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Money growth and inflation: International historical evidence on high inflation episodes for developed countries

Marco Gallegati,† Federico Giri‡ and Michele Fratianni§

Abstract

How long is the long run in the relationship between money growth and inflation? How important are high inflation episodes for the unit slope finding in the quantity theory of money? To answer these questions we study the relationship between excess money growth and inflation over time and across frequencies using annual data from 1871 to 2013 for several developed countries. Wavelet-based exploratory analysis shows the existence of a close stable relationship between excess money growth and inflation only over longer time horizons, i.e. periods greater than 16 and 24 years, with money growth mostly leading. When we investigate the sensitivity of the unit slope finding to inflation episodes using a scale-based panel data approach we find that low-frequency regression coefficients estimated over variable-length subsamples before and after WWII are largely affected by high inflation episodes. Taken together the results that inflationary upsurges affect regression coefficients but not the closeness of the long-run relationship call for a qualification of the Quantity Theory of Money and suggests that policymakers should not lose interest on monetary developments.

JEL codes: C22, E40, E50, N10.

Key words: Quantity theory of money, Time-frequency analysis, Low frequency relationships, High inflation episodes.

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1 Introduction

Lucas' (1980) unit slope findings of inflation on money growth using low-frequency regressions has favored a remarkable proliferation of empirical studies on the quantity theory of money (QTM) using both time-series of specific countries and international cross-country datasets. Notwithstanding many studies report a close relationship between a cross-section of long-term averages for monetary growth and inflation (Whiteman, 1984; Geweke, 1986; Stock and Watson, 1988; Dwyer and Hafer, 1988, 1999; Barro, 1990; King and Watson, 1992; Christiano and Fitzgerald, 2003), the empirical validation of QTM is subject to several problems.

The first refers to the extremely vague definition and the wide range of frequencies used to identify the medium- and the long-term. If this range of frequencies is better defined in its lower limit, generally identified with the upper limit of business cycle fluctuations (8 to 10 years), much more vague is the definition of its upper limit, whose range can vary from 20 to 50 years and sometimes longer (e.g. Blanchard, 1997; Rotenberg, 1999; and Comin and Gertler, 2003). This indeterminacy emerges in studies that examine the money growth-inflation relationship using very different pre-determined frequency bands: 8 to 20 years, and 20 to 40 years (Christiano and Fitzgerald, 2003), 8 to 40 years (Haug and Dewald, 2004), up to periods of 30 years and longer (Benati, 2009).

Second, the presence of countries with high rates of money growth and inflation may cast doubts on the finding observed in most studies of a close relationship between monetary growth and inflation (e.g., Barro, 1993; Mc-Candless and Weber, 1995; Lucas, 1996). Moreover, De Grauwe and Polan (2005), Sargent and Surico (2011), and Teles et al. (2016) document that the long-run link between money growth and inflation has weakened in low inflation countries during the post-WWII years, especially after the Great Inflation period. More to the point, the unit slope of inflation with respect to money growth appears to be dependent on the presence of a substantial number of high-inflation observations in the sample, which may reflect two types of bias: the presence of countries with high rates of money growth inflation countries, and the occurrence of high-inflation episodes in low inflation countries. Furthermore, the prevalence of empirical works on the unit slope hypothesis relying on post-WWII data may unduly restrict the universality of the money growth-inflation relationship.[1]

So far, very few papers have employed pre-WWII data in their empirical work. We concur with Christiano and Fitzgerald (2003:22) that "much can be learned by incorporating data from the first half of the [20th] century into the analysis of inflation and monetary policy". Indeed, in the first half of the 20th century the inflation rate has been lower on average, more volatile and less persistent than in the second half of the century. Between late 19th and early (mid)-20th century the price level displays a very large amount of variation over time around a trendless or slightly declining trend. Otherwise, after WWII prices rise continuously as a result of the change in the underlying process of price determination, the effect being the emergence of a strong positive trend (van Ewjik, 1982).[2]

The aim of this contribution is twofold: to shed light on the length of the long-run relationship between excess money growth and inflation, and to investigate the sensitivity of the estimated coefficients in the money growth inflation relationship to the presence of high money and inflation episodes in the estimation sample. To do that, and differently from previous contributions, we use the new historical macro database recently developed by Jorda et al. (2017). By covering 16 developed countries and spanning 140 years this international historical dataset is

1 Few studies use long historical data going back to the beginning of the 20th century: for the US, see Christiano and Fitzgerald (2003) and for a sample of industrial countries, see Dewald (2003), Haug and Dewald (2004), and, more recently, Sargent and Surico (2011).

2 Gallegati et al. (2017) document a shift in the pattern of inflation around WWII, suggesting a data generating process of inflation resembling a segmented trend stationary process.

well suited for investigating both issues, as it contains several high inflation and monetary growth episodes experienced by a group of low inflation countries in the 1910s, the 1940s, and the 1970s (Dewald, 2003).

Following Neumann and Greiper (2004), Rua (2012), and Mandler and Scharnagl (2014), we exploit the ability of wavelet methods to capture the time-frequency relationship between money growth and inflation. The wavelet transform uses a set of local basis functions that are dilated or compressed through a scale or dilation factor and shifted along the signal through a translation or location parameter. This property may be particularly useful when dealing with complex, non-stationary signals, such as historical time series, since their secular movements are likely to exhibit structural changes due to shocks such as wars, economic and/or financial crises or changes in monetary regimes.[3] Moreover, by using relatively short rolling windows at shortest time scales and relative long windows at longest time scales, the wavelet transform allows to identify both short lived high-frequency phenomena and long-lasting features pertaining to the very long run.[4] Finally, a fundamental benefit of wavelet analysis is that it is more robust than other techniques in a "messy world",[5] and, in contrast to band-pass filtering approach, allows researchers not to be committed to any particular class of models.

The results of wavelet-based exploratory analysis show that the length of the long-run relationship is quite long: strong stable co-movements are evident for the Anglo-Saxon countries over 24-year time horizons, and for the rest of the countries

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3 This is evident, for example, in the new dataset recently assembled by Schularick and Taylor (2012) where two very different patterns in the long-run trends of money and credit aggregates relative to GDP are evident with a trend break occurring around 1950.

4 By decomposing a signal into a set of time scale components, each associated to a specific frequency band and with a resolution matched to its scale, the wavelet transform, in contrast to frequency domain methods, attains an optimal trade-off between time and frequency resolution levels (Lau and Weng 1995, Mallat 1998).

