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## 1 Introduction

Labour markets across the euro area have performed well over the last years despite various economic disturbances and crises. Especially during the recovery from the Covid-19 pandemic employment rates and labour market tightness have risen to high, even record, levels. However, the evolution of hours of work has been weaker, lagging behind the rise of employment. A noteworthy feature in European labour markets is that employment and labour market tightness have remained at high levels even though the business cycle has more recently weakened due to the energy crisis, high inflation and war related uncertainty. This potential detachment of the business cycle and labour market tightness on the one hand, and the relative weakness in hours of work on the other, may alter the transmission of shocks in the economy and is a challenge for macroeconomic policy.

In this paper we present a New Keynesian model that accounts for the role of labour market frictions and wage rigidity in shaping business cycle fluctuations in a small open economy. The focus is on the labour market mechanisms that are particularly relevant for the Finnish economy, although the mechanisms and the qualitative results of the model apply more generally to small member states of a monetary union. We illustrate the properties of the model by presenting how shocks after the Covid-19 pandemic explain inflation and economic recovery in the euro area and Finland, with a specific emphasis on factors that

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are critical to explaining how strongly wages react to the acceleration in inflation. Although matching frictions and wage rigidity are key elements for wage dynamics in the model, it importantly features both the intensive (hours) and extensive margin (employment) of labour adjustment. This makes the model particularly well-suited to study key features of the Finnish labour markets in recent years, high labour market tightness and labour shortages on the one hand, and a fall of hours of work per employee on the other. Bick et al. (2022) and Lee et al (2023) document these to be common patterns in many European economies and the U.S. in recent years, which makes the results of the model applicable beyond the Finnish economy.

According to our simulations, reduction in working hours in particular could explain recent labour market tightness and also increase wage pressures significantly. Increased public consumption may also have had effects that accelerated inflation. The increase in public consumption also seems to have tightened the labour market and somewhat accelerated inflation. According to model simulations, wage pressures could also increase in the future due to the recent drop in the price of energy, which could increase overall demand and thus increase the demand for labour.

The purpose of our model is to complement the existing analytical tools of the Bank of Finland by enabling detailed analysis of labour markets in a DSGE setting. There is a long tradition of DSGE modelling at the Bank of Finland. The Aino model (Kilponen and Ripatti, 2006) was one of the first models of that kind that was operative for both policy simulations and forecasting. The Aino 2.0 model (Kilponen et al. 2016) featured a monopolistically competitive banking sector as well as short-term corporate lending. The most recent vintage, the Aino 3.0 (Silvo and Verona, 2020) builds on the previous work and introduces a housing market and credit-contrained households into the model. Setting our study apart from the Aino model, we introduce a small open economy New Keynesian model including labour market frictions and wage rigidity.

There are a number of studies that feature a similar labour market structure and rigidity in wage setting. Models with these features are better equipped to reproduce empirically observed business cycles (see, e.g., Guerra-Salas et al., 2021, Christiano et al., 2016, Obstbaum, 2011, Shimer, 2010, and Gertler and Trigari, 2009). Gertler et al. (2008) has been the closest original reference for our present approach. They develop and estimate a medium-scale DSGE model that allows for labour market frictions and staggered nominal wage contracting. They find that the model with wage rigidity provides a better description of the data than a flexible wage version. We employ the model developed in Obstbaum (2011), which features a similar labour market structure as in Gertler et al. (2008) but extends the model to account for monetary union membership and for distortionary taxes. Obstbaum (2011) found that a calibrated model for the Finnish economy with these features provided model moments that best corresponded to data moments for the time period 1994-2010. The model of Obstbaum (2011) is estimated using Bayesian Maximum Likelihood methods in (Obstbaum 2012).

The rest of the paper is organised in the following way. In Section 2, we present the structure of the model, with a specific emphasis on the labour market. In Section 3, we present the current parametrization of the model. In Section 4, we illustrate the properties of the model through a simulation exercise describing recent shocks that Finland and the euro area have faced. Finally in Section 5, we discuss potential further extensions and conclude.

## 2 The model

## 2.1 General features

The model is based on Obstbaum (2012). The model considers a small monetary union member state and builds in this respect on Galí and Monacelli (2005). As in Corsetti, Meier and Müller (2009), however, we close the model by assuming a debt-elastic interest rate instead of complete asset markets. The home country is modelled along standard New-Keynesian practise comprising households, firms and a public sector. Following Gertler, Sala and Trigari (2008) we incorporate labour market frictions and, staggered Nash wage bargaining. However, firms only employ one worker, employment can be adjusted along both the extensive and the intensive margin (hours of work), and there are distortionary labour taxes as in Obstbaum (2011).

One advantage of the staggered bargaining approach is that wage rigidity gets the explicit interpretation of longer wage contracts. Lengthening the duration of wage contracts makes wages in each period less responsive to economic conditions, and shifts adjustment to the labour quantity side.

Wages can only be negotiated at specific intervals but hours can be renegotiated in each period. As a result, hours are more volatile than the number of workers in the model. Hours are more volatile also in the data due to e.g. overtime hours. A specific feature of the Finnish labour market is the furlough system which accounts for a large part of employment adjustment in downturns. As furloughed workers are accounted as workers in the statistics but their hours are reduced or even zero, this adds to the volatility of hours per workers. However, the volatility of hours implied by the model is somewhat larger than what is observed in the data. This could be addressed by introducing an adjustment cost in the determination of hours but the focus is here instead on the determination of wages.

The wage and price setting decisions are separated from each other. Labour market frictions arise in the intermediate good sector. The wholesale firms buy intermediate goods and re-sell them to the final goods sector. Wholesale firms operate under monopolistic competition and set prices subject to Calvo rigidities. Final goods are produced from domestic and imported intermediate inputs under perfect competition.

The government's policy instruments include a lump-sum tax, a proportional wage tax paid by the employees, wage taxes paid by the employers in the form of social security contributions, unemployment benefits and other government transfers as well as a consumption tax.

#### 2.2 Preferences

As in similar kinds of models, we adopt the representative household approach. This implies perfect consumption insurance, a key assumption needed to embed the MP model in a GE framework. Household members perfectly insure each other against variations in labour income due to their labour market status. This tackles the problem whereby households are identical but not all of their members are employed. As a result, the employment and

unemployment rates are identical at the household level and across the population at large (see e.g. Merz, 1995).

The representative household maximizes the expected lifetime utility of its members

$$\int_{0}^{1} E_{0} \left\{ \sum_{t=0}^{\infty} \beta^{t} \left[ \frac{(C_{i,t} - \varkappa C_{t-1})^{1-\varrho}}{1-\varrho} - \delta n_{t} \frac{(h_{i,t})^{1+\phi}}{1+\phi} \right] \right\} di$$
 (1)

where  $C_{i,t}$  is final good consumption in period t by household member  $i, \varkappa \in (0,1)$  indicates an external habit motive,  $C_{t-1}$  stands for aggregate consumption in the previous period,  $h_{i,t}$ are hours worked by household member i, and  $\delta$  is a scaling parameter for the disutility of work. The inverses of  $\varrho$  and  $\varphi$  are the elasticities of intertemporal substitution and of labour supply respectively. The household's (real) budget constraint is

$$(1 + \tau_t^c) C_t + I_t + A(\nu_t) K_{t-1}^p + \frac{B_t^f}{P_t}$$

$$= n_t \frac{w_t}{P_t} h_t (1 - \tau_t) + (1 - n_t) b + \frac{TR_t}{P_t}$$

$$+ R_{t-1}^f p\left(b_{t-1}^f\right) \frac{B_{t-1}^f}{P_t} + r_t^k \nu_t K_{t-1}^p + D_t$$
(2)

The left-hand side of the equation describes the expenditures of the household. Consumption  $C_t$  is subject to a proportional tax  $\tau_t^c$ . As an alternative to consumption, the household may choose to invest  $I_t$ . As households own the capital stock, they also bear the cost of capital utilization  $A(\nu_t)K_{t-1}^p$ . In addition, the household can nominal one-period foreign bonds  $B_t^f$  which are both denominated in the monetary union currency.

The right hand side describes the household's income sources which consist of after-tax real wage  $n_t \frac{w_t}{P_t} h_t (1 - \tau_t)$ , unemployment benefits  $(1 - n_t) b$ , lump-sum transfers  $\frac{TR_t}{P_t}$ , capital rental payments from firms  $r_t^k \nu_t K_{t-1}^p$  and profit from firm ownership  $D_t$ . Income is also received in the form of repayment of last period's foreign bond purchases. The interest rate paid or earned on foreign bonds by domestic households  $R_t = R_{t-1}^f p\left(b_{t-1}^f\right)$  consists of the common currency union gross interest rate  $R_{t-1}^f$  which, for the small member state is taken to be exogenous, and a country-specific risk premium  $p\left(b_{t-1}^f\right)$ . The risk premium is assumed to be increasing in the aggregate level of foreign real debt as a share of domestic output  $(-b_t^f = -\frac{B_t^f}{DN})$ .

The capital utilization rate  $\nu_t$  transforms physical capital into effective capital according to

$$K_t = \nu_t K_{t-1}^p \tag{3}$$

<sup>&</sup>lt;sup>1</sup>This is the debt-elastic interest rate assumption which is one of the mechanisms suggested by Schmitt-Grohé and Uribe (2003) to close a small open economy model. Note that with the current notation a negative (positive) deviation of the stock of foreign bonds from the steady state zero level implies that the home country as a whole becomes a net borrower (lender), and faces a positive (negative) risk premium.

Effective capital is rented to firms at the rate  $r_t^k$ . The cost of capital utilization per unit of physical capital is  $A(\nu_t)$ . It is assumed that in the steady state  $\nu_t = 1$  and A(1) = 0.  $A'(1)/A''(1) = \eta_{\nu}$ . The capital accumulation equation is

$$K_t^p = \left(1 - \delta^k\right) K_{t-1}^p + \epsilon_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{4}$$

where  $\delta^k$  is the capital depreciation rate and  $\epsilon^I_t$  is an investment specific shock. The investment adjustment cost function  $S(\cdot)$  is assumed to have the following properties in the steady state:  $S(\gamma_z) = S'(\gamma_z) = 0$  and  $S''(\gamma_z) \equiv \eta_z > 0$ , where  $\eta_z$  is the economy's steady state growth rate. We assume  $\epsilon^I_t$  follows the exogenous stochastic process

$$\log(\epsilon_t^I) = (1 - \rho_I)\log(\epsilon^I) + \rho_I\log(\epsilon_{t-1}^I) + \varsigma_t^I$$
, where  $\rho_I \in (0, 1), \varsigma_t^I \stackrel{iid}{\sim} N(0, \sigma_I^2)$ 

We leave aside for a moment the labour supply decision, which will be dealt with in the section describing the labour market, below. Optimal allocations of consumption and financial assets are characterized by the following conditions

$$\Lambda_t = \frac{\lambda_t}{(1 + \tau_t^c)} \tag{5}$$

$$\Lambda_t = \beta E_t \left[ \Lambda_{t+1} \frac{R_{t-1}^f p\left(b_{t-1}^f\right)}{\pi_{t+1}} \right]$$
 (6)

where  $\lambda_t = (C_t - \varkappa C_{t-1})^{-\varrho}$  is the marginal utility of consumption and  $\pi_{t+1} = \frac{P_{t+1}}{P_t}$  is CPI inflation.

The nominal rate of return in the domestic economy is  $R_t = R_t^f p\left(b_t^f\right)$ , where the risk premium on foreign bond holdings  $p\left(b_t^f\right)$  follows the function

$$p\left(b_t^f\right) = \exp\left[-\gamma_{bf}\left(b_t^f - \overline{b}\right)\right], \text{ with } \gamma_{bf} > 0$$
 (7)

This should ensure the stability and determinacy of equilibrium in a small member state of the monetary union model <sup>2</sup>. In the steady state, the risk premium is assumed to be equal to one, and the domestic and foreign interest rates are the same. After loglinearization the above arbitrage relation gets the form

$$\widehat{R}_t = \widehat{R}_t^f - \gamma_{bf} \widehat{b}_t^f$$

Furthermore, the first order conditions for capital utilization  $\nu_t$ , investment  $I_t$ , and physical capital  $K_t^p$  are respectively

<sup>&</sup>lt;sup>2</sup>As Galí and Monacelli (2005) point out, along with accession to the monetary union the small member state no longer meets the Taylor principle since variations in its inflation that result from idiosyncratic shocks will have an infinitesimal effect on union-wide inflation, and will thus induce little or no response from the union's central bank. According to the Taylor principle, in order to guarantee the uniqueness of the equilibrium, the central bank would have to adjust the nominal interest rates more than one-for-one with changes in inflation (see e.g. Woodford, 2003)

$$r_t^k = A'(\nu_t) \tag{8}$$

$$Q_t \epsilon_t^I \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] = Q_t \epsilon_t^I S'(\frac{I_t}{I_{t-1}}) \left( \frac{I_t}{I_{t-1}} \right) - \beta E_t Q_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \epsilon_{t+1}^I S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 + 1 \quad (9)$$

$$Q_{t} = \beta E_{t} \frac{\Lambda_{t+1}}{\Lambda_{t}} \left[ \left( 1 - \delta^{k} \right) Q_{t+1} + r_{t+1}^{k} \nu_{t+1} - A(\nu_{t+1}) \right]$$
(10)

where  $Q_t$  is Tobin's Q, the present value of an additional unit of capital divided by the cost of acquiring one unit of capital, or the ratio of the Lagrange multipliers for the capital accumulation equation and for the consumer budget constraint. The discount factor is the same for all optimizing agents in the economy and is hereafter defined as  $\beta_{t,t+s} = \beta^s \frac{\Lambda_{t+s}}{\Lambda_t}$ .

## 2.3 The labour market

## 2.3.1 Unemployment, vacancies and matching

The labour market brings together workers and intermediate good firms. The measure of successful matches  $m_t$  is given by the matching function

$$m_t(u_t, v_t) = \sigma_m u_t^{\sigma} v_t^{1-\sigma} \tag{11}$$

where  $u_t$  and  $v_t$  are the aggregate measures of unemployed workers and vacancies.  $m_t$  is the flow of matches during a period t, and  $u_t$  and  $v_t$  are the stocks at the beginning of the period. The matching function is increasing in both vacancies and unemployment, concave, and homogeneous of degree one. The Cobb-Douglas form implies that  $\sigma$  is the elasticity of matching with respect to the stock of unemployed people, and  $\sigma_m$  represents the efficiency of the matching process. The probabilities that a vacancy will be filled and that the unemployed person finds a job are respectively

$$q_t^F = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma} \tag{12}$$

$$q_t^W = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma} \tag{13}$$

so the probability of a firm to fill a vacancy  $q_t^F$  is decreasing and the probability of an unemployed worker finding a job  $q_t^W$  is increasing in labour market tightness  $\theta_t = \frac{v_t}{u_t}$ .

