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Inflation dynamics and forecast: frequencies matter*

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Abstract

We use a New Keynesian Phillips Curve (NKPC) to study in-sample inflation dynamics and to forecast inflation out-of-sample in the frequency domain. In-sample, inflation expectations dominate medium-to-long-run cycles, energy inflation dominate short cycles and also longer cycles once expectations became anchored. While statistically significant, unemployment is never economically relevant. Out-of-sample, forecasts from a low-frequency NKPC significantly outperform several benchmark models. The low-frequency component of unemployment is key for such remarkable forecasting performance. Hence, while unemployment is of little relevance in-sample, it remains crucial in predicting inflation out-of-sample due to its role at cycles longer than typical business cycles.

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1 Introduction

Policymakers typically see inflation characterized by i) a trend strongly influenced by inflation expectations that, in turn, are shaped by monetary policy, and ii) deviations from that trend caused by persistently high or low resource utilization, as well as temporary movements in energy prices and other shocks (see e.g. Yellen, 2016).

In the economics literature, this general description of inflation dynamics is referred to as the expectations-augmented Phillips curve. The empirical micro-founded New Keynesian Phillips Curve (NKPC) (see e.g. Coibion and Gorodnichenko, 2015) relates inflation (π_t) to inflation expectations (π^e_{t+1}), a measure of economic activity (such as the unemployment gap, $ugap_t$), and measures of supply shocks (such as energy inflation, en_t):

$$\pi_t = \alpha_1 \pi^e_{t+1} + \alpha_2 \, ugap_t + \alpha_3 \, en_t + \varepsilon_t \ , \tag{1}$$

where ε_t the error term.

The NKPC and the policymakers' views of inflation dynamics are close but not equal. For example, agents' expectations of next-period inflation (π^e_{t+1} in the NKPC) are not long-run expectations and do not necessarily align with the central bank's inflation target. Energy inflation may affect expectations and on occasion may have persistent impacts. The unemployment gap may result from various types of shocks and its fluctuations may exhibit varying persistence over time. Hasenzagl, Pellegrino, Reichlin, and Ricco (2022) attempt to reconcile the two views with a multivariate semi-structural time-series model with endogenous trend inflation, natural unemployment, and potential output.

In this paper, we take a different approach. We do a frequency-domain decomposition of the time series of inflation and its NKPC determinants, and then use linear regression and forecast methods to study the in-sample dynamics of inflation and to forecast inflation out-of-sample. Specifically, we decompose the time series into four cyclical components: high frequency (less than 2 years), business cycle frequencies (2 to 8 years), medium frequencies (8 to 16 years), and low frequencies (more than 16 years). In-sample, we expect the low-frequency (long-run) component of inflation to be closely related with inflation expectations – especially with their low-frequency component – and less so with resource utilization and energy inflation. At the other end of the spectrum, we expect high-frequency (short-run) movements of inflation to be closely related to energy inflation and to a lesser extent with cycles of unemployment and expectations. At intermediate frequencies, unemployment is expected to play a key role, while the relevance of inflation expectations and supply shocks possibly shifting during specific episodes. Out of sample, we expect that accurately forecasting the low-frequency component of inflation is the key to a robust inflation forecast – a view that comports with most of the literature. Beyond allowing us to check that consensus from a novel perspective, our flexible approach reveals the specific NKPC determinants of inflation and their frequencies that contribute most to improving inflation forecasts.

Our method has a number of convenient features, as well as empirical and methodological contributions. First, decomposition of the time series into a set of frequency-domain components is more granular than the standard trend-cycle decomposition. This allows us to make the economically relevant distinction between business cycles, medium-run, and long-run fluctuations. Second, each time series is decomposed into frequency components that precisely add up to the original time series, thus providing assurance that we are neither ignoring nor overlapping information. Third, our approach involves only finite analytical computations, thus eliminating the need to approximate the ideal filter as in standard band-pass filters. Fourth, the in-sample and the out-of-sample exercises are conducted in a consistent framework based on the same NKPC, frequency-domain components, and frequency-domain tools. Fifth, our approach gives us the statistical and economic relevance of each NKPC determinants of inflation dynamics (in-sample) for each frequency of inflation along time. Sixth, our flexible forecast method allows for forecasting (out-of-sample) each specific frequency of inflation using information of expectations, slack, and supply shocks from the same and possibly other frequencies. Furthermore, and in contrast with standard forecasting models, our method allows for uncovering which frequencies of inflation are important for a good forecast of inflation along time.

In the first part of the analysis we provide new stylized facts on inflation dynamics by answering to the following two questions. Does the ability of the NKPC to explain inflation dynamics vary across cyclical frequencies? What is the contribution of each NKPC determinant in explaining inflation across cycles and along time? Notably, the full-sample estimates of the NKPC slope (α_2 in equation 1) for the two frequency bands comprising cycles between 2 and 16 years are quite similar to the standard time-series estimate, suggesting that slack explains inflation beyond the business cycle, while the estimate for cycles longer than 16 years is not significant, confirming the natural rate hypothesis. Regarding the contribution of each NKPC determinant in explaining inflation over time and across frequencies, we find that expectations dominate at medium- and long-run cycles (longer than 8 years), while energy inflation dominates at short cycles but is still relevant at business-cycle, medium-run, and even long-run cycles in some specific episodes. Most importantly, we show that unemployment fails to account for a substantial part of the variation of inflation – even at business-cycle frequencies.

In the second part of this paper, we use a frequency-domain approach to make out-of-sample forecasts of inflation. Here, we answer the following questions. Can frequency-domain NKPC-based forecasts outperform those of standard time-series benchmarks and, in particular, those of a standard time-series NKPC? Which NKPC determinants of inflation matter the most? Are all frequencies of inflation equally important to the forecast? We show that to obtain a good inflation forecast it is crucial to have a good forecast of its low frequency, i.e. cycles longer than 16 years. In fact, the model that ignores all the other frequencies of inflation consistently outperforms both the standard benchmarks and the forecasts with the time-series NKPC over the 22 years of our out-of-sample period. Second, besides the low-frequency component of inflation expectations, it is highly relevant to use the low frequency of the unemployment gap as a predictor. Other frequencies of the predictors only provide marginal improvements to the inflation forecasts. Finally, we show that forecasts can be improved even further by excluding the low frequency of inflation from the forecast around recessions, when other frequencies become more relevant. Ultimately, we determine that all frequencies of inflation and of its predictors matter.

The rest of the paper is organized as follows. In section 2, we discuss the studies that laid the groundwork for this work and our contribution to the literature. In section 3, we present the data and the method. Sections 4 and 5 present the results of our in-sample analysis of inflation dynamics and out-of-sample forecasts. Section 6 concludes.

2 Related literature and our contribution

This paper relates to the literature that deals with specification of the NKPC, frequency-domain analyses of Phillips curves, and frequency-domain approaches to forecasting. In this section, we review the literature on these topics and indicate how our contributions relate to each of them.

2.1 The New Keynesian Phillips Curve

Empirically, the history of the Phillips curve is one of "seemingly stable relationships falling apart upon publication" (Stock and Watson, 2010), as well as of strong specifications and sampling uncertainty (Mavroeidis, Plagborg-Moller, and Stock, 2014).

In this paper, we use an empirical NKPC (equation 1) in the spirit of Coibion and Gorodnichenko (2015), Fuhrer (2017), and Coibion, Gorodnichenko, and Kamdar (2018).[1] This modern version of the NKPC derives from a vast literature that gradually refines the micro-founded full-information rational expectations NKPC (described by e.g. Woodford, 2003). It replaces the labor share (e.g. Gali and Gertler, 1999)

[1] Given our purpose of describing and forecasting inflation with an integrated framework that explores the frequency-domain information in the data, we follow the vast literature focusing on reduced-form Phillips curves, where identification depends upon the ability of expectations and energy inflation to control for changes in the position of the curve. Alternative and more sophisticated identification strategies that have recently gained popularity include the use of instrumental variables or cross-sectional data to isolate the effect of demand shocks on real activity (see e.g. McLeay and Tenreyro, 2020, Barnichon and Mesters, 2021, and Hazell, Herreno, Nakamura, and Steinsson, 2022).

with the unemployment (or output) gap (e.g. Rudd and Whelan, 2007 and King and Watson, 2012) as proxy for marginal costs, adds controls for supply shocks (in the spirit of Gordon, 2011), and replaces rational expectations with survey expectations (e.g. Roberts, 1995).[2]

Household survey inflation expectations based on the Michigan surveys of consumers are used in the latest versions of the empirical NKPC for theoretical and empirical reasons. Household expectations have been shown to be the closest possible to firm expectations, which, while relevant according to micro foundations, are still unavailable for long sample periods (see Coibion and Gorodnichenko, 2015, Coibion, Gorodnichenko, and Kumar, 2018, and Pfajfar and Roberts, 2022). Moreover, the Michigan surveys of household inflation expectations feature intrinsic inertia due to the micro-founded inefficiency with which agents revise their expectations, and thus dispense of *ad hoc* inertial mechanisms (Fuhrer, 2017 and Coibion, Gorodnichenko, and Kamdar, 2018).[3]

While equation (1) has often been used to study the dynamics of inflation, it has rarely been used to forecast inflation out-of-sample. The literature of inflation forecasting based on Phillips curves mostly relies on some version of the Friedman-Phelps accelerationist specification (see e.g. Stock and Watson, 1999, Canova, 2007 and Dotsey, Fujita, and Stark, 2018). The notable exception is Berge (2018), who forecasts US inflation with several models, including specifications of the NKPC with expectations of inflation taken from the Michigan surveys of consumers. The key difference between our paper and Berge (2018) is that we run our forecasts in the frequency domain. This approach provides empirical and methodological innovations to the literature of Phillips curve-based inflation forecasting.

[2] For long samples such as ours, it could be argued that the NKPC should be modified along the lines of Ascari and Sbordone (2014) to account for trend inflation. Nevertheless, we follow the NKPC tradition of Coibion and Gorodnichenko (2015) and many others for two reasons. First, our focus is empirical. Specifically, we seek to assess an empirical reduced form of the NKPC that efficiently uses the available proxies for the triangular determinants of inflation. Second, our focus is policy oriented. Specifically, we want to study inflation with a Phillips curve similar to those typically used by monetary policymakers.

