B + \frac{B\frac{dh(t)}{h(t)}}{h(t)-B} - \lambda_t\tilde{k}$ for r_B, $r_G(t) + \frac{dh(t)}{h(t)} - \lambda_t\tilde{k}$ for r_G, and $r - \frac{dh(t)}{h(t)}$ for r into the portfolio fraction of risky bonds stated in (102d). If the basic convenience assumption $\sigma_G = \sigma_B\rho_{BG}$ is used, the price-level-dependent portfolio fraction ξ_2 becomes:

(136) $$\xi_2(t) = \bar{\xi}_2 + f_1\left[\frac{dh(t)}{h(t)-B} - \lambda_t\tilde{k}\right]$$

where $$f_1 = \frac{1}{\sigma_B^2\rho_{BG}^2}$$

$\bar{\xi}_2$ = general equilibrium level of ξ_2

According to (136), the portfolio fraction ξ_2 increases if the inflation rate rises and decreases if there are expectations of a downward discrete jump in the price level. Since the fraction ξ_1 remains constant, the fraction of riskless government bonds (or speculative money) must adjust accordingly.

Equipped with (136), we are ready to analyze price and output adjustments under the interest rate targeting regime. When the only shock affecting the economy is the anticipated discrete demand shift with uncertain timing, we can further simplify equation (136). In such a case, the actual inflation rate is zero before the shock, as long as ξ_1 remains constant at $\bar{\xi}_1$. Thus, the time path of ξ_2 becomes very simple in the case that we are studying and is[23]:

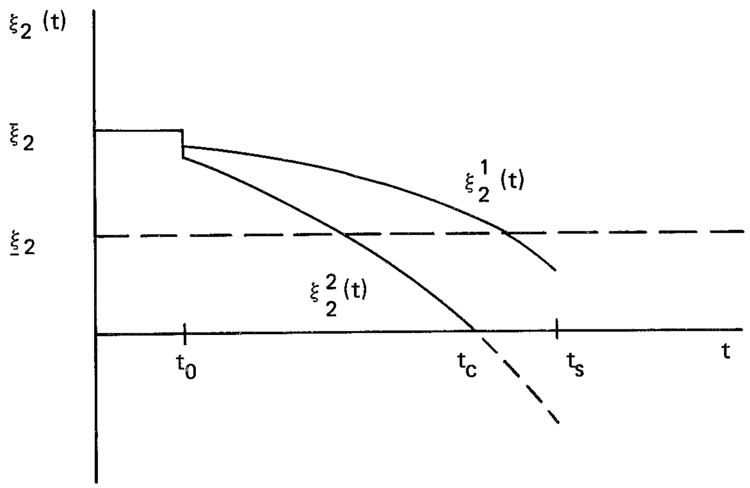
(137) $$\xi_2(t) = \bar{\xi}_2 - f_1\lambda_t\tilde{k}$$

The time path of ξ_2 is drawn in Figure 12, where it is assumed that the shock hits at the last possible moment t_s.

[23]Note that if there were a mixture of shocks - say the decisions of some agents were smoothly distributed over a time interval and the rest were anticipated to decide at the same unknown time within an interval - then the general expression (136) for $\xi_2(t)$ would become relevant. In such a case, the dynamics of $\xi_2(t)$ could potentially become complicated.

Fig. 12.

PORTFOLIO FRACTION ξ_2 UNDER THE INTEREST RATE TARGETING REGIME



Two alternative time paths for ξ_2) are drawn in the figure, one for a small parameter value k (curve $\xi_2^1(t)$) and the other for a large k (curve $\xi_2^2(t)$). The latter leads to a faster substitution from risky to riskless bonds and, at time t_c, all risky bonds are converted into riskless bonds. The substitution possibilities are then exhausted at time t_c, given that the government will not finance the ongoing speculation. The time path with a small value of k, on the other hand, implies a positive value of ξ_2 at time t_s, so that all of the adjustment capacity has not been exhausted during the speculation period. In practical situations, there could be a positive constraint for ξ_2 as well. Such a constraint, like $\underline{\xi}_2$ in the figure, could arise if e.g. the government only adjusts the nominal terms of the maturing risky bonds when they are re-issued. If such an institutional constraint were to exist, then the adjustment capacity would be insufficient in the case of the small shock as well.

A question of considerable importance is then: what happens if no more substitution among public sector securities is possible? The answer lies in the fact that, once such a situation emerges, there are still two investment alternatives open to investors. While the investors cannot speculate any more on the demand shock by substituting from risky to riskless bonds, they can still continue speculation by shifting capital from shares into riskless bonds. But the latter speculation is approximately characterized by the price level equation (132) and the shift coefficient equation (133). What happens, then, at the time when there are no more substitution possibilities among the public sector assets is that investors shift their investments discretely from the private to the public sector to the extent that the portfolio fraction $\xi_1(t)$ drops discretely on the time path implied by (133) evaluated at that time. Likewise, the price level jumps discretely on the time path (132). The shifts are indeed discrete, because the investment opportunity set changes discretely. From the time of the shift onwards, equations (132) and (133) govern the dynamics of the price level and real variables.

As a final point concerning the interest rate targeting regime, one should note that, even in cases where k is so small that ξ_2 as determined by (137) will not come up against any constraint within the interval (t_0, t_s), there can be real effects. This is due to the fact that (137) will characterize the behaviour of investors only up to time t_s-2h, $h \to dt$. If the shock has not started by time t_s-h, or one unit of measurement before the last possible time for the shock to enter, agents know with certainty the timing of the shock, i.e. that it will start at t_s. We know from the analysis of section 3.4 that under such circumstances the money targeting and the interest rate targeting policies are indifferent. Under both policies, the underlying factor in the behaviour of investors is that it is optimal for them to make the price level fall by a factor $\sqrt{(1-k)}$ rather than by 1-k. Under the interest rate targeting regime, they can only carry this out by moving all risky bonds and a fraction $\sqrt{(1-k)}$ of capital invested in firms into riskless bonds.

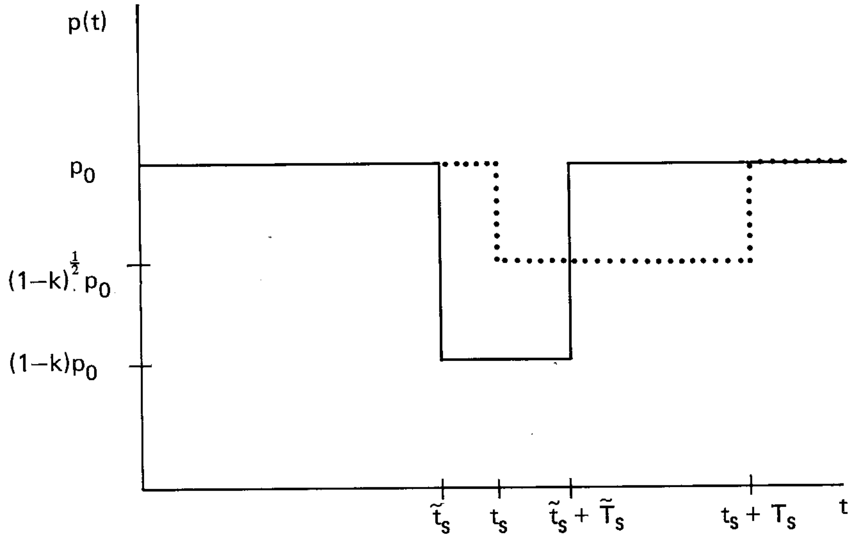
In principle, there can be three different types of adjustment processes in the economy under the interest rate targeting policy, given the anticipated demand shock with uncertain timing. The price level and output adjustments in the three cases are illustrated in Figure 13.

In panels (a) and (b), two alternative paths for the price level and the portfolio fraction $\xi_1(t)$ are drawn for a small shock. The shock is considered to be small in this context if the speculation caused by the shock does not imply a level of the portfolio fraction ξ_2 smaller than the exogenous constraint $\underline{\xi}_2$ during the interval (t_0, t_s) in which the actual shock is known to hit. The solid lines portray the case in which the shock hits at $\tilde{t}_s < t_s$. They show that the price level jumps temporarily to the level $(1-k)p_0$ and that the portfolio fraction $\bar{\xi}_1$ remains constant at ξ_1. Thus the paths are identical to those in the unanticipated case. The interest rate targeting policy has neutralized the shock. The broken lines, on the other hand, show the case in which the shock starts and stops at the last possible moments, i.e. at times t_s and t_s+T_s, respectively. The price level jumps temporarily down to $\sqrt{(1-k)}p_0$ and the portfolio fraction ξ_1 drops as well. These effects are similar to those in the anticipated case with certain timing. For both cases in panels (a,b), the adjustment paths of the portfolio fraction ξ_2 are also drawn. In both cases, ξ_2 drops discretely at time t_0. If the shock stops before time $\tilde{t}_s+T_s$, or before the latest possible time if it started at $\tilde{t}_s$, then there is a discrete jump in ξ_2 at the time when the shock stops. This is so because the price level returns to p_0 after the shock stops and $\bar{\xi}_2$ is the optimal ξ_2 for investors at that price level.

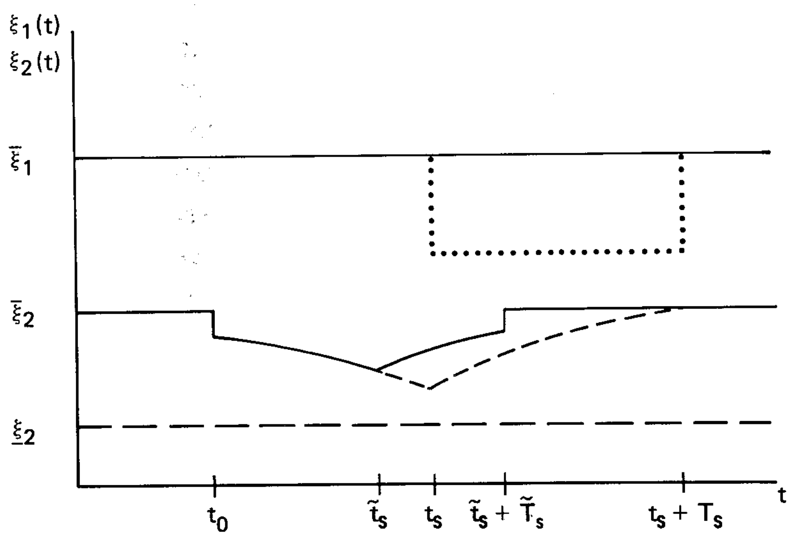
Fig. 13.

PRICE LEVEL AND PORTFOLIO FRACTION $\xi_1(t)$ WHEN INVENTORS SPECULATE ON A SAVING SHOCK WITH UNCERTAIN TIMING UNDER THE INTEREST RATE TARGETING POLICY REGIME

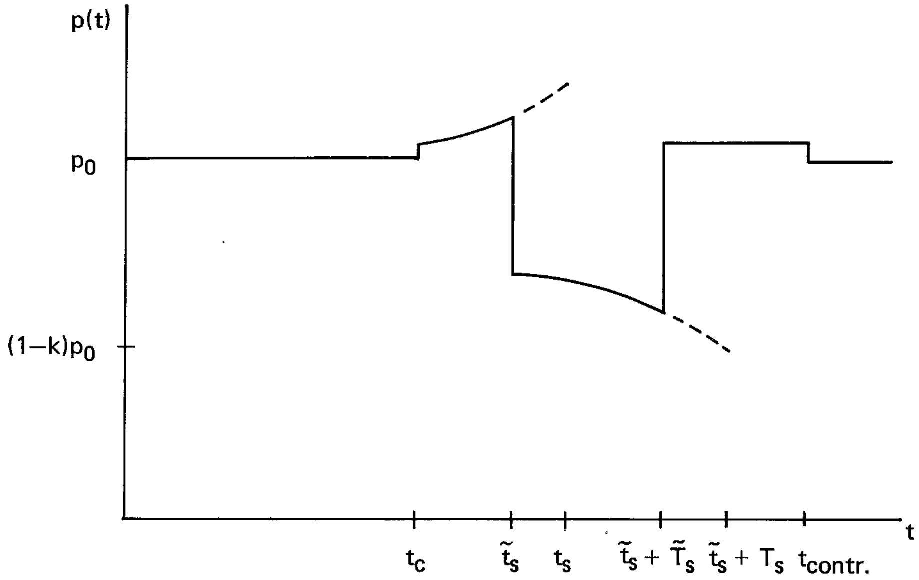
a) Price level (small shock)



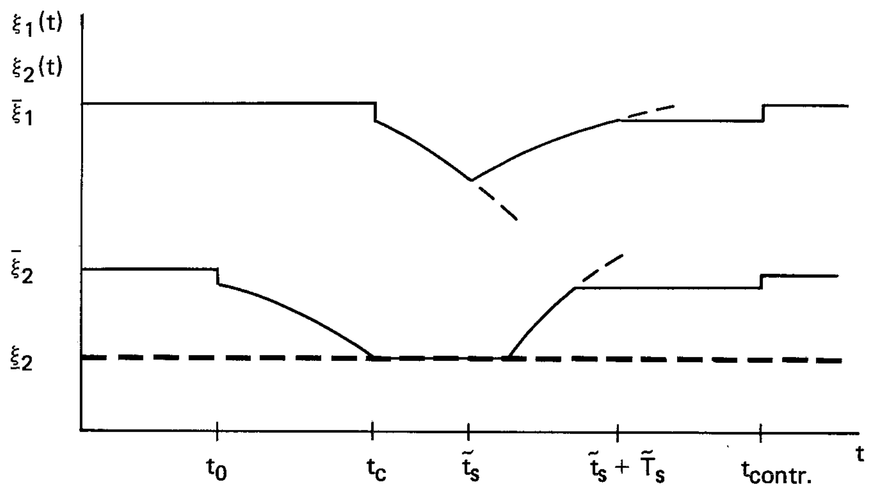
b) Portfolio fraction $\xi_1(t)$ (small shock)



c) Price level (large shock)



d) Portfolio fraction $\xi_1(t)$ (large shock)



In panels (c) and (d), the time paths of the price level and the portfolio fraction $\xi_1(t)$ are drawn in the case where the value of parameter k is so large that the portfolio fraction ξ_2 comes up against the constraint $\underline{\xi}_2$ at time $t_c < t_s$, or before the shock starts. At time t_c, investors observe the discrete change in investment opportunities and reallocate their portfolios so that the fraction ξ_1 jumps down to the path implied by (133) and the price level up to the path determined by (132). Output decreases and the price level inflates until time $\tilde{t}_s$, at which point the shock starts and the price level falls by the factor 1-k. Investors then start to speculate on the end of the saving period. The portfolio fraction ξ_1 increases and the price level deflates. At time $\tilde{t}_s+\tilde{T}_s$, saving ends and the price level jumps up by factor $\frac{1}{1-k}$. If, as in our case, the shock ends before the last possible moment $\tilde{t}_s+T_s$, then the price level does not fall as far as the level $(1-k)p_0$. Consequently, the new no-shock price level is above the stationary state price level p_0. Output is, respectively, below its stationary state time path. Investors will not adjust their portfolios to the stationary state allocation, because doing so would decrease the price level and cause them larger capital losses than gains. The final adjustment to the stationary state equilibrium occurs at the next wage contracting time t_{contr}.

The consequences of the shock could also be illustrated by means of demand-supply diagrams, but they would clearly only be replicas of the previous figures and are not therefore shown here.

A general conclusion concerning the neutrality of alternative monetary policies emerges from the analysis. The money targeting policy regime is always neutral, i.e. it cannot eliminate the real effects of any anticipated shocks. This is so because it has no impact whatever on the basic distortion, whose effect is felt through variations in investment opportunities. From this point of view, the money targeting policy can be seen as the passive policy or the no-policy option of the government. The interest rate targeting policy, on the other hand, affects the basic distortion by partially regulating investment opportunities. Hence, this type of policy is not neutral relative to small anticipated shocks. However, large shocks cause real effects under the interest rate targeting regime as well. The size of the shock must be understood in a special way here. A shock is large in the conventional sense, if parameter k takes a large value. However, a shock is also large if the probability λ_t is large. Thus, an intuitively appealing interpretation arises: the less confusion there is about the timing of shocks, the more likely it is that there will be real effects under this type of monetary policy.

Again it is clear that Keynesian stabilization policy can eliminate all real effects, i.e. it is always nonneutral. If the government announces that, if a shock occurs, it will always depart from the balanced budget in the way described in section 3.4, there will never be anticipations of variations in the price level and hence no real effects either.

3.7 Neutrality of policy: Connections with Fischer and Leijonhufvud

Our analysis is linked to the conventional policy neutrality debate on the rational expectations models in a slightly different way than usual. We construct an economy with a distortion by introducing contract wage rigidity. We also allow for the possibility that an automatic stabilizing factor - the real balance effect - possibly neutralizes the real consequences of demand disturbances. Under this general setting, we study whether demand disturbances will have effects on the level of output and unemployment when the government follows alternative policy regimes. We say that a policy regime is nonneutral if there are no real fluctuations under the regime.

Because of the slight difference in emphasis, our analysis is not phrased directly in the same way, e.g. in terms of the money supply rules, as are the 'classic' writings on the theme by Lucas

(1972), Sargent-Wallace (1975), Phelps-Taylor (1977) and Fischer (1979). It should be clear, however, that, given the distortion in our model, at least some 'standard' nonneutrality propositions must be implied by the model.

Two properties of our model serve to locate our analysis in relation to the models in the literature. First, we have built the property of a vertical aggregate supply curve into the model. Real effects can only result if the aggregate supply curve shifts. Hence, our model differs strongly from the standard Lucas-Sargent-Wallace approach, in which confusions among firms about local and global price shocks generate a short-term negatively sloped Phillips-curve which makes unanticipated events have real effects. Second, a mechanism analogous in a stationary economy to the Tobin effect of some models of growth economies is a central driving force of the real effects in our model.[24] Thus, variations in the anticipated inflation rate change the allocation of capital between real and nominal assets. This feature of our model immediately identifies the analysis by Fischer (1979) as being closely related and potentially comparable to ours.

Comparisons of the results of the two studies are only relevant as long as they concern anticipated events. This follows from the fact that, in our model, only shifts of the supply curve can cause real effects. Such shifts can, as was seen in sections 3.3 - 3.6, only be realized if an event was unanticipated at the wage contracting time but is anticipated by investors before it actually occurs. It is clear from our preceding analyses that anticipated events in the above sense, which cause variations in the price level - be they either changes in money supply or demand shocks under the money targeting regime - will have real effects. Qualitatively, this conclusion is identical to Fischer's.

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[24]The Tobin effect, introduced in Tobin (1965) and Mundell (1963), is an important component of the recent studies of e.g. Fischer (1979), Singleton (1983), and Summers (1981).

However, there are two important differences between the studies concerning the dynamic paths of the relevant variables. First, if the anticipated discrete shock with certain timing in our analysis is taken to correspond to Fischer's anticipated temporary shock, then the studies differ in that the price paths generated in our experiment will never have the geometric lead and lag structure typical of the price paths in Fischer's study. In particular, an agent who started speculating on a shock with certain timing at the time when it is announced, would behave suboptimally in our model. The only way in which one could obtain gradually adjusting price paths in our model is by introducing adjustment costs into portfolio adjustments. To us, this seems to suggest that, in a model with similar structure to that of Fischer, the phrase 'money is nonneutral' is in fact identical to another phrase 'portfolio adjustments are costly'.[25] Note that the above conclusion points out the different sources of nonneutralities in the two types of models: our model does not assume adjustment costs, but money is nonneutral in it. The origins of the nonneutrality in our model are the separation of capital and labor supply decisions and the rigidity of the contract wage. These assumptions make variations in income distribution become influential in the economy. In a technical sense, this is the same thing as saying that the Tobin effect is the driving force of the model.

Another difference between the dynamic paths generated by the two models arises if the case of the anticipated discrete shock with uncertain timing is compared to the case of the temporary anticipated shock in Fischer. The former event generates gradually adjusting time paths for both the price level and real variables. However, the crucial feature of this case is that the price level

[25]One comment by Fischer gives some indirect support to our interpretation. While discussing anticipated temporary shocks, he states "For neutral money, neither anticipated nor unanticipated changes in the money stock have any effect on subsequent price levels" (Fischer (1979) p. 241).

always overshoots, which never happens in Fischer's case. Thus, if the actual shock decreases the equilibrium price level, the speculative behaviour before the shock makes the price level gradually inflate. The overall effect will always be in the direction implied by the discrete event itself, and is thus analogous to the price response in Fischer. The gradual dynamics in our model in this case, even without adjustment costs, can be understood by noting that confusion about the timing of the event creates opportunity costs of not speculating before the event, since the expected value of speculative actions is positive. These opportunity costs are exactly analogous to standard adjustment costs, and it would be a surprise if the two did not result in similar dynamic paths in a given model.

As to overshooting in the price level, it seems that Fischer's analysis precludes this phenomenon, because he requires that 'the expected real return on capital has a greater influence on the demand for capital than on the demand for real balances relative to the influence of the expected rate of inflation on the respective asset demands' (Fischer, p. 232). This requirement guarantees that the coefficient of the expected price level is positive in the equation for the current price level. However, in our view, the requirement seems to ignore the fact that if investors substitute in favour of capital in the current period in anticipation of higher inflation in the future, they flood the goods market in the current period. With a fixed money stock, this would decrease the current price level. In conclusion, we would prefer changing the direction of the requirement which would guarantee overshooting. However, this might change other results of the model as well, and would make other parts of the discussion above potentially irrelevant.

One of the main results of our analysis was that it provided rankings of alternative policy regimes relative to their efficiencies in neutralizing real fluctuations caused by demand shocks. Viewed in this light, our results can be interpreted in terms of the notion of corridor first introduced by Leijonhufvud (1973).[26] According to his view, an economy can adjust by means of automatic stabilizing mechanisms to small shocks, but adjustment to large shocks in full employment requires stabilization policy measures. Furthermore, as regards the shocks requiring policy measures, the smaller shocks can be absorbed by monetary policy, but the larger ones require the use of traditional Keynesian fiscal policy.

In our specific set-up, unanticipated discrete shocks can never make optimizing investors speculate on variations in income distribution. Hence, the change in the price level implied by the shock changes the real value of transactions balances so that the goods markets remain in equilibrium and no unemployment results. In this sense, the automatic stabilizing mechanism in our economy - the real balance effect - neutralizes the unanticipated shocks so that no real fluctuations emerge.

If, on the other hand, the demand shocks are anticipated, stabilization policy is always needed to neutralize the real effects. Owing to the specific type of distortion in our model, the money targeting policy regime is useless for stabilization, and hence the interest rate targeting regime is the relevant monetary policy to consider hence. Our analysis showed that the interest rate targeting policy is effective in neutralizing relatively small anticipated shocks. However, If the shocks are very large, then only the traditional Keynesian fiscal policy can eliminate the real consequences of them. The size of the shock must be understood in the specific extended sense characterized above in section 3.6. A shock is large in the standard meaning if the

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[26]There are only a few papers known to this author in which the notion or corridor is even referred to. Besides Leijonhufvud (1973; also published in his essay collection in 1981), there are only a comment by Grossman (1974) and two papers by Howitt (1978, 1979) which deal with the issue. The only formalized examples of the phenomenon familiar to this author are in Howitt (1978). See also the book review by Howitt (1983).

magnitude of the parameter characterizing the shock - in our case the magnitude of k - is large. But a shock is also large if the probability distribution of its timing is dense. In the extreme, a shock, the timing of which is known with certainty, is so large that, independently of the value of parameter k, real effects will always result under monetary policy.

In conclusion, the economy that we have constructed seems to have the corridor property, although in a slightly different spirit than in Leijonhufvud's original description. If we accept the view that the likelihood of an event being anticipated increases with the value of the shock-size parameter k, then the resemblance would be even closer. A basic difference is, of course, that we have derived the results in a fairly neoclassical framework, in which only the contract wage is a fixed price. Leijonhufvud's remarks are mostly made in a fix-price environment.

3.8 Reality and robustness of the results: critical remarks

The reality and robustness of the results can be questioned in two respects. One could, on one hand, raise some doubts on the empirical relevance of the key assumptions that generate the results. On the other hand, one could critisize some theoretical and technical choices that we have made.

In our model, the distortion that is the driving force of the nonneutrality and the corridor results is due to two assumptions. First, input supply decisions are separated. Second, fixed contract wages are negotiated over finite time-intervals between employers and trade unions representing workers. These assumptions make variations in income distribution influential in the economy. If a shock that affects the price level enters within a contract period, investors can respond optimally to the implied change in income distribution, while employees are tied to the wage contract.

Of the two assumptions, the separation of input supply decisions is an institutional fact in some Nordic countries. Actual allocation of capital among alternative investment opportunities is mainly carried out by often cartellized or oligopolistic financial intermediaries. Labor supply behaviour is decided by trade unions, which represent a very large proportion of the labor force. The assumption of rigid nominal wages over the contract period is not entirely accurate in Nordic labor markets. Wage drift has been substantial at times, but never in such proportions that the nominal wages were actually fully flexible. Some rigidity in nominal wages over the contract period is guaranteed merely by the fact that, e.g. in the public sector, wage drift is negligible.

Separation of labor and capital supply decisions as constructed in our model implies a sharp departure from the standard neoclassical assumption. In the neoclassical model, labor supply decisions are based on intertemporal substitution between leisure and work.[27] the labor supply decision of an agent then becomes dependent on his capital income. In our model, such intertemporal substitution possibilities are completely eliminated by the above separation. Thus, labor supply is always fixed and unemployment is caused by investment decisions, hence being involuntary. Our choice for the unemployment mechanisms possibly reflects the common view among practicing economists in the Nordic countries that hardly any of the standard macroeconomic hypotheses are less plausible than the notions of intertemporal substitution and the implied voluntary unemployment. The standard critique of the neoclassical labor market model would claim that weekly working hours, i.e. the issue of intertemporal substitution, are set for years or decades by legislation. An individual has basically a choice between working zero hours or working the agreed full time. Flexibilities in

[27]This has been the standard approach at least since the seminal work by Lucas-Rapping (1969) and is the assumption adopted recently by, among many others, Kydland-Prescott (1982).

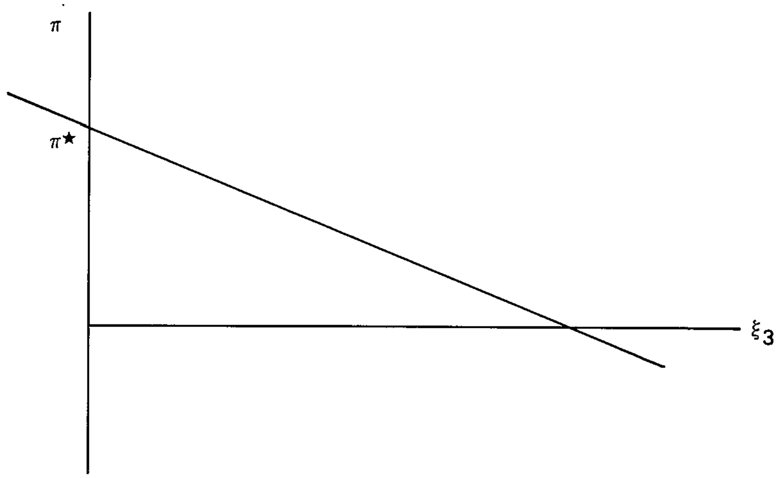
full-time working in the form of overtime or part-time work are insignificantly small. Few individuals are believed to have any flexibility in labor supply under such circumstances.

An empirical example of macroeconomic adjustment processes in which variations in income distribution are believed to play an important role is the phenomenon called the devaluation cycle. This phenomenon is analyzed more closely in the following chapter, but we briefly mention here the main features of the cycle. Exogenous events and wage contracts may make the profitability of firms decrease. Indirectly, this is associated with increasing pressures against the exchange rate. If the process continues so long that a devaluation is carried out, the profitability of firms improves substantially. Because returns on shares and income distribution vary accordingly, it is likely that firms, or their owners, act so as to accelerate the process and to shorten the cycle. Such acts will have real effects, as will be seen in the following chapter.

