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Implicit Interest as Price Discrimination in the Bank Deposit Market

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Abstract

In this paper the theory of nonlinear multiproduct pricing is applied to the problem of determining the terms of liquid bank deposits such as cheque accounts. The profit maximizing interest rate and service charge schedules are characterized within a model of a monopoly bank with a heterogeneous clientele. It is shown that the practice of "implicit interest", meaning below-cost pricing of payment services parallel with large interest margins on deposits, may well be part of the optimal price discrimination strategy. It is also shown that necessary conditions for that kind of cross subsidization to be optimal exist in an inventory theoretic model of the demand for cheque account services. It is argued that the failure to take (second-degree) price discrimination into account invalidates much of the previous research on demand deposit pricing.

Tiivistelmä

Tässä tutkimuksessa sovelletaan epälineaarisen monituotehinnoittelun teoriaa pankkitalletusten, erityisesti sekki- ja ns. käyttelytilien hinnoitteluun. Voiton maksimoivat talletuskorko- ja palvelumaksuasteikot johdetaan monopolipankin ja heterogeenisen asiakaskunnan tapauksessa. Osoitetaan, että pankin kannalta parhaaseen hintastrategiaan voi näissä oloissa kuulua "implisiittisen koron" maksaminen, eli pankkipalvelujen tarjoaminen alihintaan samaan aikaan kun ylläpidetään suurta talletuskorkomarginaalia. Edelleen näytetään, että välttämättömät ehdot tällaiselle ristisubventiolle ovat voimassa varastoteoreettisessa pankkipalvelujen kysyntämallissa. Aikaisemmassa talletusten hinnoittelua koskevassa teoriassa ei ole juuri käsitelty hintadiskriminaation mahdollisuutta, mitä voidaan tämän tutkimuksen valossa pitää vakavana puutteena.

Contents

1 Introduction

The purpose of this paper is to explore the possibilities of applying the theory of multiproduct nonlinear tariff design to the pricing of cheque accounts. This endeavour is motivated by the need to improve our under-standing of some features of deposit pricing by banks, especially the relatively low interest rates on liquid deposits and the apparent underpricing (cross-subsidization) of payment services. Neither of these features is uncommon in the deposit banking industry.

The market for liquid bank deposits such as cheque accounts consists actually of two submarkets: the market for deposited funds, and the market for payment services performed by the bank when deposits are used as a means of payment. Price formation in these two markets is an interesting subject of study, both because the nature of competition in the retail deposit market shapes the whole banking industry in a powerful way, and also because the observed pricing practices are not trivially explained by the standard competitive price theory.

A salient feature of (retail) cheque account pricing is the prevalence of cross subsidies between the deposited funds market and the payment services market. This is apparent in that the rate of interest on cheque account deposits is usually significantly lower than the rate of interest on CD's, for example. At the same time, service charge revenues from payment services are small compared to the large costs of these services, including branch networks, large computer systems etc. (this is easily verified from OECD (1993), for example; see also Vittas, et al. (1988)). This observed cross subsidization rises immediately issues of efficiency and distributional effects of the banks' conduct. The purpose of this paper is to suggest a framework which may be used in analyzing these problems.

During the decades of deposit rate regulation, the cross subsidies on depositors' payment services were easy to explain as "implicit interest" although some disagreement prevailed as to how this "implicit interest" was determined and what kind of effect it had on the demand for checkable deposits (cf. Barro and Santomero (1972), Benjamin Klein (1974) and Becker (1975)). Many economists believed that the cross subsidies would disappear when deposit pricing would be deregulated (cf. Saving (1979) and Fisher (1983)). After the international wave of deregulation in the 1980's, significant interest margins continue to exist, however, and while banks have tried to raise service charges, the share of fees in banks' net income has not become dominant.

One explanation for the viability of "implicit interest" even in unregulated markets relies on a tax minimization argument. In many fiscal systems, interest income is taxable, but the benefit of free or underpriced banking services is not taxed (nor are service charges related to payment services deductible in taxation). That kind of system encourages banks to compete for deposits with tax free implicit interest instead of taxable explicit interest (Walsh (1983)). It should be noted, however, that the tax distortion is not present in corporate taxation.[1] Also, we sometimes observe banks paying interest to customers who

[1] This has been pointed out to me by Hans-Werner Sinn.

are simultaneously paying service charges to banks. This is in contradiction with complete tax arbitrage (see Tarkka (1992)). So, other explanations than the tax distortion argument should probably be explored as well.

In this paper, imperfect competition is considered as an alternative explanation. There are already a number of studies which have applied models of imperfect competition to explain deposit pricing, starting from Michael Klein (1971). Those which have taken the pricing of payment services explicitly into account include the spatial competition model of Baxter, Cootner, and Scott (1977), Mitchell's (1988) monopoly model of service charge determination, and Whitesell's (1992) monopoly model of both demand deposit interest and service charges. Unfortunately, all of these studies share an important shortcoming: they arbitrarily restrict the analysis to the case of uniform pricing, although the uniformity assumption is in contradiction with observed pricing practice and may be seriously questioned on theoretical grounds.

In practice, banks operating in deposit markets where pricing is not regulated typically apply complex nonlinear pricing schemes on cheque accounts. The nonlinearities come in many forms. There may be fixed, monthly or yearly account maintenance fees, for instance. On the deposited funds dimension, customers with higher balances on their accounts are often favoured with better interest rates or lower service charges or both. On the payment services dimension, it is not uncommon that charges depend on the number of cheques written etc. These practices are reviewed by Hörngren (1988), Vittas et al. (1988) and Davis and Korobow (1987), for example.

Turning to theory, uniform (non-discriminatory) pricing is generally suboptimal from a monopolist's point of view. Only if the products can be costlessly resold on a "second-hand" market, or if customers can freely form coalitions to purchase a service jointly, can we safely assume that each unit of a monopolist's output must be sold at the same price. In retail banking, where each household and firm typically has its own bank account, the conditions for price discrimination are clearly present. In assessing the previous theoretical literature, it should be recalled that the possibility of price discrimination invalidates the analytical results obtained under the uniform price assumption even when all customers are identical.