[3] See Aguiar-Conraria, Martins, and Soares (2023) for details on the foundations and empirical advantages of using the Michigan households surveys of expected inflation, as well as a thorough review of the specification of the empirical NKPC.

2.2 The Phillips curve in the frequency domain

Analyzing the Phillips curve in the frequency domain is important given that the controversies surrounding its ability to explain inflation dynamics (see e.g. Del Negro, Lenza, Primiceri, and Tambalotti, 2020) have often been associated with frequency-dependent phenomena, such as the low-frequency anchoring of inflation expectations or the success of monetary policy in taming business cycle fluctuations. Moreover, some authors (e.g. Cogley and Sbordone, 2008) highlight the interaction between low-frequency and high-frequency variations in inflation. Likewise, theoretical literature using Dynamic Stochastic General Equilibrium models emphasizes the relevance of taking into account the low- to medium-frequency fluctuations of inflation (Del Negro, Giannoni, and Schorfheide, 2015), and capturing interactions between macro variables at different frequencies (see e.g. Comin and Gertler, 2006, Angeletos, Collard, and Dellas, 2020, and Beaudry, Galizia, and Portier, 2020).

Another group of studies relies on frequency-dependent regression models (Ashley and Verbrugge, 2009, 2023) to assess the (non)linearity of the Phillips curve. They find that accelerationist Phillips curves exhibit different slopes over the business cycle, with inflation reacting differently to persistent and non-persistent fluctuations of the unemployment gap.

Our paper is not the first one to explore the idea that the Phillips curve differs across frequencies. Reinbold and Wen (2020), for example, identify demand shocks and improve the identification of a static Phillips curve across frequencies using spectral analysis. Other researchers use band-pass filters and focus on the Phillips relation at business-cycle frequencies (e.g. King and Watson, 1994). Overall, this literature finds a significant relationship between inflation and unemployment at business-cycle frequencies, but not at shorter or longer cycles in the context of static or accelerationist Phillips curves.

The present investigation most closely relates to studies that use wavelet tools to analyze the dynamics of inflation in the framework of Phillips curves. Using the continuous wavelet transform, Aguiar-Conraria, Martins, and Soares (2023) show that there is considerable variation of the coefficients of a NKPC identical to (1), both across frequencies and over time. Using the discrete wavelet transform, Gallegati,

Gallegati, Ramsey, and Semmler (2011) not only find a structural break in the mid-1990s and a significant slope in the US accelerationist wage Phillips curve at business cycle frequencies, but also at cycles longer than 8 years.

Our paper contributes to this literature in several respects. First, our use of the Maximal Overlap Discrete Wavelet Transform using an empirical NKPC is novel. Second, we innovate by using such framework to consistently study both the in-sample dynamics and the out-of-sample forecast of inflation. Third, we are the first to build inflation forecasts from its frequency-specific forecasts with a flexible model that identifies the specific NKPC determinants and their frequencies that improve the inflation forecast. This is a key contribution of our paper. In particular, we show that besides confirming the well-known result that accurately forecasting the low frequency of inflation is, on average, crucial for a successful forecast of inflation (e.g. Faust and Wright, 2013, Clark and Doh, 2014, Stella and Stock, 2015, Chan, Koop, and Potter, 2016, and Chan, Clark, and Koop, 2018), our method allows for detecting episodes in which such low frequency is not important, and episodes in which other frequencies matter more.[4]

2.3 Frequency-domain forecasts

Our approach extends the use of discrete wavelet methods to forecast out-of-sample economic and financial time series. This area of research includes Rua (2011) and Rua (2017), who forecasts GDP growth and inflation using a factor-augmented wavelets approach; Zhang, Gençay, and Yazgan (2017) and Faria and Verona (2018, 2020, 2021), who focus on forecasting stock market returns; Kilponen and Verona (2022), who forecast aggregate investment rate; and Crowley and Hudgins (2021), who use a large-scale wavelet-based control model to simulate optimal forecasts for several macroeconomic variables.

Our paper relates closely to the work of Faria and Verona (2018, 2021) on improving forecasts of a variable of interest by summing the forecasts of its frequency components rather than directly forecasting

[4] Rather than decomposing the aggregate time series into frequencies, a recent strategy involves splitting the price index into its components and empirically identifying the specific price categories sensitive to the degree of resource utilization. This cyclically sensitive inflation is then used in a Phillips curve framework to analyze inflation dynamics and forecast inflation “in parts” (see Tallman and Zaman, 2017, Zaman, 2019 and Stock and Watson, 2020).

the aggregate. In those papers, each frequency component of the variable of interest is forecasted using only information from the same frequency component of the predictors. We contribute to this literature by allowing (but not imposing) that each frequency component of inflation may depend on other frequency components of the predictors. For instance, the business-cycle frequencies or medium-term frequencies of, say, the unemployment gap, are allowed to affect the low-frequency fluctuations of inflation. Our generalization of the wavelet-based approach to forecasting is potentially useful for many economics problems, and seems particularly valuable in the case of inflation in light of the different patterns of variance across frequencies of inflation, expectations, unemployment, and energy inflation.

3 Data and method

Our data are US quarterly time series for 1978Q1–2021Q4 of inflation, the unemployment gap, expectations of inflation, and energy inflation. Let P_t be the consumer price index (CPI) provided by the US Bureau of Labor Statistics in quarter t. The annualized quarter-on-quarter inflation rate is computed as $\pi_t = 400\ln(P_t/P_{t-1})$. Energy inflation is the annualized quarterly rate of growth of the respective component of the CPI. Inflation expectations are the median expected changes in prices on average over the next 12 months reported by households in the Michigan surveys of consumers. The unemployment gap is the difference between the quarterly average of the civilian unemployment rate provided by the US Bureau of Labor Statistics and a linear trend.

To analyze the Phillips curve in the frequency domain, we use wavelet tools to decompose our time series into individual components that can be associated with fluctuations at different frequencies. The wavelet method used here allows for decomposing any time series into a trend (or permanent) component and cyclical (or transitory) movements in a manner similar to the traditional time series trend-cycle decomposition approach (e.g. Beveridge and Nelson, 1981), or filtering methods as the Baxter and King (1999) bandpass filter.[5]

[5] This section provides a brief description of the theory directly relevant to our empirical analysis. Percival and Walden

By using the Maximal Overlap Discrete Wavelet Transform Multi-Resolution Analysis (MODWT MRA) with the Haar filter, any variable X_t can be decomposed as:

$$X_t = \sum_{j=1}^{J} D_{j,t} + S_{J,t} \ , \tag{2}$$

where $D_{j,t}$ are the wavelet coefficients at scale j, and $S_{J,t}$ is the scaling coefficient. These coefficients are given by

$$D_{j,t} = \frac{1}{2^j} \left[\sum_{i=0}^{2^{(j-1)}-1} X_{t-i} - \sum_{i=2^{(j-1)}}^{2^j-1} X_{t-i} \right] \tag{3}$$

and

$$S_{J,t} = \frac{1}{2^j} \sum_{i=0}^{2^j-1} X_{t-i} \ . \tag{4}$$

Equations (2)-(4) show that the original series X_t, exclusively defined in the time domain, can be decomposed in different time-series components, each defined in the time domain and representing the fluctuation of the original time series in a specific frequency band.

As an example, when J=2 the MODWT MRA with the Haar wavelet filter produces two details ($D_{1,t}$ and $D_{2,t}$) and a smooth ($S_{2,t}$) time series:

$$D_{1,t} = \frac{X_t - X_{t-1}}{2} \tag{5}$$

$$D_{2,t} = \frac{X_t + X_{t-1} - (X_{t-2} + X_{t-3})}{4} \tag{6}$$

$$S_{2,t} = \frac{X_t + X_{t-1} + X_{t-2} + X_{t-3}}{4} \ . \tag{7}$$

As equations (5)-(7) show, the wavelet coefficients resulting from the MODWT MRA with the Haar filter are simply differences and averages of moving averages.

In this paper we compute a J=5 level decomposition.[6] As we use quarterly data, the first component

(2000) provide a more detailed analysis of wavelet methods. Verona (2020) discusses the advantages of wavelet filters over other bandpass filtering techniques.

[6] The number of observations in the sample period defines the maximum number of frequency bands that can be used (J), which ultimately determines the lowest frequency band isolated. In our case, in the out-of-sample forecast part of the paper,

(D_1) captures fluctuations with a period between 2 and 4 quarters, while the components D_2, D_3, D_4 and D_5 capture fluctuations with periods of 1–2, 2–4, 4–8, and 8–16 years, respectively. Finally, the smooth component S_5 captures fluctuations with a period longer than 16 years. Subsequently, the high-frequency (HF) component of each variable (e.g. inflation, π_t) is computed as $\pi_t^{HF} = D_1 + D_2$ and captures fluctuations with a period less than 2 years, the business cycle-frequency (BCF) component (π_t^{BCF}) is computed as $\pi_t^{BCF} = D_3 + D_4$ so that it captures fluctuations between 2 and 8 years, whereas its medium-frequency (MF) and low-frequency (LF) components correspond to D_5 and S_5, respectively.[7]

The original time series of our variables and their frequency components are reported in Figures A.1 to A.4 in the online appendix. These figures highlight that the original time series are the result of the aggregation of several underlying frequency components that exhibit quite different dynamics. Table 1 reports the variance decomposition by frequency of our variables. About half of the volatility of inflation occurs at the LF band, while a quarter of its volatility is due to HF fluctuations. Almost two-thirds of the variance of inflation expectations occurs at the LF band, and almost all their variance occurs at the three lower frequency bands (BCF, MF, and LF). The unemployment gap exhibits a more even distribution of its variance across the three lower frequency bands. In contrast, two-thirds of the energy inflation variance is concentrated in the HF band.

4 Inflation dynamics

In this section, we assess the in-sample dynamics of inflation through the lenses of the NKPC across cyclical frequencies. Our focus in the first sub-section is on the estimates and statistical significance of the NKPC coefficients. In the second sub-section we turn our attention to the economic relevance of each NKPC determinant of inflation.

the initial in-sample period (used to make the first out-of-sample forecast) has N=88 observations, so $J \leq \log_2 N \simeq 6.5$.

