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Bank of Finland
Research Unit

PO Box 160
FIN-00101 Helsinki

Phone: +358 9 1831

Email: research@bof.fi
Website: www.suomenpankki.fi/en/research/research-unit/

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The *Aino 3.0* model*

Aino Silvo† Fabio Verona‡

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Abstract

In this paper we present *Aino 3.0*, the latest vintage of the dynamic stochastic general equilibrium (DSGE) model used at the Bank of Finland for policy analysis. *Aino 3.0* is a small-open economy DSGE model at the intersection of the recent literatures on so-called TANK ("Two-Agent New Keynesian") and MONK ("Mortgages in New Keynesian") models. It aims at capturing the most relevant macro-financial linkages in the Finnish economy and provides a rich laboratory for the analysis of various macroeconomic and macroprudential policies. We show how the availability of a durable consumption good (housing), on the one hand, and the presence of credit-constrained households, on the other hand, affect the transmission of key macroeconomic and financial shocks. We also illustrate how these new transmission channels affect model dynamics compared to the previous model vintage (the *Aino 2.0* model of Kilponen et al., 2016).

Keywords: business cycles, small open economy, credit constraints, housing market, long-term debt

JEL codes: E21, E32, E44, F41, R31

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†Bank of Finland – Monetary Policy and Research Department (aino.silvo@bof.fi)

‡Bank of Finland – Monetary Policy and Research Department, and University of Porto – *CEF.UP* (fabio.verona@bof.fi)

1 Introduction

The housing sector in Finland is an important driver of the Finnish business cycle. Residential investment activity is strongly pro-cyclical and also much more volatile than output (Figure 1). As a result, the share of residential investment in GDP tends to increase in booms and decrease in recessions (Figure 2). Housing is also a major component of household balance sheets: two thirds of gross household wealth is held in residential real estate.[1]

At the same time, household indebtedness, which has grown secularly in the past 20 years in an environment of low interest rates (Figure 3), is seen as one of the key macroeconomic vulnerabilities in the Finnish economy. Recent macroeconomic research has shown that credit booms and rapid increases in household indebtedness are strong and reliable predictors of recessions (see *e.g.* Gourinchas and Obstfeld (2012), Jordá et al. (2016), Martin and Philippon (2017), Mian et al. (2017) and Drehmann et al. (2018)).

Much of the current macroprudential policy debate, in Finland as well as internationally, revolves around the implementation of demand-side policy tools that affect households' ability to borrow. Such tools include loan-to-value (LTV), debt-to-income (DTI), and debt-service-to-income (DSTI) requirements. These demand-side policies complement capital-based macroprudential policies (*e.g.* bank capital and risk-weight requirements) that affect the loan supply by banks. Currently, Finnish legislation enables the use of a potentially counter-cyclical loan-to-collateral (LTC) limit on the demand side, as well as various potentially counter-cyclical requirements on bank capital on the supply side.

Structural models provide the proper tools to evaluate the general equilibrium effects of household indebtedness and house price booms and busts, as well as to analyze the (relative) effectiveness of different macroprudential policies and instruments in stabilising business and credit cycles. To address these issues, we develop a dynamic stochastic general equilibrium (DSGE) model, *Aino 3.0*, which builds on its predecessor, the *Aino 2.0* model for the Finnish economy (Kilponen et al., 2016).

The *Aino 2.0* model was developed in the aftermath of the global financial crisis of 2007–2009. It added

[1] According to the Statistics of Finland's Household's Wealth Survey (available at http://www.stat.fi/til/vtutk/index_en.html), in 2016, residential real estate property made up on average two thirds of Finnish households' total gross wealth. The figure includes all housing wealth of households, *i.e.* dwellings used as main residence of the households, holiday homes as well as other residential real estate owned by the households. Housing wealth used as primary residence comprises on average half of households' total gross wealth.

a monopolistically competitive banking sector and short-term corporate lending to the previous vintage of the *Aino* model, as the focus in macroeconomic research and policy analysis shifted to financial factors as drivers of business cycles. The Finnish economy is characterised by an oligopolistic banking sector and high reliance on bank funding by firms. These features are well captured by the *Aino 2.0* model. Likewise, the current model version also features a monopolistically competitive banking sector in the spirit of Gerali et al. (2010) within a small open economy setting similar to Adolfson et al. (2008) and Christoffel et al. (2008).

In this new version of the *Aino* model, we build on the previous model development work and introduce a housing market along the lines of Iacoviello (2005) in the *Aino 2.0* model. Our model thus belongs to a large body of quantitative research that studies the interaction of house prices and aggregate economic activity when some households are credit-constrained. Prominent contributions in this literature include Iacoviello and Neri (2010), Kiyotaki et al. (2011), Iacoviello and Pavan (2013), Justiniano et al. (2015) and Guerrieri and Iacoviello (2017). More generally, a large and growing literature explores the implications of a limited form of wealth heterogeneity — through the presence of credit-constrained households — in New Keynesian models often referred to as TANK or "two-agent New Keynesian" models (Debortoli and Gali, 2018). *Aino 3.0* belongs to this broad class of models.

In the model, there are patient households, who are savers (*i.e.* lenders) in equilibrium, and impatient households, who in equilibrium need to borrow to finance their housing expenditures. Households' borrowing is subject to a borrowing constraint that restricts the nominal loan amount to a fraction of the market value of the housing being purchased. Banks provide corporate loans to entrepreneurs and mortgage loans to households by combining deposits, collected from households, with their own capital.

As opposed to most quantitative macroeconomic research on the interactions of house prices and aggregate consumption, we realistically model mortgage loans as multi-period loans following the novel framework proposed by Kydland et al. (2016). As a consequence, our flexible framework allows us to study the effects of different average loan maturities and amortization schemes on the aggregate economy. In this way, our work also contributes to the growing research on the effects of long-term mortgage debt on business cycle dynamics and on the transmission of monetary policy. Recent contributions to this literature include Garriga et al. (2017), Gelain et al. (2018a), Gelain et al. (2018b), Garriga et al. (2019), Bluwstein et al.

(2020), Grodecka (2020) and Kaplan et al. (2020).[2]

Entrepreneurial loans are modelled as one-period loans, in line with the empirically observed short maturity of most corporate loans. Finally, banks in the model are subject to a capital requirement. Hence, as the model features both mortgage and corporate loans, the model allows the analysis risk-weight requirements associated with mortgage loans. This also allows us to distinguish between the regulatory capital-asset ratio and the actual bank leverage.

Given the two-agent setup and the richly modelled financial sector, our model is well suited to analyse the business cycle implications of household debt and the housing sector. A large and growing body of academic research has shown that household (de)leveraging is a mechanism that can amplify business cycles. Furthermore, a rapid increase in household indebtedness ratios is a strong predictor of future recessions.[3] In a recession, indebted households cut down their consumption to be able to service their debt. Falling demand pushes down house prices, which reduces collateral values. This, in turn, forces households to reduce consumption even further in order to retain positive equity in their housing. The feedback loop from asset prices back to consumption thus exacerbates the recession.

Aino 3.0 also allows to study the effects of several macroprudential policies on the Finnish economy, as it embeds a large set of macroprudential instruments. These include potentially countercyclical capital requirements, risk weighting on mortgage loans, a loan-to-value ratio requirement on mortgage loans, and different mortgage repayment schedules. Furthermore, the model allows for an in-depth analysis of the monetary policy transmission mechanism in the presence of credit-constrained households, as it features both a direct interest rate channel – through variable lending rates – and an indirect balance sheet channel – through collateral values and lending spreads faced by borrowers.

As discussed above, the current model captures the key endogenous feedback mechanisms related to household (de)leveraging, and thus the macroeconomic risk posed by elevated household indebtedness

[2]Drehmann et al. (2018) also point out the importance of long-term debt and debt service in understanding the mechanisms through which household borrowing can translate into macroeconomic instability and financial crises.

[3]In a series of influential papers, Mian and Sufi (2009), Mian and Sufi (2010), Mian and Sufi (2011) and Mian et al. (2013) document the household leveraging and deleveraging cycle before and during the financial crisis of 2007–2009 in the United States. Martin and Philippon (2017) examine the European experience in the financial crisis and the subsequent sovereign debt crisis. Empirical work by Jordá et al. (2016) and Mian et al. (2017) shows that mortgage credit booms are a reliable predictor of economic crises in advanced economies, and that the magnitude and composition of credit booms have important implications for business cycles.

on the real economy, which is identified as the main macro-financial risk in the Finnish context. A key caveat of the model is the absence of default risk both on the lender and the borrower side. Historically, however, the non-performing loan rate on mortgages has been very low in Finland. Corporate loans have been somewhat riskier, and in the Scandinavian financial crisis of the 1990's, they caused large losses for the banking sector. In contrast, the Finnish economy was spared from widespread defaults in the global financial crisis of 2007–2009.

A second caveat of the model concerns the absence of a rental market: households do not have a choice over their occupancy status. Consequently, the model does not allow to study spillover effects between rental and owner-occupied property markets, which may, at least in some situations, have important macroeconomic implications.

The remainder of the paper is organised as follows. The next section describes the model economy, with an emphasis on the new features of the model compared to its predecessor, the *Aino 2.0*. Section 3 discusses the model calibration and its fit to observed data. Section 4 presents responses of the model to key macroeconomic and financial shocks, and discusses the implications of the credit constraints and the presence of a housing asset on the aggregate dynamics. Finally, Section 5 concludes.

2 The model

This section outlines the model economy. We place particular emphasis on the new features of the *Aino 3.0* model compared to the previous model vintage. We start by describing the household sector. We then describe the structure of goods production, the export and the import markets, the banking sector, and finally the government sector, as well as the market equilibrium. The exposition of the model in this section heavily relies on Kilponen et al. (2016).

Figure 4 sketches the general structure of *Aino 3.0*. The sectors and flows plotted in blue are new elements compared to the *Aino 2.0* model. The structure of the model otherwise builds on that of *Aino 2.0* and can, given a suitable parametrisation, nest the *Aino 2.0* model.

In comparison with *Aino 2.0*, the current model vintage includes housing as an asset available to households, produced by housing investment good producers and housing producers on the supply side. On the

financial side, banks now also lend to households in the form of mortgage loans, in addition to corporate lending already present in *Aino 2.0.*

On the demand side, there are two types of households in the current model vintage: patient and impatient ones. Both types of households buy consumption goods and supply labour services monopolistically. Both types can also buy and hold housing, which yields them utility through the consumption of housing services. Patient households allocate their savings between three types of bonds – euro-denominated foreign bonds ("euro area bonds"), foreign bonds denominated in foreign currency ("rest-of-the-world bonds"), and euro-denominated domestic government bonds.[4] They can also deposit their savings at a bank. Impatient households, instead, are assumed to only have access to bank loans or deposits, and are excluded from the bond market.

The model economy is a single good economy, with no distinction made between traded and non-traded goods. Instead, varieties of the single intermediate good are produced by domestic intermediate good producers. They are then combined with imported intermediate goods to produce final goods. The final goods are produced in four sectors by four types of retailers, describing final use: consumption good retailers, housing good retailers, investment good retailers and exporters. The domestic intermediate good producers as well as the exporters and the foreign importers operate under monopolistic competition, and their pricing decisions are subject to Calvo (1983) pricing frictions.[5]

The general government purchases consumption and investment goods from the private sector. It collects labour and capital income taxes, real estate taxes, firms' social security contributions and indirect consumption taxes. The government also conducts macroprudential policy and sets the (potentially time-varying) parameters of the macroprudential requirements. Finally, we assume that the government runs a balanced budget, and makes lump-sum (net) transfers to households to balance the budget in each period.

Finally, we model Finland as a small member of a monetary union. The conduct of monetary policy and the behaviour of the external sector both reflect this assumption. Monetary policy is set fully exogenously

[4]We divide net foreign bond holdings into intra and extra euro area bonds in order to capture the shares of intra and extra euro area countries in Finnish exports.

[5]Following Kilponen et al. (2016), and in contrast to many other models, we assume that some foreign importers price their product in the local currency (euro) and the rest in their own currency (the producer currency). This assumption reflects the fact the euro area is an important trade partner for Finland and enables flexible control of the degree of exchange rate pass-through to domestic inflation (see Freystatter, 2012).

with regard to developments in the domestic economy. Foreign interest rates, exchange rates, external demand and foreign competitors' prices are all taken as exogenously given by agents in the domestic economy.

2.1 Households

The economy is populated by a continuum of households, indexed by $h \in (0,1)$. There are two types of households: a constant share ω_h of households are patient (type P), and a share $1-\omega_h$ are impatient (type I). The households' preferences differ in terms of their subjective discount factor: the discount factor of patient types is β^P and that of the impatient types is β^I, with $\beta^P > \beta^I$. In equilibrium, the patient households will be savers (*i.e.* lenders) in the economy, and the impatient households will be borrowers.

2.1.1 Patient households

Patient households consume goods and housing services, supply labour, and save into domestic and international bonds as well as housing and bank deposits. It is assumed that the patient households own all firms in the economy and are the only ones to have access to the bond market (both domestic and international).[6]

They maximize their lifetime utility over the sequences of consumption $\left(C^P_{h,t}\right)$, holdings of housing $\left(H^P_{h,t+1}\right)$, bank deposits $(D_{h,t+1})$, and euro area $\left(B^{€,\mathrm{P}}_{h,t+1}\right)$, rest-of-the-world $\left(B^{\$,P}_{h,t+1}\right)$ and domestic $\left(B^P_{h,t+1}\right)$ bond holdings. Households supply differentiated labour services $L^P_{h,t}$ to firms and act as wage setters in the monopolistically competitive labour markets. Labour services are demand-determined in equilibrium, since households commit to supply any given demand of labour at the equilibrium wage.

Preferences of patient households are given by:

$$E_t \sum_{s=t}^{\infty} \left(\beta^P\right)^{s-t} \left[\zeta^C_s \log\left(C^P_{h,s} - b^P_c C^P_{s-1}\right) + \zeta^H_s j^P \log\left(H^P_{h,s} - b^P_H H^P_{s-1}\right) - \frac{\zeta^L_s}{1+\sigma_l}\left(L^P_{h,s}\right)^{1+\sigma_l}\right], \qquad (1)$$

[6] Since patient households are (in equilibrium) not constrained by a collateral constraint, their maximisation problem is essentially identical to the problem of the representative household in Kilponen et al. (2016), with the addition of housing consumption.

where ζ_t^C is a consumption preference shock, ζ_t^H is a housing preference shock, ζ_t^L is a labour disutility shock, $b_c^P > 0$ is the degree of external habit formation in consumption, $j^P > 0$ is a scaling parameter, $b_H^P > 0$ is the degree of external habit formation in housing, $\sigma_l > 0$ is the inverse of the Frisch labor supply elasticity, and $L_{h,t}^P$ are total hours supplied by patient workers to the intermediate goods producers. Note that the preference and labour disutility shocks are not type-specific, but instead common to both types of households.[7]

A household h of type P faces the following nominal periodic budget constraint:

$$\begin{aligned}
&\left(1+\tau_s^C\right) P_s^C C_{h,s}^P + B_{h,s+1}^P + B_{h,s+1}^{€,\mathrm{P}} + S_s B_{h,s+1}^{\$,P} + P_s^H H_{h,s+1}^P + D_{h,s+1} \qquad \forall\, s = t, ..., \infty \\
&= \left(1-\tau_s^W\right) W_s^P L_{h,s}^P + \left(1-\tau_s^H\right) P_s^H \left(1-\delta_s^H\right) H_{h,s}^P + R_{s-1}^D D_{h,s} \\
&+ R_{s-1} B_{h,s}^P + \Gamma_{A^*}\left(A_s^*, \zeta_{s-1}^{€}\right)\left(R_{s-1}^{€} B_{h,s}^{€,P} + R_{s-1}^{\$} S_s B_{h,s}^{\$,P}\right) + \mathcal{D}_{h,s}^P \\
&- \mathcal{TR}_{h,s}^P + \mathcal{S}_{h,s}^P. \qquad (2)
\end{aligned}$$

The left-hand side of the budget constraint features nominal spending on consumption (τ_t^C is the tax rate on consumption), deposits ($D_{h,t+1}$), domestic bonds ($B_{h,t+1}$), foreign assets denominated in euro ($B_{h,t+1}^{€}$), and foreign assets denominated in the foreign ("rest of the world") currency ($B_{h,t+1}^{\$}$). On the right-hand side of the budget constraint, the income of the household consists of nominal after-tax wage income $\left(1-\tau_t^W\right) W_t^P L_{h,t}^P$, the nominal value P_t^H of the undepreciated housing stock $\left(1-\delta_t^H\right) H_{h,t}^P$ held by the household at the beginning of the period, net of the real estate tax rate (τ_t^H), the nominal gross return on deposits ($R_{t-1}^D D_{h,t}$), lump-sum net transfers from the government $\mathcal{TR}_{h,t}$, distributed profits from firms owned by the household $\mathcal{D}_{h,t}$, as well as the nominal gross returns R_{t-1}, $R_{t-1}^{€}$ and $S_t R_{t-1}^{\$}$ on the domestic, euro and foreign bond holdings, respectively. S_t is the nominal exchange rate vis-à-vis foreign currency, defined as €/\$.[8] $\mathcal{S}_{h,t}$ denotes state-contingent securities. We assume that they are type-specific and allow patient households to insure against wage income risk between themselves, guaranteeing that each patient household chooses the same allocations in equilibrium. The euro area interest rate ($R^{€}$) and foreign

[7] Habit formation in the consumption of both non-housing goods and housing services are assumed to introduce some sluggishness into the responses of consumption and housing investment to various shocks. In addition, habit persistence in housing services consumption faciliates the match of the behaviour of mortgage debt and household indebtedness in the model to their empirical counterparts.

[8] In practice, this exchange rate is a trade-weighted effective exchange rate index, describing the relative strength of the euro with respect to the basket of currencies.

interest rate ($R^{\$}$) are modeled as exogenous first order processes.

We assume a debt-elastic premium on foreign bond holdings $\Gamma_{A^*}(\cdot)$ to provide a well defined steady state for the net foreign assets, following Schmitt-Grohe and Uribe (2003). We assume the following functional form for premium on the foreign bond holdings:

$$\Gamma_{A^*}\left(A^*_{t+1}, \zeta^{€}_t\right) = \exp\left\{-\phi_a\left(A^*_{t+1} - A^*\right) + \zeta^{€}_t\right\}$$

$$A^*_{t+1} \equiv \frac{B^*_{t+1}}{P_t Y_t} = \frac{B^{€}_{t+1}}{P_t Y_t} + \frac{S_t B^{\$}_{t+1}}{P_t Y_t},$$

where A^*_{t+1} denotes the ratio of total net foreign assets, denominated in euro to the nominal income held by households, and ϕ_a is the elasticity of the domestic bond return with respect to the net foreign asset position. This creates an endogenous wedge between the domestic and foreign bond rates. The shock $\zeta^{€}_t$ represents an exogenous domestic risk premium shock.

Note that, anticipating the fact that the patient household is a saver in equilibrium, it is not constrained by a collateral constraint on borrowing such as (14) in equilibrium as long as β^P is sufficiently greater than β^I, although in principle the collateral constraint also applies to the patient household (see the discussion in Iacoviello, 2005 on this issue). It can thus be left out of the maximisation problem of the patient household.

The first-order conditions of the patient household's problem are:

$$C^P_{h,t}: \quad \psi^P_{h,t} = \frac{\zeta^C_t}{1+\tau^C_t}\frac{1}{C^P_{h,t} - b^P_c C^P_{t-1}} \tag{3}$$

$$B^P_{h,t+1}: \quad \beta^P E_t \frac{\psi^P_{h,t+1}}{\psi^P_{h,t}}\frac{P^C_t}{P^C_{t+1}} R_t = 1 \tag{4}$$

$$B^{€,P}_{h,t+1}: \quad \beta^P E_t \frac{\psi^P_{h,t+1}}{\psi^P_{h,t}}\frac{P^C_t}{P^C_{t+1}} \Gamma_{A^*}\left(A^*_{t+1}, \zeta^{€}_t\right) R^{€}_t = 1 \tag{5}$$

$$B^{\$,P}_{h,t+1}: \quad \beta^P E_t \frac{\psi^P_{h,t+1}}{\psi^P_{h,t}}\frac{P^C_t}{P^C_{t+1}}\frac{S_{t+1}}{S_t} \Gamma_{A^*}\left(A^*_{t+1}, \zeta^{€}_t\right) R^{\$}_t = 1 \tag{6}$$

$$D_{h,t+1}: \quad \beta^P E_t \frac{\psi^P_{h,t+1}}{\psi^P_{h,t}} \frac{P^C_t}{P^C_{t+1}} R^D_t = 1 \tag{7}$$

$$H^P_{h,t+1}: \quad \beta^P E_t \left[\frac{j^P \zeta^H_{t+1}}{\psi^P_{h,t} \left(H^P_{h,t+1} - b^P_H H^P_t \right)} + \frac{P^H_{t+1}}{P^C_{t+1}} \frac{\psi^P_{h,t+1}}{\psi^P_{h,t}} \left(1 - \tau^H_{t+1}\right) \left(1 - \delta^H_{t+1}\right) \right] = \frac{P^H_t}{P^C_t} \equiv Q^H_t \tag{8}$$

where $\phi^P_{h,t}$ is the multiplier on the budget constraint (2), and $\psi^P_{h,t} = P^C_t \phi^P_{h,t}$.

2.1.2 Impatient households

Impatient households consume goods and housing services, supply labour, and borrow from banks to finance their housing purchases. It is assumed that impatient households do not have access to the bond market (neither the domestic nor the international one), and are only able to borrow or save through the domestic banking sector. They do not own any firms.

They maximize their lifetime utility over the sequences of consumption $\left(C^I_{h,t}\right)$, holdings of housing $\left(H^I_{h,t+1}\right)$, and new mortgage loans $\left(BL^{H,new}_{h,t+1}\right)$.[9] They supply differentiated labour services $L^I_{h,t}$ to firms and act as wage setters in the monopolistically competitive labour markets.

Preferences of impatient households are given by:

$$E_t \sum_{s=t}^{\infty} (\beta^I)^{s-t} \left[\zeta^C_s \log\left(C^I_{h,s} - b^I_c C^I_{s-1}\right) + \zeta^H_s j^I \log\left(H^I_{h,s} - b^I_H H^I_{s-1}\right) - \frac{\zeta^L_s}{1+\sigma_l} \left(L^I_{h,s}\right)^{1+\sigma_l} \right], \tag{9}$$

where $b^I_c > 0$ is the degree of external habit formation in consumption, $j^I > 0$ is a scaling parameter, $b^I_H > 0$ is the degree of external habit formation in housing, and $L^I_{h,t}$ are total hours supplied by I-type workers to intermediate goods producers. Note that we assume that the exogenous shocks ζ^C_t, ζ^H_t and ζ^C_t are not type-specific, but instead experienced by all households equally.

[9] We drop the superscript I on the variables BL^H, $BL^{H,new}$ and MP^H related to mortgage loans for the sake of space, as the patient households (type P) will not in equilibrium have these loans.

A household h of type I faces the following nominal periodic budget constraint:

$$\begin{aligned}
&\left(1+\tau_s^C\right) P_s^C C_{h,s}^I + P_s^H H_{h,s+1}^I \\
&= \left(1-\tau_s^W\right) W_s^I L_{h,s}^I + \left(1-\tau_s^H\right)\left(1-\delta_s^H\right) P_s^H H_{h,s}^I + BL_{h,s+1}^{H,new} - MP_{h,s}^H \qquad \forall\, s=t,...,\infty \\
&-\mathcal{TR}_{h,s}^I + \mathcal{S}_{h,s}^I.
\end{aligned} \tag{10}$$

The home loans are multi-period loans. $BL_{h,t+1}^{H,new}$ is the flow of new nominal loans taken in period t. In each period, the household makes a nominal mortgage payment $MP_{h,t}^H$ on the existing stock of nominal debt BL_t^H. The payment consists of amortisation of existing debt and the interest payments, and is defined as:

$$MP_{h,t}^H = \left[\gamma_{h,t}^H + \left(1-\tau_t^{rH}\right) r_{t-1}^H\right] BL_{h,t}^H, \tag{11}$$

where r_t^H is the variable interest rate on home loans, $\gamma_t^H \leq 1$ is the effective amortisation rate of the outstanding debt stock, and τ_t^{rH} is a tax deduction on the interest rate. In each period, the effective amortisation rate is assumed to evolve according to:

$$\gamma_{h,t+1}^H = \left(1 - \frac{BL_{h,t+1}^{H,new}}{BL_{h,t+1}^H}\right)\left(\gamma_{h,t}^H\right)^{\alpha_M} + \frac{BL_{h,t+1}^{H,new}}{BL_{h,t+1}^H}\kappa, \tag{12}$$

where $0 < \kappa \leq 1$ is the initial amortisation rate of new loans, and $0 \leq \alpha_M \leq 1$ governs the evolution of the effective amortisation rate (see Kydland et al., 2016 for details). Setting $\alpha_M = 0$ and $\kappa = 1$ yields the one-period loan framework, so that it is nested within this more general formulation. Conversely, setting $\alpha_M = 1$ gives the constant amortisation (decaying coupon) framework with an amortisation rate $\gamma_t^H = \kappa$. The nominal stock of outstanding debt then evolves according to:

$$BL_{h,t+1}^H = \left(1-\gamma_{h,t}^H\right) BL_{h,t}^H + BL_{h,t+1}^{H,new}. \tag{13}$$

The household's new borrowing is subject to a collateral constraint that restricts the nominal loan amount to a fraction of the market value of the new housing being purchased:

$$BL_{h,t+1}^{H,new} \leq \theta_t^H P_t^H \left[H_{h,t+1}^I - \left(1-\delta_t^H\right) H_{h,t}^I\right], \tag{14}$$

where $\theta_t^H \leq 1$ is the time-varying maximum loan-to-value ratio on new housing purchases.[10]

By substituting (13) into (14), the collateral constraint can also be expressed in terms of the debt stock, instead of the flow of new loans:

$$BL_{h,t+1}^H \leq \theta_t^H P_t^H \left[H_{h,t+1}^I - \left(1-\delta_t^H\right) H_{h,t}^I\right] + \left(1-\gamma_{h,t}^H\right) BL_{h,t}^H. \tag{15}$$

By substituting (11) and (13) in to the budget constraint (10), the constraint can be expressed in terms of the debt stock:

$$\begin{aligned}
&\left(1+\tau_s^C\right) P_s^C C_{h,s}^I + P_s^H H_{h,s+1}^I \\
&= \left(1-\tau_s^W\right) W_s^I L_{h,s}^I + \left(1-\tau_s^H\right)\left(1-\delta_s^H\right) P_s^H H_{h,s}^I + BL_{h,s+1}^H - \left[1+\left(1-\tau_s^{rH}\right) r_{s-1}^H\right] BL_{h,s}^H \\
&-\mathcal{TR}_{h,s}^I + \mathcal{S}_{h,s}^I \qquad \forall\ s = t, ..., \infty.
\end{aligned} \tag{16}$$

Finally, by substituting (13) into (12), the latter can also be expressed in terms of the debt stock:

$$\begin{aligned}
\gamma_{h,t+1}^H &= \left(1-\gamma_{h,t}^H\right)\left(\gamma_{h,t}^H\right)^{\alpha_M} \frac{BL_{h,t}^H}{BL_{h,t+1}^H} + \left[1-\left(1-\gamma_{h,t}^H\right)\frac{BL_{h,t}^H}{BL_{h,t+1}^H}\right]\kappa \\
&= \kappa + \left(1-\gamma_{h,t}^H\right)\left[\left(\gamma_{h,t}^H\right)^{\alpha_M} - \kappa\right]\frac{BL_{h,t}^H}{BL_{h,t+1}^H}.
\end{aligned} \tag{17}$$

The impatient household maximises (9) subject to (15), (16) and (17). Given the assumption that β^P is sufficiently greater than β^I, the collateral constraint is always binding for the impatient household near the steady state of the economy (*i.e.* equation (15) holds with equality). For the remainder of this paper, we proceed under the assumption that the constraint always strictly binds.[11]

[10] This formulation of the borrowing constraint implies that the collateral requirement applies to the flow of new mortgage loans, and that they are indeed used to finance new housing purchases. Many models assume that the loan amount is constrained by the value of the whole stock of housing owned by the household, which would imply a home-equity line of credit against the household's real estate wealth to finance any consumption by the household; see Kydland et al. (2016) for a discussion on this distinction, and Garriga et al. (2017) and Grodecka (2020) for similar formulations of the collateral constraint in the context of long-term mortgage debt as the one used here.