This paper approaches the "implicit interest" issue and other aspects of the cheque account pricing problem from a price discrimination perspective. It presents a model of deposit pricing by a monopoly bank, which practices second-degree price discrimination among heterogenous depositors. The pricing problem of a discriminating multiproduct monopoly selling to customers differentiated by an unobservable characteristic was first solved by Roberts (1979) and Mirman and Sibley (1980).[2] In this paper the model is applied to the deposit pricing problem and the previously neglected possibility of marginal cross subsidies is considered.

It turns out that this kind of model can explain several commonly observed features of cheque account pricing. Firstly, the price dis-crimination model easily produces a large spread between the security market rate (net of intermediation costs) and the deposit rate of interest. Secondly, cases in which payment services are supplied at prices below marginal cost are shown to be

[2] Wilson (1993) presents a thorough survey of the relevant techniques.

possible. This is perhaps particularly interesting, since sufficient conditions for profit maximizing nonlinear tariffs to include marginal subsidies are not well established in the literature. Moreover, a model of the demand for money is presented which suggests that conditions for cross subsidization may be actually inherent to the transaction deposit market. Third, nonlinearities in deposit pricing get a clear microeconomic interpretation in the model, and may even be used to construct empirical tests of the importance of price discrimination in banking.

The paper is organized as follows. The microeconomic assumptions concerning the demand for deposit services are presented in section 2. The profit maximization problem of a monopoly bank is solved in section 3, yielding a first-order characterization of the optimal nonlinear tariff. The validity of the insights provided by the first-order approach especially on the cross subsidization issue is analyzed in section 4. An inventory theoretic analysis of the demand for banking services is presented in section 5, suggesting that incentives to marginal subsidization may be inherent in banking. Results of the paper are summarized and evaluated in section 6.

2 The demand for bank services

We model the demand for deposit services using a liquidity cost approach, as defined in Feenstra (1986), for example. This approach explains the demand for money by assuming that agents hold money in order to reduce liquidity costs (or transactions costs) as implied by the available transactions technology. In this analytical tradition, money balances enter the cost function as an input, reducing transactions costs incurred from earning and spending a given flow of income. In this section, this approach is generalized to the case of two (monetary) banking services: deposit-taking and payment services.

It is assumed that there is a continuum of depositors, identical in all respects except income level Y. Income level can thus serve as a "type parameter" in the language of price discrimination models. The distribution of Y in the population is according to the continuous cumulative distribution function F(Y), where $Y \in [Y^{min}, Y^{max}]$. We require finite densities, so F(0) = 0 and f(Y) = dF(Y)/dY for all Y.

Depositors incur internal transactions costs which depend upon their income level and the amount of deposit services they use. A generic form of the transactions cost function may be written as G(Y, D, N), where D denotes the average deposit balance, and N denotes the volume of transactions made with the account (number of cheques and transfers per unit of time, for example). Below, variables D and N will be called simply "deposits" and "payment services", respectively.

The shape of the cost function is obviously crucial for the demands for the two services. We assume that this function is twice continuously differentiable, strictly increasing in the income level, and strictly decreasing in both banking services, whenever the income level is positive. Denoting partial derivatives by subscripts, we may summarize these properties by writing:

(i) $G_Y(Y, D, N) > 0$

(ii) $G_D(Y, D, N) \leq 0$ and $\{G_D(Y, D, N) < 0$ for all $Y > 0\}$

(iii) $G_N(Y, D, N) \leq 0$ and $\{G_N(Y, D, N) < 0$ for all $Y > 0\}$

We also assume that transactions costs are convex in both banking services:

(iv) $G_{DD}(Y, D, N) > 0$

(v) $G_{NN}(Y, D, N) > 0$

(vi) $G_{DD}(Y, D, N)\, G_{NN}(Y, D, N) - [G_{ND}(Y, N, D)]^2 > 0$

Basically, the convexity assumptions (iv–vi) ensure that well-behaved demand functions exist in the standard neoclassical sense at least if both services (D and N) are available at given marginal prices (i.e. under a linear price system).

The depositors are assumed to choose the amount of deposits to hold (as an average stock) and the volume of payments they make with their deposits (as the average number of transactions per a unit of time) by minimizing total liquidity costs, which are defined as the sum of internal costs G(Y, D, N) and financial costs of holding deposits D and using payment services N. The financial part of total liquidity costs depends on the interest rate paid on deposits, service charges on payment services, and the alternative rate of return on the depositor's funds.

Let us assume that the alternative rate of return, determining the depositor's opportunity cost of funds, is the security market rate r. Then, collecting the net amount of deposit interest less service charges to a general tariff function P(D, N), the decision problem of a depositor of type Y can be written as

$$\min_{D,N} \; [G(Y,D,N) + P(N,D) + rD] \tag{1}$$

Restricting the tariff function to be independent of the customer's type Y means that we rule out first-degree price discrimination. By allowing the tariff function to be nonlinear we are able to take second-degree price discrimination into account.

In the case of uniform pricing of depositors' services, the deposit rate and the service charge would be constants. In that case, the tariff function would be linear, i.e. $P(D, N) = sN - iD$, where s is the average service charge and i the interest rate on the average deposit balance. It is well established that linear tariffs are not optimal for a monopolist except under restrictive assumptions, however (cf. Phlips (1983)).

In the case of nonlinear tariffs, it may not be possible to separate the tariff into additive components which could be unambiguously interpreted as price schedules for individual products. So, considering cheque account pricing, it is perfectly possible that intramarginal interest payments and service charges can not be defined in a meaningful sense. Generally, this will be the case whenever the tariff function is not additively separable.[3] Marginal prices, by contrast, can be defined whenever the tariff function is differentiable. So, we define the following concepts:

1. The marginal interest rate on deposited funds is the negative of the partial derivative of the tariff function with respect to deposits: $-P_D(D,N)$.

2. The marginal service charge is the partial derivative of the tariff function with respect to the use of payment services $P_N(D, N)$.

Let us now turn to analyze the solution of the representative depositor's cost minimization problem. The first order conditions for depositor optimum are as follows:

$$G_D(Y,\hat{D},\hat{N}) + P_D(\hat{D},\hat{N}) + r = 0 \tag{2}$$

$$G_N(Y,\hat{D},\hat{N}) + P_N(\hat{D},\hat{N}) = 0 \tag{3}$$

Here, hats over the variables denote optimal values of D and N. It should be noted that the optimal values are not constants but functions of Y. If the tariff function is linear, the second-order sufficient conditions for a minimum are assured by the convexity assumptions (ii)–(vi) above. In the general nonlinear case, the local second-order conditions require that the sum G(.) + P(.) is locally convex in D and N (see Wilson (1993) pp. 318–319). We will return to this issue in more detail below, after the profit-maximizing tariff function has been characterized.

