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PRODUCTION AND EMPLOYMENT IN THE BOF4 QUARTERLY MODEL OF THE FINNISH ECONOMY

ABSTRACT

This paper contains a description of the production and labour demand sections of the BOF4 model. CES value added production functions are developed and estimated for the five sectors of the economy. From these the marginal product of labour and capital functions are derived which are important factors in determining investments and the prices of gross outputs in each of the sectors. The demand for labour hours is defined as a gradual adjustment process towards the level determined by the inverted production functions. An input-output model is used to determine output and value-added deflators by sectors as well as the prices of the different demand components.

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1 INTRODUCTION

The BOF4 model of the Finnish economy is a quarterly econometric model developed at the Bank of Finland for forecasting and policy analysis. The present paper appears in a series intended to cover all sectors of the present version of BOF4. This report describes a part of the supply side of the model, namely the production functions and technology, the determination of sectoral outputs and labour demand. Exports and imports have been described in Tarkka and Willman (1988). Consumption and investment, the rest of the supply side i.e. labour supply, wages and prices, the government sector and the financial markets will be described in forthcoming reports.

First, the technological assumptions of the model and the estimation of the sectoral CES-production functions are described. Production functions are used to calculate the marginal costs of production and the marginal products of fixed capital, which drive the price and investment equations of the model. Second, we describe the use of an input-output model to determine the relation between production prices and value-added deflators on the other hand and output by sectors on the other. Inverted production functions are used to solve for required labour inputs. The modelling of the demand for work hours and the determination of employment consists the third part of the paper.

¹The earlier version of the model, BOF3, is documented in Tarkka and Willman (1985).

2 PRODUCTION FUNCTIONS AND TECHNOLOGY

2.1 The General Modelling Strategy

In BOF4, production is disaggregated into the following five sectors:

Table 1

Sector	Code in BOF4	1985 share in GDP
- Agriculture ' - Forestry - Manufacturing - Private services etc Government services	1 3 4 2 G	4.4 % 3.7 % 25.4 % 49.8 % 16.8 %

Manufacturing includes mining, and the private services sector includes construction, electricity, gas and water sectors. Fishing is included in agriculture.

For each sector we have estimated a production function. These production functions are not directly parts of the model, but they are used in the derivation of the factor demand equations (i.e. investment and work hours equations) and in estimating marginal costs of production; the marginal costs in turn enter the pricing equations. This two-stage strategy of modelling the supply side helps to ensure the consistency of the the equations in the model with each other and with the chosen technology assumptions.

2.2 The CES Value Added Production Functions

The production functions are assumed to be "nested" in the sense that labour and capital are separable from the other inputs. This allows us to model the production of real value added in the different sectors as functions of labour and capital inputs only. The value added production functions are of the "CES" type with Hicks-neutral technical progress and constant returns to scale:

(1)
$$Q = A \cdot \exp(\gamma \cdot TREND) \cdot (a \cdot K^{-\rho} + (1-a) \cdot L^{-\rho})^{-1/\rho}$$

Here Q is the real value added originating from the sector in question, K is the stock of fixed capital and L is the labour input (measured in hours worked). The impact of technical progress is captured by the inclusion of the linear time trend TREND. A, γ , a and ρ are parameters; of these, γ measures the rate at which technical progress increases the efficiency of production and ρ governs the elasticity of substitution between labour and capital, a indicates the weighting of capital and labour and A is a factor reflecting the scale of operation.

Although the functional form is assumed to be the same for each of the five sectors of the model, separate parameter values are estimated for each of them. For the agricultural sector the CES function was estimated directly by nonlinear least squares (in a form solved for the labour/capital ratio). For the other sectors, we have used a two-step procedure broadly similar to that described in Klein and Bodkin (1967). This method circumvents the problem of multicollinearity by assuming that the observed labour share in the value added is consistent with the cost-minimizing optimum. This allows us to write the following loglinear regression equation for each sector:

$$\log((PQ-WL)/WL) = \log(a/(1-a)) - \rho \cdot \log(K/L)$$

Here WL denotes labour costs (wage bill plus employers' social security payments) and PQ-WL is the gross operating surplus of the

sector in question (total value added at factor cost minus labour costs).

From the regression coefficients of the above equation we can solve for parameters a, ρ and, further, the elasticity of substitution $s=1/(1+\rho)$. If ρ , the coefficient of the capital-labour ratio in equation (2) were zero, the factor shares explained by the equation would be independent of the capital-labour ratio and the technology would be of the Cobb-Douglas type (with unit elasticity of substitution). The regression (2) is hence a kind of test of the CES specification against the more restrictive Cobb-Douglas specification.

After solving for the estimates of a and ρ , these are substituted into the original production function (1) and the scale parameter A and the rate of technical progress γ are estimated by loglinear least squares. After this second step, we have obtained all of the required parameter estimates.

The estimation was carried out using annual data for the period 1960 - 1985. The factor share data was adjusted for the labour input of the self-employed persons so that the wage bill was multiplied by the ratio of total hours worked to the hours worked by employees. In estimating the CES production function for the private services sector, the housing sector (letting and operating of dwellings and use of owner-occupied dwellings) was deducted from both the value added and the capital stock concept.

