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THE OPTIMAL CURRENCY DISTRIBUTION OF A CENTRAL  
BANK'S FOREIGN EXCHANGE RESERVES

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## ABSTRACT

The purpose of this study is to establish a currency distribution for foreign exchange reserves of a central bank that would minimize the risks related to the increased exchange rate volatility. The study also addresses issues, such as the preservation of value of reserves, the numeraire currency, and the central bank's attitude towards risk. The portfolio choice problem is solved through the mean-variance framework of expected returns. The uncertainty under consideration is solely due to exchange rate changes in case of nominal returns, and additionally due to changes in the deflator in case of real returns, because the reserves are assumed to be invested in one-month Eurocurrency deposits. Three alternative expectations hypotheses of the exchange rate behavior, namely (i) Random Walk, (ii) Open Interest Parity and (iii) modified interest parity (allowing for the existence of a risk premium) are employed for isolating the unexpected component of the exchange rate changes.

If the objective of the central bank is to stabilize the value of reserves with respect to the numeraire, the currency index in this case, it is shown how well the objective is achieved by investing the reserves into only a few of the currencies in the actual index. The time period for estimation of the variance-covariance matrix was selected with the help of statistical tests such that stability over time is assured.

## TIIVISTELMÄ

Tämän tutkimuksen tarkoituksena on määritellä sellainen keskuspankin valuuttavarannon jakauma, joka minimoi valuuttakurssien muutoksista aiheutuvat riskit. Tutkimus käsittelee myös seuraavia asioita: varannon arvon säilytys, numeraire-valuutta, keskuspankin suhtautuminen riskiin.

Sijoitussalkun valinta on ratkaistu odotettujen tuottojen keskiarvo-variassi -analyysiä käyttäen. Tutkimuksen kohteena oleva epävarmuus johtuu vain valuuttakurssien muutoksista nimellistuottojen ja lisäksi hintadeflaattorin muutoksista reaalisten tuottojen kohdalla, sillä tutkimuksessa oletetaan, että varanto on sijoitettu yksinomaan yhden kuukauden eurotalletuksiin, joiden tuotto on riskitön. Valuuttakurssien odotettuja muutoksia on estimoitu kolmen eri odotushypoteesin vallitessa: (i) Random Walk, (ii) avoin korkopariteetti ja (iii) modifioitu korkopariteetti (sallitaan riskipreemion olemassaolo).

Jos oletuksena on, että keskuspankin tavoitteena on varannon arvon vakaana säilyttäminen suhteessa numeraireen (tässä tapauksessa valuuttakorin) tutkimus näyttää miten hyvin tavoite saavutetaan sijoittamalla varanto vain muutamain valuuttakorin valuuttoihin ja hyväksymällä minimiriski. Variassi-kovariassi -matriisin estimoinnin ajanjakso valittiin siten, että matriisin stabiilisuus ajassa on taattu.

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## 1 THE ALLOCATION PROBLEM

For an international investor or a holder of large quantities of foreign currencies, the increased volatility of the exchange rates poses a major risk. The purpose of this study is to establish a currency distribution for foreign exchange reserves of a central bank that would minimize the risks arising from the exchange rate volatility.

The decision making process can be based on either economic or policy dictated arguments. The economic arguments are based on utility theory and more closely the maximization of utility subject to budget constraints. The non-economic arguments have often to do with institutional constraints and administrative arrangements. From the analytical point of view the economic arguments are more easily acceptable. Economic criteria make possible a unified treatment of profitability, risks and transactions costs related to the investment decision. The current study will consider only the economic arguments for solving the currency distribution problem.

The issue of the currency distribution of foreign exchange reserves of a central bank has already been widely researched (compare the literature survey of section 1.1), but as the operating environment of the banks is constantly evolving it was decided to undertake a new study with the latest data. Furthermore, as the previous studies left several issues unresolved or unanswered, we will attempt to address some of them here. The issues of most interest to us are the optimal distribution, the numeraire, the deflator, the role of the vehicle currency, the currency index and finally the technical issues relating to the empirical estimation of the optimal portfolio.

In the following we have a brief overview of the structure of the paper including a literature survey on the topic. Section 1 deals with the allocation problem of foreign exchange reserves, the sources and

uses of reserves as well as the issue of preservation of value of reserves and the numeraire. It also discusses a central bank's attitude towards risk. While the current study applies portfolio theory for selection of the optimal currency distribution, other investment planning frameworks are highlighted also.

Investment is nothing but delayed consumption, and a rational investor forms expectations about the return on investment. Section 2 specifies some expectations hypotheses on exchange rates and examines the nature of their forecast errors. Since the import price index is chosen as a deflator of the foreign reserves, the expectations model of the import price index as well as empirical estimation and test results are delineated in section 2.1. The modelling of exchange rate expectations, based on various hypotheses are introduced in sections 2.2 through 2.4.

Sections 3 and 4 are the empirical part of the paper, where we perform and discuss various tests on the hypotheses introduced in section 2. Sections 3 and 4 are the main part of the paper, because it is on the basis of these tests that we can draw conclusions that hopefully aid us in making the ultimate choice for an optimal portfolio. These tests tell us about (i) the efficiency of the expectations models (i.e. the randomness of residuals), (ii) the stability of the parameters (variances, correlations and risk premia), and finally guide us in (iii) the choice of the estimation period on the basis of apparent stability of parameters over time.

Finally, optimal portfolios under various hypotheses are presented in section 5. Lastly section 6 concludes the paper.

## 1.1 Literature Survey

The survey is limited to studies dealing with the investment of foreign currency holdings by central banks. The studies may be divided on the one hand into descriptive studies on the actual behavior of the central banks, and on the other hand into normative studies with recommendations.

The actual currency composition of the reserve portfolio of individual countries and how it was established are of course as confidential information not disclosed, but several studies conducted at the IMF, among them Heller and Knight (1978), Dooley et al (1988) and Dellas (1989) do find on the basis of indirect empirical evidence several factors that influence the central banks in the decision making process. During the period 1970 - 1976, in the study by Heller and Knight, two factors were found to be decisive: (i) the country's exchange rate regime and (ii) the pattern of its international trade. The weight of the peg currency tends to be big, as also the share of the main trading partners' currencies. The more recent study by Dooley et al, covering the data during the period 1976 - 1985, suggests that factors such as country's exchange rate arrangements, trade flows with the reserve currency countries and the currency denomination of debt-service payments are decisive for allocation of gross holding of reserve assets, thus supporting the Heller-Knight findings. For net foreign asset positions, however, risk and return considerations are also important.

Ben-Bassat (1980) presents a model for selecting an optimal portfolio for semi-industrial and developing countries with the emphasis on certain underlying motives. He finds that the choice of portfolio is related to the objectives served by holding the reserves such as protection against the instability of balance of payments flows and the preservation of value of reserves. He concludes that the composition of reserves in the semi-industrialized and developing countries is influenced by risk and return considerations and by currency composition of imports. This means that the actual portfolios resemble an efficient portfolio estimated through mean-variance approach. He adds that the major industrial countries' reserve composition is influenced more by the stability considerations with respect to the international monetary system and less by profit motive.

Another normative paper published at the IMF by Healy (1981) argues that an appropriate objective for many central banks is to minimize the variance of real returns; hence the optimal portfolio is the minimum variance portfolio. The emphasis lies on real returns deflated by the import price deflator. This is important in particular for

developing countries as they must often draw on reserves to finance essential imports in case export earnings have fallen short for instance due to low commodity prices.

A recent study by Dellas (1989), that examines the behavior of official foreign reserves of both industrial and developing country groups in the period 1977 - 1984 finds evidence that central banks apply mean-variance approach in solving their portfolio choice problem. This however does not lessen the practical critique directed against mean-variance framework voiced among others by Jorion (1985). Among the frequently mentioned shortcomings of this classical approach two are most common: namely (i) instability of optimal portfolio weights over time and (ii) poor out-of-sample performance of the portfolio (due to the assumption of constant variance-covariance matrix of the mean-variance analysis).

Perhaps the most critical comments directed against mean-variance framework are voiced by Michaud (1989). He argues that mean-variance optimizers tend to "operate in such a manner that they magnify the errors associated with input estimates". He adds that equal weighting may in many cases outperform unconstrained mean-variance optimization. Healy acknowledges the shortcomings of the mean-variance technique, in particular the instability of the variance-covariance matrix, but finds the instability insufficient for rejection of the minimum variance framework. The minimum variance portfolio estimated by Healy performed consistently better than alternative portfolios (dollar portfolio, SDR portfolio and import weighted portfolio) by achieving lower variance of returns. Portfolio based on historical price and exchange rate data enables the central bank to reduce the variability of the real purchasing power of the reserves.

Lehmussaari (1987) of the Bank of Finland applied mean-variance framework and the import price deflator in estimation of optimal portfolios. He tested and contrasted several exchange rate determination hypotheses and found the Random Walk model the most efficient during the time period under his examination. In fact, Lehmussaari's work is a starting point for the current study.

## 1.2 The Functions of the Reserves

The foreign exchange reserves held by a central bank are a residual, arising from a disequilibrium situation, in which the supply of foreign exchange exceeds the demand for it. For instance the banking sector due to a favorable interest rate differential borrows from overseas and exchanges the borrowed funds into domestic currency at the central bank increasing the foreign exchange reserves. If the central bank has a target for the exchange rate, it may not let the rate adjust but instead cover the disequilibrium with its own transactions.

In this way, the central bank uses foreign reserves for intervention in the foreign exchange markets in order to achieve the goals set for the foreign exchange policy such as a stable domestic currency. The reserves could also be used for financing of imports or for payment of government's foreign debt, in the case the country finds itself cut off from other sources of foreign exchange. The central banks in the developing countries must often facilitate trade or service debt through reserves. Among the industrial countries, however, aside from interventions the motive for holding foreign exchange reserves is precautionary rather than for transactions purposes.

## 1.3 The Preservation of Value of Reserves

An important aspect of foreign reserves management is the issue of preservation of the real value of the reserves. This brings up a question of the choice for an appropriate deflator. Should it be the import price index or some other price index measuring the international purchasing power of reserves? There is no obvious first best answer to this question. Many studies dealing with the issue of a deflator among them Lehmussaari (1987) and Healy (1981) have opted for the import price index, under the assumption that central banks seek to stabilize the purchasing power of their foreign exchange reserves over an import basket implying that reserves could be used if necessary for financing of imports or servicing of debt. The reserves need not ever be used for financing of imports, but still the import

price index is very appropriate as a deflator, provided that we are interested in the real value of the reserves in the purchasing power sense. Alternatively, real value could be obtained in each currency by deflating the amount of reserves in various currencies by the consumer price indices, respectively. The underlying assumption being that the agent is maximizing purchasing power over consumption baskets denominated in various currencies, compare Macedo (1983).

What about the return from investment of foreign reserves? Are we only interested in the return for the sake of it, or is the return used for something specific? If that specific is defined, then the preservation of value should be defined in terms of that specific expenditure. The returns from investment can be spent on either domestic or foreign goods and services. From a national point of view, the returns from foreign investment must ultimately be spent on imports. Thus it seems appropriate to deflate the returns from a central bank's foreign investment with the import deflator. Finally, if only an optimal distribution in nominal terms is of interest to us, then the issue of a deflator becomes of course superfluous.

#### 1.4 A Central Bank as an Investor

The reserves held at the central bank can be considered a part of national wealth and should be invested wisely. The level of reserves is highly volatile and the amount of future reserves is unpredictable. This unpredictability sets some limitations as to how and in which assets one should invest the funds. First some simplifying assumptions.

It is assumed that the central bank is a small agent in the international capital markets (small is equivalent to a price taker in the foreign exchange markets). This is an important assumption because it excludes large central banks whose primary objective is the maintenance of a stable international monetary system (large is equivalent to a price setter). The investment problem is further simplified by the open interest parity condition (the hypothesis will be tested during the course of the study) requiring the following

assumptions: (i) the capital is perfectly mobile and (ii) the number of risk-neutral investors is sufficient to equalize the expected yield of all comparable assets. In that case the central bank has no control over the expected yield of its reserve portfolio because every reserve asset has the same expected return. This means that the specification (denomination) of the reserve asset is irrelevant with respect to the expected return (because it is the same for all reserve assets). However, the variance of returns is important for the portfolio choice because it is influenced by the specification of assets.

In the current system of managed floating there are risks associated with holding of any currency, hence it is assumed that a small central bank is risk-averse. Macedo (1983) has extended the portfolio choice model of a risk-averse international investor to include even national investors of infinite risk-aversion and argues that the latter will also hold a diversified currency portfolio. This invalidates the usual assumption of domestic currency being a 'preferred monetary habitat' or riskfree asset by definition as opposed to foreign currency assets.

The current study applies portfolio theory for selection of the optimal currency distribution for foreign reserves. The optimal size of reserves is neither discussed nor researched in this paper, although it is also an interesting issue, because the expected return on a portfolio is not dependent on the composition of the portfolio or its size.

Within the framework of portfolio theory, the following assumptions are made: the joint distribution of the exchange rate and price changes is multivariate normal. From this follows that the probability distribution of portfolio returns is normal. Any normal distribution can be characterized completely by two components, its mean and the variance. Given these assumptions, the investor's portfolio choice problem is solved through the means, variances and covariances of asset returns. As the central bank cannot affect the expected return it should find a portfolio that minimizes the overall variance of returns. An efficient portfolio is defined as a combination of assets that minimizes the risk associated for a given level of expected return. An efficient frontier contains a combination of efficient

portfolios. Whenever expected returns to assets differ, the optimization under minimum variance framework does not have a unique solution, because the optimal distribution depends on the amount of risk accepted.

It is possible to define a globally optimal portfolio i.e. a portfolio for a country  $x$  that is independent of any uniquely national characteristics such as the import weights under certain conditions: (i) when PPP holds and (ii) when the country is seeking to minimize the variance of real portfolio. A locally optimal portfolio of country  $x$  in real terms differs from both the globally optimal portfolio and from country  $y$ 's portfolio, when the trading partners of  $x$  and  $y$  differ substantially and because the PPP does not generally hold in the short run (Isard (1977)). The variation of optimal portfolios between  $x$  and  $y$  is due to variation of consumption bundles reflected in the import weights that generate the deflator (for further details see Healy (1981)).

Besides applied portfolio theory, one could think of other mainly arbitrary investment strategies such as the distribution of the currency basket, to which the domestic currency is pegged or a major reserve currency like the U.S. dollar. Other alternatives could be the import or the export currency weights, respectively. The SDR weights<sup>1</sup> render a readily available distribution into five currencies. If the ultimate objective is the stability of reserves in nominal markka terms, then the currency index provides an ideal distribution.

The length of the investment horizon becomes important if the variances and covariances and expected rates of return are different depending on the length of the horizon. This brings up the issue of mean reversion of returns, typical in the stock market. It is found that long run volatilities are often smaller than short run

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<sup>1</sup>The composition of the SDR basket is such that it in general ensures stability in the purchasing power over imported goods and services, see Polak (December 1979, IMF Staff Papers). The weights of the current SDR basket, effective January 1, 1986, are calculated from the exports of goods and services of the five member countries having the largest exports of goods and services during the period 1980 - 1984 and are revised every five years; source: IMF Press Release No. 85/44.

volatilities (see Poterba & Summers (1988)), which means that a long horizon is less risky as opposed to a short horizon.<sup>2</sup>

The only analytical alternative to mean-variance approach would be the minimization of transaction costs, that is an outcome of risk neutrality assumption.

For accounting purposes the markka is conventionally used as a numeraire of the foreign reserves by the Bank of Finland. The amount of reserves expressed in markka can increase for two reasons: (1) as a result of actual transactions such as purchase of foreign currency or accrual of interest, or (2) following a depreciation of the domestic currency. The increase in nominal markka terms of reserves resulting from a devaluation is however misleading, since the amount of reserves in terms of foreign currencies has remained unchanged. In order to eliminate the effects of devaluations as well as revaluations, the returns in nominal terms have in this study been expressed with respect to the currency index. The point being that if the reserves were invested according to the weights of the currency index, and as the expected nominal change of the currency index is zero in terms of the index itself, it follows that the nominal return of the index weighted portfolio is just the basket interest rate. Furthermore, the expected rates of return in terms of the index of the various currencies are just equal to the nominal basket interest rate; plus the possible risk premium; the unexpected gains and losses are just excess returns over the basket, with a possible allowance for the risk premium (see equation 7, page 23).

Alternatively, the numeraire could be a major reserve currency such as the USD. A reason of selecting USD as a numeraire would be the objective of seeking to stabilize the reserves in terms of the vehicle currency and avoidance of transactions costs. Ben-Bassat (1980)

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<sup>2</sup>For testing of mean reversion in the current framework, the returns could be calculated on a 12-month horizon, and the obtained volatilities (variances) contrasted with the volatilities of returns on a one-month horizon. If there is no mean reversion present, the one-month variance is 1/12 times 12-month variance. Similar test could be developed for covariances.

estimated optimal portfolios for Israel, using data during 1972 - 1976 period, and obtained very different composition of portfolios when estimated in terms of the import basket or in terms of USD (see Appendix table 1). He emphasizes the point that the selection of the target currency or the numeraire is not merely a theoretical issue. In the case of Israel, the optimal low return-low risk portfolio turned out to be similar to the invoicing currency composition of Israeli import basket. This outcome is expected from a minimum variance portfolio, when the objective is to stabilize the purchasing power of reserves over an import basket or using the import price index as a deflator.

### 1.5 The Data Requirements

1. Choice of allowed investment currencies. Five out of the 14 currencies from the Bank of Finland's currency basket were chosen; these are U.S. dollar USD, British pound GBP, German mark DEM, French franc FRF, and Japanese yen JPY. The choice of currencies was limited to five based on a preliminary study of 8 major currencies (among the 14 in the currency index). Dutch guilder NLG, Belgian franc BEC, and Swiss franc CHF are excluded on the basis of that study, because the correlations of returns are very high between NLG, BEC and CHF and DEM and FRF respectively. Currencies that maintain a fixed bilateral exchange rate (e.g. DEM and NLG are nearly fixed) are identical assets in terms of diversification, hence one of them can be dropped from the portfolio consideration. The weights in minimum variance portfolios containing currencies, whose returns are highly correlated such as the currencies within the EMS are highly unstable. For instance an "optimal" portfolio could have (depending on the time period used for estimation of the covariance matrix) large short positions in DEM and BEC and equally large investments in FRF and NLG. In the limiting case of perfectly correlated components the efficient portfolios become totally indeterminate. We found it advisable to include in the exercise only those currencies for which the correlation of returns does not in general exceed 0.80. Furthermore, CHF was eliminated also due to insufficient liquidity conditions for assets denominated in CHF. All exchange rates are expressed in markka per local currency, end of month data, starting from January 1980 to December 1988. Source: Bank of Finland, official noon quotations.