Finally, for conditions (2) and (3) to represent a true cost minimum, so-called participation conditions must also be fulfilled. These relate to the question whether it is optimal to hold deposits and use payment services at all. The answer is positive if the following condition holds:

$$G(Y,\hat{D},\hat{N}) + P(\hat{D},\hat{N}) + r\hat{D} - G(Y,0,0) < 0 \tag{4}$$

The participation conditions are crucial for the analysis of price discrimination, for they determine the upper limit to the revenue which can be extracted from any single individual. From the tariff design point of view, participation conditions are therefore usually called participation constraints. In the present context, the economics behind participation constraints relates to alternative means of payment. That deposits are not held and payment services (produced by banks) are not used given a positive income level and transactions volume is obviously possible only due to the alternative of using currency as the only exchange medium. In constructing the transactions cost function G(Y, D, N),

[3] In practice, tax minimization reasons could cause one classification of the intramarginal tariff to be preferred to another. See Tarkka (1992) for an analysis of tax arbitrage in the deposit market.

the role of currency is not explicitly taken into account, however. This simplification, which is made for tractability, does not affect our results in a material way.[4]

3 The monopoly bank and its pricing behaviour

We consider a case in which deposits and payment services are supplied by a monopoly bank serving a given population of depositors. The bank invests the deposited funds in securities which yield interest at an exogenously given rate r. In the process, it incurs proportional intermediation costs of δ units per a unit of intermediated funds, so that the net rate of return on investment is r−δ. For simplicity, the marginal production costs of payment services are assumed to be constant, denoted by c. The interest paid to depositors and the service charges levied on them are described by the nonlinear tariff function P(D, N), as described above. Under these assumptions, the profit of the bank is given by

$$\Pi = \int_{Y^{Min}}^{Y^{Max}} f(Y)[P(D,N) + (r-\delta)D - cN]dY \tag{5}$$

The bank maximizes this expression subject to the constraint that the consumers choose D and N as defined by formulas (2) and (3) above, provided that the sufficient conditions for depositor optima are satisfied.

Mirrlees (1971) demonstrated how this kind of nonlinear tariff problem can be solved by transforming it to a standard control problem. The idea is to treat the type parameter (Y) as the independent variable; the maximand or minimand of the individuals (here the total liquidity cost Γ) as the state variable, and the quantities (here D and N) as the control variables. Once the profit maximizing assignments D(Y) and N(Y) are found, it may be possible to use the first-order conditions for individual optima to solve for the marginal tariffs.

In the first instance, the Mirrlees approach requires eliminating the tariff altogether from the profit function. This can be done by using the fact that in the optimum of the representative depositor, total liquidity cost Γ(Y) can be expressed as follows:

$$\Gamma(Y) = G[Y, \hat{D}(Y), \hat{N}(Y)] + P[\hat{D}(Y), \hat{N}(Y)] + r\hat{D}(Y) \tag{6}$$

whence

[4] If currency holdings were explicitly taken into account, the cost minimization problem of a representative individual could be written as: min C(Y,M,D,N) + P(D,N) + rD + rM, where minimization would now happen with respect to deposits D, payment services N and currency holdings M. By optimizing with respect to M one gets $M^* = M(Y,D,N,r)$, so that if we set G(Y, D, N) ≡ C(Y, M(Y,D,N,r), D, N), the problem is formally equivalent to the one in the body of the text.

$$P[\hat{D}(Y),\hat{N}(Y)] = \Gamma(Y) - G[Y,\hat{D}(Y),\hat{N}(Y)] - r\hat{D}(Y) \tag{7}$$

To construct the Hamiltonian function for the problem at hand, the "equation of motion" for $\Gamma(Y)$ is needed. Totally differentiating the definition of total liquidity cost (1) and using the first order conditions for depositor optimum (2) and (3) the equation of motion is obtained as the following envelope condition:

$$\Gamma_Y = G_Y(Y,D,N) \tag{8}$$

The Hamiltonian is now

$$H = f(Y)\{\Gamma(Y) - G[Y,D(Y),N(Y)] - cN - \delta D\} + \lambda(Y)G_Y(Y,D,N) \tag{9}$$

First-order conditions for the optimal quantity assignments D(Y) and N(Y) include the following equations:

$$f(Y)[G_D(Y,D,N) + \delta] = \lambda(Y)G_{DY}(Y,D,N) \tag{10}$$

$$f(Y)[G_N(Y,D,N) + c] = \lambda(Y)G_{NY}(Y,D,N) \tag{11}$$

$$\Gamma_Y(Y) = G_Y(Y,D,N) \tag{12}$$

$$\lambda_Y = -f(Y) \tag{13}$$

We must also consider transversality conditions of the problem. The starting point for the state variable can be fixed by referring to the participation constraint as follows. Observe that, in the monopolist's optimum, the participation constraint must be binding for some depositors – otherwise some freely available profits were lost. Moreover, if the net participation benefit from being a customer of a bank, i.e. the difference
G(Y, 0, 0) – G(Y, D(Y), N(Y)) + P(D(Y), N(Y)) + rD, is increasing in Y, the participation constraint can only be binding in the lowest point of the income distribution. Here, this monotonicity property is assumed; that it is actually possible will be demonstrated by a quadratic example in the next section. By (13), the costate variable λ is decreasing in Y; it will be zero at Y^{max}. Together, the transversality conditions are:

