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by

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Abstract

The paper attempts to formalize dealer behaviour in the foreign exchange market. The three key assumptions used in the analysis are price sensitivity of customer orders and profit maximization by dealers who are constrained by the closed position target for the end of the day. As a market maker the dealer provides liquidity services to customers by making two-way prices and by standing ready to trade at these prices on immediate demand. The paper begins with a simple one-period model assuming a monopoly dealer. This is extended to sequential pricing behaviour under transactions uncertainty (stochastic order flow). The role of information is examined separately. While the assumption of a monopolistic market structure is admittedly highly unrealistic in the case of the foreign exchange market, the same model structure also works well in a competitive environment. Competition enters the analysis in the form of customer flows and interdealer transactions. The overall picture of the market structure and price dynamics that emerges from the analysis is broadly consistent with observable facts.

The results are consistent with the efficient market hypothesis in the sense that there is no possibility that any of the customers or other dealers could predict the dealer's forthcoming quotations on the basis of the dealer's past pricing behaviour. Under pure transactions uncertainty and in the absence of new information, the dealer is reluctant to make a large adjustment to prices, because frequent price revisions are generally revenue-reducing. This does not apply to situations where new information hits the market. Any new information on forthcoming customer orders will cause a prompt change in the quoted price. Price adjustments are, however, limited by the sensitivity of customers to small price differentials. Because dealers can send orders to each other in the interbank market, the sensitivity of incoming orders to small price differentials is large. As a result, the prices quoted by different dealers cannot deviate much from each other. The price making power of each dealer is reflected in the spread. Competition reduces the spread and it approaches zero once interdealer transactions are allowed for. Under pure transactions uncertainty, the spread tends to be constant. Under competitive conditions, when the spread is very small, prices quoted by all dealers are practically equal and the share of interbank transactions in the total volume of trade is very large.

Intraday volatility of exchange rate quotations stems from two sources, one representing normal order flow and the other the arrival of new information. Price uncertainty would be small if transactions uncertainty was its only source and if order flows were purely stochastic. Any new information on forthcoming customer orders increases price uncertainty. In addition to temporal price uncertainty, instantaneous price uncertainty is present because the dealer has to make a binding two-way price without knowing what prices his competitors are quoting at the same moment. This prevents the spread from diminishing indefinitely. In times of increasing uncertainty the spread tends to widen as a result of heterogenous and asymmetric information.

Contents

1 Introduction

The foreign exchange market is an arrangement which facilitates international monetary exchange in a world where there is no single universally accepted means of payment (Einzig, 1969, p. 15, and Hirsch, 1969, p. 198). The market exists to serve the currency conversion needs of the inhabitants of different countries who are parties to economic contracts with each other. Merchants involved in exporting and importing, investors diversifying their portfolios internationally, speculators seeking to profit from exchange rate fluctuations and tourists travelling abroad, all of them constantly face situations in which they need to convert one currency into another.

A distinct feature of the foreign exchange market is that it is almost entirely intermediated. The ultimate buyers and sellers of currencies seldom meet face to face. Rather, they place buy and sell orders with commercial banks, whose foreign exchange departments stand ready to trade on the basis of incoming orders on immediate demand.

The buy and sell orders received by any single bank do not usually match in any short period. For many banks the flow of buy orders for a given currency may systematically exceed the flow of sell orders. This mismatch of market orders gives rise to the interbank market, where individual banks can acquire the currencies currently oversold or sell the currencies currently overbought.

Altogether, the total turnover in the foreign exchange market, adjusted for interbank double counting, was estimated to be USD 600 to 700 billion a day in April 1989 (BIS, 1990). Given that the market is open 24 hours a day, this means that some USD 20 to 25 billion changes hand in any single hour. Of this volume of trade only around 10 per cent is directly linked with customers' needs, while the remaining 90 per cent represents professional trading between banks.

The object of trade in the foreign exchange market is money, which is "the most fluid and most easily movable of all commodities" (Hirsch, 1969, p. 197). This is another way of saying that the transaction costs of trading money are low. Low transaction costs imply that the effective price paid by the buyer is close to the effective price received by the seller. In the foreign exchange market this means that the *bid-ask spread* is very small. Under competitive conditions and without any artificial obstacles to foreign exchange transactions, the spread should not exceed the marginal cost of trading in currencies. When this requirement is fulfilled, the foreign exchange market is *functionally* efficient, using Tobin's terminolgy (Tobin, 1984).

Another important aspect of the efficiency of the foreign exchange market is related to information and arbitrage. According to the most widely used definition, a market is said to be efficient if it is impossible to predict future price movements on the basis of publicly available information (Samuelson, 1965, and Fama, 1970). To this one may want to add a further requirement that in an efficient market it is, on average, impossible to gain from spatial price differentials which may prevail across marketplaces. When these requirements are fulfilled the market is *information-arbitrage* efficient, again using Tobin's terminology. This type of efficiency is not entirely independent of functional efficiency, because information costs depend on transaction costs. In the foreign exchange market, in particular, transacting is one way of acquiring information. Low transaction costs increase the volume of trade and improve the quality of information. Indeed, it will be argued below that the impressive size of the interbank market is, to a large extent, understandable in terms of the exchange of information.

When the bid-ask spread is very small and exchange rate quotations are practically equal everywhere, non-bank customers can regard their domestic money balances as being equivalent to international means of payment (McKinnon, 1979, pp. 4-13). Under these conditions, international monetary exchange does not essentially differ from monetary exchange in general.

There is a vast and growing literature on the microstructure of the financial markets.[1] This literature has paid attention to the important role played by market makers in providing liquidity services to customers. The typical empirical reference in the microstructure literature has been the behaviour of stock dealers, *i.e.*, the specialists who make the market for selected shares by quoting two-way prices and by standing ready to transact at these prices on immediate demand. The foreign exchange market has seldom been used as an empirical reference in the theoretical microstructure literature despite the fact that, in terms of the number of participants and the volume of trade, it is the largest financial market in the world. Because the foreign exchange market is based on market making, the theoretical results of the analysis of stock dealer behaviour should be applicable to the analysis of foreign exchange dealing as well.[2]

The present paper is an attempt to formalize dealer behaviour in the foreign exchange market. While the insights and techniques have been borrowed from the microstructure theory of stock dealer behaviour, the analysis is carried out with far fewer technical and institutional assumptions. The research strategy is to start off with a minimum number of assumptions and then to proceed step by step towards more complicated and more realistic descriptions. The three key assumptions used are the *price sensitivity* of customer orders and *profit maximization* by the dealers, which is constrained by the *closed position target* for the end of the day.

The theme of the paper is set out in *Section 2*, which describes the basic structure of the dealership market using the classical characterization by Demsetz (1968). The rest of the paper can be regarded as variations on this single theme. *Section 3* formalizes this setup in a static one-period case assuming that

[1] Baumol (1965) was perhaps the first to draw attention to the important role stock dealers play in the securities market. The most frequently cited reference, however, is Demsetz (1968). More recent contributions include Tinic (1972), Garman (1976), Stoll (1978), Amihud and Mendelson (1980), Ho and Stoll (1980 and 1981), Zabel (1981), Glosten and Milgrom (1985), Grossman and Miller (1988), Glosten (1989), Dennert (1989) and Hagerty (1991). For surveys, see Cohen, Maier, Schwartz and Whitcomb (1979) and Stoll (1985).

[2] For the importance of the microstructure aspects for the analysis of the foreign exchange market, see *e.g.* Goodhart (1988), Bingham (1991) and Bossaerts and Hillion (1991). The present study has gained valuable insights from discussions with foreign exchange dealers in a number of banks, from dealer handbooks (*e.g.* Riehl and Rodrigues, 1983, and Tygier, 1983), as well as from empirical studies on exchange rate behaviour at high or ultra-high frequencies (Glassman, 1987, Goodhart, 1989, Wasserfallen, 1989, Goodhart and Demos, 1990a and 1990b, Goodhart and Fiuglioli, 1988 and 1991, Goodhart *et al.*, 1991, and Müller *et al.*, 1991).

the dealer has a short-term monopoly. *Section 4* introduces transactions uncertainty and thereby dynamics into the model in the form of a sequence of stochastic customer orders. *Section 5* varies the dynamic theme by paying attention to the role of new information on forthcoming customer orders.

While the assumption of a monopolistic market structure is admittedly highly unrealistic in the case of the foreign exchange market, the same model structure also works well in a competitive environment, as shown in *Sections 6* and *7*. Introducing inter-dealer transactions and customer flows into the model reduces the monopoly power of each individual dealer by making their quotations interdependent. As a result, the spread diminishes and approaches zero at the same time as the volume of interbank trade increases rapidly.

In addition to transactions uncertainty, price uncertainty enters the picture in the form of simultaneous quotations made by numerous dealers who do not know what prices their rivals are quoting at the same moment. This gives rise to an arbitrage opportunity for any third party who is able to observe a pair of inconsistent quotations. A defensive strategy to reduce the risk of quoting inconsistently would be to widen the spread. This is shown in *Section 8*. *Section 9* takes us back to a dynamic framework by focusing on the intraday pricing behaviour of a foreign exchange dealer in the interdependent market. Although this variation of the model repeats many features of those of the earlier sections, it also enables us to discuss some further implications, such as asymmetric information related to forthcoming news and the integration of foreign exchange markets across time zones.

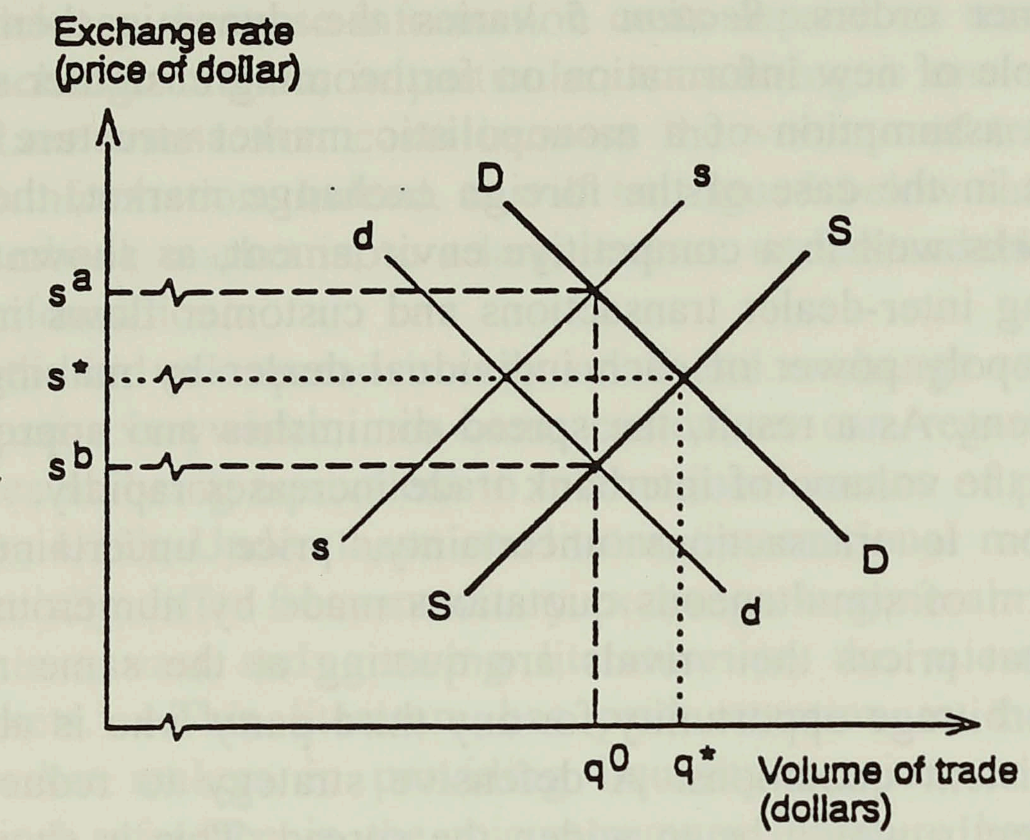
The theoretical analysis of the paper is based on a number of restrictive assumptions, which moreover vary from section to section. It is, therefore, difficult to draw any straightforward testable empirical predictions from the analytical results as such. However, the overall picture of the market structure and price dynamics that emerges from the analysis is broadly consistent with observable facts. This is discussed briefly in *Section 10*, which sums up the main results of the paper.

2 On the Dealership Market

Security dealers are specialists who differ from brokers in that, rather than matching individual buy and sell orders directly, they stand ready to buy and sell on immediate demand by trading for their own account. The availability of dealer services saves customers from waiting, and hence lowers the costs of making transactions. In the words of Demsetz (1968), security dealers produce immediacy (liquidity) services. The same applies to the foreign exchange market.