In the beginning of each period, a fraction of matches will be terminated with an exogenous probability  $\rho_t \in (0,1)$ . The separation rate evolves according to the autoregressive process

$$\log(\rho_t) = (1 - \rho_\rho) \log(\rho) + \rho_\rho \log(\rho_{t-1}) + \epsilon_t^\rho, \text{ where } \rho_\rho \in (0, 1), \ \epsilon_t^\rho \stackrel{iid}{\sim} N\left(0, \sigma_\rho^2\right)$$

Labour market participation is characterised as follows. The size of the labour force is normalised to one. The number of employed workers at the beginning of each period is

$$n_t = (1 - \rho_t) n_{t-1} + m_{t-1} \tag{14}$$

where the first term on the right hand side represents those workers who were employed

already in the previous period and whose jobs have survived beginning-of-period job destruction, and the second term covers those workers who got matched in the previous period and become productive in the current period. After the exogenous separation shock, the separated workers return to the pool of unemployed workers and start immediately searching for a job. The number of unemployed is  $u_t = 1 - n_t$ .

In the steady state an equal amount of jobs are created and destructed:

$$JC = JD \iff m = \rho n.$$
 (15)

#### 2.3.2 Wage bargaining

Job creation takes place when a worker and a firm meet and agree to form a match at a negotiated wage. The wage that the firm and the worker choose must be high enough that the worker wants to work in the job, and low enough that the employer wants to hire the worker. These requirements define a range of wages that are acceptable to both the firm and the worker. The unique equilibrium wage.is, however, the outcome of a bargain between the worker and the firm.

The structure of the staggered multiperiod contracting model follows Gertler, Sala and Trigari (2008) but includes also the intensive margin of adjustment of the labour input (hours worked per worker) as well as distortionary taxes. For comparison, the period-by-period bargaining outcome is presented in the appendix. The idea of staggered wage bargaining is analogous to Calvo price setting. Rigidity is created by assuming that a fraction  $\gamma$  of firms are not allowed to renegotiate their wage in a given period. As a result, all workers in those firms receive the wage paid the previous period  $w_{t-1}$  partially indexed to inflation. The constant probability that firms are allowed to renegotiate the wage is labeled  $1 - \gamma$ . Accordingly,  $\frac{1}{1-\gamma}$  is the average duration of a wage contract. Thus, the combination of wage bargaining and Calvo price setting allows to give an intuitive interpretation to the source of wage rigidity instead of more or less ad hoc formulations. Period-by-period bargaining corresponds to the special case of  $\gamma = 0$ .

As in the standard Mortensen-Pissarides model, it is assumed that match surplus, the sum of the worker and firm surpluses, is shared according to efficient Nash bargaining. In the baseline model, wages and hours are negotiated simultaneously. The firm and the worker choose the nominal wage and the hours of work to maximize the weighted product of their net return from the match. When wages are rigid, it is assumed that as new matches become productive, they enter the same Calvo scheme for wage-setting than existing matches. This is an important assumption for wage rigidity to have an effect on job creation. Gertler and

Trigari (2009) argue that after controlling for compositional effects there are no differences in the flexibility of new and existing worker's wages.<sup>3</sup>

The contract wage  $w_t^*$  is chosen to solve

$$\max \left[ H_t(r) \right]^{\eta_t} \left[ J_t(r) \right]^{1-\eta_t} \tag{16}$$

subject to the random renegotiation probability.  $H_t(r)$  and  $J_t(r)$  are the matching surpluses of renegotiating workers and firms respectively, and  $0 \le \eta_t \le 1$  is the relative measure of workers' bargaining strength. The value equations describing the worker's and the firm's surplus from employment are the key determinants of the outcome of the wage bargain. We assume that the bargaining power of workers  $\eta_t$  is subject to shocks  $\epsilon_t^{\eta}$  as follows

$$\log(\eta_t) = (1 - \rho_\eta) \log(\eta) + \rho_\eta \log(\eta_{t-1}) + \epsilon_t^\eta$$
, where  $\rho_\eta \in (0, 1)$ ,  $\epsilon_t^{\eta} \stackrel{iid}{\sim} N(0, \sigma_\eta^2)$ 

Workers The value to the renegotiating worker of being employed consists of after-tax labour income, the disutility from working, expressed in marginal utility terms, and the expected present value of his situation in the next period. In the case of non-renegotiation, the past nominal wage is partially indexed to CPI inflation  $[\pi_t^{\varepsilon_w}(\pi^{1-\varepsilon_w})]$  as in Smets and Wouters (2003) or Christoffel, Kuester and Linzert (2009).

$$W_{t}(r) = \frac{w_{t}^{*}}{P_{t}} h_{t} (1 - \tau_{t}) - \frac{g(h_{t})}{\Lambda_{t}} + E_{t} \beta_{t,t+1} \left\{ (1 - \rho_{t+1}) \left[ \gamma W_{t+1} \left( w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1 - \varepsilon_{w}} \right) \right] \right) + (1 - \gamma) W_{t+1}(w_{t+1}^{*}) \right] + \rho_{t+1} U_{t+1} \right\}$$

$$(17)$$

The value to the worker of being unemployed is

$$U_t(r) = b + E_t \beta_{t,t+1} \left[ q_t^W W_{x,t+1} + \left( 1 - q_t^W \right) U_{t+1} \right]$$
(18)

where the first term on the RHS is the value of the outside option to the worker, i.e. the unemployment benefit b, and the second term gives the expected present value of either working or being unemployed in the following period. Unemployed workers do not need to take into account the probability of job destruction even if they get matched because of the timing assumption. A match that has not yet become productive cannot be destroyed. Note that the value for the worker who is currently unemployed to move from unemployment to employment next period is  $W_{x,t+1}$ , the expected average value of being employed. New matches are subject to the same bargaining scheme as existing matches, and therefore the

<sup>&</sup>lt;sup>3</sup>E.g. Pissarides (2009) and Kilponen and Vanhala (2014) argues the opposite: that wages of newly hired workers are volatile unlike wages for ongoing job relationships. This would mean that there is wage rigidity, but not of the kind that leads to more volatility in unemployment fluctuations.

new worker does not have a priori knowledge of whether the firm he will start working for will be allowed to renegotiate its wage<sup>4</sup>.

Combining these value equations gives the expression for the renegotiating worker's surplus

$$H_{t}(r) = W_{t}(r) - U_{t}(r)$$

$$= \frac{w_{t}^{*}}{P_{t}} h_{t} (1 - \tau_{t}) - \frac{g(h_{t})}{\Lambda_{t}} - b$$

$$+ E_{t} \beta_{t,t+1} \left\{ (1 - \rho_{t+1}) \left[ \gamma H_{t+1} (w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right]) + (1 - \gamma) H_{t+1} (w_{t+1}^{*}) \right] - q_{t}^{W} H_{x,t+1} \right\}$$

$$(19)$$

**Intermediate firms** For the renegotiating firm, the value of an occupied job is equal to the profit of the firm in the current period net of payroll taxes  $s_t$  and capital rental payments  $r_t^k k_t$ , and the expected future value of the job

$$J_{t}(r) = x_{t} f(k_{t}, h_{t}) - \frac{w_{t}^{*}}{P_{t}} h_{t} (1 + s_{t}) - r_{t}^{k} k_{t}$$

$$+ E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) \left[ \gamma J_{t+1} (w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1 - \varepsilon_{w}} \right) \right]) + (1 - \gamma) J_{t+1} (w_{t+1}^{*}) \right]$$
 (20)

where  $x_t$  is the relative price of the intermediate sector's good.

As in Gertler, Sala and Trigari (2008) capital is introduced into the model assuming that there is a perfect rental market for capital goods. In the present framework with only one worker per firm, as capital is costly, the firm only rents it when the job becomes active (the firm finds a worker). The capital rented by firms becomes a part of the value of an occupied job (see Pissarides, 2000, ch. 1).  $f(k_t, z_t h_t) = k_t^{1-\alpha} (z_t h_t)^{\alpha}$  is the per worker production function, where  $k_t = \frac{K_t}{n_t}$  is the capital-labour ratio. The corresponding aggregate production function is  $f(K_t, n_t z_t h_t)$ . The marginal product of an extra hour of work in the match is  $mpl_t = \alpha k_t^{1-\alpha} (z_t h_t)^{\alpha-1} = \alpha \frac{f(k_t, z_t h_t)}{h_t}$ , and the marginal product of capital is  $mpk_t = (1-\alpha) \frac{f(k_t, z_t h_t)}{k_t}$ . The firm rents as much capital as is necessary to maximize the value of the job. The maximization of  $J_t(r)$  w.r.t.  $k_t$  yields the equilibrium condition for an individual firm's capital stock

$$x_t m p k_t = r_t^k (21)$$

<sup>&</sup>lt;sup>4</sup>Accordingly, the average surplus from working is  $H_{x,t+1} = \gamma H_{t+1}(w_t \left[\pi_t^{\varepsilon_w} \left(\pi^{1-\varepsilon_w}\right)\right]) + (1-\gamma) E_t H_{t+1}(w_{t+1}^*)$ . If the worker starts working in a firm that is not allowed to renegotiate, he will get last period's average wage partially indexed to inflation. This is because in the one firm - one worker setup of this paper also firms in new matches are new, they cannot have negotiated a contract wage in the previous period.

Labour-augmenting productivity  $z_t$  is identical for all matches and follows

$$\log(z_t) = (1 - \rho_z)\log(z) + \rho_z\log(z_{t-1}) + \epsilon_t^z, \text{ where } \rho_z \in (0, 1), \ \epsilon_t^z \stackrel{iid}{\sim} N\left(0, \sigma_z^2\right)$$

The value to the firm of an open vacancy is

$$V_{t} = -\kappa_{t} + E_{t}\beta_{t,t+1} \left\{ q_{t}^{F} \left[ \gamma J_{t+1} \left( w_{t} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) + (1-\gamma) J_{t+1} \left( w_{t+1}^{*} \right) \right] + \left( 1 - q_{t}^{F} \right) V_{t+1} \right\}$$
(22)

The value of a vacancy consists of an exogenous hiring cost  $\kappa_t$ , and of the expected value from future matches. Vacancy costs are subject to shocks

$$\log(\kappa_t) = (1 - \rho_{\kappa}) \log(\kappa) + \rho_{\kappa} \log(\kappa_{t-1}) + \epsilon_t^{\kappa}$$
, where  $\rho_{\kappa} \in (0, 1)$ ,  $\epsilon_t^{\kappa} \stackrel{iid}{\sim} N(0, \sigma_{\kappa}^2)$ 

The introduction of capital does not affect the value of a vacant job  $V_t$  since firms only rent capital upon finding a worker. In equilibrium, all profit opportunities from new jobs are exploited so that the equilibrium condition for the supply of vacant jobs is  $V_t = 0$ . With each firm having only one job, profit maximization is equivalent to this zero-profit condition for firm entry. Setting the equation for  $V_t$  as zero in every period gives:

$$\frac{\kappa_t}{a_t^F} = E_t \beta_{t,t+1} \left[ \gamma J_{t+1} \left( w_t \left[ \pi_t^{\varepsilon_w} \left( \pi^{1-\varepsilon_w} \right) \right] \right) + (1-\gamma) J_{t+1} (w_{t+1}^*) \right]$$
(23)

This vacancy posting condition equates the marginal cost of adding a worker (real cost times mean duration of vacancy) to the discounted marginal benefit from a new worker. After taking into account the free entry condition, the firm surplus reduces to  $J_t$ .

For later use, it is useful to note that the total real profits of the intermediate sector firms, which are paid to the families that own them, is

$$D_t^I = \int_0^{n_t} \left[ x_t^{\alpha} k_{it}^{1-\alpha} \left( z_t h_{it} \right)^{\alpha} - \frac{w_{it}}{P_t} h_{it} \left( 1 + s_t \right) - r_t^k k_{it} \right] di - \kappa_t v_t.$$
 (24)

Multiperiod bargaining set up Unlike with period-to-period bargaining, in the presence of staggered contracting, firms and workers have to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Accordingly, the first order condition for wage-setting is given by:

$$\eta_t \Delta_t J_t(r) = (1 - \eta_t) \Sigma_t H_t(r)$$
(25)

where the partial derivatives of the surplus equations w.r.t. the wage  $\Delta_t = P_t \frac{\partial H_t(r)}{\partial w_t}$  and

 $\Sigma_t = -P_t \frac{\partial J_t(r)}{\partial w_t}$  denote the effect of a rise in the *real* wage on the worker surplus and (minus) the effect of a rise in the real wage on the firm's surplus respectively (see Appendix for details).

$$\Delta_{t} = h_{t} (1 - \tau_{t}) + E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) \gamma \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1 - \varepsilon_{w}} \right) \right] \pi_{t+1}^{-1} \Delta_{t+1}$$
 (26)

$$\Sigma_{t} = h_{t} (1 + s_{t}) + E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) \gamma \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1 - \varepsilon_{w}} \right) \right] \pi_{t+1}^{-1} \Sigma_{t+1}$$
 (27)

These expressions can be interpreted as the discounting factors for the worker and the firm (respectively) for evaluating the value of the future stream of wage payments. As wage contracts extend over multiple periods, agents have to take into account also the future probabilities of not being allowed to renegotiate the wage, or of not surviving exogenous destruction. In the one firm - one worker setup, used in this paper, the discounting factors would be equal across agents unless distortionary taxes were breaking this symmetry<sup>5</sup>. With staggered bargaining, labour taxes enter the discounting factor equations of the agents implying that workers and firms also take into account the future path of taxation in their negotiating behaviour. As is apparent from the loglinearized forms of the discounting factors, presented in the Appendix, both the worker's and the firm's marginal tax rate effectively reduce the worker's relative bargaining power, and consequently his share of the surplus. This effect on the division of match surplus is amplified by staggered bargaining. In the limiting case of period-by-period bargaining,  $\gamma = 0$ , the partial derivatives of the surpluses w.r.t. the wage reduce to  $\Delta_t = h_t (1 - \tau_t)$ , and  $\Sigma_t = h_t (1 + s_t)$ , and the first order condition accordingly reduces to its period-by-period counterpart  $\eta (1 - \tau_t) J_t = (1 - \eta) (1 + s_t) H_t$ .

