Labour Markets, Wage Indexation and Exchange Rate Policy

By
JOUKO VILMUNEN
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Contents

1 Introduction

This study presents and discusses an extension to an open economy of the macrocontracting approach to the Phillips curve of Gray (1976) and Fischer (1975, 1977a and b) for the purpose of analyzing the interaction between foreign exchange intervention and wage indexation. As the labour market model incorporated in this type of approach is only weakly justified by casual reference to actual contracting processes, an attempt is made to scrutinize the wage formation process more rigorously by placing wage determination within a (partial equilibrium) union-firm bargaining framework. The modelling strategy applied here involves a novel feature in that the bargaining approach explicitly allows for exogenous uncertainty, which will be seen to have some important implications for the steady state behaviour of the economy under consideration. To derive a more tractable bargaining model, explicit functional forms for the union utility function and firms' production technology are assumed so that the resulting wage equation can be employed relatively easily in the type of macroframework used in much of this study.

Bearing in mind that, in the 1970s, the flexible wage rational expectations macromodel generated some very strong policy neutrality results, it is not surprising that economists have sought to construct macromodels that somehow justify a role for stabilization policy. The Gray—Fischer macrocontracting framework can be seen in this light, but there are no compelling reasons to invoke such economic-historical motives for the approach. As Fischer (1977b) forcefully argues against Barro (1976), the assumed wage and employment processes incorporated in the approach are at least seemingly consistent with actual labour market practice. Further, this approach can only partially justify a role for stabilization policy, since even though the approach is empirically valuable, it is theoretically largely *ad hoc* in the sense that the wage and employment patterns in these macrocontracts cannot be easily rationalized on the basis of the first principles of rational optimizing behaviour as regards the agents involved.

But, by the same token, our knowledge of wage formation is still very incomplete. This applies to attempts to provide solid choice-theoretic foundations for the macrocontracting models as well as for the microeconomics of wage formation itself. For example, the prediction generated by models in the optimal labour contract literature as to the efficiency of the contract outcome are highly sensitive to the information

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asymmetries involved as well as to the restrictions on the relative risk aversion of workers *vis-à-vis* that of firms (see e.g. Haley, 1990, p. 129, Nickell, 1990, p. 408). Further complications and uncertainties arise when we seek a theoretical rationale in e.g. the optimal contract literature for the policy predictions generated by these macrocontract models (see e.g. Cooper, 1987, pp. 59—65).

In first part of this study, we shall stay strictly within the macro-contracting framework in order to compute and characterize the relevant policy optima of foreign exchange intervention and wage indexation. The approach is simple enough and leads us to characterize the policy mix as a mechanism for dealing with the signal extraction problem introduced by imperfect information on the underlying shocks to the economy. Later, we shall make a distinct departure from the contracting approach by adopting a union-firm bargaining approach to wage determination. The focus of the analysis changes, since policy non-neutralities emerge as a result of the assumed wage formation under uncertainty. Specifically, the study is organized as follows.

The study is divided into three distinct parts. Chapter 2 remains stictly within the standard macrocontracting approach. It begins with a description and discussion of the labour market model incorporated in the macroframework of Gray and Fischer (sections 2.1 and 2.2). Then a "representative" small open economy model producing one internationally traded good is constructed in order to compute and characterize the optimal policy mix of foreign exchange intervention and wage indexation within Gray's framework for the determination of optimal wage indexation. The relevant policy optima for the case of a single instrument is also computed and discussed. Section 2.3 introduces endogenous terms-of-trade changes, but only to the extent that that they illustrate how Gray's model of the determination of optimal wage indexation can be extended to this case as well. The novel features here is the suggestion that wages be indexed to a price index derived from the labour market equilibrium condition. Details of the computation and characterization of the relevant policy optima are relegated to the references cited therein.

Section 2.4 provides a further extension of the contracting approach to a two-sector economy (nontraded and traded goods). We extend Aizenman's (1986) analysis to the case where the country in question is not constrained in its choice of the exchange rate regime to flexible exchange rates. Once again the relevant policy optima are computed and discussed, but now the focus of the analysis is on the dependence of the (stabilization) policy regimes on the degree of openness of the economy, or to put it differently, the dependence of the policy regimes on the strength of the relative price adjustment generated by the sectoral

structure of the economy. The general conclusion drawn is that monetary policy is not a perfect substitute for relative price changes in terms of aggregate adjustment of the economy.

After summarizing the main findings of chapter 2 and criticizing the labour market model incorporated in the macrocontracting approach, chapter 3 sets out to model wage formation from an entirely different perspective. The Nash bargaining solution to labour market bargaining between employee union(s) and firms is outlined and the basic bargaining approaches to wage and employment determination, i.e. efficient bargains and labour demand equilibrium models, are described, the chief motivation being that the existence of (strong) unions in modern labour markets should be formally incorporated in a model of wage and employment determination. The description ends with the suggestion that a "right to manage" model of wage and employment determination is a viable alternative as a model of actual bargaining processes. Section 3.3 builds upon explicit functional forms and restrictions on the static Nash bargaining solution under uncertainty to derive a wage equation suitable for the macroanalysis of chapter 4. The novel feature in section 3.3 is the introduction of uncertainty in a "right to manage" model, which, coupled with the possibility of indexed wages, seems to have important implications for the steady state behaviour of the economy. The principal implication is that non-neutrality of monetary policy, in the form of foreign exchange intervention, with respect to steady state employment and output emerges as a consequence of (exogenous) uncertainty impinging on the bargaining outcome.

Thus, our model will not possess the natural rate property. The contract wage process in our bargaining framework under uncertainty explicitly models the feedback from the expected "state of the economy" in the subsequent contract period to the contract wages themselves, and this generates the dependence of the steady state of our open economy on the parameters of the money and actual wage process. This is in sharp contrast to the standard models of chapter 2, where this feedback is abstracted away by imposing the natural rate on the model through the choice of the contract wage as the solution to the *ex ante* non-stochastic equilibrium in the labour markets[1]. The natural rate property in standard models clearly conveys the message that the authorities can pursue, at least approximately, welfare improving stabilization policy without imposing (notable welfare) costs on the economy in terms of the steady state level of aggregate economy activity. This is a typical feature of

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[1] Together with the log-linear macromodel and i.i.d. shocks, (stable) rational expectations are static in these models.

models incorporating the natural rate, in that they postulate a "core equilibrium" for the private economy, which cannot be affected by public policy actions, and interventionist policy can at best influence the variability of the (aggregate) economy around this equilibrium level[2]. The modelling strategy in chapters 3 and 4 clearly suggests that, as far as policy implications are concerned, we cannot measure the desirability or otherwise of policy actions merely in terms of the stability of the aggregate economy, since these policy actions will impinge on the steady state level of aggregate economic activity.

The above-mentioned policy implications are further explored in chapter 4, where the macromodel of chapter 2 is restructured so as to incorporate wage and employment determination from the static Nash bargaining framework of section 3.3. Assuming the degree of wage indexation to be exogenously given, the analysis in chapter 4 starts by discussing the relationship between exchange rate variability and steady state real wages, employment and output, with a heavy emphasis on fixed and flexible exchange rates. Tentative conclusions are drawn as to the effects on the steady state of introducing more competitive labour pricing, since the logic of the model suggests that there may exist a trade-off between mark-up and risk pricing as labour pricing becomes more competitive, especially under non-indexed wages. There thus seems to be an interesting relationship not only between indexation and aggregate price risk, as effects due to uncertainty will be called, but also between indexation and competition.

Chapter 4 continues by extending Gray's model of optimal wage indexation to the present case of wage indexation and foreign exchange intervention. This time, however, the conventional loss function representing policy makers' preferences in Gray's analysis has to be respecified, since a policy mix of wage indexation and foreign exchange intervention has direct steady state effects, which must be duly accounted for in policy makers' preferences. Chapter 4 suggests that a reasonable approximation to the policy makers' objective in the present context consists of maximizing steady state output (or employment) subject to the constraint that losses due to imperfect information are as small as

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[2] Many of the so-called real business cycle models are explicitly constructed on the premise of the natural rate (see e.g. Kydland & Prescott, 1982, Lucas's lectures, 1987, McCallum's survey, 1986). Whereas e.g. Lucas concludes on the basis of a Kydland–Prescott type dynamic macromodel (ibid. p. 105) that welfare gains from activist stabilization policy are likely to be small, Baily (1978, p. 47), by employing a different approach, forcefully argues that "private economic behaviour will be altered by the existence of active policy". Needless to say, the critical and highly substantive issue is the overall (i.e. theoretical and empirical) validity of the Nntural rate property.

possible. This enables us to derive and analyze the relevant policy optima of wage indexation and foreign exchange intervention in much the same wage as in more standard open economy models of optimal wage indexation and foreign exchange intervention. Finally, chapter 5 provides a summary, and considers possible important extensions of the themes taken up in this study.

2 Wage Indexation and Foreign Exchange Intervention in an Open Economy: the Gray—Fischer Approach

2.1 Introduction

In this chapter we shall review the basic macroeconomic theory of wage indexation and foreign exchange intervention by constructing a simple stochastic general equilibrium macromodel incorporating the dual assumption of short-run wage rigidity and a foreign exchange intervention rule on the part of the monetary authority. The analysis based on this single commodity macromodel follows the standard extensions of the Gray—Fischer framework or macroeconomic contracting approach to open economies (see e.g. Aizenman & Frenkel, 1985a, b and Turnovsky, 1986). After section 2.3 we shall comment briefly on how the model's predictions change once we relax the assumption of perfect commodity arbitrage (i.e. purchasing power parity). This also gives us a chance to discuss the problem of the appropriate price index to which wages should be indexed.

The analysis of the single commodity macromodel is followed by an extension of the macroeconomic contracting approach to an open economy consisting of two sectors, an open sector and a closed sector. We shall extend Aizenman's (1985) analysis of wage indexation under flexible exchange rates by allowing the exchange rate system to be chosen by the monetary authority via an intervention rule. The analysis focuses on the effects of relative price movements on wage rigidity and foreign exchange intervention from the point of view of macroeconomic stability. Since the variability of relative prices is directly related to the openness of the economy, the analysis proceeds in terms of how the openness of the economy affects indexation and intervention and the relationship between the two.

Since the labour market model in these two macroeconomic models is the same, we shall start the analysis by describing labour market behaviour. The chapter concludes with an evaluation of the macroeconomic contracting approach to questions concerning macroeconomic stability.

2.2 The labour market model

Labour market behaviour in the Gray—Fischer model is contract-based. However, care should be taken in interpreting these labour contracts, at least from the point of view of the modern microeconomic theory of labour contracts (see e.g. Azariadis, 1975). The labour contracts in the model are extremely simplified; the determination of wages and employment is not based on individual optimization or, less extremely, the "microeconomics" of the wage and employment rule are not explicitly spelled out.

The contracting parties are a risk neutral employer(s) and employee(s). This assumption abstracts completely from allocation of risk as part of (optimal) labour contracts, which is so prevalent e.g. in implicit contracts à la Azariadis. The main justification for this assumption seems to be that the Gray—Fischer analysis is concerned solely with the macroeconomic stability implications of wage rigidity; the determinants of wage rigidity, such as risk sharing between a risk averse employee and a risk neutral employer, are not specified but stem from the existence of labour contracts.

Contracts are drawn up before uncertainty is resolved, i.e. before random shocks affecting production and employment decisions are realized. Since wages and employment are not determined by spot market equilibrium, contracts have to specify how wages and employment are determined. The Gray—Fischer model of labour markets assumes that, given wages, employment is determined by the demand for labour, i.e. by the marginal productivity condition. Actual wages are determined by two factors; the base wage together with an indexation scheme, whereby wages are indexed to changes in the price level(s). The base wage is determined by the *ex ante* non-stochastic equilibrium in the labour markets; let W_t denote nominal wages in period t, P_t product prices in period t and C_t consumer prices in period t. Then the base or contract wages W_t^c, determined at the end of period t-1 for period t, solve

$$N^d[W_t^c/P_0] = N^s[W_t^c/C_0],\qquad(2.1)$$

where N^d and N^s denote, respectively, demand for and supply of labour, and where $_0$ emphasizes that the equality of demand and supply is evaluated at non-stochastic values of the relevant variables, i.e. at values where all the random variables take the value zero. Given contract wages, actual wages are given by the wage formula

$$W_t = f[P_t/P_0, C_t/C_0; W_t^c, b_1, b_2],\qquad(2.2)$$

where the function f is homogenous of degree b_1+b_2 in (P_t,C_t), or homogenous of degree zero in (P_t,C_t,P_0,C_0). The (exogenous) parameters b_i are indexation coefficients.[1] Given the structure of the labour market model, the focus of the analysis is on how the introduction of uncertainty affects the behaviour of the aggregate economy or how uncertainty is transmitted via the wage formula (2.2) into aggregate economic fluctuations, particularly fluctuations in aggregate employment and output.

Given this kind of normalization, it is immediately evident that the non-stochastic equilibrium of the economy is completely independent of indexation arrangements in the labour markets (and foreign exchange intervention for that matter).[2] We shall return below (see section 2.4) to this strict separation of level and variability of an aggregate variable around that level, which is characteristic of a model incorporating an assumption on the natural rate.

For now, we take the labour market model as given and analyze the behaviour of the aggregate economy subject to short-run wage rigidity induced by the labour markets. The main emphasis in what follows will be the interaction between wage rigidity and foreign exchange intervention from the point of view of macroeconomic fluctuations defined generically as fluctuations in aggregate output around the full information equilibrium output.

2.2.1 Comments on the Gray—Fischer labour market model

Before turning to the analysis of the macroeconomic implications of wage rigidity and foreign exchange intervention, we briefly discuss the criticism directed against Gray—Fischer labour contracts. Although these

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[1] Note that in the Gray—Fischer analysis the justification for wage indexation is not spelled out; potential indexation arrangements are taken as an "institutional datum". That these models do not provide any rationale for wage indexation stems from the structure of the macromodels employed, in which only labour markets do not clear. According to Blinder (1989), the rationale for the existence of indexation in contracts stems from the "inherently incomplete market structure", whereby market opportunities for hedging are limited because of the imperfectness or incompleteness or even non-existence of certain (insurance) markets. This kind of analysis should, however, be interpreted with care, since (indexed) labour contracts can go a long way towards explaining the non-existence of certain markets, notably insurance markets for labour, as e.g. Wright (1989) persuasively argues.

[2] Permanent shifts in e.g. exchange rate policy can be envisaged to have these steady state effects, but the point is that these shifts cannot be scrutinized within the Gray—Fischer macromodel.

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sticky price models provide a very simple approach to analyzing the interaction of the real sector and financial markets, the labour market behaviour embedded in them seems to have some serious drawbacks and to be at odds with rational maximizing individual behaviour. Thus to find some (theoretical) support for these labour market constructs would be more than comforting.

As argued above, the Gray—Fischer contracts consist of two (interrelated) elements, the wage rule and the employment rule, which are related to each other by the neoclassical labour demand function. Actual wages are assumed to be determined by the wage indexation formula, while contract or base wages are modelled as being determined by the *ex ante* labour market equilibrium. Thus, after shocks to the economy have occurred and actual wages been computed, employment is determined by the demand for labour.[3]

As such, these macrocontracts[4] are *ad hoc*, and focus on the consequences of specific contracting structures without providing any firm justification for them. An obvious remedy for this is to develop the microeconomic base for macrocontracting models. This has proved difficult, especially as far as the employment rule is concerned. The reason is intuitively obvious. The optimal (implicit) contracts literature[5] describes state-contingent compensation levels and, independently, state-contingent employment levels. The crucial point however, is that the employment rule need not correspond to points on either the labour supply or labour demand curves. Thus, in terms of employment rules, the optimal contracts literature does not seem to support the assumption on employment determination contained in the Gray—Fischer model. The same point is made by Barro (1977).[6]

Cukierman (1980) explores the significance of the labour demand employment rule for the implications of essentially Gray's macromodel

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[3] Thus the model implicitly posits no constraints from the side of labour supply (e.g. unemployed workers).

[4] This aspect of the Gray–Fischer contracts should be emphasized, as is evident from the title of Gray's original contribution (1976): Wage indexation; a macroeconomic approach.

[5] See Azariadis (1975, 1978), Baily (1974), Gordon (1974) and Grossman & Hart (1981, 1983). For surveys, see Rosen (1985), Stiglitz (1986) and Haley (1990).

[6] Gottfries (1990) analyzes optimal labour contracts in a simple macromodel with insiders and outsiders. When insiders have job security this has important implications for labour contracts. Optimal labour contracts leave employment to be determined by the firm. Note further that in his model small costs associated with writing state-contingent labour contracts can be prohibitive for indexed contracts.

by adopting an apparently *ad hoc* employment adjustment rule in states of labour market disequilibrium, according to which actual employment is a weighted average of the demand for and supply of labour:

$$N = \Theta N^d + (1 - \Theta) N^s. \tag{2.3}$$

While arbitrary, it preserves linearity in a rational expectations model and includes as a special case the min-rule for actual employment. Furthermore, a simple aggregation argument can be invoked to give (2.3) as an aggregate employment function; Θ is the proportion of those labour "submarkets" where excess supply of labour prevails and employment is determined by the labour demand function. Incorporating (2.3) into a macromodel, Cukierman concludes that the employment function is critical for Gray's results concerning the degree of wage indexation in the presence of specific exogenous shocks impinging on the economy.[7] This is intuitive enough, but it is difficult to imagine what sort of pressure Cukierman's specification and results put on Gray's formulation, given that both are based on "equally" arbitrary formulations.

As far as the rule for wage determination is concerned, there are at least two elements contained in it, i.e. indexation or the appropriate indexation' rule, and the wage patterns implied by the particular indexation rule employed in these contracts. As for the former, the problem arises since typically only price level information is utilized in adjusting wages in these models. This may imply that information, including information provided by the model at hand, is inefficiently used or that implicit restrictions on information utilization are not work. This may sound dubious, especially in the context of rational expectations. This is the point made by Karni (1983), who notes that an indexation rule dominating that of the Gray—Fischer rule can be found, once the contracting parties are allowed unrestricted use of the information of the model they entertain at the time of contracting.[8] These indexed contracts

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[7] This feature of Gray's model is somewhat dependent upon the specification of the equilibrium in labour markets as well as upon the highly aggregative structure of her model, since e.g. Gray (1983) demonstrates that full wage indexation in the presence of only demand shocks is no longer a desirable state of affairs if, in a multisector economy, there are also sector-specific disturbances present in the economy. To continue, the destabilizing effects on aggregate output of full wage indexation in the presence of only supply shocks seems to vanish in e.g. efficiency wage models; see Waller (1989).

[8] Essentially, this implies that the underlying shocks to equilibrium real wages become *de facto* observable via the use of the equilibrium condition for labour markets, which is sufficient to generate equilibrium real wages via the indexation rule. For an application of this logic to wage indexation in an open economy, see Vilmunen & Haaparanta (1988).

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may, however, be highly complex and even unenforceable in practice, since information on variables relevant for the wage indexation rule becomes available with a different lag, so that the simple price indexation rule may be an "optimal compromise".[9]

Karni's argument is directly relevant for Barro's (1976) analysis of indexation in a rational expectations model. In a Lucas-type economy, where the Phillips curve is due to confusion or misperception on the part of the rational agents involved between local, market-specific shocks and aggregate shocks, Barro considers *ex post* price indexation, whereby prices are *ex post* adjusted for a number of observable state variables. The model's information structure is rather delicate, generating imperfect information typical of Lucas's (1972) island economies; information flows are "noisy", being contaminated by a mixture of local and aggregate shocks in a way that makes it impossible for agents to disentangle local shocks from information available to them.[10] What is crucial, however, is that the resulting rational expectations equilibrium fully reflects this lack of perfect information; it is an equilibrium, albeit one of imperfect information. There is no way agents can improve upon their state of affairs, given the information available.

In this set up, Barro demonstrates that the type of indexation he considers does nothing to clarify the information signals relevant for real decisions. In this sense indexation is inconsequential or redundant in a rational expectations model; it only gives a rule for *ex post* transfer payments across markets. Barro's analysis has problems, many of them detailed in Karni (ibid., pp. 288–289). The main practical problem is that the length of the contract coincides with the time lag in obtaining information about aggregate variables (the price level in Barro's specific instance). This can be objected to on empirical grounds, since the length of most wage contracts exceeds the time lag involved in obtaining information on aggregate variables (Karni, ibid., p. 288).

At a more theoretical level, Barro's analysis simply begs the main question, as far as the role of wage indexation in alleviating the friction introduced by labour contracts is concerned. Barro's indexation duplicates the equilibrium that would obtain if labour services were contracted for after the occurrence of the stochastic disturbances. As argued above, in

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[9] The wording is taken from Blanchard (1979). He argues that in general it would be desirable to correct indexed wages for changes in relative prices, but if these corrections are available only at high costs, it may be an "optimal compromise" to index wages only to the price level. Further, the correlation between the price level and individual prices may be so high as to reduce the need for additional adjustment of wages due to changes in individual prices.

[10] Lucas (1973) summarizes some international evidence bearing on his island model.

this instance the relevant equilibrium notion fully reflects the imperfectness of information; nevertheless it is a general equilibrium that corresponds to spot labour markets. The perception that not all information is current has implications for the nature of the frictionless equilibrium, but is quite irrelevant for the issue of the role of indexation in alleviating the frictions due to labour contracts (Karni, ibid., pp. 288–289). In this sense, then, Barro's analysis seems to have little bearing on the original problem posed by the Gray—Fischer analysis.

Now, what of the second question posed above, that is can we find any support in the optimal contract literature for the wage patterns implied by the Gray—Fischer macrocontracts? In this respect, Cooper (1987, pp. 60 - 65) tentatively gives a positive answer by constructing an overlapping generations model in which an equal number of risk neutral and risk averse agents are born each period. Uncertainty in this economy is generated by both real and nominal shocks to the system as in Lucas (1972). Under certain conditions concerning the expectation of the inverse of the next period's price level conditional on the current period price level and workers' attitudes towards risk, compensation (wage) patterns vary with the price level as predicted by the macrocontracting models of Gray and Fischer, i.e. "the degree of indexation" falls as the variance of the real productivity shock increases relative to the variance of the monetary shock. This sounds comforting, but we should be ready to acknowledge, that much research on the microfoundations of macrocontracts still remains to be done before we will be able to say more definitive and conclusive about the robustness of the predictions generated by these macromodels of wage and employment contracts.

So a reasonable conclusion from the above analysis seems to be as follows. The macrocontracting models of Gray and Fischer seem to contain many of the ingredients encountered in actual labour contracts, albeit in a highly simplified form, but justifying these specific contract structures in terms of the first principles of rational optimizing behaviour has proved a difficult task. The crucial problem is not to construct optimal contracts, optimality being defined in some precise manner. Rather, the relevant question is "why are contracts encountered in practice astonishingly simple?", a question motivating e.g. Gottfries' (1990) work. It is this question to which an answer should be given on the basis of the first principles, if we are also to have empirically meaningful theories of wage and employment determination in a contract economy. Further, these foundations seem to be almost a *sine qua non* for a meaningful welfare analysis of the implications of interactions between policy and contract structures.

2.3 A small open economy subject to wage rigidity and foreign exchange intervention

The framework presented below is a standard stochastic IS-LM model of a small open economy subject to short-run wage rigidity induced by labour contracts. The model is widely used in the indexation-intervention literature and is summarized in e.g. Aizenman & Frenkel (1985a, b), Turnovsky (1984, 1986) and Vilmunen (1989). The log-linear model consists of the following equations:

$$\ln Y_t = \alpha \ln N_t + \mu_t, \quad 0 < \alpha < 1 \tag{2.4}$$

$$\ln N_t^d = \beta(\ln P_t - \ln W_t) + \beta\mu_t, \quad \beta = 1/(1-\alpha) > 1 \tag{2.5}$$

$$\ln N_t^s = \varepsilon(\ln W_t - \ln P_t) \tag{2.6}$$

$$\ln W_t = \ln W_t^c + b(\ln P_t - \ln P_0) \tag{2.7}$$

$$\ln M_t - \ln P_t = \ln Y_t - k i_t + u_t, \quad k \geq 0 \tag{2.8}$$

$$\ln M - \ln M_0 = -\delta(\ln S_t - \ln S_0), \quad \delta \leq \infty \tag{2.9}$$

$$\ln P_t = \ln P_t^* + \ln S_t \tag{2.10}$$

$$i_t = i_t^* + \ln S_{t,t+1} - \ln S_0, \tag{2.11}$$

where
Y_t = real aggregate output in period t
N_t = aggregate employment in period t
P_t = price of domestic aggregate output in period t
W_t = actual nominal wages in period t
M_t = nominal money stock in period t
S_t = exchange rates in period t (price of foreign currency in terms of domestic currency)
i_t = domestic interest rates in period t
P_t^* = foreign price level in period t
i_t^* = foreign interest rates in period t
M_0 = long-run target level of the money stock set by the monetary authority
S_0 = long-run target level of exchange rates held by the monetary authority
W_t^c = contract wage for period t negotiated at the end of period t-1
P_0 = non-stochastic equilibrium value of the domestic price level.

Equation (2.4) gives short-run aggregate output as a function of labour input. The production function displays diminishing returns and μ represents aggregate shocks to productivity, whose distributional characteristics together with other random processes will be specified shortly. Demand for labour is given by equation (2.5). It is a derived demand based on profit maximization by a representative competitive firm. Demand for labour depends negatively on the actual (log of the) real wage $\ln W_t - \ln P_t$. Furthermore, demand for labour is subject to random shocks due to productivity disturbances.

Supply of labour, equation (2.8), is optional, and is more properly called *ex ante* labour supply. It is needed for the benchmark case of full information equilibrium, where spot market equilibrium prevails in every market. Supply for labour is a stable function of the real wage, and is not subject to any preference or shocks. Since the spot labour markets are missing or not active, the labour supply function becomes redundant for the contract economy once the exogenous shocks are realized.

Actual wages are driven by the wage scheme (2.9), according to which actual wages equal contract wages corrected for changes in the price level due to random shocks. The elasticity of wages with respect to the price level around the non-stochastic equilibrium is the indexation coefficient b. When b=0 nominal wages and when b=1 real wages are completely rigid. Needless to say, the wage formula is the critical element in the model for understanding the results derived from the model. It is critical in its form, since wages are assumed to be indexed only to the price level. For example, Karni (1983) has demonstrated that the Gray–Fischer contracts are dominated by a contract where wages are also indexed to real income. Optimal indexation would involve no efficiency loss, since the dependence of wages on real income would make the underlying productivity shock observable (via the labour market equilibrium). The main motivation for using this rather simplistic indexation rule seems to be its popularity in practical indexation arrangements in various countries (see, however, Simonsen's discussion on wage indexation in Brazil, 1983).

Equations (2.8) and (2.9) describe money market behaviour. Money markets are assumed to adjust continuously to equilibrium, an assumption embodied in equations (2.8) and (2.9). Equation (2.8) gives the conventional stochastic LM function according to which demand for real balances depends on real income and interest rates. The parameter k is the semi-elasticity of money demand with respect to domestic interest rates i. We have, for simplicity, set the income elasticity of money demand equal to one. This is a critical assumption in the ensuing analysis.

Equation (2.9) describes the foreign exchange intervention rule utilized by the monetary authority. According to this rule, changes in the money stock around the long-run target level $M^°$ depend on changes in exchange rates around their long-run level $S^°$. The elasticity of money supply with respect to changes in exchange rates, $-\delta$, is the foreign exchange intervention parameter. The above equation implicitly defines a whole spectrum of exchange rate regimes, from fixed exchange rates ($\delta=\infty$) via dirty floating to clean floating ($\delta=0$) and perhaps "over and above" clean floating.

Devereuex (1988) calls this simple intervention rule a "restricted" intervention rule, and justifies its use on the grounds of the existence of transaction costs and the complexity of more sophisticated intervention rules. A more sophisticated intervention rule whereby changes in the money stock depend on a number of observables could easily be adopted. This might be especially recommended, if the main focus of the analysis is on the design of optimal monetary policy from the point of view of macroeconomic stability in an open economy subject to wage rigidity and various stochastic disturbances. Aizenman & Frenkel (1985b) and Turnovsky (1983a, 1986), for example, favour a more sophisticated policy rule in their analysis.[11] This would, however, involve a detailed description of the information flow concerning the timing of events and observability of various variables to the participants in the economy, as Turnovsky's analysis demonstrates. Typically, information on the real variables becomes available with a lag, while asset market information is available almost instantaneously. This enables the monetary authority to improve upon the simple rule above, but we do not wish to refine the structure of the model in this direction, since the main point of the analysis is highlighted when the more restricted intervention rule is

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[11] For a discussion of the design of more sophisticated policy rules and their applications in a macrocontext, see Fischer (1977a), Karni (1983) for closed economies, and Turnovsky (1983a, 1986), Marston (1983), Marston & Turnovsky (1984), and Aizenman & Frenkel (1985b) for open economies. What these studies convey to us is that in many cases it is possible in principle to design a single instrument so that the efficiency losses due to suboptimal labour markets can be reduced considerably, even eliminated completely. This means that the other instrument is redundant or can be used for other objectives. One of the instruments is not, however, redundant if it provides information about shocks influencing equilibrium real wages not contained in the other instrument. Granted that the design of such a sophisticated set of policies is possible, it is still relevant to ask why we so seldom observe them in practice. For example, evidence on labour contracts suggests that the simple price indexation rule applied in this study, perhaps adjusted for productivity (see Simonsen, 1983), is the best we observe. Clearly, the implementation of more sophisticated policy rules hinges upon the costs associated with the use of these rules; do the marginal gains in efficiency outweigh the marginal costs?

applied. We shall, however, return to the question of the availability of information to agents shortly.

Equations (2.10) and (2.11) are the two parity conditions widely used in open economy macroeconomic models. Under the purchasing power parity (PPP) rule, the level of exchange rates is such that the costs of a representative commodity bundle are equalized throughout the world. It is an arbitrage condition, which finds its application in well developed and internationally intergrated commodity markets. A small open economy takes its prices as given; i.e. they are determined in international commodity markets. The uncovered interest parity condition (UIP) (2.11) can be supported by a similar arbitrage argument as is applied to risk neutral investors in well developed and internationally intergrated asset markets. According to UIP, domestic interest rates are equal to foreign interest rates plus a premium for an expected depreciation of the domestic currency. Of the two parity conditions, it is PPP which is most consistently rejected in empirical studies.[12] This does not mean that the debate PPP is over, since estimating and testing PPP involves a whole range of problems ranging from data quality and selection of proper price indexes to the specification of the equation to be estimated and estimation and testing methodology itself. The inclusion of PPP along with UIP leads to a very tractable macromodel, as Marston (1983) has argued, and it is advisable to include at least one of them in a macromodel in order to maintain analytical tractability.

Expectations concerning exchange rates, as well as other variables, are rational. Thus

$$X_{t,t+s} = E[X_{t+s} : I_t]$$

for all variables and $s \geq 0$ and t, where I_t denotes information available to agents in period t. The information set I_t includes the structure of the model and the history of all variables dated t and earlier. Muth's (1961) expectations formation hypothesis is the norm in modern macroeconomics, and it derives its authority from the idea that information is a scarce commodity and thus subject to optimization. In general, the adoption of rational expectations may involve major theoretical and

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[12] This applies to both the absolute version and the relative version, according to which changes in exchange rates are such that changes in price levels are equalized around the world. For a discussion of these issues, see e.g. Dornbusch (1985); for a discussion of time series characterizations of real exchange rates and testing of versions of PPP, see Corbae & Ouliaris (1988), Enders (1988), Giovannetti (1988) and Taylor (1988), to mention just a few.

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computational problems.[13] This is one reason why assuming a linear structure in one's model is so important.

Finally, equation (2.12) specifies the stochastic structure of the model according to which shocks to foreign prices and interest rates and to the domestic money market and productivity form a stationary serially uncorrelated vector whose components are also uncorrelated with each other:

$$\ln P_t^* = \ln P_0^* + v_{1t}$$

$$i_t^* = i_0^* + v_{2t}$$

$$(2.12)$$

$$(\mu_t, u_t, v_{1t}, v_{2t})' \sim N(0, \Sigma)$$

$$\Sigma = \text{diag}[\sigma_\mu^2, \sigma_u^2, \sigma_1^2, \sigma_2^2],$$

where 0 is a 4-vector of zeros. The model thus contains four random shocks, two of which are of domestic origin. Furthermore, the important distinction is between productivity shocks and other shocks, which is also of key importance in Gray's original contribution.

The model determines domestic output, exchange rates, the domestic price level and interest rates as functions of the exogenous stochastic disturbances. We shall, however, proceed by transforming the model, using Gray's normalization to express the model in deviation form. Denoting the deviation of (the log of) a variable from its (log) non-stochastic equilibrium value by a small case letter the model can be written in the following form:[14]

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[13] Although Muth's idea is intuitively appealing, the justification for the concept of rational expectations equilibrium seems to be more complex than Muth's vision of it as a direct extension of the usual rationality postulates for individual economic behaviour, as the recent literature has recognized. Evans & Honkapohja (1990), in analyzing learning, convergence and stability with multiple rational expectations equilibria, argue that this is primarily due to the interdependence of the stochastic process of the endogenous variables and the perceptions of that process formed by economic agents. The complexity of the associated inference problem has potentially far-reaching consequences, one of them being the likelihood of indeterminacy and the possibility of a high dimensionality of equilibria (Evans & Honkapohja, ibid. p. 1; see also the references therein).

[14] The time index has been dropped from the subscript for the sake of simplicity.

$$y = \alpha n + \mu \qquad (2.4')$$

$$n^d = \beta(p - w) + \beta\mu \qquad (2.5')$$

$$n^s = \epsilon(w - p) \qquad (2.6')$$

$$w = bp \qquad (2.7')$$

$$m - p = y - ki + u \qquad (2.8')$$

$$m = -\delta s \qquad (2.9')$$

$$p = p^* + s \qquad (2.10')$$

$$i = i^* + s_{t,t+1} - s. \qquad (2.11')$$

The intepretation of the model runs in terms of deviation from the non-stochastic equilibrium, where the deviation is driven by serially uncorrelated random processes. We can immediately see that indexation and intervention affect the way shocks are transmitted into fluctuations in key macroeconomic variables, notably aggregate employment and output. Before analyzing the transmission of shocks in the economy, we solve for rational exchange rate expectations and full information output.

2.3.1 Exchange rate expectations

We shall work with the model form (2.4') - (2.11'). From the money market equilibrium (2.8'), we obtain, after substituting from equations (2.4'), (2.5'), (2.7') and (2.9') - (2.11'), an equation for the changes in exchange rates

$$[1 + k + \delta + \alpha\beta(1 - b)]s = ks_{t,t+1} - \tau, \qquad (2.13)$$

where $\tau = [u + \beta\mu + (1 + \alpha\beta(1 - b)v_1 - kv_2]$.

From equation (2.13) we obtain a first order difference equation about expectations of future changes in exchange rates conditional on period t information

$$s_{t,t+i} - ds_{t,t+i+1} = 0 \qquad\qquad (2.13')$$

$$d = k/[1 + k + \delta + \alpha\beta(1 - b)].$$

We write the solution this homogenous equation, with one degree of freedom, in the form

$$s_{t,t+i} = Cd^i, \text{ for some real } C \quad (\text{all } i \geq 0). \qquad\qquad (2.13'')$$

The parameter C in the solution is arbitrary, but the interesting point is that the solution and the convergence of exchange rate expectations depend on the indexation coefficient, and more critically on the intervention coefficient δ. The intervention parameter is critical for the convergence of exchange rate expectations. Specifically, if

$$-[1 + \alpha\beta(1 - b)] > \delta > -[1 + \alpha\beta(1 - b) + 2k],$$

then expectations fail to converge. Otherwise expectations converge, but become indeterminate, depending on the choice of intervention parameter δ.[15] So, we have a continuum of solutions to exchange rate expectations. Because of this possible indeterminacy and instability,[16] we choose the solution obtained by setting $C \equiv 0$. While this choice is arbitrary, it is simple and has the virtue of yielding a solution the form of which is independent of the choice of intervention parameter δ (see,

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[15] So stability considerations do not alone suffice to tie the solution to $s_{t,t+i} = 0$, if abs(d) < 1. The problem is intrinsic to the solution of a general first order stochastic difference equation; the solution, without further side conditions, always has one degree of freedom. Thus, as McCallum (1983) so forcefully argues, indeterminacy is not specific to rational expectations, but to the method of solution. He has further suggested a criterion of minimal state representation for providing a unique solution, according to which the solution should not be based on extraneous variables. After all, he argues, in a rational expectations model an arbitrary factor enters the solution function because it is rationally expected to enter it. Whether this argument substantially suffices to set C = 0 in the present context is not entirely clear and perhaps problematic. But, given the stochastic structure, and the intuitive appeal of static expectations in the present model, this choice gains in plausibility.

[16] The procedure of picking the stable root is typical of rational expectations models (see, also, fn. 4) and is often justified on the grounds that the instability that would otherwise occur would be inconsistent with observed behaviour (agents can see when they are on an unstable path, and instantaneously adjust expectations to a stable path). Alternatively, it is sometimes justified more formally by appealing to transversality conditions from appropriate optimizing models, which, provided the underlying utility functions satisfy suitable restrictions, ensure that the expected price movements remain bounded; see, for example, Brock (1974).

also, Turnovsky 1983, 1984). All in all, this particular choice of the solution to exchange rate expectations means that they are static. Intuitively, this choice increases the plausibility of this particular solution, given the stochastic structure of the model.[17]

2.3.2 Full information output

The full information output is based on the spot market equilibrium of the labour markets. Thus, from equations (2.5') and (2.6'), we have that spot equilibrium changes in real wages satisfy

$$(w - p)^f = \beta\mu/(\varepsilon + \beta), \qquad (2.14)$$

which implies that the full information output is given by

$$
\begin{aligned}
y^f &= \varepsilon\alpha\beta\mu/(\varepsilon + \beta) + \mu \\
&= [\beta(1 + \varepsilon)/(\varepsilon + \beta)]\mu.
\end{aligned}
\qquad (2.15)
$$

From equations (2.14) and (2.15) we can immediately deduce a form of classical dichotomy, i.e. that only productivity disturbances affect equilibrium real wages and real output (and of course employment). The money market, or more generally demand-side disturbances, have no effect on full information equilibrium. This distinction between sources of uncertainty will be of great importance when we turn to discuss wage indexation, especially from the point of view of signal extraction.