The dealership market can be illustrated by means of a simple diagram. In *Figure 1* the *DD*-curve represents the flow demand for foreign currency (the flow supply of domestic currency). The *SS*-curve represents the flow supply of dollars (the flow demand for domestic currency). The curves refer to the currency conversion needs of the residents of different countries during a given

Figure 1. *The Dealership Market: An Illustration*



short period.[3] The exchange rate s is defined as the price of one unit of foreign currency (*dollar*) in terms of domestic currency (*marks*).

If those who need to convert one currency into another could communicate without costs, the two curves would in each short period determine the equilibrium exchange rate s^* and the equilibrium volume of trade q^*. Because such costless communication is not possible, commercial customers place their orders with foreign exchange dealers who stand ready to trade on immediate demand.

A dealer does not purchase dollars (or marks) in order to hold them or to spend them on goods, but to sell them back to customers at a higher price. Therefore, every part of the dealer's supply curve (*ss*-curve) lies above the customers' supply curve by a given distance. Similarly, every part of the dealer's demand curve (*dd*-curve) lies below the customers' demand curve by the same distance.

The distance between the buying price and the selling price is called the *bid-ask spread*. For a given spread, the equilibrium selling price s^a, the *ask rate*, is determined by the intercept of the dealer's supply curve and the customers' demand curve. This is the price at which the dealer is willing to exchange dollars for marks. Similarly, the equilibrium buying price s^b, the *bid rate*, is determined by the intercept of the dealer's demand curve and the customers' supply curve. This is the price the dealer is prepared to pay in order to buy dollars in exchange for marks. When transactions are made at these prices, the incoming buy and sell orders match as long as the structure which generates the customer orders remains constant.

Two further aspects have to be taken into account in interpreting the diagramme. First, the *DD*-curve and the *SS*-curve do not represent always-present market orders. Rather, they represent the average arrival rates of incoming orders generated by a given macroeconomic environment. In any short period, the actual rate of incoming orders may differ from the average

[3] This illustration is borrowed from Demsetz (1968). For an application to the foreign exchange market, see Levich (1979).

one. Secondly, as a market maker, the dealer quotes a two-way price upon request and generally stands ready to transact at these prices if the customer decides to buy or to sell. Therefore, the *DD*- and *SS*-curves correspond to the dealer's expectations about their average levels, and the *dd*- and *ss*-curves are set accordingly. Equilibrium prevails when expectations are correct on the average, but they need not be correct in any short time interval. This is where transactions uncertainty enters the picture. Price uncertainty stems from the fact the dealer's expectations about the average flow of orders may prove to be wrong. This may happen because new information hits the market unexpectedly, or because successive customer orders begin to depart from the expected path.

Figure 1 also shows that the average volume of currency conversions q^o depends negatively on the bid-ask spread. This results from the sensitivity of customers to the levels of exchange rates. Using the industry analogy, it means that the demand for dealer services depends negatively on its price, which is represented by the bid-ask spread.

While the demand side of the market for dealer services is straightforward, the supply side is more difficult to handle analytically. It requires a theory of the behaviour of an individual dealer, including the identification of relevant costs associated with dealing activity. Leaving aside the fixed costs related to capital equipment and highly specific human skills, the major variable cost arises from the opportunity cost of holding currencies (including trading balances in domestic currency). These are needed for the dealers to be able to make transactions on immediate demand.

Finally, the specification of the market conditions under which the dealers operate is very important, because they determine to what extent each individual dealer can exercise his potential price-setting power. The decisive factors in this respect are the absence of exchange controls, the degree of competitition between the dealers and the functioning of the interbank market.

3 Simple One-Period Model

In order to introduce the notation and to present the structure of the problem, the dealer's pricing behaviour is first analyzed in a simple deterministic one-period case. It will be seen that the analysis is, in fact, a fairly straightforward application of standard microeconomic theory. We start off from a highly unrealistic assumption that the dealer operates in isolated circumstances and is able to fully exploit the monopoly power created by the price sensitivity of customers' buy and sell orders.[4] The dealer is assumed to maximize his trading income subject to the constraint that his expected foreign exchange position is closed, or meets some other fixed target, at the end of the period.

In accordance with the discussion of the dealership market in Section 2, the price-dependent arrival rates of customer orders are written as follows (*cf. Figure 1*):

[4] The behaviour of stock dealers on the New York Stock Exchange was the most frequently cited empirical reference in early microstructure literature. In their case the assumption of a monopoly is realistic, because stock dealers on the NYSE are specialized in trading in particular shares, see Demsetz (1968), Ho and Stoll (1981) and Zabel (1981).

(1) $$p(t) = a(t) - cs^a(t)$$

(2) $$q(t) = b(t) + cs^b(t).$$

The first equation expresses the customer demand (arriving sell orders) as a function of the ask rate $s^a(t)$. The second equation shows the customer supply (arriving buy orders) as a function of the bid rate $s^b(t)$ quoted by the dealer at the beginning of a short period t. The ask and bid rates are defined as the price of one unit of foreign currency (*dollar*) in terms of domestic money (*marks*). The amounts bought, $q(t)$, and sold, $p(t)$, by the dealer have the dimension of dollars per discrete unit of time. This time span will in the following be called a *trading session.*

The shift variables $a(t)$ and $b(t)$ are assumed to be known for the time being. Assuming that $a(t)-b(t)>0$ guarantees that in equilibrium both the price and the volume of trade are positive. The parameter $c>0$ stands for customer sensitivity to exchange rates. Nothing is lost in generality if it is assumed to be equal on both the buy and the sell side.[5]

It is convenient to redefine the ask rate and the bid rate in terms of the *mid-rate*, $s(t)=[s^a(t)+s^b(t)]/2$, and the *half-spread*, $z(t)=[s^a(t) - s^b(t)]/2$.[6] The trading income per session, defined in terms of domestic money, is equal to the value of sales minus the value of purchases, or

(3) $$R(t) = [\, s(t) + z(t)]p(t) - [\, s(t) - z(t)]q(t)$$

$$= z(t)[\, p(t) + q(t)] + s(t)[\, p(t) - q(t)].$$

From the latter expression it is seen that trading income is equal to the spread times the volume of trade[7], adjusted for any cash inflow or outflow arising from the net sales or purchases of the foreign currency. Redefining the parameters, the trading income can be expressed in the following form:

(4) $$R(t) = \alpha(t)s(t)+\beta(t)z(t) - \delta[\, s^2(t)+z^2(t)],$$

where $\alpha(t)=a(t)-b(t)>0$, $\beta(t)=a(t)+b(t)>0$, and $\delta=2c>0$.

Maximizing the trading income in terms of domestic currency without any constraints would lead to substantial net sales of foreign currency. This is not possible over successive trading sessions, assuming that the dealer intends to stay in the market, because the source of his income is two-way trade, buying foreign currency from customers and selling it back at a higher price. This observation gives an important reason why the dealer has to be concerned about net sales, *i.e.*, changes in his foreign exchange position. The change in the position, in turn, is equal to net sales during a trading session:

[5] *Cf.* Suvanto (1982a), where the price sensitivities differ on each side of the market.

[6] The half-spread is a measure of the transaction cost. A simultaneous purchase and sale of one dollar involves *two* transactions and costs the amount of the full spread.

[7] See *Figure 1*. Note that the spread is $2z(t)$ and the average volume of trade is $[p(t)+q(t)]/2$.

(5) $$x(t) - x(t+1) = p(t) - q(t) = \alpha(t) - \delta s(t),$$

where $x(t)$ is the foreign exchange inventory at the beginning of the trading session and $x(t+1)$ at the end. Note that net sales are measured in terms of flows of foreign currency over the trading session. Hence the foreign exchange position is a stock variable representing the dealer's inventory of dollars at a given moment.

Define the *equilibrium quotation* $[s^o(t),\ z^o(t)]$ as that combination of $s(t)$ and $z(t)$ that will maximize the trading income subject to the constraint that net sales must be equal to zero. It is obtained by solving the following problem:

(6) $$\max_{s(t),\, z(t)} \{\alpha(t)s(t) + \beta(t)z(t) - \delta[\, s^2(t)+ z^2(t)]\}$$

$$\text{s.t. } \alpha(t) - \delta s(t) = 0,$$

which gives

(7) $$s^o(t) = \alpha(t)/\delta, \quad z^o(t) = \beta(t)/2\delta.$$

It is seen that in periods of high demand (high $a(t)$, therefore both $\alpha(t)$ and $\beta(t)$ are high) the equilibrium mid-rate is high and the spread is wide. Similarly, in periods of high supply (high $b(t)$, therefore $\alpha(t)$ is low and $\beta(t)$ is high) the equilibrium mid-rate is low but again the equilibrium spread is wide.

This result brings out the distinction between the demand for and the supply of foreign exchange by customers, on the one hand, and the demand for dealer services, on the other. Whereas the mid-rate is adjusted to balance the customer orders, the spread is the price of the dealer's liquidity services. It increases with the volume of trade (and hence with $\beta(t)$), reflecting the dealer's monopoly power in the market.

At the quotation $[s^o(t),\ z^o(t)]$ the trading income becomes

(8) $$R^o(t) = \beta^2(t)/4\delta,$$

which is equal to the equilibrium spread $2z^o(t)$ times the equilibrium volume of trade; *cf.* equation (3). The latter is obtained by dividing the sum of expected sales and purchases at the quotation $[s^o(t),\ z^o(t)]$ by two, and it is equal to $\beta(t)/4$.

Assume next that the dealer has a given inventory $x(t)$ of dollars and that he wants to go to some target level $x(t+1)=x^*$ during the trading session. This implies that the dealer has to announce a quotation that differs from the equilibrium one in order to attract net sales by the amount $x(t)-x^*$. Maximizing $R(t)$ with respect to $s(t)$ and $z(t)$ and subject to the constraint that $x(t)-x^*-\alpha+\delta s(t)=0$ leads to the following state-dependent quotation:

(9) $$s(t) = s^o(t) - (1/\delta)[\, x(t) - x^*], \quad z(t) = z^o(t).$$

Only the mid-rate is state-dependent. It is below the equilibrium rate if the current position is above the target and above it if the current position falls short of the target. The spread, in turn, is independent of both the initial and the target position, and it is always chosen to maximize the return on the one-period equilibrium volume of trade. This result will appear throughout the subsequent analysis.

The *Lagrange coefficient* associated with the net sales constraint is

(10) $$\mu(t) = s(t) - (2/\delta)\,[\,x(t) - x^{*}].$$

It is one of the key variables and is interpreted as the *shadow price* of one unit of foreign currency. It is the price the dealer would be willing to pay or offer for one unit of foreign currency in order to reduce the position constraint marginally.[8]

Several straightforward extensions are possible within the framework of this simple one-period model. For instance, one can incorporate the dealer's transaction costs into the analysis. Assuming a constant cost per dollar bought or sold will widen the spread, although not by the full amount. It does not, however, affect the mid-rate rule. The extra cost is thus shared by the dealer and his customers. On the other hand, assuming that customers bear a constant transaction cost per dollar transacted will lead to a narrower spread. Again the mid-rate rule remains unchanged and the extra cost is shared by both counterparts.[9] As these extensions do not affect the structure of the problem, they are ignored below.

4 Sequential Pricing Decisions under Transactions Uncertainty

Let us now introduce transactions uncertainty into this simple one-period model. Transactions uncertainty arises because, as a market maker, the dealer sets prices in advance and the customers then decide whether to buy or sell at these prices and in what amounts during a short trading session. The simplest way to introduce transactions uncertainty is to assume that the shift variables $a(t)$ and $b(t)$ in eqs. (1) and (2) are random variables with a constant mean: $a(t)=a+u(t)$ and $b(t)=b+v(t)$, where $u(t)$ and $v(t)$ are serially uncorrelated, independent of each other and have zero expectation.

As a result, net sales are now stochastic:

(11) $$x(t) - x(t+1) = \alpha - \delta s(t) - w(t),$$

where $\alpha=a-b>0$ and $w(t)=v(t)-u(t)$. The expected trading income is equal to the right-hand side of eq. (4), except that $\alpha(t)$ and $\beta(t)$ are replaced by α and β, respectively, $\beta=a+b>0$. Maximizing the expected trading income subject to the constraint that expected net sales are zero, *i.e.*, $x(t)-\mathbf{E}_t\{x(t+1)\}=0$, gives the following equilibrium (zero net sales) quotation:

(12) $$s^{o} = \alpha/\delta, \quad z^{o} = \beta/2\delta.$$

[8] This can be seen by inserting the quotation (9) into the revenue function (4) and differentiating it with respect to $[x(t)-x^{*}]$.

[9] These examples are discussed in Suvanto (1982b).

This quotation is used as a reference price in much of the subsequent discussion. The corresponding expected trading income is denoted by R^{o}.