$$\Gamma(Y^{min}) = G(Y^{min},0,0); \qquad \lambda(Y^{max}) = 0 \tag{14}$$

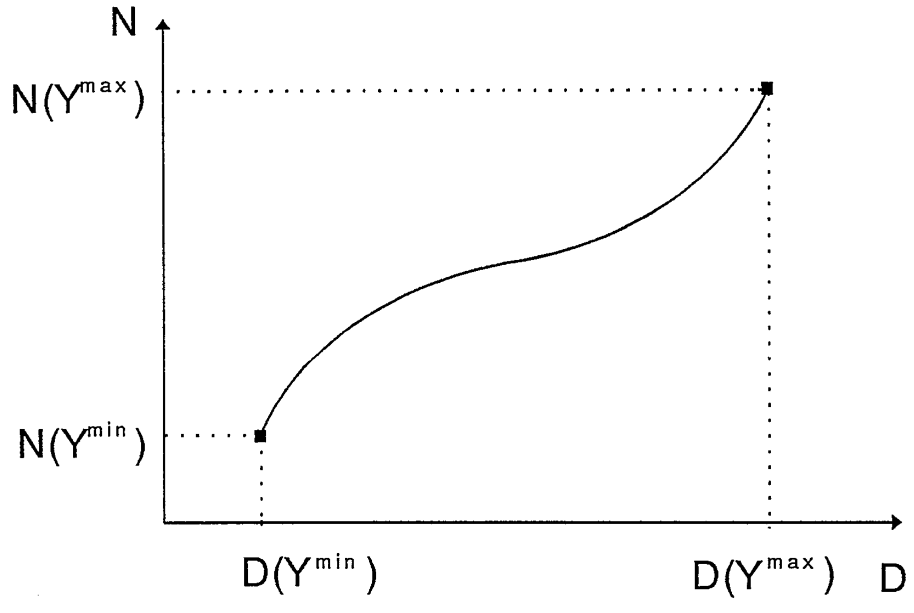
The terminal point constraint enables us to integrate the equation of motion for λ to obtain

$$\lambda(Y) = 1 - F(Y) \tag{15}$$

Equations (10), (11) and (15) implicitly define the assignments of deposits and payment services to depositors at different points of the Y-distribution as functions of Y. Recall that these assignments are voluntarily chosen by the depositors on the basis of a nonlinear tariff which is implicit in the solution but will be characterized below.

In the well-behaved case in which both of the functions $D = D(Y)$ and $N = N(Y)$ defined by (10), (11) and (15) are monotonous (both strictly increasing, for example), it happens that all customers will be induced to choose bundles of D and N such that the lie on a single locus in the D, N surface (Figure 1). By their position on this locus, the customers reveal their type Y. This property of the solution for nonlinear tariff design problems is typical for problems where customers are heterogenous with respect to a single type parameter (Mirman and Sibley (1980); Wilson (1993) p. 327–331).

Figure 1. **An equilibrium locus of (D,N) bundles**



What kind of tariff leads to the voluntary choice of the quantity assignments derived above? By applying the depositors' first order optimality conditions to the results (10) and (11) and using (15), we are able to characterize the optimal tariff with the following equations:

$$-P_D(D,N) = (r - \delta) + \frac{[1 - F(Y)]}{f(Y)} G_{DY}(Y,D,N) \tag{16}$$

$$P_N(D,N) = c - \frac{[1 - F(Y)]}{f(Y)} G_{NY}(Y,D,N) \tag{17}$$

The formula (16) gives the marginal interest rate on deposited funds; (17) gives the marginal service charge. These conditions are analogous to the marginal price functions derived by Mirman and Sibley. These conditions are useful, of course, only if the depositors' first order conditions suffice to define global optima. Provided that this is the case, the following properties of the optimal tariff emerge:

i) The depositors at the top end of the income distribution (for whom $Y = Y^{max}$) are charged marginal prices which correspond to marginal costs. This follows from the fact that under the assumptions which we made on the distribution F(Y), we get $[1 - F(Y^{max})]/f(Y^{max}) = 0$. In the market for deposits, this implies the result $-P_D(D(Y^{max}), N(Y^{max})) = r - \delta$, meaning that, for largest deposits, the marginal deposit rate equals the bank's marginal net rate of return on investment. As regards pricing in the payment services market, we obtain the result $P_N(D(Y^{max}), N(Y^{max})) = c$, implying that (again for the top-bracket customers) the marginal service charge equals the marginal cost of payment services.

ii) The depositors who are located at the interior points or at the low end of the income distribution ($Y^{min} \leq Y < Y^{max}$) are charged marginal prices which generally deviate from the marginal cost. Because the hazard rate $f(Y)/[1-F(Y)]$ is always positive for these depositors, the sign of the price-cost margin on service i (i = D, N) is necessarily opposite to that of the cross derivative G_{Yi}. So, if $G_{YD} > 0$, we get marginal deposit rates below $r-\delta$; this is the case which seems to prevail in actual markets. In the payment services market, correspondingly, the pricing is "above cost" ($P_N > c$) if $G_{YN} < 0$. However, the optimality conditions suggest below-cost pricing of payment services ($P_N < c$) if $G_{YN} > 0$.

Note that while equations (16) and (17) may be used to characterize the optimal tariff, they alone do not suffice to determine the precise form of the tariff. This is apparent from the fact that the right hand sides of these equations contain the type parameter Y, whereas the tariff by definition has to be independent of Y. However, once explicit assumptions are made on the functional form of the cost function G(.) and the form of the distribution F(Y), the actual tariff implied by these may be derived. Instructive examples of how this is done are given by Spulber (1981) and Wilson (1993), for instance.

In accordance with intuition, the above results suggest that the monopoly bank exercising second-degree price discrimination will generally not set the interest paid on deposits equal to its marginal rate of return on investment, nor will it generally set the service charge on payment services equal to the marginal cost of producing these services. Different cases can arise depending on the signs of the cross derivatives of the transactions cost function. The most interesting one of these, cross subsidization of payment services, occurs if $G_{YD} < 0$ and $G_{YN} > 0$.

All of these predictions of profit maximizing pricing behaviour are, of course, conditional on the compatibility of the first-order optimality conditions with the sufficient conditions for customers' cost minimization. This matter is especially interesting in the case of predicted cross subsidization, which has usually been excluded by certain regularity assumptions which have been adopted for the purpose of ensuring the validity of the second-order conditions for customers' local optima. For this reason, the next section of this paper is dedicated to the analysis of the validity of the first order conditions.

4 The existence of cross subsidizing optima

The control theoretic approach to tariff design relies heavily on the first order necessary conditions for profit and utility maximization. However, the first order conditions are not necessarily relevant for all shapes of the cost function G(Y, D, G) or the type distribution F(Y). The question must be posed, do the first order conditions which were derived above satisfy the sufficient conditions for global optima for the customers and the monopoly bank? In other words, is there a genuine, incentive compatible, separating equilibrium? Can class or classes of cost functions and income distributions be indicated for which the first order conditions do satisfy the relevant sufficient conditions as well?