Given that the probability of wage adjustment is i.i.d., and all matches at renegotiating firms end up with the same wage  $w_t^*$ , the evolution of the nominal average hourly wage in the economy can be expressed as a convex combination of the contract wage and the average wage across the matches that do not renegotiate, after taking into account the indexation scheme.

$$w_{t+1} = (1 - \gamma) w_{t+1}^* + \gamma \int_0^{n_t} \frac{w_{it}}{n_t} \left[ \pi_t^{\varepsilon_w} \left( \pi^{1 - \varepsilon_w} \right) \right] di$$
 (28)

Wage dynamics The staggered bargaining framework has implications on the behavior of workers and firms. To describe wage dynamics in the presence of staggered contracting, we will develop loglinear expressions for the relevant wage equations in the same way as in Gertler, Sala and Trigari (2008). The contract wage is solved by first linearizing the first order condition

<sup>&</sup>lt;sup>5</sup>In Gertler and Trigari (2009), this is not the case. Differences in the worker's and the firm's optimization perspectives, a "horizon effect", arises because large firms take into account possible changes in future hiring rates. The effect of distortionary taxes is different. Proportional tax rates influence the *division* of the total surplus from a job in equilibrium, irrespective of the bargaining horizon (see Pissarides, 2000, chapter 9).

$$\widehat{J}_t(r) + \widehat{\Delta}_t = \widehat{H}_t(r) + \widehat{\Sigma}_t - \frac{1}{1 - n}\widehat{\eta}_t \tag{29}$$

and then plugging into the FOC the value equations and discounting factors for the worker and the firm respectively in their loglinearized form. The latter, as well as the derivation of the contract wage, are presented in detail in the Appendix. The resulting contract wage is

$$\widehat{w}_{t}^{*} = \left[1 - \iota\right] \widehat{w}_{t}^{0}\left(r\right) + \iota E_{t}\left(\widehat{\pi}_{t+1} - \varepsilon_{w}\widehat{\pi}_{t}\right) + \iota E_{t}\widehat{w}_{t+1}^{*}$$

$$(30)$$

where  $\iota = \overline{\beta} (1 - \overline{\rho}) \gamma$ . This is the optimal wage set at time t by all matches that are allowed to renegotiate their wage. As is usual with Calvo contracting, it depends on a wage target  $w_t^0(r)$  and next period's optimal wage. As the probability of not being able to renegotiate the wage approaches zero  $\gamma \longrightarrow 0$ ,  $\iota \longrightarrow 0$ , and the contract wage,  $w_t^*$ , approaches the period-by-period Nash wage.

Unlike in the more conventional set up of New Keynesian models, where Calvo wage contracting is combined with a monopolistic supplier of labour, the target wage here also includes a spillover effect that brings about additional rigidity on top of that implied by the Calvo scheme alone. Gertler and Trigari (2009) show how these spillover effects result from wage bargaining. The target wage can then be decomposed into two parts

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[ \widehat{w}_{t+1} - \widehat{w}_{t+1}^* \right]$$
(31)

where  $\varphi_H \Gamma = \frac{(1-\eta)\beta q^w}{(1-\iota)}$  is the spillover effect<sup>6</sup>. The spillover coefficient is positive, indicating that whenever the expected average market wage  $E_t \widehat{w}_{t+1}$  is higher than the expected contract wage  $E_t \widehat{w}_{t+1}^*$ , (reflecting unusually good labour market conditions), this raises the target wage in the negotiations. Thus, wage rigidity and the resulting employment dynamics are not only a product of the Calvo-type rigidity in wage setting, but also of the spillover effects from the Nash bargaining process.

The spillover-free component of the target wage is of the same form than the period-by-period negotiated wage, adjusted for the multiperiod discounting factors.

$$\widehat{w}_{t}^{0} = \varphi_{x} \left( \widehat{x}_{t} + \widehat{mpl}_{t} \right) - \varphi_{k} (\widehat{r}_{t}^{k} + \widehat{k}_{t}) + \varphi_{m} \widehat{mrs}_{t} + \varphi_{H} E_{t} \left( \widehat{q}_{t}^{W} + \widehat{H}_{t+1} \left( w_{t+1}^{*} \right) + \widehat{\beta}_{t,t+1} \right)$$

$$- \varphi_{h} \widehat{h}_{t} - \varphi_{s} \widehat{s}_{t} + \varphi_{\tau} \widehat{\tau}_{t} + \varphi_{\tau^{c}} \widehat{\tau}_{t}^{c} + \varphi_{D} E_{t} \left[ \widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1} \right] + \varphi_{\eta} \widehat{\eta}_{t} + \widehat{P}_{t}$$

$$(32)$$

Increases in the productivity of the match, in the marginal rate of substitution of the worker/consumer, in the value of the worker's outside option, in the labour tax of the worker or the consumption tax, and in the bargaining power of the worker raise the target wage in the negotiations, whereas, increases in the cost of capital, in working hours per worker and

<sup>&</sup>lt;sup>6</sup>In Gertler and Trigari's (2006) original framework, there is also an indirect spillover effect because the expected hiring rate of the large renegotiating firm affects the bargaining outcome. In the present one worker per firm setup that effect disappears.

in the firms' social security contributions lower the target wage. Finally, combining all the relevant elements of the wage bargaining outcome yields a second-order difference equation for the evolution of the average wage (see Appendix for detailed derivation)

$$\widehat{w}_{t} = \lambda_{b} \left( \widehat{w}_{t-1} + \varepsilon_{w} \widehat{\pi}_{t-1} - \widehat{\pi}_{t} \right) + \lambda_{0} \widehat{w}_{t}^{0} + \lambda_{f} E_{t} \left( \widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t} \right)$$
(33)

Due to staggered contracting,  $\widehat{w}_t$  depends on the lagged wage  $\widehat{w}_{t-1}$ , the spillover-free target wage  $\widehat{w}_t^0$ , and the expected future wage  $E_t\widehat{w}_{t+1}$ .

#### 2.3.3 Determining hours of work

While matches are restrained to renegotiate the wage only with a given exogenous probability, hours per worker can be renegotiated at each point in time. With efficient Nash bargaining, optimal hours of work can be found from the following first order condition obtained by differentiating the Nash maximand w.r.t hours

$$(1 - \tau_t) x_t f_{h,t} = (1 + s_t) \frac{g'(h_t)}{\Lambda_t}$$

where  $f_{h,t}$  is, as before, the marginal product of the labour input i.e. hours, and which, using the expressions for the production and utility functions, can be written as

$$(1 - \tau_t) x_t m p l_t = (1 + s_t) m r s_t (1 + \tau_t^c)$$
(34)

This optimality condition equates the value of marginal product to the marginal rate of substitution between work and leisure, and resembles, thus, to the corresponding condition in a competitive labour market. However, with labour market frictions, while the hourly wage is such that the marginal cost to the worker from working is equal to the marginal gain to the firm, neither of these measures needs to be equal to the wage. It is important to observe that the optimality condition for hours determines the optimal hours per worker, i.e. the intensive margin of labour adjustment. This individual labour input of a worker is determined *irrespective of the wage*. But the model also allows for labour adjustment in the number of workers, as defined by the vacancy posting condition and the matching function.

## 2.4 Final good firms

There are two types of final goods firms. One produces private consumption goods and the other type of final goods firm produces public consumption goods<sup>7</sup>.

<sup>&</sup>lt;sup>7</sup>This is a standard assumption in New Open Economy Macro Models that assess fiscal policy. E.g. in Obstfeld and Rogoff's (1996) extension of the Redux model, government spending is introduced as a basket of public consumption goods aggregated in the same way as for private consumption.

#### 2.4.1 Private consumption good

The private consumption good is a composite of intermediate goods distributed by a continuum of monopolistically competitive wholesale firms at home and abroad. Wholesale firms, their products and prices are indexed by  $i \in [0,1]$ . Final good firms operate under perfect competition and purchase both domestically produced intermediate goods  $y_{H,t}(i)$  and imported intermediate goods  $y_{F,t}(i)$ . They minimize expenditure subject to the following aggregation technology

$$C_{t} = \left[ (1 - W)^{\frac{1}{\varpi}} \left( \left[ \int_{0}^{1} y_{H,t} \left( i \right)^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}} \right)^{\frac{\varpi - 1}{\varpi}} + W^{\frac{1}{\varpi}} \left( \left[ \int_{0}^{1} y_{F,t} \left( i \right)^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}} \right)^{\frac{\varpi - 1}{\varpi}} \right]^{\frac{\varpi}{\varpi - 1}}$$

$$(35)$$

where  $\varpi$  measures the trade price elasticity, or elasticity of substitution between domestically produced intermediate goods and imported intermediate goods in the production of final goods for given relative prices, and W is the weight of imports in the production of final consumption goods. The parameter  $\varepsilon > 1$  is the elasticity of substitution across the differentiated intermediate goods produced and distributed within a country.

The optimization problem determining the allocation of expenditure between the individual varieties of domestic and foreign intermediate goods yields the following demand curves facing each wholesale firm

$$y_{H,t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t}$$
(36)

$$y_{F,t}(i) = \left(\frac{p_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} Y_{F,t}$$
(37)

where  $P_{H,t}$  and  $P_{F,t}$  are the aggregate price indexes for the domestic and foreign intermediate goods respectively

$$P_{H,t} = \left[ \int_{0}^{1} p_{H,t} \left( i \right)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$
(38)

$$P_{F,t} = \left[ \int_{0}^{1} p_{F,t} \left( i \right)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$
(39)

To determine the optimal allocation between the domestic and imported intermediate goods, the final good firm minimizes costs  $P_{H,t}Y_{H,t}+P_{F,t}Y_{F,t}$  subject to its production function or aggregation constraint. This yields the demands for the domestic and foreign intermediate good bundles by domestic final good producers

$$Y_{H,t} = (1 - W) \left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t \tag{40}$$

$$Y_{F,t} = W \left(\frac{P_{F,t}}{P_t}\right)^{-\varpi} C_t \tag{41}$$

where  $P_t$  is the home country's aggregate price index, or consumption price index

$$P_{t} = \left( (1 - W) P_{H,t}^{1-\varpi} + W P_{F,t}^{1-\varpi} \right)^{\frac{1}{1-\varpi}} \tag{42}$$

At the level of individual intermediate goods the law of one price holds<sup>8</sup>. That, together with the assumption that the weight of the home country good in the foreign consumer price index is infinitesimally small, implies that  $P_{F,t}$  is equal to the foreign CPI  $P_t^f$  (see Galí-Monacelli, 2005).

## 2.4.2 Public consumption good

The public consumption good is composed of only domestic intermediate goods  $g_t(i)$ . This assumption implies, contrary to e.g. the Redux model, full home bias in government spending. This simplifying assumption can be supported by the observation from input-output tables that the use of foreign intermediate goods in government spending is significantly lower than in private consumption.

$$G_{t} = \left[ \int_{0}^{1} g_{t} \left( i \right)^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}} \tag{43}$$

Each wholesale firm i selling intermediate goods to the public consumption good producer faces the following demand schedule

$$g_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} G_t \tag{44}$$

<sup>&</sup>lt;sup>8</sup>Note, however, that due to home bias in consumption the basket of consumed goods may differ in the two areas, and therefore purchasing power parity does not hold.

## 2.5 Wholesale firms and price setting

The wholesale firms buy the homogeneous intermediate goods at nominal price  $p_{H,t}x_t$  per unit and transform them one-to-one into the differentiated product. As in most models that incorporate labour market matching into the NK framework, the price setting decision is separated from the wage setting decision to maintain the tractability of the model<sup>9</sup>. Price rigidities arise at the wholesale level while search frictions and wage rigidity only affect directly the intermediate goods sector.

There is Calvo-type stickiness in price-setting and the relative price of intermediate goods  $x_t$  coincides with the real marginal cost faced by wholesale firms. In each period, the wholesale firm can adjust its price with a constant probability  $1-\xi$  which implies that prices are fixed on average for  $\frac{1}{1-\xi}$  periods. The wholesale firm's optimization problem is to maximize expected future discounted profits by choosing the sales price  $p_{H,t}(i)$ , taking into account the pricing frictions and the demand curve they face. It is assumed that the wholesale firm sells the home-country intermediate goods for the same price for domestic and foreign final goods producers, and for the domestic government.

The first order condition for the pricing decision of a wholesale firm that reoptimizes at t is

$$E_{t} \sum_{s=0}^{\infty} \xi^{s} \beta_{t,t+s} \left[ \left( \frac{p_{H,t}(i)}{P_{H,t+s}} \right) y_{t+s} \left( i \right) - x_{t+s} y_{t+s} \left( i \right) \right] = 0$$
 (45)

where  $y_t(i)$  is the demand of firm i's product by domestic private consumption good firms, foreign private consumption good firms and the domestic government as outlined in the previous section

$$y_{t}(i) = y_{H,t}(i) + y_{H,t}^{f}(i) + g_{t}(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{t}^{D}$$

where  $Y_t^D$  stands for total demand for domestic intermediate goods. All wholesale firms are identical except that they may have set their current price at different dates in the past. However, in period t, if they are allowed to reoptimize their price, they all face the same decision problem and choose the same optimal price  $p_{H,t}^*$ . Using the definition of the discount factor and rearranging, the FOC can be rewritten as

$$E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[ (1-\varepsilon) \left( \frac{p_{H,t}^*}{P_{H,t+s}} \right) + \varepsilon x_{t+s} \right] \left( \frac{1}{p_{H,t}^*} \right) \left( \frac{p_{H,t}^*}{P_{H,t+s}} \right)^{-\varepsilon} Y_{t+s}^D = 0$$
 (46)

which can be solved for  $\frac{p_{H,t}^*}{P_{H,t}}$  to yield the following pricing equation

$$\frac{p_{H,t}^*}{P_{H,t}} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} x_{t+s} \left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon} Y_{t+s}^D}{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{P_{H,t+s}}{P_{H,t}}\right)^{\varepsilon - 1} Y_{t+s}^D}$$
(47)

<sup>&</sup>lt;sup>9</sup>A number of extensions merge the intermediate and retail sectors so that there are interactions between wage and price setting at the level of the individual firm. E.g. Christoffel et al. (2009) assess the implications of that specification for inflation dynamics.

where  $\frac{\varepsilon}{\varepsilon-1} = \mu$  is the flexible-price markup. This is the standard Calvo result. In the absence of price rigidity, the optimal price would reduce to a constant markup over marginal costs. Log-linearizing the FOC around the steady state yields the New Keynesian Phillips Curve where domestic inflation depends on marginal costs and expected future inflation

$$\widehat{\pi}_{H,t} = \nu \widehat{x}_t + \beta E_t \widehat{\pi}_{H,t+1} \tag{48}$$

where  $\nu = \frac{(1-\xi)(1-\xi\beta)}{\xi}$ . Total real profits of the wholesale sector firms are

$$D_t^R = \int_0^{n_t} \left[ \left( \frac{p_{H,t}(i)}{P_{H,t}} - x_t \right) y_t(i) \right] di.$$
 (49)

