Bank of Finland Research Discussion Papers
12 • 2019

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Bank of Finland
Research

Bank of Finland Research Discussion Papers
Editor-in-Chief Esa Jokivuolle

Bank of Finland Research Discussion Paper 12/2019
12 July 2019

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ISBN 978-952-323-280-8, online
ISSN 1456-6184, online

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The Phillips Curve at 60: time for time and frequency

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May 18, 2019

Abstract

We estimate the U.S. New Keynesian Phillips Curve in the time-frequency domain with continuous wavelet tools, to provide an integrated answer to the three most controversial issues on the Phillips Curve. (1) Has the short-run tradeoff been stable? (2) What has been the role of expectations? (3)Is there a long-run tradeoff?

First, we find that the short-run tradeoff is limited to some specific episodes and short cycles and that there is no evidence of nonlinearities or structural breaks. Second, households' expectations captured trend inflation and were anchored until the Great Recession, but not since 2008. Then, inflation over-reacted to expectations at short cycles. Finally, there is no significant long-run tradeoff. In the long-run, inflation is explained by expectations.

Keywords: Phillips Curve; Inflation; Unemployment; Business Cycles; Continuous Wavelet Transform; Partial Wavelet Gain.

JEL codes: C49, E24, E32.

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1 Introduction

Six decades after its birth – Phillips (1958), Samuelson and Solow (1960) –, five decades after being augmented with inflation expectations and the Natural Rate hypothesis – Friedman (1968), Phelps (1967) –, four decades since it started receiving the microeconomic foundations for price-setting and expectations that led to its modern New Keynesian formulation – Woodford (2003) –, and almost two decades after Mankiw (2001) stated that it was inexorable but mysterious, the Phillips Curve remains inexorable and mysterious.[1]

Inexorable because of its undisputed status as the primary model for inflation. It is a building block of modern macro models – e.g., Smets and Wouters (2007) –, and is omnipresent in macroeconomics textbooks, applied macro analyses, as well as in the conduct and analysis of monetary policy. It has motivated a vast academic literature and the constant interest of policymakers – see the surveys by Mavroeidis, Plagborg-Møller, and Stock (2014) and Gordon (2011). Moreover, following the behavior of inflation during and after the Great Recession, it has received a renewed interest – see, e.g., Coibion and Gorodnichenko (2015), Blanchard (2016), Ball and Mazumder (2019), Stock and Watson (2018).

Mysterious because dissension between at least two traditions marks its history: the backward-looking triangle model and the micro-founded New Keynesian Curve – Gordon (2011). Moreover, empirically, its history is one of seemingly stable relationships falling apart upon publication – Stock and Watson (2010). Furthermore, the New Keynesian Phillips Curve (NKPC) features high specification uncertainty and high sampling uncertainty – Mavroeidis, Plagborg-Møller, and Stock (2014). Overall, macroeconomists recurrently discuss its merit – recent examples are Blanchard (2018), Hall and Sargent (2018), and Mankiw and Reis (2018).

There are three main controversial issues on the Phillips Curve: two regarding the short-run Curve – Mavroeidis, Plagborg-Møller, and Stock (2014) – and the other regarding the long-run Curve – e.g., Benati (2015):

1. Has the (short-run) Phillips slope been stable? If not, was it because the Phillips Curve is nonlinear (featuring different coefficients at different phases of the business cycle), or

[1] In this paper we focus on the price inflation NKPC and do not address the Wage Phillips Curve, even though this continues to be a lively strand of research, especially having in mind the ongoing structural changes in demography and the functioning and structure of the labor market.

because the Phillips relation has been subject to gradual changes (say, during the Great Moderation) or to structural breaks (e.g., the Great Recession)?

2. What is the role of expectations of inflation in the (short-run) Phillips Curve? To what extent do they capture the changing patterns of inflation dynamics – namely persistence, forward-looking behavior, changing trends, and anchoring to the policy target?

3. Is the long-run Phillips Curve truly vertical, as most macroeconomists believe after the Natural rate hypothesis? Alternatively, is there any exploitable long-run trade-off, at least for certain levels of inflation or unemployment?

All these questions have simultaneously a time and a frequency domain dimension. Assessing the slope of the short-run Phillips Curve under different macroeconomic conditions calls for estimation of a time-varying coefficient at business cycles frequencies. To assess the role of inflation expectations, we should also estimate its coefficient in the Phillips Curve across time and frequencies as well as detecting if they lead or lag inflation. This is so because whether expectations are anchored, whether they reflect inflation trends or whether they are forward-looking or backward-looking all have different implications at different frequencies. Assessing the Phillips tradeoff in the long-run calls for estimation of the Phillips slope at the lowest possible frequencies.

We contribute to the literature on the Phillips Curve by providing answers to these three key research questions in an integrated way, with a state-of-the-art New Keynesian Phillips Curve (NKPC). Our approach is novel as we rely on the continuous wavelet transform to look at the data. Wavelets give us a natural way to distinguish short from long-run relations, including correlations and lead/lags relationships – see Aguiar-Conraria, Martins, and Soares (2012) and Aguiar-Conraria and Soares (2014). Moreover, Aguiar-Conraria, Soares, and Sousa (2018) and Aguiar-Conraria, Martins, and Soares (2018) showed that wavelet analysis could also be used to estimate equations in the time-frequency domain. In this paper we estimate, for the first time, an empirical NKPC (U.S. data for 1978:I-2018:IV) simultaneously allowing for variation in coefficients along time and across frequencies as well as in the timing of each Phillips Curve relationship, therefore providing new answers to the three questions above.

The paper is structured as follows. In Section 2, we present and motivate the specification of our empirical NKPC, discuss the literature on the three Phillips Curve controversies, and further clarify our contributions to the literature. In Section 3, we briefly describe our methodology. In Section 4, we present the data and perform a preliminary time-frequency analysis of each time series. In Section 5, we present our results of estimation of the Phillips Curve in the time-frequency domain. Section 6 concludes, presenting in a systematic way the stylized facts uncovered in the paper and comparing these with the current literature on the U.S. Phillips Curve.

2 The Phillips Curve at 60

We study a New Keynesian Phillips Curve (NKPC) specified in the spirit of Coibion and Gorodnichenko (2015), Fuhrer (2017) and Coibion, Gorodnichenko and Kamdar (2018):

$$\pi_t = \alpha_1 \pi^e_{t+1} + \alpha_2 \left(u_t - u^N_t\right) + \alpha_3 SS_t \,, \tag{1}$$

where π_t is the inflation rate, π^e_{t+1} is the average expected rate of inflation for the following period by households (given by a survey of consumers), $\left(u_t - u^N_t\right)$ is the difference between the rate of unemployment and the long-term Natural Rate (unemployment gap), and SS_t is the rate of inflation for energy (meant to proxy for supply shocks).

2.1 State-of-the-art NKPC

This empirical NKPC is state-of-the-art, firstly and mainly in what regards expectations. The micro-foundations for price setting and expectations initially led to a purely forward-looking rational-expectations NKPC – as described by Woodford (2003). Yet, it did not take long until the literature became aware that estimation under rational expectations was prone to substantial econometric problems.[2] Moreover, such specification was not consistent with the inflation persistence observed in earlier post-War monetary regimes as well as with changing trends and the anchoring of inflation in the transition to more recent regimes – Fuhrer and

[2] See the survey by Mavroeidis, Plagborg-Møller, and Stock (2014).

Moore (1995), Benati (2008). Galí and Gertler (1999) and followers tackled the problem with hybrid forward-and-backward-looking NKPCs. Still, the use of survey data for inflation expectations pioneered by Roberts (1995, 1997) gradually emerged as the most successful approach to measure the expectational component of NKPCs. There are several reasons for this success.

First, survey-based expectations exhibit an intermediate degree of rationality, in line with agents' actual behavior and with the estimates for real-world episodes such as disinflations – e.g., Roberts (1998), and Mankiw, Reis, and Wolfers (2003).

Second, Adam and Padula (2011) showed that survey-based expectations could be used in the NKPC provided that agents abide by a weak form of rationality (the law of iterated expectations), which Coibion, Gorodnichenko, and Kamdar (2018) confirm for U.S. surveys.

Third, with some exceptions, see, e.g., Nunes (2010), the literature showed that survey-based expectations dominate backward-looking (accelerationist) and forward-looking (rational) expectations in nested empirical models – e.g., Fuhrer (2012), Fuhrer, Olivei, and Tootell (2012), and Coibion, Gorodnichenko, and Kamdar (2018).

Fourth, the literature has recently shown that survey-based expectations additionally feature two highly useful properties. On the one hand, they capture changes in trend inflation, in the inflation target, and overall structural breaks in inflation dynamics – Coibion, Gorodnichenko, and Kamdar (2018). Consequently, there is no need to model trend inflation with time-series methods as done in alternative literature – e.g., Ascari and Sbordone (2014), and Mertens (2016). On the other hand, survey-based expectations feature intrinsic inertia due to the micro-founded inefficiency with which agents revise expectations, thus bringing inertia into the NKPC and NK models overall. Therefore one can avoid standard ad-hoc mechanisms such as lags of inflation or autocorrelated shocks in the NKPC, or habit formation, adjustment costs and auto-correlated shocks in NK models, as argued by e.g., Fuhrer (2017), Fuhrer (2018), and Coibion, Gorodnichenko, and Kamdar (2018).

Finally, faced with the absence of data from expectations actually formed by firms, as the theory would require, recent research established that households expectations as measured by consumers surveys are the closest to expectations made by firms – Coibion and Gorodnichenko (2015), Coibion, Gorodnichenko and Kumar (2018), and Pfajfar and Roberts (2018).