A comprehensive account of the estimation results is given in appendix 1. The most important characteristics of the estimated production functions are the elasticities of substitution and the rates of technical progress. These are given in the following table:

Table 2

Sector	elasticity of substitution	rate of technica progress (% p.a.				
	$s = \frac{1}{1+\rho}.$	Υ				
- Agriculture - Forestry - Manufacturing	0.97 1.48 0.88	0 0.3 3.0				
- Private services etc Government services	0.62 0.51	3.2 0.7				

The production functions which are actually used in BOF4 are quarterly versions of the estimated annual production functions. The conversion of annual CES functions into quartely ones is done by multiplying the capital stock by four in the estimation of the annual functions. The finally obtained quarterly functions are:

Agriculture:

(3) GDP1 =
$$0.424 \cdot (0.6663 \cdot \text{KF1}^{-.028} + 0.3337 \cdot \text{LH}^{-.028})^{(-1/.028)}$$

Forestry:

(4) GDP3 =
$$9.042 \cdot e^{.003 \cdot TREND} (0.1611 \cdot KF3^{.326} + 0.8389 \cdot LH3^{.326})^{(1/.326)}$$

Manufacturing:

(5) GDP4 =
$$2.268 \cdot e^{.031 \cdot TREND} (0.5535 \cdot KF4^{-.143} + 0.4465 \cdot LH4^{-.143})^{(-1/.143)}$$

Private services etc. (excl. housing):

(6) GDP2 =
$$1.073 \cdot e^{.032 \cdot TREND} (0.8882 \cdot KF2^{-.605} + 0.1118 \cdot LH2^{-.605})^{(-1/.605)}$$

Government services:

(7) GDPG =
$$0.807 \cdot e^{.007 \cdot TREND} (0.9864 \cdot (KFCG + KFLG)^{-.978} + 0.0136 \cdot LHG^{-.978})^{(-1/.978)}$$

See appendix 3 for a list of symbols.

From these functions it is straightforward to derive the inverted production functions (the labour requirement (LH $_i$) functions), the marginal product of labour (MPL $_i$), the marginal product of capital (MPK $_i$), and the marginal cost of production (SMC $_i$) functions used in BOF4.

The marginal product of labour implied by the equation (1), can be written as

(8)
$$\partial Q/\partial L = (1-a) \cdot A \cdot \exp(\gamma \cdot TREND) \cdot [a \cdot (K/L)^{-\rho} + (1-a)]^{-(\rho+1)/\rho}$$

This formula is used in developing the sectoral short-run marginal cost of production functions. These are of the form

(9)
$$SMC = (1-a_Q) \cdot P^m + (a_Q) \cdot W/(\partial Q/\partial L),$$

where P^{m} is the weighted sum of input prices from other sectors and raw materials and the parameter a_{Q} describes the division of costs between material inputs and value added. The marginal product of labour (8) could also be expressed in terms of Q/L on the right hand side. This form was actually used in the earlier versions of the SMC functions. (See BOF4: Equations, TU 6/87). Having production in the denominator of the SMC function reduces costs (and prices) in the short run when GDP rises because labour hours react slowly to changes in production. This feature was regarded as undesirable in simulations and so the MPL; functions were written in the form (8).

The marginal product of capital derived from equation (1) is

(10)
$$(\partial Q/\partial K) = a \cdot A^{-\rho} \cdot \exp(-\rho \cdot \gamma \cdot TREND) \cdot (\frac{GDP}{KF_{-1}})^{\rho+1}.$$

Using the parameters estimated from the production functions we can calculate time series for (10) to be used in the investment equations.

2.3 The Leontief Outer Production Functions

In the technology assumption used in the BOF4 model, the CES value added production functions are nested in a Leontief outer or "gross" production function in each sector of production. This means that the proportions of real value added in real output are assumed to be fixed, as are the proportions of non-primary inputs - meaning imports and purchases from other sectors used in production. These assumptions make it possible to use an input-output table to evaluate the cost structure of the different sectors in the model.

Since gross outputs and intersectoral flows of goods are not explicitly included in the model as variables, the outer production functions are not needed as such. Rather, the Leontief functions assigned for each sector are used in the actual model structure in two ways.

First, the cost functions implied by the outer production functions are used to determine the relationships between producer prices and value added deflators. This is possible because the knowledge of input coefficients enables us to compute the share of the producer price needed to cover costs of non-primary inputs and so the value added deflator is "residually" determined. This is explained more closely in the report describing the modelling of prices and wages in BOF4 (forthcoming; see also Tarkka & Willman (1985)).

Second, the information on input coefficients is used in the determination of the value added by sectors. This is explained in more detail in the next section. The idea is familiar from

input-output models: assume that the structure of aggregate demand is known, and also that each component of aggregate demand uses gross output of the different sectors in fixed proportions. This makes it possible to calculate the gross outputs of different sectors used to satisfy final demand. However, since gross outputs are also used as intermediate inputs in production, information on the input coefficients is needed before the gross output and the real value added produced in each sector can be solved for.

In estimating the input coefficients the Finnish input-output table of 1982 is used. The results are shown in the following table, where

a ₁	is	the	coefficient	of	inputs	from	agriculture,
a3					inputs	from	forestry,
a4					inputs	from	manufacturing,
a2					inputs	from	the private service sector,
aĞ							the govt. services sector,
aMR					importe	d rav	w materials (excl. fuels),
aMF					importe	d fue	els,
aQ					capital	and	labour (i.e. value added).

Table 3

The Input Coefficients in 1982.

		Sec	ctor:		
	Agric. (1)	Forestry (3)	Manufact. (4)	Priv.serv. (2)	Govt. (G)
a1	0.0648	0.0096	0.0652	0.0014	0.0000
ag	0.0079	0.0307	0.0393	0.0013	0.0000
ад	0.2684	0.0223	0.3004	0.1413	0.0000
a ₂	0.1132	0.0307	0.1082	0.2582	0.0000
ag	0.0000	0.0000	0.0000	0.0000	0.0000
aMR	0.0359	0.0074	0.1115	0.0495	0.0000
aMF	0.0095	0.0148	0.0700	0.0148	0.0000
aQ	0.5003	0.9200	0.3054	0.5335	1.0000
Sum	1.0000	1.0000	1.0000	1.0000	1.0000
	···				

The cost functions in the different sectors have the following form:

(11)
$$C = X \cdot (a_1 \cdot P1 + a_3 \cdot P3 + a_4 \cdot P4 + a_2 \cdot P2 + a_6 \cdot PG + a_{MR} \cdot PMR + a_{MF} \cdot PMF + a_0 \cdot PGDP),$$

where C is the total cost of production, X is gross output, a_i:s are input-output coefficients, P1, P3, P4, P2, PG, PMR and PMF are the prices of respective inputs and PGDP is the value added deflator. This is the form of the total cost functions. It is useful mainly because it can be inverted to a formula for the value added deflator.