#### 2.6Fiscal policies

The public sector's role in this economy is to collect taxes and use them to finance unemployment benefits and lump-sum transfers as well as government spending  $G_t$ . We make the simplifying assumption that the government budget is balanced in each period with the help of lumpsum transfers. In real terms, the government budget constraint thus reads as

$$\frac{TR_t}{P_t} = n_t \frac{w_t}{P_t} h_t(\tau_t + s_t) + \tau_t^c C_t - \frac{P_{H,t}}{P_t} G_t - bu_t$$
 (50)

The per period lumpsum transfers thus depend positively on tax revenue from labour taxes or consumption taxes. On the other hand, transfers have to be cut if government spending or unemployment benefit expenditure increase. Government spending is subject to shocks

$$\log(G_t) = (1 - \rho_G)\log(\overline{G}) + \rho_G\log(G_{t-1}) + \epsilon_t^G$$
, where  $\rho_G \in (0, 1)$ ,  $\epsilon_t^G \stackrel{iid}{\sim} N\left(0, \sigma_G^2\right)$ 

#### Equilibrium 2.7

For each intermediate good, supply must equal total demand. The demand for good i is, as shown previously,  $y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t^D$ , where  $Y_t^D$  is total demand for domestic intermediate. diate goods by domestic and foreign final goods firms and the domestic government. Using the expressions for the demands for domestic intermediate good bundles derived previously, this can be written as

$$y_t(i) = \left(\frac{p_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left\{ (1 - W) \left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^f}\right)^{-\varpi} C_t^f + G_t \right\}$$
(51)

Following Galí and Monacelli (2005) defining an index for aggregate domestic demand  $Y_t^D = \left[\int\limits_0^1 y_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ allows us to rewrite this as}$ 

$$Y_t^D = (1 - W) \left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t + W \left(\frac{P_{H,t}}{P_t^f}\right)^{-\varpi} C_t^f + G_t$$

Aggregate demand for domestic intermediate goods has to equal their aggregate supply minus the resources lost to vacancy posting and the costs of capital utilization, leading to the home economy's aggregate resource constraint

$$Y_{t} = (1 - W) \left(\frac{P_{H,t}}{P_{t}}\right)^{-\varpi} C_{t} + W \left(\frac{P_{H,t}}{P_{t}^{f}}\right)^{-\varpi} C_{t}^{f} + G_{t} + A(\nu_{t}) K_{t-1}^{p} + \kappa_{t} \nu_{t}$$
 (52)

where the demand for domestic intermediate goods by foreign final goods firms follows the AR(1) process

$$\log(C_t^f) = (1 - \rho_{C_t^f}) \log(\overline{C_t^f}) + \rho_{C_t^f} \log(C_{t-1}^f) + \epsilon_t^{C_t^f}$$
, where  $\rho_{C_t^f} \in (0, 1)$ ,  $\epsilon_t^{C_t^f} \stackrel{iid}{\sim} N\left(0, \sigma_{C_t^f}^2\right)$ 

While the above resource constraint equation states that in equilibrium domestic output has to equal its usage as consumption, exports and government spending, market-clearing in the intermediate good sector also requires

$$Y_t = K_t^{1-\alpha} \left( n_t z_t h_t \right)^{\alpha} \tag{53}$$

The net foreign asset position (in real terms) is determined by the trade balance - the difference between domestic output and domestic consumption.

$$\frac{B_t^f}{P_t} - R_{t-1}^f p\left(b_{t-1}^f\right) \frac{B_{t-1}^f}{P_t} = \frac{P_{H,t}}{P_t} Y_t - C_t - I_t - \frac{P_{H,t}}{P_t} G_t - A\left(\nu_t\right) K_{t-1}^p - \frac{P_{H,t}}{P_t} \kappa_t v_t \tag{54}$$

This relation is obtained by combining the consumers' budget constraint, the government's budget constraint and the economy's aggregate resource constraint as well as the equation for total real dividends accrued to households, i.e. the sum of the profits in the intermediate and wholesale sectors

$$D_{t} = \frac{P_{H,t}}{P_{t}} Y_{t} - n_{t} \frac{w_{t}^{*}}{P_{t}} h_{t} (1 + s_{t}) - r_{t}^{k} K_{t} - \frac{P_{H,t}}{P_{t}} \kappa_{t} v_{t}.$$
 (55)

## 3 Model parametrization

The current model parametrization is mainly based on Obstbaum (2012) and uses a combination of estimation and calibration. The parameters of the model have been estimated and calibrated to fit the Finnish economy, but the interpretation and logic of the results can be generalized more broadly to a small member state of a monetary union. The current parametrization is to be viewed as a starting point for further development of the model. For a detailed discussion on the estimation and calibration, we refer to Obstbaum (2012).

The model has been estimated in Obstbaum (2012 in log-linearized form using Bayesian Maximum Likelihood methods described in e.g. An and Schorfheide (2007). Parameters are either matched to the sample means of steady state values of the observed variables using Finnish data in the period 1994Q1-2010Q3 ("great ratios") or taken from evidence found in micro level studies. The parameters for which such empirical observations are not available, are fixed using standard ranges of parameter values in the business cycle and labour market matching literature. Used parameter values are summarized in Table 1.

Table 1.	Fixed	parameter	values
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Parameter	Value	Explanation
Preferences		
β	.992	Time-discount factor
$\phi$	5	Labour supply (Frish) elasticity of $0.2$
$\varrho$	1.5	Risk aversion
$\varkappa$	0.6	External habit persistence
Labour market		
$\alpha$	0.67	Labour elasticity of production
$\sigma$	0.6	Elasticity of matches w.r.t. unemployment
$\sigma_m$	0.6	Efficiency of matching
ho	0.06	Exogenous quarterly job destruction rate
$\eta$	0.6	Bargaining power of workers
b	0.4	Unemployment benefits
z	1.1	Technology, targets output $Y=1$
$\gamma$	0.7	Calvo wage parameter
$arepsilon_w$	0	Wage indexing parameter
Capital		
$\delta_k$	0.025	Capital depreciation rate
$\eta_k$	2.4	Capital adjustment cost elasticity
$\gamma_{b^f}$	0.05	Debt-elasticity of interest rates
$\eta$	0.7	Sensitivity of capital utilization rate
Wholesale sector		
$\varepsilon$	6	Elasticity of substitution, implies a markup of 20 percent
$\xi$	0.75	Calvo price parameter
Final goods sector		
$\overline{\omega}$	2	Trade price elasticity
W	0.1	Import content of final goods production

Table 2. Steady state ratios

Variable	Value	Description
$\overline{Y}$	1	Output
C	0.57	Private consumption
I	0.22	Private investment
G	0.2	Government spending
$\kappa v$	0.01	Total vacancy costs
u	0.1	Unemployment rate
n	0.9	Employment
qw	0.54	Probability of finding a job
qf	0.7	Probability of finding a worker
$b/(wh(1-\tau))$	0.64	Net replacement rate
nwh	0.5	Wage bill
$ au^C$	0.11	Consumption tax
au	0.15	Labour tax rate on employee
s	0.12	Employers' social security contribution
$TR$ / $ au^{LS}$	0.04	Lump-sum transfers

The steady state values of key model variables implied by the current parametrization can be found in Table 2. The steady state equations of the model are, in turn, provided in Appendix A.1. In the steady state, output is normalized to one, so that GDP components can be interpreted directly as percent shares of GDP.

## 4 Model simulation: Demand and supply shocks have raised inflation significantly

In this section we present simulations based on our model. We examine the effects of shocks that the Finnish and euro area economies have faced after the Covid-19 pandemic. As mentioned earlier, the parameters of the model are based on the Finnish economy, but the results can be generalized more broadly to the small member state of the monetary union. Since monetary policy is exogenous in the simulations, we do not examine or take into account the role of monetary policy in balancing the shocks.

We illustrate the recent developments in the Finnish and euro area economies through four calibrated shocks presented in Table 3. The calibration is based on developments in the Finnish and euro area economies after the Covid-19 pandemic until the end of 2022. It should be noted, that instead of evaluating exact quantitative effects, the main focus in the simulations is to analyse the transmission channels of the factors that affect the real economy and accelerate inflation. We pay specific attention to the effects of the shocks on the tightness of the labour market and further to price and wage pressures.

<sup>&</sup>lt;sup>10</sup>Simulated effects of different shocks are sensitive to the chosen parameter values and to the persistence of the shocks. For example, in the case of import price and working hours shocks higher wage indexation

Table 3. Shocks used in the simulation

shock	size	persistence
public demand growth, $ ho_G$	3%	0.8
export demand growth, $ ho_{Cf}$	1%	0.95
import price increase, $\rho_{PM}$	20%	0.95
marginal rate of substitution between consumption and hours worked, $\rho_{MRS}$	-2%	0.8

First, we examine the effects of public demand growth (shock size 3% and quarterly persistence 0.8). Recent crises increased public demand by an exceptional amount. After the Covid-19 pandemic broke out, governments supported households and companies in many ways in order to prevent bankruptcies and widespread unemployment. With the war of aggression initiated by Russia and the resulting energy crisis, new support measures were introduced and existing ones were extended. As a result, debt levels increased and the balance of public finances deteriorated.

Second, we employ a shock on export growth (shock size 1% and quarterly persistence 0.95). Euro area external demand has recovered from the collapse in exports triggered by the pandemic. Finnish exports have also recovered, but remain modest compared to pre-crisis forecasts.

The third shock concerns import price increases (shock size 20% and quarterly persistence 0.95). Energy prices and, with them, also import prices, began to rise strongly already at the end of 2021, when the pandemic receded and the economy got back on track. The rise in import prices was clearly accelerated by the faster-than-anticipated recovery following the pandemic and simultaneous problems with the availability of goods and materials, as well as bottlenecks in international freight traffic. This led to significant supply and demand imbalances. The Russian attack on Ukraine led to an energy crisis in Europe, as a result of which energy and import prices rose even further.

Finally, the fourth shock is a decrease of the marginal rate of substitution between consumption and hours worked (size of the shock -2% and quarterly persistence of 0.8). This shock decreases working hours per employee initially by roughly 1.3%. The Covid-19 pandemic caused a strong shock to the euro area labour market. Both total working hours and hours per employee fell significantly. Pre-pandemic levels in total working hours were reached in the first half of 2022. On the other hand, the average working hours per person have remained more clearly and permanently below the pre-pandemic level. During 2022, the number of working hours has remained somewhat unchanged and at the end of the year, the number of working hours per employee in the euro area was still 2% below the pre-pandemic level. In Finland, the dynamics of working hours in recent years differs from the development of the euro area. The total number of working hours in Finland decreased after the outbreak of the pandemic, but unlike in the euro area, the number of working hours per employee fell only temporarily. The total number of working hours in the economy improved quickly in Finland, like in the euro area, returning to the pre-pandemic level in early 2021. However, the number of working hours per employee have decreased modestly since 2021.

parameter,  $\epsilon_W = 0.7$ , would affect the responses of nominal and real wages by some 0.1-0.2 percentage points. Compared to no indexation, this would slight increase volatility of real variables and lead to slightly higher unemployment. However, this would not have impact on the qualitative interpretation of the impacts of the shocks.

All the shocks presented above, which describe the developments observed in Europe and Finland, also accelerate inflation according to model simulations. The increase in public consumption has accelerated inflation, but changes on the supply side, i.e. a decrease in average working hours and increased import prices, are also reflected in the inflation rate. In 2022, inflation in the euro area averaged 8.4%, ranging from 5.9% in France to 19.4% in Estonia. In Finland, inflation was 7.2%. The shocks in our simulation are able to partly explain the acceleration in inflation. However, factors that accelerated inflation, which have not been taken into account in the model simulations, are, for example, the effects of global supply chain disruptions caused by the Covid-19 pandemic, the spillover of import prices also to the prices of domestic value added and possibly further to wages (see the simulation below), and the recovery of price levels from the low price levels of the pandemic year 2020. In this simulation, the inflationary effects of the shocks appear to be rather short-lived, but the exact timing and magnitude of the effects of the shocks are sensitive to the estimated and calibrated model parameters.

## 4.1 Findings of the simulation

The findings of the simulation exercise are presented in this Section. We first discuss the impulse responses of each individual shock (Figures 1–4) All the examined shocks initially accelerate inflation, but their effects on the real economy, i.e. the labour market and total output, are different.

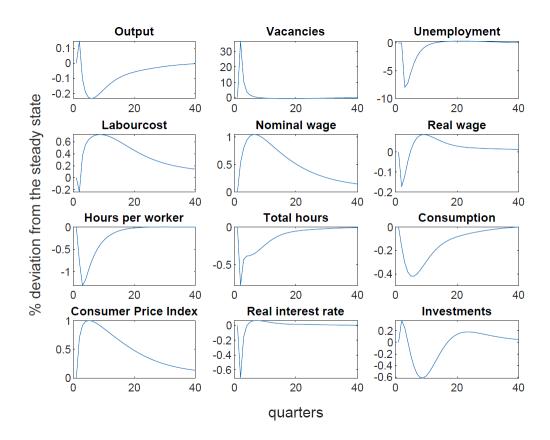
We first examine the effects of the two supply shocks, i.e. the fall in average working hours (induced by a shock on the marginal rate of substitution between consumption and hours worked) given in Figure 1 and the rise in import prices in Figure 2. Especially the increase in import prices would seem to significantly slow down economic growth. The decrease in average working hours also has a persistent effect of decreasing overall production.

There is an important difference in the effects of the two supply shocks on the labour market. In the model simulation, the decrease in the supply of working hours significantly tightens the labour market and induces wage pressures, whereas the import price shock mitigates both labour market tightness and wage pressures.

The effects of the reduction in working hours per employee may be influenced by a substitution effect. Thereby employers respond to the lower supply of hours per worker (intensive margin) by posting more vacancies to employ a higher number of workers (extensive margin) to compensate for the lower hours per worker. The rise in vacancies and fall in unemployment leads to a tightening of the labour market ( $\frac{v}{u}$  increases), which in turn tends to raise wages. The fall in output due to the reduced labour input at the intensive margin is at least partly compensated by an increase of labour input at the extensive margin. The reduction in output remains thus relatively muted, although in comparison to the import price shock the shock itself is also smaller.<sup>11</sup>

<sup>&</sup>lt;sup>11</sup>The exact effects on the labour market and employment are also sensitive to both the chosen parameters of the model and the labour market characteristics. For example, it is not certain how well skilled labour can be replaced by increasing the number of jobs, nor how shorter working hours affect the international competitiveness of companies.