In the following the stochastic version of the one-period model is extended to a situation where the position is allowed to fluctuate, but the quotation can be changed from session to session in order to control the expected position. Following Zabel (1981), a trading day is divided into T trading sessions, which are of equal length and which together make a *trading day*.[10] The length of the day and the number of sessions within the day are exogenously given. Above we defined a target level x^{*} for the dealer's dollar holdings at the end of a short trading period. In the following the position constraint is assumed to apply only for the end of the day, not for each trading session separately.

The end-of-day position target x^{*} can be interpreted as the closed position. The interpretation is formally correct if it is assumed that the dealer has borrowed an amount x^{*} of dollars, which appears on the liability side of his balance sheet. A closed overnight position target can be justified by risk considerations. For notational reasons the closed position is in the following defined as $x^{*} = 0$.[11]

In order to facilitate settlements, the same amount has been placed as liquid demand deposits with foreign commercial banks (*nostro accounts*) and therefore appears on the asset side of the balance sheet. The actual trading balances $x(t)$, in turn, are constantly fluctuating.[12] The amount of borrowing, as well as the average trading balances, are fixed over a longer period. Therefore, the net interest costs are fixed, and do not affect the dealer's short-run pricing decisions.

Although the position may be closed at the beginning and at the end of the trading day, the intraday position $x(t)$ fluctuates throughout the day as a result of the deals done during successive dealing sessions. In this interpretation, $x(t) > 0$ is a *long position*; *i.e.*, the dealer has overbought dollars and the dollar-denominated assets exceed liabilities. Similarly, $x(t) < 0$ is a *short position*; *i.e.*, the dealer has oversold dollars and the dollar-denominated assets fall short of liabilities. With an open position the dealer may lose or gain if something happens that changes the average arrival rates of customers' buy and sell orders in such a way that it would call for a new equilibrium mid-rate. Therefore, the assumption of a closed overnight position target involves a substantial degree of risk aversion.

Assume that the dealer opens the day with a given position $x(0) = 0$ and he wants to be in a closed position at the end of the day. At the beginning of each trading sesssion $(t,\ t+1)$, the dealer is free to announce a new quotation at which he stands ready to trade on the basis of incoming buy and sell orders during that short period. His objective is to maximize the expected trading

[10] The assumption that the trading periods are of equal length is not essential in itself. What matters for the subsequent analysis is that the number of trading periods per day is given and that the degree of transactions uncertainty is the same for each period.

[11] The formal analysis does not depend on how the target position x^{*} is interpreted, as long as it is fixed. Therefore, the end-of-day position target need not be equal to the closed position. Speculation with an open overnight position is not, however, analyzed in this paper.

[12] As the mirror image, the dealer's trading balances in domestic currency are fluctuating as well, although in the opposite direction.

income during the day subject to the system constraint as well as to the constraint that the expected position at the end of the day is closed. The system constraint, or the state equation, describes how the position evolves in discrete time.

This formulation of the problem leads to a straightforward application of dynamic programming techniques (cf. Bertsekas, 1981, *Ch. 2*). The dealer's problem is to find a feedback controller of the form $\{ f_i [x(i)] \; ; \; i = 0,1, ..., T-1\}$, which tells us that if at moment t the state is $x(t)$, then the control $f_t [x(t)]$ should be applied. The control $f_t : \mathbb{R} \mapsto \mathbb{R}^2$ is a mapping from the state space (the position) into the control space (the quotation) such that $f_t [x(t)] = [s(t), z(t)]$. The state equation is of the form $x(t+1) = x(t) - [\, p(t) - q(t) - w(t)]$.

Assuming a finite horizon and perfect information on the current state, the dynamic programming algorithm can be written as follows:

(13) $$J_t [x(t)] = \max_{s(t),\, z(t)} \mathbf{E}_t \{R(t) + J_{t+1} [x(t+1)]\} \quad \text{(value function)}$$

(14) $$J_T [x(T)] = 0 \quad \text{(terminal value)}$$

(15) $$x(t+1) = x(t) - \alpha + \delta s(t) + w(t) \quad \text{(system constraint)}$$

(16) $$\mathbf{E}_t\, x(T) = 0 \quad \text{(terminal state constraint)},$$

where $t = 0, 1, 2, ..., T-1$, and $\mathbf{E}_t$ stands for the expectation at moment t. The one-period revenue function $R(t)$ in equation (13), written with stochastic disturbances, is

(17) $$R(t) = \alpha s(t) + \beta z(t) - \delta[\, s^2(t) + z^2(t)] + z(t)[\, u(t) + v(t)] + s(t)[\, u(t) - v(t)].$$

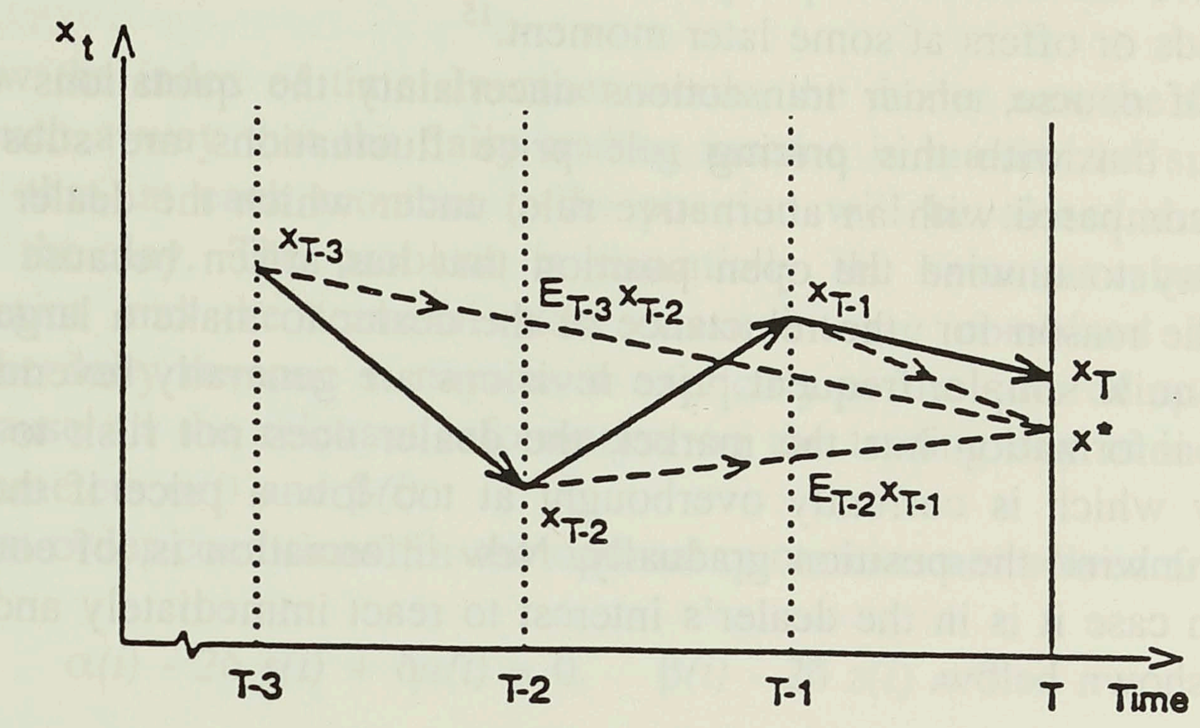
The value function $J_t [x(t)]$ gives the expected revenue from moment t until the end of the day when optimal control is applied at each future moment $t+1$, $t+2$, ..., $T-1$. No cost is attached to the possible open position at the end of the day, which makes the terminal value $J_T [(x(T)]$ equal to zero. This assumption can be dropped without affecting the results if the horizon is extended indefinitely over many trading days, provided that the requirement of the closed expected position at the end of each day is maintained.

Following standard dynamic programming techniques, the sequential pricing rule is derived by solving first the optimal quotation for the last period ($T-1$, T) and proceeding recursively backward in time.[13] The resulting feedback controller, or the sequence of functions expressing the optimal quotation for each period (t, $t+1$) as a function of the current state, has the form:

(18) $$s(t) = s^o - (1/\delta)\, x(t)/(T-t), \quad z(t) = z^o,$$

[13] The solution is presented in a technical appendix, which is available on request from the author.

Figure 2. *Expected and Actual Position*



Explanations **: x^* is the end of day target position, x_t is the the realized position, and E_t is an expectation (a plan) at moment *t*.**

where s^o and z^o are equal to the one-period equilibrium quotation derived above; *cf.* equation (12). As in the one-period case, only the mid-rate is adjusted to steer the position. The spread remains constant at the level that maximizes the one-period return on the equilibrium volume of trade.

Inserting the quotation (18) into the net sales equation (11) and noting that $\alpha - \delta s^o = 0$ gives the following change (increase) in the foreign exchange position in session (t, $t+1$):

(19) $$x(t+1) - x(t) = - x(t)/(T-t) + w(t).$$

The first term on the right-hand side expresses the planned, or expected, change in the position. Hence, according to the optimal pricing rule (18), the dealer quotes in such a way that he can expect to close a fraction $1/(T-t)$ of the possible open position both during the current session and in each of the remaining sessions of the trading day. This result is illustrated in *Figure 2*.[14]

Constant planned net sales for each of the remaining trading sessions implies a constant planned quotation. The actual quotation is adjusted in proportion to the latest realized transactions shock:

(20) $$s(t+1) - s(t) = - (1/\delta)\, w(t)/(T-t-1).$$

Unexpected net purchases, $w(t)>0$, lead to a downward adjustment of the quotation, and *vice versa*. Note that the coefficient of proportionality increases as t approaches $T-1$.

The fact that the planned or expected price is constant for the remainder of the day corresponds to the familiar market efficiency in competitive models

[14] The pricing rule follows a Walrasian *tatonnement* process in real time: $dp/dt = f(D-S)$, $f' > 0$, where $D - S$ refers to the excess demand that has already occured (realized net sales). Unlike in the case of an continuous auction system, $f(0)$ need not be equal to zero in all trading periods, *cf.* Zabel (1981).

(*cf.* Samuelson, 1965, Fama, 1970, and Amihud and Mendelson' 1980, p. 32). The fact that intra-day quotations are martingales implies, *inter alia*, that customers have no reason to postpone a transaction in anticipation of obtaining better bids or offers at some later moment.[15]

Of course, under transactions uncertainty the quotations will never be constant, but with this pricing rule price fluctuations are substantially restrained, compared with an alternative rule, under which the dealer would immediately try to unwind the open position that has arisen because of earlier net sales. The reason for the reluctance of the dealer to make a large price adjustment is quite simple: frequent price revisions are generally revenue-reducing. If no new information hits the market, the dealer does not rush to sell back the currency which is currently overbought at too low a price if there is enough time to unwind the position gradually. New information is, of course, possible, in which case it is in the dealer's interest to react immediately and promptly, as will be shown below.

5 New Information and Jumps in Quotations

It is to be noted that the constancy of the planned quotation and the constancy of the spread result from the assumption that transactions uncertainty is characterized by serially uncorrelated disturbances of the customers' buy and sell orders. There are no jumps in the shift variables α and β describing the average flows of customer orders. This is equivalent to saying that no new information hits the market changing the dealer's perception about the average arrival rates of future customer orders. If that happens in the middle of the day, the quoted price will react immediately. For instance, if the demand curve jumps up, the equilibrium mid-rate also jumps up (because α goes up) and the spread widens (because β goes up).

It is instructive to make a clear distinction between stochastic transaction disturbances and jumps in the model parameters. For that purpose let us forget, for a while, the stochasticity of customer orders, and assume that the future flows of customer orders are known to the dealer, but not to customers. The sequential problem can be reformulated in the following form:

(21) $$\max_{s(i),\, z(i)} \sum_{i=0}^{T-1} \{\alpha(i)s(i) + \beta(i)z(i) - \delta[\, s^2(i) + z^2(i)]\}$$

$$\text{s.t.} \quad x(t) - \sum_{i=t}^{T-1} [\alpha(i) - \delta\, s(i)] = 0,$$

where i is now the index of time. In other words, the dealer sets the sequence of prices in such a way that the daily trading income is maximized subject to the constraint that, at each moment t, the position will be closed during the remainder of the day. The problem is essentially the same as above. The system constraint and the terminal state constraint are combined into one expression. The daily revenue is expressed explicitly as the sum of one-period revenues, because all the relevant information on the trading day is incorporated in the parameters $\alpha(i)$ and $\beta(i)$.