5 A "messy world" is one in which the parameterization of approximating models need to be changed over time and it is usual that the distributions relevant in one time period are not statistically similar in another time period.

over 16-year time horizons. Moreover, in the 2nd half of the post-WWII period there is evidence of a widespread tendency of the long-run relationship to shift towards lower frequencies. When the relationship between inflation and excess money growth is estimated parametrically using separate panel datasets, each composed by data at different time horizons (e.g. Gallegati et al., 2016),[6] upward and downward shifts in the low-frequency regression coefficients are clearly detected in both the pre- and post-WWII periods. These shifts in the values of the estimated coefficients of inflation on excess money growth, as the unit slope finding, are critically dependent on high-inflation episodes. Our headline finding is that allowing contemporaneously for time and frequency variations in the data is essential for a proper verification of the QTM and useful for understanding the current conduct of monetary policy. That inflationary upsurges affect regression coefficients, but not the closeness of the long-run relationship between money growth and inflation, call for a qualification of the quantitative theory of money and suggest that policymakers should not lose interest on monetary policy developments.

The paper is divided in four sections. Section 2 applies wavelet coherence analysis to detect the length of the long run relationship in the QTM. Section 3 tests the QTM unit slope hypothesis applying a "scale-based" panel regression approach to variable-length sub-samples in the pre- and post-WWII periods. Section 4 offers the main conclusions of the paper.

6 Recently developed panel unit root and panel co-integration techniques also allow estimating the long-run and short-run relationships between variables (Im *et al.*, 2003, and Pedroni, 2001, respectively).

2 The length of the long run in the QTM? Wavelet-based exploratory analysis

How long is the long run? That is, at what time horizon, if any, does a link between excess money growth and inflation exist? To identify the length of this long-run relationship, we shall employ wavelet-based exploratory analysis.[7] Descriptive techniques generally find limited application in empirical works because they require subjective judgments in interpreting results. With wavelets methods it is possible to preserve the main advantages of exploratory data analysis, i.e. looking for flexible ways to examine data without preconceptions, and at the same time to avoid its main disadvantages. Indeed, in contrast to standard exploratory techniques, wavelet tools allow the researcher to gain insights on the underlying structure of the data without relying on a subjective visualization process.

The application of the wavelet transform requires the specification of the wavelet function (and the treatment of boundary conditions). Within the relatively large family of wavelets (e.g., Daubechies, Haar, Mexican hat), the Morlet wavelet is the most widely used function because of its optimal joint time-frequency concentration, as it pertains to the minimum uncertainty value of the corresponding Heisenberg box. The Morlet wavelet consists of a complex exponential modulated by a Gaussian window and is defined as

$$\psi(t) = \pi^{-1/4} e^{i\omega_0 t} - e^{-t^2/2},$$

where ω_0 is a dimensionless frequency that is set equal to 6. This specification provides a good balance between time and frequency localization (Grinsted et al. 2004); it also simplifies the interpretation of the wavelet analysis because its scale, s, is inversely related to the frequency, f=1/s, and as a complex wavelet it can shed

7 A brief technical introduction to wavelet analysis is provided in the Appendix.

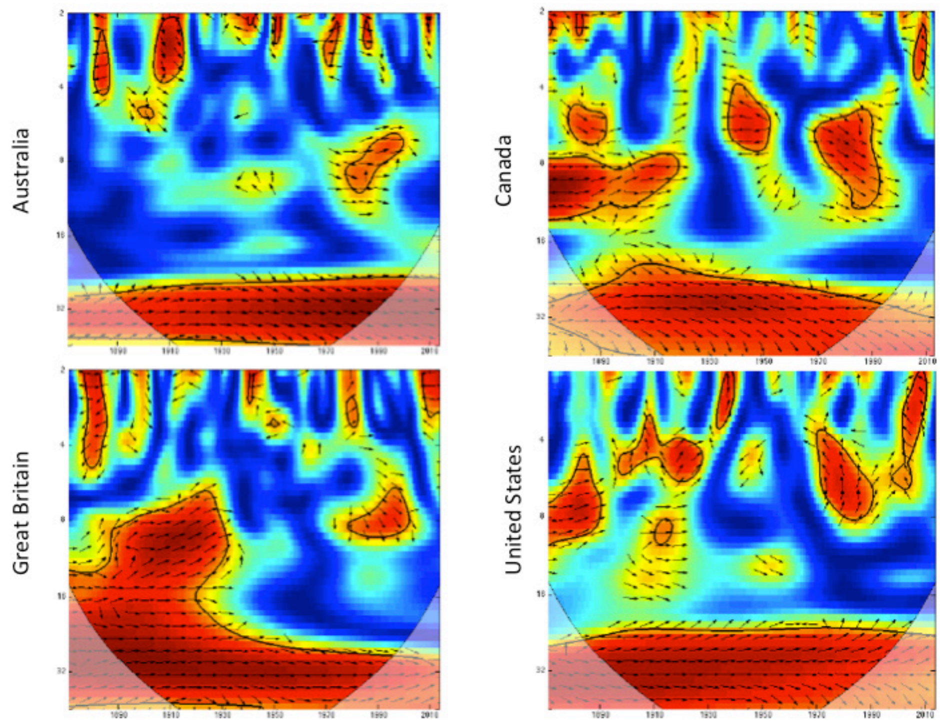
information on both amplitude and phase.

The continuous wavelet transform, like other types of transforms, suffers from a distortion at the boundaries because the finiteness of the time series impacts on the wavelet transform coefficients at the beginning and at the end of the series. The affected region is called the "cone of influence", an area where results are to be interpreted carefully (Percival and Walden, 2000). Since the effective support of the wavelet at scale *s* is proportional to s, these edge effects increase with s so that the number of wavelet coefficients affected by the boundary conditions tends to increase as the wavelet scale increases.

We need to identify, in the time-frequency domain, those time scales where the hypothesized relationship is statistically significant and its evolution over time. CWT bivariate tools, such as the wavelet coherence and phase difference analysis, allow measuring the local correlation and the wavelet phase the lead/lag relationship between two variables in time-frequency space, respectively (Grinsted et al. 2004; Aguiar-Conraria and Soares, 2010).

The squared wavelet coherence, which is analogous to the squared correlation coefficient in linear regression and can be used to assess how the degree of co-movement of two time series is changing across frequencies and over time, is visualized by using contour plots where the color of each point measures the amount of signal energy contained at a specific scale and location. Time is recorded on the horizontal axis and periods, with the corresponding scales of the wavelet transform on the vertical axis. By reading across the graph at a given value of the wavelet scale, one sees how the power of the projection varies over time while reading down the graph at a given point in time one sees how the power varies with the wavelet scale (Ramsey et al., 1995). The color code for power ranges from dark blue (low power) to dark red (high power), with regions with warmer colors corresponding to areas of high power, that is regions with wavelet transform coefficients of large modulus. By examining the contour plot it is thus possible to identify regions in the time-frequency space corresponding to areas of strong local correlation, i.e. regions of high coherence, and to assess whether the strength of the co-movements change across frequency bands (on the vertical axis) and over time (on the horizontal axis). The statistical significance of the wavelet power coefficients, represented by a black contour line, is assessed against the null hypothesis of an auto-regressive process of the first order, using Schulte's (2015) cumulative area-wise significance test.[8] The phase information is graphically coded by arrow orientation: a right (left) arrow means that two variables are in-phase (anti-phase). A right arrow pointing up (down) means that inflation is leading (lagging). A left arrow pointing up (down) means that inflation is lagging (leading).