2. The interest rates. One month Eurodeposit rates for the above currencies, end of period data. The Eurodeposit rates were chosen rather than local rates due to better comparability of Eurocurrency deposits in terms of credit risk, maturity and issuer. Comparability is essential, since our aim is not to replicate all of the aspects of asset choice, but to focus on the currency distribution. Interest rates are needed for testing of expectations hypotheses such as interest rate parity or for the presence of a risk premium. Source: Data Resources, DRIFACS PLUS, London close, bid.
3. The currency index of the markka (end of period data) to be used as a numeraire of nominal returns. Source: Bank of Finland.
4. The import price index (monthly average data rather than end of month data was chosen, because an average is a closer measure of inflation than end of period data would be as consumption takes place on each day of the month) to be used as a deflator for calculation of real returns (real in the sense of purchasing power over import basket as opposed to real return that is calculated as a difference of nominal return in currency  $i$  and change in the consumer price index in currency  $i$ ). The latter real return measures purchasing power over consumption basket payable in currency  $i$ ). Source: Bank of Finland.
5. A weighted eight-currency interest rate, calculated from the one month Eurodeposit rates with weights derived from the currency index (an ideal interest basket would comprise interest rates for all 14 currencies, however the major eight currencies are a close approximation, since there is no Eurodeposit data available for all 14 currencies dating back to January 1980). The basket interest rate is used as a benchmark for the return from the minimum variance portfolio. Ideally, these two returns should be almost identical, in nominal terms, provided that the currency index is the numeraire (compare graph 13.). The interest basket weights are: USD 9.7, GBP 16.8, SEK 24.2, DEM 24.2, NLG 5.9, CHF 3.3, FRF 8.3, JPY 7.6. Sources: Interest rates for all except Swedish krona from Data Resources, DRIFACS PLUS, London close, bid; for Sweden from Riksbanken, Sweden.

## 2 THE EXPECTATION HYPOTHESES AND THE FORECAST ERRORS

In an investment situation that is forward looking, the investor necessarily needs to form expectations about the return of the investment. Changes can be either fully anticipated (and are reflected in today's prices) or trend-like and thus predictable, or fully unanticipated. Our aim is to isolate the unanticipated component of the exchange rate changes. The investor, in this case the central bank, is assumed to be risk-averse. Also, more expected return is preferred to less. Given the mean return and the variance of the available currencies, the investor chooses a portfolio that has the largest expected return, given the variance, and a portfolio that has the smallest possible variance, given the expected return. If the central bank's objective is maximization of expected nominal return, then it should invest all reserves into an asset with the highest expected nominal return (it is usually a currency of high inflation).

The expected real return is a function of the expected change in the exchange rate, the interest rate as well as the expected change in the deflator:

$$(1) \quad E[R(t+1) | t] = \frac{E[e(t+1) | t]}{e(t)} (1+r(t)) - \frac{E[p(t+1) | t]}{p(t)}$$

where  $R$  = real return in terms of purchasing power over import basket

$e$  = exchange rate in markka per foreign currency

$r$  = nominal return of foreign currency

$p$  = import deflator

In order to calculate the expected real return and ultimately the forecast errors (the unanticipated component in returns) one needs to specify the expectations hypotheses for both the exchange rate and the deflator, while the interest rate is known for the one month time horizon. In the case of perfectly anticipated import prices, the changes in real returns are solely due to the unanticipated exchange rate changes.

In summary, depending on the objective function of the central bank, either nominal or real return calculations are appropriate. Hence both nominal and real return alternatives will receive equal treatment in the study i.e. minimum variance portfolios are calculated in both nominal and real terms.

## 2.1 The Estimation of Expected Import Prices

For the calculation of expected return in real terms, the expectations model for the import price index must be specified. We have chosen an autoregressive model, under the assumption that expectations about price changes are formed on the basis of the information about the price movements during the previous periods.

Before modeling the import prices, the time series of import prices needs to be transformed into a stationary process. An autocorrelation function is utilized in the decision making process. The log of the import price index appears to be first-order homogeneous non-stationary. Thus, after first-differencing the log of the price series, the resulting series will be stationary (graph 1). According to both AIC and Schwarz criteria an autoregressive process of ARIMA(1,1,0) was selected as the most suitable description of the log-differenced import prices:

The Model: Estimation period from January 1980 to December 1988.

$$(2) \quad \Delta \ln P(t) = A + B \Delta \ln P(t-1) + e(t)$$

where  $\Delta \ln P(t) = \ln P(t) - \ln P(t-1)$

so that  $E[\ln P(t+1)] = A + (1 + B)\ln P(t) - B\ln P(t-1)$

$$A = 0.001269 \quad (t = 1.18826)$$

$$B = 0.417555 \quad (t = 4.72728)$$

$$DW = 2.1030$$

$$R^2 = 0.1741$$

$$SE = 0.0109$$

$$F = 22.35 \quad (1,106)$$

Two diagnostic tests were performed on the import price equation (2), namely a Chow test for testing of a structural change and another for testing for any autocorrelation of residuals. In the Chow test, the two subperiods of equal lengths extend from January 1980 to June 1984 and from July 1984 to December 1988. The hypothesis that the estimated coefficients are equal for the two subperiods cannot be rejected at 5 per cent level of significance.<sup>3</sup> It was also found, that the residuals are not autocorrelated.

How well the model predicts the price movements can be seen from graph 2 of the appendix showing both the actual and the predicted import price series. The difference between the actual series at time  $t$  and the predicted series at time  $t$  is the error of the forecast of import prices formed at  $t-1$ .

## 2.2 The Modeling of the Exchange Rate Expectations: Various Hypotheses

There are various models on exchange rate determination that can be utilized for forming expectations about the future exchange rate changes. The models fall into several categories, such as arbitrage models, time series models, and structural models. The random walk hypothesis, the open interest rate parity and covered interest rate parity as well as the forward market hypothesis can be applied for testing the arbitrage models. The applicability of time series models can be tested by ARIMA techniques (not done here). The monetary and portfolio balance models of exchange rate determination belong to the group of structural models and are not tested now.

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<sup>3</sup>F-statistic of 1.5973 as opposed to critical value of 3.94 for  $p = 5\%$ .

### 2.3 Random Walk Hypothesis

On the basis of rational expectations models, it is often assumed that the exchange rates follow a random walk, and today's exchange rate reflects all the available information, so that the expected change in the exchange rate is zero (3).

$$(3) \quad E[e(t+1)/e(t) | t] = 0$$

Lehmussaari (1987), after having tested forward rate and interest rate parity hypotheses as well as the ARIMA model, chose Random Walk hypothesis on the basis of highest accuracy among the alternatives during his estimation period. Notably, the information in the current interest rates is irrelevant under this hypothesis. It is commonly assumed that the exchange markets are efficient, although various studies that have tested the hypothesis, have been unable to approve or disapprove it, Levich (1985).

Under the Random Walk hypothesis of the exchange rate determination the expected returns from investing into various currencies will coincide with the interest rates. The currency with the highest nominal interest rate would be the most attractive one.

The Random Walk hypothesis was tested on the series of exchange rate changes vis-a-vis the currency index (4).

$$(4) \quad ERW_i = \{ [e_i(t+1)/e_i(t)] \cdot [\text{index}(t)/\text{index}(t+1)] - 1 \} \cdot 100$$

where  $ERW_i$  = return in per cent per month of currency  $i$   
 $e_i$  = exchange rate of currency  $i$  in markka per  $i$   
 index = currency index of markka

The returns expressed in (4) are in per cent per month and adjusted for the changes in the currency index, that is used as a numeraire. The null hypothesis is that the exchange rate changes are not autocorrelated, in other words the expected change of the exchange rate is zero. The null hypothesis cannot be rejected at 5 per cent level of significance on the basis of the two tests performed, namely

the autocorrelation function test and the Ljung-Box test. The results are shown in the appendix (table 2).

#### 2.4 The Open Interest Parity Hypothesis

The interest rate parity theory has been adopted here as one of the options suitable for formulation of the expected exchange rate changes. It is assumed that under the conditions of perfect capital mobility and perfect substitutability of various currencies, the expected depreciation of the exchange rates is equal to the differential between domestic and foreign interest rates of comparable assets (i.e. assets with equal risk). Note that when the expected returns of all currencies are equal, the efficient portfolio frontier collapses to just one point, the minimum variance portfolio (under nominal interest parity).

The open interest parity condition is written as

$$(5) \quad E[e(t+1)/e(t) | t] = [1+r(t)]/[1+r^*(t)]$$

where  $e$  = the exchange rate  
 $r$  = the domestic nominal interest rate  
 $r^*$  = the foreign nominal interest rate

Under the assumptions of perfect capital mobility, resulting e.g. from the presence of at least some risk neutral investors in the market, no or negligible transactions costs (and no government intervention) the above condition should hold.

Equation (5) can be easily tested with actual interest and exchange rate data and if the outcome is like (6)

$$(6) \quad E[e(t+1)/e(t) | t] - (1+r(t))/(1+r^*(t)) = B \quad \text{where } B \neq 0$$

it suggests that the difference between domestic and foreign interest rates and the expected exchange rate changes includes a systematic component  $B$ , called a risk premium. It arises from the markets'

attitude towards the risk associated with foreign investment. Part of the risk can be explained by uncertainty in inflationary expectations and/or in expected depreciation. The existence of a risk premium makes the open interest parity theory too restrictive.

If the existence of the risk premium is established, the characteristics of the premium are of interest. Is it constant, or does it change over time? Does it follow a random walk or is it explainable with some structural variables? Some of the questions are answered in the following.

For the sake of clarity, let us summarize the hypotheses introduced in this section. We are calculating expected returns in both nominal and real terms under three different hypotheses: (i) random walk hypothesis, (ii) open interest parity hypothesis, and (iii) a modified interest parity hypothesis allowing for the existence of a risk premium. In the following section 3 we test the open interest parity and the risk premium hypothesis. We also test for the constancy of the risk premium, provided that the premium exists.

### 3 NOMINAL AND REAL RETURNS: TESTING OF HYPOTHESES

After having calculated monthly returns over one month horizon in both nominal and real terms, adjusted for the changes in the currency index and as deviations from the eight-currency interest basket, we performed tests on the hypotheses discussed above.

#### Case 1. Nominal return, no risk premium

The nominal return forecast errors made by the open interest parity (zero risk premium) model are calculated as follows:

$$(7) \quad NR_i = (1+r_i) \left[ \left( \frac{e_i(t+1)}{e_i(t)} \right) \cdot \left( \frac{\text{index}(t)}{\text{index}(t+1)} \right) \right] - (1+r_b)$$

where  $NR_i$  = nominal return forecast error of currency  $i$  in per cent per month

- $r_i$  = one month euro-deposit rate of currency  $i$  in per cent per month  
 $e_i$  = the exchange rate of currency  $i$ , in markkaa per  $i$   
 index = Bank of Finland's currency index  
 $r_b$  = eight-currency interest basket in per cent per month

It is emphasized that the NR, as it is expressed in (7) does not specify the expected returns from investments into individual currencies, but only the return differentials from individual currencies to the basket interest rate, adjusted for the changes in the currency index. We are, however, able to circumvent this problem by noting that the nominal change of the currency index is zero (in terms of the index itself) and so the nominal return of the index weighted portfolio is just the basket interest rate. The expected rates of return of the various currencies in terms of the index are just equal to the nominal basket interest rate; plus the possible risk premium. The unexpected gains and losses are just excess returns over the basket, again with a possible allowance for the risk premium.

Next, it was tested whether NR follows a Random Walk Process. The Random Walk property of NR implies that the expectations model is efficient or the residuals (forecast errors) are white noise. This was tested through the autocorrelation function and the Ljung-Box test. The null hypothesis of zero autocorrelation could not be rejected at 5 per cent level of significance on the basis of these tests (see appendix table 3).

If the open interest rate parity condition were to hold, NR should be zero on average i.e. the expected value of NR is zero. This was tested by regressing NR on a constant, the null hypothesis being that the coefficient  $B$  equals zero. The measured  $B$ s are of course not zeros, but the low  $t$ -statistics for the coefficients  $B$  render evidence that  $B$ s do not significantly deviate from zero i.e. there is no clear evidence of risk premia (table 4).

Table 4. NOMINAL RETURN, TEST ON RISK PREMIA  
(January 1980 - November 1988)

$$NR_i(t) = B_i + v(t)$$

NR <sub>i</sub>	B <sub>i</sub>	t-statistics; critical value of 1.98 for p = 5 %
USD	0.14	0.537
GBP	0.02	0.100
DEM	-0.21	-1.566
FRF	-0.11	-0.743
JPY	0.44	1.872

In addition to the single-equation t tests, the null hypothesis of B equals zero was tested by a joint test, namely the Hotelling T square test<sup>4</sup>, the results of which are shown in table 5. Again the null hypothesis of B = 0 could not be rejected at 10 per cent level of significance.

Table 5. HOTELLING T SQUARE JOINT TEST ON RISK PREMIA

Nominal return	T square	critical value	
January 1980 - November 1988	0.0511	0.093	p = 10 %
January 1985 - November 1988	0.2160	0.238	p = 10 %

These tests of course test the the zero risk premia only against the specific alternative of constant risk premia.

#### Case 2. Nominal return, constant or variable risk premium

Although our tests render statistical evidence for a zero risk premium, we proceeded to test for the constancy of the risk premium.

<sup>4</sup>For a reference see E. Malinvaud (1966) p. 205 - 207.

The issue of the risk premium is continuously under scrutiny with three schools of thought prevailing: (i) zero risk premium, (ii) nonzero constant risk premium and (iii) time-varying risk premium. Under the assumption that the risk premium were to exist, we constructed a cumulative sum of recursive residuals (CUSUM) test<sup>5</sup> for testing whether the premium is constant. And if it is not constant, the test will suggest when the possible break did occur in 1980 - 1988. A recursive regression was performed (backwards) by regressing NR on a constant, the obtained coefficients being estimates of the risk premia. The results of the CUSUM test are depicted in the appendix graph 3. Ideally the cumulated residuals from the recursive regression should fluctuate around zero or at least between given significance limits. Admittedly that is not exactly true in the case of the USD and the GBP, but according to the test the null hypothesis of constancy of the risk premium cannot be rejected at 5 % level of significance. Also when considering only the last four years, the fluctuations diminish. The importance of the chosen time period will become clearer later on in the paper.

### Case 3. Real return, no risk premium

The forecast error in real terms is calculated simply as the difference between the nominal return forecast error NR as in (7) and the import price forecast error MPEV, as expressed in (8)

$$(8) \quad \text{RR}_i = \text{NR}_i - \text{MPEV}$$

where  $\text{RR}_i$  = real return forecast error of currency  $i$  in per cent per month  
 $\text{NR}_i$  = nominal return forecast error of currency  $i$  in per cent per month  
 $\text{MPEV}$  = forecast error of import prices, in per cent per month

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<sup>5</sup>For more details see Brown, Durbin, and Evans (1975).

Random Walk test as well as the tests on the existence of the risk premium were performed for real return forecast errors also: the results shown in the appendix table 6 and tables 7 and 8 below.

It is evident from table 6, that real return forecast errors follow a random walk process.

Table 7. REAL RETURN, TEST ON THE RISK PREMIA  
(January 1980 to November 1988)

$$RR_i(t) = B_i + v(t)$$

RRi	Bi	t-statistics; critical value of 1.98 for p = 5 %
USD	0.14	0.546
GBP	0.02	0.082
DEM	-0.21	-1.150
FRF	-0.11	-0.553
JPY	0.44	1.742

Table 8. HOTELLING T SQUARE JOINT TEST ON RISK PREMIA

Real return	T square	critical value
January 1980 - November 1988	0.0498	0.093 p = 10 %
January 1985 - November 1988	0.2351	0.238 p = 10 %

The test results in tables 7 and 8 are such that the null hypothesis of  $B = 0$  cannot be rejected at 5 % level of significance in table 7 and at 10 % level of significance in table 8. It should be noted that in particular during the shorter time period from January 1985 to November 1988 the statistical evidence of a zero risk premium is weaker on the basis of the Hotelling T square test than of the t-test. Hence the testing of the constancy of the risk premium was also

performed through CUSUM test, results shown in graph 4. The constancy of the risk premium cannot be rejected on the basis of this graphical analysis.

#### 4 STABILITY OF EXPECTATIONS MODELS AND VARIANCES AND COVARIANCES OF RETURNS

In order to answer some of the critique against mean-variance approach, the additional tests performed in this section are meant to check whether both the variances and covariances of returns are constant over time. This is important from the point of view of corroboration of the chosen models, and also for the choice of a data period with an unchanged exchange rate process.

First, the CUSUMSQUARE test on constancy of the variances was performed under two different hypotheses: (1) nominal returns with constant risk premium, and (2) real returns with constant risk premium. The test results are shown in graphs 5 to 8. The normalized cumulative sum of squared recursive residuals (cumulated backwards) should ideally follow a steady, though fluctuating path from zero to one without crossing the significance limits. A crossing of the limits would suggest that there is some instability, which in case of exchange rates could be explained by discrete devaluations. Even if the limits are crossed, the information of the test is useful for selection of the time period during which we can assume the variance to be stable.

As a general rule, these graphical tests are good in pointing to a possible problem, not an exact proof of them. It was unnecessary to test for the constancy of variances under the zero risk premium hypothesis, because the regression technique applied here would produce results identical to those depicted on graphs 5 through 8. The reason is that in the case of constant risk premium, which is measured by the mean of nominal return forecast error and the mean of real return forecast error, respectively, these means are deducted from the forecast errors.