The first-order conditions for the optimal quotations are solved from

(22) $$\alpha(i) - 2\delta\, s(i) + \delta\mu(t) = 0, \qquad \beta(i) - 2\delta\, z(i) = 0,$$

where $\mu(t)$ is the Lagrange coefficient associated with the position constraint at moment t. The spread depends only on each period's volume of trade $\beta(i)$, $z(i) = \beta(i)/2\delta$, whereas the mid-rate depends on each period's excess demand $\alpha(i)$, as well as on the position constraint. The mid-rate is obtained from the first equation as a function of the Lagrange coefficient:

(23) $$s(i) = [\alpha(i)\,/\delta + \mu(t)]\,/2.$$

Inserting this into the position constraint, adding up over the remaining trading sessions and solving for $\mu(t)$ gives

(24) $$\mu(t) = [\delta(T-t)]^{-1}\,\{\sum_{i=t}^{T-1} \alpha(i) - 2x(t)]\}\,.$$

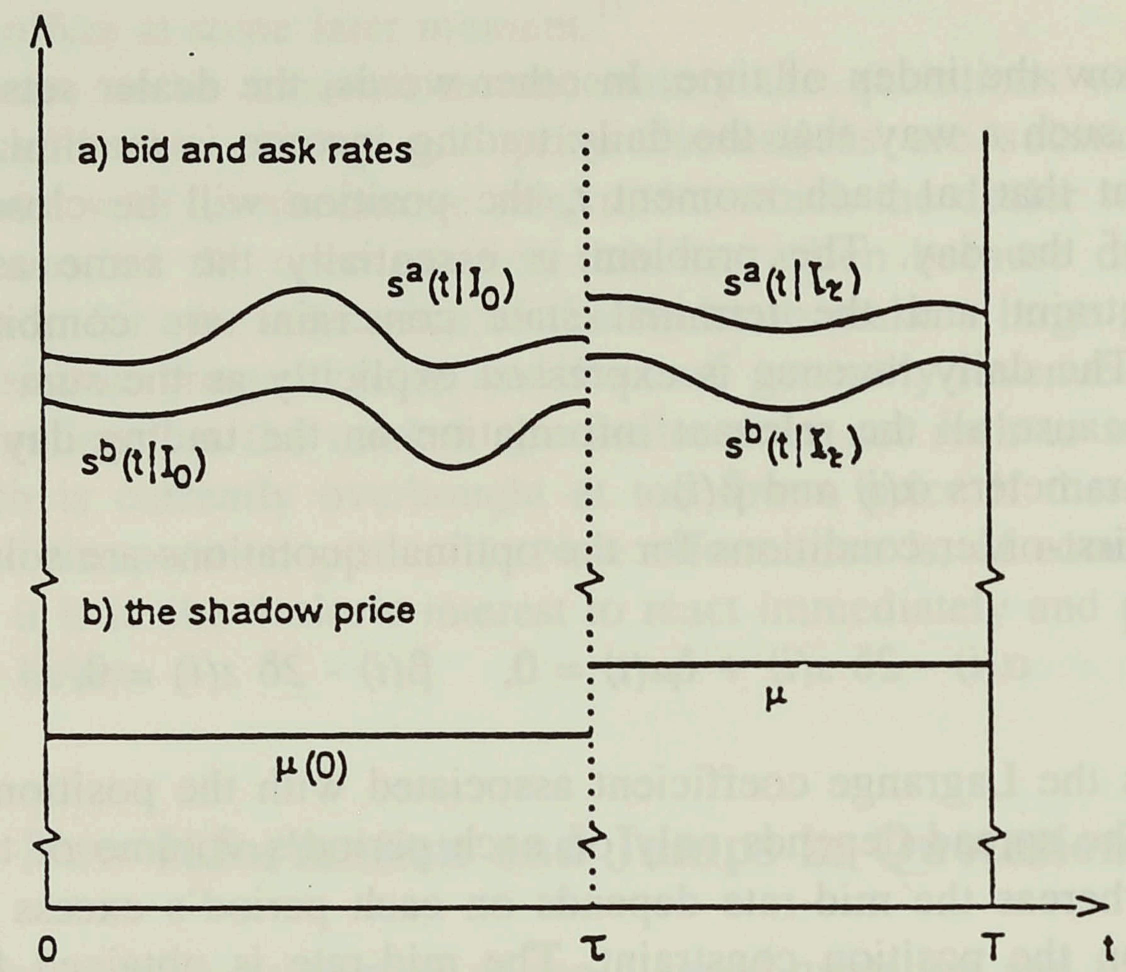
It is obvious from (24) that the shadow price $\mu(t)$ contains all the relevant information on the remainder of the day. It is transmitted to quoted prices via equation (23). If there is no new information, $\mu(t)$ will remain constant for the remainder of the day. It can be shown that $\mu(t+1) = \mu(t)$ by shifting (24) forward and inserting into it the net sales in period $(t, t+1)$; *i.e.*, $x(t) - x(t+1) = \alpha(t) - \delta s(t)$.

What is interesting in this result is that the mid-rate and the spread do change, although the shadow price remains constant. In periods of high customer demand, the mid-rate is high and the spread is wide, implying that the ask price rises more than the bid price. Correspondingly, in periods of high customer supply the mid-rate is low, but the spread again is wide, implying that the bid rate declines more than the ask rate.

It is also obvious from equation (24) that $\mu(t)$ reacts to new information. Assume that at moment t the dealer learns that at some later time $i>t$ during the remainder of the day, there will be high demand. From eq. (24) it is seen that $\mu(t)$ will jump up immediately, as will the mid-rate according to equation (23). The jump does not depend on whether high demand will materialize very soon or at some later period. An equal change in any of the remaining $\alpha(i)$'s affects the shadow price by an equal amount. The spread, however, will rise

[15] The empirical evidence on the behaviour of exchange rates within a trading day is scarce, although expanding. A study by Wasserfallen (1989) using intra-day data (drawn at five-minute intervals) on one single bank's quotations seems to confirm the weak-form efficiency, *i.e.*, the absence of serial correlation within a day. Other studies with hourly data on indicative market quotations (keyed by different banks into electronic informations systems) show significant negative first-order autocorrelation around the time of large jumps in prices; see Ito and Roley (1986) and Goodhart and Giugale (1988). Negative autocorrelation appears to be fairly common in indicative market quotations at ultra-high frequencies, but disappears after ten-minute intervals; see Goodhart and Fiuglioli (1991). Müller *et al.* (1991) analyze continuous 'tick-by-tick' data over a three-year period (taken from Reuters' page FXFX) which shows significant intra-day heteroscedasticity with a regular 'seasonal' pattern.

Figure 3. *New Information and Jumps in Quotations*



only when the increase in demand actually takes place.

Because the mid-rate rises and remains at a higher level throughout the remainder of the day, the increase in the volume of trade will be less than what is implied by the increase in $\alpha(i)$'s, because a higher price reduces other customers' demand.

Figure 3 illustrates the implications of the model for the behaviour of the exchange rate during the day. The curves $s^a(t \mid \mathbf{I}_0)$ and $s^b(t \mid \mathbf{I}_0)$ illustrate the expected time paths of the ask and bid rates conditional on information $\mathbf{I}_0$, available at time $t=0$. Neither the mid-rate nor the spread need be constant. They depend on the dealer's information on the timing of customer orders within the day. At time τ, new information hits the market: both the ask and bid rates jump and thereafter move along the new time paths $s^a(t \mid \mathbf{I}_\tau)$ and $s^b(t \mid \mathbf{I}_\tau)$. This example highlights the distinction between anticipated changes and unanticipated jumps. The distinction becomes even clearer in the lower panel of *Figure 3*, where the time path of the shadow price of foreign exchange associated with the position target is depicted. At time $t=0$, the shadow price is equal to $\mu(0)$. If there is no new information, it will remain constant. At time τ, however, new information hits the market, $\mu(0)$ jumps to $\mu(\tau)$ and remains in the new position unless any further news arrives during the remainder of the day.

In the above model it is important, however, that the information of the sequence of $\alpha(i)$'s and $\beta(i)$'s is the dealer's private information. If the customers had the same information about the trading day, they would know the dealer's forthcoming quotations during the remainder of the day and would therefore choose a different timing for their purchases and sales. As a result, the values of $\alpha(i)$ and $\beta(i)$ would alter until all the predictable changes in the quotations disappeared. If this happens then $\alpha(i)=\alpha$ and $\beta(i)=\beta$ for each i. The

latter situation would lead us back to the stochastic version of the model discussed above, implying a constant spread and a constant planned quotation.

The distinction between the stochasticity of the arrivals of customer orders and unexpected news is also important for understanding some real world phenomena. As was shown above, one result of the optimal pricing strategy under pure transactions uncertainty was the dampening of short-term price fluctuations. On the other hand, the dealer's reaction to new information is a prompt change in prices, which reduces the amount of currency flows that could potentially take place if the customers were better informed than the dealer. As noted by Begg (1989, pp. 28-29), by avoiding taking positions himself and by immediately changing prices to the level that chokes off most of the potential currency flows, the dealer restores the business to smaller-sized and more balanced trade in both directions and reverts to servicing the currency conversion needs of liquidity-oriented commercial customers.

6 Introducing Inter-Dealer Transactions

Although the circumstances referred to above are far from those characterizing the near-perfect foreign exchange market in reality, the results of Sections 3 to 5 are applicable to more complicated and more realistic situations, such as customer flows and inter-dealer competition. In the following we analyze such situations by starting from a simple one-period model into which we now introduce inter-dealer transactions.

Assume that the global foreign exchange market is composed of a large number of local markets with one foreign exchange dealer in each. Assume further that transaction and information costs prevent non-dealer customers from trading outside their local market. The latter assumption, which will be relaxed later on, implies that each dealer can control the expected buy and sell orders of his local customers by changing quotations. To that extent the results of earlier sections are directly applicable. The complication that arises is that we now allow for dealers to send buy and sell orders to each other.

Any one dealer can, whenever he so desires, place an order in the interbank market. Note that in an interbank transaction the roles of the two counterparties differ. The one who takes the initiative for a transaction by requesting a quotation acts in the role of a customer, while his counterparty, who makes a price, acts in the role of a market maker. In order to distinguish between different types of transactions, let us call the interbank transactions the dealer makes on his own initiative *cover transactions*.

For the time being, it is assumed that the share of any one dealer in total trade is so small that he can disregard any systematic reactions by others to his own actions. Quantitative characteristics, especially those describing the customers' average buy and sell orders may differ from one local market to the next, but qualitatively the situation is the same in each local market. As above, each dealer maximizes his trading income during the day subject to price-dependent customer orders and to the constraint that the the expected foreign exchange position at the end of the day is closed (or at some other well-defined target level).

Let us first consider a dealer who is able to initiate a transaction with other dealers, but who never receives orders from other dealers. The justification for this, admittedly artificial, assumption might be, for instance, the small size of the local market, or that the dealer applies too wide a spread.

Omitting the stochastic terms, the expected buy and sell orders of the customers in a local market j are written as follows (*cf.* eqs. 1 and 2.):

$$p_j(t) = a_j(t) - c[s_j(t) + z_j(t)] \tag{25}$$

$$q_j(t) = b_j(t) + c[s_j(t) - z_j(t)], \tag{26}$$

where $s_j(t)+z_j(t)$ is the ask rate and $s_j(t)-z_j(t)$ the bid rate quoted by the dealer. The shift parameters $a_j(t)$ and $b_j(t)$ take into account the local characteristics of the market j and c describes the price sensitivity of customer orders.

Let the dealer's initial position be equal to $x_j(t)$ and define the *closed* position at the end of the period as $x_j(t+1) = 0$. The size of a possible cover sale in the interbank market is denoted by $P_j(t) - Q_j(t)$, with $P_j(t) > 0$ representing a sale and $Q_j(t) > 0$ representing a purchase. When $P_j(t) = Q_j(t) = 0$, no cover transaction is made. Note that, due to the positive spread in the interbank market, a simultaneous sale and purchase in the interbank market is not profitable, except in rare cases when quotations are inconsistent (see below).

Although the dealer may not know exactly what prices other dealers will quote if he places an order with them, it is assumed that, being in constant touch with the market, he knows at least the average bid and ask rate quoted in the outside markets.[16] He can take this information into account when making his own price and deciding upon a possible wholesale transaction. Hence, he knows that he can buy foreign exchange in the interbank market at a price that most likely is not higher than $S^a(t)$ (the average outside ask rate) and sell foreign exchange there at a price that most likely is not lower than $S^b(t)$ (the average outside bid rate).

The dealer's net revenue is the expected trading income from customer deals adjusted for the cash outflow (inflow) of a possible cover purchase (sale):

$$R_j(t) = \alpha_j(t)s_j(t) + \beta_j(t)z_j(t) - \delta[s_j^2(t) + z_j^2(t)], \tag{27}$$
$$+ S^b(t)P_j(t) - S^a(t)Q_j(t),$$

where $\alpha_j(t) = a_j(t)-b_j(t)$, $\beta_j(t) = a_j(t)+b_j(t)$, $\delta=2c$. The first three terms indicate the net revenue from customer deals and the remaining terms take into account the contribution of a cover transaction.