Figure 1 -Wavelet coherence between inflation and excess money growth for Australia (top left), Canada (top right), Great Britain (bottom left) and United States (bottom right)



8 The statistical significance of the results obtained through wavelet coherence analysis was first assessed by Torrence and Compo (1998) using pointwise significance test.

Figure 2: Wavelet coherence between inflation and excess money growth for Denmark (top left), Finland (top right), Norway (bottom left) and Sweden (bottom right)

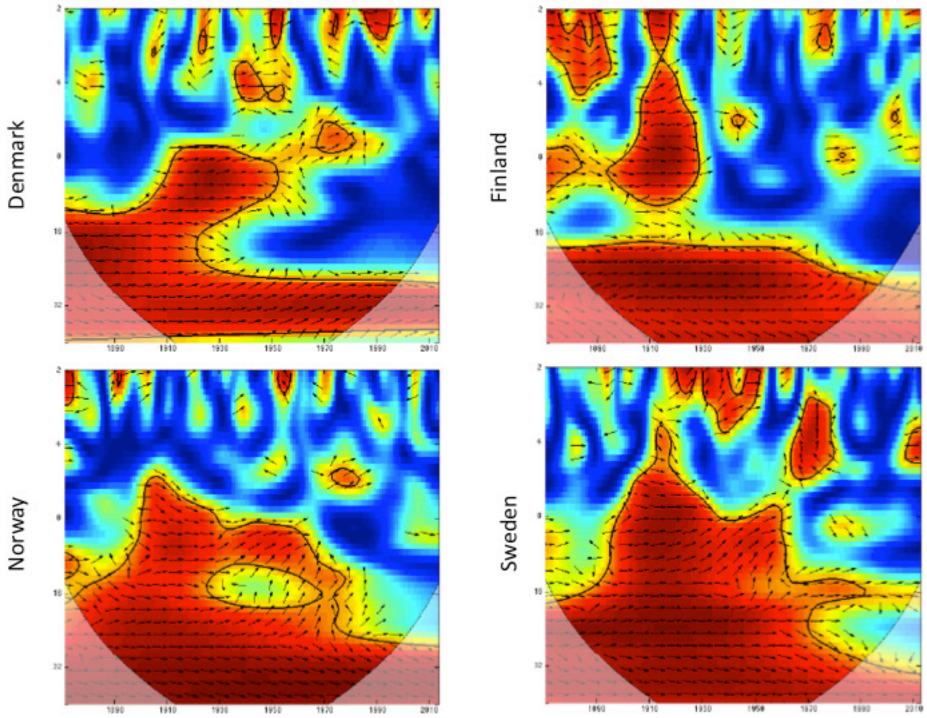


Figure 3 - Wavelet coherence between inflation and excess money growth for Spain (top left), Italy (top right), Portugal (bottom left) and Japan (bottom right)

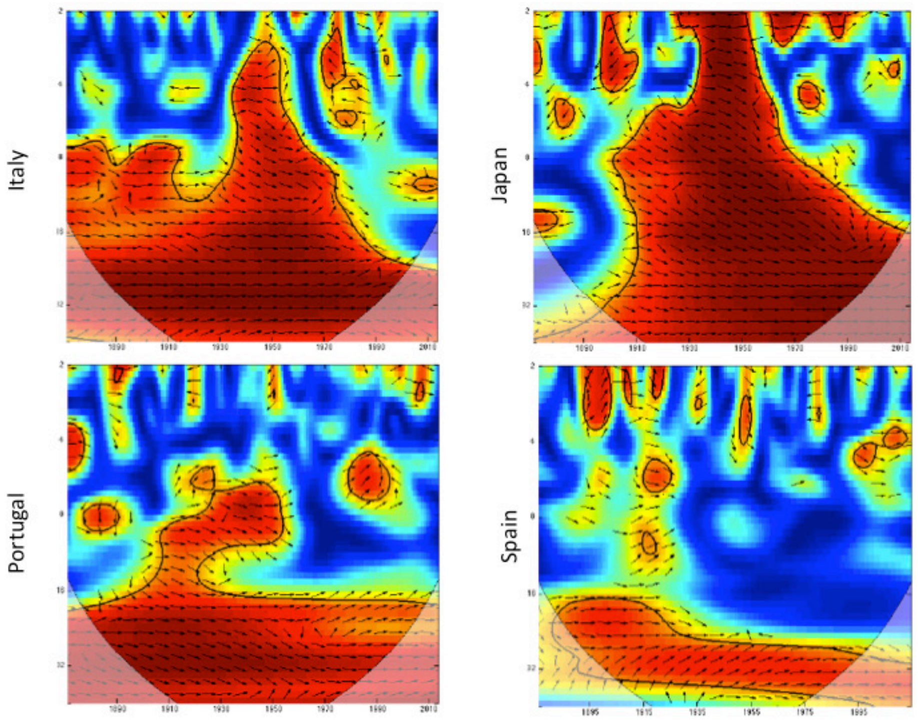
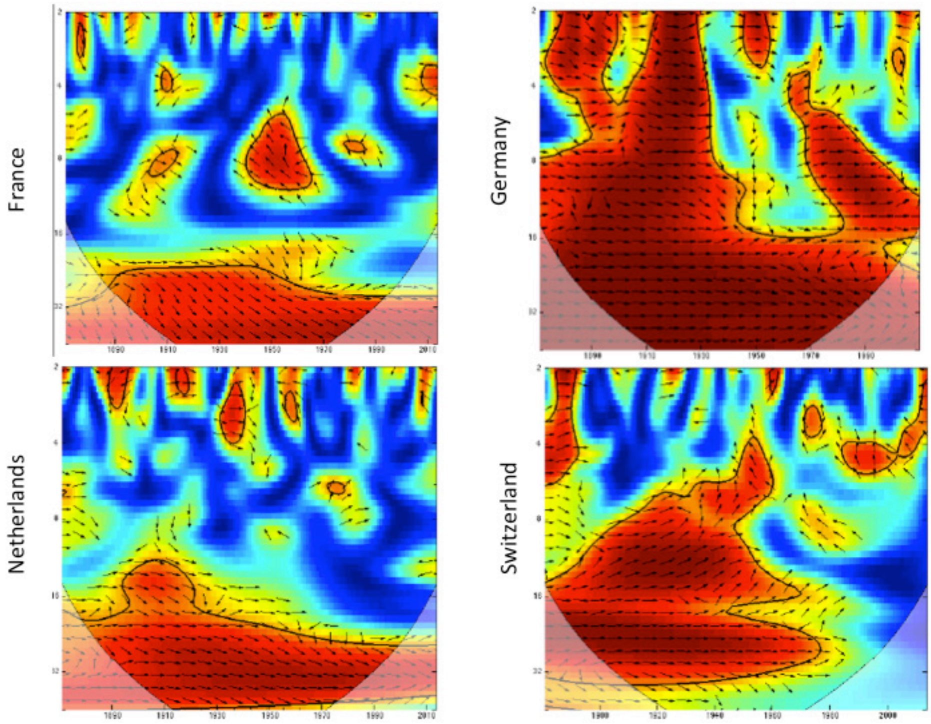