Our NKPC is also state-of-the-art as regards the proxy for slack. According to the microeconomic foundations of the NKPC, the primary driving variable of inflation should be (besides inflation expectations) real marginal costs – Woodford (2003). These were typically proxied with labor's share – Galí and Gertler (1999). However, the labor share correlates negatively with the output gap – Rudd and Whelan (2007) – and most of its recent variation occurs at low frequencies and not at the typical business cycle frequencies – King and Watson (2012). Therefore, virtually all empirical NKPCs currently use the unemployment gap (or the output gap).

Finally, we control for possible supply shocks, by including energy inflation in the NKPC. This component may seem more reminiscent of Gordon's triangle model of inflation – e.g., Gordon (2011, 2013) – and not much of the fundamental NKPC. However, the theory of the NKPC admits a role for cost-push shocks – Woodford (2003). It is true that a substantial part of the empirical literature on the NKPC does not include such a component,[3] but it is also true that many authors do, e.g. Coibion and Gorodnichenko (2015). We argue (and confirm) that an appropriate proxy for supply shocks is empirically relevant and contributes to a consistent estimate of the NKPC.

Equipped with this state-of-the-art NKPC and our set of continuous wavelet tools, we provide new answers to the three most controversial issues in the current literature of the Phillips Curve. We now discuss the literature background on these three issues and further clarify the novelty of our contribution.

2.2 Three controversial issues

2.2.1 Has the (short-run) Phillips slope been stable?

The literature has challenged the stability of the (short-run) Phillips slope – i.e., the sensitivity of inflation to the unemployment or output gap – focusing on two key aspects: nonlinearities and time-variation. Both have a theoretical background in New Keynesian models, in which several microeconomic foundations have been put forth to justify that price adjustments are costly and therefore infrequent – e.g., models of capacity constraints, menu costs, efficiency

[3] Part of it because of the focus on explaining core inflation, which does not include changes in prices of energy and other commodities.

wages, downward nominal wage rigidity.

It has long been established that the lower the average rate of inflation and its variability, the less frequently prices are adjusted and the flatter is the Phillips Curve – Ball, Mankiw, and Romer (1988). Likewise, downward nominal wage rigidity is more binding in recessions, so that wages and inflation do not fall much although unemployment is rising. Naturally, the deflation repressed during recessions may imply that during recovery wages and inflation react with a lag – Daly and Hobijn (2014).

Regarding nonlinearities, there is a vast literature on the convexity of the U.S. Phillips Curve since the 1990s. Several alternative specifications and methods led to different conclusions – see Clark, Laxton, and Rose (2001) and Hamilton (2001), for papers with opposing results. More recently, several authors found that the U.S. Phillips Curve seems convex: some found that it has been flatter when inflation was below some threshold – e.g. Carrera and Ramírez-Rondán (2017), and López-Villavicencio and Mignon (2015) –, others when the economy was in a recession and early recovery – Daly and Hobijn (2014) – and yet others when macroeconomic volatility (uncertainty) was lower – Castelnuovo and Pellegrino (2018). However, there is also recent research in which it was not possible to reject linearity – e.g., Berger, Everaert and Vierke (2016), and Doser, Nunes, Rao, and Sheremirov (2018).

The existence of structural breaks in the short-run Phillips Curve slope has received much interest recently. The Great Moderation of 1984-2007 resulted in low and stable inflation associated with a credible monetary policy regime and anchored expectations. Given this, NK models would predict that the frequency of price adjustments would fall and that nominal rigidities would become more relevant. Therefore, the NKPC would become flatter.

Consistent with the theory, Benati (2007) found a positive correlation between the trend of inflation and the sensitivity of inflation to the output gap. Also in line with the NK theory, many authors found that the slope of the U.S. Phillips Curve fell since the early- or mid-1980s – e.g., Roberts (2006), Benati (2007), Blanchard, Cerutti, and Summers (2015), Blanchard (2016), and Chin (2018). However, others found no significant change in the U.S. Phillips Curve slope associated with the Great Moderation – see Watson (2014), Coibion and Gorodnichenko (2015), Barnichon and Mesters (2019).

More recently, the apparent missing deflation in the Great Recession and the missing inflation in the subsequent recovery were taken by many as evidence of a breakdown of the NKPC. However, there is no evidence of any structural change in the Phillips Curve slope in the late 2000s – see, e.g. Jorda, Marti, Nechio, and Tallman (2019). The latest fall in the slope with some supporting evidence dates to 1997 – e.g., Pfajfar and Roberts (2018) and Jorda, Marti, Nechio, and Tallman (2019).

2.2.2 What is the role of expectations of inflation in the (short-run) Phillips Curve?

The controversy about the slope of the short-run Phillips Curve is related to our second research question: the measurement and the role of inflation expectations – e.g., Pfajfar and Roberts (2018). The more clear commitment of monetary policy to price stability apparent in the increase of its response to inflation is associated with the Great Moderation and should have led to more stable inflation expectations and a smaller impact of real fluctuations on inflation – see, e.g., Boivin, Kiley, and Mishkin (2010).

Many authors found that the anchoring of inflation expectations from the early-to-mid-1980s until the late 1990s gradually led to a fall in inflation persistence – e.g., Watson (2014) – and even to a loss of any role of inflation lags in the Phillips Curve – e.g., Blanchard, Cerutti, and Summers (2015), Blanchard (2016, 2018). By then, survey data of inflation expectations became the state-of-the-art for empirical analyses of the Phillips Curve. Notably, survey-based expectations dominate backward-looking (accelerationist) and forward-looking (rational) expectations in nested empirical models – e.g., Fuhrer (2012), Fuhrer, Olivei, and Tootell (2012), and Coibion, Gorodnichenko, and Kamdar (2018).

With survey expectations, it became clear that the NKPC found support in the data and did not collapse in the Great Recession and the ensuing recovery. Ball and Mazumder (2019) solved the missing deflation puzzle and found a stable slope by replacing backward-looking expectations with expectations anchored to a constant level – the one apparent in the expectations made by professional forecasters since 1998.[4] Coibion and Gorodnichenko

[4]Ball and Mazumder (2019) made another adjustment to their specification that is also necessary for their results: replacing the total unemployment gap by the short-run unemployment gap. This is motivated by the abnormal difference between total and short-run unemployment in the specific period of 2008-2015 – and by the stronger pressure implied by the latter over wages and therefore prices. Gordon (2013) also uses this line of research in the context of his triangle model of inflation. We stick to the usual unemployment gap, as we

(2015) showed that households expectations as measured by consumers surveys are closer to expectations formed by firms – as confirmed by Coibion, Gorodnichenko, and Kumar (2018) – and dominate all other sources of expectations data such as professional forecasters surveys, Greenbook forecasts and expectations derived from financial markets – as confirmed by Coibion, Gorodnichenko, and Kamdar (2018). Most important, they found that households expectations are not as anchored as alternatives – a result that recently received micro-foundations, as Fuhrer (2018) showed that each consumer partially indexes his expectation to past aggregate expectations. It was their reaction to the increase in oil prices in 2009-11 that prevented deflation, in the context of a stable NKPC slope.[5]

Assessing the fall in the estimates of the NKPC slope at 1997 (about 40%), Pfajfar and Roberts (2018) showed that, when one uses households expectations, the decrease in the frequency of price adjustments may fully explain such fall. This is consistent with the theory of Ball, Mankiw, and Romer (1988) and with the microdata in Nakamura, Steinsson, Sun, and Villar (2018). Moreover, they showed that using households expectations adequately controls for the degree of attention paid by households to macroeconomic conditions, whereby they only revise expectations when unusual changes in macro conditions occur.

Overall, there is the need to estimate the NKPC (with households expectations) using an empirical approach that can detect periods of anchored expectations and periods in which expectations decouple from the inflation target due to salient shocks, i.e., periods in which expectations should relate to inflation broadly on a one-to-one basis at medium-to-long run frequencies, and periods in which expectations relate more strongly with inflation at higher frequencies. Our approach allows for detecting such changes continuously along time and across frequencies, including possible changes in the timing of the relation.

Moreover, by delivering estimates of the NKPC slope continuously both in time and frequency, our approach may discriminate between the literature that suggests a gradual fall in the slope during the Great Moderation and the literature suggesting a structural break at

seek to uncover time and frequency variation in a general state-of-the-art NKPC.

[5] Bianchi and Melosi (2017) suggest an alternative explanation for the increase in inflation expectations in 2009-11 that prevented deflation. They argue that uncertainty about the policy regime, in a context of a binding zero lower bound, may lead agents to anticipate an expansionary fiscal policy and a rise in inflation to control the public debt ratio. The bottom line is that there are various possible reasons for inflation expectations to depart from the anchor.

1997. It may also assess the literature that points to a fall in the slope during recessions, during periods of low inflation or low macro volatility. By also allowing for possible changes in the timing of the relation between the unemployment gap and inflation, our approach may shed light on whether the downward nominal rigidity hypothesis – predicting that the gap should lead inflation during recessions/recoveries – is more relevant than the alternatives, associated with the frequency of price revisions. To assess the slope of the (short-run) NKPC, we focus on business cycle frequencies. Nevertheless, given the evidence that business cycles have elongated in recent decades, there is the need to obtain time-varying estimates of the slope within business cycle frequencies as well as at adjacent frequencies.[6]

2.2.3 Is the long-run Phillips Curve vertical?

Is the long-run Phillips Curve truly vertical, or is there any exploitable long-run tradeoff? After Friedman (1968) and Phelps (1967), the Natural rate hypothesis is part of the core of macroeconomics. However, there is some dispute about it.