There are some remarks to be made concerning the theoretical choices that we have adopted. The most important and consequential choice in the theoretical arguments is the approach followed in modelling investor behaviour. The portfolio behaviour in our model is a variant of the mean-variance tradition, which has been heavily critisized in general. The particular property of the mean-variance analysis, which proves to be crucial in our analysis, is the linearity of the optimal portfolio rules in the mean rates of return on assets. Besides being very convenient analytically, this property is the fundamental technical factor that generates the corridor property for the economy. Linearity implies that there are rates of return on each asset at which implied demands for those assets would become negative, i.e. agents would like to sell short in those assets. To illustrate the issue, we draw the demand function for the riskless bond as a function of the inflation rate in Figure 14.

Fig. 14.

DEMAND FOR RISKLESS BONDS AND THE INFLATION RATE



Note that the figure is drawn under the interest rate targeting regime. If the inflation rate (or $\lambda_t k$ for an expected upwards jump in the price level) rises above Π^*, then agents want to borrow at the riskless rate from to government. This creates a problem if one recalls that riskless bonds were specified so as to be exact analogues of speculative money, the only difference being that bonds pay a positive interest rate. Thus, at inflation Π^*, 'conventional' speculative money would not be held at all. The linearity of the demand function then makes room for the interpretation that there would be a very high inflation rate at which all money holdings would fall to zero, an empirically implausible situation. Such an interpretation is incorrect, however. Immediately an investor starts converting more than his speculative money holdings into other assets, he begins to give up his transactions balances. Through the quantity theory equation, given an interest inelastic velocity of money, this will cause decreases in the price level because output remains on its equilibrium time path. Therefore, such portfolio adjustments are not optimal. The inflation rate cannot escape the level Π^* and there is a discontinuity in the demand function for speculative money at that inflation rate.

What the foregoing discussion points out is that the separation between transactions and speculative money demand is very important. This is why we have replaced speculative money in the analysis by a dominant asset, the riskless bond. The question whether the demand for the riskless asset can become negative or not is then purely one of an institutional constraint. If the government also lends at the riskless rate, then the linear demand function for riskless bonds continues into the negative region as well as inflation rates above Π^*; otherwise it does not.

While the linearity of asset demand functions does not pose a problem in terms of empirically plausible portfolio holdings, it is, of course, a restrictive assumption in the general sense. However, in general, any dynamic analysis requires sufficiently linear functional forms. For instance, the portfolio demand equations in Fischer (1979) are linear in rates of return as well as in the inflation rate.

Another theoretical approach worth some discussion here is our decision to study discrete shocks in a continuous-time model. Even though the choice may seem implausible at first glance, it has several merits. First, it is likely that individual agents always respond discretely to any changes in the environment. In addition, changes in the environment are basically discrete events - even rates of change adjust discretely at times. The basic issue is then whether the decisions of different agents are so spread out over some time-interval that the aggregate decision is smoothed to an approximately continuous adjustment. Another relevant question is whether agents anticipate a discrete or smooth aggregate shock. Nothing conclusive can be said here on these questions. Second, the analysis of discrete events seems to be considerably simpler than one might have expected. In fact, it seems that speculation on discrete events does not necessarily increase the nonlinearity from which smooth aggregate shocks would arise. Actually, it would be hard to find simpler price and output dynamics in an analogous model with smooth aggregate shocks. Third, discrete shocks with uncertain timing seem to generate quite plausible dynamics in relevant variables. This would become especially clear if one added other shocks to the model which partly affected within the same time-interval, or if one made the speculation interval flexible, i.e. allowed the latest possible time for the shock to start to vary. Augmented by these two features, our model could generate price and output paths with rare and moderate discrete jumps. Costs of adjustment in portfolio reallocations would further smooth out discrete jumps in the time paths.

3.9 Conclusion

The main results of the analysis were broadly discussed and evaluated in the two previous sections. Therefore, only few concluding remarks are made here.

The presence of rationally behaving investors in the asset markets seems to have a potentially destabilizing effect on the economy. When anticipated demand shocks hit the economy, such agents respond by adjusting their portfolios optimally. These responses force the government to adopt the interest rate targeting monetary policy regime. Otherwise, crowding in and crowding out periods may result, implying variations in the rate of unemployment in our framework.

The monetary policy reactions of the government may not be sufficient to guarantee full employment. If the anticipated demand shocks are very large either in the sense of having a large absolute magnitude or in the sense of having little uncertainty concerning their timing, real effects result under monetary policy. Only traditional Keynesian fiscal policy can neutralize such effects. Thus the economy has the corridor property: The economy can adjust to very large demand shocks in full employment only by means of Keynesian stabilization policy, while adjustment through monetary policy is possible in the case of moderately large anticipated shocks and through the real balance effect in the case on unanticipated shocks.

4 SPECULATION, FIXED EXCHANGE RATES AND THE SMALL OPEN ECONOMY

4.1 Introduction

It is sometimes suggested that countries operating under fixed exchange rate regimes experience periodic exchange rate changes which are generated by the responses of the agents in these countries to exogenous shocks. These endogenous exchange rate adjustments are thought to have the tendency to repeat themselves, i.e., to be cyclical in nature.

The specific phenomenon of endogenous exchange rate cycles has received little attention in the literature. Two categories of studies can be distinguished. In the Finnish experience, cyclically recurrent devaluations have been an empirical fact. This observation has given rise to a number of studies, including studies by Paunio (1969), Korkman (1978, 1980), and Kouri (1979), in which recurrent devaluations are an important building block in the general macroeconomic adjustment process. The lack of flexibility in the wage-price adjustment mechanism due to economywide wage contracting is, in a sense, replaced by the adjustment forms generated by the devaluation cycle. In its most simplified interpretation, the devaluation cycle can be related to variations in factor income distribution and to the underlying struggle for income shares in the economy between trade unions and, indirectly, the shareholders of firms. Thus, in the course of the cycle, but before devaluation, wage contracts are such that the real profitability of the firms decreases, causing losses to the shareholders. On the other hand, devaluation generates capital gains to portfolio holders who have positive net positions in foreign assets. After the devaluation, the profitability of the firms improves since their home currency revenues rise. Thus the cumulative relative income gains of employed labor in the course of the cycle up to the devaluation are partially, fully, or more than fully eliminated by the relative losses after the devaluation. Associated with these cyclical trends in the profitability of firms are the real adjustments. Thus, typically, unemployment increases in the course of the cycle before the devaluation and decreases after the devaluation. According to this view, the fundamental reason for the recurrent devaluations is the loss of international competitiveness in the open sector of the home country.[28] This, it is claimed, is due to the tendency for nominal wages to be set too high in wage contracts.

The idea of endogenous exchange rate jumps has also been studied in a more restricted sense. The works of Krugman (1979), Turnovsky (1980) and Obstfeld (1982) view such jumps as consequences of exchange rate crises caused by optimizing investors who speculate on the timing and magnitude of the exchange rate jump. If, for instance, an economy has suffered substantial reserve losses, speculating agents launch an attack on the remaining reserves in order to maximize capital gains perceived to emerge as a consequence of the devaluation. This attack ultimately forces the authorities to carry out the devaluation. In this approach, general macroeconomic aspects are largely ignored, both to avoid technical problems and because there is not motivation for such considerations since all prices in the economy except the exchange rate are perfectly flexible.

In the following, we analyze the macroeconomic adjustment process of a small open economy in which there is wage contracting. Our main interest is the nature and the role of exchange rate jumps in

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[28] A clear description of the stylized facts of the devaluation cycle can be found in Kouri (1979), pp. 137 - 142.

this process. Two different wage contract rules are studied. It will be shown that if the wage contract exerts any rigidity on the adjustments of firms, i.e. if nominal wages adjust more slowly than prices, then endogenous exchange rate jumps are an essential element of the general macroeconomic adjustment process. Speculators are assigned an important role in the unfolding of the endogenous cycle. In particular, it will be shown that at some point in the macroeconomic adjustment process, speculation can become so potent that, unless it is countered by strong exogenous shocks, it becomes self-fulfilling.

The analysis also shows that the economy has the same likelihood of experiencing both self-fulfilling revaluation and devaluation speculation only if there is fully flexible labor supply, even at the domestic full-employment level. This implies internationally mobile labor and an infinitely elastic labor supply at the home country wage rate. If there is any rigidity in labor supply at the full-employment level, then the economy is biased towards experiencing endogenous devaluation cycles more often than revaluation cycles. In this light, jumps in the exchange rate provide the economy with an ultimate adjustment channel when nominal wages, labor supply and the amount of reserves fail to adjust.

The contract wages are set either according to an ad hoc rule, in which the rate of increase in the real contract wage is always set equal to the expected rate of growth of productivity, or according to the feedback rule. In the latter, the rate of increase in the contract wage is set so as to maximize trade union preferences, which give weight to both nominal wage rate increases and unemployment. It turns out that if the union follows the ad hoc policy, then the economy is more prone to exchange rate cycles than if the union follows the feedback policy. This conclusion is independent of whether the economy is biased or neutral relative to the two types of exchange rate cycles. On the other hand, the former type of economy is more responsive to exchange rate policy than the latter; a devaluation leads to a steadier recovery in the ad hoc wage setting economy than in the feedback wage setting economy.

Technically, the analysis of this chapter is developed on the basis of the open economy model of sections 2.4 - 2.5. The important departure from that model in the technical sense is that the analysis here is carried out in the discrete-time format. The economic forces generating the results are the same as in the closed economy analysis of the previous chapter. The separation of input supply decisions and the contract wage rigidity make variations in income distribution influential in the economy. This amounts to making the Tobin effect operate in the allocation of capital. The portfolio choices of investors then give rise to the possibly systematic tendencies towards reserves and unemployment.

In the general sense, our work can be related to the main body of recent studies on open economy macroeconomics. Hence, our reserves determination model is a fixed exchange rate regime analogue to the exchange rate determination models under flexible exchange rates. Our model makes extensive use of portfolio theoretic arguments and is therefore an intellectual relative of the Branson-Kouri portfolio balance approach. But in addition to the monetary side, our model includes the real side of the economy as well. Thus, our work also parellels the branch of literature started by Dornbusch (1976). In comparison to Dornbusch, the real side of the economy is more complete and is based on a general equilibrium framework with optimizing consumers and firms. Interestingly, although not surprisingly, an analogue of the overshooting hypothesis can result in our model, but is not a necessary outcome in it.

We proceed as follows. First, we describe the basic research question in a discrete-time open economy variant of our general framework developed in sections 2.4 - 2.5. The discrete-time variant is a somewhat linearized version of the basic model, but still contains seriously nonlinear features. Consequently, only approximate qualitative properties of the model can be discussed. Secondly, using the qualitative properties of the nonlinear model, we develop a strictly linear framework, in which the research questions can be analytically answered.

4.2 Existence of the endogenous exchange rate cycle: the nonlinear model case

We follow a two-stage strategy in our analysis. In the first stage, we briefly study the exchange rate cycle in general terms in a discrete-time nonlinear model. In the second stage, we construct a linear model in which questions relating to the macroeconomic adjustment process are investigated in some detail. The reason for the chosen strategy is to be found in technical difficulties. The nonlinear model is an essentially direct conversion of the continuous time analysis developed above in sections 2.4 - 2.5. Hence, the model also has the strong choice-theoretic foundations of the basic framework. However, the model is inherently too nonlinear for a satisfactory study of the type of problems that we are interested in here. Therefore, we hope to be able first to derive enough information on the local behaviour of the nonlinear model, so that we can form a linear model with the same local properties. In doing so, we hope that some of the strengths of the general equilibrium choice-theoretic approach will be transmitted to the conclusions.

4.2.1 Description of the nonlinear model

The notion of the small open economy is the general set-up for our analysis. The agents of the home country are price takers in all the international markets. In particular, the world market prices of the imported and the exported goods are p^*_{mt} and p^*_{xt}, respectively. As we want to abstract from variations in the terms of trade and instead concentrate on variations in the world price level, we assume that goods are physically measured in units such that $p^*_{mt} = p^*_{xt} = p^*_t$. The world price level p^*_t is determined by the quantity theory of money and is given by

$$(138) \qquad p^*_t = (1+\dot{m}^*_t-\nu^*)^t p^*_0$$

where $\dot{m}^*_t$ = rate of growth of money supply abroad
ν^* = rate of growth of productivity abroad.

We assume that the world money supply rule follows the following stochastic process

$$(139) \qquad \dot{m}^*_t = \nu^* + \Pi^* + u_t$$

$u_t \sim$ i.i.d. $N(0,\sigma^2_u)$
Π^* = world inflation rate

Using (2), we can write the world price level equation as follows:

$$(138') \qquad p^*_t = \prod_{j=0}^{t} \lfloor 1+\Pi^*+u_j \rfloor p^*_0$$

If the exchange rate is assumed for the time being to be at its initial level s_0, the corresponding domestic price level becomes:

$$(140) \qquad p_t = \prod_{j=0}^{t} \lfloor 1+\Pi^*+u_j \rfloor s_0 p^*_0$$

$$\equiv \prod_{j=0}^{t} \lfloor 1+\Pi^*+u_j \rfloor p_0$$

Note that while the analogues of price paths (138') and (140) in continuous-time representations are perfectly plausible for all states of nature, price paths (138') and (140) imply that there always exist some states of nature with positive probabilities in which the price levels would fall to zero. However, we ignore the problem because it clearly is associated with the formulation, since the formulation applied in (138') and (140) is only valid for relatively small shocks u_j. In the general discrete-time representation, log-normally distributed additive shocks would have to be used to formulate uncertainty so as to avoid the inconsistency for any state of nature. However, the adopted formulation is commonly used in the background of macroeconomic models and serves well in setting up the linear model.

In addition, the stochasic process of the nominal world interest rate is fully exogenous to the home country agents. The rest of the world behaves in a non-strategic way with respect to the home country, so that home government policy actions do not generate retaliatory reactions in the rest of the world. The home country government tries to operate in a fixed exchange rate regime by means of direct intervention, i.e. by being prepared to make any offered currency trades at the official exchange rate.

As in the continuous-time model, we assume specifically that the home country and the rest of the world have a permanent currency exchange arrangement so that the reserves grow autonomously at the rate of growth of productivity ν. Similarly, the critical devaluation and revaluation limits $\underline{R}_t$ and $\bar{R}_t$ are assumed for simplicity to grow at rate ν. If the actual level of reserves falls below $\underline{R}_t$, then the home country will devalue the currency, while if it rises above $\bar{R}_t$, then the home currency will be revalued. In both cases, the country follows the k-percent policy described in section 2.4, which is not very restrictive, as will be seen in section 4.3.3.

The monetary and fiscal policies in the home country assumed in the following are simple. The world price level fixes the domestic price level completely in the case of the small open economy. The quantity theory equation holds in the home country. In order not to cause serious disturbances in the economy, the home country follows the monetarist money supply rule. The quantity of money then grows at the rate

$$\dot{m}_t = \nu^* + \Pi^* \tag{141}$$

or at the same rate as in the rest of the world. Note that in (141) there is an implicit assumption that the home country rate of growth of productivity ν equals ν^*. As in the closed economy analysis, we assume that the velocity of money adjusts so as to absorb random variations about the mean demand for nominal transactions balances.

The home country always hires fractions α_3 of the aggregate capital stock and the labor force in order to provide public services and goods. If there are no shocks, then the government borrows a fixed fraction $(\alpha_3-\xi_3)$ of the aggregate capital stock from abroad.[29] As, by assumption, the portfolio fraction ξ_3 remains constant at ξ_{30} independently of shocks (see note 29), the government's borrowing from abroad to finance this permanent gap remains constant except for changes in valuation. The government runs a balanced budget so that it finances all the public expenditure, like wages and the debt service of the public sector, by raising taxes.

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[29] Recall from the analysis of section 2.4 that α_3 is the preference weight of public expenditure in the preferences of consumers, whereas ξ_{30} is the portfolio fraction of government bonds in the portfolios of investors. For simplicity, we assume that $\xi_{10} = \alpha_1+\alpha_4$ so that the portfolios of investors exhibit 'correct' general equilibrium allocation of capital to the private sector in the initial state (i.e. imply full employment in that state). Also, we assume that substitution in the portfolios of consumers only occurs between shares and foreign bonds so that ξ_3 always remains constant. In the continuous-time model of section 2.4 this implies the assumptions that $\sigma^* = \rho_{B^*}\sigma_B$ if $\nu^* > \nu$, or $\sigma_B = \rho_{B^*}\sigma^*$ if $\nu > \nu^*$. Neither of the assumptions restricts the generality of the analysis to a nonnegligible extent.

The implications for the tax rule, disposable income and the capital account of the above assumptions will be studied later. Finally, we state that the riskless rate of return on government bonds issued only to domestic investors is r_G, while the interest rate on foreign borrowing is the world interest rate specified below.

Two different periodic sets of events must be distinguished in the input markets as well as in the whole economy. Production, consumption and investment decisions are made for each period. In addition, exogenous shocks can hit the economy in each period. By contrast, a wage contract is negotiated between the firms and a trade union only every n^{th} period. The outcome of the negotiations specifies the contract wage for n periods. This distinction would empirically correspond to the notion that, while wage contracts are negotiated for, e.g., one year at a time, a number of other economically significant decisions can be made much more frequently, e.g., monthly or quarterly.

The firms have a stochastic Cobb-Douglas technology defined by:

$$(142) \qquad y_t = \prod_{j=0}^{t} (1+\nu+\eta_j) L_t^{\beta} K_t^{1-\beta}$$

where ν = mean growth rate of productivity

η_t = productivity shock

$\eta_t \sim$ i.i.d. $N(0,\sigma_\eta^2)$

Technology (142) implies that the marginal productivities of both labor and capital are stochastic. Note that the remarks on price paths made above after equation (140) also apply where appropriate to the technology.

Similarly, as in the basic models of sections 2.3 - 2.4, the firms choose the levels of inputs by maximizing expected logarithmic utility, or the preferences of investors, over the contract period. The workers are represented in the labor market by a trade union. The trade union negotiates a wage contract with the firms for each contract period. For simplicity, the wage-setting rule is specified in the nonlinear model by the following statements:

(143a) $\dot{w} = \nu + \Pi^*$

(143b) $w_0 = \frac{\beta(1-d_L)L_0^{\beta}K_0^{1-\beta}}{L} = \frac{BL_0^{\beta}K_0^{1-\beta}}{L}$

where $\dot{w}$ = growth rate of the contract wage

w_0 = initial wage level

d_L = risk premium paid in wages as the price of avoiding variations in the marginal productivity of labor

$B = \beta(1-d_L)$ = income share of labor corrected for the risk premium

The rule states that the expected rate of increase in the real wage rate equals the mean rate of growth of productivity.

In the linear analysis, another, more realistic, wage-setting rule will be applied as well.

It is important to notice that the specific wage-setting rule does not respond to excess supply in the labor markets, if such situations emerge within the contract periods. For their factor services, the shareholders receive the mean nominal interest rate $r_B = \nu + \Pi^*$ plus the profit effects of price level shocks and of all the productivity variation in technology. For also bearing the productivity risks of the workers, the investors receive the risk premium d_L.[30]

[30]For derivation of the risk premium, see appendix 1.

Again we assume on the grounds of the same arguments as in section 2.3 that the firms keep the capital-labor ratio constant in the short-run despite variations in the price level. To utilize the short-run fixed-coefficient property of the technology implied by the above assumption, we define next the concept of norm production. The norm production in period t is the output in private sector and if capital and labor are only employed in the private sector and are used in quantities corresponding to the long-run optimal capital-labor ratio, given the wage-setting rule and the productivity growth. Formally, we can express the norm production $\tilde{y}_t$, conditional on the wage-setting rule specified in (143), as:

$$(144) \qquad \tilde{y}_t = \prod_{j=0}^{t} (1+\nu+\eta_j) H^{\beta} \tilde{K}^{1-\beta}$$

where $H = \ell H_0$ is the full employment level of labor supply
$\tilde{K}$ = total capital stock owned by the investors

Using the norm production concept and the fixed-coefficient property of the technology, we can now express the aggregate private sector production as:

$$(145) \qquad y_t = \xi_{1t} \tilde{y}_t$$

ξ_{1t} = portfolio fraction of shares for period t

Formulation (145) will be extensively used in the analysis. Note that if the wage-setting rule remains unchanged throughout time, then the representation (145) is valid globally.

Because of the preferences, the analysis of the demand side of the economy is simple if only we can develop a shock-dependent representation for aggregate real wealth. However, with Cobb-Douglas preferences and the assumption of an exogenous price level set by the world price level, it is more convenient to carry out the analysis in terms of the periodic accumulation of real wealth, or, real income. Under these assumptions, both consumption and imports are fixed fractions of real income.

In order to determine the real income we state first the expressions for the nominal prices of assets and the nominal wage rate:

(146a) $$Q_{Bt} = \prod_{j=0}^{t} \lfloor 1+r_B+\alpha_u u_j+\alpha_\eta \eta_j \rfloor Q_B$$

(146b) $$Q_t^* = \prod_{j=0}^{t} \lfloor 1+r_B^*+\eta_j^* \rfloor Q_0^*$$

(146c) $$Q_t = \lfloor 1+r_G \rfloor^t Q_0$$

(146d) $$w_t = \nu \lfloor 1+\nu+\Pi^* \rfloor^t Q_{HO}$$

where $r_B = \nu + \Pi^*$ = nominal rate of return on shares
$r_B^* + \eta_t^*$ = nominal world interest rate,
$\eta_t^* \sim N(0,\sigma_{\eta^*}^2)$ i.i.d.

In (146a), the coefficients α_u and α_η of the price and productivity shocks are the simplest linear approximations of the respective nonlinear factors in the continuous-time model of section 2.4.[31]

Corresponding real variables are obtained by dividing expressions (146) by the price level given in (148). Assuming that ρ, or the rate of time preference of consumers, equals ν, defining ξ_1, ξ_2, and ξ_3 as the portfolio fractions of shares, foreign bonds and government bonds, respectively, and applying directly the

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[31]For the derivation of the coefficients, see appendix 2, equations (A11) - (A20).

definition for disposable income in (82'), we can write the real disposable income as follows:

(147)
$$\begin{aligned}\tilde{I}_t &= \xi_{1t}\nu \prod_{j=0}^{t}\lfloor 1+\nu+\alpha_u u_j+\alpha_\eta \eta_j\rfloor \frac{Q_{B0}}{p_0} \\ &+ \xi_{2t}\nu \prod_{j=0}^{t}\lfloor 1+r^*-\Pi^*-u_j+\eta_j^*\rfloor \frac{s_0 Q_0^*}{p_0} \\ &+ \xi_{3t}\nu \prod_{j=0}^{t}\lfloor 1+r_G-\Pi^*-u_j\rfloor \frac{Q_0}{p_0} + (\xi_{1t}+1-\xi_{10}) \\ &\cdot \nu \prod_{j=0}^{t}\lfloor 1+\nu-u_j\rfloor \frac{Q_{H0}}{p_0} - \tau_{Rt} \\ &\equiv \xi_{1t}I_{Bt} + \xi_{2t}I_t^* + \xi_{3t}I_t + (\xi_{1t}+1-\xi_{10})I_{Ht} - \tau_{Rt}\end{aligned}$$

where $I_{Bt} \equiv \nu \prod_{j=0}^{t}\lfloor 1+\nu+\alpha_u u_j+\alpha_\eta \eta_j\rfloor \frac{Q_{B0}}{p_0}$

= real income on shares if all capital is invested in shares at the general equilibrium rate of return

$I_t^* \equiv \nu \prod_{j=0}^{t}\lfloor 1+r^*-\Pi^*-u_j+\eta_j^*\rfloor \frac{s_0 Q_0^*}{p_0}$

= real income on foreign bonds if all capital is invested in foreign bonds

$I_t \equiv \nu \prod_{j=0}^{t}\lfloor 1+r_G-u_j\rfloor \frac{Q_0}{p_0}$

= real income on government bonds if all capital is invested in government bonds

$I_{Ht} \equiv \nu \prod_{j=0}^{t}\lfloor 1+\nu-u_j\rfloor \frac{Q_{H0}}{p_0}$

= real wage income

Expression (147) specifies the real disposable income as a weighted average of different income components net of real taxes, where the weights are appropriate portfolio fractions. The expression can be simplified appreciably by investigating the net tax function τ_{Rt}. The government collects taxes to pay wages and interest payments abroad and to domestic investors, and runs a balanced budget. The government is assumed for simplicity to face the same world interest rate as the private investors. Therefore, the tax function must be of the following form:

$$(148) \qquad \tau_{Rt} = (\alpha_3 - \xi_{30})I_t^* + \xi_{30}I_t + (1-\xi_{10})I_{Ht}$$

By noting that $\alpha_3 - \xi_{30} = \xi_{20}$,[32] the real income (147) can be rewritten in the following relatively simple form:

$$(149) \qquad \tilde{I}_t = \xi_{1t}(I_{Bt}+I_{Ht}) + (\xi_{2t}-\xi_{20})I_t^*$$

Expression (149) indicates that the real disposable income in the home country includes the real value of domestic output and the interest revenue on the net foreign lending in excess of the initial holdings of foreign bonds.

With the Cobb-Douglas preferences, real consumption and imports are fixed fractions of the real income and are thus given by:

$$(150a) \qquad c_t = \alpha\tilde{I}_t$$

$$(150b) \qquad c_{mt} = (1-\alpha)\tilde{I}_t$$

where α and $1-\alpha$ are the Cobb-Douglas preference weights for consumption of the private good and the imported good, respectively.

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[32]This can be seen from the following sequence of identities: $\alpha_3 = 1-(\alpha_1+\alpha_4) = 1-\xi_{10} = \xi_{20}+\xi_{30}$.

Exports can be obtained simply by subtracting the consumption of the private good from the production of it. Formally, exports are then given by

$$(151) \qquad x_t = \xi_{1t}\tilde{y}_t - \alpha\tilde{I}_t$$

Besides the autonomous component, the actual level of real reserves at the end of each period is affect by the two traditional channels, namely, the current account and net capital flows. The real current account surplus for period t, denoted by S_t, is defined as the net exports during that period valued at the initial foreign price $p_0^* = 1$. Using (151) and (150b), we obtain directly that the real current account surplus is given by

$$(152) \qquad S_t = s_0\lfloor \xi_{1t}\tilde{y}_t - \tilde{I}_t \rfloor$$

The determination of the real net flow of capital is relatively simple as well. The stock of capital exports from the home country is given by the fraction of total capital held in foreign bonds. The stock of capital imports is given by the residual demand for capital by the government. As was mentioned above, this residual demand is determined by two factors. First, the government always employs a fraction α_3 of the home economy's total inputs in public production. On the other hand, the government only receives the fraction of capital corresponding to the portfolio fraction of government bonds directly from the home country investors. The rest of the capital the government borrows from abroad. The real stock of capital exports, D_t, and capital imports, F_t, at the end of period t are then given by the following expressions:

(153a) $$D_t = \frac{s_0}{p_t^*} \xi_{2t} Q_t^* B$$

(153b) $$F_t = \frac{s_0}{p_t^*} (\alpha_3 - \xi_{3t}) Q_t B$$

In (153a,b), the stocks are valued in the home currency and at the initial price level.

The real net inflow of capital during period t, or the capital account surplus, C_t, consists of the difference between the one period changes in stocks (153b) and (153a). The net interest income from abroad is included in the disposable income. Then we can write the capital account surplus in the following form:

(154) $$C_t = F_t - F_{t-1} - [D_t - D_{t-1}]$$

The 'endogenous' change in actual real reserves during the period t is given by the sum of the current account surplus and the capital account surplus. The autonomous change caused by the exchange arrangement gives the trend in the real reserves. Thus, using (152) and (154), we obtain the following expression for the changes in real reserves:

(155) $$R_t - R_{t-1} = R_t^P - R_{t-1}^P + p_0 \xi_{1t} \tilde{y}_t - \tilde{I}_t + F_t - F_{t-1} - [D_t - D_{t-1}]$$

where R_t^P = autonomous real reserves at the end of period t

At the initial contracting time, the portfolio fractions of investors are at their general equilibrium values, derived from utility maximization, which we denote by ξ_{10}, ξ_{20} and ξ_{30}. Within the contract period, the portfolio fractions are affected by productivity and world price shocks, and exchange rate jump expectations. In section 2.5, we showed that devaluation expectations are analogous to deflation of the world price level in that both raise the expected rate of return on foreign bonds. The difference between the two shocks is that devaluation expectations have no effects at all on the actual return or income as such; only if a devaluation actually occurs will the expected real income increases be realized as capital gains. This is due to the exogeneity of the world financial markets to the home country investors: the real returns on foreign assets do not respond at all to exchange rate speculation in the home country.

