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The equity risk premium and the low frequency of the term spread*

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Abstract

We extract cycles in the term spread (TMS) and study their role for predicting the equity risk premium (ERP) using linear models. The low frequency component of the TMS is a strong and robust out-of-sample ERP predictor. It obtains out-of-sample R-squares (versus the historical mean benchmark) of 2.09% and 22.9% for monthly and annual data, respectively. It forecasts well also during expansions and outperforms several variables that have been proposed as good ERP predictors. Its predictability power comes exclusively from the discount rate channel. Contrarily, the high and business-cycle frequency components of the TMS are poor out-of-sample ERP predictors.

Keywords: equity risk premium, term spread, predictability, frequency domain

JEL classification: C58, G11, G12, G17

1 Introduction

The equity risk premium (ERP) plays a crucial role in economics and finance. Hence, forecasting it has been a flourishing area of research for many years (see *e.g.* the literature reviews of Rapach and Zhou, 2013 and Harvey, Liu, and Zhu, 2016). Within the large set of ERP predictors considered in the literature, one that has since long time generated particular interest is the term spread (TMS) of US treasury securities – the difference between the long-term government bond yield and the T-bill. The attractiveness of the TMS is that it is easy to compute from publicly available data, is closely linked with the business cycle (see *e.g.* Wheelock and Wohar, 2009), and is continuously watched by professional forecasters and policymakers alike.[1] In their seminal studies, Chen, Roll, and Ross (1986), Campbell (1987) and Fama and French (1989) find that the term structure of interest rates predicts the ERP. However, as it happens with many other predictors, the predictability power of the TMS is rather poor when the forecasting exercise is done out-of-sample (Goyal and Welch, 2008). This is particularly relevant as an out-of-sample test is more appropriate to assess genuine return predictability in real time. Accordingly, our key contribution in this paper is to evaluate the ERP out-of-sample forecasting performance of the TMS and of its frequency components.

In a recent paper, Dew-Becker and Giglio (2016) derive frequency-specific risk prices that capture the price of risk of fluctuations in consumption growth at different frequencies. This allows to measure the relative importance of economic fluctuations at different frequencies and to assess whether they are priced in risky asset markets. Similarly, Fama and French (1989) propose a frequency-domain analysis of the ERP predictability, suggesting that different variables may track different frequency components of the ERP. In particular, they claim that the default spread and the dividend yield track long-term business conditions, while

[1] For instance, in September 2016 the Bank of Japan supplemented its quantitative and qualitative monetary easing with an explicit yield curve control (Nakaso, 2017).

the TMS tracks variation in expected returns in response to shorter-term business-cycle fluctuations.[2]

Motivated by this conjecture, in this paper we consider three economically-motivated frequency components of the TMS as potential ERP predictors: the high frequency component, the business cycle frequency component, and the low frequency component. We first explore the in-sample ERP predictability of the TMS and of its frequency components, and then explore the economic source of this predictability using the Campbell and Shiller (1988) present value identity and the log linearization of stock returns (Cochrane, 2008, 2011). The predictability power of the TMS comes from both the discount rate and cash-flow channels. The analysis using the different frequency components of the TMS allows to unveil that the predictability power of the TMS that operates through the cash-flow channel is concentrated at business-cycle frequencies, while the one operating through the discount rate channel is concentrated at low frequencies.

When evaluating the ERP out-of-sample forecasting performance of the TMS and of its frequency components, we find that the low frequency component of the TMS is a strong and robust out-of-sample predictor of the ERP for forecasting horizons ranging from one month to two years. Its outperformance versus the historical mean (HM) benchmark is remarkably good for the one-month horizon, increases with the forecasting horizon, and is consistently stable throughout an out-of-sample period comprising 27 years of monthly data. It also performs well during expansions. Differently, the remaining frequency components of the TMS are poor ERP out-of-sample predictors. This finding is aligned with existing evidence showing that low frequency fluctuations in the economy – long-run risks – are significantly

[2] A similar reasoning underlies the Ferreira and Santa-Clara (2011) sum-of-parts method for forecasting stock returns, where different parts of stock market returns (the dividend-price ratio, earnings growth, and price-earnings ratio growth) capture different frequencies of stock returns. More recently, Bandi, Perron, Tamoni, and Tebaldi (2018) and Faria and Verona (2018) achieve relevant statistical and economic improvements in ERP predictability by explicitly taking into account the frequency dependence between the ERP and its predictors.

priced in the equity market, while business cycle and higher frequency fluctuations are not (*e.g.* Dew-Becker and Giglio, 2016).

We then evaluate the forecasting performance of the predictors from an economic point of view by means of an asset allocation analysis. For a mean-variance investor who allocates his or her wealth between equities and risk-free bills, we find significant utility gains when making the forecasts using the low frequency component of the TMS. For example, considering a trading strategy based on one-month-ahead forecasts, the annual rate of return that an investor would be willing to accept instead of holding the risky portfolio is 591 basis points.

Finally, we run three additional exercises. First, we evaluate the forecasting performance of the low frequency component of the TMS for different sub-samples. We find that our results are robust to the sub-period considered and we document that forecasting during good growth periods using the low frequency component of the TMS *also* outperforms the HM benchmark in a statistically significant way. This contrasts with large evidence in the literature that return predictability is usually concentrated in recessions. Second, we demonstrate that the filtering method used to extract the low frequency component of the TMS is crucial, as methods based on the more popular band-pass and Hodrick-Prescott (HP) filters underperform the one using wavelets. Third, we show that the low frequency component of the TMS also outperforms several variables that have recently been proposed as good ERP predictors.

We review related literature in the remaining part of this section. Section 2 introduces wavelet filtering methods. Section 3 presents the data and the method to construct the predictors. Section 4 presents the in-sample (IS) predictability analysis and section 5 the out-of-sample (OOS) forecasting results. Section 6 reports the results of the asset allocation exercise. Section 7 discusses possible interpretations of the results. Robustness analyses are done in section 8. Finally, section 9 concludes.

Related literature

This paper is primarily related with the literature that analyzes the ERP forecasting properties of the TMS. Fama and French (1989) find that the ERP on US stocks is positively related to the slope of the yield curve of US Treasury securities. Asprem (1989) studies the relationship between the US TMS and the returns on stocks of ten European countries. Boudoukh, Richardson, and Smith (1993) and Ostdiek (1998) show how risk premia on US stocks and the world stock portfolio are negative in periods preceded by inverted yield curves. McCown (2001) finds empirical evidence about the relationships between the yield curves of larger economies (US, Germany, and Japan) and risk premia of stocks for eight industrialized countries. Resnick and Shoesmith (2002) and Nyberg (2013) find that the US TMS is a powerful predictive variable for bear equity markets in the US. While this literature mainly analyzes the IS predictability of the TMS, our main focus is on its OOS forecasting power.

The second strand of literature to which this paper relates focuses on finding good OOS predictors of the ERP. As pointed out initially by Goyal and Welch (2008), most ERP predictors perform poorly OOS on US data up to 2008. Since Goyal and Welch (2008), several new predictors have been developed and tested, most notably macro, financial market- and behavioral-related variables.[3] Our paper contributes to this literature as the different frequency components of the TMS are used as ERP predictors.

[3] With regards to macro variables, Cooper and Priestley (2009, 2013) use the output gap and the world business cycle, respectively, Favero, Gozluklu, and Tamoni (2011) consider a demographic variable (the proportion of middle-aged to young population), Li, Ng, and Swaminathan (2013) study the aggregate implied cost of capital, Chava, Gallmeyer, and Park (2015) study the predictive power of bank lending standards, and Moller and Rangvid (2015, 2018) study different US-based macroeconomic variables and global economic growth, respectively, by focusing on their fourth-quarter growth rate. Financial market variables include the variance risk premium (Bollerslev, Tauchen, and Zhou, 2009), lagged US market returns for the OOS predictability of stock returns of other industrialized countries (Rapach, Strauss, and Zhou, 2013), the stock-bond yield gap (Maio, 2013), technical indicators (Neely, Rapach, Tu, and Zhou, 2014), the government bond volatility index (Pan and Chan, 2017), option-implied state prices (Metaxoglou and Smith, 2017), risk neutral variance of the equity market return measured from index option prices (Martin, 2017), and generalized financial ratios (Algaba and Boudt, 2017). Behavioral-related variables include the investment sentiment indexes (Huang, Jiang, Tu, and Zhou, 2015) and information on short-interest positions (Rapach, Ringgenberg, and Zhou, 2016).

In doing this, our paper is also related to the literature that focuses on the spectral properties of financial asset returns. Frequency domain tools have long been used in economics (*e.g.* Granger and Hatanaka, 1964 and Engle, 1974). In finance, the interest in using frequency domain tools has been growing in more recent years. For example, Dew-Becker and Giglio (2016) develop a frequency domain decomposition of innovations to the pricing kernel, Chaudhuri and Lo (2016) apply spectral analysis techniques to quantify stock-return dynamics across multiple time horizons and propose a spectral portfolio theory, and Bandi, Perron, Tamoni, and Tebaldi (2018) explore a model where returns and predictors are linear aggregates of components operating over different frequencies, and where predictability is frequency-specific.

Finally, this paper relates to the literature that uses wavelets methods to forecast (out-of-sample) economic and financial time series.[4] Examples include Zhang, Gencay, and Yazgan (2017) and Faria and Verona (2018), both focused on stock market returns predictability; Rua (2011, 2017), who propose a factor-augmented wavelets approach to forecast GDP growth and inflation; and Kilponen and Verona (2016), who forecast aggregate investment using the Tobin's Q theory of investment.