Among the individual currencies depicted in graph 5, DEM and FRF and JPY show some shifts in the slope of the normalized CUSUMSQUARES line such that the lower significance limits are crossed. This change in the slope could be due to greater volatility of exchange rates in the early 1980s prior to the Plaza Accord in September of 1985. Overall, the normalized CUSUMSQUARES increase rather monotonically from zero to one, with the exception of FRF, that shows a change in the slope in early 1982. On the basis of these graphs, the final estimation period for the optimal portfolios was shortened to start only from January 1985. Graph 6 differs from graph 5 due to a shorter time period of the regression, as indicated on the graphs; the same applies for graphs 7 and 8. A quick comparison between charts 5 and 7 reveals that the normalized CUSUMSQUARES of real returns fluctuate less than those of the nominal returns.

Another test on the stability of variances was performed, namely a test on autocorrelation of squared residuals for both nominal and real returns, results shown in table 9. The residuals are not serially correlated: in other words the null hypothesis of constant variances cannot be rejected at 5 per cent level of significance. This test indicates that our model belongs to the group of constant variance models as opposed to the ARCH alternatives, which are characterized by time-varying (autocorrelated) variances.

Thirdly, the stability of the covariances of returns was tested using a recursive regression technique that is a novelty in the application of this technique.<sup>6</sup> An auxiliary regression for each pair of currencies was performed by having a return of one currency regressed on the return of another currency, and the CUSUM test applied on the recursive residuals, the results shown in graphs 9 and 10. The null hypothesis of stable covariances cannot be rejected at 5 % level of significance.

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<sup>6</sup>The application of a CUSUM test for testing the stability of covariances has not been found in the empirical work yet. According to Assistant Professor M. Rahiala, from the Department of Statistics of the University of Helsinki the CUSUM test is applied correctly, which he proves in his memorandum of 19.3.1989.

## 5 OPTIMAL PORTFOLIOS UNDER VARIOUS HYPOTHESES

According to modern portfolio theory, to state it in a nutshell, it is possible to choose a portfolio such that the total risk of it is smaller than the risk of single instruments included in the portfolio. Furthermore, the total risk of a portfolio can be reduced by diversification. The investor, however, has to make a choice between the level of risk and expected return.

### 5.1 Minimum Variance Portfolios.

The portfolio choice is based on mean-variance analysis of the expected returns. Given the expected returns, and their variances and correlations, the frontier of efficient portfolios can be calculated. For each point on the efficient frontier the expected return and risk are fixed. It is not possible to raise the expected return of a portfolio located on the frontier without increasing the risk as well. The location of the minimum variance portfolio on the frontier is a point, at which the variance is minimized, accepting any resulting expected return. Lowering the expected return below this point would be associated with higher variance of return.

A number of minimum variance portfolios has been calculated with the help of a computer soft ware of Bank of Finland<sup>7</sup> and are shown in tables 10 through 15. Based on our tests on the variances and covariances, the time period from January 1985 to November 1988 was chosen for estimating the variance-covariance matrices. The minimum variance portfolios on tables 10 through 15 differ due to different expectations hypotheses, and with respect to the choice of either real or nominal investment criterion, as indicated on the tables.

As for the currency distributions between either four or five currencies, and based on nominal returns under different hypotheses,

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<sup>7</sup>For details see Bank of Finland Discussion Papers, VP 1/84 by O-P Lehmussaari and Juha Tarkka.

TABLE 10

MINIMUM VARIANCE PORTFOLIOS, JANUARY 1985 - NOVEMBER 1988  
(Random Walk Hypothesis, nominal return)

4 currencies		5 currencies		
USD	14.7	USD	13.3	
GBP	21.7	GBP	20.7	
DEM	57.7	DEM	45.9	EMS total 59.7
JPY	5.9	FRF	13.8	
		JPY	6.3	

standard deviation 0.49 %  
(1.70 %)

standard deviation 0.47 %  
(1.63 %)

Standard deviations and correlations

s		USD	GBP	DEM	FRF	JPY
3.15 (10.91)	USD	1				
2.02 (6.99)	GBP	-.10	1			
1.10 (3.81)	DEM	-.55	-.39	1		
1.04 (3.60)	FRF	-.24	-.27	.58	1	
2.17 (7.52)	JPY	.03	-.22	.10	-.04	1

Annual volatility inside parentheses.

TABLE 11

MINIMUM VARIANCE PORTFOLIOS, JANUARY 1985 - NOVEMBER 1988  
(Nominal return, zero risk premium)

4 currencies		5 currencies		
USD	13.1	USD	12.2	
GBP	22.0	GBP	20.7	
DEM	57.0	DEM	45.2	EMS total 59.6
JPY	7.9	FRF	14.4	
		JPY	7.6	

standard deviation 0.49 %  
(1.70 %)

standard deviation 0.47 %  
(1.63 %)

Standard deviations and correlations

s		USD	GBP	DEM	FRF	JPY
3.21 (11.11)	USD	1				
2.07 (7.17)	GBP	-.13	1			
1.07 (3.71)	DEM	-.49	-.41	1		
1.05 (3.64)	FRF	-.28	-.25	.58	1	
2.14 (7.41)	JPY	.06	-.22	.02	-.01	1

Annual volatility inside parentheses.

TABLE 12

MINIMUM VARIANCE PORTFOLIOS, JANUARY 1985 - NOVEMBER 1988  
(Nominal return, constant risk premium)

4 currencies		5 currencies		
USD	13.5	USD	12.5	
GBP	21.5	GBP	20.6	
DEM	57.7	DEM	45.8	EMS total 59.7
JPY	7.3	FRF	13.9	
		JPY	7.3	

standard deviation 0.48 %  
(1.66 %)

standard deviation 0.47 %  
(1.63 %)

Standard deviations and correlations

s		USD	GBP	DEM	FRF	JPY
3.06 (10.60)	USD	1				
2.05 (7.10)	GBP	-.10	1			
1.07 (3.71)	DEM	-.51	-.41	1		
1.04 (3.60)	FRF	-.26	-.27	.58	1	
2.11 (7.31)	JPY	.12	-.24	.03	-.03	1

Annual volatility inside parentheses.

the results in tables 10, 11 and 12 below are rather consistent. With the addition of the fifth currency, the FRF, the shares of the other currencies are reduced. The reduction is largest in DEM, because FRF is a good substitute for DEM due to high correlation between DEM and FRF within the EMS. The figure EMS total is just the sum of shares for DEM and FRF. As is evident in the five currency portfolio, the EMS total is close to the DEM share in the four currency portfolio. The addition of a new currency to the portfolio reduces expectedly the total risk.

Portfolios based on real returns under different hypotheses are displayed in tables 13, 14 and 15 below. Again, the currency distributions do not differ too much across alternative expectations hypotheses. The share of the dollar is now about 4 - 6 % higher than in the "nominal" portfolios, because the dollar denominated prices have a large weight in our import price index. Interestingly, the

TABLE 13

MINIMUM VARIANCE PORTFOLIOS, JANUARY 1985 - NOVEMBER 1988  
(Random Walk Hypothesis, real return)

4 currencies		5 currencies		
USD	20.8	USD	19.6	
GBP	23.4	GBP	22.5	
DEM	52.6	DEM	42.0	EMS total 54.4
JPY	3.1	FRF	12.4	
		JPY	3.5	

standard deviation 1.11 %  
(3.85 %)

standard deviation 1.11 %  
(3.84 %)

Standard deviations and correlations

s		USD	GBP	DEM	FRF	JPY
3.10 (10.74)	USD	1				
2.22 (7.69)	GBP	-.05	1			
1.63 (5.65)	DEM	-.27	.08	1		
1.53 (5.30)	FRF	-.07	.14	.81	1	
2.46 (8.52)	JPY	.10	.03	.41	.32	1

Annual volatility inside parentheses.

TABLE 14

MINIMUM VARIANCE PORTFOLIOS, JANUARY 1985 - NOVEMBER 1988  
(Real return, zero risk premium)

4 currencies		5 currencies		
USD	19.4	USD	19.4	
GBP	22.3	GBP	22.3	
DEM	53.6	DEM	53.4	EMS total 53.6
JPY	4.7	FRF	0.2	
		JPY	4.7	

standard deviation 1.08 %  
(3.74 %)

standard deviation 1.08 %  
(3.73 %)

Standard deviations and correlations

s		USD	GBP	DEM	FRF	JPY
3.15 (10.91)	USD	1				
2.29 (7.93)	GBP	-.08	1			
1.56 (5.40)	DEM	-.25	.06	1		
1.57 (5.44)	FRF	-.09	.17	.81	1	
2.41 (8.35)	JPY	.11	.02	.35	.34	1

Annual volatility inside parentheses.

TABLE 15

MINIMUM VARIANCE PORTFOLIOS, JANUARY 1985 - NOVEMBER 1988  
(Real return, constant risk premium)

4 currencies		5 currencies		
USD	17.8	USD	17.6	
GBP	23.3	GBP	23.0	
DEM	52.5	DEM	49.9	EMS total 53.0
JPY	6.4	FRF	3.1	
		JPY	6.3	

standard deviation 1.08 %  
(3.74 %)

standard deviation 1.07 %  
(3.71 %)

Standard deviations and correlations

s		USD	GBP	DEM	FRF	JPY
3.06 (10.60)	USD	1				
2.23 (7.72)	GBP	-.03	1			
1.54 (5.33)	DEM	-.22	.03	1		
1.53 (5.30)	FRF	-.04	.13	.80	1	
2.34 (8.11)	JPY	.18	-.03	.33	.30	1

Annual volatility inside parentheses.

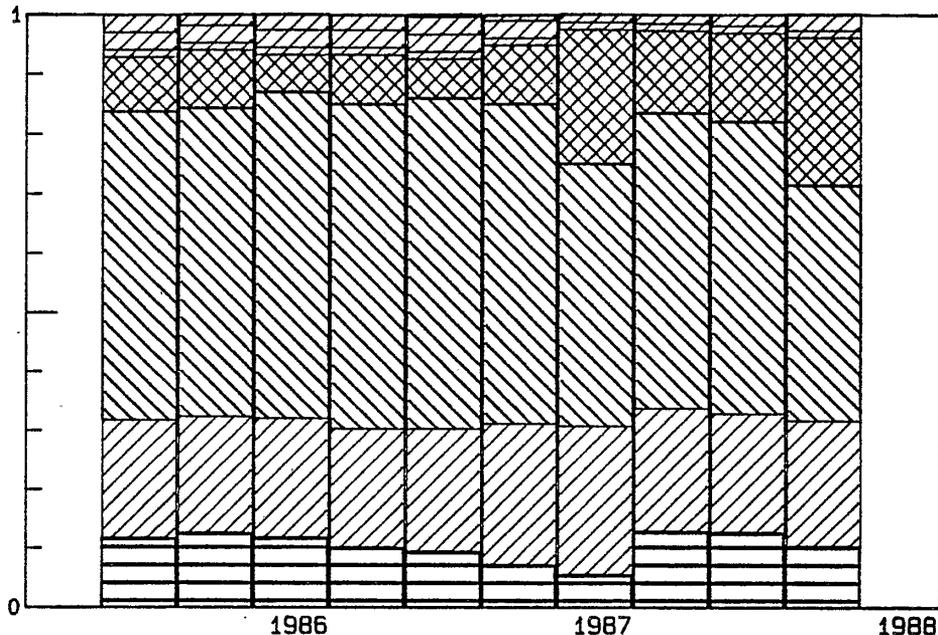
share of the pound varies the least, between 20.6 and 23.4 per cent under the various hypotheses, both nominal and real.

On the basis of the statistical tests performed in section 4 the null hypothesis of constant variances and covariances could not be rejected. In order to reduce the remaining skepticism regarding the stability of the composition of minimum variance portfolios we performed a further test as follows: we calculated so called adjusted portfolio shares, based on sample size of 30 months, starting from January 1984 and re-estimating a new variance-covariance matrix at the beginning of each quarter while maintaining the sample size constant. The optimal portfolio shares are depicted in graph 11 (nominal) and graph 12 (real). The only major shift occurred in October 1987, after the stock market crash, that induced a large temporary drop in the interest rates, in particular in the USA. The instability of the variance-covariance matrix is obviously not a serious problem,

GRAPH 11. NOMINAL OPTIMAL PORTFOLIO SHARES, ADJUSTED AT THE BEGINNING OF EACH QUARTER, TEN MATRICES OF SAMPLE SIZE 30 MONTHS, STARTING JANUARY 1984 TO SEPTEMBER 1988<sup>1</sup> (CONSTANT RISK PREMIUM HYPOTHESIS)

USD.WGHT  
GBP.WGHT  
DEM.WGHT  
FRF.WGHT  
JPY.WGHT

86Q2 88Q3  
86Q2 88Q3  
86Q2 88Q3  
86Q2 88Q3  
86Q2 88Q3

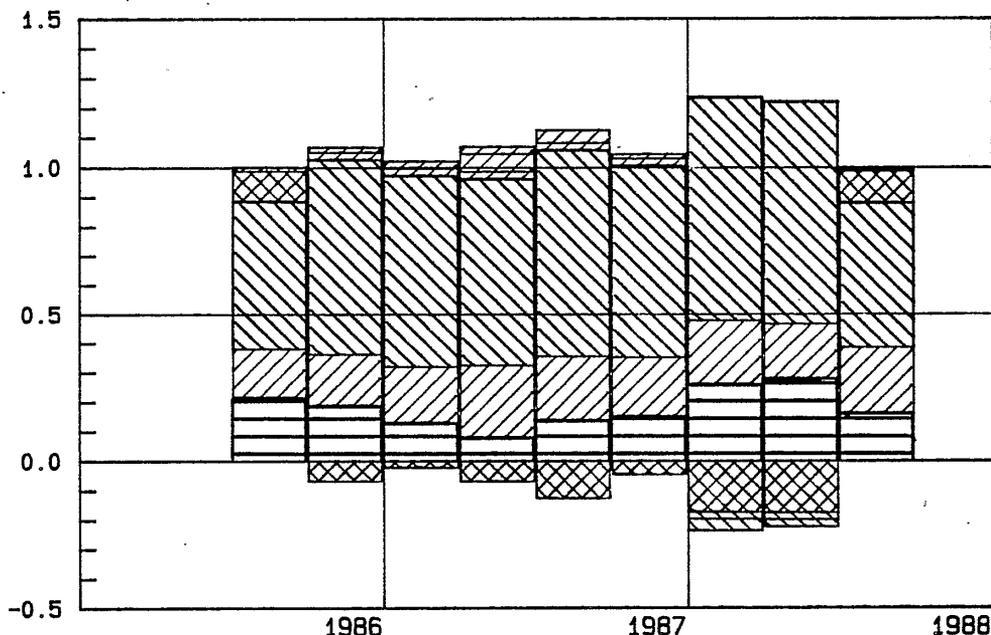


<sup>1</sup> January 1984 - June 1986, April 1984 - September 1986, July 1984 - December 1986, October 1984 - March 1987, January 1985 - June 1987, April 1985 - September 1987, July 1985 - December 1987, October 1985 - March 1988, January 1986 - June 1988, April 1986 - September 1988.

GRAPH 12. REAL OPTIMAL PORTFOLIO SHARES, ADJUSTED AT THE BEGINNING OF EACH QUARTER, TEN MATRICES OF SAMPLE SIZE 30 MONTHS, STARTING JANUARY 1984 TO SEPTEMBER 1988<sup>1</sup> (CONSTANT RISK PREMIUM HYPOTHESIS)

USDR.WGHT  
GBPR.WGHT  
DEMR.WGHT  
FRFR.WGHT  
JPYR.WGHT

86Q3 88Q3  
86Q3 88Q3  
86Q3 88Q3  
86Q3 88Q3  
86Q3 88Q3



<sup>1</sup> January 1984 - June 1986, April 1984 - September 1986, July 1984 - December 1986, October 1984 - March 1987, January 1985 - June 1987, April 1985 - September 1987, July 1985 - December 1987, October 1985 - March 1988, January 1986 - June 1988, April 1986 - September 1988.

although one should be aware of it. No model will insulate against such unexpected events as the crash was, even though there had been wide speculation about the US stock market being overpriced at that time. The results on graph 12 are more cumbersome to digest by an investor who does not conventionally have liabilities, since the optimal shares imply a short position in some of the currencies at times.

The nominal return criterion implies that the variance of the index weighted portfolio is zero. Thus if the complete set of 14 currencies would be allowed, a zero variance would be attainable. Our results show how well we are able to replicate the full index with just a few currencies (graph 13). It is thus not surprising that the weights resemble the index weights, adjusted for EMS linkages and the krona baskets (see table 17, page 39).

In order to evaluate the performance of different portfolios, we constructed graphs 13 through 16 showing cumulative returns of an investment of 100 in January 1985 and reinvested at the beginning of each month into a minimum variance portfolio, nominal returns as well as the individual currency returns (graph 15 and 16). Graph 16 is identical to graph 15, except that it depicts real returns. The JPY has yielded the highest cumulative return since January 1985, and the USD the lowest (see graphs 15 and 16). Graphs 13 and 14 contain cumulative nominal and real returns from a minimum variance portfolio of five currencies, from a SDR weighted portfolio and the currency index weighted (eight currency) portfolio.

The performance of alternative portfolios is presented in table 16 below. The criteria used for comparison are the mean return of the portfolio, and the standard deviation and the variance of returns. The minimum variance portfolio (MVP) outperforms the other alternatives, the equal weighted portfolio and the index weighted portfolio on the basis of standard deviation and variance. This is actually what is expected from a MVP. The fact that the return from the index weighted portfolio seems to exceed that of the MVP is actually irrelevant, because the actual return does not enter into the calculation of the minimum variance portfolio. Michaud's rather casual comment about

equal weighted portfolio outperforming in many cases a MVP does not hold, as was shown also by Lehmuussaari (1987), pages 117 - 118.