The portfolio decisions of investors are also affected by inertia. This is because we assume that there are costs associated with aggregate portfolio adjustments.[33] For convenience, we adopt here the technically simplest possible representation for the inertial portfolio adjustment. Let then f_2 be the partial derivative of the portfolio fraction of shares of an optimizing agent with respect to a unit change in the mean real rate of return on shares.[34] The

[33]Empirically, it is quite plausible to assume that portfolio adjustments are costly, because only relatively small fractions of the total private sector portfolios - mainly some very short-term financial investment - are fully liquid assets. We used the costless adjustment assumption in the theoretical analysis of the third chapter and it might be preferable here as well for this reason alone. However, in the succeeding analysis we actually need some friction in adjustments, because the idea of the speculative exchange rate cycle implies instability of the dynamics to be studied. Under rational expectations, such an unstable system would explode at the very moment when speculation grows so strong that the instability becomes understood by all investors. Thus, we need rigidity in adjustments to prevent momentary jumps in reserves. Even though the friction could possibly be constructed in an analogous way to section 3.5, the partial adjustment mechanism is the simplest technical formulation in linear discrete time analysis.

[34]Note from portfolio rule (97a) that

$$f_2 = \frac{1}{\sigma_B^2(1-\rho_{B*}^2)}$$

portfolio fraction of shares of an optimizing investor for period t, operating in frictionless conditions, would be

$$\xi_{1t}^{0} = \xi_{10} + f_2 z_t \tag{156}$$

where z_t = arbitrary anticipated change in the mean rate of return

But the investors are also affected by friction in portfolio adjustments. The simplest available representation of such effects is to assume that the portfolio fraction ξ_{1t} is determined by the partial adjustment mechanism as follows:

$$\tilde{\xi}_{1t} - \tilde{\xi}_{1t-1} = a(\tilde{\xi}_{1t}^{0} - \tilde{\xi}_{1t-1}) \tag{157}$$

$$a \in (0,1)$$
$$\tilde{\xi}_{1t} = \xi_{1t} - \xi_{10}$$
$$\tilde{\xi}_{1t-1} = \xi_{1t-1} - \xi_{10}$$
$$\tilde{\xi}_{1t}^{0} = \xi_{1t}^{0} - \xi_{10}$$

The reduced form solution for the stochastic difference equation (157) takes the form

$$\tilde{\xi}_{1t} = \tilde{\xi}_{10} + a\ f_2 \frac{1}{[1-(1-a)L]} z_t = \tilde{\xi}_{10} + a\ f_2 \sum_{j=0}^{\infty} (1-a)^j z_{t-j} \tag{158}$$

If the definition of the variable $\tilde{\xi}_{1t}$ is directly applied to equation (158), we obtain the solution for the portfolio fraction ξ_{1t} as:

$$\xi_{1t} = \xi_{10} + a\ f_2 \sum_{j=0}^{\infty} (1-a)^j z_{t-j} \tag{159}$$

With the simplifying assumption on the variance-covariance structure between return on shares and foreign bonds (see note 29), the substitution in porfolios only occurs between shares and foreign bonds. We can then write the portfolio fraction of foreign bonds directly as:

$$\xi_{2t} = \xi_{20} + a\ f_2 \sum_{j=0}^{\infty} (1-a)^j z_{t-j} \tag{160}$$

Thus, the investment fraction of government bonds remains constant at ξ_{30} independently of the shocks.

The building blocks of our nonlinear model have now been specified in terms of end-of-period magnitudes of the relevant variables.

4.2.2 Basic properties of the nonlinear model

The basic logic of the properties of the open economy model is almost exactly analogous to that of the closed economy case of the previous chapter. The wage contracts fix the nominal wage for several periods (in the present simplified case, for all periods). If any exogenous anticipated events hit the economy during the contract period, they change the mean real rates of return on different assets in a non-neutral way. Consequently, investors adjust their portfolio composition in an optimal manner. Given the specific construction of the model, capital moves between the domestic private sector and the rest of the world. Employment and output vary accordingly in the home country.

The open economy case differs from the closed economy setting in two respects. First, all transactions between the home country and the rest of the world involve changes in the reserves of the home country. As the home country tries to maintain the fixed exchange rate, and yet has limited autonomous reserves, strong speculation may strengthen the tendencies towards real fluctuations in the home economy. Secondly, financial connections between the home country and the rest of the world, i.e. borrowing and lending, can involve genuine capital gains or losses to the home country.

All the responses of the variables of the model are due to either portfolio substitution effects of capital gains effects. Portfolio effects alone determined variations in the real variables and capital flows. Because of the latter, the capital account effects on reserves are relatively easy to detect. On the other hand, income formation is affected simultaneously by both types of effects. Therefore, the current account effects on reserves are much less clearcut and more diffucult to model than the capital account effects.

An increase in either the rate of growth of productivity or the world inflation rate which was unanticipated at the time of wage contracting makes investors reallocate their portfolios away from foreign bonds towards the shares of the domestic firms. The reason for this is that the firms' profits fall since the nominal labor costs are fixed by the contract and hence independent of the shocks. A productivity shock of a given size (say a one percentage point increase in the rate of growth of productivity) causes a larger portfolio effect than a world price shock of equal magnitude. The reason is that the price shock changes the real rate of return on shares only because a part of factor payments, i.e. wages, is paid at a fixed rate. Thus, price variations only cause income redistribution. The productivity shock, by contrast, increases the real rate of return on shares by increasing output and therefore total revenues. Hence, positive productivity shocks both generate more disposable income in the aggregate and re-distribute income in favor of investors.

To understand the shock-dependence of real income, which is necessary for understanding the current account effects of the two types of shocks, let us write the domestic real income presented in (149) in the following form:

$$(161) \qquad \tilde{I}_t = \xi_{1t}(I_{Bt}+I_{Ht}-I^*_t) + \xi_{10}I^*_t$$

In obtaining (161), we have used (159) and (160) and the definition $\xi_1+\xi_2+\xi_3 = 1$.

Let us first investigate the world price shocks. Denoting the first order difference operator by Δ, we can decompose the change in real disposable income from the previous period in the following way:

$$(162) \qquad \Delta\tilde{I} = \Delta\xi_{1t}(I_{Bt-1}+I_{Ht-1}-I^*_{t-1}) + \xi_{1t-1}(\Delta I_{Bt}+\Delta I_{Ht}-\Delta I_t) + \xi_{10}\Delta I^*_t$$

In the decomposition, the first term is the direct portfolio substitution effect. Income changes because capital moves from the home country abroad or vice versa and labor is consequently laid off or hired. The following two terms represent the direct income effects; income changes because different types of income are making capital gains and losses on account of the shocks. The second order effects are ignored.

The decomposition can be simplified by recalling that, since price variations only cause variations in income distribution, the capital gains possibly made by the shareholders are exactly equal to the capital losses made by the workers, i.e. $\Delta I_{Bt} = -\Delta I_{Ht}$. Applying this fact, the decomposition can be written as follows:

$$(163) \qquad \Delta\tilde{I}_t = \Delta\xi_{1t}(I_{Bt-1}+I_{Ht-1}-I^*_{t-1}) + (\xi_{10}-\xi_{1t-1})\Delta I^*_t$$

The sign of (163) is ambiguous. If there is acceleration in world inflation, investors adjust their portfolios towards shares and hence $\Delta\xi_{1t}$ is positive. A positive $\Delta\xi_{1t}$ implies increases in real national income because more capital and labor is employed in domestic production. On the other hand, it implies decreases in real income because less capital is earning revenue abroad. We assume that the overall effect of a positive $\Delta\xi_{1t}$ is positive, i.e. real national income is assumed to increase when unemployment decreases in the home country. The second term is unambiguously negative. Because of the fixed coefficient short-term technology, zero is the minimum of $\xi_{10}-\xi_{1t-1}$ if no labor mobility is allowed. The real returns on foreign bonds decrease as world inflation accelerates. Hence, ΔI_t^* is negative, making the second term negative.[35] This negative effect may or may not make the aggregate change in income negative.

The ambiguity of the sign of the aggregate change in real national income is, fortunately, not a very serious difficulty as regards the total current account effect on reserves. This can be seen by noting that the change in production due to the price shock is:

$$(164) \qquad \Delta y_t = \Delta\xi_{1t}\tilde{y}_{t-1} = \Delta\xi_{1t}(I_{Bt-1}+I_{Ht-1})$$

In the case of an acceleration in inflation, the change in output is positive, as more inputs are employed in firms. Furthermore, the change in output is larger than the corresponding change in real income. This becomes formally obvious if one compares (163) and (164) and recalls the definition of income in (149). Intuitively, it is due to the fact that output increases by the

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[35]The interpretation of the second term is simple. The fraction $\xi_{10}-\xi_{1t-1}$ indicates, if positive, that the economy is out of the full employment equilibrium. Then, the revenues derived from the fraction $\xi_{2t-1}-\xi_{20}$ invested in foreign bonds are not taxed by the government to cover the interest payments on the public debt raised abroad, but are subject to capital losses.

full impact of the shift of inputs into firms, while income only changes to the extent that new alternative uses of inputs generate more income than the old ones. By the above conclusions, exports must always increase more than imports when a world price shock causes increases in private production. Hence, the current account effect of such a shock on reserves must be unambiguously positive. Because of the positive portfolio effect, the capital account effect of the given shock on reserves is positive as well.

Next, let us briefly study the income effects of productivity shocks. In the case of such shocks, foreign investments or labor income are not subject to capital gains or losses at all. Hence, the change in real national income becomes:

$$(165) \qquad \Delta\tilde{I}_t = \Delta\xi_{1t}(I_{Bt-1}+I_{Ht-1}-I^*_{t-1}) + \xi_{1t-1}\Delta I_{Bt}$$

In (165), using the convention adopted above in the context of (163), both terms are of the same sign. Shifts of capital into the domestic private sector can only happen, if the productivity shock is positive, or, of ΔI_{Bt} is positive.

Output changes in the case of a productivity shock in the following way:

$$(166) \qquad \Delta y_t = \Delta\xi_{1t}\tilde{y}_{t-1} + \xi_{1t-1}\Delta\tilde{y}_t$$

The crucial point to observe here is that, since shareholders receive all the additional revenues caused by the productivity shock, then $\Delta I_{Bt} = p_a\Delta y_t$ must hold. Hence, in the current account, the capital gains effect on the demand side and the productivity effect on the supply side neutralize each other. The total current account effect on reserves of a productivity shock is then a pure portfolio effect, which in our case is positive by the same argument, as in the case of the price shock. The capital account effect is again positive.

In conclusion, we can say that the nonlinear model is in fact too nonlinear for analytical purposes. To see this clearly, let us convert the nonautonomous change in reserves in equation (155) into the difference operator notation as follows:

$$(167) \qquad \Delta R_t^n = \Delta(\xi_{1t}\tilde{y}_t) - \Delta\tilde{I}_t + \Delta F_t - \Delta D_t$$

Applying equations (163 - 166) and by recalling that $\Delta F_t = 0$ and $\Delta D_t = -\Delta\xi_{1t}D_{t-1}$ when changes in valuation are neglected, equation (167) can be written in the following form:

$$(168) \qquad \Delta R_t^n = \Delta\xi_{1t}[p_0\tilde{y}_{t-1}-(I_{Bt-1}+I_{Ht-1}-I_{t-1}^*)+D_{t-1}] + (\xi_{10}-\xi_{1t-1})\Delta I_t^*$$

In equation (168), the first term in the square brackets represents the production effect and the second term the income effect on the current account. The third term represents the capital flow effect. These three terms are due to the fact that capital moves between domestic private sector production and the rest of the world, and, consequently, the unemployment rate varies in the home country. The last term, on the other hand, is the capital gains effect on the current account and is the 'disequilibrium' effect on reserves. The capital gains effect is always positive if the country can only be a net investor in the world market, i.e. if the home country is limited to the full employment level of domestic activity by the immobility of labor and by the fixed-coefficient production technology. If the country were a net borrower in the world market, then this effect would be positive.[36]

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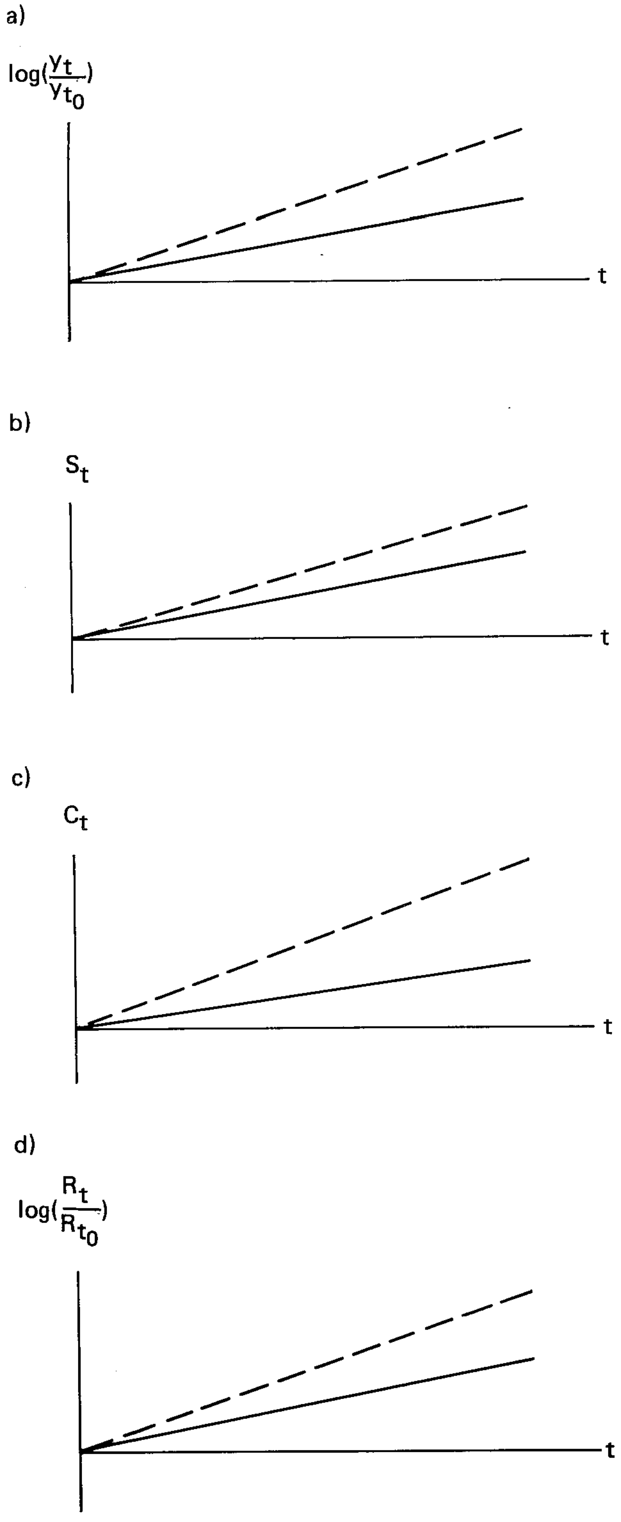
[36]Note that fluctuations in the nominal world interest rate would generate other capital gains effects, which would follow the opposite sign pattern relative to the gains arising from the capital movements. We will abstract away from such capital gains effects.

If equation (168) were inserted into equation (155) in an appropriate form, it would be seen that the change in reserves could potentially include dynamics from several additively associated sources. This indicates that the underlying dynamics, even with the simplest available assumptions, would be of multiple order. Moreover, incorporating speculation explicitly into the analysis would further complicate the dynamics. It is therefore necessary for further analysis to develop a simpler, preferably fully linear, model which includes the local properties of the nonlinear model as accurately as possible. In section 4.3, we will proceed along these lines. It will then be seen that ignoring the capital gains effects facilitates the linearization to the extent that even a simple notion of speculation can be included in the analysis.

A most illustrative way of presenting the properties of the nonlinear model is by means of a set of figures. In Figure 15, two series of shocks - a sequence of increases in world inflation and in productivity growth from their mean rates - hit the economy. Despite the serious nonlinearity of the reserves equation, it is assumed that the time series of the logarithmic deviations from the trend paths of production and reserves and the current account and capital account surpluses can be easily constructed. For the purposes of illustration, it is assumed that the time paths of the deviations are linear. In the four panels of Figure 15, the solid lines represent the responses of the model to the set of world price shocks and the broken lines the corresponding responses to the productivity shocks.

Fig. 15.

LOCAL PROPERTIES OF THE NONLINEAR MODEL



4.2.3 Existence of speculative exchange rate cycles

The foregoing discussion on the basic properties of the nonlinear model made it clear that feasible sets of unanticipated productivity or world price level shocks can, at least in principle, lead to such responses that implied reserve losses or gains cause endogenous devaluations or revaluations. The reason for this is that contract wage rigidity reduces the ability of the economy to automatically absorb shocks in full employment equilibrium. Here we examine what role speculation plays in endogenous exchange rate changes in the nonlinear model and under what circumstances speculation can be the immediate cause of exchange rate jumps. By immediate cause we mean that, at least during the subperiod ending in an exchange rate jump, there are no exogenous shocks at work in the economy.

The intuitive characterization of the issues associated with the question is fairly simple. Uncertainties in productivity and the world price level make the level of real reserves at the end of each period a random variable. In our case, the distribution of reserves is likely to be some time-dependent variant of the log-normal distribution, but here we allow for a general distribution, the density and distribution functions of which are denoted by $f_t(R_t)$ and $F_t(R_t)$, respectively. Note that in the general case the density f_t and the distribution F_t are time-dependent, i.e. the moments characterizing them vary endogenously over time. The only restriction for the distribution and the devaluation and revaluation limits $\underline{R}_t$ and $\bar{R}_t$ is that, if the economy is on its trend path, then the probabilities of devaluation and revaluation are equally large. Formally, this can be specified in the following way:

$$(169) \qquad F_t(\underline{R}_t) = 1 - F_t(\bar{R}_t)$$

If a shock now enters the economy, it shifts the actual level of reserves so that the probability λ of a devaluation or a revaluation occurring during the next period becomes positive. This magnifies the pure shock-induced reserve effect, as speculating investors adjust their portfolios. If the density function of reserves has the conventional single-peaked form and the peak is between the limits $\underline{R}$ and $\bar{R}$, then the sequence of changes in the probabilities λ is increasing in the deviations from the trend path.[37] Hence, it becomes possible that, after a sufficient amount of cumulative shocks, the implied increase in the probabilities λ becomes so large that no additional exogenous shocks are needed, and the reserves still continue to deviate more and more from the trend path. Thus expectations of an exchange rate jump may become self-fulfilling: changes in reserves caused by speculation continue until the exchange rate jumps.

To present the argument more formally, a critical observation has to be made: during a devaluation cycle, capital flows out while the opposite is true during a revaluation cycle. This observation is particularly useful because the devaluation and revaluation cycles can be given simple definitions in terms of the portfolio fraction ξ_{1t}. The definitions are:

(170a) if $E_{t-1}(\xi_{1t}-\xi_{1t-1}) < 0$, then a devaluation cycle is in operation

(170b) if $E_{t-1}(\xi_{1t}-\xi_{1t-1}) > 0$, then a revaluation cycle is in operation

Variations in the portfolio fraction ξ_{1t} are in our formulation a function of variations in the expected yield differential between

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[37] This claim ignores the fact that the time-dependence of higher - than -first moments of the distribution F_t would possibly imply stricter conditions for guaranteeing the increasing sequence of the probabilities λ.

shares and foreign bonds. This differential depends, on one hand, on exogenous world price level and productivity shocks. On the other hand, we showed in section 2.5 that this differential also depends on exchange rate jump expectations. Specifically, we saw that if a k-percent devaluation is expected to occur during period t with a probability λ_t, then the yield differential between returns on shares and foreign bonds diminishes by a factor λ_{tk}. Denoting the exogenous shocks by z_t, the solution for the portfolio fraction ξ_{1t} can be written as:

$$(171) \quad \begin{aligned} \xi_{1t} &= \xi_{10} - a\ f_2[z_t+(1-a)z_{t-1}+(1-a)^2 z_{t-2}+\ldots+(1-a)^n z_{t-n} \\ &\quad +\lambda_t k+(1-a)\lambda_{t-1}k+(1-a)^2\lambda_{t-2}k+\ldots+(1-a)^{n-1}\lambda_{t-n+1}k] \\ &= \xi_{10} - a\ f_2[z_t+Z_{t-1}+\lambda_t k+\Lambda_{t-1}k] \end{aligned}$$

where

$$Z_{t-1} = (1-a)z_{t-1} + (1-a)^2 z_{t-2} + \ldots + (1-a)z_{t-n}$$

$$\Lambda_{t-1} = (1-a)\lambda_{t-1} + (1-a)^2\lambda_{t-2} + \ldots + (1-a)^{n-1}\lambda_{t-n+1}$$

Forming the corresponding solution for ξ_{1t-1}, we can express the change in the fraction ξ_{1t} in the following form:

$$(172) \quad \xi_{1t} - \xi_{1t-1} = -a\ f_2[z_t - \frac{a}{1-a} Z_{t-1} + \lambda_t k - \frac{a}{1-a} \Lambda_{t-1} k]$$

On the basis of (172) and using definition (170a,b), the two types of cycles can be distinguished formally by conditions

$$(173a) \quad \text{If } z_t + \lambda_t k > \frac{a}{1-a} \tilde{Z}_{t-1}; \qquad \tilde{Z}_{t-1} > 0,$$

then a devaluation cycle is in operation

$$(173b) \quad \text{If } z_t + \lambda_t k > \frac{a}{1-a} \tilde{Z}_{t-1}; \qquad \tilde{Z}_{t-1} < 0,$$

then a revaluation cycle is in operation

where $\tilde{Z}_{t-1} = Z_{t-1} + \Lambda_{t-1}k$, or the cumulative present effect of past shocks.

The role of speculation in the course of the cycle is formally seen if we set $z_t = 0$ in conditions (173a,b). Considering only the devaluation cycle case, the condition becomes:

$$(174) \qquad \lambda_t > \frac{a}{(1-a)k} \tilde{Z}_{t-1}$$

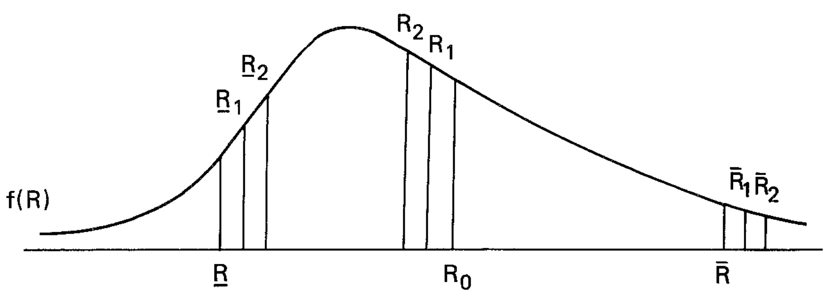
The condition says that, if the endogenously generated probability of devaluing is larger than the cumulative effect of the past shocks multiplied by $\frac{1}{k}$, then the devaluation cycle continues without the contribution of present exogenous shocks. Furthermore, if the probability λ_t satisfies condition (174), then the reserves decrease further during the t^{th} period. This makes the probability λ_{t+1} larger than the probability λ_t. Hence, no $t+1^{st}$ period exogenous shocks are needed for the cycle to proceed, and so on. Speculation based on devaluation expectations makes the reserves fall, so that ultimately the expectations are fulfilled and the devaluation takes place.

Note the important fact that the argument (170 - 174) is only relevant for illustration. In the whole argument, the probabilities λ_t are treated parametrically, on the assumption that they are exogenously generated and in linear relationship with the portfolio fraction ξ_{1t}. In general this cannot be the case, and the essence of the idea of speculative exchange rate cycles is that the probabilities λ_t are endogenously generated by the model and hence in a nonlinear relationship with the portfolio fraction ξ_{1t}. To gain some insight into the difficulties associated with endogenizing the probabilities λ_t in condition (174), let us consider a special assumption on the distribution F_t. With this assumption, the generation of probabilities λ_t can be approximated in terms of actual and expected reserves.

Specifically, let us assume that the density f_t and the distribution F_t are time-independent, so that the higher-than-first moments of F_t are constant. In such a case, the effects of shocks on reserves become relatively straightforward. Such time-independent functions are denoted by f and F in the formalizations below. In Figure 16, the density function of the reserves and the reaction limits $\underline{R}$ and $\bar{R}$ are drawn in the initial equilibrium and after some reserve losses.

Fig. 16.

APPROXIMATION OF PROBABILITIES λ_t



If the economy loses reserves, then the location of the density shift to the left so that the mean of the distribution moves to the left to position R_1 at the end of period one. This move can also be represented by moving the limits to the right by the amount R_0-R_1 to positions $\underline{R}_1$ and $\bar{R}_1$. The probability mass to the left of $\underline{R}_1$ is larger than the corresponding mass to the right of $\bar{R}_1$. If another fall in reserves occurs in the second period, the limits move to positions $\underline{R}_2$ and $\bar{R}_2$. In general, for period t-1 the probability of devaluation is ex-post, i.e. knowing the actual reserves R_{t-1} given by

$$(175) \qquad \lambda_{t-1} = F(\underline{R}+R_0-R_{t-1}) - [1-F(\bar{R}+R_0-R_{t-1})]$$
$$= F(\underline{R}_{t-1}) - [1-F(\bar{R}_{t-1})]$$

The first order approximation of the probability λ_t can now be written as

$$(176) \qquad \lambda_t = \lambda_{t-1} + f(\underline{R}_{t-1})E_{t-1}\Delta R_t + f(\bar{R}_{t-1})E_{t-1}\Delta R_t$$
$$= \lambda_{t-1} + (\underline{\omega}_t+\bar{\omega}_t)E_{t-1}\Delta R_t$$

where $\underline{\omega}_t = f(\underline{R}_{t-1})$

$\bar{\omega}_t = f(\bar{R}_{t-1})$

If the right hand side of equation (176) is substituted into (174) for λ_t, we obtain

$$(177) \qquad \lambda_{t-1} + (\omega_t+\bar{\omega}_t)E_{t-1}\Delta R_t > \frac{a}{(1-a)k}\tilde{Z}_{t-1}$$

Equation (177) defines implicitly the cumulative reserve losses required by the end of period t-1 in order for the endogenous changes in the probabilities λ to become large enough for a speculative devaluation cycle to get under way from period t on. Both the weighting functions $\underline{\omega}_t$, $\bar{\omega}_t$ and the reserve losses ΔR_t are nonlinear functions in our model, which makes further analysis of the equation without linearizations impossible. It should be understood that condition (177) is in fact a condition for the dynamics of reserves. Roughly speaking, condition (177) states that there is a critical level of reserves, smaller than the trend reserves, such that, if actual reserves fall below this level, then the endogenous dynamics in reserves becomes unstable, characterizing the presence of self-fulfilling devaluation speculation. This is the line of argument that we will follow when we establish conditions for the existence of self-fulfilling speculation in the linear model.

4.3 Cyclical fluctuations in a small open economy: the linear model case

4.3.1 The basic linear model

In the following three subsections, the linear open economy model will be presented, its properties characterized and its macroeconomic implications pointed out. As far as possible, we try to keep the exposition on the intuitive level. With this purpose in mind, we will discuss the key assumptions of the model and only state the results of the formal analysis. The derivations of the results are carried out in appendix 2, the relevant parts of which will be referred to in the following sections.