[4] Crowley (2007) and Aguiar-Conraria and Soares (2014) provide excellent reviews of economic and finance applications of wavelets tools. The seminal works of Ramsey and Lampart (1998a,b) apply wavelets to study the relationship between macroeconomic variables (consumption versus income and money supply versus income, respectively). More recently, wavelets methods have been applied to test for the (IS) frequency dependence between two or more variables (Kim and In, 2005, Gencay, Selcuk, and Whitcher, 2005, Gallegati, Gallegati, Ramsey, and Semmler, 2011 and Gallegati and Ramsey, 2013), to study the comovements and lead-lag relationship between variables at different frequencies (Rua and Nunes, 2009, Rua, 2010, Aguiar-Conraria and Soares, 2011, and Aguiar-Conraria, Martins, and Soares, 2012), and to analyze optimal fiscal and monetary policy (Crowley and Hudgins, 2015, 2017).

2 Time-frequency decomposition of economic time series using wavelet filtering methods

2.1 Wavelet filtering methods

Wavelets have long been popular in many fields such as geophysics, engineering, medicine, and biomedical engineering. Real-life applications include the FBI algorithm for fingerprint data compression (Onyshczak and Youssef, 2004). Notably, Yves Meyer, a French mathematician, received the 2017 Abel Prize "for his pivotal role in the development of the mathematical theory of wavelets."[5,6]

Wavelet multiresolution analysis (MRA) is a useful tool to analyze the time and frequency properties of a time series. Using a wavelet filter, any time series y_t – regardless of its time series properties – can be decomposed as

$$y_t = \sum_{j=1}^{J} y_t^{D_j} + y_t^{S_J} \quad , \tag{1}$$

where $y_t^{D_j}$, $j = 1, 2, \ldots, J$, are the J wavelet detail components and $y_t^{S_J}$ is the wavelet smooth component. Equation (1) shows that the original series y_t, exclusively defined in the time domain, can be decomposed in different time series components, each defined in the time domain and representing the fluctuation of the original time series in a specific frequency band. In particular, for small j, the j wavelet detail components represent the higher frequency characteristics of the time series (*i.e.* its short-term dynamics). As j increases, the j wavelet detail components represent lower frequencies movements of the series. Finally, the wavelet's smooth component captures the lowest frequency dynamics (*i.e.* its long-term behavior).

[5] The Abel Prize is, with the Fields Medal, considered to be the highest honor a mathematician can receive. These awards have often been described as the mathematician's "Nobel Prize".

[6] Our presentation here is limited to basic facts that are directly relevant to our empirical analysis. A more detailed analysis of wavelets methods can be found in Appendix A and in Percival and Walden (2000).

In this paper, we perform wavelet decomposition analysis by applying the maximal overlap discrete wavelet transform (MODWT) MRA. This methodology i) is not restricted to a particular sample size; ii) is translation-invariant, so that it is not sensitive to the choice of starting point for the examined time series; and iii) does not introduce phase shifts in the wavelet coefficients, *i.e.* peaks or troughs of the original time series are perfectly aligned with similar events in the MODWT MRA. This last feature is especially relevant in a forecasting exercise.[7]

2.2 The advantages of using wavelets

Traditional econometric techniques (time series and spectral/frequency analysis) impose strong assumptions about the data generating process. In particular, they usually require the data to be stationary, *i.e.* its mean and variance do not change over time and do not follow any trends. However, several economic and financial time series are hardly stationary as they exhibit trends and patterns such as structural breaks, clustering and long memory.

Unlike Fourier analysis, wavelets are defined over a finite window in the time domain, which is automatically and optimally resized according to the frequency of interest. That is, using a short time window allows to isolate the high frequency features of a time series, while looking at the same signal with a large time window reveals its low frequency features. Hence, by varying the size of the time window, it is possible to capture simultaneously both time-varying and frequency-varying features of the time series. Wavelets are thus extremely useful when dealing with non-stationary time series, regardless of whether the non-stationarity comes from the level of the time series (*i.e.* from long-term trend or jumps) and/or from higher order moments (*i.e.* from changes in volatility).

[7] Papers using the MODWT MRA decomposition include Galagedera and Maharaj (2008), Xue, Gencay, and Fagan (2013), Bekiros and Marcellino (2013), Barunik and Vacha (2015), Caraiani (2015), Bekiros, Nguyen, Uddin, and Sjo (2016), Berger (2016), Kilponen and Verona (2016), Zhang, Gencay, and Yazgan (2017) and Faria and Verona (2018).

Wavelet filtering methods allow a decomposition of a time series into different frequency bands. To obtain the decomposition, an appropriate cascade of wavelet filters is applied. This is essentially equivalent to filtering by a set of band-pass filters so as to capture the fluctuations of the time series in different frequency bands.

Skeptics of the wavelet methodology may ask the question of why not just use the more popular Baxter and King (1999) or Christiano and Fitzgerald (2003) band-pass filters, which also permit the isolation of fluctuations in different frequency bands. The band-pass filter is a combination of a Fourier decomposition in the frequency domain with a moving average in the time domain. It is optimized by minimizing the distance between the Fourier transform and an ideal filter. Like a short-time Fourier transform, it applies an "optimal" Fourier filtering on a sliding window in the time domain with constant length regardless of the frequency being isolated. Wavelet filtering, in contrast, provides better resolution in the time domain as the wavelet basis functions are both time-localized and frequency-localized. Guay and St.-Amant (2005) observe that the band-pass filter is not an ideal filter, as it is a finite representation of an infinite moving-average filter, and that it performs well at business-cycle frequencies but not at low and high frequencies. Moreover, Murray (2003) points out that the band-pass filter may introduce spurious dynamic properties.[8]

3 Data and predictors

We use monthly data from January 1973 to December 2017 and focus on the predictability of the S&P500 index excess returns (ERP), measured as the log return on the S&P500 index (including dividends) minus the log return on a one-month Treasury bill. Data for the S&P500 index total return is from CRSP and for the one-month Treasury bill is from the

[8] Some observations at the beginning and at the end of the sample have to be discharged when using the Baxter and King (1999) filter.

FRED2 database. The TMS (TMS_{TS}) is computed as the difference between the US 10-year government bond yield and the 3-month T-bill time series, and the time series is obtained from the New York Federal Reserve Bank website.

Although the relationship between stock market returns, economic growth, and the TMS has been extensively studied in previous research (see references in section 1), we are particularly motivated by the conjecture of Fama and French (1989) that the TMS tracks variation in expected returns in response to business cycles. So, besides the time series of the TMS, we evaluate three frequency components of the TMS as individual ERP predictors. The first, denoted TMS_{HF}, captures the high-frequency fluctuations of the series (*HF* stands for high frequency). The second, denoted TMS_{BCF}, broadly corresponds to business cycle fluctuations. The third, denoted TMS_{LF}, captures the low-frequency fluctuations of the series (*LF* stands for low frequency).

To compute those frequency components, we start by running a J=6 level MODWT MRA to the time series of the TMS using the Haar wavelet filter with reflecting boundary conditions (as done by *e.g.* Manchaldore, Palit, and Soloviev, 2010 and Malagon, Moreno, and Rodriguez, 2015).[9] As we use monthly data, the first component ($TMS_t^{D_1}$) captures oscillations of the TMS between 2 and 4 months, while components $TMS_t^{D_2}$, $TMS_t^{D_3}$, $TMS_t^{D_4}$, $TMS_t^{D_5}$ and $TMS_t^{D_6}$ capture oscillations of the TMS with a period of 4-8, 8-16, 16-32, 32-64 and 64-128 months, respectively. Finally, the smooth component $TMS_t^{S_6}$ captures oscillations of the TMS with a period exceeding 128 months (10.6 years).[10]

Subsequently, the high-frequency component (TMS_{HF}) is computed as $TMS_{HF,t} = \sum_{i=1}^{3} TMS_t^{D_i}$, the business cycle frequency component ($TMS_{BCF,t}$) is computed as $TMS_{BCF,t} = \sum_{i=4}^{6} TMS_t^{D_i}$,

[9] Results are robust using different wavelet filters (like *e.g.* Daubechies). As regards the choice of J, the number of observations dictates the maximum number of frequency bands that can be used. In our case, $N = 204$ is the number of observations in the in-sample period, so J is such that $J \leq \log_2 N \simeq 7.7$.

[10] In the MODWT, each wavelet filter at frequency j approximates an ideal high-pass filter with passband $f \in \left[1/2^{j+1}\,,\,1/2^{j}\right]$, while the smooth component is associated with frequencies $f \in \left[0\,,\,1/2^{j+1}\right]$. The level j wavelet components are thus associated to fluctuations with periodicity $\left[2^{j}\,,\,2^{j+1}\right]$ (months, in our case).

whereas the low-frequency component (TMS_{LF}) corresponds to $TMS_t^{S_6}$.

To illustrate the rich set of different dynamics aggregated (and thus hidden) in the original time series, Figure 1 plots the time series of the TMS and its three frequency components under analysis. As expected, the lower the frequency, the smoother the resulting filtered time series. We note that, by summing these three frequency components, we get the exact original time series of the TMS.