When evaluating the relevant marginal cost of production, a short-run variable cost concept is needed instead of the total cost given by (11). If the stock of fixed capital is treated as predetermined at each instant, the short-run marginal costs do not include gross operating surplus. Hence the short-run marginal costs can be expressed by eq. (9). These matters are discussed in more detail in a later report of the modelling of prices.

3 THE SUPPLY SIDE: PRODUCTION AND EMPLOYMENT

The supply side in BOF4 consists of two types of equations. On the one hand, the production and employment equations determine how a given aggregate demand is satisfied by production taking place in the different sectors of the economy. On the other hand, the labour supply, wage and price equations describe the behavioural aspects of supply, namely the decisions of workers, unions and firms. The first, technological aspect of the supply side in BOF4 is discussed in the present chapter.

3.1 The Determination of Output by Sectors

The determination of the sectoral composition of real value added starts from the composition of aggregate demand given by the equations on the demand side of the model. The structure of demand is converted to the corresponding structure of output and indirect taxes (less subsidies) by means of an input-output model. Estimates of this conversion matrix (except for inventories) are based on the input-output coefficient matrix presented in table 4.

Note that general government is a peculiar production sector in that its output is used only for public consumption. This implies that no other demand component but public consumption affect, directly or indirectly, general government production. Equation Q.6 (in appendix 2), which defines the production of general government sector (equalling its value-added), states that 67 per cent of the the increase in public consumption is directed at the production of general government sector.

¹A more profound description of such a model can be found in Tarkka & Willman (1985), pp. 156 - 158.

Equations Q.1 - Q.5 convert the rest of the final demand and imports of inputs into value-added of four private sectors and into indirect taxes less subsidies. Denote the conversion matrix contained by these equations by B. Now, after cancelling the row of general government production and the column of public consumption originating from the government sector in table 4, the matrix B can be expressed in terms of coefficients presented in table 4 as follows:

$$B = \begin{bmatrix} F^{d}(I_{4}-A)^{-1}D(I_{10}+\gamma^{d})^{-1} \\ [\pi(I_{4}-A)^{-1}D+\gamma](I_{10}+\gamma^{d})^{-1} \end{bmatrix}$$

where

F is the row vector of value added coefficients

A is the matrix of inter industry input coefficients

D is the matrix of the coefficients of imported inputs and final demand components

γ is the row vector of idirect tax coefficients levied on production

 π is the row vector of indirect tax coefficients levied on imported inputs and final demand

I4 is the four by four identity matrix

I₁₀ is the ten by ten identity matrix

and superscript d is used to denote diagonal matrixes formed by row vectors F and γ_{\bullet} .

However, as to the conversion of the change in inventories into sectoral production and indirect taxes, the input-output estimates obtained by this way were not accepted. Estimates based on the input-output study are point estimates of a single year (in this case 1982), and can deviate, especially in the case of inventories, quite considerably from their longer-run averages. That is why the parameters of this row were estimated by using the OLS method.

Table 4.

The matrix of input-output coefficients in 1982

Bying Industries

Final Demand

	agric.	priv.	forest.	manuf.	govt.	imported	pı	iv. comsump	tion	public c	onsumpt.	inve	stment	ex	ports	Change in
Selling Industries	(1)	services (2)	(3)	(4)	(G)	inputs	durabl.	non-dur.	services	priv. origin	govt. origin	prod. inv.	housing inv.	goods	services	inven- tories
agriculture (1)	.0766	.0016	.0095	.0769	.0000	0478	.0000	.0626	.0002	.0038	.0000	.0000	.0000	.0192	.0000	2.5554
priv. services (2)	.1096	.2730	.0306	.1141	.0000	1191	.4427	.4178	.9938	.5956	.0000	.8738	1.0000	.0000	1.0000	2.0997
forestry (3)	.0076	.0013	.0056	.0439	.0000	0151	.0000	.0160	.0003	.0104	.0000	.0087	.0000	.0017	.0000	.6156
manufacturing (4)	.2901	.1740	.0340	.4710	.0000	8180	.5573	.5035	.0057	.3902	.0000	.1175	.0000	.9791	.0000	-4.2707
government (G)	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	1.0000	.0000	.0000	.0000	.0000	.0000
indirect taxes minus subsidies	.0316	.0357	.0039	0162	.0000	.0000	.1370	.2513	0036	.0486	.0000	.0423	.0197	0228	.0197	4715
value added	.4845	.5144	.9164	.3103	1.0000	-	_	-	-	-	-	-	- ;	-	-	-
sum	1.0000	1.0000	1.0000	1.0000	1.0000	-1.0000	1.1370	1.2513	.9964	1.0486	1.0000	1.0423	1.0197	.9772	1.0197	.5285

In calculating the conversion estimates for the imported final goods the following additional assumptions were needed:

- The tax content of imported and domestically produced services is the same.
- Indirect taxes are levied 23.22 per cent and 9.71 per cent of the imports of consumption goods and investment goods, respectively.
- Of the imports of consumption goods 34 per cent is included in the private consumption of durables, 49 per cent in the private consumption of non- and semi-durables and 17 per cent in public consumption.
- All imports of investment goods are included in the private nonresidential investment and public investment.