The dealer is assumed to maximize the one-period net revenue subject to the constraint that at the end of the period the position is closed; *i.e.*, $x_j(t) - P_j(t) + Q_j(t) - p_j(t) + q_j(t) = x_j(t+1)=0$. The corresponding Lagrangian

$$L_j(t) = R_j(t) + \mu_j(t)[x_j(t) - \alpha_j(t) + \delta s_j(t) - P_j(t) + Q_j(t)] \tag{28}$$

[16] This is a somewaht tricky assumption, because if he knows the average he should know something about the distribution and be able to find a better price than the average. In practice, all market participants are fully informed about the indicative prices, which, though frequently revised, are continously displayed on *Reuters* and *Telerate* (*cf.* Goodhart and Demos, 1990a, pp. 333-335). For the purposes of the subsequent analysis one could regard these indicative prices as representing estimates of the average transaction prices for each moment.

is discontinuous and therefore non-differentiable with respect to $P_j(t)$ and $Q_j(t)$ at those points where either a cover sale or a cover purchase becomes profitable. Taking this into account, the differentiation of (28) with respect to the four decision variables, $s_j(t)$, $z_j(t)$, $P_j(t)$ and $Q_j(t)$, gives the following first-order conditions:

(29) (i) $\alpha_j(t) - 2\delta s_j(t) + \delta\mu_j(t) = 0$

(ii) $\beta_j(t) - 2\delta z_j(t) = 0$

(iii) $\mu_j(t) = S^b$ when $P_j(t) > 0,\ Q_j(t) = 0$
$\mu_j(t) = S^a$ when $P_j(t) = 0,\ Q_j(t) > 0$
$S^b(t) < \mu_j(t) < S^a(t)$ when $P_j(t) = Q_j(t) = 0.$

The spread, $z_j(t) = \beta_j(t)/2\delta$, depends only on the local market characteristics, but is independent of the position. The mid-rate is state-dependent and depends on the size of a possible cover transaction, in addition to the initial position, as follows:

$$s_j(t) = s^o_j(t) - (1/\delta)[x_j(t) - P_j(t) + Q_j(t)]. \tag{30}$$

The corresponding shadow price is

$$\mu_j(t) = s_j(t) - (2/\delta)[x_j(t) - P_j(t) + Q_j(t)]. \tag{31}$$

Because $S^a>S^b$, simultaneous cover purchases and sales are out of the question. Whether the cover transaction is a sale, a purchase, or neither, depends on the initial position, as well as on the outside prices, as follows:

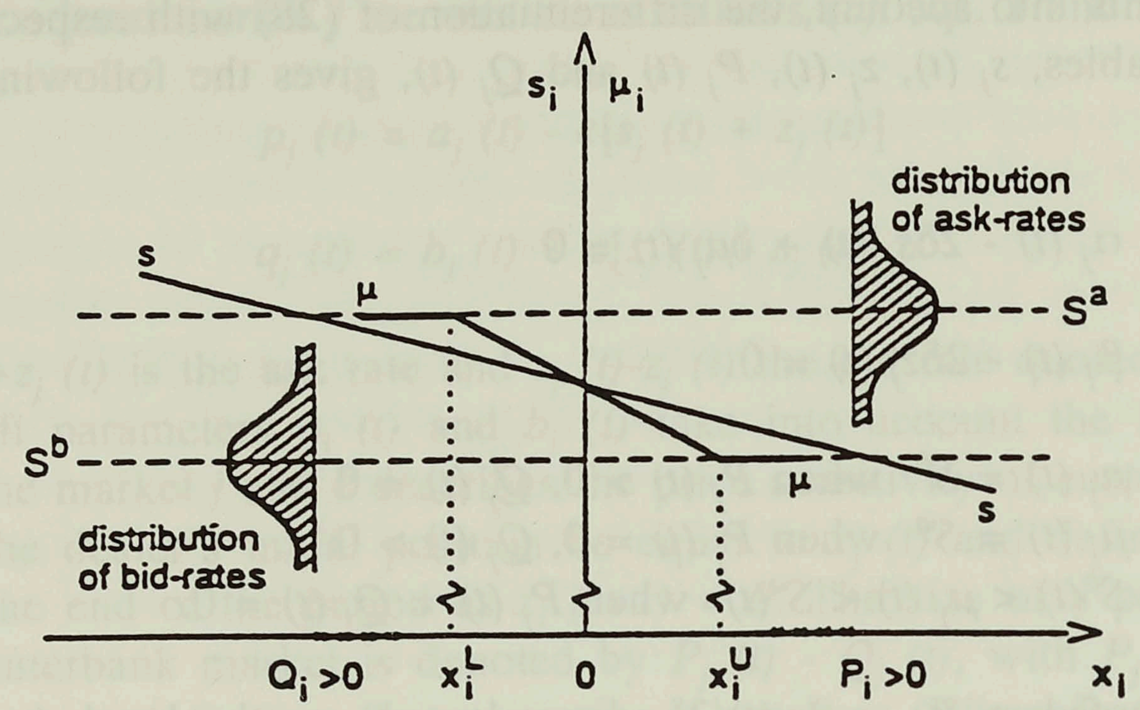
$$P_j(t) > 0,\ Q_j(t) = 0 \ \text{ if } x_j(t) > (\delta/2)[s^o_j(t) - S^b(t)] \equiv x^u_j(t) \tag{32}$$
$$P_j(t) = 0,\ Q_j(t) > 0 \ \text{ if } x_j(t) < (\delta/2)[s^o_j(t) - S^a(t)] \equiv x^l_j(t)$$
$$P_j(t) = Q_j(t) = 0 \ \text{ if } x^l_j(t) \le x_j(t) \le x^u_j(t),$$

where $x^u_j(t)$ is that level of an open (*long*) position above which a cover sale becomes profitable. The reason for this is the fact that at $x_j(t) > x^u_j(t)$ the shadow price of foreign exchange would become lower than the bid price available in the interbank market. Similarly, $x^l_j(t)$ is the size of the open (*short*) position below which a cover purchase becomes profitable, because for $x_j(t) < x^l_j(t)$ the shadow price would become higher than the ask price available in the interbank market. These position limits result from the availability of outside quotations and they determine the range for the prices quoted to customers. In terms of the mid-rate, this is

$$[s^o_j(t) + S^b(t)]/2 \le s_j(t) \le [s^o_j(t)+S^a(t)]/2. \tag{33}$$

Figure 4 illustrates the result. Both the mid-rate (*ss*-line) and the shadow price (*μμ*-line) are depicted as a function of the initial position that has to be

Figure 4. *Cover Transactions in the Interbank Market*



squared during one trading session. Within the range $[S^b(t), S^a(t)]$ the two lines are downward sloping, with the $\mu\mu$-line declining twice as fast as the *ss*-line. The limits $x^l_j(t)$ and $x^u_j(t)$ are determined at those points where the $\mu\mu$-line intercepts the bid and ask rates available in the interbank market. Beyond these points the $\mu\mu$-line becomes horizontal. The corresponding limits to the mid-rate are obtained from the *ss*-line at these position levels. If the initial position is above $x^u_j(t)$ it is adjusted by a cover sale of the size $x_j(t) - x^u_j(t)$ and the remaining $x^u_j(t)$ is closed by net sales to customers. If $x_j(t)$ is below $x^l_j(t)$, the position is adjusted by a cover purchase of the size $x^l_j(t) - x_j(t)$ and the remaining $x^l_j(t)$ is squared by net purchases from commercial customers.

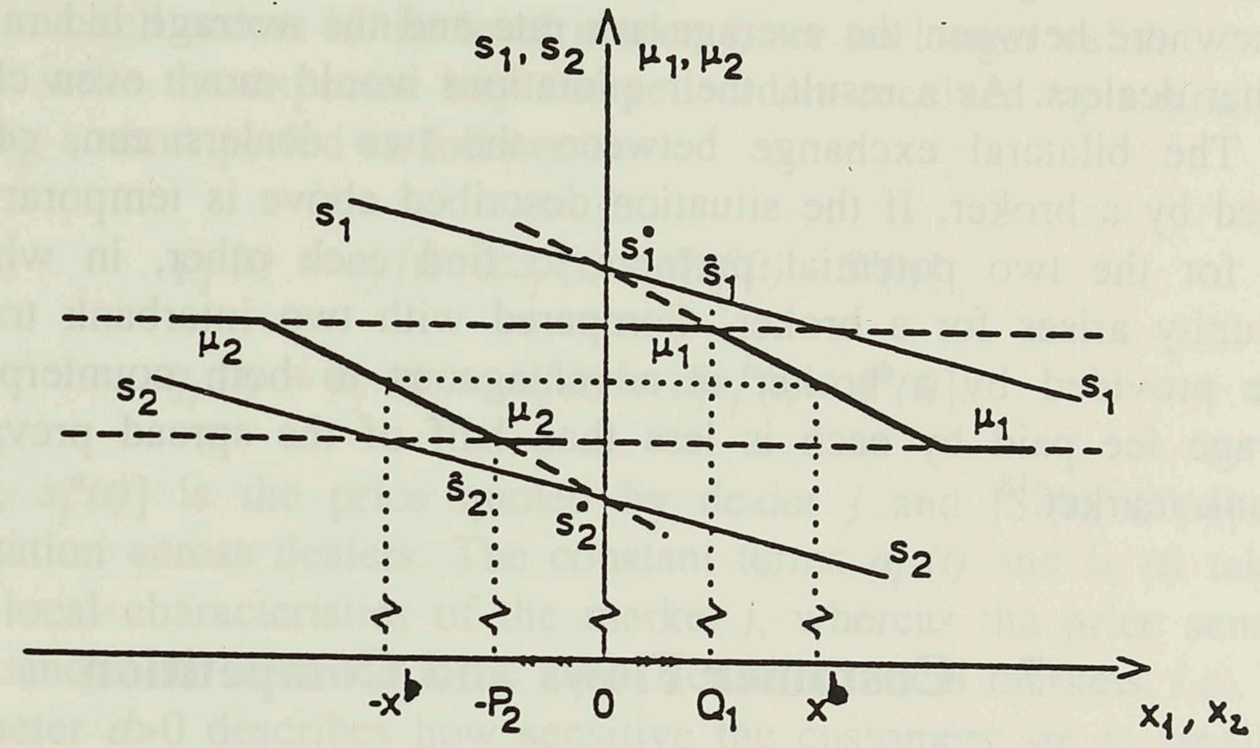
As mentioned above, the price available in the interbank market is not well defined. At each moment, there are several dealers quoting prices without full knowledge of how others are quoting at the same moment. Therefore, the prices quoted by individual dealers may at any moment t differ across dealers. In *Figure 4* this is taken into account by drawing a (non-observed) frequency distribution of the bid and ask prices in the market.

Above it was assumed that $S^b(t)$ and $S^a(t)$ represent average or indicative rates which are common knowledge to all dealers. If better prices are possible, it is in the dealer's interest in each situation to try to observe the best one of the available prices by requesting a binding quote from a number of counter-parties. When such a request proves successful, the dealer concludes a cover deal before the position reaches either of the limits derived above.[17]

As in the case of *Section 3*, the spread, $z_j(t) = \beta_j(t)/2\delta$, is always chosen to maximize the return on the equilibrium volume of customer trade. It is independent of the initial position as well as of the outside quotations. Because it depends on the characteristics of the local market, it may differ from one market to another. This result follows from the assumption that dealer j operates in an isolated environment in the sense that he does not receive interbank orders from other dealers and that local customers are unable to *shop*

[17] It is, of course, possible that he can find an ask rate that is lower than some bid rate, implying an arbitrage profit opportunity. We return to this situation below.

Figure 5. *Interbank Transactions and Integration of Local Markets*



around. These assumptions will be relaxed in *Section 7*.

The most important result, however, is the integration of local markets brought about by the cover facility in the interbank market. The possibility of placing interbank orders with other dealers tends to draw local quotations closer to the global average. At the same time the local characteristics become less important as far as the mid-rate is concerned.

Consider, for example, two local markets, each having only a small share of global trade. Let us assume that in the first market (market *1*) one currency is relatively scarce in the sense that the local customers' demand is high relative to what they supply. In the second market (market *2*) this currency is relatively abundant. *Figure 5* illustrates the point. Assuming that initially the dealers in the two markets have a closed position and assuming full isolation, they would quote the mid-rates s_1^o and s_2^o, respectively, the latter being much below the former because of the differences in local characteristics. Having the possibility to trade in the interbank market, dealer *1* finds it profitable to place a sell order in this market in order to purchase an amount Q_1 at the average ask rate (the shadow price is equal to S^a). The size of the purchase would be larger if the dealer were able to receive an even lower offer price. By lowering his own quotation somewhat to $\hat{s}_1$ or below, he can sell the resulting long position to his customers and make a higher profit compared to the case of complete isolation. Similarly, dealer *2* finds it profitable to sell an amount $-P_2$ in the interbank market at the average bid rate (the shadow price is equal to S^b). At a bid higher than S^b the sale would be even larger. As a result dealer *2*'s quotation is drawn upwards to $\hat{s}_2$, or above, if he is able to observe a higher bid price in the interbank market.