The summary statistics of the predictors (and of the ERP) and their correlations are reported in Panel A and B of Table 1, respectively. The monthly excess market return has a mean of 0.43% and a standard deviation of 4.44%, which implies a monthly Sharpe ratio of 0.10. The excess market return has little autocorrelation, while the predictors are quite persistent (with the exception of the TMS_{HF}). The frequency components of the TMS are low correlated.

4 In-sample predictability

Let r_t be the ERP for month t and h the forecasting horizon. For each variable x_t, the predictive regression is

$$r_{t:t+h} = \alpha + \beta x_t + \varepsilon_{t:t+h} \quad \forall t = 1, ..., T - h \ , \tag{2}$$

where $r_{t:t+h} = (1/h)\,(r_{t+1} + \cdots + r_{t+h})$. The objective of the IS analysis is to estimate equation (2) by OLS in order to test the significance of estimated β coefficients. As there are some concerns about the statistical inferences from equation (2) (related with Stambaugh, 1999 and Campbell and Yogo, 2006 bias), to make reliable inferences we follow Rapach, Ringgenberg, and Zhou (2016) and use a heteroskedasticity- and autocorrelation-robust t-statistic and compute a wild bootstrapped p-value to test $H_0 : \beta = 0$ against $H_A : \beta > 0$ in (2). To enhance comparisons across predictors, we also standardize each predictor to have a unitary

standard deviation before estimating equation (2). After accounting for lags and overlapping observations, we thus have 540, 538, 535, 529 and 517 to estimate equation (2) for one-month-ahead (h=1), one-quarter-ahead (h=3), one-semester-ahead (h=6), one-year-ahead (h=12), and two-years-ahead (h=24) forecasting horizons.

Panel A of Table 2 reports, for each predictor and forecasting horizon, the OLS estimate of β in equation (2), its t-statistic, and the R^2 of the regression.

Starting with the monthly horizon (h=1), the high and business-cycle frequencies of the TMS (TMS_{HF} and TMS_{BCF}) are not statistically significant, whereas the TMS_{TS} and the TMS_{LF} are significant at the 10% and 5% level, respectively. Overall, the R^2s are rather small, which is expectable due to the large unpredictable component in monthly data. Campbell and Thompson (2008) argue, however, that a monthly R^2 of about 0.5% represents an economically relevant degree of return predictability. The monthly R^2s of the statistically significant predictors are indeed slightly above that threshold.

Looking at longer forecasting horizons ($h \geq 3$), the estimated βs for the TMS_{TS} and the TMS_{LF} are similar to the ones obtained for h=1. Those two predictors continue to be statistically significant (at least at the 10% level) at all forecasting horizons. As it is common in this literature, the fit of the regression increases as the forecasting horizon increases. The high frequency component is never significant, while the business cycle frequency component becomes significant at longer horizons ($h \geq 12$).

Thus, the TMS and its low frequency component are statistically significant IS predictors of the ERP for all forecasting horizons. In what follows we provide the economic source of ERP predictability of the TMS and of its frequency components.

Channels of predictability

The value of a stock is the discounted value of its expected cash flows. Stock return can thus result from change in the discount rate, change in the expectations of cash flows, or both. That is, a variable that predicts lower stock market return should either predict an increase in the discount rate or a decrease in cash flow expectations, or both (Baker and Wurgler, 2006, 2007). In the following analysis, we use the log dividend-price ratio (DP) as the proxy for the discount rate channel, supported by the evidence that changes in aggregate DP ratio comes primarily from changes in discount rates (Cochrane, 2008, 2011). Regarding the cash flow channel, we follow an extensive literature and use the log dividend growth (DG) as its proxy (see *e.g.* Campbell and Shiller, 1988, Menzly, Santos, and Veronesi, 2004 and Cochrane, 2008, 2011).

Most existing attempts to disentangle variation in expected discount rates versus variation in future cash flows start with the Campbell and Shiller (1988) log linearization of stock returns (Cochrane, 2008, 2011):

$$R_{t+1} = \kappa + DG_{t+1} - \rho DP_{t+1} + DP_t \ , \tag{3}$$

where R_{t+1} is the one-month-ahead stock market return, and κ and ρ are positive log-linearization constants.

Equation (3) implies that, if a variable has forecasting power of the next-period market return (beyond that of DP_t), then it must predict DP_{t+1}, DG_{t+1}, or both. As DP_{t+1} and DG_{t+1} are proxies for the discount rate and cash flow channels, respectively, evaluating their predictability from a variable provides insight into the economic source of the eventual market return predictability power of that variable.

We use the approach in Huang, Jiang, Tu, and Zhou (2015) to investigate whether the

discount rate channel or the cash flow channel (or both) play a role in the market return predictive ability of the TMS and of its frequency components under analysis. To do so, we estimate two bivariate predictive regressions for the TMS and for each of its frequency components:

$$Y_{t+1} = \varrho + \delta X_t + \psi DP_t + \vartheta_{t+1} \ , \tag{4}$$

where $X = TMS_{TS}$, TMS_{HF}, TMS_{BCF}, TMS_{LF} and $Y = DP$, DG. To make reliable inferences, we use a heteroskedasticity- and autocorrelation-robust t-statistic and compute a wild bootstrapped p-value to test $H_0 : \delta = 0$ against $H_A : \delta < 0$ and $H_0 : \psi = 0$ against $H_A : \psi > 0$ in (4).[11]

Table 3 reports the results. The lagged DP ratio has a strong predictive power for the one-month-ahead DP ratio, with very high persistence as given by the auto-regressive coefficient of 0.99, and no forecasting power for the one-month-ahead DG. This supports the claim of Cochrane (2008, 2011) that the dividend-price ratio captures the time change in discount rates. The TMS_{TS} has predictive power for both the discount rate and cash-flow proxies, as both slope estimates are statistically significant. This suggests that the predictability power of the TMS_{TS} is from both channels. Interestingly, this analysis also unveils that the predictability power of the TMS operating through the cash-flow channel is concentrated at the business cycle frequency, while the one operating through the discount rate channel is concentrated at the low frequency.

[11] Data is from Goyal and Welch (2008) updated database and the sample period is 1973-2016.

5 Out-of-sample forecasting

As it has been emphasized in the literature (*e.g.* Goyal and Welch, 2008 and Huang, Jiang, Tu, and Zhou, 2015), an OOS exercise is more relevant to evaluate effective return predictability in real time while avoiding the IS over-fitting issue, eventual small-sample size distortions and the look-ahead bias concern.

The OOS forecasts are produced using a sequence of expanding windows. We use an initial sample (1973:01 to 1989:12) to make the first OOS forecast. The sample is then increased by one observation and a new OOS forecast is produced. This is the procedure until the end of the sample. The full OOS period runs from 1990:01 to 2017:12.

The h-step-ahead OOS forecast of the excess returns, $\hat{r}_{t:t+h}$, is computed as

$$\hat{r}_{t:t+h} = \hat{\alpha}_t + \hat{\beta}_t x_t \ , \tag{5}$$

where $\hat{\alpha}_t$ and $\hat{\beta}_t$ are the OLS estimates of α and β in equation (2), respectively, using data from the beginning of the sample until month t.[12]

The OOS forecasting performance of each predictor is evaluated using the Campbell and Thompson (2008) R^2_{OS} statistic. As is standard in the literature, the benchmark model is the historical mean (HM) forecast $\overline{r}_t$, which is the average excess return up to time t. The R^2_{OS} statistic measures the proportional reduction in the mean squared forecast error for the predictive model ($MSFE_{PRED}$) relative to the historical mean ($MSFE_{HM}$) and is given by

$$R^2_{OS} = 100\left(1 - \frac{MSFE_{PRED}}{MSFE_{HM}}\right) = 100\left[1 - \frac{\sum_{t=t_0}^{T-h}\left(r_{t:t+h} - \hat{r}_{t:t+h}\right)^2}{\sum_{t=t_0}^{T-h}\left(r_{t:t+h} - \overline{r}_t\right)^2}\right] \ ,$$

[12] As the MODWT MRA at any given point in time uses information of neighboring data points (both past and future), we recompute the frequency components at each iteration of the OOS forecasting process to make sure we only use current and past information when making the forecasts. As a result, our forecasting method does not suffer from a look-ahead bias.

where $\hat{r}_{t:t+h}$ is the excess return forecast from the model using each of the alternative predictors. A positive (negative) R^2_{OS} indicates that the predictive model outperforms (underperforms) the HM in terms of MSFE.

As in Rapach, Ringgenberg, and Zhou (2016), the statistical significance of the results is evaluated using the Clark and West (2007) statistic. This statistic tests the null hypothesis that the MSFE of the HM model is less than or equal to the MSFE of the predictive model against the alternative hypothesis that the MSFE of the HM model is greater than the predictive model ($H_0 : R^2_{OS} \leq 0$ against $H_A : R^2_{OS} > 0$).

Columns two to six of Panel A in Table 4 show the R^2_{OS} of each predictor for the entire OOS period (1990:01-2017:12). As in the IS analysis, five forecasting horizons h are considered.

For forecasting horizons up to six months, the TMS_{TS} is a poor OOS predictor of the ERP (negative R^2_{OS}). However, it outperforms the HM benchmark (positive and statistically significant R^2_{OS}) at the one-year- and two-years-ahead forecasting horizons.

