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# Testing Nonlinearities with Finnish Historical Time Series***

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# Abstract

This paper contains a set of tests for nonlinearities in economic time series. The tests correspond both to standard diagnostic tests and some new developments in testing nonlinearities. The latter test procedures make use of models in chaos theory, so-called long memory models and some asymmetric adjustment models. Empirical tests are carried our with Finnish monthly data for ten macroeconomic time series covering the period 1920–1993. Test results support unambiguous the notion that there are nonlinearities in the data. Nonlinearities are detected not only in a univariate setting but also in some preliminary investigations dealing with a multivariate case. Certain differences seem to exist between nominal and real variables in nonlinear behaviour.

# Tiivistelmä

Tässä tutkimuksessa testataan taloudellisiin aikasarjoihin liittyviä epälineaarisuuksia. Testit koostuvat sekä tavanomaista diagnostisista testeistä että eräistä uusista epälineaarisuuksien olemassaoloa selvittävistä testimenetelmistä. Jälkimmäiset testit liittyvät kaaosteorian sovellutuksiin, ns. pitkän muistin malleihin ja epäsymmetrisen sopeutumisen malleihin. Empiiriset analyysit tehdään kymmenellä Suomea koskevalla kuukausisarjalla, jotka kattavat ajanjakson 1920–1993. Testit tulevat kiistatta sitä oletusta, että aikasarjoissa on epälineaarisuuksia. Näitä ominaisuuksia ilmenee sekä yksittäisten muuttujien suhteen mutta myös tutkittaessa muuttujien välisiä riippuvuuksia. Nimellisten ja reaalisten aikasarjojen välillä näyttää olevan jonkin verran eroja epälineaarisuuksien määrässä ja luonteessa.

# Contents

# 1 Introduction

This paper examines several long Finnish time series. The purpose of the examination is to find out whether there are any signs of nonlinearities in these series. Thus, we carry out a set of tests analogously to Lee, White and Granger (1993). At this stage, most of these tests are applied to univariate models although a multivariate application would obviously be more interesting. When scrutinizing the series we pay special attention to the distinction between nominal and real series. This can be motivated by the fact that nonlinearities are presumably quite different with nominal and real variables. (For an extensive survey to the litterature, see Mullineux and Peng (1993).) Thus, it is of some interest of compare a typical real series, say industrial production, and a nominal series, say stock prices, in this respect.

Most monetary series – like relative prices, changes in price level and money aggregates – show some form of nonlinear behaviour. Prices are often more volatile than the real series, since they have a role of clearing device in the market. Monetary phenomenon are based upon valuations that could be adjusted without any relevant cost. In the market clearing situation it is often – but not necessarily always – easier to change the price than the quantity. Although prices could easily move into both directions, crises in the market produce large negative changes. Therefore it may be no surprice that real exchange rate, stock prices or inflation seem to adjust asymmetrically to shocks.

This affects the volatility of these series. Another major observation about the origin of "price shocks" relates to their unstable variance in time. It has been verified that in many cases price changes – e.g. in the stock market – cluster significantly. Forecasting price changes is therefore a harder task for economic agents than forecasting smoother real variables.

Nowadays, a general response to situations of changing volatility (heteroskedasticity) is to use an ARCH model specification. It may well be, however, that the ARCH model is not the proper framework. It may well be that prices, for instance, have the so-called long memory property, thus containing permanent components. In particular, the long memory property shows up in high and persistent serial correlation over long lags between absolute values of the (linearly filtered) series. Obviously, this kind of long memory is at variance with a linear structure and therefore it may be useful to consider it also here.

However, in many cases also real economic variables vary in a nonlinear way. Obvious evidence of nonlinear adjustment could be seen e.g. from the apparent and persistent tendency to cycles in most important production variables (see, e.g., Pfann and Palm (1993) for details). Whether these nonlinearities in real series araise from the generating process of a series itself or random shocks is largely an empirical question. So far no agreement has emerged on the subject whether real or monetary phenomenon are responsible about business cycles. We hope that our estimates about the nonlinearity of these series could shed some light on this issue as well.

Although the analysis mainly deals with univariate models, some preliminary work is done to identify nonlinear relationships between variables. In this context, we do not follow any specific hypothesis concerning the relationships between variables. By contrast, we simply make use of a cross-correlation analysis with respect different moments of our variables. Thus, the analyses represent some sort of first step towards a generalized Granger tests for nonlinear relationships. This analysis gives us a general idea of the magnitude and nature of these relationships. An obvious next step is to go back to theory and think about how the findings coincide with different theoretical approaches.

The structure of the paper is very straightforward. First, we have a look at the data in section 2, then we briefly present the test statistics in section 3 and in section 4 we go through the test results for univariate models. In section 4, we consider the long memory property in the context of our (filtered) series, in section 5 we scrutinize the results from a cross-correlation analyses between different moments of these series and, finally, in section 6 we present some concluding remarks. Needless to say, the paper is very preliminary and one should consider the results with some caution, at least.

## 2 The data

The data are monthly Finnish data covering the period 1920M1–1993M6. (In some cases, however, the period was somewhat shorter, i.e. 1922M1–1993M3.) Thus, there are typically 882 observations in each series. The following ten series are analyzed in this connection.

Industrial production (ip)
Bankruptcies (bank)
Terms of trade (tt)
The real exchange rate index (fx)
Yield on long-terms government bonds (r)
The consumer price index (cpi)
The wholesale price index (wpi)
Banks' total credit supply (credit)
Narrow money (M1)
The UNITAS (Helsinki) stock exchange index (sx)

The first four series are real and the subsequent six nominal. The data are presented in Figure 1. For presentational convenience, most of the series have been presented in an transformed form. Thus, they are presented in logs and in some cases the series have also been deflated by the CPI. To get some idea of the timing of changes in these variables the recession periods are marked by shaded areas.

Otherwise, the details of the data are presented in Virén (1992). We only point out that ip, bank, credit and M1 series are seasonally adjusted. This is simply because of data reasons – only seasonally adjusted data were available for the prewar period 1920–1938. As for the World War II (1939–1945), the data are treated in the same way as for the peace years.



## 3 The test statistics

Testing nonlinearities is preferred to be started by estimating linear model and analysing the respective residuals. Although economic relationships are most likely to be nonlinear, there is also danger of unnecessary complication, if the difference to a linear model is small.

The need for nonlinear model depends also on the purpose of the model. For short-run forecasting linear models may do the thing, but for long-run forecasts or explanation of apparent nonlinear features a more proper modelling is needed. Since testing linearity is widely covered in Granger and Teräsvirta (1993), we give here only few basic standpoints. The linearity tests could be divided into two groups, depending on whether a specific nonlinear alternative exists or not. Since our data does not refer to any specific nonlinear formulation, we concentrate on testing against the general nonlinear alternative.

As it was mentioned above, here we analyze only univariate models. A some sort of basic specification is a linear AR(4) which turned out to a reasonably good approximation for all time series. In specifying the order of the autoregressive models, we used model selection criterions (SC, HQ, AIC). In order to study the dynamic dependencies between variables, we though that in the first place it would be best to filter the original series with the linear autoregressive model of the same order. Thus, the residuals are not severely autocorrelated. A few exceptions do exist, however, for higher order autocorrelation (for the lag 12, for instance). Anyway, we prefer the parsimonious AR(4) model to more sophisticated specifications.

Dealing with nonlinearities is often easier after the linear dependencies in a time series have already taken care of. Therefore nonlinear adjustment can be found from a series property filtered with autoregressive (linear) model. However, empirical problems do emerge at this point. It often happens, especially in multivatiate analysis, that filtering is almost too effective, since all the significant relationships between variables are removed. Therefore too long autoregressive lag models that also affect the asymmetricity in the series should be avoided.

### Standard diagnostic tests

Given the autoregressive model, we compute the following sets of tests: First some basic statistics on residuals of this linear AR(4) model (see Table 1). These statistics include the coefficients of skewness and kurtosis in addition to the median. Quite obviously, we intend to discover possible asymmetries with these data. The second set of tests consist of traditional specification tests for functional misspecification/nonlinearity. The tests (reported in Table 2) consists of Engle' s (1982) ARCH test in terms of lagged squared residuals, Ramsey's (1969) RESET test in terms of higher-order powers of the forecast value of $x_t$, White's (1980) heteroskedasticity/functional form misspecification test in terms of all squares and cross products of the original regressors, The Jarque and Bera

(1980) test for normality of residuals and, finally, Tsay's (1986) nonlinearity test in terms of squared and cross-products of lagged values $x_t$.[1]

**BDS-test for chaotic process**

In addition to these "traditional" test statistics we also computed the BDS (Brock, Dechert and Scheinkman) test statistic (see Table 3) and Ramsey's (1990) irreversibility $G_{1,2}$ test. The BDS test comes from an analysis of chaos model and it is intended to a test for detecting general stochastic nonlinearity (see, e.g., Brock, Hsieh and LeBaron (1991), Frank and Stengos 1988 and Medio (1992) for details). The key concept here is the correlation dimension, which could be applied in finding the topological properties of series. For purely random variable, the correlation dimension increases monotonically with the dimension of the space and the correlation dimension remains small even when the topological dimension of the space (embedding dimension) increases (Brock, Hsieh and LeBaron (1991)).

BDS tests is designed to evaluate hidden patterns of systematic forecastable nonstationary in time series. The test was originally constructed to have high power against deterministic chaos, but is was find out that it can be used to test other forms of nonlinearities as well (Brock, Scheinkman and LeBaron (1991)).

BDS test could be applied also as a test for adequacy of a specified forecasting model. This could be accomplished by calculating the BDS test for the standardized forecast errors. Then BDS test is used as a specification test. If no forecastable structure exists among forecast errors, the BDS test should not alarm. BDS test has been found useful as a general test for detecting forecastable volatility.

For a single series $x_t$ for which $x_{t,m}$ is the set of m adjacent values of this time series $x_{t+j}$, j=0, ..., m−1 the m-correlation integral $C_m(\varepsilon)$ is defined as

$$C_m(\varepsilon) = \lim_{T\to\infty} T^{-2}[\text{pairs } (i,j) \text{ for which } |x_i - x_j| < \varepsilon, ..., |x_{i+m-1} - x_{j+m-1}| < \varepsilon]$$

The idea is that for chaotic series, the subsequent values of $x_i$ and $x_j$ will be very close. If the time series is a stochastic sequence, this does not happen. Now defining the correlation dimension d(m) as

$$d(m) = \lim_{\varepsilon\to\infty} \frac{\partial \log C_m(\varepsilon)}{\partial \log \varepsilon}$$

it will be seen, that for truly chaotic process $C_m(\varepsilon) \approx \varepsilon^d$, if $\varepsilon$ is small. This means that correlation dimension is independent of m if the process is chaotic. Otherwise, if the process is truly stochastic the correlation dimension will increase linearly with m.

[1] As for the properties of these test statistics see e.g. Petruccelli (1990) and Lee, White and Granger (1993).



The purpose of the correlation measure is to describe the complexity of the true series and measure the nonlinear dimension (degrees of freedom) of the process. Tests of chaos concentrate on low-dimensional deterministic chaos prosesses, since there is no efficient way to tell the difference between high-dimensional chaos and randomness. Here, we do not use/estimate the correlation dimension.

Instead, we use a simpler test procedure by calculating the BDS test statistic.

$$BDS(m,\varepsilon) = \sqrt{T}(\hat{C}_m(\varepsilon) - [\hat{C}_1(\varepsilon)]^m)/\sigma(m,\varepsilon),$$

Where $\sigma(m,\varepsilon)$ is an estimate of the standard deviation. BDS tests whether $C_m(\varepsilon)$ is significantly greater than $C_1(\varepsilon)^m$, and when this happens nonlinearity is present. Under the null hypothesis of $x_t$ following i.i.d., and for fixed m and $\varepsilon$, $C_{m,T}(\varepsilon) \to C(\varepsilon)^m$, as $T\to\infty$, and SDB(m,$\varepsilon$) has the standard normal distribution. The power of the test will depend critically on the choice of $\varepsilon$.