[7] In the MODWT, each wavelet component at frequency j approximates an ideal high-pass filter with passband $f \in \left[1/2^{j+1}, 1/2^j\right]$. Hence, they are associated to fluctuations with periodicity $\left[2^j, 2^{j+1}\right]$ (quarters, in our case).

4.1 NKPC coefficients across frequencies

We start by analyzing differences in the estimates of the NKPC across cyclical frequencies.

Table 2 shows linear regressions of the NKPC. The first row displays the estimates obtained with the original time-series data. These are overall consistent with the literature (see e.g. Coibion and Gorodnichenko (2015). The second to fifth rows present estimates of the NKPC for the four frequency bands, i.e. estimates of OLS regressions

$$\pi_t^f = \alpha_1^f \pi_{t+1}^{e,f} + \alpha_2^f ugap_t^f + \alpha_3^f en_t^f + \varepsilon_t^f \ , \tag{8}$$

where f=HF, BCF, MF, LF.[8]

The estimate of the NKPC slope (α_2^f) is the same for BCF and MF, both being equal to the time-series estimate. Notably, the slope is more precisely estimated for the BCF and MF cycles separately than for the aggregate time series. The estimate of the slope for the HF NKPC is somewhat larger, but only marginally statistically significant. The LF estimate of the slope is small and not statistically significant, suggesting that there is no Phillips tradeoff in the long run.[9]

The coefficient associated with expectations differs substantially across frequencies. Inflation reacts about one-to-one to changes in expectations at BCF and MF, and slightly less so at LF. In turn, inflation expectations have a much smaller impact on inflation at HF, as theory predicts. Overall, the standard time-series estimate appears to be an artifact as it averages out heterogeneous estimates across different cycles, especially at the extremes of the frequencies.

The estimates of the energy inflation coefficients are equal to the time-series estimates for the HF, BCF

[8] This setup is akin to the band spectrum regression proposed by Engle (1974).

[9] The estimate of the NKPC slope for the LF cycles is not exactly an estimate of the slope of the long-run Phillips curve, but an approximation comprising cycles longer than 16 years. Estimates of the long-run Phillips curve slope vary markedly in the literature. Benati (2015), for example, shows that any estimate of the long-run Phillips curve faces vast uncertainty as it would require an infinite amount of data. Blanchard (2018) sees "the macroeconomic and the microeconomic evidence as suggestive but not conclusive evidence against the natural rate hypothesis." (page 99). Our results are consistent with Aguiar-Conraria, Martins, and Soares (2023), who do not find any significant slope at the lower-end of their frequencies (cycles of around 16 years).

and MF bands, while the estimate for the LF cycles is larger than the others.

Thus, the NKPC explains the dynamics of inflation rather differently for each of the four frequency bands, which confirms the advantage of assessing inflation dynamics separately for different cycles.

4.2 Time- and frequency- varying NKPC determinants of inflation

We now move from statistical significance to economic relevance, conducting a full-sample decomposition of inflation using the coefficient estimates of the NKPC in Table 2 and the actual data. Figure 1 shows the contribution of each NKPC determinant to explain inflation from 1978Q1 to 2021Q4.

The top graph relates to the time-series NKPC. It shows that the good fit of that model ($R^2 = 0.86$) comes largely from expected inflation (blue line) and somewhat less from energy inflation (green line). Notably, the explanatory contribution of unemployment (red line) is visibly quite limited throughout the entire sample period.

The results for each of the four frequency components are reported in the graphs in the second and third row. As expected, over the short run (HF) the NKPC explains inflation almost entirely through energy inflation. Expectations and unemployment do not capture much of the HF movements of inflation.

Over the long run (LF), inflation co-moves very closely with expectations, but energy also plays a substantial role in some specific episodes. In particular, during the oil crises and their aftermath, until around 1987, energy prices are key for the ability of the NKPC to better fit actual inflation. More recently, in the run-up to the great financial crisis and the ensuing recession, energy inflation has led the NKPC to overestimate inflation. At low frequencies, unemployment does not contribute visibly to a NKPC explanation of inflation.

There is also a prominent role of expectations apparent over the medium run (MF). Since the late 1990s, however, there have been several episodes in which expectations do not explain medium-run inflation as well as energy inflation. This shift may be related to the anchoring of expectations and their consequent limitation in explaining MF fluctuations of inflation.

Results for BCF are similar to those of MF cycles in spite of the higher volatility of all variables. Expectations and energy inflation contribute most to explaining inflation, while the contribution of unemployment is overall marginal. Since the late 1990s, the contribution of expectations in explaining inflation over the business cycle has often fallen short of the contribution of energy prices. Again, we view this result as consistent with the anchoring of expectations consistently dated in the literature around 1999 (see e.g. Jorgensen and Lansing, 2022).

Overall, our in-sample results confirm the key role of expectations in explaining LF fluctuations of inflation, but add a new result: in some specific episodes, LF fluctuations of energy inflation contributed substantially to explain the LF oscillations of inflation. More generally, our results suggest a somewhat larger than expected role of energy inflation, which are highly relevant not only at HF but also at BCF and even at MF, once expectations became anchored. Our results clearly show that unemployment has had a limited role in explaining inflation. This is rather surprising given that slack should be a key determinant of inflation in the context of the NKPC, especially at BCF and MF.

In the next section, we ask whether the NKPC model is useful in forecasting inflation out-of-sample and dig deeper into the contribution of unemployment in inflation forecasting.

5 Inflation forecasts

In this section, we assess the performance of our frequency-domain approach to the NKPC to forecast inflation out-of-sample (OOS). This contrasts with the purely time-series benchmarks that dominate the literature and the standard time-series NKPC. The first sub-section is methodological. After briefly reviewing how forecasts are computed with the benchmark models, we describe our method. In the second sub-section, we present our results in two steps. First, we compare the overall precision of our forecasts with those of the benchmarks. Second, we show which cyclical frequencies and which NKPC determinants of inflation actually improve the forecasts. In the final sub-section, we check which frequencies of inflation matter the most in inflation forecasting.

Our OOS forecasting exercise targets the annualized h-period average inflation rate, computed as $\pi_t^h = \frac{1}{h}\sum_{i=0}^{h-1}\pi_{t-i} = \frac{400}{h}\ln\left(P_t/P_{t-h}\right)$. We focus on the 4-quarter and 8-quarter (h=4 and 8) average inflation rates as they are the most relevant for policymakers.

Our OOS forecasts are direct forecasts produced with a sequence of expanding windows. We start by obtaining the first OOS forecasts with the sample 1978Q1–1999Q4. The sample is then increased by one observation and a new set of OOS forecasts is produced. This procedure is repeated until the end of the sample. Hence, the full OOS period runs from 2000Q1 to 2021Q4.

5.1 Forecasting models

5.1.1 Time-series benchmarks

As is common in the literature on inflation forecasting, we compare the forecasting performance of the NKPC against two time-series models. First, the random walk model of Atkeson and Ohanian (2001) (AO model), for which the h-period-ahead forecast is given by $\hat{\pi}_{t+h}^h = \frac{1}{4}\sum_{p=0}^{3}\pi_{t-p}^h$. Second, the univariate unobserved components model with stochastic volatility of Stock and Watson (2007) (UCSV model), which is given by $\pi_t = \tau_t + \varepsilon_t$ and $\tau_t = \psi_{t-1} + \eta_t$, where ε_t and η_t feature stochastic volatility. The forecasts of the UCSV model are computed as $\hat{\pi}_{t+h}^h = \tau_t$.[10]

5.1.2 Time-series NKPC

Ultimately, we want to assess the merit of our wavelet-based NKPC in forecasting US inflation relative to the time-series NKPC. This is the most relevant benchmark for our forecasting model (with the time-series benchmark models used as minimum thresholds in accordance with the literature). Forecasts with the NKPC are obtained as follows.

[10] Following Chan (2018), we use non-centered parameterization in the UCSV model.

At each step of the recursive OOS period, for each h we first estimate a regression

$$\pi_t^h = c^h + \alpha_1^h \pi_{t+1}^e + \alpha_2^h ugap_t + \alpha_3^h en_t + \varepsilon_{t+h}^h \ , \tag{9}$$

and then compute the h-step ahead forecasts as

$$\hat{\pi}_{t+h}^h = \hat{c}^h + \hat{\alpha}_1^h \pi_{t+1}^e + \hat{\alpha}_2^h ugap_t + \hat{\alpha}_3^h en_t \ . \tag{10}$$

To avoid any look-ahead bias in the predictive regression forecast based on the NKPC, we compute the unemployment gap by fitting a linear trend to the unemployment data up to the quarter when the forecast is made. We denote this model as NKPC_TS.

5.1.3 Wavelet-based NKPC

Our wavelet-based NKPC forecast method builds on the NKPC and on the filtered data obtained with the MODWT MRA decomposition.[11]

For each frequency component *f*, *f*=(HF, BCF, MF, LF), we have the following Phillips curve:

$$\pi_t^{h,f} = c^{h,f} + \alpha_1^{h,f} \pi_{t+1}^{e,f} + \alpha_2^{h,f} ugap_t^f + \alpha_3^{h,f} en_t^f + \varepsilon_{t+h}^{h,f} \ , \tag{11}$$

in which each frequency component of inflation ($\pi_t^{h,f}$) depends only on the same frequency component *f* of the predictors. However, we want to generalize this specification by allowing other frequency components of the predictors into the NKPC of inflation at frequency *f*. This methodological contribution should prove highly useful, given that inflation and its predictors have different relative powers across

[11] In the OOS exercise, we use a two-sided version of the Haar filter. To ensure that our method does not have a look-ahead bias, we recompute the frequency components recursively at each iteration of the OOS forecasting process using data from the start of the sample through the quarter at which the forecasts are made. Our forecasts are thus made only with current and past information. When using a two-sided filter, some assumptions as regards how to deal with boundary observations have to be made. The literature suggests several types of boundary treatment rules to deal with boundary effects (e.g. periodic rule, reflection rule, zero padding rule, and polynomial extension). Here, we use a reflection rule, whereby the original time series are reflected symmetrically at the boundaries before applying the filter.

cyclical bands. Moreover, theory and practice both suggest that they may interact with each other across frequencies.