The results for the different frequency components of the TMS allow us to uncover some interesting features about the OOS predictive power of the TMS. The high and business-cycle frequencies of the TMS (TMS_{HF} and TMS_{BCF}) perform rather poorly as OOS ERP predictors. This result is hardly surprising given their poor IS performance. In contrast, the TMS_{LF} has a remarkable OOS forecasting power for all forecasting horizons under analysis. Its R^2_{OS} ranges between 2.09% for h=1 and 31.9% for h=24.

To evaluate the consistency over time of the OOS performance of the predictors, we look at the dynamics of the difference between the cumulative square forecasting error for the HM forecasting model and the cumulative square forecasting error when the TMS_{TS} and the TMS_{LF} are used as ERP predictors. The results are plotted in Figure 2 and should be read as follows. When the line rises (falls), the predictive regression using the TMS_{TS} (black line) and the TMS_{LF} (blue line) outperforms (underperforms) the HM. A forecasting model/variable

that consistently outperforms the HM over time would then feature an upward-sloping curve. Furthermore, the R^2_{OS} is positive when the end point is above the zero line. For all forecasting horizons, the TMS_{LF} consistently outperforms the HM benchmark during the entire OOS period (excluding the first five years for h=1, 6 and 12).

From a statistical point of view, these results show that the low frequency component of the TMS is a remarkably good predictor of the ERP for forecasting horizons from one month to two years. This is an improvement with respect to previous results using wavelet filtering methods in out-of-sample forecasting exercises (see *e.g.* Rua, 2011 and Kilponen and Verona, 2016), which find improved predictability only at very short horizons.

6 Asset allocation analysis

To analyze the economic value of the different predictive models from an asset allocation perspective, we consider a mean-variance investor allocating his or her wealth between equities and risk-free bills. At the end of month t, the investor optimally allocates

$$w_t = \frac{1}{\gamma}\frac{\hat{R}_{t+h}}{\hat{\sigma}^2_{t+h}} \tag{6}$$

of the portfolio to equity for the period from t to t+h. In equation (6), γ is the investor's relative risk aversion coefficient, $\hat{R}_{t+h}$ is the model prediction of excess return at time t for the period t+h, and $\hat{\sigma}^2_{t+h}$ is the forecast of the variance of the excess return. As in Rapach, Ringgenberg, and Zhou (2016), we assume a relative risk aversion coefficient of three, use a ten-year moving window of past excess returns to estimate the variance of the excess return, and constrain the weights w_t to a range between -0.5 and 1.5. These constraints introduce limits to the possibilities of short-selling and leveraging the portfolio.

We assume that the rebalancing frequency of the portfolio is equal to the forecasting horizon

h. Taking the semester horizon (h=6) as an example, the procedure is as follows. The investor uses the model prediction of excess returns over the next six months and the rule (6) to define the equity weight for the next six months. Then, at the end of that semester, the investor updates the model prediction of excess returns and determines the new weight using the non-overlapping return forecasts.

The average utility (or certainty equivalent return, CER) of an investor that uses the portfolio rule (6) is given by $CER = \overline{RP} - 0.5\gamma\sigma^2_{RP}$, where $\overline{RP}$ and σ^2_{RP} are the sample mean and variance of the portfolio return, respectively. We report the annualized utility gain from using the predictive models associated with different predictors. The utility gain is computed as the difference between the CER for an investor that uses the predictive model to forecast excess returns and the CER for an investor who uses the HM benchmark for forecasting. The difference can be interpreted as the annual portfolio management fee that an investor is willing to pay for access to the alternative forecasting model versus the historical average forecast. The analysis of different forecasting/rebalancing horizons (from one month to two years) allows us to take into account the perspective of agents with different profiles, as they include those with short- and medium-term approaches (*e.g.* some mutual funds) and those with longer-term horizons (*e.g.* central banks, pension and sovereign wealth funds).

Reported results in columns seven to eleven in Panel A of Table 4 show that the performance of the TMS_{LF} is strong also from an economic point of view. The CER gains obtained are remarkable and range from 453 basis points (h=24) to 659 basis points (h=3). From a practical standpoint, this means that the information contained in the TMS_{LF} may be useful to investors with different profiles regarding their forecasting and rebalancing horizons.

To complement this analysis, Figure 3 provides a dynamic perspective of the portfolio and cumulative wealth for an investor that uses a trading strategy (for h=1) based on the ERP forecast using the HM model (black dashed line), the TMS_{TS} (black solid line), and the TMS_{LF} (blue line). Panel A presents the dynamic equity weights (resulting from equation

6). Three results stand out. First, changes in the equity exposure of the TMS_{LF} portfolio are much smoother than those using the TMS_{TS}. Second, a trading strategy based on the TMS_{LF} is unaffected by the lower bound on the equity weight (-0.5), but is quite often constrained by the upper bound (1.5). Third, the strategy based on the TMS_{LF} displays excellent market timing in the three business cycle recessions. The exposure of the TMS_{LF} based portfolio to the equity market smoothly decreases before the occurrence of a recession (leading the portfolio to enter the recession with a rather low exposure to the risky asset), starts to increase in the late stage of the recession period, and continues to increase, smoothly, at the beginning of the subsequent expansionary period.

Panel B in Figure 3 shows the log cumulative wealth for an investor that begins with \$1 and reinvests all proceeds. Consistent with the results reported in Table 4 and Figure 2, the strategy based on the TMS_{LF} clearly outperforms those based on the HM and on the time series of the TMS. In particular, an investor who put \$1 at the end of December 1989 would have accumulated approximately \$43 (\$8.6 / \$6.9) by the end of December 2017 using a strategy based on the TMS_{LF} (TMS_{TS} / HM).

7 Interpretation of the results

We now discuss several possible interpretations of the predictive power of the TMS_{LF} .

In section 4 we show that the ERP predictability power of the TMS_{LF} comes from the discount rate channel. Accordingly, high (low) TMS_{LF} predicts high (low) future returns, because it predicts low (high) discount rates. This implies an increased (decreased) appetite for risk-taking, triggering an increased (decreased) future equity exposure. This lead-lag pattern is illustrated in Figure 4. Let us start from the most left point A in the Figure, where both the TMS_{LF} and the optimal equity exposure are at their maximum. After point A, the TMS_{LF} starts to decrease while the optimal equity exposure still stays at its maximum

for approximately four more years (until point B), after which the equity exposure starts to decrease. At point C the TMS_{LF} reaches its relative minimum, while the equity exposure continues to decrease and reaches its relative minimum immediately before the beginning of the recession. It stays around that level until the very end of the recession (point D), after which it starts to increase again. After point D, approximately the same lead-lag pattern restarts. So this Figure shows the good market timing of the TMS_{LF} as ERP predictor.

Another possible explanation comes from Dew-Becker and Giglio (2016). They find that low frequency shocks in the consumption growth are significantly priced in the U.S. equity markets, which affects the one-period innovation in the stochastic discount factor, whereas business-cycle and higher frequencies are not priced. This supports the existence of aversion to low frequency fluctuations by investors in the equity market, and is related to our findings in two ways. First, it is the dynamics of the low-frequency component of macroeconomic variables – rather than their business cycle or higher frequencies components – that is really relevant for the equity markets evolution, either from a cross-sectional or from a forecasting perspective. Second, this relationship works through the discount rate channel.

Finally, in Figure 5 we plot the dynamics of the one-month ahead ERP forecast using the TMS_{LF} and the high frequency, business-cycle and low frequency components of the ERP (top, middle and bottom graph, respectively). It is clear that the ERP predictability power of the TMS_{LF} comes essentially from its ability to capture the low frequency dynamics of the ERP. In fact, the correlation between the forecast with the TMS_{LF} and the low frequency component of the ERP is 0.62, while the correlation between the forecast with the TMS_{LF} and the other frequency components are much lower. This is consistent with empirical evidence that there are low-frequency, decades-long shifts in asset values relative to measures of macroeconomic fundamentals in the U.S. (see *e.g.* Bianchi, Lettau, and Ludvigson, 2017).

8 Extensions and robustness analysis

8.1 Different sample periods

We first test the robustness of the results by evaluating the forecasting performance of the low frequency component of the TMS in different sample periods.

First, we divide the OOS period into two sub-periods: from January 1990 to December 2006, which broadly corresponds to the great moderation period, and from January 2007 onwards, which includes the great financial crisis and its aftermath. Panel A of Table 5 presents the R^2_{OS} and CER gains for the TMS_{TS} and the TMS_{LF}. For both sub-sample periods and all forecasting horizons considered, the TMS_{LF} has strong statistical and economic performances.

Second, we evaluate the one-month-ahead return forecasts based on the TMS_{TS} and TMS_{LF} during periods of bad, normal, and good economic growth. These regimes are defined as the bottom, middle, and top-third of sorted growth rates of industrial production in the US, respectively.[13] This analysis is motivated by the fact that, while there is common agreement in the literature that return predictability is usually concentrated in recessions,[14] there is an ongoing debate about OOS returns predictability during expansions and good times. Henkel, Martin, and Nardari (2011), Ferreira and Santa-Clara (2011) and Neely, Rapach, Tu, and Zhou (2014) find no return predictability during expansions, whereas Dangl and Halling (2012) and Huang, Jiang, Tu, and Zhou (2016) find statistically significant levels of OOS predictability during expansions and good times when using models with time-varying or state-dependent coefficients.

We report the R^2_{OS} and the CER gains for each regime in Panel A of Table 6. Overall, the

[13] Data on US industrial production was downloaded from Federal Reserve Economic Data at http://research.stlouisfed.org/fred2/.