This is the mechanism by which local foreign exchange markets become integrated, even though some of them may be isolated from each other. The two dealers need not meet directly, nor through a broker or some other middleman; rather, they meet through the interbank market made up of other dealers standing ready to trade with dealers *1* and *2* on immediate demand. If these two dealers were symmetrically related to the *average market* and if they were able to meet directly, they would find it even more profitable to bypass the interbank market altogether and exchange an amount x^b bilaterally at a price which is somewhere between the average ask rate and the average bid rate quoted by the other dealers. As a result, their quotations would move even closer to each other. The bilateral exchange between the two dealers can, of course, be arranged by a broker. If the situation described above is temporary, it may be costly for the two potential partners to find each other, in which case an opportunity arises for a broker. Compared with two interbank transactions, a service provided by a broker is advantageous to both counterparties if the brokerage fee paid by each is less than half of the spread prevailing in the interbank market.[18]

7 Customer Flows and Competition

Although the two dealers in the above example could place orders in the interbank market, both remained in a relatively isolated environment and, in addition, both could fully exploit the monopoly power in relation to local customers. We can go further and relax the assumption that transaction and information costs prevent local customers from trading outside their local market. Instead of prohibitively high transaction costs, we can assume that local customers face greater transaction costs when trading outside than with the local dealer. The customers may also differ in this respect. In fact, we can assume a continuum of customers ordered according to their relative transaction costs. At one end, there are well-informed customers with low transaction costs. These customers choose to go outside once they observe a quotation that is only marginally better than the one quoted by the local dealer. At the other end, there are those customers for whom it is hardly ever possible to approach any of the outside dealers when they need to convert one currency into another. The allowance of *customer flows* of this kind introduces inter-dealer competition into the model.

Though admittedly vaguely defined, these plausible economic frictions leave each local dealer with some freedom to make his own prices and hence to control the expected net sales by his own decisions. Accordingly, we can continue analyzing the behaviour of an individual dealer *as if* he had a short-term monopoly.[19] As a market maker, the dealer announces his quotation upon request and stands ready to trade with any incoming order without knowing in advance whether the customer is a potential buyer or a seller and without knowing whether the same customer is asking quotations from other dealers at the same moment. By these assumptions the possibility of price discrimination is excluded.

[18] This is one justification for the use of broker services in the foreign exchange market. These are rather common in practice, especially in cases when larger-than-normal amounts are being exchanged. According to the BIS Survey, 20 to 50 per cent of the total net turnover in the foreign exchange market in April 1989 was arranged through brokers (BIS, 1990, p. 10).

[19] This assumption is similar to that of Phelps and Winter (1970), who analyzed atomistic competition with slow diffusion of information on prices between local market places.

For the sake of symmetry, we should allow for the possibility that the same dealers themselves may receive orders from outside and also that the other dealers are eager to find the best available prices for such transactions. These assumptions imply that equations (25) and (26), which describe the customers' buy and sell orders in the local market *j*, are no longer valid.

Let us write the expected buy and sell orders received by a single local dealer *j* during a short period as follows:

$$p_j(t) = a_j(t) - cs_j^a(t) - d[s_j^a(t) - S^a(t)] \tag{34}$$

$$q_j(t) = b_j(t) + cs_j^b(t) + d[s_j^b(t) - S^b(t)], \tag{35}$$

where $[s_j^b(t), s_j^a(t)]$ is the price quoted by dealer j and $[S^b(t), S^a(t)]$ is the average quotation across dealers. The constant terms $a_j(t)$ and $b_j(t)$ take into account the local characteristics of the market j, whereas the price sensitivity parameters c and d are assumed to be equal across all local markets, *i.e.*, for all j. The parameter $d>0$ describes how sensitive the customers are to small price differentials, while the parameter $c>0$ shows how sensitive the non-dealer customers are, in the aggregate, to a uniform change in prices everywhere.[20] If the customers' information and transaction costs of trading outside the local market are low, then d is large relative to c. It will be even larger if the dealer does not regard interbank orders from the other dealers as purely random events, but instead recognizes the fact that he is more likely to receive orders from outside if his quotation differs from the global average.

As above, the dealer's expected trading income per period from customer trade adjusted for a possible cover transaction is

$$\begin{aligned} R_j(t) &= s_j^a(t)p_j(t) - s_j^b(t)q_j(t) + S^b(t)P_j(t) - S^a(t)Q_j(t) \\ &= [\alpha_j(t) + \eta S(t)]s_j(t) + [\beta_j(t) + \eta Z(t)]z_j(t) - (\delta+\eta)[s_j^2(t) + z_j^2(t)] \\ &\quad + [S(t) - Z(t)]P_j(t) - [S(t) + Z(t)]Q_j(t), \end{aligned} \tag{36}$$

where $\alpha_j(t) = a_j(t) - b_j(t)$, $\beta_j(t) = a_j(t) + b_j(t)$, $\delta=2c$, $\eta=2d$, $s_j(t) = [s_j^a(t) + s_j^b(t)]/2$, $z_j(t) = [s_j^a(t) - s_j^b(t)]/2$, $S(t) = [S^a(t) + S^b]/2$ and $Z(t) = [S^a(t) - S^b(t)]/2$. It is seen that the equation is formally similar to equation (27); only the parameters and their interpretation are different.

Assuming that the dealer maximizes the one-period revenue subject to the condition that the end-of-period position is closed leads to the following problem:

$$\underset{s_j,\, zj,\, P_j - Qj}{\textbf{Max}} \quad R_j(t); \tag{37}$$

$$\text{s.t. } x_j(t) - [P_j(t) - Q_j(t)] - [\alpha_j(t) + \eta S(t) - (\delta+\eta)s_j(t)] = 0,$$

[20] This is seen by adding equations (4.4) and (4.5), respectively, over all N markets, $i=1, ..., N$, and taking the average.

where the last term (within the brackets) of the system constraint is equal to net sales to customers associated with the mid-rate $s_j\ (t)$. The resulting Lagrangian is non-differentiable with respect to $P_j\ (t) - Q_j\ (t)$ at those points where this becomes either positive or negative. Applying the average interbank prices $[S^b(t),\ S^a(t)]$ to cover transactions, the first-order conditions for the maximum can be written as follows:

(38) (i) $s_j\ (t) = [\alpha_j\ (t) + \eta S(t)]/2(\delta+\eta) + (1/2)\mu_j\ (t)$

(ii) $z_j\ (t) = [\beta_j\ (t) + \eta Z(t)]/2(\delta+\eta)$

(iii) $\mu_j\ (t) = S^b(t)$, when $P_j\ (t) > 0$
$\mu_j\ (t) = S^a(t)$, when $Q_j\ (t) > 0$
$S^b\ (t) \le \mu_j\ (t) \le S^a(t)$, when $P_j\ (t) = Q_j\ (t) = 0$.

When $P_j\ (t) = Q_j\ (t) = 0$, the mid-rate is obtained directly from the system constraint,

$$s_j\ (t) = [\alpha_j\ (t) + \eta S(t) - x_j\ (t)]/(\delta+\eta). \tag{39}$$

The corresponding $\mu_j\ (t)$ is

$$\mu_j\ (t) = [\alpha_j\ (t) + \eta S(t) - 2x_j\ (t)]/(\delta+\eta). \tag{40}$$

It is seen that the local quotation $[s_j\ (t),\ z_j\ (t)]$ now depends directly on quotations elsewhere $[S(t),\ Z(t)]$, a relationship which follows from the fact that customers (including dealer customers) are sensitive to price differentials across market makers. The spread is no longer entirely dependent on local conditions or on the local dealer's price making power.

Recall that local quotations were also dependent on the prices quoted elsewhere in the previous example, but in that case the result was entirely due to the fact that the local dealer could on his own initiative adjust his position by making cover transactions in the interbank market. In the present case, with customer flows and with the possibility of interbank orders arriving from other dealers, local quotations are drawn even closer to each other; *i.e.*, the spread narrows, as does the dispersion of prices. In the limiting case, as $\eta \to \infty$, the mid-rate becomes the same everywhere, $s_j\ (t) \to S(t)$ and the spread approaches zero, $z_j\ (t) \to Z(t)/2 \to 0$. The latter result follows from the fact that $z_j\ (t) \to Z(t)/2$ must hold for each j, which is possible only if $Z(t)$ approaches zero.

The conditions (38) determine whether or not a cover transaction is profitable. The answer depends on the initial position and on what price offers the dealer obtains in the interbank market. If the initial position $x_j\ (t)$ is such that without a cover transaction $\mu_j\ (t)$ would go above S^a, a cover purchase $Q_j\ (t) > 0$ is profitable until $x_j\ (t) + Q_j\ (t)$ reaches the point $x^l_j\ (t)$ at which $\mu_j\ (t) = S^a\ (t)$ (or the lowest observed ask rate). Similarly, if the initial position would, without a cover transaction, bring $\mu_j\ (t)$ below $S^b\ (t)$, a cover sale $P_j\ (t) > 0$ becomes profitable until $x_j\ (t) - P_j\ (t)$ reaches the point $x^u_j\ (t)$ at which $\mu_j\ (t) = S^b\ (t)$ (or the highest observed bid rate). Between these limits no cover transaction is profitable.

The problem is formally equivalent to the previous example. Therefore, *Figure 4* serves to illustrate the result in the present case as well. Because $\delta+\eta>\delta$, the *ss*-curve and the $\mu\mu$-curve both decline less steeply, and therefore the position, which is not covered in the interbank market, is allowed to fluctuate in a wider range:

$$x^l_j\ (t) \equiv (1/2)(\delta+\eta)[s^o_j\ (t) - S^a(t)] \le x_j\ (t) \le (1/2)(\delta+\eta)[s^o_j\ (t) - S^b(t)] \equiv x^u_j\ , \tag{41}$$

where $s^o_j\ (t) \equiv [\alpha_j\ (t) + \eta S]/(\delta+\eta)$ is the equilibrium mid-rate that would balance dealer j's customer orders (non-dealer and dealer alike) on the average. Note that it is possible that $s^o_j \ge S^a(t)$, in which case a closed initial position $x_j\ (t) = 0$ would imply an immediate cover purchase and a downward adjustment of the price, a process in which foreign currency flows from the interbank market to dealer j's customers; *cf. Figure 5*. Or *vice versa*, $s^o_j\ (t) \le S^b(t)$ would imply an immediate cover sale and an upward adjustment of the price.

The less steep $\mu\mu$ -curve should reduce the likelihood of profitable cover transactions, because any open position can be easily unwound by marginal changes in quoted prices. On the other hand, greater price sensitivity reduces the size of the average spread and the dispersion of quotations, which increases the likelihood of profitable cover transactions in the interbank market. In the limiting case as $\eta \to \infty$ the spread becomes zero and $s_j\ (t)$ cannot deviate from $S(t)$, implying that it does not matter whether an open position is covered in customer trade or in the interbank market.

The price differentials and the resulting interbank transactions disappear in aggregation across all market makers. The global average price is

$$S(t) = A(t)/\delta - (1/\delta)X(t), \qquad Z = B(t)/(2\delta+\eta), \tag{42}$$

where $A(t) = \Sigma_n\ \alpha_n(t)/N$, $B(t) = \Sigma_n\ \beta_n(t)/N$, and $X(t) = \Sigma_n\ x_n\ (t)/N$, the summation being taken over all N dealers $n=1,\ 2,\ ...,\ N$. The aggregated open position $X(t)$ is inherited from earlier global transaction shocks and is likely to be comparatively small in proportion to the global volume of trade. The effect of any local transaction shock on the global average price is small, because individual positions can be traded in the interbank market, whereby local shocks are diffused globally. Because the extent of genuine currency conversion needs by the non-bank public is likely to be limited in any short period, it is the interbank trade which grows most rapidly as the spread becomes increasingly narrow.

One consequence of having a narrower spread and larger volume of interbank trade is that non-bank customers benefit less and less from shopping around. In the limiting case, with zero spread and prices exactly the same everywhere, it would not matter with which dealer a customer places his order. However, interbank transactions would at the same time lose their economic meaning, because then it would not matter on which side of the market each dealer operates. In fact, the whole notion of the dealership market in producing liquidity services would lose its *raison d'être*.[21]

8 Arbitrage and Uncertainty

As long as there are economic frictions that make customer orders less than infinitely elastic with respect to price differentials and as long as the dealers do not have complete information about how others are quoting at the same moment, each single dealer is left with some price-making power, which can be used for position adjustment purposes. However, because each dealer quotes prices independently of, and simultaneously with, the other dealers, the quotations are bound to differ across the market makers at any particular moment. This implies the possibility of arbitrage profit opportunities from time to time.[22]

An arbitrage opportunity arises if one dealer quotes an ask rate that is lower than the bid rate quoted by some other dealer at the same moment. In this situation, any third party who is quick enough to observe the pair of inconsistent quotations has an opportunity to make an immediate riskless profit by buying from the former and simultaneously selling to the latter. The arbitrager can be any third dealer or a well-informed non-dealer customer.