TABLE 16 PERFORMANCE OF VARIOUS PORTFOLIOS

(Standard deviations and covariance matrix are calculated for the period from January 1985 to November 1988)

Nominal return, constant risk premium<sup>1</sup>

	Four currencies			Five currencies		
	Mean	s	$\sigma^2$	Mean	s	$\sigma^2$
MVP	-0.05	0.48	0.23	-0.03	0.47	0.22
Equal weights	-0.09	0.89	0.79	-0.05	0.69	0.48
Index <sup>2</sup> weights	-0.04	0.58	0.34	-0.02	0.52	0.27

	Portfolio shares				Portfolio shares				
	USD	GBP	DEM	JPY	USD	GBP	DEM	FRF	JPY
MVP	13.5	21.5	57.7	7.3	12.5	20.6	45.8	13.9	7.3
Equal weights	25.0	25.0	25.0	25.0	20.0	20.0	20.0	20.0	20.0
Index <sup>2</sup> weights	16.6	28.9	41.6	12.9	14.5	25.3	36.4	12.5	11.3

<sup>1</sup> Nominal return is expressed as deviations from the interest rate basket, in per cent per month. Nominal return calculation is based on the constant risk premium hypothesis.

<sup>2</sup> Index weights are derived from the currency basket.

Healy contrasted the sample variances of optimal MVPs calculated for 4 countries to sample variances of SDR-, USD-, and import portfolios, and found the MVPs superior in all cases. The return from an equal weighted portfolio may exceed that of a MVP, but it is associated with a higher exposure to risk and by change only.

Graphs 17 and 18 in the appendix show cumulative returns of out-of-sample minimum variance portfolios and SDR weighted portfolios. Again, if the objective is the stability of return, then the MVP (minimum variance portfolio) is certainly less risky than the SDR weighted alternative, with variance of the nominal MVP nearly three times smaller than that of the nominal SDR portfolio. The variance of the real MVP is also significantly smaller than the variance of the real SDR portfolio.

## 5.2 Efficient Portfolio Frontiers

In the case when expected returns of investment assets differ, there is a trade off between risk and return of the portfolio. In our study, this trade off exists in the random walk and constant risk premium model. The frontier of efficient portfolios was calculated for both nominal and real returns, shown in table 17 below. Note that the required return is not an absolute return but a deviation from the eight currency interest basket return, expressed in per cent per month. If we stay in the minimum variance portfolio strategy, the currency shares based on comparable expectations hypotheses, i.e. either nominal or real, do not differ significantly, nor do the standard deviations. The variance of nominal return of a portfolio consisting of all 14 currencies of the currency index is zero, compare graph 13. In case B, second row, we can expect a nominal return equivalent to the basket return from a portfolio consisting of only five currencies along with the annual volatility of 1.6. If we are aiming for higher return relative to the basket return, the expectations hypotheses play a role. Among the nominal return alternatives A and B, as the return requirement is increased, the shares of USD, DEM and JPY fall in case A and the shares of USD and DEM fall in case B, while the share of FRF increases in both cases. As for the increase in volatility, when the return requirement is raised for instance to 2 per cent per annum above the basket return in nominal terms, case B renders the least increase of volatility from 1.6 of the MVP to 2.2. In case of real return, case C, Random Walk hypothesis is associated with higher volatility as opposed to constant risk premium hypothesis, case D for the same level of required return:

for a return of 2 per cent per annum above the basket return a difference in volatility of 4.5 and 3.7. In summary, under both Random Walk Hypotheses, nominal and real, cases A and C, a required return in excess of the basket return by 2 per cent per annum is achieved only by acceptance of higher risk in comparison to the constant risk premium hypotheses, cases B and D.

TABLE 17 FRONTIER OF EFFICIENT PORTFOLIOS

(Standard deviations and covariance matrices are calculated for the period from January 1985 to November 1988)

	Required return <sup>1</sup>	Standard deviation	USD	GBP (shares in per cent)	DEM	JPY	FRF
<b>A. Nominal return, Random Walk Hypothesis</b>							
1	-0.06 (-0.7) <sup>2</sup>	0.47 (1.6) <sup>2</sup>	13.3	20.7	45.9	6.3	13.8
2	0.02 (0.2)	0.55 (1.9)	12.3	27.2	22.9	4.2	33.4
3	0.10 (1.2)	0.74 (2.6)	11.3	33.8	-0.1	2.1	52.9
4	0.18 (2.1)	0.97 (3.4)	10.3	40.3	-23.0	0.0	72.4
<b>B. Nominal return, constant risk premium</b>							
1	-0.03 (-0.4)	0.47 (1.6)	12.5	20.6	45.8	7.3	13.9
2	0 (0)	0.47 (1.6)	10.8	21.3	42.4	8.8	16.7
3	0.08 (1.0)	0.53 (1.8)	5.6	23.5	31.9	13.7	25.3
4	0.17 (2.0)	0.63 (2.2)	0.3	25.7	21.5	18.5	34.0
<b>C. Real return, Random Walk Hypothesis</b>							
1	-0.03 (-0.4)	1.11 (3.8)	19.6	22.5	42.0	12.4	3.5
2	0.05 (0.6)	1.14 (3.9)	18.6	29.1	19.0	1.4	31.9
3	0.13 (1.6)	1.24 (4.3)	17.6	35.6	-3.9	-0.7	51.4
4	0.21 (2.6)	1.39 (4.8)	16.6	42.2	-26.9	-2.8	71.0
<b>D. Real return, constant risk premium</b>							
1	0.16 (2.0)	1.07 (3.7)	17.6	23.0	49.9	6.3	3.1
2	0.24 (3.0)	1.09 (3.8)	12.4	25.2	39.5	11.1	11.7
3	0.33 (4.0)	1.13 (3.9)	7.1	27.4	29.0	16.0	20.5
4	0.41 (5.0)	1.20 (4.2)	1.8	29.7	18.4	20.9	29.2

<sup>1</sup> The returns are expressed in per cent per month as deviations from the interest rate basket; weights derived from the currency index weights. The first row of returns represents the return from the minimum variance portfolio.

<sup>2</sup> The figures in parentheses are returns per annum or annual volatilities, respectively.

## 6 SUMMARY AND CONCLUSIONS

1. We have estimated various minimum variance portfolios consisting of foreign currency eurodeposits in both nominal and real terms under different expectations hypotheses. In terms of the overall approach, we have chosen the currency index to serve as the numeraire for nominal returns and the import price index for measuring the real returns. The holding period is set to one month. We have not dealt with the issue of investment horizon or the maturity structure. The effects of transactions costs are excluded also.
2. The shares of the currencies in the various MVPs are in general of the expected magnitudes, taking into account the large share of the EMS countries in Finnish imports and the weight of their currencies in the currency index. The indirect link through the Swedish krona obviously strengthens the influence of the EMS even further.
3. The performance of the MVPs has been contrasted with alternative portfolios such as SDR-, index- and equal-weighted portfolios (table 16 and graphs 17 and 18). It has been shown that the MVPs outperform the alternatives in terms of a lower standard deviations of return. If the objective of the central bank is to stabilize the value of reserves with respect to the numeraire (the currency index), it has been shown that the objective is well achieved by investing the reserves into only five currencies as opposed to 14 in the actual index. If the objective of the bank would be of different nature, it would have to be specified explicitly, before the estimation of an optimal portfolio would be feasible.
4. As for the stability of the portfolio shares, it was shown not to be a problem, once an appropriate time period without statistically significant changes in the variance-covariance structure has been chosen. An attempt to was made to design tests that would point out the proper time period. Consequently, both the CUSUM and CUSUMSQUARE tests were utilized in the selection of the final estimation period of 47 months, extending from January 1985 to November 1988. The out-of-sample portfolios were estimated with a minimum sample size of 30 months. The standard deviations of the out-of-sample portfolios are

expected to be slightly higher than the standard deviations of in-sample portfolios (compare tables 12 and 15 and graphs 17 and 18), but the differences seem to be minor.

5. The evidence for the existence of exploitable risk-return trade offs in the foreign exchange markets is weak at best. We have, however, computed illustrative examples of efficient portfolio frontiers also.

6. For further research are left issues such as i) the case of a time-varying risk premium estimable through Kalman filtering or ii) the testing for mean reversion of return or iii) estimation of an optimal portfolio under some new objective function. The current MV framework and the empirical tests could also be applied for bond yield data, provided that comparable data in terms of risk is available. This would make it possible to study the diversification with respect to maturities as well as currencies.

## APPENDIX

TABLE 1 COMPOSITION OF ISRAELI OPTIMUM PORTFOLIOS FOR SELECTED RETURNS,<sup>1</sup> 1972 - 1976 (PER CENT) ACCORDING TO BEN-BASSAT

	In import terms			In dollar terms		
	Selected optimum combinations					
Mean return	9.0	12.0	14.0	8.0	12.0	14.0
Standard deviation	3.3	10.5	19.5	2.3	20.2	30.0
Currency composition of selected combinations						
Dollar	46.0	50.3	30.3	96.4	62.0	44.6
Canadian dollar	-	-	-	1.9	0.9	0.4
Sterling	21.5	-	-	-	-	-
Deutschemark	10.6	-	-	0.5	-	-
Guilder	4.8	19.4	16.3	-	-	-
Swiss franc	1.8	19.3	43.4	0.2	24.2	36.0
French franc	7.1	6.0	-	-	-	-
Yen	7.9	-	-	0.3	-	-
Gold	0.3	5.0	10.0	0.7	12.9	19.0
Total	100.0	100.0	100.0	100.0	100.0	100.0

<sup>1</sup>Based on monthly data.

The rate of return on gold was estimated as the change in the price of gold in the dollar-denominated market, converted into terms of the Israeli import currency basket.

Source: Ben-Bassat (1980).

TABLE 2      RANDOM WALK TEST ON EXCHANGE RATES,<sup>1</sup>  
 JANUARY 1980 - NOVEMBER 1988

Autocorrelation functions

Lags	USD	GBP	DEM	FRF	JPY
1	.047	.021	-.062	-.008	-.082
2	.109	.031	.034	.084	.016
3	.044	.017	.025	-.037	-.013
4	.122	-.021	-.020	-.041	.108
5	.125	-.150	.012	-.044	-.030
6	-.071	-.087	.067	.002	-.114
7	.173	.147	.132	-.015	-.007
8	-.051	-.102	-.011	.187	.083
9	.085	-.060	-.188	.021	-.122
10	-.038	-.005	.154	.085	.002
11	.186	.060	.036	-.000	-.019
12	-.039	-.001	-.046	-.058	-.056

Critical value = 0.20 for 5 % level of significance

Ljung-Box test on multiple autocorrelation

Lags	USD	GBP	DEM	FRF	JPY
1-12	15.149	8.396	10.834	6.916	6.809

Critical value = 21.03 for 5 % level of significance

<sup>1</sup> The exchange rate changes as indicated by equation (4).

TABLE 3      RANDOM WALK TEST ON NOMINAL RETURNS,<sup>1</sup>  
 JANUARY 1980 - NOVEMBER 1988

Autocorrelation functions

Lags	USD	GBP	DEM	FRF	JPY
1	.084	.052	-.071	.016	-.058
2	.134	.049	.028	.049	.037
3	.072	.031	.017	-.049	.004
4	.143	-.013	-.023	-.079	.129
5	.139	-.145	.003	-.084	-.018
6	-.051	-.081	.058	-.030	-.100
7	.184	.140	.116	-.022	-.007
8	-.039	-.103	-.023	.134	.083
9	.092	-.065	-.203	-.044	-.124
10	-.025	-.013	.148	.038	-.008
11	.190	.050	.045	-.073	-.028
12	.020	-.009	-.047	-.112	-.064

Critical value = 0.20 for 5 % level of significance

Ljung-Box test on multiple autocorrelation

Lags	USD	GBP	DEM	FRF	JPY
1-12	17.926	8.309	10.900	7.048	6.918

Critical value = 21.03 for 5 % level of significance

<sup>1</sup> The nominal returns as indicated by equation (7).

TABLE 6      RANDOM WALK TEST ON REAL RETURNS,<sup>1</sup>  
 JANUARY 1980 - NOVEMBER 1988

Autocorrelation functions

Lags	USD	GBP	DEM	FRF	JPY
1	-.062	-.059	.095	.087	.007
2	.131	-.001	.036	.054	-.033
3	.011	.021	.120	.035	.086
4	.041	.001	.129	.107	.135
5	.131	-.159	-.028	-.125	-.031
6	-.073	-.164	.147	.048	-.240
7	.151	.185	.118	.013	.031
8	-.089	-.121	-.025	.074	.020
9	.090	-.119	-.045	.011	-.189
10	-.025	.011	.101	.094	-.037
11	.074	.113	.126	.019	.022
12	-.029	.110	.042	-.035	-.095

Critical value = 0.20 for 5 % level of significance

Ljung-Box test on multiple autocorrelation

Lags	USD	GBP	DEM	FRF	JPY
1-12	10.648	17.006	12.646	6.637	15.644

Critical value = 21.03 for 5 % level of significance

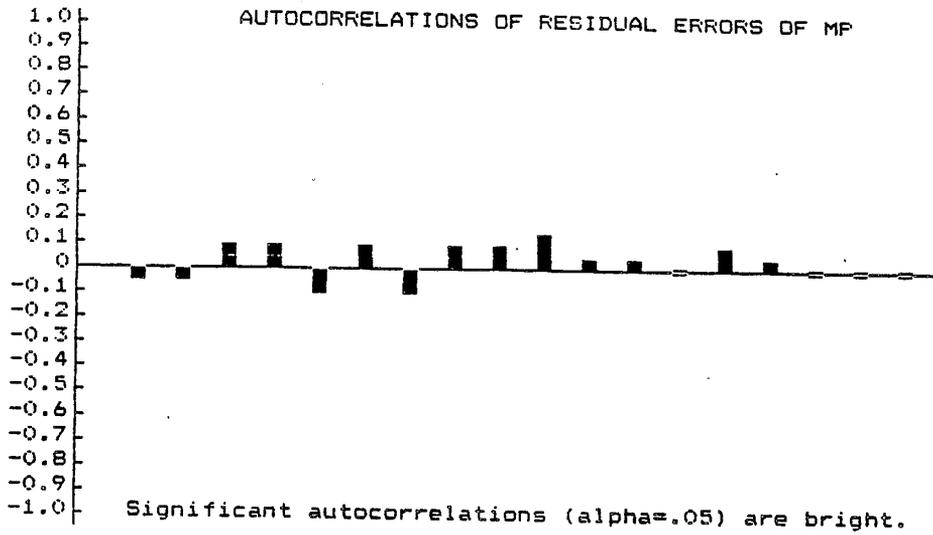
<sup>1</sup> The real returns as indicated by equation (8).

TABLE 9 TEST ON AUTOCORRELATION OF SQUARED RESIDUALS,  
JANUARY 1985 - NOVEMBER 1988

LAG	Autocorrelation function				
	USD	GBP	DEM	FRF	JPY
<b>A. Nominal returns</b>					
0	1.000	1.000	1.000	1.000	1.000
1	0.010	-0.122	-0.085	0.216	-0.189
2	-0.145	0.043	-0.158	0.095	0.086
3	-0.198	-0.073	0.048	-0.095	-0.108
4	0.042	0.065	0.144	-0.289	0.145
5	-0.011	-0.053	-0.138	-0.233	-0.211
6	-0.128	0.231	0.069	0.190	0.373
7	0.077	-0.041	-0.075	0.053	-0.233
8	0.044	-0.080	-0.126	0.131	0.105
9	-0.105	0.035	0.050	0.181	-0.174
10	0.072	0.193	-0.191	-0.184	0.299
11	0.011	-0.119	-0.249	-0.142	-0.096
12	-0.078	0.217	-0.088	-0.078	-0.136
<b>B. Real returns</b>					
0	1.000	1.000	1.000	1.000	1.000
1	0.037	-0.109	-0.053	0.139	-0.152
2	-0.103	0.094	-0.170	-0.028	-0.021
3	-0.206	-0.058	-0.005	-0.060	0.040
4	0.015	0.064	-0.085	-0.135	0.201
5	0.008	-0.038	0.095	-0.154	-0.088
6	-0.055	0.463	0.095	0.070	0.265
7	-0.020	-0.105	-0.094	-0.095	-0.191
8	0.196	-0.047	-0.232	0.232	-0.029
9	-0.069	-0.017	-0.018	0.044	-0.221
10	0.079	0.180	-0.038	-0.066	0.238
11	-0.173	-0.100	-0.157	-0.174	-0.025
12	-0.085	0.350	-0.036	0.003	-0.132

Critical value = 0.29 for 5 % level of significance

GRAPH 1. IMPORT PRICE EQUATION: TEST OF AUTOCORRELATION OF RESIDUAL ERRORS (January 1980 - December 1988)

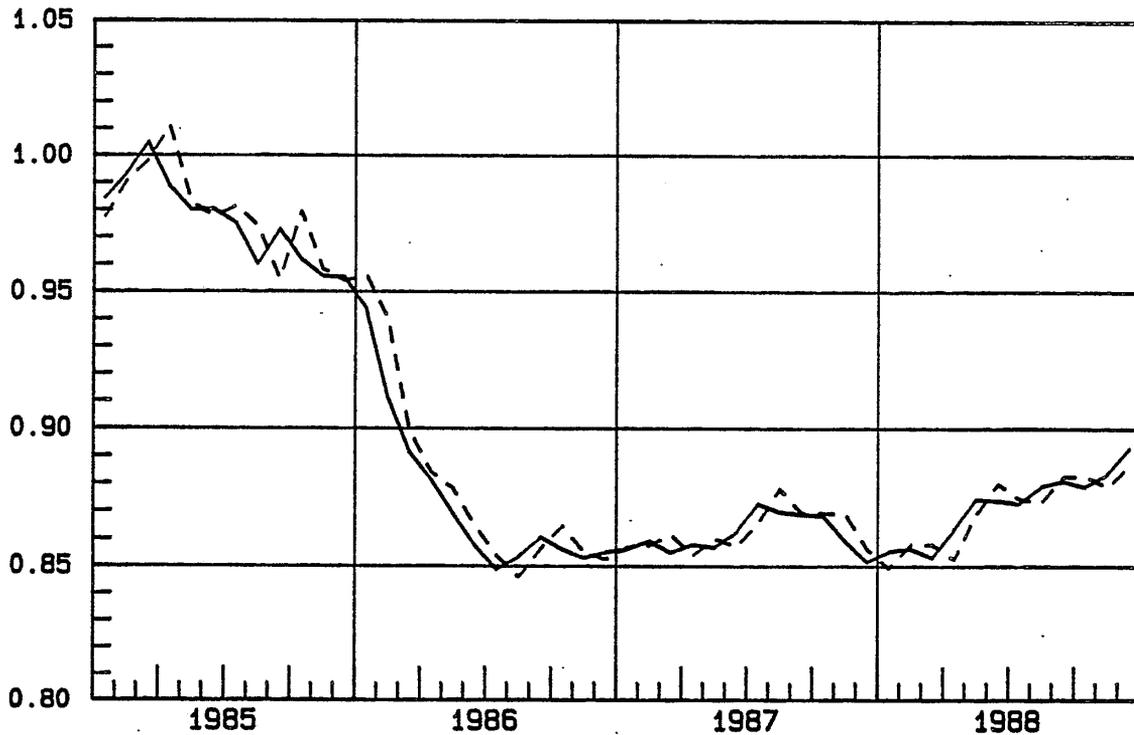


AUTOCORRELATIONS OF LAGGED RESIDUAL ERRORS

Lag:	1	2	3	4	5	6	7	8	9	10	11	12
MP	-.03	-.04	.11	.11	-.09	.12	-.10	.09	.10	.13	.03	.04
	-.02	.09	.04	.02	.00	-.00						