2.3.3 Actual real wages and ouput

Actual wages are given by the indexation scheme (2.7'), and since actual employment is given by the demand for labour (2.5'), changes in actual aggregate output are obtained by substituting the demand for labour function into the output equation (2.4'). Thus,

$$y = \alpha\beta(1 - b)p + \beta\mu. \qquad (2.16)$$

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[17] Note that the stable forward solution to exchange rate expectations is consistent with either "leaning with a wind" ($\delta < 0$) or "leaning against the wind" ($\delta > 0$) exchange rate policy. An interesting implication of this fact is that the corresponding stable backward solution is consistent only with a "leaning with the wind" policy.

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Equation (2.16) gives the contract-based Phillips curve; changes in the price level p are transmited into changes in aggregate output via the wage formula (2.7'). The slope of the Phillips curve is directly related to the degree of wage indexation. At one extreme we have completely rigid nominal wages (b=0), while at the other we have completely rigid real wages (b=1), implying a vertical Phillips curve in the (y,p)-plane. As in the original closed economy Gray—Fischer model, neither of these polar cases need be optimal in the sense of minimizing the variability of aggregate output around the full infromation output y^f. For example, in the case of rigid real wages we have

$$ |y^f| = |[ß(1+\varepsilon)/(\varepsilon+ß)]\mu| < |ß\mu| = |y|, $$

so that if supply shocks are the dominant source of aggregate fluctuations in the economy, wage indexation tends to increase their effects on aggregate employment and output. Of course, the classical dichotomy would be clearly evident in the contract model with full wage indexation if there were only money market or demand disturbances in the economy. Thus fully rigid real wages completely insulate output from money market disturbances.

On the other hand, under partial indexation money market disturbances have real effects. Furthermore, monetary policy in the form of foreign exchange intervention has (second order) real effects achieved by influencing the price level via the intervention rule. Thus (partial) wage indexation provides leeway for systematic monetary policy or, more generally, the contracting approach provides one macroeconomic channel for the interaction of financial markets and the real sector of the economy. We shall analyze the effects of indexation and intervention more thoroughly in next section in the context of optimal wage indexation and foreign exchange intervention.

2.3.4 Optimal wage indexation and foreign exchange intervention from the point of view of macroeconomic stability

One of the chief motivations for Gray's analyses was to show that the ultimate effects of wage indexation on aggregate fluctuations depend on the source of the exogenous disturbances; while full wage indexation insulates the real sector from monetary disturbances, it increases the effects of productivity shocks on aggregate employment and output. This distinction between sources of disturbances is also relevant in an open economy context, but the ultimate effects of the various disturbances also

depend on the exchange rate regime chosen by the country concerned. In particular, the efficiency consequences induced by wage rigidity can be better monitored by the use of active monetary policy. This is the theme taken up in this section.

On the assumption that the labour markets are the only ones not clearing continuously, Gray has suggested an efficiency or welfare criterion based on an approximation of the welfare loss induced by non-clearing labour markets to analyze the determination of optimal wage indexation. This approximation is derived from basic utility maximization by Aizenman & Frenkel (1985b, appendix). Its extension to an open economy context requires that the optimal mix of wage indexation and foreign exchange intervention, (b^*, δ^*), be such that

$$(b^*, \delta^*) = \text{argmin}_{(b, \delta)} E\{(y - y^f)^2 : I_{t-1}\}. \tag{2.17}$$

Thus the optimal mix (b^*, δ^*) is such that the (conditional) expected squared difference between actual and full information output is minimized. Since $E[y : I_{t-1}] = 0 = E[y^f : I_{t-1}]$, the criterion requires that the (conditional) variance of $y - y^f$ be minimized. Using the production function (2.4'), we see that the loss function (2.17) indicates that we should minimize the expected squared difference between actual and full information equilibrium employment, which from the labour demand function and labour market equilibrium reduces to minimizing

$$H = BE\{[(w - p) - (w - p)^f]^2 : I_{t-1}\}, \tag{2.17'}$$

where B depends on the model's parameters, excluding b and δ.

From equations (2.7') and (2.14) we have $(w - p) = -(1 - b)p$ and $(w - p)^f = \beta\mu/(\varepsilon + \beta)$. On the other hand, PPP (2.10') and the exchange rate equation (2.12) together give

$$\begin{aligned}
p = v_1 + s &= v_1 - \tau/[1 + k + \delta + \alpha\beta(1 - b)] \\
&= \{kv_2 + (k + \delta)v_1 - (u + \beta\mu)\}/[1 + k + \delta + \alpha\beta(1 - b)].
\end{aligned} \tag{2.18}$$

The loss function (2.17') has to be minimized with respect to b and δ. Holding δ fixed for the moment, the loss function is first minimized with respect to the indexation coefficient b. Under the stated assumptions the loss function is the expected residual sum of squares from regressing equilibrium real wages $(w - p)^f$ on actual real wages $-(1 - b)p$. Setting $(1b)/[1 + k + \delta + \alpha\beta(1 - b)] \equiv \Gamma$ and $\{kv_2 - (k + \delta)v_1 - (u + \beta\mu)\} \equiv \Phi$, we have

$$\Gamma^* = -\text{cov}\{\Phi, \beta\mu/(\varepsilon + \beta)\}/\text{var}(\Phi), \tag{2.19}$$

where Γ^* denotes the value of Γ minimizing the loss function holding δ fixed. Using the definition of Γ and making some manipulations, we have that the optimal degree of wage indexation b^* is given by

$$b^* = b^*(\delta) = 1 - \frac{[1+k+\delta]}{[\Omega+1+\varepsilon]}, \qquad (2.20)$$

where

$$\Omega = \left[k^2\frac{\sigma_2^2}{\sigma_\mu^2}+(k+\delta)^2\frac{\sigma_1^2}{\sigma_\mu^2}+\frac{\sigma_2^u}{\sigma_\mu^2}\right]\left[\frac{(\beta+\varepsilon)}{\beta^2}\right]$$

From equation (2.20) we can immediately see that

$$\frac{\partial b^*}{\partial\sigma_x^2}>0, \quad x = 1,2,u;\ \frac{\partial b^*}{\partial\sigma_\mu^2}<0;\ \frac{\partial b^*}{\partial\varepsilon}>0,$$

These comparative static results demonstrate that the results obtained by Gray generalize to an open economy context; i.e. dominant productivity shocks decrease and dominant demand-side shocks increase the optimal degree of wage indexation, given the degree of intervention δ. This is intuitively obvious, since equilibrium changes in real wages are driven only by productivity shocks.

More specifically, when there are only productivity shocks or demand-side shocks, actual real wages, as implied by the corresponding indexation coefficient (2.20), coincide with equilibrium real wages. Finally, for a more elastic labour supply function, the bulk of the equilibrium adjustment in the labour markets takes place through changes in employment, so that real wages are relatively more rigid. This translates into a requirement that the optimal degree of wage indexation increases with the elasticity of labour supply function. All these results are familiar from Gray's analysis.

Differentiating the loss function with respect to δ and setting the resulting expression equal to zero we obtain

$$\frac{\partial H}{\partial\Gamma}\frac{\partial\Gamma}{\partial\delta}+2(k+\delta)\sigma_1^2\Gamma^2 = 0. \qquad (2.21)$$

Using the envelope theorem, i.e. assuming that the degree of wage indexation is at its optimal value, we have $(\partial H/\partial \Gamma) = 0$. Thus, the optimal degree of foreign exchange intervention δ^* is given by

$$\delta^* = -k. \qquad\qquad (2.22)$$

Alternatively, (2.22) can be obtained directly by inspecting the value of Φ in the loss function and noting that in order to minimize the loss function we need to set $\delta^* = -k$. This is an extremely simple intervention rule, since it is completely independent of the stochastic structure of the exogenous disturbances. This feature is a consequence of the simple exchange rate intervention rule employed in the model, and it is not present in models with more sophisticated intervention rules (see e.g. Turnovsky, 1986). It should be further noted that under this particular intervention rule changes in the domestic price level are independent of foreign price level shocks p^*.

A second interesting feature about the optimal intervention rule is that it implies an exchange rate policy of "leaning with the wind". Thus when exhange rates are above (below) the steady state level, the monetary authority increases (decreases) the supply of money above (below) the steady state level. There are two reasons for this result. Furthermore, there is one important implication regarding the effects of wage indexation on price level variability so prevalent in the standard contracting approach to the Phillips curve.

First, by adopting a policy of "leaning with the wind" the monetary authority is able to generate efficiency improving changes in actual real wages relative to equilibrium changes. Secondly, the optimal intervention policy can at least partly be justified by noting that it maximizes the absolute correlation between the price level and the underlying productivity shock. Thus efficiency improving changes in actual wages are generated by optimal intervention policy because of the improved information content of the price level with respect to productivity shocks.[18]

As far as the implication mentioned above is concerned, one can observe from the price level equation (2.18) that indexation increases price level variability. That wage indexation destabilizes the price level is often quoted in practical debates as the most important factor

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[18] An analogous argument explains the welfare superior performance of flexible exchange rates in Flood & Marion's (1982) analysis of exchange rate regimes under optimal wage indexation. A particular exchange rate regime can be optimal relative to specific shocks, but overall flexible exchange rates are optimal across the alternatives they analyze.

explaining the non-existence of widespread indexation arrangements in actual labour contracts.[19] However, care should be taken when judging the behaviour or performance of the model in this respect, since there are at least two factors greatly affecting a serious welfare evaluation of increased indexation; on the one hand, increased indexation increases price level variability, but, on the other, increased indexation makes "it easier to live" with price level instability. It is quite conceivable (Cecchetti & Ball, 1989) that the latter "insurance" effect dominates in terms of welfare.

We can derive a further aspect of the optimal wage indexation coefficient by manipulating the expression (2.20). By setting $\delta=0$ and assuming that $v_1 \equiv 0 \equiv v_2$ in (2.20), we obtain the closed economy version of the indexation optimum, i.e.

$$b_c^* = 1 - \frac{[1+k]}{[\Omega+1+\varepsilon]} \tag{2.20'}$$

$$\Omega = \frac{\sigma_u^2(\beta+\varepsilon)}{\beta^2\sigma_\mu^2}.$$

This is the same optimum derived by Aizenman (1983, eq. (30)) in analyzing wage contracts with incomplete and costly information. The optimal degree of wage indexation depends now on the magnitude of the monetary shock relative to the productivity shock.[20]

On the other hand, setting $\delta = \delta^* = -k$, the optimal degree of wage indexation reduces to

$$b^*\big|_{\delta=\delta^*} = 1 - \frac{1}{[\Omega+1+\varepsilon]} \tag{2.20"}$$

$$\Omega = \left[k^2\frac{\sigma_2^2}{\sigma_\mu^2} + \frac{\sigma_u^2}{\sigma_\mu^2}\right]\left[\frac{(\beta+\varepsilon)}{\beta^2}\right].$$

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[19] For a macroeconomic analysis of wage indexation and price level variability, see e.g. Fischer (1983b).

[20] Note that if, in the closed economy, there are only aggregate productivity shocks, the optimal degree of wage indexation reduces to $1 - (1+k)/(1+\varepsilon)$, which may well be negative.

Upon comparing equations (2.20') and (2.20"), we can deduce that $b_c^* < b^*|_{\delta=\delta'}$, so that at the optimum, real wages are more rigid in the open economy. This may to some extent be a model-specific result, since, as Aizenman & Frenkel (1985b, p. 412, fn. 11) note this result reflects the specification of the nature of the model's stochastic shocks whereby the openness of the economy does not increase the exposure to foreign real shocks. In principle, the relative importance of real shocks may be greater in the open economy if, for example, it faces shocks to the price of imported raw materials. These may affect equilibrium real wages in the home country in such a way as to make the optimal degree of wage indexation in the open economy less than in the corresponding closed economy.

2.3.5 Optimality of a single instrument

In practice, it is frequently the case that the parties to labour contracts have committed themselves only to a certain prespecified degree of wage indexation, which need not be based on any efficiency calculation, or that the monetary authority actually commits itself to a certain degree of foreign exchange intervention. It is then of interest to ask what is the optimal degree of intervention (in the former case) or of indexation (in the latter case) as regards fluctuations in aggregate output and employment.

2.3.5.1 Optimal wage indexation under a given exchange rate regime

Treating the degree of foreign exchange intervention as exogenously given, the optimal degree of wage indexation is given by equation (2.20). The relationship between indexation and intervention embodied in (2.20) implies that for a fixed exchange rate regime ($\delta=\infty$), real wages should be completely rigid, i.e. $b^*=1$. This policy mix induces a welfare loss equal to

$$H[\text{fix}] = \left[\frac{\beta^2}{(\beta + \varepsilon)^2} \right] \sigma_\mu^2. \tag{2.23}$$

In a fixed exchange rate system, exchange rates do not adjust to shocks, so the only shock affecting the price level and thus actual real wages is the foreign price shock (via PPP), whose influence on real wages can be eliminated by setting b=1. This, however, leaves no room for adjusting

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real wages to productivity shocks, whose influence is transmitted in full to employment and output fluctuations. This is captured by the value of the loss function in (2.23). On the other hand, the introduction of a small degree of exchange rate flexibility lowers the optimal degree of wage indexation, i.e.

$$\frac{\partial b^*}{\partial \delta}\bigg|_{\delta=\infty} > 0.$$

If the economy is constrained in its choice to flexible exchange rates ($\delta=0$), the optimal degree of wage indexation is

$$b^*\big|_{\delta=0} = 1 - \frac{(1+k)}{[\Omega+1+\varepsilon]} \tag{2.24}$$

$$\Omega = \left\{ k^2 \left[\frac{\sigma_2^1 + \sigma_2^2}{\sigma_\mu^2} \right] + \frac{\sigma_u^2}{\sigma_\mu^2} + \frac{\sigma_u^2}{\sigma_\mu^2} \right\} \left[\frac{\beta+\varepsilon}{\beta^2} \right].$$

Thus, under flexible exchange rates, real wages are not completely rigid. Note, however, that flexible exchange rates are fully consistent with nominal wage rigidity, i.e. $b^*=0$, even though this may require very special parameter values ($\Omega+\varepsilon = k$). The efficiency loss under flexible exchange rates is equal to

$$H[\text{flex}] = H[\text{fix}] + F \tag{2.23}$$

$$F = - \frac{\left[\frac{\beta}{\beta+\varepsilon} \right] \sigma_\mu^2}{\left[1 + \frac{\Omega}{\beta+\varepsilon} \right]} < 0,$$

where Ω is given in (2.24). This means that under a flexible exchange rate regime efficiency losses are smaller than under a fixed exchange rate regime, when wages are optimally indexed, a result also derived by Flood & Marion (1982). The rationale for this result is that under fixed exchange rates real wages do not adjust, not even for productivity shocks, which imply equilibrium changes. Flexible exchange rates do not fully insulate the real economy from demand disturbances, but in terms of the overall welfare of efficiency, this is outweighed by the option of

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adjusting actual real wages, employment and output to productivity shocks.[21]

The relationship between the exogenously given degree of intervention and optimal wage indexation is depicted in figure 2.1 (see Aizenman & Frenkel, 1985a, p. 12).

Figure 2.1   **Optimal degree of wage indexation for a given degree of intervention**
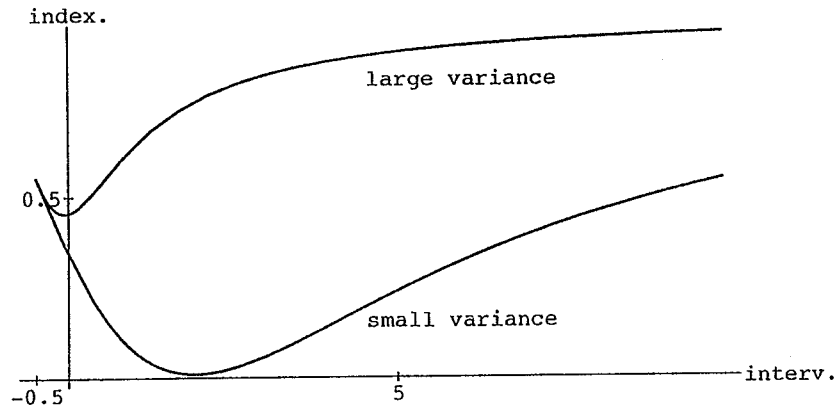
"Small (large) variance" refers to small (large) variance of the foreign price shock

As it stands, the relationship between the two is by no means monotonic. In fact, we can calculate from equation (2.20) that

$$\text{sign} \left\{ \frac{\partial b^*}{\partial \delta} \right\}$$

$$= \text{sign} \left\{ \left[\frac{\beta + \varepsilon}{\beta^2} \right] [2(k + \delta) + (k + \delta)^2] \left[\frac{\sigma_1^2}{\sigma_\mu^2} \right] \right. \tag{2.26}$$

$$\left. - \sigma_\mu^2 [k^2 \sigma_2^2 + \sigma_u^2] \left[\frac{\beta + \varepsilon}{\beta^2} \right] - (1 + \varepsilon) \right\},$$

---

[21] Note, however, that unless the (semi)elasticity of money demand is zero, the flexible exchange rate regime is not the optimal choice in terms of aggregate real fluctuations. The (unconstrained) optimal policy mix (b*,δ*) is such that δ*=-k.

from which we deduce that $(\partial b^*/\partial \delta)|_{\delta=\delta^*} < 0$. We already know that $(\partial b^*/\partial \delta)|_{\delta=\infty} > 0$, so the dependence of optimal wage indexation on the exchange rate regime has to change its sign in the interval $[-k,\infty]$. Actually, we can see from equation (2.3.25) that

$$\frac{\partial b^*}{\partial \delta}\Big|_{\delta=0} > 0, \text{ for } \sigma_1^2 \to \infty$$

$$< 0, \text{ for } \sigma_1^2 \to 0.$$

The economic rationale for this is that if the foreign price is very volatile, money market disturbances under flexible exchange rates reflect mainly foreign price shocks. If the monetary authority adopts a policy of "leaning with the wind" ($\delta<0$), exchange rates are used to neutralize the effects of foreign price shocks on the price level. This implies a lower degree of optimal wage indexation. If, on the other hand, the monetary authority "leans against the wind" ($\delta>0$), it fights the current change in exchange rates, so that the ultimate effects of the foreign price shocks on the price level tend to be stronger. This calls for higher optimal indexation to mitigate the effects of foreign price shocks on actual real wages. The case where foreign prices are relatively constant and money market disturbances reflect mainly foreign interest and money demand shocks can be handled by noting that their effects on the price level, and real wages, come about only through exchange rate changes.

2.3.5.2 Optimal degree of intervention given the degree of wage indexation

If the degree of wage indexation in labour contracts is given, the optimal degree of foreign exchange intervention can be derived by minimizing the loss function H with respect to δ. After appropriate manipulations, this gives

$$\delta^* = \delta^*(b) = \frac{(1-b)[k^2\sigma_2^2 + \sigma_u^2 + \beta^2\sigma_\mu^2] - \phi\pi\beta^2\sigma_\mu^2}{(1-b)\phi\sigma_1^2 + \pi\beta^2\sigma_\mu^2} - k, \qquad (2.27)$$

where $\phi = [1 + \alpha\beta(1-b)]$ and $\pi = 1/(\beta+\varepsilon)$.

As can be immediately seen from equation (2.27), if there are only money market and/or foreign interest rate shocks impinging on the economy, the optimal exchange rate regime, given b, is that of

completely fixed rates. This is the only way their effects on the domestic price level, and thus output, can be eliminated by means of exchange rate policy. For the same reason, if there are only foreign price shocks impinging on the domestic economy, the optimal intervention rule coincides with the unconstrained optimum, i.e. $\delta^*(b) = -k = \delta^*(b^*)$.

For domestic productivity shocks, the optimal intervention rule is given by $\delta^*(b) = (1+\epsilon)(1-b)-(1+k)$, which is consistent with either "leaning with the wind" or "leaning against the wind" exchange rate policy. The implied exchange rate policy depends heavily on wage rigidity. For elastic labour supply, small changes in real wages are needed to restore labour market equilibrium following productivity shocks. If nominal wages are rigid because of a low degree of indexation, it is optimal for the monetary authority to try to reduce price level variability by dampening exchange rate variability. This implies that $\delta^*(b) > 0$, i.e. an exchange rate policy of "leaning against the wind". On the other hand, for relatively rigid real wages, high price level variability is needed for real wage adjustments. This is achieved by increasing exchange rate variability, i.e. an intervention policy with $\delta^*(b) < 0$.

Figure 2.2 plots the relationship between the exogenously given b and the corresponding optimal $\delta^*(b)$ (see also Aizenman & Frenkel, 1985a, p. 14). As can be seen from (2.27), the nature of the relationship depends on the magnitude of the variance of the foreign price shock, σ^2_1. For small σ^2_1, the relationship is approximately linear (figure 2.2a), while for large σ^2_1, figure 2.2b plots a representative graph of the relationship.

Figure 2.2   **Optimal degree of intervention for a given degree of wage indexation**

See note to fig. 2.1

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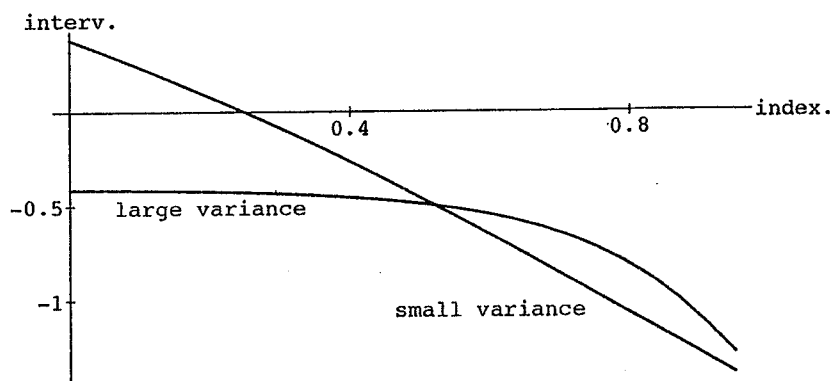
Figure 2.2 also shows the unconstrained optimum policy mix $(\delta^*, b^*(\delta^*))$, where $\delta^* = -k$ and $b^*(\delta^*)$ is the corresponding value for optimal indexation from equation (2.20). Note that the extreme policy mix $(\delta^*(b), b) = (-(1+k), 1)$ should be interpreted as a limiting case, as Aizenman & Frenkel (ibid., p. 14) argue; when the degree of wage indexation approaches one, the corresponding optimal degree of intervention approaches the value $-(1+k)$. Economically, the interpretation of this result is that full indexation introduces complete real wage rigidity. Consequently, changes in the price level which can be brought about through changes in the exchange rate will be inconsequential since, owing to complete real wage rigidity, such changes in the price level induce equiproportionate changes in nominal wages (Aizenman & Frenkel, ibid., p. 14).[22]

2.4   Introducing terms-of-trade changes

The previous analysis was based on the assumption of a continuously holding PPP relation between domestic and foreign price levels. The status of the PPP relation in making open economy macromodels analytically simple and tractable should not be underestimated, but its overall validity, both theoretical and empirical, is admittedly problematic (see the references cited in fn. 11, section 2.3). In the context of our specific macromodel considered so far, in particular in the context of the money supply rule employed, we have seen that the inclusion of PPP leads to a very strong prediction about the nature of the unconstrained optimal intervention policy; the optimal intervention rule is such that changes in the domestic price level are independent of shocks to foreign prices. Furthermore, this particular intervention rule is completely independent of the distributional characteristics of the exogenous disturbances.

What if we relax the assumption of continuous PPP? What is the nature of the optimal policy mix in this case, and more importantly, is it amenable to analytical characterization? First of all, the PPP relation fits nicely into our open economy framework, in that we assumed a small, competitive open economy facing well integrated international commodity markets. Clearly, one can think of reasons why PPP does not hold

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[22] That full wage indexation and the implied real wage rigidity is an effective constraint on activist monetary policy is also clearly brought out by e.g. Fischer's (1983b) analysis of supply shocks, wage stickiness and accommodation.

even in the small open economy context,[23] but a particularly simple extension of the model is generated if we assume that domestic and foreign goods are not perfect substitutes,[24] so that the demand for domestic goods by domestic citizens depends on the price of domestic goods relative to the price of foreign goods in terms of domestic currency. This relative price will henceforth be called the terms of trade, denoted by T. Thus $T = P/(SP^*)$, where P denotes domestic product prices, P^* foreign prices and S exchange rates.

To be more specific, we shall assume that percentage changes in the demand for domestic goods (around the non-stochastic equilibrium) take the following form:

$$y_t = \Theta_1(p_t - p_t^* - s_t) - \Theta_2[i_t - (c_{t,t+1} - c_t)] + d_t$$

$$\Theta_1 \leq 0, \; \Theta_2 \geq 0.$$

(2.28)

The non-positivity of Θ_1 implies that the Marshall-Lerner condition is assumed to hold. In addition to depending negatively on the terms of trade, changes in the demand for domestic goods depend negatively on changes in the expected real interest rate $[i_t - (c_{t,t+1} - c_t)]$, where $c_t = ap_t + (1-a)(p_t^* + s_t)$ denotes changes in consumer prices. d_t is a (NID-)demand disturbance capturing unexpected shifts in the demand for domestic goods.[25]

Introducing endogenous terms-of-trade changes into our model of the interaction between wage indexation and foreign exchange intervention complicates the analysis further, due to the fact that the real wages relevant for labour demand and supply decisions do not generally coincide; demand for labour depends (negatively) on the producer's real wage W/P, while supply of labour depends (positively) on the worker's real wage W/C. More formally,

$$N^d = N^d(W/P) \quad \text{and} \quad N^s = N^s(W/C).$$

(2.29)

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[23] These could include differences in the structure of the economies engaged in international trade, perhaps with different productivity conditions in the open and closed sectors of the economies, differential preferences of domestic and foreign citizens etc.

[24] For example, because to the domestic country produces a differentiated good.

[25] In the original Gray model, an interest-elastic aggregate demand schedule is not postulated, and the goods market equilibrium is a residual determined by the money market equilibrium, where money demand is specified by the quantity equation. Cukierman (1980), however, demonstrates that this particular specification of macromodel is not critical for Gray's results on the effects of wage indexation on macroeconomic stability. In many senses, the critical feature of Gray's formulation is the employment rule.

Combining these decision rules with the spot labour market equilibrium requirement gives equilibrium employment as an increasing function of the terms of trade T:

$$N = N(T), \quad N' > 0, \qquad (2.30)$$

which, with the production function $Y = F(N)$, $F' > 0$, gives aggregate output as an increasing function of the terms of trade:

$$Y = G(T), \quad G' > 0. \qquad (2.31)$$

So changes in the terms of trade T produces changes in (equilibrium) aggregate output. Furthermore, and this is important for the present analysis, changes in the terms of trade T change the producer's real wage relative to the worker's real wage, since $[W/C]/[W/P] = T^{1-a}$. On purely analytical grounds, this may cause some problems, since when we come to discuss wage indexation, we face the problem of choosing a proper price index for the wage formula. Should wages be indexed to product prices P or consumer prices C, or to both? The form of the indexation rule is important, since the macroeconomic implications of wage indexation (and intervention for that matter) depend on the precise form of the indexation rule; different price indexes have differential macroeconomic effects, and ultimately the desirability or otherwise of a particular price index depends on the source of the (exogenous) disturbance impinging on the economy (see e.g. Marston, 1983a).

Various price indexation rules have been employed in an open economy context (see Marston, 1984, Turnovsky, 1983, Haaparanta, 1986, Marston & Turnovsky, 1985a, b), the most common ones being product and consumer price indexing, or an indexation rule, whereby wages are indexed separately to domestic product prices and foreign prices (in domestic currency, see Turnovsky, 1983). Haaparanta (ibid.) introduces terms-of-trade indexation into his model and analyzes how its introduction is related to macroeconomic stability, either in the sense of insulating the economy against all disturbances, or of generating minimal departures from the full information equilibrium of the economy, as in Gray's original model of optimal wage indexation. Furthermore, he poses the question of the optimality of the weights in the consumer price index, and derives conditions under which these weights are optimal.

Despite their merits and empirically plausible nature, these indexation rules make the closed form application of the Gray—Fischer framework of optimal wage indexation intractable; no tractable closed form expression for the optimal policy mix of wage indexation and foreign exchange intervention can be derived from a macromodel employing

these conventional indexation rules, unless the structure of the exogenous disturbances is heavily restricted.[26] The root cause of this problem is, as noted above, the existence of two real wages in the labour market, and the use of these conventional indexation rules implies that price information relevant for only one of labour the demand or supply decisions is actually utilized.

There is, however, theoretically an elegant and precise way to solve this problem. It starts by posing the question "what is the relevant price index for the labour market (spot) equilibrium?" Since there are two prices which affect the labour market equilibrium wage rate, there has to be some combination or average of the two to summarize price level information in the economy. Let us consider once again the labour market equilibrium, but now in a more specific form. Suppose labour demand and supply are given by

$$N^d = A_d[P/W]^\beta \qquad\qquad (2.32)$$

$$N^s = A_s[W/C]^\varepsilon, \qquad\qquad (2.33)$$

where the A's are constants and ß and ε are the elasticities of labour demand and supply with respect to real wages. ß generally depends on the parameters specific to the (decreasing returns to scale) production technology. Now, solve for the labour market equilibrium wage rate W, denoted by W*;

$$W^* = CQ, \qquad\qquad (2.34)$$

where

$$C = \frac{A_d}{A_s} \text{ and } Q = P^\pi(SP^*)^{1-\pi}$$

$$\pi = \frac{\beta + a\varepsilon}{\beta + \varepsilon}.$$

What is relevant here is that in deriving the equilibrium wage rate, we have defined a new price index Q, dubbed the equilibrium price index or optimum weight index by Vilmunen (1989); Q is a weighted geometric average of domestic product and foreign prices, where the weight for the domestic product price depends on labour demand and supply elasticities and a, the weight domestic product prices have in the consumer price

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[26] The typical mode of analysis has been to focus on the optimal policy mix under specific disturbances, as exemplified by Turnovsky (ibid.) and Marston (ibid.).

index C.[27]. For labour market equilibrium, Q is the relevant price index in that it summarizes price level information relevant for labour demand and supply decisions. Thus, it seems only natural that actual wages should be indexed to this particular price level, i.e. we should write

$$\frac{W_t}{W_t^c} = \left(\frac{Q}{Q_0}\right)^b .$$ (2.35)

where W_t^c denotes contract wages for period t, determined by the non-stochastic equilibrium of the labour market (cf. (2.34)), Q_0 is the non-stochastic equilibrium value of Q, and b denotes the degree of indexation.

This is exactly the approach taken by Devereuex (1988) and Vilmunen (1989) in analyzing the optimal mix of wage indexation and foreign exchange intervention in an open economy subject to endogenous terms of trade changes. The main contributions of this approach to indexed wages is that the extension to an open economy of the Gray—Fischer framework for deriving policy mix optima can be readily applied under the hypothesis that wages are indexed according to the formula (2.35). Thus, the interaction between wage indexation and foreign exchange intervention need not be analyzed separately in cases where only a subset of the exogenous disturbances is present in the economy, a notable feature in e.g. Turnovsky's (ibid.) approach.

Furthermore, and at more substantial level, there is a direct correspondence between the present approach and the one followed in the previous section, where PPP was assumed to hold; there the problem was that of choosing a policy mix such that changes in actual real wages [W/P] were as informative as possible about changes in equilibrium real wages [W/P]*. According (2.35) actual real wages are [W/Q] and equilibrium real wages [W/Q]*. The point is, of course, that the relevant real wage for the labour market as a whole is defined by [W/Q].[28]

We shall not go into the details of the result on the relationship between wage indexation and foreign exchange intervention in an open economy consisting of the macromodel in section 2.2 restructured so as to incorporate equations (2.28) and (2.35). For these details we refer to Devereux (ibid.) and Vilmunen (ibid.).[29] There is, however, one feature

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[27] To gain some intuitive understanding of the price index Q, direct calculation reveals that it is equal to the consumer price index corrected for (equilibrium) terms of trade, i.e. $Q = CT^w$, where $w = \beta(1-a)/(\beta+\varepsilon)$.

[28] Which, of course, reduces to [W/P] if PPP holds.

[29] For results from more conventional wage indexation models, see e.g. Marston (ibid.), Turnovsky (ibid.) and Haaparanta (ibid.).

of the optimal policy mix of indexation and foreign exchange intervention that deserves special attention. This is that there emerges a nice separation between the optimal degree of wage indexation and foreign exchange intervention in that the latter is independent of the distributional information associated with domestic supply or productivity disturbances,[30] while the former is affected by the whole variance-covariance structure of the disturbances.[31]

The intuitive reason for this independence is the wage indexation scheme and the fact that only domestic productivity shocks are hypothesized to induce unexpected shifts in equilibrium real wages [W/Q]*. Due to the signal extraction problem, the price index Q provides only imperfect information on the underlying sources of exogenous disturbances, so that the single instrument of wage indexation cannot perfectly adjust actual real wages (and employment and output) to its market-clearing level. Optimal foreign exchange intervention attempts to reduce the influence of demand disturbances on the price index Q, thereby making the price index more informative about the underlying domestic productivity shock. Thus the efficiency of the optimal indexation rule in achieving market-clearing is improved (see, also, Devereuex, ibid., p. 389).

Otherwise, the optimal degree of wage indexation behaves much as in the previous section in that it represents a trade-off between optimal policy for all demand disturbances (full real wage rigidity) and optimal policy for domestic supply disturbances (partial wage indexation). For a given degree of exchange intervention, the actual degree of wage indexation depends upon the size of the variance of the domestic supply shock relative to a weighted average of the variances of the demand disturbances. Finally, for the case of only two sources of exogenous variability, a demand disturbance coupled with the presence of domestic supply shocks, the optimal policy mix eliminates the efficiency loss completely, a result familiar from the analysis of e.g. Aizenman & Frenkel (1985b), where the potential for the elimination of the efficiency loss depends on the relationship between the number of independent indicators policy makers have and the number of independent sources of exogenous disturbances.[32]

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[30] The optimal degree of intervention can imply either a "leaning with the wind" or "leaning against the wind" exchange rate policy.

[31] Later in section 2.4 we shall see that this same separation emerges from our two sector economy model.

[32] For specific results on the optimal mix in the case of two shocks, see e.g. Devereux (ibid., p. 391, table 1).

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2.5 Wage rigidity, foreign exchange intervention and the degree of openness

In this section we shall further extend the contracting approach to an open economy. This time the focus of the analysis is different. The open economy framework applied so far has been a highly aggregative, one commodity model that has served the purpose of highlighting how the stochastic behaviour of certain key macroeconomic variables is affected once short-run wage rigidity is introduced into the model. Thus the only relative price subject to changes is the real wage, and any possible role assumed by other relative prices in the adjustment of the aggregate economy is abstracted away by postulating a one commodity model.

But certainly other relative prices also adjust after shocks to the economy have occurred. Changes in relative prices imply reallocation of resources in the economy, and to analyze the behaviour of the aggregate economy, from the point of view of stability or otherwise, there is a clear need to take into account these allocational effects of relative price changes.[33]

We shall take a step towards reconsideration of the implications of wage indexation (and intervention) by postulating a two-sector economy, where the open sector is fully competitive and internationally integrated and the closed sector's price adjusts to eliminate any excess demand for its product. From the point of view of macroeconomic stability, this framework opens up a new vista by making it possible to consider the effects of changes in the relative price of the closed sector on macroeconomic stability. Changes in the relative price are closely related to the degree of openness of the econoomy, as measured by certain substitutability parameters in production and consumption and/or the size of the open sector. This means that by analyzing the effects of changes in relative price on macroeconomic stability we are in effect analyzing how the degree of openness affects macroeconomic stability. More precisely, we shall analyze how the degree of openness of the economy affects the relationship between the degree of wage indexation and foreign exchange intervention.

This section extends Aizenman's (1986) framework for the degree of wage indexation and openness of the economy. Aizenman builds his model on the assumption that the economy under consideration is constrained in its choice of exchange rate regime to flexible exchange rates. Vilmunen (1989) relaxes this constraint by allowing the exchange

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[33] To be more specific, changes in price variables other than real wages.

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rate to be determined by an intervention rule, whereby changes in the money stock are tied to changes in the exchange rate, a rule applied in earlier sections (see 2.2–2.3). Some of the key results obtained by Aizenman, especially those related to the efficiency considerations under flexible exchange rates, need to be reconsidered in the light of this more general setting. Furthermore, a nice separation of the policy instruments emerges from this model in that the degree of foreign exchange market intervention depends only on the stochastic structure of foreign variables, while the degree of indexation focuses mainly on domestic variables, especially on domestic productivity shocks.

2.5.1 A two-sector economy with wage indexation and foreign exchange intervention

Consider a two-sector economy producing traded and non-traded goods under perfect capital mobility, and assume that it can freely choose its exchange rate regime. Consumers are assumed to have identical homothetic preferences, generating a price index given by[34]

$$P_t = [P_{1,t}]^\alpha [P_{2,t}]^\beta \quad (\alpha + \beta = 1), \tag{2.36}$$

where $P_{1,t}$ and $P_{2,t}$ are money prices of non-traded and traded goods at time t, and α and β denote the share of non-traded and traded goods, respectively. Real aggregate output is given by

$$Y_t = \frac{Y_{1,t} P_{1,t} + Y_{2,t} P_{2,t}}{P_t}, \tag{2.37}$$

where Y_1 and Y_2 represent the output of non-traded and traded goods, respectively. The demand for non-traded goods is given by

$$D_t = [\frac{P_{2,t}}{P_{1,t}}]^\tau Y_t \exp[C - \phi(i_t - \pi_t)], \tag{2.38}$$

where τ denotes (compensated) demand elasticity, i is the money interest rate, and π is the expected inflation:

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[34] This price index is the "price dual" corresponding to Cobb–Douglas preferences, where the parameters α and β correspond to budget shares of the two sectors' products, respectively.

$$\pi_t = \frac{E_t P_{t+1} - P_t}{P_t}, \tag{2.39}$$

where E_t is the conditional expectation operator corresponding to the use of information available at time t, which is assumed to include the structure of the model and the values of all variables dated t and earlier[35]. Thus, expectations in equation (2.39), as elsewhere in the following, are assumed to be rational. A higher real interest rate is assumed to discourage current consumption, and ϕ in equation (2.38) is the interest rate semi-elasticity of demand for non-traded goods.