Figure 4 - Wavelet coherence between inflation and excess money growth for France (top left), Germany (top right), Netherlands (bottom left) and Switzerland (bottom right)



The issue of missing data and extreme values (such as outliers and spike signals) is commonplace in historical datasets covering long time span. With two World Wars and the hyperinflation period in Germany in early 1920s the dataset developed by Jorda et al. (2017) makes no exception. Five countries present missing data for money and/or prices, Denmark (1946-50), France (1914-20), Germany (1923-25 and 1939-48), Netherlands (1942-45) and Spain (1936-41), and four hyperinflation episodes (triple-digit values) in Finland (1919), Germany (1923-25), Italy (1944) and Japan (1945) are included in the sample.[9]

Figures 1 to 4 show the estimated wavelet coherence[10] between the rate of inflation and excess money growth over the period 1871-2013.[11] Annual growth rates of

9 We decide to replace missing values using an interpolation technique. We expect this solution not to affect the ability of wavelet coherence analysis to answer our question on the length of the long-run relationship between inflation and money growth.

10 The analysis has been performed using the MatLab package developed by Grinsted et al. (1994). "http://www.mathworks.com/matlabcentral/fileexchange/52325cumulativeareawisetestinginwaveletanalysis"

11 As evidenced by Assenmacher-Wesche and Gerlach (2007) and Teles and Uhlig (2013) correcting monetary growth for real output allows to depure the money growth-inflation relationship from the effects of shifts

consumer prices indices, broad (typically M2 or M3) monetary aggregates and real output for 16 countries are used (Jordà et al. 2017). These countries are divided in four groups: the Anglo-Saxon group consisting of Australia, Canada, the UK and the US; the Northern group consisting of Denmark, Finland, Norway, and Sweden; the Euro-Mediterranean group consisting of Italy, Spain and Portugal plus Japan; and the Euro-core group consisting of France, Germany, and the Netherlands plus Switzerland.

Two main findings are evident from the visual inspection of the panels in Figures 1 to 4. The first is the presence for Finland, Germany, Italy and Japan of high coherence regions at shorter time scales coincident with the hyperinflationary episodes previously mentioned. The latter is the clear evidence of a strong, stable and significant medium to the long-run relationship between excess money growth and inflation, with excess money growth mostly leading inflation. Notwithstanding this common pattern of strong long-term co-movements throughout the sample period some differences emerge. The Anglo-saxon countries exhibit strong stable comovements at frequencies corresponding to periods greater than 24 years (Great Britain makes partial exception by also displaying significant comovements at frequencies greater than 8 years until the end of WWI). By contrast, for all other countries in the sample such strong stable comovements extend to frequencies corresponding to 16 years, with comovements going back to frequencies corresponding to 8 years between late XIXth and mid-XXth century for Norway, Sweden, Italy, Netherlands, Germany and Switzerland. Specific patterns are displayed by Japan, Spain and Switzerland: in the first case the relationship is limited to the longest run until the early XXth century, while for Spain comovements shift from 16 to 32 years starting from the early XXth century. For Switzerland, the relationship first stretches towards higher frequencies (periods of 8 years and shorter), then gradually shifts to lower frequencies (periods

in trend output.

between 16 and 32 years) until the late 1960s, to finally disappearing at the end of the XXth century. Finally, for Germany from the 1970s there is clear evidence of a high coherence region in the 4-16 frequency range that tend to progressively shift towards lower frequencies, i.e. 8-16 frequency range, throughout the sample.

In sum for the developed countries of this sample the QTM relationship is not only strong, but stable, in the sense that it holds consistently in the medium to long run; however, since the 1970s a shift of the relationship towards lower frequencies is common to most countries of the sample.[12]

What policy lessons can we draw from the evidence in Figures 1-4? First, the QTM relationship holds in the medium to long run, a critical qualifier one needs to add to Friedman's (1963) statement that "inflation is always and everywhere a monetary phenomenon." Second, the observed strong correlation in the medium to long-term between excess money growth and inflation justifies the policy of monitoring and targeting monetary aggregates with the aim of stabilizing medium-to-long-run prices. This is the significance of the monetary pillar in the European Central Bank's monetary policy strategy: the intent there is to capture the effects of money growth at the lower frequencies. However, our evidence is supportive of Svensson's (1999, p. 215) criticism that "this long-run correlation is irrelevant at the horizon relevant for monetary policy" and provides a strong rationale for the inflation targeting policy framework adopted by most central banks during the last 25 years. Citing Woodford (2008: 1561), there is no compelling reason "for assigning an important role to tracking the growth of monetary aggregates when making decisions about monetary policy." Based on this evidence, the decision of the European Central Bank to assign a prominent role to money in the conduct of the Euro-area monetary policy stands out as an

12 The findings on the Euro-core countries concur with those reported by Rua (2012) and Mandler and Scharnagl (2014). Rua (2012) finds significant cross-spectral coherencies between 12 and 16 years, while Mandler and Scharnagl (2014) present evidence of a very weak relationship at low frequencies, but significant covariability at higher frequencies over short sub-samples.

exception that can be rationalized as a remaining umbilical cord of the Deutsche Bundesbank.[13]

3 QTM and high inflation episodes: a "scale-based" panel regression analysis

While a powerful tool for investigating the strength of the time-frequency variation in the money growth-inflation relationship the CWT cannot provide either numerical estimates or uncertainty measures around such estimates. To overcome this deficiency, a discretized version of the CWT, the Maximal Overlap Discrete Wavelet Transform (MODWT), can be applied. The MODWT is a compromise between the CWT, with its continuous variations in scale, and the DWT, where the number of scale variations J is discrete and the number of wavelet coefficients halves at each sequential level. The MODWT is highly redundant, meaning that it returns at each scale a number of coefficients equal to the length of the original series, but its transformations at each scale are not orthogonal. This is offset by the gain that the transform leaves the phase invariant. Furthermore, the transform is not restricted by the dyadic expansion used by DWT, and thus is also applicable to data sets of length not divisible by 2^J.