BDS test statistic is complicated since it depends on the embedding dimension (m) and the chosen distance ($\varepsilon$) related to standard deviation of the data. The selection of m is important in small samples especially when m is large, since increasing m means that the number of nonoverlapping sequences will become smaller. And when sample is less than 500 the asymptotic distribution may be different than the sampling distribution of the BDS statistic. The selection of $\varepsilon$ is even more crucial and a failure to detect non-normality in calculating BDS with small $\varepsilon$ is a consequence of too few observations. Brock, Sheinkman and LeBaron (1991, p. 52) suggests that for 500 or more observations, the embedding dimension m should be smaller or equal to 5, whereas $\varepsilon$ should be 0.5–2 times the standard deviation of the data. In the empirical application, some alternative values of the dimension parameter m and the distance parameter $\varepsilon$ are used.

The problem with BDS test is however, that it does not have a simple interpretation. Nonlinearity based on BDS test could be a result from chaos or nonlinear stochastic process. However, BDS test was originally designed to test whether data generating process of a series is deterministic (chaotic) or not (Granger & Teräsvirta (1993), p. 63). Since the BDS test is based on the null hypothesis that the observations (here AR(4) residuals) are i.i.d., a rejection merely reveals that this is not the case. The specific form of nonlinearity is therefore an open question.

As for the practical implementation of the test, it is here done by using the residuals of the AR(4) model as inputs. The use of the autoregressive filter is based on the invariance property of chaotic equations shown by Brock (1986). Brock showed that if one carried out a linear transformation of chaotic data, then both the original and the transformed data should have the same correlation dimension and the same Lyapunov exponents. Some alternative values for the dimension parameter m and distance parameter $\varepsilon$ are applied.

### The Ramsey irreversibility test

The irreversibility test, which has been derived by Ramsey and Rothman (1988), deals with the concept of time reversibility.[2] Time irreversibility is concept which useful in analyzing possible asymmetries (nonlinearities) in economic time series, for instance, in output series. According to conventional Mitchell–Keynes business cycle hypothesis cyclical upturns are longer, but less steep, than downturns (see also the "plucking model" of Friedman (1993)) If one traces out the behaviour of cycles in reverse time it can be seen that the symmetric cycle is time reversible and the asymmetric cycle is time irreversible.

Ramsey and Rothman (1988) propose that the presence of time irreversibility checked by estimating a symmetric bicovariance function in terms of $x_t$. The test statistic which is obtained from this bicovariance function is of the following type:

$$G^k_{i,j} = T^{-1}\sum_{t=1}^{T}\left[(x_{t-1})^i(x_{t-k})^j - (x_t)^j(x_{t-k})^i\right], \qquad k=1,2,\ldots,K.$$

If the time series is time reversible, $G^k_{i,j}=0$ for all k. As for the choice of exponents, i and j, we assume here that i = 2 and j = 1 (here we just follow Ramsey (1990)). In addition, we experiment with the pair i = 3 and j = 1. The maximum lag length K is se at 120. To ensure stationarity, we use also here the AR(4) residuals instead of the original time series. The significance of the G statistic is tested by computing the confidence limits according to the following formula for the variance of $G^k_{1,2}$:

$$\mathrm{Var}\left[G^k_{1,2}\right] = \left(\frac{2}{(T-k)}\right)\left[\mu_4\mu_2 - \mu_2^3\right],$$

where $\mu_2 = E\left[x_t^2\right]$ and $\mu_4 = E\left[x_t^4\right]$. Assuming that the data are independent and identically distributed $N(0,\sigma^2)$, the right hand side of the above formula can be simplified to be $\left(\frac{4}{(T-1)}\right)\left[\mu_2^3\right]$. This is clearly a crude approximation because the normality assumption does not hold, nor are the variables uncorrelated. However, it is not all clear how the variance terms should be computed when $x_t$ is not IID but follows e.g. some general ARMA(p,q) model (see Ramsey and Rothman (1988) for various experiments). Here the test statistics and the respective confidence limits are displayed in Figure 2.

[2] A stationary time series $\{x_t\}$ is time reversible if for any positive integer n, and for every $t_1, t_2, \ldots, t_n, \varepsilon$ z, where z is the set of integers, the vectors $(x_{t1}, x_{t2}, \ldots, x_{tn})$ and $(x_{-t1}, x_{-t2}, \ldots, x_{-tn})$ have the same joint probability distributions. A stationary time series which is not time reversible is said to be irreversible. Notice, that by definition, a non-stationary series is time irreversible. See e.g. Tong (1983) for further details.



### A nonlinear adjustment equation

Instead of just computing test statistics for nonlinearity, it would be tempting to estimate a general nonlinear time series model and compare its properties with a linear model. Unfortunately, such general nonlinear model does not exist nor is there any agreement of a reasonable approximation which could be used to capture the possible nonlinear elements of the data. Still, the situation is not completely hopeless. There some interesting candidates for a nonlinear specification. The first which deserves to be mentioned is the threshold model specification introduced by Tong (see e.g. Tong (1983)). Another specification which is clearly worth mentioning is the nonlinear employment (output) equation introduced by Pfann (1992). This (estimating) equation takes the following form:

$$x_t = a_0 + a_1 t + a_2 x_{t-1} + a_3 x_{t-2} + a_4(x_{t-1}x_{t-2}) + a_5(x_{t-1}^3 x_{t-2}) + a_6(x_{t-1} - x_{t-2})^3 + \mu_t,$$

where $\mu$ is the random term. According to Pfann (1992) and Pfann and Palm (1993), the parameter of the nonlinear terms can be unambiguously signed in the case employment equations. Thus, $a_4$ should be positive (if hiring costs are larger than firing costs, or in general, if the cycle spends more time rising to a peak than time falling to a trough). Moreover, parameter $a_5$ is expected to be negative if the asymmetry (skewness) of magnitude (i.e. the magnitude of troughs exceeds the magnitude of peaks) is negative and parameter $a_6$ also negative is the asymmetry (skewness) of duration (i.e., it takes longer for a series to rise from a trough to a peak than to fall from a peak to a trough).

Although this model may make more sense with (productive) input and output series we also apply it to all ten Finnish series partly to see whether the real and nominal series can be discriminated on the basis of this equation. The results are reported in Table 4. This table also includes a comparison of this model with a linear alternative.[3]

# 4 Empirical test results

The message of the empirical analyses is quite clear and systematic: the data do not give much support to linear models. Thus, all tests statistics in reported in Table 2 and 3 indicate that at least a linear AR(4) model is trouble.[4] According to Table 2, the residuals from the AR(4) model suffer from heteroskedasticity

[3] Here, we merely replicate the experiments by Pfann (1992). Thus, we take the same detrending procedure (see the second term on the right hand side) and the same lag structure. Obviously, extending the lag length beyond 2 would enormously complicate the model.

[4] In addition of the test statistics reported in Table 2 we also computed the Keenan (1985) and McLeod-Li (1983) test statistics. Both of these turned out be highly significant. Thus the marginal significance levels were in all cases well below 5 per cent. The test statistics were also computed for the post Second Word War period. Results were quite similar to those reported in Table 2. Thus the war itself cannot explain why the results are favourable to nonlinearities.

and non-normality. The ARCH(7) statistic significant for all variables (perhaps excluding the interest rate). Thus, even with real series like industrial output an autoregressive conditional heteroskedasticity effect can be discerned. This is something new. Nobody is surely surprised to find an ARCH effect in stock prices but here a similar result applies to other variables as well.

Nonnormality is clearly a severe problem. It is quite obvious that normality is violated because of outlier observations. Clearly, some observations can classified as outliers and it might well be that these observations contribute to the rejection of linearity. This can be seen from Figures 2 and 3 which contain the time series and frequency distributions for the AR(4) residuals. In accordance with Table 1, the main problem seems to be excess kurtosis, not so much excess skewness. Although the normality assumption is rejected, the graphs suggest that the distributional problems not, after all, be so severe as the Jarque–Bera normality test statistic suggests.

Unfortunately, there is no obvious remedy to nonnormality and outlier observations. One alternative is, of course, to use robust estimators and examine whether the results (e.g., the properties of residuals) change importantly due to the change in estimators. In fact, we did do this but it turned out that the results with the least absolute deviations estimator were qualitatively very similar to the OLS results. Another possibility is to reconsider the relevant sampling distributions of the nonlinearity tests statistics in the light of observed behaviour of OLS residuals. Here, we have not yet worked out this alternative.

After these considerations, some comments on the RESET and TSAY nonlinearity test statistics merit note. Both tests do suggest that the (linear) functional form is misspecified for most of the variables. The results are, however, very systematic. Thus, for instance, industrial production and bankruptcies, on the one hand, and narrow money and credit supply, on the other hand, behave in a different way in these tests. Moreover, the test results do not allow from drawing a line between real and nominal variables.

As far as the BDS test statistic is concerned, the results are much more systematic and alarming from the linearity point of view. The null hypothesis that the series is a random i.i.n. variate is rejected from all series with all standard significance levels. Obviously, this does not automatically imply nonlinearity, but surely the latter hypothesis must be taken more seriously.[5]

A similar result emerges with Ramsey's (1990) irreversibility tests statistics reported in Figure 4.1. Although, the confidence limits are only indicative some signs of nonlinearities can be discerned with all series. Somewhat surprisingly, stock prices do not seem to be the most striking example of this sort of nonlinearities. Thus, for instance, the test results for industrial production tell more about nonlinearities than the results for the stock index (see Figure 4.2). Also bankruptcies and banks' total credit supply seem to be more obvious candidates. Perhaps, this is something which is in accordance with the observed nature of indebtedness and the relationship between indebtedness, credit supply and bankruptcies (see, for instance, Stiglitz and Weiss (1981) and Bernanke (1983)).

[5] One may suspect that the results with the BDS test result from outlier observations. In turns out, however, that this is not the case. We eliminated all outlier observations (±1*SD or alternatively ±2*SD) from the AR(4) residuals, but the values of the BDS test statistic changed only marginally.



Can anything else be said about the nature of nonlinearities? Tables 1 and 4 suggest that this is the case. Table 1 indicates that the real series and the nominal series behave in a very different way. The nominal series do not show up any signs of negative skewness. Moreover, the nonlinear adjustment equations (reported in Table 4) behave very badly, for instance, in terms of stationarity.[6] It is particularly interesting to compare the behaviour and industrial production and stock prices. Industrial output is characterized by clear negative skewness (in magnitude) while there is no apparent skewness in stock prices. With industrial production, positive residuals are much smaller and obviously more numerous than negative residuals. Intuitively, this makes sense since capacity constraints limit increasing production while a decrease in orders or bankruptcies may lower production more rapidly. With stock prices, there is no difference between positive and negative residuals. Thus, adjustment of stock prices does not contain significant asymmetries. See Figure 5 for details; notice that positive and (absolute values of) negative AR(4) residuals are presented here in an ascending order.

Thus, if anything can be learned from this exercise, it is the fact that nonlinearities seem to exist with the long Finnish times but there seems to be clear differences between nominal and real variables. Thus, it is perhaps futile to analyze all sort of nonlinearities using a single model as a frame of reference.

# 5 Long-memory properties in historical time series

In time series, a long-term memory property is said to be present if absolute values of a stationary variable $r_t$ has significant autocorrelations for long lags i.e. $\rho(|r_{t-k}|, |r_t|) \neq 0$, when k is large. This property was first noted for speculative price series by Taylor (1986) and called thereafter also the Taylor effect (see Granger and Ding (1993)). In practice, this property implies that the simple random walk model does not hold for stock prices, even if the price changes are serially uncorrelated. Residuals from linear model with zero mean would account for the expectation of the series, but leave the higher moments unadjusted.