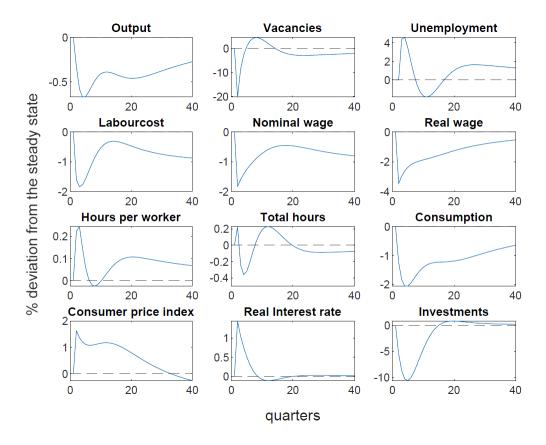
Figure 1. Effects of a shock reducing average working hours



An interesting question is whether the decrease in working hours per employee is a permanent or temporary phenomenon. A recovery to pre-crisis levels would reduce tightness in the labour market, support growth, as well as reduce wage and inflation pressures.

The import price shock seems to have persistent negative effects on real economy according to the model simulations (Figure 2). This is partly due to the persistence of the shock itself, but also due to its negative effect on household real income that contributes to a strong response in private demand, i.e. consumption and investment. The weaker expected aggregate demand (output) reduces labour demand, manifesting itself in reduced vacancy creation by firms and a rise in unemployment. The consequent fall in labour market tightness also contributes to weaker wages. As the effect of the import price shock on inflation and thus real income is relatively short-lived, the labour market starts to recover manifesting itself through a recovery in vacancy creation. Due to labour market rigidities, the unemployment rate normalizes slowly, only two years after the shock. This is also reflected in the weakness of wages over several years. Note that one reason for the very different effects of the two supply shocks is because the shock to intensive margin of labour supply can at least partly be compensated by an adjustment at the extensive margin, whereas the shock to import prices involves no such channel of substitution.

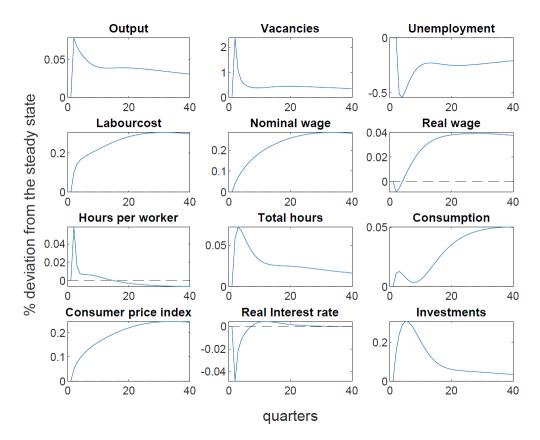
Figure 2. Effects of a positive import price shock



In the model simulations, the increase in export demand has a small but long-lasting positive effect on output (Figure 3). The increase in export demand increases domestic wealth and leads to a persistent acceleration of private consumption. The higher output arising from the export demand shock has a positive overall effect on labour markets. Firms post more job vacancies and unemployment falls, leading to an increase in labour market tightness. This tends to have a positive effect on wages, which also contributes to higher private consumption and output. Also hours of work increase in response to the improved macroeconomic conditions. Overall, however, the effects of the shock remain small, because the examined export demand shock (1%) is quite small.

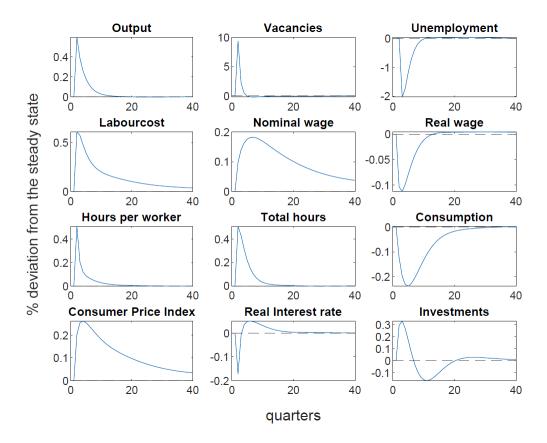
The shock to public consumption has a rather short-lived positive effect on output. However, this effect is reduced by the fact that public consumption is financed by lump sum taxes, which directly displace private consumption. The negative effects of raising taxes on growth would be more persistent if they were, for example, implemented through the taxation of work and thus cause negative behavioural effects. The labour market effects are qualitatively similar to those arising from the export demand shock. In response to higher demand, firms post more vacancies, unemployment falls and labour market tightness rises. We observe a mild positive effect on wages. Overall, the effects of public consumption depend on the coefficients of fiscal policy, the magnitude of which has been much debated (see, e.g. Ramey, 2019 and Batini et al., 2014). The fiscal multiplier in our model is similar in magnitude to those used in similar models, i.e. in the short run 0.7 at most.

Figure 3. Effects of a positive export demand shock



The demand shocks thus seem to have contributed to the acceleration of inflation and the tightening of the labour market by increasing the demand for labour. In the simulations, the effects of the now materialized demand shocks on the tightness of the labour market seem to be smaller than the effect of the decrease in working hours per employee.

Figure 4. Effects of a increase in public consumption



## 5 Concluding remarks

In this paper we present a New Keynesian DSGE model for the Finnish economy that incorporates labour market frictions and wage rigidity. We slightly modify the staggered wage bargaining framework of Gertler and Trigari (2009) to describe the rigidity in wage determination.

The strength of the model is that it describes the tightness of the labour market, i.e. unemployment and open vacancies, as well as the determination of wages as the end result of negotiations between the worker and the firm. Thus, it enables the analysis of the effects of different shocks, especially in relation to labour market tightness and wage pressures. In these regards, it is a valuable addition to the model toolbox currently being used at the Bank of Finland.

In our empirical exercise we present simulations using four shocks that describe the economic development in the aftermath of the Covid-19 pandemic. These four shocks include positive shocks on public consumption and export demand, as well as a rise in import prices

and a decrease in working hours per employee. The findings illustrate that all of the aforementioned shocks have contributed to the increase in inflation, but their effects on the real economy differ from each other.

The purpose of this article has been to introduce the model and illustrate its' dynamics in an empirical exercise. It should be noted that model simulations are always sensitive to selected parameter values and used model assumptions. They are also sensitive to, for example, the persistence of simulated shocks. This paper acts as a starting point for further work on the model, that includes e.g. the estimation of the model parameters using up-to-date data.

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## A Appendix

## A.1 Steady state of the model economy

Euler equation

$$\beta = \frac{1}{R}$$

Marginal utility of consumption

$$\lambda = (C - \varkappa C)^{-\varrho}$$

Marginal utilility of wealth

$$\Lambda = \frac{\lambda}{(1 + \tau^c)}$$

Interest rate on foreign bonds

$$R^f = R$$

Capital rental rate

$$r^k = xmpk = (R - 1) + \delta^k$$

where

$$mpk = (1 - \alpha) k^{-\alpha} (zh)^{\alpha} = (1 - \alpha) \frac{y}{k}$$

Tobin's Q

$$Q = 1$$

FOC of retail firm

$$x = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon}$$

Matches

$$m = \sigma_m u^{\sigma} v^{1-\sigma}$$

Employment

$$\rho n = m$$

Unemployment

$$u = 1 - n$$

Probability of finding a worker

$$q^F = \frac{m}{v}$$

Probability of finding a job

$$q^W = \frac{m}{u}$$

Labour market tightness

$$\theta = \frac{v}{u}$$

FOC for hours

$$(1 - \tau) xmpl = (1 + s) mrs (1 + \tau^c)$$

where

$$mpl = \alpha k^{1-\alpha} (zh)^{\alpha-1} z = \alpha \frac{y}{h} \text{ and } mrs = \frac{\delta h^{\phi}}{\lambda}$$

Economy-wide resource constraint

$$Y = C + I + G + \kappa v$$
, in the symmetric steady state

Government budget constraint

$$(1-R)B = G + bu + TR - nwh(\tau + s) - \tau^{c}C$$

Market clearing / aggregate output

$$Y = K^{1-\alpha} \left( nzh \right)^{\alpha}$$

Wage

$$w = \frac{\eta}{(1+s)} \left[ \frac{xmpl}{\alpha} - \frac{r^k k}{h} \right] + \frac{(1-\eta)}{(1-\tau)} \left[ \frac{mrs(1+\tau^c)}{(1+\phi)} + \frac{b}{h} + \beta \frac{q^w}{h} H \right]$$

Job creation condition

$$\kappa = q^F \beta J$$

where the firm surplus

$$J = \frac{1}{1 - \beta (1 - \rho)} \left[ x k^{1 - \alpha} (zh)^{\alpha} - wh (1 + s) - r^k k \right]$$

Worker surplus

$$H = \frac{1}{1 - \beta \left(1 - \rho - q^{W}\right)} \left[ wh \left(1 - \tau\right) - \frac{mrsh \left(1 + \tau^{c}\right)}{\left(1 + \phi\right)} - b \right]$$

Worker discount factor

$$\overline{\Delta} = \frac{\overline{h}(1-\overline{\tau})}{1-\overline{\beta}(1-\overline{\rho})\gamma}$$

Firm discount factor

$$\overline{\Sigma} = \frac{\overline{h}(1+s)}{1-\overline{\beta}(1-\overline{\rho})\gamma}$$

## A.2 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation around a deterministic steady state.

Euler equation

$$\widehat{\Lambda}_t = E_t \left( \widehat{\Lambda}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1} \right)$$

Shadow value of wealth

$$\widehat{\Lambda}_t = \widehat{\lambda}_t - \frac{\overline{\tau}^c}{(1 + \overline{\tau}^c)} \widehat{\tau}_t^c$$

Marginal utility of consumption

$$\widehat{\lambda}_t = -\frac{\varrho}{(1-\varkappa)} \left( \widehat{C}_t - \varkappa \widehat{C}_{t-1} \right)$$

Interest rates

$$\widehat{R}_t = \widehat{R}_t^f - \gamma_{bf} \widehat{b}_t^f$$

Capital utilization

$$\widehat{\nu}_t = \eta_{\nu} \widehat{r}_t^k$$

Investment

$$\widehat{I}_t = \frac{1}{(1+\beta)}\widehat{I}_{t-1} + \frac{1/\eta_k}{(1+\beta)}\left(\widehat{Q}_t + \widehat{\epsilon}_t^I\right) + \frac{\beta}{(1+\beta)}E_t\widehat{I}_{t+1}$$

Capital accumulation

$$\widehat{K}_{t}^{p} = \left(1 - \delta^{k}\right) \widehat{K}_{t-1}^{p} + \frac{\overline{I}}{\overline{K}^{p}} \left(\widehat{I}_{t} + \widehat{\epsilon}_{t}^{I}\right)$$

Effective capital

$$\widehat{K}_t = \widehat{\nu}_t + \widehat{K}_{t-1}^p$$

Tobin's Q

$$\widehat{Q}_{t} = \beta \left( 1 - \delta^{k} \right) E_{t} \widehat{Q}_{t+1} + \left[ 1 - \beta \left( 1 - \delta^{k} \right) \right] E_{t} \left( \widehat{r}_{t+1}^{k} + \widehat{\nu}_{t+1} \right) - \left( \widehat{R}_{t} - E_{t} \widehat{\pi}_{t+1} \right)$$

Matching function

$$\hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \,\hat{v}_t$$

Employment dynamics

$$\hat{n}_t = (1 - \bar{\rho}) \, \hat{n}_{t-1} + \frac{\bar{m}}{\bar{n}} \hat{m}_{t-1} - \bar{\rho} \hat{\rho}_t$$

Unemployment

$$\widehat{u}_t = -\frac{1 - \overline{u}}{\overline{u}} \hat{n}_t$$

Transition probabilities

$$\hat{q}_t^F = \hat{m}_t - \hat{v}_t$$

$$\hat{q}_t^W = \hat{m}_t - \hat{u}_t$$

labour market tightness

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t$$

FOC for hours worked

$$\hat{x}_t = m\hat{r}s_t - m\hat{p}l_t + \frac{\overline{\tau}}{(1-\overline{\tau})}\hat{\tau}_t + \frac{\overline{s}}{(1+\overline{s})}\hat{s}_t + \frac{\overline{\tau}^c}{(1+\overline{\tau}^c)}\hat{\tau}_t^c$$

where

$$m\hat{p}l_t = \hat{z}_t - (1 - \alpha)\,\hat{h}_t$$

and

$$m\hat{r}s_t = \phi\hat{h}_t - \hat{\lambda}_t$$

Capital rental rate

$$\hat{r}_t^k = \hat{x}_t + m\hat{p}k_t$$

New Keynesian Phillips Curve

$$\widehat{\pi}_{H,t} = \nu \widehat{x}_t + \beta E_t \widehat{\pi}_{H,t+1}$$

where  $\widehat{\pi}_{H,t} = \widehat{P}_{H,t} - \widehat{P}_{H,t-1}$  is domestic inflation

First order condition for wage setting

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t - \frac{1}{1 - \eta} \widehat{\eta}_t$$

Firm surplus

$$\widehat{J}_{t}(w_{t}^{*}) = \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}} \left(\widehat{x}_{t} + \widehat{mpl}_{t} + \widehat{h}_{t}\right) - \frac{\overline{wh}\left(1 + \overline{s}\right)}{\overline{J}} \left(\widehat{w}_{t}^{*} - \widehat{P}_{t} + \widehat{h}_{t}\right) - \frac{\overline{whs}}{\overline{J}} \widehat{s}_{t} 
- \frac{\overline{r}^{k}\overline{k}}{\overline{J}} \left(\widehat{r}_{t}^{k} + \widehat{k}_{t}\right) - \overline{\beta}\overline{\rho}E_{t}\widehat{\rho}_{t+1} + \overline{\beta}\left(1 - \overline{\rho}\right)E_{t} \left(\widehat{J}_{t+1}\left(w_{t+1}^{*}\right) + \widehat{\beta}_{t,t+1}\right) 
- \frac{\overline{\beta}\left(1 - \overline{\rho}\right)\gamma}{1 - \overline{\beta}\left(1 - \overline{\rho}\right)\gamma} \frac{\overline{wh}\left(1 + \overline{s}\right)}{\overline{J}} E_{t} \left(\widehat{w}_{t}^{*} + \varepsilon_{w}\widehat{\pi}_{t} - \widehat{w}_{t+1}^{*} - \widehat{\pi}_{t+1}\right)$$