In this section we perform a parametric analysis of the QTM relationship on a scale-by-scale basis. First, we decompose the inflation rate and excess money growth variables for each country using the MODWT. We apply the LA(8) Daubechies (1992) wavelet filter (with reflecting boundary conditions and) for a number of levels J=4. A J=4 level decomposition produces four wavelet detail vectors, D_1,...,D_4, each associated with a specific frequency range (2-4, 4-8, 8-16 and 16-32 years, respectively), and one wavelet smooth vector, S_4, capturing

[13] The evidence provided in Figure 4 for Germany rationalizes the attention of German's monetary authority for monetary targeting.

fluctuations greater than 32 years. By sequentially adding the detail level components D_4, D_3, D_2 to the lower "smooth" component S_4 we get three additional levels of approximation, S_3, S_2 and S_1. The higher the index, the smoother is the function: S_1 captures fluctuations greater than 4 years, S_2 greater than 8 years, and S_3 greater than 16 years. Table 1 presents the frequency domain interpretation in terms of periods for each detail and approximation level when annual data are used.

Table 1: Frequency interpretation of detail and approximation levels

Detail level, D_j	Years	Approximation level, S_J	Years
D_1	*2-4*		
D_2	*4-8*	S_1	*from 4 to ∞*
D_3	*8-16*	S_2	*from 8 to ∞*
D_4	*16-32*	S_3	*from 16 to ∞*
		S_4	*from 32 to ∞*

Following Assenwacher-Wesche and Gerlach (2009) we estimate the inflation equation using the low frequency components of inflation and excess money growth corresponding to periodicities greater than 4, 8 and 16 years ($S_1, S_2, S_3,$). In addition, we also estimate equations at different frequency ranges, that is D_1, D_2, D_3, D_4. As in Gallegati et al. (2016, 2017) the (approximation and detail level) components of inflation and excess money growth of each country are first stacked into separate panel datasets, one for each approximation and detail level component. Then the inflation equation is estimated on a "scale-by-scale" basis using

$$\pi[S_j]_{it} = \alpha_i + a_j\, emg[S_j]_{it} + e_{j,it}$$

and

$$\pi[D_j]_{it} = \alpha_i + b_j\, emg[D_j]_{it-1} + e_{j,it}$$

where $[S_j]_{it}$, and $[D_j]_{it}$ represent the j-level approximation and detail components of inflation rate, π, and excess money growth, *emg*, for country *i* at time t, with J=1,2,....,4, and α_i individual effects with i=1,...,N.[14]

[14] With cross-sectional units such as this group of developed countries, the individual effects can be treated as fixed constant parameters rather than to be drawn from a distribution as in the random effect model.

Table 2: Full sample approximation and detail levels panel regressions: 1871-2013

$$\pi[S_j]_{it} = \alpha_i + a_j EMG[S_j]_{it} + e_{j,it} \quad \text{and} \quad \pi[D_j]_{it} = \alpha_i + b_j EMG[D_j]_{it-1} + e_{j,it}$$

Approx. and detail level	S_3 > 16 yrs	S_2 > 8 yrs	S_1 > 4 yrs	D_4 16-32 yrs	D_3 8-16 yrs	D_2 4-8 yrs	D_1 2-4 yrs
12 countries							
a_{it}, b_{it}	**0.822** *(0.116)*	**0.700** *(0.122)*	**0.621** *(0.121)*	**0.560** *(0.111)*	**0.264** *(0.118)*	*0.102* *(0.093)*	*0.022* *(0.060)*
R^2	0.747	0.594	0.484	0.590	0.099	0.016	0.001
plus Finland							
a_{it}, b_{it}	**0.911** *(0.141)*	**0.798** *(0.153)*	**0.783** *(0.195)*	**0.685** *(0.178)*	**0.382** *(0.165)*	*0.548* *(0.397)*	*0.150* *(0.137)*
R^2	0.714	0.583	0.493	0.539	0.175	0.177	0.025
plus Italy							
a_{it}, b_{it}	**0.901** *(0.127)*	**0.794** *(0.139)*	**0.770** *(0.177)*	**0.874** *(0.235)*	**0.735** *(0.362)*	*0.656* *(0.350)*	*0.218* *(0.148)*
R^2	0.721	0.592	0.496	0.604	0.258	0.179	0.026
plus Japan							
a_{it}, b_{it}	**0.855** *(0.123)*	**0.748** *(0.132)*	**0.719** *(0.167)*	**1.184** *(0.313)*	**1.569** *(0.617)*	**1.370** *(0.620)*	**1.676** *(0.548)*
R^2	0.691	0.553	0.454	0.593	0.496	0.261	0.562

Note: HAC (heteroskedastic consistent) robust standard errors in parenthesis. 5% significance level in bold. The 12 countries included in the initial estimation sample are: Australia, Canada, Denmark, France, Great Britain, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the US. a_{it} and b_{it} are parameter estimates of approximation and detail level regressions, respectively.

Table 2 presents the panel regression results for the inflation equation estimated over the 1871-2013 period using different subsets of countries. In particular, the first row excludes those countries that experienced a triple-digit increases of inflation and money growth during war years.[15] The following rows show the results when these countries are then included, one at a time.

The estimates of Table 3 resemble those reported in Assenmacher-Wesche and Gerlach (2009). The smoother is the series, the larger are the regression slopes of filtered data and the explanatory power of the regressions tends to rise monotonically when higher frequency components are progressively excluded. Interestingly, the results point to instability of the regression coefficients across different panels, with the inclusion of countries experiencing hyperinflation

[15] During war periods several countries suffered hyperinflation. Triple-digit inflation rates were experienced by Finland in 1918, and by several countries in the mid-1940s including France? Germany, Italy (peak in 1944) and Japan (from 1946 to 1949).

episodes determining a significant increase of the values of estimated coefficients at both the approximation and detail levels, without a corresponding increase of the explanatory power.

Beyond hyperinflation episodes at the individual country level, the inflation-money growth relationship is likely to be affected by episodes of inflation acceleration at the global level. Dewald (2003) reports that sustained inflationary trends, measured by ten-year inflation rate averages, have occurred in almost every country during wartime periods in the 1910s and the 1940s, as well as after the oil supply shock of the early 1970s; and these high rates of inflation were generally accompanied by equally high rates of money growth. According to Benati (2009) such infrequent inflationary upsurges are responsible for the one-to-one correlation between the long-term components of inflation and money growth.

In order to see whether high-inflation episodes create a potential bias in favor of the unit-slope hypothesis, in what follows we estimate the QTM relationship for different approximation and detail level components using moving variable-length sub-samples windows for the pre and post-WWII periods.[16] In particular, in the pre-WWII period we use sub-sample windows with a fixed starting point, 1871, while the ending point is allowed to move 5-years forward starting from 1905 to 1940. By contrast, in the post-WWII period we use sub-sample windows with end-point fixed, 2013, while the starting point is allowed to move 5-years forward starting from 1955 to 1990. We expect that adding or deleting observations pertaining to inflationary periods will lead to significant upward changes in parameter estimates. Moreover, to address the problem of hyperinflation for countries like Italy and Japan we apply the MODWT separately to pre- and post-WWII periods data. The aim of this procedure is to verify the role of high inflation

[16] This procedure allows to manage the inevitable trade-off between identification of time variation in the data and estimation of long-run relationship which requires a comparatively long window.

episodes for the unit slope result in the QTM while avoiding the distortionary effects stemming from the presence of hyperinflationary periods at the country level.