[14] Using a general equilibrium model, Cujean and Hasler (2017) explore the issue of why the return predictability tends to concentrate in bad times.

TMS_{TS} is never significant while the TMS_{LF} is statistically significant in all sub-samples – even in good growth periods. In bad growth periods, the TMS_{LF} delivers R^2_{OS} of 2.87% and CER gains of 752 basis points. These values are higher than in the full sample case, thus confirming that return predictability and utility gains are higher in bad times.

8.2 Comparison with alternative filtering methods

We evaluate the importance of the filtering method used to extract the low frequency component of the TMS by using two alternative filters. The first one is the Christiano and Fitzgerald (2003) asymmetric band-pass filter, assuming a unit root with drift. The frequency bands of the filter are chosen so as to extract exactly the same frequency components as in our previous analysis: the high frequency (TMS_{BP-HF}), the business-cycle frequency (TMS_{BP-BCF}) and low frequency (TMS_{BP-LF}) components. The second filter is the one-sided Hodrick and Prescott (1997) filter, which is used to isolate the business-cycle component (TMS_{HP-CY}) from its low-frequency component (TMS_{HP-TR}).[15]

Panels B of Tables 2, 4, 5 and 6 report the R^2, R^2_{OS} and CER gains for TMS_{BP-HF}, TMS_{BP-BCF}, TMS_{BP-LF}, TMS_{HP-CY} and TMS_{HP-TR} for the full sample and for the different sub-sample periods. We only discuss the results for the TMS_{BP-LF} and TMS_{HP-TR}, as our main interest is to compare the performance of the TMS_{LF} with that of predictors with similar characteristics.

As regards the IS analysis (Table 2), the low-frequency components of the TMS obtained using the alternative filtering methods are statistically significant (at least at the 10% level) for all forecasting horizons. Looking at the OOS results (Tables 4 to 6), the TMS_{BP-LF} is a poor ERP predictor as it is never statistically significant. Regarding the TMS_{HP-TR}, it features positive and statistically significant R^2_{OS}s for all forecasting horizons when looking

[15] See Mehra (2004). As we use monthly data, we set the smoothness parameter of the HP filter to 129600 as suggested by Ravn and Uhlig (2002) and de Jong and Sakarya (2016).

at the entire sample period. However, its forecasting performance across different sub-sample periods is mixed. It performs poorly during the great moderation period for the one-month horizon and in periods of normal and good growth, but performs reasonably well in the other periods/horizons. Despite being a good ERP predictor, the TMS_{HP-TR} lacks robustness across sample periods.

These results show that the methodology used to extract the low frequency component of the TMS is crucial for the quality of the ERP forecasting exercise. Wavelet filtering methods enable the extraction of a low-frequency component with forecasting performance clearly superior to that obtained using alternative filters.

8.3 Comparison with other predictors

We evaluate the OOS ERP predictability performance of some variables which have been recently proposed as good ERP predictors. We consider two financial market variables – the excess bond premium (EBP, Gilchrist and Zakrajsek, 2012) and the yield gap (Maio, 2013), a macro variable – the output gap (Cooper and Priestley, 2009), a technical indicator based on financial market variable (TI-MA(2,12), Neely, Rapach, Tu, and Zhou, 2014), and a behavioral-related variable – the short-interest positions (SII, Rapach, Ringgenberg, and Zhou, 2016).

Due to data availability, the sample period starts in January 1973 and ends in December 2014. The OOS period spans from January 1990 to December 2014. Panel C of Table 4 reports the results. For comparison, we also report the results for the TMS_{LF} for this sample period. Overall, none of these alternative predictors outperforms the TMS_{LF}. Except for the SII, these variables are not good predictors of the ERP during the OOS under consideration. As shown by Rapach, Ringgenberg, and Zhou (2016), the SII is a good predictor up to a one-year horizon. Its forecasting power, however, deteriorates significantly at the two-year horizon.

Panels C of Tables 5 and 6 report the results for the OOS periods as in sub-section 8.1. As shown in Table 5, the SII exhibits an unstable performance as its success as ERP predictor is a fairly recent phenomenon. In fact, it strongly underperforms the HM benchmark during the first sub-sample period (up to 2006), while it features an outstanding performance in the second sub-sample period (from 2007 onwards).

9 Concluding remarks

In this paper we show that the low frequency component of the term spread, when extracted using wavelet filtering methods, has a remarkably robust empirical equity premium OOS forecasting power. Its OOS forecasting performance is strong for forecasting horizons ranging from one month to two years. It is also consistently stable throughout an OOS period comprising 27 years of monthly data. Importantly, it performs well in expansions and outperforms several variables that have recently been proposed as good ERP predictors.

In line with the literature claiming that stock market predictability comes from time variation of discount rates, we find that the excess stock returns predictability power of the low frequency component of the term spread operates through the discount rate channel. We also show that the methodology used to extract the low frequency component of the term spread is relevant. In particular, wavelet filtering methods allow the extraction of a predictor that is clearly superior versus predictors extracted using alternative filtering methods.

From a practical standpoint, the term spread (and its low frequency component obtained through a wavelets decomposition) is easy to compute from publicly available data, making it straightforward for financial market practitioners to use reported findings in their asset allocation decision process. Additionally, Bianchi, Lettau, and Ludvigson (2017) show evidence that "breaks" in the mean of the consumption-wealth variable (as introduced by Lettau and Ludvigson, 2001) are strongly associated with low frequency fluctuations in the real value

of the Federal Reserve's primary policy rate, with low policy rates associated with high asset valuations, and vice versa. Therefore, for policymakers interested in targeting valuation regimes, defined by the level of equity markets and excess returns, the low frequency of the term spread can be a very promising variable to look at.

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Table 1: Summary statistics and correlations

Panel A reports the summary statistics for the equity risk premium (ERP) and the predictors. Panel B reports the correlation coefficients for the predictors. Predictors are the original time series of the term spread TMS_{TS} and the three frequency components TMS_{HF}, TMS_{BCF} and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS of less than 16 months, between 16 and 128 months and greater than 128 months, respectively. The database contains 540 monthly observations from 1973:01 to 2017:12.

Panel A: Summary statistics						
Variable	Mean	Median	1st percentile	99th percentile	Std. dev.	AR(1)
ERP (%)	0.43	0.85	-11.7	10.5	4.40	0.05
TMS_{TS} (%, ann.)	1.58	1.75	-2.38	3.67	1.35	0.95
TMS_{HF} (%, ann.)	0.00	-0.01	-1.03	1.67	0.42	0.60
TMS_{BCF} (%, ann.)	0.00	0.09	-2.00	1.53	0.95	0.99
TMS_{LF} (%, ann.)	1.58	1.76	0.56	2.37	0.52	1.00

Panel B: Correlations				
Variable	TMS_{TS}	TMS_{HF}	TMS_{BCF}	TMS_{LF}
TMS_{TS}	1			
TMS_{HF}	0.47	1		
TMS_{BCF}	0.89	0.22	1	
TMS_{LF}	0.61	0.02	0.31	1

Table 2: In-sample predictive regression results

This table reports the β estimation by OLS of the predictive model (2) and the corresponding R^2 statistic (in percentage), for the various forecasting horizons ($h = 1, 3, 6, 12, 24$) and different predictors. The predictors in Panel A are the original time series of the term spread TMS_{TS}, and the three frequency components TMS_{HF}, TMS_{BCF}, and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS time series less than 16 months, between 16 and 128 months, and greater than 128 months, respectively. The predictors in Panel B are the high frequency, business-cycle and low frequency components (TMS_{BP-HF}, TMS_{BP-BCF} and TMS_{BP-LF}) of the TMS obtained using the BP filter and the cycle (TMS_{HP-CY}) and the low frequency component (TMS_{HP-TR}) of the TMS obtained using the one-sided HP filter. Each predictor variable is standardized to have a standard deviation of one. Brackets below the β estimates contain the heteroskedasticity- and autocorrelation-robust t-statistics for $H_0 : \beta = 0$ versus $H_A : \beta > 0$. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively, accordingly to wild bootstrapped p-values. The sample period runs from 1973:01 to 2017:12, monthly frequency.