GRAPH 2. ACTUAL AND PREDICTED IMPORT PRICES: January 1985 - December 1988<sup>1</sup>

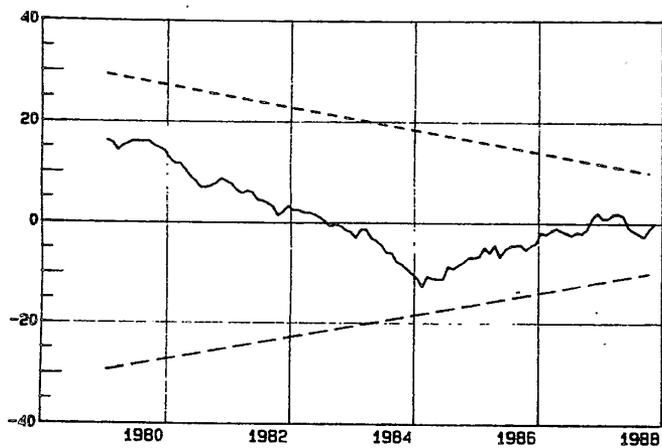
ACTUAL IMPORT PRICES  
 PREDICTED IMPORT PRICES



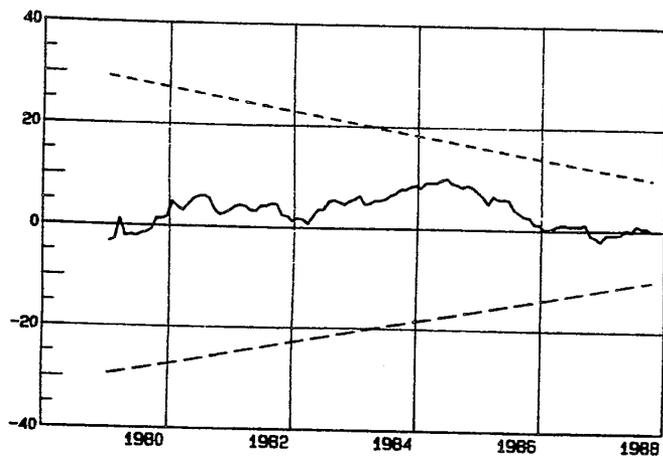
<sup>1</sup> The estimation period for the price equation is January 1980 - December 1988.

GRAPH 3. CUSUM TEST ON CONSTANCY OF RISK PREMIUM OF NOMINAL RETURN

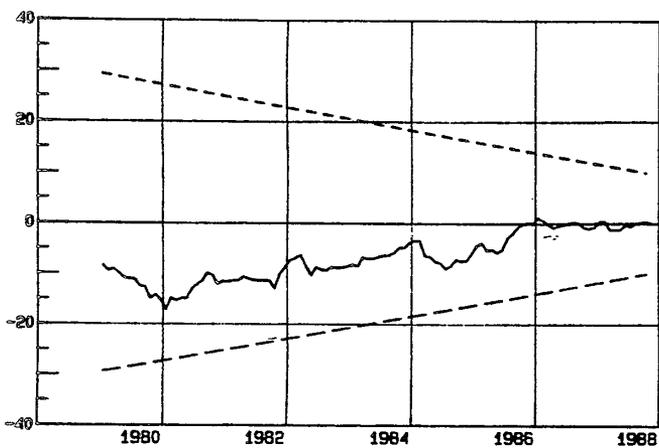
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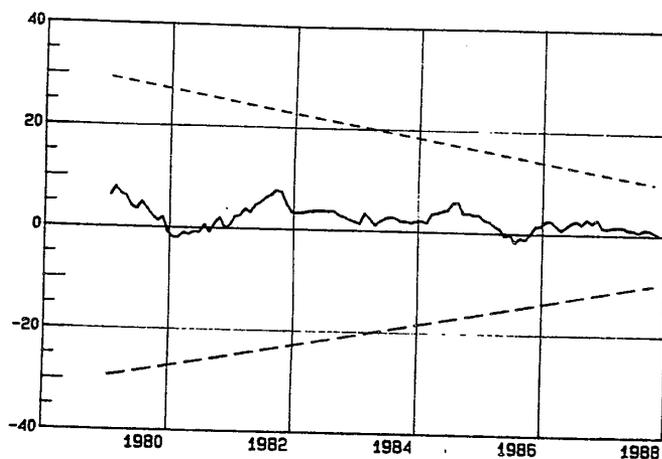
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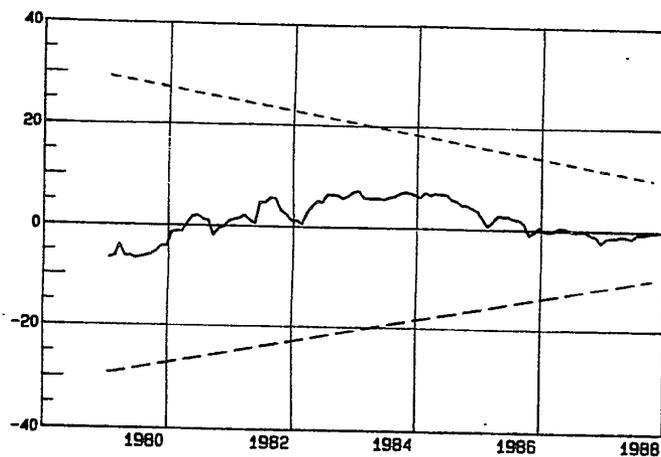
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*JPY*

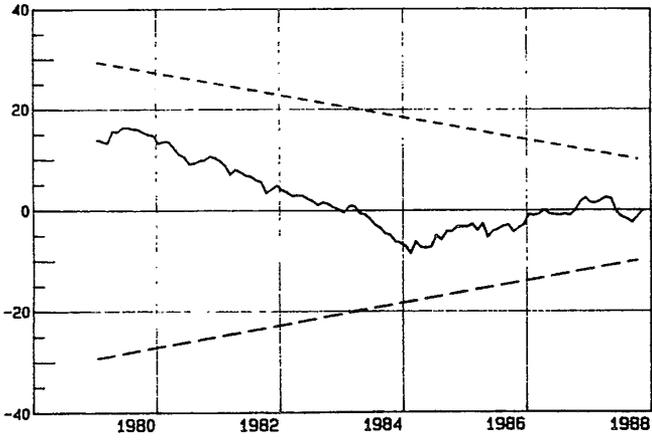


*FRF*

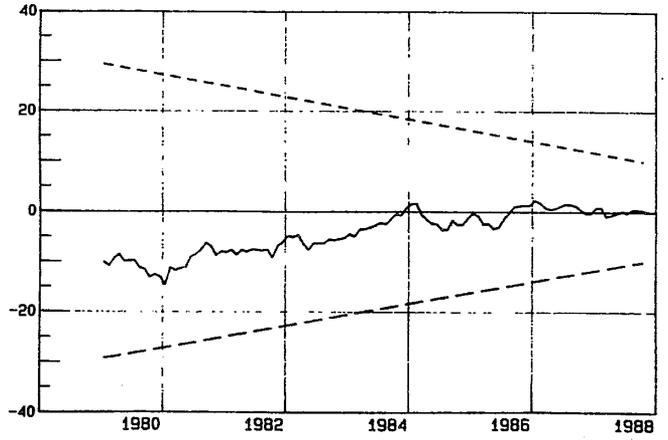


GRAPH 4. CUSUM TEST ON CONSTANCY OF RISK PREMIUM OF REAL RETURN

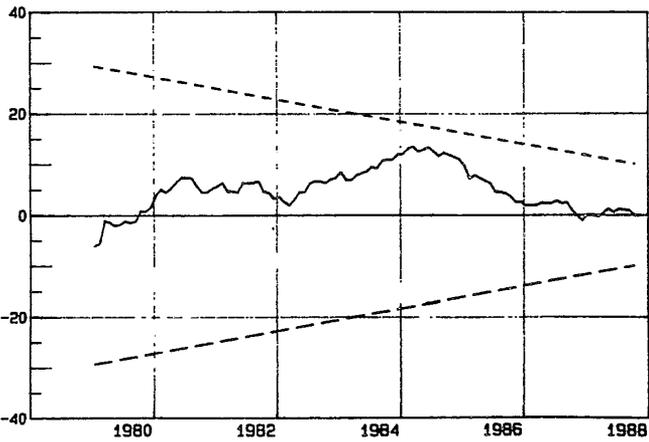
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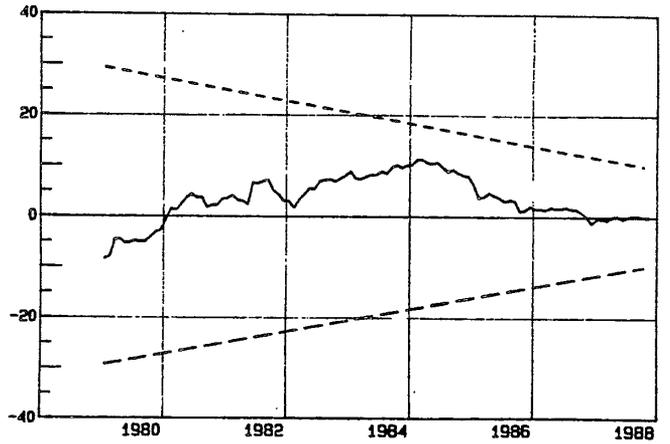
GBP



DEM

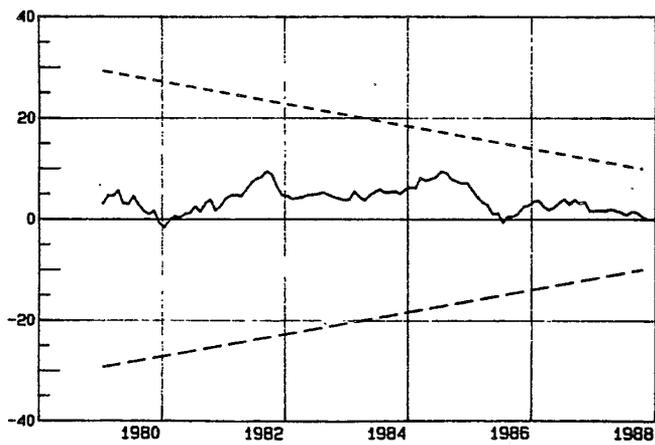


FRF



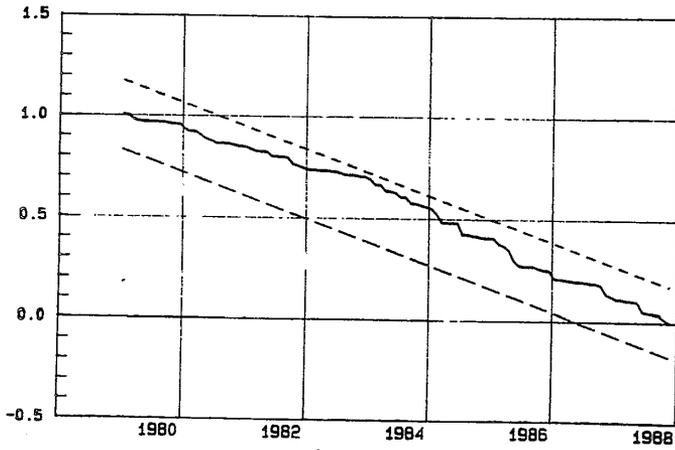
1970 - 1988

JPY

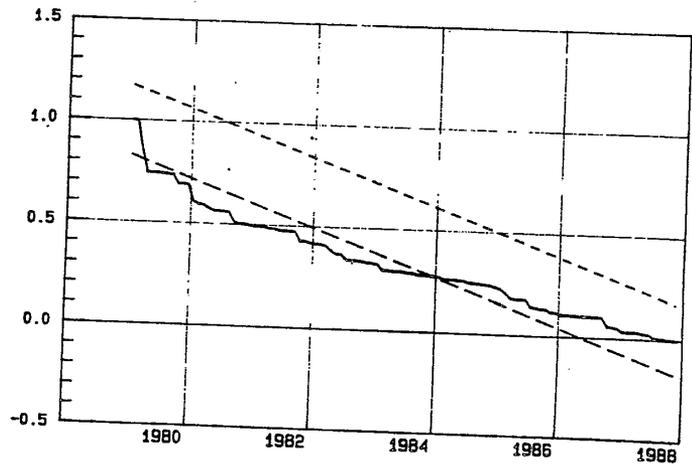


GRAPH 5. CUSUMSQUARE TEST ON CONSTANCY OF VARIANCES OF NOMINAL RETURNS, January 1980 - November 1988

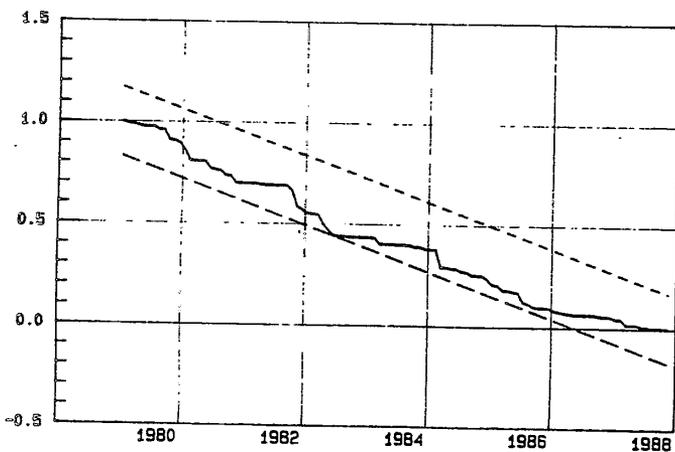
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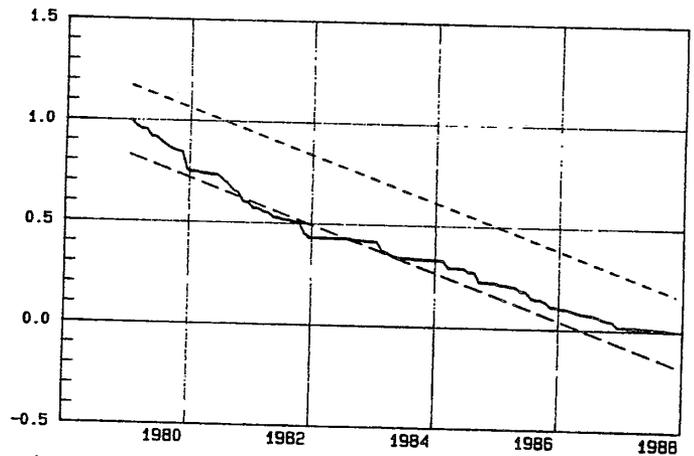
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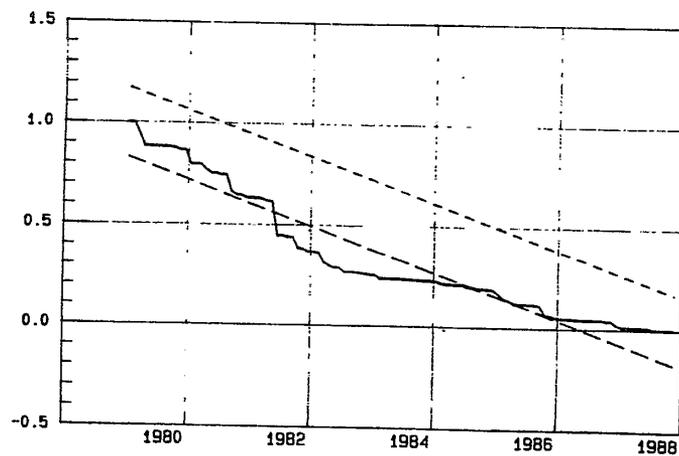
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*JPY*

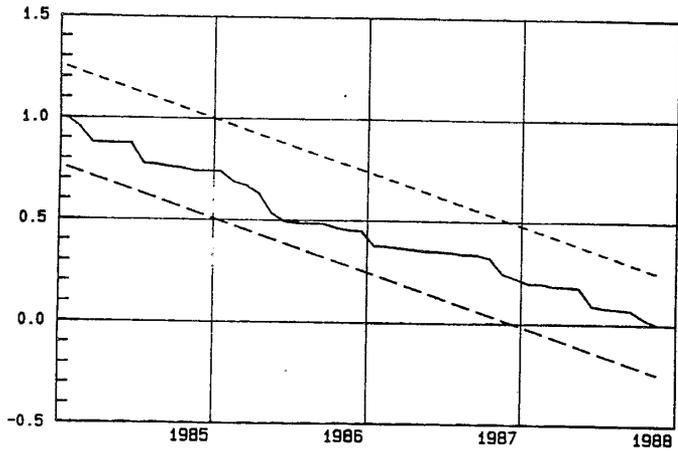


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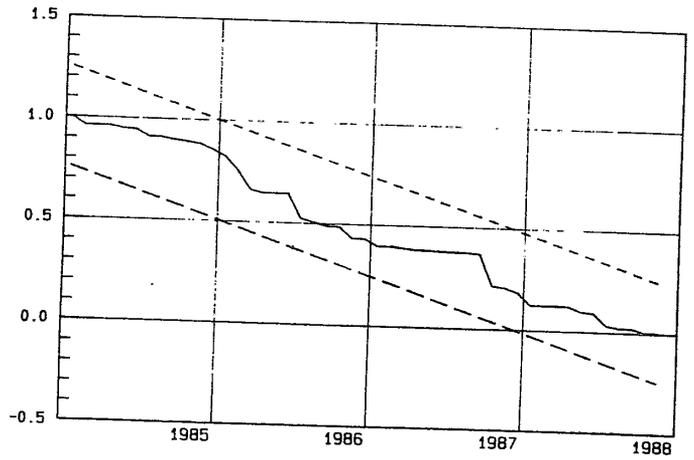