Tables 3 and 4 show the results for the inflation equation estimated over moving subsample windows with fixed starting-points (pre-WWII) and fixed end-points (post-WWII periods), with approximation and detail levels regressions being presented in the upper and lower panel of each table, respectively. The results point to instability of the coefficient of excess money growth across sub-samples, with upward and downward shifts in the estimated values observed both in the pre- and post-WWII periods. A similar pattern is also provided by the explanatory power of the panel regressions. This is evident at both approximation and detail levels.

In particular, Table 3 shows that between the sample ending in 1910 and that ending in 1920, and thus after the sharp increase in inflation during the 1910s, a *dramatic* upward shift is estimated at all approximation levels S_1 S_2 and S_3 with the excess money growth parameter reaching unity values at lower frequencies (greater than 16 years). Significant upward shifts of the estimated coefficients are also evident at the detail levels D_4 and D_3.[17] Thereafter, the estimated coefficients and explanatory power remain remarkably stable around these highs until the end of the pre-WWII period. The same pattern, upward shift followed by stability, is also followed by the explanatory power of the regressions.

[17] The coefficients of excess money growth increase from lower to higher detail levels, with the higher detail level, D_4, providing the highest estimated coefficient values, i.e. about 0.88 in sub-samples ending in early-to-mid 1920s. The estimated coefficients at the detail levels D_3 to D_4 are generally significant at the 5% level, whereas those at the D_2 level are never significant.

Table 3: *Approximation (upper) and detail (lower) levels panel regressions*

Pre-WWII estimates (1870-)

$\pi[S_j]_{it} = \alpha_i + a_j\, emg[S_j]_{it} + e_{j,it}$

	-1905	-1910	-1915	-1920	-1925	-1930	-1935	-1940
a$_{J[S3]}$ > 16 yrs	0.406 (0.212)	**0.460** (0.211)	**0.763** (0.224)	**1.003** (0.179)	**1.013** (0.164)	**0.985** (0.163)	**0.981** (0.159)	***0.899*** *(0.157)*
R²	0.347	0.345	0.509	0.706	0.734	0.730	0.728	0.693
a$_{J[S2]}$ > 8 yrs	0.267 (0.206)	0.291 (0.205)	**0.530** (0.209)	**0.926** (0.194)	**0.930** (0.178)	**0.923** (0.173)	**0.918** (0.169)	**0.850** *(0.161)*
R²	0.153	0.160	0.326	0.631	0.645	0.639	0.637	0.603
a$_{J[S1]}$ > 4 yrs	0.170 (0.180)	0.169 (0.176)	0.216 (0.147)	**0.939** (0.272)	**0.940** (0.247)	**0.934** (0.239)	**0.926** (0.232)	**0.868** *(0.220)*
R²	0.073	0.068	0.093	0.537	0.546	0.545	0.542	0.515

$\pi[D_j]_{it} = \alpha_i + b_j\, emg[D_j]_{it-1} + e_{j,it}$

	-1900	-1905	-1910	-1915	-1920	-1925	-1930	-1935
b$_{J[D4]}$ 16-32 yrs	0.465 (0.363)	0.540 (0.348)	**0.819** (0.332)	**0.828** (0.268)	**0.885** (0.227)	**0.882** (0.215)	**0.826** (0.195)	**0.801** (0.183)
R²	*0.185*	*0.241*	*0.440*	*0.484*	*0.580*	*0.584*	*0.596*	*0.594*
b$_{J[D3]}$ 8-16 yrs	**0.447** *(0.086)*	**0.466** *(0.072)*	**0.635** *(0.132)*	**0.799** *(0.184)*	**0.815** *(0.171)*	**0.781** *(0.156)*	**0.737** *(0.151)*	**0.689** *(0.153)*
R²	*0.184*	*0.197*	*0.319*	*0.448*	*0.507*	*0.510*	*0.484*	*0.439*
b$_{J[D2]}$ 4-8 yrs	*-0.004* *(0.144)*	*0.001* *(0.136)*	*0.010* *(0.127)*	*0.408* *(0.312)*	*0.844* *(0.492)*	*0.819* *(0.445)*	*0.806* *(0.429)*	*0.763* *(0.418)*
R²	*0.001*	*0.001*	*0.001*	*0.129*	*0.320*	*0.299*	*0.294*	*0.274*

Note: HAC (heteroskedastic consistent) robust standard errors in parenthesis. 5% significance level in bold. All countries, except Germany, are countries are included in the estimation sample.

Table 4 show the estimation results for the approximation and detail levels in the post-WWII period. There are strong similarities between the pre- and post-WWII estimates. Similarities are evident with respect to the tendency of the estimated coefficients and the explanatory power of the regressions to increase monotonically for the approximation components when higher frequency components are progressively excluded and for the detail components when moving from higher to lower frequency bands. Similar, although opposite in sign, is the shift of the estimated coefficients in the post-WWII period after the end of the inflationary period in the 1970s. When observations from the great inflation of the 1970s are excluded from the estimation sample a rapid decline in the values of the estimated coefficients in the S_1-S_3 regressions is observed. Indeed, in a decade, that is from early-1980s to early-1990s, the estimated coefficient values of excess money growth fell to about one-third of their initial values. Estimated coefficients corresponding to the sub-samples including the 1970s reach values between 0.89 and 0.92 at the S_3 approximation level (values which are well beyond the unit slope), values between 0.77 and 0.80 at the S_2 level, and values between 0.65 and 0.69 at the S_{12} level. Comparable is also the stability pattern of the estimated coefficients and explanatory power across sub-samples including the 1970s (the only high inflation period in the post-WWII era). The main difference between the the pre- and post-WWII periods is that while estimation results are qualitatively similar, they tend to weaken in the post-WWII years.[18] This is consistent with the conclusions reached by other scholars: for example, Begg et al. (2003), Sargent and Surico (2011) and Teles et al. (2016) document that the long-run link between money growth and inflation has become looser after WWII.