[11] We calibrate the parameter ω_h to match the share of credit-constrained households in Finnish household-level data (see section 3.1 for more details.) Therefore, while the assumption of an always binding borrowing constraint for the impatient households is not without loss of generality, it is not unrealistic in the case we consider.

The first-order conditions of the impatient household's maximisation problem are:

$$\psi_{h,t}^{I,2} = \frac{\zeta_t^C}{1+\tau_t^C} \frac{1}{C_{h,t}^I - b_c^I C_{t-1}^I} \tag{18}$$

$$\frac{P_t^H}{P_t^C} = \beta^I E_t \left[\frac{j^I \zeta_{t+1}^H}{\left(\psi_{h,t}^{I,2} - \psi_{h,t}^{I,1}\theta_t^H\right)\left(H_{h,t+1}^I - b_H^I H_t^I\right)} + \frac{P_{t+1}^H}{P_{t+1}^C} \frac{\psi_{h,t+1}^{I,2}\left(1-\tau_{t+1}^H\right) - \psi_{h,t+1}^{I,1}\theta_{t+1}^H}{\psi_{h,t}^{I,2} - \psi_{h,t}^{I,1}\theta_t^H}\left(1-\delta_{t+1}^H\right) \right] \tag{19}$$

$$\psi_{h,t}^{I,1} = \psi_{h,t}^{I,2} - \frac{\gamma_{h.t+1}^H - \kappa}{BL_{h,t+1}^H} P_t^C \phi_{h,t}^{I,3} + \beta^I E_t \left[\frac{P_t^C}{P_{t+1}^C} \left(\left(1-\gamma_{t+1}^H\right)\psi_{h,t+1}^{I,1} - \left(1+\left(1-\tau_{t+1}^{rH}\right) r_t^H\right)\psi_{h,t+1}^{I,2} \right) + \frac{\gamma_{h,t+2}^H - \kappa}{BL_{h,t+1}^H} P_t^C \phi_{h,t+1}^{I,3} \right] \tag{20}$$

$$\phi_{h,t}^{I,3} = \beta^I E_t \left\{ \left[\alpha_M \left(\gamma_{h,t+1}^H\right)^{\alpha_M - 1} - \left(\alpha_M + 1\right)\left(\gamma_{h,t+1}^H\right)^{\alpha_M} + \kappa \right] \frac{BL_{t+1}^H}{BL_{t+2}^H} \phi_{h,t+1}^{I,3} - \frac{BL_{t+1}^H}{P_{t+1}^C} \psi_{h,t+1}^{I,1} \right\}, \tag{21}$$

where $\phi_{h,t}^{I,1}$ is the multiplier on the collateral constraint (15), $\phi_{h,t}^{I,2}$ is the multiplier on the budget constraint (16) and $\phi_{h,t}^{I,3}$ is the multiplier on the law of motion of the effective amortisation rate γ_t^H (17), and the multipliers are redefined as $\psi_{h,t}^{I,i} \equiv P_t^C \phi_{h,t}^{I,i}$ for $i = 1, 2$.

2.1.3 Financing cost minimisation problem of the impatient household

In the model, there is a continuum of banks, indexed by $z \in [0,1]$, and each bank z has some market power in providing its intermediation services and thus pricing its loans. The banking sector is described in more detail in Section 2.9. We assume that the borrower household is able to refinance its total stock of mortgage debt in each period, such that a household who chooses a total mortgage debt stock equal to BL_{t+1}^H would allocate its debt among different banks so as to minimise the total repayment due. At the end of period t, the household decides on how much to borrow from bank z, $bl_{t+1}^H(z)$, by solving the following problem:

$$\min_{bl_{t+1}^H(z)} \int_0^1 r_t^H(z)\, bl_{t+1}^H(z)\, dz$$

$$s.t.\;\; BL_{t+1}^H = \left\{ \int_0^1 \left[bl_{t+1}^H(z) \right]^{\frac{\varepsilon_t^H - 1}{\varepsilon_t^H}} dz \right\}^{\frac{\varepsilon_t^H}{\varepsilon_t^H - 1}},$$

where $r_t^H(z)$ is the interest rate on mortgage loans charged by the z-th bank and $\varepsilon_t^H > 1$ is the time-varying interest rate elasticity of the demand for mortgage loans, which measures the degree of competition in the

banks' mortgage lending activities. The first order condition yields the following optimal demand schedule for mortgage loans by the household:

$$bl_{t+1}^{H}(z) = \left(\frac{r_t^H(z)}{r_t^H}\right)^{-\varepsilon_t^H} BL_{t+1}^{H}, \tag{22}$$

where r_t^H is the nominal average mortgage loan rate prevailing in the market at time t, defined as:

$$r_t^H = \left\{\int_0^1 \left[r_t^H(z)\right]^{1-\varepsilon_t^H} dz\right\}^{\frac{1}{1-\varepsilon_t^H}}. \tag{23}$$

As expected, the loan demand schedule has a negative slope: when the interest rate set by the z-th bank increases relative to the average rate, the household wants to borrow less funds from that particular bank.

2.1.4 Wage setting

Each household h of type k ($k = P, I$) supplies differentiated labour services $L_{h,t}^k$ to intermediate good producing firms in monopolistically competitive labour markets. There are two representative labour unions, one for each type k, that bundle together the labour of each respective type.

Household h of type k optimally resets its nominal wage $W_{h,t}^k$ in a given period t with probability $1-\xi_w$, $\xi_w \in [0,1]$. This probability is assumed to be the same for both types. Those households that cannot reset their wage contract are subject to the following wage indexation scheme:

$$W_{h,t}^k = \mu_t \Pi_{t-1} W_{h,t-1}^k, \;\; k = P, I, \tag{24}$$

where $\Pi_{t-1} = P_{t-1}/P_{t-2}$ is the past inflation rate of intermediate goods. Household labour is transformed into a homogeneous input good L_t^k via the following production functions:

$$L_t^P = \left[\int_0^{\omega_h} \left(L_{h,t}^P\right)^{\frac{1}{\lambda_{w,t}}} dh\right]^{\lambda_{w,t}} \quad , \quad L_t^I = \left[\int_{\omega_h}^1 \left(L_{h,t}^I\right)^{\frac{1}{\lambda_{w,t}}} dh\right]^{\lambda_{w,t}}, \tag{25}$$

where $\lambda_{w,t} \geq 1$ is the gross time-varying wage markup. This mark-up is assumed to be the same for both types. The demand for labour in period $t+s$ for the household of type k that resets its wage in period t

is determined by:

$$L^k_{h,t+s|t} = \left(\frac{W^k_{h,t+s|t}}{W^k_{t+s}} \right)^{\frac{\lambda_{w,t+s}}{1-\lambda_{w,t+s}}} L^k_{t+s}, \;\; k = P, I, \tag{26}$$

where W^k_{t+s} is the aggregate wage of type k households in period t and $W^k_{h,t+s|t}$ is the wage for the household h of type k that last reset its wage in period t. Notice that for the household that last optimised its wage in time t, the wage in $t+s$ can be expresssed also as $W^k_{h,t+s|t} = \prod_{k=1}^{s} \left(\mu_{t+k}\Pi_{t+k-1}\right) W^{k\star}_{h,t}$, where $W^{k\star}_{h,t}$ is the optimal wage rate chosen in period t by all households of type k able to reset their wage contracts. We can the express the labour demand as:

$$L^k_{h,t+s|t} = \left(\frac{\prod_{k=1}^{s} \left(\mu_{t+k}\Pi_{t+k-1}\right) W^{k\star}_{h,t}}{W^k_{t+s}} \right)^{\frac{\lambda_{w,t+s}}{1-\lambda_{w,t+s}}} L^k_{t+s}, \;\; k = P, I.$$

Each household h that can reoptimise its wage contract in period t maximises its lifetime utility ((1) and (9), respectively) subject to its periodic budget constraint ((2) and (10), respectively), demand for labour (26), and the wage indexation scheme (24). The maximisation problem is:

$$\max_{\{W^{k\star}_{h,t}\}} E_t \sum_{s=0}^{\infty} \left(\xi_w \beta^k\right)^s \left[\phi^k_{h,t+s} \left(1 - \tau^W_{t+s}\right) \mu^s_t \Pi^s_t W^{k\star}_{h,t} L^k_{h,t+s} - \frac{\zeta^L_{t+s}}{1+\sigma_l} \left(L^k_{h,t+s} \right)^{1+\sigma_l} \right], \quad k = P, I, \tag{27}$$

where $\mu^s_t = \prod_{k=1}^{s} \mu_{t+k}$ and $\Pi^s_t = \prod_{k=1}^{s} \Pi_{t+k-1}$, and $\phi^I_{h,t+s} = \phi^{I,2}_{h,t+s}$ denotes the marginal utility of consumption of the impatient household. In a symmetric equilibrium the marginal utility of consumption as well as labour demand are equal across households of type k, *i.e.* $\phi^k_{h,t+s} = \phi^k_{t+s}$, $L^k_{h,t+s} = L^k_{t+s}$.

The first-order condition of this problem can be written as:

$$E_t \sum_{s=0}^{\infty} \frac{\left(\xi_w \beta^k\right)^s}{1 - \lambda_{w,t+s}} \left[\begin{array}{l} \psi^k_{\Lambda,t+s} \left(1 - \tau^W_{t+s}\right) L^k_{t+s} \wp^k_{t+s} \left(\frac{\Pi^s_t \ell^k_t \wp^k_t}{\Pi^{C,s}_t \wp^k_{t+s}} \right)^{\frac{1}{1-\lambda_{w,t+s}}} \\ -\zeta^L_{t+s} \lambda_{w,t+s} \left[\left(\frac{\Pi^s_t \ell^k_t \wp^k_t}{\Pi^{C,s}_t \wp^k_{t+s}} \right)^{\frac{\lambda_{w,t+s}}{1-\lambda_{w,t+s}}} L^k_{t+s} \right]^{1+\sigma_l} \end{array} \right] = 0, \quad k = P, I. \tag{28}$$

where $\ell^k_t = \frac{W^{k\star}_{h,t}}{W^k_t}$, $\wp^k_t = \frac{W^k_t}{\Lambda^P_{l,t} P^C_t}$ and $\psi^k_{\Lambda,t} = \Lambda^P_{l,t} P^C_t \phi^k_{h,t}$.

This yields two wage Phillips curves, one for each type k. As long as labour is separable in the utility

function, the analytical form of the optimality condition is unaffected by the household type k, i.e. the first-order conditions are symmetric.

2.2 Production of domestic intermediate goods

2.2.1 Composite intermediate goods

Differentiated domestic intermediate goods are produced in monopolistically competitive markets, described below in more detail. These individual brands of goods, denoted by $Y_t(j)$, $j \in [0,1]$, are then aggregated into a homogeneous composite intermediate good Y_t by using to the following Dixit and Stiglitz (1977) aggregator:

$$Y_t = \left[\int_0^1 Y_t(j)^{-\rho_t^z} dj\right]^{-\frac{1}{\rho_t^z}}. \tag{29}$$

We allow for a time-varying markup captured by the parameter ρ_t^z, to capture changes in the gross operating surplus and the labour share observed over time.[12] The cost minimisation of intermediate good inputs in the production of the composite intermediate good Y_t yields the following conditional demand function for each individual intermediate good of type j:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1}{1+\rho_t^z}} Y_t. \tag{30}$$

The corresponding price index for the composite domestic intermediate good can be obtained by substituting (30) into (29) and integrating over firms:

$$P_t = \left[\int_0^1 P_t(j)^{\frac{\rho_t^z}{1+\rho_t^z}} dj\right]^{\frac{1+\rho_t^z}{\rho_t^z}}. \tag{31}$$

[12] See Ripatti and Vilmunen (2001) for a discussion.

2.2.2 Cost minimisation problem of an intermediate goods producer

Differentiated domestic intermediate goods, $Y_t(j)$, are produced by combining homogeneous capital services K_t and labour services L_t using the following Harrod-neutral CES production function:[13]

$$Y_t\left(j\right) = \left[\delta_Y \left(\Lambda_{k,t} K_t\right)^{-\rho_Y} + \left(1-\delta_Y\right)\left(\Lambda_{l,t} L_t\right)^{-\rho_Y}\right]^{-1/\rho_Y}. \tag{32}$$

The elasticity of factor substitution is given by $\sigma_Y \equiv (\rho_Y+1)^{-1}$, where ρ_Y is the substitution parameter. δ_Y is a quasi factor input share parameter, and $\Lambda_{k,t}$ and $\Lambda_{l,t}$ denote capital-augmenting and labour-augmenting technological progress, respectively. As Kilponen et al. (2016), we assume that along the balanced growth path, technological change is labour-augmenting. Following Adolfson et al. (2005), we decompose labour-augmenting technological change $\Lambda_{l,t}$ into a permanent $\Lambda^P_{l,t}$ and a temporary $\Lambda^T_{l,t}$ technology component, such that $\Lambda_{l,t} = \Lambda^P_{l,t}\Lambda^T_{l,t}$. The gross growth rate of the permanent component of labour-augmenting technological change, denoted by $\mu_t = \Lambda^P_{l,t}/\Lambda^P_{l,t-1}$, follows a stationary first-order autoregressive process.

The intermediate good producing firms minimise the total nominal costs of production $R^K_t K_t + \left(1+\tau^F_t\right) W_t L_t$ subject to the technology constraint (32) by adjusting capital and labour inputs. The capital rental rate price R^K_t and nominal labour costs $\left(1+\tau^F_t\right) W_t$ are taken as given in this optimisation. τ^F_t denotes the firm's mandatory social security contribution. The cost minimisation yields the following optimality condition for nominal marginal costs:

$$\mathcal{MC}_{\mathrm{t}}\left(j\right) = \left[\delta_Y^{\frac{1}{\rho_Y+1}} \left(\frac{R^K_t}{\Lambda_{k,t}}\right)^{\frac{\rho_Y}{\rho_Y+1}} + \left(1-\delta_Y\right)^{\frac{1}{\rho_Y+1}} \left(\frac{\left(1+\tau^F_t\right) W_t}{\Lambda_{l,t}}\right)^{\frac{\rho_Y}{\rho_Y+1}}\right]^{\frac{\rho_Y+1}{\rho_Y}}. \tag{33}$$

Since all firms j face the same input prices and production technology, their marginal costs are also equalised in equilibrium, such that $\mathcal{MC}_{\mathrm{t}}\left(j\right) = \mathcal{MC}_{\mathrm{t}}$.

The total labour input used by firm j, $L_t\left(j\right)$, is a composite of the two types of labour aggregates, $L^P_t\left(j\right)$

[13] For a discussion of the empirical fit of various forms of the production function in the Finnish case, see Ripatti and Vilmunen (2001) and Kilponen et al. (2016).

and $L_t^I(j)$, sold by the two labour unions representing the two household types:

$$L_t(j) = \left(\frac{L_t^P(j)}{\omega_h}\right)^{\omega_h} \left(\frac{L_t^I(j)}{1-\omega_h}\right)^{1-\omega_h}. \tag{34}$$

where:

$$L_t^P(j) = \left[\int_0^{\omega_h} L_t^P(j,h)^{\frac{1}{\lambda_{w,t}}} dh\right]^{\lambda_{w,t}},$$

$$L_t^I(j) = \left[\int_{\omega_h}^1 L_t^I(j,h)^{\frac{1}{\lambda_{w,t}}} dh\right]^{\lambda_{w,t}}.$$

Note that by assumption, the wage income share of the patient type, ω_h, is exactly equal to the population share of the patient type. The optimal demand for labour services of household h of type k is given by the schedule:

$$L_t^k(j,h) = \left(\frac{W_{h,t}^k}{W_t^k}\right)^{\frac{\lambda_{w,t+s}}{1-\lambda_{w,t+s}}} L_t^k(j) \quad , k = P, I. \tag{35}$$

The optimal use of the two labour aggregates $L_t^P(j)$ and $L_t^I(j)$ is found by minimising the total wage bill $W_t^P L_t^P(j) + W_t^I L_t^I(j)$ with respect to (34). This implies that from the firms' perspective, labour supplied by patient and impatient households are perfect substitutes. The corresponding first-order condition is:

$$\frac{L_t^P(j)}{L_t^I(j)} = \frac{\omega_h}{1-\omega_h}\frac{W_t^I}{W_t^P}. \tag{36}$$

The conditional factor demands for each labour type in aggregate are:

$$L_t^P(j) = \frac{\omega_h W_t L_t(j)}{W_t^P}, \tag{37}$$

$$L_t^I(j) = \frac{(1-\omega_h) W_t L_t(j)}{W_t^I}. \tag{38}$$

Substituting the solution back to (34), for a given level of expenditure on total labour input $W_t L_t(j)$, yields the aggregate wage index

$$W_t = \left(W_t^P\right)^{\omega_h} \left(W_t^I\right)^{1-\omega_h}, \tag{39}$$

where:

$$W_t^P = \left[\int_0^{\omega_h} W_t^P(h)^{\frac{1}{1-\lambda_{w,t}}} dh\right]^{1-\lambda_{w,t}}$$
$$= \omega_h^{1-\lambda_{w,t}} \left[\xi_w \left(\mu_t \Pi_{t-1} W_{t-1}^P\right)^{\frac{1}{1-\lambda_{w,t}}} + (1-\xi_w)\left(W_t^{P*}\right)^{\frac{1}{1-\lambda_{w,t}}}\right]^{1-\lambda_{w,t}}$$

and

$$W_t^I = \left[\int_{\omega_h}^1 W_t^I(h)^{\frac{1}{1-\lambda_{w,t}}} dh\right]^{1-\lambda_{w,t}}$$
$$= (1-\omega_h)^{1-\lambda_{w,t}} \left[\xi_w \left(\mu_t \Pi_{t-1} W_{t-1}^I\right)^{\frac{1}{1-\lambda_{w,t}}} + (1-\xi_w)\left(W_t^{I*}\right)^{\frac{1}{1-\lambda_{w,t}}}\right]^{1-\lambda_{w,t}}.$$

2.2.3 Pricing of domestic intermediate goods

An intermediate good firm sells its differentiated good at price $P_t(j)$. We assume that price contracts are staggered as in Calvo (1983). The intermediate good firm j re-optimises its price in each period with probability $1-\zeta$, $\zeta \in [0,1]$. Since there is a continuum of intermediate producers, $1-\zeta$ also represents the share of producers changing their price in each period. Following Christoffel et al. (2008), we allow for partial indexing, such that firms that cannot optimise their prices index them to the geometric average of past inflation and the steady state inflation rate according to:

$$P_t = \Pi_{t-1}^{\theta} \bar{\Pi}^{1-\theta} P_{t-1}, \tag{40}$$

where $\Pi_{t-1} \equiv P_{t-1}/P_{t-2}$ and $\bar{\Pi}$ denotes the gross steady-state inflation rate. θ is the indexation parameter.

Let $P_t^{\circ}(j)$ denote the price level set by those intermediate goods producers who received the price-change signal in period t. Given that with probability ζ^s the price $P_t^{\circ}(j)$ is still in effect at date $t+s$ ($s \geq 0$), the intermediate-goods producer solves the following problem:

$$\max_{\{P_t^{\circ}(j)\}} E_t \sum_{s=0}^{\infty} \zeta^s M_{t,t+s} \left[P_{t+s|t}^{\circ}(j) - \mathcal{MC}_{t+s}(j)\right] Y_{t+s|t}(j) \tag{41}$$

subject to (40) and subject to the sequence of conditional demand functions for its products:

$$Y_{t+s|t}(j) = \left(\frac{P^{\circ}_{t+s|t}(j)}{P_{t+s}} \right)^{-\frac{1}{1+\rho^{z}_{t+s}}} Y_{t+s|t}.$$

Following the assumption that the patient households own all firms, the nominal stochastic discount factor (the pricing kernel) $M_{t,t+s} = \left(\beta^{P}\right)^{s} U'(C^{P}_{t+s}) P^{C}_{t} / \left[U'(C^{P}_{t}) P^{C}_{t+s}\right]$ is obtained from the patient household's consumption Euler equation. P^{C}_{t} is the price index of composite consumer goods. $Y_{t+s|t}(j)$ denotes demand in period $t+s$ faced by an intermediate good firm j that last reset its price at time t.

Assuming a symmetric equilibrum where $P^{\circ}_{t}(j) = P^{\circ}_{t}$, as a result of this optimisation problem and given the aggregate price index (31), the aggregate price level of intermediate goods evolves according to:

$$P_t = \left[\zeta \left(\Pi^{\theta}_{t-1} \bar{\Pi}^{1-\theta} P_{t-1} \right)^{\frac{\rho^{z}_{t}}{1+\rho^{z}_{t}}} + (1-\zeta) P^{\circ}{}^{\frac{\rho^{z}_{t}}{1+\rho^{z}_{t}}}_{t} \right]^{\frac{1+\rho^{z}_{t}}{\rho^{z}_{t}}}. \tag{42}$$

2.3 Production of final consumption, capital investment and housing investment goods

Final goods meant for domestic consumption are produced by domestic retailers operating under perfect competition. One retailer specialises in the production of consumption goods, one in capital investment goods and one in housing investment goods. The consumption good retailer combines composite domestic intermediate goods Y^{C}_{t} and imported consumption goods M^{C}_{t} to produce a composite consumption good C_t, purchased and consumed by households. The investment good retailer combines the domestic and the foreign intermediate goods Y^{I}_{t} and M^{I}_{t}, respectively, to produce a composite investment good I_t, which is purchased either by capital good producers or the government. Similarly, the housing investment good retailer combines the domestic and the foreign intermediate goods Y^{H}_{t} and M^{H}_{t}, respectively, to produce a composite housing investment good I^{H}_{t}. This good can be thought of as "structures" that the housing producer will use to produce new housing units.

The production of consumption, capital investment and housing investment goods is based on the following

CES production functions, respectively:

$$C_t = \left\{ \delta_c \left(\Lambda_{cy,t} Y_t^C \right)^{-\rho_c} + (1-\delta_c) \left[\left(1 - \Gamma_{cm}\left(\cdot\right)\right) \Lambda_{cm,t} M_t^C \right]^{-\rho_c} \right\}^{-1/\rho_c} \tag{43}$$

$$I_t = \left\{ \delta_i \left(\Lambda_{iy,t} Y_t^I \right)^{-\rho_i} + (1-\delta_i) \left[\left(1 - \Gamma_{im}\left(\cdot\right)\right) \Lambda_{im,t} M_t^I \right]^{-\rho_i} \right\}^{-1/\rho_i} \tag{44}$$

$$I_t^H = \left\{ \delta_{ih} \left(\Lambda_{hy,t} Y_t^H \right)^{-\rho_h} + (1-\delta_{ih}) \left[\left(1 - \Gamma_{hm}\left(\cdot\right)\right) \Lambda_{hm,t} M_t^H \right]^{-\rho_h} \right\}^{-1/\rho_h}, \tag{45}$$

where $\Lambda_{cy,t}$, $\Lambda_{cm,t}$, $\Lambda_{iy,t}$, $\Lambda_{im,t}$, $\Lambda_{hy,t}$ and $\Lambda_{hm,t}$ represent factor-specific preference shifters, and

$$\Gamma_{cm}\left(\cdot\right) = \frac{\gamma_{cm}}{2}\left(\frac{M_t^C/C_t}{M_{t-1}^C/C_{t-1}} - 1 \right)^2, \quad \Gamma_{im}\left(\cdot\right) = \frac{\gamma_{im}}{2}\left(\frac{M_t^I/I_t}{M_{t-1}^I/I_{t-1}} - 1 \right)^2,$$

$$\Gamma_{hm}\left(\cdot\right) = \frac{\gamma_{hm}}{2}\left(\frac{M_t^H/I_t^H}{M_{t-1}^H/I_{t-1}^H} - 1 \right)^2$$

are the external adjustment cost functions with $\gamma_{cm} > 0$, $\gamma_{im} > 0$ and $\gamma_{hm} > 0$. The interpretation of the CES production function parameters ρ_c, δ_c, ρ_i, δ_i, ρ_h and δ_{ih} is analogous to the case of the intermediate good producing firms. Since final consumption, capital investment and housing investment goods are produced under perfect competition, their prices (P_t^C, P_t^I and P_t^{IH}, respectively) are each a function of domestic intermediate goods' and imported goods' prices:

$$P_t^C = \left\{ \delta_c^{\sigma_c} \left(P_t / \Lambda_{cy,t} \right)^{\sigma_c \rho_c} + (1-\delta_c)^{\sigma_c} \left[P_t^M / \left(\Lambda_{cm,t} \Gamma_{cm}^{\dagger} \right) \right]^{\sigma_c \rho_c} \right\}^{\frac{1}{\sigma_c \rho_c}}$$

$$P_t^I = \left\{ \delta_i^{\sigma_i} \left(P_t / \Lambda_{iy,t} \right)^{\sigma_i \rho_i} + (1-\delta_i)^{\sigma_i} \left[P_t^M / \left(\Lambda_{im,t} \Gamma_{im}^{\dagger} \right) \right]^{\sigma_i \rho_i} \right\}^{\frac{1}{\sigma_i \rho_i}}$$

$$P_t^{IH} = \left\{ \delta_{ih}^{\sigma_h} \left(P_t / \Lambda_{hy,t} \right)^{\sigma_h \rho_h} + (1-\delta_{ih})^{\sigma_h} \left[P_t^M / \left(\Lambda_{hm,t} \Gamma_{hm}^{\dagger} \right) \right]^{\sigma_h \rho_h} \right\}^{\frac{1}{\sigma_h \rho_h}},$$

where $\sigma_c = \frac{1}{\rho_c + 1}$, $\Gamma_{cm}^{\dagger} \equiv 1 - \Gamma_{cm} - \Gamma_{cm}' M_t^C$, $\sigma_i = \frac{1}{\rho_i + 1}$, $\Gamma_{im}^{\dagger} = 1 - \Gamma_{im} - \Gamma_{im}' M_t^I$, $\sigma_h = \frac{1}{\rho_h + 1}$ and $\Gamma_{hm}^{\dagger} = 1 - \Gamma_{hm} - \Gamma_{hm}' M_t^H$.