3.2 The Demand for Labour Hours

In BOF4, two measures of labour input are used: work hours (LH $_{i}$) and paid labour inputs (LW $_{i}$). The latter are defined as the wage bill at constant (base-year) wages.

(12)
$$LW_{i} = 100 \cdot \frac{YW_{i}}{WR_{i}}$$

where LW_i is paid labour inputs, YW_i is the wage bill and WR_i is the wage rate all referring to the sector in question. Since the wage rate indices do not reflect changes in paid holidays nor changes in overtime compensation, paid labour inputs overestimate the real hours worked. Furthermore, work hours include the effort of self-employed persons, whereas paid labour input depends only on "bought" labour. On the other hand, by using the concept of paid labour input the sectoral wage bill can be expressed as the product of the paid labour input and the wage rate, which is more consistent from the point of view of the whole model.

The demand for hours in each production sector is explained by the inverted CES production functions. Thus, the effects of capital

deepening and technical progress on the demand for labour are captured in a way consistent with the technological assumptions of the whole BOF4 model. However, the inverted production function approach is incapable of taking into account the dynamics of labour demand. In particular, the observed procyclical movement of productivity cannot be explained by the inverted production function approach alone. (For a survey of labour demand equations see Fair (1969), also Intriligator (1978) ch. 8.5.)

To overcome this notorious problem, we have allowed a slow adjustment in the demand for work hours towards the equilibrium dictated by the inverted production functions. The adjustment mechanisms are obtained as results of an iterative specification search method, starting form unconstrained dynamics and dropping one by one, those lags of the explanatory variables which were statistically insignificant. The conformity of the equations in the long run with the inverted production function is ensured by the so-called error correction terms.

The dynamic properties of the demand for hours equations are described in the following tables, in which the elasticities of work hours with respect to output and the capital stock are presented. In table 6 the elasticity of labour demand is expressed in terms of the elasticity of productivity holding output constant.

Table 5. The Dynamic Elasticities of Work Hours With Respect to Changes in Output in the BOF4 Model.

sector	immediate	one-year	fifth-year
	elasticity	elasticity	elasticity
agriculture	0.11	0.20	1.05
forestry manufacturing	0.56	0.66	1.97
	0.45	0.64	1.36
priv. services etc. general government	0.45	0.51	1.04
	0.00	0.70	1.08
total economy	0.33	0.54	1.13

Table 6. The Dynamic Elasticities of Productivity With Respect to Changes in the Stock of Fixed Capital in the BOF4 Model.

sector	immediate elasticity	one-year elasticity	fifth-year elasticity
agriculture	0.00	0.06	0.61
forestry	0.00	0.16	1.16
manufacturing	0.00	0.07	0.42
priv. services etc.	0.00	0.03	0.13
general government	0.00	0.05	0.07
total economy	0.00	0.05	0.26

Changes in the sectoral composition of the value added also have an effect on the average labour productivity, since the productivities differ across sectors, as indicated by the following table:

Table 7. The Productivity of Labour in Different Sectors of the BOF4 Model in 1985, (average for the economy = 100)

agriculture forestry manufacturing priv. services etc. general government	34 169 116 108 100	
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After the demand for labour hours is determined in the model, these are used to solve for paid labour inputs. This is done by simple linkage equations (L.6 to L.10 in the list of equations).

3.3 Employment

The equation for employment (see L.12 in appendix 2) determines the number of employed persons on the basis of the total demand for work hours. The elasticity of employment with respect to the demand for hours is given in the following table.

Table 8. The Dynamic Elasticity of Employment With Respect to Changes in the Demand for Work Hours.

	immediate	one-year	fifth-year
	elasticity	elasticity	elasticity
total economy	0.14	0.45	0.95

In the long run the elasticity is constrained to unity. In addition, the equation contains a trend variable which causes a decrease in hours per person at a rate of 0.86 per cent per annum.

A simple linear relationship (L.11) is used to convert the Labour Force Survey statistics to correspond to the SNA concept of employment.

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APPENDIX 1

THE ESTIMATION OF THE CES VALUE ADDED PRODUCTION FUNCTIONS

For sectors 2, 3, 4 and G the following loglinear equation is estimated with annual data. The capital stock is multiplied by four to get parameter estimates equivalent to quarterly equations. (A list of variables is in appendix 3).

(A1)
$$\log((PQ - WL \cdot Z)/WL \cdot Z) = \log(a/(1-a)) - \rho \cdot \log((4 \cdot K/L))$$

Where Z is the correction term to account for the self-employed in sectors 2, 3 and 4. From this the value for a (the distribution parameter) and ρ (the parameter which governs the elasticity of substitution between labour and capital: $s = 1/(1+\rho)$) are solved for and substituted into:

(A2)
$$Q = A \cdot \exp(\gamma \cdot TREND) \cdot [a \cdot (4 \cdot K)^{-\rho} + (1-a) \cdot L^{-\rho}]^{-1/\rho}.$$

Dividing by $[a \cdot (4 \cdot K)^{-\rho} + (1-a) \cdot L^{-\rho}]^{-1/\rho}$ and taking logs gives

(A3)
$$\log(Q/[a\cdot(4K)^{-\rho} + (1-a)\cdot L^{-\rho}]^{-1/\rho}) = \log A + \gamma \cdot TREND$$

From this second stage equation we get estimates for A and γ . Using the estimated parameters for a, ρ , A and γ the final production functions can be written as equations (3) - (7) in the main text. See tables A1 and A3 below.