We are interested in both the real and monetary adjustments in the economy when exogenous shocks introduce disturbances into the economy. In the analysis, we will characterize real fluctuations in terms of the dynamics of the unemployment rate. Monetary responses in the open economy are summarized by the time path of real reserves. It will turn out later that this choice of 'indicator' variables is very convenient from the analytical point of view. For the unemployment rate, we can find an exact linear representation, while the dynamics in reserves are so complicated that for linearization we have to simplify the determination of reserves in the nonlinear model by invoking some ad hoc assumptions.

An important and useful observation is that in our model all real variables, including real reserves, grow at the mean rate of growth of productivity ν if the economy is on the trend path. This fact makes it possible in principle for us to aim at deriving a linear representation in the logarithmic or percentage deviations of reserves from the trend path. Hence, we 'save' one order of dynamics in the linearization and also need fewer ad hoc assumptions in the derivation. Moreover, logarithmic deviations from the trend path is precisely the concept of reserves that has the simplest connection with the unemployment rate.

In section 4.2.3, we mentioned that all first order effects of shocks on reserves are either portfolio substitution effects or capital gains effects. Among the two types of effects, the portfolio effects are not very nonlinear, as the dynamics of the portfolio fractions was already linearized in equations (159 - 160). On the other hand, the capital gains effects are multiplicative in shocks, as can be seen from the last term of equation (168) above. There is no convenient approximation which could be used to linearize such effects. Thus, the current account effects of shocks on reserves are inherently nonlinear, given the capital gains effects.

To avoid further difficulties in attempts to linearize the capital gains effects, we choose to ignore them entirely. This assumption is a good approximation if the economy is not very far from the initial general equilibrium $\xi_{1t} = \xi_{10}$. The capital gains effects arising from variations in the world price level can potentially be relatively large if the economy deviates greatly from the initial general equilibrium. Recall from the remarks below (168) that the income effects associated with the capital gains effects are negative if international labor mobility is not allowed, but can also be positive if it is allowed. Hence, the decision to ignore such effects implies that the approximation of the current account effects of shocks on reserves will be an underestimate in the former case and can be either an under- or an overestimate in the latter case (see also note 36).

Given the assumption that the capital gains effects are negligible, reserves can deviate from the trend path only because of portfolio substitution effects. Such effects arise by our assumptions from changes in output, changes in disposable income and changes in gross capital exports due to portfolio adjustments. Formally, we can state on the basis of equation (168) that the deviation of reserves from the trend path at the end of period t is:

$$(178) \qquad R_t - R_t^P = (\xi_{1t}-\xi_{10})E_0\left[p_0\tilde{y}_t-(I_{Bt}+I_{Ht}-I_t^*)+D_t\right]$$

In equation (178), the contributions of terms p_0y_t and D_t are clear. The term $I_{Bt} + I_{Ht} - I_t^*$ represents the income effects on the current account, and its origin can be most easily seen from equation (163). As was mentioned above after (163), this type of term represents the income changes in the current account, because some income earned on foreign lending will be lost if capital is shifted into domestic firms. The capital that is shifted then generates new income for both labor and capital in the domestic production process. The net effect is the difference between the two effects with opposite signs. Note that when the capital gains effects are ignored then the three terms in the square brackets are actually evaluated at the trend values.

Equation (178) can be developed into a more convenient form by defining the following fractions:

(179a) $\gamma_I \equiv \dfrac{E_0 I_t^*}{p_0 E_0 \tilde{y}_t}$

= unit opportunity national income earned by domestic capital if invested abroad[38]

(179b) $\gamma_C \equiv \dfrac{\xi_{20} \dfrac{s_0}{E_0 p_t^*} E_0 Q_t^* \tilde{K}}{\xi_{10} p_0 E_0 \tilde{y}_t}$

= ratio of the home currency value of the mean gross lending abroad to the mean value of output

where $\gamma_I \in (0,1)$ by assumption; $\gamma_C > 0$

Using (179a-b) and recalling that $p_0 E_0 \tilde{y}_t = E_0(I_{Bt}+I_{Ht})$, we can write (178) in the following form:

(180) $R_t - R_t^p = (\xi_{1t}-\xi_{10})[\gamma_I+\gamma_C]\xi_{10} p_0 E_0 \tilde{y}_t$

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[38]Intuitively, γ_I has an appealing interpretation. If one unit of capital is invested in domestic private firms, it generates one unit of mean output, which has the real trend value p_0. On the other hand, if one unit of capital is invested abroad it generates an amount equivalent to

$$\frac{\nu p_0}{E_0 p_t^*} E_0 Q_t^*$$

of national income in the home country. This is approximately equal to $\gamma_I p_0 < p_0$ if measured in terms of the domestic opportunity income. The national income earned abroad can be considered smaller than the corresponding income earned in the home country, because the latter generates both labor income and capital income, while the former only generates capital income. Implicitly, we are assuming that if labor moves abroad with capital, then only the capital income is repatriated, while the labor income is spent abroad.

One more definition is needed before we are ready to state the expression for the logaritmic deviation of reserves from the trend path. Let then γ_R be defined as follows:

(181) $$\gamma_R \equiv \frac{\xi_{10} p_0 E_0 \tilde{y}_t}{R_t^P}$$

= ratio of the mean value of output to the autonomous reserves

Using (180) and (181), the percentage or logarithmic deviation of reserves from the trend, denoted by r_t, is given by:

(182) $$r_t \equiv \frac{R_t - R_t^P}{R_t^P} = (\xi_{1t} - \xi_{10})[\gamma_I + \gamma_C]\gamma_R$$

Note that in (182) γ_I represents the current account effects due to the fact that domestic resources move between two alternative uses that generate different amounts of national income per unit of resources. The capital flows effects are captured by the term γ_C. Note that, in general, both γ_I and γ_C can be time-dependent, but for convenience we will treat them as constants in the following. Finally, parameter γ_R, a constant, scales the variations that are actually measured about the trend value of output into percentage deviations about the autonomous reserves.

A representation for the unemployment rate is easily obtained. The short-term fixed-coefficient technology implies directly that, if the portfolio fraction ξ_{1t} is below the initial general equilibrium level ξ_{10}, then a fraction $\xi_{10} - \xi_{1t}$ of the domestic labor force is unemployed. The definition for the unemployment rate, denoted by un_t, is then:[39]

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[39]Note that if $\xi_{10} < \xi_{1t}$, then (183) specifies the labor imports as a fraction of the total domestic labor force.

(183) $$un_t = \xi_{10} - \xi_{1t}$$

Comparison of (182) and (183) indicates that there is a linear relationship between the logarithmic deviation of reserves from the trend path and the unemployment rate. Specifically, the relationship is:

(184) $$un_t = - \frac{1}{\gamma_R(\gamma_I + \gamma_C)} r_t$$

Thus, once we have derived a dynamic representation for one of the variables, the corresponding representation for the other is implied by (184).

The logic in the dynamics of reserves and the unemployment rate is clear from (182) and (183). The driving force of the time paths of the two variables lies in the choices of investors, which are fully characterized by the time path for the portfolio fraction ξ_{1t}. The real rates of return for the different investment opportunities vary relative to each other, as different shocks distort the economy. The ultimate reason for the distortions is to be found in the rigidity of the contract wage and the exchange rate.

We will consider four types of shocks which affect the decisions of investors. The two purely exogenous shocks are random variations in the world price level and in domestic productivity. Endogenous shocks are generated by wage-setting, as to alternative trade union contract policies are considered. One of the two policies implies contract wage-setting in which the mean rate of growth of the contract wage may systematically differ from the mean rate of growth of productivity. The fourth type of shock is the effects of exchange rate speculation on the decisions of investors.

The general dynamic representation for the portfolio fraction ξ_{1t} with the partial adjustment mechanism included was presented above in (159). In this representation, the arbitrary rate-of-return shocks z_t were left unspecified. To obtain a linear formulation for the shocks z_t in terms of the four types of shocks, we apply directly portfolio rule (97a) in which exchange rate speculation is explicitly formulated. Specifically, we obtain the following representation for the 'frictionless' portfolio fraction ξ^0_{1t}:

$$(185) \qquad \xi^0_{1t} = f_2 \lfloor r_B(u_t, \eta_t, \dot{w}_t - \dot{w}^*) - r^*_B + \lambda_t k \rfloor$$

where $f_2 = \dfrac{1}{\sigma^2_B(1-\rho^2_{B^*})}$

$\dot{w}$ = rate of increase in the contract wage in period t

$\dot{w}^* = \nu + \Pi^*$

λ_t = probability for a revaluation to occur within period t

From the continuous time analysis of the second and third chapters, we know that the mean real rate of return on shares is nonlinear in variations in the inflation rate. The same holds for the other two types of shocks as well. Hence, we are obliged to develop simple forms which specify linear relationships between the shocks and the real rate of return on shares. Such forms are derived in the appendix (see expressions (A11 - A20) in appendix 2), and here we only state the final result in terms of the equation of motion for the portfolio fraction ξ^0_{1t}:

$$(186) \qquad \xi^0_{1t} - \xi_{10} \equiv \tilde{\xi}^0_{1t} = f_2 \lfloor \frac{B}{(1-B)} (u_t - (\dot{w}_t - \dot{w}^*)) + \frac{1}{(1-B)} \eta_t + \lambda_t k \rfloor$$

In (186), the expression in the square brackets corresponds to the arbitrary shocks z_t in (159). Therefore, the specific solution for the time path of the portfolio fraction ξ_{1t}, in which the friction in adjustments is included, becomes:

$$(187)\qquad \xi_{1t} = \xi_{10} + af_2 \frac{1}{[1-(1-a)L]} \left[\frac{B}{(1-B)}(u_t-(\dot{w}_t-\dot{w}^*)) + \frac{1}{(1-B)}\eta_t + \lambda_t k\right]$$

The linear coefficients of the inflation, contract wage and productivity shocks are intuitively quite appealing when one recalls that B and (1-B) are the risk-premium corrected income shares of labor and capital. The coefficient of the productivity shocks is larger than that of the other two shocks because the former shocks include variations in total income in addition to implying variations in income distribution. The latter shocks involve only income redistribution.

Solution (187) implies by (182) the following representation for reserves:

$$(188)\qquad r_t = \frac{\Gamma}{[1-(1-a)L]}\left[\frac{B}{(1-B)}(u_t-(\dot{w}_t-\dot{w}^*) + \frac{1}{(1-B)}\eta_t + \lambda_t k\right]$$

where $\Gamma = \gamma_R(\gamma_I+\gamma_C)af_2$

In (188), the contract wage effects and effects of speculation need to be formalized before further analysis is possible. The latter effects will be specified here, while discussion on the former will be postponed till the following section.

The behaviour of investors underlying (188) is based on the fact that u_t and η_t are anticipated shocks at the beginning of period t. But, in that case, (188) is a deterministic relation and there can be no role in it for speculation. To make (188) genuinely stochastic, we have to distinguish between unanticipated and anticipated shocks in the aggregate. In both cases, individual agents have anticipations concerning u_t and η_t. In the aggregate unanticipated case, individual agents do not know the anticipations of other agents and their best estimates are that the aggregate anticipations of price and productivity shocks are draws from probability distributions u_t and η_t, respectively. In the aggregate anticipated case, agents either have uniform anticipations about the shocks, which all agents know, or then all agents know what the aggregate weighted anticipation is if individual anticipations differ.

According to assumptions (139) and (143), the exogenous shocks are normally distributed independent random variables, i.e. $u_t \sim N(0,\sigma_u^2)$ and $\eta_t \sim N(0,\sigma_\eta^2)$. If the shocks are unanticipated in the aggregate, then by (188) and by the standard theorem on the linear combinations of independent, normally distributed random variables, the distribution of r_t is:

$$(189) \qquad r_t \sim N(\mu_t, H[(\frac{B}{1-B})^2 \sigma_u^2 + (\frac{1}{1-B})^2 \sigma_\eta^2]) \equiv N(\mu_t, \sigma_r^2)$$

where μ_t = time-dependent mean of r_t
H = constant implied by the solution for r_t

The exchange rate speculation of investors is based on the distribution (189) and on how the devaluation and revaluation limits $\underline{r}_0$ and $\bar{r}_0$, defined in the appendix, are located relative to the mean μ_t. The important properties of the distribution (189) are that the variance is constant and that higher order moments disappear. Then, the approximation procedure for the probabilities λ_t proposed in section 4.2.3, equation (176), can be applied in the present analysis. The key idea in the procedure is that the drift in the mean of the distribution (189) can be replaced by making the limits $\underline{r}_0$ and $\bar{r}_0$ time-dependent, and by evaluating the probabilities relative to the trend path $E_0(r_t) = 0$. The approximation is derived in the appendix (see equations A23 - A31 in appendix 2) and the derivation is not repeated here. Instead, we state directly that the approximation rule of the probabilities λ_t is the following:

$$(190) \qquad \lambda_t = \lambda_{t-1} + f(E_{t-1}r_t - r_{t-1}) + \frac{1}{2} f'(E_{t-1}r - r_{t-2})$$

where $f = f(\underline{r}_{t-1}) = f(\underline{r}_0 + r_{t-1})$ if $r_{t-1} < 0$

$\quad = f(\bar{r}_{t-1}) = f(\bar{r}_0 + r_{t-1})$ if $r_{t-1} > 0$

$f' = f'(\underline{r}_{t-1})$ if $r_{t-1} < 0$

$f' = f'(\bar{r}_{t-1})$ if $r_{t-1} > 0$

and f and f' are the density function of the distribution (189) and its derivative.

In the analysis, f and f' will be treated as constant evaluations of the density and its derivative, but in the description of the results of the analysis the actual time-dependent f and f' prove to be very useful. The limitations of the rule given in (190) are discussed in the appendix.

The form (190) is not a closed form approximation of the probabilities λ_t, as the past probabilities λ_{t-1} appear in it. In cases where the past effects of speculation affect the present state of reserves, the closed form representation is needed. By iterating (190) form the initial state, it can be seen that the closed form solution is a sum of the infinite sums of the last two terms in (190). However, by considerations presented in the appendix, an unweighted sum of the last terms in (190) is likely to be an overestimate of the past effects. Therefore, the closed form solution for (190) to be used in the relevant contexts is:

$$(190')\quad \lambda_t = f\,E_{t-1}r_t + \frac{1}{2}f'\,\frac{1}{(1-v_0L)}[E_{t-1}r_t - r_{t-2}]$$

where $v_0 \in (0,1)$

$$f\,E_{t-1}r_t = \sum_{j=0}^{t} f(E_{t-j-1}r_{t-j} - r_{t-j-1})^{40}$$

Equipped with the approximation rule (190), we can write the general expression for the logarithmic deviation of reserves from the trend path in the following form:

$$(191)\quad r_t = \frac{\Gamma}{[1-(1-a)L]}\cdot\Big\{\frac{B}{(1-B)}[u_t - (\dot{w}_t - \dot{w}^*)] + \frac{1}{(1-B)}\eta_t + k[\lambda_{t-1} + f(E_{t-1}r_t - r_{t-1}) + \frac{1}{2}f'(E_{t-1}r_t - r_{t-2})]\Big\}$$

The dynamic representation for the unemployment rate is directly defined by the identity in (184), in which r_t is replaced by (191).

The description of the basic linear model is now complete. In the following section, the technical properties of the basic model and its extensions will be studied under two alternative contract wage-setting policies.

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[40]The right hand side of the equality is exactly the following:

$$\sum_{j=0}^{t} f(E_{t-j-1}r_{t-j} - r_{t-j-1}) = f[E_{t-1}r_t + (E_{t-2}r_{t-1} - r_{t-1}) + (E_{t-3}r_{t-2} - r_{t-2}) + \ldots (E_0r_1 - r_1) - r_0]$$
$$= f[E_{t-1}r_t + z_{t-1} + z_{t-2} + \ldots + z_1 - r_0]$$

where z_i are i.i.d. expectations errors.

If the system starts from equilibrium, then $r_0 = 0$. For i.i.d. expectations errors, the sum of the errors converges asymptotically to zero. Therefore, the form used in (190') is asymptotically justified.

4.3.2 Analytical properties of the basic linear model

The dynamics in reserves and unemployment can be characterized in terms of different parameters implicitly included in the structure of a second order stochastic difference equation such as equation (191). We will characterize the dynamics under two alternative trade union contract wage-setting policies, which are very different in nature.

According to the first contract wage-setting policy, the trade union always sets the contract wage so that the expected rate of change of the contract wage equals the sum of the mean rate of growth of productivity and the mean world inflation rate. Specifically, then, we have that for the first policy

(192) $$\dot{w}_t = \nu + \Pi^* = \dot{w}^* \quad \forall\, t$$

Hence, under the first policy there is no feedback from the unemployment rate or any other variables of the model to contract wage-setting, and the term $\dot{w}_t - \dot{w}^*$ disappears from the determination of reserves and unemployment.

The first policy, while a commonly proposed guideline for contract wage-setting in public debates, is very ad hoc in nature and also unsatisfactory from the theoretical point of view, as it contains no feedback from labor market disequilibria to the wage-setting. As an alternative for the ad hoc rule, we assume that the union sets the contract wage so as to maximize the following preferences:

$$(193)\qquad \max_{\{\dot{w}-\dot{w}^*\}} G_t \equiv q_1(\dot{w}_t-\dot{w}^*) - q_2[\tfrac{1}{2} E_{t-1}(un_t+un_{t+1})] - q_3[\tfrac{1}{2} E_{t-1}(un_t+un_{t+1})]^2$$

where $q_1,\ q_2,\ q_3 \geqslant 0$

$$\dot{w}_t - \dot{w}^* = \dot{w}_{t+1} - \dot{w}^*$$

According to the second policy, there is a continuous feedback from the labor market disequilibria to the contract wage-setting. The union sets a wage contract for two periods at a time, and for convenience we assume that the rate of growth of the contract wage is the same for both subperiods. It is shown in the appendix that the first two terms in (193) resemble the static Stone-Geary preferences in the corresponding level variables. The third quadratic term represents the feedback mechanism in wage-setting. In particular, it will be seen later that the first two terms in preferences (193) generate a natural rate of unemployment in the economy. The role of the quadratic term is then to correct the contract wage in each contract so that the expected unemployment rate is stabilized about the natural rate.

The magnitudes of parameters q_1, q_2, and q_3 determine what is the actual natural rate of unemployment implicitly chosen by the union that behaves according to the preferences given in (193). Yet another interpretation for the preferences is that the first two terms represent the degree of money illusion in the trade union behaviour. It turns out - see appendix 2, equations (A64 - A65) - that the maximization problem in (193) produces the following general wage-setting rule:

$$(194)\qquad \dot{w}_t - \dot{w}^* = V_0 + \frac{1}{2}\frac{(1-B)}{B}(1+L^{-1})(k\lambda_{t-1}+S_t+\tilde{\varepsilon}_t)$$

where $V_0 = \frac{(1-B)^2}{2q_3(f_2B)^2} \cdot (q_1 - \frac{q_2f_2}{(1-B)})$

$$S_t = E_{t-1}k[f(E_{t-1}r_t - r_{t-1}) + \frac{1}{2}(E_{t-1}r_t - r_{t-2})]$$

$$L^{-1} : L^{-1}S_t = S_{t+1}$$

$$\tilde{\varepsilon}_t = E_{t-1}[\frac{B}{(1-B)} u_t + \frac{1}{(1-B)} \eta_t]$$

$$\equiv \frac{B}{(1-B)} \tilde{u}_t + \frac{1}{(1-B)} \tilde{\eta}_t$$

= shocks anticipated at contracting time t-1 to occur in period t

Intuitively, the wage-setting rule given in (194) is quite easy to interpret. The term V_0 represents the autonomous money illusion component in the rate of increase in the contract wage. The rest of the terms represent adjustments in the rate of increase in the contract wage needed to stabilize systematic fluctuations about the natural rate of unemployment caused by structural speculation and anticipated shocks. All the feedback terms enter as arithmetic averages of the values that the respective terms adopt in the two subperiods which the contract covers.

Next, contract the wage-setting rules in (192) and (194) are applied to the general equation for reserves, equation (191) and the implied equation for the unemployment rate. To distinguish between the rules, we will throughout use the superscripts A when we are referring to specific values associated with policy rule (192) and superscripts F when we want to identify the regime implied by rule (194). The first rule will be called the ad hoc rule and the second rule the feedback policy.

The time paths of the logarithmic deviations of reserves from the trend path and the unemployment rate implied by the ad hoc contract wage-setting rule are given by the following expressions:

$$(195) \qquad r_t^A = \left[\frac{H_1^A}{(1-K_1^A L)} - \frac{H_2^A}{(1-K_2^A L)}\right]\left[T\left(\frac{B}{(1-B)} u_t + \frac{1}{(1-B)} \eta_t\right)\right]$$

$$(196) \qquad un_t^A = -\frac{af_1}{\Gamma}\left[\frac{H_1^A}{(1-K_1^A L)} - \frac{H_2^A}{(1-K_2^A L)}\right]\left[T\left(\frac{B}{(1-B)} u_t + \frac{1}{(1-B)} \eta_t\right)\right]$$

Both expressions are essentially differences of two infinite sums of past price and productivity shocks scaled by appropriate constants. The signs of the two equations are mutually plausible: when r_t is negative, un_t is positive, and vice versa. A more detailed characterization of the dynamics in reserves and unemployment requires information on parameters H and K. First, it can be shown (see A49 - A54 in appendix 2) that $0 < K_1^A < 1$ and $K_1^A < K_2^A$; and that both H_1^A and H_2^A are negative. From these facts, it follows that the second infinite sum in (195) and (196) is larger in absolute value than the first sum. Also, the sign of the second sum is positive: if u_t and η_t are positive, the accumulated second sum will be positive. Then, the aggregate effects of world price and productivity shocks on reserves and unemployment have plausible signs. Negative price and productivity shocks will cause losses in reserves and increases in unemployment.

A question of fundamental interest is whether the dynamics of reserves exhibits properties that are characteristic of self-fulfilling exchange rate speculation. To formulate the question properly, recall that exchange rate speculation is specified in terms of the probabilities λ_t in our model. The approximation of these probabilities introduces the second order dynamics in reserves into the model. Hence, if the implied dynamics becomes unstable for some plausible parameter values, meaning that reserves will hit any finite limit in finite time, we can say that a process of self-fulfilling speculation is going on in the economy. In technical terms, the existence of self-fulfilling speculation can be characterized by imposing a simple condition on the dominating parameter K_2^A: if $K_2^A > 1$, then the dynamics in reserves is unstable and ultimately the home country is forced either to devalue or to revalue. It is shown in the appendix (A55 - A56 in appendix 2) that the following condition can be established:

$$(197) \qquad K_2^A = 1 \iff f(1+v_0) + f' = \frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}$$

where $\quad \tilde{\Gamma}a = \Gamma$

In condition (197), the evaluations f and f' of the density of reserves and its derivative are treated as fixed points. If these fixed values are replaced in condition (197) by the appropriate variable values $f(\bar{r}_{t-1})$ and $f'(\bar{r}_{t-1})$, and some properties of the normal distribution and the definitions of the limits $\bar{r}_{t-1}$ and $\underline{r}_{t-1}$ are used, condition (197) can be developed into the following, intuitively appealing form (see A57 - A58 in appendix 2):

$$(198) \qquad K_2^A \geqslant 1 \iff \frac{r_{t-1}}{\sigma_r} \geqslant \lfloor \frac{1}{f_2}\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] - (1+v_0)] \equiv r^*$$

= revaluation speculation limit

$$\frac{r_{t-1}}{\sigma_r} \leqslant -\lfloor \frac{1}{f_2}\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] - (1+v\ )] \equiv r^{**}$$

= devaluation speculation limit

According to the condition in (198), if standardized reserves rise more than r* per cent above the trend path, then self-fulfilling revaluation speculation begins. Similarly, if reserves fall more than r** per cent below the trend path, self-fulfilling devaluation speculation begins.

The properties of condition (198) are quite plausible. The speculation limits are negatively related to all of the structural parameters. Thus, increases in the parameters make self-fulfilling speculation a more likely phenomenon in the economy. This is intuitively natural for the following reasons. First, increases in the 'shock-multiplier' $\tilde{\Gamma}$ magnify the income and capital flows effects of shocks on reserves. Second, increases in parameter a represent falls in portfolio-adjustment costs, and hence make investors more sensitive to shocks. Third, an increase in parameter k makes exchange rate speculation more profitable, thus enhancing the motivation for speculation. Fourth, increases in the evaluation of the density, f, increase the weight of speculation in the decision making of investors.

Condition (198) can also be stated in closed form, i.e. by developing Taylor expansions about some initial point for f and f' in condition (197), and then solving for the required reserves limits. The derivation of the closed form condition is presented in appendix 2 (A59 - A61) and here we only state the result:

$$(199) \qquad K_2^A \geq 1 \iff \frac{r_{t-1}}{\sigma_r} \geq \beta_0\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] - \beta_1 \equiv r^{*A}$$

$$\frac{r_{t-1}}{\sigma_r} \leq -\beta_0\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] + \beta_1 \equiv r^{**A}$$

where $\beta_0, \beta_1 > 0$; r^{*A} and r^{**A} are speculation limits

Not that the closed form conditions have the same properties relative to parameters as the implicit condition (198). The closed form conditions turn out to be useful later in comparisons with the feedback wage-setting model.

The time paths of the logarithmic deviations of reserves and the unemployment rate generated in the feedback wage-setting economy - corresponding to the respective paths (195) and (196) in the ad hoc wage-setting economy - are specified by the following relationships:

$$(200) \qquad r_t^F = \left[\frac{H_1^F}{(1-K_1^F L)} - \frac{H_2^F}{(1-K_2^F L^{-1})}\right](T'\left[\frac{B}{(1-B)}(u_t-\tilde{u}_t) + \frac{1}{(1-B)}(n_t-\tilde{n}_t)\right] - \Gamma V_{0t})$$

$$(201) \qquad un_t^F = \frac{af_2}{\Gamma}\left[\frac{H_1^F}{(1-K_1^F L)} - \frac{H_2^F}{(1-K_2^F L^{-1})}\right](T'\left[\frac{B}{(1-B)}(u_t-\tilde{u}_t) + \frac{1}{(1-B)}(n_t-\tilde{n}_t)\right] - \Gamma V_{0t})$$

Again, (200) and (201) are differences of two infinite sums, one summing effects from the past and one from the future. Given that K_1^F and K_2^F are real,[41] the above equations have analogous technical properties to those of equations (195) and (196). In such a case, $0 < K_1^F < K_2^F$, H_1^F and H_2^F are both negative, and T' is positive. Hence, the infinite sum defined by the lead operator is the dominating sum in both equations. The forward looking sum contributes positively to the deviations of reserves: positive future surprises increase r_t and negative future surprises decrease it. Therefore, the dynamics in reserves has expected properties relative to shocks.

[41]Equations (200) and (201) are not 'as well behaved' as equations (195) and (196). In particular, equations (200) and (201) may exhibit implausible fluctuating dynamics for the parameter values suggested in appendix 2, p. 258, unless the economy is relatively rigid in adjusting, i.e. unless parameter a is small. The problems with plausible dynamics are probably due to the fact that, analytically, we are restricted to two periods wage setting at most. As past systematic effects of speculation are eliminated from equations (200) and (201), the only role of speculation in these equations is in capitalizing future surprises, and is thus relatively weak. If wages were fixed for more periods than two at a time, the range of plausible behaviour of the equations under study would possibly be extended. If the role of speculation is magnified in the model by e.g. changing the parameters in an appropriate way (see appendix 2, p. 227), the above equations become much better behaved.