More formally and generally, we first estimate the following system of equations at each step of the OOS period:

$$\pi_t^{h,f} = c^{h,f} + \alpha_1^h \pi_{t+1}^{e,f} + \alpha_2^h ugap_t^f + \alpha_3^h en_t^f + \varepsilon_t^{h,f} \ . \tag{12}$$

where $\pi_t^{h,f}$, $\pi_{t+1}^{e,f}$, $ugap_t^f$ and en_t^f are 4x1 vectors of observables, $c^{h,f}$ is a 4x1 vector of intercepts and $\varepsilon_t^{h,f}$ is a 4x1 vector of residuals, while α_1^h, α_2^h and α_3^h are 4x4 matrices of coefficients.[12] We consider two cases of this wavelet-based model.

In a first case, matrices α_m^h, with $m = (1,2,3)$, are restricted to be diagonal. In this specification, we consider that only the components of the predictors at frequency f are used to forecast the frequency component of inflation at frequency f. We denote this model as NKPC_WAV_diag.

In a second specification, we allow for interactions between inflation and its predictors across frequencies. In particular, all the coefficients in matrices α_m^h are allowed to be different from 0. We denote this model as NKPC_WAV_all.

The choice between NKPC_WAV_diag and NKPC_WAV_all is ultimately empirical. NKPC_WAV_all is a generalization of NKPC_WAV_diag, including 12 predictors for each frequency f of inflation, rather than 3, and it should lead to better in-sample fit. It remains to be seen, however, whether relations between inflation and its predictors across frequencies are empirically so relevant that the improved in-sample fit does not harm the OOS performance. Thus, the more parsimonious NKPC_WAV_diag may achieve better OOS performance. Finding which specification predicts inflation most accurately and robustly is a key contribution of our approach.

After regressing equation (12), the forecasts of each frequency component of inflation are computed as:

[12] As in the forecast with the NKPC_TS model, the unemployment gap is recomputed at each step of the OOS period by fitting a linear trend to the data up to the quarter when the forecast is made.

$$\hat{\pi}^{h,f}_{t+h} = \hat{c}^{h,f} + \hat{\alpha}^h_1 \pi^{e,f}_{t+1} + \hat{\alpha}^h_2 ugap^f_t + \hat{\alpha}^h_3 en^f_t \ . \tag{13}$$

Then, given that $\pi^h_t = \pi^{h,HF}_t + \pi^{h,BCF}_t + \pi^{h,MF}_t + \pi^{h,LF}_t$, the h-quarter-ahead inflation forecast is given by the sum of the h-quarter ahead forecasts given by each frequency component of the NKPC. That is,

$$\hat{\pi}^h_{t+h} = \hat{\pi}^{h,HF}_{t+h} + \hat{\pi}^{h,BCF}_{t+h} + \hat{\pi}^{h,MF}_{t+f} + \hat{\pi}^{h,LF}_{t+h} \ . \tag{14}$$

Following the literatures of wavelet-based forecasts (see Faria and Verona, 2018, 2021) and inflation forecasting (e.g. Faust and Wright, 2013 and Chan, Clark, and Koop, 2018), and motivated by the observation that about half of the variance of inflation occurs at LF (see Table 1), we also consider a forecasting model than takes into account only forecasts of the LF component of inflation. That is, we assess the forecasting performance of the model

$$\hat{\pi}^h_{t+h} = \hat{\pi}^{h,LF}_{t+h} \ . \tag{15}$$

As before, we consider a model strictly comprising the forecasts computed using only the LF of the predictors, which we denote as NKPC_WAV_diag (LF), and a model in which the whole elements of the last row of matrices α^h_m are allowed to be non-null, which we denote NKPC_WAV_all (LF). This is the generalized version of our LF forecast model. It allows for influence from the HF, BCF, and MF fluctuations of the predictors into the LF of inflation.

5.2 Results

5.2.1 Forecast precision

We now evaluate the ability of the wavelet-based NKPC to forecast inflation OOS in the period from 2000Q1 until 2021Q4. Following common practice, we assess the forecast accuracy by computing the root mean squared forecast error (RMSFE) for each model and computing the statistical significance of the differences in RMSFEs across methods. Table 3 reports our results. Panel (a) shows the RMSFEs of the AO model and the UCSV models, the standard benchmarks for inflation forecasts. Panels (b) and (c) report the RMSFEs of the time-series- and wavelet-based NKPCs, respectively, relative to that of the AO model. A value below 1 indicates that the model outperforms the AO benchmark. Panel (d) reports the RMSFEs of the wavelet-based NKPC models relative to those of the time-series NKPC. Asterisks indicate statistical significance according to the Diebold and Mariano (1995) test (with the West, 1996 correction) of relative predictive accuracy at the 10 % (*) and 5 % (**) levels.

Despite being much more sophisticated than the AO model, the rather limited performance of the univariate UCSV model is in line with the literature (see e.g. Jarocinski and Lenza, 2018 and Banbura and Bobeica, 2022).[13]

Similarly, panel (b) confirms that the traditional time-series NKPC underperforms the AO model.

The first two lines of panels (c) and (d) indicate that the forecasts of the wavelet-based NKPC models that include the forecasts of all frequencies of inflation (equation 14) are usually not statistically significantly better than those of the AO and the NKPC_TS models.

Consistent with our conjecture and with the literature, the third and fourth lines of panels (c) and (d) show that focusing solely on the forecast of the LF component of inflation (equation 15) produces forecasts that are substantially and statistically more accurate than those of both the AO and the NKPC_TS model.

In particular, the generalized LF model, NKPC_WAV_all (LF), produces slightly better forecasts than the NKPC_WAV_diag (LF) model. That is, allowing influences from higher frequencies (HF, BCF and MF) of the predictors produces the closest forecasts to actual inflation. The RMSFEs of the NKPC_WAV_all (LF) model are 85 % and 81 % of those of the AO benchmark (for h=4 and h=8, respectively), which are both statistically significant at the 5 % level; and they are 77 % and 55 % of those of the time-series

[13] Some literature has improved the forecast performance of the UCSV model but departing from the original model by adding extra time-series ingredients (e.g. Chan, 2013) or by formulating multivariate UCSV models (e.g. Stella and Stock, 2015).

NKPC (statistically significant at the 10 % level for h=4, and at the 5 % level for h=8).

Overall, the wavelet-based NKPC forecasts of the LF of inflation are substantially and significantly better forecasts of inflation than those of time-series benchmarks and the standard time-series NKPC. Our frequency-domain approach to the NKPC thus effectively resurrects the forecast ability of the Phillips curve. The gain in forecast accuracy is about 15 to 20 % with respect to the AO model and about 23 % (45 %) regarding the time-series NKPC forecasts at the 4-quarter (8-quarter) horizon.

Figure 2 shows actual inflation (black lines), the forecasts obtained with the NKPC_WAV_diag (LF) model (blue solid lines), and those from the time-series NKPC (blue dashed lines). The performance of the NKPC_TS deteriorates markedly after the Great Recession, with forecasts consistently overshooting actual inflation. Our in-sample results suggest that this poor forecasting performance stems from a combination of two factors: the steadiness of inflation expectations, which play a prominent role in that model, and the minor relevance of the unemployment gap, indicated by the low estimate of the slope.

The good performance of our model comes from the smoothness of its forecasts. The method avoids excessively sharp fluctuations while capturing the essence of the evolution of inflation over time. In the next sub-section, we analyze which NKPC determinants of inflation drive the forecast performance of our model.

5.2.2 Forecast determinants

In Table 4, we present in further detail the RMSFEs of our LF NKPC models relative to those of the AO model. In the first three columns, we show the relative RMSFEs that would be obtained if we restricted the NKPC predictors to the LF of expectations (first column), of expectations and unemployment (second column), or of expectations and energy inflation (third column). The last column shows the relative RMSFE of the NKPC_WAV_diag (LF) model, which forecasts the LF of inflation using the three LF components of the predictors.

The comparison of the first and the second columns of table 4 allows us to assess the contribution of the LF component of the unemployment gap above that of the LF component of expectations. At the 4-quarter-ahead horizon, it decreases the relative RMSFE from 96 % to 86 %. At the 8-quarter-ahead forecast horizon, it cuts the relative RMSFE from 113 % to 81 %. More importantly, the forecast gains given by the LF of the unemployment gap are not just economically relevant, but statistically significant at the 5 % level for both forecasting horizons. The LF of energy inflation, on the other hand, adds little to the forecasts.

In Figure 2, besides the forecasts from the NKPC_WAV_diag (LF) model (blue solid lines), we plot the forecasts from that model restricted to $\pi^{e,LF}$ (red lines), from that model restricted to $\pi^{e,LF}$ and en_t^{LF} (yellow lines), and from that model restricted to $\pi^{e,LF}$ and $ugap_t^{LF}$ (green lines).

Figure 2 shows that the forecasting gains given by the inclusion of the low-frequency of the unemployment gap essentially start in 2010 for h=4 and in 2011 for h=8. The key role of the LF of unemployment may be clearly understood by looking with further detail at the wake of the Great Recession. Inflation (h=4) falls substantially from the 5.2 % peak in 2008Q3 to almost 0 % in 2015Q1. In the same period, the LF of unemployment increases from -0.9 % to 1.6 %, and features the largest and the most persistent increase in the unemployment gap across all frequency bands. The size and persistence of such increase is key for improving the prediction of the marked and persistent fall in 4-quarter and 8-quarter inflation rates in the years through 2015.

Overall, and differently from the in-sample analysis, we find that the unemployment gap is of crucial importance to forecast inflation with a frequency-domain NKPC. Hence, the Phillips tradeoff is still relevant.