The demand for money balances takes the following form

$$M_t^d = P_t Y_t \exp[-ki_t + u_t], \quad k \geq 0, \tag{2.40}$$

while the supply of money is based on the now familiar intervention rule

$$M_t^s = M_0 [\frac{S_t}{S_0}]^{-\delta}, \tag{2.41}$$

where M_0 and S_0 denote the target (steady state) values of the money stock and exchange rates, respectively, and δ measures the degree of foreign exchange intervention (or the elasticity of money supply changes with respect to exchange rate changes). Money markets are assumed to clear continuously, so the two money market equations above are supplemented with the money market equilibrium condition

$$M_t^d = M_t^s. \tag{2.42}$$

The law of one price (PPP) is assumed to hold for traded goods:

$$P_{2,t} = S_t P_{2,t}^*, \tag{2.43}$$

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[35] Note that the informational assumptions imply *inter alia* that employment decisions are based, as earlier, on the actual values of the productivity shock, and not on its observed values. The latter leads to a signal extraction inference problem, whereby the producers infer the values of the productivity shock from observed information. A modification of the model along these lines does not lead to any substantial changes in the model's implications. It only introduces in an additional parameter describing the precision of information in observed variables regarding the underlying productivity shock, so we will continue to assume that the productivity shock is observed by the producers before making employment decisions.

where S_t is the exchange rate at time t (domestic money price of one unit of foreign currency) and $P^*_{2,t}$ denotes the price of traded goods in internationally fully integrated commodity markets. Capital is perfectly mobile, and in the absence of risk aversion, an (approximate) arbitrage condition links the domestic and foreign interest rates:

$$i_t - i_t^* = \frac{E_t S_{t+1} - S_t}{S_t}. \qquad (2.44)$$

To model short-run wage rigidity, we refer to the contracting approach outlined in section 2.1. We shall not repeat the whole approach here, but merely write down the relevant equations. The supply of labour is given by

$$N_t^s = A_s[\frac{W_t}{P_t}]^\varepsilon, \qquad (2.45)$$

where W_t is the money wage at time t and ε denotes the labour supply elasticity. Labour is the only mobile factor and sectoral outputs are given by

$$Y_{1,t} = A_1[N_{1,t}]^\Theta \exp(\mu_t)$$

$$Y_{2,t} = A_2[N_{2,t}]^\Theta \exp(\mu_t), \qquad (2.46)$$

where $N_{i,t}$ denotes labour employed in sector i (i=1,2). Note that i) output labour elasticities are assumed to be equal in both sectors and ii) there are no sector-specific productivity shocks, i.e. the same productivity shock impinges on both sectors.

The assumptions amount to saying that, apart from scale factors A_i, the sectors share a common short-run production technology. As Aizenman argues (ibid., p. 541 fn. 1), relaxing the former assumption has no systematic effects on the results, but it simplifies that the computations involved. The common shock assumption implies the sectoral production possibility sets are not affected by idiosyncratic (i.e. sector-specific) shocks so that changes in the relative price of non-traded goods do not reflect changes in production possibilities. It further implies that the analysis focuses only on aggregate productivity shocks, not on sector-specific shocks. That there is only one aggregate productivity shock and that the output labour elasticities are equal may be highly counterfactual assumptions, but if, for example, sector-specific productivity shocks are introduced into the model, the simple indexation

scheme, whereby nominal wages are indexed to the aggregate price level, may not be fully justified.

Producers are maximizing profits, so that the demand for labour functions are given by

$$N_{1,t} = [A_1]^h [\frac{P_{1,t}}{W_t}]^h \exp(h\mu_t) \tag{2.47}$$

$$N_{2,t} = [A_2]^h [\frac{P_{2,t}}{W_t}]^h \exp(h\mu_t),$$

where $h = 1/(1-\Theta)$. Actual wages are indexed to the aggregate price level according to the indexation scheme

$$W_t = W_t^c [\frac{P_t}{P_0}]^b, \tag{2.48}$$

where b denotes the degree of indexation, W_t^c the contract wage for period t and P^o is the aggregate price level in non-stochastic equilibrium.

To close the model, we have to specify the stochastic structure of the various disturbances. For foreign prices, we assume $P_{2,t} = \exp(p_{2,t}^*)$, and for the vector process $X_t = (\mu_t, u_t, p_{2,t}^*, i_t^*)'$ we assume

$$X_t \sim NID(0,\Sigma)$$

$$\Sigma = diag[\sigma_\mu^2, \sigma_u^2, \sigma_1^2, \sigma_2^2].$$

As in the earlier sections, in order to analyze the effects of the various shocks we shall normalize the model by defining the non-stochastic equilibrium of the economy as the equilibrium in the economy where the values of all the random variables are zero, i.e. $X_t \equiv 0$. We assume that in this equilibrium prices are given by: $P_{0,1} = P_{0,2} = S_0 = 1$. Furthermore, let lowercase letters denote the percentage deviation of a variable from its non-stochastic equilibrium value, i.e. $x_t \equiv (X_t-X_0)/X_0$ for a variable X. Given the stationarity of the model's stochastic structure, we delete the time subscript from the variables, i.e. (x_t, x_{t+k}) is replaced by (x, x_{+k}). The above model is cast in a deviation form by log-linearizing it around its non-stochastic equilibrium. This is equivalent to the use of a first-

order approximation of a Taylor expansion around the non-stochastic equilirium.[36]

Substituting from equations (2.47) into equations (2.46), we can represent the percentage deviation of sectoral outputs in the form

$$y_1 = h^*(p_1 - w) + h\mu$$

$$y_2 = h^*(p_2 - w) + h\mu,$$

(2.46')

where $h^* = \Theta h$. Aggregating and substituting in the equation for changes in actual wages, $w = bp$, $p = \alpha p_1 + \beta p_2$, we obtain an equation for the Phillips curve relating changes in aggregate output to changes in the price level:

$$y = h^*(1 - b)p + h\mu.$$

(2.49)

In equation (2.49) $h^*(1-b)$ corresponds to the Phillips-curve slope, which depends critically on the degree of wage indexation b. Full wage indexation completely eliminates the dependence of output changes on (unexpected) price level changes, while a lower degree of wage indexation enhances the output effects of (unexpected) price level changes.

Since markets for non-traded goods clear continuously, equilibrium in this sector implies that

$$y_1 = -\tau(p_1 - p_2) + y - \phi(i + p).$$

(2.50)

Equilibrium in the money market is given by[37]

$$m - p = y - ki + u; \quad i = i^* - s.$$

(2.51)

Note that the real interest rate is $i + p$, since an unexpected price level increase induces andexpectation that the price level will fall next period by p (see, also, Aizenman, ibid., p. 543). Equations (2.46'), (2.49) and

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[36] This approximation is useful, if the variances of the disturbances are sufficiently small. Furthermore, for the aggregate output equation, first order Taylor approximations with respect to the sectoral outputs only are employed.

[37] In equations (2.50) and (2.51) we have implicitly used the fact that rational expectations are static here, i.e. $E_t p_{+k} = 0 = E_t s_{+k}$, for k > 0. The argument in favour of these choices is given in section (2.2.1) and is not repeated here.

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(2.50) provide the short-term equilibrium condition, which implies that changes in the relative price of non-traded goods have to satisfy

$$p_1 - p_2 = -\frac{\phi}{\alpha} \frac{(i^* + p_2^*)}{\Phi}, \qquad (2.52)$$

where $\Phi = \phi + (\tau + \beta h^*)/\alpha$. A rise in the foreign interest rate, or a transitory increase in the foreign price, leads to a higher real interest rate, which reduces the current demand for both goods. Since the market for non-traded goods clears continuously, the relative price of non-traded goods should fall. Greater substitutability of the two goods in consumption (τ) or production (h^*) reduces the needed adjustment in relative prices. Notice that changes in the relative price of non-traded goods are independent of the productivity shocks impinging on the economy, as argued earlier. This is due to assumption that there are no sector-specific productivity shocks affecting each sector's production possibilities. Thus, changes in the relative price of non-traded goods reflect only shocks affecting the demand for each sector's goods.

Continuous equilibrium in the money market implies that equilibrium changes in exchange rates satisfy

$$s = D^{-1}\{-[1 + h^*(1 - b)][1 - \frac{\phi}{\Phi}]p_2^*$$

$$+ [k + (\frac{\phi}{\Phi})(1 + h(1 - b))]i^* - (u + h\mu)\}, \qquad (2.53)$$

where $D \equiv [1 + k + \delta + h^*(1 - b)]$.

The equation for equilibrium changes in exchange rates has the familiar form, but now the reduced form equation is heavily influenced by the parameter related to the sectoral structure of the economy, i.e. by the parameters determining the degree of openness of the economy. Openness can be measured either by the share of traded goods ß, or by the substitutability in consumption (τ) and production (h^*).[38]

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[38] Note that $d\log[Y_1/Y_2]/d\log[P_1/P_2] = h^*$.

2.5.2 Optimal degree of wage indexation and foreign exchange intervention and openness

To derive some welfare implications of wage indexation and intervention, we shall employ the welfare criterion suggested by Gray (1976). To this end, we have to solve for the fully flexible or full information equilibrium of the economy. First of all, the loss function employed is

$$H = E\{(y - y^f)^2 : I_{t-1}\}$$

$$= AE\{(n - n^f)^2 : I_{t-1}\} \quad n = \alpha n_1 + \beta n_2 \qquad (2.54)$$

$$= BE\{[(w - p) - (w - p)^f]^2 : I_{t-1}},$$

where I_{t-1} denotes the information available to the agents at the end of period t-1 (i.e. at the time contracts are written), A and B are constants not depending on b and δ, and where the superscript "f" indicates the full information equilibrium value of a variable. From the full information labour market equilibrium, equations (2.45) and (2.47), we obtain

$$(w - p)^f = \left[\frac{h}{h + \varepsilon}\right]\mu \qquad (2.55)$$

$$y^f = \left[\frac{1 + \varepsilon}{h + \varepsilon}\right]h\mu. \qquad (2.56)$$

Since actual real wages are given by -(1-b)p, the loss function to be minimized reduces to

$$H = E\{[(1 - b)p + [\frac{h}{h + \varepsilon}]\mu]^2 : I_{t-1}\} \qquad (2.54')$$

To obtain a reduced form equation for the changes in the domestic price level we compute

$$p = \alpha p_1 + \beta p_2$$

$$= D^{-1}\{[k + \delta][1 - \frac{\phi}{\Phi}]p_2^* + [k - [k + \delta](\frac{\phi}{\Phi})]i^* \qquad (2.57)$$

$$- (u + h\mu)\}.$$

From equation (2.57) we see that the ultimate effects various shocks have on the changes in the domestic price level depend heavily on the degree of indexation, the exchange rate regime and on the sectoral structure of the economy or openness of the economy. Only under flexible exchange rates do foreign prices and interest rates exert an equal effect on the domestic price level. The rationale for this is that, under flexible exchange rates and static expectations, exchange rates adjust so that foreign price and interest rate shocks have an equal effect on domestic interest rates. On the other hand, given the choice of (b,δ), shocks to foreign prices and interest rates exert smaller effects on the price level due to adjustment in relative prices, but unless exchange rates are flexible, exchange rates provide an additional channel for foreign interest rate shocks to affect domestic price level variability. This effect is magnified in a more closed economy. This feature is present in the structure of the model, since it envisages an economy fully open as regards its money and capital markets, but whose openness as regards commodity markets varies (as measured by Φ). Thus, in a relatively more closed economy, foreign disturbances reflect mainly capital market disturbances. Finally, as previously observed, a higher degree of wage indexation, *ceteris paribus*, increases price level variability.

Substituting in the equation for changes in the domestic price level, the loss function takes the form

$$H = BE\{[\Gamma\Omega + [\frac{h}{h+\varepsilon}]\mu]^2 : I_{t-1}\}, \qquad (2.54'')$$

where

$$\Gamma = \frac{(1-b)}{[1+k+\delta+h^*(1-b)]}$$

$$\Omega = [1-\frac{\phi}{\Phi}][\delta+k]p_2^* + [k-(k+\delta)(\frac{\phi}{\Phi})]i^* - (u+h\mu).$$

As before, $-\Gamma\Phi$ represents changes in actual real wages, while $h/(h+\varepsilon)\mu$ denotes changes in equilibrium real wages. The second moment of the difference between these two, arising from imperfect information, is proportional to the variance of the deviation $(y-y^f)$. The problem is to choose those values of b and δ, the optimal policy (b*,δ*), that minimizes H. Fix the value of δ for the moment. Since the loss function is the mean squared error resulting from regressing equilibrium real wages on actual real wages, the optimal degree of wage indexation b*, given δ, can be found from the regression coefficient Γ^*:

$$\Gamma^* = \frac{-\operatorname{cov}\left[\Omega, \dfrac{h\mu}{h+\varepsilon}\right]}{\operatorname{var}(\Omega)}$$

(2.58)

$$= \frac{\left[\dfrac{h^2}{h+\varepsilon}\right]\sigma_\mu^2}{K},$$

where

$$K = \sigma_\mu^2\{1 + h^2 + (\frac{\sigma_u^2}{\sigma_\mu^2}) + [(k+\delta)(1-\frac{\phi}{\Phi})]^2(\frac{\sigma_1^2}{\sigma_\mu^2}) + [(-\delta\frac{\phi}{\Phi}) + k[1-\frac{\phi}{\Phi}]]^2(\frac{\sigma_2^2}{\sigma_\mu^2})\}.$$

Using the definition of Γ, we finally obtain an expression for the optimal degree of wage indexation

$$b^* = 1 - \frac{1+k+\delta}{K'+1+\varepsilon},$$

(2.58')

where $K' = [\frac{h+\varepsilon}{h^2}][(\frac{K}{\sigma_\mu^2}) - (1+h^2)]$.

As can be seen from this equation, in addition to its dependence on the stochastic structure of the economy and on the exchange rate regime, b^* also depends on the openness of the economy, as fully reflected in the parameter K'.[39] Certain special cases are of interest here.

First, setting $\delta=0$, i.e. under flexible exchange rates, the parameter K' reduces to

$$K' = [\frac{h+\varepsilon}{h^2}]\{\frac{\sigma_u^2}{\sigma_\mu^2} + k^2[1-\frac{\phi}{\Phi}]^2\frac{\sigma_1^2+\sigma_2^2}{\sigma_\mu^2}\},$$

---

[39] A direct inspection of equations (2.58) and (2.58') reveals that the openness of the economy is fully captured in the parameter K' as far as the share of the traded goods sector (β) and substitution in consumption (τ) are concerned. Since, however, $h^* = \theta h$, substitution in production affects the various policy instruments and their comparative statics properties independently of K'. As we move on in our analysis, this qualification should be borne in mind. Thus if, for example, we postulate that $\Phi \to \infty$, this limit process is strictly speaking feasible only for $\tau \to \infty$ and/or $\alpha = 1-\beta \to 0$.

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so that the corresponding optimal degree of wage indexation reduces to the one obtained by Aizenman (ibid., p. 545, eq. (24)). On the other hand

$$b^* \rightarrow 1 - \frac{1+k+\delta}{K''+1+\varepsilon}$$

$$K'' = [\frac{h+\varepsilon}{h}2]\{\frac{\sigma_u^2}{\sigma_\mu^2} + k^2\frac{\sigma_2^2}{\sigma_\mu^2} + (k+\delta)^2\frac{\sigma_1^2}{\sigma_\mu^2}\},$$

when $\Phi \rightarrow \infty$, which is equivalent to the optimal degree of wage indexation obtained previously using the one-commodity open economy framework (see section 2.3.4, eq. (2.54) as well as Aizenman & Frenkel, 1985a, b). To interpret this, we note that the condition $\Phi \rightarrow \infty$ describes a "limit economy" which is completely open, i.e. it depicts a small open economy producing one homogenous product and facing fully intergrated international commodity and money markets.[40]

Furthermore, setting $(\tau,\beta,\delta) = (0,0,0)$, which describes a closed economy, implies that b* is given by

$$b_c = 1 - \frac{1+k}{[\frac{h+\varepsilon}{h^2}][\frac{\sigma_u^2}{\sigma_\mu^2} + 1 + \varepsilon]},$$

which is exactly the same as the equation derived earlier (see eq. (2.54')).

Note further that the effects on the optimal degree of indexation (and, of course, on the optimal degree of intervention, as we will shortly see) of openness is critically dependent on the interest rate semi-elasticity of the demand for non-traded goods (ϕ). To be more precise, if $\phi = 0$, b* is independent of our openness measure Φ. In this case demand for non-traded goods is independent of the real interest rate i + p. But this implies that the equilibrium condition for the non-traded goods sector is independent of the real interest rate, so that no changes in the equilibrium relative price of non-traded goods is generated as a result of shocks to real interest rates (cf. equation (2.52)). If, on the other hand, $\phi \rightarrow \infty$ shocks to real interest rates are completely accommodated by equilibrium

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[40] As explained above Φ parameterizes the openness of the economy, so that higher values of Φ mean that the economy is more open. This intuitive picture can be more formally checked by inspecting the constituents of Φ; higher values of Φ come from greater substitutability (τ) between traded and non-traded goods or a greater share of the traded goods sector (β).

changes in the relative price of non-traded goods. Thus shocks to domestic real interest rates have no aggregate affects (cf. eq. (2.57)).[41]

Differentiating the loss function H with respect to δ, we obtain the necessary condition for the minimum implicitly defining the optimal degree of foreign exchange intervention δ^*:

$$\frac{\partial H}{\partial \Gamma} \frac{\partial \Gamma}{\partial \delta} + 2E\{[\Gamma\Omega + \frac{h}{(h+\varepsilon)}\mu]\Gamma\frac{\partial\Omega}{\partial\delta} : I_{t-1}\} = 0. \qquad (2.59)$$

Since this expression is evaluated at the optimal b,[42] the first term on the left-hand side of equation (2.59) disappears. Finally, solving the remaining equation for δ, we obtain

$$\delta^* = -k[1 - \frac{\phi}{\Phi}]\{[1 - \frac{\phi}{\Phi}]\sigma_1^2 - \frac{\phi}{\Phi}\sigma_2^2\}F^{-1}$$

$$F = \{[1 - \frac{\phi}{\Phi}]^2\sigma_1^2 + (\frac{\phi}{\Phi})^2\sigma_2^2\}. \qquad (2.59')$$

From (2.59') we can immediately see that the use of the simple intervention rule for determining exchange rates produces at the optimum a rather surprising result: the optimal degree of foreign exchange intervention, δ^*, is independent of the stochastic structure of the productivity and money market disturbances, i.e. domestic disturbances. There is thus a certain degree of specialization between indexation and intervention in that intervention policy focuses only on foreign disturbances while indexation also takes into account disturbances in domestic productivity and money markets. Note, however, that the optimal degree of intervention depends heavily on the sectoral structure of the economy, since

$$\delta^* \to -k,$$

as $\Phi \to \infty$, i.e. in a fully open economy the optimal degree of intervention is completely independent of the stochastic structure of the

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[41] Note that even for the case $\phi = \infty$ shocks to the foreign interest rate impinge on the aggregate domestic economy through exchange rates, unless the country happens to choose flexible exchange rates ($\delta=0$), as can be readily verified from equation (2.57).

[42] That is, an argument based on envelope theorem dictates that the first term disappears.

economy. This is the result we also derived earlier. We also argued that strict adherence to the PPP rule coupled with the simple money supply rule used by the monetary authority produces this result.[43]

That optimal intervention is independent of domestic shocks is a consequence of the particular choice of the form of the policy instrument in use. Indexation attempts to adjust the real wage w - p (and, accordingly, employment and output) to its market-clearing value. Because price level variability reflects both demand and supply shocks, in general the single instrument of wage indexation cannot accomplish the needed adjustment, i.e. the price level is not a sufficient statistic for all of the underlying disturbances. The indexation scheme carries out a trade-off between the costs of allowing the demand disturbances to affect output (or the real wage) suboptimally and the benefits of optimally adjusting output to supply shocks. The optimal intervention rule actively reduces the influence of foreign shocks on the price level, thereby improving the efficiency of the wage indexation rule in achieving the market-clearing real wage, i.e. intervention increases, at the aggregate level, the economic relevance of supply shocks embodied in the domestic "signal" $(u+h\mu)$.

Note, however, that the optimal policy mix (b^*, δ^*) is designed to improve the adjustment of the aggregate economy, but the relative price adjustment, due to its allocational effects, also reduces the aggregate effects of the foreign shocks. Since relative price adjustment is stronger in a more closed economy, the aggregate relevance of changes in relative prices is directly related to the openness of the economy.

Note further that the optimal intervention rule implies that the corresponding exchange rate policy can be either "leaning with the wind" or "leaning against the wind", but that in a relatively more open economy, optimal exchange rate policy is likely to be one of "leaning with the wind". The rationale for this is that the need to adjust exchange rates to maintain PPP increases with the openness of the economy.

A feature worth noticing in the optimal mix (b^*, δ^*) is that in cases where there is only one foreign shock in addition to one domestic shock, the real wage implied by the corresponding optimal mix is the equilibrium real wage. This result serves to supplement Tinbergen's theorem concerning the relationship between targets and instruments of economic policy. The target here is the elimination of a distortion due to wage rigidity, and the instruments consist of wage indexation and foreign

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[43] Note that for interest inelastic demand for non-traded goods, i.e. $\phi = 0$, produces this particular intervention rule, while infinite (semi-)elasticity ($\phi = \infty$) produces flexible exchange rates. This latter result follows from the fact that flexible exchange rates fully insulates, the domestic aggregate real economy from shocks to foreign interest rates, if the relative price of non-traded goods fully accommodates shocks to domestic real interest rates.

exchange intervention. The wage indexation scheme and intervention rule are able to eliminate the distortion only if they provide enough information about the determination of the equilibrium real wage. In our economy the equilibrium real wage depends only on the productivity shock. In cases where only one foreign and one domestic productivity shock impinge on the economy, optimal intervention is capable of eliminating its influence on the domestic price level, which accordingly signals only domestic productivity shocks. Since the optimal degree of wage indexation in this case optimally captures the effects of domestic productivity shocks on actual wages, this results in actual real wages being equilibrium real wages. To put it briefly, in these two shock cases (w,s) is a sufficient statistic for the underlying shocks. In cases where there are more shocks, more instruments or "independent indicators" (Aizenman & Frenkel, 1985b, p. 412) are needed.

Note further that the optimal degree of wage indexation need not behave properly with respect to openness, if, in addtion to domestic productivity and/or money market shocks, there are only foreign interest rate shocks, since the (intuitively acceptable) restriction $0 \leq b^* \leq 1$ is likely to be violated even in a relatively closed economy for reasonable parameter values. The reason for this is, as Devereuex (1988, p. 387, fn. 5, but see also Turnovsky, 1983) observes, that the market-clearing nominal wage and the price level can be negatively correlated for some parameter values. This problem is accentuated in an open economy, since the optimal choice for the exchange rate regime is completely fixed rates, implying an unbounded (negative) degree of wage indexation. But this case is somehow degenerate, since if only foreign interest shocks (in addition to domestic shocks) impinge on the domestic economy, and the monetary authority reacts by fixing the exchange rates, then the price level becomes a constant in an open economy, and provides no valuable information as regards the underlying shocks.

Aizenman derived some comparative statics results concerning the relationship between openness and the optimal degree of wage indexation, and it is of interest to ask whether these results are still valid under this more general specification. More precisely, Aizenman concluded (ibid. p. 545) that the optimal degree of wage indexation under flexible exchange rates increases with openness, i.e. $(db^*/d\Phi) > 0$. This implies that we are likely to expect more rigid real wages in a more open economy, a result which follows from the fact that (sectoral) adjustment due to relative price changes is weaker in a more open economy. This implies that for more open economies more of the after-shock adjustment takes place at the aggregate level, enhancing real wage rigidity due to absence of the cushioning (Aizenman, ibid. p. 546) of the foreign shocks provided by relative price changes.

Aizenman's result needs some qualifications, once the domestic economy is not constrained in its choice of flexible exchange rates. From equation (2.58') we obtain that

$$\frac{\partial b^*}{\partial \Phi} = [\frac{\partial b^*}{\partial \delta}]_\Phi [\frac{\partial \delta}{\partial \Phi}] + [\frac{\partial b^*}{\partial \Phi}]_\delta. \qquad (2.60)$$

The last term on the right-hand side of equation (2.60) gives the effect of openness on the optimal degree of wage indexation keeping δ fixed. From (2.58') we can see that, given δ, our measure of openness affects the optimal degree of wage indexation only through K', so that

$$\frac{\partial b^*}{\partial \Phi}\Big|_{\delta=\delta^*} = \frac{\partial K'}{\partial \Phi}\Big|_{\delta=\delta^*} = \frac{\{2\sigma_1^2\sigma_2^2 k^2(\frac{\phi}{\Phi})(1-\frac{\phi}{\Phi})(\frac{\phi}{\Phi^2})\}}{[\sigma_\mu^2 F^2]} > 0.$$

so that we will face more rigid real wages in more open economies given that exchange rate policy follows optimal intervention rule δ^*.

The first term on the right-hand side of the above equation is composed of two terms, the first of which gives the effect on the optimal degree of wage indexation of changes in the degree of foreign exchange intervention given the degree of openness Φ. The second term measures the effect on b^* of adjusting the degree of intervention δ to openness. Once again from equation (2.58') we deduce that

$$\text{sign}[(\frac{\partial b^*}{\partial \delta})\Big|_\Phi]$$

$$= -\text{sign}\{[k-(k+\delta)(\frac{\phi}{\Phi})][k+(2+k+\delta)(\frac{\phi}{\Phi})](\frac{\sigma_2^2}{\sigma_\mu^2})$$

$$-[k+\delta][1-\frac{\phi}{\Phi}]^2[2+k+\delta](\frac{\sigma_1^2}{\sigma_\mu^2}) + (\frac{\sigma_u^2}{\sigma_\mu^2}) + 1 + \varepsilon\}.$$

If evaluated at $\delta = \delta^*$, we have

$$\frac{\partial b^*}{\partial \delta}\Big|_{\Phi, \delta=\delta^*} = -\{[\frac{H}{\sigma_\mu^2 F}][k^2[(1-\frac{\phi}{\Phi})\sigma_1^2\sigma_2^2] + H(\frac{\sigma_u^2}{\sigma_\mu^2}) + 1 + \varepsilon\} < 0,$$

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where $H = (h+\varepsilon)/h^2$. So, given Φ a marginal reduction in exchange rate variability around the optimum, δ^* will increase real wage rigidity. Finally, if the monetary authority follows the optimal policy rule δ^*, the effects on the degree of intervention of openness is given by

$$\frac{\partial \delta^*}{\partial \Phi} = k[(\frac{\phi^2}{\Phi})^3]\sigma_2^2 \frac{(\frac{\phi}{\Phi})^2\sigma_2^2 - [1 - (\frac{\phi}{\Phi})^2]\sigma_1^2}{F^2} < > 0.$$

Thus, the (optimal) degree of intervention can either increase or decrease with openness. For the above expression to be negative, we need either a sufficiently open economy (high Φ) or sufficiently flat distribution of foreign price shocks (high σ_1^2). If these sufficient conditions are met, then under the optimal exchange rate policy δ^*, the optimal degree of wage indexation will increase with increases in the openness of our economy. Figure 2.3 illustrates the possibilities involved; we have plotted the optimal degree of wage indexation against openness, $b^*[\delta^*(\Phi),\Phi]$ say, given that exchange rates are optimally adjusting for openness.

Figure 2.3   **Optimal degree of wage indexation and openness**
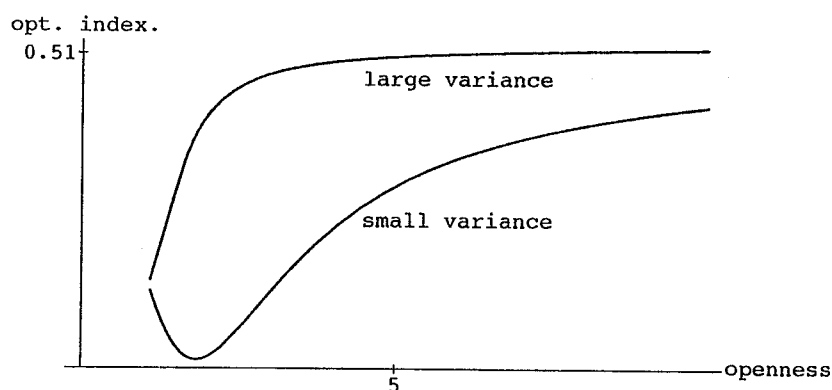
Although overall the effects of openness on the optimal degree of wage indexation under optimal intervention are ambiguous, it seems worthwhile to try to seek some understanding of the positive relationship between openness and optimal indexation noted above. For more closed economies changes in the equilibrium relative price of non-traded goods cushions the effects of shocks to foreign interest rates and prices on the pricelevel more powerfully. Intervention policy is dominated by the attempt to neutralize the effects of foreign interest rate shocks on the

domestic price level, since the money market effects of interest rate shocks on the aggregate economy for a given b are reinforced in a more closed economy. This calls for an intervention policy to reduce exchange rate variability, i.e. an exchange rate policy of "leaning against the wind". Finally, flexibility of real wages should be increased because of productivity shocks, i.e. the optimal degree of wage indexation should fall in a more closed economy.

Substituting the optimal degree of wage indexation and intervention, equations (2.58') and (2.59'), respectively, into the loss function (2.54"), we obtain an expression for the efficiency loss, once the degree of indexation and intervention are optimally adjusted for openness and the stochastic structure of the economy.

$$
H = B[\frac{h}{h+\varepsilon}]^2\sigma_\mu^2\{1 - \frac{1}{A(\Phi)}\} + [\frac{h}{h+\varepsilon}]^2\sigma_\mu^2, \qquad (2.61)
$$

where $A(\Phi)$ is an increasing function of Φ, and $A(\Phi) \geq 1$ for all values of Φ. This implies that by differentiating the loss function with respect to Φ, we have

$$
\frac{\partial H}{\partial \Phi} > 0, \qquad (2.62)
$$

i.e. efficiency losses due to suboptimal labour markets increase with openness, when indexation and intervention follow the optimal policy mix (b*,δ*). Aizenman (ibid., p. 546) derives the same result, under the assumption that the economy is constrained to flexible exchange rates. The idea Aizenman wants to highlight in his analysis is the stabilizing effect of relative price changes *vis-à-vis* the aggregate economy, and (2.62) does seem to suggest that the welfare effects of the use of the single instrument of wage indexation do not depend critically on the exchange rate regime. This is not to say that flexible exchange rates are, under all possible contingencies, the optimal exchange rate regime for a country to choose.

On the contrary, flexible exchange rates are the optimal choice only under special circumstances (cf. (2.59')). The above result merely suggests that, in terms of aggregate efficiency, the use of the simple intervention rule determining money supply is not a substitute for relative price changes, i.e. sectoral adjustment brought about by changes in relative prices has a strong stabilizing effect on the aggregate economy.

2.5.3 Optimality of a single instrument

As above, it is instructive to analyze the behaviour of the aggregate economy when only one of the two instruments can be adjusted optimally, given that the other instrument is fixed, for example through an administrative process. Labour market participants certainly take into account the prevailing exchange rate regime while contracting. On the other hand, certain countries (see e.g. Simonsen's description of the Brazilian experience, 1983) have adopted a specified, publicly available wage indexation rule, which should be considered as a constraint on monetary policy.[44] In this section we shall investigate separately the aggregate effects of the optimal use of one of the instruments, taking the other as exogenously given.

2.5.3.1 Optimal wage indexation under a given exchange rate regime

To fix ideas, we shall first take the degree of foreign exchange intervention as given, e.g. through an administrative process. This implies that exchange rates are not optimally adjusted for openness and stochastic shocks, so that the burden of adjustment falls entirely on wage indexation. Of particular interest is the rigidity of real wages as a function of openness, once the constraint on exchange rate variability is taken into account. This question is addressed by analyzing the optimal degree of wage indexation, equation (2.58'), for a given δ. Equation (2.58') is reproduced here for completeness

$$b_\delta^* = 1 - \frac{1+k+\delta}{K'+1+\varepsilon}. \qquad (2.58')$$

The sectoral structure of the economy is completely embedded in K', which is given by

$$K' = [\frac{h+\varepsilon}{h^2}]\{\frac{\sigma_u^2}{\sigma_\mu^2} + [(k+\delta)(1-\frac{\phi}{\Phi})]^2\frac{\sigma_2^2}{\sigma_\mu^2} + [k-(k+\delta)\frac{\phi}{\Phi}]^2\frac{\sigma_1^2}{\sigma_\mu^2}\}.$$

---

[44] It is not argued that monetary policy is designed to improve aggregate adjustment due to suboptimal labour markets or that the policy rule is the one considered here. We are merely saying that the labour market practice is a constraint on monetary policy, so that when evaluating the ultimate effects of monetary policy, the repercussions or feedback effects from labour markets should be duly accounted for.

By differentiating K' with respect to Φ we find that

$$\frac{\partial K'}{\partial \Phi} = \frac{C\phi}{\Phi^2}\{(k+\delta)^2(1-\frac{\phi}{\Phi})\sigma_1^2 - \frac{\phi}{\Phi}\sigma_2^2] + k(k+\delta)\sigma_2^2\}.$$

In principle, any conceivable value can be given to δ, but if we restrict ourselves to the range $\delta \geq -k$, we deduce that the sign of this derivative hinges upon the inequalities

a) $k > (k+\delta)[(\frac{\phi}{\Phi}) - (1 - \frac{\phi}{\Phi})(\frac{\sigma_1^2}{\sigma_2^2})]$

b) $k < (k+\delta)[(\frac{\phi}{\Phi}) - (1 - \frac{\phi}{\Phi})(\frac{\sigma_1^2}{\sigma_2^2})].$

Given the restriction $\delta \geq -k$, we can immediately see from these inequalities that the relationship between openness and indexation hinges upon the characteristics of the unconstrained intervention rule δ^*. If $\delta^* < 0$, i.e. the optimal exchange rate policy for our economy is one of "leaning with the wind", real wage rigidity increases with increases in openness. For real wage rigidity to fall with increases in openness, it is not, according to inequality b), sufficient for the optimal exchange rate policy to "lean against the wind" ($\delta^* > 0$). The reduction in exchange rate variability brought about by the optimal intervention rule has to be sufficiently large in order to induce a fall in the degree of wage indexation. It is hard to formalize further the conditions needed to make this statement more precise, but note that the likelihood of a negative relationship between optimal indexation and openness decreases as our economy becomes more open.[45]

Notice that if the actual degree of intervention happens to coincide with the (unconstrained) optimal degree of intervention δ^*, then, as we

---

[45] Inequalities a) and b) jointly define a "critical" value of Φ, Φ^* say, as a result of which the dependence of wage indexation on openness changes its sign. This value is given by

$$\Phi^* = \frac{\phi(k+\delta)(\sigma_1^2+\sigma_2^2)}{k(\sigma_1^2+\sigma_2^2)+\delta\sigma_1^2}$$

Thus $(\partial b^*/\partial \Phi) <> 0$ as $\Phi <> \Phi^*$. Note that under flexible exchange rates ($\delta = 0$) Φ^* equals ϕ, from which Aizenman's comparative statics result immediately follows, since $\Phi \geq \phi$.

showed in the preceding section, $(db*_\delta/d\Phi) > 0$. Notice further that by setting $\delta=0$, i.e. flexible exchange rates, then

$$\frac{\partial b_\delta^*}{\partial \Phi}\Big|_{\delta=0} = \frac{C\phi}{\Phi^2}k^2[1 - \frac{\phi}{\Phi}][\sigma_1^2 + \sigma_2^2] > 0.$$

Thus, under flexible exchange rates real wage rigidity unambiguously increases with increases in openness (cf. Aizenman, ibid.). The rationale for this is that under flexible exchange rates the effects of shocks to foreign prices and interest rates on the domestic interest rates are equal. The optimal degree of wage indexation under flexible exchange rates reduces to

$$b^*(\text{flex}) = 1 - \frac{1+k}{K'+1+\varepsilon}, \tag{2.58'}$$

where

$$K' = [\frac{h+\varepsilon}{h^2}]\{\frac{\sigma_u^2}{\sigma_\mu^2} + [k(1 - \frac{\phi}{\Phi})]^2\frac{\sigma_1^2 + \sigma_2^2}{\sigma_\mu^2}\}.$$

Thus only the sum of the variances of the foreign shocks relative to domestic productivity shocks matter, not the individual variances. The corresponding efficiency loss under flexible exchange rates is given by

$$H[\text{flex}] = [\frac{h}{h+\varepsilon}]^2\sigma_\mu^2[1 - \frac{1}{K''+1}], \tag{2.63}$$

where

$$K'' = \frac{K'}{h+\varepsilon}.$$

On the other hand from equation (2.58') we can see that under fixed exchange rates ($\delta=\infty$), the optimal degree of wage indexation is one, i.e.

$$b^*(\text{fix}) = 1. \tag{2.58'}$$

The corresponding loss is now

$$H[fix] = [\frac{h}{h+\varepsilon}]^2 \sigma_\mu^2. \tag{2.64}$$

From (2.63) and (2.64) we deduce that

$$H[flex] - H[fix] = \frac{-\left[\dfrac{h}{h+\varepsilon}\right]^2 \sigma_\mu^2}{K'' + 1} < 0. \tag{2.65}$$

Thus, given the degree of openness of the economy, efficiency losses are always higher under a fixed exchange rate regime than under flexible exchange rates, confirming the overall conclusions of Flood & Marion (1982) concerning the welfare properties of various exchange rate regimes. Note that under flexible exchange rates and partial indexation nominal shocks affect the real equilibrium of the economy, while under fixed exchange rates and full indexation they do not. However, full indexation is a hindrance to real wage adjustment due to productivity shocks. The complete rigidity of real wages under fixed exchange rates induces costs that outweigh the costs due to nominal shocks under flexible exchange rates and partial wage indexation.

One further interesting point about equation (2.65) is that losses under fixed exchange rates relative to flexible rates decrease as the economy becomes more open. To gain some understanding of this, we note that in the above calculations the degree of wage indexation is optimally adjusting for openness. From the preceeding discussion we know that the degree of wage indexation also increases with increases in openness under flexible exchange rates. Accordingly, it is the rigidity of the real wages in more open economies, rather than a particular exchange rate regime, which explains why the difference in welfare losses under flexible and fixed exchange rate regimes shrinks with increases in openness.