When equations (200) and (201) are well-behaved, then it is also possible that periods of self-fulfilling speculation occur in the feedback wage-setting economy. Again, the condition for self-fulfilling speculation to exist is that $K_2^F \geqslant 1$ must hold. In terms of observable variables, this condition can be formulated as follows (see (A74 - A75) in appendix 2):

$$(202) \qquad K_2 \geqslant 1 \iff \frac{r_{t-1}}{\sigma_r} \geqslant \tilde{\beta}_0 \frac{2}{a(\tilde{\Gamma}k+1)} - \tilde{\beta}_1 \equiv r^{*F}$$

$$\frac{r_{t-1}}{\sigma_r} \leqslant -\tilde{\beta}_0 \frac{2}{a(\tilde{\Gamma}k+1)} + \tilde{\beta}_1 \equiv r^{**F}$$

where $\tilde{\beta}_1, \tilde{\beta}_2 > 0$; r^{*F} and r^{**F} are the revaluation and devaluation speculation limits

According to condition (202), self-fulfilling speculation begins in the economy if standardized reserves at time t-1 are f^{*F} per cent above the trend path or r^{**F} per cent below it. In the former case, agents are expecting a revaluation and in the latter case a devaluation. The properties of the conditions in (202) are qualitatively plausible. The required deviations of reserves from the trend path that would trigger self-fulfilling speculation decrease if either the ability of the economy to adjust increases (a increases), or the shock-multiplier $\tilde{\Gamma}$ or the size of exchange rate jump k increase. We can also conclude that the limits in (202) are likely to be larger than the corresponding limits in (199) (see parameter values β_0 and β_1 in (A61) and $\tilde{\beta}_0$ and $\tilde{\beta}_1$ in (A75), and remark 3 on p. 227 in appendix 2). Hence, under practically any circumstances, the feedback wage-setting economy is less likely to experience self-fulfilling speculation generated by random shocks than the ad hoc wage-setting economy. The intuitive reason for this result is that, in the feedback wage-setting economy, the wage-setting provides the economy with an additional channel for adjusting to exogenous disturbances, while in the ad hoc wage-setting economy, the only ways for the economy to adjust are through variations in employment and capital flows.

On the other hand, there could be an autonomous, natural unemployment rate in the feedback wage-setting economy caused by the money illusion included in the preferences of the trade union. In equations (200) and (201), this shows up if the term ΓV_{0t} is positive. The lead-lag polynomial multiplying permanent, constant shocks ΓV_0 implies convergence in the long-run to a constant which, as defined by equation (201), is the natural rate of unemployment.[42] In principle, the term ΓV_0 could be negative as well, indicating negative money illusion in the preferences of the union. This alternative seems implausible to us, since it implies a negative natural rate of unemployment, i.e. permanent labor imports.

According to the discussion above, the two types of economies differ substantially in the ways in which cycles of exchange rate jumps are generated in them. In the ad hoc economy, a favourable sequence of random shocks may cause such deviations of reserves from the trend path that strong speculation on an exchange rate jump begins, ultimately causing the jump. The economy 'inherits' the past disturbances, as there is no mechanism in it which would neutralize the past effects. The exchange rate cycle is the only way in which this type of economy adjusts to sequences of

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[42]Note that, for plausible results in all cases, the permanence of shocks ΓV_0 implies a specific restriction on the lead-lag structure in (200) and (201). If, namely, $K_2^F > 1$ occurs, the lead sum becomes immediately infinite, because the sum goes to infinity. This is not very plausible for practical interpretations. Consequently, we have to assume that, for instance, the lead sum of terms ΓV_0 represents the sum of the terms of the first 10 periods into the future and the lag sum the sum of the terms of the first 10 periods into the past, respectively. With such an assumption, the autonomous component in (200) and (201) never becomes infinite.

disturbances. In the feedback economy, there is an efficient mechanism for eliminating past and anticipated future shocks. The contract wage increases are set precisely so as to neutralize such shocks, and therefore, only shocks that were unanticipated at the contracting time, i.e. shocks $u_t-\tilde{u}_t$ and $\eta_t-\tilde{\eta}_t$, remain effective. Even these expectation errors will be taken into account in the following contract negotiations, as they will affect the expected unemployment rate for the subsequent wage contract period. In particular, future wage contracts will always be fully indexed to permanent shocks which start within contract periods. In principle, we can distinguish four different types of reasons for the realization of self-fulfilling speculation in the feedback wage-setting economy. First, the 'average' wage-setting over two subperiods may generate systematic unanticipated averaging erros into equation (200). As a practical reason, we ignore this contribution. Second, new exogenous shocks may drive reserves to the jump limit. Third, the autonomous money illusion shock may, given the assumption in note 15, gain sufficient weight to cause the jump. Fourth - and genuinely in the spirit of self-fulfilling speculation - investors may generate an unanticipated shock by themselves, by actually speculating, if they believe that the exchange rate jump can be endogenously enforced within a contract period. For this to happen, the reserves must be driven close enough to the jump limit by other shocks, so that a maximum one period portfolio shift can cause the jump.

Both equations (195) and (196), and equations (200) and (201) for reserves and the unemployment rate are symmetric about the domestic full employment equilibrium, if it is assumed that shocks V_{0t} are zero. Given this assumption, it is equally likely in both types of economies that devaluations will be experienced. The symmetricity about the domestic full employment equilibrium implies perfectly flexible labor supply when the entire domestic labor force is employed. This is an implausible implication, as the additional labor supply must be in the form of either overtime work by domestic labor or imports of labor. Neither costless overtime working nor internationally frictionless labor mobility seem empirically valid assumptions.

To introduce a simple notion of less than fully flexible labor supply at the domestic full employment level, we define the elasticity of the negative unemployment rate with respect to the rate of growth of the wage rate $\dot{w}_t$ to be the following at or above the full employment level:

$$(203) \quad \frac{\Delta\xi_{1t}}{\xi_{10}} \Big/ \frac{\Delta\dot{w}_t}{\dot{w}^*} = \frac{(\xi_{1t}-\xi_{10})}{\xi_{10}} \Big/ \frac{(\dot{w}_t - \dot{w}^*)}{\dot{w}^*} \sim \frac{\xi_{1t} - \xi_{10}}{\dot{w}_t - \dot{w}^*} \equiv \delta$$

given that $\xi_{1t} \geqslant \xi_{10}$

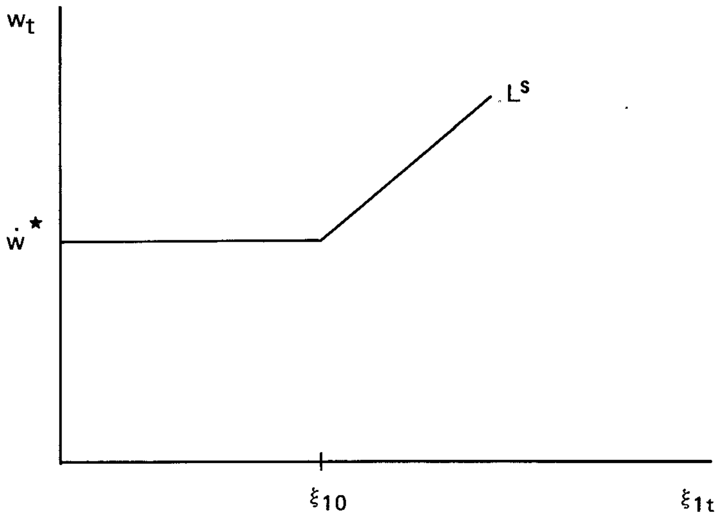
where $\delta \geqslant 0$

The above partially flexible labor supply assumption is illustrated in Figure 17. The key assumption in (203) is the constant elasticity of labor supply over the whole relevant range.

For our purposes, it is not necessary to specify the source of the additional labor supply, but the negative unemployment rate could well be interpreted either as the supply of overtime work by domestic labor, or then as the residual labor supply function of the rest of the world.

Fig. 17.

LABOR SUPPLY FUNCTION



It is clear that if the economy starts at full employment and a positive world price or productivity shock hits the economy, output will not rise by as much as it would if the economy started from an unemployment situation. When firms hire more labor, they bid up wages, which affects the rate of return on shares negatively. Hence, investors will not be willing to increase their investments in firms by as much as they would if wages did not respond. The connections between the labor supply assumption (203) and the determination of reserves and the negative unemployment rate are derived in appendix 2 (see A63 - A65), and only the outcomes will be presented here. Thus, under assumption (203), given that $\xi_{1t} \geqslant \xi_{10}$, the deviations of reserves and the negative unemployment rate are given in the ad hoc wage-setting economy by the following equations:

(204) $$r_t\Big|_{\xi_{1t}\geq\xi_{10}} = \frac{\Gamma}{af_2} H(L)^A\left[\frac{B\delta u_t+\delta\eta_t}{(1-B)\delta+BH(L)^A}\right] \equiv \tilde{r}_t^A$$

(205) $$\xi_{1t}-\xi_{10}\Big|_{\xi_{1t}\geq\xi_{10}} = H(L)^A\left[\frac{B\delta u_t+\delta\eta_t}{(1-B)\delta+BH(L)^A}\right] \equiv un_t^A$$

where $$H(L) = \frac{af_2}{\Gamma}\left[\frac{H_1^A}{(1-K_1^A L)} - \frac{H_2^A}{(1-K_2^A L)}\right]T$$

The corresponding equations for the feedback wage-setting economy are:

(206) $$r_t\Big|_{\xi_{1t}\geq\xi_{10}} = \frac{\Gamma}{af_2} H(L)^F\left[T'\left(\frac{B\delta(u_t-\tilde{u}_t)+\delta(\eta_t-\tilde{\eta}_t)}{(1-B)\delta+BH(L)^F}\right)-\Gamma V_{0t}\right] \equiv \tilde{r}_t^F$$

(207) $$\xi_{1t}-\xi_{10}\Big|_{\xi_{1t}\geq\xi_{10}} = H(L)^F\left[T'\left(\frac{B\delta(u_t-\tilde{u}_t)+\delta(\eta_t-\tilde{\eta}_t)}{(1-B)\delta+BH(L)^F}\right)-\Gamma V_{0t}\right] \equiv un_t^F$$

where $$H(L)^F = \frac{af_2}{\Gamma}\left[\frac{H_1^F}{(1-K_1^F L)} - \frac{H_2^F}{(1-K_2^F L^{-1})}\right]$$

In the technical sense, the important property of equations (203 - 207) is that, as is seen in appendix 2, world price shocks u_t have smaller coefficients than $\frac{B}{1-B}$ in the above equations. Similarly, productivity shocks η_t have smaller coefficients than $\frac{1}{1-B}$ in (203 - 207). This is precisely an indication of the property that, if assumption (203) is adopted, then the responses of the economy are no longer symmetric. Specifically, the asymmetric economy will, on the average experience unemployment, reserves below the trend level and devaluation speculation more often than the corresponding phenomena on the other side of the full employment equilibrium. The feedback effect from wages dampens the effects of all shocks when the economy is above the full employment equilibrium. Note that the above properties are independent of which wage-setting regime is studied.

4.3.3 Predictions of the linear model: the macroeconomic adjustment process of a small open economy

We have now completed the technical analysis and characterization of the dynamic adjustment processes operating in a small open economy. From now on, we shall dispense with technical exercises and try to provide an intuitive description of the most important aspects and properties of the stylized open economy that we have constructed.

The general logic of cyclical fluctuations is quite simple in our model. Wage contracts fix the nominal wage for a number of periods at a time, while all other types of economical decisions can be made in each period. When exogenous world price and productivity shocks hit the economy, the profitability of firms and the real rate of return on shares change. Hence, investors will observe incentives to adjust their portfolios. The implied flows of capital between the home country and the rest of the world are the driving force of real and nominal effects in our model.

Consider first a sequence of decelerations in the world inflation rate or, alternatively, a sequence of negative productivity shocks. Both types of shocks reduce the profitability of firms, which are paying fixed nominal wages to employees. Consequently, investors will perceive shares as a less favourable investment and start shifting capital into foreign assets. The outflow of capital has several implications. First, output and employment fall. Second, income falls, proportionately less than output, as the foreign investments are earning opportunity income abroad. These two effects together generate a negative reserves effect: both exports and imports decrease, but the former decrease more. The negative current account effect is reinforced by the negative capital account effect. As capital flows out and no capital imports by the government for counter-cyclical purposes are allowed, reserves are lost. When the sequence of negative profitability shocks continues, investors will understand that the probability of the home country government being forced to devalue the currency becomes nonnegligible. This observation will further magnify the effects on output, employment, income, capital outflow and reserves as the investors start speculating on the devaluation.

When the home country eventually devalues, the profitability of firms improves as long as wages remain nonindexed to the devaluation. Investors again begin to invest more in domestic firms and a reverse process to the one described above may result.

An analogous chain of events takes place if a sequence of positive profitability shocks hits the economy. Investors import capital (or, rather, repatriate it) and output and employment increase. Income increases proportionately less than output. The subsequent current and capital account effects on reserves are positive. Unless inflexibilities in labor supply prevent the process from continuing, the accumulation of reserves reinforced by revaluation speculation will make the home country revalue. This decreases the profitability of firms, turning the cycle towards recession.

The general logic of events can be further clarified and deepened by making a few remarks on some more limited issues. First, and independently of the different model cases and regimes, two types of cyclical fluctuations can be distinguished in the economy. If the exogenous shocks that distort the economy are relatively small and money illusion has no weight in the contract wage-setting, then unemployment, output and reserves fluctuate mildly about the respective full employment general equilibrium levels, and no jumps in the exchange rate will occur. If, on the other hand, the exogenous shocks are large, then the real variables and reserves may fluctuate with considerable amplitude, and exchange rate jumps are observed. Cycles of the former type will be called stable cycles and those of the latter type speculative cycles.[43] The two types of cycles are illustrated in Figure 18, in which the stylized time paths of the logarithmic deviations of reserves from the trend path and the unemployment rate are drawn. The solid lines represent reserves and the broken lines the unemployment rate. In both panels of Figure 18, the revaluation and devaluation limits $\bar{r}_0$ and $\underline{r}_0$ and the revaluation and devaluation speculation limits r^* and r^{**} are drawn. Actual exchange rate jumps occur when reserves hit $\bar{r}_0$ or $\underline{r}_0$. Self-fulfilling speculation begins when reserves hit either r^* or r^{**}. The latter two points indicate the losses or gains of reserves that have to be decumulated or accumulated before $K_2 > 1$ can result. In the stable cycle, reserves will never rise to r^* or fall to r^{**}, and hence self-fulfilling speculation never gets started.

Another property of some interest is the partial irrelevance of the size of the exchange rate jump percentage. The size of k is important in that it directly affects the dynamics of the model. But, as to the actual change in the exchange rate, it is not important. The country could, for instance, apply a one percent devaluation rule mechanically every time a devaluation is carried out. If such a devaluation were not large enough to reverse the trend in reserves, unstable speculation would continue. The country would be forced to devalue again, and so on. Thus, the markets would search for the cumulative devaluation percentage that would be large enough to reverse the trend in reserves.

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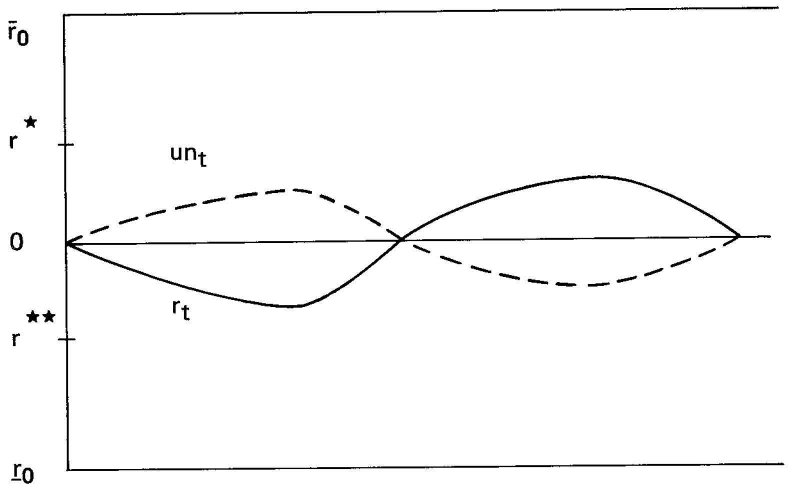
[43]The terminology is due to the fact that the two cases are distinguished by the stability condition on reserves. During stable cycles, reserves will always follow stable time paths, i.e. $K_2 < 1$. During speculative cycles, on the other hand, $K_2 > 1$ will always hold at the latest at the time when the exchange rate jumps, and the dynamics of reserves are unstable.

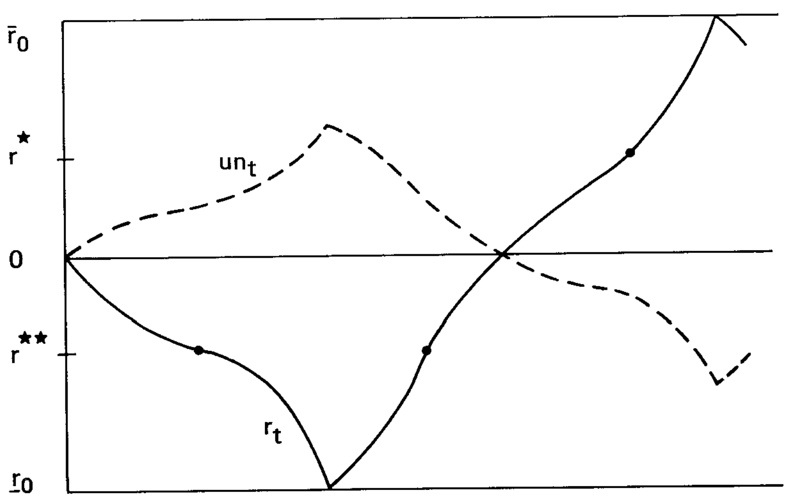
Fig. 18.

STABLE AND SPECULATIVE CYCLES

a) Stable cycle



b) Speculative cycle



In our model, average deviations in reserves and real variables from the full employment general equilibrium levels are caused by two types of factors. If money illusion enters into the trade union's preferences, reserves will be lower and the unemployment rate higher, on the average, than their respective full employment general equilibrium values. Similarly, a country in which there are rigidities in labor supply at the domestic full employment level is asymmetric with respect to symmetric shocks. Of course, it is possible even in the asymmetric cases to observe e.g. revaluations and periods of full employment, but these are generated by feasible stochastic events, which may not have very high probabilities.

The two different wage-setting policies, while they have rather similar analytical properties, may possibly differ in their economic implications. The essential feature of the ad hoc wage-setting economy is that it has no mechanism for eliminating cumulative effects of past and present shocks. Thus, with a feasible sequence of shocks, such an economy is bound to end up in a speculative cycle, and possibly experience relatively large real effects as well. On the other hand, this type of economy is also responsive to policy shocks. A devaluation or a revaluation will have rapid and strong effects on real variables in this kind of economy, the reason being that devaluation and revaluation affect the profitability of firms and are not neutralized by actions of other agents.

The feedback wage-setting economy is efficient in eliminating past shocks. It also has the basic autonomous component of a natural rate of unemployment caused by the money illusion. Still, one might suggest that such an economy performs better relative to exogenous shocks than the ad hoc wage-setting economy. An argument in support of this suggestion is the fact that, even with the permanent money illusion shock, the feedback economy rarely experiences shocks large enough to bring about the deviations of reserves needed to make the self-fulfilling speculation start. The weakness of the feedback economy may lie in the fact that it could actually be 'too' efficient in eliminating any anticipated and past shocks, such as a change in the exchange rate. It might well be that in such an economy wage contracts are indexed to changes in the exchange rate at the first possible moment. Then, effects of e.g. a devaluation on the profitability of firms would be neutralized, and the desired adjustments in employment and output would not happen.

To see the above point more clearly, consider equations (195), (196), (200), and (201) when they are subjected to a sequence of specific shocks. For simplicity, assume that the shocks are announced at the beginning of the period in which they occur, so that they are anticipated one period before, but unanticipated earlier. Suppose that wage contracts are negotiated at times $t \pm 2i$, $i = 0,1,2$. Let us study the simple special case in which deflationary world price shocks u_{t-3}, u_{t-1}, and u_t hit the economy. Let all other shocks except the permanent money illusion shock be equal to zero. Given these shocks, the unemployment rate and the deviations of reserves from the trend path at the end of period t are the following in the ad hoc and feedback wage-setting economies:

$$(208) \qquad r_t^A = - H_1^A T \cdot \frac{B}{(1-B)} [u_t + K_t^A u_{t-1} + (K_1^A)^3 u_{t-3}]$$

$$+ H_2^A T \frac{B}{(1-B)} [u_t + K_1^A u_{t-1} + (K_1^A)^3 u_{t-3}]$$

$$(208b) \qquad un_t^A = - \frac{af_2}{\Gamma} r_t^A$$

(209a) $$r_t^F = -\tilde{V}_{0t} - (H_1^F - H_2^F)T'\frac{B}{(1-B)}u_t$$

where $$\tilde{V}_{0t} = \left[\frac{H_1^F}{(1-K_1^F L)} - \frac{H_2^F}{(1-K_2^F L^{-1})}\right]T'V_{0t}$$

(209b) $$un_t^F = -\frac{af_2}{\Gamma} r_t^F$$

The two economies differ fundamentally in their responses to the above shocks. The shocks u_{t-1} and u_{t-3}, which were known at the wage contracting time, are neutralized completely in the feedback wage-setting economy, but have full effects in the ad hoc economy. On the other hand, the money illusion shocks V_{0t} create a structural deviation of reserves from the trend path and a 'natural' rate of unemployment in the feedback wage-setting economy.

To make the experiment interesting, let us assume that for both types of economies the deviations of reserves from the trend path at time t are smaller than the respective devaluation speculation limits, thus causing self-fulfilling devaluation speculation to start. For convenience, let us also assume that a k-percent devaluation occurs at time t+1 in both economies and that investors observe it before their decisions concerning the period t+2. The reserves at times t+2 and t+3 in the two types of economies are then the following:

(210a) $$r_{t+2}^A = (H_1^A - H_2^A)\, T\,\frac{B}{(1-B)}\,k - (K_1^A)^2 r_t^{A-} + (K_2^A)^2 r_t^{A+}$$

(210b) $$r_{t+3}^A = (H_1^A - H_2^A)\, T\,\frac{B}{(1-B)}\,k + T\,\frac{B}{(1+B)}[H_1^A K_1^A - H_2^A K_2^A]k - (K_1^A)^3 r_t^{A-} + (K_2^A)^3 r_t^{A+}$$

where $r_t^{A-} = H_1^A T \frac{B}{(1-B)}[u_t + K_1^A u_{t-1} + (K_1^A)^3 u_{t-3}]$

$$r_t^{A+} = H_2^A T \frac{B}{(1-B)}[u_t + K_1^A u_{t-1} + (K_1^A)^3 u_{t-3}]$$

$$\text{(211a)} \quad r_{t+2}^F = -\tilde{V}_{0t+2} - H_2^F T' \frac{B}{(1-B)} k + H_1^F T' \frac{B}{(1-B)}[k-(K_1^F)^2 u_t]$$

$$\text{(211b)} \quad r_{t+3}^F = -\tilde{V}_{0t+3} + H_1^F K_1^F T' \frac{B}{(1-B)} k - H_1^F T' \frac{B}{(1-B)} (K_1^F)^3 u_t$$

$$= -\tilde{V}_{0t+3} + K_1^F H_1^F T' \frac{B}{(1-B)}[k-(K_1^F)^2 u_t]$$

Equations (210a,b) clearly indicate that the ad hoc economy is well-behaved in the recovery from the period of high unemployment and low profitability preceding the devaluation. By well-behaved we mean that, if the devaluation leads to increases in the reserves in the first period after the devaluation, then the reserves will continue rising in the subsequent periods as well. Ultimately, the reserves will overshoot to a new level above the trend path level, unless new shocks change the environment.

On the other hand, according to equations (211a,b), the feedback economy is not at all well-behaved during the recovery. Reserves may increase in the first period after the devaluation if the devaluation percentage k is large enough. In the first period after the devaluation, there are two effects which can potentially resist the direct effect of devaluation on reserves. First, the autonomous money illusion shock again has a negative effect on reserves. Second, the feedback wage-setting policy implies - besides indexing wages fully to anticipated shocks - that wages are subsequently adjusted so as to eliminate partially the effects of unanticipated events as well. Thus, on one hand, wages are adjusted downwards to eliminate the expansionary effect of the price shock u_t on unemployment. But, on the other hand, wages are also adjusted upwards to capitalize, as wage increases, part of the decrease in unemployment caused by the devaluation in the first period, when the union had no possibility to respond to it. The difference $k-(K_1^F)^2 u_t$ represents the aggregate response of wages to unanticipated shocks, and it is likely to be positive. But in such a case the reponse of the contract wage to unanticipated shocks has a negative effect on reserves in the aggregate.

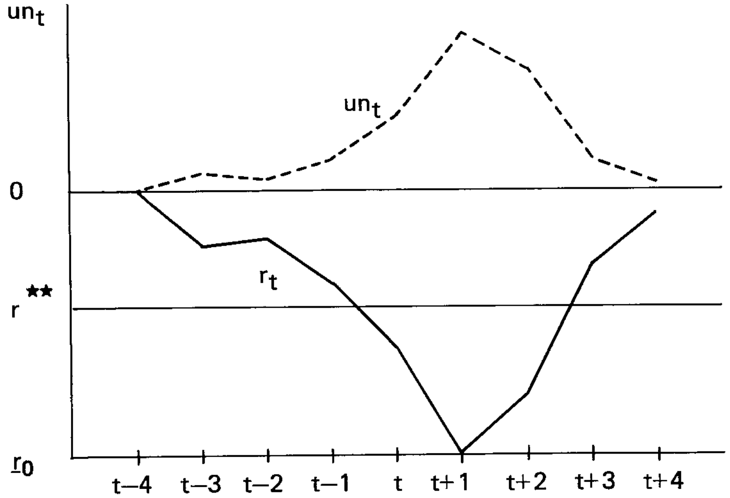
Even if reserves increase in the first period after the devaluation, they may, and are likely to, decrease again later. This is because in the following wage contract the wage rate is fully indexed to the devaluation. Hence, the positive profitability shock due to the devaluation can only be temporary, and exists only in one period in our special case. After the first period, only the autonomous shock and the response of wages to unanticipated shocks remain, and both of these are likely to have negative effects on reserves.

The time paths of reserves and unemployment are drawn in Figure 19 for the two types of economies and assuming the shocks discussed above. In panel (a), the consistent recovery of reserves and employment after the devaluation is shown in the ad hoc wage setting economy. In panel (b), the corresponding alternative time paths of reserves and unemployment are presented in the feedback wage-setting economy. It is observed in all alternatives that a sustained recovery after the devaluation is unlikely. If the increase in reserves in the first period after the devaluation is so small that the economy remains in the speculative regime ($r^{**} > r_{t+2}$), then $K_2^F > 1$ holds, and in the subsequent periods the money illusion shocks will drive the economy to a new devaluation. If the reserves increase so much in period t+2 that the economy returns to the stable regime ($r^{**} < r_{t+2}$, $K_2^F < 1$), several alternative cases are possible. If the shock $k-(K_1^F)^2 u_t$ is positive, the reserves begin to fall again from the second period after the devaluation and finally converge on some stationary level if no

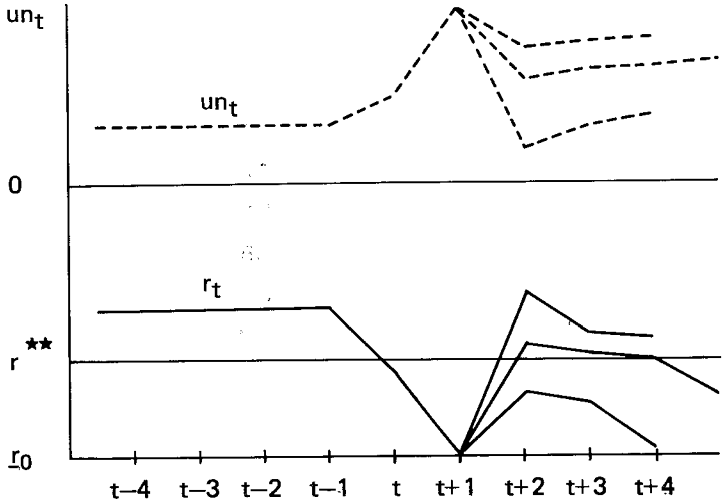
Fig. 19.