5.2.3 Robustness checks

We submitted our OOS forecast procedure to several robustness checks. First, we considered alternative proxies for slack: the unemployment rate (in levels) and the output gap (computed as the difference between real GDP and its linear trend). Second, we forecast personal consumption expenditure (PCE) inflation using the corresponding component for energy price inflation. Third, we experimented with alternative wavelet filters such as Daubechies and Coiflets of different lengths. Fourth, we computed the forecasts using rolling window estimates rather than expanding window estimates (with a window size of 88 quarters, the same as for our initial in-sample period). Fifth, we considered expanding windows starting in 1985Q1, to avoid the period of disinflation and skip the potential structural break at the start of the Great Moderation.[14]

Results are reported in Table 5. Overall, the results of these analyses indicate that our conclusions are qualitatively – and often quantitatively – robust to all these changes.

Finally, we evaluate the importance of the filtering method used to extract the frequency components by using the Christiano and Fitzgerald (2003) asymmetric band-pass filter, assuming a unit root with drift. The frequency bands of the filter are chosen so as to extract exactly the same frequency components as in our wavelet analysis. Results, reported in the last columns in Table 5, are clearly worse than those using wavelet filters.[15]

5.3 Frequencies matter

We have shown that the key to obtaining a good forecast of inflation is to forecast well the LF of inflation. Are the other frequencies of inflation then useless?

To analyse whether this is the case or not, we consider the h-quarter-ahead inflation forecast given by

$$\hat{\pi}_{t+h}^{h} = \kappa_{HF}\hat{\pi}_{t+h}^{h,HF} + \kappa_{BCF}\hat{\pi}_{t+h}^{h,BCF} + \kappa_{MF}\hat{\pi}_{t+f}^{h,MF} + \kappa_{LF}\hat{\pi}_{t+h}^{h,LF} \quad . \tag{16}$$

We then check whether setting κ_f to 0 (rather than 1) for some frequencies f, at some periods, yields

[14] In this particular check, given the reduction of the sample size, we use a J=4 level MODWT decomposition. Therefore, the results reported in the last two lines of panels c) and d) of Table 5 relate to forecasts of a smooth component of inflation that captures fluctuations with a period longer than 8 years. These are not strictly comparable to those reported in the paper and in the other robustness checks, where the low-frequency component of inflation corresponds to cycles with a period longer than 16 years.

[15] Other robustness checks are briefly described in section 3.2 in the online appendix.

better forecasts than simply setting $\kappa_{HF} = \kappa_{BCF} = \kappa_{MF} = 0$ and $\kappa_{LF} = 1$ as in the best models seen so far in the paper.[16]

In practice, we optimize the forecast by grid searching the (0 versus 1) weights that minimizes the RMSFE in each quarter of the OOS period.

The optimal combinations of frequency forecasts for h=4 (h=8) are reported in the top left (right) plot in Figure 3. For each frequency (HF, BCF, MF, and LF) and at each point of the OOS, a blue dot denotes that that frequency forecast is included in the forecasting model. The forecasts for h=4 (h=8) are reported in the bottom left (right) plot in Figure 3. Black lines denote actual inflation, red lines the forecasts using the NKPC_WAV_diag (LF) model, and blue lines the forecasts with this optimized model. Two results stand out. First, selecting the right frequencies leads to forecasts that track inflation remarkably well. Using this optimized model, we obtain relative RMSFEs (vs AO) of 0.53 for h=4 and 0.45 for h=8. This model significantly improves the forecasts of inflation even when compared with our best NKPC_WAV model. Second, in periods when inflation features large swings (positive or negative, usually around recessions), it is possible to improve the forecast of those swings if the LF forecast of inflation is ignored. Indeed, it is important to use other frequency forecasts of inflation around periods of recession.

This analysis strengthens the case for analysing inflation with the Phillips curve in the frequency domain, clearly showing that all frequencies matter, not just the low frequency of inflation (and its predictors).[17]

6 Concluding remarks

In this paper we applied a frequency-domain approach to an empirical New Keynesian Phillips Curve to study the in-sample dynamics of inflation and forecast inflation out-of-sample in a consistent framework.

[16] This exercise is similar to the one in Faria and Verona (2018), which shows that the forecasting of stock market returns can be improved by disregarding forecasts for certain frequencies.

[17] The results in this sub-section raise the challenge of picking relevant frequencies in real-time. Such exercise is, however, beyond the scope of this paper and is left for future work.

With regard to the dynamics of inflation, we find that the unemployment gap is statistically significant at business-cycle and medium-run cyclical frequencies, but not at long-run cycles. Unemployment is, however, not economically relevant even at business and medium-run cycles. The dynamics of inflation is explained essentially by inflation expectations and energy inflation, with inflation expectations dominating in long cycles and energy inflation in short cycles. There are, however, some qualifications in what regards the role of energy inflation. First, at business and medium-run cycles, it became dominant after expectations became anchored in 1999. Second, in some specific episodes, fluctuations of energy inflation in long cycles contributed substantially to the in-sample prediction of the long-run oscillations of inflation given by the NKPC model.

With regard to inflation forecast, we show that the key to obtaining a good inflation forecast starts with a good forecast for the low frequency of inflation. Unlike for the in-sample analysis, the low-frequency component of the unemployment gap turns out to be crucial in realizing these forecasting gains. Finally, our flexible frequency-domain approach further indicates that around business cycle recessions forecasts of other frequencies of inflation prove to be more relevant than those of its low frequency. Hence, all frequencies of inflation and of its predictors matter.

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Figures and tables

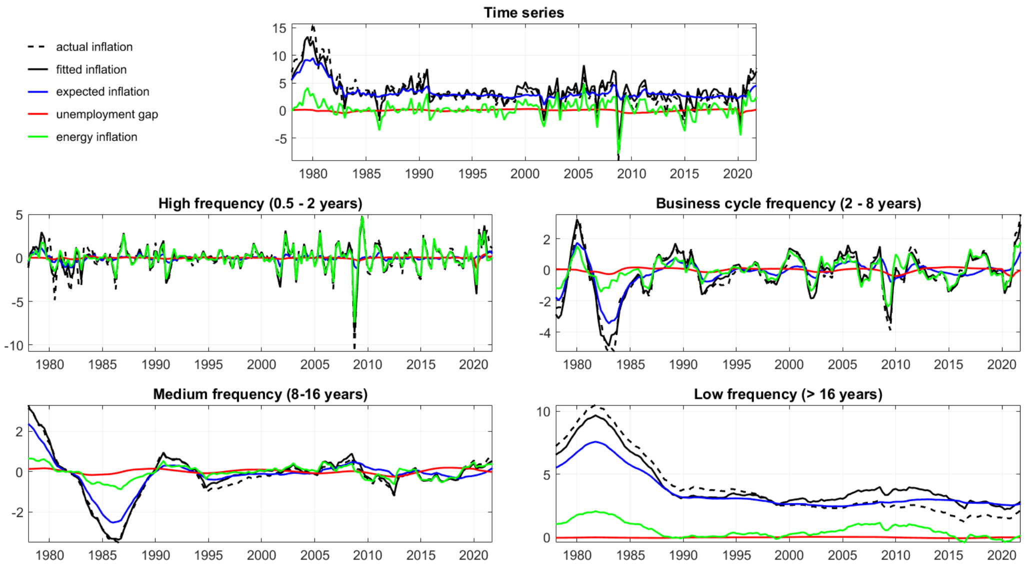

Figure 1: Phillips curve-based decomposition of inflation

Notes: The lines show the contributions of each variable for the period 1978Q1–2021Q4. Constant terms omitted. Black dashed lines: actual inflation. Black lines: fitted inflation. Blue lines: contribution of inflation expectations. Red lines: contribution of output gap. Green lines: contribution of energy inflation. Top graph: decomposition of the time series of inflation. Remaining graphs: decomposition of four frequency-specific time series of inflation.

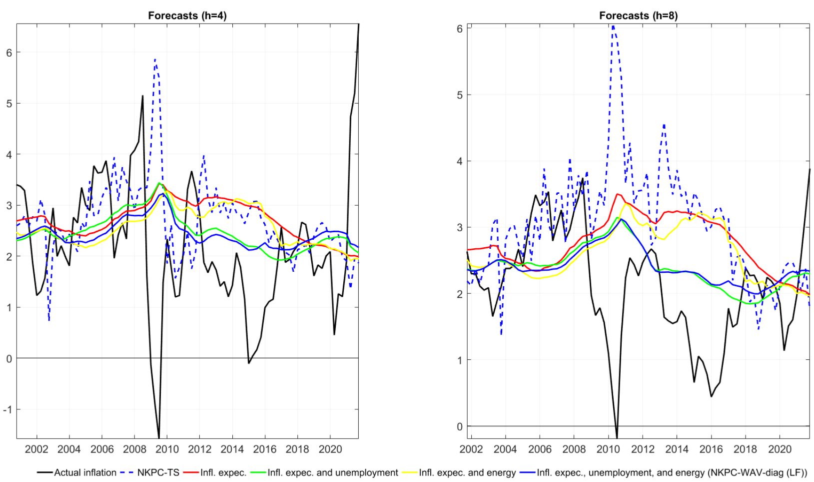

Figure 2: New Keynesian Phillips Curve forecasts for different forecasting horizons: time series vs. wavelet

Forecasts for h=4 (left) and h=8 (right). Black lines: actual inflation. Blue dashed lines: forecasts with the NKPC_TS model. Red lines: forecasts using $\pi^{e,LF}$. Green lines: forecasts using $\pi^{e,LF}$ and $ugap^{LF}$. Yellow lines: forecasts using $\pi^{e,LF}$ and en^{LF}. Blue lines: forecasts with the NKPC_WAV_diag (LF) model. The out-of-sample period is 2000Q1–2021Q4.