For instance if we consider stock price changes, it seems intuitively appealing to observe that they are uncorrelated, but this does not explain anything about the heteroskedasticity found in them. Statistically stock prices could be martingales with non-constant innovation variance (see e.g. Spanos

[6] With consumer and wholesale prices there seems to be positive skewness indicating that prices tend to increase faster than to decrease, which obviously makes sense. The behaviour of long-term interest rate may only reflect this same fact. The real exchange rate, in turn, is characterized by gradual deterioration of competitiveness and once-for-all devaluations of the currency. Money and credit seem to behave in the same way as stock prices in terms of skewness although the estimations results are somewhat different. With bankruptcies, the results represent some sort of puzzle. Industrial output and bankruptcies do not seem to be just mirror images – quite the contrary. Thus, there are some (although not very significant) signs of negative skewness indicating that peaks in bankruptcies are smaller than the corresponding troughs. This clearly indicates that bankruptcies are perhaps more related to financial and institutional variables than just to demand and output.

(1986)). However, from the economic point of view the problem is to find out whether residual variance from linear model follow conditional heteroscedasticity (ARCH), generalized version of it (GARCH), asymmetric power ARCH (A-PARCH as defined in Ding, Granger and Engle (1993)) or some other form of heteroskedasticity appropriate for the particular time series. However, univariate models could be helpful in identification and prediction of the type of heteroskedasticity, but likely insufficient for understanding these prosesses.[7]

Heteroskedasticity in residuals shows already that stronger forms of rational expectations rationality, which imply efficient use of all information, does not hold for higher moments of the process. In fact expectation error are not white noise, but rather innovation processes with non-constant variance. The long-memory phenomenon puts emphasis also to the long-term cyclical swings often accounted in economic time series. These cyclical swings could relate to business cycles or even Kutznets and Kontrajev cycles or tendency to generate serious financial crises as those withnessed in 1930's and 1980's. However, as Granger and Ding (1993) emphasize, that caution in interpretation should be maintained, since it is not the series themselves but their absolute values, that have the long-memory property.

If the efficient market hypothesis would hold strictly, the random walk property implies that $r_t$ is an i.i.d process. In addition any transformation of $r_t$, like $|r_t|$ or $r_t^2$ should also be i.i.d process (Ding, Granger, Engle (1993), s. 87). The sample autocorrelations of i.i.d process will have finite variance $1/\sqrt{(T)}$ and larger correlations for $|r_t|$ will indicate long-memory property. Ding, Granger and Engle (1993) show that, if $|r_t|^d$ is taken for yardstick in measuring the strengthness of autocorrelation for long lags, the long-memory property is strongest around d = 1.

In the same way as Ding, Granger and Engle (1993), we found out that all variables in our data set showed clear evidence of long-memory, thus the sample autocorrelations for absolute values of residuals were greater than the autocorrelations of squared residuals. This resemblance could indicate that economic time series have characteristics of models, not fully described and understood so far.

Series, which had $|r_t|$ well above $r_t^2$ were industrial production, bankruptcies, bank loans and both price price indexes. A little bit different were series like terms of trade and real exchange rate, money supply and stock prices, which mostly shared the same characteristics. This could due to rare, but large discrete changes in these series e.g. like the effects of devaluations. The results from these long-memory tests performed to AR(4)-residuals of our time series are presented in table 5 below. Figures of sample autocorrelation functions for the absolute values of the AR(4) residuals are shown in Figures 6.

Among other things these results indicate that linear filtering with AR(4) model is not sufficient to remove dependence on faraway past in these series, even though model selection criteria would suggest in most times 4th order

[7] Granger and Teräsvirta (1993, p. 51–53) note that a series may have short-memory in mean, and long-memory in variance, but not so likely the opposite i.e. long-memory in mean with short-memory in variance. Short-memory in mean is often found in stationary series, whereas long-memory is present in integrated "level" series.



autoregressive polynomial should be long enough. Despite the fact that these series have dominant long-run features like unit roots and trends, parsimonous linear models seem unable to account for this task. Observations refer therefore to conclusion that trends in economic time series are most likely stochastic rather than deterministic. Nonlinearities are hereby faced again.

The main message is however, that long-memory property is very persistently present in all of the real and monetary series. In addition there seems to be no difference between real and monetary variables about how fast autocorrelations would die out for long lags.

# 6 Testing dependencies between residual moments

The purpose of applying first an autoregressive model to the series is to remove the potential trend component from series. Removing deterministic or stochastic long term trend could be done by other means as well e.g. differencing or modelling by structural time series models and thereafter eliminating the trend component. We proceed by calculating dependency measures of different transformations of these AR(4) residuals.[8] Different moments of residual series and absolute values of residuals are considered as transformations. Therefore we calculate dependence tests from cross-autocorrelations between these univariate residuals as a first step in searching for dynamic relationships.

As could be seen this procedure looks like an extension of the Granger causality test. However, we start by calculating Portmanteau test statistics without conditioning on past observations of the transformed residuals of the series itself. Portmanteau tests give us potential evidence about the direction and strengthness of the dynamic dependencies between variables. If relationship is one-sided it simplifies greatly the identification of the sources of shocks in these series.

To test whether residuals of the autoregressive model satisfies properties of independent white noise series could be accomplished with calculating Portmanteau (Q) statistic. This test is designed to pick up departures from randomness among the k first auto- or crosscorrelations. Test has the following form

$$Q = T(T+2)\sum_{k=1}^{M}(T-k)^{-1}r_k^2,$$

where $r_k^2$ are the squared correlation of the residuals.

This modification of the basic Box-Pierce statistic was first presented in Ljung and Box (1978). The test statistic is asymptotically $\chi^2(M)$ distributed when the original residuals are independent. There is no clear solution in choosing M, but in our case a too small values could result in a failure to detect dependencies between important higher order lags. As could be guessed,

[8] We also computed the same measures with respect to the ARCH-model residuals. The results turned out to be so close to the results with squared OLS residuals that we do not report them.

increasing M will on the other hand lead to lower power of the test (Harvey (1981), p. 211).

The Portmanteau statistic could be applied also to the higher moments or absolute values of stationary series as a general test against non-randomness. McLeod and Li (1983) have shown that for squared residuals have the same standard asymptotic variance (1/T) as the original series if the residuals are random.

In the following tests we assumed lag order to be 60 (5 years) to be large enough to pick up long term dependencies between different moments of residuals. In our application economic theory has rather little to say about the lags between shocks leading to variation in other variables.

Tables 6.1–6.3 present estimated Q statistics for the different moments of the residuals from autoregressive models. Tests are presented in significant levels and separated on basis of one sided dependence tests. The causal interpretation of these tests is based on the idea, that future cannot cause the past. These positions could be interpreted as follows; the first position shows the Q tests, with positive lags 1–24, where the second variable is lagged. Therefore the first position could be interpreted as the second variable causing the first variable. The second position shows the Q test with second variable lagged on negative lags 1–24 and therefore telling whether the first variable (column) causes the second (row) variable. The most evident thing, we can see from these tables is that there exist quite a lot very significant relationships between both real and monetary variables. In particular, we would stress the very significant test values for bankruptcies and banks' credit supply. Also stock prices deserve to be mentioned. All of these variables seem to be related to other variables so that causation goes to both directions. So, for instance, volatility shocks may have a rather complicated propagation mechanism in the economy. Moreover, the regularities seem to be rather robust in the sense, that significant dependencies exist in same positions of different moments of the residuals. It is also clear for some variables that there is tendency of the significant correlations to dilute when we move up to higher moments. But this is not always the case.[9]

In economics few phenomenon mostly regarding uncertainty consider relationship between expectations and variances. Since the estimation of variance includes also assessment about the expectation, it is not quite clear what interpretation should be made between causality found between higher moments, if no relation is not found between expectations.

Table 6.4 presents the Portmanteau tests calculated for the absolute values of the univariate AR(4) residuals. The main observation in these tests is analogous to those in long-memory tests, namely that almost all the group correlations are highly significant. The structure of correlations seems to be very similar to the structure of correlations between the second (and third) moments of the AR(4) residuals. Now, only the correlations are somewhat higher. In fact, most of the correlations are highly significant and the

[9] As noticed earlier, these tests could be seen as a preliminary analysis (necessary condition) in comparison with Granger causality tests, since in predictive Granger causality conditioning is done with respect to the past history of the dependent variable. Granger causality test is defined as excess predictive power of the explanatory variable in addition to the past of the variable itself.



uncorrelated pairs of variables can be counted very easily: terms of trade and interest rate represent variables which are only loosely related to other variables.

Calculating the contemporaneous correlations between variables does not have any dynamic causal interpretation as it indicates only instantaneous dependency (positive or negative) within a month. As could be seen from table 6.5 about one third of the off-diagonal correlations are significant at 5 per cent level. Some of the correlations are harder to interpret than others. Consumer prices correlate, in addition to wholesale prices, with monetary variables like credit, money aggregate, stock prices and the real exchange rate. Inflation is however not instantaneously correlated with the real variables.[10]

Altogether, the correlations between higher moments of the AR(4) residuals – in the same way as between the absolute values – are so strikingly high that further analysis in a multivariate nonlinear set-up is clearly required. The first step is simply to find out why volatility changes are so much related. In addition, one has to think about a possible explanation to the observed strong co-skewness between variables. Finally, one has also to take into account the fact that the long memory property seems to apply also to the co-movements of different series – both nominal and real. It seems at least that a (multivariate) ARCH model is not a sufficient or a proper specification to account for these features of the data.

# 7 Concluding remarks

The empirical analyses which are presented in this paper have given strong and unambiguous support to the existence of nonlinearities in Finnish historical time series. The univariate case is very clear but it seems that nonlinearities may be even stronger and more important in the multivariate set-up. Obviously this calls for further research in this area.

It seems well possible that nonlinearities may change some widely accepted assumptions or results. Thus, for instance, the neutrality of money may not be so good approximation as is looks like in the context of linear models. It may also be that the conventional symmetric adjustment mechanisms represent a very poor framework for dynamic specification. Finally, it may be that the importance of certain variables (and unimportance of the other variables) in the propagation mechanism of nominal and real shocks in the economy will change a lot if nonlinearities are taken into account. The Finnish data suggest that, for instance, bankruptcies is such a neglected variable.

[10] On the other hand it is interesting to note that wholesale prices do correlate with both real and monetary variables. Industrial production correlates only with wholesale prices and bankruptcies, but in both cases the sign of the correlation seems to be the opposite than expected. It is also hard to interpret why interest rate correlates positively with stock prices. According to present value formulae, the relation should be just opposite.

Table 1. **Descriptive statistics for the residuals of a linear AR(4) model**

| | skewness | kurtosis | median | med(–) | med(+) | stand.dev. |
|---|---|---|---|---|---|---|
| ip | –0.64 | 4.98 | .267 | –.008 | .587 | .056 |
| bank | –0.59 | 4.41 | .226 | –1.154 | 2.424 | .312 |
| tt | 0.69 | 25.59 | .039 | –.081 | .146 | 2.284 |
| fx | 2.76 | 34.07 | –.250 | –.325 | –.192 | 3.909 |
| r | 0.29 | 20.25 | –.157 | –.157 | –.157 | .256 |
| cpi | 2.70 | 24.86 | –.134 | –.178 | –.092 | .014 |
| wpi | 1.07 | 22.18 | –.129 | –.181 | –.069 | .015 |
| credit | 0.09 | 8.15 | .003 | –.046 | .046 | .010 |
| M1 | 0.88 | 17.01 | .034 | –.129 | .129 | .025 |
| sx | –0.19 | 5.26 | .039 | –.262 | .290 | .049 |

Skewness and kurtosis denote the coefficients of skewness and kurtosis, respectively. Median denotes the sample median, med(–) and med(+) denote the endpoints of the confidence interval for the median. In the case of log transformation, the values of the median, med(–) and med(+) have been multiplied by 100. ip denotes (log) industrial production, bank (log) bankruptcies, tt terms of trade, fx the real exchange rate index, r yield on long-terms government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money and sx the (log) UNITAS stock price index. The sample period is (with some exceptions) 1920M5–1993M6.