Some authors provided evidence in favor of a long-run Phillips curve bending from vertical to a negative slope at low rates of inflation. The seminal paper is Akerlof, Dickens, and Perry (1996). Recent examples of models and simulations include Benigno and Ricci (2011) and Daly and Hobijn (2014). Others followed the model with downward nominal rigidity and near-rationality – suggested by Akerlof, Dickens, and Perry (2000) –, which predicts a negatively sloped long-run Phillips Curve for inflation rates close to the target, and then a positively sloped Curve as it converges to the vertical long-run relation at sufficiently high inflation rates. For example, Svensson (2015) estimated a significantly negative slope for 2000:IV–2011:II. Pure time-series analyses had also produced estimates of negatively sloped long-run Phillips Curves – e.g., King and Watson's (1994). However, such results are not consensual: Benati (2015) found that the evidence of cointegration between inflation and unemployment cannot be considered reliable and that in structural VARs it is not possible to reject the null

[6] Crowley and Hughes Hallett (2018) established that the Great Moderation has been characterized by a transfer of volatility of output growth from the short-end of business cycles (cycles with up to 4 years) to long cycles (longer than 16 years) and maybe even intermediate-frequency business cycles (between 4 and 8 years) and medium-run cycles (8 to 16 years). Such volatility transfer seems associated with the stronger reaction of monetary policy to inflation since the mid-1980s. It is an expression of the trade-off known as "design limit" in optimal control theory, studied by Brock, Durlauf, and Rondina (2008, 2013), whereby feedback policy rules that reduce variance at some frequencies induce increases in variance at others.

of a vertical long-run Phillips Curve.

In contrast, other authors found evidence and explanations for a positively-sloped long-run Phillips curve. Beyer and Farmer (2006) argued that a positive long-run inflation-unemployment relation explains the Great Inflation in an NK model, and obtained estimates of positive cointegration coefficients. Russell (2011) identified eight monetary regimes between 1952 and 2004, and, assuming that inflation was stationary around shifting means, obtained an estimate of a long-run Phillips curve with a small but significant positive slope. Berentsen, Menzio, and Wright (2011) showed that the Hodrick-Prescott trends of inflation and unemployment were positively related, and designed a model with search and bargaining in the labor and goods markets that leads to higher unemployment at higher inflation rates. Haug and King (2014) found that in cycles with a period larger than eight years, inflation and unemployment were positively correlated, with the inflation leading by about three years.

Pure time series methods cannot assess the slope of the long-run NKPC in the recent monetary policy regime, in which inflation does not seem to have a unit root – Benati (2015). An alternative is to use frequency domain methods to isolate very low frequency cycles and then estimate the NKPC slope for those frequencies – Haug and King (2014), Berentsen, Menzio and Wright (2011).

The theoretical foundations for non-vertical long-run Phillips Curves typically depend on the average level of inflation, and in particular on its difference to the inflation target, which varies along time. Therefore, there is the need to estimate the NKPC at very low frequencies allowing for variations in the slope, which is one significant advantage of our continuous time-frequency domain approach.

It is not possible to uncover precise long-run relations from finite samples. Here lays another advantage of our method: as we estimate the NKPC continuously in time and frequencies, we can detect the pattern of association between inflation and the unemployment gap as we move closer to the long-run. In other words, even knowing that we will not reach the long-run, we obtain good indications about it – with, recall, time variation allowing for estimating different long-run slopes of the NKPC under different levels of inflation.

2.3 Our contribution

Overall, the three most controversial issues on the Phillips Curve have both time- and frequency-domain implications. However, there is no literature on the Phillips Curve with a time-frequency domain approach that answers these issues. The paper that is closer to that goal is Gallegati, Gallegati, Ramsey, and Semmler (2011), who use the maximal overlap discrete wavelet transform. Their approach however consists of decomposing the time series into several frequency bands, which is equivalent to decompose the variables with band-pass filters. For each frequency band, they run least squares regressions of the Phillips Curve to detect non-linearities over the full sample period (1948:II-2009:II). They also split the sample at 1992:IV to identify structural changes in the coefficients. Despite quite significant contributions, their approach does not allow for continuously time-varying parameters, does not provide information about correlation at specific frequencies within each band, and does not allow for changes in the structure of leads and lags. Moreover, they study a wage Phillips Curve augmented with price inflation and productivity growth, not a NKPC.

We are the first to apply the Continuous Wavelet Transform to study the NKPC in the time-frequency domain. Specifically, we apply a set of continuous wavelet tools developed recently to a state-of-the-art empirical NKPC, and provide coherent answers to the three most controversial issues on the Phillips Curve – short-run slope, the role of expectations, and long-run slope – in the U.S. 1978:I-2018:IV.

We estimate the partial wavelet coherencies, partial phase-difference diagrams, and partial wavelet gains between inflation and each of the independent variables of equation (1). The partial wavelet coherency, proposed by Aguiar-Conraria and Soares (2014), is a concept akin to the partial correlation. For example, the partial wavelet coherency between inflation and the unemployment gap tells us, for each moment in time and each frequency, the degree of association between those two variables, after controlling for inflation expectations and energy inflation. Therefore, we will have a direct estimation of how this degree of association evolves with time and frequency. On its turn, the partial phase-difference between these two variables will tell us which one is leading at different frequencies and different points in time. It is possible that one variable leads the other at high frequencies and that the relationship is reversed at low frequencies. We will also capture variations that occur across time. Finally, Aguiar-Conraria, Soares, and Sousa (2018) and Aguiar-Conraria, Martins, and Soares (2018) showed how the partial wavelet gain could be used to directly estimate α_1, α_2, and α_3 in equation (1) as a function of time and frequency. This way we have a direct estimation of those coefficients for different points in time and frequency.

The stylized facts and answers uncovered in this paper would be close to impossible to obtain with the traditional econometric tools.

3 Methodology

In this section, we give a brief description of the continuous wavelet tools used in our analysis. For technical details, we refer the reader to Aguiar-Conraria and Soares (2014), Aguiar-Conraria, Soares, and Sousa (2018) and Aguiar-Conraria, Martins, and Soares (2018). For an intuitive explation of the Continuous Wavelet Transform, the reader may rely on Aguiar-Conraria, Magalhães, ans Soares (2012).

3.1 The Continuous Wavelet Transform

For all practical purposes, a *wavelet* is simply a small wave: a *wave*, in the sense that it is a function $\psi(t)$ whose graph oscillates up and down the t-axis (integrating to zero) and *small* meaning that it rapidly decays as $t \to \pm\infty$. To obtain the continuous wavelet transform (CWT) of a time-series $y(t)$ (w.r.t. a given wavelet ψ), we generate a family of continuously translated and dilated versions of ψ, $\psi_{\mathtt{t},\mathtt{s}} = \frac{1}{\sqrt{|\mathtt{s}|}}\psi\left(\frac{t-\mathtt{t}}{\mathtt{s}}\right)$, and compare y with all these wavelets, thus obtaining a function of two-variables $(\mathtt{t},\mathtt{s})$:

$$\mathcal{W}_y(\mathtt{t},\mathtt{s}) = \frac{1}{\sqrt{|\mathtt{s}|}}\int_{-\infty}^{\infty} x(t)\overline{\psi}\left(\tfrac{t-\mathtt{t}}{\mathtt{s}}\right)dt\,, \tag{2}$$

where the over-bar denotes complex conjugation. Note that the parameters $\mathtt{s}$ and $\mathtt{t}$ control, respectively, the width and the location along the t-axis of the function $\psi_{\mathtt{t},\mathtt{s}}$.

If we want to obtain some phase information about the cycles present in a time-series — which will be essential to assess the lead/lag relationships between two series —, it is necessary to work with a complex-valued wavelet ψ.

In our computations, ψ was chosen as the following particular member of the so-called *Morlet family*, introduced by Grossmann and Morlet (1984): $\psi(t) = \pi^{-\frac{1}{4}}e^{i6t}e^{-t^2/2}$; this is the complex wavelet most used in Economics, due to its interesting properties. In particular, when such wavelet is used, one can consider that the Fourier frequency f satisfies $f \approx \frac{1}{\mathtt{s}}$, greatly facilitating the interpretation of the results. Due to this relation between scale and frequency, we will usually refer to the $(\mathtt{t},\mathtt{s})$-plane as the time-frequency plane.

Similarly to the terminology used in the Fourier case, the (*local*) *wavelet power spec-*

trum (WPS) of y, is defined as $WPS_y(\mathtt{t},\mathtt{s}) = \mathcal{W}_y(\mathtt{t},\mathtt{s})\overline{\mathcal{W}_y}(\mathtt{t},\mathtt{s}) = |\mathcal{W}_y(\mathtt{t},\mathtt{s})|^2$. The wavelet power spectrum gives us a measure of the variance distribution of the time-series in the time-frequency plane. When we average the wavelet power over all times, we obtain the so-called *global wavelet power spectrum (GWPS)*, $GWPS_y(\mathtt{s}) = \int_{-\infty}^{\infty} |\mathcal{W}_y(\mathtt{t},\mathtt{s})|^2 \, d\mathtt{t}$, which gives us, essentially, the same information as the Fourier power spectrum.