Worker discount factor

$$\widehat{\Delta}_{t} = (1 - \iota) \widehat{h}_{t} - \frac{(1 - \iota) \overline{\tau}}{(1 - \overline{\tau})} \widehat{\tau}_{t} + \iota E_{t} \left( \widehat{\beta}_{t, t+1} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{\pi}_{t+1} + \widehat{\Delta}_{t+1} \right) - \overline{\beta} \overline{\rho} \gamma E_{t} \widehat{\rho}_{t+1}$$

Worker surplus

$$\widehat{H}_{t}(w_{t}^{*}) = \frac{\overline{wh} (1 - \overline{\tau})}{\overline{H}} \left( \widehat{w}_{t}^{*} - \widehat{P}_{t} + \widehat{h}_{t} \right) - \frac{\overline{wh}}{\overline{H}} \widehat{\tau}_{t} - \frac{\overline{mrsh} (1 + \overline{\tau}^{c})}{(1 + \phi) \overline{H}} \left[ \widehat{mrs}_{t} + \widehat{h}_{t} \right] \\
- \frac{\overline{mrsh}\overline{\tau}^{c}}{(1 + \phi) \overline{H}} \widehat{\tau}_{t}^{c} - \overline{\beta} \overline{q}^{W} E_{t} \left( \widehat{q}_{t}^{W} + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1} \right) \\
+ \overline{\beta} (1 - \overline{\rho}) E_{t} \left( \widehat{H}_{t+1} \left( w_{t+1}^{*} \right) + \widehat{\beta}_{t,t+1} \right) - \overline{\beta} \overline{\rho} E_{t} \widehat{\rho}_{t+1} \\
+ \frac{\overline{\beta} (1 - \overline{\rho}) \gamma}{1 - \overline{\beta} (1 - \overline{\rho}) \gamma} \frac{\overline{wh} (1 - \overline{\tau})}{\overline{H}} E_{t} \left( \widehat{w}_{t}^{*} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{w}_{t+1}^{*} - \widehat{\pi}_{t+1} \right)$$

Firm discount factor

$$\widehat{\Sigma}_{t} = (1 - \iota) \widehat{h}_{t} + \frac{(1 - \iota) \overline{s}}{(1 + \overline{s})} \widehat{s}_{t} + \iota E_{t} \left( \widehat{\beta}_{t,t+1} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{\pi}_{t+1} + \widehat{\Sigma}_{t+1} \right) - \overline{\beta} \overline{\rho} \gamma E_{t} \widehat{\rho}_{t+1}$$

Optimal contract wage

$$\widehat{w}_{t}^{*} = \left[1 - \iota\right] \widehat{w}_{t}^{0} \left(r\right) + \iota E_{t} \left(\widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t}\right) + \iota E_{t} \widehat{w}_{t+1}^{*}$$

Target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[ \widehat{w}_{t+1} - \widehat{w}_{t+1}^* \right]$$

Spillover-free target wage

$$\widehat{w}_{t}^{0} = \varphi_{x} \left( \widehat{x}_{t} + \widehat{mpl}_{t} \right) - \varphi_{k} (\widehat{r}_{t}^{k} + \widehat{k}_{t}) + \varphi_{m} \widehat{mrs}_{t} + \varphi_{H} E_{t} \left( \widehat{q}_{t}^{W} + \widehat{H}_{t+1} \left( w_{t+1}^{*} \right) + \widehat{\beta}_{t,t+1} \right)$$

$$- \varphi_{h} \widehat{h}_{t} - \varphi_{s} \widehat{s}_{t} + \varphi_{\tau} \widehat{\tau}_{t} + \varphi_{\tau^{c}} \widehat{\tau}_{t}^{c} + \varphi_{D} E_{t} \left[ \widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1} \right] + \varphi_{\eta} \widehat{\eta}_{t} + \widehat{P}_{t}$$

Average wage

$$\widehat{w}_{t} = (1 - \gamma) \, \widehat{w}_{t}^{*} + \gamma \left( \widehat{w}_{t-1} - \widehat{\pi}_{t} + \varepsilon_{w} \widehat{\pi}_{t-1} \right)$$

or

$$\widehat{w}_t = \lambda_b \widehat{w}_{t-1} + \lambda_0 \widehat{w}_t^0 + \lambda_f E_t \widehat{w}_{t+1}$$

Vacancy posting condition

$$\widehat{\kappa}_{t} - \widehat{q}_{t}^{F} = E_{t} \left( \widehat{J}_{t+1} \left( r \right) + \widehat{\beta}_{t,t+1} \right) + \frac{\gamma}{1 - \iota} \frac{\overline{wh} \left( 1 + \overline{s} \right)}{\overline{J}} E_{t} \left( \widehat{w}_{t+1}^{*} + \widehat{\pi}_{t+1} - \widehat{w}_{t} - \varepsilon_{w} \widehat{\pi}_{t} \right)$$

Trade balance

$$\widehat{TB}_{t} = \widehat{Y}_{t} - \overline{C}\widehat{C}_{t} - \overline{I}\widehat{I}_{t} - \overline{G}\widehat{G}_{t} - \overline{r}^{k}\overline{K}\widehat{\nu}_{t} - \overline{\kappa v}\left(\widehat{\kappa}_{t} + \widehat{v}_{t}\right) + \left(\overline{C} + \overline{I}\right)\left(\widehat{P}_{H,t} - \widehat{P}_{t}\right)$$

Economy-wide resource constraint

$$\widehat{Y}_{t} = (1 - W) \, \overline{C} \widehat{C}_{t} + W \overline{C}^{f} \widehat{C}_{t}^{f} + \overline{G} \widehat{G}_{t} + \overline{\kappa v} \left( \widehat{\kappa}_{t} + \widehat{v}_{t} \right) - \left[ (2 - W) \, \overline{C} \varpi W \right] \left( \widehat{P}_{H,t} - \widehat{P}_{t} \right)$$

Consumer price index

$$\widehat{P}_t = (1 - W)\,\widehat{P}_{H,t} + W\widehat{P}_t^f$$

Evolution of debt / Government budget constraint

$$\widehat{bb}_{t} = \overline{Rb}(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{\pi}_{t}) + \overline{G}(\widehat{P}_{H,t} - \widehat{P}_{t} + \widehat{G}_{t}) + b\overline{u}\widehat{u}_{t} + \overline{TR}(\widehat{TR}_{t} - \widehat{P}_{t}) \\
- \overline{nw}\overline{h}(\overline{\tau} + \overline{s})(\widehat{n}_{t} + \widehat{w}_{t} - \widehat{P}_{t} + \widehat{h}_{t}) - \overline{nw}\overline{h}\overline{\tau}\widehat{\tau}_{t} - \overline{nw}\overline{h}\overline{s}\widehat{s}_{t} - \overline{\tau}^{c}\overline{C}(\widehat{\tau}_{t}^{c} + \widehat{C}_{t})$$

Market clearing / aggregate output

$$\widehat{Y}_t = (1 - \alpha) \, \widehat{K}_t + \alpha \left( \widehat{n}_t + \widehat{z}_t + \widehat{h}_t \right)$$

## A.3 Period-by-period Nash bargaining

In the standard MP model, it is assumed that total match surplus,  $S_t = (W_t - U_t) + (J_t - V_t)$ , the sum of the worker and firm surpluses is shared according to efficient Nash bargaining where wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of the worker's and the firm's net return from the match.

$$\max_{w,h} (H_t)^{\eta} (J_t)^{1-\eta}$$

where  $0 \le \eta \le 1$  is the relative measure of workers' bargaining strength.

The worker surplus gets the following form.

$$H_{t} = W_{t} - U_{t} = \frac{w_{t}^{*}}{P_{t}} h_{t} (1 - \tau_{t}) - \frac{g(h_{t})}{\Lambda_{t}} - b + E_{t} \beta_{t,t+1} (1 - \rho_{t+1} - q_{t}^{W}) H_{t+1}$$

and the firm surplus is (after taking into account the free entry condition  $V_t = 0$ )

$$J_{t} = x_{t} f(k_{t}, h_{t}) - \frac{w_{t}^{*}}{P_{t}} h_{t} (1 + s_{t}) - r_{t}^{k} k_{t} + E_{t} \beta_{t, t+1} (1 - \rho_{t+1}) J_{t+1}$$

The first-order condition for wage-setting is

$$\eta \frac{\partial H_t}{\partial w_t} J_t = (1 - \eta) \frac{\partial J_t}{\partial w_t} H_t$$

$$\iff \eta (1 - \tau_t) J_t = (1 - \eta) (1 + s_t) H_t$$

which would, without taxes, correspond to the simple surplus splitting result where the total surplus from the match is shared according to the bargaining power parameter  $\eta$ .

The optimality condition for wage-setting can be rewritten as a wage equation that includes only contemporaneous variables by substituting the value equations into the Nash FOC, and making use of the expressions for the production and utility functions.

$$\frac{w_t^*}{P_t} = \frac{\eta}{(1+s_t)} \left[ \frac{x_t m p l_t}{\alpha} - \frac{r_t^k k_t}{h_t} \right] + \frac{(1-\eta)}{(1-\tau_t)} \left[ \frac{m r s_t (1+\tau_t^c)}{(1+\phi)} + \frac{b}{h_t} + \frac{q_t^W}{h_t} E_t \beta_{t,t+1} H_{t+1} \right] + \frac{\eta}{h_t} E_t \beta_{t,t+1} (1-\rho_{t+1}) J_{t+1} \left[ \frac{1}{(1+s_t)} - \frac{(1-\tau_{t+1})}{(1-\tau_t)} \frac{1}{(1+s_{t+1})} \right]$$

where  $w_t$  is the nominal hourly wage in a match.

The wage equation is a convex combination of what the worker contributes to the match (the first square brackets) and what he has to give up in terms of disutility from supplying hours of work plus a term that accounts for possible changes in tax rates over time. Since workers and firms are homogeneous and all matches adjust their wages every period, they will all choose the same wage. The economy's wage bill is this wage rate times the total number of hours worked in the economy. It is clear from the wage equation that the introduction of taxes works to decrease the worker's relative effective bargaining power from  $\eta$  to  $\frac{\eta}{(1+s_t)}$ . Consequently, economic conditions get a smaller weight in wage determination.

## A.4 Dynamics with wage rigidity

The derivation of the wage under staggered contracting follows Gertler, Sala and Trigari (GST) (2008). The Nash first order condition is in this case

$$\eta_t \Delta_t J_t \left( w_t^* \right) = \left( 1 - \eta_t \right) \Sigma_t H_t \left( w_t^* \right)$$

where the effect of a rise in the real wage on the worker's surplus is

$$\Delta_{t} = P_{t} \frac{\partial H_{t}(w_{t})}{\partial w_{t}}$$

$$= h_{t} (1 - \tau_{t}) + E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) \gamma P_{t+1} \frac{\partial H_{t+1}(w_{t} \left[\pi_{t}^{\varepsilon_{w}} \left(\pi^{1-\varepsilon_{w}}\right)\right])}{\partial w_{t}}$$

$$= h_{t} (1 - \tau_{t}) + E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) \gamma \left(\left[\pi_{t}^{\varepsilon_{w}} \left(\pi^{1-\varepsilon_{w}}\right)\right] h_{t+1} (1 - \tau_{t+1}) + E_{t} \beta_{t+1,t+2} \gamma P_{t+2} \frac{\partial H_{t+2} \left(w_{t} \left[\pi_{t+1}^{\varepsilon_{w}} \left(\pi^{1-\varepsilon_{w}}\right)\right] \pi_{t}^{\varepsilon_{w}} \left(\pi^{1-\varepsilon_{w}}\right)\right]\right)}{\partial w_{t}}\right) \dots$$

$$= E_{t} \sum_{s=0}^{\infty} \beta_{t,t+s} \zeta_{t,t+s} \gamma^{s} \left[\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\varepsilon_{w}} \left(\pi^{1-\varepsilon_{w}}\right)^{s}\right] h_{t+s} (1 - \tau_{t+s})$$

$$\iff \Delta_{t} = h_{t} (1 - \tau_{t}) + E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) \gamma \left[\pi_{t}^{\varepsilon_{w}} \left(\pi^{1-\varepsilon_{w}}\right)\right] \pi_{t+1}^{-1} \Delta_{t+1}$$

where  $\zeta_{t,t+s}$  denotes the probability of match survival from period t to period t+s. And similarly for the firm

$$\Sigma_{t} = -P_{t} \frac{\partial J_{t}\left(w_{t}\right)}{\partial w_{t}} = h_{t}\left(1 + s_{t}\right) + E_{t} \beta_{t,t+1}\left(1 - \rho_{t+1}\right) \gamma \left[\pi_{t}^{\varepsilon_{w}}\left(\pi^{1 - \varepsilon_{w}}\right)\right] \pi_{t+1}^{-1} \Sigma_{t+1}$$

The dynamic contract wage equation is solved by first linearizing the FOC for wage setting, and then substituting the linearized worker and firm surplus equations as well as the above discount factors in their loglinearized form (see GST, 2008 for more details).