GRAPH 6. CUSUMSQUARE TEST ON CONSTANCY OF VARIANCES OF NOMINAL RETURNS, January 1985 - November 1988

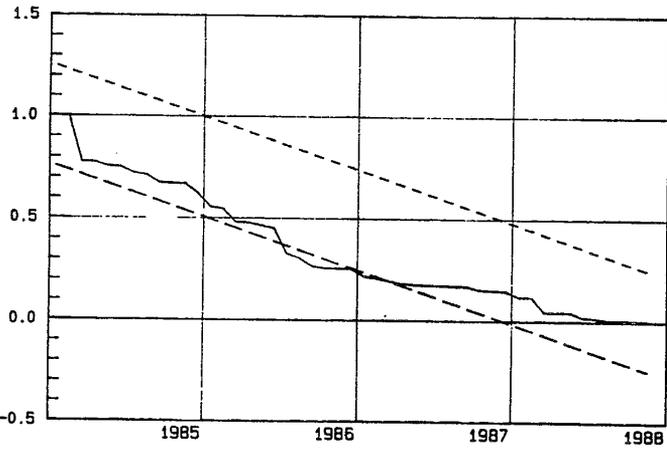
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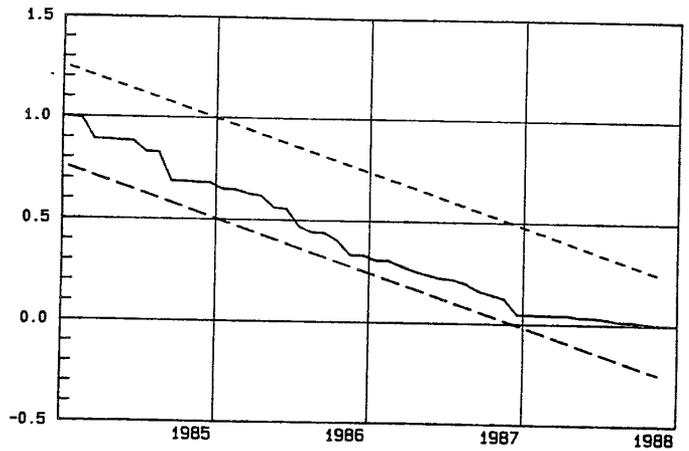
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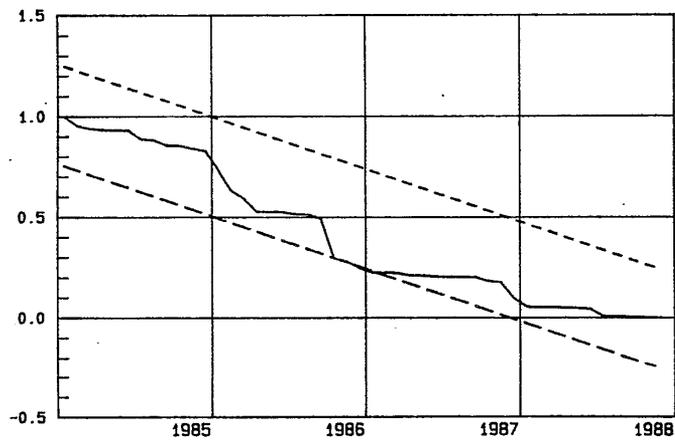
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*JPY*

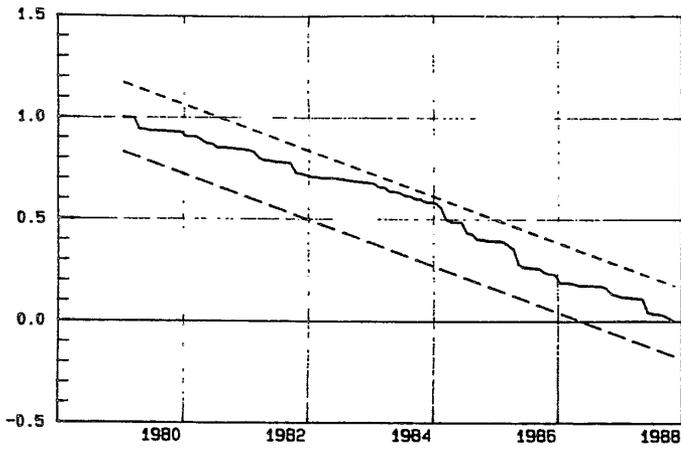


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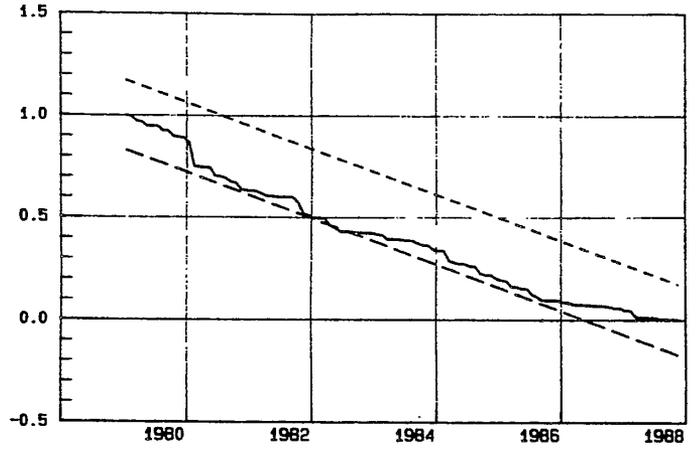


GRAPH 7. CUSUMSQUARE TEST ON CONSTANCY OF VARIANCES OF REAL RETURNS, January 1980 - November 1988

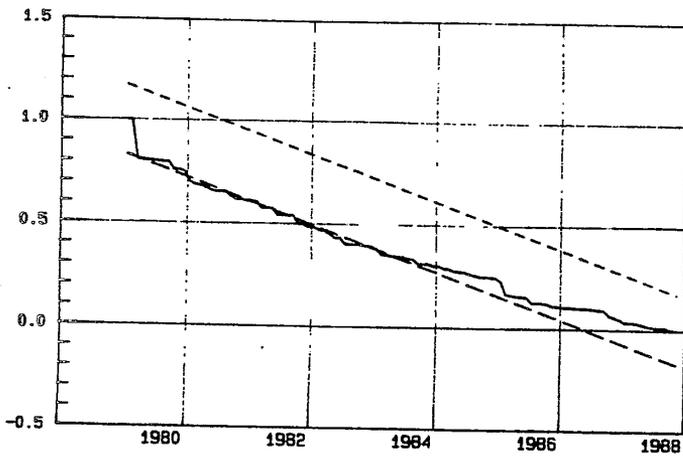
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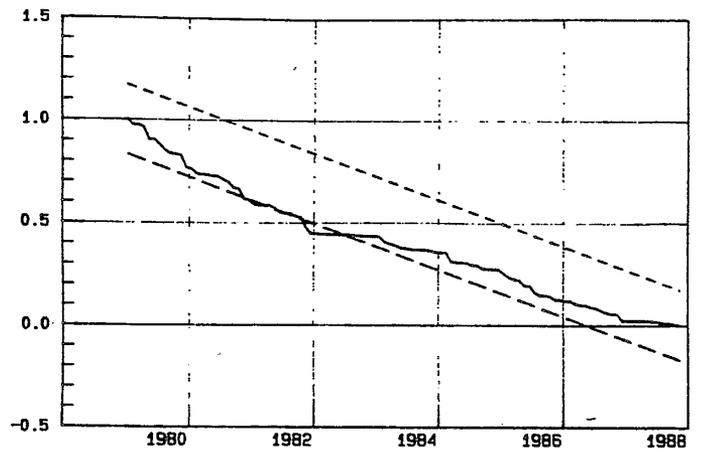
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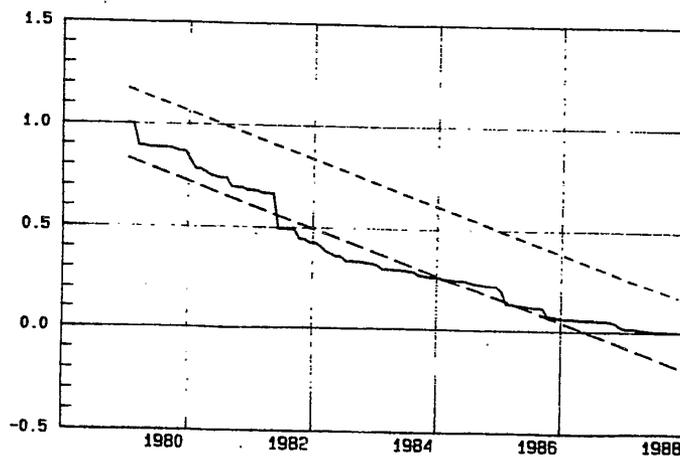
*DEM*



*JPY*

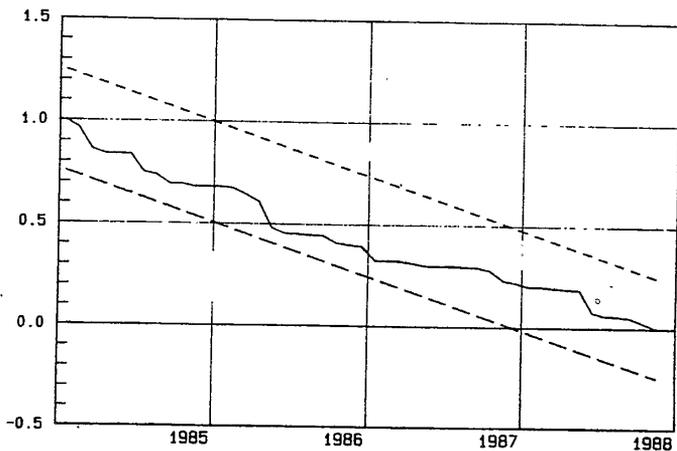


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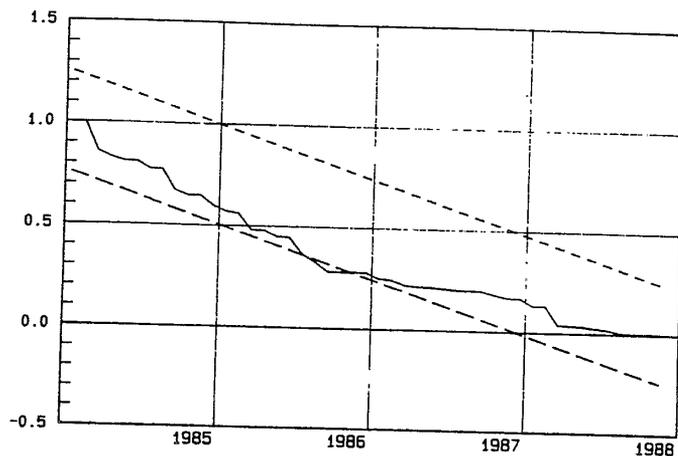


GRAPH 8. CUSUMSQUARE TEST ON CONSTANCY OF VARIANCES OF REAL RETURNS, January 1985 - November 1988

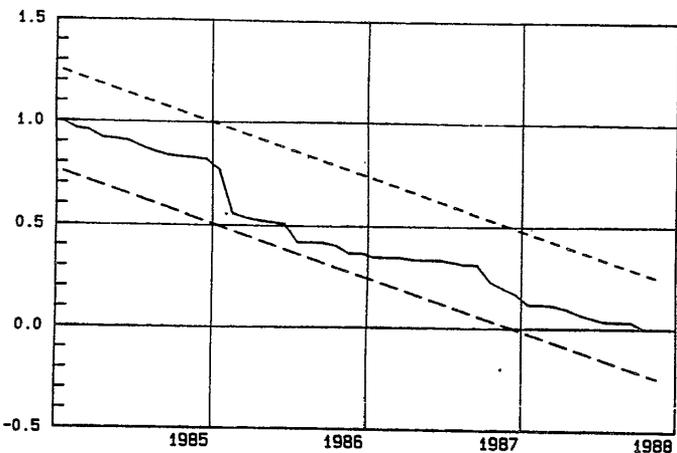
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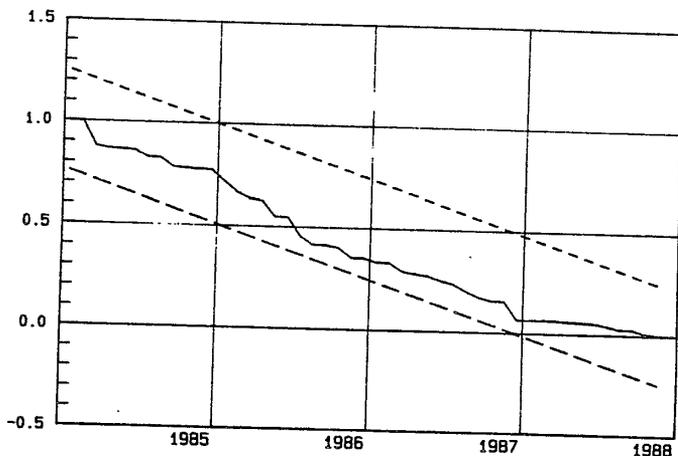
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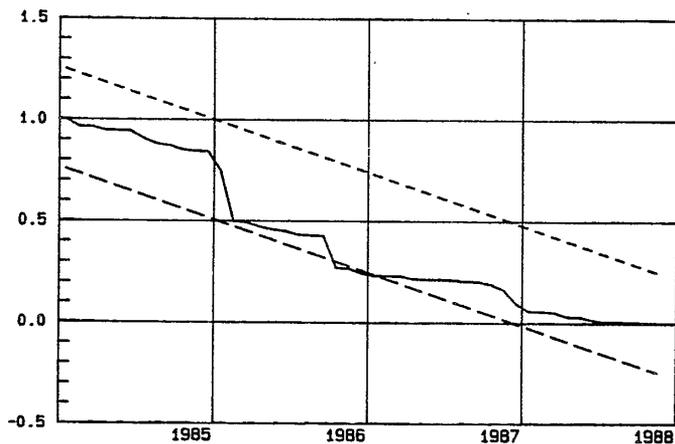
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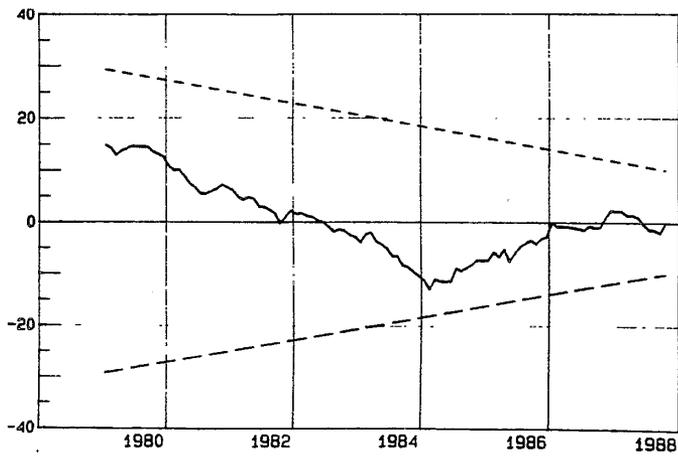
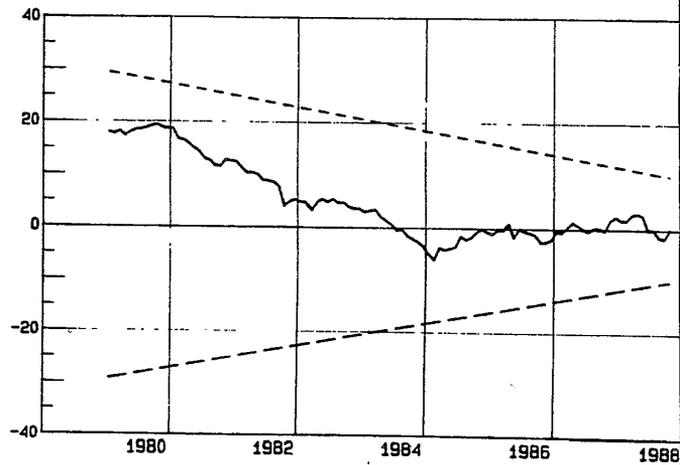
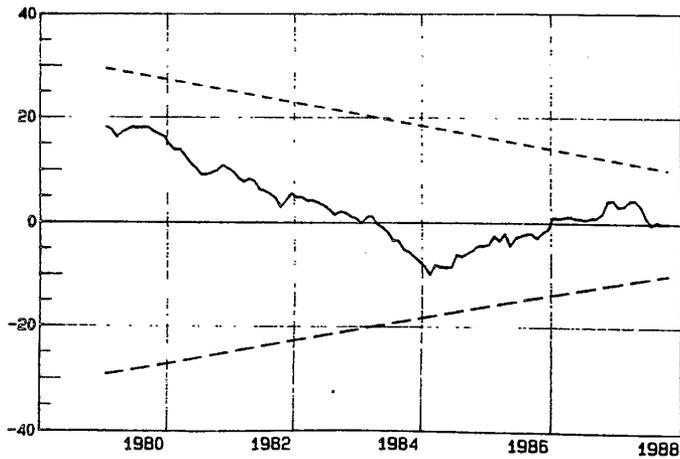
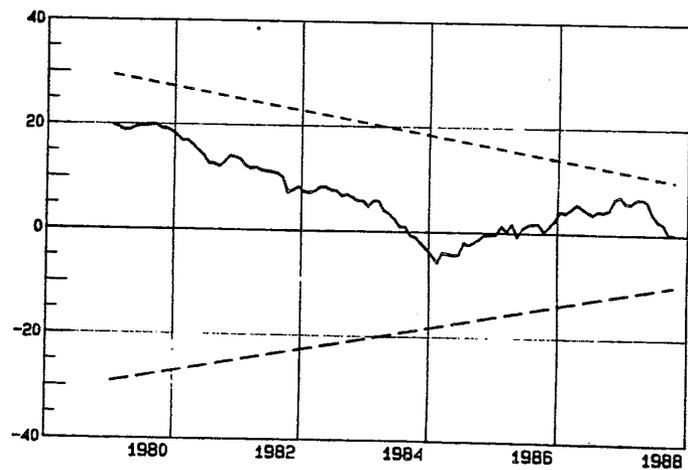
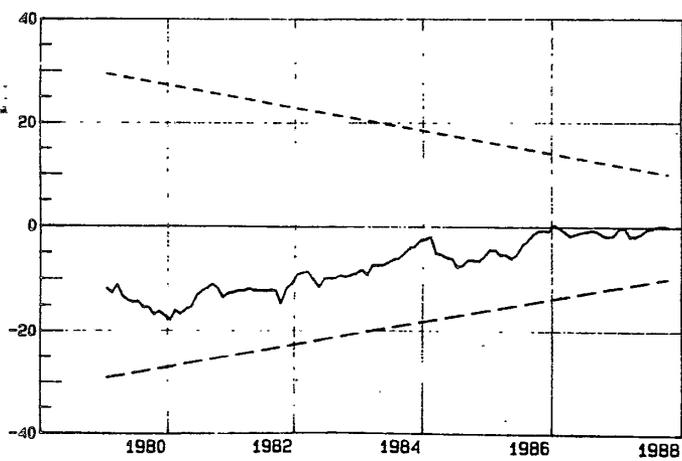
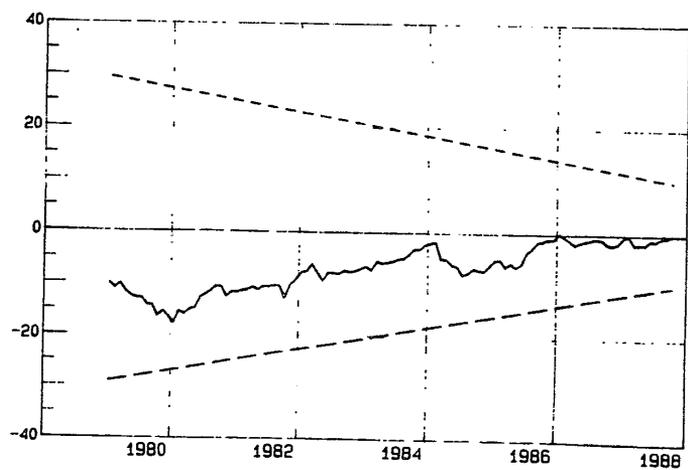
*JPY*