The conditions for validity of the first order approach in problems of economic design have been discussed mostly in the principal-agent literature (Rogerson (1985); Jewitt (1988)), where it has been established that sufficient conditions are not trivially fulfilled. In nonlinear pricing literature, the validity of the first-order approach has usually been guaranteed by very strong regularity assumptions. The most important of such assumptions is the sorting condition. In single-product problems, this condition requires that the size of the customers' optimal purchases and their marginal valuation of the good are both increasing functions of their type (see Tirole (1988), pp. 154–157). When multiproduct pricing has been analyzed, an additional assumption of "symmetry" has been utilized, stipulating that the marginal benefit from each good is strictly increasing in the customer's type (see e.g. Roberts (1979); Mirman and Sibley (1980); Wilson (1993), p. 318).

In the present context, the symmetry assumption would require $G_{YD} < 0$ and $G_{YN} < 0$. A direct and obvious consequence of the symmetry assumption would be the impossibility of marginal subsidies. However, while the symmetry assumption is convenient for tractability, it is not necessary. The sufficient conditions for optimality may well be fulfilled even when symmetry is violated. Below, it will be demonstrated that a separating equilibrium characterized by first order conditions such as those derived above, can be incentive compatible even when it involves marginal cross subsidies. The argument goes in two stages. First, the second-order necessary condition for a separating equilibrium (for which the first order approach is valid) is developed; second, a quadratic example is considered under which the second-order condition is easy to check and also guarantees global optimality.

The analysis of incentive compatibility of the separating equilibrium suggested by the first-order conditions is generally quite complicated in multiproduct pricing problems. However, the task is greatly simplified in models such as the one developed in this paper, where customer hetero-geneity is described by a single type parameter. In this simple case, the incentive compatibility problem may be analyzed as a unidimensional problem. This is based on representing the customers' choices by the type they "reveal" through their demand behaviour. From this perspective, it is sufficient for incentive compatibility if it is optimal for each individual to reveal just her own type through her demand behaviour. In particular, if the customers' preference functions can be shown to be globally concave with respect to their revealed type, the first-order conditions do define a global optimum within the set of points belonging to the optimal assignment locus. The area outside the locus need not be similarly investigated, for there the tariff can always be designed so that all customers stay on the locus (Mirman and Sibley (1980)).

Specifically, in the present context, the first order condition defines a global maximum at least if the total liquidity cost given by G(Y, D(Z), N(Z)) + P(D(Z), N(Z) + r·D(Z) is strictly globally convex with respect to Z. Here Z is the "revealed Y" indicating the customer's choice of position on the {D(Y), N(Y)} -locus. The checking of this condition is facilitated by the following result (Roberts (1979), p. 82).

Define K(Y, Z) ≡ G(Y, D(Z), N(Z)) + P(D(Z), N(Z)) + r·D(Z). This is the total liquidity cost incurred by the customer of type Y when she behaves lika a customer of type Z. Note that the first order conditions now read $K_Z(Y, Y) = 0$. Totally differentiating this yields the result $K_{ZZ}(Y, Y) = -K_{YZ}(Y, Y)$. In terms of the present model, this result implies that $K_{ZZ}(Y, Y) > 0$ and the problem is thus locally convex if $K_{YZ}(Y, Y) < 0$, a condition which can be written out as

$$G_{DY}(Y, D(Y), N(Y)) \cdot \frac{\partial D(Y)}{\partial Y} + G_{NY}(Y, D(Y), N(Y)) \cdot \frac{\partial N(Y)}{\partial Y} < 0 \tag{18}$$

Global convexity requires $K_{ZZ}(Y, Z) > 0$ for all Z, not just Z = Y; It can be shown that this is satisfied if $K_{YZ}(Y, Z) < 0$ (for proof, see Tirole (1988) p. 156n). In the present model, this condition amounts to

$$G_{DY}(Y, D(Z), N(Z)) \cdot \frac{\partial D(Z)}{\partial Z} + G_{NY}(Y, D(Z), N(Z)) \cdot \frac{\partial N(Z)}{\partial Z} < 0 \tag{19}$$

According to condition (19), the first order conditions characterize a global optimum at least if a) assignments grow with the type, in the sense that $\partial D/\partial Z$ and $\partial N/\partial Z$ are both positive and b) cross derivatives G_{DY} and G_{NY} are both negative for all Y. The part (b) of the condition is the symmetry assumption mentioned above.

However, convexity under normality with respect to type is possible also when cross derivatives are of opposite signs. This is due to the fact that, if the goods D and N are complementary, the assignments may be increasing in type even though the direct effects of the customer type on their marginal valuation is asymmetric. In other words, one of the terms in (19) can be positive without violating the inequality. In this case, convexity depends on the relative magnitudes of the relevant derivatives, however.

The easiest way to demonstrate that convexity of the consumer's problem is compatible with asymmetric valuation effects is to use a specific example. Consider the case in which the internal cost function G(.) is quadratic, with the properties (i)–(vi) above, and the distribution F(Y) has an increasing hazard rate. In the quadratic case, the derivatives of the assignment functions are the following:

$$\partial D/\partial Z = (1 + h(Z)/H(Z)^2)\frac{G_{DN}G_{NY} - G_{DY}G_{NN}}{\Delta}$$

$$\partial N/\partial Z = (1 + h(Z)/H(Z)^2)\frac{G_{DN}G_{DY} - G_{NY}G_{DD}}{\Delta} \qquad (20)$$

where $H(Z) = f(Z)/[1 - F(Z)]$

$h(Z) = \partial H(Z)/\partial Z$

and $\Delta = G_{DD}G_{NN} - G_{DN}G_{DN}$

These results can now be substituted in the inequality (19). Under the assumptions on the shape of the G(.) function (assumption (vi) above) we have $\Delta > 0$. Further, the assumption of increasing hazard rate means that $(1 + h(Z)/H(Z)^2) > 0$. Then, the global sufficiency condition (19) can be reduced to the following form:

$$(G_{DY})^2G_{NN} + (G_{NY})^2G_{DD} - 2G_{DY}G_{NY}G_{DN} > 0 \qquad (21)$$

This can be broken into two cases, depending on whether the cross derivatives G_{DY} and G_{NY} are of different sign or not:

$$G_{DN} > \frac{(G_{DY})^2G_{NN} + G_{DD}(G_{NY})^2}{2G_{DY}G_{NY}}, \quad \text{if } G_{DY}G_{NY} < 0 \qquad (22)$$

$$G_{DN} < \frac{(G_{DY})^2G_{NN} + G_{DD}(G_{NY})^2}{2G_{DY}G_{NY}}, \quad \text{if } G_{DY}G_{NY} > 0 \qquad (23)$$

Thus, in the quadratic case with increasing hazard rate in the depositors' income distribution, the convexity of the problem relates to the complementarity of the products as captured by the derivative G_{DN}. If cross derivatives G_{DY} and G_{NY} are of opposite signs, G_{DN} must not be too negative (condition 22); in the other case, it must not be too positive (condition 23).