Tables 2.4.1a—1b simulate the dependence of the optimal degree of wage indexation on the degree of intervention and openness, assuming particular values for the parameters in the model.[46]

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[46] The parameter structure corresponding to figure 2.2 is given by $(\theta,\varepsilon,k) = (0.8,1,0.5)$ with unit variances for the shocks except for foreign interest rates, which is larger.

Table 2.4.1a | Optimal degree of wage indexation under fixed and flexible exchange rates as the degree of openness Φ varies

Values of Φ	b*(flex)	H(flex)	b*(fix)	H(fix)	b*(δ*)
.1	0.2986	0.0512	1	0.62	0.2986
1.0	0.3608	0.0773	1	0.62	0.5559
5.0	0.3712	0.0819	1	0.62	0.5813
10.0	0.3725	0.0825	1	0.62	0.5839
∞	0.3741	0.0832	1	0.62	0.5865

b*(δ*) refers to the optimal degree of wage indexation, given the optimal degree of intervention δ*.

Table 2.4.1b | Optimal degree of wage indexation for an open and "closed" economy as the degree of intervention varies

Values of δ	$b^*_{\delta,c}$	$b^*_{\delta,o}$	$H_c(b^*_\delta;\delta)$	$H_o(b^*_\delta;\delta)$
-.6	0.5865	0.5865	0.0676	0.0676
-.1	0.3435	0.4032	0.0517	0.0785
0	0.2986	0.3741	0.0512	0.0832
1.0	0.0233	0.2338	0.0946	0.1617
10.0	0.7127	0.7435	0.5586	0.5648
∞	1.0000	1.0000	0.6200	0.6200

The subscripts c and o refer to relatively closed and open economies. The optimal degree of intervention is 0 and -.6, respectively.

From table 2.4.2a we can tentatively draw the conclusion that, under the assumption of equal variances, wage indexation tends to be too low under flexible exchange rates and certainly too high under fixed exchange rates compared to the optimal policy mix (b*(δ*),δ*). What is also of some interest is the rather narrow range of optimal wage indexation under the optimal policy mix, and expecially if the degree of intervention is exogenously fixed.

Table 2.4.2b immediately reveals the power of the adjustment induced by relative price changes. For moderate values of the degree of intervention, the optimal degree of wage indexation drops much more

sharply in a closed economy than in an open economy. For unit elastic money supply with respect to exchange rate changes, nominal wages are almost fully rigid in a relatively closed economy, whereas in an open economy the optimal degree of wage indexation is still 0.2338. This means that some 23 % of the price level changes are transmitted into changes in nominal wages in an open economy. Once the variability of the exchange rate decreases as a result of an exchange rate policy of strongly "leaning against the wind", the difference between the optimal degree of wage indexation in closed and open economies disappears.

Tables 2.4.1 immediately reveal the power of the adjustment induced by relative price changes. For moderate values of the degree of intervention, the optimal degree of wage indexation drops much more sharply in a closed economy than in an open economy. For unit elastic money supply with respect to exchange rate changes, nominal wages are almost completely rigid in a relatively closed economy, whereas in an open economy the optimal degree of wage indexation is still relatively high. This means that a relatively high percentage of price level changes is transmitted into changes in nominal wages in an open economy. Once the variability of exchange rate decreases as the result of an exchange rate policy of strongly "leaning against the wind", the difference between the optimal degree of wage indexation in a closed and open economy disappears.

2.5.3.2 Optimal degree of intervention given the degree of wage indexation

We shall now turn to the case where labour market arrangements produce a given degree of wage indexation and the monetary authority takes this into account in designing optimal intervention policy for the two-sector economy in question. We can quite reasonably argue that this is the relevant case in practice, since labour market contracts are typically long-term contracts, and monetary policy has the time (and resources) to react to various disturbances. We shall not pursue this argument further, but shall return to it in the next chapter, when we approach labour market contracting from an entirely different angle. Here we shall merely derive the optimal degree of intervention given the degree of wage indexation.

The optimal degree of foreign exchange intervention for a given b can be derived by minimizing the loss function H with respect to δ. Solving the first order conditions for δ we obtain

$$\delta^*(b) = \delta_b^* = -k + \frac{\Gamma_1}{\Gamma_2}, \tag{2.66}$$

where

$$\Gamma_1 = (1-b)\{k\sigma_2^2[k + (\frac{\phi}{\Phi})(1 + h^*(1-b))] + [\sigma_u^2 + h^2\sigma_\mu^2]\} - [\frac{h}{h+\varepsilon}]\sigma_\mu^2$$

$$\Gamma_2 = \{(1-b)[(1 + h^*(1-b))][(1 - \frac{\phi}{\Phi})^2\sigma_1^2 + (\frac{\phi}{\Phi})^2\sigma_2^2] + k(\frac{\phi}{\Phi})\sigma_2^2]$$

$$+ [\frac{h}{h+\varepsilon}]\sigma_\mu^2\}.$$

Expression (2.66) is very involved and the nature of the dependence of δ^* on wage indexation and/or openness is not readily available. We can simplify the analysis a bit by considering the two polar cases of full wage indexation and no wage indexation. Under the former the optimal degree of intervention approaches the limit

$$\delta_{b=1}^* = -(1+k), \tag{2.67}$$

while formally the associated efficiency loss is equal to the amount generated under fixed exchange rates. Note, however, that this value for δ^* is a limiting value, which implies an unbounded variability for exchange rates and the price level (cf. equations (2.53) and (2.57)). This result is purely formal, since with full wage indexation, changes in the price level brought about through changes in exchange rates are inconsequential (see, also, Aizenman & Frenkel, 1985b, p. 410 fn. 8).

With completely rigid nominal wages, δ^* reduces to

$$\delta_{b=0}^* = -k + \frac{\Gamma_1}{\Gamma_2}, \tag{2.68}$$

where

$$\Gamma_1 = \{k\sigma_2^2[k + (\frac{\phi}{\Phi})h] + [\sigma_u^2 + h^2\sigma_\mu^2]\} - [\frac{h}{h+\varepsilon}]\sigma_\mu^2$$

$$\Gamma_2 = \{h[(1 - \frac{\phi}{\Phi})^2\sigma_1^2 + (\frac{\phi}{\Phi})^2\sigma_2^2] + k(\frac{\phi}{\Phi})\sigma_2^2 + [\frac{h}{h+\varepsilon}]\sigma_\mu^2\}.$$

As in the case where the degree of wage indexation is b (not necessarily 0 or 1), the effects of openness on the optimal degree of intervention are

in principle ambiguous, but for a sufficiently open economy to start with, $\delta^*_{b=0}$ (and δ^*_b) will decrease (algebraically) with further increases in openness.

There is no point in comparing the these two polar cases in terms of efficiency loss, since the case of full wage indexation is rather pathological. Instead, we shall simulate the behaviour of δ^* under the assumption of full nominal wage rigidity and near real wage ridigity as the degree of openness Φ varies. This is the message of tables 2.4.2, which relate δ^* to the degree of wage indexation and openness of the econonomy. Specific parameter values are as in tables 2.4.1.

Table 2.4.2a   **Optimal degree of intervention with nominal and real wage rigidity as the degree of openness varies**

Values of Φ	$\delta^*_{b=0}$	$\delta^*_{b=1}$	$H(\delta^*_{b=0})$	$H(\delta^*_{b=1})$
.1	4.005	-0.777	0.2811	0.3763
1.0	2.719	-0.792	0.3078	0.3729
5.0	2.329	-0.794	0.3082	0.3723
10.0	2.282	-0.794	0.3081	0.3723
∞	2.235	-0.794	0.3080	0.3722

$\delta^*_{b=1}$ corresponds to the case where the degree of wage indexation is 0.95.

Table 2.4.2b   **Optimal degree of intervention in a "closed" and open economy as the degree of wage indexation b varies**

Values of b	$\delta^*_{b,c}$	$\delta^*_{b,o}$	$H_c(\delta^*_b;b)$	$H_o(\delta^*_b;b)$
0	4.005	2.235	0.2811	0.3080
.2	3.943	2.383	0.2872	0.3018
.4	3.624	2.406	0.2992	0.3023
.6	2.826	2.092	0.3189	0.3135
.8	1.206	0.979	0.3487	0.3398
.95	-.777	-.794	0.3763	0.3722

The subscripts c and o refer to a relatively closed and open economy respectively.

Tables 2.4.2a–2b give evidence on the observation that real wage rigidity leaves no room for monetary policy to improve the aggregate performance of the economy. If real wages are almost completely rigid, the degree of intervention is almost a constant regardless of the degree of openness, and the gains in efficiency brought about by an exchange rate policy of "leaning with the wind" are marginal as the economy opens. By way of contrast, if nominal wages are completely rigid, a "leaning against the wind" exchange rate policy is followed in a closed and open economy, but the inflexibility of exchange rates is less pronounced in a more open economy.

Notice further that the degree of intervention seems to shift uniformly downwards as the degree of openness of the economy increases.[47] When a relatively high degree of wage indexation is reached, exchange rate policy rapidly changes regime from "leaning against the wind" to "leaning with the wind". The rationale for this is that intervention policy fights against real wage rigidity, inducing price level variability through exchange rate variability. From the last two columns, we can see that in a more open economy more rigid real wages and more variable exchange rates generate relatively less efficiency loss, an observation made earlier in the context of optimal policy mix. Finally, by way of a curiosity, table 2.4.3b indicates that for this equal variance case, flexible exchange rates go with relatively rigid real wages.

2.6 Summary of chapter two and discussion

In this section, we briefly summarize the implications of the open economy Gray–Fischer frameworks presented above, which we call standard (macro)models of wage indexation and foreign exchange intervention. We then pave the way for the topic to be taken up in the next chapter, where we shall take a closer look at wage formation under uncertainty. The ensuing approach is based on imperfectly competitive labour markets, which are due the to presence and influence of strong trade unions, an institutional feature prevalent in many OECD countries, notably in the Nordic countries. Some new and interesting features of wage formation emerge from the suggested model, which are further explored in the macromodel of chapter 4.

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[47] It should be recalled that "openness" in the present context is measured as the exposure of the domestic product market to international influence, so that a relatively closed economy means that the size of the open sector is relatively small or that substitutability between traded and non-traded goods is relatively low.

For the moment, we take the Gray—Fischer model of labour markets or contracts as given. Later we shall return to this part of the framework. Wage rigidity relative to spot wages emerges in these models largely as a aresult of two interrelated elements in the models; a simple indexation rule and the fact that the price index employed in the indexation rule provides, in general, only imperfect information on the underlying disturbances impinging on the economy or the (spot) equilibrium in the labour market. This basic setting gives rise to the signal extraction problem, which is the driving force in the determination of the optimal policy mix of wage indexation and foreign exchange intervention. Also, this imperfect information resulting from non-optimal wage formation generates efficiency losses which are amenable to reduction by proper choice of the policy mix. The exact amount by which losses can be reduced depends in general on the relation between the number of policy instruments employed and the number of independent stochastic sources of exogenous disturbances.

As far as the (optimal) degree of wage indexation is concerned, the traditional closed economy Gray—Fischer analysis differentiates between sources of exogenous uncertainty; pure demand-side shocks entail complete real wage rigidity and supply shocks entail partial wage indexation. Optimal wage indexation resides between these two (equilibrium) cases. This differentiation between sources of exogenous uncertainty is relevant in an open economy context as well, although the exact relationship between exogenous variability and the optimal degree of wage indexation is heavily dependent upon the prevailing exchange rate regime, hypothesized to be chosen by the monetary authority via a foreign exhange intervention rule.

Thus to understand the behaviour of actual real wages in these models a clear distinction should be drawn between disturbances to the equilibrium real wage and other disturbances. If the former are absent from the economy, which in the present context means that no exogenous shifts occur in the economy's production possibilities, then equilibrium real wages are constant and consequently actual wages should be perfectly indexed to the price level (complete real wage rigidity), no matter what the prevailing exchange regime or the degree of openness is (cf. equations (2.54) and (2.58')).

As far as the relationship between the optimal degree of wage indexation and foreign exchange intervention is concerned, not much in the way of full generality can be asserted on the basis of the models employed, even admitting their simplicity. The structure of the optimal mix (b^*, δ^*) is deeply rooted in the signal extraction approach assumed to drive the determination of the optimal mix. Certain weak theoretical predictions seem, however, to be suggested, especially when we come to

consider the optimality of one instrument in sections 2.2 and 2.4. First, taking the degree of foreign exchange intervention as exogenously determined and evaluating the behaviour of wages around complete fixity of exchange rates, we can conclude that the introduction of (minor) exchange rate flexibility ($\delta < \infty$) increases the flexibility of real wages, i.e. the optimal degree of wage indexation falls. The increase in real wage rigidity can be very sharp, depending somewhat critically on the behaviour of foreign prices. More precisely, if foreign prices are almost constant, the increase in real wage flexibility due to the introduction of exchange rate flexibility is very sharp, but further increases in exchange rate flexibility may in fact lower real wage flexibility. If, on the other hand, foreign prices are highly volatile, i.e. shocks to foreign prices are big, real wage flexibility seems to increase rather monotonically with increases in exchange rate flexibility. Once again, however, there could be a sharp fall in real wage flexibility around the unconstrained optimum degree of foreign exchange intervention. Over all, then, we have a rather non-monotonic relationship between the exchange rate and real wage flexibility.

Take now the other constrained case, where the degree of wage indexation is exogenously determined. If real wage flexibility increases due to a fall in the degree of wage indexation, then this seems to produce a fall in exchange rate variability. This relationship always prevails around the unconstrained optimum degree of wage indexation. The optimal degree of intervention responds to increased real wage flexibility in this monotonic fashion, if shocks to foreign prices are small enough, but non-monotonicities emerge in cases where foreign prices are highly volatile. Thus, the model in section 2.2 produces the possibility that after some particular point exchange rate variability due to optimal intervention policy starts (marginally) to increase with further increases in real wage flexibility. This possibility emerges because shocks to foreign prices are the dominant signal in the observed price level variability, and intervention policy is accordingly adjusted.

How does openness of the economy (section 2.4) change this picture of the relationship between exchange rate variability and wage patterns?[48] Openness as such, or the structure of the economy, greatly affects the qualitative picture of the optimal policy mix or the optimality of a single instrument. This is not, perhaps, so surprising, taking into account the fact that adjustments in relative prices heavily influence aggregate adjustment in the economy. At the most general level, we must conclude from section 2.4 that monetary policy is in general not a perfect substitute for relative price changes from the point of view of aggregate adjustment. The model in section 2.4 is constructed so that relative price adjustment enhances

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[48] And by implication employment and output patterns.

macroeconomic stability, and once the economic relevance of relative price adjustment diminishes due to increased openness of the economy, monetary policy cannot fully offset, in terms of efficiency, the cushioning effect produced by relative price changes.

Some weak prediction can once again be deduced from the two-sector model of section 2.4. The optimal policy mix is such that, given a sufficiently open economy to start with, intervention policy is likely to be one of "leaning with the wind". The increased exchange rate variability implied by the optimal policy mix goes with increased real wage rigidity as the economy becomes more open. Further, the increased real wage rigidity for more open economies seems to suggest that the differences in efficiency losses under fixed and flexible exchange rates diminish. The intuition behind this result is that, although flexible exchange rates increase the economic relevance of productivity shocks, this flexibility becomes more inconsequential as regards aggregate adjustment as the economy becomes more open as a result of increased real wage rigidity. In terms of efficiency, the advantage of flexible exchange rates over fixed rates does not vanish for an economy like the one represented by the model in section 2.4, since there is in general some real wage adjustment under flexible exchange rates. Finally, increased exchange rate variability is in general implied by the optimal intervention policy in response to increased real wage rigidity, a general feature somewhat dependent upon the degree of openness of the economy.

Extensions of the basic Gray—Fischer framework to an open economy subject to endogenous terms-of-trade changes have traditionally proved to be less successful because in such an economy labour demand and supply decisions are generally dependent on different real wage measures, the producer's and worker's real wage or the consumption real wage, respectively, so that equilibrium aggregate output depends on the terms of trade. Under conventional wage indexation rules, whereby wages are indexed to producer or consumer prices, the closed form solution to the optimum posed by Gray is not available. The analysis of the optimal policy mix typically reduces to the derivation of the optimal combination subject to the presence of only a subset of the original exogenous disturbances.

Section 2.3 argues that the Gray—Fischer framework can readily be extended to this case by indexing wages to a price index "relevant for the labour markets as a whole", meaning that we should search for a price index summarizing price level information relevant for both labour demand and supply. This price index is derived from the labour market spot equilibrium condition, the index being a weighted average of producer prices and foreign prices in domestic currency. The weight for the producer's price is a function of the elasticities of labour demand and supply schedules and the weight given to producer prices in the consumer

price index. This index was called the "optimum weight index" and was considered to be a consumer price index corrected for terms of trade. It was suggested that this index be used to index wages, which implies that the formal framework of section 2.2 for deriving the optimal policy mix can be invoked. References for detailed analyses based on this approach are Devereux (ibid.) and Vilmunen (ibid.).

We shall now return to the specifics of the Gray—Fischer model of the labour markets. Although the Gray—Fischer contracting model can be seen as a highly convenient approach to modelling the interaction between the real sector and financial markets, it si still true that these contracts lack firm choice-theoretic foundations. The need for firm foundations is more than obvious, given the great potential for monetary policy to affect wage, employment and output patterns in an economy represented by these standard contracting models. Historically speaking, the heavy emphasis the Gray—Fischer framework places on the macroeconomic approach incorporated these highly simplified labour contracts may seem justified, given the then available knowledge on the implications of alternative models of labour contracts within a macro context.[49] It seems, however, that this does not dispense with the need to employ alternative models of wage formation, perhaps more deeply rooted in the tradition of rational maximizing behaviour.

What, then, are these alternative models of wage formation? To simplify the classification scheme, we identify the following alternative approaches to wage formation; implicit contract theory, efficiency wage theory and trade union models of wage formation. Implicit contract theory[50] starts with the presumption that e.g. employees are constrained in their access to capital markets, so that possibilities for intertemporal smoothing of consumption in the face of uncertain and perhaps erratic income streams are severely restricted. Firms (shareholders) are typically assumed to have almost free access to capital markets so that they have better opportunities to insure against idiosyncratic risk by appropriate portfolio management. Given this set-up, employees typically seek insurance through labour contracts with firms, i.e. there are assumed to exist expected gains from optimal risk sharing between employees and employers.

Optimal (implicit) labour contracts thus characterize optimal wage and, independently, employment patterns, where, at least in some basic versions

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[49] From this historical perspective it seems reasonable to assert that the strong neutrality results derived from various rational expectations flexible price models in the 1970s have had an influence on the form the Gray—Fischer approach originally took.

[50] For a survey of implicit contract theory, see e.g. Rosen (1986), Stiglitz (1986) and Haley (1990).

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of the theory, wages serve to allocate risk optimally (see e.g. Cooper, 1987, pp. 6—18). Thus wages do not typically serve as signals for eliciting supplies of, and demand for, labour.

Implicit contract theory, however, suffers from some drawbacks as far as wage stickiness and/or unemployment are concerned. First, it is clearly an attempt to explain real wage rigidity, not nominal wage rigidity. Second, although overemployment is a frequent outcome of many of the implicit contract models, the outcome is critically dependent upon the assumptions that are made regarding access to information and firms' attitudes towards risk; implicit contracts with risk neutral firms and perfect information are efficient, but do not generate unemployment. Alternatively, implicit contracts with risk averse firms and imperfect information generate unemployment, but are not efficient. Third, the model explains underemployment (work sharing), not unemployment; it does not provide a compelling reason why firms prefer to lay off workers (see Haley, 1990, p. 129). Finally, the predicted form of the labour contract is much more complicated than those actually observed (Oswald, 1986). This has led to some scepticism about the value of these models as a foundation for a theory of wages and employment, perhaps most trenchantly expressed in Stiglitz (1986).

The basis of the efficiency wage[51] hypothesis is the assumption that worker productivity is a function of the wage paid. From this we conclude that the amount of labour available to the firm for production is not simply the number of labour units, but rather the number of efficiency labour units (Fisher, ibid., p. 3). Another fundamental idea underlying efficiency wage models is that firms set wages and gain some benefit from paying higher wages which offset direct costs (Nickell, 1990, p. 408).[52] Firms face a two-stage optimum problem, where in the first stage wages are set so as to minimize the (wage) cost per unit of efficiency labour subject to a constraint that wages are at least at some exogenous level W*. In the second stage, with wages thus determined, firms choose labour and capital input so as to maximize profits.

We shall not delve into the logic of efficiency wage models, any further, particularly the justifications given for the assumption that worker productivity or effort is positively related to the wage paid. Various explanations have been forward, and we refer the reader to Stiglitz (ibid.), Haley (ibid.), Fisher (ibid.) and Nickell (1990) for more detailed analysis. One must nevertheless feel somewhat uncomfortable about how integral a part of rational, optimizing behaviour these ratinalizations of the positive

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[51] For surveys, see Stiglitz (ibid.), Haley (ibid.) and Fisher (1989).

[52] This second feature of efficiency wage models clearly implies a departure from e.g. competitive models of wage setting, where markets set the wage rate.

relationship between productivity and effort really are.[53] Rather, we note some difficulties of the efficiency wage hypothesis as a theory of (sticky) wages and (un)employment.

It seems that efficiency wage models offer insights into the importance of relative wage structures in preserving the proper incentives, but it does not really explain why all firms would not uniformly lower their wages in response to a fall in aggregate demand (Haley, ibid. p. 138). On the other hand, the findings of MacLeod & Malcolmson (1986) on incentive compatibility in efficiency wage models when no restrictions are placed on the form of the contract between the firm and the worker and Chiswick (1986) on the interaction between labour quantity and quality suggest that efficiency wage models are really not capable of explaining unemployment. The intuition is that for the efficiency wage hypothesis to apply, firms must be able to vary the quantity of labour input over some relevant range without affecting its price. This can only occur if there is already unemployment (Haley, ibid., p. 137). Finally, although there exists some empirical evidence supporting the efficiency wage hypothesis, it is not overwhelming (on this, see Nickell, ibid. and the references therein).

Both of the above approaches to wage formation can be further criticized on the grounds that they seem to ignore some (economically and otherwise) important institutional features so very prevalent in many labour markets in modern industrialized nations; one cannot help feeling that trade unions, both employee and employer, should be explicitly incorporated into all models of wage formation in modern industrialized countries. Both employees and employers have, to varying degrees, joined unions, and more often than not contractual arrangements, notably wages and employment, are agreed upon in negotiations between employee and employer organizations. From the modelling point of view, we enter into the worlds of bargaining between firms and workers or their unions. The resulting approach to wage (and employment) formation will be called trade union models (or bargaining models).

Or course, the idea that unions, a particular form of imperfect competition, critically influence wage and employment formation and dynamic adjustment to various shocks, in an economy is not new, but the idea that union-based labour market behaviour is an integral part of an analytical structure representing an economy is. That is, the methodological tone of voice is new; trade union models are treated as viable models of labour market behaviour, and much analytical work has been done on them

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[53] Efficiency wage models were first applied to the modelling of wage formation in developing countries where the logic goes that higher wages would improve nutritional standards and increase worker productivity (Haley, ibid., p. 129).

and at a growing pace.[54] In addition to these theoretical developments in the bargaining relationships in labour markets, empirical applications of these theoretical constructs been steadily increasing in number (for surveys on empirical applications, see fn. 28). It seems that the basic logic of these models is now well understood, even though both theoretical and empirical problems certainly still exist, and certain extension of these models, notably dynamic trade union models,[55] are still in their infancy.

In the next chapter we shall employ a trade union model to wage formation in an open economy subject to exogenous uncertainty. The choice of this approach to wage formation does not imply that we take trade union models to be unquestionably superior to e.g. implicit contract theory or the efficiency wage hypothesis referred to above. Rather, the choice rests on institutional considerations; we think that these trade union models conform well with modern labour market institutions and practices in general, most notably in highly industrilized countries like Finland, Sweden and Norway. Thus our approach is mainly a reflection of these actual labour market institutions.

In what follows we shall invoke the static generalized Nash bargaining solution in a bargaining game over wages between employee trade unions and (competitive) firms or their representatives. Employee unions are supposed to have some, but not necessarily complete, power over the determination of wages, but none over employment. The resulting bargaining model is called the "right to manage model" (Nickell & Andrews, 1983). We shall make an extension of this framework to the case where there exists exogenous uncertainty and wages are determined before this uncertainty is resolved. We shall try to justify this choice of approach as we proceed in the next chapter, where we shall in addition try to operationalize and modify the Nash bargaining framework in order to make it suitable for an analysis of uncertainty in the present context. These macroeconomic implications are discussed in chapter 4, where we once again consider the macromodel of this chapter under the maintained hypothesis on bargaining-based wage formation.

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[54] See the special issue of the Scandinavian Journal of Economics, 1985, vol. 87, no. 2, especially Oswald's and Pencavel's survey. Further references include McDonald & Solow (1981), Farber (1985).

[55] See Minford (1983) for an empirical applications to an open economy of a dynamic monopoly union, Lockwood & Manning (1988) for a theoretical analysis of dynamic wage and employment bargaining with adjustment costs, and Espinosa & Rhee (1989) on efficient wage bargaining as a repeated game.

3 Bargaining, Trade Unions and Wage Formation in an Open Economy

3.1 Introduction

In this chapter we shall we return to wage and employment formation, taking as given the existence of (strong) trade unions in the labour markets. First, we provide some theoretical background concerning issues relating to unions and bargaining (section 3.2). The ensuing description is restricted to the presentation of the static Nash bargaining solution of wage and employment determination. Section 3.3 presents a model for the determination of the contract wage by developing a workable form of the bargaining structure or Nash programme while section 3.4 derives and comments on the contract wage. Macroeconomic implications of the wage formation process are postponed to chapter 4.

3.2 Bargaining between trade unions and firms

We shall consider a union or unions representing a given number of workers. Unions negotiate with employers or employer organizations. In countries like Finland, unionization has not been confined solely to employees. Rather, the degree of unionization is nowadays relatively high among enmployers as well, in Finland, and more often than not the level of bargaining between employer and employee organizations takes place on a nationwide basis (see Erikson, Suvanto & Vartia, 1989).

Basically, we can distinguish two approaches to union-firm bargaining, depending on the scope of bargaining, i.e. on which variables are subject to bargaining. These approaches or models are the efficient bargain model and the labour demand equilibrium model (LDEM). The latter includes the "right to manage model" (Nickell & Andrews, 1983) and the "pure monopoly union model", which can further be seen as a special case of the right to manage model.

The distinguishing feature between these two types of models is that in the efficient bargain model both employment and wages are

subject to bargaining, whereas in the labour demand models only wages are subject to bargaining, and demand for labour by firms, which is based on marginal productivity considerations, serves as a binding constraint for unions. The difference between the right to manage model and the pure monopoly union model is one of the degree of bargaining power; in a pure monopoly union model, the union is all powerful, being able to impose on firms its desired wage subject to the labour demand constraint, whereas in the right to manage model the union is less powerful, so that there is genuine scope for bargaining over wages.

The efficient bargain model and the LDEM can be formally derived from a two-person bargaining framework by invoking the generalized or asymmetric Nash bargaining solution (NBS, Nash, 1950, 1953). Let us denote by X the set of feasible outcomes of the bargaining game.[1] Then an element x* ε X is a Nash bargaining solution to a two-person bargaining game if

$$x^* = \operatorname*{argmax}_{x \varepsilon X} [U(x) - U^*]^{\Theta} [V(x) - V^*]^{1-\Theta}, \tag{3.1}$$

where U and V are the utility functions of the union and firm, repectively, U* and V* are the threat or disagreement points of the bargainers, and Θ denotes the bargaining power of the union. Thus the static Nash bargaining solution is found by maximizing the weighted geometric average of the utility rents or surpluses of the players above the threat points, with weights equal to the bargaining powers of the players. The axiomatic justification for the symmetric (Θ=½) NBS is given in Nash (1950).[2] Extensions to n-person games are provided by Harsanyi (1959), while Harsanyi and Selten (1972) have extended the NBS to take account of the unequal bargaining positions of the players. The strategic justification for the static NBS, based on Selten's refinement of Nash equilibria in noncooperative games, i.e. on

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[1] The interpretation of X depends on physical considerations as well as on variables subject to bargaining.

[2] Nash's (ibid.) axiomatic derivation is a cooperative justification for the NBS. In his 1953 article he provides a noncooperative justification for the NBS by considering his famous demand game.

the notion of subgame perfect equilibrium, is given by Binmore et al. (1986)[3],[4].

The threat points in the Nash product in (3.1), U* and V*, respectively, represent each side's options during a dispute or their options should bargaining break down. These options are influenced by various factors in the bargaining environment such as each side's financial resources and access to "solidarity funds" as well as long-run opportunity costs of labour and capital (Dorwick, 1989, p. 1127, Binmore et al., ibid.). These factors are treated here as exogenous (i.e. unmodelled).

From a technical point of view, the bargaining power of the union, Θ, determines the location of the NBS on the boundary of the set of feasible outcomes (i.e. picks one of the Pareto-optimal outcomes). In other words, in the efficient bargain model it determines a particular point on the contract curve, while in the right to manage model it determines a point on the labour demand schedule. From a substantive point of view, it can be interpreted either as depending (negatively) on a union's time preference or a union's subjective probability of the game breaking down before next "bargaining" round (Binmore et al., ibid., Hoel, ibid.). It is treated as exogenously given.[5]

Efficient bargains

If $x = (W,N)$, where W denotes wages and N employment, the NBS to the efficient bargain lies on the contract curve, i.e. on the curve representing common tangency points of the union's and firm's

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[3] Roth (1985) argues that, in addition to the works of Selten on the notion of perfect equilibrium, Harsanyi's (1967, 1968a, b) work on games of imperfect information has had an influence on the progress of the strategic approach to bargaining.

[4] The paradigm of the strategic approach is Rubinstein's (1982) perfect information alternating offer model, from which the static NBS (with a linear utility function) can be derived as the time lag between successive offers vanishes. An extension of the Rubinstein model to nonlinear utility functions is provided by Hoel (1986); see also Canning's (1989) analysis of a finite version of the Rubinstein model.

[5] Svejnar (1986) argues that Θ should be made to depend on various exogenous or institutional factors which affect the bargaining outcome but are not themselves subject to bargaining (e.g. legal restrictions on the bargaining process, indexation arrangements, price and wage controls etc.). Thus we should write $\Theta = \Theta(z)$, where z represents these institutional factors. Furthermore he argues that the bargaining solution should reflect, locally, the union's risk aversion or fear of disagreement, a notion which he defines in terms of the union's utility function. He then replaces Nash's symmetry axiom of equal bargaining powers by "equality of bargaining powers relative to fear of disagreement".

indifference curves. The tangency condition for the firm is given in terms of the isoprofit curve, i.e. those wage-employment combinations giving constant profits. Since[6]

$$0 = U_W dW + U_N dN$$
$$0 = \pi_W dW + \pi_N dN,$$

(3.2)

the equation for the contract curve is given by

$$\frac{U_N}{U_W} = \frac{\pi_N}{\pi_W}$$

(3.3)

i.e. by the equality of marginal substitution of wages and employment in the union's utility and firm's profits; see points A and B in figure 3.1. The slope of the contract curve depends on the functional forms of the union's utility function and the firm's revenue function. For "conventional" functional forms (e.g. a utilitarian union with a CRRA utility function for a representative member and a concave production function for a profit-maximizing firm) the contract curve is upward sloping, as in figure 3.1.

Figure 3.1.   **Efficient contrast**
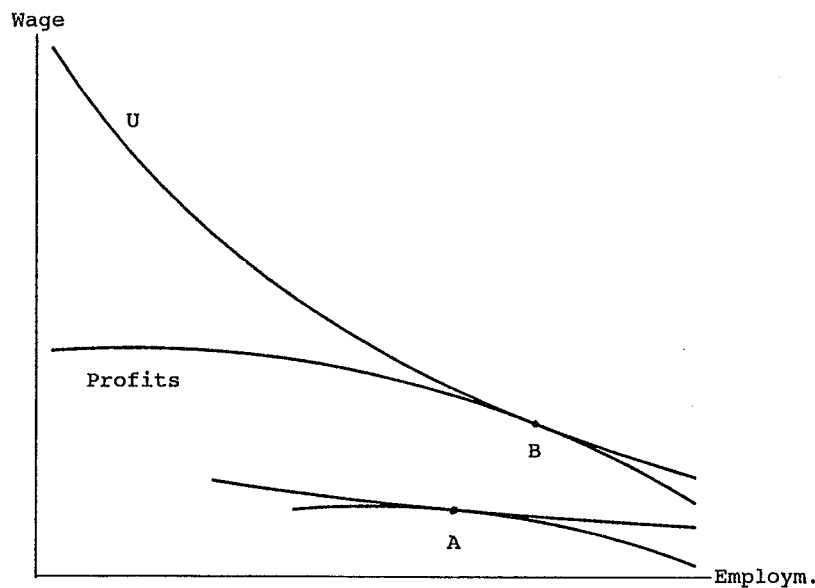
"Profits" refers to those wage-employment levels giving constant profits and "U" to the union's indifference curve

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[6] Here we have implicitly written V(W,N) = v(π(W,N)) for some v.

Efficient bargains are (by construction) Pareto-optimal, i.e. wages and employment in these bargains are such that no welfare improving adjustments in wages and employment exist for both union and firm. In this respect they are very similar to the classical pure exchange problem, where two agents engage in an exchange in an expectation of mutual gains from the exchange. This optimality property results from the maximization of joint utility, i.e. internalizing the effects of one agent's choices on the other agent. Optimality is an attractive feature, but problems, both theoretical and empirical, attach to these efficient bargains, to which we shall return later. At this point we merely note that efficient bargains (or NBS) have the property that

$$U - U^* = \Theta(U - U^* + V - V^*), \qquad (3.4)$$

which characterizes the distribution of the utility rents at the optimum, i.e. the bargaining power of the union(s) corresponds to the share of labour in the total surplus created by the wage-employment bargain.[7]

Labour demand equilibrium models

In the right to manage model (Nickell & Andrews, 1983) the scope of bargaining is restricted to wages alone and the union faces a binding contstraint in the firm's demand for labour function. In terms of the generalized Nash maximand, the right to manage model solves

$$W^* = \text{argmax}_W\{[U(W,N(W)) - U^*]^\Theta[V(W,N(W)) - V^*]^{1-\Theta}\}, \qquad (3.5)$$

where $N(W)$ is the profit-maximizing level of employment, given wages W, i.e

$$N(W) = \text{argmax}_N\pi(W,N). \qquad (3.6)$$

The distinguishing feature of the right to manage model is that the solution to it lies on the neoclassical labour demand curve (see figure 3.2). This is taken care of by the constraint (3.6), which guarantees that employment is determined according to the marginal product of labour.

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[7] This property provides some intuition for understanding the role played by the NBS. In principle any point on the contract curve represents an optimal outcome. To pick a specific point on it, we need some specific sharing rule for selecting that particular point. The NBS solution wages and employment, i.e. an efficient bargain represents such a sharing rule.

Figure 3.2   **Right to manage model**
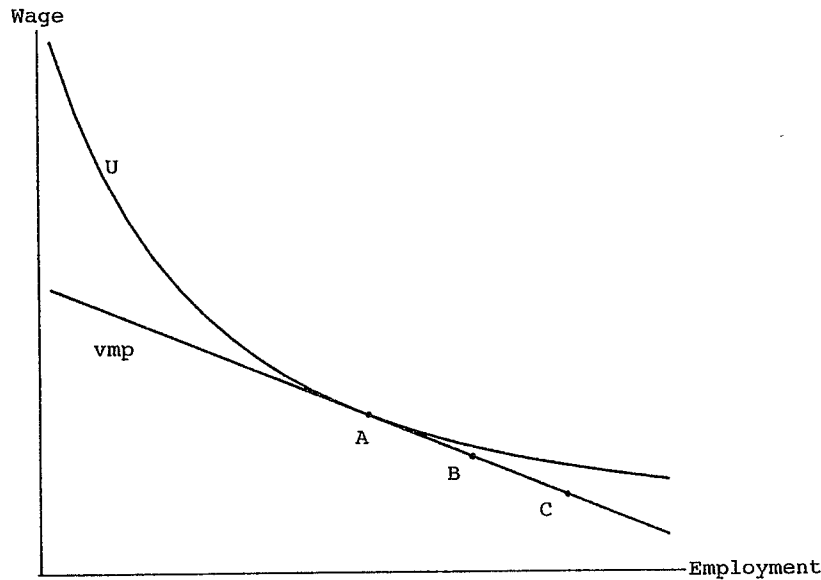
"Vmp" refers to the value of the marginal product of labour.
See also note to fig. 3.1.

The first order condition for the maximum in (3.5) is

$$\phi(W) = \Theta[V - V^*][U_W + U_N N_W]$$

$$+ (1 - \Theta)[U - U^*]V_\pi[\pi_W + \pi_N N_W] = 0, \tag{3.7}$$

where N_W denotes the partial derivative of the labour demand function with respect to the wage W, i.e. $N_W = 1/pf''(N) < 0$ for a profit-maximizing firm with a concave production function. Moreover, as is clear from the definition of profits, $\pi_W = -N$ and $\pi_N = pf'(N) - W$, which is zero by the constraint. Thus the term $\pi_N N_W$ drops out of the above equation.[8]

Some special cases are of interest. For a pure monopoly union, $\Theta=1$, which according to (3.7) implies that either $V = V^*$ or $U_W + U_N N_W = 0$ (or both). The latter condition is the familiar tangency condition for the union's indifference curve and firm's labour demand schedule. The condition $V = V^*$ and $U_W + U_N N_W \neq 0$ formally satisfies the above first order condition for maximum. Substantially it says that the monopoly union cannot force firms to accept profit levels below their

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[8] Second order conditions require that $\phi'(W) < 0$ at the optimum. To implement the unique static Nash bargaining solution, convexity of the set of feasible utility outcomes with a concave Pareto-optimal boundary would be an attractive feature.

fall-out profits. Equilibrium in the "pure" monopoli case corresponds to the point A in figure 3.2, giving the maximum wages an all-embracing monopoly union can obtain from the bargain.