First order condition

$$\widehat{J}_t(w_t^*) + \widehat{\Delta}_t = \widehat{H}_t(w_t^*) + \widehat{\Sigma}_t - \frac{1}{1 - \eta}\widehat{\eta}_t$$

where the loglinear forms of the discount factors are

$$\widehat{\Delta}_{t} = (1 - \iota) \widehat{h}_{t} - \frac{(1 - \iota) \overline{\tau}}{(1 - \overline{\tau})} \widehat{\tau}_{t} + \iota E_{t} \left( \widehat{\beta}_{t, t+1} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{\pi}_{t+1} + \widehat{\Delta}_{t+1} \right) - \overline{\beta} \overline{\rho} \gamma E_{t} \widehat{\rho}_{t+1}$$

$$\widehat{\Sigma}_{t} = (1 - \iota) \widehat{h}_{t} + \frac{(1 - \iota) \overline{s}}{(1 + \overline{s})} \widehat{s}_{t} + \iota E_{t} \left( \widehat{\beta}_{t, t+1} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{\pi}_{t+1} + \widehat{\Sigma}_{t+1} \right) - \overline{\beta} \overline{\rho} \gamma E_{t} \widehat{\rho}_{t+1}$$

and the expressions for  $\widehat{J}_t(w_t^*)$  and  $\widehat{H}_t(w_t^*)$  can be found as follows

## A.4.1 Worker surplus

The worker surplus can be written as

$$H_{t}(w_{t}^{*}) = \frac{w_{t}^{*}}{P_{t}} h_{t} (1 - \tau_{t}) - \left[ \frac{g(h_{t})}{\Lambda_{t}} + b + q_{t}^{W} E_{t} \beta_{t,t+1} H_{x,t+1} \right]$$

$$+ E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) H_{t+1}(w_{t+1}^{*})$$

$$+ \gamma E_{t} \beta_{t,t+1} (1 - \rho_{t+1}) \left[ H_{t+1}(w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) - H_{t+1}(w_{t+1}^{*}) \right]$$

In the last term, evaluate the expression  $E_t\left[H_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])-H_{t+1}(w_{t+1}^*)\right]$ 

$$E_{t} \left[ H_{t+1} \left( w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) - H_{t+1} \left( w_{t+1}^{*} \right) \right]$$

$$= E_{t} \left[ \frac{w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right]}{P_{t+1}} - \frac{w_{t+1}^{*}}{P_{t+1}} \right] h_{t+1} \left( 1 - \tau_{t+1} \right) + \gamma E_{t} \beta_{t+1,t+2} \varsigma_{t+1,t+2} \left[ H_{t+2} \left( w_{t}^{*} \left[ \pi_{t+1}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right] \right) - H_{t+2} \left( w_{t+1}^{*} \left[ \pi_{t+1}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) \right]$$

When linearized, this expression gets the following form

$$E_{t}\left[\widehat{H}_{t+1}\left(w_{t}^{*}\left[\pi_{t}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right]\right)-\widehat{H}_{t+1}\left(w_{t+1}^{*}\right)\right]$$

$$=\frac{\overline{wh}\left(1-\overline{\tau}\right)}{\overline{H}}E_{t}\left[\widehat{w}_{t}^{*}+\varepsilon_{w}\widehat{\pi}_{t}-\widehat{w}_{t+1}^{*}-\widehat{\pi}_{t+1}\right]$$

$$+\overline{\beta}\left(1-\overline{\rho}\right)\gamma E_{t}\left[\widehat{H}_{t+2}\left(w_{t}^{*}\left[\pi_{t+1}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\left[\pi_{t}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right]\right]\right)-\widehat{H}_{t+2}\left(w_{t+1}^{*}\left[\pi_{t+1}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right]\right)\right]$$

Iterating forward this can be further simplified to yield

$$E_{t} \left[ \widehat{H}_{t+1} \left( w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) - \widehat{H}_{t+1} \left( w_{t+1}^{*} \right) \right]$$

$$= \frac{1}{1 - \overline{\beta} \left( 1 - \overline{\rho} \right) \gamma} \frac{\overline{wh} \left( 1 - \overline{\tau} \right)}{\overline{H}} E_{t} \left[ \widehat{w}_{t}^{*} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{w}_{t+1}^{*} - \widehat{\pi}_{t+1} \right]$$

With the help of the above expression, the loglinear formulation of the worker surplus is found to be

$$\begin{split} \widehat{H}_{t} &= \frac{\overline{wh} \left(1 - \overline{\tau}\right)}{\overline{H}} \left(\widehat{w}_{t}^{*} - \widehat{P}_{t} + \widehat{h}_{t}\right) - \frac{\overline{wh}}{\overline{H}} \widehat{\tau}_{t} - \frac{\overline{mrsh} \left(1 + \overline{\tau}^{c}\right)}{\left(1 + \phi\right) \overline{H}} \left[\widehat{mrs}_{t} + \widehat{h}_{t}\right] \\ &- \frac{\overline{mrsh}}{\overline{\tau}^{c}} \widehat{\tau}_{t}^{c} - \overline{\beta} \overline{q}^{W} E_{t} \left(\widehat{q}_{t}^{W} + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\ &+ \overline{\beta} \left(1 - \overline{\rho}\right) E_{t} \left(\widehat{H}_{t+1} \left(w_{t+1}^{*}\right) + \widehat{\beta}_{t,t+1}\right) - \overline{\beta} \overline{\rho} E_{t} \widehat{\rho}_{t+1} \\ &+ \frac{\overline{\beta} \left(1 - \overline{\rho}\right) \gamma}{1 - \overline{\beta} \left(1 - \overline{\rho}\right) \gamma} \frac{\overline{wh} \left(1 - \overline{\tau}\right)}{\overline{H}} E_{t} \left(\widehat{w}_{t}^{*} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{w}_{t+1}^{*} - \widehat{\pi}_{t+1}\right) \end{split}$$

where as shown in Gertler and Trigari (2006) up to a first order approximation  $\widehat{H}_{x,t+1} = \widehat{H}_{t+1}(w_{t+1})$ .

## A.4.2 Firm surplus

The firm surplus can be written as

$$J_{t}(w_{t}^{*}) = x_{t}f(h_{t}) - \frac{w_{t}^{*}}{P_{t}}h_{t}(1+s_{t}) - r_{t}^{k}k_{t} + E_{t}\beta_{t,t+1}(1-\rho_{t+1})J_{t+1}(w_{t+1}^{*}) + \gamma E_{t}\beta_{t,t+1}(1-\rho_{t+1})\left[J_{t+1}(w_{t}^{*}\left[\pi_{t}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right]) - J_{t+1}(w_{t+1}^{*})\right]$$

In the last term, evaluate the expression  $E_t\left[J_{t+1}(w_t^*\left[\pi_t^{\varepsilon_w}\left(\pi^{1-\varepsilon_w}\right)\right])-J_{t+1}(w_{t+1}^*)\right]$ 

$$E_{t} \left[ J_{t+1} \left( w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) - J_{t+1} \left( w_{t+1}^{*} \right) \right]$$

$$= -E_{t} \left[ \frac{w_{t}^{*} \left[ \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right]}{P_{t+1}} - \frac{w_{t+1}^{*}}{P_{t+1}} \right] h_{t+1} \left( 1 + s_{t+1} \right)$$

$$+ \gamma E_{t} \beta_{t+1,t+2} \varsigma_{t+1,t+2} \left[ J_{t+2} \left( w_{t}^{*} \left[ \pi_{t+1}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \pi_{t}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) - J_{t+2} \left( w_{t+1}^{*} \left[ \pi_{t+1}^{\varepsilon_{w}} \left( \pi^{1-\varepsilon_{w}} \right) \right] \right) \right]$$

When linearized this expression gets the following form

$$E_{t}\left[\widehat{J}_{t+1}(w_{t}^{*}\left[\pi_{t}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right])-\widehat{J}_{t+1}(w_{t+1}^{*})\right]$$

$$=-\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}E_{t}\left[\widehat{w}_{t}^{*}+\varepsilon_{w}\widehat{\pi}_{t}-\widehat{w}_{t+1}^{*}-\widehat{\pi}_{t+1}\right]$$

$$+\overline{\beta}\left(1-\overline{\rho}\right)\gamma E_{t}\left[\widehat{J}_{t+2}(w_{t}^{*}\left[\pi_{t+1}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\pi_{t}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right]\right)-\widehat{J}_{t+2}(w_{t+1}^{*}\left[\pi_{t+1}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right])\right]$$

Iterating forward this can be further simplified to yield

$$E_{t}\left[\widehat{J}_{t+1}\left(w_{t}^{*}\left[\pi_{t}^{\varepsilon_{w}}\left(\pi^{1-\varepsilon_{w}}\right)\right]\right)-\widehat{J}_{t+1}\left(w_{t+1}^{*}\right)\right]$$

$$=-\frac{1}{1-\overline{\beta}\left(1-\overline{\rho}\right)\gamma}\frac{\overline{wh}\left(1+\overline{s}\right)}{\overline{J}}E_{t}\left[\widehat{w}_{t}^{*}+\varepsilon_{w}\widehat{\pi}_{t}-\widehat{w}_{t+1}^{*}-\widehat{\pi}_{t+1}\right]$$

Finally, as with worker surplus, the following loglinear formulation of the renegotiating firm's surplus can be found with the help of the above expression

$$\widehat{J}_{t} = \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}} \left( \widehat{x}_{t} + \widehat{mpl}_{t} + \widehat{h}_{t} \right) - \frac{\overline{wh} (1 + \overline{s})}{\overline{J}} \left( \widehat{w}_{t}^{*} - \widehat{P}_{t} + \widehat{h}_{t} \right) - \frac{\overline{whs}}{\overline{J}} \widehat{s}_{t} 
- \frac{\overline{r}^{k} \overline{k}}{\overline{J}} \left( \widehat{r}_{t}^{k} + \widehat{k}_{t} \right) - \overline{\beta} \overline{\rho} E_{t} \widehat{\rho}_{t+1} + \overline{\beta} (1 - \overline{\rho}) E_{t} \left( \widehat{J}_{t+1} \left( w_{t+1}^{*} \right) + \widehat{\beta}_{t,t+1} \right) 
+ \frac{\overline{\beta} (1 - \overline{\rho}) \gamma}{1 - \overline{\beta} (1 - \overline{\rho}) \gamma} \frac{\overline{wh} (1 + \overline{s})}{\overline{J}} E_{t} \left( \widehat{w}_{t+1}^{*} + \widehat{\pi}_{t+1} - \widehat{w}_{t}^{*} - \varepsilon_{w} \widehat{\pi}_{t} \right)$$

#### A.4.3 The Contract wage

Inserting the expressions for the worker and firm surpluses, as well as those for the discount factors, into the FOC yields (after collecting the wage terms to the left-hand side and using the Nash FOC for next period)

$$\Rightarrow \left[\frac{wh(1-\overline{\tau})}{\overline{H}} + \frac{wh(1+\overline{s})}{\overline{J}}\right] \widehat{w}_{t}^{*} \\ + \frac{\iota}{(1-\iota)} \left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right] E_{t} \left(\widehat{w}_{t}^{*} + \varepsilon_{w}\widehat{\pi}_{t} - \widehat{w}_{t+1}^{*} - \widehat{\pi}_{t+1}\right) \\ = \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}} \left(\widehat{x}_{t} + \widehat{mpl}_{t}\right) - \frac{\overline{\tau}^{k}\overline{k}}{\overline{J}} \left(\widehat{r}_{t}^{k} + \widehat{k}_{t}\right) + \frac{\overline{mrsh}(1+\overline{\tau}^{c})}{(1+\phi)\overline{H}} \left(\widehat{mrs}_{t}\right) + \frac{\overline{mrsh}\overline{\tau}^{c}}{(1+\phi)\overline{H}} \widehat{\tau}_{t}^{c} \\ + \overline{\beta} \left(1-\overline{\rho}\right) E_{t} \left[\widehat{J}_{t+1}\left(w_{t+1}^{*}\right) + \widehat{\beta}_{t,t+1}\right] - \overline{\beta} \left(1-\overline{\rho}\right) E_{t} \left[\widehat{J}_{t+1}\left(w_{t+1}^{*}\right) + \widehat{\beta}_{t,t+1} + \widehat{\Delta}_{t+1} - \widehat{\Sigma}_{t+1} + \frac{1}{1-\eta}\widehat{\eta}_{t}\right] \\ + \overline{\beta}\overline{q}^{W} E_{t} \left(\widehat{q}_{t}^{W} + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1}\right) \\ - \left\{ \left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right] - \frac{\overline{x}\overline{mplh}}{\alpha\overline{J}} - \frac{\overline{mrs}}{1+\phi} \frac{\overline{h}(1+\overline{\tau}^{c})}{\overline{H}} \right\} \widehat{h}_{t} \\ - \left[\frac{\overline{whs}}{\overline{J}} + \frac{(1-\iota)\overline{s}}{(1+\overline{s})}\right] \widehat{s}_{t} + \left[\frac{\overline{wh\tau}}{\overline{H}} - \frac{(1-\iota)\overline{\tau}}{(1-\overline{\tau})}\right] \widehat{\tau}_{t} + \left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right] \widehat{P}_{t} \\ + \iota E_{t} \widehat{\Delta}_{t+1} - \iota E_{t} \widehat{\Sigma}_{t+1} + \frac{1}{1-\eta} \widehat{\eta}_{t}$$

where  $\iota = \overline{\beta} \left(1 - \overline{\rho}\right) \gamma$ . Dividing by the term  $\left[\frac{\overline{wh}(1-\overline{\tau})}{\overline{H}} + \frac{\overline{wh}(1+\overline{s})}{\overline{J}}\right] = \frac{\overline{wh}(1+\overline{s})}{\eta \overline{J}} = \frac{\overline{wh}(1-\overline{\tau})}{(1-\eta)\overline{H}}$ , and using the steady state equations for  $\overline{\Delta}$  and  $\overline{\Sigma}$ , and for the Nash FOC allows us to rewrite the contract wage equation in the following simpler form

$$\Rightarrow \widehat{w}_{t}^{*} + \frac{\iota}{1 - \iota} E_{t} \left( \widehat{w}_{t}^{*} + \varepsilon_{w} \widehat{\pi}_{t} - \widehat{w}_{t+1}^{*} - \widehat{\pi}_{t+1} \right)$$

$$= \varphi_{x} \left( \widehat{x}_{t} + \widehat{mpl}_{t} \right) - \varphi_{k} \left( \widehat{r}_{t}^{k} + \widehat{k}_{t} \right) + \varphi_{m} \widehat{mrs}_{t} + \varphi_{H} E_{t} \left( \widehat{q}_{t}^{W} + \widehat{H}_{x,t+1} + \widehat{\beta}_{t,t+1} \right)$$

$$- \varphi_{h} \widehat{h}_{t} - \varphi_{s} \widehat{s}_{t} + \varphi_{\tau} \widehat{\tau}_{t} + \varphi_{\tau^{c}} \widehat{\tau}_{t}^{c} + \varphi_{D} E_{t} \left( \widehat{\Delta}_{t+1} - \widehat{\Sigma}_{t+1} \right) + \varphi_{\eta} \widehat{\eta}_{t} + \widehat{P}_{t}$$

$$= \widehat{w}_{t}^{0}(r)$$

where  $\widehat{w}_{t}^{0}(r)$  is the target wage in the bargain, and its coefficients are

$$\varphi_{x} = \frac{\overline{x}\overline{mpl}\eta}{\alpha\overline{w}(1+\overline{s})}, \quad \varphi_{k} = \frac{\overline{r}^{k}\overline{k}\eta}{\overline{wh}(1+\overline{s})}, \quad \varphi_{m} = \frac{\overline{mrs}(1-\eta)(1+\overline{r}^{c})}{(1+\phi)\overline{w}(1-\overline{\tau})}, \quad \varphi_{H} = \frac{(1-\eta)\overline{H}}{\overline{wh}(1-\overline{\tau})}\overline{\beta}\overline{q}^{W},$$

$$\varphi_{h} = \left\{1 - \frac{\overline{x}\overline{mpl}\eta}{\alpha\overline{w}(1+\overline{s})} - \frac{\overline{mrs}(1-\eta)(1+\overline{r}^{c})}{(1+\phi)\overline{w}(1-\overline{\tau})}\right\}, \quad \varphi_{s} = \frac{\eta\overline{s}}{(1+\overline{s})}\left[1 + \frac{(1-\iota)\overline{J}}{\overline{wh}(1+\overline{s})}\right],$$

$$\varphi_{\tau} = \frac{(1-\eta)\overline{\tau}}{(1-\overline{\tau})}\left[1 - \frac{(1-\iota)\overline{H}}{\overline{wh}(1-\overline{\tau})}\right], \quad \varphi_{\tau^{c}} = \frac{\overline{mrs}(1-\eta)\overline{\tau^{c}}}{(1+\phi)\overline{w}(1-\overline{\tau})},$$

$$\varphi_{D} = \frac{\left[\iota - \overline{\beta}(1-\overline{\rho})\right]\eta\overline{J}}{\overline{wh}(1+\overline{s})}, \quad \text{and} \quad \varphi_{\eta} = \frac{\left[1 - \overline{\beta}(1-\overline{\rho})\right]\overline{H}}{\overline{wh}(1-\overline{\tau})}$$

The target wage  $\widehat{w}_t^0(r)$  is of the same form than the period-by-period negotiated wage, adjusted for the new bargaining weights. The equation for the contract wage can be further rewritten as

$$\frac{1}{(1-\iota)}\widehat{w}_{t}^{*} = \widehat{w}_{t}^{0}(r) + \frac{\iota}{(1-\iota)}E_{t}(\widehat{\pi}_{t+1} - \varepsilon_{w}\widehat{\pi}_{t}) + \frac{\iota}{(1-\iota)}E_{t}\widehat{w}_{t+1}^{*}$$

$$\iff \widehat{w}_{t}^{*} = [1-\iota]\widehat{w}_{t}^{0}(r) + \iota E_{t}(\widehat{\pi}_{t+1} - \varepsilon_{w}\widehat{\pi}_{t}) + \iota E_{t}\widehat{w}_{t+1}^{*}$$

This is the optimal contract wage set at time t by all matches that are allowed to renegotiate their wage. As is usual with Calvo-type contracting, it depends on a wage target  $w_t^0(r)$  and next period's optimal wage.