Let us now focus our attention to a particularly interesting case which can be called "normality with respect to type". In economic terms, this implies that the customers' cost or preference functions are such that if goods are available at fixed marginal prices, the demands for both of the goods are increasing functions of the customer's type.[5]

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[5] The properties of the G(.)-function which imply normality with respect to type are mathematically equivalent to the properties of production functions which give rise to positively sloped expansion paths.

It is easy to show that, given the assumptions (i) to (vi) above, normality with respect to type requires that following inequalities hold:

$$G_{DY}G_{DN} - G_{DD}G_{NY} > 0 \tag{24}$$

$$G_{NY}G_{DN} - G_{NN}G_{DY} > 0 \tag{25}$$

Clearly, these inequalities must hold if G_{DN} and G_{NY} and G_{DY} are all negative. In that case, increasing the income level Y makes both deposits D and payment services N more efficient cost-saving factors at the margin. However, normality with respect to type is possible also in the asymmetric cases when the cross derivatives G_{DY} and G_{NY} have different sign. When $G_{NY} > 0$ and $G_{DY} < 0$, for example, normality with respect to type requires $(G_{DY}/G_{NY}) \cdot G_{NN} < G_{DN} < (G_{NY}/G_{DY}) \cdot G_{DD} < 0$.

Consider again the case of quadratic transactions costs, and an income distribution with increasing hazard rate. Assume also that the necessary condition for cross subsidization holds, i.e. $G_{DY}G_{NY} < 0$. Then, for both incentive compatibility and normality w.r. to type conditions to be fulfilled, the value of the cross derivative G_{DN} must be a negative number in the range

$$0 > \frac{G_{NY}G_{DD}}{G_{DY}} > G_{DN} > \frac{G_{NN}(G_{DY})^2 + G_{DD}(G_{NY})^2}{2G_{DY}G_{NY}} \tag{26}$$

Such a range exists, and cross subsidization is thus compatible with normality with respect to type and incentive compatibility, if

$$\frac{(G_{NY})^2}{G_{NN}} < \frac{(G_{DY})^2}{G_{DD}} \tag{27}$$

The interpretation of conditions (26) and (27) is that cases in which profit-maximizing nonlinear tariffs display cross subsidies between goods which are normal with respect to type may occur only if the goods are complementary enough, and the asymmetricity in the effects of the type on the marginal valuations of the goods is not too extreme.

To conclude the analysis of the quadratic case, we must check the validity of the assumption that the participation constraint holds only at the point $Y = Y^{min}$. This assumption was used in solving the maximum principle conditions (10)–(13). We will see that this assumption actually amounts to a requirement on the shape of the internal cost function G. This can be demonstrated as follows.

It was noted in the previous section that the assumption that the participation constraint is binding only at the low end of the type distribution is certainly valid if the net participation benefit $B = G(Y, 0, 0) - G(Y, D(Y), N(Y)) + P(D(Y), N(Y)) + rD$ is increasing in Y. Now, differentiating B with respect to Y and applying the envelope theorem yields $B_Y = G_Y(Y, 0, 0) - G_Y(Y, D, N)$. When G is quadratic, this can be developed into $B_Y = -(G_{DY}D + G_{NY}N)$, and the condition $B_Y > 0$ becomes equivalent to $G_{DY}D + G_{NY}N < 0$. This is clearly ensured for all $D = D(Y)$ and $N = N(Y)$ if it holds for $D^{min} = D(Y^{min})$ and $N^{min} = N(Y^{min})$, and the convexity condition (18) holds, too. On this basis, it is immediately obvious that the assumption concerning the participation constraint valid if the marginal valuation effects of the type are symmetric ($G_{DY} < 0$ and $G_{NY} < 0$) and the minimum assignments D^{min} and N^{min} are positive. However, the assumption can clearly be valid also in the asymmetric case. Consider for example the case in which $G_{DY} < 0$ and $G_{NY} > 0$, leading to marginal subsidies on the payment services. In that case, the net benefit of participation is increasing at Y^{min} if the cost function is such that $-G_{DY}/G_{NY} > N^{min}/D^{min}$. Coefficients of the linear terms of the quadratic G(.) function provide sufficient degrees of freedom for this condition to be satisfied without violating the other regularity conditions referred to above. If the convexity conditions developed above also hold, net participation benefits are increasing for all Y and the required assumption concerning the participation constraint is valid.

5 An Inventory Theoretic Cost Function

In this section we look at a specific example. We derive a transactions cost function in a model of multiple means of payment and show that it naturally displays the properties which in the above analysis led to cross subsidization of payment services. This is seen by examining its cross derivatives with respect to the relevant variables. Unfortunately we have not been able to perform the whole of the tariff design exercise with the explicit transactions cost function we have derived, due to the problems of intractability.

The cost function is derived in the Baumol-Tobin tradition, with two extensions. First, we assume positive transaction costs associated with the spending of money, unlike in the original Baumol-Tobin framework.[6] Second, we assume two means of payments, which are substitutable with each other: currency and bank deposits.