*FRF*



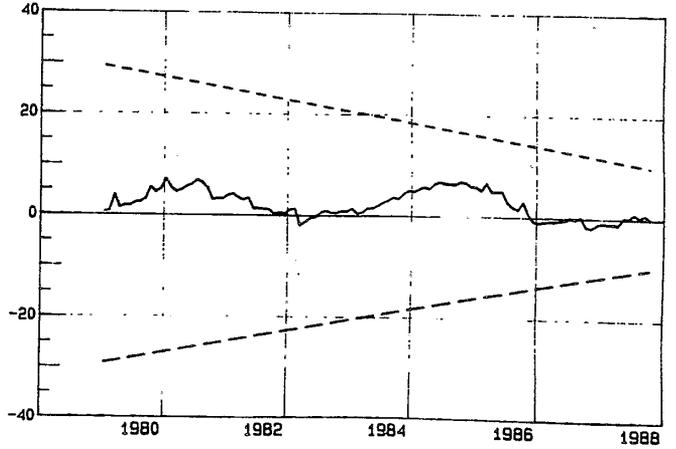
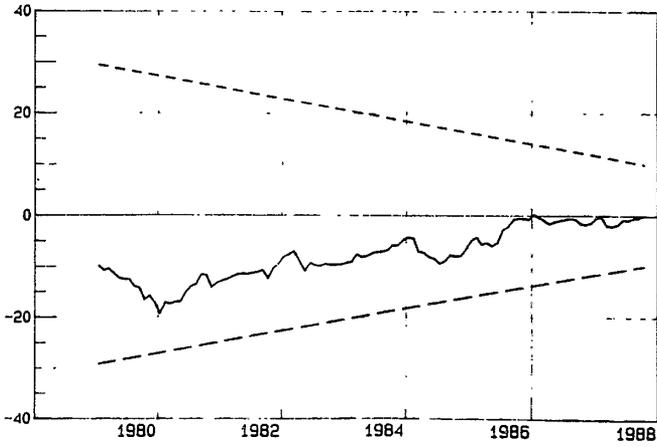
GRAPH 9. CUSUM TEST ON CONSTANCY OF COVARIANCES OF NOMINAL RETURNS

*GBP.USD**DEM.USD**JPY.USD**FRF.USD**DEM.GBP**FRF.GBP*

GRAPH 9. cont.

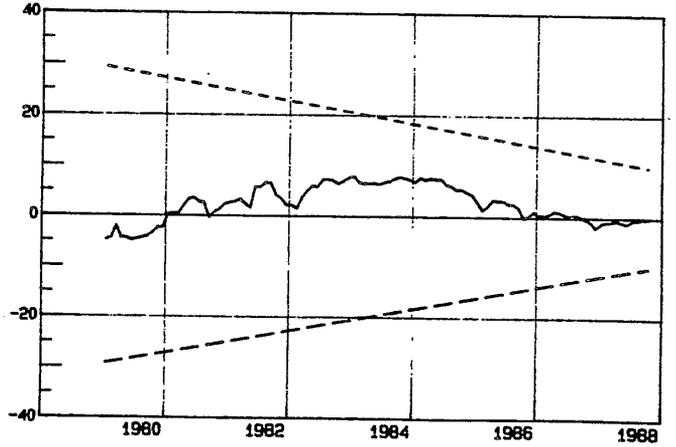
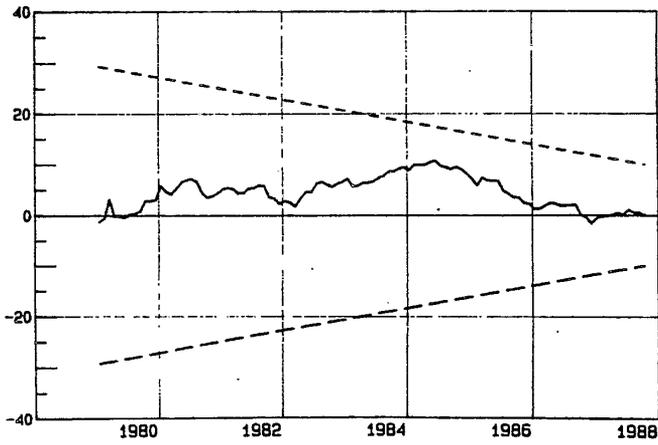
*JPY.GBP*

*FRF.DEM*

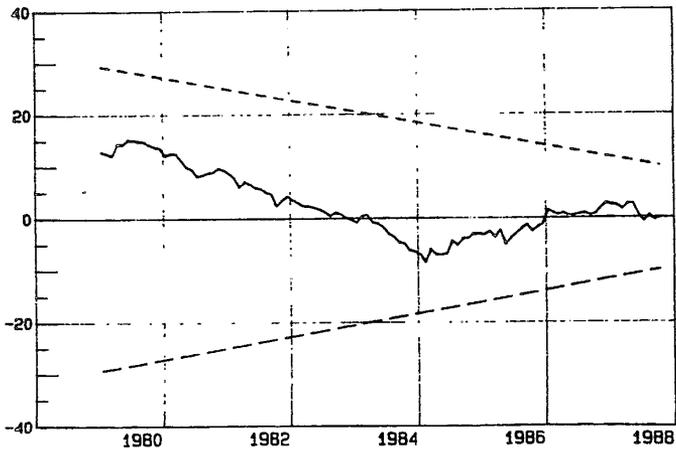
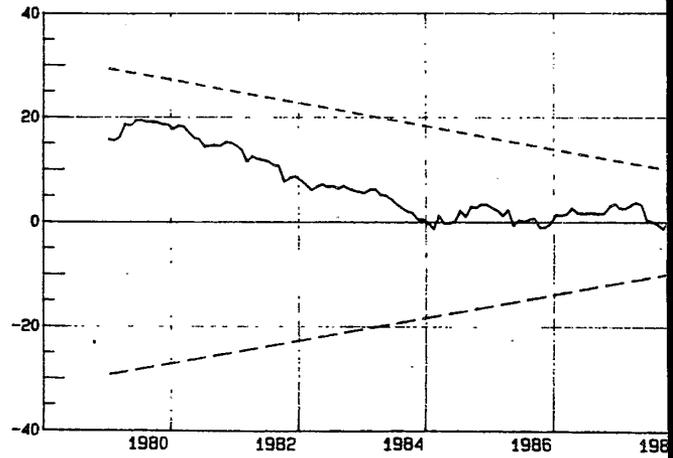
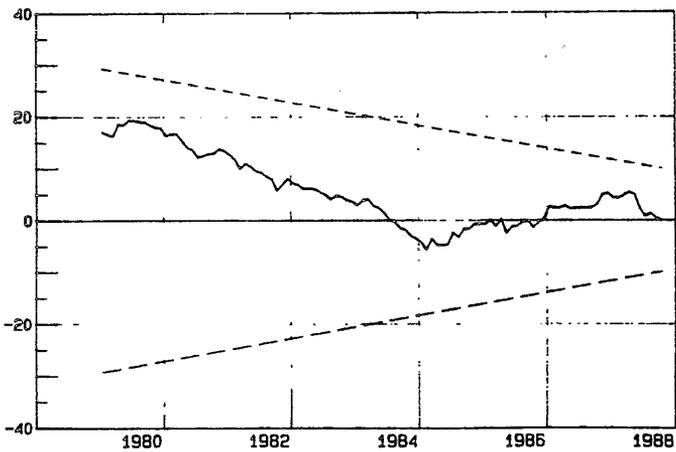
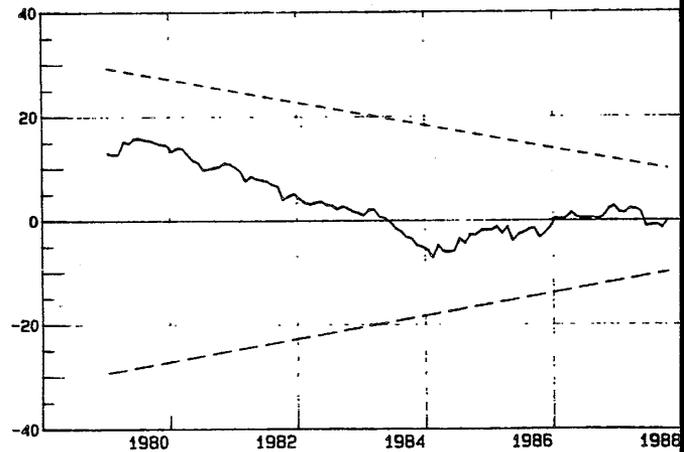
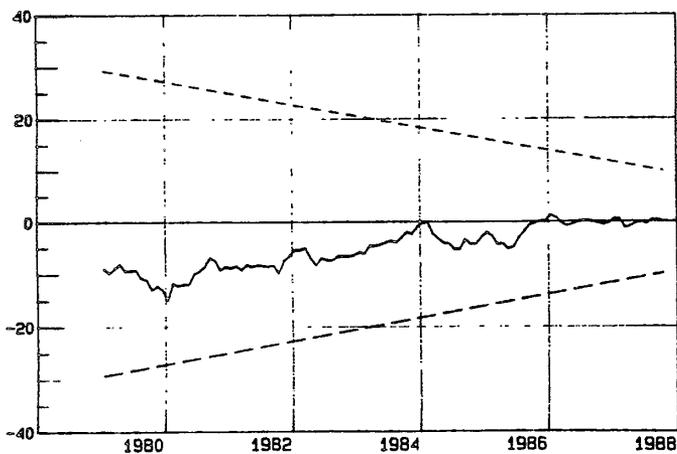
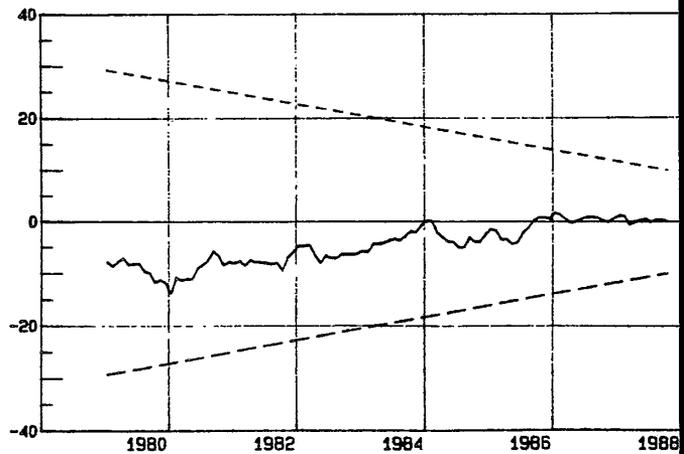


*JPY.DEM*

*JPY.FRF*

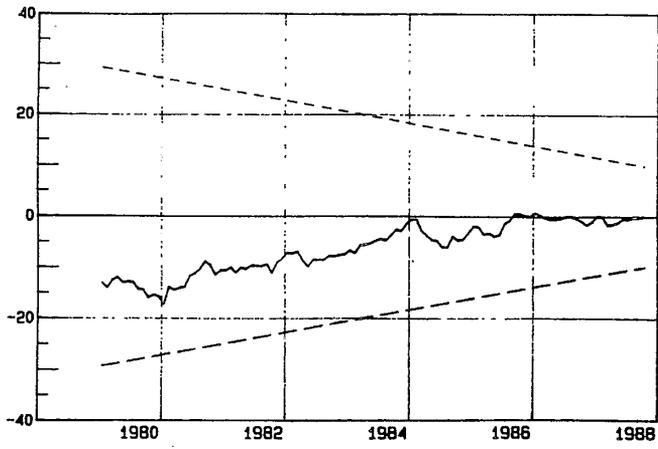


GRAPH 10. CUSUM TEST ON CONSTANCY OF COVARIANCES OF REAL RETURNS

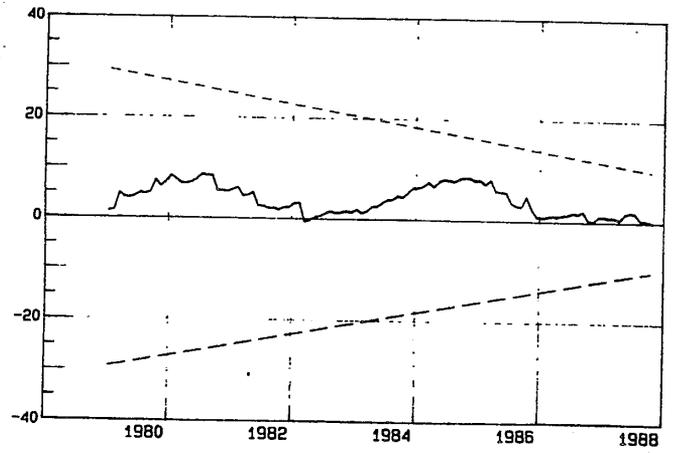
*GBP.USD**DEM.USD**JPY.USD**FRF.USD**DEM.GBP**FRF.GBP*

GRAPH 10. cont.