## A.4.4 The spillover effect

To derive the spillover effect, consider the worker surplus with optimal (contract) wage versus the expected average market wage in the same way as above

$$E_{t}\widehat{H}_{t+1}(w_{t+1}) = E_{t}\widehat{H}_{t+1}(w_{t+1}^{*}) + \frac{\overline{wh}(1-\overline{\tau})}{(1-\iota)\overline{H}}E_{t}(\widehat{w}_{t+1}-\widehat{w}_{t+1}^{*})$$

Denoting  $\frac{\overline{wh}(1-\overline{\tau})}{(1-\iota)\overline{H}} = \Gamma$  and substituting the above expression in the target wage equation gives

$$\widehat{w}_{t}^{0}(r) = \varphi_{x} \left( \widehat{x}_{t} + \widehat{mpl}_{t} \right) - \varphi_{k} \left( \widehat{r}_{t}^{k} + \widehat{k}_{t} \right) + \varphi_{m} \widehat{mrs}_{t}$$

$$+ \varphi_{H} E_{t} \left( \widehat{q}_{t}^{W} + \widehat{H}_{t+1}(w_{t+1}^{*}) + \widehat{\beta}_{t,t+1} + \Gamma E_{t} \left[ \widehat{w}_{t+1} - \widehat{w}_{t+1}^{*} \right] \right)$$

$$+ \varphi_{h} \widehat{h}_{t} - \varphi_{s} \widehat{s}_{t} + \varphi_{\tau} \widehat{\tau}_{t} + \varphi_{\tau^{c}} \widehat{\tau}_{t}^{c} + \varphi_{D} E_{t} \left[ \widehat{\Sigma}_{t+1} - \widehat{\Delta}_{t+1} \right] + \varphi_{\eta} \widehat{\eta}_{t} + \widehat{P}_{t}$$

$$\iff \widehat{w}_{t}^{0}(r) = \widehat{w}_{t}^{0} + \varphi_{H} \Gamma E_{t} \left[ \widehat{w}_{t+1} - \widehat{w}_{t+1}^{*} \right]$$

where the target wage  $\widehat{w}_t^0(r)$  - the wage the firm and its worker would agree to if they are allowed to renegotiate, and if firms and workers elsewhere remain on staggered multiperiod wage contracts - is a sum of the wage that would arise if all matches were negotiating wages period-by-period  $\widehat{w}_t^0$  and the spillover effect  $\varphi_H \Gamma E_t \left[ \widehat{w}_{t+1} - \widehat{w}_{t+1}^* \right]$ .

## A.4.5 Evolution of the average wage

To derive the appropriate loglinear expression for the evolution of the average wage, first collect the necessary elements from previous calculations

1) The contract wage

$$\widehat{w}_{t}^{*} = \left[1 - \iota\right] \widehat{w}_{t}^{0}(r) + \iota E_{t} \left(\widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t}\right) + \iota E_{t} \widehat{w}_{t+1}^{*}$$

2) The average wage

$$\widehat{w}_{t} = (1 - \gamma) \, \widehat{w}_{t}^{*} + \gamma \left( \widehat{w}_{t-1} - \widehat{\pi}_{t} + \varepsilon_{w} \widehat{\pi}_{t-1} \right)$$

3) The target wage

$$\widehat{w}_t^0(r) = \widehat{w}_t^0 + \varphi_H \Gamma E_t \left[ \widehat{w}_{t+1} - \widehat{w}_{t+1}^* \right]$$

First, insert the target wage in the contract wage equation

$$\widehat{w}_{t}^{*} = [1 - \iota] \left( \widehat{w}_{t}^{0} + \varphi_{H} \Gamma E_{t} \left[ \widehat{w}_{t+1} - \widehat{w}_{t+1}^{*} \right] \right) + \iota E_{t} \left( \widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t} \right) + \iota E_{t} \widehat{w}_{t+1}^{*}$$

Then update the average wage equation by one period and take expectations

$$E_t \widehat{w}_{t+1} = (1 - \gamma) E_t \widehat{w}_{t+1}^* + \gamma \left( \widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t \right)$$

$$\iff E_t \widehat{w}_{t+1}^* = \frac{1}{(1-\gamma)} \left( E_t \widehat{w}_{t+1} - \gamma \left( \widehat{w}_t - E_t \widehat{\pi}_{t+1} + \varepsilon_w \widehat{\pi}_t \right) \right)$$

Use this expression to eliminate  $E_t \widehat{w}_{t+1}^*$  from the contract wage equation

$$\widehat{w}_{t}^{*} = \left[1 - \iota\right] \left(\widehat{w}_{t}^{0} + \varphi_{H} \Gamma E_{t} \widehat{w}_{t+1} - \varphi_{H} \Gamma \left[\frac{1}{(1 - \gamma)} \left(E_{t} \widehat{w}_{t+1} - \gamma \widehat{w}_{t} + \gamma E_{t} \left(\widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t}\right)\right)\right]\right) + \iota E_{t} \left(\widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t}\right) + \iota \left[\frac{1}{(1 - \gamma)} \left(E_{t} \widehat{w}_{t+1} - \gamma \widehat{w}_{t} + \gamma E_{t} \left(\widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t}\right)\right)\right]$$

$$\widehat{w}_{t}^{*} = (1 - \iota) \widehat{w}_{t}^{0} + (1 - \iota) \varphi_{H} \Gamma E_{t} \widehat{w}_{t+1} - (1 - \iota) \varphi_{H} \Gamma \frac{1}{(1 - \gamma)} E_{t} \widehat{w}_{t+1}$$

$$+ [1 - \iota] \varphi_{H} \Gamma \frac{\gamma}{(1 - \gamma)} (\widehat{w}_{t} - E_{t} \widehat{\pi}_{t+1} + \varepsilon_{w} \widehat{\pi}_{t}) + \iota (E_{t} \widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t})$$

$$+ \frac{\iota}{(1 - \gamma)} E_{t} \widehat{w}_{t+1} - \frac{\iota \gamma}{(1 - \gamma)} (\widehat{w}_{t} - E_{t} \widehat{\pi}_{t+1} + \varepsilon_{w} \widehat{\pi}_{t})$$

$$\iff \widehat{w}_{t}^{*} = (1 - \iota) \, \widehat{w}_{t}^{0} + \left[ (1 - \iota) \, \varphi_{H} \Gamma \frac{\gamma}{(1 - \gamma)} - \iota \frac{\gamma}{(1 - \gamma)} \right] (\widehat{w}_{t} - E_{t} \widehat{\pi}_{t+1} + \varepsilon_{w} \widehat{\pi}_{t})$$

$$+ \iota \left( E_{t} \widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t} \right)$$

$$+ \left[ (1 - \iota) \, \varphi_{H} \Gamma - (1 - \iota) \, \varphi_{H} \Gamma \frac{1}{(1 - \gamma)} + \iota \frac{1}{(1 - \gamma)} \right] E_{t} \widehat{w}_{t+1}$$

Denote  $\zeta = (1 - \iota) \varphi_H \Gamma$ , and use the above equation to eliminate  $\widehat{w}_t^*$  from the average wage equation (equation 2)

$$\widehat{w}_{t} = (1 - \gamma) (1 - \iota) \widehat{w}_{t}^{0} + (\zeta \gamma - \iota \gamma) (\widehat{w}_{t} - E_{t} \widehat{\pi}_{t+1} + \varepsilon_{w} \widehat{\pi}_{t})$$

$$+ (1 - \gamma) \iota (E_{t} \widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t})$$

$$+ [(1 - \gamma) \zeta - \zeta + \iota] E_{t} \widehat{w}_{t+1} + \gamma (\widehat{w}_{t-1} - \widehat{\pi}_{t} + \varepsilon_{w} \widehat{\pi}_{t-1})$$

$$[1 - \gamma(\zeta - \iota)] \widehat{w}_{t} = (1 - \gamma)(1 - \iota) \widehat{w}_{t}^{0} - \gamma(\zeta - \iota) (E_{t}\widehat{\pi}_{t+1} - \varepsilon_{w}\widehat{\pi}_{t}) + (1 - \gamma)\iota(E_{t}\widehat{\pi}_{t+1} - \varepsilon_{w}\widehat{\pi}_{t}) + [(1 - \gamma)\zeta - \zeta + \iota] E_{t}\widehat{w}_{t+1} + \gamma(\widehat{w}_{t-1} - \widehat{\pi}_{t} + \varepsilon_{w}\widehat{\pi}_{t-1})$$

Finally, after dividing by  $[1 - \gamma (\zeta - \iota)]$ , the dynamic average wage equation can be expressed as

$$\iff \widehat{w}_{t} = \lambda_{b} \left( \widehat{w}_{t-1} - \widehat{\pi}_{t} + \varepsilon_{w} \widehat{\pi}_{t-1} \right) + \lambda_{0} \widehat{w}_{t}^{0} + \lambda_{f} E_{t} \left( \widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \varepsilon_{w} \widehat{\pi}_{t} \right)$$

where 
$$\lambda_b = \frac{\gamma}{[1 - \gamma(\zeta - \iota)]}$$
,  $\lambda_0 = \frac{(1 - \gamma)(1 - \iota)}{[1 - \gamma(\zeta - \iota)]}$ , and  $\lambda_f = \frac{\iota - \gamma\zeta}{[1 - \gamma(\zeta - \iota)]}$ ,  
with  $\zeta = (1 - \iota)\varphi_H\Gamma$ ,  $\iota = \overline{\beta}(1 - \overline{\rho})\gamma$ ,  $\Gamma = \frac{\overline{wh}(1 - \overline{\tau})}{(1 - \iota)\overline{H}}$ ,  $\varphi_H = \frac{(1 - \eta)\overline{H}\overline{\beta}\overline{q}^W}{\overline{wh}(1 - \overline{\tau})}$  as previously denoted.

## **BoF Economics Review**

2020	No 1	Crowley, Patrick M.; Hudgins, David: How Effective is the Taylor rule? : Some Insights from the
2020	140 1	Time-Frequency Domain
	No 2	Ambrocio, Gene; Juselius, Mikael: Dealing with the costs of the COVID-19 pandemic – what are the fiscal options?
	No 3	Kilponen, Juha: Koronaviruskriisi leikkaa syvän loven Suomen talouteen
	No 4	Laine, Olli-Matti; Lindblad, Annika: Nowcasting Finnish GDP growth using financial variables : a MIDAS approach
	No 5	Kortelainen, Mika: Yield Curve Control
	No 6	Ambrocio, Gene: European household and business expectations during COVID-19 : Towards a v-shaped recovery in confidence?
	No 7	Nissilä, Wilma: Probit based time series models in recession forecasting : A survey with an empirical illustration for Finland
	No 8	Grym, Aleksi: Lessons learned from the world's first CBDC
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	No 2	Nyholm, Juho; Voutilainen, Ville: Quantiles of growth : household debt and growth vulnerabilities in Finland
	No 3	Juselius, Mikael; Tarashev, Nikola: Could corporate credit losses turn out higher than expected?
	No 4	Nelimarkka, Jaakko; Laine, Olli-Matti: The effects of the ECB's pandemic-related monetary policy measures
	No 5	Oinonen, Sami; Vilmi, Lauri: Analysing euro area inflation outlook with the Phillips curve
	No 6	Pönkä, Harri; Sariola, Mikko: Output gaps and cyclical indicators : Finnish evidence Analysing euro area inflation outlook with the Phillips curve
	No 7	Hellqvist, Matti; Korpinen, Kasperi: Instant payments as a new normal : Case study of liquidity impacts for the Finnish market
	No 8	Markkula, Tuomas; Takalo, Tuomas: Competition and regulation in the Finnish ATM industry
	No 9	Laine, Tatu; Korpinen, Kasperi: Measuring counterparty risk in FMIs
	No 10	Kokkinen, Arto; Obstbaum, Meri; Mäki-Fränti, Petri: Bank of Finland's long-run forecast framework with human capital
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	No 2	Gulan, Adam; Jokivuolle, Esa; Verona, Fabio: Optimal bank capital requirements: What do the macroeconomic models say?
	No 3	Oinonen, Sami; Virén, Matti: Has there been a change in household saving behavior in the low inflation and interest rate environment?
	No 4	Nyholm, Juho; Silvo, Aino: A model for predicting Finnish household loan stocks
	No 5	Oinonen, Sami; Virén, Matti: Why is Finland lagging behind in export growth?
	No 6	Mäki-Fränti, Petri: The effects of age and cohort on household saving
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