For the agricultural sector where no technical progress could be identified (γ = 0 in equation A2) the CES value added production

 $^{^{1}}$ See the list of variables in appendix 3 for a more accurate explanation.

function (A2) was estimated directly by nonlinear least squares in a form solved for the labour/capital - ratio:

(A4)
$$L/K = \left[\frac{A^{\rho}}{(1-a)} \cdot \left(\frac{Q}{K}\right)^{-\rho} + \frac{a}{1-a}\right]^{\frac{1}{\rho}}$$

The estimation results are

$$\frac{LH1}{KF1} = \begin{bmatrix} 2.9253 & (\frac{GDP1}{KF1}) & -0.0281 & (2.69) \\ (19.91) & -2.69) & -1.9967 \\ (28.01) & -1.9967 \end{bmatrix}$$

$$\bar{R}^2 = 0.444 \qquad DW = 0.430 \qquad SE = 0.00129$$

Estimation period 1960 - 1985. The solved parameters are A = .4242, ρ = 0.0281 and a = 0.6663. Numbers in brackets are standard deviations.

Note that here the stock of capital is not multiplied by four since it appears on the both sides of the equation.

PRODUCTION FUNCTION ESTIMATION RESULTS FOR SECTORS 2, 3, 4 and G

TABLE A1	EDULATION AT	(First stage)
IABLE AT	FOUNTION AT	(rirst stage)

	\bar{R}^2	DW	SE	estimation period	Solved a	from the ρ	equation S
Private services $log(\frac{GDPV2 - Q1V83}{(1+SOCCR2) \cdot YW2 \cdot ZL2} - 1) = 2.0726 - 0.6050 log(4 \cdot \frac{KF2}{LH2})$ etc. (excl. housing) log($\frac{GDPV2 - Q1V83}{(1+SOCCR2) \cdot YW2 \cdot ZL2} - 1) = 2.0726 - 0.6050 log(4 \cdot \frac{KF2}{LH2})$	0.756	1.129	0.118	1960 - 1985	0.8882	-0.605	0.623
Forestry $log(\frac{GDPV3}{(1+SOCCR3)\cdot YW3\cdot ZL3} - 1) = -1.6501 + 0.3261 log(4 \cdot \frac{KF3}{LH3})$	0.789	1.170	0.138	1960 - 1985	0.1611	0.326	1.484
Manufacturing $log(\frac{GDPV4}{(1+SOCCR4)\cdot YW4\cdot ZL4} - 1) = 0.2148 - 0.1429 log(4 \cdot \frac{KF4}{LH4})$	0.044	1.042	0.148	1960 - 1985	0.5535	-0.143	0.875
Government services $log(\frac{GDPVG}{(1+SOCCRG) \cdot YWG} - 1) = 4.2834 - 0.9781 log(4 \cdot \frac{(KFCG+KFLG)}{LHG})$ (0.8955) (0.1281)	0.696	0.905	0.043	1960 - 1985	0.9864	-0.978	0.506
\cdot							
TABLE A3 EQUATION A3 (Second stage)						Solved	l from
		\bar{R}^2	DW		timation period		uation Y
Private services etc. (exc1. housing) $\log \left[\frac{\text{GDP2} - \text{QIV83/(.01 \cdot PGDP2)}}{(.8882(4 \cdot \text{KF2})^{605} + .1118 \text{LH}^{605})^{(-17.605)}} \right] = 0.0702 + 0.0316 \cdot \text{TREND} = 0.0702 + 0.0$)	0.988	0.390	0.026 19	60 - 1985	1.073	0.032
Forestry $\log \left[\frac{\text{GDP3}}{(.1611(4 \cdot \text{KF3}) \cdot 3261 + .8389 \text{LH3} \cdot 3261)(-1/.3261)} \right] = 2.2019 + 0.0029 \cdot \text{TRE}$	ND	0.041	0.842	0.076 19	60 - 1985	9.042	0.003
Manufacturing $\log \left[\frac{\text{GDP4}}{(.5535(4 \cdot \text{KF4})^{143} + .4465 \text{LH4}^{143})^{(-1/.143)}} \right] = 0.8191 + 0.0306 \cdot \text{TRE}$ $(0.0145) (0.0010)$	ND	0.976	0.557	0.037 19	60 - 1985	2.268	0.031
Government services $\log \left[\frac{\text{GDPG}}{(.9864(4 \cdot (\text{KFCG+KFLG}))^{978} + .0136 \text{LHg}^{978})^{(-17.978)}} \right] = -0.2147 + 0.$	0071 - TDE	ND 0 027	0.024	0.014 10	co 100c		0.007

List of symbols, see appendix 3. Standard deviations are given in brackets under the parameters.

APPENDIX 2

LIST OF EQUATIONS

List of output and employment equations for the BOF4 quarterly model of the Finnish economy, May 1988 version.

Notation used:

Values of parameter estimates are ordinary least squares estimates.

Standard errors of parameter estimates are in parentheses below the coefficients.

When standard error is not shown, the parameter in question is fixed a priori e.g. on the basis of input-output studies.

Weights of Almon lags are denoted by ai, bi, etc.

Variables with a subscript are lagged. Subscripts refer to number of lags in quarters.

Δ is the difference operator.

 Δ^n denotes difference over n quarters.

log denotes natural logarithms.

Units:

Values are in millions of FIM.

Volumes are in millions of FIM at 1985 prices.

Price indices take the value 100 in 1985.

Interest rates are in per cent.

Energy is in 1000 toe.

Labour force figures are in 1000 persons.