Table 2. **Diagnostic test statistics for a linear AR(4) model 1920M5–1993M3**

| | ARCH | RESET1 | RESET2 | Func. form | WHITE | J–B | TSAY |
|---|---|---|---|---|---|---|---|
| ip | 18.56 | 0.26 | 12.76 | 2.56 | 12.74 | 930 | 7.53 |
| bank | 16.99 | 5.51 | 19.30 | 10.59 | 14.51 | 734 | 33.41 |
| tt | 10.33 | 7.76 | 3.81 | 5.68 | 3.85 | 24234 | 30.07 |
| fx | 26.50 | 8.71 | 4.42 | 7.28 | 17.08 | 43160 | 83.10 |
| r | 2.10 | 2.29 | 1.98 | 3.41 | 2.03 | 14876 | 16.55 |
| cpi | 13.11 | 51.86 | 21.95 | 22.30 | 17.08 | 3986 | 101.21 |
| wpi | 18.07 | 23.30 | 8.12 | 12.15 | 11.41 | 3678 | 100.50 |
| credit | 10.33 | 0.00 | 13.63 | 1.76 | 16.82 | 3769 | 33.16 |
| M1 | 27.99 | 15.45 | 34.93 | 10.83 | 34.96 | 10600 | 163.31 |
| sx | 51.84 | 17.44 | 42.08 | 7.02 | 34.82 | 12544 | 44.76 |
| 5 % | 2.02 | 3.85 | 1.70 | 2.61 | 1.65 | 3.8 | 18.31 |
| 1 % | 2.66 | 6.66 | 2.10 | 3.80 | 2.01 | 6.0 | 22.21 |

ARCH denotes the Engle's ARCH test statistic (with 7 lags), RESET1 test statistic adds the second power of the fitted value as an additional regressor RESET2 includes both the second and third powers of y. Func. form is the F-test of the second power of the explanatory variables and their cross-terms included into the regression. White denotes White' heteroskedasticity/ functional form test statistic, J–B the Jarque-Bera test statistic for residual normality and TSAY Tsay's nonlinearity test statistic for 4 lags. 1 % and 5 % denote the critical values of the respective test statistics. ip denotes (log) industrial production, bank (log) bankruptcies, tt terms of trade, fx the real exchange rate index, r yield on long-terms government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money and sx the (log) UNITAS stock price index. The sample period is (with some exceptions) 1920M5–1993M6.



Table 3. **BDS test statistics for the residuals of a linear AR(4) model**

| | m=2 ε=0.5 | m=3 ε=0.5 | m=4 ε=0.5 | m=10 ε=0.5 | m=2 ε=1.0 | m=10 ε=1.0 |
|---|---|---|---|---|---|---|
| ip | 12.3 | 17.7 | 22.3 | 181.8 | 10.7 | 40.0 |
| bank | 9.0 | 11.2 | 13.0 | 41.7 | 10.3 | 31.7 |
| tt | 11.5 | 14.4 | 17.7 | 92.4 | 8.7 | 21.3 |
| fx | 15.7 | 17.7 | 19.5 | 41.6 | 17.1 | 18.6 |
| r | 13.4 | 16.4 | 18.4 | 42.3 | 8.6 | 14.5 |
| cpi | 10.7 | 14.6 | 16.1 | 75.9 | 11.3 | 27.9 |
| wpi | 8.1 | 10.4 | 12.5 | 46.7 | 10.7 | 18.5 |
| credit | 10.7 | 14.4 | 18.3 | 120.4 | 11.4 | 35.0 |
| M1 | 22.6 | 34.1 | 54.3 | 18.6 | 13.7 | 56.1 |
| sx | 7.8 | 8.5 | 9.7 | 22.4 | 9.1 | 25.2 |

The test statistic is BDS = $T^{½}[C_m(\varepsilon)-C_1(\varepsilon)^m]/\delta_m(\varepsilon)$, where T = the number of observations, $C_n$ = the correlation integral = $T^{-2}$*[number of pairs (i,j) such that $| y_i - y_j | < \varepsilon$, $| y_{i+1} - y_{j+i} | < \varepsilon$,..., $| y_{i+m-1} - y_{j+m-1} | < \varepsilon$] so that $y_i,...,y_{i+m-1}$ and $y_j,...,y_{j+m-1}$ are two segments of the series $y_t$ of length m and $\delta_m(\varepsilon)$ is the respective standard deviation. Under the null that the series is independently and identically distributed, BDS has a limiting standard normal distribution. Here, ε = 0.5 corresponds to ε = 0.5*{the standard deviation of the residual series}. ε = 1.0 is defined in the same way. ip denotes (log) industrial production, bank (log) bankruptcies, tt terms of trade, fx the real exchange rate index, r yield on long-terms government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money and sx the (log) UNITAS stock price index. The sample period is (with some exceptions) 1920M5–1993M6.

Table 4. **Estimation results of a nonlinear AR model**

| | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | SEE | DW | F3 |
|---|---|---|---|---|---|---|---|---|---|---|
| ip | .319 | .098 | .580 | .157 | .055 | −.771 | −.525 | .056 | 2.09 | 2.68 |
| | (3.21) | (2.89) | (10.33) | (2.42) | (2.74) | (2.63) | (0.59) | | | |
| bank | .926 | .070 | .271 | .156 | .097 | −.744 | −.013 | .325 | 2.23 | 3.70 |
| | (3.64) | (1.30) | (3.24) | (1.20) | (2.74) | (2.24) | (0.35) | | | |
| tt | .218 | .689 | 1.103 | −.619 | .343 | −.499 | −.570 | .023 | 2.06 | 10.31 |
| | (2.71) | (1.26) | (13.57) | (4.88) | (2.76) | (3.06) | (4.08) | | | |
| fx | .211 | −.966 | 1.132 | −.598 | .295 | −.301 | −.230 | .038 | 1.89 | 15.28 |
| | (2.97) | (1.51) | (16.46) | (5.38) | (3.19) | (3.91) | (4.77) | | | |
| r | .458 | .062 | .894 | .274 | −.016 | .031 | .029 | .259 | 1.95 | 1.47 |
| | (0.70) | (1.36) | (9.66) | (1.65) | (0.69) | (0.50) | (1.40) | | | |
| cpi | −.132 | .025 | 1.408 | −.406 | −.003 | .048 | 1.168 | .014 | 2.13 | 3.80 |
| | (0.81) | (2.57) | (34.48) | (9.88) | (2.66) | (2.12) | (0.44) | | | |
| wpi | −.161 | .024 | 1.553 | −.487 | −.007 | .020 | −13.560 | .015 | 2.13 | 10.83 |
| | (3.09) | (2.33) | (37.24) | (10.63) | (3.13) | (2.83) | (3.54) | | | |
| credit | −.017 | .020 | 1.460 | −.454 | −.001 | .001 | −102.35 | .011 | 2.16 | 11.60 |
| | (0.94) | (2.04) | (34.58) | (10.66) | (1.26) | (0.93) | (4.88) | | | |
| M1 | −.030 | .058 | .738 | .278 | −.002 | .006 | 6.256 | .025 | 2.00 | 8.20 |
| | (1.44) | (2.21) | (17.43) | (6.35) | (2.19) | (1.71) | (3.54) | | | |
| sx | .000 | .001 | 1.284 | −.309 | .000 | .000 | .158 | .049 | 1.97 | 0.53 |
| | (0.05) | (4.08) | (32.04) | (7.70) | (0.32) | (0.07) | (0.17) | | | |

The estimating equation is of the form: $x_t = a_0 + a_1 t + a_2 x_{t-1} + a_3 x_{t-2} + a_4(x_{t-1}x_{t-2}) + a_5(x^3_{t-1}x_{t-2}) + a_6(x_{t-1} - x_{t-2})^3 + \mu_t$, where $\mu$ is the random term. If we restrict $a_4 = a_5 = a_6 = 0$, we end up with a standard linear model. F3 represents a F test statistic for this restriction. The corresponding 5 % (1 %) critical value(s) is 2.64 (3.86). ip denotes (log) industrial production, bank (log) bankruptcies, tt terms of trade, fx the real exchange rate index, r yield on long-terms government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money and sx the (log) UNITAS stock price index. The sample period is (with some exceptions) 1920M5–1993M6. Coefficient $a_5$ has been divided by 1000.



Table 5 **Long-memory tests for AR(4) residuals of the historical time series, Period: 1922/M1–1993/M6**

| | Significance level of the Ljung-Box Q(60) statistic for residual transformation | | | First order autocorrelation coefficients for residual transformations | | |
|---|---|---|---|---|---|---|
| Variable | $r_t$ | $\lvert r_t \rvert$ | $r_t^2$ | $r_t$ | $\lvert r_t \rvert$ | $r_t^2$ |
| ip | .000 | .000 | .000 | −.006 | .289** | .137** |
| bank | .000 | .000 | .000 | −.000 | .208** | .084* |
| tt | .004 | .000 | .000 | .016 | .187** | .036 |
| fx | .528 | .000 | .027 | −.013 | .392** | .095* |
| r | .037 | .000 | .000 | −.002 | .247** | .058 |
| cpi | .000 | .000 | .000 | −.013 | .388** | .302** |
| wpi | .003 | .000 | .000 | −.007 | .324** | .180** |
| credit | .000 | .000 | .000 | −.008 | .351** | .317** |
| M1 | .000 | .000 | .000 | −.004 | .423** | .346** |
| sx | .001 | .000 | .000 | .000 | .268** | .182** |

* = significant at 5 per cent level (±2/√T) = 0.068
** = significant at 1 per cent level (2.58/√T) = 0.088

Table 6.1 **Ljung-Box test statistics for the cross-correlation coefficients of the AR(4) untransformed residuals of different variables**

| | first variable | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| second variable | ip | bank | tt | fx | r | cpi | wpi | credit | M1 | sx |
| ip | .000 | | | | | | | | | |
| | (.000) | | | | | | | | | |
| bank | .033 | .000 | | | | | | | | |
| | (.346) | (.000) | | | | | | | | |
| tt | .867 | .140 | .000 | | | | | | | |
| | (.029) | (.003) | (.000) | | | | | | | |
| fx | .119 | .000 | .012 | .053 | | | | | | |
| | (.001) | (.000) | (.989) | (.053) | | | | | | |
| r | .407 | .465 | .001 | .999 | .287 | | | | | |
| | (.253) | (.031) | (.219) | (.368) | (.287) | | | | | |
| cpi | .081 | .002 | .076 | .011 | .136 | .000 | | | | |
| | (.000) | (.000) | (.983) | (.000) | (.999) | (.000) | | | | |
| wpi | .131 | .239 | .000 | .021 | .012 | .000 | .005 | | | |
| | (.001) | (.000) | (.676) | (.000) | (.765) | (.004) | (.005) | | | |
| credit | .013 | .000 | .330 | .000 | .375 | .000 | .000 | .000 | | |
| | (.007) | (.000) | (.854) | (.000) | (.809) | (.000) | (.000) | (.000) | | |
| M1 | .987 | .463 | .061 | .564 | .354 | .496 | .768 | .023 | .000 | |
| | (.652) | (.018) | (.033) | (.954) | (.009) | (.071) | (.000) | (.082) | (.000) | |
| sx | .848 | .726 | .036 | .066 | .594 | .000 | .019 | .000 | .035 | .000 |
| | (.540) | (.000) | (.559) | (.000) | (.238) | (.005) | (.001) | (.000) | (.568) | (.000) |

Numbers denote the marginal significance levels of the Ljung-Box test statistic with 24 lags of the cross-correlation function. The first line denotes the test statistic in terms of the positive lags of the first variable (numbers inside parentheses refer to negative lags)