On the other hand, if $\Theta = 0$, so that only profit maximizing matters, equation (3.7) implies that $U = U^*$, or $W = W^*$, in which case the union is pushed to a wage bargain corresponding to the "competitive" wage level.[9] This is illustrated by point C in figure 3.2. Now, the solution to the right to manage model lies along the segment AC of the labour demand curve, depending on the bargaining power of the union. A typical solution point for a given Θ is the point B in figure 3.2. Furthermore, by differentiating the first order conditions, and solving for $\partial W / \partial \Theta$, we obtain

$$\frac{\partial W}{\partial \Theta} = - \frac{\Theta^{-1}[U - U^*]V_\pi \cdot N}{\phi'(W)}, \qquad (3.8)$$

where

$$\phi'(W) = \Theta[V - V^*][U_{WW} + 2U_{NW}N_W + U_{NN}N^2_W + U_N N_{WW}]$$

$$- NV_\pi(U_W + U_N N_W) + (1 - \Theta)[V_{\pi\pi}N^2 - V_\pi N_W] < 0$$

by the second order conditions for the maximum of the Nash maximand, and $\pi_W = -N$ from the profit function. So the solution wage is an increasing function of the union's bargaining power. This is intuitive enough, and something to be expected.

Even though efficient bargain and labour demand models are the main approaches employed in modelling trade union behaviour,

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[9] Note that if we take these polar cases, i.e. W^* and W^m, say, as our bounds on the set of feasible wage levels, we obtain a compact set $[W^*, W^m]$, so that the maximum in (3.10) exists assuming continuity of the Nash maximand. Note further that to justify the vague notion of "more competitive labour pricing as Θ falls (to zero)" requires an additional assumption, which is only implicit in much of the literature on bargaining in labour markets between unions and competitive firms. This is that as the bargaining power of firms rises, i.e. Θ falls, any collusive or cooperative behaviour of firms generating possible monopsony pricing has to be assumed away. We shall not make an exception to this implicit assumption in what follows.

Manning's (1987) sequential 2-stage bargaining framework[10] suggests that we are not in fact constrained in our choice of bargaining models to efficient bargain and (generally) inefficient labour demand models. The choice is simply not so dichotomous. Two problems are involved here. First, in searching for possible causes of inefficiencies in a unionized economy, we ought to focus on possible differences in the union's bargaining power over different decisions of the firm. These differences impose a particular form of a constraint on the union's decision over variables of interest. Secondly, the standard models of union-firm bargaining should be seen as special cases of a more general sequential bargaining game, where the imposition of special constraints on the bargaining parameters generates these standard models. In this respect, all standard models are somewhat special, although the monopoly union models seem even more extreme.[11]

In what follows we shall propose an operationalization of the static asymmetric Nash bargaining framework in order to generate a wage equation in an open economy subject to union-firm bargaining. Furthermore, in applying the bargaining framework, we shall explicitly incorporate uncertainty into the partial equilibrium wage in the labour markets (see e.g. Oswald, 1985, Andersen & Sørensen, 1989) by assuming that the wage is agreed before uncertainty is resolved, i.e. before random disturbances are realized. Basically, uncertainty is generated by stochastic shocks to the price level and production technology. We shall make further comments on certain features of the framework, while imposing specific constraints on the Nash maximand and the functional forms embodied in it.

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[10] The efficient bargain and labour demand models can be incorporated into the unified sequential bargaining framework proposed by Manning (ibid.). Manning formulates the labour market bargain as a sequential two-stage bargaining game between employees and employers, where either wages or employment is determined in the first stage and the other in the second stage of the bargaining game. The solution of the first-stage bargain is constrained by the choice in the second stage of the bargaining game. The union's bargaining power need not be equal in both stages of the game, which clearly increases the generality of the model. Manning proves (ibid., p. 126) that a sufficient condition for an efficient bargain is that there should be no wedge between the union's bargaining power in the two stages of the sequential bargain, and goes further in suggesting, but not formally proving, that any inefficiency induced by trade union activities is not likely to be the result of union power *per se*, but rather of the difference in its power with respect to particular issues.

[11] Note that the only constraint facing a monopoly union is the demand for labour function of the firms. It is certainly true that the main (theoretical) virtue of the labour demand model, especially in its simplest (pure monopoly) form, is its simplicity, even in a dynamic context, which is not to be expected from a bargaining framework in general. Dynamic bargaining models, although rare, have started to appear; see e.g. Lockwood & Manning, 1988, Espinosa & Rhee, 1989. For a dynamic monopoly union model in an open economy, see Minford, 1983.

3.3 Indexed contract wages: an operationalization of the asymmetric Nash bargaining model

In this section, we employ the right to manage model to generate a wage equation in our unionized economy. Not choosing an efficient bargain approach or even a more general version due to Manning, despite its optimality properties, involves restrictions that have to be justified somehow. These considerations are both theoretical and empirical. On the theoretical side, we note that there are potential problems as to the *ex post* enforceability of efficient bargains.

These stem from the fact that, although efficient bargains are Pareto-optimal, so that no welfare improving adjustment in wages and employment exists for both parties, there exists for each wage-employment combination on the contract curve a wage-employment combination on the labour demand schedule that gives firms the same profits as the bargain on the contract curve, but where production efficiency is also guaranteed. Thus, *ex post* firms have incentives to renege on the negotiated bargain, i.e. to adjust wages and employment so as to reflect production efficiency. This means that the enforceability of efficient bargains presents a problem, which is not solved by reference to the Pareto-optimality of efficient bargains alone. Enforceability means that the union is strong enough to force the employer to stick to the wage-employment combination that has already been committed to (Holmlund, 1989, p. 18).

From the empirical point of view, however, "casual empiricism" concerning actual bargaining processes suggest that in practice employers seem to exercise substantial discretion over employment decisions. At this point it is worth making a distinction between two things; is the employment level *per se* a variable subject to bargaining and are negotiations related to employment restricted to work rules, working conditions etc. in actual bargaining processes? We rarely see the first of these in practice. From an intuitive point of view, this seems understandable, since it is difficult to imagine how, in a market economy, a particular employment level could be enforced for each firm, especially if these employment levels are negotiated centrally (Hoel, 1990).

As such, empirical evidence concerning either union preferences or the efficiency of labour contracts is mixed (for a survey, see Farber, 1985, pp. 29–44). Thus, the first preliminary evidence as to the inefficiency of labour contracts is found in Ashenfelter & Brown

(1983) in their empirical study of International Typographical Union data. However, McCurdy & Pencavel (1984), working on the ITU data, reject a version of the labour demand model, thus suggesting that "in the case of ITU the wage-employment bargain is not characterized properly by a union selecting a wage to maximize its objective function subject to the constraint that is imposed by the labour demand schedule" (Farber, ibid., p. 41).

Major problems nevertheless attach to the empirical implementation and interpretation of trade union models, in addition to the technical problems related to estimation methods etc. First, most of the studies so far on unionized labour contracts[12] simultaneously seek to estimate the parameters of the union's objective function, conditional on employment being on the labour demand schedule. To this end, certain functional forms for union preferences, notably wage bill and wage rent specifications, are rejected, largely because unions typically place more emphasis on employment than these preferences imply. The essential point is, however, that the rejection of the null of a labour demand model may be entirely due to misspecification of union preferences. This is the crux of Manning's (ibid., p. 127) observation on these tests; if we interpret a point off the labour demand curve as a result of the efficient bargain model, we run the danger of accepting misspecified preferences for the union.

Second, as to the efficiency of labour contracts themselves, we quote Farber (1985, p. 41; see also McCurdy & Pencavel, ibid.): "a rigorous test of the contract equilibrium model[13] is not possible without making more restrictive assumptions regarding the form of the union objective function and the associated marginal rate of substitution".[14] So, one must be agnostic as to whether the contract is, in fact, efficient.

Thirdly, and in summary, Farber (ibid., p. 42) suggests a plausible interpretation of the empirical test so far: a reasonable interpretation is that the structure of bargains is such that the parties negotiate over wages and a set of work rules. However, there is no presumption that

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[12] Including Farber (1978a, b), Dertouzos & Pencavel (1981), Carruth & Oswald (1983), Pencavel (1984a, b), but also Ashenfelter & Brown (ibid.) and McCurdy & Pencavel (ibid.).

[13] That is, the efficient bargain model.

[14] This particular wording refers to the form the test takes in McCurdy & Pencavel; in their test an equilibrium condition in the contract equilibrium model is identical to that in the labour demand model, exept for an additional term in the former model representing the marginal rate of substitution in the union objective function. Their test amounts to checking the importance of this term, but needless to say, the structure of union preferences imposes restrictions on the form of this term.

these work rules are sufficient to ensure that the bargain is efficient. The union has, at best, partial control over employment.[15]

So, we proceed on the assumption that the right to manage model of Nickell and Andrews is a reasonable approximation to union-firm bargaining. The ensuing bargaining model is best understood as representing bargaining between one economy-wide union and a representative competitive firm or an employer organization representing competitive firms. An implication of this institutional set-up is that the effects on the aggregate price level of centralized wage setting are taken into account in the solution to this bargaining game. This particular institutional setting gives an approximation to that prevalent in Nordic countries, where wage setting has, more often than not, been highly centralized. We shall start with the specification of the bargainers' utility function.

Union preferences

The union is assumed to have well specified preferences[16] concerning real wages and employment, represented by a (at least twice) continuously differentiable function $U = U(W,N)$, $U_i > 0$, $i=W,N$, where W is the wage rate and N denotes employment. A variety of functional forms for the union preferences U have been suggested in the literature.[17], including Stone-Geary (Dertouzos & Pencavel, ibid.,

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[15] The above-mentioned studies of union preferences also indicate that the structure of the bargain, including work rules, is situation-specific, and that there seems to be a great deal of variation in union preferences across locations and industries. Thus, one cannot escape the conclusion that these studies do not provide any convincing evidence on the efficiency of labour contracts or the validity of the labour demand model in the economy in general or in the newspaper industry in particular.

[16] Problems relating explicitly to preference aggregation (of union members) are generally bypassed. The implicit aggregation convention is given by assuming a specific form for the union utility function. For a discussion of these problems see e.g. Farber (ibid.) and Carruth & Oswald (1987). Further, the problems related to the philosophical question as to from where the union inherits its preferences are abstracted away. Farber (ibid.) suggests that it seems reasonable to posit the union as a utility maximizer, but the political process within the union can greatly affect union preferences, and should as often as possible be analyzed in the context of representing union preferences. Note that this problem of union preferences is somewhat deeper than the now classic problem associated with the preferences of a society in that in addition to the proper aggregation of divergent individual preferences, there is the problem associated with non-union, but potential union, employees.

[17] For a survey, see Oswald (1985); but also Carruth & Oswald (ibid.) and Farber (ibid.).

Pencavel, 1984a, b, McCurdy & Pencavel, ibid.) and augmented addilog (Pencavel, 1984a). By far the most popular union model is that of a utilitarian union, whose preferences are represented by[18]

$$U(W,N) = Nu(W) + (M-N)u(a), \quad M \geq N$$
$$= Mu(W), \quad M < N, \tag{3.9}$$

where M is the fixed[19] exogenous membership of the union, u is a concave twice continuously differentiable utility function of an individual or representative member and a represents outside income opportunities of an unemployed member. It is assumed that lay-offs are generated by a random device. This, together with fixed membership, implies that the union's objectives can be written as $U(W,N) = pu(W) + (1-p)u(a)$, for $M \geq N$, where $p = N/M$. The expression $pu(W) + (1-p)u(a)$ gives the expected utility of union membership, so that in this case the utilitarian union behaves as if maximizing the expected utility of a representative member.

The preference functional of a utilitarian union can be quite hard to operate because of the "kink" in its indifference curves at membership. More formally,

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[18] Note that for fixed membership, the utility function of a utilitarian union is observationally equivalent to that of a union whose preferences collapse to the preferences of a median-aged member. The main virtues of the utilitarian union model are its tractability, both in theoretical and empirical work, and the fact that it incorporates as special cases various functional forms considered in the literature, e.g. wage bill specification, wage rent specification. A Stone—Geary specification is also widely used, and its virtues stem mainly from its success in applied consumer theory. It also includes the above-mentioned functional forms as special cases. But the Stone—Geary does not seem be as well suited for dealing with union preferences under uncertainty as the utilitarian union model, so simplicity once again dictates the choice.

[19] Variable membership is thus abstracted away. It is quite conceivable that membership considerations affect union objectives (Farber, ibid.). In fact Jones & McKenna (1989) argue that it should also affect the utilitarian union's objectives. The argument is that the standard utilitarian union attaches a zero weight to the utility of employed outsiders. Hence the union treats unemployed and employed outsiders alike. But this ignores the important difference between employed and unemployed outsiders, i.e. that the former group may become insiders whilst the latter group cannot. So a union which cares about future membership will be interested in current employment of outsiders (Jones & McKenna, ibid., p. 1570). While the reformulation suggested by these authors implies that the union's indifference curves have a kink at membership, i.e. at N=M, these indifference curves are otherwise negatively sloped, a property not shared by the Carruth—Oswald (1987) formulation.

$$\frac{\partial W}{\partial N}\Big|_{-} = -[u(W) - u(a)]/Nu'(W)$$

$$\frac{\partial W}{\partial N}\Big|_{+} = 0,$$
(3.10)

so that the union's indifference curves become horizontal after membership.[20,21]

Another unattractive feature of the utilitarian union model with fixed membership, which is partially removed by a reformulation suggested by Jones & McKenna (1989), is revealed most clearly in the case of a pure monopoly union model. In this case the monopoly union determines its most desired wage and imposes it on the firm, thereby forcing the firm to accept some minimum profit level ("fall back" profits). This model has a very peculiar comparative static property in that a necessary condition for generating growth in employment beyond membership is a fall in the price of output.[22] What this result says is that procyclical movements in employment beyond membership are impossible in this model. This is the reason for the reformulation of the utilitarian union preferences to take into account the preferences of employed outsiders in Jones & McKenna (ibid.). Their models give rise to the possibility of procyclical employment growth beyond membership, a possibility depending heavily on the behaviour of the labour demand function in the range N>M.

Despite these theoretical shortcomings of the utilitarian fixed membership model, we shall exploit it below because of its analytical simplicity and tractability. Furthermore, being essentially a utility representation of a representative member, aggregation is especially

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[20] $\partial w/\partial N|_{-}$ and $\partial w/\partial N|_{+}$ denote one-sided derivatives (from the left and right, respectively).

[21] Since u is concave it can be shown that the contract curve is now upward sloping for M>N, becomes vertical at M=N and coincides with the labour demand function after N.

[22] For a proof, see Carruth & Oswald (ibid., p. 436).

simple. But to restate and make the ensuing maximizing problem work well, we explicitly assume that M≥N.[23]

Firms

The production technology of a (representative) risk neutral[24] firm is given by a Cobb—Douglas production function:

$$Y = \frac{1}{\alpha} N^{\alpha} \exp(\mu), \quad 0 < \alpha < 1, \tag{3.11}$$

where μ represents random shocks to productivity or production possibilities, and whose distribution is given by the distribution function F. Product markets are competitive,[25] and firms maximize profits

$$P\pi = PY - WN, \tag{3.12}$$

where P is the price of (domestic) output, π real profits and W denotes (actual) nominal wages. Profit maximization by the firm leads to the familiar demand for labour function, where demand for labour depends on the real wage relevant for the firm:

$$N^d = P^{\beta} W^{-\beta} \exp(\beta\mu), \quad \beta = \frac{1}{1-\alpha} > 1. \tag{3.13}$$

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[23] This assumption, together with the concavity of the utility function u, implies enough structure for the union maximization problem to be well behaved. It should, however, be emphasized that there could be e.g. some institutional constraints or conventions in real economies that imply a violation of the constraint M>N, thus giving rise to a corner solution. Such institutional constraints include union shops (USA) and open shops (UK), where every new worker must join the union (Carruth & Oswald, ibid., p. 435). These imply that membership tends to very close to employment. Note further that in this setting assuming random selection in lay-offs drives a wedge between an efficient bargain and a monopoly union model, i.e. the latter cannot be efficient. If, instead of random selection, lay-offs are by seniority ('last in, first out'), then union indifference curves are horizontal, and efficient bargains are on the labour demand curve, as demonstrated by Oswald (1985).

[24] The firm's utility function is thus an identity mapping, i.e. $V(\pi) = \pi$.

[25] For the implications of industrial structure for wage-employment bargains, see Dorwick (ibid.).

This solution to the profit-maximization problem gives us the profit function of the firm (maximal profits given P and W), obtained by substituting (3.13) into (3.12):

$$P\pi^* = P\pi(P,W) = \frac{P^\beta W^{1-\beta}\exp(\beta\mu)}{\beta-1}.$$ (3.14)

As can be seen from (3.14) the profit function is homogenous of degree one in (P,W), thus verifying the usual property of profit functions (see e.g. Varian, 1984, p. 46). For future reference, we note that real profits are given by $\pi = P^{\beta-1}W^{1-\beta}\exp(\beta\mu)/(\beta-1)$.

Uncertainty and labour contracts

A typical feature of modern labour contracts is that they are long-term contracts. For example, in Finland contract duration is typically one to two years, and sometimes even longer-term contracts are written, with possible periods of renegotiations for reviewing the terms of the current contract. Given that the contracting are forward-looking in the sense that contracts incorporate expectations concerning at least those variables greatly affecting the realized terms of the contracts, a pervasive problem facing the contracting parties at the time contracts are negotiated is uncertainty, at least in the form of exogenous disturbances impinging on the economy during the contract period. Thus there is a need to incorporate uncertainty explicitly into our model of union-firm bargaining.

Trade unions under uncertainty have not been dealt with much in the literature. Notable exceptions are Oswald (1982) and Andersen & Sørensen (1989). Oswald investigates possible alterations in the comparative statics implications of a utilitarian (monopoly) union under uncertainty. Introduction of uncertainty into the trade union model does not destroy the familiar comparative statics results (see e.g. Oswald, 1985) derived from the corresponding models under certainty.[26] Andersen & Sørensen employ the utilitarian (monopoly) union model under uncertainty to analyze the relationship between unionized wage formation and exchange rate variability, a tentative

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[26] A minor problem is caused by the effects of the "comparison wage" (Oswald, 1979) on the marginal utility of income (consumption) in that it could change in sign in such a way that the effects of the comparison wage on the marginal utility of income and the expected value thereof have the opposite sign (Oswald, 1982, p. 107).

conclusion being that exchange rate variability may have adverse effects on the steady state equilibrium of the open economy (output, employment and current account). In fact, we shall extend Andersen & Sørensen in that we impose more structure on wage formation by employing a right to manage model, and we shall be more explicit in our macromodel about the sources of aggregate price variability.

Restrictions on the generalized Nash maximand

As noted above, we shall employ a right to manage model of union-firm wage bargaining under uncertainty. Thus, the scope for bargaining is restricted to wages or to put it in Manning's terminology, in a two-stage sequential wage-employment bargain, the union has no power over employment in the second stage. Furthermore we will be more specific about the sources of exogenous shocks generating stochastic price level variability[27] in chapter 4 in the context of our macromodel. To simplify the derivation of the wage formula, we shall impose the following restrictions on the Nash product:

i) we shall exploit Svejnar's extension of the asymmetric Nash bargain by transforming the product so as to reflect explicitly the firm's bargaining power only. Furthermore, we shall modify the standard Nash maximand so that instead of writing the functional to be maximized as $[EU - U^*][E\pi - \pi^*]^\Theta$ we propose the following functional

$$E\{[U - U^*][\pi - \pi^*]^\Theta\}, \qquad (3.15)$$

where Θ denotes the firm's bargaining power: $\Theta=0$ corresponds to the monopoly union model, and as Θ tends to infinity, only (expected) profit maximization carries weight. This parameterization of the Nash maximand greatly simplifies the computations involved. The expectation operator E denotes the assumption that the labour contract will be signed before uncertainty is resolved, i.e. union-firm bargaining takes place under uncertainty.

Clearly, then, the suggested functional does not strictly conform to the standard assumptions of the Nash bargaining game, and some justification, and preferably rationalization, based on the first principles

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[27] And, of course, giving rise to the normal distribution of the logarithm of the price level assumed later in our analysis (see p. 102).

of bargaining theory to support it would be desirable. The differences between the standard Nash product and the functional (3.15) reduce to the way uncertainty is assumed to be resolved in the bargaining process. We shall not try to give any formal treatment of the problem, but merely rely on informal arguments and intuition.

The standard formulation of the static Nash bargaining game (see e.g. Osborne & Rubinstein, 1990, pp. 9—11) starts out by defining a bargaining situation as a 4-tuple consisting of the number of players, the set of possible outcomes or agreements, disagreement outcomes and each player's preferences. Players' preferences are defined on the set of lotteries over possible agreements, not just on the set of agreements themselves. There is no explicit risk in the bargaining situation thus defined. Preferences satisfy the assumptions of von Neumann and Morgenstern. Thus, agreement x is at least as good as agreement y to a player if and only if the expected utility of x exceeds that of y.

Some points deserve special attention here. In a bargaining situation thus defined, uncertainty or risk is fully captured by the notion of the set of feasible agreements (and disagreement points). This set consists of the list of all lotteries available in the game. This information is given exogenously, i.e. it is an unmodelled element of a bargaining situation. What is even more important in the present context is that this set of agreements is in no way affected by the choices of the players. In this sense no risk or uncertainty is generated in or from the bargaining situation itself, and uncertainty is resolved in it in a highly decentralized way through individual utilities. Actually, uncertainty is resolved before bargaining between the players starts.

But once contracts allow wages to be indexed, complications arise due to the fact that the strict exogeneity of lotteries with respect to the choices of the players no longer holds, i.e. the set of lotteries is no longer given. Indexation interacts with exogenous sources of uncertainty so that systematic effects on the distribution of various endogenous variables of the model economy, including of course wages, will emerge. Thus choices of the degree of wage indexation will affect the set of feasible (wage) agreements, so that uncertainty will be endogenously affected. Given this background, we can think of the union and firms bargaining over wages and indexation. We can now state that the bargaining process itself generates uncertainty so that all of the Nash axioms are not satisfied. Strictly speaking, then, it would be logically incorrect to derive wages and indexation from the standard Nash maximand.

The functional (3.15) proposes a way of resolving uncertainty in this situation in that it can be thought of as a preference functional of

an arbitrator, who averages out the distribution of excess utilities (actually, a weighted average of excess utilities) over uncertain states of nature. Thus, in a sense there is a social utility surplus, depending on the excess utilities of the players, to be divided. The parameter Θ can be interpreted as a (relative) weight given to firms or "competitiveness" in the arbitrator's preferences. Although all this, together with (3.15), may sound contrived, it should be remembered that a suggestion for resolving uncertainty in this case has to be made, since the standard Nash formulation does not give the right answer either.

Another way of arguing in favour of something like (3.15) is to think of a two-stage bargaining process between the labour market participants, where there is explicit uncertainty about whether the actual wage negotiations will take place. In the first stage the union and firms compute wages as if uncertainty had been resolved. But, owing to e.g. the possibility of an exogenous event which would affect each player's (conception of his) relative bargaining position or which might have a differential effect on each player's feasibility set, a delay in the start of the negotiations threatens to occur. The point here is that there is, for some reason, a wedge between the union's and firms' demands which cannot be negotiated in the "usual" way and which may jeopardize the negotiations. So, in the second stage, an arbitrator is needed, who resolves uncertainty by imposing on the players a wage setting rule, possibly coupled with indexation, based on maximizing (3.15). This formulation of the problem clearly has parallels with a theory of strikes and the behaviour of an arbitrator therein.

So the argument above, although informal, does suggest that a modification of the standard Nash bargaining model in the present context is somehow justified. A qualification in the ensuing analysis derives from the fact that no tractable closed-form solution to the degree of wage indexation can be derived from the labour market model, so that the contract wage equation below displays only a partial relationship between contract wages and indexation. However, we still prefer (3.15), since later in the policy analysis we shall propose a model of an optimal policy mix of wage indexation and foreign exchange intervention where the choice of the policy mix depends on wage formation. So we are back in the situation where wages and indexation jointly interact to affect the feasibility set of the bargaining game in the labour markets.

In an appendix to this chapter we prove that the differences between the standard Nash form and (3.15) are very small indeed in the present context in terms of the resulting contract wage equation. In fact, as far as the qualitative features of the relationship between contract wages, indexation and intervention are concerned, there is

essentially no difference between the two models. As far as aggregate uncertainty itself is concerned, there is, however, one important difference between the two models. The standard Nash form implies that the effects of aggregate uncertainty — aggregate price risk as we call it — on contract wages are completely independent of the bargaining power of firms or competitiveness of the labour markets. So, according to the standard Nash formulation, labour market competitiveness in the present model in no way directly interacts with random aggregate price and production variability so as to affect contract wages. This is not something to be expected intuitively, and does not greatly enhance the plausibility of the standard Nash formulation itself. The formulation (3.15), on the other hand, generates a contract wage with an interesting interaction between aggregate uncertainty, indexation and competitiveness of the labour markets. This is one reason we prefer (3.15) over the more standard formulation, but to avoid excessive renaming and new concepts, we shall still call the maintained labour market model with (3.15) a "Nash bargaining game" or simply a "bargaining game".

ii) the firm's fall back or threat point profits π^* are set equal to zero. This seems an extreme assumption, at least at a more disaggregate (e.g. industry) level, since we saw earlier that the threat point should be seen as being affected by the agent's financial resources should the bargaining break down or during a delay. Now we are saying that the firm cannot secure any (short-run) profits should the negotiation break down. A partial justification for zero fall back profits can be given by assuming the there is no second-hand market for existing capital (Holmlund, 1990, p. 259), i.e. existing capital equipment stays idle during a dispute thus incurring fixed costs. Be what may, we exploit this assumption mainly because of analytical simplicity; the computations involved are greatly simplified by assuming $\pi^*=0$;[28]

iii) the union's fall back or threat point utility U* is equal to Mu(a). So, we are assuming that each member of the union obtains u(a), where a represents exogenous, nonrandom income opportunities, in utility during a dispute or delay. Furthermore, we assume that the utility function of a representative union member belongs to the class of constant relative risk aversion (CRRA) functions given by u(x) = $x^{1-\tau}/(1-\tau)$, where τ is the coefficient of relative risk aversion, i.e. $\tau = -u''(x)x/u'(x)$. Each member's utility is taken to depend on real

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[28] See e.g. Dorwick (ibid.) on the comparative static role of fall back profits in the bargaining solution.

wages, i.e. u(w) = u[W/Q], where Q denotes the random price level index relevant for the union or union's member (e.g. consumer price index).

Given these restrictions, the Nash product can now be written as

$$W^c = \text{argmax}_w E\{[N(u(w) - u(a)]\pi^\Theta\}$$

$$\text{s.t. } \pi_N = 0. \tag{3.16}$$

W^c provides the contract wage, while actual wages[29] are possibly indexed according to the formula $W = W^c(Q/Q^e)^b$, where b is the indexation coefficient and the superscript e denotes expectations. Thus, we are explicitly allowing for wage indexation as in the standard Gray-Fischer model of wage determination.

An immediate problem arises as to the determination of the indexation coefficient, i.e. whether it is subject to bargaining or not. For the present we shall treat the indexation coefficient as an exogenous parameter not subject to bargaining.[30] The idea is that contract wages W^c are negotiated before uncertainty is resolved. Thus after shocks to the economy occur and the relevant price indexes are observed, wages are adjusted to the formula given above.[31] Later, we shall make a few comments on the optimal determination of wage indexation based on the bargaining approach.

Since the union is conceived as an economy-wide union, the effects of wages on the price levels P and Q are taken into account in deriving the contract wage W^c. Thus we write $P = P(W)$ and $Q = Q(W)$ to emphasize the dependence of the price levels on the wage rate.

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[29] Note that labour demand decisions are based on the actual wage.

[30] That the degree of wage indexation is not explicitly treated as a variable subject to bargaining is an important limitation of the present analysis in that it would be preferable to derive it from the same basic labour market model as the contract wage. Furthermore such a modelling strategy would more directly support the imposition of a preference functional like (3.15) on the bargaining game. The main reason for deriving only contract wages from the labour market model is that no workable closed-form formula for the degree of wage indexation is available from the model, if it in fact were treated as a variable subject to bargainig. One feature of this solvability problem is that uncertainty would in this case be affected by labour market decisions, since indexation typically affects aggregate price variability, a feature familiar from the standard Gray—Fischer model. Once again, then, simplicity dictates the choice.

[31] We could think of public sector authorities as making the required wage adjustment immediately after shocks occur.

Substituting the equations for real profits and employment in the Nash maximand and differentiating with respect to wages, gives the first order necessary conditions for the maximum:

$$\phi(W^c) =$$

$$[W^c]^{1-\tau}E\{P^{[\beta(1+\Theta)-\Theta]}Q^{-[(1-\tau)(1-b)+b(\beta(1+\Theta)-\Theta)]}\exp[\beta(1+\Theta)\mu]\}$$

$$-u(a)[Q^e]^{b(1-\tau)}CE\{P^{[\beta(1+\Theta)-\Theta]}Q^{-b(\beta(1+\Theta)-\Theta)}\exp[\beta(1+\Theta)\mu]\} = 0,$$

(3.17)

where

$$C = \frac{(1-\tau)[\beta(1+\Theta)-\Theta][1-\varepsilon_{P,W}]}{\{[\beta(1+\Theta)-\Theta][1-\varepsilon_{P,W}]-(1-\tau)[1-\varepsilon_{Q,W}]\}}$$

and $\varepsilon_{X,W}$ = elasticity of X with respect to W, assumed to be constant.[32,33]

The expected value in (3.17) depends, of course on the distribution of the random vector $(P,Q,\mu)'$. We assume that the random vector $(p,q,\mu)' = (\ln P,\ln Q,\mu)'$ is a normally distributed random vector with a mean vector $(Ep,Eq,0)'$ and variance-covariance matrix $\Sigma = (\sigma_{ij})$ for i,j = 1,2,3. This implies that the price levels P and Q are log-normally distributed random variables. To solve equation (3.17) for $\ln W^c$, we exploit the following property of normally distributed random variables: for a normal n-vector X, the moment generating function of the normal distribution is $M_x(t) \equiv E\{\exp[t'x]\} = \exp\{t'm + \frac{1}{2}t'\Sigma t\}$, where m is the mean vector, Σ the variance-covariance matrix of the distribution and $t = (t_1,t_2,...,t_n)'$ is an n-vector of real variables. Using this fact, we can express the logarithm of the contract wage, w^c, in the following form[34]

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[32] In the log-linear macromodel to be presented later in chapter 4, the elasticities are contstant.

[33] Actually, the f.o.c. for the maximum should be written as
$(1 - b\eta)\phi(W^c) = 0$, where η denotes the elasticity of the expected value of the consumer price index with respect to the contract wage, i.e. $\eta = \partial\ln Q^e/\partial\ln W^c$. So, we have implicitly imposed the condition $b\eta \neq 1$, which is assumed to throughout the text.

[34] We are implicitly assuming that the relevant second order condition for a maximum is satisfied, i.e. $\phi'(W^c) < 0$.

$$w^c = (1-\tau)^{-1}[\ln u(a) + \ln C] + b\ln Q^e$$
$$+ (1-b)\{Eq - \tfrac{1}{2}\{[(1-\tau)(1-b) + 2b(\beta(1+\Theta) - \Theta)]\sigma_q^2 \qquad (3.18)$$
$$- 2[\beta(1+\Theta)\sigma_{\mu q} + (\beta(1+\Theta) - \Theta)\sigma_{pq}]\}\},$$

where σ_x^2 denotes the variance of the random variable x and σ_{xy} is the covariance of the random variables x and y. Now, since $Q^e = E(Q) = \exp\{Eq + \tfrac{1}{2}\sigma_q^2\}$, equation (3.18) reduces to

$$w^c = u^* + c + Eq - \tfrac{1}{2}V(\Sigma), \qquad (3.18')$$

where $c = (1-\tau)^{-1}\ln C$; $u^* = (1-\tau)^{-1}\ln u(a)$

and
$$V(\Sigma) = \{[(1-b)[(1-\tau)(1-b) + 2b(\beta(1+\Theta) - \Theta)] - b]\sigma_q^2$$
$$- 2(1-b)[\beta(1+\Theta)\sigma_{\mu q} + (\beta(1+\Theta) - \Theta)\sigma_{pq}]\}.$$

The solution to the wage bargain, (3.18'), has an interesting structure. Note first that for $(b,\Theta) = (0,0)$ the solution reduces to the one derived by Andersen & Sørensen (ibid.) in their analysis of wage formation and exchange rate variability in a unionized open economy. Their analysis does not, however, allow for a more flexible bargaining structure owing to the presence of an all-powerful union that imposes its desired wage on firms.

Once we incorporate our unionized labour markets and the ensuing wage equation (3.18') in the standard macromodel of chapter 2, we observe that the natural rate property is lost. This is because monetary policy affects wage formation through the variance-covariance term $V(\Sigma)$. Thus, the policy implications from the present model must differ from the standard Gray—Fischer model of an open economy, as exemplified by e.g. Aizenman & Frenkel (1985a, b) and Turnovsky (1986). The crucial difference is the *ex ante* partial equilibrium in the labour markets upon which the determination of the contract wage is based. Standard analysis chooses as its *ex ante* equilibrium concept the flexible wage, non-stochastic equilibrium of the labour markets. This choice implies that the contract wage is not affected by the parameters of wage bargaining or of monetary policy rules. The assumption of risk neutral workers and firms implies that random (aggregate) variability has no utility costs and hence it need not be priced *ex ante* in contracts; there are no gains from risk sharing.

The particular choice of the *ex ante* labour market equilibrium generates a natural rate property in a model of an open economy incorporating Gray—Fischer contracting in that the steady state equilibrium level of the economy is independent of the policy instruments being used, i.e. indexation and foreign exchange intervention.[35] In the present context, there is a clear interaction between indexation and intervention and contract wages. First, given Σ, indexation directly influences the way aggregate variability Σ impinges on contract wages. Second, in a general (macroeconomic) equilibrium, Σ typically depends on the degree of indexation and intervention, so that the degree of aggregate variability is affected by indexation and intervention. Thus we see that the effects on contract wages, and thus on the steady state of the economy of a particular policy mix of wage indexation and exchange rate regime (intervention) is two-fold; the amount of aggregate variability and its impact on contract wages are both affected by the choice of the policy mix.[36]

The solution wage, (3.18'), clearly displays a "mean-variance" structure.[37] Apart from the variance-covariance structure incorporated in $V(.)$, wages are determined by price level expectations and a mark-up over the union's disagreement point.[38] The mark-up depends on the risk parameter of the utility function of a representative union member, the elasticity of labour demand with respect to real wages (thus on production technology), the bargaining parameters (Θ) and product market conditions, as summarized by the elasticities $\varepsilon_{P,W}$ and $\varepsilon_{Q,W}$. The mark-up structure is intuitively understandable and seems to

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[35] Note that the non-stochastic equilibrium and the (conditionally) expected equilibrium of the labour markets need not be the same, even if shocks are serially uncorrelated.

[36] From this argument and equation (3.18') we can clearly see the implications of the specific assumption that the degree of wage indexation b is not a variable subject to bargaining; bargaining over b in the labour markets largely means bargaining over the degree of uncertainty impinging on the economy, as argued in fn. 30 above.

[37] From this partial equilbrium point of view there are three (correlated) sources of stochastic variability; prices P and Q and the productivity disturbance μ.

[38] We cannot set the fall back utility of the union, U*, equal to zero under the assumptions of a Cobb—Douglas production technology and zero fall back profits, since in this case the Nash maximand becomes unbounded in W^e and the maximization problem is not well defined; the union's utility and the firm's profits are increasing without bound as wages tend to zero. The utility and profit gains come indirectly from unbounded increases in employment (as wages tend to zero) in the Nash maximand. We note here the restriction M≥N, which in this case cannot be met, so that for zero fall back utility and profits we have a corner solution at membership. This situtation results in indeterminate wages.

be typical of the solution to monopoly union-firm bargaining,[39] where union tries to exercise its wage setting power to set wages above the available alternative.[40]

The mark-up parameter c is, *ceteris paribus*, decreasing in the elasticity of labour demand ß and the firm's bargaining power Θ.[41] The former is due to the fact that variations in employment are larger for a given change in wages the more elastic is the labour demand schedule. This, in turn, implies that the employment-induced changes in the union's utility are quantitatively more important for a more elastic labour demand function. Increases in the firm's bargaining power mean, in the present context, that labour pricing becomes more competitive, and so the the wage equation says that firms are more able to push the union towards the threat point alternative as their bargaining power increases. This is intuitive enough.

Since an expected increase in consumer prices leads to a proportionate increase in wages, the bargainers act as if choosing the expected real (consumption) wage, $E(w^c - q)$, depending on the fall back utility or income opportunities to the unemployed, the degree of wage indexation, the firm's bargaining power and the second moment characteristics of the distribution of (p,q,μ)'. These second moment characteristics are summarized by the term $V(\Sigma)$, which will henceforth be called the price risk,[42] since it summarizes the effects of aggregate price variability on the contract wage, and this price variability introduces an "insurance" component which affects the pricing of labour.

We know from e.g. implicit contract theory that a contract between a risk averse employee and a risk neutral employer under symmetric information specifies optimal risk sharing in that an

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[39] Compare e.g. Nickell (ibid., p. 25).

[40] Taking into account the employment repercussions of wages of course; see e.g. Nickell (1990).

[41] As noted by Andersen & Sørensen (ibid. p. 265), if the competing sector is becoming too small relative to the sheltered sector ($\varepsilon_{Q,w}$ is small relative to $\varepsilon_{P,w}$), there is no well-defined solution to the maximization problem. However, the distinction between product and consumer prices is non-existent in the macromodel to be specified later, since we are focusing on a small open economy under well integrated international commodity markets (law of one price or PPP).

[42] This terminology is adopted from Andersen & Sørensen (ibid.). In the present context it would be prefererable to call V simply aggregate risk, since V now also depends on aggregate shocks to production technology, which are absent from Andersen's and Sørensen's model. The main point is not, however, giving a name to V, but the fact that it summarizes the effects on contract wages of the economy's aggregate variability.

employee accepts lower wages, if this means more certain wages, while an employer agrees to this, if, in consequence, expected profits go up. Matters are analogous here in that the risk inherent in the price of labour is priced *ex ante* in contracts. But note that even if a union member were risk averse, the union need not be, because of the employment effects of wage setting. Consequently, the effects on wages of uncertainty in the form of price variability are not determinate.