The equilibrium import price P_t^M will be defined in Section 2.5.

2.4 Export market

2.4.1 Cost minimisation problem of an export goods producer and export good aggregation

Export good producers operate in monopolistically competitive markets. An export good producer i produces a differentiated export good $X_t(i)$ using a CES production function:

$$X_t(i) = \left[\delta_x \left(\Lambda_{xy,t} Y_t^X\right)^{-\rho_x} + (1-\delta_x)\left(\Lambda_{xm,t} M_t^X\right)^{-\rho_x}\right]^{-1/\rho_x}, \tag{46}$$

where the factors of production include domestic intermediate goods Y_t^X and imported goods M_t^X. The elasticity of substitution is given by $\sigma_x \equiv (1+\rho_x)^{-1}$, where ρ_x is the substitution parameter in the production function, δ_x is a quasi factor input share parameter, and $\Lambda_{xy,t}$ and $\Lambda_{xm,t}$ denote time-varying technology shifters common to all exporting firms.

Export goods producing firms minimise total factor costs $P_t Y_t^X + P_t^M M_t^X$ by adjusting factor inputs, subject to the technology constraint (46). The export goods producer takes factor prices P_t and P_t^M as given. The resulting marginal costs are given by:

$$\mathcal{MC}_{x,t}(i) = \left[(\delta_x)^{\frac{1}{1+\rho_x}} \left(\frac{P_t}{\Lambda_{xy,t}}\right)^{\frac{\rho_x}{1+\rho_x}} + (1-\delta_x)^{\frac{1}{1+\rho_x}} \left(\frac{P_t^M}{\Lambda_{xm,t}}\right)^{\frac{\rho_x}{1+\rho_x}}\right]^{\frac{1+\rho_x}{\rho_x}}. \tag{47}$$

The differentiated domestic export goods are then passed to the export retailer firm, which aggregates the continuum of domestic export goods $X_t(i)$ into a composite export good. Analogously to the composite intermediate good, the composite domestic export good is produced using the following CES aggregation technology:

$$X_t = \left[\int_0^1 X_t(i)^{-\rho_{f,t}} di\right]^{-\frac{1}{\rho_{f,t}}}, \tag{48}$$

where $\rho_{f,t}$ is the time-varying markup in the production of export goods. We assume that domestic export retailers follow foreign currency pricing. Hence, the price indices $P_t^X(i)$ are denominated in the foreign currency. The cost minimisation subject to the aggregator (48) yields the following conditional demand

schedules for the differentiated export goods:

$$X_t(i) = \left[\frac{P_t^X(i)}{P_t^X}\right]^{-\frac{1}{1+\rho_{f,t}}} X_t. \tag{49}$$

Accordingly, the price index of the composite domestic export good is:

$$P_t^X = \left[\int_0^1 P_t^X(i)^{\frac{\rho_{f,t}}{1+\rho_{f,t}}} di\right]^{\frac{1+\rho_{f,t}}{\rho_{f,t}}}. \tag{50}$$

2.4.2 Pricing of export goods

An export firm subject to staggered pricing resets its price level when it receives a random price-change signal, which occurs with probability $1-\zeta_x$, $\zeta_x \in [0,1]$. Firms that cannot re-optimise their prices instead index them to the geometric average of past export price inflation and the steady state export price inflation:

$$P_t^X = \Pi_{x,t-1}^{\theta_x} \bar{\Pi}_x^{1-\theta_x} P_{t-1}^X, \tag{51}$$

where $\Pi_{x,t-1} = P_{t-1}^X / P_{t-2}^X$ is gross export price inflation and $\bar{\Pi}_x$ denotes the gross steady state export price inflation rate. θ_x is the indexation parameter.

Let $P_t^{\circ X}(i)$ denote the price level set by those export goods producers that received the price-change signal in period t. The export producing firm chooses its price to maximise:

$$\max_{\{P_t^{\circ X}(i)\}} \sum_{s=0}^{\infty} \zeta_x^s E_t \left\{ M_{t,t+s} \left[P_{t+s|t}^{\circ X}(i) - S_{t+s}^{-1} \mathcal{MC}_{x,t+s}(i) \right] X_{t+s|t}(i) \right\},$$

subject to (51) and to the sequence of conditional demand functions of its products:

$$X_{t+s|t}(i) = \left[\frac{P_{t+s|t}^{\circ X}(i)}{P_{t+s}^X}\right]^{-\frac{1}{1+\rho_{f,t+s}}} X_t, \tag{52}$$

where $X_{t+s|t}(i)$ denotes demand of export good firm i in period $t+s$ for a firm which reset its price last time at t.

The nominal exchange rate enters into the price optimisation problem due to the assumption of foreign

currency pricing. Analogously to the domestic intermediate goods' price index, the price index for export goods evolves according to the following equilibrium law of motion in a symmetric equilibrium:

$$P_t^X = \left\{ \zeta_x \left(\Pi_{x,t-1}^{\theta_x} \bar{\Pi}_x^{1-\theta_x} P_{t-1}^X \right)^{\frac{\rho_{f,t}}{\rho_{f,t^{+1}}}} + (1-\zeta_x) P_t^{\circ X \frac{\rho_{f,t}}{\rho_{f,t^{+1}}}} \right\}^{\frac{\rho_{f,t^{+1}}}{\rho_{f,t}}}. \tag{53}$$

2.4.3 Export demand

The producers of the composite export good face the competition from other countries producing similar export goods. We assume that each country exports differentiated export goods to the global market, operating under monopolistic competition. The following export demand function can be obtained from a CES aggregator combining the differentiated export goods from all countries:

$$X_t = \exp(e_{x,t}) \left[\frac{P_t^X}{P_t^W} \right]^{-\frac{1}{1+\rho_w}} M_t^W, \tag{54}$$

where $\sigma_w = (\rho_w + 1)^{-1}$ is the elasticity of substitution, M_t^W denotes the exogenously given global export demand, P_t^W is the corresponding world price index, reflecting competitors' prices, and $e_{x,t}$ is an exogenous external export share shock. As above, P_t^X is in terms of the foreign currency.[14]

2.5 Import market

There are three sets of import firms: an import retailer, foreign importers pricing their products in euro (local currency pricing or LCP firms, share ω_m), and foreign importers pricing their products in the foreign currency (producer currency pricing or PCP firms, share $1-\omega_m$). The import firms are owned by foreign households. Their pricing is subject to a Calvo friction and a partial indexation scheme. We denote by M_t the aggregate imported good with the corresponding aggregate price level P_t^M, and the prices of LCP and PCP firms are denoted by $P_t^{M,LCP}$ and $P_t^{M,PCP}$, respectively. The quantities produced by LCP and PCP firms are denoted by M_t^{LCP} and M_t^{PCP}, respectively. The foreign discount factor between periods t

[14] When stationarizing the model, we assume that along the balanced growth path, global demand grows at the same rate as the Finnish economy. This can be interpreted as if the Finnish economy shared the growth rate of permanent labour productivity with the rest of the world.

and $t+k$ is denoted by $R^{\star}_{t,t+k}$.[15]

2.5.1 Import retailer

An import retailer aggregates the products of the foreign importing firms. The import goods are produced by the LCP and PCP firms in a number of varieties defined over a continuum of unit mass. Brands of goods produced by LCP firms are indexed by $k \in [0, \omega_m)$ and those of PCP firms by $k \in (\omega_m, 1]$. The composite import good M_t is produced according to the following CES production function:

$$M_t = \left[\int_o^{\omega_m} M_t^{LCP}(k)^{-\rho_{m,t}} \, dk + \int_{\omega_m}^{1} M_t^{PCP}(k)^{-\rho_{m,t}} \, dk \right]^{-1/\rho_{m,t}}, \tag{55}$$

where $\rho_{m,t}$ denotes the common price markup of LCP and PCP firms.

The cost minimisation with respect to varieties leads to the following demand functions for each variety:

$$M_t^{LCP}(k) = \left[\frac{P_t^{M,LCP}(k)}{P_t^M} \right]^{-\frac{1}{1+\rho_{m,t}}} M_t \tag{56}$$

$$M_t^{PCP}(k) = \left[\frac{S_t P_t^{M,PCP}(k)}{P_t^M} \right]^{-\frac{1}{1+\rho_{m,t}}} M_t \tag{57}$$

The corresponding price index for the composite imported good can be obtained by substituting conditional factor demand functions into (55) and integrating over varieties:

$$P_t^M = \left\{ \int_0^{\omega_m} P_t^{M,LCP}(k)^{\frac{\rho_{m,t}}{1+\rho_{m,t}}} \, dk + \int_{\omega_m}^{1} \left[S_t P_t^{M,PCP}(k) \right]^{\frac{\rho_{m,t}}{1+\rho_{m,t}}} \, dk \right\}^{\frac{1+\rho_{m,t}}{\rho_{m,t}}}. \tag{58}$$

2.5.2 Foreign importers

Foreign importers face monopolistic competition in their output markets, taking into account account the demand functions (56) and (57) in their pricing decisions. We assume that all LCP and PCP firms share the same nominal marginal cost function $\mathcal{MC}_m(k)$, equal to the foreign price P_t^W. They also share the same stochastic discount factor $R^{\star}_{t,t+k}$.

[15] Under the assumption of full international consumption risk-sharing, the domestic and foreign discount factors are the same, *i.e.* $R^{\star}_{t,t+k} = M_{t,t+k}$.

The dynamics of the price $P_t^{M,j}(k)$ of an importer k ($j = LCP, PCP$) are analogous to those of the exporting and domestic intermediate good firms. We denote the probability of receiving a price-change signal as $1-\zeta_m$, $\zeta_m \in [0,1]$, common to LCP and PCP firms. The price indexation scheme is also similar to that of the exporting and domestic intermediate goods producing firms:

$$P_t^{M,j} = \Pi_{m,t-1}^{\theta_m} \bar{\Pi}_m^{1-\theta_m} P_{t-1}^{M,j}, \tag{59}$$

where $\Pi_{m,t-1} = P_{t-1}^M / P_{t-2}^M$, and $\bar{\Pi}_m$ denotes the gross steady-state import price inflation rate for LCP and PCP firms. θ_m is the common price indexation parameter for both the LCP and the PCP firms. Let $P_t^{\circ M,j}(k)$ denote the price level set by a firm k that received the price-change signal in period t. The LCP and PCP firms both maximise their expected discounted profits in home currency.

As a result, in a symmetric equilibrium, the aggregate price levels $P_t^{M,LCP}$ and $P_t^{M,PCP}$ evolve according to the following laws of motion:

$$P_t^{M,LCP} = \left\{ \zeta_m \left(\Pi_{m,t-1}^{\theta_m} \bar{\Pi}_m^{1-\theta_m} P_{t-1}^{M,LCP} \right)^{\frac{\rho_{m,t}}{1+\rho_{m,t}}} + (1-\zeta_m) \left(P_t^{\circ M,LCP} \right)^{\frac{\rho_{m,t}}{1+\rho_{m,t}}} \right\}^{\frac{1+\rho_{m,t}}{\rho_{m,t}}} \tag{60}$$

$$S_t P_t^{M,PCP} = \left\{ \zeta_m \left(S_t \Pi_{m,t-1}^{\theta_m} \bar{\Pi}_m^{1-\theta_m} P_{t-1}^{M,PCP} \right)^{\frac{\rho_{m,t}}{1+\rho_{m,t}}} + (1-\zeta_m) \left(S_t P_t^{\circ M,PCP} \right)^{\frac{\rho_{m,t}}{1+\rho_{m,t}}} \right\}^{\frac{1+\rho_{m,t}}{\rho_{m,t}}}, \tag{61}$$

where $S_t P_t^{M,PCP}$ denotes the aggregate price level of PCP firms in the domestic currency.[16]

2.6 Capital goods producers

There is a single, representative capital producer, owned by the representative patient household, that operates on perfectly competitive markets. At the end of period t, the capital producer purchases existing capital $(1-\delta) K_t$ from entrepreneurs as well as investment goods I_t^{CGP}, and combines them to produce new capital K_{t+1} using the following technology:

$$K_{t+1} = (1-\delta) K_t + \zeta_t^I F\left(I_t^{CGP}, I_{t-1}^{CGP} \right),$$

[16] For a more detailed derivation of these conditions, see Kilponen et al. (2016).

where $F\left(I_t^{CGP}, I_{t-1}^{CGP}\right) = \left[1 - \frac{\gamma_I}{2}\left(\frac{I_t^{CGP}}{I_{t-1}^{CGP}} - \mu\right)^2\right] I_t^{CGP}$ denotes capital investment net of adjustment costs, and ζ_t^I is an investment-specific productivity shock. New capital K_{t+1} produced in period t can be used in production in period $t+1$. Investment goods are purchased at price P_t^I. Let P_t^K denote the nominal price of capital. Since the marginal rate of transformation between new and old capital is assumed to be unity, the price of both new and undepreciated old capital is P_t^K.

The capital producer's profit maximisation problem is thus given by:

$$\begin{aligned} &\max_{\left\{I_t^{CGP}\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \phi_t^P \left\{P_t^K K_{t+1} - P_t^K \left(1-\delta\right) K_t - P_t^I I_t^{CGP}\right\} \\ &s.t. \quad K_{t+1} = \left(1-\delta\right) K_t + \zeta_t^I F\left(I_t^{CGP}, I_{t-1}^{CGP}\right), \end{aligned} \tag{62}$$

where ϕ_t^P is the Lagrange multiplier associated with the patient households' budget constraint. Taking the first order condition and rearranging yields:

$$\frac{P_t^I}{P_t^C} = \frac{P_t^K}{P_t^C}\zeta_t^I F_1\left(I_t^{CGP}, I_{t-1}^{CGP}\right) + \beta^P E_t\left[\frac{\psi_{t+1}^P}{\psi_t^P}\frac{P_{t+1}^K}{P_{t+1}^C}\zeta_{t+1}^I F_2\left(I_{t+1}^{CGP}, I_t^{CGP}\right)\right], \tag{63}$$

where $\psi_{h,t}^P = \phi_{h,t}^P P_t^C$ is the nominal shadow price of a unit of the consumption good, and $F_1(I_t^{CGP}, I_{t-1}^{CGP}) = \frac{\partial F(I_t^{CGP}, I_{t-1}^{CGP})}{\partial I_t^{CGP}}$, $F_2(I_t^{CGP}, I_{t-1}^{CGP}) = \frac{\partial F(I_t^{CGP}, I_{t-1}^{CGP})}{\partial I_{t-1}^{CGP}}$ denote the partial derivatives with respect to the first and second argument of the adjustment cost function, respectively.

2.7 Housing goods producers

Housing good producers ("construction firms") use final housing investment goods I_t^H to produce new housing units H_t^N, sold to households, subject to a technology with quadratic adjustment costs. Housing producers operate on perfectly competitive markets and are owned by the patient households. The problem of the representative housing producer is:

$$\begin{aligned} &\max_{\left\{I_t^H\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \phi_t^P \left\{P_t^H H_t^N - P_t^{IH} I_t^H\right\} \\ &s.t. \quad H_t^N = \zeta_t^{IH} F\left(I_t^H, I_{t-1}^H\right) \end{aligned}$$

where $F\left(I_t^H, I_{t-1}^H\right) = \left[1 - \frac{\gamma_{IH}}{2}\left(\frac{I_t^H}{I_{t-1}^H} - \mu\right)^2\right] I_t^H$ denotes housing investment net of adjustment costs, and ζ_t^{IH} is an investment-specific productivity shock in housing production.

The problem yields the optimality condition:

$$\frac{P_t^{IH}}{P_t^C} = \frac{P_t^H}{P_t^C}\xi_t^H F_1\left(I_t^H, I_{t-1}^H\right) + \beta^P E_t\left[\frac{\psi_{t+1}^P}{\psi_t^P}\frac{P_{t+1}^H}{P_{t+1}^C}\xi_{t+1}^H F_2\left(I_{t+1}^H, I_t^H\right)\right], \tag{64}$$

where $F_1\left(I_t^H, I_{t-1}^H\right) = \frac{\partial F(I_t^H, I_{t-1}^H)}{\partial I_t^H}$ and $F_2\left(I_t^H, I_{t-1}^H\right) = \frac{\partial F(I_t^H, I_{t-1}^H)}{\partial I_{t-1}^H}$ denote the partial derivatives with respect to the first and second argument of the adjustment cost function, respectively.

The aggregate housing stock then evolves according to:

$$H_{t+1} = \left(1 - \delta_t^H\right) H_t + H_t^N = \left(1 - \delta_t^H\right) H_t + \xi_t^H F\left(I_t^H, I_{t-1}^H\right), \tag{65}$$

where δ_t^H is the (time-varying) depreciation rate of the housing stock.

2.8 Entrepreneurs

The economy is populated by a continuum of identical entrepreneurs of mass one. They provide capital rental services to domestic intermediate good firms and are assumed to operate under perfect competition. Entrepreneurs have special skills in the operation and management of physical capital. To fund their activities, entrepreneurs may have to borrow from banks, as their own financial resources may not be sufficient to fully finance the capital expenditures. We assume that the entrepreneurs' subjective discount rate is the same as that of the patient households' (β^P).

2.8.1 Profit maximisation problem of the entrepreneur

At the beginning of period t, the representative entrepreneur rents capital to domestic intermediate good firms, who use it as an input in production (see section 2.2). Then, at the end of period t, he sells the undepreciated capital to capital producers at price P_t^K, makes repayments on any bank loans, and purchases new capital K_{t+1} from the capital producers at price P_t^K. At the end of period t, the entrepreneur

has available net worth, N_{t+1}, which he uses to finance his capital expenditures, $P_t^K K_{t+1}$. To finance the difference between expenditures and net worth, he borrows from the bank an amount BL_{t+1}^{NFC}, given by:

$$BL_{t+1}^{NFC} = P_t^K K_{t+1} - N_{t+1}. \tag{66}$$

The entrepreneur's profit maximisation problem is then given by:

$$\max_{\{K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \left(1-\tau_t^K\right) R_t^K K_t + (1-\delta) P_t^K K_t + \delta \tau_t^K P_t^K K_t - P_t^K K_{t+1} - r_{t-1}^{NFC} \left(P_{t-1}^K K_t - N_t\right) \right\},$$

where r_{t-1}^{NFC} is the net nominal interest rate on corporate loans, which the entrepreneur takes as given. R_t^K is the rental cost of capital, which in equilibrium is equal to the marginal productivity of capital. The first order condition is:

$$P_t^K = \beta^P E_t \left\{ \left(1-\tau_{t+1}^K\right) R_{t+1}^K + \left(1-\delta+\delta\tau_{t+1}^K\right) P_{t+1}^K - r_t^{NFC} P_t^K \right\}. \tag{67}$$

The capital Euler equation (67) equates the value of a unit of installed capital at time t, on the left-hand side, to the expected discounted return of that extra unit of capital in period $t+1$, on the right-hand side. Let $Q_t = P_t^K / P_t^C$ denote the price of capital relative to the price of the final consumption good. This price can also be interpreted as Tobin's q. Equation (67) can then be rewritten as:

$$Q_t = \frac{\beta^P}{1+\beta^P r_t^{NFC}} E_t \left[\left(1-\tau_{t+1}^K\right) \frac{R_{t+1}^K}{P_t^C} + \left(1-\delta+\delta\tau_{t+1}^K\right) Q_{t+1} \frac{P_{t+1}^C}{P_t^C} \right]. \tag{68}$$

The entrepreneur's equity at the end of period t, V_t, is given by:

$$V_t = \left[\left(1-\tau_t^K\right) R_t^K + \left(1-\delta+\delta\tau_t^K\right) P_t^K \right] K_t - \left(1+r_{t-1}^{NFC}\right) \left(P_{t-1}^K K_t - N_t\right). \tag{69}$$

The first term represents the after-tax gross return on capital owned by the entrepreneur, composed of the income from the rental activities and the proceeds from selling the undepreciated capital to capital producers. The second term represents the gross repayment of the loan taken in the previos period $(t-1)$.

To avoid a situation in which the entrepreneur accumulates enough net worth to become self-financed, we

follow *e.g.* Bernanke et al. (1999) and assume that, in each period, the entrepreneur exits the economy with probability $1-\gamma$. In that case, he rebates his equity to the patient households as a lump-sum transfer. To keep the entrepreneurial population constant, a new entrepreneur is born with probability $1-\gamma$. The entrepreneur's net worth N_{t+1} combines his equity and a transfer, W^e, received from the patient households, which corresponds to the initial net worth (seed money) necessary for a new entrepreneur's activity to start. The law of motion for the entrepreneur's net worth is then given by:

$$N_{t+1} = \gamma V_t + W^e. \tag{70}$$

2.8.2 Financing cost minimisation problem of the entrepreneur

The financing cost minimisation problem of an entrepreneur is analogous to that of an impatient household's seeking to finance its housing purchases with bank loans. An entrepreneur seeking a nominal amount of borrowing for period $t+1$ equal to BL_{t+1}^{NFC}, defined by (66), would allocate his borrowing among different banks so as to minimize the total repayment due. At the end of period t, the entrepreneur decides on how much to borrow from bank z, $bl_{t+1}^{NFC}(z)$, by solving the following problem:

$$\min_{bl_{t+1}(z)} \int_0^1 r_t^{NFC}(z)\, bl_{t+1}^{NFC}(z)\, dz$$

$$s.t.\;\; BL_{t+1}^{NFC} = \left\{ \int_0^1 \left[bl_{t+1}^{NFC}(z) \right]^{\frac{\varepsilon_t^{NFC}-1}{\varepsilon_t^{NFC}}} dz \right\}^{\frac{\varepsilon_t^{NFC}}{\varepsilon_t^{NFC}-1}},$$

where $r_t^{NFC}(z)$ is the interest rate charged by the z-th bank on corporate loans, and $\varepsilon_t^{NFC} > 1$ is the time-varying interest rate elasticity of the demand for corporate loans, which measures the degree of competition in banks' corporate lending activities. The first order condition yields the following demand schedule for loans by the entrepreneur:

$$bl_{t+1}^{NFC}(z) = \left(\frac{r_t^{NFC}(z)}{r_t^{NFC}} \right)^{-\varepsilon_t^{NFC}} B_{t+1}^{NFC},$$

where r_t^{NFC} is the nominal average loan rate on corporate loans prevailing in the market at time t, defined as:

$$r_t^{NFC} = \left\{ \int_0^1 \left[r_t^{NFC}(z) \right]^{1-\varepsilon_t^{NFC}} dz \right\}^{\frac{1}{1-\varepsilon_t^{NFC}}}.$$

2.9 Banks

Following Kilponen et al. (2016), the banking sector in this paper is modeled in the spirit of Gerali et al. (2010).[17] Each bank in the model is composed of two retail branches and one wholesale unit. The first retail branch is responsible for giving out differentiated corporate loans to entrepreneurs. The second retail branch is responsible for granting differentiated mortgage loans to impatient households. Both retail branches operate under monopolistic competition and set their offered lending rates accordingly, subject to adjustment costs. Finally, the wholesale unit manages the capital position of the whole banking group, collects deposits from households, and allocates funds to the retail branches for their lending activities. It is assumed to operate under perfect competition. The banks are owned by the patient households.

Bank capital is accumulated through retained earnings. Furthermore, the banks are subject to an exogenous target for their capital-to-assets ratio (*i.e.* the inverse of leverage) set by the regulator, deviations from which are costly. As a consequence, aggregate bank capital affects the total credit supply and, as such, generates a feedback link between the real and financial sides of the economy.

Banks provide loans to entrepreneurs and impatient households by combining deposits, collected from patient households, with their own bank capital K_{t+1}^b.[18] At the end of period t, the aggregate bank balance sheet is given by the identity:

$$BL_{t+1}^{NFC} + BL_{t+1}^{H} = D_{t+1} + K_{t+1}^b. \tag{71}$$

[17] The literature that incorporates a banking sector into a DSGE framework has bloomed in the aftermath of the global financial crisis. See Goodfriend and McCallum (2007), Christiano et al. (2008), Kobayashi (2008), Verona et al. (2013) and Silvo (2019) for examples of alternative ways of introducing a banking sector into a DSGE framework.

[18] We assume perfect competition in the market for households' deposits, and also rule out the entry and exit of banks.

2.9.1 Wholesale branch

The wholesale bank chooses the total risk-weighted volume of loans BL_{t+1}^{REG} to maximise the discounted sum of cash flows from the banking groups' activities:

$$\max_{\{BL_{t+1}^{REG}\}} r_t^b BL_{t+1}^{REG} - r_t^d D_{t+1} - \frac{\kappa_{K^b}}{2}\left(\frac{K_{t+1}^b}{BL_{t+1}^{REG}} - \nu_t^b\right)^2 K_{t+1}^b \ ,$$

subject to the balance sheet identity $BL_{t+1}^{NFC} + BL_{t+1}^{H} = K_{t+1}^b + D_{t+1}$ and given the regulatory risk-weighted assets of the bank defined as $BL_{t+1}^{REG} = BL_{t+1}^{NFC} + \phi_t^H BL_{t+1}^{H}$. The bank takes as given the net wholesale loan rate r_t^b and the net wholesale deposit rate r_t^d. Housing loans are assigned an exogenous, potentially time-varying risk weight $\phi_t^H > 0$ relative to NFC loans by the financial regulator.[19] This allows us to distinguish between the regulatory capital-asset ratio and the bank's actual leverage, defined as $K_{t+1}^b / \left(BL_{t+1}^{NFC} + BL_{t+1}^{H}\right)$. The bank has to pay a quadratic cost, proportional to the stock of bank capital outstanding and parametrised by κ_{K^b}, whenever the regulatory capital-to-risk-weighted-assets ratio $K_{t+1}^b / BL_{t+1}^{REG}$ deviates from the target value ν_t^b.