R² = corrected coefficient of determination

DW = Durbin - Watson statistic SE = standard error of estimate

rho = coefficient of first-order autocorrelation correction

The estimation period is given after the summary statistics

Q. TUOTANTO PRODUCTION

Q.1 - Q.5 Tuotannon määrä sektoreittain Production at Factor Cost by Sectors

	ΔCD	ΔCND	ΔCS	ΔCG-ΔGDPG	ΔIF+ΔIG	ΔΙΗ	ΔXG	ΔXS	ΔΜC	ΔMS	ΔΜΙ	∆MR+∆MFL	IIS
ΔGDP1	+.0507	+.0698	+.0224	+.0465	+.0281	+.0214	+.0971	+.0214	0773	0224	0311	0996	+.1134
ΔGDP2	+.3844	+.3341	+.7517	+.4977	+.6522	+.7380	+.1898	+.7380	4937	7517	7224	2497	(0.02173) +.3275
∆GDP3	+.0515	+.0563	+.0233	+.0546	+.0365	+.02220	+.0897	+.02220	0708	0233	0404	0898	(0.03804) +.1278
∆GDP4	+.3827	+.3281	. +.1609	+.3345	+.2062	+.1544	+.6614	+.1544	4529	1609	2284	5663	(0.01919) +.3841
ΔTIN	+.1307	+.2117	+.0417	+.0667	+.0770	+.0640	0380	+.0640	+.0947	0417	+.0223	+.0054	(0.03774) +.0439 (0.02215)
						R ²	Di	4	SE		aikaväl	i	(0.02220)
GDP1 Tuotannon määrä, maatalous Production at Factor Cost, Agriculture, Volume					2246	2.8	388	240.2	729	62.1 - 8			
GDP2 Tuotannon määrä, palvelukset ym. Production at Factor Cost, Services etc., Volume					4382	2.8	311	420.60	088	62.1 - 8	5.4		
GDP3 Tuotannon määrä, metsätalous Production at Factor Cost, Forestry, Volume					3219	2.4	189	212.17	714	62.1 - 8	5.4		
GDP4 Tuotannon määrä, teollisuus Production at Factor Cost, Manufacturing, Volume					5216	3.:	1411	417.20	585	62.1 - 8	5.4		
TIN Välilliset verot miinus tukipalkkiot, määrä Indirect Taxes Less Subsidies, Volume			ä 0.	0392	2.9	996	244.92	243	62.1 - 8	5.4			

CG	Total public consumption, millions of 1985 FIM
GDP1	Production at factor cost, agriculture, millions of 1985
	FIM
GDP2	Production at factor cost, services etc., millions of
	1985 FIM
GDP3	Production at factor cost, forestry, millions of 1985 FIM
GDP4	Production at factor cost, manufacturing, millions of
•	1985 FIM
GDPF	GDP at factor cost, millions of 1985 FIM
GDPG	Production at factor cost, general government, millions
	of 1985 FIM
GDPV	GDP in Purchasers' Values, FIM million
GDPV1	Production at factor cost, agriculture, FIM million
GDPV2	Production at factor cost, services and other, FIM
	million
GDPV3	Production at factor cost, forestry, FIM million
GDPV4	Production at factor cost, manufacturing, FIM million
PGDP1	Value added deflator in agriculture, 1985 = 100
PGDP2	Value added deflator in services etc., 1985 = 100
PGDP3	Value added deflator in forestry, 1985 = 100
PGDP4	Value added deflator in manufacturing, 1985 = 100

```
Q.6 Tuotannon määrä, julkinen toiminta
Production at Factor Cost, General Government, Volume

ΔGDPG = 0.66225 • ΔCG

(0.02210)
```

Q.7 Bruttokansantuotteen määrä, tuottajahintaan Gross Domestic Product at Factor Cost, Volume GDPF = GDP1 + GDP2 + GDP3 + GDP4 + GDPG

Q.8 Tuotannon arvo, maatalous
Production at Factor Cost, Agriculture, Value

GDPV1 = GDP1 • PGDP1/100

Q.9 Tuotannon arvo, palvelukset ym.

Production at Factor Cost, Services etc., Value

GDPV2 = GDP2 • PGDP2/100

Q.10 Tuotannon arvo, metsätalous
Production at Factor Cost, Forestry, Value

GDPV3 = GDP3 • PGDP3/100

Q.11 Tuotannon arvo, teollisuus
Production at Factor Cost, Manufacturing, Value

GDPV4 = GDP4 • PGDP4/100

GDP	GDP in Purchasers' Values, millions of 1985 FIM
GDPF	GDP at factor cost, millions of 1985 FIM
GDPFV	GDP at factor cost, FIM million
GDPG	Production at factor cost, general government, millions of 1985 FIM
	• • • • • • • • • • • • • • • • • • • •
GDPV1	Production at factor cost, agriculture, FIM million
GDPV2	Production at factor cost, services and other, FIM million
GDPV3	Production at factor cost, forestry, FIM million
GDPV4	Production at factor cost, manufacturing, FIM million
GDPVG	Production at factor cost, FIM million
PGDPG	Value added deflator in general government, 1985 = 100
SUB	Subsidies, millions of 1985 FIM
TIN	Indirect taxes less subsidies, millions of 1985 FIM
TIV	Central government revenue from commodity taxes, FIM million

- Q.12 Tuotannon arvo, julkinen toiminta
 Production at Factor Cost, General Government, Value

 GDPVG = GDPG PGDPG/100
- Q.13 Bruttokansantuotteen arvo, tuottajahintaan Gross Domestic Product at Factor Cost, Value
- Q.14 Bruttokansantuotteen määrä, markkinahintaan
 Gross Domestic Product in Purchasers' Values, Volume

 GDP = GDPF + TIN

GDPFV = GDPV1 + GDPV2 + GDPV3 + GDPV4 + GDPVG

Q.15 Bruttokansantuotteen arvo, markkinahintaan
Gross Domestic Product in Purchasers' Values, Value

GDPV = GDPFV + TIV - SUB

GDP1	Production at factor cost, agriculture, millions of 1985
	FIM
GDP2	Production at factor cost, services etc., millions of
	1985 FIM
KF1	Net stock of fixed capital, agriculture, millions of
	1985 FIM
KF2	Net stock of fixed capital, service etc., millions of
	1985 FIM
KH	Net stock of private residential capital, net, millions
	of 1985 FIM
LH1	Performed working hours, agriculture, millions of hours
LH2	Performed working hours, services etc., millions of hours
TREND	Linear trend: 60.1 = .25, 60.2 = .50 etc.
DQ1	Seasonal dummy, the first quarter
DQ2	Seasonal dummy, the second quarter