The ambiguity of the price risk effects on wages stems, as mentioned above, from the risk characteristics of the union. To understand what is involved here, we apply the results of Rotschild & Stiglitz (1971) concerning the effects of increasing riskiness. As the union's utility in the Nash product is composed of employment and the utility of a representative member (above the threat point), the ambiguity in the union's risk characteristics stems from the fact that the convexity/concavity properties of the relevant marginals of these components differ, thus driving the union's marginal utility of wages in different directions.

Specifically, from the employment function we can see that $N_W = -ßP^ß W^{-(1+ß)} \exp[ß\mu]$ is a concave function of P, and from the union member's utility function that $u_W = W^{-\tau}[1/Q]^{1-\tau}$ is a convex function of Q for $0 \le \tau < 1$. These push the convexity of U_W in different directions.[43]

The ultimate effects, both qualitative and quantitative, of the price risk on wages depend heavily on the degree of wage indexation, b, and firms' bargaining power, Θ. Furthermore, firms' power to influence wages through the price risk is dependent on the degree of wage indexation. For fully indexed wages (b=1), firms cannot have any extra power to price the risk, and the contract wage fully reflects only the risk due to variability in consumer prices. This source of variability is related to the distributional characteristics of consumer prices, which affect the real wages relevant to the union. Moreover, the particular way in which the distribution of consumer prices also affects contract wages under full indexation guarantees that expected consumption real wages are constant and independent of the distribution of consumer prices, since $E(W/Q) = W^c \exp\{-(Eq + \frac{1}{2}\sigma_q^2)\} = \exp\{u^* + c\}$ under full indexation.

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[43] Rather than characterize the convexity properties of the relevant marginals in terms of the price levels P and Q, as in Andersen & Sørensen (ibid.), it would be preferable to characterize these properties in terms of "purchasing power units" 1/P and 1/Q, since it is wages relative to these units which affect employment decisions and union utility. To this end, N_W is convex in 1/P, and u_W is concave in 1/Q. (Note that the convexity characterization of N_W and u_W in P and Q in Andersen & Sørensen should be reversed.)

On the other hand, under non-indexed wages (cf. Andersern & Sφrensen, ibid.), firms have more power (within Θ) to affect the pricing of risk, but only by influencing the way covariability between consumer and producer prices and productivity shocks and consumer prices affects contract wages (i.e. through the terms σ_{pq} and $\sigma_{\mu q}$ in the price risk). This feature of the solution hinges upon the structure of the (Nash) maximand and especially upon the specification of union preferences, whereby employment acts as a scale factor in the union's utility function. The ambiguity a rises because in general we would set $\sigma_{pq} > 0$ and $\sigma_{p\mu} < 0$, i.e. the covariances have differential effects on the price risk, and is directly related to the union's risk characteristics mentioned earlier.[44]

Note further that the product price and the productivity shock enter the price risk and accordingly the wage equation only through their covariances with consumer prices, regardless of the degree of wage indexation[45]. This, as in the case of non-indexed wages, hinges upon the structure of the (Nash) maximand where firms' fall out profits are zero and the product price (via employment) and productivity shocks enter the maximand as if they were scale factors. This need not be the case for non-zero fall out profits or for other specifications of the union's and firms' preferences.

Having described labour market behaviour in a unionized economy and derived a contract wage as a solution to union-firm bargaining over wages, we shall now proceed to analyze the implications of wage formation in a small open economy context. The model of the small open economy is a log-linear stochastic IS-LM model of the type utilized in chapter 2 in the context of standard models of wage indexation and foreign exchange intervention. Since the steady state equilibrium of the economy will now fully reflect the parameters of the wage process and foreign exchange intervention, we can no longer analyze the variability of aggregate output and employment around the steady state independently of the steady state level itself. This implies that policy strategies designed to adjust aggregate employment and output probably have costs in terms of the levels of these variables, e.g. output stabilizing policies may only be achieved at lower levels of employment and output.[46]

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[44] The presence of productivity shocks has a tendency to reduce wages, since N_w is a concave function of μ, thus tending to increase the concavity of U_w.

[45] Provided b is not equal to one.

[46] Of course stabilization or other policies can have (ex post) costs in terms of the utilities of the bargainers.

Appendix to chapter three

Derivation of the contract wage equation from the standard Nash formulation

Here we shall derive the contract wage equation corresponding to the standard Nash maximand, which is written as

$$[EU - U^*][E\pi - \pi^*]^\Theta. \tag{A.1}$$

The union and firms set contract wages W^c as if maximizing (A.1). The first order condition for the maximum can be written as

$$\phi(W^c) = \frac{\partial EU}{\partial W^c}[E\pi - \pi^*] + \Theta[EU - U^*]\frac{\partial E\pi}{\partial W^c} = 0. \tag{A.2}$$

Substituting in $\pi^* = 0$, the indexation scheme $W = W^c(Q/Q^e)^b$, the profit, employment and CRRA utility function and manipulating (A.2) can be expressed in the following form

$$[W^c]^{1-\tau}E\{P^\beta Q^{-(1-\tau)(1-b)+b\beta}\exp(\beta\mu)\}$$
$$-u(a)[Q^e]^{b(1-\tau)}CE\{P^\beta Q^{-b\beta}\exp(\beta\mu)\} = 0, \tag{A.3}$$

where, as in the text,

$$C = \frac{(1-\tau)[\beta(1+\Theta)-\Theta][1-\varepsilon_{P,W}]}{\{[\beta(1+\Theta)-\Theta][1-\varepsilon_{P,W}]-(1-\tau)[1-\varepsilon_{Q,W}]\}}$$

and $\varepsilon_{X,W}$ = elasticity of X with respect to W. Finally, solving equation (A.3) for the log of the contract wage gives us

$$w^c = (1-\tau)^{-1}[\ln u(a) + \ln C] + b\ln Q^e$$
$$+(1-b)\{Eq - \tfrac{1}{2}\{[(1-\tau)(1-b)+2b\beta]\sigma_q^2 \tag{A.4}$$
$$-2[\beta\sigma_{\mu q}+\beta\sigma_{pq}]\}\},$$

or

$$w^c = u^* + c + Eq - \tfrac{1}{2}V(\Sigma), \tag{A.3'}$$

where $c = (1-\tau)^{-1}\ln C$; $\quad u^* = (1-\tau)^{-1}\ln u(a)$

and $V(\Sigma) = \{[(1-b)[(1-\tau)(1-b)+2b\beta]-b]\sigma_q^2$
$\qquad\qquad -2(1-b)[\beta\sigma_{\mu q}+\beta\sigma_{pq}]\}$.

From (A.3) or (A.3') we can immediately see that the aggregate price risk, which summarizes the information about the random variability in the aggregate economy generated by shocks to consumer and producer prices and production technology, is completely independent of the parameter Θ. In this sense aggregate uncertainty is, as claimed in the text, in the above model independent of the bargaining power of firms.

4 Exchange Rate Variability and Wage Formation

4.1 Introduction

The solution to the bargaining game in the previous chapter gives us a partial equilibrium relation between contract wages and aggregate price risk. A partial equilibrium relation, since we need a macroeconomic structure in which we can characterize the structure of the aggregate price risk and in which, as a byproduct, we can analyze the steady state policy non-neutralities that emerge as a result of uncertainty. The novel feature here lies in restructuring conventional open economy macromodels (of wage indexation and foreign exchange intervention) to incorporate a unionized labour market where uncertainty impinges on the wage formation process. The construction and analysis of (union-firm) wage bargaining models is nowadays a well established research field, but these models are mainly partial equilibrium models designed either to lay theoretical foundations for unionized wage formation or to test various model specifications empirically. Furthermore, most of these models deal with wage (and employment) determination under certainty. So, the introduction of uncertainty into a wage bargaining model and its application in a conventional macroeconomic framework seems to be unexplored theoretical territory.

Two important exceptions to the previous observations are Oswald (1985) and Andersen & Sørensen (1986). Oswald explicitly introduces uncertainty into a pure monopoly union model and searches for potential changes in the comparative statics properties of the monopoly union model under uncertainty *vis-à-vis* the corresponding model under certainty. In general, no changes can be observed.[1] Andersen & Sørensen, on the other hand, build on Oswald and construct a utilitarian monopoly union model of wage determination under uncertainty, constrained by the demand for labour of a Cobb—Douglas firm, in order to analyze the steady state implications of wage formation and exchange rate variability. Under some (implicit) parameter restrictions, they conclude that, because of aggregate price risk, a reduction in exchange variability may be desirable from the point of view of aggregate output and employment.

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[1] A (minor) qualification arises from the stochastic behaviour of the marginal utility of income (consumption) with respect to the comparison wage (see Oswald, ibid.).

In this chapter we make some modifications to the framework employed by Andersen and Sørensen in order to analyze the interaction between exchange rate variability and wage formation. First of all, we relax their pure monopoly union model of wage determination as being too restrictive in that it does not allow firms to affect wages. Instead, the "right to manage" model of the previous chapter is employed, thus enabling us to control for the competitiveness of labour pricing in the ensuing analysis. Second, the macroeconomic structure is modelled as in chapter 2. Thus, we are describing a small open economy with unionized labour markets. This choice is mainly designed to maintain comparability with previous analyses of wage indexation and foreign exchange intervention.

The main focus of the ensuing analysis relates to steady state policy non-neutralities of a combination of wage indexation and foreign exchange intervention. These non-neutralities emerge in the model because of aggregate uncertainty impinging on the wage formation process. Thus, the natural rate property, a fundamental feature of the standard Gray—Fischer model, no longer holds. This property of the wage formation process has far-reaching implications for e.g. the choice of the exchange rate regime or exchange rate policy. This is especially so when we come to discuss matters in terms of the optimality of the policy instruments of indexation and intervention, since the various policy optima cannot be derived and analyzed solely on the grounds of minimizing the effects of various disturbances on the variability of aggregate real economy, an implication tentatively suggested by the standard models.

4.2. A macromodel of a small open economy with union-firm wage bargaining

We use our small open economy macromodel of chapter 2 to analyze the macroeconomic effects of union-firm bargaining. The formal (log-linear) small open macroeconomy is repeated here in equations (4.1)–(4.8)[2,3,4]

$$y_t = a + \alpha n_t + \mu_t \tag{4.1}$$

$$n_t = \beta(p_t - w_t) + \beta\mu_t, \quad \beta > 1 \tag{4.2}$$

$$w_t = w_t^c + b(p_t - p_{t-1,t})$$

$$w_t^c = u^* + c + p_{t-1,t} - \tfrac{1}{2}(1-b)V(\Sigma)$$

$$c = (1-\tau)^{-1}\log\frac{[\beta(1+\Theta)-\Theta]}{[\beta(1+\Theta)-\Theta-(1-\tau)]} \tag{4.3}$$

$$= V(\Sigma) = (1-b)[(1-\tau)-2(\beta(1+\Theta)-\Theta)]\sigma_p^2 - 2\beta(1+\Theta)\sigma_{\mu p}$$

$$m_t - p_t = y_t - \phi i_t + u_t \tag{4.4}$$

$$m_t - m_o = -\delta(s_t - s_o), \quad \delta \leq \infty \tag{4.5}$$

$$i_t - i_t^* = s_{t,t+1} - s_t \tag{4.6}$$

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[2] Note that for a random variable x $E(x) = E_y[E(x \mid y)]$, so that maximizing the expected value of x is equivalent to maximizing its conditional expected value for all choices of y. In the contract wage equation, we have conditioned the expectations concerning the information available at the end of period t-1. Since we are assuming that the period t stochastic disturbances to our economy are orthogonal to the information available at the end of period t-1, the conditional and unconditional expectations coincide.

[3] The PPP relation (4.7) implies that p = q, i.e. there is no difference between consumer and producer prices in this economy, so that $\varepsilon_{P,W} = \varepsilon_{Q,W}$. This fact has been used to remove these elasticities from the constant c.

[4] Since p = q due to PPP, we have $\sigma_{pq} = \sigma^2_p$, which has been used in the expression for the price risk.

$$p_t = p_t^* + s_t, \tag{4.7}$$

where for all x

$$x_{s,t} = E[x_t : I_s] \quad s < t. \tag{4.8}$$

Notation

y_t = log of period t real output
n_t = log of period t employment
p_t = log of period t price level
w_t = log of period t wages
w_t^c = log of period t contract wage
i_t = domestic interest rate in period t
s_t = log of period t exchange rate (domestic price of foreign currency)
m_t = log of period t money stock
m_o = log of the long-run target level of m
s_o = log of long-run equilibrium value of the exchange rate
$*$ refers to foreign variables.

As before, equation (4.1) describes the short-run (decreasing returns to scale) production technology of the economy subject to (unpredictable) random shifts in the production possibilities as represented by random productivity shocks μ. Equation (4.2) is the derived demand for labour as implied by the profit-maximizing behaviour of the firms.[5] Here, as before, ß = 1/(1-α) > 1 is the elasticity of labour demand with respect to real wages. As the notation in equation (4.2) and (4.1) suggests, actual employment is determined by the demand for labour. This is the constraint faced by the monopoly union in a right to manage model of wage determination. Labour market behaviour is completed by specifying the determination of actual wages and the outcome of the union-firm bargaining represented by the contract wage w^c in equation (4.3).[6] Actual wages are may be indexed to the price level possibly

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[5] That is n is the log of N, where N is chosen so that N = argmax$_N${Pf(N) - WN}, where f(N) = AN$^{\alpha}$exp(μ) for some A.

[6] Note that labour supply shifts in the contract wage equation are generically captured by the exogenous fall back utility of the union, i.e. by u*. Other determinants of labour supply behaviour are embodied in the union's objective function.

through an exogenous administrative process.[7] The mark-up constant in the contract wage formula does not include the elasticity terms due to the PPP rule (4.7). For the same reason the price risk now depends only on the variance of (the log of) the price level and its covariance with the productivity shock μ.[8]

Money market equilibrium behaviour is described in equations (4.4) and (4.5). Equation (4.5) gives the "restricted" (Devereux, 1988) foreign exchange intervention rule currently employed by the monetary authority. Coupled with the money market equilibrium condition (4.4), equation (4.5) determines the exchange rate system prevailing in the domestic economy. The two most commonly employed exchange rate systems of clean floating and fixed exchange rates are generated by equation (4.5) by the parameter values $\delta = 0$ and $\delta = \infty$ respectively. The most interesting point about this particular parameterization of exchange rate regimes in the present context is that there is a direct link between exchange rate variability and wage formation as in Andersen & Sørensen (ibid.). By implication there is a close relationship between the exchange rate system (or intervention policy) and the steady state of the system.

Equations (4.6) and (4.7) are the two parity conditions commonly used in standard models, the former postulating uncovered interest rate parity and the latter purchasing power parity. As we argued earlier in chapter 2, these condition essentially posit perfectly integrated capital and commodity markets, and they offer great technical advantages in small open economy models under rational expectations.

Condition (4.8) postulates rational expectations. As it stands, it says that subjectively held conditional expectations coincide with the conditional expectations generated by the model itself, just as Muth's (1961) original breathtaking analysis convincingly argues.[9] Given the structure of wages in this unionized economy, there is a potential

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[7] This determination of the degree of wage indexation need not be as arbitrary as it seems at first sight. Simonsen (1983) gives a good description of the Brazilian indexation experience, where a proper indexation formula has been sought through an administrative process. In Finland, for example, there are legal restrictions on the use of indexed contracts, so that the determination of the degree of wage indexation normally requires considerations beyond those directly related to the bargaining process in the labour market itself. A great deal of public sector "intervention" is required to achieve indexed contracts.

[8] Note that the coefficient of the price level variance is negative, unless the firm's bargaining position, as represented by its bargaining power, is very weak indeed and the coefficient of relative risk aversion, μ, is negative.

[9] Of course, rational expectations require equality of "subjective" and "objective" distributions, and not merely expected values.

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problem in solving rational expectations in this model, since the conditional moments of an endogenous variable will depend on the variance of the (log of the) price level and its covariance with the productivity shock. However, given the (log-)linearity of the model and the structure of the stochastic disturbances, to be specified shortly, and their relationship to the conditioning information (I_{t-1}), we can ascertain that the variance structure of the system is constant, thus simplifying the calculations involved.

For the stochastic structure of the disturbances, we postulate the familiar structure of serially uncorrelated normally distributed disturbances:

$$p_t^* = p_o + v_{1t}$$

$$i_t^* = i_o + v_{2t}$$

(4.9)

and

$$(\mu_t, u_t, v_{1t}, v_{2t})' \sim NID(0, \Sigma)$$

$$\Sigma = diag(\sigma_\mu^2, \sigma_u^2, \sigma_1^2, \sigma_2^2).$$

This structure implies that period t disturbances are orthogonal to the information available at the end of period $t-1$, which includes the structure of the model and all variables dated $t-1$ and earlier. In addition to being serially uncorrelated and normally distributed (and thus independent), the disturbance structure displays non-existent correlation (independence) across variables.

Substituting the labour demand function (4.2) and wage formula into the production function, we obtain the aggregate supply function under imperfect information:

$$y_t = a - \alpha\beta(w_t^c - p_{t-1,t}) + \alpha\beta(1-b)(p_t - p_{t-1,t}) + \beta\mu_t.$$

(4.10)

From (4.10) we can see that the aggregate supply function decomposes into two distinct parts. First, unexpected changes in the price level affect aggregate supply, i.e. an effect due to imperfect information (wages are set before uncertainty is resolved). This effect is familiar from the standard Gray—Fischer analysis and gives rise to a Phillips curve, where changes in prices and output are positively correlated, if wages are not fully indexed. Furthermore, this is the channel through

which monetary policy is transmitted into the real sector of the economy.

In addition to price level surprises, expected real wages affect the level of aggregate output. This effect is directly related to wage formation based on union-firm bargaining in the labour markets. In a standard Gray—Fischer framework the expected real wage effect is largely suppressed, since contract wages adjust so as to clear *ex ante* labour markets in a fully flexible wage economy.[10] This device is responsible for the natural rate property of the standard models, i.e. the steady state of the economy displays "neutrality" with respect to the policy intervention under consideration.

From (4.10) we see that since the contract wage is not independent of indexation and foreign exchange intervention, the *ex ante* or stationary equilibrium of output depends on the full range of parameters affecting wage and exchange rate formation in the economy. Thus indexation and intervention will have steady state effects as well as effects due to imperfect information (second order effects).[11]

After substituting in equations (4.5) and (4.10), the money market equilibrium condition (4.4) yields the following equation for exchange rates

$$[1 + \delta + k + \alpha\beta(1 - b)]s_t = k_t s_{t,t+1} + C + \Phi_t, \tag{4.11}$$

where

$$C = m_0 + \delta s_0 - [a - \alpha\beta(u^* + c - \tfrac{1}{2}(1 - b)V(\Sigma)] + ki_0 - [1 + \alpha\beta(1 - b)]p_0$$

$$\Phi_t = [kv_{2t} - (1 + \alpha\beta(1 - b))v_{1t} - \beta\mu_t - u_t].$$

Following the steps taken in chapter 2 in deriving the solution for exchange rate expectations, equation (4.11) implies the following (convergent) solution

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[10] It should be emphasized that this is an *ex ante* non-stochastic equilibrium. In the present context this determination of the contract wage gives rise to an approximation to a wage equation of the form $w_t^c = p_{t,t-1}^e$, where w_t^c denotes the contract or base wage for period t.

[11] This implies, among other things, that the efficiency or welfare criterion suggested by Gray, which approximates losses due solely to imperfect information, is not fully satisfactory here, even if it is accepted as a reasonable welfare criterion in itself.

$$s_{t,t+1} = \frac{C}{[1+\alpha\beta(1-b)]},$$ (4.12)

where

$$C = m_0 - [a - \alpha\beta(u^* + c - \tfrac{1}{2}(1-b)V(\Sigma))] + ki_0 - [1 + \alpha\beta(1-b)]p_0.$$

In what follows we shall take this as our solution to exchange rate expectations.[12] Note, however, that this particular choice for exchange rate expectations does not display complete independence of the solution from foreign exchange intervention, as in the standard model (e.g. Turnovsky, 1983a), since intervention affects the variance-covariance structure of the price level and productivity shock, and thus the price risk.

Perhaps the most interesting feature of the solution (4.12) is the direct link between exchange rate expectations and wage formation. An increase in the union's threat point u^* will, *ceteris paribus*, lead to an expected depreciation of the domestic currency because of the increase in the contract wage. On the other hand, if we treat the price risk $V(\Sigma)$ parametrically, an (algebraic) increase in it will lead to an expected revaluation of the domestic currency, since this increase in the price risk will lower the contract wage.

Substituting (4.12) into (4.11), we obtain the following equation for actual exchange rates

$$s_t = \frac{C}{[1+\alpha\beta(1-b]} + [1+\delta+k+\alpha\beta(1-b)]^{-1}\Phi_t.$$ (4.13)

From this equation we can see that wage rigidity (indexation) affects exchange rates for a given δ through two distinct channels. First, there is the effect on exchange rates via exchange rate expectations. This is the first term on the right hand side of equation (4.13). In addition, indexation affects the transmission of different shocks into changes in the exchange rate. The second term on the right hand side of equation (4.13) captures this effect, which is very familiar from standard analysis of wage indexation and foreign exchange market intervention (see e.g. Aizenman—Frenkel, 1985a, b). Thus, foreign exchange

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[12] In choosing this particular solution, we have imposed similar restrictions on the intervention parameter δ as in chapter 2 in order to guarantee convergenge of exchange rate expectations. Furthermore, we have excluded from the solution an additional non-convergent factor (i.e. a factor formally satisfying the expectational difference equation) justifying this exclusion by typical stability considerations under rational expectations.

intervention (δ) has a direct influence on the impact effects of disturbances on exchange rates, as well as an indirect effect via exchange rate expectations. As we have seen, exchange rate expectations depend on the price risk, which, in turn, depends on the variance-covariance structure of the price level and productivity shocks. Foreign exchange intervention thus influences exchange rate expectations through its effects on the price risk.

Using the PPP rule we can derive an expression for the domestic price level

$$p_t = p_t^* + s_t = p_0 + v_{1t} + s_t$$

$$= \frac{C'}{[1 + \alpha\beta(1-b)]} + \frac{\Phi'_t}{[1 + \delta + k + \alpha\beta(1-b)]},$$

(4.14)

where

$$C' = m_0 - [a - \alpha\beta(u^* + c - \tfrac{1}{2}(1-b)V(\Sigma))] + ki_0$$

$$\Phi'_t = [kv_{2t} + (k+\delta)v_{1t} - \beta\mu_t - u_t].$$

As it stands,[13] the price level is heavily influenced by wage formation; an increase in either the union's fall back utility u^* or the mark-up c increases the price level, since these factors increase the contract wage. On the other hand, an increase in price risk induces a fall in the price level via the contract wage effect. Note that the impact effects of different disturbances on the domestic price level is reinforced by a higher degree of wage indexation, i.e. the variance of the price level increases with increases in the degree of wage indexation, a feature familiar from the standard models.

In what follows we shall mainly focus on the effects of wage formation on the steady state output of the economy and make some comparisons across different exchange rate regimes subject to a differential degree of wage indexation. These comparisons are important, since while the standard models do seem to imply that policies designed to stabilize output variability can be conducted without affecting the steady state output level (natural rate), the present

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[13] Note that the log of the price level is a linear combination of independent normal variates, so it is also a normal variate with a (conditional as well as unconditional) mean equal to the first term on the right hand side of the price level equation (4.15) and variance equal to the variance of the second term on the right hand side of equation (4.15).

model clearly denies this possibility; wage stickiness and the exchange rate regime or intervention policies also have repecussions for the steady state of the economy, and not only for the transmission of various disturbances into real fluctuations.

4.3   Steady state effects of indexation and foreign exchange intervetion

The policy instruments (b and δ) exert their real effects through the price risk term in the wage equation (4.3). In order to analyze policy effects, we have to solve for the price risk. Since unexpected price level changes are given by the second term on the right hand side of the price level equation (4.14), the variance of the price level is given by

$$\sigma_p^2 = \Omega^{-2}\{(k+\delta)^2\sigma_1^2 + k^2\sigma_2^2 + \beta^2\sigma_\mu^2 + \sigma_u^2\}, \qquad (4.15)$$

where $\Omega = [1+k+\delta+\alpha\beta(1-b)]$, while the covariance between the productivity shock and the price level is $\sigma_{\mu p} = -\Omega^{-1}\beta\sigma_\mu^2$. From the price risk equation (4.3) we observe that the effects of foreign exchange intervention on the price risk (given b) are given by

$$\text{sign}\frac{\partial V}{\partial \delta} = \text{sign}\frac{\partial[(1-b)\alpha_1\sigma_p^2 - \alpha_2\sigma_{\mu p}]}{\partial \delta}, \qquad (4.16)$$

where $\alpha_1 = [(1-\tau)-2(\beta(1+\Theta)-\Theta)] < 0$ and $\alpha_2 = 2\beta(1+\Theta) > 0$. From (4.16) we deduce that the comparative static effect of foreign exchange intervention on (expected real) contract wages is given by

$$\text{sign}\frac{\partial(w^c - p_{t-1,t})}{\partial \delta} = -\text{sign}\frac{\partial[(1-b)\alpha_1\sigma_p^2 - \alpha_2\sigma_{\mu p}]}{\partial \delta}. \qquad (4.17)$$

Now, the effects of intervention on the price level variance cannot be signed *a priori* without further restrictions on the source of exogenous uncertainty. The problem arises because the greater fixity of exchange rates (higher δ) reduces price level variability due to shocks in domestic supply, money demand and foreign interest rates, but because of PPP, price level variability increases due to shocks to foreign

prices. Thus the effects of intervention and *a fortiori* exchange rate variability on steady state employment and output are ambiguous.

Ambiguities also arise concerning the effects of wage indexation. As such, a higher degree of wage indexation (given δ) increases price level variability and increases the covariability between the price level and productivity shock. Simultaneously, the effects on the (expected real) contract wage of a given aggregate price risk are reduced because of a higher degree of wage indexation. The net effect on the contract wage is thus ambiguous, as are the effects on steady state employment and output of a higher degree of wage indexation.

To obtain somewhat more specific information on the effects of indexation and intervention on the steady state equilibrium, we resort to the familiar special cases of fixed and flexible exchange rates. Some comparisons across these regimes are also presented.

4.3.1 Steady state output under fixed and flexible exchange rates

Substituting the price level prediction error Φ' from equation (4.15) into the output supply equation (4.10), we can write the output supply equation as follows

$$
\begin{aligned}
y_t &= a - \alpha\beta(w_t^c - p_{t-1,t}) + \alpha\beta(1-b)(p_t - p_{t-1,t}) + \beta\mu_t \\
&= a - \alpha\beta[u^* + c - \tfrac{1}{2}(1-b)V(\Sigma)] \\
&\quad + \Omega^{-1}\{\alpha\beta(1-b)[kv_{2t} + (k+\delta)v_{1t} - u_t] + (1+k+\delta)\beta\mu_t\}
\end{aligned}
\tag{4.10'}
$$

As for the mean level of aggregate output, we can make some comparisons across exchange rate regimes, once the the parameters associated with the bargaining process (especially b and Θ) are given. The mean or stationary aggregate output is given by

$$
E(y_t) = a - \alpha\beta[u^* + c - \tfrac{1}{2}(1-b)V(\Sigma)].
\tag{4.18}
$$

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4.3.1.1 Fixed exchange rates

Since the *ex ante* price risk under fixed exchange rates is given by

$$V^f(\Sigma) = (1-b)[(1-\tau) - 2(\beta(1+\Theta) - \Theta)]\sigma_1^2 < 0,$$

we have

$$E(y_{fix}) = a - \alpha\beta[u^* + c - \tfrac{1}{2}(1-b)V^f(\Sigma)]. \tag{4.19}$$

Under fixed exchange rates, the only source of domestic price level variability are direct shocks to the foreign price level (v_1), which impinge on the domestic price level through PPP. The impact effect of foreign price shocks on the price risk, as summarized by their variance σ_1^2, is unambiguously negative, since $[1-\tau] - 2[\beta(1+\Theta) - \Theta] < 0$, although their ultimate quantitative effects on the contract wage depend heavily on the degree of wage indexation b. However, under partial wage indexation (0<b<1), domestic price level variability due to foreign price shocks tends to increase the contract wage, and thus decrease the stationary equilibrium output under fixed exchange rates. Note that since the higher degree of wage indexation reduces the price risk effect on the contract wage, it follows that increased wage indexation under fixed exchange rates tends to increase the stationary equilibrium output. In the limiting case of complete wage indexation (b=1), the price risk effect on the contract wage is completely eliminated, so that the steady state aggregate output in this case is given by

$$E(y_{fix})\big|_{b=1} = a - \alpha\beta[u^* + c]. \tag{4.19'}$$

For further comparisons we resort to the implications of the bargaining parameters on the contract wage and consequently on output. As it stands, the stationary output under fixed exchange rates and complete wage indexation is completely determined by the mark-up structure of the contract wage (u* + c). Union fall back utility ("competitive wage" or shifts in labour supply) increase the contract wage and thus induces a fall in the stationary aggregate output. Furthermore, since the mark-up constant c depends negatively on firms' bargaining power Θ, we see that output increases with firms' bargaining power under fixed exchange rates (and complete wage indexation), reaching its maximum of a - αβu* when only profit maximization matters in the labour market bargaining process.

4.3.1.2 Flexible exchange rates

Under flexible exchange rates $\delta=0$, so that the stationary aggregate output is given by

$$E(y_{flex}) = a - \alpha\beta[u^* + c - \tfrac{1}{2}(1-b)V^e(\Sigma)] \qquad (4.20)$$

$$V^e(\Sigma) = \Gamma_1\{(1-b)[(1-\tau) - 2(\beta(1+\Theta) - \Theta)][k^2(\sigma_1^2 + \sigma_2^2) + \sigma_u^2] + \Gamma_2\beta^2\sigma_\mu^2\}$$

$$\Gamma_1 = \frac{1}{[1 + k + \alpha\beta(1-b)]^2} > 0$$

$$\Gamma_2 = \{2(1+\Theta)[k+1] - (1-b)(1+\tau)\} > 0.$$

As can be seen from equation (4.3.6) the price risk term is now much more involved than in the previous case of fixed exchange rates. This is understandable, since the source of the price variability is now much wider, being affected by the entire exogenous stochastic structure of the disturbances. We can, however, most conveniently classify the determinants of the price risk into two broad classes, as in the original Gray—Fischer analysis, i.e. demand- and supply-side disturbances, represented formally by $(v_1,v_2,u)'$ and μ, respectively. These sources of variability have differential effects on the stationary equilibrium of the economy, so we focus on each of them separately.

i) No productivity shocks ($\sigma_\mu^2 = 0$). From equation (4.20) we can see that in this case the price risk behaves qualitatively in the same way as under a fixed exchange rate regime. Thus under partial wage indexation (0<b<1), price variability due to demand-side disturbances tends to exert upward pressure on contract wages. This induces a fall in the stationary equilibrium output relative to the situation where these sources of variability are also non-existent. Moreover, the increased bargaining power of firms enhances this effect, as in the fixed exchange rate case. Quantitatively, the reduction in the steady state output can be larger than in a fixed exchange rate regime, if money market disturbances (v_2 and u) are an important source of variability.

Now in the standard Gray—Fischer analysis of wage indexation and foreign exchange intervention (e.g. Aizenman & Frenkel, 1985a, b), partial indexation is doomed to be inefficient in cases where only demand-side disturbances impinge on the domestic economy, since these sources of variability induce fluctuations in aggregate output not

prevalent in the corresponding full information, flexible wage equilibrium of the economy.[16].

In the present unionized economy, this kind of characterization is inadequate,[17] since indexation impinges directly on the steady state. But it can be readily verified from equations (4.3) and (4.18) that complete wage indexation eliminates the need for *ex ante* risk pricing, so that the steady state output under flexible exchange rates and full wage indexation coincides with the one under fixed exchange rates and full wage indexation.[18]

ii) No demand-side disturbances ($\sigma_x^2 = 0$, $x = v_1$, v_2, u).
This time only domestic productivity shocks impinge on the economy, and the price risk now reads

$$V^e(\Sigma) = \frac{\{2(1+\Theta)[k+1] - (1-b)(1+\tau)\}}{[1+k+\alpha\beta(1-b)]^2}\beta^2\sigma_\mu^2,$$

which is positive. This implies that the price risk exerts downward pressure on contract wages as a result of more risky union utility.[19] The steady state output is thus increased, increasingly so for more

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[16] That is, the classical dichotomy prevails in the corresponding full information flexible wage economy.

[17] The basic inefficiencies in the present model are the presence of imperfectly competitive labour markets and the non-existence of spot labour markets for (union) labour. The standard analysis draws macroeconomic implications on the basis of the second inefficiency. It is imperfect competition which generates monopoly pricing or the potential for extracting a surplus above the competitive norm (within the limits of Θ). This source of inefficiency can be thought to be more fundamental, making welfare comparisons based on the signal extraction framework inadequate.

[18] This characterizes the interdependence of wage indexation and foreign exchange intervention at the steady state level, so much advocated in standard models (Aizenmen & Frenkel, ibid., Turnovsky, 1983) in the context of stabilizing aggregate output around the (fixed) steady state by employing these instruments. Note that the Phillips curve effect is eliminated under full wage indexation in both regimes, but the behaviour of the price level differs across these regimes (cf. eq. (4.15)).

[19] The notion of "more risky union utility" needs explanation. From the specification of union utility, we can see that productivity shocks enter it through the employment function N. Direct calculation shows that N_w is a concave function of the productivity shock μ, which increases the concavity of the union's marginal utility function U_w. According to Rotschild & Stiglitz (1971), this source of increase in risk reduces the contract wage.

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powerful firms. This time, however, increased wage indexation reduces the effect of the price risk on output.

Now, from equations (4.19) and (4.20), we can see that the difference between the steady state behaviour of aggregate output under fixed and flexible exchange rates is dependent on the behaviour of the price risk under these regimes. Specifically, from equations (4.19) and (4.20) we obtain

$$E[y_{fix}] - E[y_{flex}] = \tfrac{1}{2}\alpha\beta(1-b)[V^f(\Sigma) - V^e(\Sigma)]$$

$$= \tfrac{1}{2}\alpha\beta(1-b)\{\pi_1\sigma_1^2 - \pi_2\sigma_2^2 - \pi_3\sigma_u^2 - \pi_4\sigma_\mu^2\},$$

(4.21)

where

$$\pi_1 = (1-b)[(1-\tau) - 2(\beta(1+\Theta) - \Theta)][1 - \frac{k^2}{[1+k+\alpha\beta(1-b)]^2}] < 0$$

$$\pi_2 = k^2[(1-\tau) - 2(\beta(1+\Theta) - \Theta)]\Gamma_1 < 0$$

$$\pi_3 = [(1-\tau) - 2(\beta(1+\Theta) - \Theta)]\Gamma_1 < 0$$

$$\pi_4 = \Gamma_1\Gamma_2 > 0.$$

From (4.21) we can immediately deduce that if price level variability is mainly generated in the money market by shocks to foreign interest rates and demand for money, then the steady state output (and employment) under partial indexation is higher under fixed than flexible exchange rates. This is intuitive enough, since under partial indexation these shocks exert real effects only through changes in exchange rates.

If, on the other hand, price level variability is mainly due to shocks to foreign prices and/or productivity, then the steady state employment and output are higher under flexible exchange rates than under fixed exchange rates. This is so because exchange rate variability reduces the price risk effect of shocks to foreign prices while it increases the importance due to domestic supply shocks.

The latter, as we have seen, tend to exert downward pressure on contract wages by reinforcing the variability of the utility of a representative risk averse union member. In this sense, the union's utility becomes more risky.

4.3.2 Structural features of the labour markets

The above analysis demonstrates that more competitive labour pricing, represented formally by increases in the bargaining power of firms' (Θ), tends to enhance the impact effect of various shocks on the price risk. Depending on the ultimate source of aggregate price variability, more competitive labour pricing increases or decreases steady state employment and output, as the expected real contract wage falls or rises via the price risk effect. But more competitive labour pricing also impinges on the steady state of the economy through the mark-up structure of wage, represented by the constant c in the wage formula (4.3). In the absence of aggregate prices, the effects of union wage formation or of the structure of labour markets on the aggregate economy are completely characterized by the mark-up, and, of course, the determinants of the threat point u*.

In an economic environment where no aggregate price risk exists or is indexed away, more competitive labour pricing always enhances aggregate employment and output, since c is decreasing in firms' bargaining power Θ. But when uncertainty due to aggregate price variability is accounted for in wage formation, this need not be the case. One is then naturally led to analyze the other extreme labour market model, i.e. the pure monopoly union model, where the monopoly union imposes its desired wage on firms, conditional on firms' employment decisions.

Andersen & Sørensen (1988) have employed the pure monopoly union model in their analysis of wage formation and exchange rate variability. They employ non-indexed wage contracts and allow for endogenous terms-of-trade changes. So we can check for the compatibility of our model with theirs, and compare the conclusions from the models, given that we are much more explicit about the sources of aggregate uncertainty.

We can formally represent the wage demands of the pure monopoly union by imposing the condition $\Theta = 0$ in the wage formula (4.3). This produces a price risk term of the form

$$V(\Sigma; \Theta = 0) = (1 - b)\{(1 - b)[(1 - \tau) - 2\beta]\sigma_p^2 - 2\beta\sigma_{\mu p}\}, \qquad (4.22)$$

whereas the mark-up factor c is given by

$$c = (1 - \tau)^{-1}\log\frac{\beta}{[\beta - (1 - \tau)]}. \qquad (4.23)$$

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To trace out the implications of more competitive wage setting across exchange rate regimes, we refer to tables 4.1a and b, where we tabulate the mark-up factor c and price risk under fixed and flexible exchange rates (c^f, c^e and V^f and V^e, respectively) with the maintained hypothesis of non-indexed and partially indexed wage contracts, equal size variances ($\sigma_x^2 = 1$ for all x = 1,2,u and μ) and given technological and preference parameter values.

Table 4.1a   **Labour market competitiveness and wage setting under non-indexed contracts (b=0)**

Θ	c^f-c^o	c^e-c^o	V^f-V^o	V^e-V^o
.5	-.04	-.04	-2.33	-.55
1	-.06	-.06	-5.16	-1.11
2	-.09	-.09	-9.82	-2.21
5	-.11	-.11	-23.80	-5.53

We have set τ=.5=k, α=.7. The superscript ^{o} denotes the case of a pure monopoly union, i.e. Θ=0.