Table 6.2 **Ljung-Box test statistics for the cross-correlation coefficients of the squared residuals of different variables**

| | first variable | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| second variable | ip | bank | tt | fx | r | cpi | wpi | credit | M1 | sx |
| ip | .000 | | | | | | | | | |
| | (.000) | | | | | | | | | |
| bank | .000 | .000 | | | | | | | | |
| | (.000) | (.000) | | | | | | | | |
| tt | .999 | .000 | .000 | | | | | | | |
| | (.000) | (.000) | (.000) | | | | | | | |
| fx | .517 | .000 | .999 | .000 | | | | | | |
| | (.077) | (.000) | (.999) | (.000) | | | | | | |
| r | .232 | .135 | .000 | .999 | .000 | | | | | |
| | (.000) | (.423) | (.000) | (.999) | (.000) | | | | | |
| cpi | .796 | .000 | .621 | .005 | .992 | .000 | | | | |
| | (.001) | (.000) | (.999) | (.000) | (.999) | (.000) | | | | |
| wpi | .423 | .000 | .000 | .489 | .000 | .000 | .000 | | | |
| | (.000) | (.000) | (.999) | (.000) | (.000) | (.000) | (.000) | | | |
| credit | .000 | .000 | .999 | .000 | .996 | .000 | .000 | .000 | | |
| | (.000) | (.000) | (.999) | (.000) | (.984) | (.000) | (.000) | (.000) | | |
| M1 | .044 | .826 | .024 | .999 | .004 | .904 | .804 | .814 | .000 | |
| | (.213) | (.989) | (.000) | (.999) | (.000) | (.991) | (.034) | (.685) | (.000) | |
| sx | .977 | .000 | .748 | .000 | .147 | .000 | .000 | .000 | .999 | .000 |
| | (.033) | (.000) | (.999) | (.000) | (.993) | (.000) | (.000) | (.000) | (.816) | (.000) |

Numbers denote the marginal significance levels of the Ljung-Box test statistic with 24 lags of the cross-correlation function. The first line denotes the test statistic in terms of the positive lags of the first variable (numbers inside parentheses refer to negative lags)

Table 6.3 **Ljung-Box test statistics for the cross-correlation coefficients of the third power of the AR(4) residuals of different variables**

| second variable | first variable: ip | bank | tt | fx | r | cpi | wpi | credit | M1 | sx |
|---|---|---|---|---|---|---|---|---|---|---|
| ip | .002 | | | | | | | | | |
| | (.002) | | | | | | | | | |
| bank | .090 | .000 | | | | | | | | |
| | (.000) | (.000) | | | | | | | | |
| tt | 1.000 | .000 | .000 | | | | | | | |
| | (.000) | (.001) | (.000) | | | | | | | |
| fx | .999 | .000 | 1.000 | .660 | | | | | | |
| | (.983) | (.000) | (1.000) | (.660) | | | | | | |
| r | .095 | .104 | .000 | 1.000 | .000 | | | | | |
| | (.000) | (.007) | (.000) | (1.000) | (.000) | | | | | |
| cpi | .999 | .002 | .999 | .960 | .999 | .000 | | | | |
| | (.986) | (.000) | (1.000) | (.000) | (1.000) | (.000) | | | | |
| wpi | .999 | .000 | .000 | 1.000 | .000 | .000 | .884 | | | |
| | (.000) | (.000) | (1.000) | (.000) | (.000) | (.000) | (.884) | | | |
| credit | .791 | .000 | 1.000 | .000 | 1.000 | .000 | .000 | .000 | | |
| | (.057) | (.000) | (1.000) | (.000) | (1.000) | (.000) | (.004) | (.000) | | |
| M1 | 1.000 | .999 | .309 | 1.000 | .048 | 1.000 | 1.000 | 1.000 | .000 | |
| | (.023) | (.999) | (.000) | (1.000) | (.048) | (1.000) | (.002) | (.999) | (.000) | |
| sx | 1.000 | .000 | .999 | .132 | .969 | .000 | .003 | .000 | 1.000 | .000 |
| | (.986) | (.000) | (1.000) | (.000) | (1.000) | (.000) | (.000) | (.000) | (.970) | (.000) |

Numbers denote the marginal significance levels of the Ljung-Box test statistic with 24 lags of the cross-correlation function. The first line denotes the test statistic in terms of the positive lags of the first variable (numbers inside parentheses refer to negative lags)



Table 6.4 **Ljung-Box test statistics for the cross-correlation coefficients of the absolute values of the residuals of different variables**

| second variable | first variable: ip | bank | tt | fx | r | cpi | wpi | credit | M1 | sx |
|---|---|---|---|---|---|---|---|---|---|---|
| ip | .000 | | | | | | | | | |
| | (.000) | | | | | | | | | |
| bank | .000 | .000 | | | | | | | | |
| | (.000) | (.000) | | | | | | | | |
| tt | .549 | .000 | .000 | | | | | | | |
| | (.001) | (.000) | (.000) | | | | | | | |
| fx | .000 | .000 | .009 | .000 | | | | | | |
| | (.000) | (.000) | (.269) | (.000) | | | | | | |
| r | .073 | .000 | .000 | .999 | .000 | | | | | |
| | (.000) | (.000) | (.023) | (.993) | (.000) | | | | | |
| cpi | .000 | .000 | .000 | .000 | .912 | .000 | | | | |
| | (.000) | (.000) | (.543) | (.000) | (.461) | (.000) | | | | |
| wpi | .000 | .000 | .000 | .000 | .096 | .000 | .000 | | | |
| | (.000) | (.000) | (.011) | (.000) | (.336) | (.000) | (.000) | | | |
| credit | .000 | .000 | .000 | .000 | .503 | .000 | .000 | .000 | | |
| | (.000) | (.000) | (.351) | (.000) | (.999) | (.000) | (.000) | (.000) | | |
| M1 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | |
| | (.000) | (.258) | (.000) | (.000) | (.000) | (.000) | (.000) | (.000) | (.000) | |
| sx | .961 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .962 | .000 |
| | (.001) | (.000) | (.185) | (.000) | (.021) | (.000) | (.000) | (.000) | (.769) | (.000) |

Numbers denote the marginal significance levels of the Ljung-Box test statistic with 24 lags of the cross-correlation function. The first line denotes the test statistic in terms of the positive lags of the first variable (numbers inside parentheses refer to negative lags)

Table 6.5 **Contemporaneous correlation coefficients between the untransformed residuals of univariate AR(4)-models for different variables**

| | ip | bank | tt | fx | r | cpi | wpi | credit | M1 | sx |
|---|---|---|---|---|---|---|---|---|---|---|
| ip | 1.000 | | | | | | | | | |
| bank | .046 | 1.000 | | | | | | | | |
| tt | .024 | .000 | 1.000 | | | | | | | |
| fx | −.025 | −.016 | −.053 | 1.000 | | | | | | |
| r | −.027 | .032 | .091 | −.006 | 1.000 | | | | | |
| cpi | .014 | .020 | .014 | −.137 | −.015 | 1.000 | | | | |
| wpi | −.037 | −.039 | .082 | .163 | .027 | .497 | 1.000 | | | |
| credit | .021 | −.076 | −.033 | −.032 | −.012 | .092 | .044 | 1.000 | | |
| M1 | .016 | .029 | .106 | −.015 | .012 | .065 | .056 | .068 | 1.000 | |
| sx | −.029 | −.041 | −.027 | −.103 | .197 | .199 | .079 | −.028 | .077 | 1.000 |

A value of $\pm 2/\sqrt{T}$ = 0.068 corresponds to the critical value at the 5 per cent level of significance.