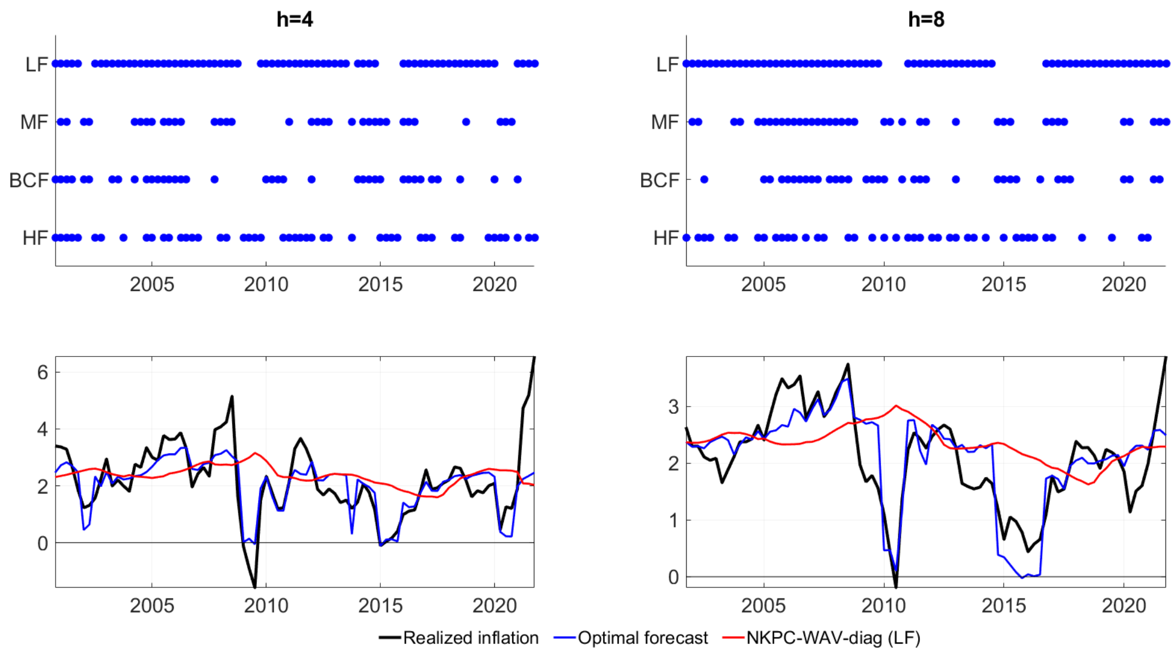

Figure 3: Importance of different frequency forecasts of inflation over the OOS period

Left (right) side graphs: h=4 (h=8). Upper side graphs: each dot indicates whether that specific frequency forecast is included in the forecast of inflation at each point of the OOS period. Lower side graphs: realized inflation (black lines), forecasts with the best model at each point of the OOS forecast period (blue lines), and forecasts using the NKPC_WAV_diag (LF) model (red lines). The out-of-sample period is 2000Q1–2021Q4.

	HF	BCF	MF	LF
Inflation	27	19	11	43
Inflation expectations	6	17	16	61
Unemployment gap	14	32	28	26
Energy inflation	64	26	5	6

Table 1: Variance decomposition by frequency

Each row presents the percentage of the variance of the corresponding time series explained by each specific frequency band; US data, sample period 1978Q1–2021Q4. HF: high frequency, cycles with periods between 2 and 8 quarters; BCF: business cycle frequency, cycles with periods between 2 and 8 years; MF: medium frequency, cycles with periods between 8 and 16 years; LF: low frequency, cycles longer than 16 years. Percentages may not add up to 100 due to rounding.

	α_1	α_2	α_3	R^2
NKPC	0.93^{***} (0.02)	-0.12^{*} (0.05)	0.08^{***} (0.005)	0.86
NKPC_HF	0.76^{***} (0.14)	-0.17^{*} (0.08)	0.08^{***} (0.004)	0.81
NKPC_BCF	1.05^{***} (0.04)	-0.12^{***} (0.02)	0.08^{***} (0.003)	0.95
NKPC_MF	1.07^{***} (0.03)	-0.12^{***} (0.01)	0.08^{***} (0.005)	0.98
NKPC_LF	0.92^{***} (0.03)	-0.03 (0.08)	0.12^{***} (0.02)	0.89

Table 2: Estimates of the New Keynesian Phillips Curve

Estimates of equation (1); US data, sample period 1978Q1–2021Q4. First row: estimates obtained with the original time-series data. Subsequent rows: estimates obtained from filtered data for different frequency bands. HF: high frequency, cycles with periods between 2 and 8 quarters; BCF: business cycle frequency, cycles with periods between 2 and 8 years; MF: medium frequency, cycles with periods between 8 and 16 years; LF: low frequency, cycles longer than 16 years. Newey and West (1987) standard errors are reported in parentheses. *,**, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Model	Forecasting horizon h=4	h=8
a) Benchmarks		
AO random walk	1.63	1.12
UCSV	1.79	1.15
b) Time-series Phillips curve (vs AO)		
NKPC_TS	1.1	1.48
c) Wavelet-based Phillips curves (vs AO)		
NKPC_WAV_diag	1.02	1
NKPC_WAV_all	1.08	1.12
NKPC_WAV_diag (LF)	0.85**	0.82**
NKPC_WAV_all (LF)	0.85**	0.81**
d) Wavelet-based Phillips curves (vs NKPC_TS)		
NKPC_WAV_diag	0.93	0.68**
NKPC_WAV_all	0.98	0.76
NKPC_WAV_diag (LF)	0.78*	0.56**
NKPC_WAV_all (LF)	0.77*	0.55**

Table 3: Relative out-of-sample root mean squared forecast errors

Panel a): Root mean squared forecast errors (RMSFEs) at different forecasting horizons (h=4 and h=8) for the AO model and the UCSV model. Panels b) and c): RMSFEs relative to those of the AO model (RMSFE_j / RMSFE_AO for model j). Panel d): RMSFEs relative to those of the NKPC_TS model (RMSFE_j / $\text{RMSFE}_\text{NKPC-TS}$ for model j). Asterisks indicate statistical significance of the Diebold-Mariano-West test of comparative predictive accuracy at the 10 % (*) and 5 % (**) levels, relative to the AO model (panels b and c) or the NKPC_TS model (panel d). The out-of-sample period is 2000Q1–2021Q4.

forecasting horizon	predictors $\pi^{e,LF}$	$\pi^{e,LF}, ugap_t^{LF}$	$\pi^{e,LF}, en_t^{LF}$	$\pi^{e,LF}, ugap_t^{LF}, en_t^{LF}$
h=4	0.96	0.86**	0.95	0.85**
h=8	1.13	0.81**	1.07	0.82**

Table 4: The importance of the low frequencies of each predictor

This table reports the RMSFEs relative to those of the AO model. Asterisks indicate statistical significance of the Diebold-Mariano-West test of comparative predictive accuracy at the 5 % (**) level relative to the AO model. The out-of-sample period is 2000Q1–2021Q4.

Model	Measure of slack				Measure of inflation		Wavelet filter	
	Unemployment rate		Output gap		PCE inflation		Daubechies length 4	
	Forecasting horizon		Forecasting horizon		Forecasting horizon		Forecasting horizon	
	h=4	h=8	h=4	h=8	h=4	h=8	h=4	h=8
a) Benchmarks								
AO random walk	1.63	1.12	1.63	1.12	1.3	0.91	1.63	1.12
UCSV	1.79	1.15	1.79	1.15	1.65	1.10	1.79	1.15
b) Time-series Phillips Curve (vs AO)								
NKPC_TS	1.08	1.38	1.06	1.51	1.15	1.58	1.1	1.48
c) Wavelet-based Phillips Curves (vs AO)								
NKPC_WAV_diag	1.01	0.99	1.06	1.12	1.03	1.02	1.05	1.03
NKPC_WAV_all	1.09	1.15	1.09	1.09	1.1	1.19	1.1	1.13
NKPC_WAV_diag (only LF)	0.86**	0.84**	0.89*	0.88	0.89	0.92	0.86**	0.84**
NKPC_WAV_all (only LF)	0.85**	0.82**	0.89*	0.84**	0.88*	0.87	0.86**	0.84**
d) Wavelet-based Phillips Curves (vs NKPC_TS)								
NKPC_WAV_diag	0.94	0.72**	1.00	0.74*	0.90	0.64**	0.96	0.70*
NKPC_WAV_all	1.01	0.83	1.03	0.73	0.96	0.75	1	0.77
NKPC_WAV_diag (only LF)	0.80*	0.61**	0.84	0.59*	0.78	0.58**	0.78*	0.57**
NKPC_WAV_all (only LF)	0.79*	0.59**	0.84	0.56**	0.77	0.55**	0.79*	0.57**

Table 5: Relative out-of-sample root mean squared forecast errors - robustness checks

Panel a): Root mean squared forecast errors (RMSFEs) at different forecasting horizons (h=4 and h=8) for the AO model and the UCSV model. Panels b) and c): RMSFEs relative to those of the AO model ($RMSFE_j$ / $RMSFE_{AO}$ for model j). Panel d): RMSFEs relative to those of the NKPC_TS model ($RMSFE_j$ / $RMSFE_{NKPC\text{-}TS}$ for model j). Asterisks indicate statistical significance of the Diebold-Mariano-West test of comparative predictive accuracy at the 10 % (*) and 5 % (**) levels, relative to the AO model (panels b and c) or the NKPC_TS model (panel d). The out-of-sample period is 2000Q1–2021Q4.

	In-sample length		In-sample start		Band-pass filter	
	Rolling window		1985:Q1			
	Forecasting horizon		Forecasting horizon		Forecasting horizon	
Model	h=4	h=8	h=4	h=8	h=4	h=8
a) Benchmarks						
AO random walk	1.63	1.12	1.63	1.12	1.63	1.12
UCSV	1.79	1.15	1.79	1.15	1.79	1.15
b) Time-series Phillips Curve (vs AO)						
NKPC_TS	1.11	1.08	1.13	1.2	1.1	1.48
c) Wavelet-based Phillips Curves (vs AO)					**c) Bandpass-based Phillips Curves (vs AO)**	
NKPC_WAV_diag	1.1	1.01	1.11	1.08	1.06	1.14
NKPC_WAV_all	1.1	1.06	1.02	1.02	1.1	1.2
NKPC_WAV_diag (only LF)	0.86**	0.83**	0.97	0.97	0.99	1.04
NKPC_WAV_all (only LF)	0.85**	0.83**	0.89*	0.89	1.02	1.1
d) Wavelet-based Phillips Curves (vs NKPC_TS)					**c) Bandpass-based Phillips Curves (vs NKPC_TS)**	
NKPC_WAV_diag	1	0.94	0.98	0.90*	0.97	0.77*
NKPC_WAV_all	1	0.98	0.90**	0.85	1.01	0.81
NKPC_WAV_diag (only LF)	0.77*	0.76	0.86*	0.81*	0.90	0.70
NKPC_WAV_all (only LF)	0.77*	0.76	0.78**	0.74**	0.93	0.74

Table 5: Relative out-of-sample root mean squared forecast errors - robustness checks (continue)

Panel a): Root mean squared forecast errors (RMSFEs) at different forecasting horizons (h=4 and h=8) for the AO model and the UCSV model. Panels b) and c): RMSFEs relative to those of the AO model ($RMSFE_j$ / $RMSFE_{AO}$ for model j). Panel d): RMSFEs relative to those of the NKPC_TS model ($RMSFE_j$ / $RMSFE_{NKPC\text{-}TS}$ for model j). Asterisks indicate statistical significance of the Diebold and Mariano (1995) test of comparative predictive accuracy at the 10 % (*) and 5 % (**) levels, relative to the AO model (panels b and c) or the NKPC_TS model (panel d). The out-of-sample period is 2000Q1–2021Q4.