The two dealers who quoted inconsistently in the first place will lose if an arbitrage operation moves their positions far off the desired path. At the next moment they either have to restore the position in the interbank market at unfavourable prices or to make a relatively large adjustment to their customer quotation. In each case the quotation will change in the direction that will eliminate, or at least reduce, the probability of inconsistent quotations immediately after the event.

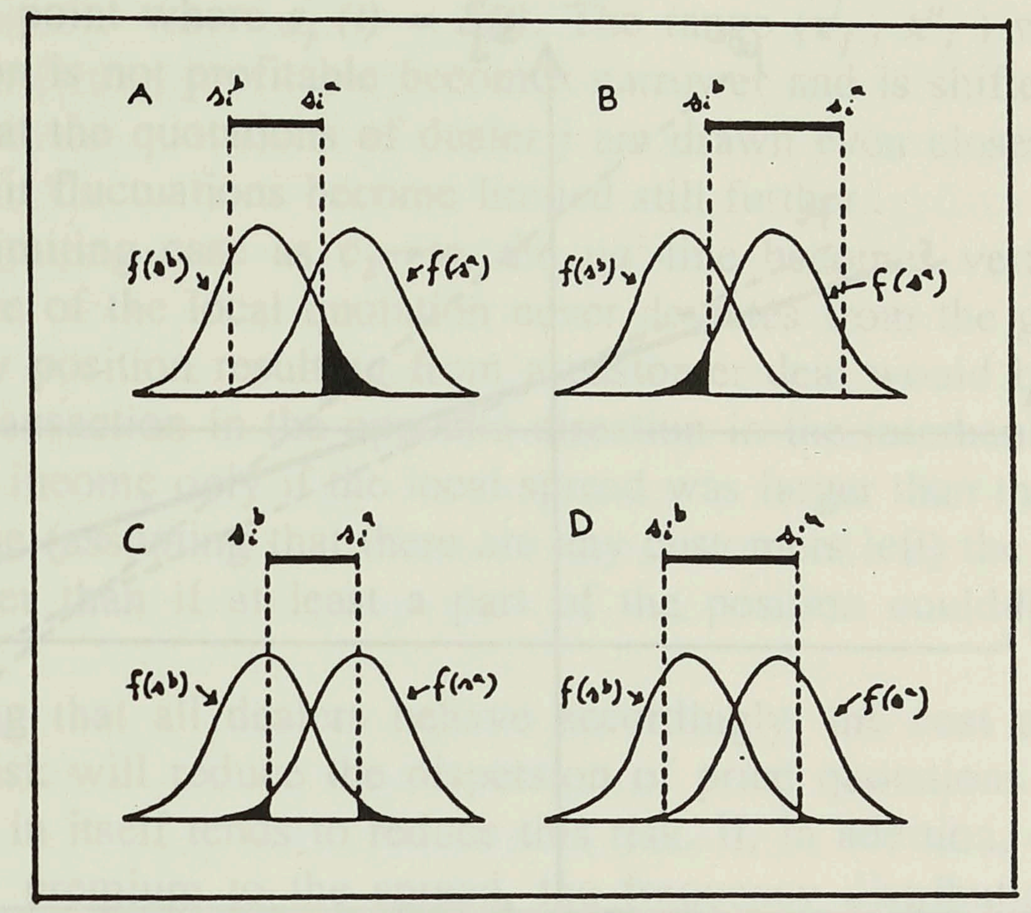
The possibility of an arbitrage opportunity was already illustrated in *Figure 4*. It shows the frequency distributions of the ask rate and the bid rate in the interbank market. As long as the two frequency distributions overlap, there is always a positive probability for an arbitrage opportunity to arise; *i.e.*, that at moment t two dealers m and n, without knowing each other's prices, quote in such a way that $s_m^a(t) < s_n^b(t)$.

Figure 6 illustrates the point still further. Its four panels depict the frequency distributions of the ask rate and the bid rate quoted in the global market as well as the price quoted by a local dealer j at the same moment. The shaded area in each panel illustrates the probability of an inconsistent quotation for a given two-way price $(s_j^b(t), s_j^a(t))$ quoted by dealer j. The larger the shaded area, the larger is the probability that there is another market participant who is able to observe an inconsistent quotation and to make a profit at the cost of dealer j. A comparison between panels C and D shows that, for a given

[21] Zero spread is equivalent to complete disappearance of monopoly profits arising from the dealers' capacity to exploit price-sensitive customer orders. The spread remains positive when the costs of producing dealer services are taken into account, even if the customers are infinitely sensitive to price differentials. This case is discussed in Suvanto (1982b).

[22] "If the markets were perfectly arbitraged all the time, there are never any profits to be made from the activity of arbitrage. But then how do arbitragers make money, particularly if there are costs associated with obtaining information about whether markets are already perfectly arbitraged?", *cf.* Grossman and Stiglitz, 1976, pp. 247-248).

Figure 6. *Probability of Inconsistent Quotations*



Explanation: In each panel, *f(.)* shows the frequency distribution of the ask-rates and bid-rates quoted by the dealers at a particular moment. The sum of the traded areas in each panel illustrates the probability of inconsistency for a given price (s_i^b, s_i^a) quoted by dealer i.

mid-rate, the wider the spread, the smaller is the probability of it being used in arbitrage between inconsistent prices.[23] A comparison between panels A and C, or B and D, shows that this probability is positively related to the deviation of the mid-rate in dealer j's quotation from the global average price.

These observations suggest that imposing a cost for the risk of inconsistency will affect the individual dealer's spread positively and will draw the local quotations even closer to the global average. This can be shown formally by postulating the following cost function:

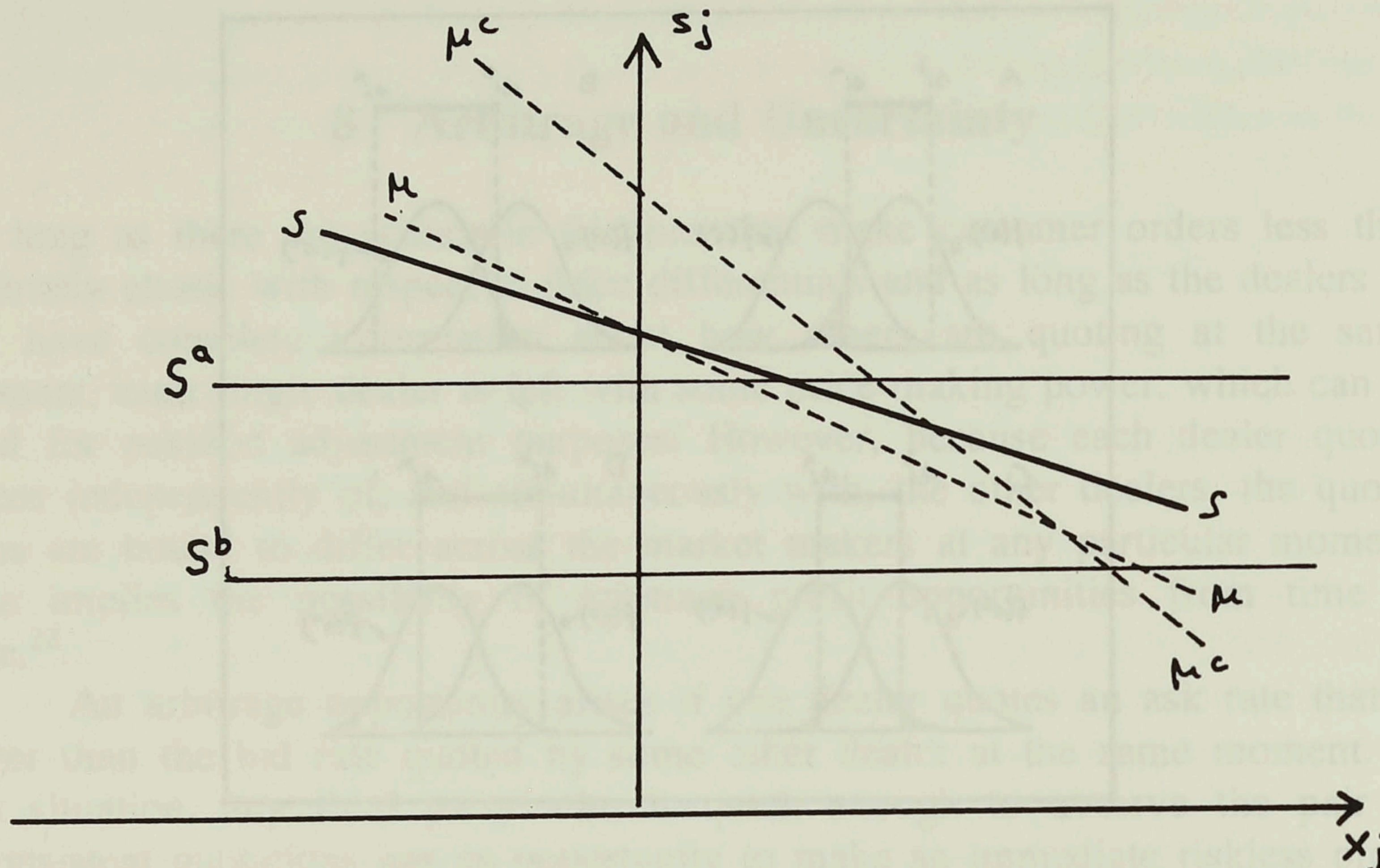
$$C_j(t) = c_1 z_j(t) + c_2 [s_j(t) - S(t)]^2, \quad c_1 > 0, c_2 > 0, \qquad (42)$$

where the cost attached to the inconsistency risk is assumed to be linear with respect to z_j and quadratic with respect to $s_j(t) - S(t)$.

Maximizing the net revenue function $R_j(t) - C_j(t)$, where $Rj(t)$ is defined in eq. (36), with respect to $s_j(t)$, $z_j(t)$ and $Pj(t) - Qj(t)$ and subject to the position constraint $x_j(t) - [\alpha_j(t) + \eta S(t) - (\delta+\eta) s_j(t)] - [(P_j(t) - Q_j(t)]$ gives the following first-order conditions:

[23] This situation is parallel to the presence of informed *insiders* in the securities market. Because of information asymmetries, a stock dealer is unable to distinguish between insiders and *liquidity traders*. Because the dealer loses on average in trade with insiders, he has to compensate for the loss by applying a spread which is wider than the marginal cost of providing the dealer services, see Glosten and Milgrom (1985), Kyle (1985), Glosten (1989), Dennert (1989) and Hagerty (1991).

Figure 7. *Pricing Rule under Inconsistency Risk*



(43) (i) $s_j(t) = [\alpha_j(t) + (\eta + 2c_2)S(t) + (\delta+\eta)\mu^c_j(t)]/2(\delta+\eta+c_2)$

(ii) $z_j(t) = [\beta_j(t) + \eta Z(t) + c_1]/[2(\delta+\eta)]$

(iii) $\mu^c_j(t) = S^b(t)$ when $P_j(t) > 0$
$\mu^c_j(t) = S^a(t)$ when $Q_j(t) > 0$
$S^b(t) < \mu^c_j(t) < S^a(t)$ when $Q_j(t) = P_j(t) = 0$,

where μ^c_j is the shadow price for the position constraint. The superscript c is added to distinguish it from μ_j in the previous case considered; *cf.* eq. (40).

It is seen that the inconsistency risk makes the spread wider, as expected. It widens as c_1 increases. However, with a very large spread, the dealer would lose all his customers.

Compared to the previous example, the mid-rate rule does not change. When no cover transaction is made, $s_j(t)$ is obtained directly from the position constraint; *cf.* eq. (39). What does, however, change is the shadow price of the position constraint and, therefore, the limits to the position at which either a cover purchase or a cover sale becomes profitable will also alter. The shadow price $\mu^c_j(t)$ is solved from the condition (iii) above and it is equal to

(44) $$\mu^c_j(t) = \mu_j(t) + [2c_2/(\delta+\eta)]\,[s_j(t) - S(t)].$$

Because both $s_j(t)$ and $\mu_j(t)$ depend on $x_j(t)$ negatively, $\mu^c_j(t)$ declines much more rapidly than $\mu_j(t)$ as $x_j(t)$ increases, implying that the range within which the position is not squared by cover transactions diminishes.