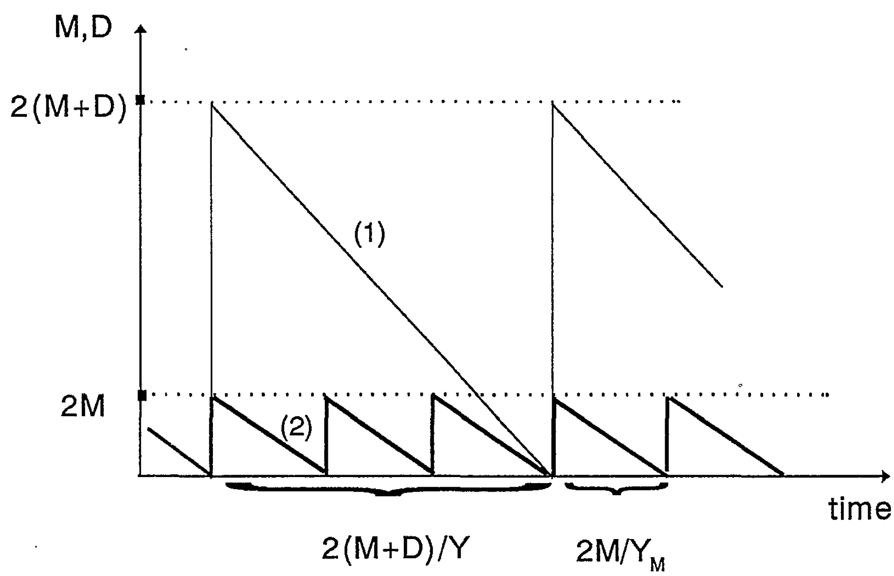
We consider the behaviour of a representative individual over some fixed time period (say a year). The individual holds currency and deposits. During the period, the individual uses a given amount of money Y in the form of payments, which she can make either with currency or with a transfer of deposit money. The total frequency of payments is X and the frequency of those paid with deposits is N. These are volume magnitudes. In value terms, we denote the part of expenditure paid with cash by Y_M and the part paid with deposit money by Y_D. By definition, $Y = Y_M + Y_D$. For simplicity, the individual's disposable income, which is also Y, is assumed to be received in the form of an investment asset ("bonds").

The payment patterns arising from the use of the income flow are assumed to be as depicted in Figure 2, where the payment flows from monies to goods are drawn as continuous for simplicity. Periodically, the individual sells some

[6] The case of discrete payments in "goods transactions", i.e. in the ultimate use of money, has been analyzed by Santomero (1979), for example. In his analysis, the payment pattern is entirely endogenous. We do not go that far, but assume an exogenous distribution of payments instead.

bonds in order to increase his money holdings (M + D). As in the Baumol-Tobin framework, the frequency of these transactions is Y/[2(M + D)], where M denotes average currency holdings and D denotes average deposit holdings. Both monies are used for making payments. It is assumed that the initial currency balance 2M is depleted to zero much quicker than the initial deposit balance 2D, meaning that $M/Y_M < D/Y_D$. Each time the currency balance reaches zero (here, as elsewhere, we follow the tradition of neglecting cumbersome integer constraints!) it is restored to the level 2M by a withdrawal from the deposit balance until the deposit balance too has been depleted and the payment cycle starts anew.

Figure 2. **The time pattern of money holdings in the two payment medium model**



1 = currency + deposits
2 = currency

The costs arising from this payment pattern are assumed to come from four different sources:

1. Holding costs of the currency (loss, inconvenience of storage and carrying), which is proportional to the amount of currency held. These costs are measured by parameter h.

2. Transactions costs of paying with deposit money, except outright service charge to the bank (including the value of time lost in making payments, for example). The cost per deposit payment is measured by parameter c.

3. Transactions costs of currency withdrawals from the bank. These are assumed to be fixed can are measured by parameter k_M.

4. Transactions costs incurred when "bonds" are sold for money. These are fixed too and are measured by parameter k_D.

Adding these cost items, the transactions costs C may be written as follows:

$$C = hM + k_M \frac{Y_M}{2M} + cN + k_D \frac{Y}{2(D+M)} \tag{28}$$

There is an exogenous size distribution of payments over some interval $[p^{min}, p^{max}]$. This distribution is defined as a function Y(x) indicating the value of payments due to the x largest transactions. From this definition it follows that if X is the total number of transactions, then $Y(X) = Y$, $Y(0) = 0$. By definition, the function must be increasing $(dY(x)/dx > 0)$ and concave $(d^2Y(x)/d^2x < 0)$. Now, given the cost structure described above, and given that both Y_M and Y_D are positive, it will be rational to pay the largest transactions with deposits and the smallest with currency. Hence, the total value of deposit payments is simply $Y_D = Y(N)$ and, further, $Y_M = Y - Y(N)$. Substituting this function for Y_M in we get

$$C = hM + k_M \frac{Y - Y(N)}{2M} + cN + k_D \frac{Y}{2(D+M)} \tag{29}$$

In order to differentiate the transactions cost function in a way comparable to the derivatives of G(.) used in the previous sections of this paper, we must allow for adjustments in the currency holdings M. The demand for currency as a function of Y, D, N and r may be derived by minimizing the transactions cost plus opportunity cost of currency holdings $r \cdot M$ with respect to M. The first order condition for optimum demand for currency is

$$\frac{\partial(C + rM)}{\partial M} = h + r - \frac{k_D Y}{2(D+M)^2} - \frac{k_M[Y - Y(N)]}{2M^2} = 0 \tag{30}$$

From this we get the following results on the adjustment of M when D, N, or Y change:

$$M_Y = \frac{\frac{k_M}{2M^2} + \frac{k_D}{2(D+M)^2}}{\Omega} > 0$$

$$M_N = \frac{-k_M\left(\frac{\partial Y_D}{\partial N}\right)M^2}{2\Omega} < 0 \qquad (31)$$

$$M_D = \frac{-k_D Y/(D+M)^3}{\Omega} < 0, \quad (-1 < M_D)$$

$$\text{Where} \quad \Omega = \frac{k_D Y}{(D+M)^3} + \frac{k_M[Y - Y_D]}{M^3} > 0$$

The reactions of the demand for currency to changes in income, bank-intermediated payments and the stock of deposits are intuitively plausible. The result that deposits do not fully crowd out currency ($M_D > -1$) is important for the cross derivatives which are evaluated below. Now we can find out the derivatives of the cost function C(Y, M(Y, D, N), D, N) with respect to its arguments:

$$\frac{\partial C}{\partial Y} = \frac{k_M}{2M} + \frac{k_D}{2(M+D)} > 0$$

$$\frac{\partial C}{\partial D} = \frac{-k_D Y}{2(D+M)^2} < 0 \qquad (32)$$

$$\frac{\partial C}{\partial N} = c - \frac{k_M\left(\frac{\partial Y_D}{\partial N}\right)}{2M}$$

These are as envisaged in assumptions (i)–(iii) provided that the transaction cost c of paying with deposit money is not too high, so that the derivative dC/dN can obtain its assumed negative value. If the demand price of payment services is to be positive, these parameter values are necessary.