Online appendix of

"Inflation dynamics and forecast: frequencies matter"*

Manuel M. F. Martins[†] Fabio Verona[‡]

This online appendix is composed of the following:

- Section 1: Data
- Section 2: Complementary analyses on inflation dynamics
- Section 3: Complementary analyses and robustness checks on inflation forecast

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* The views expressed are those of the authors and do not necessarily reflect those of the Bank of Finland. Any remaining errors are the sole responsibility of the authors.

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1 Data

Our data are US quarterly time series for 1978Q1–2021Q4 of inflation, the unemployment gap, expectations of inflation, and energy inflation. The original time series of these variables and their frequency components (extracted with the Haar wavelet filter) are reported in Figures A.1 to A.4 below.

2 Complementary analyses on inflation dynamics

2.1 Time- and frequency-varying NKPC coefficients

In this section we analyze whether the estimates of the NKPC coefficients for our cyclical frequencies are more or less stable over time than those of the standard time-series NKPC. We do so by inspecting estimates obtained from expanding windows that start with the sample 1978Q1–1999Q4, and recursively add one quarter through 2021Q4.[1]

Figure A.5 shows these estimates and their statistical (in)significance. The charts in the first column plot estimates of α_1, the second column those of the NKPC slope (α_2), and the third column the coefficients associated with energy prices (α_3). The graphs in the upper row report the results for the standard time-series NKPC, and each subsequent row shows time-varying estimates for each frequency band, from HF through LF.

We find no evidence of flattening of the time-series NKPC after 2000. The estimate of the slope features a rapid decrease since the latter stages of the Great Recession, becoming statistically insignificant, and then returning to statistical significance only at the end of the sample period. While our result is not strictly comparable with most literature on the flattening of the Phillips curve (which dates it sometime between the mid-1980s and the early 1990s – periods not covered in the Figure), it contrasts with papers that point

[1] We set 22 years as the minimum length of our samples, thus focusing on estimates from 2000Q1 onwards, because estimates for frequency bands comprising medium- and long-run cycles (MF, LF) would not be reliable for shorter samples. The choice of the initial sample period is also consistent with the forecast period used in the out-of-sample forecast part of the paper.

to a flatter curve after the Fed announced its 2 % inflation target in 2012 (e.g. Bundick and Smith, 2020). Interestingly, our results indicate that the slope of the time-series NKPC has decreased suddenly but not markedly during the Covid period. The sensitivity of inflation to expected inflation and to energy inflation changed markedly at the end of the Great Recession, but in opposite ways: the estimates of α_1 fell substantially, and then kept on gradually decreasing until around 2015, remaining statistically significant throughout; in turn, the estimates of α_3 (the coefficient associated to energy inflation) increased markedly and remained stable at high and statistically significant values.

There are considerable differences in the evolution over time of the estimates of the NKPC coefficients for the several frequency bands. Focusing on the slope (α_2), we first note that it is more precisely estimated for most frequency bands than for the aggregate time series. Of the four frequency bands, between 2000 and 2021, we found not statistically significant estimates only at LF cycles and since around 2013. At HF cycles, there is evidence of a flattening after the Great Recession and, more recently, during the Covid recession. At business cycles (BCF), the estimate suddenly fell in 2008-2009, but rapidly recovered and remained stable and significant throughout, with a full-sample estimate similar to that of the time-series NKPC. At medium-run cycles (MF) the estimate of the NKPC slope gradually became more negative since 2005, reaching a full-sample estimate equal to the time-series NKPC slope. The estimates of the slope at long-run cycles (LF) are positive and significant for most of the sample period, inconsistently with theory. Since around 2013, however, they become not statistically significant, suggesting that when enough data is taken in econometric estimates are consistent with a vertical long-run Phillips curve. Overall, these results suggest that the Phillips trade-off remains a significant is a short-run and business-cycle phenomenon, and that it is increasingly relevant also at medium-run cycles.

The estimates of the coefficient associated to energy prices (α_3) increase abruptly at the end of the Great Recession for both the short-run (HF) and business-cycle (BCF) frequencies, but also – albeit more gradually – for medium-run cycles (MF). Overall, our time-varying estimates across frequencies indicate that its time variation in the time-series NKPC is determined by these frequencies. In turn, the estimates of α_3 for long-run cycles (LF) decrease from 2005 until 2012, when they are not significant for a very short period before increasing again since 2014, to a full sample estimate above that obtained for the time-series NKPC and the other three frequency bands.

The estimates of the coefficient associated to expectations (α_1) at short-run cycles (HF) are consistently smaller than the time-series NKPC estimates, but always statistically significant. They experience a small decrease during the Great Recession, subsequently increasing since the start of 2020. At business cycles (BCF), the estimates of α_1 fell markedly during the Great Recession, remaining statistically significant throughout and indicating a one-to-one change of inflation with expectations. A similar pattern is featured by the estimates of α_1 at medium-run cycles (MF), albeit with a more gradual decrease around the Great Recession. At LF cycles, the response of inflation to expectations is fairly stable and close to one until 2012, then gradually decreasing through the end of the sample period to an estimate slightly below one.

3 Complementary analyses and robustness checks on inflation forecast

3.1 Forecast timing

To explore the timing of the outperformance of the NKPC_WAV_all (LF) model versus both the AO model and the time-series NKPC model, in Figure A.6 we show the cumulative differences between the squared forecast errors (SFE) of the NKPC_WAV_all (LF) model and those of the AO model (left-hand side charts) and the NKPC_TS (right-hand side charts) through the OOS period. The top charts show cumulative differences in SFE for 4-quarter-ahead forecasts and the bottom charts relate to the 8-quarters-ahead forecasts.

In the plots in Figure A.6, a rising line indicates the predictive regression of the NKPC_WAV_all (LF) model outperforms the alternative model at the relevant forecast horizon.

A broad conclusion drawn from Figure A.6 is that, along the 22 years of the OOS forecasting exercise, our

NKPC_WAV_all (LF) model rarely underperforms any of the alternative models. Indeed, the cumulative difference of SFEs rarely decreases for all forecast horizons and relative to both models. Furthermore, the temporal pattern of forecasting outperformance of our model is rather similar for all forecast horizons, irrespective of the alternative model: our model clearly performs better from the early stages of the recovery from the Great Recession until about 2015 in the case of the AO and until about 2016 for the time-series NKPC.

3.2 Further robustness checks

We ran several additional checks that we do not report here for the sake of brevity as the results were quite similar. In particular, we ran forecasts i) for other measures of slack (quadratic detrended unemployment, CBO detrended unemployment, quadratic detrended real GDP, CBO detrended real GDP), and ii) with the original MODWT frequency decomposition rather than using the sum of D_1 and D_2 for HF, and of D_3 and D_4 for BCF. These results are available upon request.

References

BUNDICK, B., AND A. L. SMITH (2020): "Did the Federal Reserve Break the Phillips Curve? Theory and Evidence of Anchoring Inflation Expectations," *Federal Reserve Bank of Kansas City Working Paper*, (20-11).

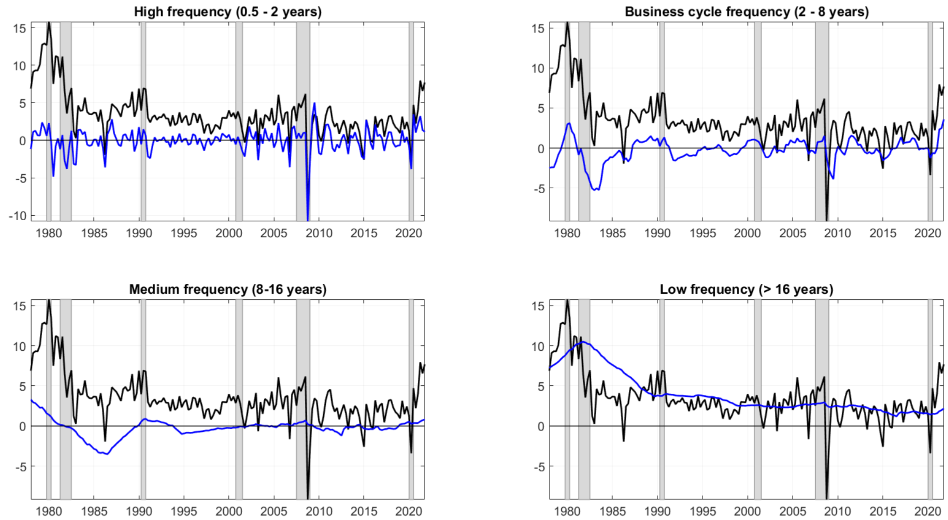

Figure A.1: Inflation: time series and frequency decomposition

Time series of inflation (black lines) and four frequency-specific time series resulting from the frequency decomposition of the inflation time series (blue lines). Gray bars denote NBER-dated recessions. US data, sample period 1978Q1–2021Q4.