3.2 Bivariate Analysis

We now describe several wavelet tools which enable us to study the relation between two variables y and x in the time-frequency domain.[7]

The *complex wavelet coherency* of y and x is given by

$$\varrho_{yx} = \frac{S(\mathcal{W}_{yx})}{\sqrt{[S(|\mathcal{W}_y|^2)\, S(|\mathcal{W}_x|^2)]}},$$

where $\mathcal{W}_{yx} = \mathcal{W}_y \overline{\mathcal{W}_x}$ is the so-called cross-wavelet power of y and x and S denotes a smoothing operator in both time and scale. The complex wavelet coherency may be written in polar form as $\varrho_{yx} = \left|\varrho_{yx}\right| e^{i\phi_{yx}}$ with $\phi_{yx} \in (-\pi, \pi]$.

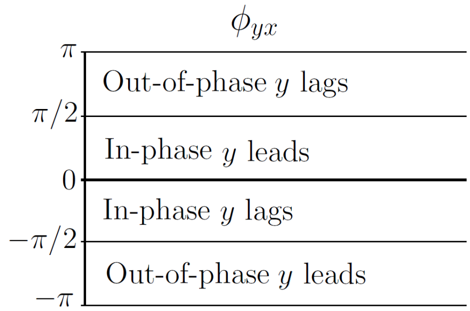

Figure 1: Interpretation of the wavelet phase-difference between series y and x.

The modulus of the complex wavelet coherency is called the *wavelet coherency* and the angle ϕ_{yx} is called the *wavelet phase-difference* between y and x.[8] The wavelet coherency can be seen as the correlation between the two variables at each time and frequency. The phase-difference ϕ_{yx} can be used to obtain an indication on whether the two series are in-phase or

[7]Since all the wavelet measures that we are going to introduce are functions of the two variables, `t` and `s`, in order to simplify the notation, unless strictly necessary, we will describe these quantities for a specific value of the argument, $(\mathtt{t},\mathtt{s})$, which will be omitted in the formulas.

[8]Recall that, given a complex number $z = a + bi$, the modulus of z is given by $|z| = \sqrt{a^2 + b^2}$ and the angle (argument) ϕ can be obtained from $\tan\phi = \frac{b}{a}$, together with the information on the signs of a and b to determine to which quadrant the angle belongs to.

out-of-phase and also on the the the lead/lag relationship between y and x; the interpretation is summarized in the diagram of Figure 1.

Recently, Mandler and Scharnagl (2014) introduced the concept of *wavelet gain* between y and x, G_{yx}, given by

$$G_{yx} = \frac{|S(\mathcal{W}_{yx})|}{\sqrt{S(|\mathcal{W}_x|^2)}}.$$

In analogy with the interpretation of the Fourier gain given by Engle (1976), we can see the gain as the regression coefficient of y on (appropriately shifted) x; more precisely, we can see the the gain as the regression coefficient of the regression of y on x (at each time and frequency), if there is no time lag between the independent and dependent variables; if there is a time lag, the gain can be interpreted as the regression coefficient if the series x was shifted the right amount to eliminate any phase shift, and the phase ϕ_{yx} is the angle by which it would have to be shifted.[9]

3.3 Mutivariate Wavelet Analysis

All the concepts introduced in the last section have generalizations to the case where we are dealing with more than two series; since the formulas are cumbersome, we do not include them here, and refer the interested reader to Aguiar-Conraria and Soares (2014) and Aguiar-Conraria, Martins and Soares (2018).

Given a series y and m series $x_i;, i = 1, \ldots, m$, to obtain a measure of the degree of linear association between the series y and the m series x_i (at each time and frequency), we can compute the *multiple wavelet coherency* between y and $x_1, \ldots, x_m$.

To estimate the interdependence between the variable y and a particular variable x_k, controlling for the effect of the other variables x_j $(j = 1, \ldots, m; j \neq k)$, we can use the *wavelet partial coherency*, the *wavelet partial phase-difference* and the *wavelet partial gain* between series y and x_k, controlling for x_j $(j = 1, \ldots, m; j \neq k)$. In particular, the wavelet partial gain between y and x_k controlling for the series x_j $(j = 1, \ldots, m; j \neq k)$ can be seen as the coefficient of x_k in the the multiple regression of y on the (appropriately shifted) m

[9] With this interpretation, a negative regression coefficient between two simultaneous series would correspond to a positive gain — the modulus of the regression coefficient — and a phase-difference of π.

variables $x_i; i = 1, \ldots, m$.

4 Data

Our data are U.S. quarterly time-series for 1978:I-2018:IV of inflation, the unemployment gap, expectations of inflation and energy inflation. Inflation is the annualized quarterly rate of growth of the consumer price index provided by the U.S. Bureau of Labor Statistics, and energy inflation is the annualized quarterly rate of growth of the respective component of the consumer price index. Inflation expectations are the median expected changes in prices on average during the next 12 months reported by households in the Michigan survey of consumers. The unemployment gap is the difference between the (quarterly average of the) civilian unemployment rate provided by the U.S. Bureau of Labor Statistics and the quarterly estimates of the (long-term) natural rate of unemployment provided by the U.S. Congressional Budget Office (CBO).[10]

Figure 2 presents the data. In addition to the standard time-series graphs – left panel –, we also present their wavelet power spectra (WPS) – middle panel – and the wavelet global power spectra (GWPS) – right panel.

For a finite time-series, the integral involved in the computation of the CWT has to be discretized, being replaced by a summation. Given that the transform is computed only for a finite set of values of the parameters `t` and `s`, the wavelet power ends up being merely a matrix, which we display as a heat map. The same kind of representation is used later for the (multiple and partial) coherencies. Computation of the transform at the beginning and end of the series involves missing values which we have to prescribe artificially causing the so-called edge effects. The region in the time-frequency plane where the CWT is affected by edge effects is called the cone-of-influence (COI). Results in this region should be interpreted carefully.

In the plots of the WPS, the color code ranges from blue (low volatility) to red (high volatility); the white lines show local maxima of the WPS. The black (gray) contours indicate

[10] Data for inflation and unemployment have been downloaded from the FRED Economic Database of the Federal Reserve Bank of St. Louis, available at https://fred.stlouisfed.org. Data for inflation expectations have been downloaded from the University of Michigan webpage of consumers surveys, available at http://www.sca.isr.umich.edu.

5% (10%) significance level. The cone-of-influence is shown with a parabola-like black line.

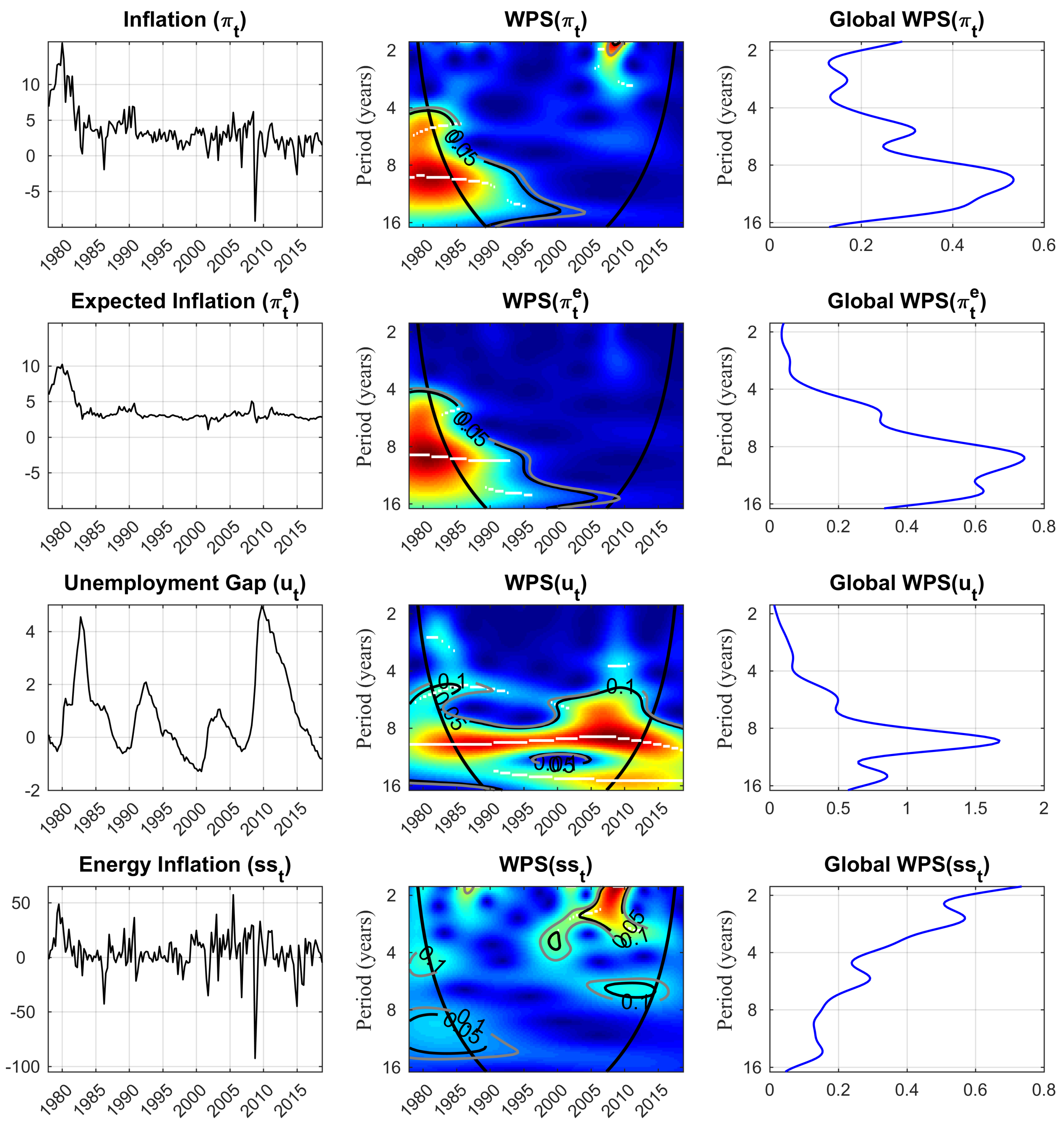

Figure 2: On the left: Plots of the time-series – inflation, expected inflation, unemployment gap and energy inflation. On the middle: The corresponding wavelet power spectra; code for power ranges from blue (low power) to red (high power). The white lines show the local maxima of the wavelet power spectrum. The black (gray) contours indicate the 5% (10%) significance level. The cone-of- influence, which indicates the region affected by edge effects, is shown with a parabola-like black line. On the right: The corresponding global wavelet power spectra.