Table 4 - *Approximation (upper) and detail (lower) levels panel regressions - Post-WWII estimates (-2013)*

$\pi[S_J]_{it} = \alpha_i + b_J\, emg[S_J]_{it} + e_{J,it}$

	1955-	1960-	1965-	1970-	1975-	1980-	1985-	1990-
$a_{J[S3]}$ > 16 yrs	**0.895** (0.051)	**0.920** (0.042)	**0.919** (0.039)	**0.923** (0.039)	**0.919** (0.048)	**0.817** (0,062)	**0.555** (0.131)	**0.260** (0.108)
***R*²**	0.801	0.837	0.853	0.873	0.879	0.832	0.643	0.489
$a_{J[S2]}$ > 8 yrs	**0.772** (0.055)	**0.798** (0.048)	**0.802** (0.047)	**0.806** (0.044)	**0.793** (0.049)	**0.638** (0.064)	**0.382** (0.103)	**0.249** (0.079)
***R*²**	0.703	0.729	0.739	0.758	0.758	0.695	0.515	0.468
$a_{J[S1]}$ > 4 yrs	**0.652** *(0.072)*	**0.680** (0.065)	**0.685** (0.063)	**0.689** (0.060)	**0.691** (0.067)	**0.562** (0.077)	**0.285** (0.082)	**0.191** (0.063)
***R*²**	0.579	0.615	0.628	0.646	0.683	0.604	0.441	0.376

$\pi[D_J]_{it} = \alpha_i + b_J\, emg[D_J]_{it-1} + e_{J,it}$

	1955-	1960-	1965-	1970-	1975-	1980-	1985-	1990-
$b_{J[D3]}$ 16-32 yrs	***0.626*** *(0.101)*	***0.592*** (0.070)	***0.571*** (0.071)	***0.556*** (0.063)	***0.517*** (0.068)	***0.444*** (0.063)	***0.432*** (0.070)	**0.347** (0.063)
***R*²**	0.419	0.464	0.474	0.507	0.508	0.487	0.489	0.483

[18] For instance, the magnitude of the estimated coefficients is comparatively higher in the pre-WWII period with values for all approximation levels greater than .90.

b$_{J[D2]}$ 8-16 yrs	***0.207*** *(0.061)*	**0.243** (0.068)	**0.218** (0.067)	**0.185** (0.069)	**0.156** (0.063)	**0.139** (0.057)	**0.149** (0.061)	**0.112** (0.064)
R²	0.085	0.120	0.106	0.082	0.073	0.089	0.122	0.096
b$_{J[D1]}$ 4-8 yrs	***0.067*** *(0.040)*	**0.067** (0.039)	**0.064** (0.038)	**0.069** (0.035)	**0.117** (0.038)	**0.064** (0.029)	**0.047** (0.031)	**0.039** (0.031)
R²	0.013	0.013	0.012	0.014	0.047	0.023	0.018	0.015

Note: HAC (heteroskedastic consistent) robust standard errors in parenthesis. 5% significance level in bold. All 16 countries are included in the estimation sample.

In sum, the main result stemming from this "scale-by-scale" panel regression analysis on variable-length sub-samples in the pre- and post-WWII periods is that the excess money growth coefficient has been subject to upward and downward shifts which are associated with the beginning of the inflationary upsurges around the time of WWI and the ending of the great Inflation period in early 1980s. In sub-samples characterized by these inflationary upsurges the estimated coefficients are boosted towards one and tend to exhibit remarkable stability. Outside these periods the coefficients are much lower.

Our findings are consistent with those reported by Sargent and Surico (2011) who examine US data for the 1900-2005 period and show substantial deviations from unit slopes, except for the 1900-28 and 1960-1983 years (and to a lesser extent between 1955 and 1975). Similarly, Sargent and Surico (2011) and Teles et al. (2016) document a looser long-run link between money growth and inflation after the great inflation of the 1970s, especially in typically low inflation countries.

4 Conclusion

Our exercise confirms the problems, risks, but also the benefits of using historical macroeconomic datasets to investigate the observed changes in the low-frequency relationships between money growth and inflation. We draw several conclusions from our empirical analysis of the quantity theory of money relationship. First, Friedman's celebrated statement that "inflation is always and everywhere a monetary phenomenon" requires the critical qualifiers "in the medium to long run", a period well beyond the business cycle frequency range. Second, the low estimated values of the excess money growth coefficient outside periods characterized by high inflation episodes suggests that the one-to-one relationship within the QTM is no more than a temporarily pattern associated with large fluctuations in the low frequency component of inflation and excess money growth typical of inflationary upsurges (Benati, 2009). All in all, that inflationary upsurges affect regression coefficients, but not the closeness of the long-run relationship, is a finding that may reconcile the validity of the quantitative theory of money and suggest that policymakers should not lose interest in money growth rates for the conduct of monetary policy.

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Appendix: some basic concepts on wavelet analysis

The wavelet transform uses a set of orthogonal basis functions which are local, not global. Thus, wavelet analysis, by dealing with local aspects of a signal, provides us with a method having the ability to handle a variety of nonstationary and complex signals.

The Continuous Wavelet Transform

The essential characteristics of wavelets are best illustrated through the development of the continuous wavelet transform (CWT). We seek functions $\psi(u)$ such that:

$$\int_{-\infty}^{\infty} \psi(u)du = 0$$

$$\int_{-\infty}^{\infty} \psi(u)^2 du = 1$$

The cosine function is a “large wave” because its square does not converge to 1, even though its integral is zero; a wavelet, a “small wave” obeys both constraints. The continuous wavelet transform (CWT) of a signal x(t) with respect to the wavelet function ψ is a function $W_x(s, u)$

$$W_{xy}(s,u) = \int_{-\infty}^{\infty} \psi_{(s,u)}(t)x(t)dt$$

where the wavelet basis, called “mother wavelet”, defined as

$$\psi_{s,u}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)$$

is a function of two parameters s and u. The first is a scaling or dilation factor that controls the length of the wavelet, the latter is a location parameter that indicates where the wavelet is centered along the signal. The set of CWT wavelet coefficients, each representing the amplitude of the wavelet function at a particular position and for a particular wavelet scale, is obtained by projecting x(t) onto the family of "wavelet daughters" $\psi_{(s,u)}$ obtained by scaling and translating the “mother wavelet” ψ by s and u, respectively.

The application of the continuous wavelet transform requires the specification of the wavelet function and the treatment of boundary conditions, as the continuous wavelet transform, with other types of transforms, suffers from a distortion problem due to the finite time series length which affects wavelet transform coefficients at the beginning and end of the data series. Wavelet transform coefficients are then calculated using the Morlet wavelet, a widely used wavelet among the numerous types of wavelet families available, i.e. Mexican hat, Haar, Daubechies, etc..

As with other types of transforms, the CWT applied to a finite length time series inevitably suffers from border distortions; this is due to the fact that the values of the transform at the beginning and the end of the time series are always incorrectly computed, in the sense that they involve missing values of the series which are then artificially prescribed; the most common choices are zero padding extension of the time series by zeros or periodization. Since the effective support of the wavelet at scale s is proportional to s, these edge effects also increase with s. The region in which the transform suffers from these edge effects is called the cone of influence. In this area of the time-frequency plane, the results are unreliable and have to be interpreted carefully (see Percival and Walden, 2000).