THE AD HOC AND FEEDBACK WAGE-SETTING ECONOMIES: DEVALUATION AND RECOVERY

a) Ad hoc wage setting economy



b) Feedback wage setting economy



more shocks emerge. This level may be above or below r**. If the level is below r**, then, at some point of time, the speculative regime resumes and the autonomous money illusion shocks cause a new devaluation. If the level is above r**, then reserves and unemployment converge gradually to new 'natural' levels.

These levels must imply reserves below and unemployment above the initial 'natural' levels. This can be easily seen by iterating equation (211b) further into the future. Shocks V_{0j} are the same for all periods j, but, in addition to the autonomous shocks, there is also the effect of unanticipated shocks in the equation for reserves. Hence, the new natural level of reserves cannot be the same as the initial level. Only in the case where $k-(K_1^F)^2 u_t$ is negative and the reserves still increase in the first period after the devaluation so that $r^{**} < r_{t+2}$ would the reserves continue rising in the subsequent periods as well. This case can be considered very unlikely, because the devaluation percentage k must be relatively large in order to make even the first period effect on reserves positive.

Now we are ready to characterize in general terms the macroeconomic adjustment processes in the two types of economies. We take the partially elastic labor supply at the full employment level to be the general constraint for both types of economies. The dynamic behaviour of the ad hoc wage-setting economy is then characterized by equations (195) and (196) when $\xi_{1t}<\xi_{10}$, or there is unemployment in the economy, and by equations (204) and (205) when $\xi_{1t}\geq\xi_{10}$. The corresponding equations in the feedback wage-setting economy are (200) and (201) if $\xi_{1t}<\xi_{10}$, and (206) and (207) if $\xi_{1t}\geq\xi_{10}$. The general implication of the partially elastic labor supply assumption is that, merely for this reason, both types of economies tend, on the average, to experience periods of unemployment, reserves below the trend level, devaluation speculation and devaluations more frequently than periods of labor imports, reserves above the autonomous level, revaluation speculation and revaluations.

In the ad hoc wage-setting economy, exogenous shocks are the immediate reason for cyclical fluctuations in reserves and unemployment. Since the contract wage is rigid, exogenous shocks cause variations in the real profitability of the firms. From the point of view of investors, the relative desirability of different investment opportunities varies, making them willing to adjust their portfolios. Thus fluctuations in reserves and unemployment replace the lack of flexibility in the labor costs of the firms as channels of adjustment in the economy. The cyclical fluctuations would occur both with or without exchange rate speculation. What speculation adds to the adjustment process is that it speeds up the cyclical fluctuations: given a specific sequence of shocks that leads to a devaluation, both with and without speculation, the devaluation occurs earlier in a speculative economy. Exchange rate changes are an efficient policy measure in the ad hoc wage-setting economy. They constitute permanent profitability shocks to firms, which guarantee consistent adjustments from the disequilibria towards the full employment general equilibrium. However, because of the permanence of devaluation and revaluation shocks, a given single exchange rate change always overcorrects the disequilibrium. A revaluation implies convergence of reserves to a level below the trend path and unemployment. Hence, only a combination of at least two exchange rate changes in opposite directions can correctly neutralize the effects of exogenous shocks in the economy. These facts are an indication of the overshooting property that our model has if the exchange rate jumps. Overshooting in reserves is a natural property in a fixed exchange rate economy with a relative price rigidity, as there is nothing ultimately different in the model relative to the rational expectations models under flexible exchange rates, in which overshooting results (see Frenkel-Rodriguez (1982)).

The nature of the macroeconomic adjustment process is sharply different in the feedback wage-setting economy. If the information sets of the trade union, investors and firms are the same, then only shocks which were unanticipated at the contract wage-setting time and occur within the contract period can cause fluctuations in reserves and unemployment. All anticipated and permanent shocks are eliminated by full indexation in the wage contracts. In this sense, the feedback wage-setting economy experiences fewer and milder fluctuations than the ad hoc wage-setting economy. On the other hand, there may be a higher average unemployment rate in the feedback wage-setting economy because of the possible weight given to money illusion in the preferences of the trade union. Non-indexed shocks can cause an exchange rate change in the feedback wage-setting economy. The sensitivity of the economy to such shocks is increased by the presence of exchange rate speculation.

Let us now ignore the money illusion for a moment. Then, the crucial general property that follows from the feedback wage-setting is that the efficiency of exchange rate changes as policy measures is substantially weakened. Exchange rate changes are permanent shocks and therefore the wage contracts concluded after an exchange rate change are fully indexed to it. Hence, devaluations and revaluations are profitability shocks to firms only in the short-run, in our specific case only for one period. Thus, exchange rate policy has desirable effects on real activity and reserves only in the first period. Moreover, in subsequent periods the exchange rate change is likely to have the opposite effect on reserves, since future wage contracts will include the response of wages to the exchange rate change because it was an unanticipated shock in the period when it occurred. The role of exchange rate speculation then becomes important if the combined effects of the exchange rate change and the unanticipated shock keep the economy in the range of destabilizing speculation. In that case, the economy is relatively sensitive to future shocks and repeated exchange rate changes may result.

The autonomous money illusion shocks make the consequences of devaluations and revaluations asymmetric. If exogenous shocks cause a revaluation and reserves decrease in the first period succeeding it, they are likely to continue decreasing further.

This is due to the fact that, while the only effect of the revaluation itself is that it was an unanticipated shock to the union, money illusion shocks are permanent negative profitability shocks, which drive the economy towards the state of the natural rate of unemployment. On the other hand, if a devaluation is not large enough to lift the economy out of the range of destabilizing devaluation speculation, then the money illusion shocks enforce a new devaluation. Thus, with money illusion, it is much harder to steer the economy towards higher levels of activity through devaluations than to bring it down from booms through revaluations. In conclusion, we can say that the partial sterilization of exchange rate policy in the feedback wage-setting economy is an indirect policy neutrality result in the Lucasian spirit. Exogenous shocks combined with a structure in which different groups of agents make decisions according to different criteria can generate states which are undesirable from the social welfare point of view. However, neutralization of such distortions by means of policy is difficult, because agents consider permanent policy changes as anticipated events. They change their decisions so that the effects of policy measures are partially eliminated. Only policies based on surprises or regulation of the wage determination can correct the distortions.

4.3.4 Comparison of the results with some other studies

From the analytical point of view, the novelty in the above analysis is the introduction of the notion of endogenously determined exchange rate speculation into an open economy model. Exchange rate speculation under fixed exchange rates has been examined in a few studies (see Krugman (1979), Turnovsky (1980), and Obstfeld (1982)), but in all of them speculation is based on deterministic information on the timing of the actions of the government. In our analysis, the probability of a discrete jump in an exchange rate within a given time-interval is endogenously determined by the whole macroeconomic model. The technique of continuous-time stochastic programming provides a means of formulating the effects of such endogenously generated probabilities on the decisions of agents. The key idea in the derivation of decision rules that take possible exchange rate jumps into account is to interpret the jump probabilities as instantaneous probabilities of a Poisson process. Turnovsky (1980) briefly mentions a case in which the agents expect the exchange rate to jump with some probability within a given time-interval, but he takes the probability as exogenous.

Exchange rate speculation interpretd in terms of an endogenously changing probability of deriving discrete capital gains or losses leads to two interesting general analytical properties. First, it was seen above that there exist limits defined in terms of losses or gains in reserves beyond which speculation becomes self-fulfilling. These limits correspond to the notion of speculative attacks by agents on reserves, which is the central feature in both Krugman's (1979) and Obstfeld's (1982) analyses. Second, the notions of plausible dynamics and stability in our model differ sharply from those in other rational expectations models in the theory of flexible exchange rates (e.g. the original contributions of Kouri (1976), Branson (1976), and Dornbusch (1976)) as well as those in the closed economy context (e.g. Taylor (1980), Fischer (1977), (1979)).

In the tradition of rational expectations or perfect foresight analyses, the general convention has been that the saddle-point solution for the dynamics is the only plausible case to be considered. While it is well-known that there is only one path which has the saddle point property and an infinite number of unstable paths, the general assumption is that agents find the stable path, because other paths would imply outcomes which are implausible given the rationality assumption. Such irrational outcomes could, for example, generate infinite capital losses for agents in finite time (for an interesting discussion on this point, see the early paper by Shell-Stiglitz (1967)).

In our model, on the other hand, the whole characterization of the self-fulfilling speculation is based on the fact that, at some point in the dynamic process, the underlying dynamics can change from the stable to the unstable regime. In particular, such a change is perfectly consistent with rational behaviour. The reason for the nonconventional view on plausible dynamics in our study is that the actions of the government support the global stability of the economy. The government always reacts by changing the exchange rate before its reserve losses or gains become too large. When the agents know this, they can indeed speculate - and, by so doing, generate the unstable dynamics - without worrying about states of nature that would e.g. imply zero wealth for them. This idea that economies are likely to contain forces which prevent the worst states of nature from occurring and guarantee that the economy stays within a 'corridor' no matter what the dynamics of the system is between the limits, should be of substantial interest in macroeconomic analysis in general. It might be fruitful and interesting to analyze e.g. the Branson-Kouri portfolio balance model under such a general assumption.

Concerning the economic predictions of our analysis, two studies, in particular, are directly comparable to ours. Both Korkman (1978) and Kouri (1979) analyze the phenomenon of the devaluation cycle observed in e.g. the Finnish economy. In both studies, the tradeables - nontradeables framework with capital accumulation is used. In our model, we ignore issues of variations in the terms of trade as well as those of capital accumulation. Thus, our model neither has the two-sector distinction nor specifies the difference between savings and investment. On the other hand, we specify rational behaviour, speculation and expectations formation in a genuinely uncertain environment, and pay special attention to trade union behaviour by assuming that the trade union is a monopoly relative to the firms in the labor market.

While all three studies charaterize the devaluation cycle in terms of variations in the profitability of firms (in the tradeables sector in Kouri and Korkman), our analysis provides some new aspects concerning the explanation of the phenomenon. First, our analysis gives a natural interpretation in terms of rigidities in labor supply at the full employment level or money illusion in wage-setting to why an economy may be biased towards the devaluation cycle. Second, our analysis distinguishes between two types of wage-setting policies, and makes clear the sharply different cyclical fluctuations implied by the two policies. Third, our analysis points out the different roles of optimizing behaviour in the open economy. In particular, the distortionary role of exchange rate speculation and the policy neutralizing role of feedback wage-setting are emphasized. Fourth, our analysis shows that, from the point of view of microeconomic theory, the origin of the cyclical fluctuations may lie in the fact that, while agents behave optimally, they may maximize different preferences so that their internally consistent decisions need not result in socially optimal aggregate outcomes.

4.4 Concluding remarks

In the foregoing analysis, an open economy model was developed. In the first stage, we converted the basic choice-theoretic framework derived in chapter one into a discrete-time nonlinear open economy model. The key economic properties of the model were determined, and the role of exchange rate speculation in it was demonstrated. In the second stage, a completely linearized version of the nonlinear model with similar local properties was developed.

It was shown that endogenous exchange rate jumps can be part of the macroeconomic adjustment process even if the contract wage-setting policy of the trade union is based on maximizing preferences in expected unemployment within the contract period and the contract wage rate. This result is due to the difference in adjustment speeds between wages and prices, and the separation of capital and labor supply decisions. As wages are set for a longer time than other economic decisions are binding, the wage contracts introduce a rigidity into the economy. The economy facing exogenous shocks ultimately adjusts the consequences of this rigidity by setting the exchange rate at a new level.

The role of speculation was seen to be crucial in generating endogenous exchange rate jumps. If exogenous shocks lead to sufficient losses or gains in reserves, then speculative behaviour can become so decisive that only strong exogenous events operating in the opposite direction can prevent the expectations behind the speculation from becoming fulfilled.

The ad hoc wage-setting policy and the feedback wage-setting policy turn out to have strictly dichotomous properties in respect to causing cyclical fluctuations. The ad hoc rule makes the economy more prone to cyclical fluctuations caused by exogenous shocks than the feedback rule. On the other hand, the ad hoc wage-setting makes the economy consistently responsive to policy measures. Under such wage determination, a devaluation leads to a steady recovery in the level of real activity. An economy in which wages are determined by the feedback rule may, by contrast, remain locked to low levels of activity after the devaluation. Such a wage-setting policy neutralizes anticipated policy measures in the standard Lucasian fashion.

Finally, the elasticity of labor supply and the weight of money illusion in the preferences of the trade union are crucial in determining whether the economy is symmetric relative to devaluation and revaluation cycles. If the labor supply is perfectly elastic at the full employment level, implying completely flexible labor mobility, and there is no money illusion in the wage-setting, then the economy is equally likely to experience both revaluations and devaluations. If labor supply is partially elastic at the full employment level, or there is money illusion in wage-setting, then devaluations are carried out more often in the economy than revaluations.

5 SUMMARY AND POSSIBLE EXTENSION

The results of the study will only be briefly reviewed here, as Chapters 2 - 4 include their own conclusions. Furthermore, sections 3.8 - 3.9 and 4.3 contain relatively detailed discussions on and interpretations of the results of the closed and open economy analyses.

In the second chapter, analytical frameworks for the study were developed. The method of continuous-time stochastic dynamic programming was applied to derive optimizing behaviour for different types of agents. The main innovations of the chapter were the derivation of an explicit general equilibrium solution for the simple basic closed economy model and the decision rules of an investor facing discrete jumps in some of the relative prices.

As macroeconomic issues of a specific type of economy were the main interest of the study, a distortion characteristic of the institutional setting of that type of economy was introduced. The distortion consisted of two parts. First, the capital and labor supply decisions were separated by assuming that the decisions are made by entirely different agents. Second, nominal wage adjustments were made sticky by assuming that one trade union, a monopoly relative to the firms, sets the contract wage concerning the whole economy for a certain contract period at a time. This type of distortion essentially makes the Tobin effect the driving force of the analysis.

In the third chapter, the case of the closed economy was investigated. The main conclusion was that an economy with the above distortion has Leijonhufvud's so-called corridor property. The economy can absorb 'small' demand shocks in full employment by means of an automatic stabilizer, in our case by means of the real balance effect, but requires actions by the policy maker to remain in full employment when subject to 'large' demand shocks. A shock was interpreted as being 'small' if it is unanticipated or anticipated in the limited sense that the timing of the shock is not precisely known. On the other hand, a shock was interpreted as being 'large' if it is anticipated in the conventional sense. Interpreted in light of the standard policy neutrality analyses, our analysis produced dynamics which differ from the dynamics obtained in analogous contexts and considered conventional (Fischer (1979)). Some hidden assumptions in Fischer's analysis were pointed out, which seem to explain the differences. The interest rate targeting policy turned out to be superior to the money targeting policy in neutralizing real effects.

In the fourth chapter, the case of the small open economy was studied. The analysis was carried out in a linearized discrete time approximation of the underlying continuous-time model. Exchange rate speculation under fixed exchange rates and the general distortion were the sources of cyclical fluctuations in real and nominal variables. It turned out that, with symmetric exogenous world price and productivity shocks, the economy is biased towards devaluations and real activity below capacity levels if labor supply is not infinitely elastic at the domestic full employment level. This would require labor which is fully mobile internationally. Money illusion in the wage-setting provides the same result. Two alternative wage-setting policies - the ad hoc policy, in which the rate of increase of the expected real wage is always set equal to the trend growth rate of productivity, and the feedback policy, in which variations of unemployment are minimized about a natural level of unemployment - turned out to predict very different cyclical fluctuations. The ad hoc policy leads to more frequent fluctuations than the feedback policy, but an economy with such wage determination is also more amenable to control by the government through exchange rate policy. Under both types of wage determination, the economy was shown to have the property that a self-fulfilling exchange rate speculation can emerge. Specifically, levels of reserves were shown to exist such that, if actual reserves deviate from the trend reserves further than specified by these limits, then speculation on the jump in the exchange rate can begin and this ultimately enforces the jump, unless exogenous events intervene.

The analytical framework and approach of our study can be applied and extended to numerous contexts and issues. Here we will mention only three possible extensions, which we consider the most interesting and immediate.

First, the closed economy model of the third chapter could be made linear and converted into the discrete-time format. Then the model would be comparable to the conventional rational expectations models. The interesting modification in the converted model in relation to conventional models would be the notion of shocks which occur with some endogenously generated probability within a given time-interval. The dynamics of the price level and real variables could be analyzed in a standard fashion. The dynamics with endogenously generated probabilities for the occurrence of shocks would possibly imply unstable dynamics for some states of nature. To support the global stability of the system, the government reaction could then be made endogenous. Then the agents would know that, if their destabilizing speculative actions take the economy far enough from the full employment equilibrium, the government is guaranteed to 'bail out' the economy. The two aspects of this extension, i.e. that in practice many events are expected to occur at an unspecified time within a given time-interval and that, for individual agents, it is natural to think that an exogenous force will always intervene in the course of events if disturbances grow too large, are certainly of great theoretical and practical interest.

Another extension of some interest would be to convert the linear open economy model into the flexible exchange rate regime. The model would then be directly comparable to other exchange rate determination models. Again, speculative behaviour could be made an important factor in exchange rate determination, if it were assumed that the government allows exchange rates to be determined in the market as long as the market rate stays within certain limits. Beyond these limits the government would intervene in the market. The dynamics of the exchange rate would most likely differ substantially from what is predicted by, for example, the portfolio balance models.

The final extension we mention concerns the assumptions of our linear fixed exchange rate open economy model. The variations in the terms of trade could be an important factor in the cyclical fluctuations of the small open economy. Therefore, including such variations in the model would generalize it further. However, the difficulty with such variations is that they might further complicate the nonlinear dynamics. In such a situation, which is likely to be the case, we would be obliged to introduce another linear approximation into the model. This would reduce the possibilities of identifying different effects. Even so the exercise would certainly be worth trying.

APPENDIX 1

Derivation of the Risk Premium d_L

The condition that implicitly defines the risk premium d_L is the formalization of the requirement that the expected discounted utility from producing over the contract period is at the competitive level for individual firms, i.e. at the level at which firms are indifferent between producing and not producing.

(A1) $$E_0 \int_0^{T_c} e^{-\rho s} \log[u(s)]\,ds = \int_0^{T_c} e^{-\rho s}[\log C]\,ds, \text{ where } C > 1$$

In (A1), profits, or u(s), is given by

(A2) $$u(s) = p_0 y(s) - \beta(1-d_L)p_0 E_0 y(s) - (1-\beta)p_0 y(s)$$
$$= \beta p_0 y_0 e^{\nu s}\left[e^{\sigma\int_0^s dz} - (1-d_L)\right]$$

The stochastic differential of profits can be written as:

(A3) $$\frac{du(s)}{u(s)} = \nu ds + \frac{\exp\{\sigma\int_0^s dz\}\sigma dz}{\exp\{\sigma\int_0^s dz\} - (1-d_L)}$$
$$+ \frac{1}{2}\frac{[\exp\{\sigma\int_0^s dz\}(\exp\{\sigma\int_0^s dz\}-(1-d_L))-\exp\{\sigma\int_0^s dz\}]\sigma^2 ds}{[\exp\{\sigma\int_0^s dz\}-(1-d_L)]^2}$$

Let us evaluate (A3) at the point

$$E_0 \exp\{\sigma \int_0^s dz\} = 1,$$

which yields

(A4) $$\frac{du(s)}{u(s)} = \left[\nu - \frac{1}{2}\frac{(1-d_L)}{d_L^2}\sigma^2\right]ds + \frac{\sigma}{d_L}\,dz.$$

(A4) can be solved for the logarithm of instantaneous profits, or

(A5) $$\log(u(s)) = \log(u(0)) + \left[\nu - \frac{1}{2}\frac{(1-d_L)}{d_L^2}\sigma^2\right]s + \frac{\sigma}{d_L}\int_0^s dz.$$

Using the fact that $u(0) = \beta p_0 y_0 d_L$, the expectation of the logarithm of instantaneous profits can be written as:

(A6) $$E_0 \log(u(s)) = \log(\beta p_0 y_0 d_L) + \left[\nu - \frac{1}{2}\frac{(1-d_L)\sigma^2}{d_L^2}\right]s.$$

Note that condition (A1) must hold for the initial riskless state in which $h = (0,T_C) \longrightarrow 0$. Then we have that the following equality must hold:

(A7) $$\log\beta p_0 y_0 d_L = \log C$$

Using (A7), the expectations of the logarithm of instantaneous profits simplifies to:

(A6') $$E_0 \log(u(s)) = \log C + \left[\nu - \frac{1}{2}\frac{(1-d_L)\sigma^2}{d_L^2}\right]s.$$

Using (A6') and assuming that the expectations operator can be included in the integral operator, the condition (A1) can now be written as:

$$\text{(A8)} \qquad \int_0^{T_c} e^{-\rho s}\left[\log C + \left[\nu - \frac{1}{2}\frac{(1-d_L)\sigma^2}{d_L^2}\right] s\right] ds = \int_0^{T_c} e^{-\rho s}[\log C]ds$$

The condition can only be satisfied if

$$\text{(A9)} \qquad \nu - \frac{1}{2}\frac{(1-d_L)\sigma^2}{d_L^2} = 0$$

From condition (A9), d_L can be easily solved as:

$$\text{(A10)} \qquad d_L = \frac{\sqrt{((\sigma^2)^2 + 8\nu\sigma^2)} - \sigma^2}{4\nu}$$

According to (A10), d_L is positive, independent of time, and, perhaps somewhat surprisingly, also independent of the length of the contract period. These facts are due to the logarithmic objective function, in which the coefficient of relative risk aversion is one. From this it follows that accumulation of profits over time does not change the risk-premium, and hence the length of the contract period becomes irrelevant.

APPENDIX 2

The complete linear model

A2.1 The structure of the linear model

In section 4.2, the discrete-time version of the basic open economy model of section 2.4 - 2.5 was outlined. It became clear that the model is seriously nonlinear in shocks and hence impossible to study by means of analytical methods.

In this appendix, we specify and analyze an ad hoc dynamic discrete-time model which has qualitative properties closely related to the local properties of the nonlinear model and which is linear in shocks. For our purposes, i.e. for the study of the macroeconomic adjustment process in an open economy with speculating investors, it is sufficient to formulate representations for the dynamic behaviour of reserves and unemployment. These two equations, the former describing 'nominal' adjustments and the latter real fluctuations in the economy, capture fully the effects of wage and exchange rate rigidity and the separation of input supply decisions.

To derive an ad hoc representation for reserves, it is useful to note that, by our assumptions, all real variables, including real reserves, grow at the rate of growth of productivity ν if no shocks affect the economy. Then it seems reasonable to model the dynamics of real reserves as percentage or logarithmic deviations from the long-term trend path.

The logarithmic deviations of reserves from the long-term trend can in principle be related to present, past and possibly future productivity and world price shocks, and correspoding deviations of portfolios from the trend composition. According to equation (155), shocks can affect reserves directly by changing output and real income, and indirectly by making investors adjust their portfolios, which cause changes in output, real income and net capital flows. In addition, these changes are reinforced by the fact that optimizing investors may start speculating on possible jumps in the exchange rate if observed or anticipated events make such jumps likely.

In the considerations of section 4.2.2, it was pointed out that real reserves follow a very nonlinear dynamic path. This is mainly due to capital gains effects. Capital gains that affect the current account are mainly caused by world price shocks. In the following analysis, we abstract from the capital gains effects on real income formation. Also, we ignore variations in the nominal world interest rate. With these assumptions, non-autonomous changes in reserves are only caused by changes in the portfolio fraction ξ_{1t}, as all other terms in equation (168) disappear. In particular, the non-autonomous change in reserves, measured in home currency, takes the following form:

$$\text{(A1)} \qquad \Delta R^n_t = \Delta\xi_{1t}(P_0\tilde{y}_{t-1}-(I_{Bt-1}+I_{t-1}-I^*_{t-1}) + D_{t-1})$$

With the simplified assumptions, the reserves can only deviate from the trend path as long as investors want to adjust their portfolios. The latter occurs if shocks are anticipated and if there is exchange rate speculation going on. Analogously to (A1), the deviation of reserves from the trend path is given by:

$$\text{(A2)} \qquad R_t - R^p_t = (\xi_{1t}-\xi_{10})E_0[P_0y_t-(I_{Bt}+I_t-I^*_t) + D_t]$$

To develop an expression for the logarithmic deviations of reserves from the mean path, we have to develop the right hand side of (A2) into a more convenient form. First, note that $P_0E_0\tilde{y}_t = E_0(I_{Bt}+I_{Ht})$. Let us define the ratio of the mean gross revenues on domestic capital if it is invested abroad and the mean value of output, denoted by γ_I, as follows:

$$\text{(A3)} \qquad \gamma_I \equiv \frac{E_0 I_t^*}{P_0 E_0 \tilde{y}_t}$$

Another interpretation would be that γ_I is the unit opportunity national income earned by domestic capital if invested abroad.

For simplicity, we assume γ_I to be time-independent, even though this would not be the case in general. Next, let us define the ratio between the home currency value of the mean gross lending abroad and the mean value of output, denoted by γ_C, as follows:

$$\text{(A4)} \qquad \gamma_C \equiv \frac{E_0 D_t}{\xi_{10} P_0 E_0 \tilde{y}_t} = \frac{\xi_{20} \frac{s_0}{E_0 P_t} E_0 Q_t^* \tilde{K}}{\xi_{10} P_0 E_0 \tilde{y}_t}$$

Again, we abstract from complexities by assuming that γ_C is time-independent.

Equipped with definitions (A3) and (A4), we can write (A2) in an alternative form:

$$\text{(A5)} \qquad R_t - R_t^P = (\xi_{1t} - \xi_{10}) \lfloor \gamma_I + \gamma_C \rfloor \xi_{10} P_0 E_0 \tilde{y}_t$$

To proceed from (A5) towards an expression for the logarithmic deviation of reserves from the trend path, we still need to define the ratio between the mean value of output and the autonomous reserves. Denoting this ratio by γ_R, the definition for it is:

$$\text{(A6)} \qquad \gamma_R \equiv \frac{\xi_{10} p_0 E_0 \tilde{y}_t}{R_t^P} = \frac{\xi_{10} p_0 \tilde{y}_0 (1+\nu)^t}{R_0^P (1+\nu)^t} = \frac{\xi_{10} p_0 \tilde{y}_0}{R_0^P}$$

Using definition (A6), the percentage or logarithmic deviation of reserves from the trend, denoted by r_t, is given by:

$$\text{(A7)} \qquad r_t \equiv \frac{R_t - R_t^P}{R_t^P} = (\xi_{1t} - \xi_{10})[\gamma_I + \gamma_C]\gamma_R$$

Note that in (A7), γ_I represents the net current account effect, while γ_C represents the capital account effect. The interpretation of (A7) is simple: If the difference $\xi_{1t}-\xi_{10}$ equals, say, -a, then reserves deviate from their trend path by $-a[\gamma_I + \gamma_C]\gamma_R$ percent. One should bear in mind that (A7) is an approximation, as the capital gains effects, shocks in the world interest rate and the actual time-dependence of coefficients γ_I and γ_C have been ignored.

While the dynamic representation for reserves is somewhat of an ad hoc nature, we can develop an exact formulation for the unemployment rate. With short-term fixed-coefficient technology, the deviations of the portfolio fraction ξ_1 from the trend level ξ_{10} also indicate labor market disequilibria. When the actual ξ_1 is above ξ_{10}, then a fraction $\xi_{1t}-\xi_{10}$ of the total labor force in the home country is imported from the rest of the world. When, on the other hand, ξ_1 is below ξ_{10}, then a fraction $\xi_{10}-\xi_{1t}$ of the domestic labor force is unemployed. Hence, the natural definition for the unemployment rate, denoted by un_t, is

$$\text{(A8)} \qquad un_t = \xi_{10} - \xi_{1t}$$

The important thing to notice in (A7) and (A8) is that the real and nominal effects, i.e. variations in the unemployment rate and reserves are proportional to each other. Specifically, the relationship between un_t and r_t is:

$$(A9) \qquad un_t = - \frac{1}{\gamma_R(\gamma_I+\gamma_C)} r_t$$

Hence, as soon as we have found a representation for the logarithmic deviation of reserves, we have a solution for the unemployment rate as well.