Predictor	h=1		h=3		h=6		h=12		h=24	
	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2	$\hat{\beta}$	R^2
PANEL A: Predictors										
TMS_{TS}	0.33	0.55	0.30	1.33	0.28	2.22	0.36	6.95	0.32	12.9
	[1.64]*		[1.71]*		[1.63]*		[2.35]**		[2.99]**	
TMS_{HF}	0.17	0.16	0.03	0.02	-0.09	0.23	0.04	0.10	0.01	0.01
	[0.91]		[0.22]		[-0.76]		[0.56]		[0.15]	
TMS_{BCF}	0.21	0.22	0.23	0.79	0.26	1.90	0.30	5.04	0.29	10.0
	[1.12]		[1.37]		[1.53]		[1.96]*		[2.15]*	
TMS_{LF}	0.33	0.56	0.33	1.61	0.33	3.00	0.34	6.26	0.32	12.1
	[1.68]**		[1.93]**		[1.99]*		[2.25]**		[3.72]***	
PANEL B: Alternative filtering methods										
TMS_{BP-HF}	0.13	0.09	-0.03	0.01	-0.16	0.74	-0.02	0.02	-0.02	0.07
	[0.71]		[-0.19]		[-1.40]		[-0.27]		[-0.83]	
TMS_{BP-BCF}	0.20	0.21	0.23	0.78	0.26	1.95	0.30	4.78	0.28	9.68
	[1.11]		[1.40]		[1.56]		[1.95]*		[2.19]**	
TMS_{BP-LF}	0.28	0.40	0.28	1.14	0.27	2.09	0.28	4.13	0.23	6.47
	[1.38]*		[1.55]*		[1.60]*		[1.77]*		[2.63]**	
TMS_{HP-CY}	-0.22	0.25	-0.26	1.00	-0.28	2.15	-0.15	1.22	-0.06	0.39
	[-1.04]		[-1.22]		[-1.36]		[-0.93]		[-0.74]	
TMS_{HP-TR}	0.51	1.35	0.51	3.93	0.51	7.30	0.49	12.9	0.38	17.4
	[2.53]***		[2.81]***		[2.74]**		[2.74]**		[2.82]**	

Table 3: Economic channel analysis

This table reports the estimation results of equation (4) considering four predictors (X): the original time series of the term spread TMS_{TS}, and the three frequency components TMS_{HF}, TMS_{BCF}, and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS time series less than 16 months, between 16 and 128 months and greater than 128 months, respectively. DP stands for the dividend-price ratio and represents the discount rate channel. DG is the dividend growth and represents the cash flow channel. The regression slopes, the heteroskedasticity- and autocorrelation-robust t-statistics for $H_0 : \delta = 0$ versus $H_A : \delta < 0$ and $H_0 : \psi = 0$ versus $H_A : \psi > 0$ in equation (2), and the R^2 (in percentage) are reported. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively, accordingly to wild bootstrapped p-values. The sample period runs from 1973:01 to 2016:12, monthly frequency.

X_t	Y_{t+1}	δ	t-stat	ψ	t-stat	R^2
TMS_{TS}	DP	-0.24	-1.60*	0.99	218***	98.9
	DG	-0.08	-3.06***	0.04	0.58	3.78
TMS_{HF}	DP	-0.30	-0.67	0.99	218***	98.9
	DG	-0.02	-0.45	0.07	1.06	0.30
TMS_{BCF}	DP	-0.24	-1.21	0.995	216***	98.9
	DG	-0.15	-3.80***	0.09	1.29*	6.08
TMS_{LF}	DP	-0.68	-1.63**	0.99	196***	98.9
	DG	-0.04	-0.59	0.05	0.65	0.41

Table 4: Out-of-sample R-squares (R^2_{OS}) and annualized CER gains

Columns two to six report the OOS R-squares R^2_{OS} (in percentage) for the excess returns forecasts at h-month horizon from the model as given by equation (5). The R^2_{OS} measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean HM. The h-month-ahead OOS forecast of excess return is generated using a sequence of expanding windows. Panel A reports the results for the original time series of the term spread TMS_{TS}, and the three frequency components TMS_{HF}, TMS_{BCF}, and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS time series less than 16 months, between 16 and 128 months and greater than 128 months, respectively. Panel B reports the results for the high frequency, business-cycle and low frequency components (TMS_{BP-HF}, TMS_{BP-BCF} and TMS_{BP-LF}) of the TMS obtained using the BP filter, and the cycle (TMS_{HP-CY}) and the low frequency component (TMS_{HP-TR}) of the TMS obtained using the one-sided HP filter. Columns seven to eleven present the annualized certainty equivalent return (CER) gains (in percent) for an investor who allocates his or her wealth between equities and risk-free bills according to the rule (6), using stock return forecasts from model in equation (5) with alternative predictors under analysis instead of the forecasts based on the HM. Panel C reports the R^2_{OS} and the CER gains obtained using alternative predictors from the literature (excess bond premium, yield gap, output gap, technical indicator based on moving averages and the short interest index). The sample period is from 1973:01 to 2017:12. The OOS period is from 1990:01 to 2017:12, monthly frequency. Asterisks denote significance of the OOS *MSFE*-adjusted statistic of Clark and West (2007). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Predictor	R^2_{OS}					CER gains				
	h=1	h=3	h=6	h=12	h=24	h=1	h=3	h=6	h=12	h=24
PANEL A: Predictors										
TMS_{TS}	-0.72	-1.99	-1.28	3.47**	15.0***	0.10	0.52	0.33	1.38	1.84
TMS_{HF}	-0.87	-1.70	0.58*	-1.63	2.37	-1.16	-0.78	0.05	-0.81	-0.03
TMS_{BCF}	-1.52	-5.01	-8.16	-7.78	5.19*	-2.25	-1.96	-2.19	-0.58	0.81
TMS_{LF}	2.09***	6.36***	12.0***	22.9***	31.9***	5.91	6.59	6.34	5.46	4.53
PANEL B: Alternative filtering methods										
TMS_{BP-HF}	-0.13	-0.28	-5.25	-0.43	-0.50	1.28	-0.14	-1.05	-0.01	-0.30
TMS_{BP-BCF}	-0.68	-2.30	-3.81	-2.58	6.46**	-1.29	-1.33	-1.24	-0.21	0.78
TMS_{BP-LF}	-0.01	0.06	0.59	1.55	8.26	0.90	1.89	1.63	1.83	2.73
TMS_{HP-CY}	0.21	0.79	1.91	-0.98	1.41	0.38	0.39	0.52	-0.82	0.64
TMS_{HP-TR}	1.24**	3.89***	8.10***	15.5***	20.5***	3.83	4.03	3.98	3.40	2.21
PANEL C: Alternative predictors (OOS period: 1990-2014)										
EBP	0.97	0.66	-7.23	-10.1	-11.6	3.90	5.09	0.14	-1.30	1.44
Yield gap	-1.13	-4.22	-8.79	-15.7	-16.2	-0.21	-0.25	-1.01	-1.29	-0.87
Output gap	-3.24	-7.40	-8.52	-5.25	-8.02	-3.73	-2.86	-1.65	-0.75	0.70
TI-MA(2,12)	1.20*	0.76	2.55	0.86	-0.34	4.78	1.76	2.53	0.67	0.21
SII	1.94***	6.52***	11.6***	13.1**	4.85	4.18	4.67	5.44	3.40	-0.02
TMS_{LF}	2.17***	6.49***	12.1***	23.1***	31.0***	6.29	6.99	6.68	5.63	4.24

Table 5: Out-of-sample R-squares (R^2_{OS}) and annualized CER gains

Columns three to six present the OOS R-squares R^2_{OS} (in percentage) for the excess returns forecasts at h-month horizon from the model as given by equation (5). Panel A reports the results for the original time series of the term spread TMS_{TS}, and the low frequency component of the term spread TMS_{LF}, obtained through wavelets decomposition capturing oscillations of the TMS time series greater than 128 months. Panel B reports the results for the low frequency component (TMS_{BP-LF}) of the TMS obtained using the BP filter, and the low frequency component (TMS_{HP-TR}) of the TMS obtained using the one-sided HP filter. Panel C reports the results obtained using alternative predictors from the literature (excess bond premium, yield gap, output gap, technical indicator based on moving averages and the short interest index). The R^2_{OS} measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean HM. The h-month-ahead OOS forecast of excess returns is generated using a sequence of expanding windows. Columns seven to ten present the annualized certainty equivalent return (CER) gains (in percent) for an investor who allocates his or her wealth between equities and risk-free bills according to the rule (6), using stock return forecasts from model in equation (5) with alternative predictors under analysis instead of the forecasts based on the HM. The sample period runs from 1973:01 to 2017:12. Two OOS forecasting periods are considered. The first runs from 1990:01 to 2006:12 and the second from 2007:01 to 2017:12, monthly frequency. Asterisks denote significance of the OOS *MSFE*-adjusted statistic of Clark and West (2007). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Sample period	Predictor	R^2_{OS}				CER gains			
		h=1	h=3	h=6	h=12	h=1	h=3	h=6	h=12
PANEL A: Predictors									
1990-2006	TMS_{TS}	-1.12	-3.13	-3.44	-2.37	0.72	1.67	0.71	0.13
	TMS_{LF}	1.66***	6.13***	13.5***	25.1***	5.51	6.57	5.62	4.72
2007-2017	TMS_{TS}	-0.16	-0.74	0.57	9.38***	-0.87	-1.28	-0.37	3.08
	TMS_{LF}	2.67***	6.62***	10.8***	20.8***	6.50	6.59	7.39	6.44
PANEL B: Alternative filtering methods									
1990-2006	TMS_{BP-LF}	0.05	0.79	2.57	4.17	1.34	2.95	2.19	3.22
	TMS_{HP-TR}	0.35	1.54*	4.29**	8.64***	2.57	3.20	2.28	1.47
2007-2017	TMS_{BP-LF}	-0.10	-0.74	-1.09	-1.09	0.24	0.30	0.90	0.04
	TMS_{HP-TR}	2.48**	6.48***	11.3***	22.4***	5.77	5.26	6.48	6.20
PANEL C: Alternative predictors (OOS period: 1990-2014)									
1990-2006	EBP	0.64	3.41**	-6.47	-10.7	2.59	4.31	-0.98	-1.23
	Yield gap	-0.99	-3.33	-8.43	-21.5	-0.89	-0.40	-1.35	-1.35
	Output gap	-4.05	-9.31	-13.0	-8.74	-5.12	-3.96	-2.60	-1.13
	TI-MA(2,12)	1.05	1.67	6.25**	0.24	4.17	2.15	2.72	-0.10
	SII	-0.15	-0.51	-0.95	-9.49	0.88	0.74	0.59	-0.38
2007-2014	EBP	1.48	-2.62	-7.90	-9.56	6.68	6.69	2.42	-1.47
	Yield gap	-1.34	-5.27	-9.11	-9.48	1.24	0.06	-0.28	-1.10
	Output gap	-1.96	-5.14	-4.50	-1.48	-0.80	-0.64	-0.11	-0.99
	TI-MA(2,12)	1.44	-0.33	-0.76	1.53	6.06	0.95	2.33	2.33
	SII	5.24***	14.9***	22.9***	37.4***	11.2	13.1	15.9	11.2