Let W_x and W_y be the continuous wavelet transform of the signals x(.) and y(.), their cross-wavelet power is given by $|W_{xy}|=|W_xW_y|$ and depicts the local covariance of two time series at each scale and frequency (see Hudgins, Friehe, and Mayer 1993). Being the product of two non-normalized wavelet spectra, the cross-wavelet can identify the significant cross-wavelet spectrum between two time series, although there is no significant correlation between them.

The wavelet coherence is defined as the modulus of the wavelet cross spectrum normalized by the wavelet spectra of each signal,

$$R_{xy}^{2} = \frac{\left|S\left(s^{-1}W_{xy}(s,u)\right)\right|^{2}}{\left|S\left(s^{-1}W_{x}(s,u)\right)\right|\left|S\left(s^{-1}W_{y}(s,u)\right)\right|}$$

where S is a smoothing operator (see Torrence and Webster, 1999). The squared wavelet coherence coefficient R^2_{xy}, ranging between 0 and 1, is analogous to the squared correlation coefficient in linear regression. It can be considered a direct measure of the local correlation between two time series at each scale (Chatfield, 1989) and used to detect the time and frequency intervals where two phenomena have strong interactions. Moreover, from the imaginary and real parts of the cross wavelet transform we can get information regarding the relative position of the two series through the phase difference, defined as:

$$\phi_{xy} = \tan^{-1}\left(\frac{\Im\left|\left(W_{xy}(s,u)\right)\right|}{\Re\left|\left(W_{xy}(s,u)\right)\right|}\right)$$

A phase-difference of zero indicates that the time series move together at the specified time-frequency. If $\varphi_{x,y} \in (0, \pi/2)$, then the series move in phase, but securities leads loan; if $\varphi_{xy} \in (-\pi/2, 0)$, then is loan that is leading; a phase-difference of π or −π indicates an anti-phase relation; if $\varphi_{xy} \in (\pi/2, \pi)$, then loan is leading, while securities is leading if $\varphi_{xy} \in (-\pi, -\pi/2)$.

The Discrete Wavelet Transform

The CWT contains a high amount of redundant information so that it is computationally impossible to analyze a signal using all wavelet coefficients. A more parsimonious representation of the evolution over time of the periodic components of a signal is provided by the discrete wavelet transform (DWT) which discretize the transform over scale and over time through the dilation and location parameters. In the DWT only a limited number of translated and dilated versions of the mother wavelet are used to decompose the original signal by selecting t and λ in a way that the information contained in the signal can be summarized in a minimum number of wavelet coefficients. The general formulation for the continuous wavelet transform can be restricted to the definition of the discrete wavelet transform (DWT) by discretizing the parameters s and u. In order to obtain an orthonormal basis a transform of the scaling parameter, $s=s^j_o$, and the Nyquist sampling rule, $u=ks^j_oT$, are used. The key difference between the CWT and the DWT lies in the fact that the DWT uses only a limited number of translated and dilated versions of the mother wavelet to decompose the original signal. When the computation is done octave by octave, i.e. $\lambda_0=2$, we get the following equation for the "mother wavelet":

$$\psi_{J,k}(t) = 2^{-J/2}\psi\left(\frac{t-2^J k}{2^J}\right)$$

This function represents a sequence of rescaleable functions at a scale of $\lambda = 2^j$, j = 1, 2, ...J, and with time index k, k = 1, 2, 3, ...$N/2^j$. The wavelet transform coefficient of the projection of the observed function f(t) for i = 1, 2, 3, ...N, $N = 2^J$ on the wavelet $\psi_{j,k}(t)$ is given by:

$$d_{j,k}(t) = \int \psi_{j,k}(t) f(t) dt$$

$$j = 1, 2, ..J \qquad (5)$$

For a complete reconstruction of a signal f(t), one requires a scaling function, φ(.), that represents the smoothest components of the signal. While the wavelet coefficients represent weighted "differences" at each scale, the scaling coefficients represent averaging at each scale. One defines the scaling function, also known as the "father wavelet", by:

$$\varphi_{J,k}(t) = 2^{-J/2}\varphi\left(\frac{t-2^J k}{2^J}\right)$$

and the scaling function coefficients vector is given by:

$$s_{J,k}(t) = \int \varphi_{J,k}(t) f(t) dt$$

By construction, we have an orthonormal set of basis functions, whose detailed properties depend on the choices made for the functions, φ(.) and ψ(.), see for example the references cited above as well as Daubechies (1992) and Silverman (1999). At each scale, the entire real line is approximated by a sequence of "non-overlapping" wavelets. The deconstruction of the function f(t) is, therefore:

$$f(t) \approx \Sigma_k s_{J,k}\varphi_{J,k}(t) + \Sigma_k d_{J,k}\psi_{J,k}(t) + \Sigma_k d_{J-1,k}\psi_{J-1,k}(t) + \ldots + \Sigma_k d_{1,k}\psi_{1,k}(t)$$

The above equation is an example of the Discrete Wavelet Transform, DWT, based on an arbitrary wavelet function, φ(.). For the DWT, where the number of observations is N, $N = 2^J$, the number of coefficients at each scale is:

$$N = N/2^J + N/2^J + N/2^{J-1} + \ldots N/4 + N/2$$

That is, there are $N/2^J$ coefficients $s_{J,k}$, $N/2^J$ coefficients $d_{J,k}$, $N/2^{J-1}$ coefficients $d_{J-1,k}$... and N/2 coefficients $d_{1,k}$. Further, the approximation can be re-written in terms of collections of coefficients at given scales as:

$$f(t) \approx S_J + D_J + D_{J-1} + \ldots D_2 + D_1 \qquad (11)$$

S_J contains the "smooth component" of the signal, and the D_j , j = 1, 2, ..J, the detail signal components at ever increasing levels of detail. S_J provides the large scale road map, D_1 shows the pot holes. The previous equation indicates what is termed the multiresolution decomposition, MRD.

Finally, we might mention the maximal overlap discrete wavelet transform (MODWT) which is a compromise between the CWT, with continuous variations in scale, and DWT where the power of the transform is highly localized. The MODWT is highly redundant so that the transformations at each scale are not orthogonal, but the offsetting gain is that applying the transform leaves the phase invariant, a very useful property in analyzing transformations, and the transform is not restricted to limitations imposed by the dyadic expansion used by the DWT. Indeed, because of the practical limitations of the DWT, wavelet analysis is generally performed by applying the MODWT, a non-orthogonal variant of the classical discrete wavelet transform that, unlike the DWT, is i) translation invariant, as shifts in the signal do not change the pattern of coefficients, ii) can be applied to data sets of length not divisible by 2^J and iii) returns at each scale a number of coefficients equal to the length of the original series.

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