Some overall patterns stand out and deserve mention. First, most of the variability of inflation, expected inflation, and the unemployment gap occurs at frequencies corresponding to cyclical periods larger than four years. Moreover, the dominant cycle is roughly similar for these three variables – a cycle with a period slightly above eight years.

Second, the unemployment rate behaves quite differently from inflation and expected inflation in that it maintains significant variability throughout the sample at least at cycles of a period around eight years. Overall, the unemployment gap featured a primary cycle with a medium-run period of about $8 \sim 12$ years during the four decades. A long business cycle with a period of $4 \sim 6$ years was significant until 1985 but then lost significance and power, disappearing by 2002. A long-run cycle with a period of $12 \sim 14$ years emerged in 1990. The power spectrum of the unemployment gap is thus consistent with the hypothesis that cycles have elongated since the Great Moderation.

Third, the wavelet power spectra of inflation and expected inflation are very similar, even though expected inflation is somewhat less volatile than inflation. Overall, the second relevant cycle on average throughout the sample has about $10 \sim 14$ years for both inflation and expected inflation. The graph of expected inflation shows the success of the 1980-86 disinflation in bringing expectations rapidly and consistently down by the mid-1980s. It then shows that expectations of inflation exhibited less variation than inflation throughout the Great Moderation, the Great Recession and afterward. Notably, households never expected negative inflation rates.

Fourth, energy inflation behaves quite differently from inflation and expected inflation; in particular, energy inflation exhibits much stronger short-run volatility, which suggests that it captures well (cost-push) shocks. The time series graph shows the peak in energy inflation following the second oil shock in the late 1970s and a trough in the initial phase of the Great Recession.

5 Results: the New Keynesian Phillips Curve in the Time-Frequency Domain

We now assess the U.S. New Keynesian Phillips Curve in the time-frequency domain, using continuous wavelet tools. Note that while we presented the NKPC in equation (1) with the standard timings, our continuous wavelet framework endogenously determines the timing of the relation between inflation and each of its determinants. Furthermore, our approach allows for variation in timings along time and across frequencies. Both are notable features of our approach, given the high specification uncertainty in the literature of the NKPC.

To facilitate the presentation, the comparison with the literature and the answer to our three research questions, we provide phase-difference and gain diagrams displaying mean values for three frequency intervals: cycles of period 2∼4 years (short-end of business cycles), cycles of period 4∼8 years (long-end of business cycles) and cycles of period 8∼16 years (medium-to-long run cycles).[11] Most literature has focused on the standard business cycles, but, in the case of the Phillips Curve, it is worthy to assess separately their short-end and their long-end. The inclusion of the medium-to-long-run frequency bands is important: by displaying the coherency for the continuum of cycles from a period of 8 years up to periods of 16 years, we can infer about the long-run Phillips Curve.[12]

To facilitate the analysis of a possible nonlinearity of the NKPC – the fall of its slope in recessions found in some literature – we include bars with the recession periods in the charts of the gains and of the lead/lags.

5.1 The overall fit of the NKPC

Figure 3 displays the multiple coherency of inflation versus expected inflation, unemployment gap, and energy inflation. Regions with 5%(10%) significance are identified with black (gray) contours.[13] Significant regions of multiple coherency mean that the three explanatory variables are jointly significant at those time-frequency locations.

[11]In the case of phase-differences, which are angular measures, the mean is a circular mean; see Zar (1996), for details.

[12]A cyclical period of 16 years is not the actual long-run. However, with 40 years of data, it is not reasonable to implement our approach for longer cycles. The cone-of-influence implies that, at the 16 years cycles, the results can be confidently interpreted only for about 1988-2007. Strictly speaking – see, e.g. Baneti (2015) – there is never enough data to assess the long-run properly. Our approach gives better indications about the long-run than most alternative methods, given that we display results for a continuum of cycles, so that we may infer the long-run relationships from the pattern detected as we approach our lowest frequencies.

[13]Since no theoretical distribution for the wavelet multiple coherency is available, we find the significance levels with 1000 Monte-Carlo simulations. In the next subsections, we do the same for partial coherencies.

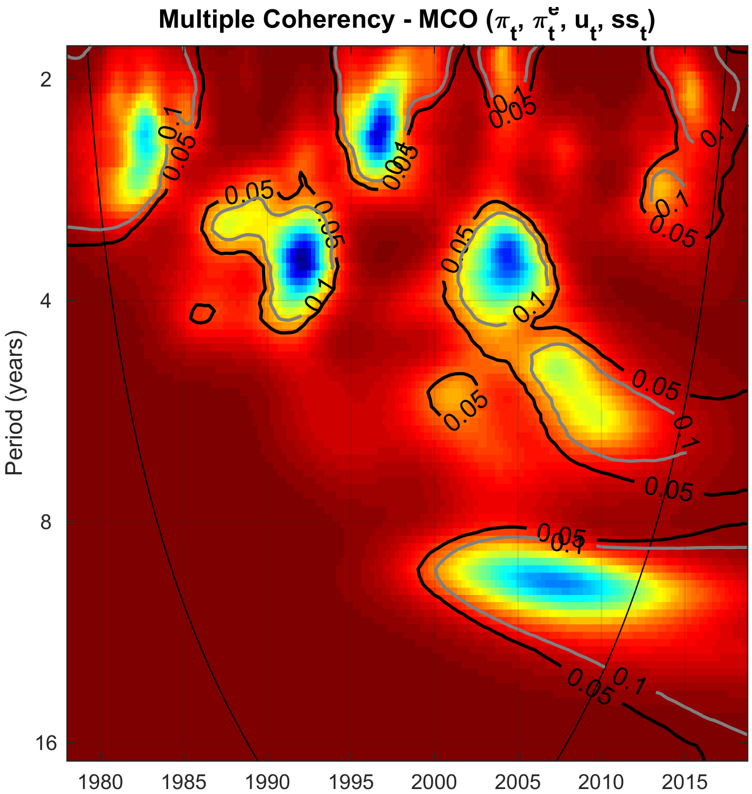

Figure 3: Multiple wavelet coherency between inflation and the three explanatory variables in the Phillips Curve – expectations of inflation, unemployment gap and energy inflation. The black (gray) contour designates the 5% (10%) significance level. The color code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). The cone -of-influence, indicating the region affected by edge effects, is shown with a parabola-like black line.

Overall, the empirical NKPC is an excellent model for inflation. The episode in which it has more difficulties in explaining inflation is between 2000 and 2015. For those years, the model captures very well inflation at the short-end of business cycles, but at the long-end of business cycles and at medium-to-long run cycles ($8 \sim 14$ years) there are some areas of low coherency. This episode starts roughly at the time when some literature has identified a fall in the NKPC slope – e.g., Pfajfar and Roberts (2018), and Jorda, Marti, Nechio, and Tallman (2019) – a hypothesis to be assessed below when we inspect each coefficient of the NKPC.

The model explains very well inflation in the long-run, as the multiple coherency is statistically significant in the frequencies approaching the longest cycle assessed (16-years cycles). Our analyses in the next sub-sections will tell whether that is due to a significant long-run Phillips tradeoff or inflation expectations (or energy inflation).

5.2 Energy inflation

Before focusing on our research questions, we analyze the role of our proxy for cost-push shocks. Figure 4 displays our results for the relation between inflation and energy inflation.

The coherency between inflation and energy inflation is widespread significant across time and frequencies. Phase-differences fluctuate closely around zero suggesting that both variables are almost contemporary and that the estimate of the coefficient or energy inflation is positive, as expected. The coefficients are quite stable, especially in the two lower frequency bands. Our results confirm that including energy inflation as a proxy for cost-push shocks is relevant, as this proxy for cost-push shocks significantly explains inflation in several time-frequency regions.