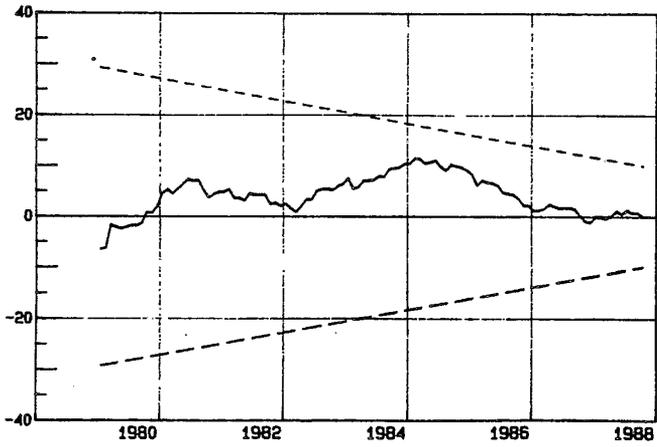
*JPY.GBP*



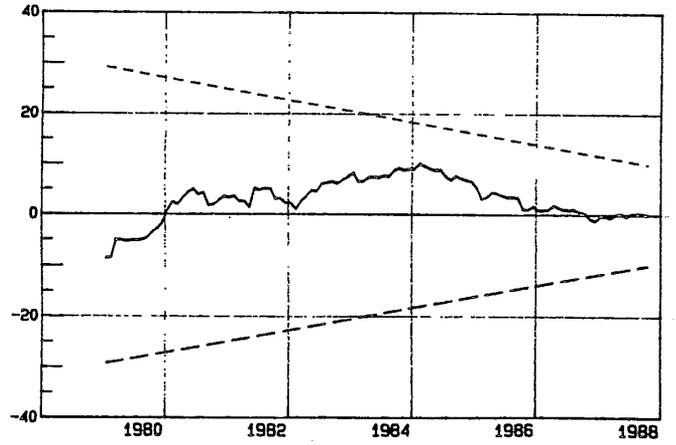
*FRF.DEM*



*JPY.DEM*



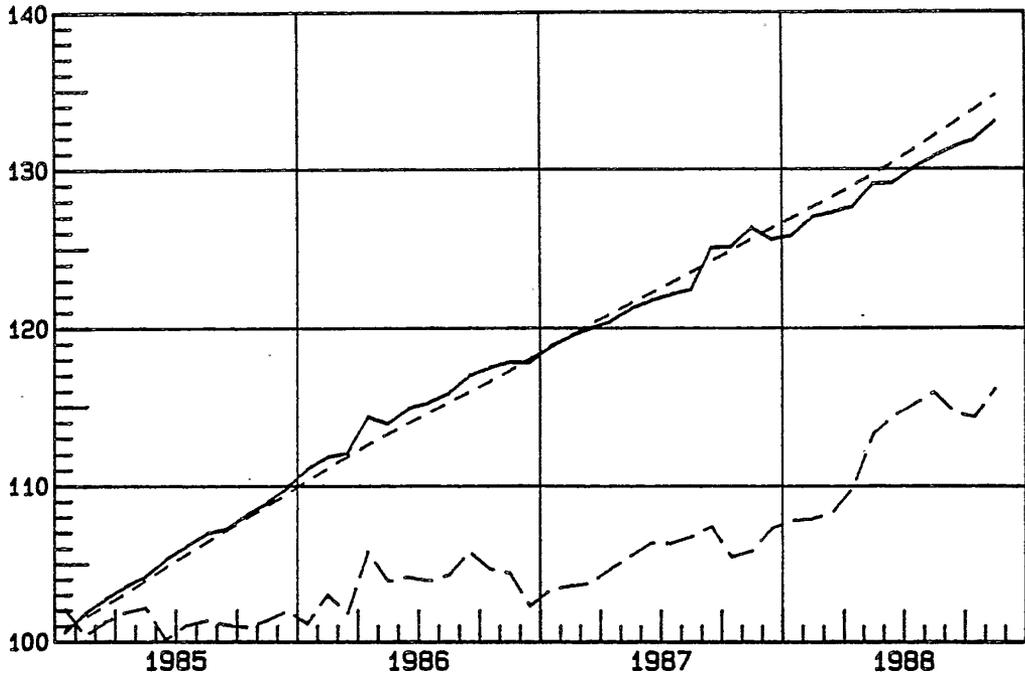
*JPY.FRF*



GRAPH 13. CUMULATIVE IN-SAMPLE NOMINAL RETURNS OF A MINIMUM VARIANCE PORTFOLIO,<sup>1</sup> SDR-WEIGHTED, AND CURRENCY BASKET WEIGHTED PORTFOLIOS

MINIMUM VARIANCE PORTFOLIO  
 CURRENCY BASKET PORTFOLIO  
 PORTFOLIO WITH SDR WEIGHTS

—————  
 - - - - -  
 - - - - -

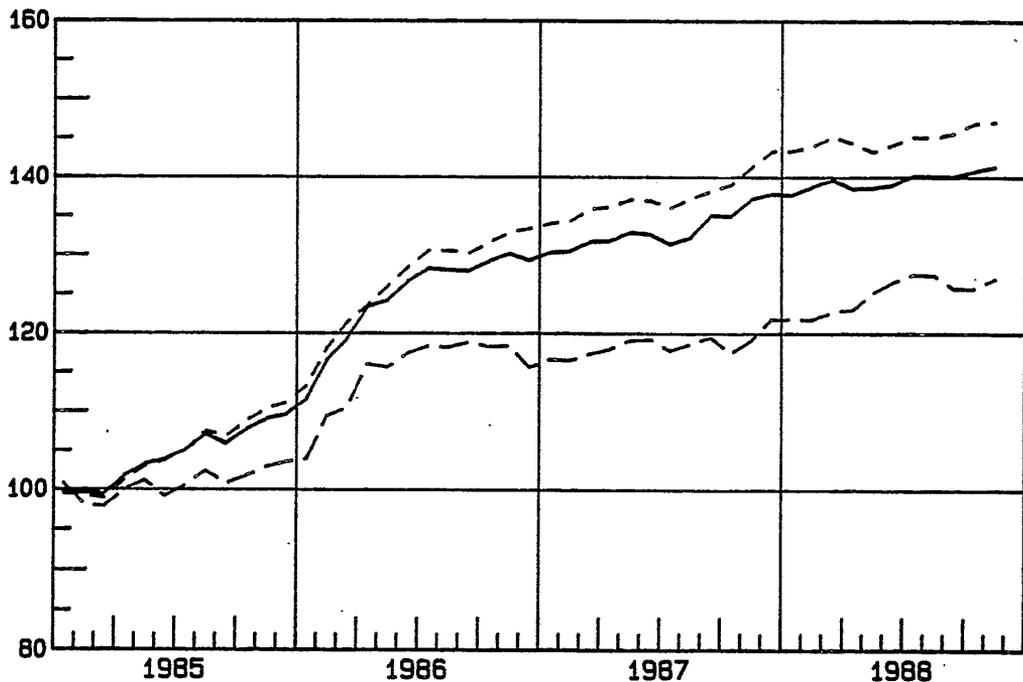


<sup>1</sup> Covariance matrix from January 1985 to November 1988, constant risk premium hypothesis, weights: USD 12.5, GBP 20.6, DEM 45.8, FRF 13.9, JPY 7.3. SDR weights: USD 42, GBP 12, DEM 19, FRF 12, JPY 15; and currency basket weights (for the weights see section 1.5).

GRAPH 14. CUMULATIVE IN-SAMPLE REAL RETURNS OF A MINIMUM VARIANCE PORTFOLIO,<sup>1</sup> SDR-WEIGHTED, AND CURRENCY BASKET WEIGHTED PORTFOLIOS

MINIMUM VARIANCE PORTFOLIO  
 CURRENCY BASKET PORTFOLIO  
 PORTFOLIO WITH SDR WEIGHTS

—————  
 - - - - -  
 - - - - -

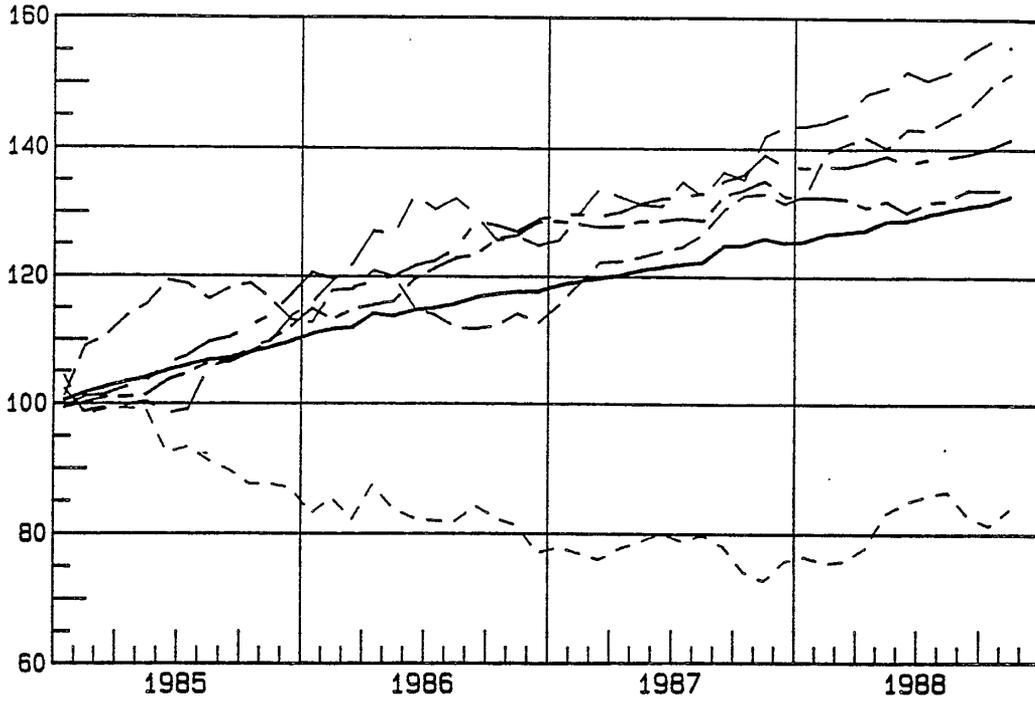


<sup>1</sup> Covariance matrix from January 1985 to November 1988, constant risk premium hypothesis, weights: USD 17.6, GBP 23.0, DEM 49.9, FRF 3.1, JPY 6.3. SDR weights: USD 42, GBP 12, DEM 19, FRF 12, JPY 15; and currency basket weights (for the weights see section 1.5).

GRAPH 15. CUMULATIVE IN-SAMPLE NOMINAL RETURNS OF A MINIMUM VARIANCE PORTFOLIO, AND THE INDIVIDUAL CURRENCIES IN THE PORTFOLIO (CONSTANT RISK PREMIUM HYPOTHESIS, COVARIANCE MATRIX FROM JANUARY 1985 TO NOVEMBER 1988)<sup>1</sup>

MINIMUM VARIANCE PORTFOLIO

USD  
GBP  
DEM  
FRF  
JPY

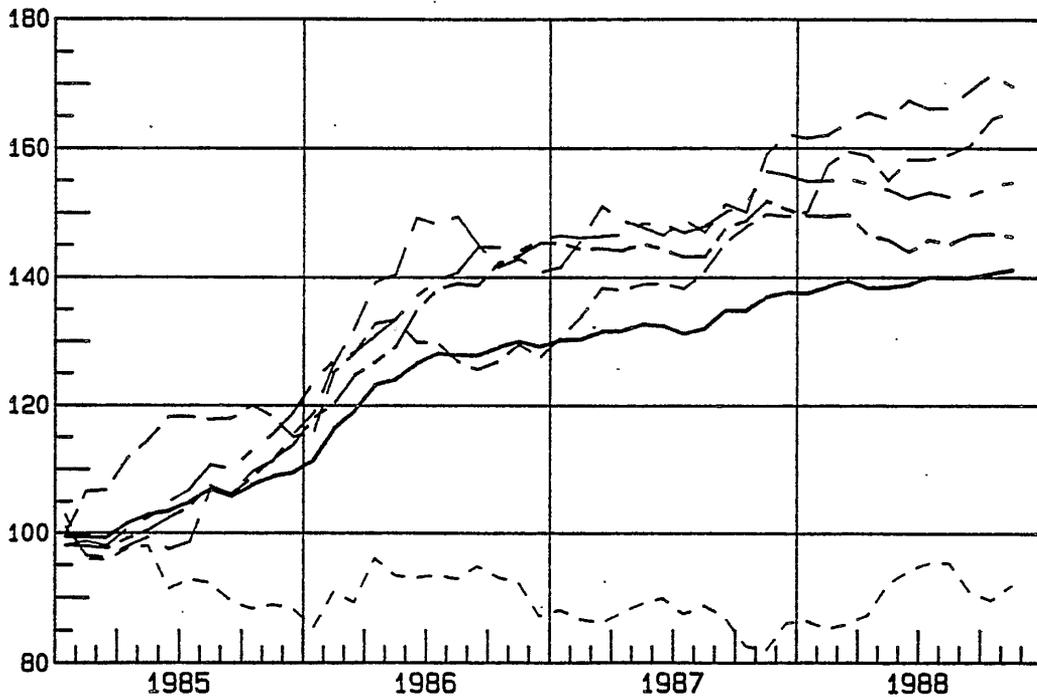


<sup>1</sup> Portfolio shares: USD 12.5, GBP 20.6, DEM 45.8, FRF 13.9, JPY 7.3.

GRAPH 16. CUMULATIVE IN-SAMPLE REAL RETURNS OF A MINIMUM VARIANCE PORTFOLIO, AND THE INDIVIDUAL CURRENCIES IN THE PORTFOLIO (CONSTANT RISK PREMIUM HYPOTHESIS, COVARIANCE MATRIX FROM JANUARY 1985 TO NOVEMBER 1988)<sup>1</sup>

MINIMUM VARIANCE PORTFOLIO

USD  
GBP  
DEM  
FRF  
JPY



<sup>1</sup> Portfolio shares: USD 17.6, GBP 23.0, DEM 49.9, FRF 3.1, JPY 6.3.

GRAPH 17. CUMULATIVE OUT-OF-SAMPLE NOMINAL RETURNS OF A MINIMUM VARIANCE PORTFOLIO (SOLID LINE) AND A SDR-WEIGHTED PORTFOLIO (DASH LINE),<sup>1</sup> INITIAL INVESTMENT IN JANUARY 1987, AND REINVESTED AT THE BEGINNING OF EACH QUARTER

MINIMUM VARIANCE PORTFOLIO  
PORTFOLIO WITH SDR WEIGHTS

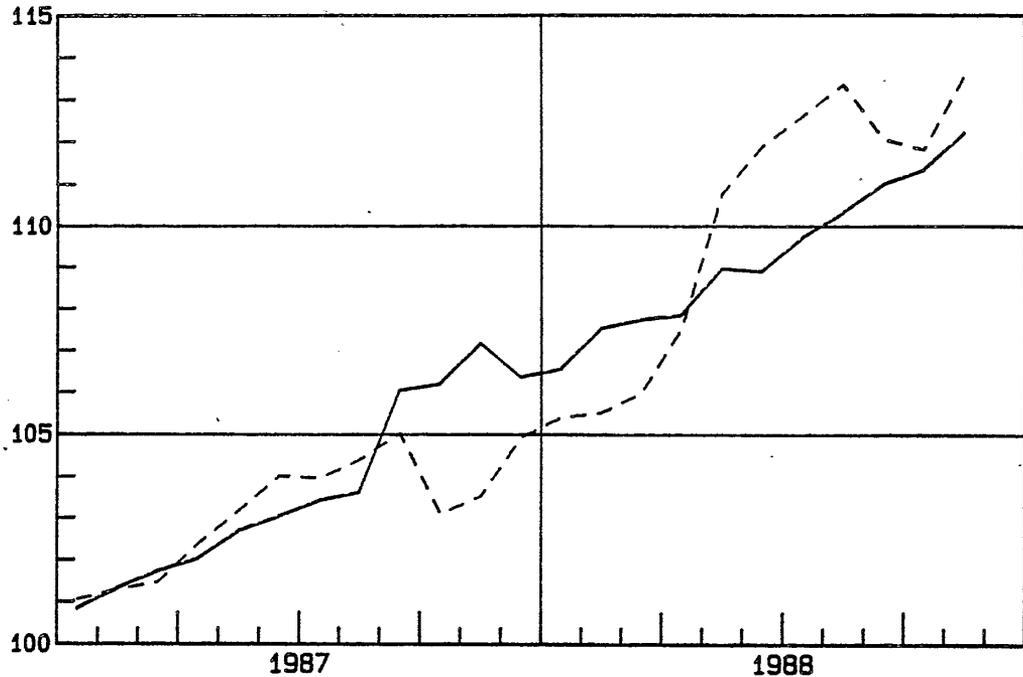
$$\sigma^2 = 0.317$$

$$\sigma^2 = 0.900$$

$$s = 0.563$$

$$s = 0.949$$

—————  
- - - - -



<sup>1</sup> Minimum variance portfolio weights (covariance matrix July 1984 to December 1986): USD 10.9, GBP 20.1, DEM 54.7, FRF 7.4, JPY 6.9 (constant risk premium hypothesis); SDR weights: USD 42, GBP 12, DEM 19, FRF 12, JPY 15.

GRAPH 18. CUMULATIVE OUT-OF-SAMPLE REAL RETURNS OF A MINIMUM VARIANCE PORTFOLIO (SOLID LINE) AND A SDR-WEIGHTED PORTFOLIO (DASH LINE),<sup>1</sup> INITIAL INVESTMENT IN JANUARY 1987, AND REINVESTED AT THE BEGINNING OF EACH QUARTER

MINIMUM VARIANCE PORTFOLIO  
PORTFOLIO WITH SDR WEIGHTS

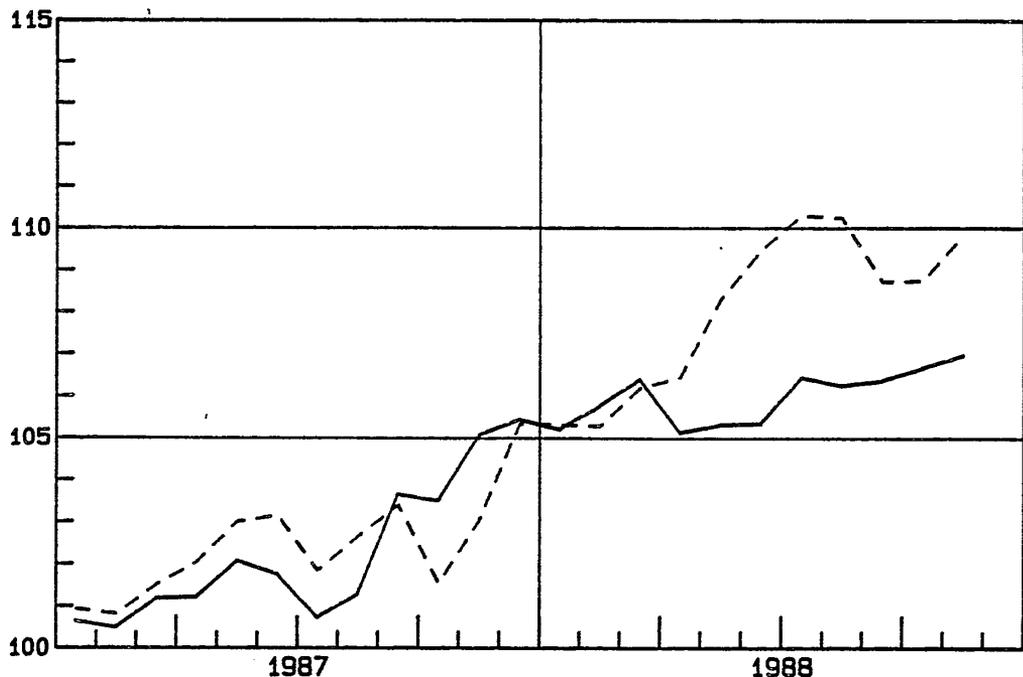
$$\sigma^2 = 0.586$$

$$\sigma^2 = 0.948$$

$$s = 0.766$$

$$s = 0.974$$

—————  
- - - - -



<sup>1</sup> Minimum variance portfolio weights (covariance matrix July 1984 to December 1986): USD 17.2, GBP 17.7, DEM 66.6, FRF -6.0, JPY 4.5 (constant risk premium hypothesis); SDR weights: USD 42, GBP 12, DEM 19, FRF 12, JPY 15.

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