Figure 1. **Historical Finnish time series**

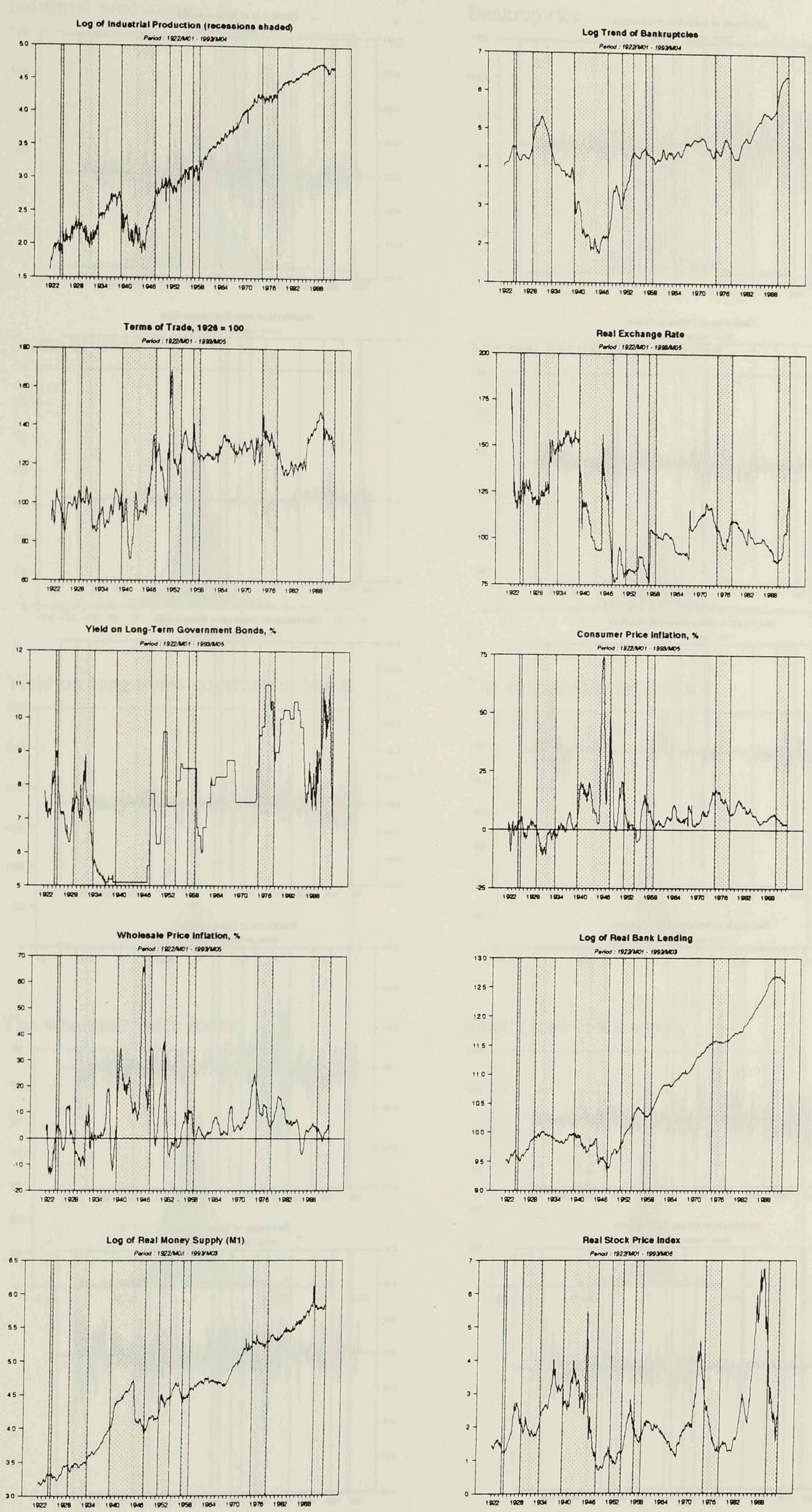


Figure 2. **Time series of AR(4) residuals**

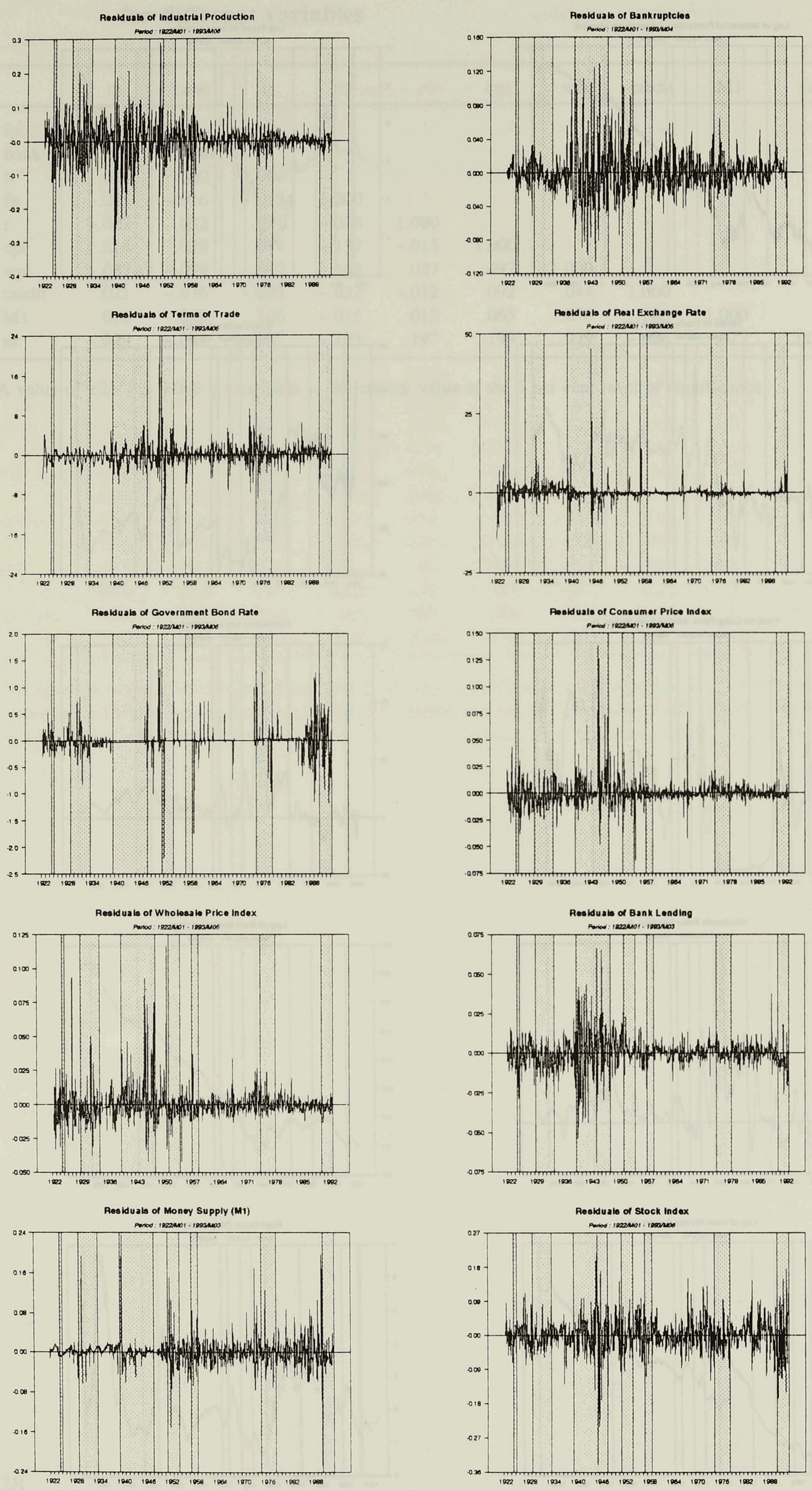
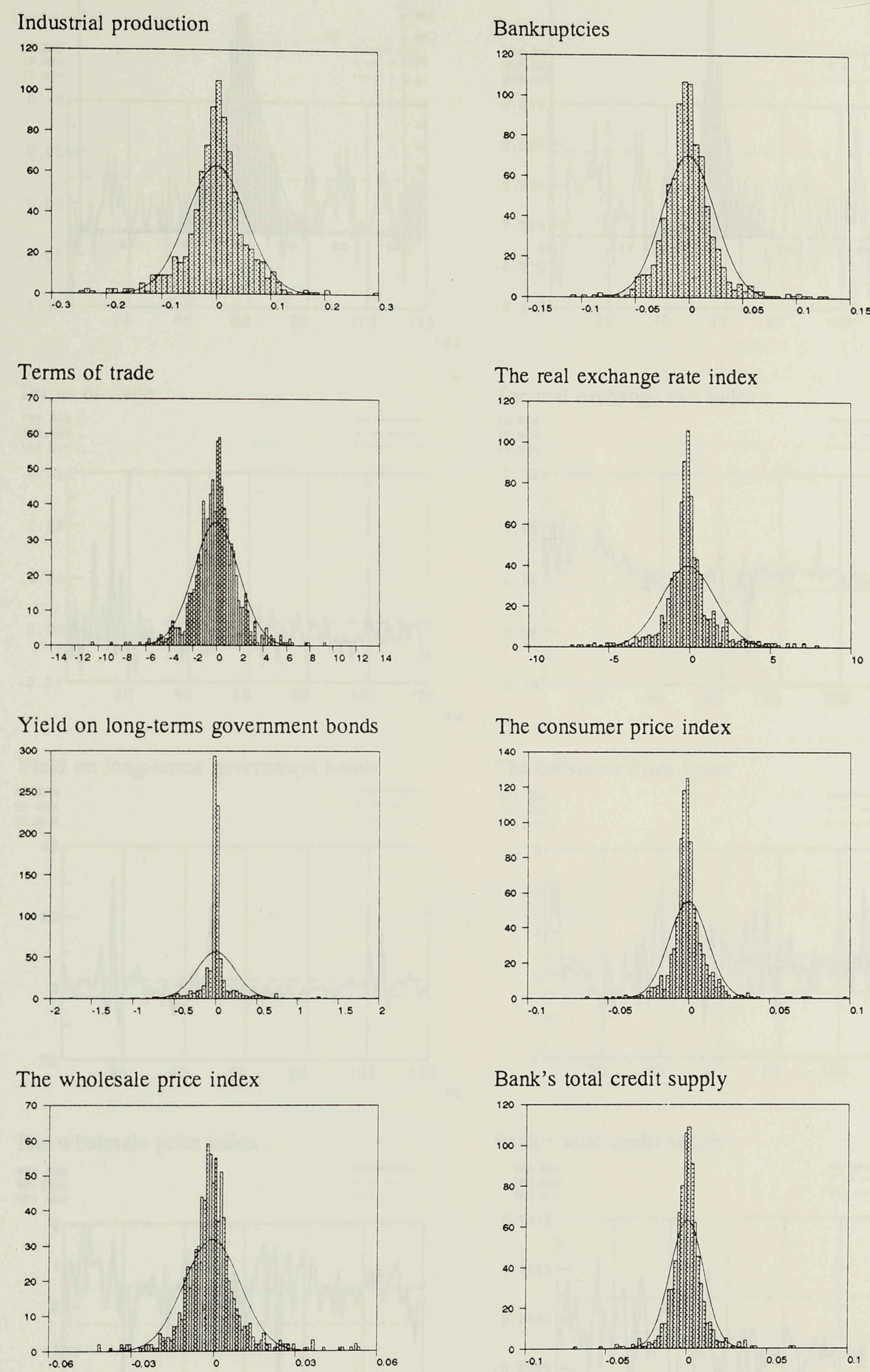
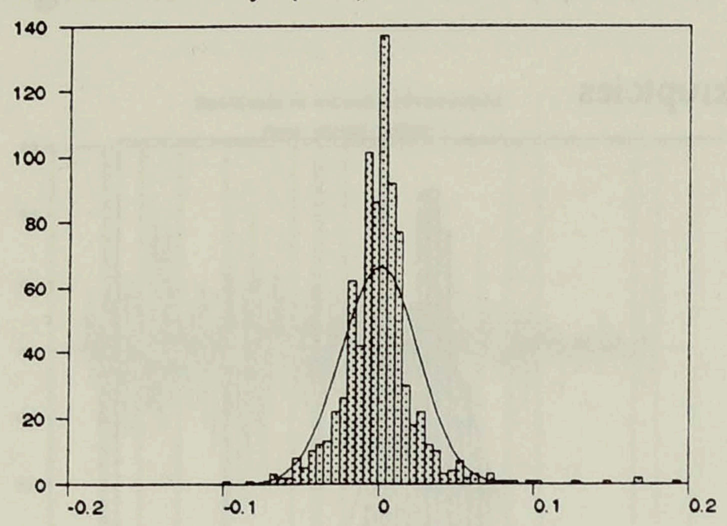



Figure 3. **Frequency distribution of AR(4) residuals**



Narrow money (M1)