The maximisation problem yields the following first order condition, which links the spread between wholesale rates on loans and on deposits to the bank's regulatory capital-to-asset position:

$$r_t^b = r_t^d - \kappa_{K^b}\left(\frac{K_{t+1}^b}{BL_{t+1}^{REG}} - \nu_t^b\right)\left(\frac{K_{t+1}^b}{BL_{t+1}^{REG}}\right)^2 . \tag{72}$$

To close the model, we assume that banks have access to unlimited wholesale funds at the net rate $r_t^{FI} \equiv R_t - 1$, which is the net rate of return on domestic government bonds.[20] Hence, by arbitrage, $r_t^d = r_t^{FI}$, and the condition above becomes:

$$S_t^w \equiv r_t^b - r_t^{FI} = -\kappa_{K^b}\left(\frac{K_{t+1}^b}{BL_{t+1}^{REG}} - \nu_t^b\right)\left(\frac{K_{t+1}^b}{BL_{t+1}^{REG}}\right)^2 , \tag{73}$$

where S_t^w is the bank's funding spread prevailing at the wholesale level. The left-hand side of the equation represents the marginal benefit from increasing lending, equal to the funding spread. The right-hand side

[19] A similar risk weight is used *e.g.* in Angelini et al. (2014).

[20] Note that, by this assumption, an increase in the risk premium on government bonds is also fed into the banks' funding costs, reflecting the link between bank's and sovereign's financing costs.

gives the marginal cost of doing so, which equals the cost of deviating from the regulatory target ν_t^b.

2.9.2 Retail corporate loan branch

The retail corporate loan branch branch maximises its profits from its lending activities, taking as given the return r_t^b payable to the wholesale unit. It grants differentiated loan products and sets the interest rate on them. In choosing the interest rate, the branch faces quadratic adjustment costs for changing the rates it charges on its loans over time. These costs are parametrised by κ_b^{NFC} and are proportional to the aggregate returns on corporate loans.[21]

At the end of period t, a branch z sets is loan rate $r_{t+\tau}^{NFC}(z)$ to maximise the objective:

$$E_t \sum_{\tau=0}^{\infty} \left(\beta^P\right)^\tau \phi_{t+\tau}^P \Big[r_{t+\tau}^{NFC}(z)\, bl_{t+1+\tau}^{NFC}(z) - r_{t+\tau}^b bl_{t+1+\tau}^{NFC}(z)$$

$$-\frac{\kappa_b^{NFC}}{2}\left(\frac{r_{t+\tau}^{NFC}(z)}{r_{t-1+\tau}^{NFC}(z)}-1\right)^2 r_{t+\tau}^{NFC} BL_{t+1+\tau}^{NFC}\Big]$$

subject to the demand curve on corporate loans offered by the branch z:

$$bl_{t+1+\tau}^{NFC}(z) = \left(\frac{r_{t+\tau}^{NFC}(z)}{r_{t+\tau}^{NFC}}\right)^{-\varepsilon_{t+\tau}^{NFC}} BL_{t+1+\tau}^{NFC}. \tag{74}$$

Deriving the first-order condition, imposing a symmetric equilibrium and rearranging yields:

$$1-\varepsilon_t^{NFC}\frac{r_t^{NFC}-r_t^b}{r_t^{NFC}}-\kappa_b^{NFC}\left(\frac{r_t^{NFC}}{r_{t-1}^{NFC}}-1\right)\frac{r_t^{NFC}}{r_{t-1}^{NFC}}+\beta^P E_t\left[\frac{\phi_{t+1}^P}{\phi_t^P}\kappa_b^{NFC}\left(\frac{r_{t+1}^{NFC}}{r_t^{NFC}}-1\right)\left(\frac{r_{t+1}^{NFC}}{r_t^{NFC}}\right)^2\frac{BL_{t+2}^{NFC}}{BL_{t+1}^{NFC}}\right]=0. \tag{75}$$

[21] This assumption is a modeling shortcut for introducing sticky loan rates. It allows to have an incomplete short-run pass-through of policy rates to retail loan rates. Empirical studies (see *e.g.* de Bondt et al., 2005 and Gropp et al., 2014) have found that the pass-through from money-market rates to retail lending rates is far from complete in several euro area countries. In Finland, the pass-through is expected to be relatively fast – but still incomplete – due to the fact that a majority of loan contracts are in terms of variable interest rates (see *e.g.* Kauko, 2005).

2.9.3 Retail mortgage loan branch

The problem of the retail mortgage loan branch is analogous to that of the retail corporate loan branch. A branch z maximises its profits, taking as given the return r_t^b payable to the wholesale unit and subject to quadratic adjustment costs. These costs are parametrised by κ_b^H and are proportional to aggregate returns on mortgage loans.

At the end of period t, branch z maximises, over $r_{t+\tau}^H(z)$, the objective:

$$E_t \sum_{\tau=0}^{\infty} \left(\beta^P\right)^\tau \phi_{t+\tau}^P \left\{ \left[r_{t+\tau}^H(z) - r_{t+\tau}^b \right] bl_{t+1+\tau}^H(z) \right.$$

$$\left. -\frac{\kappa_b^H}{2} \left(\frac{r_{t+\tau}^H(z)}{r_{t-1+\tau}^H(z)} - 1 \right)^2 r_{t+\tau}^H BL_{t+1+\tau}^H \right\}$$

subject to the demand curve:

$$bl_{t+1+\tau}^H(z) = \left(\frac{r_{t+\tau}^H(z)}{r_{t+\tau}^H} \right)^{-\varepsilon_{t+\tau}^H} BL_{t+1+\tau}^H.$$

Deriving the first-order condition, imposing a symmetric equilibrium and rearranging yields the optimality condition:

$$1 - \varepsilon_t^H \frac{r_t^H - r_t^b}{r_t^H} - \kappa_b^H \left(\frac{r_t^H}{r_{t-1}^H} - 1 \right) \frac{r_t^H}{r_{t-1}^H} + \beta^P E_t \left[\frac{\phi_{t+1}^P}{\phi_t^P} \kappa_b^H \left(\frac{r_{t+1}^H}{r_t^H} - 1 \right) \left(\frac{r_{t+1}^H}{r_t^H} \right)^2 \frac{BL_{t+2}^H}{BL_{t+1}^H} \right] = 0.$$

We make the assumption that the borrower household can freely renegotiate the allocation of its total stock of debt into different banks in each period, or in other words, refinance its mortgage loans. Because the household commits to being a customer of a given loan branch for one period only (and the interest rate is variable), from the bank's point of view the mortgage loans are like one-period loans. The bank gets r_{t-1}^H on its entire stock of mortgage loans (BL_t^H), regardless of whether they are new or old mortgage loans. From the household's point of view this is not the case, as the old debt stock limits the amount of new borrowing in each period.

2.9.4 Bank profits

Overall bank profits are the sum of net earnings from the wholesale unit and the two retail branches. Period t profits are given by:

$$J_t^B = r_{t-1}^{NFC} BL_t^{NFC} + r_{t-1}^H BL_t^H - r_{t-1}^{FI} D_t$$

$$-\frac{\kappa_{K^b}}{2}\left(\frac{K_{t+1}^b}{BL_{t+1}^{NFC} + \phi_t^H BL_{t+1}^H} - v_t^b\right)^2 K_{t+1}^b$$

$$-\frac{\kappa_b^{NFC}}{2}\left(\frac{r_t^{NFC}}{r_{t-1}^{NFC}} - 1\right)^2 r_t^{NFC} BL_{t+1}^{NFC} - \frac{\kappa_b^H}{2}\left(\frac{r_t^H}{r_{t-1}^H} - 1\right)^2 r_t^H BL_{t+1}^H. \tag{76}$$

Bank capital is accumulated out of retained earnings:

$$K_{t+1}^b = \left(1 - \delta^b\right) K_t^b + \frac{J_t^b}{\varepsilon_t^{Kb}}, \tag{77}$$

where the bank capital depreciation rate δ^b could capture either the costs associated with managing bank capital and conducting overall banking activity (as in Gerali et al., 2010), or with the dividend policy of the bank (as in *e.g.* Brubakk and Gelain, 2014). Furthermore, since in this model neither the borrowers nor the banks default endogenously, an exogenous financial shock (ε_t^{Kb}) is introduced to capture unanticipated losses on the bank's balance sheet.

2.10 Fiscal authority

The fiscal authority collects taxes and issues euro-denominated domestic bonds B_t to finance government spending on goods and services. These public purchases are the sum of government demand for domestic intermediate goods and investment goods. We assume full home bias in public sector consumption demand, while in the case of investment goods, the public sector purchases final investment goods. Consequently, the relevant price index for public investment is P_t^I, while for public sector consumption, the relevant price index is that of domestic intermediate goods P_t.

Public purchases of domestic intermediate goods and investments goods are determined exogenously, so that:

$$P_t^G G_t = P_t C_t^G + P_t^I I_t^G,$$

where C_t^G denotes exogenous public consumption spending and I_t^G denotes exogenous public investments. Given public spending, the fiscal authority's nominal budget constraint takes the form:

$$\begin{aligned} &P_t^G G_t + B_t = \tau_t^C P_t^C C_t^H + \left(\tau_t^W + \tau_t^F\right) W_t L_t \\ &+\tau_t^K \left(R_t^K K_t - P_t^I \delta K_t\right) + \tau^H P_t^H (1 - \delta_t^H) H_t \\ &-(1-\omega_h)\tau_t^{rH} r_{t-1}^H B L_t^H + \mathcal{TR}_t + \frac{B_{t+1}}{R_t} \ . \end{aligned}$$

The government's net lump sum transfers $\mathcal{TR}_t$ close the fiscal authority's budget constraint in each period. Finally, we assume that all distortionary tax rates τ_t^s, $s = C, W, F, K, H, rH$, are constant and set to their steady-state values.

2.11 Market equilibrium

2.11.1 Factor markets

The domestic intermediate good firms' cost minimisation problem implies the following equilibrium aggregate labour supply schedule and nominal wage index (abstracting away from wage dispersion):

$$L_t = \left(\frac{L_t^P}{\omega_h}\right)^{\omega_h} \left(\frac{L_t^I}{1-\omega_h}\right)^{1-\omega_h}, \tag{78}$$

$$W_t = \left(W_t^P\right)^{\omega_h} \left(W_t^I\right)^{1-\omega_h}. \tag{79}$$

In equilibrium, the composite supply of differentiated labour services L_t is equal to the intermediate goods producing firms' labour demand L_t^F, such that:

$$L_t = L_t^F,$$

with $\int_0^1 L_t(j)dj \equiv L_t^F$.

Market clearing in capital markets implies that capital services provided by capital good producers are equal to the total demand by intermediate good firms:

$$K_t = \int_0^1 K_t(j)dj.$$

2.11.2 Intermediate goods market

Each intermediate good producing firm acts as price setter in the monopolistically competitive intermediate goods market. Since the domestic intermediate goods are used as inputs to produce consumption, capital investment, housing investment and export goods in competitive markets, the supply of differentiated goods must be equal to the total demand. Hence, the corresponding equilibrium condition states that:

$$\int_0^1 Y_t\left(j\right) dj = Y_t^C + Y_t^I + Y_t^H + Y_t^X, \tag{80}$$

where the right-hand side corresponds to the conditional factor demands of consumption, investment and export goods producing firms. The left-hand side is the total supply of the intermediate goods producing firms. Aggregating over the continuum of intermediate goods producing firms, we obtain:

$$\int_0^1 Y_t\left(j\right) dj = \Delta_{p,t} Y_t \ , \tag{81}$$

where $\Delta_{p,t} = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1}{1+\rho_t^z}} dj$ is the price dispersion term. In nominal terms, it follows that:

$$P_t Y_t = \int_0^1 P_t\left(j\right) Y_t\left(j\right) dj = P_t(Y_t^C + Y_t^I + Y_t^H + Y_t^X) \ . \tag{82}$$

2.11.3 Export market

Finnish export goods $X_t\left(i\right)$ are aggregated by the foreign retailer firm, such that the total supply of Finnish export goods is given by $\int_0^1 X_t\left(i\right) di$. In market equilibrium, the total supply of Finnish exports

will be equal to their demand:

$$\int_0^1 X_t(i)\,di = \int_0^1 \left(\frac{P_t^X(i)}{P_t^X}\right)^{-\frac{1}{1+\rho_{f,t}}} X_t di = \Delta_{px} X_t = X_t, \tag{83}$$

where $\Delta_{px} = \int_0^1 \left(\frac{P_t^X(i)}{P_t^X}\right)^{-\frac{1}{1+\rho_{f,t}}}$ is the export price dispersion term and X_t is the total export demand, given by equation (54). This also trivially holds in nominal terms, such that $P_t^X X_t = \int_0^1 P_t^X(i) X_t(i)\,di$, given the properties of the price index P_t^X and conditional export demand $X_t(i)$.

2.11.4 Import market

We assume that the supply of foreign goods used as inputs in the production of domestic and exported final goods is fully elastic and matches the total demand. The total supply of imported composite goods is equal to the sum of demands for imported intermediate inputs in the production of consumption, investment, housing investment and export goods:

$$M_t = M_t^C + M_t^I + M_t^H + M_t^X.$$

In nominal terms this is equivalent to:

$$P_t^M M_t = P_t^M \left(M_t^C + M_t^I + M_t^H + M_t^X\right),$$

where the corresponding price index for the imported composite goods is obtained by integrating over the prices set by the LCP and PCP firms (see equation (58)).

2.11.5 Domestic final goods market

The final goods market for the composite consumption, capital investment, housing investment and export goods are fully competitive. Hence, market clearing in the final goods markets implies that, in addition to the export good producer's equilibrium condition (83), the total supply of consumption, capital investment and housing investment goods is equal to their total demand, respectively. For the consumption goods,

total supply must equal total demand by the households:

$$C_t = C_t^H,$$

where total private consumption demand is given by $C_t^H = \int_0^1 C_{h,t}^k dh = \omega_h C_t^P + (1 - \omega_h) C_t^I$.

For capital investment goods, total demand is given by the sum of private and public demands, respectively:

$$I_t = I_t^{CGP} + I_t^G.$$

As for the housing investment goods, market clearing requires that the supply of housing investment goods I_t^H equals their demand by the housing goods producer as input in the production of new housing units H_t^N, given by the optimality condition (64).

2.11.6 Housing market

Housing market clearing requires that housing supply equals the demand by both patient and impatient households:

$$H_t = \int_0^1 H_{h,t}^k dh = \omega_h H_t^P + (1 - \omega_h) H_t^I. \tag{84}$$

2.11.7 Banking sector

The banks' balance sheet identity requires that the banks' total assets, composed of corporate and mortgage loans, equal its total liabilities, comprising bank capital and households' deposits:

$$BL_{t+1}^{NFC} + BL_{t+1}^H = K_{t+1}^b + D_{t+1}. \tag{85}$$

2.11.8 Bond market

The fiscal authority's budget constraint determines the supply of domestic bonds. Under the assumption that the fiscal authority balances its budget at all times, the bonds are assumed to be in zero net supply

in every period, and the market clearing condition becomes:

$$B_t = \int_0^1 B_{h,t} dh = 0.$$

As for the internationally traded bonds, we assume that the supply of foreign bonds is fully elastic, matching the demand of domestic residents for holdings of foreign bonds:

$$\int_0^1 B_{h,t}^{€} dh + \int_0^1 B_{h,t}^{\$} dh = B_t^{€} + B_t^{\$}.$$

2.11.9 Nominal aggregate resource constraint and government budget constraint

Combining the market clearing conditions for the domestic final goods markets and the intermediate goods markets results in the following representation of the economy's nominal resource constraint:

$$P_t Y_t = P_t^C C_t^H + P_t^I I_t^{CGP} + P_t^G G_t + P_t^{IH} I_t^H + S_t P_t^X X_t - P_t^M \left(M_t^C + M_t^I + M_t^H + M_t^X \right).$$

The government budget constraint, taking into account the bond market clearing condition $B_t = 0 \, \forall \, t$, is given by:

$$\begin{aligned} & P_t^G G_t = \tau_t^C P_t^C C_t^H + \left(\tau_t^W + \tau_t^F \right) W_t L_t \\ & + \tau_t^K \left(R_t^K K_t - P_t^I \delta K_t \right) + \tau_t^H P_t^H (1 - \delta_t^H) H_t - \tau_t^{rH} r_{t-1}^H B L_t^H + \mathcal{TR}_t. \end{aligned} \tag{86}$$

2.11.10 Net foreign assets, trade balance and terms of trade

The economy's net foreign assets equal the economy-wide net holdings of the foreign bonds, such that:

$$NFA_{t+1}^* = R_t^{€} B_t^{€} + R_t^{\$} S_t B_t^{\$} + TB_t,$$

where TB_t denotes the trade balance, given by:

$$TB_t = S_t P_t^X X_t - P_t^M \left(M_t^C + M_t^I + M_t^H + M_t^X \right).$$

In addition, the terms of trade are defined as:

$$ToT_t = S_t P_t^X / P_t^M.$$

2.12 Model solution

In order to compute the equilibrium of the model around the balanced growth path, the model variables are scaled and stationarised by dividing real variables by the price level P_t and by the permanent labour-augmenting productivity λ_t^l, and the nominal variables by the price level P_t. The model is then solved by taking a first-order linear approximation around the non-stochastic balanced growth path (steady state) of the model. The full system of log-linearised scaled model equations as well as the exogenous shock processes are reported in Appendix A.

3 Calibration and steady state properties

Calibration of the model parameters is done in two steps. In the first step, we first fix the values of some structural parameters either to conventional values found in the literature, or to values derived directly from observed data. We then choose a number of other parameters to match selected unconditional steady-state model moments as closely as possible to the corresponding long-run moments in the data; namely, key aggregate macroeconomic and financial steady-state ratios. This step is described in more detail below in Section 3.1.

In the second step, we calibrate key elasticities and shock processes driving the dynamics of the model to match unconditional second-order moments of selected macroeconomic and financial variables of the log-linearised model as closely as possible to the corresponding moments in the data. This step is described in more detail in Section 3.2. In both steps, we employ a GMM-type algorithm to minimise the distance between the unconditional model moments implied by the parameter values and the corresponding data moments. To calibrate the model, we use quarterly data from 1996Q1 to 2019Q4. Macroeconomic data are from the Finnish Quarterly National Accounts, and the data on loan volumes and interest rates are collected from the statistics on monetary financial institutions published by the Bank of Finland.

3.1 Matching of long-run moments

Along the non-stochastic balanced growth path, all scaled domestic real variables are assumed to grow at a rate of 2.16% per year (quarterly gross growth rate $\mu = 1.0054$), corresponding to the average growth rate of output in the data sample. All quantity variables are expressed in per-capita terms. Hence, the growth rate of domestic real variables, after removing growth differentials, corresponds to the growth rate of labour-augmenting productivity. The scaled nominal variables, apart from the nominal wage, are assumed to grow at a rate of 1.28% per year (quarterly gross inflation rate $\Pi = 1.0032$), corresponding to the average inflation rate in the data sample.

The discount rate of patient households (β^P) is set at 0.998 to deliver an equilibrium annual nominal interest rate of about 4.3%. The interest rate elasticity of the demand for mortgage loans (ε^H) and non-financial corporations' loans (ε^{NFC}) are set at 3.64 and 3.72, respectively, so as to match the average spreads between the respective lending rates and the risk-free rate.

Following Kilponen et al. (2016), the price markup for intermediate good producing firms Υ is set at 1.08, and for export producing firms Υ^f and importing firms Υ_m at 1.10 and 1.05, respectively. The labor markup λ_w is 1.20. The value of the depreciation rate of physical capital δ is set at 0.013.

The depreciation rate for bank capital δ^b is set to 0.0884, which ensures that the steady-state bank capital-to-asset ratio v^b is 10%.[22] Consistently with current regulation, the steady-state loan-to-value ratio on new housing purchases (θ^H) and the risk-weight requirements associated with mortgage loans (ϕ^H) are set at 0.90 and 0.15, respectively. These values correspond to the baseline values set in the macroprudential policy legislation currently in place in Finland. The parameters governing the effective amortisation rate of the mortgage loan stock are set to $\kappa = 0.0125$ and $\alpha^M = 0.99$ to match an average initial loan maturity of 20 years, as observed in the data.

The survival probability of entrepreneurs γ is set at 0.9914, which allows us to exactly match the non-financial corporations' loans-to-GDP ratio in the data. The initial transfer from households to entrepreneurs W^e is arbitrarily set to a small value of 0.01.

[22] The value of v^b could be thought of as comprising a minimum requirement established by the regulator (the 8% benchmark from the Basel regulation) plus a voluntary buffer that the bank decides to hold for precautionary reasons. In this paper we assume that the regulator can set v^b, which implies that the voluntary buffer is constant.

The tax rates on capital income (τ^K), on consumption (τ^C) and on labor income (τ^W) are set at 20%, 21% and 32%, respectively, close to their sample means. The social security contribution rate of private sector firms (τ^F) is set at 16%. The shares of government consumption ($\mathbf{s_{GCF}}$) and government investment ($\mathbf{s_{IG}}$) are set to 0.32 and 0.24, respectively, based on their sample means. The real estate tax rate (τ^H) and the the tax deduction on the interest rate (τ^{rH}) are set at 0 in the baseline calibration.

The share of impatient households ($1-\omega_h$) is calibrated to match the income share of financially constrained homeowners in the 2016 wave of the Finnish Household Wealth Survey. The homeownership rate in the survey is 64%. Of all homeowners, 49% have mortgage loans. However, in the data, not all homeowners with debt may actually be financially constrained. We define a financially constrained household along the lines of Kaplan et al. (2014) and Justiniano et al. (2015): a household is defined as constrained if its total liquid assets (the sum of its deposits and its holdings of publicly traded equity, debt securities and investment fund shares) equal less than twice the household's monthly disposable income. The income share of these constrained homeowners in the data is 33% of all homeowners. Alternatively, we can define constrained households as those who self-report in the survey as, on average, not being able to save money out of their total income. Defined this way, the income share of the constrained households in the data is 35%. These values are broadly in line with corresponding values found in the literature.[23] Accordingly, we set the share of patient agents ω_h to 0.67.

We then match the rest of the parameters that affect the steady state of the model — mainly substitution parameters, quasi share parameters and steady state values of productivity shifters — by using a GMM-type algorithm to match selected steady state ratios of the model with corresponding long-run great ratios observed in the data. Table 1 shows the calibration of the parameters, and the selected steady-state ratios are reported in table 4. As seen from the table, the key aggregate ratios are matched with their empirical counterparts very well over the sample period 1996Q1–2019Q4.

[23] In similar two-type setups, Iacoviello (2005) sets (income) share of borrower households to 0.2, Iacoviello and Neri (2010) to 0.36, Gerali et al. (2010) to 0.2, Quint and Rabanal (2014) to 0.39, and Pedersen (2016) to 0.5. The first two papers use U.S. data to calibrate or estimate the share, while the latter three papers apply to the European context. In comparison, using a similar approach as we do, Justiniano et al. (2015) find a higher labor income share of 0.64 of constrained households in U.S. household-level data.

3.2 Second-order moment matching

The second step of our calibration strategy consists of setting values of the parameters that affect the model's dynamics around the steady state, but that do not enter the model's analytical steady state solution, so as to match selected second-order moments of key variables in the model to their observed counterparts in the data. This set of parameters includes various elasticities, adjustment cost parameters, and the standard deviation and autocorrelation parameters of the exogenous shock processes that drive most of the dynamics of the model variables.

We target the contemporaneous correlation structure, as well as relative volatilities, of key macroeconomic aggregates with the aggregate output cycle in the sample 1996Q1–2019Q4. The observed data is detrended by transforming it to quarterly growth rates. The rest of the parameter values not used in the calibration algorithm, mainly standard deviation and autocorrelation parameters of shock processes that have an insignificant impact on the aggregate model dynamics, are set following Kilponen et al. (2016).

Tables 2 and 3 report the parameter values, and Table 5 shows the match of the targeted second order moments of selected model variables to the corresponding data moments. Overall, in qualitative terms, the model matches very well the cyclicality and the relative volatility of key macroeconomic variables with respect to output. The only notable exception is aggregate labour supply, which is less volatile than output in the data, but more volatile in the model. This is reflected in the behaviour of the real wage, which is somewhat less volatile in the model than what is observed in the data.

Table 5 reports, in addition, the corresponding moments implied by the calibration of key financial variables and their empirical counterparts. These moments were not explicitly targeted in the calibration algorithm. Nonetheless, the model also matches very well the dynamic behaviour of the financial aggregates observed in the data. Most importantly, the model captures the cyclical behaviour of both mortgage loan-to-GDP and corporate loan-to-GDP ratios.

4 Model dynamics

In this section, we describe the model's dynamic behaviour through the analysis of impulse responses to selected key macroeconomic and financial shocks. In each case, we show the responses under three alternative calibrations:

1. the baseline calibration, where one third of households are credit-constrained ($\omega_h = 0.67$);
2. a model where there are no constrained households ($\omega_h = 1$) but there is demand and supply for housing;
3. a model where there are no constrained households and no housing consumption and production.

The last case corresponds to the structure of the *Aino 2.0* model.[24] The comparison illustrates the way in which the presence of constrained households, on the one hand, and the presence of a housing asset, on the other hand, affect the transmission of shocks in the model. In general, the dynamics of the model are amplified by both channels that we disentangle: on the one hand, through the presence of constrained households who are not able to optimally smooth their consumption over time, and, on the other hand, by the presence of a durable good (housing) from which households derive utility.

We report impulse responses to key shocks affecting the housing market and the mortgage loan market, and show how the maturity of mortgage loans affects the behaviour of the households and the dynamics of the model. The long-maturity mortgage loans are an important new feature in the model compared to previous model vintages, and they substantially affect the ability of constrained agents to adjust to various shocks. In this way, multi-period loans act as another amplifying force in the model.

4.1 Responses to key macroeconomic shocks

Figures 5, 6, 7 and 8 report key impulse responses of the model to a temporary capital productivity shock ($\varepsilon_t^{\lambda k}$), price markup shock ($\varepsilon_t^{v}$), government consumption shock (ε_t^{cG}), and external demand shock (ε_t^{mW}),

[24] Strictly speaking, *Aino 2.0* is not fully nested within the *Aino 3.0* model, since the baseline calibration of *Aino 2.0* includes public production of goods and services, which we abstract away from. If this feature is shut down in *Aino 2.0*, the structure of *Aino 3.0* nests *Aino 2.0*. The calibration of the model used in this paper differs from the estimated *Aino 2.0* model presented in Kilponen et al. (2016).

respectively.