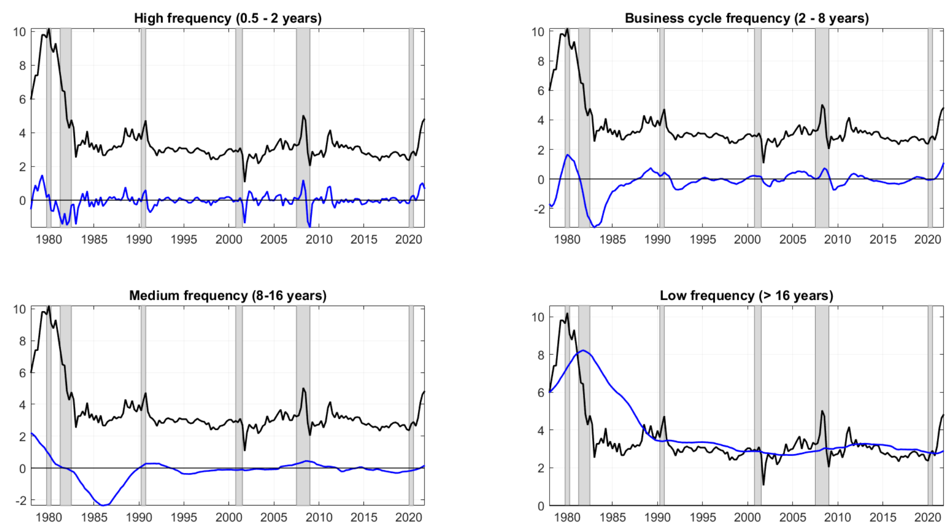

Figure A.2: Inflation expectations: time series and frequency decomposition

Time series of inflation expectations (black lines) and four frequency-specific time series resulting from the frequency decomposition of the inflation expectations time series (blue lines). Gray bars denote NBER-dated recessions. US data, sample period 1978Q1–2021Q4.

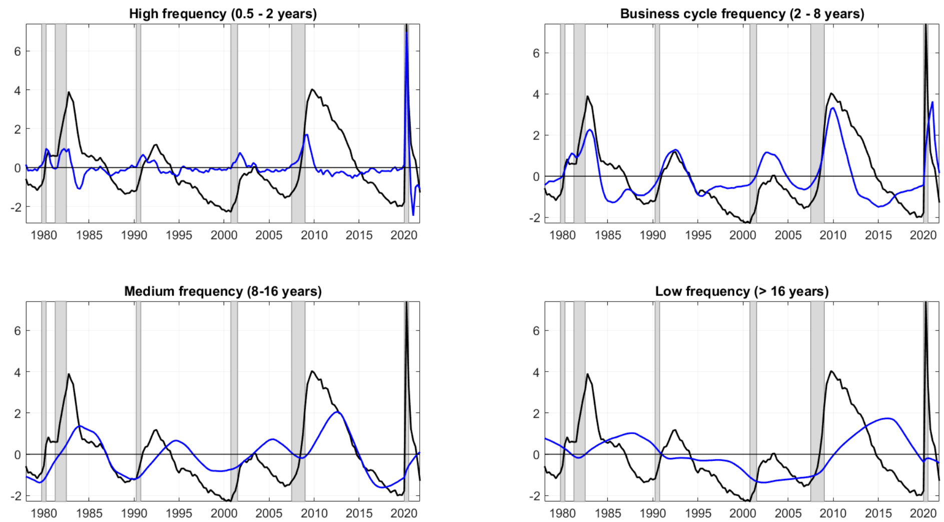

Figure A.3: Unemployment gap: time series and frequency decomposition

Time series of unemployment gap (black lines) and four frequency-specific time series resulting from the frequency decomposition of the unemployment gap time series (blue lines). Gray bars denote NBER-dated recessions. US data, sample period 1978Q1–2021Q4.

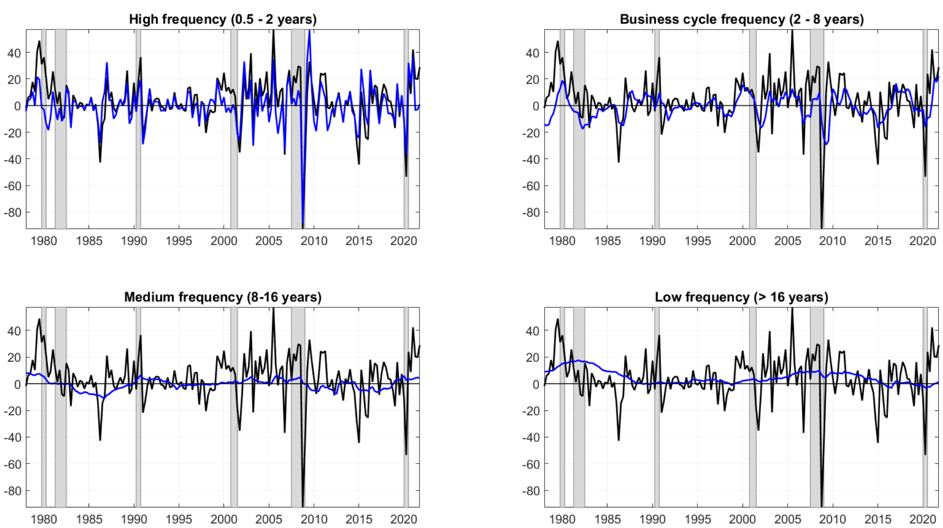

Figure A.4: Energy inflation: time series and frequency decomposition

Time series of unemployment gap (black lines) and four frequency-specific time series resulting from the frequency decomposition of the energy inflation time series (blue lines). Gray bars denote NBER-dated recessions. US data, sample period 1978Q1–2021Q4.

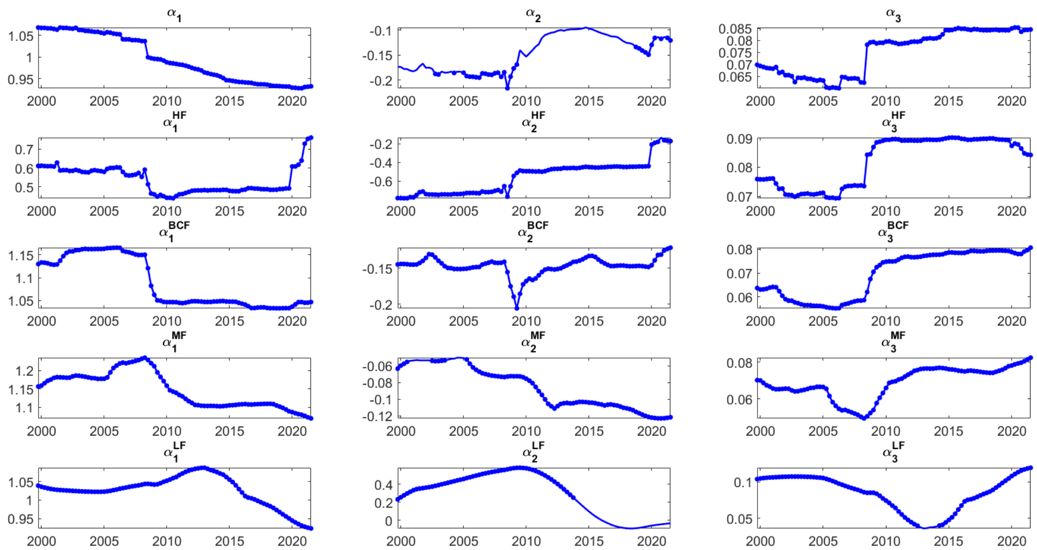

Figure A.5: Estimated coefficients of the New Keynesian Phillips Curve over time and across frequencies

Recursive estimates of the coefficients of the Phillips curve (equation (1)) for the original time series data (upper row) and filtered data for different frequency bands (remaining rows). HF: high frequency, cycles with periods between 2 and 8 quarters; BCF: business cycle frequency, cycles with periods between 2 and 8 years; MF: medium frequency, cycles with periods between 8 and 16 years; LF: low frequency, cycles longer than 16 years. Sample periods are expanding windows starting in 1978Q1–1999Q4, recursively including one additional quarter through 2021Q4. Statistically significant coefficients (at 10 %) are reported with a circled marker.

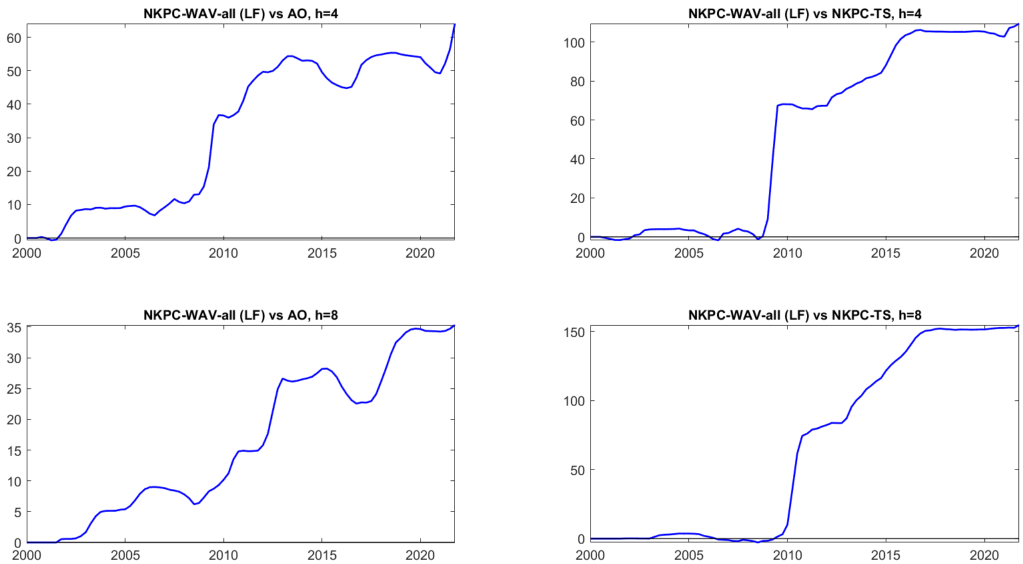

Figure A.6: Cumulative differences in squared forecast errors

Left side graphs: cumulative difference between the squared forecast errors of the NKPC_WAV_all (LF) model and those of the AO model, for h=4 (top) and h=8 (bottom). Right side graphs: cumulative difference between the squared forecast errors of the NKPC_WAV_all (LF) model and those of the NKPC_TS model, for h=4 (top) and h=8 (bottom).

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