- . TYÖLLISYYS EMPLOYMENT
- L.1 Tehdyt työtunnit, maatalous Performed Working Hours, Agriculture

$$\Delta \log LH1 = -0.00817$$
 (0.0073)
 $+ 0.10695 \cdot \Delta \log GDP1$
 (0.0868)
 $- 0.06912 \cdot \log(GDP1T_{-1}/GDP1_{-1})$
 (0.0638)

jossa GDP1T = 0.4242 · (0.6663 · KF1-.0281 + 0.3337 LH1-0.0281)35.6
 $\bar{R}^2 = 0.000$
 $DW = 2.334$
 $SE = 0.07112$
 $62.1 - 85.4$

L.2 Tehdyt työtunnit, palvelukset ym. Performed Working Hours, Services etc.

62.1 - 85.4

GDP3 Production at factor cost, forestry, millions of 1985 FIM

KF3 Net stock of fixed capital, forestry, millions of 1985 FIM

LH3 Performed working hours, forestry, millions of hours

TREND Linear trend: 60.1 = .25, 60.2 = .50 etc.

SE = 0.08015

DW = 2.310

 $\bar{R}^2 = 0.340$

D75 Dummy 60.1 - 74.4 = 1, 75.1 - 85.4 = 0GDP4 Production at factor cost, manufacturing, millions of 1985 FIM KF4 Net stock of fixed capital, manufacturing, millions of 1985 FIM LH4 Performed working hours, manufacturing, millions of hours *TREND Linear trend: 60.1 = .25, 60.2 = .50 etc.

```
Performed Working Hours, Manufacturing
\Delta \log LH4 = -0.00572
              (0.0024)
             - 0.15119 • Alog LH4_1
              (0.0643)
             + 0.44859 • Alog GDP4
              (0.0821)
             -0.13498 \cdot \log(GDP4T_{-1}/GDP4_{-1})
              (0.0539)
             - 0.20787 · △D75
              (0.0210)
             + 0.21837 · DD75_1
              (0.0262)
jossa GDP4T = 2.2685 \cdot e^{0.0306 \cdot TREND} \cdot (0.553 \cdot KF4^{-0.143} + e^{-0.143})
                0.447 · LH4-0.143)-6.997
\bar{R}^2 = 0.721
```

SE = 0.0207

62.1 - 85.4

DW = 2.347

L.4 Tehdyt työtunnit, teollisuus

62.1 - 85.4

GDPG Production at factor cost, general government; millions of 1985 FIM Net stock of fixed capital, central government. KFCG millions of 1985 FIM **KFLG** Net stock of fixed capital, local government, millions of LH1 Performed working hours, agriculture, millions of hours Performed working hours, general government, millions of LHG hours LW1 Paid labour input, agriculture, millions of 1985 FIM TREND Linear trend: 60.1 = .25, 60.2 = .50 etc.

L.5 Tehdyt työtunnit, julkinen toiminta Performed Working Hours, Central and Local Government $\Delta \log LHG = -0.00267$ (0.0127)- 0.76273 • log(GDPGT_1/GDPG_1) (0.1035) $iossa GDPGT = 0.80682 \cdot e^{0.0071 \cdot TREND} \cdot (0.986 \cdot e^{0.0071} \cdot TREND)$ $(KFCG + KFLG)^{-0.978} + 0.014 \cdot LHG^{-0.978})^{-1.022}$ $\bar{R}^2 = 0.359$

SE = 0.12424

DW = 1.916

L.6 Ansiotyöpanos, maatalous Paid Labor Input. Argiculture

$$\bar{R}^2 = 0.930$$
 DW = 1.437 SE = 0.0113 76.1 - 85.4

```
LH2 Performed working hours, services etc., millions of hours
LH3 Performed working hours, forestry, millions of hours
LW2 Paid labour input, services etc., millions of 1985 FIM
LW3 Paid labour input, forestry, millions of 1985 FIM
```

```
L.7 Ansiotyöpanos, palvelukset ym.
      Paid Labor Input, Services etc.
      \Delta \log LW2 = + 0.00118
                   (0.00102)
                 + 0.67917 • Alog LW2_1
                  (0.11294)
                 + 0.95488 • Alog LH2
                  (0.02964)
                 - 0.70972 · Δlog LH2_1
                  (0.10979)
      \bar{R}^2 = 0.970
                      DW = 0.527
                                        SE = 0.0061
                                                          76.1 - 85.4
L.8 Ansiotyöpanos, metsätalous
      Paid Labor Input, Forestry
      \Delta \log LW3 = + 0.00166
                  (0.00201)
                 + 0.79961 • Alog LW3_1
                  (0.10022)
                 + 0.98501 • Alog LH3
                  (0.02836)
                 - 0.80135 · Alog LH3_1
                  (0.10031)
```

DW = 0.452

SE = 0.0123

76.1 - 85.4

 $\bar{R}^2 = 0.976$

LH4	Performed working hours, manufacturing, millions of hours
LHG	Performed working hours, general government, millions of
	hours
LW4	Paid labour input, manufacturing, millions of 1985 FIM
LWG	

L.9 Ansiotyöpanos, teollisuus Paid Labor Input, Manufacturing

$$\bar{R}^2 = 0.742$$
 DW = 0.818 SE = 0.0123 76.1 - 85.4

L.10 Ansiotyöpanos, julkinen toiminta
Paid Labor Input, Central and Local Government

$$\bar{R}^2 = 0.900$$
 DW = 1.476 SE = 0.0347 76.1 - 85.4

LE Employment, 1000 persons

LES Employment (Labour Force Survey), 1000 persons

LH Performed working hours total, millions of hours

TREND Linear trend: 60.1 = .25, 60.2 = .50 etc.