Table 4.1b   **Labour market competitiveness and wage setting under partially indexed contracts (b=.5)**

Θ	c^f-c^o	c^e-c^o	V^f-V^o	V^e-V^o
.5	-.04	-.04	-1.17	.01
1	-.06	-.06	-2.58	.03
2	-.09	-.09	-4.91	.06
5	-.11	-.11	-11.90	.14

See table 4.1a for parameter values.

Given equal variances, the price risk term is negative. From table 4.1a we can see that under non-indexed wages more competitive labour pricing tends to increase the price risk effect on wages to the extent that the gains from a lower mark-up are dominated by losses due to aggregate price variability. Thus if pricing aggregate price uncertainty is highly relevant in wage formation, there may be no gains in terms of lower wages to be expected from more competitive labour pricing. Notice that the assumption of equal variances for the shocks is not critical for this result, but that of non-indexed contracts is. The source of price variability becomes relevant once we compare fixed and

flexible exchange rates, as argued earlier. Thus it is the comparison between the last two columns in Table 4.1a which is affected by the source of aggregate price variability.

Table 4.1b captures in an elementary fashion the effects of indexed contracts on wage formation. The results are interesting, at least as far as flexible exchange rates are concerned (last column in table 4.1b). If fixed exchange rates are accompanied by (partially) indexed contracts, more competitive wage formation need not bring any gains in terms of lower wages; this is the case if aggregate price variability due to foreign prices is high enough. But the introduction of flexible exchange rates changes the picture. From the last column of Table 4.1b we can see that the price risk effect on wages under flexible exchange rates falls relative to the pure monopoly case as labour pricing becomes more competitive. Intuitively, the reason for this is that higher indexation increases (in absolute terms) the covariability of the price level and supply shock. It also increase the variability of the price level, but the effects of price level variability, as measured by σ^2_p, on the price risk are reduced by the higher degree of wage indexation. In effect, higher indexation enhances the price risk effect due to supply shocks, while it reduces the price risk effect due to price level variability *per se*.

The results in Table 4.1b are yet another instance where the distinction between price level variability *per se* and the covariability of the price level between different sources of exogenous shocks, so relevant for the optimal wage indexation schemes advocated since the original Gray—Fischer analysis, is relevant. It is relevant because of the signal extraction problem faced by economic agents in the presence of uncertainty about the sources of variability in observable variables. What is of interest in the present context is that a higher degree of wage indexation *per se* works exactly along these lines; once we investigate the components of the price risk, we observe that higher wage indexation tends to reinforce the effects on the price risk of exogenous stochastic signals; signals which indicate exogenous changes in employment and output opportunities.

4.3.3 Variability versus level; policy makers' objectives and optimality of a single instrument

The final theme taken up here concerns the effects on the steady state output of stabilization policies which the monetary authority conducts by employing specific monetary feedback rules. In the standard models of chapter 2, and generally in models incorporating the natural rate, stabilization policies designed to stabilize aggregate real fluctuations generated by imperfect information are neutral in terms of their steady state effects. This conveys the impression that stabilization policies can be exercised without repercussions on the steady state level of real variables. The calculation e.g. in Lucas (1985) belongs to this tradition. McCallum (1986, p. 412), in commenting on Lucas's business cycle calculation, finds the assumption of a natural rate attractive, but hastens to emphasize that it need not be entirely innocuous in the context of the discussion in Lucas. As McCallum further notes (ibid., p. 412), the assumption is a highly substantive one, not merely a matter of convention or terminology.[20]

The present model clearly displays the absence of the natural rate. It is thus of interest to trace out the effects stabilization policy, in the form of an optimal policy mix of the foreign exchange intervention rule and wage indexation, has on the steady state equilibrium of the economy. To analyze these effects on the steady state, we need to specify the objectives of the monetary authority.[21] Assigning objectives to the monetary authority in a macroeconomic context involves a great deal of ad hockery, but we shall consider the following objectives for deriving an optimal policy mix of foreign exchange intervention and wage indexation; δ and b are chosen so as to maximize H

$$H = (\alpha\beta)^{-1}d_1E[y] - \tfrac{1}{2}(\alpha\beta)^{-2}d_2E\{[y - y^f]^2\}, \qquad (4.24)$$

where d_i, i=1,2, are weights in the preferences of the monetary authority attached to the steady state effects of the policy mix (d_1) and to the inefficiencies due to imperfect information (d_2). The logic of

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[20] Note that the assumption of the natural rate is separate from the postulate of rational expectations. Note further that "all pervasive market clearing" is not at issue here, but rather whether this clearing pertains to auction type markets, or to ones with nominal contracts that have allocational effects (see McCallum, ibid. p. 412).

[21] Thus implicitly postulating that the monetary authority sets the degree of wage indexation.

equation (4.24) is that the optimal policy mix is chosen to maximize the steady state aggregate output (and employment). Costs are probably incurred as a result of such a policy mix in terms of aggravating inefficiencies due to imperfect information, i.e. due to the fact that labour contracts are signed before uncertainty is resolved. The second term in equation (4.24) measures this loss, which is familiar from the standard framework (see chapter 2), where y^f denotes the full information aggregate output, i.e. aggregate output resulting from bargaining under perfect information about the underlying disturbances. The normalization factor ($\alpha\beta$) is made purely for convenience.

Now, we have to solve for the full information output. The static Nash bargaining solution under perfect information corresponding to equation (3.15) is, after appropriate manipulations, given by the following real wage formula:

$$w_t - p_t = u^* + c, \qquad (4.25)$$

where w denotes the (log of the) nominal wage, p the (log of) the price level and u^* and c are as before (cf. equation 4.3). From equaiton (4.25) we can immediately see that real wages are a mark-up over union's threat point or threat point alternative(s), and that lower real wages result in more competitive labour market institutions (i.e. as Θ icreases). Equation (4.25) implies that (the log of) employment under perfect information is given by

$$n_t = -\beta[u^* + c] + \beta\mu_t, \qquad (4.26)$$

which gives rise to the following aggregate output equation under perfect information

$$y_t^f = a - \alpha\beta[u^* + c] + \beta\mu_t. \qquad (4.27)$$

Upon comparing this to equation (4.10), which describes actual aggregate output, we note that two factors give rise to differences between the two. First, full information output (4.27) is independent of price level surprises. In equation (4.10) the positive correlation between actual output and the price level is generated by imperfect information, which is absent from equation (4.27) because of spot bargaining. Second, the steady state full information output corresponding to equation (4.27) is given by $a - \alpha\beta[u^* + c]$, which differs from the one corresponding to equation (4.10) by factors

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directly related to aggregate uncertainty, i.e. by the price risk V (cf. equation 4.18).

From equations (4.10) and (4.27) we obtain the following expression for the difference between actual and full information output

$$y - y^f = \alpha\beta(1 - b)(p - p_{t-1,t}) + \tfrac{1}{2}\alpha\beta(1 - b)V(\Sigma), \qquad (4.28)$$

where we have dropped the time subscript for simplicity. From equation (4.28) we can immediately verify the interesting fact that for fully indexed wages, actual aggregate output equals full information output, in sharp contrast to the prediction of the standard model in chapter 2. The intuition here is that spot or full information bargaining produces constant real wages whereas in the model of chapter 2 full information equilibrium real wages are affected by stochastic shifts in productivity.

The objective function H can now be written in the following form

$$H = B + \tfrac{1}{2}d_1(1 - b)V - \tfrac{1}{2}d_2(1 - b)^2\{\sigma_p^2 + \tfrac{1}{4}V^2\}, \qquad (4.24')$$

where B is a constant excluding parameters δ and b. In what follows we shall restrict our attention to only one of the instruments, δ or b, in achieving the prescribed objectives given in equation (4.24). We start with the degree of foreign exchange market intervention.

Optimal exchange rate variability given the degree of wage indexation

Fix the degree of wage indexation b, and differentiate (4.24') with respec to δ. After manipulation, the necessary first order conditions for the optimum can be written in the following form:

$$[d_1 - \tfrac{1}{2}d_2(1 - b)V]\frac{\partial V}{\partial \delta} = d_2(1 - b)\frac{\partial \sigma_p^2}{\partial \delta}. \qquad (4.29)$$

This equation implicitly defines the optimal degree of foreign exchange intervention, $\delta^* = \delta^*(b,\Theta;\Sigma)$, say, in order to emphasize its dependence on the degree of wage indexation, the bargaining power of

the firms and the variance structure of the disturbances.[22].We shall not, however, employ this implicit relationship explicitly. Rather, we note that according to equation (4.29) the optimal degree of intervention is such that its effects on the aggregate price risk, when properly weighted, should equal its effects on aggregate price level variability. This is interesting, since intervention affects the price risk through price level variability and the correlation between the price level and productivity shocks.

Now, we know, for example from equation (4.17), that, for given b,

$$\frac{\partial[w^c - p_{t-1,t}]}{\partial\delta} = -\tfrac{1}{2}(1-b)\frac{\partial V}{\partial\delta}, \tag{4.30}$$

so that, under the restriction that $[d_1 - \tfrac{1}{2}d_2(1-b)V]$ does not vanish,[23] we obtain

$$\frac{\partial[w^c - p_{t-1,t}]}{\partial\delta} = \frac{-\tfrac{1}{2}d_2(1-b)^2}{[d_1 - \tfrac{1}{2}d_2(1-b)V]}[\frac{\partial\sigma_p^2}{\partial\delta}]. \tag{4.30'}$$

A priori, the effects of optimal foreign exchange intervention on expected real wages, and thus on steady state output and employment, are ambiguous. Apart from the degree of wage indexation, these effects depend on the weights attached to the different objectives in the monetary authority's preferences (d_i), the sign of the aggregate price risk induced by the optimal degree of intervention and the policy-induced change in aggregate price level variability. There is, however, one case where we can compute unambiguous effects. This case emerges if we assume that no weight in the monetary authority's preferences is given to the loss due to imperfect information, i.e. $d_2 = 0$. From equation (4.24') it follows that in this case the optimal degree of foreign exchange intervention obeys

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[22] It is assumed that the relevant second order conditions for the maximum are satisfied. Equation (4.29) produces, for a given b, a third order polynomial in δ, whose roots can in principle be solved and checked for the maximizing value. We will not follow this route, but will employ equation (4.29) in analyzing comparative statics.

[23] This places implicit restrictions on the behaviour of the price risk and/or weights d_i and b at the optimum. However, were this restriction to hold, we can see from equation (4.29) that in this case the optimum essentially requires that $[\partial\sigma_p^2/\partial\delta] = 0$, unless of course wages are fully indexed at the corresponding optimum.

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$$\frac{\partial V}{\partial \delta} = 0, \tag{4.29'}$$

or in terms of equation (4.16)

$$(1-b)\alpha_1 \frac{\partial \sigma_p^2}{\partial \delta} = \alpha_2 \frac{\partial \sigma_{p\mu}}{\partial \delta}. \tag{4.29''}$$

When solved for the optimal degree of foreign exchange intervention, we obtain

$$\delta^* = -k + \frac{[(1-b)(\Theta - \tau) + (1+\Theta)(1-\beta(1-b))]\beta^2\sigma_\mu^2 + \alpha_1\sigma_m^2}{\alpha_1[1+\alpha\beta(1-b)]\sigma_1^2 - (1+\Theta)\beta^2\sigma_\mu^2}, \tag{4.31}$$

where $\sigma_m^2 = [k^2\sigma_2^2 + \sigma_u^2]$.

From equation (4.31) the following comparative statics results[24] can be derived:

$$\frac{\partial \delta^*}{\partial \sigma_x^2} > 0, \text{ for } x = 2,u$$

$$\frac{\partial \delta^*}{\partial \sigma_x^2} <> 0, \text{ for } x = 1,\mu$$

$$\frac{\partial \delta^*}{\partial b} <> 0$$

$$\frac{\partial \delta^*}{\partial \Theta} < 0.$$

So, exchange rate variability due to optimal intervention decreases as shocks to money demand or foreign interest rates become more pronounced. Increased random variability in foreign prices or domestic productivity can either increase or reduce exchange rate variability

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[24] It can be shown the the relevant second order condition for the maximum requires that the denominator be negative, which is satisfied, since $\alpha_1 < 0$.

due to optimal adjustment in the degree of intervention. The intuition here relies on a simple application of the signal extraction principle determining the comparative statics behaviour of the degree of intervention. Consider of first the foreign interest and domestic money market shocks. If no other random shocks impinge on the economy, fixed rates are optimal. Adding other sources of random variability implies that the optimal degree of intervention is finite. A change in the distribution of foreign interest rates and domestic money disturbances to display more variability reinforces the signal in the observed price level and exchange rate variability, so that the optimal response to this increased source of variability is to increase the degree of intervention. By implication, the implied optimal intervention policy thus reduces the price risk effects of the price level variability due to these shocks, and thereby induces a fall in expected real wages. This fall in wages enhances steady state output and employment.

Similarly, the optimal intervention rule in the presence of random shocks in foreign prices only would be $\delta^* = -k$. The presence of other random shocks marks a departure from this simple intervention rule, either downwards ($\delta^* < -k$) or upwards ($\delta^* > -k$). If the former prevails, for example, because the price risk is dominated by price level variability, then a more pronounced foreign price signal increases the optimal degree of intervention towards $-k$, while if the latter prevails, δ^* falls towards $-k$. Analogous qualitative reasoning reveals that exchange rate variability induced by the adjustment in the degree of intervention can either increase or decrease as a result of to increased random variability of productivity, depending to a large extent on the channel through which productivity shocks mainly affect the aggregate price risk. Note that exchange rate variability due to optimal adjustment in intervention is likely to increase more the more indexed labour contracts are, reflecting the fact that the insensitiveness of the aggregate price risk to price level variability increases with indexation.

Finally, the last two comparative statics derivatives give us the change in the optimal degree of foreign exchange intervention for given (marginal) changes in the degree of indexation and firms' bargaining power. Changes in the degree of indexation have, in general, ambiguous effects on the degree of optimal intervention. To understand why, note first that in the absence of random shocks to foreign prices, the relationship between indexation and intervention is monotonic so that increases in the degree of indexation will induce a reduction in the optimal degree of intervention. If the distribution of foreign prices displays sufficiently little variability, this result still roughly holds. But if foreign prices are highly variable (high σ_1^2),

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then, once we take into account the fact that δ^* now lies in the neighbourhood of -k, it is possible that increased indexation induces an increase in δ^* for small values of the indexation coefficient. For sufficiently high degrees of indexation, however, there will once again be a positive relationship between exchange rate variability due to optimal intervention and wage rigidity (indexation). Note, however, that, overall, increased indexation will reduce the effects of the aggregate price risk on expected real wages, so that it becomes increasingly difficult for the monetary authority to influence the steady state as indexation continuously increases.

That more competitive labour pricing implies more exchange rate variability due to optimal adjustment in the degree of intervention is interesting, but this result clearly hinges upon the assumption that there are stochastic shifts in the production possibilities (i.e. the distribution of μ is non-degenerate). From equation (4.31) we can immediately deduce that in the absence of stochastic shocks to productivity, δ^* is independent of the bargaining power parameter Θ. In this case, the labour demand schedule is non-stochastic and the problem faced by the monetary authority is to minimize price level variability due to shocks to foreign prices, interest rates and domestic money demand. Even though the price level, and thus price risk, is minimized in this case, the monetary authority can do nothing to counteract the fact that contract wages increase as labour pricing becomes more competitive.

Thus there is a non-trivial relationship between intervention and labour market structure (competitiveness) only when there are stochastic shifts to production possibilities, and this relationship largely results from the fact that price level variability now contains information about stochastic shifts in production possibilities,[25] represented by the covariance term $\sigma_{p\mu}$ in the price risk V. Since the relative significance of this covariance term for the behaviour of the price risk, and thus for expected real wages and steady state employment and output, increases with more competitive labour pricing, the monetary authority can counteract the positive effects of price level variability on expected real wages by increasing exchange rate variability (i.e. reducing δ^*), since more flexibility in exchange rates will increase the economic significance of shocks to productivity. This explains this particular comparative statics result.[26]

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[25] And thus about the shift in the constraint facing the monopoly union.

[26] This distinction between productivity and other shocks resembles the one drawn in standard models with competitive labour market institutions (cf. chapter 2). The reason is, of course, that these shocks induce changes in the set of feasible wage-employment pairs, while other shocks can be analyzed within a fixed feasibility set.

As noted above, a tractable closed-form solution to the optimal degree of intervention corresponding to the unrestricted objective function is not readily available, so we resort to simulation and visual characterization. These give us a more concrete, albeit more restricted, idea of the behaviour of the optimal intervention rule. Tables 4.2a - b cross-tabulate δ^* against the degree of indexation b and bargaining power Θ for some parameter values. The maximizing value of δ corresponding to the objective function (4.24) is generated by a step search procedure, whereby the relevant maximum is sought in a stepwise, hill-climbing manner. The tables differ with respect to the assumptions concerning the relative strength of the variability of various shocks and weights attached to the different objectives in the monetary authority's preferences. Table 4.2a displays the case where foreign interest and domestic money demand shocks are dominant. The column labelled (1) roughly represents the case just analyzed ($d_1=1$, $d_2=0.001$), while column (2) gives δ^* in the case where the minimization of losses due to imperfect information is dominant ($d_1=0.001$, $d_2=1$). Table 4.2b reproduces 4.2a under the maintained hypothesis that shocks to foreign prices and domestic productivity are dominant. Figure 4.1a, on the other hand, once again visually relates δ^* to b and θ in three dimensions, while figure 4.1b and c relate δ^* directly to b for low and high θ, i.e for highly monopolized and competitive wage setting.[27]

Table 4.2a **Optimal degree of foreign exchange intervention given b and Θ**

b						
	0		.4		.8	
	1	2	1	2	1	2
Θ						
0	2.94	6.24	2.55	8.88	0.50	11.10
1	2.63	5.76	2.16	8.51	0.24	13.19
5	2.33	4.34	1.87	7.32	0.06	16.33

Parameter values; $\sigma_z^2 = \sigma_u^2 = 0.1$, $\sigma_1^2 = \sigma_\mu^2 = 0.01$, $\alpha = 0.7$, $\tau = 0.5$, $k = 0.5$, $(d_1,d_2) = (1,0.001)$ in (a) and $(d_1,d_2) = (0.001,1)$ in (b).

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[27] The parameter values in figure 4.1a - c are slightly different from those in tables 4.2 in that the weights in the objective function are equal ($d_i = 1$), as are the various variances.

Table 4.2b	Optimal degree of foreign exchange intervention given b and Θ

b						
	0		.4		.8	
	1	2	1	2	1	2
Θ						
0	0.53	3.03	0.26	4.90	-0.68	10.43
1	0.33	3.78	0.03	6.25	-0.81	15.15
5	0.17	4.68	-0.13	7.70	-0.91	21.64

Parameter values; see table 4.2a, except that

$$\sigma_2^2 = \sigma_u^2 = 0.01, \quad \sigma_1^2 = \sigma_\mu^2 = 0.1.$$

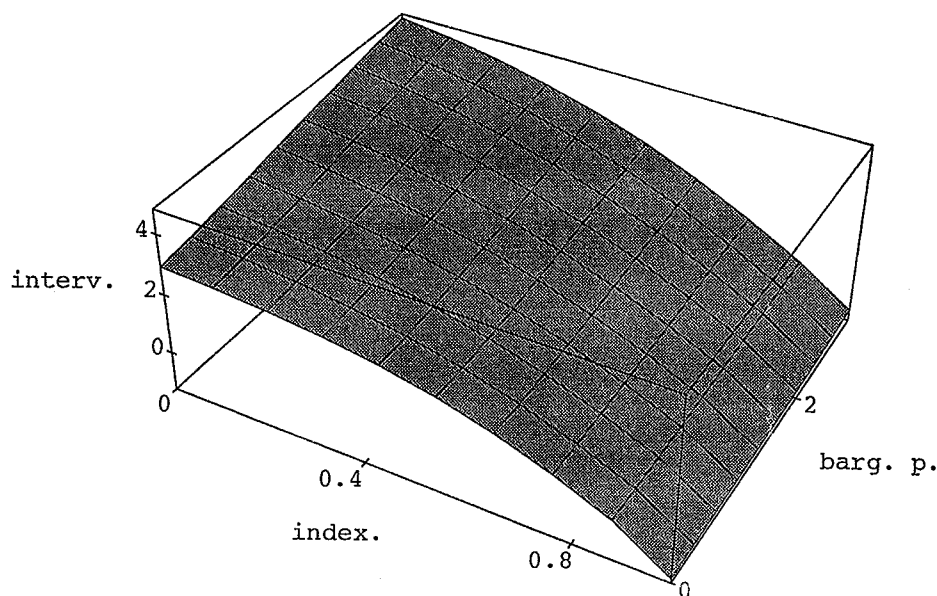**Figure 4.1a** **Optimal degree of intervention for a given degree of indexation and bargaining power of firms**

Figure 4.1b   **Intervention and bargaining power of firms**

interv.

4

2

b low

b high

barg. p.

2
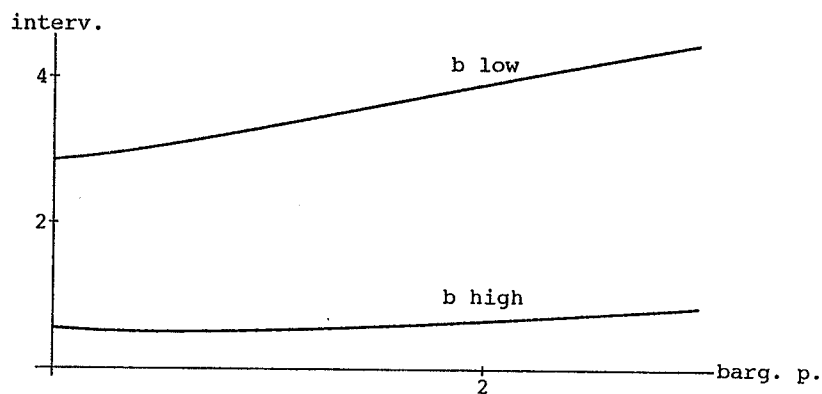
The curves are indexed by the degree of indexation.

Figure 4.1c   **Intervention and indexation**
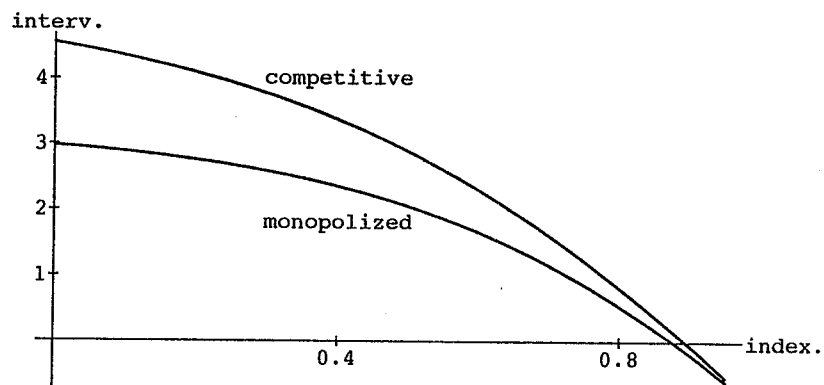
interv.

4

3

2

1

competitive

monopolized

0.4 0.8 index.

The curves are indexed by the degree of competitiveness
of the labour markets.

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What is of great interest in these two tables is the difference between columns (1) and (2), corresponding to output maximization and minimization of losses due to imperfect information. The former generally favours more, and the latter less, variable exchange rates induced by intervention. The intuition here is that as far as output maximization is concerned, exchange rate variability provides information about stochastic shifts in production possibilities. This source of random variability exerts downward pressure on contract wages, and thus provides a channel for the monetary authority to boost steady state output and employment by generating sufficient variability in exchange rates.

That imperfect information tends to favour more fixed exchange rates follows directly from the model structure in the sense that the model's full information real wage corresponding to spot bargaining is constant.[28] Thus divergences between the actual and full information output are driven by price level surpises and the aggregate price risk. This implies that to minimize costs due to imperfect information, intervention policy should be so designed as to minimize both price level variability and (absolute) covariability between the price level and domestic productivity shocks. But because of the assumption of continuous PPP, it is clear that, apart from shocks to foreign prices, random disturbances can only generate price level variability via exchange rate variability. This implies that if the distribution of foreign prices is degenerate, a fixed exchange rate system will eliminate all losses due to imperfect information irrespective of the values of b and Θ.[29] If, on the other hand, only shocks to foreign prices generate (aggregate) random variability, the optimum $\delta^* = -k$ once again emerges from the model, again independently of b and Θ.

So, "interesting" intervention schedules can be generated in cases where $d_1 = 0$ when domestic productivity shocks μ occur together with shocks to foreign prices. Such a (representative) possibility is shown in table 4.2b, column (2). According to it, exchange rate variability falls due to optimal adjustment in intervention as wage indexation increases and as labour pricing becomes more competitive. This is so despite the fact that the effects of shocks to foreign prices on price level variability increase as the degree of intervention increases due to

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[28] As long as the union's threat point is non-stochastic, which is our assumption.

[29] It is, of course, quite clear that from the point of view of minimization of losses due to imperfect information the choice of the exchange rate system via intervention is inconsequential if wages are fully indexed.

reduced flexibility of exchange rates and continuous PPP. But since indexation reinforces exchange rate and thus price level variability due to domestic productivity shocks, the monetary authority optimally responds to increased indexation by reducing exchange rate variability to eliminate this source of random variability. Note further that increased indexation generally reduces the effects of shocks to foreign prices on the price risk and that contract wages become less responsive to any given aggregate price risk as indexation increases.[30]

Qualitatively, the same reasoning applies to increases in firms' bargaining power Θ. So, the "interesting" case is again that where domestic productivity shocks occur together with foreign price shocks. As for the loss due to imperfect information and the aggregate price risk, the relative importance of shocks to domestic productivity increases as labour pricing becomes more competitive.[31,32] Thus, once again optimal intervention policy seeks to minimize losses due to domestic productivity shocks. Note, also, that the reduction in exchange rate variability due to optimal adjustment in intervention as Θ rises is more pronounced for wage contracts with a higher degree of indexation, since increased indexation typically reduces the effects on aggregate price risk of shocks affecting only price level variability (i.e. affecting only σ_p^2).

Optimal degree of wage indexation for a given degree of foreign exchange intervention

We shall now treat the degree of foreign exchange intervention as exogenously given. This implies that the currently prevailing exchange rate regime is exogenously chosen with no necessary reference to the basic optimization procedure outlined above. We follow the procedure adhered to above, i.e. we check for the optimal degree of wage indexation in cases where only output maximization and loss

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[30] It is possible that, for a sufficiently low degree of indexation and intervention, a marginal increase in the degree of wage indexation actually reinforces the price risk effects of foreign price shocks.

[31] It is not entirely clear how robust this feature of the model is, since the Cobb—Douglas technology implies that the derived demand for labour is a convex function of the real wage.

[32] This case is the only one where more than one local extremum emerged. The one tabulated corresponds to a global extremum. This supports the argument that it is domestic productivity shocks which ultimately determine the nature of the optimal intervention rule.

minimization are concerned, since in general the optimal degree of wage indexation is a combination corresponding to these extremes.

We start with the assumption that $d_1 = 0$, so that only minimization of losses due to imperfect information counts in the monetary authority's preferences. From equation (4.24) we can immediately deduce that, no matter what the actual exchange rate regime and degree of competitiveness of labour pricing are, the optimal degree of wage indexation corresponds to fully rigid real wages, i.e. $b^* = 1$, if we let b^* denote the optimum. No matter what the exchange regime or firms' bargaining power are, divergences between full information and actual real wages occur as a result of price level surprises and the price risk if wages are only partially indexed. This result follows directly from a specific feature of the labour market model: that is, that the full information equilibrium real wages corresponding to spot Nash bargaining are constant or nonstochastic, or, what amounts to the same thing, full information nominal wages are fully indexed to the price level, and no indexing to specific shocks, notably productivity shocks, occurs.

That real wages should be completely rigid independently of the prevailing exchange rate regime or competitiveness of the labour markets is a very strong implication of the present model. Needless to say, the robustness of this result should be checked against changes in the underlying preferences of the union and the firm. It is not so much the assumption of a utilitarian union alone that is critical for the constancy of the full information real wage, but rather the assumed functional forms together with zero lock-out profits for the firm(s). Zero lock-out profits abstract from the interaction between the firm's minimum profit constraint and shocks to production possibilities, so that (stochastic) profits behave like a scale factor in the Nash maximand.

But taken as it is, complete real wage rigidity should be compared to the results derived earlier in chapter 2 within the open economy framework of optimal wage indexation and foreign exchange intervention. There, complete real wage rigidity results from an optimal policy either because of fixed exchange rates or the absence of domestic productivity shocks. The former obtains, since no information about changes in the full information real equilibrium of the economy is provided by the exchange rate, and thus by movements in the price level; under PPP only shocks to foreign prices generate movements in the domestic price level and their effects are eliminated by full wage indexation. The latter obtains, since in the absence of shocks to production possibilities, full information equilibrium real wages are non-stochastic, or constant in the particular

model of chapter 2. In the present context, full information equilibrium real wages are constant, so that in the corresponding full information equilibrium of the economy shocks to production possibilities are fully absorbed by stochastic shifts in employment.[33] In the contract economy, this adjustment is achieved by full wage indexation and letting employment adjust to shocks to production possibilities.

If, on the other hand, we set $d_2 = 0$, so that only steady state output maximization matters, we can immediately conclude that fully rigid real wages are the optimum, if there are no stochastic shocks to domestic productivity. This is because in this case, under partial indexation, the aggregate price exerts upward pressure on expected real wages. The effect of the price risk on wages can be eliminated, and thus the steady state output maximized, by full indexation of wages.

The general case, with stochastic shifts in production possibilities, can be derived by differentiating the objective function (4.24), subject to the restriction $d_2 = 0$, with respect to b and solving the implied first order condition for a maximum for b. This gives us

$$b^* = 1 + \frac{[1 + k + \delta]\beta^2(1 + \Theta)\sigma_\mu^2}{\{\alpha_1[(k + \delta)^2\sigma_1^2 + \sigma_m^2 + \beta^2\sigma_\mu^2] + \alpha\beta(1 + \Theta)\beta^2\sigma_\mu^2\}}, \qquad (4.32)$$

which is valid for values of δ not equal to $-(1+k)$. From equation (4.32) we can immediately deduce that $b^* \to 1$ as $\delta \to \infty$, i.e. with fixed exchange rates, wages will be fully indexed. This is intuitively obvious, since fixed rates neutralize all shocks excluding foreign price shocks. Since foreign price shocks exert upward pressure on wages via aggregate price risk, full wage indexation will neutralize this additional source of variability, so that steady state output (and employment) is maximized. If, on the other hand, we let δ approach $-(1+k)$, we can immediately see from equation (4.32) that once again $b^* \to 1$, so that the relationship between the degree of intervention and optimal wage indexation is non-monotonic in the range from $-(1+k)$ to fixed rates. To be more precise, the following behaviour of the optimal degree of wage indexation in the neighbourhood of $\delta=0$ can be derived

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[33] Assuming, of course, that these stochastic shifts in employment occur in the interior of the set of feasible solutions of our labour demand model, i.e. $N \le M$ with probability one.

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$$\text{sign} \frac{\partial b^*}{\partial \delta} \Big|_{\delta=0} = \text{sign} \{ \alpha_1 (\sigma_m^2 + \beta^2 \sigma_\mu^2)$$

$$+ \alpha\beta(1 + \Theta)\beta^2 \sigma_\mu^2 - \alpha_1 k(2 + k)\sigma_1^2 \}. \tag{4.33}$$

This implies that

$$\frac{\partial b^*}{\partial \delta} \Big|_{\delta=0} > 0, \text{ as } \sigma_1^2 \rightarrow \infty$$

$$< 0, \text{ as } \sigma_1^2 \rightarrow 0.$$

Once again, the comparative statics behaviour of b* can be explained by signalling. The appropriate question is then: from where does the price level variability originate? Answering this question amounts to imposing a particular structure the aggregate price risk. Thus, if, for example, shocks to foreign prices are sufficiently large (in terms of σ_1^2), from which it, follows that a proper intervention rule is close to δ=-k, a marginal reduction in exchange rate variability (increase in δ) due to intervention relative to flexible rates increases the optimal degree of indexation. This is because the reduced flexibility in exchange rates exchanges the effects of shocks to foreign prices on price level variability, and thus on aggregate price risk. In order not to increase expected real wages, the degree of indexation should increase.[34]

The degree of optimal wage indexation, given δ, falls as labour pricing becomes more competitive, i.e. $(\partial b^*/\partial \Theta) < 0$, a comparative statics result completely explained by the presence of stochastic productivity shocks. If the latter are absent, b* = 1 as explained above. If there are nontrivial stochastic shifts in production possibilities, these tend to be reinforced in more competitive labour market environments in a way that exerts downward pressure on expected real wages. The reduced degree of optimal wage indexation as firms' bargaining power increases simply means more leverage to domestic productivity shocks, thus enhancing increases in steady state output and employment.

In general, then, the optimal degree of wage indexation is some combination of the case analyzed above, corresponding to $d_1 = 0$ and $d_2 = 0$, respectively. The shape of the schedule describing the dependence of the optimal degree of wage indexation on the degree of foreign exchange intervention is mainly determined (apart from the

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[34] This comparative statics behaviour of the optimal degree of wage indexation qualitatively resembles that analyzed in chapter 2.

weighting implicitly defined by the choice of (d_1, d_2)) by the restriction $d_2 = 0$. In particular, $b^* = 1$, if $\delta = \infty$ or $\delta \rightarrow -(1+k)$ or there are no stochastic shocks to domestic productivity. To give a sense of concreteness, we simulate the optimal degree of wage indexation in table 4.2c, where we have cross-tabulated b^* against the degree of intervention (δ) and firms' bargaining power (Θ). Figure 4.2a, on the other hand, plots b^* against δ and θ in three dimensions, i.e. the information in table 4.2c, while figure 4.2b and c relates b^* directly to Θ and δ respectively. The curves in figure 4.26 are indexed by the degree of competitiveness of the labour markets, and by the degree of exchange rate flexibility in figure 4.2c.

Table 4.2c   **Optimal degree of wage indexation for given values of δ and Θ**

Θ	δ						
	-1	-0.5	0	0.5	1.0	2.0	4.0
0	0.90	0.70	0.69	0.62	0.58	0.57	0.66
1	0.88	0.74	0.62	0.53	0.49	0.49	0.60
3	0.86	0.70	0.57	0.47	0.43	0.44	0.57
5	0.85	0.69	0.55	0.45	0.40	0.42	0.56

Parameter values; see table 4.2a except that $\sigma_x^2 = 0.1$ for all $x=1,2,u,\mu$ and $d_1 = d_2 = 1$.

That the schedule describing the dependence of the optimal degree of wage indexation on the degree of intervention inherits its shape mainly from the case where $d_2 = 0$ is clearly apparent in table 4.2c. Increasing d_2 from $d_2 = 0$ results in an increase in b^* for each Θ and δ.[35]

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[35] An increase in τ, the degree of risk aversion of a representative union member, also increases b^*. This follows basically from the fact that the utility function of the union member, being essentially an indirect utility function, is a convex function of the price level, or that the marginal utility of income (wage) is a convex function of the price level. The effects on b^* of an increase in α, the elasticity of output with respect to labour, on the other hand, are twofold. First, b^* increases for low values of δ and the minimum degree of wage indexation, i.e. the turning point of the schedule relating δ and b^* occurs at higher values of δ. This is intuitively understandable, since the constraint associated with the Nash maximand, i.e. the labour demand schedule, allocates more variation to employment and less to wages as α increases.

Figure 4.2a   **Optimal degree of wage indexation for a given degree of intervention and bargaining power of firms (competitiveness)**
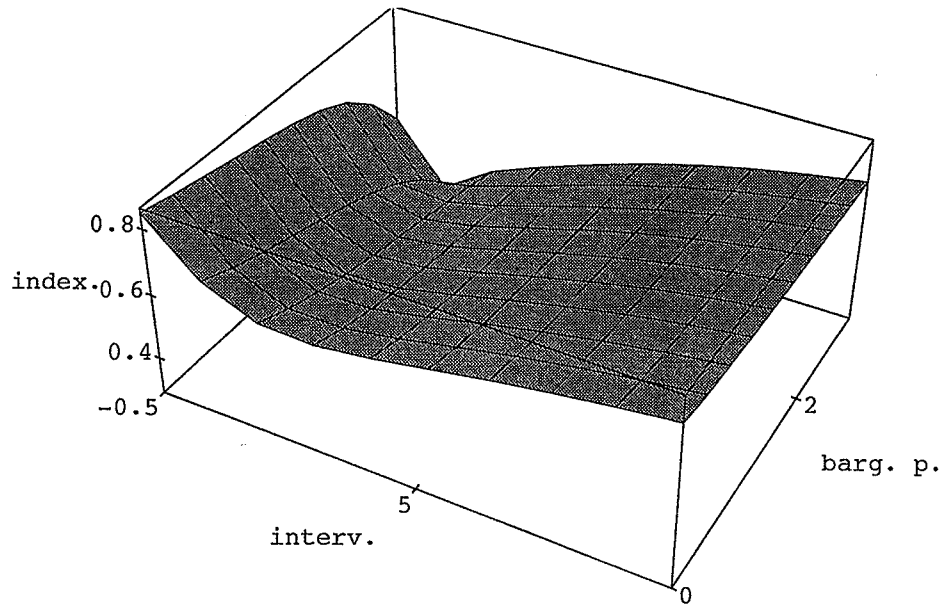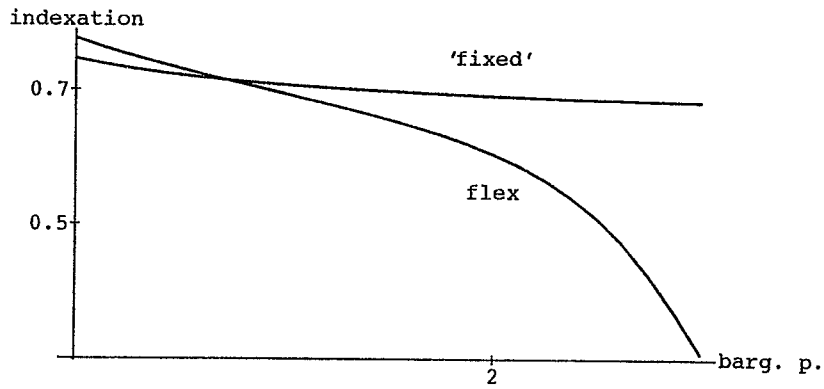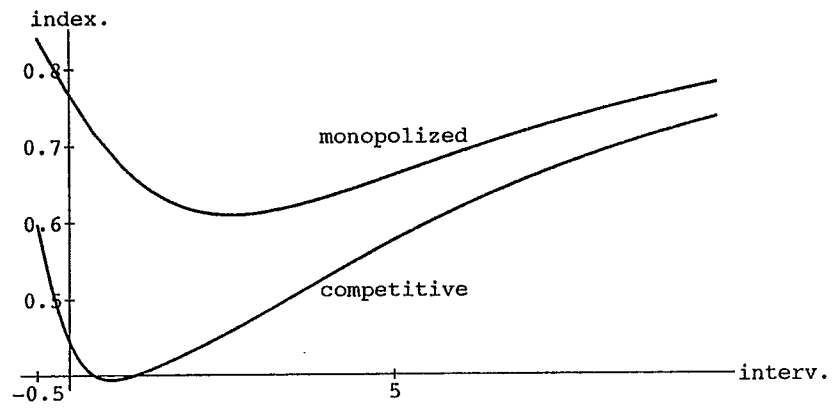

Figure 4.2b   **Wage indexation and bargaining power of firms**

The curves are indexed by the degree of intervention.