In (156) - (159) above we derived the general relationship between shocks in the mean rate of return on shares and the portfolio fraction ξ_{1t}. This relationship left the shocks completely unspecified. In the following analysis, shocks of four types will be considered. Random world price and domestic productivity shocks are the two purely exogenous disturbances in the analysis. The third type of shock is caused by contract wages. As we will study different contract wage setting rules, we allow for the contract wage rate increase to deviate from the mean growth rate of productivity under some circumstances. The above three types of shocks affect the portfolio fraction ξ_{1t} - if they are anticipated - by affecting the mean real rate of return on shares. The fourth shock is due to exchange rate speculation. If the probability to devalue or to revalue increases, the effective mean rate of return on foreign bonds changes accordingly, making the portfolio fraction ξ_{1t} change. This effect was formally stated in section 2.5 in the portfolio rules (97).

The general relationship between the portfolio fraction ξ_{1t} and all the four types of shocks can be specified by applying directly portfolio rule (97a), and using the assumption that $\sigma_B^* = \rho_{B*}\sigma_B$. Under these circumstances, we can write the general expression for

Under these circumstances, we can write the general expression for the portfolio fraction ξ_1^0, with exchange rate speculation included and under the assumption of costless adjustments, as follows

$$(A10) \qquad \xi_{1t}^0 = f_2[r_B(u_t, \eta_t, \dot{w}_t - \dot{w}^*) - r_B^* + \lambda_t k]$$

where $f_2 = \dfrac{1}{\sigma_B^2(1-\rho_{B*}^2)}$

$\dot{w}_t$ = rate of increase in the contract wage in period t

$\dot{w}^* = \nu + \Pi^*$ = rate of increase in the contract wage according to the ad hoc rule, when $w_t = \nu + \Pi^* \; \forall \; t$

λ_t = probability that a revaluation occurs within period t

In order to proceed, we need to specify linear relationships between the mean nominal rate of return on shares r_B and the three (anticipated) shocks u_t, η_t, and $\dot{w}_t - \dot{w}^*$. To do this, it is most convenient to resume the continuous-time exposition for the moment. Let us again write first the definition for the mean real price of a share

$$(A11) \qquad \bar{Q}_{BR}(t) = (1-B)\frac{y(t)}{\nu K} = (\bar{p}(t)\bar{y}(t) - B\bar{p}(t)\bar{y}(t))\frac{1}{\bar{p}(t)\nu K}$$

where the bars refer to means of the respective variables

Consider then a shift function $\psi(t)$ which can enter (A11) in three different ways, corresponding to the three types of anticipated shocks. In the first case, there is an anticipated multiplicative shift $\psi(t)$ in the world price level within a wage contract period, while production is assumed to stay on the mean path. In this case, (A11) takes the form:

$$(A12) \qquad \bar{Q}_{BR}(t) = \frac{(\psi(t)-B)}{\psi(t)}\frac{\bar{y}(t)}{\nu K}$$

The real rate of return on shares in case (A12) is:

$$\text{(A13)} \qquad \frac{d\bar{Q}_{BR}(t)}{\bar{Q}_{BR}(t)} = \frac{Bd\psi(t)}{\psi(t)(\psi(t)-B)} + \nu$$

If we approximate (A13) by assuming that $\psi(t)$ is close to one, we obtain

$$\text{(A14)} \qquad \frac{d\bar{Q}_{BR}(t)}{\bar{Q}_{BR}(t)} = \nu + \frac{B}{1-B}\, d\psi(t)$$

(A14) is the linear approximation that we will use to formulate the effect of anticipated price shocks on the real rate of return on shares.

Secondly, let us assume that $\psi(t)$ represents a productivity shift which is anticipated by investors, but is not indexed in wage contracts. In this case, the expression for the real price of a share becomes:

$$\text{(A15)} \qquad \bar{Q}_{BR}(t) = (\psi(t)-B)\, \frac{\bar{y}(t)}{\nu K}$$

The real rate of return on shares is the following:

$$\text{(A16)} \qquad \frac{d\bar{Q}_{BR}(t)}{\bar{Q}_{BR}(t)} = \frac{d\psi(t)}{\psi(t)-B} + \nu$$

The approximation of (A16) corresponding to (A14) is:

$$\text{(A17)} \qquad \frac{d\bar{Q}_{BR}(t)}{\bar{Q}_{BR}(t)} = \nu + \frac{1}{1-B}\, d\psi(t)$$

(A17) is the linear approximation of the relationship between the real rate of return on shares and anticipated productivity shocks.

Consider finally the case that the actual nominal wage shifts. The real price of a share is then:

$$\text{(A18)} \qquad \bar{Q}_{BR}(t) = (1-B\psi(t))\,\frac{\bar{y}(t)}{\nu K}$$

The rate of return on a share is now:

$$\text{(A19)} \qquad \frac{d\bar{Q}_{BR}(t)}{\bar{Q}_{BR}(t)} = -\,\frac{Bd\psi(t)}{1 - B\psi(t)} + \frac{d\bar{y}(t)}{\bar{y}(t)}$$

The linear approximation of (A19) is:

$$\text{(A20)} \qquad \frac{d\bar{Q}_{BR}(t)}{\bar{Q}_{BR}(t)} = \nu - \frac{B}{1-B}\,d\psi(t)$$

The form (A20) will be used in modelling the effects of contract wage shocks on the real rate of return on shares.

Now we are ready to return to the discrete-time exposition. If we assume that the shift $d\psi(t)$ corresponds to the different discrete shocks and apply directly the linear approximations (A14), (A17), and (A20) to specify the relationship between anticipated shocks and the real rate of return on shares, we obtain the following stochastic difference equation for the optimal 'frictionless' portfolio fraction ξ^0_{1t}:

$$\text{(A21)} \qquad \xi^0_{1t} - \xi_{10} \equiv \tilde{\xi}^0_{1t} = f_2\left[\frac{B}{(1-B)}(u_t-(\dot{w}_t-\dot{w}^*)) + \frac{1}{(1+B)}\,\eta_t+\lambda_t k\right]$$

We observe immediately from (A21) that the sum in the square brackets corresponds to z_t in (156) above. Then we obtain the solution for ξ_{1t} immediately by substituting the sum in the square brackets in (A21) into (159) for z_t. The solution is:

$$\text{(A22)} \quad \xi_{1t} = \xi_1 + af_2 \frac{1}{[1-(1-a)L]} \left[\frac{B}{(1-B)}(u_t-(\dot{w}_t-\dot{w}^*)) + \frac{1}{1-B}\eta_t+\lambda_t k\right]$$

The interpretation of (A22) is relatively simple as long as it is recalled that B and 1-B are the risk premium corrected income shares of labor and capital. An anticipated increase of one percentage point in the world price level or an equivalent decrease in the difference $\dot{w}_t-\dot{w}^*$ increases the mean real rate of return on shares by the percentage $\frac{B}{1-B}$. An identical productivity shock, on the other hand, increases the return by the percentage $\frac{1}{1-B}$. The reason for the fact that a productivity shock has a larger effect than price and wage shocks is that the former represents a gain in the aggregate, while the latter two essentially involve income redistribution.

An operational formulation of (A22) still requires an approximation of the probabilities λ_t. Before developing the approximation, it is worth making a few points concerning the stochastic nature of (A22). According to assumptions (139) and (142), the exogenous shocks u_t and η_t are both normally distributed about zero and independent, identical random variables for all periods t. On the other hand, (A22) was derived assuming that shocks u_t and η_t are anticipated at the beginning of period t. If this is always the case in the aggregate and investors hold similar anticipations, (A22) is a deterministic relationship and agents know with certainty when the possible jump in the exchange rate is going to happen. To obtain a meaningful definition of the probabilities λ_t, we have to distinguish between unanticipated and anticipated events in the aggregate. In the former case, we assume that all individual investors have anticipations about the shocks u_t and η_t. Thus, they behave according to (A22) as individuals. We also assume that individual investors do not know each other's anticipations, but behave as if the anticipations of individual investors were distributed so that the aggregate anticipated price and productivity shock would have the distributions u_t and η_t. Specifically, this assumption means that the individual investors believe that the weighted aggregate anticipations constructed from the anticipations of individual investors are distributed according to distributions u_t and η_t, respectively. Thus, whatever aggregate anticipations result, individual investors interpret them to be draws from random variables u_t and η_t. An aggregate anticipated event, on the other hand, is an event about which either all investors have identical and correct anticipations in advance, or about which all investors know the aggregate weighted anticipation. Such an event causes a certain shift in the portfolio fraction ξ_1 as well as in reserves.

The form of (A22) indicates that, under aggregate unanticipated events, the deviation of reserves from the trend path in period t has a normal distribution which is a linear combination of the normally distributed random variables u_t and η_t. Hence, the distribution has constant parameters and we can develop a relatively simple approximation for the probabilities λ_t along the lines presented in section 4.2.3.

The first step is to specify expressions for the counterparts of devaluation and revaluation limits that are comparable to the logarithmic deviation concepts. By our assumptions, the two limits are:

(A23a) $\underline{R}_t = (1+\nu)^t \underline{R}_0$

(A23b) $\bar{R}_t = (1+\nu)^t \bar{R}_0$

The limits grow at the same rate as the mean of reserves. The logarithmic deviations of the limits from the trend reserves are:

(A24a) $\underline{d} \equiv \log(\frac{\underline{R}_t}{R_t}) = \log \underline{R}_0 - \log R_0$

(A24b) $\bar{d} \equiv \log(\frac{\bar{R}_t}{R_t}) = \log \bar{R}_0 - \log R_0$

To simplify matters, we assume that $-\underline{d} = \bar{d} \equiv d$. Then the initial devaluation and revaluation limits that correspond to the logarithmic deviation of reserves from the trend can be defined as follows:

(A25a) $\underline{r}_0 = \nu - d$

(A25b) $\bar{r}_0 = \nu + d$

where $\underline{r}_0$ and $\bar{r}_0$ are the devaluation and revaluation limits, respectively

If the reserves deviate from the trend path, the probability distribution of the logarithmic deviation of reserves for the following period is no longer concentrated about ν, but, instead, about r_{t-1}. Shifts of this type can be taken into account by making the limits vary accordingly over time and by fixing the mean of the distribution at ν. The particular time-dependent limits are of the following form:

(A26a) $\underline{r}_t = \underline{r}_0 - r_t$

(A26b) $\bar{r}_t = \bar{r}_0 - r_t$

According to (A26a), the devaluation limit $\underline{r}_t$ increases, thus making devaluation more likely if reserves fall below the trend path. The revaluation limit $\bar{r}_t$ decreases, hence raising the probability for revaluation, if reserves rise above the trend level.

Having derived the limits $\underline{r}_t$ and $\bar{r}_t$, we can apply the approximation for the probabilities λ_t suggested in (176) in the context of the nonlinear analysis. In the linear analysis, a construction of type (176) cannot be directly applied as the specification of the speculative behaviour. The problem is that a straightforward linearization of the weighting function $\underline{\omega}_t+\bar{\omega}_t$ would lead to quadratic expressions in logarithmic deviations of reserves. This fact is an indication of the fundamental nonlinearity that speculation can generate in the determination of reserves. Somehow, this nonlinearity must be captured in any linear approximation that could be considered satisfactory.

Let us use the notation that $F(r_t^0)$ and $f(r_t^0)$ are the distribution and density functions of r_t evaluated at r_t^0. With this notation, and applying (A26a), (A26b), and (176), the probability in period t for devaluation, or λ_t can be written as:

(A27) $$\lambda_t = F(\underline{r}_{t-1}) - [1-F(\bar{r}_{t-1})] + f(\underline{r}_{t-1})(E_{t-1}\Delta r_t) + \frac{1}{2}f'(\underline{r}_{t-1})(E_{t-1}\Delta r_t)^2 + f(\bar{r}_{t-1})(E_{t-1}\Delta r_t) + \frac{1}{2}f'(\bar{r}_{t-1})(E_{t-1}\Delta r_t)^2$$

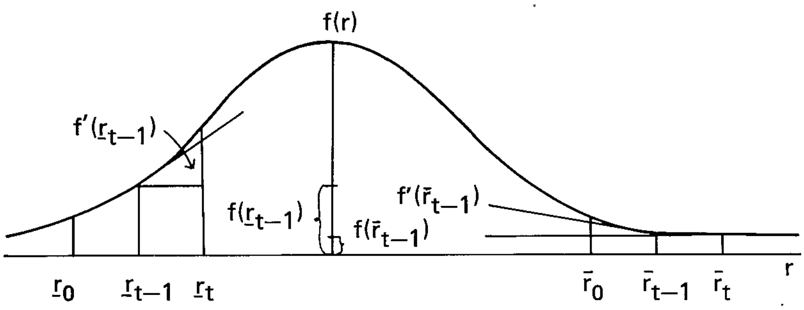
(A27) can be simplified to the form:

(A28) $$\lambda_t = \lambda_{t-1} + [f(\underline{r}_{t-1})+f(\bar{r}_{t-1})](E_{t-1}\Delta r_t) + \frac{1}{2}[f'(\underline{r}_{t-1})+f'(\bar{r}_{t-1})](E_{t-1}\Delta r_t)^2$$

To see the assumptions needed for the linearization of (A28), Figure 20 is illustrative:

Fig. 20.

LINEARIZATION OF λ_t



The figure demonstrates that if the devaluation probability becomes positive, then the values of both the density function and its derivative at the adjusted devaluation limit point $\underline{r}_{t-1}$ become much larger than the corresponding values at the revaluation limit point $\bar{r}_{t-1}$. For linearity in (A28), we prefer ignoring the smaller values and use a combined fixed evaluation point for both the value of the density function and its derivative. Let such fixed values be f and f'. These assumptions allow (A28) to be written as:

(A29) $$\lambda_t = \lambda_{t-1} + f(E_{t-1}r_t - r_{t-1}) + \frac{1}{2}f'(E_{t-1}r_t - r_{t-1})^2$$

(A29) is still nonlinear in the second degree term and it is difficult to find a good linear approximation for the squared term. We suggest the following approximation:

(A30) $$(E_{t-1}r_t - r_{t-1})^2 = E_{t-1}r_t - r_{t-2}$$

Note that approximation (A30) is quite accurate, if $\Delta r_t \sim 2 \sim \Delta r_{t-1}$. Moreover, it captures the nonlinearity, at least to a certain extent, since it introduces second order dynamics into the analysis. Furthermore, the approximation overestimates the nonlinearity for slowly accumulating reserves, i.e. for $\Delta r_t < 2$, and underestimates it for rapidly accumulating reserves, i.e. for $\Delta r_t > 2$. Hence, when we use (A30), the resulting effect of speculation on reserves is too strong for economies with slow trends in reserves and too weak for economies with strong trends in reserves. Applying (A30), we obtain the final form for (A28):

(A31) $$\lambda_t = \lambda_{t-1} + f(E_{t-1}r_t - r_{t-1}) + \frac{1}{2}f'(E_{t-1}r_t - r_{t-2})$$

Approximation (A31) will be used in the analysis as the specification of the effects of speculation on real reserves. Note that equation (A31) is not a closed from solution, as the probability λ_{t-1}, capturing the accumulated effects from the past, appears in it. In analytical contexts in which the systematic effects from the past are eliminated by the structure of the model, form (A31) is a valid representation. One of the two specific cases to be studied is of this type. However, in the other example, the past effects exercise an influence in the model. Therefore, we have to develop an approximate closed form solution for (A31). Iterating equation (A31), we obtain the following forms (see note 40 in Chapter 4):

$$\lambda_t = \lambda_0 + \sum_{j=0}^{t}\left[f(E_{t-j-1}r_{t-j}-r_{t-j-1}) + \frac{1}{2}f'(E_{t-j-1}r_{t-j}-r_{t-j-2})\right]$$

$$= \lambda_0 + f(E_{t-1}r_t-r_0) + \frac{1}{2}f' \sum_{j=0}^{t}(E_{t-j-1}r_{t-j}+r_{t-j-2})$$

Assuming that the initial probability λ_0 and the initial deviation r_0 are equal to zero, we have that

$$\lambda_t = f(E_{t-1}r_t) + \frac{1}{2}f' \sum_{j=0}^{t}(E_{t-j-1}r_{t-j}-r_{t-j-2})$$

The problem with the form obtained is that the second term is likely to seriously overestimate the sum of $(E_{t-j-1}\Delta r_t)^2$, which it is meant to approximate, because in the early stages of the process reserves are likely to deviate only slightly from the trend path. To remedy this problem, we suggest the following corrected approximation:

(A31') $$\lambda_t = f(E_{t-1}r_t) + \frac{1}{2}f' \frac{1}{(1-v_0L)} (E_{t-1}r_t-r_{t-2})$$

where $v_0 \in (0,1)$

In (A31'), low weights are given to the second differences far back in the past. The form (A31') is the closed form solution for the probabilities λ_t that will be used in appropriate contexts.

It should be understood that approximations (A31) and (A31') are not very general. In defending their use, however, we can say that any linear approximation of probabilities of combined events that are generated by the normal distribution and move over time are going to be unsatisfactory. It will be seen that, even with the simple notion of the dynamic motion of probabilities, the result of the analysis are intuitively plausible and relatively easy to interpret in terms of the parameters in (A31) and (A31').

Substituting (A31) into (A22) for λ_t, we obtain the following expression for the portfolio fraction ξ_{1t}:

$$\text{(A32)} \qquad \xi_{1t} = \xi_{10} + \frac{af_2}{[1-(1-a)L]}\{\frac{B}{(1-B)}[u_t-(\dot{w}_t-\dot{w}^*)] + \frac{1}{(1-B)}\eta_t + k[\lambda_{t-1}+f(E_{t-1}r_t-r_{t-1}) + \tfrac{1}{2}f'(E_{t-1}r_t-r_{t-2})]\}$$

By using (A32) in (A7) in an appropriate way, the following representation for the logarithmic deviations of reserves from the trend path results:

$$\text{(A33)} \qquad r_t = \gamma_R[\gamma_I+\gamma_C]\frac{af_2}{[1-(1-a)L]}\{\frac{B}{(1-B)}[u_t-(\dot{w}_t-\dot{w}^*)] + \frac{1}{(1-B)}\eta_t + k[\lambda_{t-1}+f(E_{t-1}r_t-r_{t-1}) + \tfrac{1}{2}f'(E_{t-1}r_t-r_{t-2})]\}$$

The dynamics of equation (A33) is studied in the following two sections under two different contract wage-setting rules. Once the dynamics of r_t is made clear, then the dynamics in real variables will follow directly from (A9).

A2.2 The dynamics of reserves and unemployment under the ad hoc contract wage-setting rule

In this section, we analyze the dynamics of reserves and unemployment when the contract wage always increase at the rate $\nu+\Pi^*$, or at the trend rate of growth of productivity corrected by the trend world inflation rate. Under this assumption, $\dot{w}_t = \dot{w}^*$ always so that the contract wage-setting never introduces shocks into the economy.

Under the ad hoc wage-setting rule, the equation for the logarithmic deviation of reserves from the trend path, or equation (A33), can be written in the following form:

$$\text{(A34)} \qquad r_t = \gamma_R[\gamma_I+\gamma_C]\,\frac{af_2}{[1-(1-a)L]}\,\{\frac{B}{(1-B)}u_t+\frac{1}{1-B}\eta_t$$
$$+k[\lambda_{t-1}+f(E_{t-1}r_t-r_{t-1})+\frac{1}{2}f'(E_{t-1}r_t-r_{t-2})]\}$$

Imbedded in (A34) is a second order stochastic difference equation governing the behaviour of expected deviations of reserves from the trend path. To find the solution for the equation, we take expectations dated at t-1 on (A34), obtaining:

$$\text{(A35)} \qquad E_{t-1}r_t = \frac{\Gamma k}{[1-(1-a)L]}\,[\lambda_{t-1}+f(E_{t-1}r_t-E_{t-1}r_{t-1})$$
$$+\frac{1}{2}f'(E_{t-1}r_t-E_{t-1}r_{t-2})]$$

where $\Gamma = \gamma_R[\gamma_I+\gamma_C]af_2$

In (A35), the term λ_{t-1} still appears, including the accumulated effects of the lagged values of r_t. Therefore, we have to replace λ_t by the closed form approximation (A31'). By doing so, we obtain the following equation, in which λ_{t-1} does not appear:

$$\text{(A36)} \qquad E_{t-1}r_t = \frac{\Gamma k}{[1-(1-a)L]}\,[fE_{t-1}r_t+\frac{1}{2}f'\,\frac{1}{(1-v_0L)}(E_{t-1}r_t-r_{t-2})]$$

Rearranging terms and using the standard lag operator notation, (A36) can be written in the following form:

(A37) $\left[1-\Gamma k(f+\frac{1}{2}f') - (1-a+v_0-\Gamma kfv_0)L + (v_0(1-a)+\frac{1}{2}\Gamma kf')L^2\right]$
$\cdot E_{t-1}r_t = \varepsilon_t$

where ε_t is an arbitrary autonomous shock to be replaced later by actual shocks

The solution for (A34) is found by studying the lag polynomial in (A37). The polynomial can be factorized as follows:

(A38) $$1 - \frac{(1-a+v_0-\Gamma kfv_0)}{1-\Gamma k(f+\frac{1}{2}f')}L + \frac{v_0(1-a)+\frac{1}{2}\Gamma kf'}{1-\Gamma k(f+\frac{1}{2}f')}L^2 = (1-K_1L)(1-K_2L)$$

Following Sargent (1979), the values of the undetermined coefficients K_1 and K_2 are found by solving the characteristic equation:

(A39) $$Z^2C_2 - ZC_1 + 1 = 0$$

where $C_2 = \frac{v_0(1-a)+\frac{1}{2}\Gamma kf'}{1-\Gamma k(f+\frac{1}{2}f')}$, $C_1 = -\frac{1-a+v_0-\Gamma kfv_0}{1-\Gamma k(f+\frac{1}{2}f')}$

$Z = \frac{1}{K}$

The solutions for K_1 and K_2 implied by (A36) are:

(A40) $$K_1 = \frac{D_0}{D_1 + \sqrt{\lfloor D_1^2-D_2\rfloor}}$$

(A41) $$K_2 = \frac{D_0}{D_1 - \sqrt{\lfloor D_1^2-D_2\rfloor}}$$

where $D_0 = 2(v_0(1-a)+ \frac{1}{2}\Gamma kf')$

$D_1 = 1 - a + v_0 - \Gamma kfv_0$

$D_2 = 4(v_0(1-a)+ \frac{1}{2}\Gamma kf'v_0)(1-\Gamma k(f+ \frac{1}{2}f')$

Using the factorization, the solution for (A37) can be written as:

$$\text{(A42)} \qquad E_{t-1}r_t = \frac{1}{(1-K_1L)(1-K_2L)} \cdot \frac{1}{(1-\Gamma k(f+ \frac{1}{2}f')} \varepsilon_t$$

The coefficient operator on the right hand side of (A39) can further be written as follows:

$$\text{(A43)} \qquad \frac{1}{(1-K_1L)(1-K_2L)} = \frac{A_0}{(1-K_1L)} + \frac{A_1}{(1-K_2L)}$$

The undetermined coefficients A_0 and A_1 in (A40) can be solved by cross-multiplying the numerators by denominators and then equating the coefficients of terms of the same degree on both sides. The solution are

$$\text{(A44)} \qquad A_0 = \frac{K_1}{K_1 - K_2}$$

$$\text{(A45)} \qquad A_1 = - \frac{K_2}{K_1 - K_2}$$

By applying (A44) and (A45), the solution (A42) can finally be written in the closed form as follows:

$$\text{(A46)} \qquad E_{t-1}r_t = \left[\frac{K_1}{(K_1-K_2)} \frac{1}{(1-K_1L)} - \frac{K_2}{(K_1-K_2)} \frac{1}{(1-K_2L)}\right] \cdot \left[\frac{1}{(1-\Gamma k(f+ \frac{1}{2}f'))}\right] \varepsilon_t$$

In (A46), we have the solution for the expectation of the logaritmic deviation of reserves at the end of the t^{th} period formed one period earlier, given an autonomous arbitrary shock ε_t.

Naturally, then, the actual logarithmic deviation of reserves from the trend path becomes:

$$\text{(A47)} \qquad r_t = \left[\frac{K_1}{(K_1-K_2)}\frac{1}{(1-K_1L)} - \frac{K_2}{(K_1-K_2)}\frac{1}{(1-K_2L)}\right]$$

$$\cdot\left[\frac{\Gamma}{(1-\Gamma k(f+\frac{1}{2}f')}\left(\frac{B}{(1-B)}u_t + \frac{1}{(1-B)}\eta_t\right)\right]$$

$$\equiv \left[\frac{H_1}{(1-K\ L)} - \frac{H_2}{(1-K\ L)}\right]\left[T\left(\frac{B}{(1-B)}u_t + \frac{1}{(1-B)}\eta_t\right)\right]$$

where the notation used in the latter form is obvious.

Applying the definition for the unemployment rate presented above in (A9), we can write the present period unemployment rate as:

$$\text{(A48)} \qquad un_t = -\frac{af_2}{\Gamma}\left[\frac{H_1}{(1-K_1L)} - \frac{H_2}{(1-K_2L)}\right]T\left(\frac{B}{(1-B)}u_t + \frac{1}{(1-B)}\eta_t\right)$$

To see the properties of the solutions for reserves and the unemployment rate, we need to study parameters K_1, K_2, H_1, H_2, and T. First, let us write the solution for K in the following general form:

$$\text{(A49)} \qquad K = \frac{D_0}{D_1 \pm \sqrt{[D_1^2-D_2]}}$$

From the form (A49), we can see that K_1 and K_2 are real if $D_1^2-D_2 \geqslant 0$. After some algebraic steps, the following condition can be established:

(A50) $D_1^2 - D_2 = 1 + (a-v_0)^2 + 2av_0 + R + 2(1-a-v_0)\Gamma kfv_0$

where $R > 0$

The smallest feasible value of (A50) is obtained at $a = v_0 = 1-\varepsilon$, where ε is very small. With these values, the condition becomes:

(A51) $D_1^2 - D_2 \sim 3 + R - 2\Gamma kf$

The most negative estimate of (A51) is obtained at $R = 0$. The implication of such an estimate is:

(A52) $D_1^2 - D_2 < 0 \iff \Gamma kf \gg \frac{3}{2}$

That condition (A52) is unlikely to hold for plausible parameter values will be seen after we establish another condition which more precisely specifies the nature of the dynamics in equation (A48) and (A49).

If, namely, K_1 and K_2 are real, then $0 < K_1 < K_2$ holds always if $D_2 > 0$. This is the case if

(A53) $4(v_0(1-a)+ \frac{1}{2}\Gamma kf'v_0)(1-\Gamma k(f+ \frac{1}{2}f') > 0$

$\iff 1 > \Gamma k(f+ \frac{1}{2}f') \iff f' < 2\left\lfloor\frac{1}{\Gamma k} -f\right\rfloor$

In (A53), $\Gamma k = \gamma_R(\gamma_C-\gamma_I)af$

We suggest the following values for parameters:

$\gamma_R = 20$ (ratio of trend value of output to trend reserves)

$\gamma_C = 0.2$ (ratio of gross trend foreign lending to trend value of output)

$\gamma_I = 0.4$ (opportunity unit national income earned by domestic capital if employed abroad)

$f_2 = 1$ (partial derivative of the portfolio fraction ξ_1 with respect to a change in the mean return differential)

$k = 0.05$ (devaluation or revaluation percentage)

$a = 0.5$ (adjustment coefficient)

The suggested parameter values generate a value $\Gamma k = 0.3$. Hence, for these parameter values, $D_1^2 < D_2$ can never occur as $f \in (0,1)$. Also $D_2 > 0$ holds always.

In conclusion, then, $0 < K_1 < K_2$. With the notation of (A53), H_1 and H_2 can be written as follows:

(A54a) $$H_1 = \frac{K_1}{K_1 - K_2} < 0$$

(A54b) $$H_2 = \frac{K_2}{K_1 - K_2} < 0$$

The signs of K and H provide us with two important conclusions:

1 Because K_2 is larger than K_1, the second infinite sum dominates the time paths of reserves and unemployment.

2 Because H_1 and H_2 are both negative, then the sign of the aggregate effect must be positive, i.e. a positive shock on reserves exercises a positive effect through the

infinite sums as well. This is due to the fact that H_2 enters with the negative sign in the infinite sum that is defined by the dominating parameter K_2.