Figure 7 illustrates the result in the case where the characteristics of the local market j would imply a price level above the global average; *i.e.*, $\alpha_j > \Sigma_n \alpha_n$. It shows the mid-rate rule (*ss*-line) as well as two lines for the shadow price, one for the case where the inconsistency risk is not taken into account

($\mu\mu$-line) and the other for the case when this risk matters ($\mu\mu^c$-line). As shown above, the latter line declines more rapidly than the former and the two lines intercept at the point where $s_j(t) = S(t)$. The range (x^l_j, x^u_j) within which a cover transaction is not profitable becomes narrower and is shifted to the right. This implies that the quotations of dealer j are drawn even closer to the global average and their fluctuations become limited still further..

In the limiting case as $c_2 \to \infty$, the $\mu\mu^c$-line becomes vertical, implying that the mid-rate of the local quotation never deviates from the global average. In this case any position resulting from a customer deal would be immediately covered by a transaction in the opposite direction in the interbank market. This would generate income only if the local spread was larger than the average, and even in that case (assuming that there are any customers left) the income would be much smaller than if at least a part of the position could be sold to the customers.

Assuming that all dealers behave accordingly, the cost attached to the inconsistency risk will reduce the dispersion of price quotations between local markets, which in itself tends to reduce this risk. If, in addition, all dealers add a precautionary premium to the spread, the frequency distributions of the ask rate and the bid rate in *Figure 6* will become narrower and the distance between the means will increase. However, as long as the dealers quote prices without full knowledge of how others are quoting at the same moment, and as long as the spread is relatively narrow, the probability that occasional arbitrage profit opportunities will arise remains positive.

9 Dynamic Extension and the Integration Across Time Zones

The analysis of *Section 7* shows how the local foreign exchange markets become globally integrated as a result of customer flows and interdealer transactions. A small local transaction shock may be absorbed by the local dealer's position with only minimal subsequent price reactions. On the other hand, a position resulting from a large local transaction shock can always be reversed in the interbank market, whereby its effects will be diffused globally. As the capacity of the interbank market to absorb such shocks is large, compared to that of any single dealer, the price reactions across time zones will remain small.

In the following the analysis is extended to a dynamic framework. For this we use the jump version of the model presented in Section 5. Let the customer orders during a short trading session (t, $t+1$) be represented by eqs. (34) and (35). The trading day is composed of a sequence of such periods, $t=0,...,T-1$. The dealer j maximizes his daily trading income adjusted for possible cover transactions subject to the constraint that any existing position $x(t)$ has to be reversed during the remainder of the day. The problem is written as follows:

$$\text{(45)}\quad \mathbf{Max} \sum_{i=0}^{T-1} \{[\alpha_j(i) + \eta S(i)]s_j(i) + [\beta_j(i) + \eta Z(i)]\, z(i)$$

$$- (\delta + \eta)[s_j^2(i) + z_j^2(i)] + S^b(i)P_j(i) - S^a(i)Q_j(i)\}$$

$$\text{s.t.}\quad x_j(t) - \sum_{i=t}^{T-1} [\alpha_j(i) + \eta S(i) - (\delta+\eta)s_j(i) + P_j(i) - Q_j(i)] = 0,$$

where the variables and the parameters are the same as defined in Section 7. The choice variables are $s_j(i)$, $z_j(i)$, $P_j(i)$ and $Q_j(i)$, $i = 0,\ldots,T-1$.

Following the already familiar procedures, the solutions for the mid-rate and the spread, obtained from the first-order conditions, are

$$\text{(46)}\quad s_j(i) = [\alpha_j(i) + \eta S(i)]/2(\delta+\eta) + (1/2)\mu_j(t),$$

$$\text{(47)}\quad z_j(i) = [\beta_j(i) + \eta Z(i)]/2(\delta+\eta),$$

where $\mu_j(t)$ is the shadow price at moment t associated with the end-of-day position constraint. Insert eq. (26) into the position constraint and solve for $\mu_j(t)$ to obtain

$$\text{(48)}\quad \mu_j(t) = \{\sum_{i=t}^{T-1} [\alpha_j(i) + \eta S(i)] + 2\sum_{i=t}^{T-1} [P_j(i) - Q_j(i)]$$

$$- 2x_j(t)\}\,/(\delta+\eta)(T-t).$$

The mid-rate and the spread may fluctuate over the day, depending on the sequence of customers' excess demand and the volume of customer trade, as long as these are dealer j's private information. In the case of the mid-rate, the price at moment t depends on the whole sequence of $\alpha_j(i)$, $i>t$, whereas the spread is adjusted only in those trading sessions when the change in the volume of trade $\beta_j(i)$ actually takes place. The mid-rate and the spread also depend on the prices quoted by other dealers, *i.e.*, on $S(i)$ and $Z(i)$, although the sequence of future global average prices can hardly be regarded as dealer j's private information.

This model combines the results of *Section 5* and *Section 7*. In particular, the shadow price $\mu_j(t)$ is constant for given information about the day in the present case as well. Although the mid-rate may fluctuate over the day, its fluctuations are limited both because on some occasions it is profitable to adjust the position by undertaking cover transactions in the interbank market and because the customers (including other dealers) will react to price differentials. As above, no cover transaction is profitable as long as $\mu_j(t)$ remains between $S^a(t)$ and $S^b(t)$, *i.e.*, between the average (or the best observed) bid and ask prices quoted in the interbank market. These limits together with equation (46) determine the limits within which the mid-rate is allowed to fluctuate; that is

$$\text{(49)}\quad [s^o_j(t) + S^b(t)]/2 < s_j(t) < [s^o_j(t) + S^a(t)]/2,$$

where $s^o_j(t) = [\alpha_j(t) + \eta S(t)]/(\delta+\eta)$ is the one-period equilibrium mid-rate that would imply zero net sales to customers.

The shadow price - and along with it the mid-rate - will jump immediately each time dealer j receives new information on the customers' net demand during the remainder of the day. Assume, for instance, that at moment t dealer j learns that there will be more sell orders coming from certain customers at some later moment of the day, $i > t$. The shadow price and the mid-rate will jump upwards immediately and will remain at a higher level throughout the remainder of the day. The upward movement in the price will imply more sales to and less purchases from the other customers. Note that the equilibrium one-period mid-rate $s^o_j(t)$ does not change, although the quoted price $s_j(t)$ will jump. It will not, however, jump above the upper limit indicated in inequality (47), because beyond that point it will be advantageous for the dealer to initiate a position in the interbank market by making a cover sale.

An unexpected change in customer orders or in the average interbank quote both affect the dealer j's quote, although only after the event. Assume that a quote ($s_j(t)$, $z_j(t)$) has already been made and the deals concluded. If either the net sales to customers, $\alpha_j(t)$, or the prices quoted by other dealers, $S(t)$, proved to be higher than expected, the dealer would find his position $x(t+1)$ to be excessively short, thus forcing him to raise the price subsequently and/or to cover the position by a purchase in the interbank market. It is seen that transactions uncertainty (related to the stochasticity of customer orders) and price uncertainty (related to the prices quoted by competitors) both have similar effects.

The above results can be applied to a situation where all operators know that an important piece of news will be announced later in the day. If all participants have similar expectations concerning the contents of the announcement, this effect will already be fully reflected in prices in advance. There will be no price effects when the announcement is made, as long as its contents do not differ from expectations. On the other hand, the price reaction will be prompt, if the announcement contains important surprises.

It is, of course, possible that there are differences in opinion concerning the contents of the forthcoming announcement. If this is the case, some participants will be disappointed at the time of the announcement and observe their positions to move in an unwanted direction, which can be reversed only at a cost. A defensive strategy applicable for such situations would be to widen the spread prior to the announcement. This would reduce the risk that the dealer would receive orders from other dealers who have a different opinion and who may even have some inside information on the contents of the forthcoming news; *cf.* footnote 22.

We have already seen in *Section 7* how at any particular moment the prices quoted in different local markets tend to be equalized through interdealer transactions and customer flows. This means that the local markets are integrated spatially. The fact that different local markets are located in different time zones implies that the operating hours of different dealers differ. Some dealers are closing business at the same time as others are opening. In so far as the business hours overlap, the markets in different time zones become spatially integrated. This tends to draw the quotations in two time zones close to each other in the same way as spatial integration draws quotations close to each other across the local markets at any particular moment.

In the same way as local transaction shocks are transmitted spatially within a given time-zone with only negligible effects on the global average price, aggregate transactions shocks are transmitted from one time-zone to another. Assume, for example, that the dealers in time zone *1* have, on the aggregate, overbought a large amount of a given currency late in the afternoon. The aggregate position is therefore excessively long, which would call for a large reduction in prices if the position had to be unwound through customer trade before the end of the day. Assuming that time zone *2* has already opened, the dealers in time-zone *1* are likely to place interbank orders in the opening zone in order to square their positions, as a result of which the reduction in prices is much smaller or there is none at all. The dealers in the opening zone find their position to be long, but they need to reduce their prices only marginally, because they have plenty of time to steer their position in normal customer trade and in the interbank market during the day. Any regular pattern in the exchange of positions across the time-zones during hours when these overlap is taken into account and is fully reflected in the shadow price.

10 Conclusions

The above analysis contains variations on one theme, *i.e.*, the price making behaviour of a foreign exchange dealer who maximizes his net revenue knowing that customer orders are price-sensitive and recognizing the fact that, in order to remain a market maker, he must aim at balancing purchases and sales of foreign exchange in the longer run, although the long run may be as short as one day. These few behavioural assumptions suffice to generate theoretical predictions which are of empirical relevance and broadly consistent with observable facts.

The results are consistent with the efficient market hypothesis in the sense that there is no possibility that any of the customers could predict the dealer's forthcoming quotations on the basis of the dealer's past pricing behaviour. Under pure transactions uncertainty and in the absence of new information, the dealer is reluctant to make a large adjustment to prices, because frequent price revisions are generally revenue-reducing. The necessity to reverse a position that has arisen unexpectedly forces the dealer to change a price in the 'wrong' direction. This result holds equally for a monopoly dealer and for the dealer operating in a competitive environment.

The reluctance of the dealer to make frequent price adjustments does not apply to situations where new information hits the market. Whenever this happens, the quoted price will react immediately. Any new information on forthcoming customer orders will cause a prompt change in the quoted price. Price adjustments are, however, limited by the sensitivity of customers to small price differentials. Because dealers can send orders to each other in the interbank market, the sensitivity of incoming orders to small price differentials is likely to be very large. As a result, the prices quoted by different dealers cannot deviate much from each other.

The price making power of each dealer is reflected in the spread. In monopolist conditions the spread is wide, because the dealer attempts to extract the full rent which can be reaped because of the price-sensitivity of liquidity-oriented commercial customers. The spread becomes narrower when customers

have a possibility to shop around by requesting quotations from more than one market maker, and approaches zero (or the marginal cost of producing dealer services) once interdealer transactions are allowed for. Under pure transactions uncertainty, the spread tends to be constant. This need not be the case if the dealer has private information on customer orders and if he has some monopoly power in price making. In this case the spread widens as the volume of customer trade increases. Customer flows and interdealer transactions, however, greatly reduce the scope for changes in the spread for this reason. The empirical implication is that the spread tends to be large when markets are poorly organized and when there are only few market makers and customers tend to do business with a single bank. These circumstances are likely to promote collusive behaviour. As a result, the share of interbank transactions in the total volume of trade is small. Under competitive conditions, when the spread is very small, prices quoted by all dealers are practically equal and the share of interbank transactions in the total volume of trade is very large.

Price uncertainty is always present in foreign exchange dealing, although in some cases it may be difficult to distinguish between price and transactions uncertainty. Price uncertainty would be small if transactions uncertainty was its only source and if order flows were purely stochastic, apart from the average price sensitivity of customer orders. Any new information on forthcoming customer orders, whether privately or commonly observed, increases price uncertainty. The intraday volatility of exchange rate quotations stems from two sources, one representing normal order flow (customer orders and their cover operations) and the other the arrival of new information. The empirical implication is that the return distribution at ultra-high frequency is likely to exhibit leptokurtosis.

In addition to temporal price uncertainty, instantaneous price uncertainty is present because the dealer has to make a binding two-way price without knowing what prices his competitors are quoting at the same moment. If the price quoted by one dealer happens to deviate from the prices made by others, the position of the former may move in an unwanted direction, which has to be unwound at a loss. This prevents the spread from diminishing indefinitely. In times of increasing uncertainty the spread tends to widen.

If dealers have uniform expectations concerning future events, this information should already be embodied in prices. If dealers have different views, or different information, as regards the contents of a forthcoming news announcement, some dealers will experience losses at the moment when the new information is released. Widening of the spread is a defensive action to avoid losses arising from heterogenous and asymmetric information.

Although the time horizon in the dynamic variations of the model is constrained to one trading day, the main results can be generalized to longer horizons as well. The formal analysis applies when allowance is made for open overnight positions, as long as the end-of-day position target is fixed. In the interdependent market the effects of local transactions shocks and arrivals of new information are quickly diffused spatially across dealers through interdealer transactions in a given time-zone. Overlapping time-zones extend spatial integration temporally as positions are exchanged across zones.

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