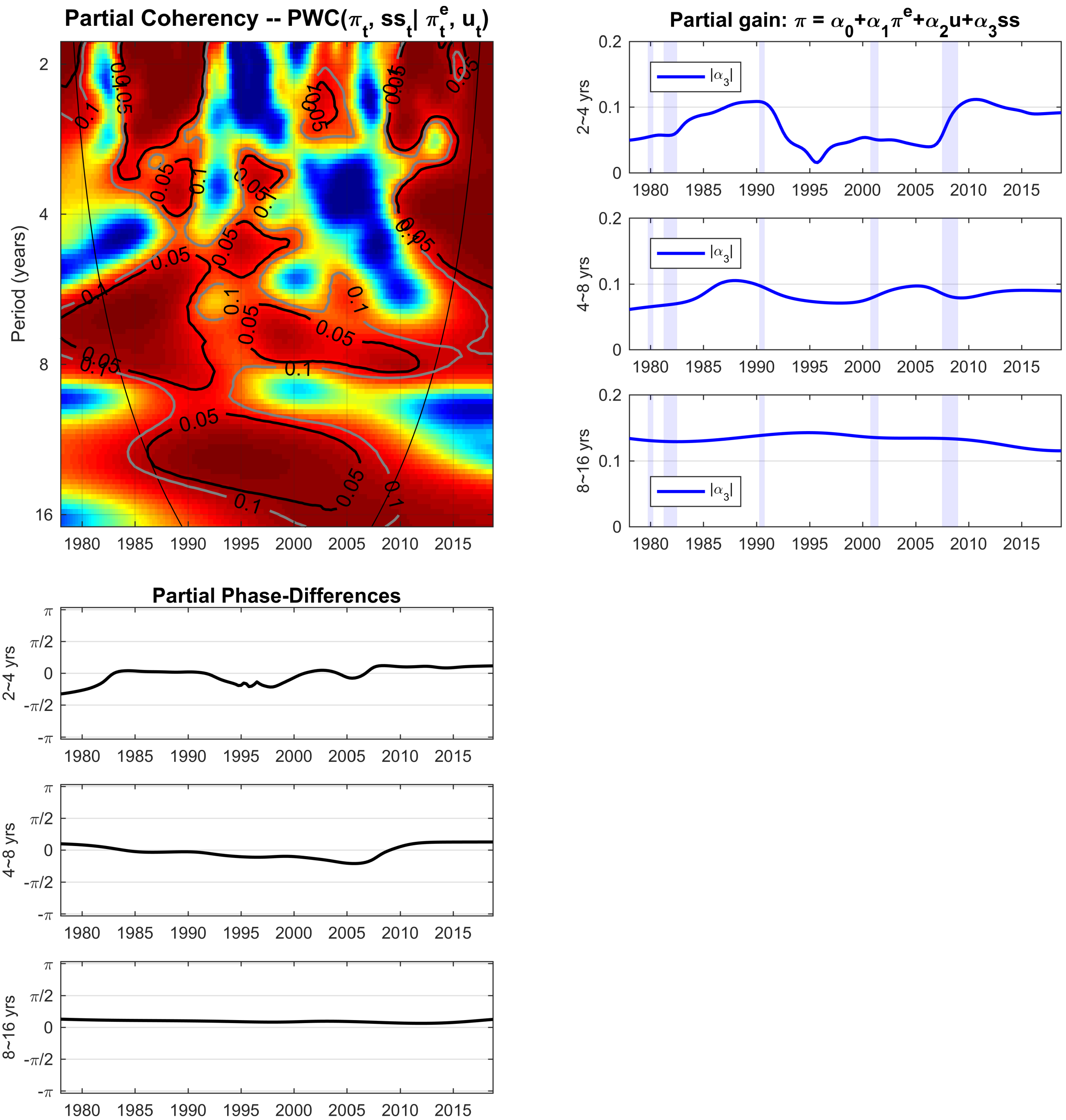

Figure 4: Wavelet measures between inflation and energy inflation controlling for the effects of the expected inflation and unemployment gap. On the left side top: Partial coherency. The color code for coherency ranges from blue (low coherency - close to zero) to red (high coherency - close to one). The black (gray) contours indicate the 5% (10%) significance level. The cone-of-influence, indicating the region affected by edge effects, is shown with a parabola-like black line. On the left side bottom: partial phase-differences for the three frequency bands. On the right top: partial gains for the three frequency bands.

5.3 Expected inflation

We start by addressing the role of expectations in the NKPC. Figure 5 summarizes our results for the relation between inflation and inflation expectations, controlling for the effects of the unemployment gap and energy inflation. Our results lend support to the literature that has concluded that survey-based households' expectations of inflation perform very well in explaining inflation, but add new results to the literature, providing a refined answer to our second research question.

At the medium-to-long run cycles (8 $\sim$ 16 years) the coherency between inflation and expected inflation is significant in most of the time-frequency space. However, from the early 2000s onwards, coherency gradually loses significance, and by 2008 it is significant only at the very long cycles. The gain produces an estimate for the coefficient of expected inflation in the NKPC equal to one. Phase-differences very close to zero inform that the coefficient is positive and that the relation is broadly contemporary.

The results for medium-to-long run cycles (8 $\sim$ 16 years) are therefore entirely consistent with the role of expectations in the theoretical NKPC. They suggest that in the long-run there is a one-to-one relation between expected inflation and inflation, as the theory predicts. These results further suggest that expectations have been solidly anchored to the inflation target until 2008 – as average or long-run inflation has converged to the implicit inflation target during the Great Moderation, and expectations relate one-to-one in the long run with such target.

At the long-end of business cycles (4$\sim$8 years), the coherency is significant for most cycles until the early 1990s. From 1990 to about 2003, the frequencies for which the coherency is significant gradually decrease, disappearing in the shorter cycles of this band. From the beginning of the sample until 2003, the gain produces an estimate for the coefficient of expected inflation in the NKPC systematically equal to one, and the phase-difference – consistently close to zero – tells that the estimate for the coefficient is positive and that it describes a broadly contemporary relationship. Hence, the results for the long-end of business cycles (4 $\sim$ 8 years) are entirely consistent with the role of expectations in the theoretical NKPC, until 2003.

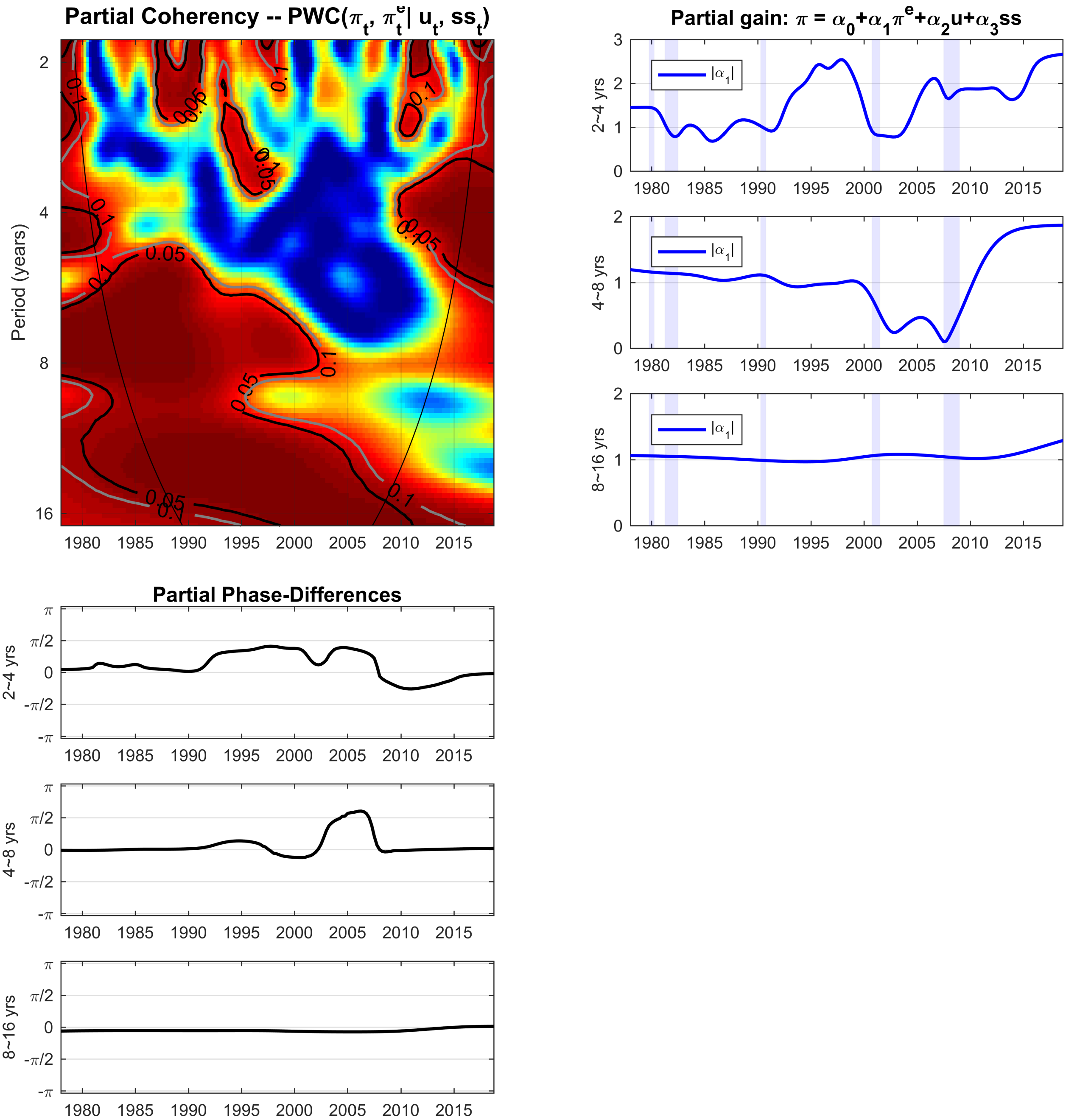

Figure 5: Wavelet measures between inflation and expected inflation controlling for the effects of the unemployment gap and energy inflation. On the left side top: Partial coherency. The color code for coherency ranges from blue (low coherency - close to zero) to red (high coherency - close to one). The black (gray) contours indicate the 5% (10%) significance level. The cone-of-influence, indicating the region affected by edge effects, is shown with a parabola-like black line. On the left side bottom: partial phase-differences for the three frequency bands. On the right top: partial gains for the three frequency bands.

From 2008 onwards, the coherency between inflation and inflation expectations at long business cycles (4∼8 years) becomes again significant; however, this phenomenon is very different from the one identified for 1990-2003. Significant coherencies appear at the shortest cycles of this band and although spread gradually to longer cycles, comprise only cycles with a period of 4$\sim$6 years at the end of the sample, indicating that this is not related with anchored expectations but with a short-run phenomenon, surely the financial crisis and Great Recession. Inflation overshoots expected inflation as the estimate of the coefficient approaches two.