The solid black lines show the responses of the baseline *Aino 3.0* model. The dotted grey lines show the responses of the *Aino 3.0* model when we set the share of patient (unconstrained) agents ω_h to unity, *i.e.* when we shut down the impact of the constrained agents on the model dynamics, and in addition abstract away from housing consumption and production. This latter case corresponds to the structure of the *Aino 2.0* model. The dashed black lines represent the intermediate case in which there are no constrained households ($\omega_h = 1$), but there is demand for housing from patient households and supply for housing.

The main observation, standing out from all four figures, is that the presence of constrained households or the availability of a housing asset does not have a significant impact on the transmission of these macroeconomic shocks, with the exception of private consumption and housing investment. Most macroeconomic aggregates behave very similarly in all three model versions. This suggests that the housing market or the consumption response of credit-constrained agents does not amplify the transmission of rather standard macroeconomic shocks related to supply, external demand or public demand. As illustrated below, however, this is not the case for domestic demand shocks, especially interest rate shocks and shocks to the collateral constraint, which directly affect the consumption possibilities of the constrained agents.

4.2 Responses to a domestic risk premium shock

Figures 9 and 10 report the responses of the model to a 25-basis-point increase in the domestic risk premium over the Euro Area interest rate ($\varepsilon_t^{\zeta^{\text{€}}}$), or in other words, to an increase in domestic interest rates. The former figure again compares the transmission of the shock in the baseline *Aino 3.0* model (solid black lines), a model specification where the channel of transmission through the presence of constrained agents is shut down (dashed black lines), and the specification where, in addition, the housing asset is absent (dotted grey lines). The latter figure shows how the mortgage loan maturity affects the transmission of shocks to the short-term interest rates.

Figure 9 shows that in this case, the macroeconomic aggregates respond in a different manner depending on the specification. The initial responses of most variables are rather similar under the three model specifications. In particular, the initial impacts of the shock on output and inflation are almost the same.

Subsequently, the responses are amplified by the presence of constrained households, as the responses of the baseline model specification (in solid black lines) diverge from the two alternative specifications.

The protracted contraction in output is mostly driven by the presence of constrained households, and not by the presence of the housing asset. This is due to a stronger negative response of private consumption and total hours worked after the initial impact. As constrained households are not able to optimally adjust their consumption as a response to the shocks, their consumption responses are more volatile than those of the unconstrained agents. Moreover, in order to maintain the market equilibrium, unconstrained households (lenders) need to absorb part of the required adjustment in consumption and savings that the constrained agents would like to make but cannot. This holds true even in an open economy, where part of the adjustment can spill over abroad through foreign trade, and leads to strong substitution effects across the consumption, savings, and labour supply decisions of the two types of households. This is a feature common to all TANK models.

In response to the interest rate premium shock, the constrained households reduce their consumption as well as their demand for housing, as mortgage borrowing becomes more expensive. The increase in interest rates also tightens the collateral constraint, as house prices decrease. This amplifies the responses of the constrained households relative to those of the unconstrained. Finally, the impatient households react to the tightening of their collateral and budget constraints by increasing their wage demands, which leads to less demand for labour and a contraction in total hours worked. In contrast, as housing becomes cheaper, saver households increase their housing purchases, which counteracts the reduction in housing demand by borrower households.

From Figure 10, it is evident that the mortgage loan maturity has an important impact on the transmission of the interest rate shock. The solid black lines show the responses under our baseline calibration, where we target an average mortgage loan maturity of 20 years. The dash-dotted black lines show the responses when the average maturity is 10 years, and the grey lines when mortgage loans are one-period (one-quarter) loans, conventional to most DSGE models. As the amount of unamortised old debt limits the ability of constrained agents to borrow more and consequently to adjust their consumption, a longer average maturity substantially slows down the adjustment of the economy back to its steady state, and as a result, renders the responses of macroeconomic aggregates more volatile.

4.3 Housing preference shock

Figure 11 reports the impulse responses to a positive housing preference shock ($\varepsilon_t^{\zeta H}$). The shock is common to both types of households. The size of the shock is such that house prices increase by 1% on impact. We report the responses under three alternative calibrations for the average mortgage loan maturity (20-year, 10-year and 1-quarter loans).

The shock causes an expansion in output, as it creates a boom in housing and, to a less extent, in capital investment. The response of total private investment is for the most part driven by boom the housing construction. As a result, there is an expansion in both mortgage and corporate lending. However, as there is relatively less demand for consumption goods other than housing services, aggregate private consumption contracts, and the shock is mildly deflationary. The shock also induces impatient households to work more, for them to be able to finance new housing purchases, which sustains the boom in output.

The longer the maturity, the stronger and the more persistent these responses are. In this case, the long-maturity loans create spillovers to capital investment. The relatively stronger demand for housing initially decreases the relative price of capital as there is less relative demand for savings into assets other than housing by patient households. However, this effect is offset by the increase in the capital rental rate, induced by the increase in labour supply and the corresponding decrease in real wages. With one-period mortgage loans, this margin of adjustment is much weaker, and the spillovers to corporate lending and capital investment are small.

4.4 An anticipated change in the loan-to-value ratio

Finally, Figure 12 shows the impulse responses to a permanent anticipated 5 percentage point tightening in the regulatory loan-to-value ratio (θ_t^H). The change in policy is announced in period t, and it comes into effect in period $t+4$, *i.e.* one year after the announcement. We again report the responses under three alternative calibrations for the mortgage loan maturity (20-year, 10-year and 1-quarter loans).

The announcement initially causes an increase in the demand for housing and mortgage loans by the impatient agents before the tightening comes into effect. Correspondingly, there is an initial increase in

mortgage lending rates. In contrast, patient households reduce their housing demand, which dominates the aggregate response of total housing investment.

In addition to increasing mortgage borrowing, the impatient households also initially increase their labour supply to finance house purchases before the tighter regulation come into force. This has an expansionary effect on output. As there is strong substitution away from other consumption into consumption of housing services by impatient households at the same time, the initial impact of the announcement is also mildly deflationary. After the tighter LTV limit is imposed, the economy slowly adjusts to a new steady-state equilibrium, where output, consumption, investment, hours worked, and asset prices are all at a permanently lower level. In the long run, aggregate output is about 0.3% lower than before the tighter policy is imposed in the baseline specification.

Remarkably, both the short-run anticipatory effects and the long-run effects are entirely driven by the long maturity of the mortgage loans. If mortgage loans are one-period loans, the long-run impact of the policy is negligible. This result highlights the importance of realistically modelling the maturity structure of mortgage loans when analysing the effects of demand-side macroprudential policies.

5 Conclusions

In this paper, we have presented the latest vintage of the Bank of Finland's macroeconomic model for the Finnish economy, the *Aino 3.0* model. It is a large-scale New-Keynesian small open economy DSGE model that takes Finland as a small member of a monetary union. It builds on its predecessor, *Aino 2.0*, by introducing a housing asset and housing production, long-term mortgage lending by banks, and credit-constrained households. It aims at capturing some of the key macro-financial linkages and vulnerabilities in the Finnish economy; namely, high household indebtedness and a volatile residential construction sector. Compared to the previous model vintages, it also enriches the modelling of the financial sector and enables the analysis of the impacts of a wide range of macroeconomic and macroprudential policies.

We have illustrated that the presence of credit-constrained households affects the dynamic responses of the model economy especially after interest rate shocks. The constrained agents are very sensitive to changes in interest rates, which renders the economy more volatile in response to such shocks and consequently

also affects the transmission of monetary policy. The dynamic behaviour of the macroeconomic aggregates is also affected by the average maturity of mortgage loans. The longer the maturity, the longer it takes for the households to adjust to shocks, as they are constrained by the slow adjustment of their debt stock. The introduction of long-term debt thus creates important (de)leveraging effects in the model. Besides the business cycle behaviour of the macroeconomic aggregates, the long-maturity mortgage debt also affects the adjustment of the economy to permanent changes in macroprudential regulation. This last observation stresses the importance of realistically modelling mortgage maturities when evaluating long-term consequences of macroprudential regulation.

The work presented in this paper could be expanded and refined in many ways. The current version of the model is calibrated, which imposes some limitations to its application in analysis. In future work, the model could benefit from estimating its parameters using Bayesian methods. Furthermore, as most other research with collateral constraints on borrowing, we assume that the constraint is always binding when solving for the model equilibrium. Solving the model under the assumption of an occasionally binding credit constraint would open up possibilities for even richer analysis of macro-financial linkages and macroprudential regulation, but for computational reasons this would likely require the simplification of the model structure in other dimensions.

In addition, while the model already incorporates various macroprudential tools regulating bank capital on the supply side, it only features a single demand-side tool: the loan-to-value ratio. Policymakers have, in the past years, paid much attention to other demand-side regulations that could directly curb overheated household borrowing, observed in many advanced economies. However, macroeconomic research on this topic is still scarce. The model could be modified to include alternative demand-side policies, such as debt-to-income or debt service-to-income requirements instead of, or in combination with, the loan-to-value requirement on mortgage loans.

Finally, the modelling of various aspects relevant to a small open economy, such as the labour market and the structure of the monetary union, could be further improved upon. The main focus in the current model version has been on the macro-financial linkages in the domestic economy. International financial linkages and possibilities of spillovers in the financial sector would also be worthwhile to explore further, especially in the Nordic context, where the financial markets are tightly inter-connected and rather concentrated.

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A The full model equations

This appendix lists the full set of linearised model equations and the shock processes. Before log-linearising the model, we scale all real variables by the permanent component of labour productivity ($\Lambda_{l,t}^P$) in order to render the variables stationary. For any real variable X_t, $\widetilde{X}_t = X_t/\Lambda_{l,t}^P$ is the corresponding stationary variable, denoted by upper-case letters. The corresponding lower-case letter $\tilde{x}_t = \frac{X_t - X}{X}$ denotes the **percentage** deviation of a stationarised variable $\tilde{X}_t$ from its steady state value $\tilde{X}$. Furthermore, $\hat{x}_t = X_t - X$ denotes the **absolute** deviation of a variable from its steady state. Tax rates, net foreign assets, the trade balance-to-output ratio, the effective loan amortisation rate γ_t^H, the loan-to-value ratio θ_t^H, the capital-to-asset ratio requirement ν_t, the mortgage loan risk weight ϕ_t^H and the time varying housing depreciation rate δ_t^H are expressed in **absolute** deviations when linearising the model.

A.1 The linearised equilibrium conditions

Banking sector

- NFC loans:

$$\tilde{bl}_{t+1}^{NFC} = \frac{Q\widetilde{P}^C\tilde{K}}{\tilde{BL}^{NFC}}\left(\tilde{q}_t + \tilde{p}_t^C + \tilde{k}_{t+1}\right) - \frac{\tilde{N}}{\tilde{BL}^{NFC}}\tilde{n}_{t+1} \tag{87}$$

- Housing loan stock:

$$\begin{aligned}\tilde{bl}_{t+1}^{H} = {} & \frac{\tilde{P}^H\tilde{H}^I}{\tilde{BL}^H}\left[1-\left(1-\delta^H\right)\mu^{-1}\right]\left(\theta^H\tilde{p}_t^H + \hat{\theta}_t^H\right) \\ & + \frac{\theta^H\tilde{P}^H\tilde{H}^I}{\tilde{BL}^H}\left[\tilde{h}_{t+1}^I - \left(1-\delta^H\right)\mu^{-1}\left(\tilde{h}_t^I - \tilde{\mu}_t\right) + \mu^{-1}\hat{\delta}_t^H\right] \\ & + \left(1-\gamma^H\right)(\mu\Pi)^{-1}\left(\tilde{bl}_t^H - \tilde{\mu}_t - \tilde{\pi}_t\right) - (\mu\Pi)^{-1}\hat{\gamma}_t^H\end{aligned} \tag{88}$$

- New housing loans:

$$\frac{\tilde{BL}^{H,new}}{\tilde{BL}^H}\tilde{bl}_t^{H,new} = \tilde{bl}_{t+1}^H - \left(1-\gamma^H\right)(\mu\Pi)^{-1}\left(\tilde{bl}_t^H - \tilde{\mu}_t - \tilde{\pi}_t\right) + (\mu\Pi)^{-1}\hat{\gamma}_t^H \tag{89}$$

- Effective amortisation rate:

$$\hat{\gamma}_{t+1}^{H} = \mathbf{a_{G1}} \left(\mu\Pi\right)^{-1} \left(\tilde{bl}_{t}^{H} - \tilde{bl}_{t+1}^{H} - \tilde{\mu}_{t} - \tilde{\pi}_{t}\right) + \mathbf{a_{G2}} \left(\mu\Pi\right)^{-1} \hat{\gamma}_{t}^{H} \tag{90}$$

- Risk-weighted total loans:

$$\tilde{bl}_{t+1}^{reg} = \frac{\tilde{BL}^{NFC}}{\tilde{BL}^{REG}} \tilde{bl}_{t+1}^{NFC} + \frac{\phi^{H} \tilde{BL}^{H}}{\tilde{BL}^{REG}} \tilde{bl}_{t+1}^{H} + \frac{\tilde{BL}^{H}}{\tilde{BL}^{REG}} \hat{\phi}_{t}^{H} \tag{91}$$

- Bank balance sheet:

$$\tilde{BL}^{NFC} \tilde{bl}_{t+1}^{NFC} + \tilde{BL}^{H} \tilde{bl}_{t+1}^{H} = \tilde{D}\tilde{d}_{t+1} + \tilde{K}^{b} \tilde{k}_{t+1}^{b} \tag{92}$$

- Tobin's Q:

$$\begin{aligned} \tilde{q}_{t} = & \frac{1}{1 + \beta^{P} r^{NFC}} \left\{ \beta^{P} \Pi \left[1 - \delta + \delta\tau^{K}\right] E_{t} \tilde{q}_{t+1} - \beta^{P} r^{NFC} \tilde{r}_{t}^{NFC} \right. \\ & \left. + \frac{\beta^{P}\Pi}{Q} \left[\frac{\tilde{R}^{K}}{\widetilde{P}^{C}} \left(1 - \tau^{K}\right) \left(E_{t} \tilde{r}_{t+1}^{K} - E_{t} \tilde{p}_{t+1}^{C}\right) - \left(\frac{\tilde{R}^{K}}{\widetilde{P}^{C}} - \delta Q \right) E_{t} \hat{\tau}_{t+1}^{K} \right] \right\} \\ & + E_{t} \tilde{\pi}_{t+1} + E_{t} \tilde{p}_{t+1}^{C} - \tilde{p}_{t}^{C} \end{aligned} \tag{93}$$

- Entrepreneur's net worth:

$$n_{1}\tilde{n}_{t+1} = n_{2}\tilde{k}_{t} + n_{3}\tilde{r}_{t}^{K} + n_{4}\tilde{q}_{t} + n_{5}\tilde{q}_{t-1} + n_{6}\tilde{n}_{t} + n_{7}\tilde{r}_{t-1}^{NFC} + n_{8}\tilde{\pi}_{t} + n_{9}\tilde{\mu}_{t} + n_{10}\hat{\tau}_{t}^{K} + n_{11}\tilde{p}_{t}^{C} + n_{12}\tilde{p}_{t-1}^{C} \tag{94}$$

where $c_{1}^{*} = \tilde{R}^{K}\left(1 - \tau^{K}\right) + \left(1 - \delta + \delta\tau^{K}\right) Q\widetilde{P}^{C}$, $n_{1} = \Pi\tilde{N}\mu/\gamma$, $n_{2} = \Pi\tilde{K}c_{1}^{*} - \left(1 + r^{NFC}\right)\tilde{K}Q\widetilde{P}^{C}$, $n_{3} = \Pi\tilde{K}\tilde{R}^{K}\left(1 - \tau^{K}\right)$, $n_{4} = \Pi\tilde{K}Q\widetilde{P}^{C}\left(1 - \delta + \delta\tau^{K}\right)$, $n_{5} = -Q\tilde{K}\widetilde{P}^{C}\left(1 + r^{NFC}\right)$, $n_{6} = \tilde{N}\left(1 + r^{NFC}\right)$, $n_{7} = -r^{NFC}\left(Q\tilde{K}\widetilde{P}^{C} - \tilde{N}\right)$, $n_{8} = \Pi\tilde{K}c_{1}^{*} - \frac{\Pi\mu}{\gamma}\left(\tilde{N} - \tilde{W}^{e}\right)$, $n_{9} = -\frac{\Pi\mu}{\gamma}\left(\tilde{N} - \tilde{W}^{e}\right)$, $n_{10} = \Pi\tilde{K}\left(\delta Q\widetilde{P}^{C} - \tilde{R}^{K}\right)$, $n_{11} = n_{4}$ and $n_{12} = n_{5}$.

- Net interest rate on NFC loans:

$$\tilde{r}_t^{NFC} = \frac{\kappa_b^{NFC}}{\varepsilon^{NFC} - 1 + (1+\beta^P)\,\kappa_b^{NFC}} \tilde{r}_{t-1}^{NFC} + \frac{\beta^P \kappa_b^{NFC}}{\varepsilon^{NFC} - 1 + (1+\beta^P)\,\kappa_b^{NFC}} E_t \tilde{r}_{t+1}^{NFC}$$
$$+ \frac{\varepsilon^{NFC} - 1}{\varepsilon^{NFC} - 1 + (1+\beta^P)\,\kappa_b^{NFC}} \tilde{r}_t^b - \frac{1}{\varepsilon^{NFC} - 1 + (1+\beta^P)\,\kappa_b^{NFC}} \tilde{\varepsilon}_t^{NFC} \tag{95}$$

- Net interest rate on housing loans:

$$\tilde{r}_t^{H} = \frac{\kappa_b^{H}}{\varepsilon^{H} - 1 + (1+\beta^I)\,\kappa_b^{H}} \tilde{r}_{t-1}^{H} + \frac{\beta^I \kappa_b^{H}}{\varepsilon^{H} - 1 + (1+\beta^I)\,\kappa_b^{H}} E_t \tilde{r}_{t+1}^{H}$$
$$+ \frac{\varepsilon^{H} - 1}{\varepsilon^{H} - 1 + (1+\beta^I)\,\kappa_b^{H}} \tilde{r}_t^b - \frac{1}{\varepsilon^{H} - 1 + (1+\beta^I)\,\kappa_b^{H}} \tilde{\varepsilon}_t^{H} \tag{96}$$

- Net wholesale loan rate:

$$\tilde{r}_t^b = \tilde{r}_t^{FI} - \frac{\kappa_{K^b}}{R-1} \left(\left(v^b\right)^3 \left(\tilde{k}_{t+1}^b - \tilde{bl}_{t+1}^{reg} \right) - \left(\nu^b\right)^2 \hat{\nu}_t^b \right) \tag{97}$$

- Bank capital:

$$\tilde{k}_{t+1}^b = \frac{1-\delta^b}{\Pi\mu} \left(\tilde{k}_t^b - \tilde{\pi}_t - \tilde{\mu}_t - \tilde{\varepsilon}_t^{Kb} \right) + \left(1 - \frac{1-\delta^b}{\Pi\mu} \right) \tilde{j}_t^b \tag{98}$$

- Bank profits:

$$\tilde{j}_t^b = \frac{r^{NFC} \tilde{BL}^{NFC}}{r^{NFC} \tilde{BL}^{NFC} + r^H \tilde{BL}^H - (R-1)\,\tilde{D}} \left(\tilde{r}_{t-1}^{NFC} + \tilde{bl}_t^{NFC} \right) + \frac{r^H \tilde{BL}^H}{r^{NFC} \tilde{BL}^{NFC} + r^H \tilde{BL}^H - (R-1)\,\tilde{D}} \left(\tilde{r}_{t-1}^{H} + \tilde{bl}_t^{H} \right)$$
$$- \frac{(R-1)\,\tilde{D}}{r^{NFC} \tilde{BL}^{NFC} + r^H \tilde{BL}^H - (R-1)\,\tilde{D}} \left(\tilde{r}_{t-1}^{FI} + \tilde{d}_t \right) - \tilde{\pi}_t - \tilde{\mu}_t \tag{99}$$

Housing market

- Demand for domestic intermediate good in housing investment:

$$\tilde{y}_t^H = \tilde{i}_t^H + \sigma_h \left(\tilde{p}_t^{IH} - \rho_h \tilde{\lambda}_{hy,t} \right) \tag{100}$$

- Demand for imported intermediate good in housing investment:

$$\tilde{m}_t^H = \tilde{i}_t^H + \frac{\sigma_h \gamma_{hm}}{1+\sigma_h \gamma_{hm}} \left(\tilde{m}_{t-1}^H - \tilde{i}_{t-1}^H\right) + \frac{\sigma_h}{1+\sigma_h \gamma_{hm}} \left(\tilde{p}_t^{IH} - \tilde{p}_t^M - \rho_h \tilde{\lambda}_{hm,t}\right) \tag{101}$$

- Final housing investment good production (housing supply):

$$\tilde{i}_t^H = \frac{1}{1+\beta^P} \left(\beta^P E_t \tilde{i}_{t+1}^H + \tilde{i}_{t-1}^H + \beta^P E_t \tilde{\mu}_{t+1} - \tilde{\mu}_t\right) - \frac{\tilde{p}_t^{IH} - \tilde{p}_t^H - \tilde{\xi}_t^H}{\gamma_{ih} \mu^2 \left(1+\beta^P\right)} \tag{102}$$

- Price of final housing investment good:

$$\tilde{p}_t^{IH} = -\frac{\mathbf{s_{YIH}}}{\tilde{P}^{IH}} \tilde{\lambda}_{hy,t} + \frac{\mathbf{s_{MIH}} \tilde{P}^M}{\tilde{P}^{IH}} \left\{\tilde{p}_t^M - \tilde{\lambda}_{hm,t} + \gamma_{hm} \left[\left(\tilde{m}_t^H - \tilde{i}_t^H\right) - \left(\tilde{m}_{t-1}^H - \tilde{i}_{t-1}^H\right)\right]\right\} \tag{103}$$

- Housing Q:

$$\tilde{q}_t^H = \tilde{p}_t^H - \tilde{p}_t^C \tag{104}$$

- Patient household housing demand:

$$\begin{aligned} \frac{\tilde{h}_{t+1}^P}{1-b_H^P \mu^{-1}} &= E_t \tilde{\zeta}_{t+1}^H + \frac{b_H^P \mu^{-1}}{1-b_H^P \mu^{-1}} \left[\tilde{h}_t^P - \mu_t\right] - \tilde{Q}^H \frac{\psi_\Lambda^P \left(1-b_H^P \mu^{-1}\right) \tilde{H}^P}{\beta^P j^P} \left[\tilde{\psi}_{\Lambda,t}^P + \tilde{q}_t^H\right] \\ &+ \tilde{Q}^H \frac{\psi_\Lambda^P \left(1-b_H^P \mu^{-1}\right) \tilde{H}^P}{j^P \mu} \Big[\left(1-\tau^H\right) \left(1-\delta^H\right) \left(E_t \tilde{q}_{t+1}^H + E_t \tilde{\psi}_{\Lambda,t+1}^P - E_t \tilde{\mu}_{t+1}\right) \\ &- \left(1-\delta^H\right) E_t \hat{\tau}_{t+1}^H - \left(1-\tau^H\right) E_t \hat{\delta}_{t+1}^H \Big] \end{aligned} \tag{105}$$

- Impatient household housing demand:

$$\begin{aligned}
&\frac{\tilde{h}^I_{t+1}}{1-b^I_H\mu^{-1}} = E_t\tilde{\zeta}^H_{t+1} + \frac{b^I_H\mu^{-1}}{1-b^I_H\mu^{-1}}\left[\tilde{h}^I_t - \mu_t\right] \\
&- \tilde{Q}^H\frac{\left(1-b^I_H\mu^{-1}\right)\tilde{H}^I}{\beta^I j^I}\left[\left(\psi^{I,2}_\Lambda - \psi^{I,1}_\Lambda\theta^H\right)\tilde{q}^H_t + \psi^{I,2}_\Lambda\tilde{\psi}^{I,2}_{\Lambda,t} - \theta^H\psi^{I,1}_\Lambda\tilde{\psi}^{I,1}_{\Lambda,t} - \psi^{I,1}_\Lambda\hat{\theta}^H_t\right] \\
&+ \tilde{Q}^H\frac{\left(1-b^I_H\mu^{-1}\right)\left[\psi^{I,2}_\Lambda\left(1-\tau^H\right) - \psi^{I,1}_\Lambda\theta^H\right]\tilde{H}^I}{j^I\mu}\left[\left(1-\delta^H\right)\left(E_t\tilde{q}^H_{t+1} - E_t\tilde{\mu}_{t+1}\right) - E_t\hat{\delta}^H_{t+1}\right] \\
&+ \tilde{Q}^H\frac{\left(1-b^I_H\mu^{-1}\right)\tilde{H}^I}{j^I\mu}\left(1-\delta^H\right)\left[\left(1-\tau^H\right)\psi^{I,2}_\Lambda E_t\tilde{\psi}^{I,2}_{\Lambda,t+1} - \theta^H\psi^{I,1}_\Lambda E_t\tilde{\psi}^{I,1}_{\Lambda,t+1} - \psi^{I,1}_\Lambda E_t\hat{\theta}^H_{t+1} - \psi^{I,2}_\Lambda E_t\hat{\tau}^H_{t+1}\right]
\end{aligned} \tag{106}$$

- Housing market clearing:

$$\tilde{h}_{t+1} = \omega_h\frac{\tilde{H}^P}{\tilde{H}}\tilde{h}^P_{t+1} + \left(1-\omega_h\right)\frac{\tilde{H}^I}{\tilde{H}}\tilde{h}^I_{t+1} \tag{107}$$

- Evolution of aggregate stock of housing:

$$\tilde{h}_{t+1} = \frac{1-\delta^H}{\mu}\left(\tilde{h}_t - \tilde{\mu}_t\right) + \left(1 - \frac{1-\delta^H}{\mu}\right)\left(\tilde{i}^H_t + \tilde{\xi}^H_t\right) - \frac{\hat{\delta}^H_t}{\mu} \tag{108}$$

Private consumption and capital investment

- Budget constraint multiplier of patient household:

$$\tilde{\psi}^P_{\Lambda,t} = \tilde{\zeta}^C_t - \frac{\widetilde{c}^P_t - \mu^{-1}b^P_c\widetilde{c}^P_{t-1}}{1-\mu^{-1}b^P_c} - \frac{\mu^{-1}b^P_c}{1-\mu^{-1}b^P_c}\tilde{\mu}_t - \frac{1}{1+\tau^C}\hat{\tau}^C_t \tag{109}$$