Table 6: Out-of-sample R-squares (R^2_{OS}) and annualized CER and SR gains

The sample period runs from 1973:01 to 2017:12. We divide the OOS in periods of bad growth, normal growth, and good growth. These regimes are defined as the bottom, middle, and top-third of sorted growth rates of industrial production in the US, respectively. This table reports, for the three regimes, the OOS R-squares R^2_{OS} (in percentage) for the excess returns forecasts at the one-month horizon ($h = 1$) from the model as given by equation (5). Panel A reports the results for the original time series of the term spread TMS_{TS}, and the low frequency component of the term spread TMS_{LF}, obtained through wavelets decomposition capturing oscillations of the TMS time series greater than 128 months. Panel B reports the results for the low frequency component (TMS_{BP-LF}) of the TMS obtained using the BP filter, and the low frequency component (TMS_{HP-TR}) of the TMS obtained using the one-sided HP filter. Panel C reports the results obtained using alternative predictors from the literature (excess bond premium, yield gap, output gap, technical indicator based on moving averages and the short interest index). The R^2_{OS} measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean HM. The one-month-ahead OOS forecast of excess return is generated using a sequence of expanding windows. The table also reports the annualized certainty equivalent return (CER) gains (in percent) for an investor who allocates his or her wealth between equities and risk- free bills according to the rule (6), using stock return forecasts from model in equation (5) with alternative predictors under analysis instead of the forecasts based on the HM. Asterisks denote significance of the OOS *MSFE*-adjusted statistic of Clark and West (2007). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Predictor	Bad growth		Normal growth		Good growth	
	R^2_{OS}	CER gains	R^2_{OS}	CER gains	R^2_{OS}	CER gains
PANEL A: Predictors						
TMS_{TS}	0.57	0.92	-2.61	-1.70	-0.82	1.03
TMS_{LF}	2.87***	7.52	2.17**	4.95	1.16**	5.24
PANEL B: Alternative filtering methods						
TMS_{BP-LF}	-0.10	1.24	-0.44	-0.81	0.37	2.30
TMS_{HP-TR}	2.51***	6.29	0.12	1.33	0.62	3.84
PANEL C: Alternative predictors (OOS period: 1990-2014)						
EBP	-0.30	5.97	3.26**	4.39	0.77	1.37
Yield gap	-1.87	-0.39	0.20	0.46	-1.22	-0.67
Output gap	-2.23	-4.18	-5.23	-4.37	-2.97	-2.69
TI-MA(2,12)	3.47*	10.3	2.49*	5.58	-2.23	-1.46
SII	2.88**	4.98	0.13	1.73	2.16*	5.79
TMS_{LF}	3.00***	7.88	2.13**	5.55	1.29**	5.45

Figure 1: Time series of the term spread and of its frequency components

This figure reports the original time series of the term spread TMS_{TS} (black line) and of its three frequency components TMS_{HF}, TMS_{BCF} and TMS_{LF} obtained through wavelets decomposition capturing oscillations of the TMS less than 16 months (green line), between 16 and 128 months (red line), and greater than 128 months (blue line), respectively. Gray bars denote NBER-dated recessions. Sample period runs from 1973:01 to 2017:12, monthly frequency.

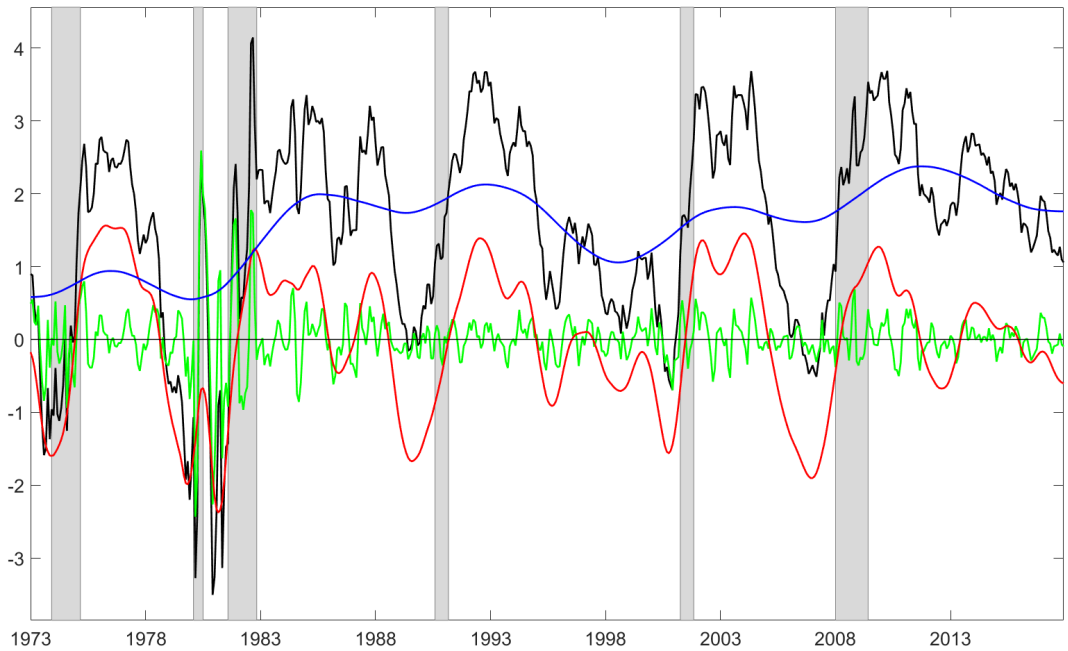


Figure 2: Cumulative sum of squared forecast errors

This figure reports the difference between the cumulative square forecasting error for the HM forecasting model and the cumulative square forecasting error for the predictive regression based on the model (5) for the original time series of the term spread TMS_{TS} (black line) and the low frequency component of the term spread TMS_{LF} (blue line). Gray bars denote NBER-dated recessions. The sample period runs from 1973:01 to 2017:12. The OOS forecasting period runs from 1990:01 to 2017:12, monthly frequency.

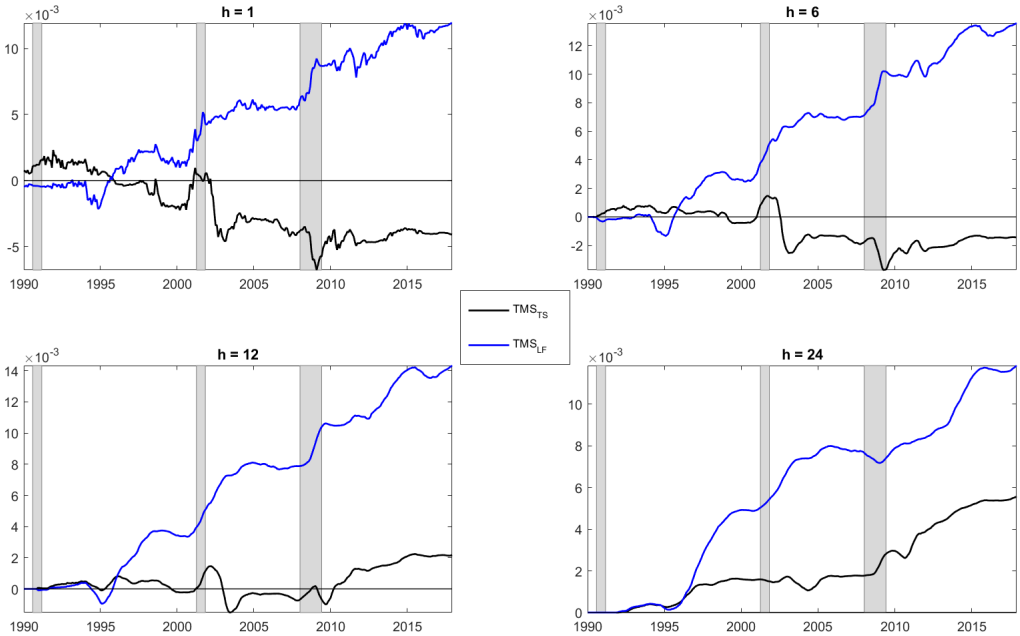
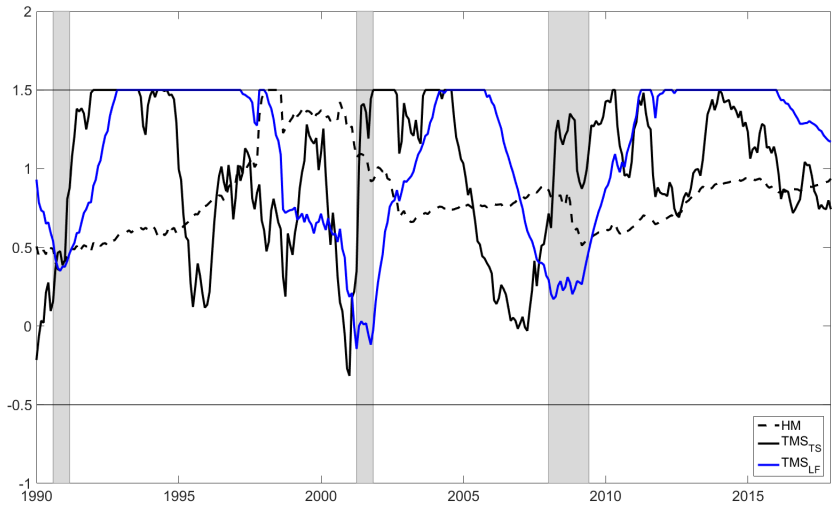