The Unitas stock exchange index

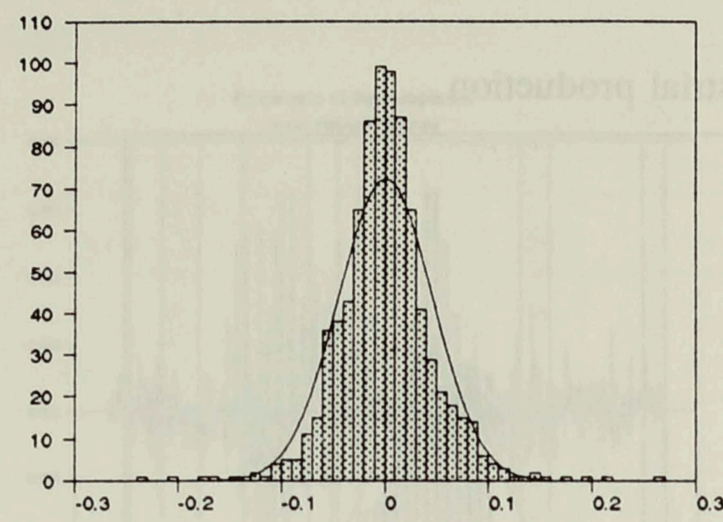
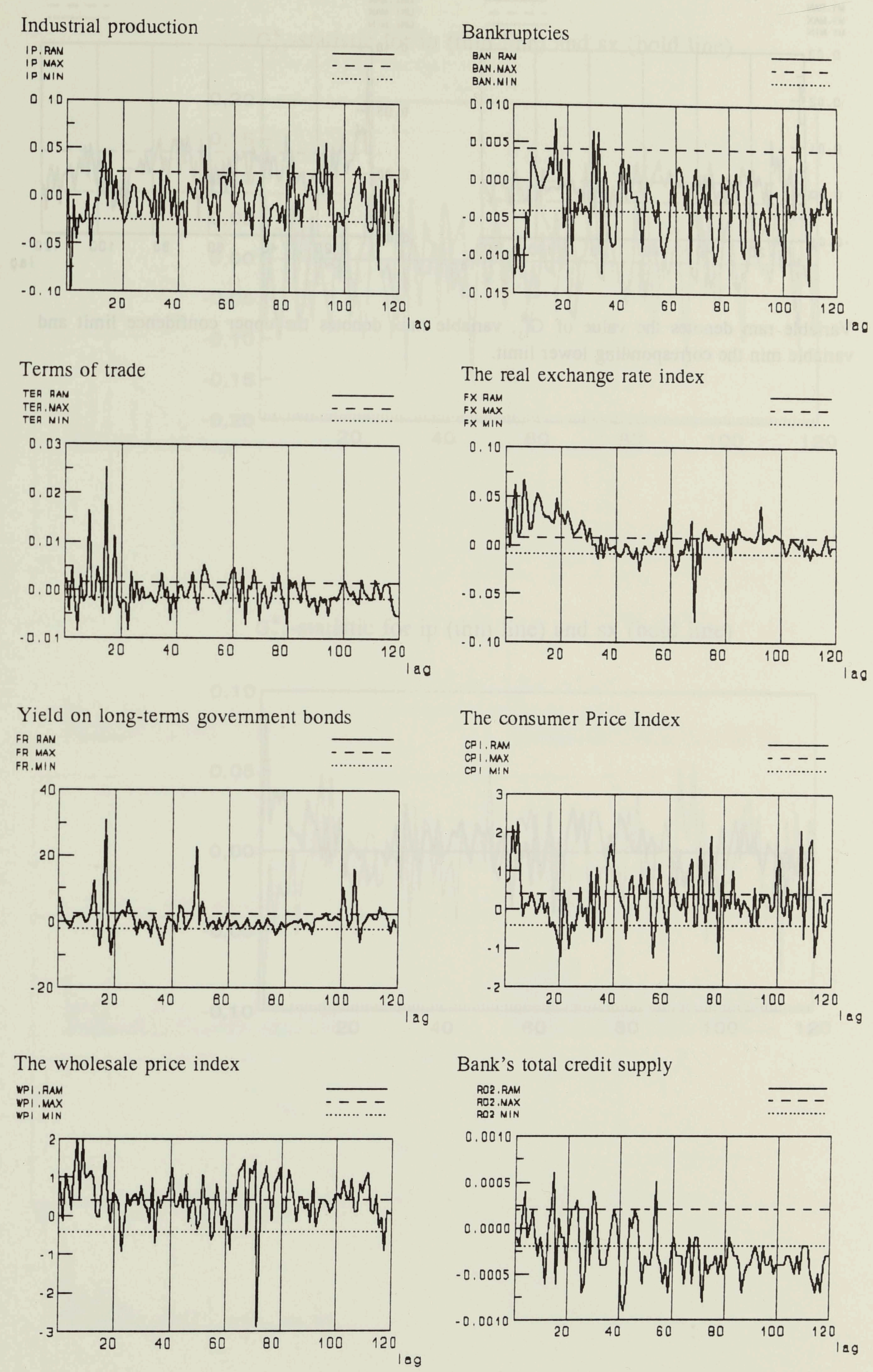
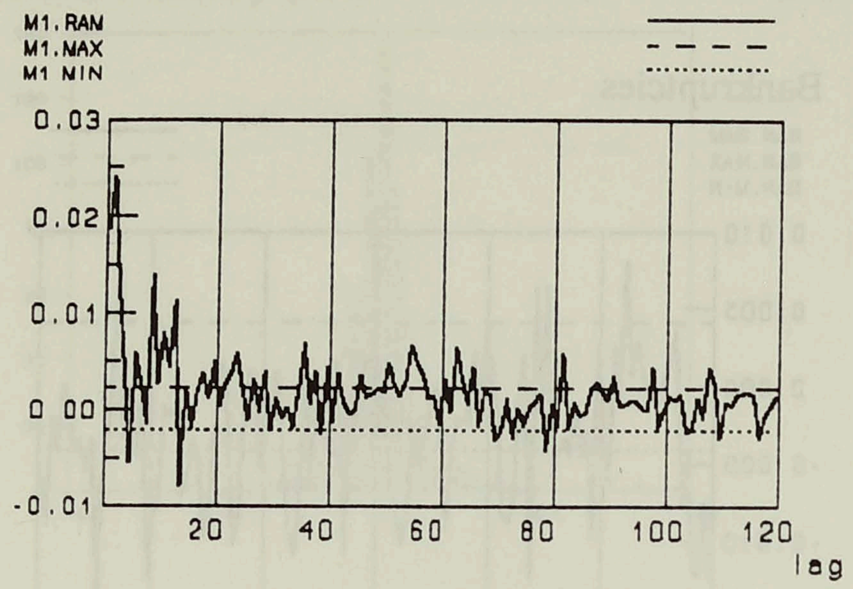




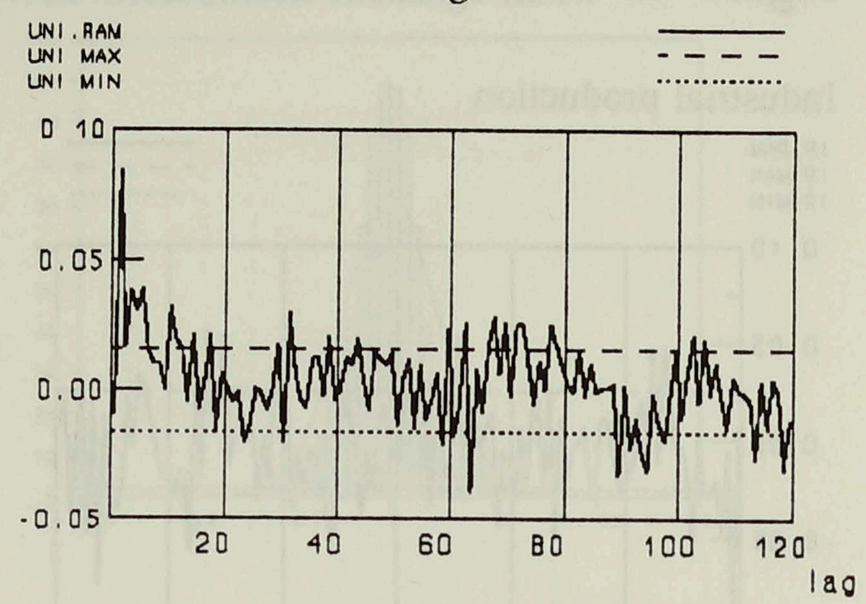
Figure 4.1 **Ramsey irreversibility test statistics**



Narrow money (M1)




The Unitas stock exchange index

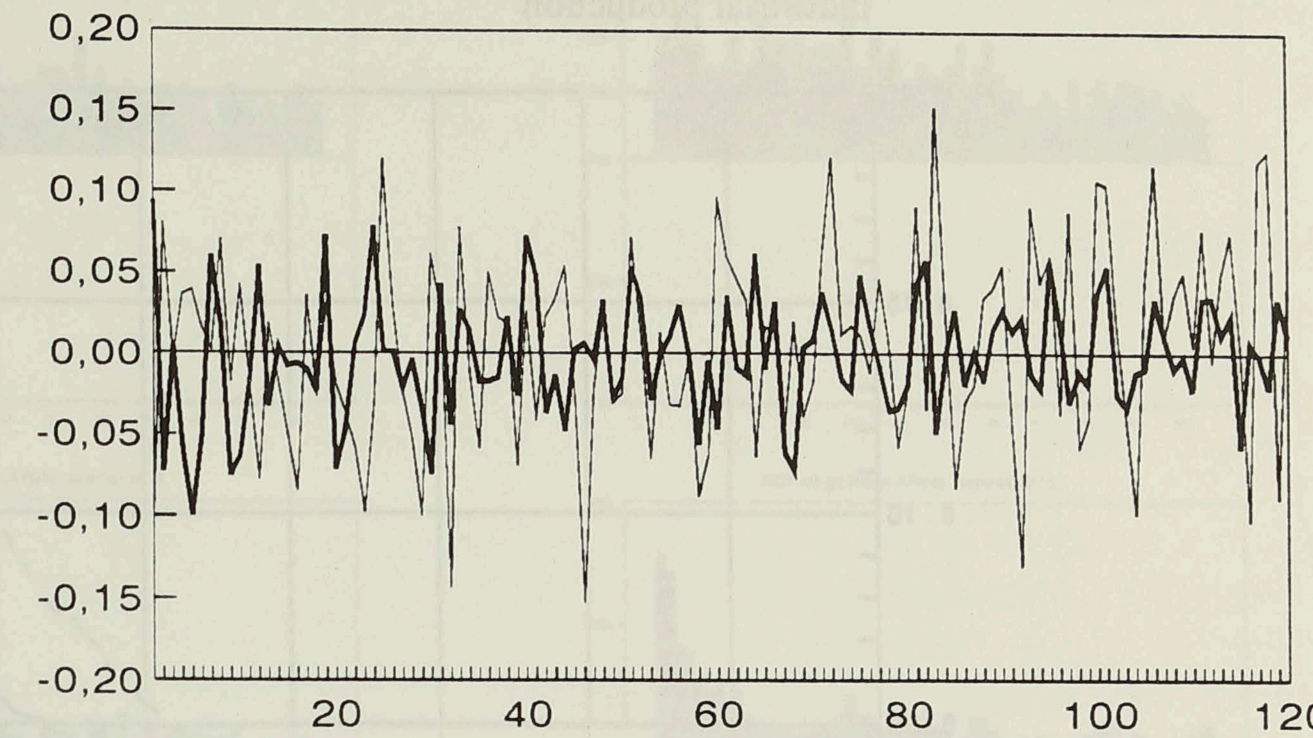



Variable ram denotes the value of $G^k_{i,j}$, variable max denotes the upper confidence limit and variable min the corresponding lower limit.



Figure 4.2 **Ramsey irreversibility test statistics for ip and sx**

$G^k_{3,1}$-statistic for ip (thin line) and sx (bold line)




$G^k_{2,1}$-statistic for ip (thin line) and sx (bold line)