The relation between inflation and inflation expectations is much more unstable at the short-end of business cycles (2$\sim$4 years), with only a few regions of significant coherency. The estimated coefficient is not stable either. Interestingly, whenever coherency is significant, inflation overshoots expected inflation as the estimate of the coefficient is higher than one.

5.4 Unemployment gap

We now reach the main focus of the paper – the short and long-run NKPC slopes – and answer to our first and third research questions. Figure 6 summarizes our results for the relation between the unemployment gap and inflation.

For cycles with a period above eight years, the coherency between inflation and the unemployment gap is very weak and never statistically significant.[14] These results provide strong evidence that, in the long-run, inflation does not correlate with the unemployment gap. Hence, the answer to one of our questions is straightforward: in our sample, there is no long-run Phillips tradeoff.

At the long-end of business cycles ($4 \sim 8$ years), the coherency is also weak throughout most of the sample period. It is statistically significant only for a specific episode – cycles of period $4 \sim 5$ years, from 2008 onwards. The gain delivers an estimate of the NKPC slope for that time-frequency region that starts close to 0.4 during the Great Recession and then gradually decreases to about 0.2 at the end of the sample (a value that upholds since 2015, when the cone of influence becomes binding). The phase-differences are located in the interval $(-\pi/2, -\pi)$ implying that the estimate for the coefficient is negative, as expected, and that the unemployment gap lags behind inflation.

[14] Given the lack of statistical significance, we refrain from analyzing the gain and the phase-differences for these cycles.

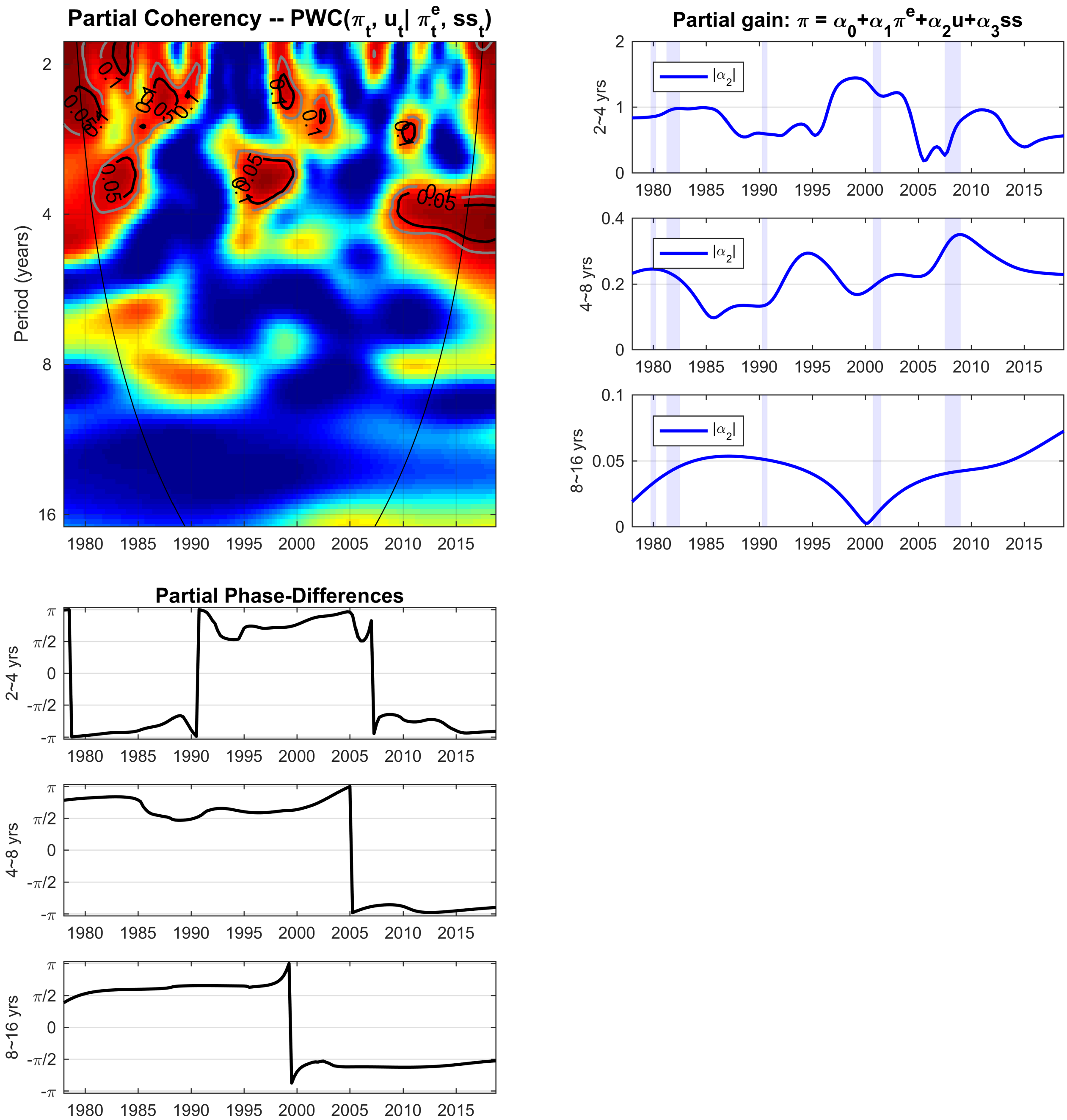

Figure 6: Wavelet measures between inflation and unemployment gap controlling for the effects of the expected inflation and energy inflation. On the left side top: Partial coherency. The color code for coherency ranges from blue (low coherency - close to zero) to red (high coherency - close to one). The black (gray) contours indicate the 5% (10%) significance level. The cone-of-influence, indicating the region affected by edge effects, is shown with a parabola-like black line. On the left side bottom: partial phase-differences for the three frequency bands. On the right top: partial gains for the three frequency bands.

These estimates for the slope of the short-run NKPC are quantitatively broadly in line with many in the literature – although not strictly comparable, as we focus on specific frequencies and allow for continuous time and frequency variation – but are surprising in two respects. First, the slope is only significant for a short period (2008-2015) and a narrow range of cycles (period $4 \sim 5$ years). Second, we find that inflation slightly led the unemployment gap, when most of the literature mentions some slowness of inflation in reacting to unemployment in the crisis and the recovery.

The relation between inflation and the unemployment gap is much stronger at the short-end of business cycles ($2 \sim 4$ years) but not as pervasive as expected. The coherency is statistically significant in three episodes: 1978-1992, 1994-2003, and 2008-2018.

During 1978-1992, the coherency is significant for a wide range of frequencies. The gain delivers an estimate of the slope of one until 1986, which then falls to about 0.6 since around 1988. The phase-differences are located in the interval $(-\pi/2, -\pi)$ implying that the estimate for the coefficient is negative, as expected, and that the unemployment gap lags inflation.

In 1994-2003, the coherency is also significant for a wide range of frequencies. The gain delivers an estimate of the slope that increases from about 0.6 to around 1.4 in the late 1990s and then falls to about 1.1 during the early 2000s. In contrast with the previous period (and with the following) the phase-differences are located in the interval $(\pi/2, \pi)$, implying that the estimate for the coefficient is negative, as expected, and that the unemployment gap leads inflation.

After 2008, the coherency is significant for a much smaller range of frequencies (period $3 \sim 4$ years, but especially close to four years). The gain delivers an estimate of the slope that increases from about 0.2 before the Great Recession to one at the beginning of the 2010s, then falling to about 0.5 since 2015. The phase-differences are located in the interval $(-\pi/2, -\pi)$ implying that the estimate for the coefficient is negative, as expected, and that the unemployment gap lags inflation.

Overall, these estimates of the slope are quantitatively larger than those described in most literature, which, taken together with the lack of coherency at most other cyclical frequencies, suggests that the estimates in the literature amalgamate estimates from frequencies at which the slope is not significant, and are therefore downward biased.

Regarding nonlinearities, we do not detect a flatter NKPC during recessions and recoveries (or periods with lower inflation). During the Great Recession and its recovery, the gain increased substantially at the $2 \sim 4$ year cycles and also somewhat at the $4 \sim 8$ year cycles. Throughout the recession of the early 1990s, it maintained its value, at the $2 \sim 4$ year cycles.

During the recession of the early 2000s, the gain fell only slightly, at the $2 \sim 4$ year cycles.

Regarding structural stability, we focus on the three main hypotheses in the literature: has the short-run Phillips Curve flattened since the 1980s, given the Great Moderation? Has it flattened since the late 1990s, given a strengthening of the anchoring of expectations? Has it flattened since the late 2000s, given the Great Recession?

During the Great Recession and the subsequent recovery, the gain increased at the $2 \sim 4$ and at the $4 \sim 8$ year cycles (in the latter, it fell somewhat during the recovery, but remained at very high levels); therefore, we find no flattening of the NKPC.

To assess the possible structural breaks in the 1980s and the late 1990s, we focus on the gain for the $2 \sim 4$ year cycles at 1978-1992, 1994-2003 and 2008-2018 (and also $4 \sim 8$ year cycles in the latter). Overall, our estimates do not suggest that the NKPC flattened since the 1980s: while it fell below one since 1987, later it was above 1 in 1997-2003, and after 2008 was not far from one. Finally, while the gain was at its maximum in 1997-2000 – between 1.4 and 1.1 – inspection of the estimates for the significant periods before and after 1997 does not suggest that the NKPC has flattened at 1997.