From the preceding remarks on the magnitudes of constants γ_R, γ_I, γ_C, f_2, k, f, and f', it is clear that the parameter T in (A47) and (A48) is positive. In conclusion, then, this fact and results (A49) and (A54a,b) tell us that the logarithmic deviation of reserves from the trend path has plausible properties. If there is an anticipated acceleration in the world inflation rate or an anticipated rise in the rate of growth of productivity, neither of which are indexed in the wage contracts, then reserves become larger than the trend reserves. Similarly, labor is imported, i.e. the unemployment rate becomes negative. The opposite effects result if the shocks in question are negative.

Through approximation (A31), exchange rate speculation is imbedded in equations (A47) and (A48). Therefore, a question of some interest is whether equations (A47) and (A48) become unstable at any plausible parameter values. If this is the case, we can claim that speculation is self-fulfilling, i.e. that speculation leads to the realization of the event that agents are speculating on. Self-fulfilling speculation exists in the linear model if the dominating parameter K_2 grows larger than one.

The simple condition for K_2 to be larger than one is obtained from the constants of the characteristic equation (A39) as follows (see Sargent (1979)):

(A55) $$K_2 \geqslant 1 \iff C_1 + C_2 \geqslant 1$$

$$\iff \frac{-1 + a - v_0 + \Gamma k f v_0 + v_0(1-a) + \frac{1}{2}\Gamma k f'}{1 - \Gamma k(f + \frac{1}{2}f')} \geqslant 1$$

Manipulating the last inequality in an appropriate way, we obtain the following condition:

(A56) $$K_2 = 1 \iff f(1+v_0) + f' = \frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}$$

where $\Gamma = \frac{\Gamma}{a}$

Condition (A56) can be further developed by recalling that f and f' are evaluations of the density and its derivative of one of the random variables $\bar{r}_{t-1}$ and $\underline{r}_{t-1}$, in particular, evaluations of the moving jump limit which is closer to ν. The effective exchange rate jump limits $\bar{r}_{t-1}$ and $\underline{r}_{t-1}$ have normal distributions with means $\bar{r}_{t-2}$ and $\underline{r}_{t-2}$ and variance σ_r^2 given by:

(A57) $$\sigma_r^2 = H\lfloor (\frac{B}{1-B})^2 \sigma_u^2 + (\frac{1}{1-B})^2 \sigma_\eta^2 \rfloor$$

where $H = \lfloor (H_1 - H_2)T \rfloor^2$

Note that the random variables $\frac{\bar{r}_{t-1} - \bar{r}_{t-2}}{\sigma_r} = \frac{r_{t-1}}{\sigma_r}$ have standard normal distributions.

The standard normal density function and its derivative are related in the following way:

$$f(\bar{r}_{t-1}) = \frac{1}{\sigma_r\sqrt{2\Pi}} \exp\{-\frac{1}{2}(\frac{\bar{r}_{t-1} - \bar{r}_{t-2}}{\sigma_r})^2\}$$

$$f'(\bar{r}_{t-1}) = -f(\bar{r}_{t-1})\frac{(\bar{r}_{t-1} - \bar{r}_{t-2})}{\sigma_r}$$

Using this fact in condition (A56), we can solve for the following condition in terms of $\bar{r}_{t-1}$ and the ultimate parameters:

$$\text{(A58)} \qquad K_2 \geqslant 1 \iff \frac{\bar{r}_{t-1} - \bar{r}_{t-2}}{\sigma_r} = \frac{r_{t-1}}{\sigma_r} \geqslant \left[\frac{1}{f_2}\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] - (1+v_0)\right] \equiv r^*$$

$$\frac{\underline{r}_{t-1} - \underline{r}_{t-2}}{\sigma_r} = \frac{r_{t-1}}{\sigma_r} \leqslant \left[\frac{1}{f_2}\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] - (1+v_0)\right] \equiv r^{**}$$

The interpretation of condition (A58) is very simple and appealing. If the standardized reserves rise more than r* percent above or fall more than r** percent below the trend path, then agents perceive the speculative prospects very favourable. They start speculating so strongly on the possible revaluation or devaluation that their actions actually cause the jump in the exchange rate even if no other exogenous shocks emerge. The properties of condition (A58) are quite plausible. The 'threshold' deviation of reserves from the trend path required to generate $K_2 > 1$, or the self-fulfilling exchange rate speculation, decreases with the 'shock-multiplier' $\tilde{\Gamma}$ and the fixed devaluation and revaluation percentage k. It also decreases with parameter a, so that a rigidly adjusting economy (small a) is less prone to self-fulfilling speculation than a flexible economy (large a). Note that in the present case, r* and r** are symmetric relative to the trend path, which is due to the assumption of flexible labor mobility under which the results were derived. The responses of the limits r* and r** are also negative with respect to the technical parameter v_0. This should be expected, as it increases the weight of speculation in the approximation (A31'), thus making speculative behaviour a more important factor in the determination of reserves.

Condition (A56) can be developed into a more explicit form by approximating f and f' on the left hand side by a first order

Taylor approximation developed about a convenient point. The left hand side of (A56) can be written in the following form

$$\text{(A59)}\qquad f(1+v_0) + f' \sim \left[f_0+f_0'\left(\frac{r_{t-1}}{\sigma_r} - \frac{r_0}{\sigma_r}\right)\right](1+v_0) + f_0' + f_0''\left(\frac{r_{t-1}}{\sigma_r} - \frac{r_0}{\sigma_r}\right)$$

Let the convenient initial point be $r_0 = -\sqrt{2}\sigma_r$, so that $r_{t-1} < 0$ is assumed. Then we have that

$$f_0 = \frac{1}{\sqrt{2\Pi}}\, e^{-\frac{1}{2}\left(\frac{2\sigma_r^2}{\sigma_r^2}\right)} = \frac{1}{\sqrt{2\Pi e}} \sim 0.13$$

$$f_0' = \frac{\sqrt{2}\sigma_r}{\sigma_r}\, f_0 = \frac{1}{\sqrt{\Pi e}}$$

$$f_0'' = \left(\frac{\sqrt{2}\sigma_r}{\sigma_r}\right)^2 f_0 + f_0$$

$$= 3f_0 = \frac{3}{\sqrt{2\Pi e}}$$

Substituting these values into the approximation (A59), we obtain the following expansion:

$$\text{(A60)}\qquad f(1+v_0) + f' \sim \left[\frac{1}{\sqrt{2\Pi e}} + \frac{1}{\sqrt{\Pi e}}\left(\sqrt{2} - \frac{r_{t-1}}{\sigma_r}\right)\right](1+v_0) + \frac{1}{\sqrt{\Pi e}} + \frac{3}{\sqrt{2\Pi e}}\left(\sqrt{2} - \frac{r_{t-1}}{\sigma_r}\right)$$

After some manipulation in (A61) and after substituting the outcome into condition (A56), we can establish the following conditions, which are approximate closed forms corresponding to conditions (A58):

$$\text{(A61)} \qquad K_2 \geq 1 \iff \frac{r_{t-1}}{\sigma_r} \geq \beta_0\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] - \beta_1$$

$$\frac{r_{t-1}}{\sigma_r} \leq -\beta_0\left[\frac{2}{a\tilde{\Gamma}k} - \frac{(1-v_0)}{\tilde{\Gamma}k}\right] - \beta_1$$

where $$\beta_0 = \frac{\sqrt{2\Pi}e}{\sqrt{2}(1+v_0) + 3}$$

$$\beta_1 = \frac{3(1+v_0) + 4\sqrt{2}}{\sqrt{2}(1+v_0) + 3}$$

Conditions (A61) add no new information to what is obtained from the conditions in (A58). Parameters β_0 and β_1, which scale the structural part in conditions (A61), depend on the initial point about which the expansion is developed.

The analysis under the ad hoc contract wage-setting is now completed. Two remarks are, however, in place here before moving on to analyze the alternative union policy. First, throughout the analysis, we treat f and f' parametrically as fixed evaluations of the density and its derivative. This is necessary to keep the dynamics of reserves technically manageable. In this sense, the dynamics of reserves and unemployment can only be analyzed qualitatively under alternative regimes in which f and f' adopt different but fixed values.

Second, the analysis was carried out under the assumption that labor is completely mobile internationally. The situation changes drastically, however, if labor mobility is limited. Consider first the case in which there is no mobility of labor at all, nor any possibility of overtime work. If the economy is in equilibrium under such circumstances, i.e. $\xi_{1t} = \xi_{10}$, and investors anticipate further improvements in the profitability of domestic firms, they will not continue increasing investments in the domestic production. If they decided to do so, their actions would only lead to the firms bidding for workers from each other. This would lead to the following wage shock in period t+1:

$$(A62) \qquad \tilde{\dot{w}}_{t+1} - \dot{w}^{*} = u_{t+1} + \frac{1}{B}\eta_{t+1}$$

where $\tilde{\dot{w}}$ = actual rate of increase in the wage rate

By (A62), the shareholders' gain from their action would be zero in period t+1. Furthermore, if the rise in wages is indeed wage drift and the contract wage is always set according to the ad hoc rule, then the action of investors causes a permanent jump downwards in the profitability of firms in future periods. Hence, it is not optimal for the investors to continue investing in domestic firms under the prevailing circumstances. But, our economy can never gain more reserves than what is implied by the autonomous generation of reserves. Regardless of the normally distributed exogenous shocks, it will never be optimal for investors to speculate on revaluation, since by doing so they would eliminate the causes of speculation.

Consider now a more realistic case in which partially flexible labor supply in the form of e.g. labor imports is allowed at the domestic full employment level. In particular, let us assume that the elasticity of the (negative) unemployment rate (rate of labor imports) with respect to the percentage deviation between the rate of increase in the actual wage rate $\dot{w}_t$ and $\dot{w}^{*}$ is the following at and above the full employment level:

$$(A63)\qquad \left.\frac{\Delta\xi_{1t}}{\xi_{10}}\Big/\frac{\Delta\dot{w}_t}{\dot{w}^*}\right|_{\xi_{1t}\geq\xi_{10}} = \left.\frac{\xi_{1t}-\xi_{10}}{\xi_{10}}\Big/\frac{\dot{w}_t-\dot{w}^*}{\dot{w}^*}\right|_{\xi_{1t}\geq\xi_{10}}$$

$$\sim \left.\frac{\xi_{1t}-\xi_{10}}{\dot{w}_t-\dot{w}^*}\right|_{\xi_{1t}\geq\xi_{10}} \equiv \delta$$

where $\delta > 0$

When investors observe a shock that makes them invest more in domestic firms and there is full employment in the domestic economy, they will have to take (A63) into account. Their investment decisions start bidding up wages, and consequently the real rate of return on shares will not rise by as much as it would if the unemployment rate were positive. Given a sequence u_t of price shocks, the negative unemployment rate can be written as follows:

$$\xi_{1t} - \xi_{10} = H(L)\left[\frac{B}{(1-B)}(u_t-(\dot{w}_t-\dot{w}^*))\right]$$

where $H(L) = \frac{af_2}{\Gamma}\left[\frac{H_1}{(1-K_1L)} - \frac{H_2}{(1-K_2L)}\right]T$

Applying (A63) to the unemployment rate, we obtain:

$$\delta(\dot{w}_t-\dot{w}^*) = H(L)\left[\frac{B}{(1-B)}(u_t-(\dot{w}_t-\dot{w}^*))\right]$$

The solution for $\dot{w}_t - \dot{w}^*$ of the above equation is:

$$\dot{w}_t - \dot{w}^* = \frac{BH(L)}{(1-B)\delta + BH(L)}\, u_t$$

The result is the additional rate of increase in the wage rate which the firms have to pay in order to hire the implied amount of labor. The expression for the unemployment rate that takes the partially flexible labor supply into account is obtained by substituting the solution for the additional rate of increase in the wage rate into the equation for the unemployment rate. The substitution gives the following result:

$$\text{(A64a)} \quad \xi_{1t} - \xi_{10} = H(L)\left[\frac{B}{(1-B)}\left(1 - \frac{BH(L)}{(1-B)\delta + BH(L)}\right)u_t\right]$$

$$= H(L)\left[\frac{B\delta}{(1-B)\delta + BH(L)}\, u_t\right]$$

The corresponding equation for reserves is obtained directly by applying the definition between the unemployment rate and reserves:

$$\text{(A64b)} \quad r_t = \frac{\Gamma}{af_2}\, H(L)\left[\frac{B\delta}{(1-B)\beta + BH(L)}\, u_t\right]$$

The first form of (A64a) indicates clearly that the coefficient of the price shocks u_t is smaller than $\frac{B}{1-B}$. This is the expected result when labor supply is partially flexible, and the wage rate must adjust if employment changes.

The coefficient polynomial for productivity shocks under the partially flexible labor supply can also be derived easily along the lines applied above. The equation for the negative unemployment rate when only a sequence η_t of productivity shocks occurs is:

$$\xi_{1t} - \xi_{10} = H(L)\left[\frac{\delta}{(1-B)\delta + BH(L)}\, \eta_t\right]$$

Again, the coefficient for shocks η_t is smaller than $\frac{1}{1-B}$, or the coefficient in the case when some labor is unemployed.

Equations (A64a,b) and the above information on the case of productivity shocks can be combined to formulate the general expressions for reserves and negative unemployment when labor supply is governed by (A63). The equations are:

$$\text{(A65a)} \quad r_t \Big|_{\xi_{1t} \geq \xi_{10}} = \frac{\Gamma}{af_2} H(L)\left[\frac{B\delta u_t + \delta \eta_t}{(1-B)\delta + BH(L)}\right] \equiv \tilde{r}_t$$

$$\text{(A65b)} \quad \xi_{1t} - \xi_{10} \Big|_{\xi_{1t} \geq \xi_{10}} = H(L)\left[\frac{B\delta u_t + \delta \eta_t}{(1-B)\delta + BH(L)}\right] \equiv un_t$$

The important property of equations (A65a,b) is that symmetric shocks will no longer cause symmetric deviations in reserves and the unemployment rate about the full employment equilibrium. Instead, reserves will, on average, be below the trend path if labor supply is partially flexible above the full employment level. Furthermore, the economy will, on average, experience unemployment. Finally, the economy is more likely to experience devaluations than revaluations because positive shocks in (A65a) will be scaled down by the wage effect.

A2.3 The dynamics of reserves and unemployment under optimizing contract wage-setting policy

In the final section of this appendix, we briefly analyze the dynamics of reserves and unemployment when the trade union follows an optimizing contract wage-setting policy. We assume that the union sets a contract wage for two periods at a time, while investors can respond to shocks in each period. The contract specifies one unique rate of growth $\dot{w}$ of the contract wage for both subperiods. The union sets the contract wage by considering both the nominal wage increase and the unemployment rate during the contract period. Specifically, we state that the union sets the rate of increase in the contract wage relative to the mean growth rate of productivity so as to maximize the following objective function:

$$\text{(A66)} \quad \max_{\{\dot{w}_t-\dot{w}^*\}} G_t \equiv q_1(\dot{w}_t-\dot{w}^*) - q_2[\tfrac{1}{2}E_{t-1}(un_t+un_{t+1})] - q_3[\tfrac{1}{2}E_{t-1}(un_t+un_{t+1})]^2$$

where q_1, q_2, and q_3 are nonnegative constants

Preferences (A66) consist of two parts. The first two terms are somewhat analogous to the static Stone-Geary preferences under certainty. To see the connection, let us compare the static Stone-Geary preferences in the corresponding level variables with our dynamic formulation:

$$\max(w-w^*)^{q_1}(L^*-L)^{-q_2} \iff \max\{q_1\log(w-w^*) - q_2\log(L^*-L)$$

$$\max\left[\left(\frac{\dot{w}_t}{\dot{w}_t^*}\right)^{q_1}\left(\frac{L_t^*}{L_t}\right)^{-q_2}\right] \iff \max\left[\left(\frac{(1+\dot{w}_t)w_{t-1}^*}{(1-\dot{w}^*)w^*_{t-1}}\right)^{q_1}\left(\frac{L_t^*}{(1-un_t)L_t^*}\right)^{-q_2}\right]$$

$$\iff \max\{q_1[\log(1+\dot{w}_t)-\log(1+\dot{w}^*)]+q_2\log(1-un_t)\}$$

$$\iff \max[q_1(\dot{w}_t-\dot{w}^*)-q_2un_t]$$

For simple demonstration, it has been assumed in the latter chain of equivalences that there are deviations between the target and actual value only in the present period.

The last term, on the other hand, represents the quadratic element technically necessary for maximization with respect to the control variable. The economic meaning of the term is that expected variations in the unemployment rate about some natural rate cause frictional disutility to the union leaders. The natural rate of unemployment is specified by the first two terms of preferences (A66). If the expected unemployment rate falls below the natural rate, then the contract wage rate increases are too small to maximize preferences, and vice versa. In effect, the third term corrects contract wage-setting so that the expected unemployment rate is stabilized about the natural rate.

To proceed, let us apply equation (A32), which by (A8) defines the unemployment rate under an arbitrary contract wage policy. Equations (A32) and (A8) imply the following forms for the second and third terms in (A66):

$$\text{(A67a)}\quad q_2[\tfrac{1}{2}E_{t-1}(un_t+un_{t+1})] = -\frac{q_2af_2}{[1-(1-a)L]} \cdot [-\frac{B}{(1-B)}(\dot{w}_t-\dot{w}^*) + \frac{(1-L^{-1})}{2}(k\lambda_{t-1}+S_t-\tilde{\varepsilon}_t)]$$

$$\text{(A67b)}\quad q_3[\tfrac{1}{2}E_{t-1}(un_t+un_{t+1})]^2 = \frac{q_3(af_2)^2}{[1-(1-a)L]^2}[(\frac{B}{1-B})^2(\dot{w}_t-\dot{w}^*)^2 + \frac{(1-L^{-1})^2}{4}(k\lambda_{t-1}+S_t-\tilde{\varepsilon}_t)^2] - \frac{B}{(1-B)}(\dot{w}_t-\dot{w}^*)(1-L^{-1})(k\lambda_{t-1}+S_t-\tilde{\varepsilon}_t)]$$

where $L^{-1}S_t \equiv S_{t+1}$

$$S_t \equiv E_{t-1}k[f(E_{t-1}r_t-r_{t-1}) + \tfrac{1}{2}f'(E_{t-1}r_t-r_{t-2})]$$

$$= k[f(E_{t-1}r_t-E_{t-1}r_{t-1}) + \tfrac{1}{2}f'(E_{t-1}r_t-E_{t-1}r_{t-2})]$$

$$\tilde{\varepsilon}_t = E_{t-1}[\frac{B}{1-B}u_t + \frac{1}{1-B}\eta_t]$$

$$\equiv \frac{B}{1-B}\tilde{u}_t + \frac{1}{1-B}\tilde{\eta}_t = \text{anticipated shocks}$$

If expression (A67a,b) are substituted into (A66) for the respective terms and the appropriate first order condition is formed, the following solution for the contract wage-setting rule is obtained:

(A68) $$\dot{w}_t - \dot{w}^* = \frac{(1-B)^2}{2q_3(f_2B)^2}\left(q_1 - \frac{a_2f_2B}{1-B}\right) + \frac{1}{2}\frac{(1-B)}{B}(1+L^{-1})(k\lambda_{t-1}+S_t-\tilde{\varepsilon}_t)$$

According to the rule given in (A68), the rate of growth of the contract wage is set below the mean productivity growth rate if the cumulative effects of speculation and exogenous shocks, represented by the second terms, have run down reserves sufficiently far below the trend path, thus implying unemployment. The first term can be either positive or negative, depending on the magnitude of parameters. If the union applies relatively greater weight to nominal wage increases than expected unemployment, then the term is positive. Otherwise, the opposite is true.

We proceed by deriving the solution for the time path of r_t, or the logarithmic deviation of reserves from the trend path. If (A68) is substituted into the general equation for reserves, equation (A33), the following results:

(A69) $$r_t = \frac{\Gamma}{[1-(1-a)L]}\left\{\frac{B}{(1-B)}u_t + \frac{1}{1-B}\eta_t - V_{0t} - \frac{1}{2}(1+L^{-1})(k\lambda_{t-1}+S_t-\tilde{\varepsilon}_t) + k\lambda_{t-1} + k[f(E_{t-1}r_t-r_{t-1}) + \frac{1}{2}f'(E_{t-1}r_t-r_{t-2})]\right\}$$

where $$V_0 = \frac{(1-B)}{2q_3f_2^2B}\left(q_1 - \frac{q_2f_2B}{1-B}\right)$$

$$\Gamma = \gamma_R(\gamma_I+\gamma_C)af_2$$

The expectation of (A69) formed at t-1 is:

(A70) $$E_{t-1}r_t = \frac{\Gamma}{[1-(1-a)L]}\left\{-V_{0t} - \frac{1}{2}(1+L^{-1})(k\lambda_{t-1}+S_t) + k\lambda_{t-1} + S_t\right\}$$

Note that in (A69) and (A70) we have time-indexed V_0, as it is a permanent shock with a constant magnitude due to union preferences rather than a constant.

Equation (A70), comparable to equation (A35) in the analysis with the ad hoc wage-setting rule, seems a relatively complicated difference equation in the present form. Fortunately, it can be simplified by observing some properties. First, by approximation rule (A31), $k\lambda_{t-1}+S_t = k\lambda_t$. Second, $L^{-1}k\lambda_{t-1} = k\lambda_t$ as well. Third $L^{-1}S_t = S_{t+1}$. Using these facts in (A70), it simplifies to:

$$\text{(A71)} \quad E_{t-1}r_t = \frac{\Gamma}{[1-(1-a)L]}\{-V_{0t} - \frac{1}{2}S_{t+1}\}$$

$$= \frac{\Gamma}{[1-(1-a)L]}\{-V_{0t} - \frac{1}{2}k[f(E_{t-1}r_{t+1}-E_{t-1}r_t) + \frac{1}{4}f'(E_{t-1}r_{t+1}-E_{t-1}r_{t-1})]\}$$

Note, in particular, that no λ_{t-1} appears in (A71). This implies that the union's contract wage-setting policy completely eliminates the systematic effects of past speculation on the unemployment rate. Instead, there is an autonomous shock V_{0t} in the equation, which will have systematic effects on reserves and unemployment. The term V_{0t} can, in a sense, be interpreted as the money illusion component in wage contracting.

Equation (A71) is the second order difference equation that defines the dynamics of reserves under the feedback wage-setting policy. It is exactly analogous to equation (A36) in the case of the ad hoc union policy. The solution procedure in solving (A71) is the same as in solving (A36) and will not be repeated here. The solution for reserves implied by (A71) is the following:

$$\text{(A72)} \qquad r_t = \left[\frac{H_1}{(1-K_1L)} - \frac{H_2}{(1-K_2L^{-1})}\right](T'\left[\frac{B}{(1-B)}(u_t-\tilde{u}_t) + \frac{1}{(1-B)}(\eta_t-\tilde{\eta}_t)\right]-\Gamma V_{0t})$$

where

$$K_{1,2} = \frac{\Gamma k\left[f+\frac{1}{2}f'\right]}{1-a+\frac{1}{4}\Gamma kf' \pm \sqrt{(1-a+\frac{1}{4}\Gamma kf')^2-2(1-\frac{1}{2}\Gamma kf)(\Gamma k(f+\frac{1}{2}f'))}}$$

$$\equiv \frac{D_0}{D_1 \pm \sqrt{\left[D_1^2-D_2\right]}}$$

$$H_1 = \frac{K_1}{K_1 - K_2} < 0$$

$$H_2 = \frac{K_2}{K_1 - K_2} < 0$$

$$T' = \frac{\Gamma}{1 - \frac{1}{2}\Gamma kf'} > 0$$

The corresponding solution for the unemployment rate is:

$$\text{(A73)} \qquad un_t = -\frac{af_2}{\Gamma}\left[\frac{H_1}{(1-K_1L)} - \frac{H_2}{(1-K_2L^{-1})}\right](T'\left[\frac{B}{(1-B)}(u_t-\tilde{u}_t) + \frac{1}{(1-B)}(\eta_t-\tilde{\eta}_t)\right]-\Gamma V_{0t})$$

Structurally, the solutions (A72) and (A73) differ from the corresponding solutions (A47) and (A48) derived under the ad hoc wage-setting rule in two respects. First, in (A72) and (A73), there is one lag operator and one lead operator, with the latter providing the potential self-fulfilling speculation, as will be seen later. This reflects the fact that wage-setting neutralizes past destabilizing shocks, but cannot eliminate unanticipated shocks that occur within the contract period. Second, there is a potential autonomous tendency in reserves towards the devaluation or revaluation limits caused by the autonomous component V_0 in contract wage-setting.

The technical properties of the dynamics of reserves as well as the existence of the speculative cycle can be shown in exactly the same way as in the previous section A2.2. Therefore, we only list here the important properties and results together with some remarks.

1 Equations (A72) and (A73) are very different in nature from the corresponding equations (A47) and (A48) in the ad hoc wage-setting case. For the parameter values stated above on p. 258, it is even possible that the dynamics in reserves could be fluctuating. To guarantee nonfluctuating dynamics in (A72) and (A73), relatively small values of parameter a would be needed, given the other parameters. Thus rigid adjustments in portfolios guarantee plausible properties for the dynamics of the feedback wage-setting model. The problems arising in connection with plausible dynamics reflects the fact that, in principle, only one period shocks can trigger speculation when the wage is set for two periods at a time. Past shocks are neutralized in the new wage contracting. If wages were set for several periods at a time, these problems would probably disappear, but, analytically, we are restricted to two-period wage-setting.

2 If speculative motive is made stronger in the economy by changing the basic parametrization, the feedback wage-setting model becomes closely analogous to the ad hoc wage-setting model. The condition in the feedback wage-setting model for the speculation to become self-fulfilling is the following:

$$\text{(A74)} \qquad K_2 \geqslant 1 \iff \tilde{\Gamma} k = \frac{2}{af} - 1$$

If the parametrization is changed so that $\gamma_R = 50$, $\gamma_I = 0.5$, $\gamma_C = 0.4$, $k = 0.2$, $f = 0.25$, $a = 0.8$, then the condition is satisfied and speculation is self-fulfilling. In the economy with the above specifications, the probabilities of exchange rate jumps are always relatively high and capital gains from speculation large.

3 Condition (A74) can also be solved for the approximate reserve gains or losses by a similar approximation procedure as was applied above in (A59 - A61). The approximate critical reserve limits implied by the procedure (with the same initial point used in the Taylor expansion) are the following:

$$\text{(A75)} \qquad K_2 \geq 1 \iff \frac{r_{t-1}}{\sigma_r} \geq \tilde{\beta}_0 \frac{2}{a(\tilde{\Gamma}k+1)} - \tilde{\beta}_1$$

$$\frac{r_{t-1}}{\sigma_r} \leq -\tilde{\beta}_0 \frac{2}{a(\tilde{\Gamma}k+1)} + \tilde{\beta}_1$$

where $\tilde{\beta}_0 = \sqrt{\Pi} e$

$\tilde{\beta}_1 = \frac{3}{\sqrt{2}}$

Conditions (A75) indicate the fact that the feedback wage-setting economy is less likely to experience self-fulfilling speculation than the ad hoc wage-setting economy. In (A75), the structural part is smaller in magnitude than in (A61), but the coefficient $\tilde{\beta}_0$ is much larger than β_0 in (A61). Coefficients $\tilde{\beta}_1$ and β_1 are roughly of the same magnitude. Hence, limits (A75) imply larger accumulated losses of reserves than limits (A61) in most cases. However, if the shock-multiplier Γ and the jump size parameter k are very large, implying very strong effects of speculation on the economy, then the forward-looking feedback wage-setting economy may require smaller limits.

4 The economy with the feedback wage-setting rule is symmetric about the full employment equilibrium. This economy could also be made asymmetric by introducing the notion of labor mobility defined in section A2.2. Such a change would make the present economy more prone to devaluation cycles than revaluation cycles.

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