- Budget constraint multiplier of impatient household:

$$\tilde{\psi}^{I,2}_{\Lambda,t} = \tilde{\zeta}^C_t - \frac{\widetilde{c}^I_t - \mu^{-1}b^I_c\widetilde{c}^I_{t-1}}{1-\mu^{-1}b^I_c} - \frac{\mu^{-1}b^I_c}{1-\mu^{-1}b^I_c}\tilde{\mu}_t - \frac{1}{1+\tau^C}\hat{\tau}^C_t \tag{110}$$

- Multiplier on collateral constraint, impatient household:

$$\tilde{\psi}^{I,1}_{\Lambda,t} = \frac{\psi^{I,2}_{\Lambda}}{\psi^{I,1}_{\Lambda}}\tilde{\psi}^{I,2}_{\Lambda,t} - \beta^I \frac{\psi^{I,2}_{\Lambda}}{\psi^{I,1}_{\Lambda}}\left[\frac{1+\left(1-\tau^{rH}\right)r^H}{\mu\Pi}\left(E_t\tilde{\psi}^{I,2}_{\Lambda,t+1} - E_t\tilde{\mu}_{t+1} - E_t\tilde{\pi}_{t+1} - E_t\tilde{p}^C_{t+1} + \tilde{p}^C_t\right)\right.$$
$$\left. + \frac{\left(1-\tau^{rH}\right)r^H\tilde{r}^H_t - r^H E_t\hat{\tau}^{rH}_{t+1}}{\mu\Pi}\right] - \frac{\gamma^H-\kappa}{\tilde{BL}^H}\frac{\psi^{I,3}}{\psi^{I,1}_{\Lambda}}\left[\tilde{\psi}^{I,3}_t - \tilde{bl}^H_{t+1} - \beta^I\left(E_t\tilde{\psi}^{I,3}_{t+1} - E_t\tilde{p}^C_{t+1} + \tilde{p}^C_t - \tilde{bl}^H_{t+1}\right)\right]$$
$$+ \beta^I\frac{1-\gamma^H}{\mu\Pi}\left(E_t\tilde{\psi}^{I,1}_{\Lambda,t+1} - E_t\tilde{\mu}_{t+1} - E_t\tilde{\pi}_{t+1} - E_t\tilde{p}^C_{t+1} + \tilde{p}^C_t\right)$$
$$+ \beta^I\frac{\psi^{I,3}}{\psi^{I,1}_{\Lambda}\tilde{BL}^H}E_t\hat{\gamma}^H_{t+2} - \left(\frac{\psi^{I,3}}{\psi^{I,1}_{\Lambda}\tilde{BL}^H} + \frac{\beta^I}{\mu\Pi}\right)\hat{\gamma}^H_{t+1} \tag{111}$$

- Multiplier on amortization rate, impatient household:

$$\tilde{\psi}^{I,3}_t/\beta^I = \frac{\mathbf{a_{G2}}}{\mu\Pi}\left(\tilde{bl}^H_{t+1} - E_t\tilde{bl}^H_{t+2} - E_t\tilde{\mu}_{t+1} - E_t\tilde{\pi}_{t+1} + E_t\tilde{\psi}^{I,3}_{t+1} - E_t\tilde{p}^C_{t+1} + \tilde{p}^C_t\right)$$
$$+ \frac{\mathbf{a_{G3}}}{\mu\Pi}\hat{\gamma}^H_{t+1} - \frac{\psi^{I,1}_{\Lambda}\tilde{BL}^H}{\psi^{I,3}\mu\Pi}\left(\tilde{bl}^H_{t+1} - E_t\tilde{p}^C_{t+1} + \tilde{p}^C_t - E_t\tilde{\mu}_{t+1} - E_t\tilde{\pi}_{t+1} + E_t\tilde{\psi}^{I,1}_{\Lambda,t+1}\right) \tag{112}$$

- Total private consumption:

$$\tilde{c}^H_t = \frac{\omega_h\tilde{C}^P}{\tilde{C}^H}\tilde{c}^P_t + \frac{\left(1-\omega_h\right)\tilde{C}^I}{\tilde{C}^H}\tilde{c}^I_t \tag{113}$$

- Patient household consumption equation:

$$\tilde{c}^P_t = \frac{E_t\tilde{c}^P_{t+1}}{1+\mu^{-1}b^P_c} + \frac{\mu^{-1}b^P_c}{1+\mu^{-1}b^P_c}\tilde{c}^P_{t-1} + \frac{E_t\hat{\mu}_{t+1} - \mu^{-1}b^P_c\hat{\mu}_t}{1+\mu^{-1}b^P_c}$$
$$\frac{1}{1+\tau^C}\frac{1-\mu^{-1}b^P_c}{1+\mu^{-1}b^P_c}\left(E_t\hat{\tau}^C_{t+1} - \hat{\tau}^C_t\right) - \frac{1-\mu^{-1}b^P_c}{1+\mu^{-1}b^P_c}\left(\frac{R-1}{R}\tilde{R}_t + E_t\tilde{\zeta}^C_{t+1} - \tilde{\zeta}^C_t - E_t\tilde{\pi}^C_{t+1}\right) \tag{114}$$

- Impatient household consumption:

$$\begin{aligned}
&\left(1+\tau^{C}\right)\tilde{P}^{C}\tilde{C}^{I}\tilde{c}_{t}^{I}=\tilde{W}^{I}\tilde{L}^{I}\left[\left(1-\tau^{w}\right)\left(\tilde{w}_{t}^{I}+\tilde{l}_{t}^{I}\right)-\hat{\tau}_{t}^{w}\right]\\
&+\frac{\tilde{P}^{H}\tilde{H}^{I}}{\mu}\left[\left(1-\tau^{H}\right)\left(1-\delta^{H}\right)\left(\tilde{p}_{t}^{H}+\tilde{h}_{t}^{I}-\tilde{\mu}_{t}\right)-\left(1-\delta^{H}\right)\hat{\tau}_{t}^{H}-\left(1-\tau^{H}\right)\hat{\delta}_{t}^{H}\right]\\
&+\tilde{BL}^{H}\tilde{bl}_{t+1}^{H}-\frac{\tilde{BL}^{H}}{\mu\Pi}\left\{\left[1+\left(1-\tau^{rH}\right)r^{H}\right]\left(\tilde{bl}_{t}^{H}-\tilde{\mu}_{t}-\tilde{\pi}_{t}\right)+\left(1-\tau^{rH}\right)r^{H}\tilde{r}_{t}^{H}-r^{H}\hat{\tau}_{t}^{rH}\right\}\\
&-\tilde{P}^{H}\tilde{H}^{I}\left(\tilde{p}_{t}^{H}+\tilde{h}_{t+1}^{I}\right)-\tilde{P}^{C}\tilde{C}^{I}\hat{\tau}_{t}^{C}-\left(1+\tau^{C}\right)\tilde{P}^{C}\tilde{C}^{I}\tilde{p}_{t}^{C}-\tilde{TRt}\tilde{r}_{t}^{I}
\end{aligned}\tag{115}$$

- UIP:

$$\frac{R-1}{R}\tilde{R}_{t}=\tilde{r}_{t}^{€}+\tilde{\zeta}_{t}^{€}-E_{t}\left(\phi_{a}\hat{a}_{t+1}^{*}\right)\tag{116}$$

- Investment equation:

$$\begin{aligned}
\tilde{i}_{t}^{CGP}&=\frac{\beta^{P}E_{t}\tilde{i}_{t+1}^{CGP}+\tilde{i}_{t-1}^{CGP}}{1+\beta^{P}}-\frac{\tilde{p}_{t}^{I}-\tilde{p}_{t}^{C}-\tilde{q}_{t}}{\gamma_{I}\mu^{2}\left(1+\beta^{P}\right)}\\
&+\frac{\beta^{P}E_{t}\hat{\mu}_{t+1}-\hat{\mu}_{t}}{1+\beta^{P}}
\end{aligned}\tag{117}$$

- Capital accumulation equation:

$$\tilde{k}_{t+1}=\frac{1-\delta}{\mu}\left(\tilde{k}_{t}-\hat{\mu}_{t}\right)+\left(1-\frac{1-\delta}{\mu}\right)\tilde{i}_{t}^{CGP}\tag{118}$$

Labor supply and wages

- Wage Phillips curve patient households:

$$\begin{aligned}
\tilde{w}_{t}^{P}-\tilde{p}_{t}^{C}&=\frac{\tilde{w}_{t-1}^{P}-\tilde{p}_{t-1}^{C}}{1+\beta^{P}}-\frac{\tilde{p}_{t}^{C}-\tilde{p}_{t-1}^{C}+\tilde{\pi}_{t}-\tilde{\pi}_{t-1}}{1+\beta^{P}}\\
&+\frac{\beta^{P}}{1+\beta^{P}}\left(-\tilde{p}_{t}^{C}+E_{t}\tilde{\pi}_{t+1}-\tilde{\pi}_{t}+E_{t}\tilde{w}_{t+1}^{P}\right)\\
&+\kappa_{\mathbf{w}}^{\mathbf{P}}\left\{\frac{1}{1+\tau^{w}}\hat{\tau}_{t}^{w}+\sigma_{l}\tilde{l}_{t}^{F,P}-\tilde{\psi}_{\Lambda,t}^{P}+\hat{\lambda}_{w,t}+\tilde{p}_{t}^{C}-\tilde{w}_{t}^{P}\right\}
\end{aligned}\tag{119}$$

where $\kappa_{\mathbf{w}}^{\mathbf{P}}=\frac{(1-\xi_{w})\left(1-\xi_{w}\beta^{P}\right)}{\xi_{w}(1+\beta^{P})\left[1-\sigma_{l}\frac{\lambda_{w}}{1-\lambda_{w}}\right]}$

- Wage Phillips curve impatient households:

$$\begin{aligned}\tilde{w}_t^I - \tilde{p}_t^C = & \frac{\tilde{w}_{t-1}^I - \tilde{p}_{t-1}^C}{1+\beta^I} - \frac{\tilde{p}_t^C - \tilde{p}_{t-1}^C + \tilde{\pi}_t - \tilde{\pi}_{t-1}}{1+\beta^I} \\ & + \frac{\beta^I}{1+\beta^I}\left(-\tilde{p}_t^C + E_t\tilde{\pi}_{t+1} - \tilde{\pi}_t + E_t\tilde{w}_{t+1}^I\right) \\ & + \kappa_{\mathbf{w}}^{\mathbf{I}}\left\{\frac{1}{1+\tau^w}\hat{\tau}_t^w + \sigma_l\tilde{l}_t^{F,I} - \tilde{\psi}_{\Lambda,t}^I + \hat{\lambda}_{w,t} + \tilde{p}_t^C - \tilde{w}_t^I\right\} \end{aligned} \tag{120}$$

  where $\kappa_{\mathbf{w}}^{\mathbf{I}} = \frac{(1-\xi_w)\left(1-\xi_w\beta^I\right)}{\xi_w(1+\beta^I)\left[1-\sigma_l\frac{\lambda_w}{1-\lambda}\right]}$

- Average real wage index:

$$\tilde{w}_t = \omega_h\tilde{w}_t^P + (1-\omega_h)\,\tilde{w}_t^I \tag{121}$$

- Household labour supply:

$$\tilde{l}_t^{F,P} + \tilde{w}_t^P = \tilde{l}_t^{F,I} + \tilde{w}_t^I \tag{122}$$

- Aggregate labour supply:

$$\tilde{l}_t^F = \omega_h\tilde{l}_t^{F,P} + (1-\omega_h)\,\tilde{l}_t^{F,I} \tag{123}$$

Domestic intermediate goods producer

- Combined first order condition:

$$\sigma_Y\tilde{r}_t^K = \tilde{l}_t^F + \hat{\mu}_t - \tilde{k}_t + \sigma_Y\left[\tilde{w}_t + \frac{\hat{\tau}_t^F}{1+\tau^F} + \rho_Y\left(\hat{\lambda}_{l,t}^T - \hat{\lambda}_{k,t}\right)\right] \tag{124}$$

- Marginal costs:

$$\widetilde{mc}_t = \alpha_k\left(\tilde{r}_t^K - \hat{\lambda}_{k,t}\right) + \alpha_l\left[\tilde{w}_t + \frac{\hat{\tau}_t^F}{1+\tau^F} - \hat{\lambda}_{l,t}^T\right] \tag{125}$$

  where $\alpha_k = \frac{\tilde{K}}{\tilde{Y}\mu}\frac{\tilde{R}^K}{\widetilde{\mathcal{MC}}}$ and $\alpha_l = \frac{L}{\tilde{Y}}\frac{\left(1+\tau^F\right)\tilde{W}}{\widetilde{\mathcal{MC}}}$

- Production function:

$$\tilde{y}_t = \alpha_k\left(\hat{\lambda}_{k,t} + \tilde{k}_t - \hat{\mu}_t\right) + \alpha_l\left(\hat{\lambda}_{l,t}^T + \tilde{l}_t^F\right) \tag{126}$$

- Domestic intermediate good inflation (price Phillips curve):

$$\tilde{\pi}_t = \frac{\theta}{1+\beta\theta}\tilde{\pi}_{t-1} + \frac{\beta}{1+\beta\theta}E_t\tilde{\pi}_{t+1} + \kappa_{\mathbf{p}}\left(\widetilde{mc}_t + \tilde{v}_t\right) \tag{127}$$

  where $\kappa_{\mathbf{p}} = \frac{(1-\zeta\beta)(1-\zeta)}{\zeta(1+\beta\theta)}$

- Aggregate nominal resource constraint:

$$\begin{aligned}\tilde{Y}\tilde{y}_t = {} & \widetilde{P}^C\tilde{C}^H\left(\widetilde{p}_t^C + \tilde{c}_t^H\right) + \widetilde{C}^G\widetilde{c}_t^G + \widetilde{P}^I\left[\left(\widetilde{I}^{CGP} + \widetilde{I}^G\right)\widetilde{p}_t^I + \widetilde{I}^{CGP}\widetilde{i}_t^{CGP} + \widetilde{I}^G\widetilde{i}_t^G\right] \\ & + \widetilde{P}^{IH}\tilde{I}^H\left(\widetilde{p}_t^{IH} + \tilde{i}_t^H\right) + RS\widetilde{P}^X\tilde{X}\left(\tilde{rs}_t + \widetilde{p}_t^X + \tilde{x}_t\right) \\ & - \widetilde{P}^M\left[\left(\widetilde{M}^C + \widetilde{M}^I + \widetilde{M}^X + \widetilde{M}^H\right)\widetilde{p}_t^M + \widetilde{M}^C\widetilde{m}_t^C + \widetilde{M}^I\widetilde{m}_t^I + \widetilde{M}^X\widetilde{m}_t^X + \widetilde{M}^H\widetilde{m}_t^H\right]\end{aligned} \tag{128}$$

Consumption goods retailer

- Demand for imported intermediate consumption good:

$$\begin{aligned}\tilde{m}_t^C = {} & \frac{\sigma_c\gamma_{cm}}{1+\sigma_c\gamma_{cm}}\left(\widetilde{c}_t^H - \widetilde{c}_{t-1}^H + \tilde{m}_{t-1}^C\right) + \frac{1}{1+\sigma_c\gamma_{cm}}\widetilde{c}_t^H \\ & - \frac{\sigma_c}{1+\sigma_c\gamma_{cm}}\left(\tilde{p}_t^M - \tilde{p}_t^C\right) - \frac{\sigma_c\rho_c}{1+\sigma_c\gamma_{cm}}\hat{\lambda}_{cm,t}\end{aligned} \tag{129}$$

- Price of consumption goods:

$$\begin{aligned}\tilde{p}_t^C = {} & -\frac{\mathbf{s_{YCZ}}}{\widetilde{P}^C}\hat{\lambda}_{cy,t} \\ & + \mathbf{s_{MCZ}}\frac{\widetilde{P}^{MC}}{\widetilde{P}^C}\left\{\tilde{p}_t^{MC} - \hat{\lambda}_{cm,t} + \gamma_{cm}\left[\left(\tilde{m}_t^C - \widetilde{c}_t^H\right) - \left(\tilde{m}_{t-1}^C - \widetilde{c}_{t-1}^H\right)\right]\right\}\end{aligned} \tag{130}$$

- Demand for domestic intermediate consumption good:

$$\widetilde{Y}^C\tilde{y}_t^C = \frac{\mathbf{s_{YCZ}}}{\widetilde{P}^C}\tilde{C}^H\left[\widetilde{c}_t^H + \sigma_c\left(\tilde{p}_t^C - \rho_c\hat{\lambda}_{cy,t}\right)\right] + \tilde{C}^G\widetilde{c}_t^G \tag{131}$$

Investment goods retailer

- Demand for imported intermediate investment good:

$$\tilde{m}_t^I = \frac{\sigma_i \gamma_{im}}{1+\sigma_i \gamma_{im}} \left(\tilde{\tilde{i}}_t - \tilde{\tilde{i}}_{t-1} + \tilde{m}_{t-1}^I \right) + \frac{1}{1+\sigma_i \gamma_{im}} \tilde{\tilde{i}}_t - \frac{\sigma_i}{1+\sigma_i \gamma_{im}} \left(\tilde{p}_t^M - \tilde{p}_t^I \right) - \frac{\sigma_i \rho_i}{1+\sigma_i \gamma_{im}} \hat{\lambda}_{im,t} \tag{132}$$

- Price of investment goods:

$$\tilde{p}_t^I = -\frac{\mathbf{s_{YII}}}{\widetilde{P}^I} \hat{\lambda}_{iy,t} + \mathbf{s_{MII}} \frac{\widetilde{P}^M}{\widetilde{P}^I} \left\{ \tilde{p}_t^M - \hat{\lambda}_{im,t} + \gamma_{im} \left[\left(\tilde{m}_t^I - \tilde{\tilde{i}}_t \right) - \left(\tilde{m}_{t-1}^I - \tilde{\tilde{i}}_{t-1} \right) \right] \right\} \tag{133}$$

- Demand for domestic intermediate investment good:

$$\tilde{y}_t^I = \tilde{\tilde{i}}_t + \sigma_i \left(\tilde{p}_t^I - \rho_i \hat{\lambda}_{iy,t} \right) \tag{134}$$

Export and import markets

- Demand for imported intermediate export good:

$$\tilde{m}_t^X = \tilde{x}_t + \sigma_x \left(\widetilde{mc}_{x,t} - \tilde{p}_t^M - \rho_x \hat{\lambda}_{xm,t} \right) \tag{135}$$

- Demand for domestic intermediate export good:

$$\tilde{y}_t^X = \tilde{x}_t + \sigma_x \left(\widetilde{mc}_{x,t} - \rho_x \hat{\lambda}_{xy,t} \right) \tag{136}$$

- Exporters' marginal costs:

$$\widetilde{mc}_{x,t} = -\frac{\mathbf{s_{YXX}}}{\widetilde{mc}_x} \hat{\lambda}_{xy,t} + \mathbf{s_{MXX}} \frac{\widetilde{P}^M}{\widetilde{mc}_x} \left(\tilde{p}_t^M - \hat{\lambda}_{xm,t} \right) \tag{137}$$

- Export demand:

$$\tilde{x}_t = \widetilde{m}_t^W - \sigma_w \widetilde{p}_t^X + \hat{\epsilon}_{x,t} \tag{138}$$

- Export price inflation:

$$\tilde{\pi}_{x,t} = \frac{\theta_x}{1+\beta\theta_x}\tilde{\pi}_{x,t-1} + \frac{\beta^P}{1+\beta\theta_x}E_t\tilde{\pi}_{x,t+1} + \kappa_{\mathbf{x}}\left(\widetilde{mc}_{x,t} - \widetilde{p}_t^X - \tilde{rs}_t + \tilde{v}_{x,t}\right) \tag{139}$$

where $\kappa_{\mathbf{x}} = \frac{(1-\zeta_x\beta)(1-\zeta_x)}{\zeta_x(1+\beta\theta_x)}$

- Import goods' inflation:

$$\begin{aligned}\tilde{\pi}_{m,t} = &\frac{\theta_m}{1+\beta\theta_m}\tilde{\pi}_{m,t-1} + \frac{\beta^P}{1+\beta\theta_m}E_t\tilde{\pi}_{m,t+1} \\ &+ \kappa_{\mathbf{m}}\left(\tilde{rs}_t - \tilde{p}_t^M + \tilde{v}_{m,t}\right) + \frac{1-\omega_m}{1+\beta\theta_m}\left(\Delta\tilde{s}_t - \beta E_t\Delta\tilde{s}_{t+1}\right)\end{aligned} \tag{140}$$

where $\kappa_{\mathbf{m}} = \frac{(1-\zeta_m\beta)(1-\zeta_m)}{\zeta_m(1+\beta\theta_m)}$

- Total imports:

$$\widetilde{m}_t = \frac{\widetilde{P}^M\widetilde{M}^C}{RS\widetilde{P}^X\tilde{X}}\widetilde{m}_t^C + \frac{\widetilde{P}^M\widetilde{M}^I}{RS\widetilde{P}^X\tilde{X}}\widetilde{m}_t^I + \frac{\widetilde{P}^M\widetilde{M}^X}{RS\widetilde{P}^X\tilde{X}}\widetilde{m}_t^X + \frac{\widetilde{P}^M\widetilde{M}^H}{RS\widetilde{P}^X\tilde{X}}\widetilde{m}_t^H \tag{141}$$

Foreign trade related quantities

- Net foreign assets:

$$\begin{aligned}\hat{b}_{t+1}^* = &\frac{\hat{b}_t^*}{\beta^P} + RS\widetilde{P}^X\tilde{X}\left(\tilde{rs}_t + \widetilde{p}_t^X + \tilde{x}_t\right) - \widetilde{P}^M\widetilde{M}^C\left(\widetilde{p}_t^M + \widetilde{m}_t^C\right) \\ &- \widetilde{P}^M\widetilde{M}^I\left(\widetilde{p}_t^M + \widetilde{m}_t^I\right) - \widetilde{P}^M\widetilde{M}^X\left(\widetilde{p}_t^M + \widetilde{m}_t^X\right) - \widetilde{P}^M\widetilde{M}^H\left(\widetilde{p}_t^M + \widetilde{m}_t^H\right)\end{aligned} \tag{142}$$

- Net foreign assets per output:

$$\hat{a}_{t+1}^* = \frac{\hat{b}_{t+1}^*}{\tilde{Y}} - \tilde{y}_t \tag{143}$$

Government

- Public sector budget constraint:

$$\begin{aligned}
&\frac{\widetilde{C}^G}{\tilde{Y}}\left(\tilde{c}_t^G-\tilde{y}_t\right)+\frac{\widetilde{P}^I\widetilde{I}^G}{\tilde{Y}}\left(\tilde{p}_t^I+\tilde{i}_t^G-\tilde{y}_t\right)= \\
&\frac{\widetilde{P}^C\tilde{C}^H}{\tilde{Y}}\left[\tau^C\left(\tilde{p}_t^C+\tilde{c}_t^H-\tilde{y}_t\right)+\hat{\tau}_t^C\right]+\frac{\widetilde{W}L^F}{\tilde{Y}}\left[\hat{\tau}_t^W+\hat{\tau}_t^F+\left(\tau^W+\tau^F\right)\left(\tilde{w}_t+\tilde{l}_t^F-\tilde{y}_t\right)\right] \\
&+\frac{\tilde{K}\left(\tilde{R}^K-\delta\widetilde{P}^I\right)}{\mu\tilde{Y}}\left[\hat{\tau}_t^K+\tau^K\left(\frac{\tilde{R}^K}{\tilde{R}^K-\delta\widetilde{P}^I}\tilde{r}_t^K-\frac{\delta\widetilde{P}^I}{\tilde{R}^K-\delta\widetilde{P}^I}\tilde{p}_t^I+\tilde{k}_t-\hat{\mu}_t-\tilde{y}_t\right)\right]+tr\,\tilde{tr}_{Y,t} \\
&+\frac{\tau^H\left(1-\delta^H\right)\tilde{P}^H\tilde{H}}{\mu\tilde{Y}}\left(\tilde{p}_t^H+\tilde{h}_t-\tilde{\mu}_t-\tilde{y}_t\right)+\frac{\left(1-\delta^H\right)\tilde{P}^H\tilde{H}}{\mu\tilde{Y}}\hat{\tau}_t^H-\frac{\tau^H\tilde{P}^H\tilde{H}}{\mu\tilde{Y}}\hat{\delta}_t^H \\
&-\frac{\tau^{rH}r^H\tilde{BL}^H}{\mu\Pi\tilde{Y}}\left(\tilde{r}_{t-1}^H+\tilde{bl}_t^H-\tilde{\mu}_t-\tilde{y}_t-\tilde{\pi}_t\right)-\frac{r^H\tilde{BL}^H}{\mu\Pi\tilde{Y}}\hat{\tau}_t^{rH}
\end{aligned} \tag{144}$$

A.2 The exogenous shock processes

- Technology shocks
  - permanent labour productivity: $\hat{\mu}_t\equiv\Delta\log\left(\Lambda_t^P/\mu\right)=\rho_\mu\hat{\mu}_{t-1}+\varepsilon_t^\mu$, $\varepsilon_t^\mu\sim N\left(0,\sigma_\mu^2\right)$
  - temporary labour productivity: $\hat{\lambda}_{l,t}^T\equiv\log\left(\Lambda_{l,t}^T/\Lambda_l^T\right)=\rho_{\lambda l}\hat{\lambda}_{l,t-1}^T+\varepsilon_t^{\lambda l}$, $\varepsilon_t^{\lambda l}\sim N\left(0,\sigma_{\lambda l}^2\right)$
  - temporary capital productivity: $\hat{\lambda}_{k,t}\equiv\log\left(\Lambda_{k,t}/\Lambda_k\right)=\rho_{\lambda k}\hat{\lambda}_{k,t}+\varepsilon_t^{\lambda k}$, $\varepsilon_t^{\lambda k}\sim N\left(0,\sigma_{\lambda k}^2\right)$
  - productivity shifters in consumption goods: $\hat{\lambda}_{cy,t}\equiv\log\left(\Lambda_{cy,t}/\Lambda_{cy}\right)=\rho_{cy}\hat{\lambda}_{cy,t-1}+\varepsilon_t^{cy}$, $\varepsilon_t^{cy}\sim N\left(0,\sigma_{cy}^2\right)$ and $\hat{\lambda}_{cm,t}\equiv\log\left(\Lambda_{cm,t}/\Lambda_{cm}\right)=\rho_{cm}\hat{\lambda}_{cm,t-1}+\varepsilon_t^{cm}$, $\varepsilon_t^{cm}\sim N\left(0,\sigma_{cm}^2\right)$
  - productivity shifters in capital investment goods: $\hat{\lambda}_{iy,t}\equiv\log\left(\Lambda_{iy,t}/\Lambda_{iy}\right)=\rho_{iy}\hat{\lambda}_{iy,t-1}+\varepsilon_t^{iy}$, $\varepsilon_t^{iy}\sim N\left(0,\sigma_{iy}^2\right)$ and $\hat{\lambda}_{im,t}\equiv\log\left(\Lambda_{im,t}/\Lambda_{im}\right)=\hat{\lambda}_{cm,t}$
  - productivity shifters in housing investment goods: $\hat{\lambda}_{hy,t}\equiv\log\left(\Lambda_{hy,t}/\Lambda_{hy}\right)=0$ and $\hat{\lambda}_{hm,t}\equiv\log\left(\Lambda_{hm,t}/\Lambda_{hm}\right)=\hat{\lambda}_{cm,t}$
  - productivity shifters in export goods: $\hat{\lambda}_{xy,t}\equiv\log\left(\Lambda_{xy,t}/\Lambda_{xy}\right)=0$ and $\hat{\lambda}_{xm,t}\equiv\log\left(\Lambda_{xm,t}/\Lambda_{xm}\right)=\hat{\lambda}_{xm,t}$