Before moving to the next section, two results are noteworthy. First, the role of the unemployment gap in explaining inflation in the context of the NKPC is somewhat limited: the slope of our state-of-the-art NKPC is never statistically significant for cycles with a period above five years and is only significant in some specific time-frequency episodes. Second, though almost all episodes with a significant slope are within the range of $2 \sim 4$ years cycles, there seems to be some effect of the recent elongation of business cycles: the Phillips tradeoff has gradually evolved to slightly longer cycles along the three significant episodes (1978-1992, 1994-2003, 2008-2018). Both may explain the sampling uncertainty noted by Mavroeidis, Plagborg-Møller, and Stock (2014) and many discrepancies between results in the literature.

6 Discussion and Conclusions

At 60, the Phillips Curve remains inexorable and mysterious. Its main mysteries are the three research questions of this paper:

1. Has the slope of the short-run Phillips Curve been stable? If not, has the tradeoff been

nonlinear, or has it experienced structural breaks? If so, when and why?

2. Have survey-based expectations of inflation captured well the dynamics of inflation? What do they reveal about persistence, forward-looking behavior, trends, and anchoring?

3. Has the long-run Phillips Curve been vertical, or should we reject the Natural Rate hypothesis? Is there no long-run tradeoff even at low levels of inflation?

We argue that these three questions have a time and a frequency domain nature, as they involve possible changes of coefficients along time, with different patterns across cyclical frequencies, and possible changes in the lead or lag between inflation and each of its determinants. We contribute to the literature using a continuous time-frequency approach that provides integrated answers to these research questions, thus uncovering the central mysteries of the Phillips Curve with a new and thorough perspective.

Our 60-year-old Phillips Curve is a state-of-the-art empirical New Keynesian Phillips Curve (NKPC) that explains inflation with households' survey-based expectations of inflation, the unemployment gap, and energy inflation (a proxy for cost-push shocks). Our data are for the U.S. 1978:I-2018:IV.

We confirm that the empirical NKPC is, apart from a few episodes rather limited both in time and frequency, a good model for inflation. Overall, we show that in the longer business cycles (period $5 \sim 8$ years) and the medium-to-long run (cycles of period $8 \sim 16$ years), inflation is explained by expected inflation and energy inflation, with no significant role for the unemployment gap – the Phillips tradeoff is a very short run phenomenon. In the long run (here proxied by frequencies corresponding to cycles slightly longer than 16-years), inflation is explained solely by expected inflation – there is no long-run Phillips tradeoff. In the short end of business cycles (with a period up to about five years), all the three determinants of inflation in the NKPC are significant. However, none is significant in the whole time and frequency ranges, but only in specific and somewhat limited episodes. Expected inflation is relevant in 1987-2000 and 2008-2018; energy inflation in 1983-1992, 1994-2006, and 2008-2018; the unemployment gap in 1978-1992, 1994-2003 and 2008-2018. Overall, the only episode in which the NKPC does not explain short cycles of inflation (period $2 \sim 5$ years) is between

2006 and 2008, the abnormal episode of maturation of the extreme boom that led to the financial crisis.

Our first research question is about the significance and stability of the short-run slope of the NKPC. We first detected that the role of the unemployment gap in the context of the NKPC is limited to rather short business cycles — with a period not above five years — and is significant only in some specific time-frequency episodes. We then noted that although most episodes with a significant slope are within the range of $2 \sim 4$ years cycles, there seems to be some effect of the recent elongation of business cycles: the Phillips tradeoff has gradually evolved to slightly longer cycles along the three episodes in which it is statistically significant – 1978-1992, 1994-2003, 2008-2018. Both may explain the sampling uncertainty noted by Mavroeidis, Plagborg-Moller, and Stock (2014) and many discrepancies between results in the literature. Overall, our estimates for the $2 \sim 4$ years frequency band are quantitatively higher than those typically obtained in the literature, seemingly because those comprise frequencies that are not statistically significant.

Regarding nonlinearities, we do not detect a flatter NKPC during recessions and recoveries (or periods with lower inflation). Actually, during the Great Recession and its recovery, the gain increased substantially at the $2 \sim 4$ year cycles and also somewhat at the $4 \sim 8$ year cycles. During the recession of the early 1990s, it maintained its value, at the $2 \sim 4$ year cycles. During the recession of the early 2000s, the gain fell only slightly, at the $2 \sim 4$ year cycles. Hence, our findings differ from those of Carrera and Ramírez-Rondán (2017), López-Villavicencio and Mignon (2015), and Daly and Hobijn (2014), and are much more in line with those reported by Berger, Everaert and Vierke (2016), and Doser, Nunes, Rao and Sheremirov (2018).

Regarding structural stability, there are three main hypotheses. Has the short-run Phillips Curve flattened since the 1980s, given the Great Moderation? Has it flattened since the late 1990s, given a strengthening of the anchoring of expectations? Has it flattened since the late 2000s, given the Great Recession? During the Great Recession and the subsequent recovery, as we have just argued, we find no flattening of the NKPC, consistent with Watson (2014), Coibion and Gorodnichenko (2015), Jorda, Marti, Nechio, and Tallman (2019), and Barnichon and Mesters (2019). Throughout the Great Recession, one could even argue that our results support those of, e.g., Castelnuovo and Pellegrino (2018), pointing to an increase in the frequency of price adjustments, and therefore of the slope, in times of uncertainty.

To assess the possible structural breaks in the 1980s and the late 1990s, we focus on the gain for the 2 $\sim$ 4 year cycles at 1978-1992, 1994-2003 and 2008-2018 (and also 4 $\sim$ 8 year cycles in the latter). Overall, our estimates do not suggest that the NKPC flattened since the 1980s. It did fell below one since 1987, but later it was above one in 1997-2003, and after 2008 was not far from one. Hence, our results do not confirm those of Roberts (2006), Benati (2007), Blanchard, Cerutti, and Summers (2015), Blanchard (2016), and Chin (2018). While the gain was at its maximum in 1997-2000 – between 1.4 and 1.1 – inspection of the estimates for the significant periods before and after 1997 does not suggest that the NKPC has flattened at 1997. Hence, our results do not confirm those of Pfajfar and Roberts (2018), and Jorda, Marti, Nechio, and Tallman (2019).

Our second research question is about the role of expected inflation in the Phillips Curve. We first confirm that survey-based households' expectations of inflation perform very well in explaining inflation in the context of the NKPC. Our finding that, for all frequencies with significant coherency, households' expectations relate contemporaneously with inflation is highly suggestive that we do not need backward- or forward-looking expectational components for an accurate empirical account of inflation with the NKPC. While in line with the state-of-the-art literature — see Coibion, Gorodnichenko, and Kamdar (2018) —, our time-frequency results complement and clarify such literature. The coefficient of expected inflation has been steadily one at medium-to-long run cycles (period 8 $\sim$ 16 years). It was also one at long business cycles (period 4 $\sim$ 8 years) until 2002. These results, and in particular those for the lowest frequencies we can assess, suggest that inflation expectations explain inflation in the long-run. i.e., duly captured trend movements in inflation.

Moreover, given the proximity between average inflation and the implicit policy target, expectations seem to have been anchored until the beginning of the financial crisis. Inflation deviated from inflation expectations during the Great Recession, more specifically overshooting expectations at most business cycles frequencies (period 2 $\sim$ 4 years and 4 $\sim$ 6 years), given the estimate of about 2 for the respective coefficient. Furthermore, the decoupling of inflation from expectations at medium-to-long run oscillations (8 $\sim$ 16 years) suggests that expectations became un-anchored during the Great Recession — as found by Coibion and Gorodnichenko (2015). Our results show that the link between expectations and inflation is much less stable for the very short-end of business cycles ($2 \sim 4$ years), where we find some periods with the theoretical coefficient of about one, but also many episodes in which expectations do not significantly explain inflation, and episodes with coefficients that depart from one.

Finally, as regards the third research question — the shape of the long-run Phillips Curve and the Natural Rate hypothesis — we find no long-run tradeoff between inflation and unemployment. Our findings are in line with those of Benati (2015) and support the recommendation of Blanchard (2018) that macroeconomics should stick to the Natural Rate hypothesis unless compelling evidence of the contrary appears. They are, however, in contrast with a vast literature – e.g. Akerlof, Dickens and Perry (1996, 2000), Benigno and Ricci (2011), Daly and Hobijn (2014), Svensson (2015), Russell (2011), Berentsen, Menzio and Wright (2011), and Haug and King (2014). With this respect, we note that there are two features of our approach that strengthen our findings. First, ours is one of the most extensive available samples with low rates of inflation — the ideal condition for a bend in the long-run Phillips curve. Second, our econometric framework is highly flexible, in that it does not depend on the (non)stationarity of the data, and provides estimates that may vary along time and comprise a *continuum* of frequencies up to the lowest possible with the available sample, thus effectively proxying for the long-run.

Acknowledgments

Research at NIPE was carried out within the funding with COMPETE reference number POCI-01-0145-FEDER-006683 (UID/ECO/03182/2013), with the FCT/MEC's (Fundação para a Ciência e a Tecnologia, I.P.) financial support through national funding and by the ERDF through the Operational Programme on "Competitiveness and Internationalization – COMPETE 2020 under the PT2020 Partnership Agreement.

Research at Cef.up has been financed by Portuguese public funds through FCT (Fundação para a Ciência e a Tecnologia, I.P.) in the framework of the project UID/ECO/04105/2019.

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Bank of Finland Research Discussion Papers 2019

ISSN 1456-6184, online