Figure 4.2c　　　　　**Wage indexation and intervention**

The curves are indexed by the degree of competitiveness
of the labour markets.

5 Summary and Discussion

There is no simple relationship between the degree of wage indexation and foreign exchange intervention or an exchange rate regime either in the more conventional Gray—Fischer models (chapter 2) or in the wage bargaining model proposed in chapters 3 and 4, even though the structure of the macromodel in which the wage formation process and intervention rule are embedded is very simple. There is, however, one common denominator, which greatly influences not only the openness of the economy as in section 2.3, but also the relationship between wage rigidity and the exchange rate regime in an economy subject to the simple wage indexation and intervention rules proposed above. This is the source of the exogenous shock impinging on the economy. The source of the exogenous shock is of course the factor so much emphasized by e.g. Gray in her original model of optimal wage indexation. Generalizing a little, the crucial distinction is between shocks affecting full information equilibrium real wages in the economy and other shocks. We have chosen the strategy, much favoured in the standard Gray—Fischer analysis, that models equilibrium real wages as being driven by exogenous productivity shocks basically, because this strategy most clearly displays the relevant distinctions involved. Being as it is, the modelling strategy leads to thinking of the policy mix of wage indexation and foreign exchange intervention as being a set of instruments dealing with the signal extraction problem facing agents in an economy subject to various exogenous shocks. The problem arises because of the insufficient number of instruments relative to the number of independent stochastic sources of variability. A further implication of this signal extraction approach is that the equality of the number of independent stochastic sources of variability and the number of policy instruments will sustain the macroeconomic equilibrium in the contract economy as a macroeconomic equilibrium in the corresponding full information equilibrium.

As far as the specific relationship between the optimal degree of wage indexation and foreign exchange intervention is concerned, not much of full generality can be asserted on the basis of the models employed, even admitting their simplicity. The structure of the optimal mix (b^*, δ^*) is deeply rooted in the signal extraction approach taken to derive the determination of the optimal mix. Certain weak theoretical predictions, however, seem to be suggested, especially when we come to consider the optimality of one instrument in sections 2.3 and 2.5. First, taking the degree of foreign exchange intervention as exogenously

determined and evaluating the behaviour of wages around complete fixity of exchange rates, we can conclude that the introduction of (minor) exchange rate flexibility ($\delta < \infty$) increases the flexibility of real wages, i.e. the optimal degree of wage indexation falls. The increase in real wage rigidity can be very sharp, depending somewhat critically on the behaviour of foreign prices. More precisely, if foreign prices are almost constant, the increase in real wage flexibility due to the introduction of exchange rate flexibility is very sharp, but further increases in exchange rate flexibility may actually lower real wage flexibility. If, on the other hand, foreign prices are highly volatile, i.e. shocks to foreign prices are big, real wage flexibility seems to increase rather monotonically with increases in exchange rate flexibility. Once again, however, there could be a sharp fall in real wage flexibility around the unconstrained optimum degree of foreign exchange intervention. Overall, then, we have a rather non-monotonic relationship between exchange rate and real wage flexibility.

Take now the other constrained case where the degree of wage indexation is exogenously determined. If real wage flexibility increases as a result of a fall in the degree of wage indexation, then this seems to produce a fall in exchange rate variability. This relationship always prevails in the neighbourhood of the unconstrained optimal degree of wage indexation. The optimal degree of intervention responds to increased real wage flexibility in this monotonic fashion, if shocks to foreign prices are small enough, but non-monotonicities emerge in cases where foreign prices are highly volatile. Thus, the model in section 2.3 gives rise to the possibility that, beyond some particular point, exchange rate variability due to optimal intervention policy starts (marginally) to increase with further increases in real wage flexibility. This possibility emerges because shocks to foreign prices are the dominant signal in observed price level variability, and intervention policy is adjusted accordingly.

How does openness of the economy (section 2.5) change this picture as regards the relationship between exchange rate variability and wage patterns?[1] Openness as such, or the structure of the economy, greatly affects the qualitative picture of the optimal policy mix or the optimality of a single instrument. This is not, perhaps, so surprising, taking into account the fact that adjustments in relative prices heavily influence aggregate adjustment in the economy. At the most general level, we must conclude from section 2.5 that monetary policy is in general not a perfect substitute for relative price changes from the point of view of aggregate

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[1] And by implication employment and output patterns.

adjustment. The model in section 2.5 is constructed so that relative price adjustment enhances macroeconomic stability, and once the economic relevance of relative price adjustment diminishes as a result of increased openness of the economy, monetary policy cannot fully compensate, in terms of efficiency, the cushioning effect produced by relative price changes.

Some weak predictions can once again be deduced from the two-sector model of section 2.5. The optimal policy mix is such that, with a sufficiently open economy to start with, intervention policy is likely to be one of 'leaning with the wind'. The increased exchange rate variability implied by the optimal policy mix is accompanied by increased real wage rigidity as the economy becomes more open. Further, the increased real wage rigidity for more open economies seems to suggest that the differences in efficiency losses under fixed and flexible exchange rates diminishes. The intuition behind this result is that, although flexible exchange rates increase the economic relevance of productivity shocks, this flexibility becomes of less consequence as regards aggregate adjustment as the economy becomes more open as a result of increased real wage rigidity. In terms of efficiency, the advantage of flexible exchange rates over fixed rates does not vanish for an economy represented by the model in section 2.5, since there is in general some real wage adjustment under flexible exchange rates.

Finally, increased exchange rate variability is in general implied by the optimal intervention policy to lead to increased real wage rigidity, a general feature somewhat dependent upon the degree of openness of the economy.

Extension of the basic Gray—Fischer framework to an open economy subject to endogenous terms-of-trade changes has traditionally proved to be less successful due to the fact that in such an economy labour demand and supply decisions are generally dependent on different real wage measures, the producer's and worker's or consumption real wage, respectively, so that equilibrium aggregate output depends on the terms of trade. Under conventional wage indexation rules, whereby wages are indexed to producer or consumer prices, the closed form solution to the optimum posed by Gray is not available. The analysis of the optimal policy mix is typically reduced to the derivation of the optimal combination subject to the presence of only a subset of the original exogenous disturbances.

Section 2.4 argues that the Gray—Fischer framework can readily be extended to this case by indexing wages to a price index "relevant for the labour markets as a whole", meaning that we should search for a price index summarizing price level information relevant for both labour demand and supply. This price index is derived from the labour market

spot equilibrium condition, the index being a weighted average of producer prices and foreign prices in domestic currency. The weight for the producer's price is a function of the elasticities of labour demand and supply schedules and the weight given to producer prices in the consumer price index. This index was called the "optimum weight index" and was seen to be the consumer price index corrected for terms of trade. It was suggested that this index be used to index wages, which implies that the formal framework of section 2.3 for deriving the optimal policy mix can be invoked. For detailed analyses based on this approach the reader is referred to Devereux (ibid.) and Vilmunen (ibid.).

Building upon a partial equilibrium model of a utilitarian union and competitive Cobb—Douglas profit-maximizing firms, we derived in chapter 3 an equation for the contract wage as a solution to a static asymmetric bargaining game under uncertainty. The implied solution has a very interesting structure. First, the solution wage displays a clear mean-variance structure, which the contract wage depends not only on the (union's fall out utility, mark-up factor and) expected price level, but also on aggregate price variability. Aggregate uncertainty thus impinges on contract wages through the variance-covariance structure of the random variables, a term generically dubbed aggregate price risk. Because of the special functional form of the utility function of the utilitarian union and the maximand, output prices and stochastic productivity shocks impinge on the contract wage only through their covariability with consumer prices. Consumer prices are relevant for the utility of a representative union member, whose preferences are represented by a CRRA (indirect) utility function.

Secondly, the degree of wage indexation greatly influences the degree to which aggregate price risk is priced in *ex ante* contract wages. With fully indexed wages, aggregate price risk has no effect on expected real contract wages, which now equal real wages derived from a solution to a spot or full information Nash bargaining game. According to the latter, a simple wage formula is implied in which real wages equal a (nonstochastic) mark-up over the union's fall out utility. Thus with only partially indexed wages, an additional "mark-up" over the union's fall out utility is generated by the presence of aggregate (price) risk. An additional feature that deserves mention at this point is that there is now a direct link between the contract wage and the degree of wage indexation, although this link is only partial and not fully captured by our solution strategy, since we have not taken the degree of wage indexation to be a variable subject to bargaining. However, this dependence on the degree of wage indexation of the contract wage, intuitive as it is, is fully absent from the standard macromodel of wage indexation.

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Thirdly, firms' bargaining power greatly influences the degree of aggregate uncertainty and its effects on contract wages. As for the corresponding full information bargaining solution, firms' bargaining power influences the mark-up factor to the extent that the mark-up factor shrinks as firms become more powerful in wage setting. Thus, uncertainty introduces an additional channel through which firms are able to influence wage setting, though it is constrained by the degree of wage indexation. So, a partially characterized dependence between wage indexation and firms' bargaining power is displayed by our wage equation. As for the specific dependence of the contract wage on the firms' bargaining power, we may note that more competitive labour pricing, in the sense of increasing firms' bargaining power, tends to exert upward pressure on contract wages because of positive covariability of product and consumer prices and downward pressure because of negative covariability of consumer prices and productivity shocks. Therefore, the net result is ambiguous.

From a partial equilibrium point of view, the effects on the contract wage of exogenous uncertainty can be analyzed at a fairly general level. In the labour market model of chapter 3, the effects of exogenous uncertainty were summarized by the distribution of the vector random variable $(p,q,\mu)'$, i.e. the distribution of the log of product and consumer prices and productivity shocks. Of course, in a general equilibrium of the economy incorporating our unionized labour markets, this distribution is dependent upon the distribution of the exogenous random variables impinging on the economy as well as upon prevailing policy regimes (i.e. wage indexation, exchange rate regime etc.). To derive and characterize this dependence, a general equilibirum structure, or an approximation to it, is called for. This is the theme taken up in chapter 4, where a small open economy macroframework incorporating the bargaining game of chapter 3 in the labour markets is employed. In an attempt to scrutinize the macroeconomic structure of the aggregate price risk and its effects on wage formation, chapter 4 also sets out to analyze how aggregate uncertainty feeds back to the steady state equilibrium of the aggregate economy, notably output and employment. To maintain simplicity and comparability, the small open economy macromodel of chapter 2 is once again employed in chapter 4, but is now modified to incorporate the labour market model of chapter 3.

Whereas the standard Gray—Fischer analysis of wage indexation and foreign exchange intervention focuses on the effects of indexation and intervention on aggregate fluctuations around a fixed nonstochastic steady state, the analysis in chapter 4 builds explicitly upon the steady state itself. Thus, chapter 4 contends that aggregate fluctuations induced by various frictions in wage formation (contracting under uncertainty) should

not be analyzed independently of the notion of the steady state equilibrium corresponding to the particular process of wage formation. Two ideas are present here. First is the existence of labour contracts. It is intuitive enough to argue that the parameter pertaining to contracting should influence the steady state behaviour of the economy. As argued above, this aspect is abstracted away from the standard Gray—Fischer analysis by assuming that risk-neutral contracting parties set wages in accordance with the nonstochastic (ex ante) equilibrium of the labour markets. The argument is now that this process of determining the contract wage potentially ignores the link between contract wages and uncertainty, i.e. ex ante pricing of risk.

The second aspect is the existence of imperfectly competitive labour markets, largely due to unionized labour markets, in many modern industrialized countries, notably the Nordic countries. Genuine bargaining takes place between employees and employer organizations over the terms of labour contracts and the outcome should display this institutional setting. Furthermore, once again it is intuitive to argue that this particular bargaining structure should be displayed in the steady state behaviour of the whole economy.

All in all, chapter 4 adds to the existing literature in that it incorporates a bargainig model of the labour markets under uncertainty into an open economy macroframework mainly for the purpose of analyzing the steady state effects of wage indexation and foreign exchange intervention. We now turn to summarize the conclusions that emerge. Foreign exchange intervention influences exchange rate variability, and thus aggregate price risk or uncertainty. Given that wages are not fully indexed, expected real contract wages are dependent on the prevailing exchange regime determined by the foreign exchange intervention rule. By implication, steady state output and employment become dependent upon the prevailing exchange rate regime. Thus the potential exists for monetary policy to affect steady state output and employment as a result of contract wages being influenced by aggregate uncertainty.

Whether increased exchange rate variability induced by intervention is desirable in terms of higher steady state aggregate output and employment depends on a number of factors, including the specific source of aggregate variability and the degree of firms' bargaining power (i.e. competitiveness of labour pricing). More specifically, fixed exchange rates eliminate exchange rate variability and thus aggregate price variability due to exchange rates. But, via PPP, it tends to enhance the effect on the domestic price level of shocks to foreign prices. This is reflected in upward pressure on real expected contract wages under partial wage indexation. Thus, if shocks to foreign prices are an

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important source of aggregate (price) variability, exchange rate variability is called for to reduce the effects of these shocks, on contract wages and steady state output and employment at least as far as higher steady state output (and employment) is a desirable objective as such.

Flexible exchange rates need not, however, be a sensible choice for an exchange rate regime, if shocks to domestic money demand and/or to foreign interest rates are an important source of aggregate (price) variability. These latter shocks impinge on the domestic price level via exchange rate variability. This exchange-rate-induced price level variability increases aggregate price risk in a fashion that tends to exert upward pressure on expected real contract wages. If, on the other hand, shocks impinging on the domestic economy originate mainly from the product market, i.e. shocks to domestic productivity and/or to foreign prices, flexible exchange rates are clearly superior to fixed rates in terms of aggregate output and employment.

As noted above, the competitiveness of labour pricing (firms' bargaining power) affects contract wages in two different ways, that is through the mark-up factor and aggregate price risk. Increased competitiveness has, for a given degree of wage indexation and foreign exchange intervention, an ambiguous effect on the aggregate price risk. To be more specific, although the mark-up over the union's fall out utility shrinks as labour pricing becomes more competitive, this reduction in the mark-up must be balanced against the behaviour of the aggregate price risk, which may increase in such a way that the net result of increased competitiveness in labour pricing is an increase in expected real wages, and thus a reduction in steady state output and employment. The model indicates that this will happen particularly, if random shocks to domestic productivity are absent or at most very small relative to other shocks and the degree of wage indexation is low.

If, on the other hand, shocks to domestic productivity are pronounced, so that the constraint facing the contracting party (labour demand schedule) undergoes large stochastic shifts, a reduction in expected real wages can be expected from increased competitiveness in labour pricing as a result of both aggregate price risk and falling mark-up. An interesting implication of the model is, however, that, in terms of aggregate output and employment, there may not be any gains to be expected from increased competition in the labour markets as such, if "aggregate risk pricing" is an integral part of wage formation in a unionized economy and if the existence of the aggregate price risk is unrelated to domestic production possibilities.[2] This is analogous to the

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[2] Of course, the robustness of the model in this respect should be checked for the specification of the firms' technology and/or the union's utility function.

prediction that increased exchange rate variability as such need not enhance aggregate output and employment, although exchange rate flexibility does provide an additional channel through which to accommodate specific shocks to the domestic economy, a feature of flexible exchange rates emphasized by the standard framework of chapter 2. Structural changes in labour markets pertaining to enhanced competitiveness in labour pricing may be deemed desirable since they reduce the mark-up due to monopoly power, but the decentralization implied by the process of increasing competitiveness need not be optimal in controlling the effects of aggregate uncertainty on wage formation.[3]

Chapter 4 concludes by raising the question of the optimal choice of the degree of foreign exchange intervention and degree of wage indexation. The standard analysis of chapter 2 is most convenient in this respect, since it leads us to characterize the optimal policy mix of intervention and indexation as a policy package designed to deal with the signal extraction problem inherent in the model economy. Thus, simple time-invariant policy rules are easily derived within the extension to an open economy of the framework of Gray's optimal choice of the degree of wage indexation. Gray's framework of the optimal degree of wage indexation is not, however, applicable in the context of chapter 4, since a policy mix of intervention and indexation has direct steady state effects not present in Gray's analysis. To this end, a different type of an objective function determining the choice of the policy instruments is clearly called for.

Chapter 4 suggests that the optimal degree of intervention or indexation be chosen such that the steady state aggregate output be maximized, once losses due to imperfect information are accounted for. Thus, this objective function for the monetary authority weighs gains from increased output against potential losses due to the signal extraction problem, i.e. losses due to fact that labour contracts are signed before uncertainty is realized. Chapter 4 considers only the optimality of a single instrument, given the other, and derives explicit closed-form solutions for time-invariant policy rules under certain special cases.

If only losses due to imperfect information carry weight, full wage indexation is the optimal choice, independently of the degree of foreign exchange intervention or the exchange rate regime. This is so because real wages corresponding to the full information bargaining game are

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[3] This tentative argument is at least partially supported by the analysis in chapter 4 in that increased competitiveness coupled with sufficiently indexed wages may actually enhance steady state aggregate output and employment, since indexation controls the effects of aggregate price risk on contract wages.

constant, i.e. independent of shocks to domestic productivity, a feature characterizing the spot labour market equilibrium in chapter 2.

Thus, differences between actual real wages and full information real wages arise because of surprises in the price level and aggregate price risk. These differences can be completely eliminated by full wage indexation.

Given the degree of wage indexation, the optimal degree of foreign exchange intervention corresponding to the case where only loss minimization is relevant is to some extent shock-specific. More specifically, all other shocks except shocks to foreign prices can be fully eliminated by fixing the exchange rate, but the implied intervention rule for shocks to foreign prices requires an intervention policy of leaning with the wind. This is so because, via PPP, only an intervention rule of leaning with the wind can eliminate the effects of these shocks on the domestic price level. However, in general, as the degree of wage indexation is increased, the implied optimal exchange rate variability is reduced, since increased indexation controls the impact effects on actual wages of price level surprises and aggregate price risk due to shocks to foreign prices. An interesting implication of the simulations performed in chapter 4 is that the optimal degree of foreign exchange intervention tends to be higher, given the degree of wage indexation, in more competitive labour market environments. This is mainly due to the fact shocks to domestic productivity are enhanced in more competitive labour market environments.

Finally, explicit closed-form, time-invariant solutions to the optimal degree of intervention and indexation were derived in the case where only steady state output maximization carries weight in the monetary authority's preferences. As one would expect, the optimal degree of intervention and indexation depend on the entire parameter and stochastic structure of the model. But there is another, and perhaps more interesting, feature of the solutions, that is the close similarity of the optimal intervention and indexation rules with those derived in chapter 2. There is, however, at least one important difference relating to firms' bargaining power, which parameterically shifts the optimal response of wage indexation (intervention) to an exogenously given degree of foreign exchange intervention (degree of wage indexation). The model predicts that optimally there will be, for any given degree of intervention, less rigid real wages and that, for any given degree of wage indexation, more variable exchange rates will be observed in more competitive labour market environments. These results are explained by the presence of productivity shocks, which exert downward pressure on expected real contract wages through aggregate price risk.

Overall, then, the models of chapter 3 and 4 clearly demonstrate the effects on the solution of the bargaining game of introducing uncertainty into the model. Uncertainty arises (apart from indexation, as argued in chapter 3) as a result of the empirically plausible assumption that labour contracts are signed before uncertainty is resolved. Upon comparing the solution wage of this bargaining game with the one corresponding to the full information Nash bargaining game, we immediately observe that an additional "mark-up" due to price risk is invoked upon the full information wage. Furthermore, the solution has a simple mean-variance structure in that (random) variables correlating positively with variables affecting the union's utility tend to increase contract wages, while a negative correlation tends to reduce it. Despite this simple and elegant structure given to contract wages by the bargaining game, some final remarks are in order.

First of all, the degree of wage indexation was not taken as a variable subject to bargaining in the 'right to manage' model. This may sound like an oversimplification of reality, which seems to suggest that the degree of wage indexation is at least partly negotiable between labour market participants. From the modelling point of view, this would lead us to a two stage bargaining game, where in the first stage wages are determined and the second stage sets the degree of wage indexation given wages from the first stage. This is certainly an interesting future extension of the framework for determining wages and indexation, and would undoubtedly provide more solid foundations for the interaction between contract wages and indexation, but would also involve some deep thoughts about the Nash bargaining model under uncertainty, as the arguments given in chapter 3 seem to suggest. We have not applied this framework in the present context mainly for reasons of analytical simplicity.

In the above analysis, a combination of wage indexation and foreign exchange intervention has been conceived as a policy mix designed to enhance the macroeconomic performance of an economy. Thus the modelling strategy, especially in the last section of chapter 4, starts explicitly from the assumption that no other strategic interaction between these instruments exists. A possible modification to this set-up would be to model the interaction between these instruments as an outcome of an explicit game between the labour market participants and the monetary authority. This would most probably imply a much tighter interpretation of the instruments as strategic devices or response functions in this (hypothetical) game than those implied by our broad macroeconomic objectives. This would be an interesting modification of the theme, especially in relation to the argument put forward in the previous paragraph, and would possibly add a new dimension to our understanding

of the possible role of indexation and intervention in an institutional setting of the kind envisaged in chapters 3 and 4. We chose not to follow this modelling strategy, because of the focus on the macroeconomic implications of wage formation under uncertainty, where the degree of aggregate uncertainty is influenced by a combination of intervention and indexation. So, the interpretation of the instruments is more like a policy mix designed to mitigate the frictions due to *ex ante* contracting in labour markets and not as strategic devices giving equilibrium responses to agents involved in a game between labour market participants and the monetary authority.

References

Aizenman, J. (1983) Wage Contracts with Incomplete and Costly Information, National Bureau of Economic Research, Working Paper No. 1150, June 1983.

Aizenman, J. (1985) Wage flexibility and Openness. The Quarterly Journal of Economics, pp. 181–90.

Aizenman & Frenkel, J. (1985a) On the Tradeoff between Wage Indexation and Foreign Exchange Intervention, Weltwissenschaftliches Archiv, Bd. CXXI, pp. 1–17.

Aizenman & Frenkel, J. (1985b) Wage Indexation, Foreign Exchange Intervention, and Monetary Policy, The American Economic Review, Vol. 75, pp. 403–23.

Andersen, T. & Sørensen, J. (1988) Exchange Rate Variability and Wage Formation in Open Economies, Economics Letters, Vol. 28, pp. 263–268.

Azariadis, C. (1975) Implicit Contracts and Underemployment Equilibria, Journal of Political Economy, Vol. 83, pp. 1183–1202.

Azariadis, C. (1978) Escalator Clauses and the Cyclical Allocation of Risk, Journal of Economic Theory, Vol. 18, pp. 119–155.

Baillie, R. & Selover, D. (1987) Cointegration and Models of Exchange Rate Determination, International Journal of Forecasting, Vol. 4, pp. 43–51.

Baily, M. (1974) Wages and Employment under Uncertain Demand, Review of Economic Studies, Vol. 41, pp. 37–50.

Baily, M. (1978) Stabilization Policy and Private Economic Behavior, Brookings Papers on Economic Activity, 1:1978, pp. 11–59.

Ball, L. & Cecchetti, S. (1989) Wage Indexation and Time-Consistent Monetary Policy, National Bureau of Economic Research, Working Paper No. 2948, April 1989.

Barro, R. (1976) Indexation in a Rational Expectations Model, Journal of Economic Theory, October 1976, pp. 229–244.

Barro, R. (1977) Long-Term Contracting, Sticky Prices and Monetary Policy, Journal of Monetary Economics, pp. 305–316.

Binmore, K., Rubinstein, A. & Wolinsky, A. (1986) The Nash Bargaining Solution in Economic Modelling, The Rand Journal of Economics, Vol. 17, pp. 176–88.

Blanchard, O. (1979) Wage Indexing Rules and the Behavior of the Economy, Journal of Political Economy, pp. 798–815.

Blinder, A. (1989) Macroeconomics under Debate, Harvester Wheatsheaf, New York, Ch. 11, pp. 160–91.

Brock, W. (1974) Money and Growth: the Case of Long-run Perfect Foresight, International Economic Review, Vol. 15, pp. 750–77.

Brown, J. & Ashenfelter, O. (1983) Testing the Efficiency of Employement Contracts, Journal of Political Economy, Vol. 94, Supplement, pp. S40–87.

Canning, D. (1990) Bargaining Theory, in The Economics of Missing Markets, Information, and Games, Frank Hahn (ed.), Clarendon Press, Oxford, 1990, pp. 163–187.

Carruth, A. & Oswald, A. (1985) Miners' Wages in Post-War Britain: An Application of a Model of Trade Union Behaviour, Economic Journal, Vol. 95, pp. 1003–34.

Chiswick, C. (1986) The Efficiency Wage Hypothesis: Applying a General Model of the Interaction between Labor Quality and Quantity, Journal of Economic Development, Vol. 20, pp. 311–24.

Cooper, R. (1987) Wage and Employment Patterns in Labour Contracts: Microfoundations and Macroeconomic Implications, Harwood Academic Publishers, 1987.

Corbae, D. & Ouliaris, S. (1988) Cointegration and Tests of Purchasing Power Parity, The Review of Economics and Statistics, Vol. LXX, pp. 508–11.

Cukierman, A. (1980) The Effects of Wage Indexation on Macroeconomic Stability: A Generalization, Journal of Monetary Economics, Vol. 6, pp. 147–70.

Dertouzos, J. & Pencavel, J. (1981) Wage and Employment Determination under Trade Unionism: The International Typographical Union, Journal of Political Economy, Vol. 89, pp. 1162–81.

Devereux, M. (1988) The Optimal Mix of Wage Indexation and Foreign Exchange Market Intervention, Journal of Money, Credit and Banking, Vol. 20, pp. 381–392.

Dornbusch, R. (1985) Purchasing Power Parity, National Bureau of Economic Research, Working Paper No. 1591.

Dornbusch, R. & Simonsen, M. (1983) Inflation, Debt and Indexation, MIT Press, 1983.

Dorwick, S. (1989) Union-Oligopoly Bargaining, The Economic Journal, Vol. 99, pp. 1123–42.

Enders, W. (1988) ARIMA and Cointegration Tests of PPP under Fixed and Flexible Exchange Rate Regimes, The Review of Economics and Statistics, Vol. LXX, pp. 504–508.

Erikson, T., Suvanto, A. & Vartia, P. (1990) Wage Formation in Finland, in Wage Formation and Macroeconomic Policy in the Nordic Countries, Calmfors, L. (ed.), Oxford University Press, pp. 189–230.

Espinosa, M. & Rhee, C. (1989) Efficient Wage Bargaining as a Repeated Game, Quarterly Journal of Economics, Vol. 104, pp. 565–88.

Evans, G. & Honkapohja, S. (1990) Convergence of Recursive Learning Mechanisms to Steady States and Cycles in Stochastic Nonlinear Models, University of Helsinki, Department of Economics, Discussion Paper No. 301.

Farber, H. (1978a) Individual Preferences and Union Wage Determination: The Case of the United Mine Workers, Journal of Political Economy, Vol. 86, pp. 923—42.

Farber, H. (1978b) The United Mine Workers and the Demand for Coal: An Econometric Analysis of Union Behaviour, Research in Labor Economics, Vol. 2, pp. 1—74.

Farber, H. (1985) The Analysis of Trade Union Behavior, National Bureau of Economic Research, Working Paper No. 1502.

Fischer, S. (1975) Wage-Indexation and Macro-Economic Stability, MIT, Department of Economics, Mimeo.

Fischer, S. (1977a) Long-Term Contracts, Rational Expectations and the Optimal Money Supply Rule, Journal of Political Economy, Vol. 85, pp. 191—205.

Fischer, S. (1977b) Long-Term Contracting, Sticky Prices and Monetary Policy: A Comment, Journal of Monetary Economics, 1977 Supplement, pp. S317—23.

Fischer, S. (1983a) Supply Shocks, Wage Stickiness, and Accommodation, National Bureau of Economic Research, Working Paper No. 1119, 1983.

Fischer, S. (1983b) Indexation and Inflation, Journal of Monetary Economics, Vol. 12, pp. 519—542.

Fisher, C. (1989) Efficiency Wages: A Literature Survey, Bank of Canada, Working Paper 89-5, December 1989.

Flood, R. & Marion, P. (1982) The Transmission of Disturbances under Alternative Exchange Rate Regimes with Optimal Indexing, The Quarterly Journal of Economics, February 1982, pp. 41—66.

Giovannetti, G. (1988) Testing Purchasing Power Parity as a Long Run Equilibrium Condition, paper presented at the EEA Conference, Bologna, August 1988.

Gordon, D. (1974) A Neo-Classical Theory of Keynesian Unemployment, Economic Inquiry, Vol. 12, pp. 431—59.

Gottfries, N. (1990) A Model of Nominal Contracts, forthcoming in Review of Economic Studies, 1990.

Gray, J. (1976) Wage Indexation: A Macroeconomic Approach, Journal of Monetary Economics, pp. 221—35.

Gray, J. (1978) On Indexation and Contract Length, Journal of Political Economy, February 1978, pp. 1—18.

Gray, J. (1983) Wage Indexation, Incomplete Information and the Aggregate Supply Curve, in Inflation, Debt and Indexation, Dornbusch, R. & Simonsen, M. (eds.), pp. 25—45.

Grossman, S. & Hart, O. (1981) Implicit Contracts, Moral Hazard and Unemployment, American Economics Association Papers and Proceedings 1981, pp. 301–7.

Grossman, S. & Hart, O. (1983) Implicit Contracts under Asymmetric Information, Quarterly Journal of Economics, Vol. 93, Supplement, pp. S123–56.

Haaparanta, P. (1986) External Disturbances, Exports, and Wage Indexation, Ph. D. thesis 1986, Yale University.

Haaparanta, P. & Vilmunen, J. (1988) Wage Indexation and Macroeconomic Theory, in Money, Inflation and Economic Policy, Honkapohja, S. & Suvanto, A. (eds.), Government Printing Centre — Department of Economics, Helsinki University, pp. 161–186, published in Finnish.

Haley, J. (1990) Theoretical Foundations for Sticky Prices, Journal of Economic Surveys, Vol. 4, pp. 115–56.

Harsanyi, J. (1967, 1968a, b) Games with Incomplete Information Played by Bayesian Players, I, II and III, Management Science, Vol. 14, pp. 159–82, 320–34 and 486–503.

Hoel, M. (1986) Perfect Equilibria in Sequential Bargaining Games with Non-linear Utility Functions, Scandinavian Journal of Economics, Vol. 88, pp. 383–400.

Hoel, M. (1990) Local versus Central Bargaining with Endogenous Investment, Scandinavian Journal of Economics, Vol. 92, pp. 453–70.

Holmlund, B. B. (1989) Wages and Employment in Unionized Economies: Theory and Evidence, in Trade Union, Employment and Unemployment Duration, Holmlund, B., Löfgren, K-G. & Engström, L. (eds.), Clarendon Press, Oxford, 1989, pp. 5–104.

Jones, S. & MCkenna, C. (1989) The Effects of Outsiders on Union Contracts, European Economic Review, Vol. 33, pp. 1567–73.

Karni, E. (1983) On Optimal Wage Indexation, Journal of Political Economy, April 1983, pp. 282–92.

Lockwood, B. & Manning, A. (1988) Dynamic Wage-Employment Bargaining with Employment Adjustment Costs, The Economic Journal, Vol. 99, pp. 1143–58.

Lucas, R. (1972) Expectations and Neutrality of Money, Journal of Economic Theory, April 1972, pp. 103–24.

Lucas, R. (1973) Some International Evidence on Output-Inflation Tradeoffs, The American Economic Review, Vol. 63, pp. 326–34.

Lucas, R. (1987) Models of Business Cycle, Yrjö Jahnsson Lectures, Yrjö Jahnsson Foundation, Basil Blackwell Ltd.

Macleod, W. & Malcolmson, J. (1986) Wage Premiums and Profit Maximization in Efficiency Wage Models, Centre for Labour Economics, D.P. No. 337, London School of Economics.

Manning, A. (1987) An Integration of Trade Union Models in a Sequential Bargaining Framework, Economic Journal, Vol. 97, pp. 225–51.

Marston, R. (1984) Wages, Relative Prices and the Choice between Fixed and Flexible Exchange Rates, Canadian Journal of Economics, Vol. 15, pp. 87–103.

Marston, R. (1983) Stabilization Policies in Open Economies, National Bureau of Economic Research, Working Paper No. 1117, May 1983.

Marston, R. & Turnovsky, S. (1985a) Macroeconomic Stabilization through Taxation and Indexation: the Use of Firm-Specific Information, National Bureau of Economic Research, Working Paper No. 1586, 1985.

Marston, R. & Turnovsky, S. (1985b) Imported Material Prices, Wage Policy and Macroeconomic Stabilization, Canadian Journal of Economics, May 1985, pp. 271–84.

McCallum, B. (1986) On "Real" and "Sticky Price" Theories of the Business Cycle, Journal of Money, Credit and Banking, Vol. 18, pp. 397–413.

McDonald, I. & Solow, R. (1981) Wage Bargaining and Employment, American Economic Review, Vol. 71, pp. 896–908.

McCurdy, T. & Pencavel, J. (1986) Testing between Competing Models of Wage and Employment Determination in Unionized Markets, Journal of Political Economy, Vol. 94, pp. 53–139.

Minford, P. (1983) Labour Market Equilibrium in an Open Economy, Oxford Economic Papers (November Supplement), Vol. 35, pp. S245–74.

Muth, J. (1961) Rational Expectations and the Theory of Price Movements, Econometrica, Vol. 29, pp. 315–35.

Nash, J. (1950) The Bargaining Problem, Econometrica, Vol. 18, pp. 286–95.

Nash, J. (1953), Two-Person Cooperative Games, Econometrica, Vol. 21, pp. 128–40.

Nickell, S. (1990) Unemployment: A Survey, The Economic Journal, Vol. 100, No. 401, pp. 391–439, June 1990.

Nickell, S. & Andrews, M. (1983) Unions, Real Wages and Employment in Britain 1951–1979, Oxford Economic Papers (Supplement), Vol. 35, pp. S183–206.

Osborne, M. J. & Rubinstein, A. (1990) Bargaining and Markets, Academic Press, Inc.

Oswald, A. (1979) Wage Determination in an Economy with Many Trade Unions, Oxford Economic Papers, Vol. 31, pp. 369–85.

Oswald, A. (1982) Uncertainty and the Trade Union, Economics Letters, Vol. 9, pp. 105–111.

Oswald, A. (1985) The Economic Theory of Trade Unions: An Introductory Survey, Scandinavian Journal of Economics, Proceedings of a Conference on Trade Unions, Wage Formation and Macroeconomic Stability, Vol. 87, pp. 160–93.

Pencavel, J. (1984a) The Tradeoff between Wages and Employment in Trade Union Objectives, Quarterly Journal of Economics, Vol. 99, pp. 215–32.

Rosen, S. (1985) Implicit Contracts: A Survey, Journal of Economic Literature, Vol. XXIII, pp. 1144–75.

Rotschild, M. & Stiglitz, J. (1971) Increasing Risk II: Its Economic Consequences, Journal of Economic Theory, Vol. 3, pp. 60–84.

Simonsen, M. (1983) Indexation: Current Theory and the Brazilian Experience, in Inflation, Debt and Indexation, Dornbusch, R. & Simonsen, M. (eds.), MIT Press, pp. 99–132.

Stiglitz, J. (1986) Theories of Wage Rigidity, National Bureau of Economic Research, Working Paper No. 1442.

Svejnar, J. (1986) Bargaining Power, Fear of Disagreement and Wage Settlements: Theory and Evidence from U.S. Industry, Econometrica, Vol. 54, pp. 1055–78.

Taylor, M. (1988) An Empirical Examination of Long-Run Purchasing Power Parity using Cointegration Techniques, Applied Economics, Vol. 20, pp. 1369–1381.

Turnovsky, S. (1983) Wage Indexation and Exchange Market Intervention in a Small Open Economy, Canadian Journal of Economics, Vol. 16, pp. 574–92.

Turnovsky, S. (1984) Exchange Market Intervention under Alternative Forms of Exogenous Disturbances, Journal of International Economics, Vol. 17, pp. 279–97.

Turnovsky, S. (1986) Optimal Monetary Policy and Wage Indexation under Alternative Disturbances and Information Structures, National Bureau of Economic Research, Working Paper No. 2042, October 1986.

Varian, H. (1981) Microeconomic Analysis, W. W. Norton & Company, New York.

Vilmunen, J. (1989) Wage Indexation, Foreign Exchange Market Intervention, and the Degree of Openness of an Economy, Licentiate Thesis, University of Helsinki, Dept. of Economics, 1989 (in Finnish).

Waller, C. (1989) Efficiency Wages, Wage Indexation and Macroeconomic Stabilization, Economics Letters, Vol. 30, pp. 125–28.

Wright, R. (1988) The Observational Implications of Labor Contracts in a Dynamic General Equilibrium Model, Journal of Labor Economics, Vol. 6, pp. 530–51.