- Domestic markup shocks
  - price markup intermediate good firms: $\tilde{v}_t = \varepsilon_t^v$, $\varepsilon_t^v \sim N\left(0, \sigma_v^2\right)$
  - price markup of export good firms: $\tilde{v}_{x,t} = \rho_{vx}\tilde{v}_{x,t-1} + \varepsilon_t^{vx}$, $\varepsilon_t^{vx} \sim N\left(0, \sigma_{vx}^2\right)$
  - wage markup: $\hat{\lambda}_{w,t} = \varepsilon_t^{\lambda w}$, $\varepsilon_t^{\lambda w} \sim N\left(0, \sigma_{\lambda w}^2\right)$
- Domestic demand shocks
  - government consumption: $\widetilde{c}_t^G = \rho_{cG}\widetilde{c}_{t-1}^G + \varepsilon_t^{cG}$, $\varepsilon_t^{cG} \sim N\left(0, \sigma_{cG}^2\right)$
  - government investment: $\widetilde{i}_t^G = \rho_{iG}\widetilde{i}_{t-1}^G + \varepsilon_t^{iG}$, $\varepsilon_t^{iG} \sim N\left(0, \sigma_{iG}^2\right)$
  - household consumption preference: $\tilde{\zeta}_t^C = \rho_{\zeta C}\tilde{\zeta}_{t-1}^C + \varepsilon_t^{\zeta C}$, $\varepsilon_t^{\zeta C} \sim N\left(0, \sigma_{\zeta C}^2\right)$
  - household housing preference: $\tilde{\zeta}_t^H = \rho_{\zeta H}\tilde{\zeta}_{t-1}^H + \varepsilon_t^{\zeta H}$, $\varepsilon_t^{\zeta H} \sim N\left(0, \sigma_{\zeta H}^2\right)$
- Foreign shocks
  - foreign export demand: $\widetilde{m}_t^W = \rho_{mW}\widetilde{m}_{t-1}^W + \varepsilon_t^{mW}$, $\varepsilon_t^{mW} \sim N\left(0, \sigma_{mW}^2\right)$
  - export share: $\hat{e}_t^x = \rho_x\hat{e}_{t-1}^x + \varepsilon_t^x$, $\varepsilon_t^x \sim N\left(0, \sigma_x^2\right)$
  - nominal effective exchange rate: $\Delta\tilde{s}_t = \rho_{\Delta s}\Delta\tilde{s}_{t-1} + \varepsilon_t^{\Delta s}$, $\varepsilon_t^{\Delta s} \sim N\left(0, \sigma_{\Delta s}^2\right)$
  - foreign inflation shock: $\tilde{\pi}_t^W = \rho_{\pi W}\tilde{\pi}_{t-1}^W + \varepsilon_t^{\pi W}$, $\varepsilon_t^{\pi W} \sim N\left(0, \sigma_{\pi W}^2\right)$
  - price markup of import firms: $\tilde{v}_{m,t} = \rho_{vm}\tilde{v}_{m,t-1} + \varepsilon_t^{vm}$, $\varepsilon_t^{vm} \sim N\left(0, \sigma_{vm}^2\right)$
- Financial shocks
  - domestic risk premium: $\tilde{\zeta}_t^{€} = \rho_{\zeta €}\tilde{\zeta}_{t-1}^{€} + \varepsilon_t^{\zeta €}$, $\varepsilon_t^{\zeta €} \sim N\left(0, \sigma_{\zeta €}^2\right)$
  - euro area interest rate: $\tilde{r}_t^{€} = \rho_{r €}\tilde{r}_{t-1}^{€} + \varepsilon_t^{r €}$, $\varepsilon_t^{r €} \sim N\left(0, \sigma_{r €}^2\right)$
  - bank markup on corporate loans: $\tilde{\varepsilon}_t^{NFC} = \rho_{\varepsilon NFC}\tilde{\varepsilon}_{t-1}^{NFC} + \varepsilon_t^{\varepsilon NFC}$, $\varepsilon_t^{\varepsilon NFC} \sim N\left(0, \sigma_{\varepsilon NFC}^2\right)$
  - bank markup on mortgage loans: $\tilde{\varepsilon}_t^H = \rho_{\varepsilon H}\tilde{\varepsilon}_{t-1}^H + \varepsilon_t^{\varepsilon H}$, $\varepsilon_t^{\varepsilon H} \sim N\left(0, \sigma_{\varepsilon H}^2\right)$
  - bank capital: $\tilde{\varepsilon}_t^{Kb} = \rho_{Kb}\tilde{\varepsilon}_{t-1}^{Kb} + \varepsilon_t^{\varepsilon Kb}$, $\varepsilon_t^{\varepsilon Kb} \sim N\left(0, \sigma_{Kb}^2\right)$
  - bank's capital-to-asset ratio: $\hat{\nu}_t^b = \rho_{\nu b}\hat{\nu}_{t-1}^b + \varepsilon_t^{\nu b}$, $\varepsilon_t^{\nu b} \sim N\left(0, \sigma_{\nu b}^2\right)$

– loan-to-value ratio: $\hat{\theta}_t^H = \rho_{\theta H}\hat{\theta}_{t-1}^H + \varepsilon_t^{\theta H}$, $\varepsilon_t^{\theta H} \sim N\left(0, \sigma_{\theta H}^2\right)$

– mortgage loan risk weight: $\hat{\phi}_t^H = \rho_{\phi H}\hat{\phi}_{t-1}^H + \varepsilon_t^{\phi H}$, $\varepsilon_t^{\phi H} \sim N\left(0, \sigma_{\phi H}^2\right)$

– housing depreciation rate: $\hat{\delta}_t^H = \rho_{\delta H}\hat{\delta}_{t-1}^H + \varepsilon_t^{\delta H}$, $\varepsilon_t^{\delta H} \sim N\left(0, \sigma_{\delta H}^2\right)$

Parameter	Value	Description
		Households
β^P	0.998	Discount factor, patient households
β^I	0.969	Discount factor, impatient households
b_c^P	0.60	External habit formation in consumption, patient households
b_c^I	0.60	External habit formation in consumption, impatient households
b_H^P	0.40	External habit formation in housing, patient households
b_H^I	0.96	External habit formation in housing, impatient households
j^P	0.07	Relative housing utility weight, patient households
j^I	0.40	Relative housing utility weight, impatient households
ω_h	0.67	Share of patient households
σ_l	0.61	Frisch elasticity
λ_w	1.20	Wage markup
κ	0.0125	Initial amortisation rate of new mortgage loans
α_M	0.99	Effective amortisation rate polynomial
		Firms
μ	1.0054	Gross growth rate of permanent labor productivity
Π	1.0032	Gross steady-state inflation rate
Λ_l	1.10	Steady-state labor productivity
Λ_k	0.11	Steady-state capital productivity
δ_Y	0.30	Share parameter in the intermediate goods production function
ρ_Y	3.71	Substitution parameter in the intermediate goods production function
$\Upsilon = -1/\rho^z$	1.08	Price markup of intermediate good producing firms
δ_c	0.74	Share parameter in final consumption goods production
ρ_c	0.04	Substitution parameter in final consumption goods production
Λ_{cy}	2.20	Productivity shifter in final consumption goods production
Λ_{cm}	3.87	Productivity shifter in final consumption goods production
δ_i	0.29	Share parameter in final investment goods production
ρ_i	-0.68	Substitution parameter in final investment goods production
Λ_{iy}	5.30	Productivity shifter in final investment goods production
Λ_{im}	2.00	Productivity shifter in final investment goods production
δ_{ih}	0.59	Share parameter in final housing investment goods production
ρ_h	9.26	Substitution parameter in final housing investment goods production
Λ_{hy}	10.8	Productivity shifter in final housing investment goods production
Λ_{hm}	1.91	Productivity shifter in final consumption goods production

Table 1: Calibrated model parameters affecting the steady state (time unit of model: quarterly)

Parameter	Value	Description
		Firms
δ_x	0.64	Share parameter in final export goods production
ρ_x	1.05	Substitution parameter in final export goods production
Λ_{xy}	2.99	Productivity shifter in final export goods production
Λ_{xm}	1.32	Productivity shifter in final export goods production
$\Upsilon^f = -1/\rho^f$	1.10	Price markup of export producing firms
$\Upsilon_m = -1/\rho_m$	1.05	Price markup of foreign importing firms
		Entrepreneurs, capital goods and housing goods producers
γ	0.992	Survival probability of entrepreneurs
W^e	0.010	Transfer from households to new entrepreneurs
δ	0.013	Depreciation rate of physical capital
δ^H	0.003	Depreciation rate of the housing stock
		Banks
ε^b	3.72	Steady-state elasticity of the demand for NFC loans
ε^H	3.64	Steady-state elasticity of the demand for mortgage loans
δ^b	0.054	Depreciation rate of bank capital
v^b	0.10	Banks' steady state capital-to-assets ratio
θ^H	0.90	Steady state loan-to-value ratio requirement
ϕ^H	0.15	Risk weight on mortgage loans
		Fiscal policy
$\mathbf{s_{GCF}}$	0.32	Share of government consumption
$\mathbf{s_{IG}}$	0.24	Share of government investment
τ^W	0.32	Tax rate on labor income
τ^F	0.16	Social security contribution of employers
τ^C	0.21	Tax rate on consumption
τ^K	0.20	Tax rate on capital income
τ^{rH}	0	Tax deduction on the interest rate
τ^H	0	Real estate tax rate

Table 1 (continued)

Parameter	Value	Description
ξ_w	0.70	Calvo wage rigidity
ζ	0.70	Calvo price rigidity
θ	0.77	Indexation of prices
ζ_m	0.60	Calvo import price rigidity
θ_m	0.11	Indexation of import prices
ω_m	0.75	Share of LCP firms
ζ_x	0.52	Calvo export price rigidity
θ_x	0.32	Indexation of export prices
γ_I	0.70	Capital investment adjustment cost
γ_{IH}	1.2	Housing investment adjustment cost
γ_{cm}	0.16	External adjustment cost in consumption goods production
γ_{im}	5.1	External adjustment cost in capital investment goods production
γ_{hm}	5.1	External adjustment cost in housing investment goods production
σ_w	0.99	Elasticity of substitution in export demand
κ_b^{NFC}	0.54	NFC loan rate adjustment cost
κ_b^H	0.54	Mortgage loan rate adjustment cost
κ_{K^b}	0.01	Cost of deviation from regulatory bank capital requirement
ϕ_a	0.005	Debt elasticity of domestic interest rate

Table 2: Calibrated structural parameters affecting model dynamics only (time unit of model: quarterly)

Variable	Data	Model
C/Y	0.75	0.75
I^{CGP}/Y	0.27	0.28
X/Y	0.53	0.53
M^C/C	0.17	0.18
M^I/I^{CGP}	0.39	0.39
M^X/X	0.51	0.52
I^H/Y	0.09	0.09
Corporate loans-to-GDP ratio	1.51	1.48
Mortgage loans-to-GDP ratio	1.83	1.83
Spread, corporate loan rate - risk-free rate, annual (%)	1.57	1.56
Spread, mortgage loan rate - risk-free rate, annual (%)	1.61	1.61
Housing Q	0.87	0.87
Tobin's Q	1.00	1.00

Notes: Data sample period: 1996Q1–2019Q4. Data are in log-levels.

Table 4: Steady-state properties, the *Aino 3.0* model versus Finnish data

Shock	Description	AR parameter (ρ)	Std. dev. (σ)
ζ_C	Households' consumption preference	0.108	0.252
ζ_H	Households' housing preference	0.680	0.010
μ	Permanent labour productivity	0.990	0.004
λ_l	Temporary labour productivity	0.980	0.044
λ_k	Temporary capital productivity	0.680	0.053
ζ_I	Investment-specific technology in capital goods production	0.000	0.000
ζ_{IH}	Investment-specific technology in housing goods production	0.000	0.000
ν	Price markup, intermediate good firms	0.000	0.027
ν_x	Price markup, export good firms	0.590	0.022
ν_m	Price markup, import firms	0.245	0.526
λ_w	Wage markup	0.000	0.495
e_x	Export share	0.904	0.500
M^W	Foreign export demand	0.930	0.100
$r_{€}$	Euro area interest rate	0.942	0.001
$\zeta_{€}$	Domestic risk premium	0.793	0.0004
ε_{NFC}	Bank markup on NFC loans	0.626	0.002
ε_H	Bank markup on mortgage loans	0.626	0.002
ε_{Kb}	Bank capital	0.485	0.048
ν_b	Bank capital-to-asset ratio	0.990	0.001
θ^H	Loan-to-value ratio	0.000	0.100
ϕ^H	Mortgage loan risk weight	0.990	0.001
δ^H	Housing depreciation rate	0.000	0.000
λ_{cy}	Productivity shifter in consumption goods production	0.753	0.007
λ_{cm}	Productivity shifter in consumption goods production	0.427	0.007
λ_{iy}	Productivity shifter in capital investment goods production	0.566	0.008
λ_{im}	Productivity shifter in capital investment goods production	0.427	0.007
λ_{hy}	Productivity shifter in housing investment goods production	0.000	0.000
λ_{hm}	Productivity shifter in housing investment goods production	0.427	0.007
λ_{xy}	Productivity shifter in export goods production	0.000	0.000
λ_{xm}	Productivity shifter in export goods production	0.427	0.007
I^G	Public investment demand	0.806	0.056
C^G	Public consumption demand	0.815	0.061
DS	Nominal exchange rate	0.179	0.016
Π^W	Foreign inflation rate	0.202	0.018
$\zeta_{€}$	Domestic risk premium	0.793	0.0004

Table 3: Calibrated exogenous shock process parameters affecting model dynamics only (time unit of model: quarterly)

	Std. dev. relative to std. dev. of Y		**Contemporaneous correlation with Y**	
Variable	**Data**	**Model**	**Data**	**Model**
Targeted moments				
Capital investment (I^{CGP})	2.02	2.00	0.37	0.49
Housing investment (I^H)	2.29	2.10	0.37	0.23
Exports (X)	3.20	2.70	0.46	0.93
Imports (M)	1.99	1.65	0.44	0.93
Private consumption (C)	0.69	0.69	0.45	0.60
Hours (L)	0.68	1.32	0.51	0.98
Real wages (W)	0.61	0.26	-0.08	-0.69
Implied moments				
Inflation rate (Π)	2.25	0.33	-0.32	0.86
Mortgage loan rate (r^H)	4.03	0.94	0.47	0.36
Corporate loan rate (r^{NFC})	3.96	0.95	0.52	0.36
Mortgage loans (BL^H)	1.90	1.24	0.41	0.67
Mortgage loans-to-GDP (BL^H/Y)	1.16	0.94	-0.57	-0.18
Corporate loans (BL^{NFC})	2.25	1.59	0.34	-0.09
Corporate loans-to-GDP (BL^{NFC}/Y)	2.01	1.95	-0.34	-0.58

Notes: Data sample period: 1996Q1–2019Q4. Data are in de-meaned quarterly growth rates. Targeted moments: model moments explicitly matched to data moments using a GMM-type algorithm. Implied moments: non-targeted moments implied by the calibration.

Table 5: Second-order moments, the *Aino 3.0* model versus Finnish data.

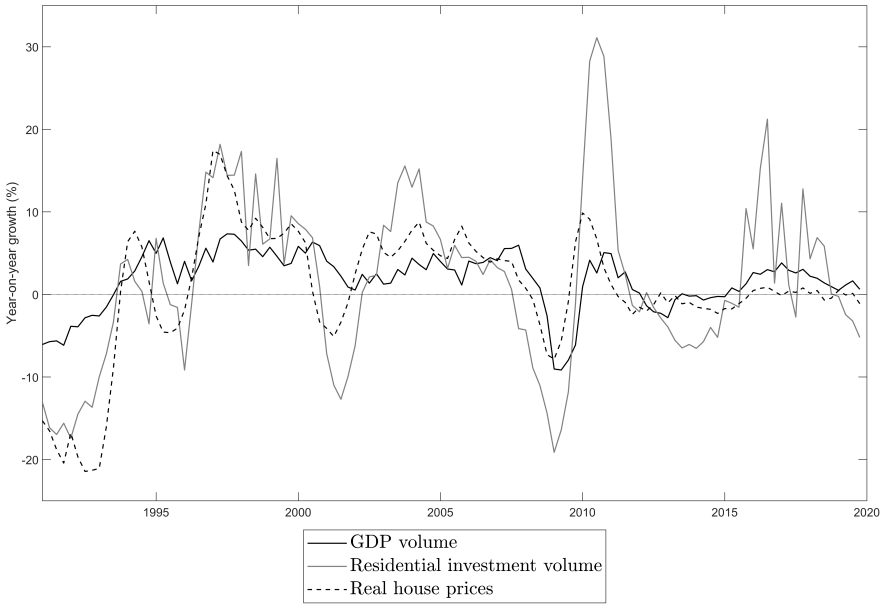

Figure 1: Real house prices and residential investment over the business cycle.
Notes: Sources: Statistics Finland and authors' calculations. Data sample 1991Q1–2019Q4.

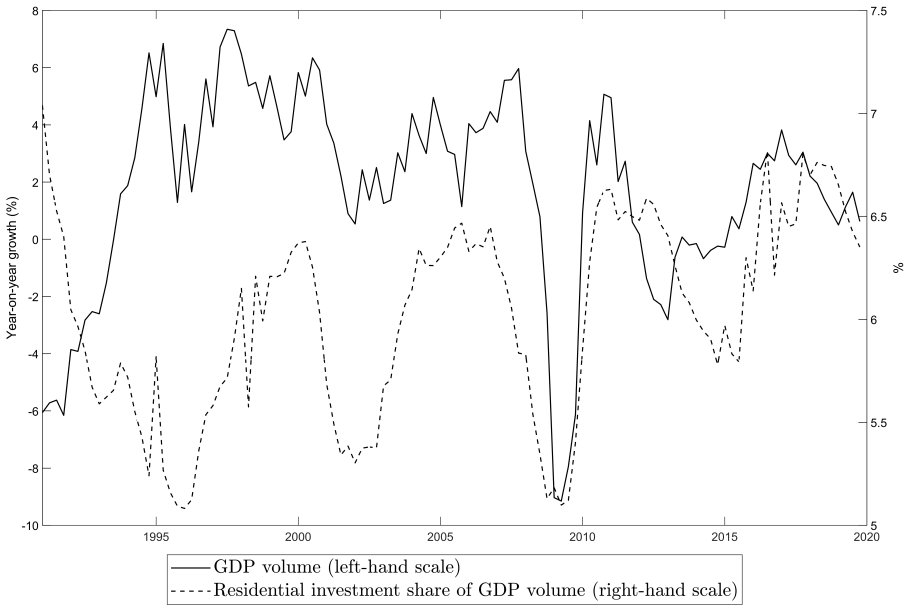

Figure 2: Residential investment as a share of GDP volume over the business cycle.
Notes: Sources: Statistics Finland and authors' calculations. Data sample 1991Q1–2019Q4.

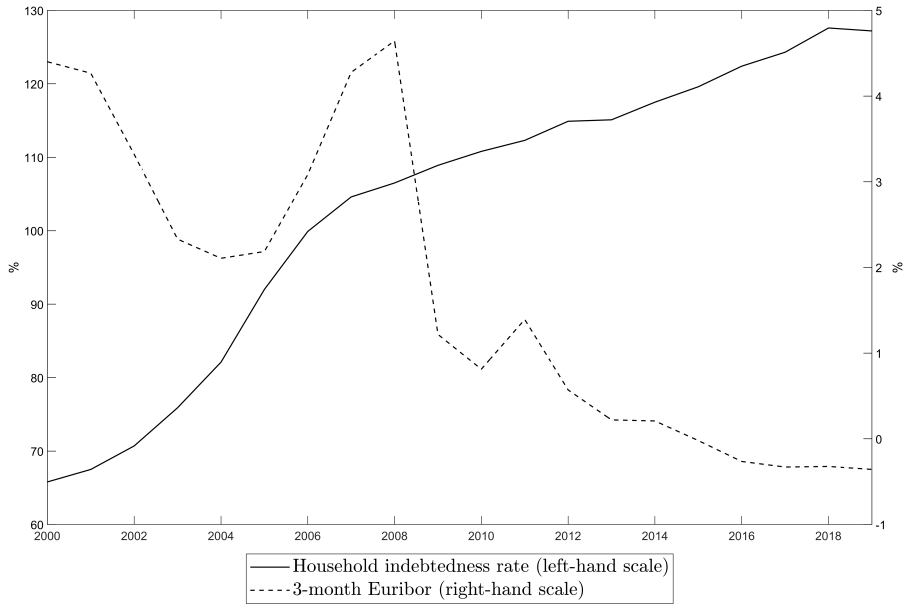

Figure 3: Household indebtedness and the 3-month Euribor rate.

Notes: The household indebtedness rate is calculated as the ratio of the total stock of gross debt of the household sector at the end of each year to the yearly disposable income of the household sector. The 3-month Euribor rate is the yearly average of the daily rates. Sources: Statistics Finland, Bank of Finland, and authors' calculations. Data sample 2000–2019.

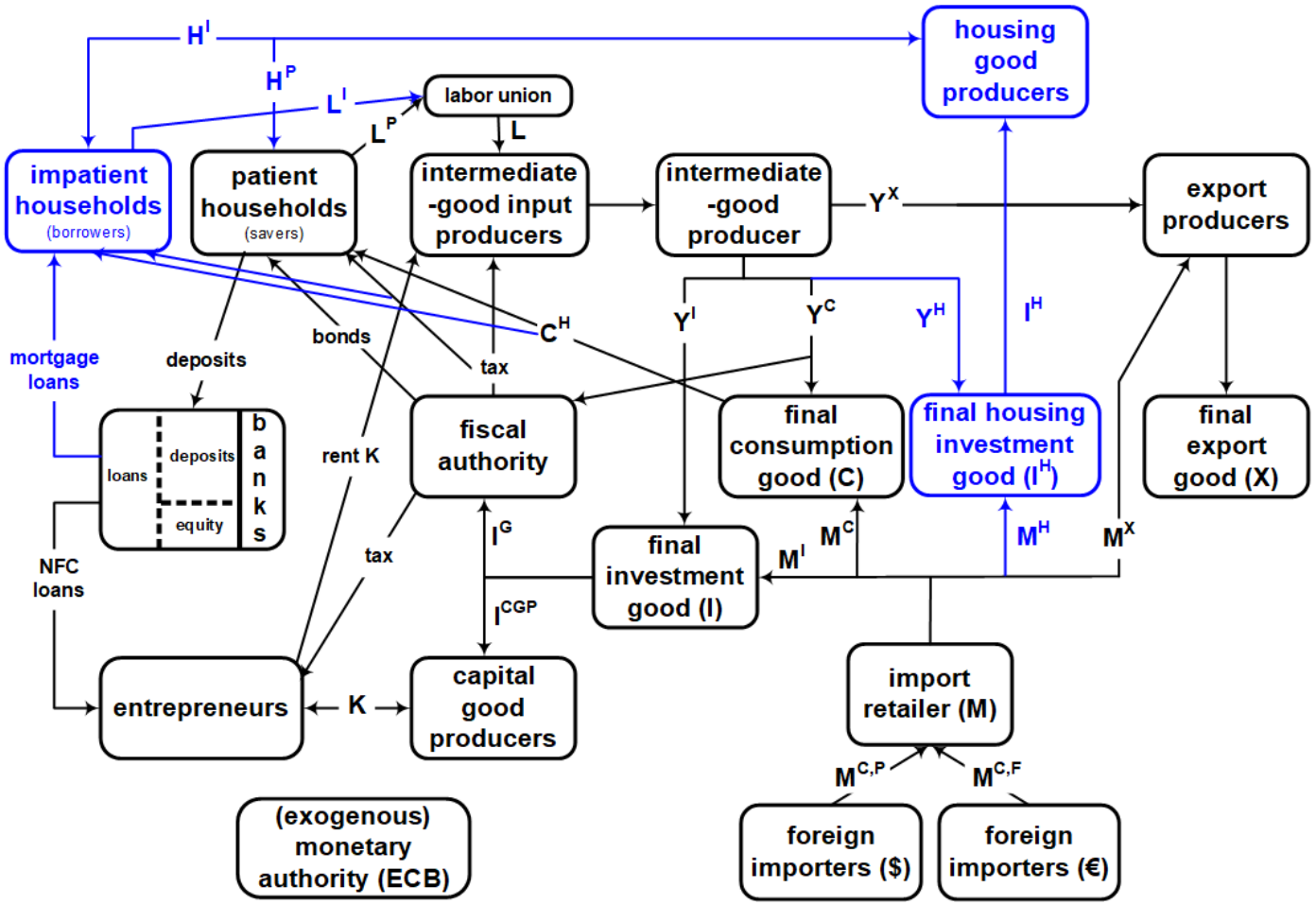

Figure 4: The structure of *Aino 3.0*. *Notes:* The elements in blue are new compared to the previous model vintage, *Aino 2.0*.