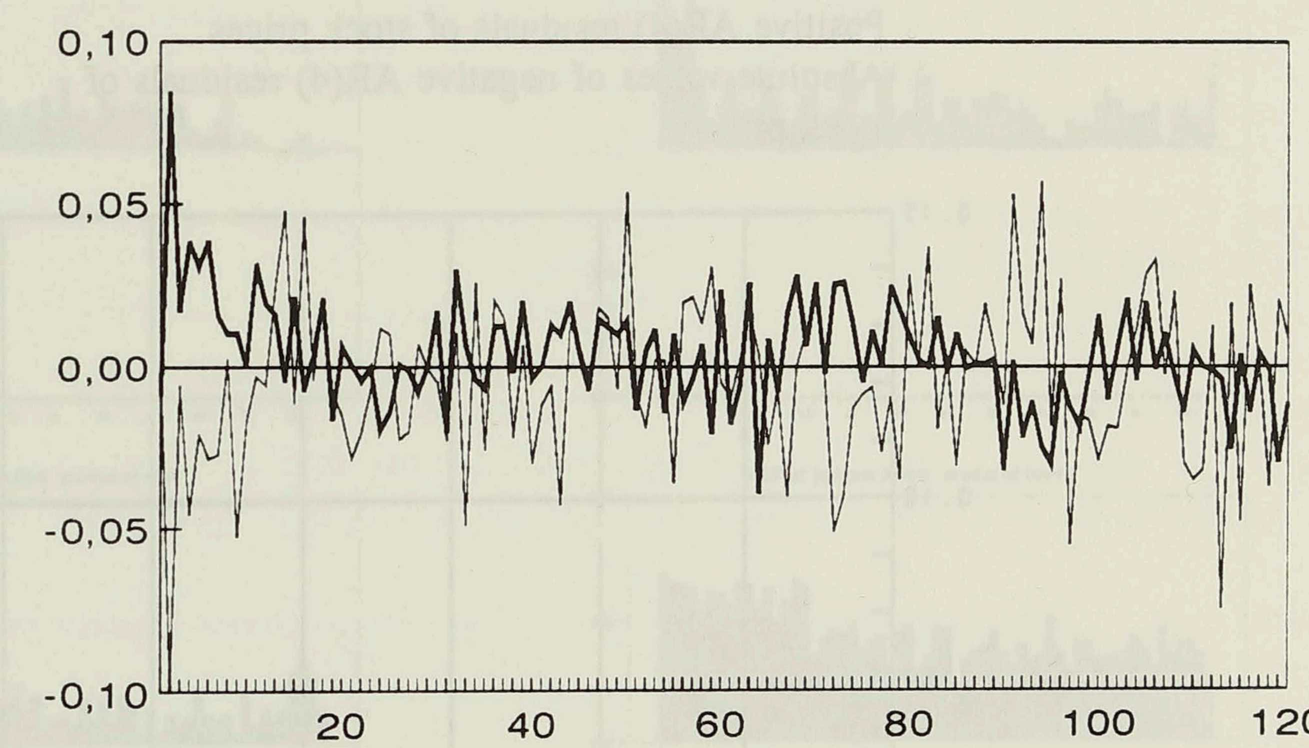
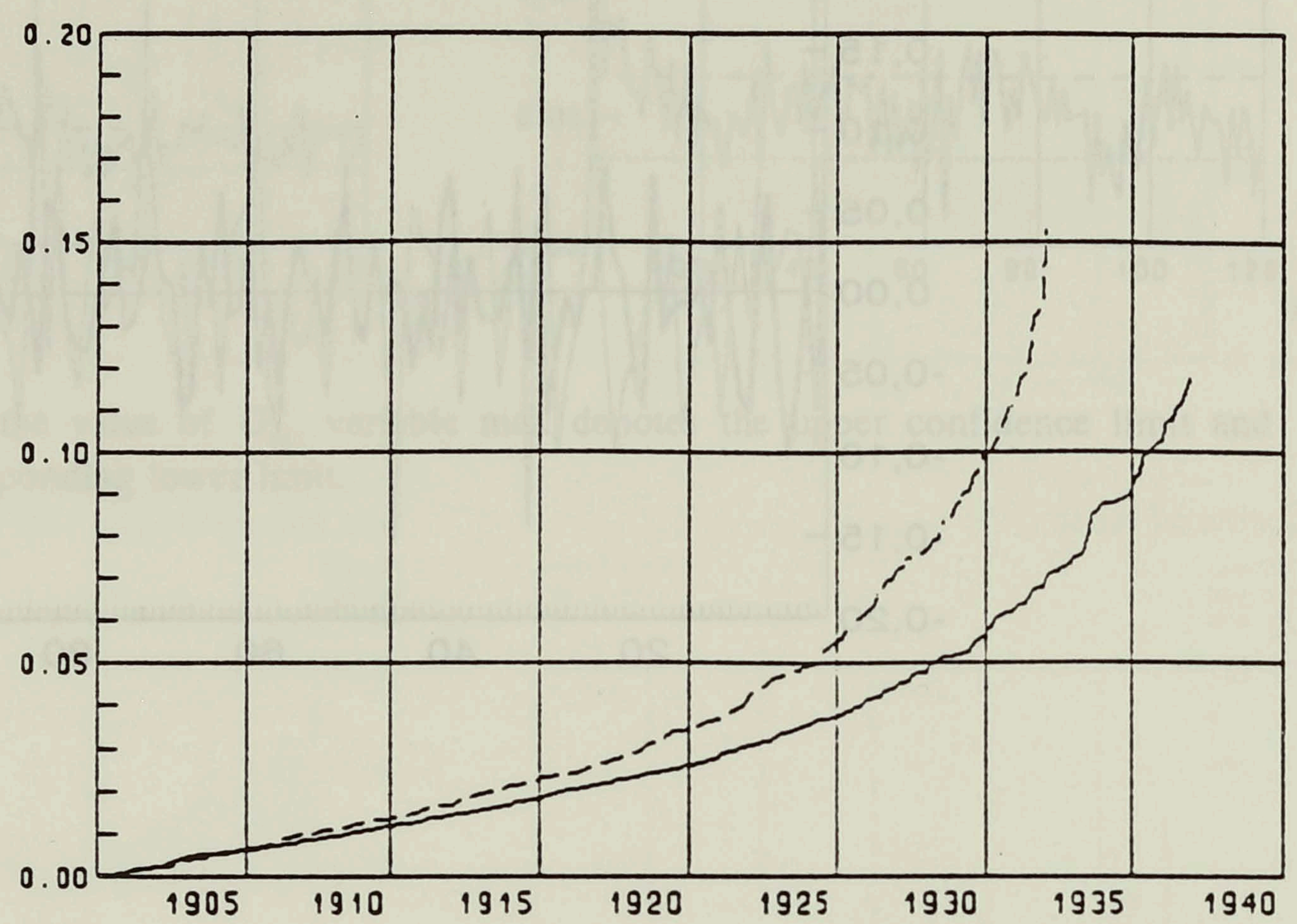


Figure 5. **Residuals for industrial production and stock prices**

Positive AR(4) residuals of industrial production ————
Absolute values of negative AR(4) residuals of industrial production — — — —




Positive AR(4) residuals of stock prices ————
Absolute values of negative AR(4) residuals of stock prices — — — —

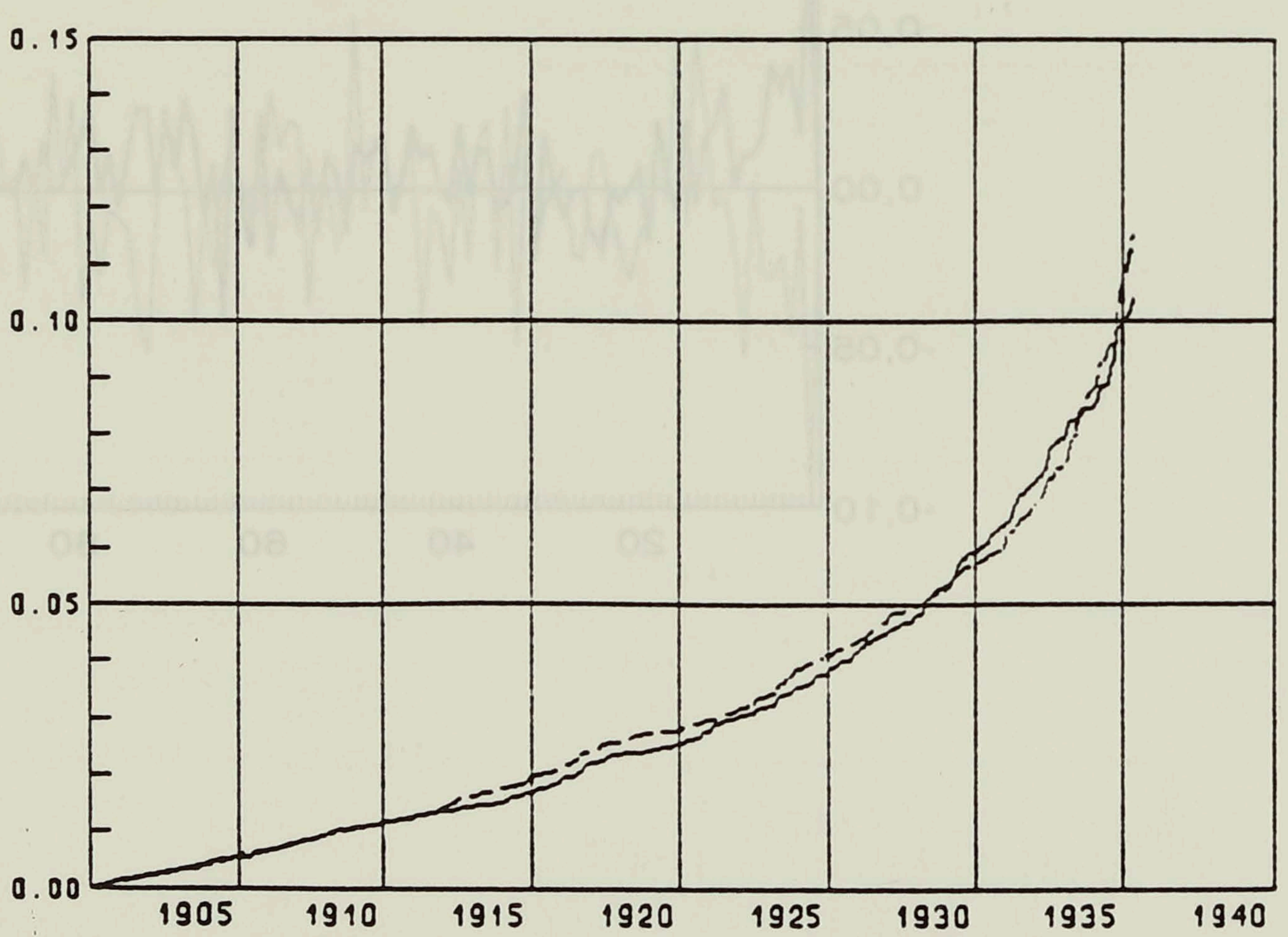
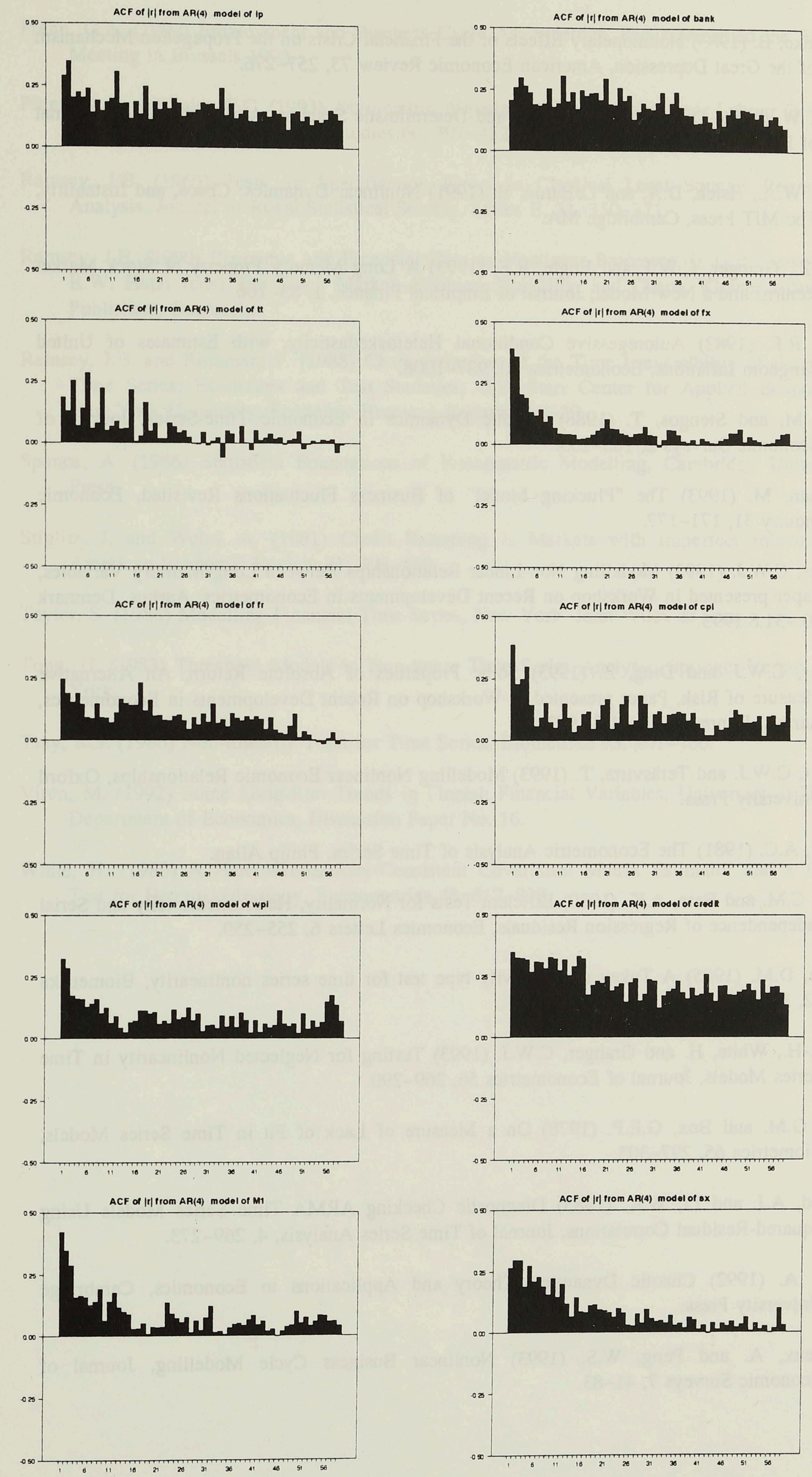





Figure 6. **Autocorrelations of absolute values of AR(4) residuals**



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