L.11 Työlliset Employment (SNA) LE = + 301.62638(92.09294) + 0.82249 • LES $\bar{R}^2 = 0.931$ RHO = 0.980SE = 3.213476.1 - 85.4 L.12 Työlliset, työvoimatutkimus Employment (Labour Force Survey) $\Delta \log LES = 0.05141$ (0.02786)+ 0.07800 • log(LH_1/LES_1) (0.04247)+ 0.14397 • Alog LH (0.03588)+ 0.13635 • Alog LH_1 (0.04364) + 0.15048 • Alog LH_2 (0.03963) + 0.18175 • Alog LH_3 (0.03558)

 $\bar{R}^2 = 0.3007$ DW =

DW = 1.7749

+ 0.07713 • Δlog LH_4 (0.03499)

+ 0.00067 • TREND (0.00033)

SE = 0.0048 62.1 - 85.4

```
C
          Total private consumption, millions of 1985 FIM
LES
          Employment (Labour Force Survey), 1000 persons
LFS
LH
          Performed working hours total, millions of hours
LH1
          Performed working hours, agriculture, millions of hours
LH2
          Performed working hours, services etc., millions of hours
LH3
          Performed working hours, forestry, millions of hours
LH4
          Performed working hours, manufacturing, millions of hours
LHG
          Performed working hours, general government, millions of
          hours
LU
          Unemployment, 1000 persons
LW
          Paid labour input, total, millions of 1985 FIM
LW1
          Paid labour input, agriculture, millions of 1985 FIM
LW2
          Paid labour input, services etc., millions of 1985 FIM
LW3
          Paid labour input, forestry, millions of 1985 FIM
LW4
          Paid labour input, manufacturing, millions of 1985 FIM
LWG
MTAX
          Personal marginal tax rate, estimate
N
          Population of working age (15-74 years), 1000 persons
PCP
          Private consumption prices, 1985 = 100
UR
          Unemployment rate, per cent
WR
          Wage rate, total, 1985 = 100
```

```
L.13 Työvoima, työvoimatutkimus
       Labour Force (Labour Force Survey)
       \Delta \log(LFS/N) = + 0.00026
                       (0.00030)
                     + 0.64817 · Δlog(LES/N)
                      (0.04672)
                     + 0.00520 • Δlog((1 - MTAX) • WR/PCP)
                      (0.00336)
                     - 0.03026 · \( \Dag(C/N) \)
                      (0.01609)
      \bar{R}^2 = 0.6871
                       DW = 2.0315
                                         SE = 0.0027
                                                          63.1 - 85.4
L.14 Tehdyt työtunnit, vhteensä
      Performed Working Hours, Total
      LH = LH1 + LH2 + LH3 + LH4 + LHG
L.15 Ansiotyöpanos, yhteensä
      Paid Labour Input, Total
```

L.15 Ansiotyöpanos, yhteensä
Paid Labour Input, Total

LW = LW1 + LW2 + LW3 + LW4 + LWG

L.16 Työttömät, työvoimatutkimus
Unemployment (Labour Force Survey)

LU = LFS - LES

L.17 Työttömyysaste, työvoimatutkimus
Unemployment Rate (Labour Force Survey)

UR = 100 • LU/LFS

APPENDIX 3

LIST OF VARIABLES

PQ-WL	gross operating surplus
WL	labour costs
K	capital stock
L	labour input (hours)
Q	real value added
TREND	linear trend

GDPG	Production at factor cost, general government, millions of 1985 FIM
GDPVG	Production at factor cost, general government, FIM million
GDPV1	Production at factor cost, agriculture, FIM million
GDPV2	Production at factor cost, services and other, FIM
	million
GDPV3	Production at factor cost, forestry, FIM million
GDPV4	Production at factor cost, manufacturing, FIM million
GDP1	Production at factor cost, agriculture, millions of 1985 FIM
GDP2	Production at factor cost, services etc., millions of 1985 FIM
GDP3	Production at factor cost, forestry, millions of 1985 FIM
GDP4	Production at factor cost, manufacturing, millions of 1985 FIM
KFCG	Net stock of fixed capital, central government, millions of 1985 FIM
KFLG	Net stock of fixed capital, local government, millions of 1985 FIM
KF1	Net stock of fixed capital, agriculture, millions of 1985 FIM

KF2	Net stock of fixed capital, service etc., millions of 1985 FIM
KF3	Net stock of fixed capital, forestry, millions of 1985 FIM
KF4	Net stock of fixed capital, manufacturing, millions of 1985 FIM
LHG	Performed working hours, general government, millions of hours
LH1	Performed working hours, agriculture, millions of hours
LH2	Performed working hours, services etc., millions of hours
LH3	Performed working hours, forestry, millions of hours
LH4	Performed working hours, manufacturing, millions of hours
PGDP2	Value added deflator in services etc., 1985 = 100
Q1V83	Production at factor cost, housing, FIM million
SOCCRG	Employers' social security contribution rate, general
	government, FIM million
SOCCR1	Employers' social security contribution rate, agriculture
SOCCR2	Employers' social security contribution rate, services etc.
SOCCR3	Employers' social security contribution rate, forestry
SOCCR4	Employers' social security contribution rate, manufacturing
YW1	Wages and salaries, agriculture, FIM million
YW2	Wages and salaries, services etc., FIM million
YW3	Wages and salaries, forestry, FIM million
YW4	Wages and salaries, manufacturing, FIM million
Z	Total hours worked per hours worked by employees in the sector; adjustment term to deduct the compensation of the
	work of the self-employed from the value-added of the sector.
ZLi	See Z
(i=2,3,4)	

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