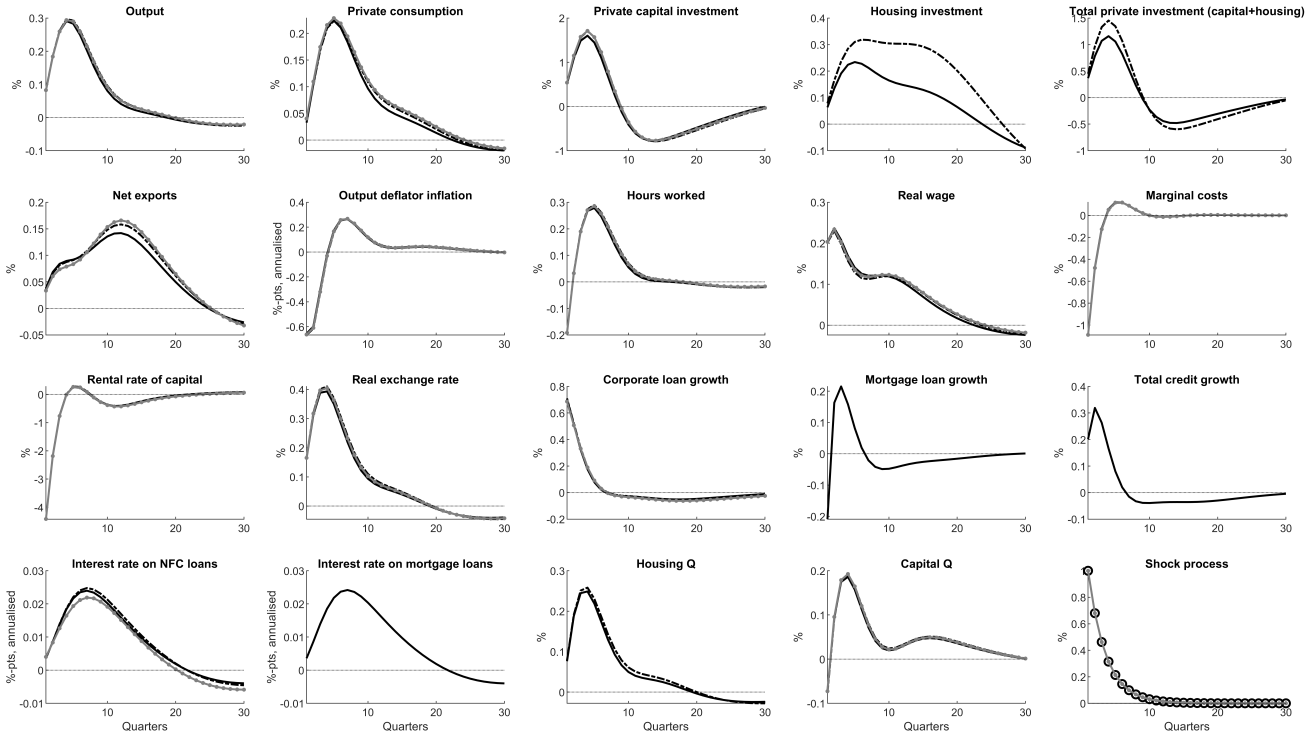

Figure 5: Impulse responses to a 1% temporary capital productivity shock

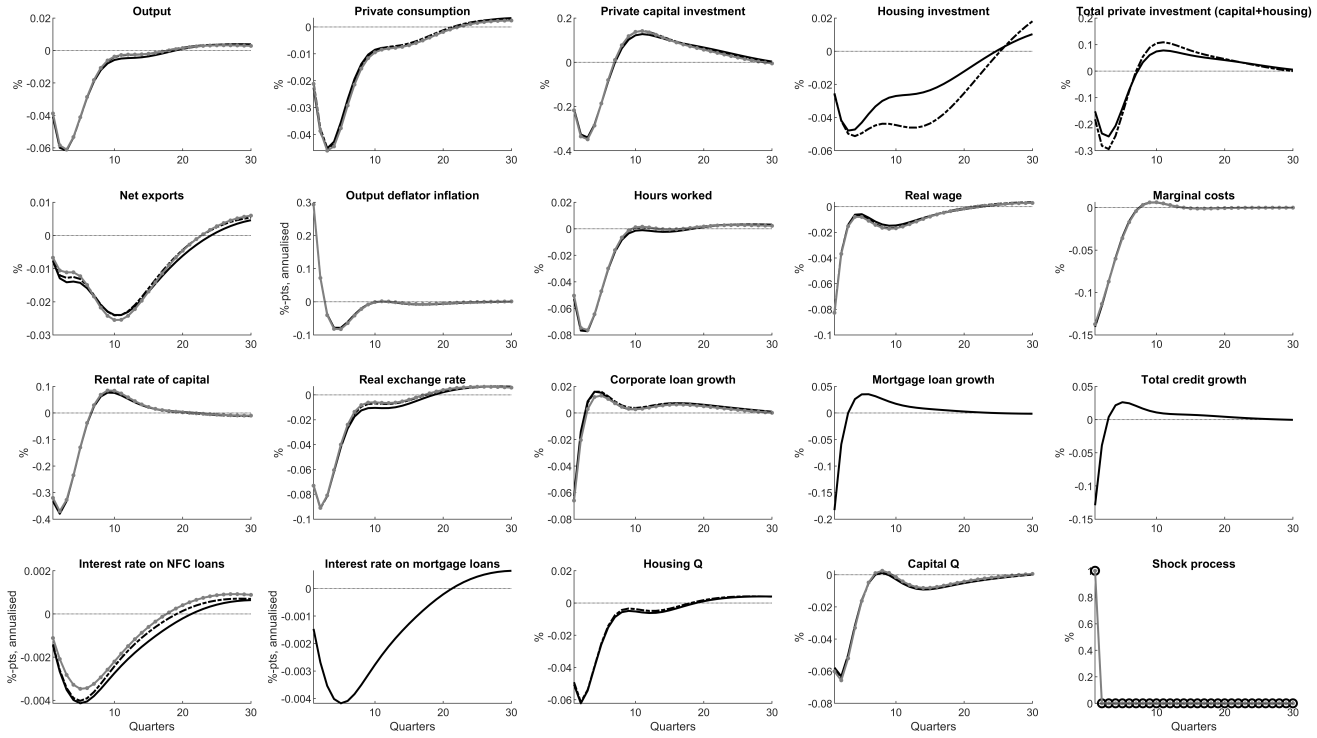

Figure 6: Impulse responses to a 1% positive price markup shock

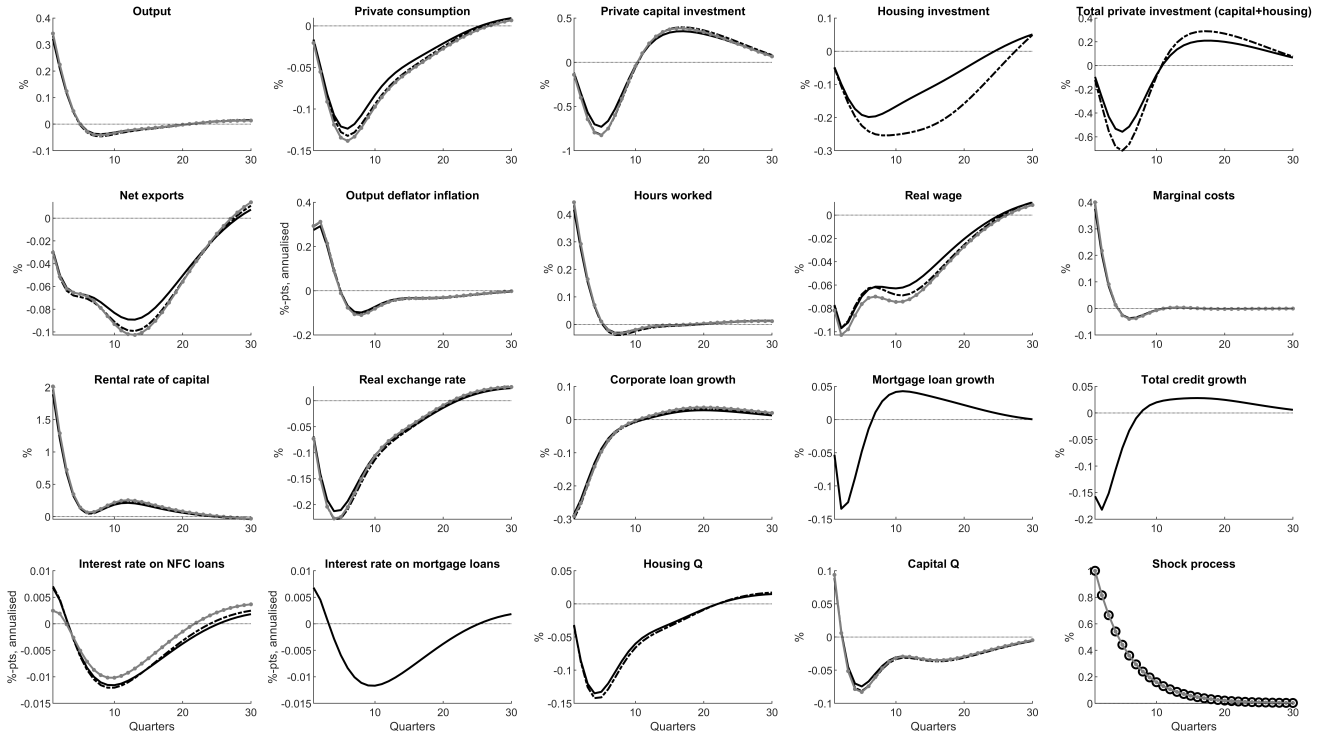

Figure 7: Impulse responses to a 1% positive government consumption shock

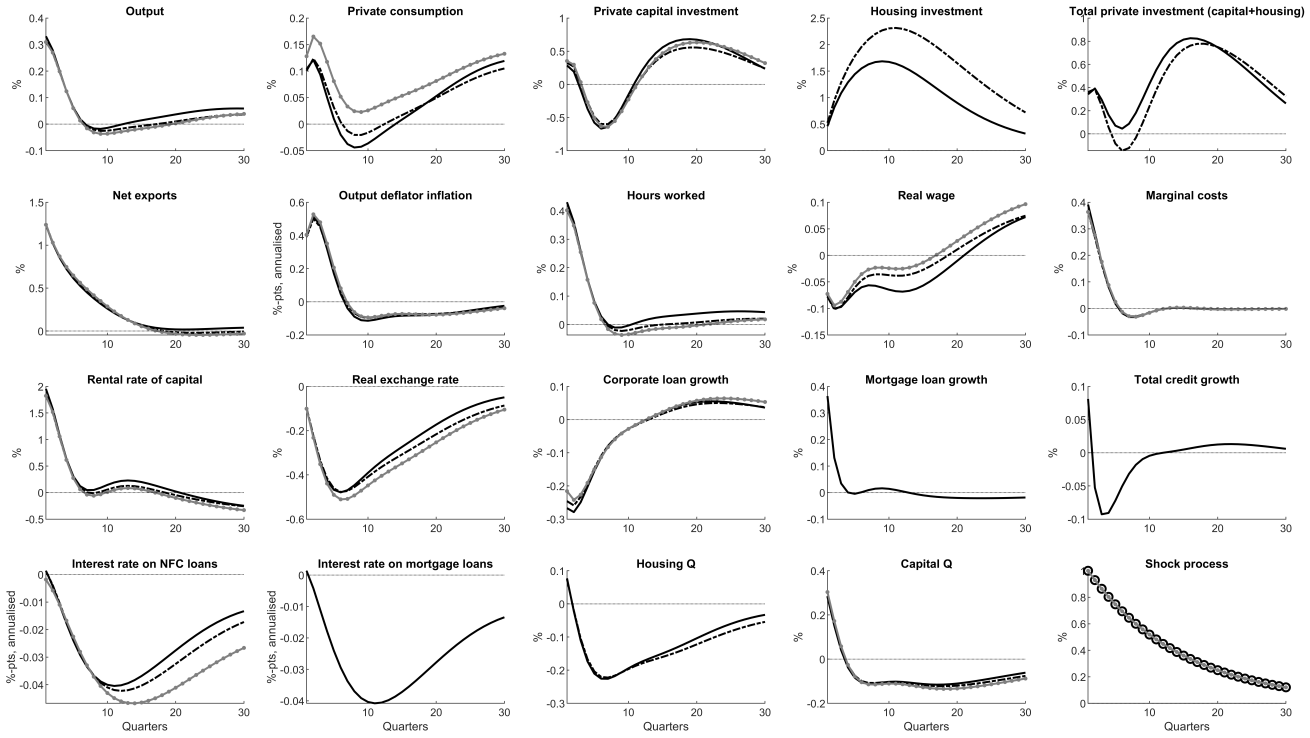

Figure 8: Impulse responses to a 1% positive external demand shock

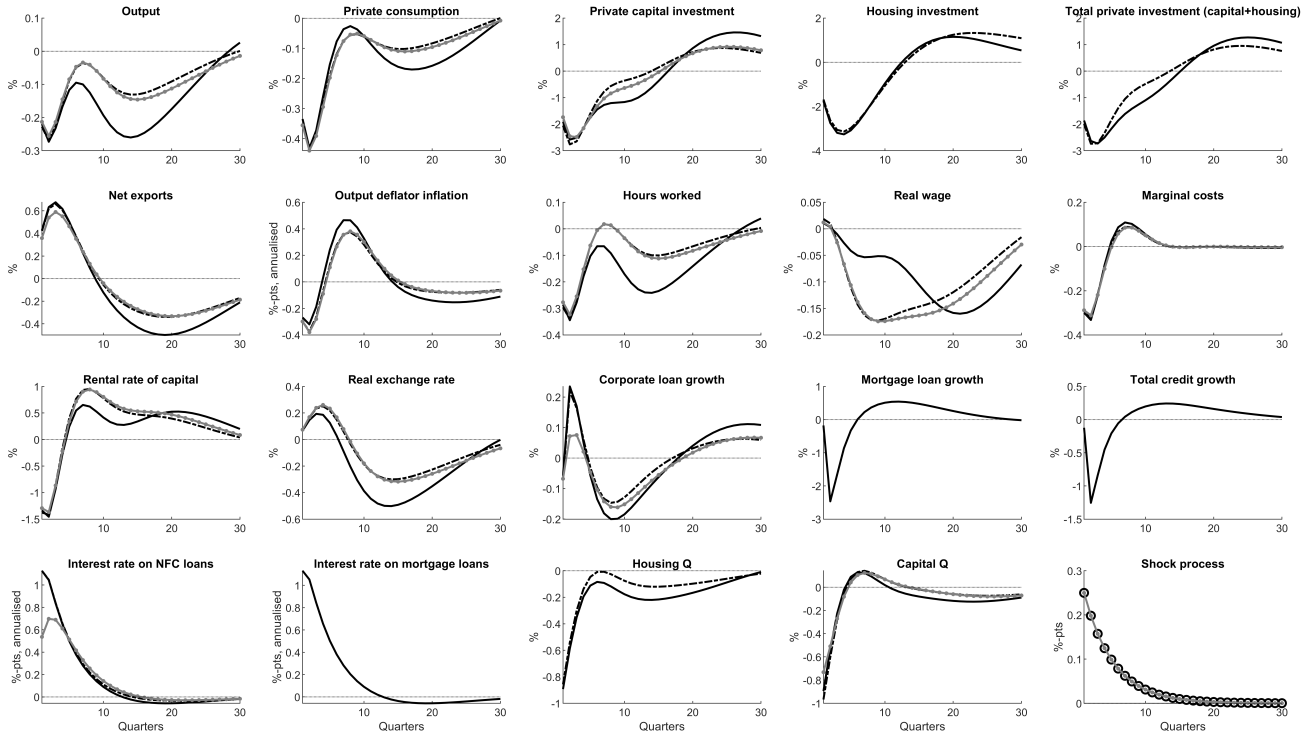

Figure 9: Impulse responses to a 25 basis point positive domestic interest rate premium shock

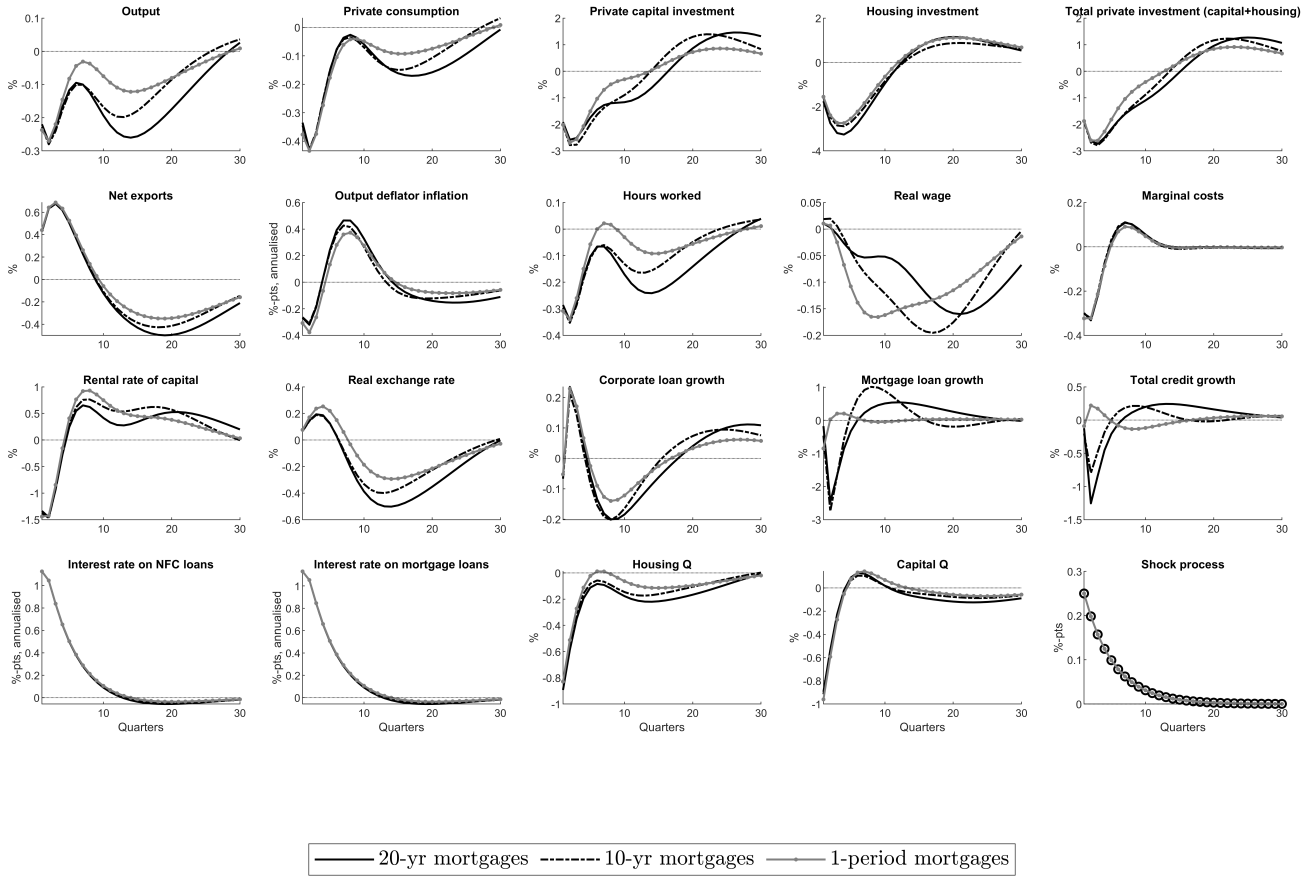

Figure 10: Impulse responses to a 25 basis point positive domestic interest rate premium shock, with alternative mortgage loan maturities

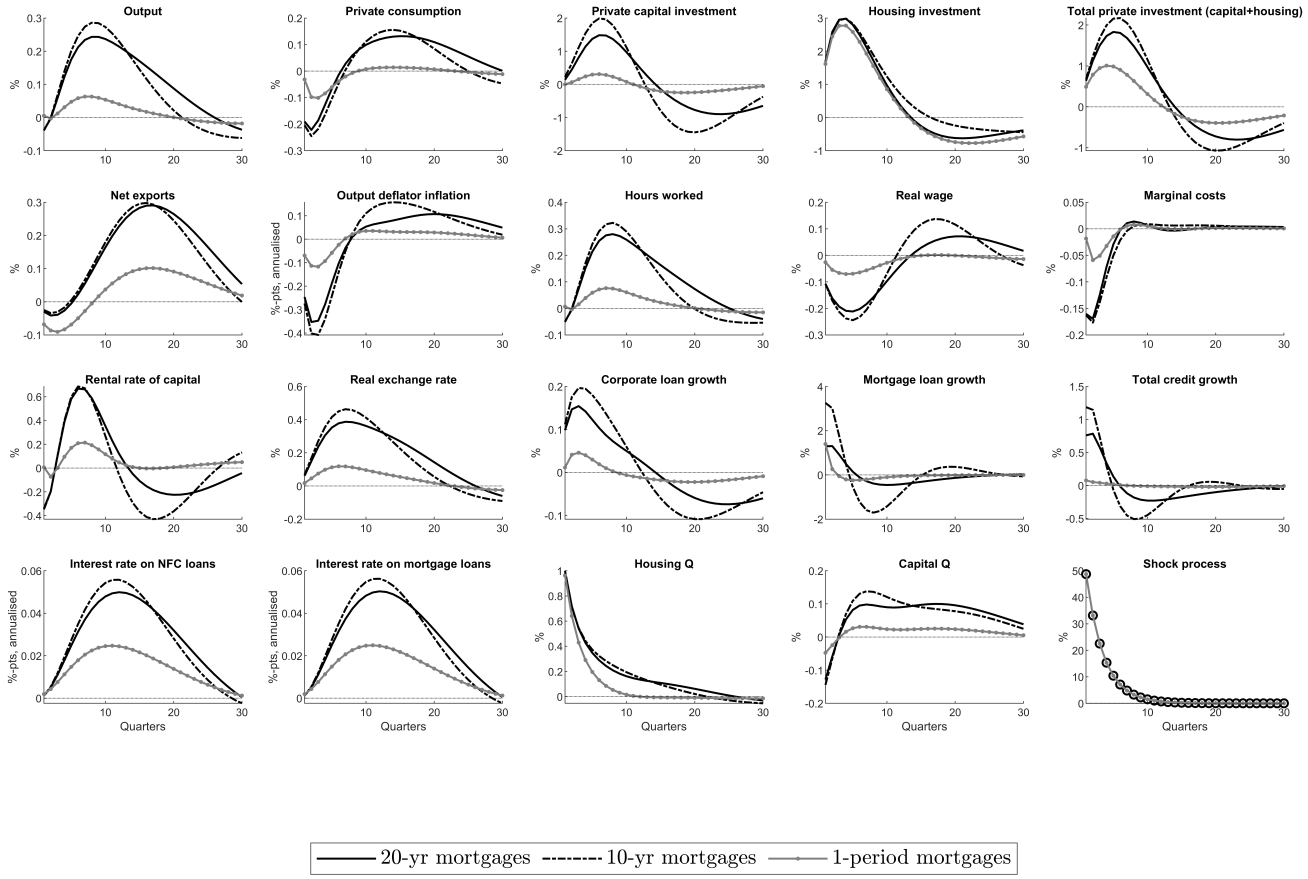

Figure 11: Impulse responses to a positive housing preference shock, with alternative mortgage loan maturities

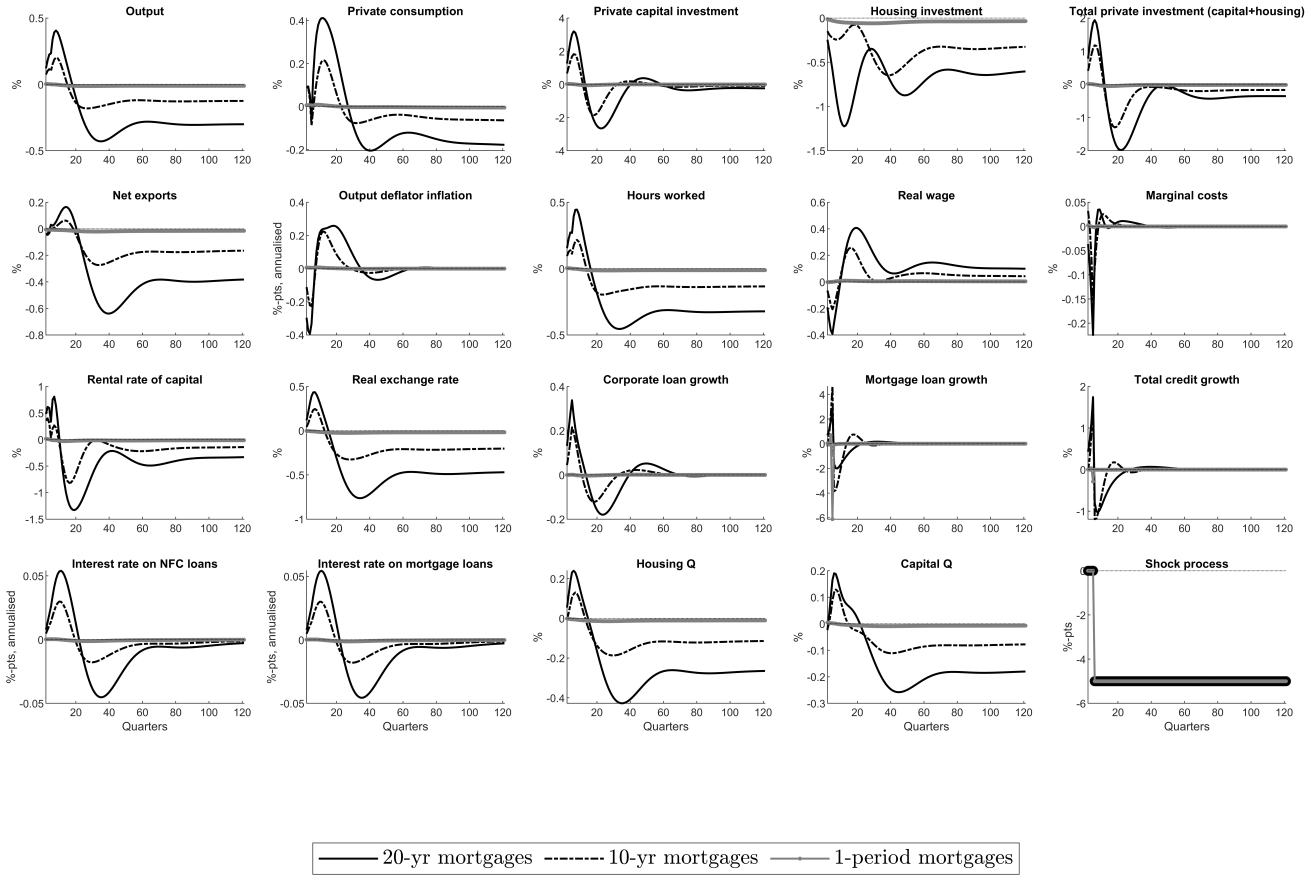

Figure 12: Impulse responses to a permanent anticipated 5 percentage point tightening in the loan-to-value (LTV) ratio, with alternative mortgage loan maturities

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