Figure 3: Equity weights and log cumulative wealth

Panel A plots the dynamics of the equity weight for a mean-variance investor who allocates monthly his or her wealth between equities and risk-free bills according to the rule (6), using stock return forecasts based on the HM benchmark (dashed black line), the original time series of the term spread TMS_{TS} (solid black line), and the low frequency component of the term spread TMS_{LF} (blue line). The equity weight is constrained to a range between -0.5 and 1.5. Panel B delineates the corresponding log cumulative wealth for the investor, assuming he or she begins with $1 and reinvest all proceeds. The investor is assumed to have a relative risk aversion coefficient of three. Gray bars denote NBER-dated recessions. Sample period runs from 1990:01 to 2017:12, monthly frequency.

A. Equity weights



B. Log cumulative wealth

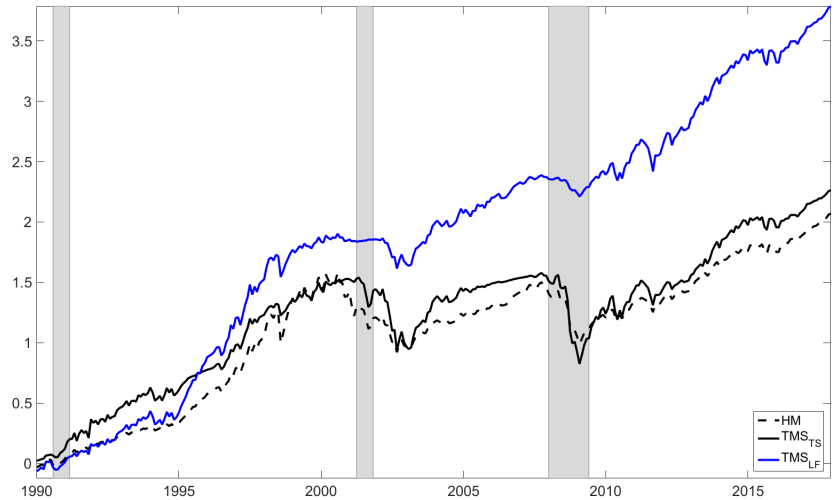


Figure 4: Equity weights and low frequency component of the term spread

This figure plots the dynamics of the low frequency component of the term spread (TMS_{LF}, black line) and the equity weight (blue line) for a mean-variance investor who allocates monthly his or her wealth between equities and risk-free bills according to the rule (6) using stock return forecasts based on the TMS_{LF}. Both series are centered to have zero mean and scaled to have standard deviation 1. The investor is assumed to have a relative risk aversion coefficient of three. Gray bars denote NBER-dated recessions. Sample period runs from 1990:01 to 2017:12, monthly frequency.

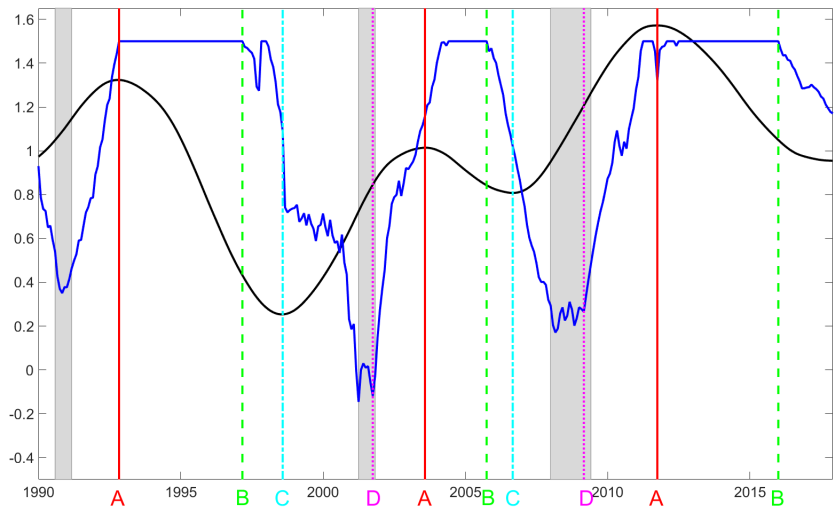
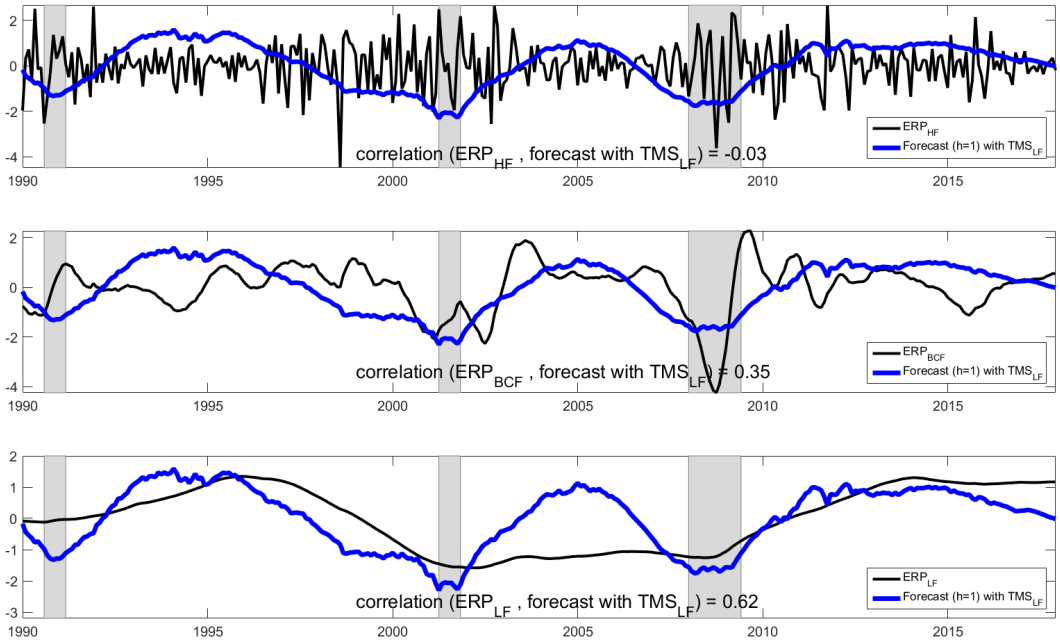


Figure 5: ERP frequency components and ERP forecast ($h = 1$) based on the TMS_{LF}

This figure plots the dynamics of the one-month ahead equity risk premium (ERP) forecast based on the low frequency component of the term spread (TMS_{LF}, blue line) and the high frequency, business-cycle and low frequency components of the ERP (top, middle and bottom graphs, respectively, black lines). The series are centered to have zero mean and scaled to have standard deviation 1. Gray bars denote NBER-dated recessions. Sample period runs from 1990:01 to 2017:12, monthly frequency.



Appendix A

The discrete wavelet transform (DWT) multiresolution analysis (MRA) allows the decomposition of a time series into its constituent multiresolution (frequency) components. There are two types of wavelets: father wavelets (ϕ), which capture the smooth and low frequency part of the series, and mother wavelets (ψ), which capture the high frequency components of the series, where $\int \phi(t)\,dt = 1$ and $\int \psi(t)\,dt = 0$.

Given a time series y_t with a certain number of observations N, its wavelet multiresolution representation is given by

$$y_t = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \quad , \qquad (7)$$

where J represents the number of multiresolution levels (or frequencies), k defines the length of the filter, $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ are the wavelet functions and $s_{J,k}$, $d_{J,k}$, $d_{J-1,k}, \ldots,$ $d_{1,k}$ are the wavelet coefficients.

The wavelet functions are generated from the father and mother wavelets through scaling and translation as follows

$$\begin{aligned} \phi_{J,k}(t) &= 2^{-J/2}\phi\left(2^{-J}t - k\right) \\ \psi_{j,k}(t) &= 2^{-j/2}\psi\left(2^{-j}t - k\right) \quad , \end{aligned}$$

while the wavelet coefficients are given by

$$\begin{aligned} s_{J,k} &= \int y_t \phi_{J,k}(t)\, dt \\ d_{j,k} &= \int y_t \psi_{j,k}(t)\, dt \ , \end{aligned}$$

where $j = 1, 2, ..., J$.

Due to the practical limitations of DWT in empirical applications, we perform wavelet decomposition analysis here by applying the maximal overlap discrete wavelet transform (MODWT). The MODWT is not restricted to a particular sample size, is translation-invariant so that it is not sensitive to the choice of the starting point of the examined time series, and does not introduce phase shifts in the wavelet coefficients (so peaks or troughs in the original time series are correctly aligned with similar events in the MODWT MRA). This last property is especially relevant in the forecasting exercise.

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