The most important of the second derivatives, which are so crucial for incentive compatibility and cross subsidization, are

$$\frac{\partial^2 C}{\partial^2 D} = \frac{k_D(1+M_D)Y}{(D+M)^3} > 0$$

$$\frac{\partial^2 C}{\partial^2 N} = k_M \left\{ \frac{M_N\left(\frac{\partial Y_D}{\partial N}\right)}{2M^2} - \frac{\frac{\partial^2 Y_D}{\partial^2 N}}{2M} \right\}$$

$$\frac{\partial^2 C}{\partial D\,\partial N} = \frac{k_D Y M_N}{(D+M)^3} < 0 \tag{33}$$

$$\frac{\partial^2 C}{\partial D\,\partial Y} = \frac{-k_D}{2(D+M)^2} - \frac{k_D^2 Y^2}{(D+M)^6 \Delta} < 0$$

$$\frac{\partial^2 C}{\partial N\,\partial Y} = \frac{\frac{k_M}{(2M^2)} + \frac{k_D}{[2(D+M)^2]}}{1 + \frac{(D+M)^3 k_M (Y-Y_D)}{M^3 k_D Y}} > 0$$

The signs of the second derivatives are unambiguous except in the second derivative with respect to N, which is ambiguous in principle. However, it must be assumed positive in order to make the cost function convex.

It can be seen from the second derivatives that deposits D and payment services N are indeed complements ($d^2C/dDdN < 0$). An especially important observation about the second derivatives is that the cross derivatives with respect to Y are of different sign, i.e. marginal valuation effects of the type are asymmetric. This is just the property of the customers' preferences which would encourage a monopoly bank to design a cross-subsidizing tariff. More precisely, the signs of the cross derivatives with respect to Y are such that, on the basis of the analysis presented in the previous sections, the bank would be induced to subsidize the payment services and extract monopoly profits from the deposit market.

An intuitive explanation of the asymmetry which arises in the model can perhaps be developed along the following lines. The marginal benefit of holding deposits is proportional to the cost of bond liquidations, and with given deposits, bond liquidation costs increase when income grows. On the other hand, the marginal benefit of payment services is determined at the currency/bank payment margin. There, an increase of income, if the use of payment services is held constant, means that more payments are done with currency. That means more cash withdrawals and a lower marginal benefit from payment services.

Although tractability problems make it impossible to proceed to a complete closed-form solution of the tariff problem within the inventory theoretic model, the result suggests that asymmetric effects of "type" may be inherent to the demand for payment media problems, a potential explanation for the prevalent cross subsidies in banking.

6 Discussion

The analysis presented in this paper is based on the observation that, given a monopolistic market structure, the market for cheque accounts obviously satisfies the conditions for price discrimination to be possible. From this starting point, we study the problem of optimal pricing of cheque accounts, emphasizing the point that the cheque account market consists of two submarkets: the market for deposited funds and the market for payment services. These cannot be considered as a single good, because depositors are free to vary the ratio of (average) deposited funds to the volume of payment services they use if they want to do so. However, the two submarkets are obviously interdependent. In the present analysis, the interaction is modelled through cross effects in the depositors' preference function (liquidity cost function) and the important feature that the depositors' participation decisions are made jointly for both markets.

Nonlinear multiproduct pricing becomes easily quite complicated mathematically. To retain tractability, we analyzed price discrimination in a model where depositors were heterogenous with respect to a single type parameter ("income") only. The analysis suggests that cross subsidies are quite possible as an outcome of profit maximizing tariff design by a monopolist. That the cross subsidizing solution is not only possible, but even likely in a banking context, was suggested by an inventory model of the parallel use of two payment media in section 5.

Heuristically, cross subsidization may occur when there are effects in the population (such as the variation of the type parameter) which increase the marginal valuation of one product and decrease the marginal valuation of another. We pointed out that, under complementarity, this asymmetry does not rule out normality with respect to type, i.e. the property that the realized demands of the two goods are positively related to each other in the population.

There is a well known analogy between nonlinear pricing and bundling. For example, in a single-product case, quantity discounts can be analyzed as instances of bundling. The analogy can be extended to multiproduct bundles as well. Adams and Yellen (1976) and Schmalensee (1984) have analyzed examples of multiproduct pricing in which independent variation of pairs of reservation prices in the customer population causes bundling to be profitable. The incentives to bundle are seen to be especially strong when reservation prices are negatively correlated – something closely related to asymmetric valuation effects of the customer's type. These bundling models are very restrictive, however, in the sense that customers' demand behaviour is assumed to be of the (0,1) -type, and the bundles were thus "fixed" by assumption. These assumptions are of course unsuitable for the analysis of banking, for instance. The analysis in the present paper seems to generalize Adams and Yellen's and Schmalensee's insights to the marginalist world of continuous demand functions and smoothly variable bundles.

A general property of optimal nonlinear tariffs is that marginal prices should equal marginal costs for those customers who purchase largest quantities (or, to be more precise, for those customers whose net benefits from participation are greatest). This property may perhaps be used to derive tests of whether price discrimination indeed is the reason why deposit pricing is often nonlinear and seems to involve cross-subsidies. On the other hand, if one takes the price discrimination model given, then marginal prices charged to largest customers may be used to measure marginal costs of bank services, which usually are not directly observable.

Full implications of the above findings have not yet been systematically explored. It is clear, however, that the results suggest that even under imperfect competition, and without interest rate regulation or distorting taxes, payment services may be supplied in excess of "first-best" allocation. However, this does not necessarily imply that nonlinear pricing of cheque accounts is socially undesirable. As pointed out by Wilson (1993), the extraction of some monopoly profits may be optimal in a "second-best" sense, if there is increasing returns to scale in banking and if banks are required to survive without subsidies. Furthermore, as pointed out by Whitesell (1992), there may be negative externalities in the use of currency, and some subsidization of paying with deposit money may thus be warranted. At this stage, however, closer investigation of these conjectures must be left to later research.

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