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Modelling a Small Open Economy Using a Wavelet-Based Control Model

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Abstract. This paper develops a wavelet-based control system model that can be used to simulate fiscal and monetary strategies in an open economy context in the time-frequency domain. As the emphasis on real exchange rate stability is increased, the model simulates the effects on both the aggregate and decomposed trade balance under both constant and depreciating real exchange rate targets, and also the effects on the real GDP expenditure components. This paper adds to recent research in this area by incorporating an external sector via the use of a real effective exchange rate as a driver of output. The research is also the first to analyze exchange rate effects within a time-frequency model with integrated fiscal and monetary policies in an open-economy applied wavelet-based optimal control setting. To demonstrate the usefulness of this model, we use post-apartheid South African macro data under a political targeting design for the frequency range weights, where we simulate jointly optimal fiscal and monetary policy under varying preferences for real exchange rate stability.

Keywords: Discrete Wavelet Analysis; Exchange Rate, Fiscal Policy; Monetary Policy; Optimal Control, South Africa

JEL classifications C61 . C63 . C88 . E52 . E61 . F47

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1. Introduction

Wavelet-based optimal control macro models have recently been used in a series of papers to simulate economic policy for both the U.S. and euro area economies. Crowley and Hudgins (2015) employed a Maximal Overlap Discrete Wavelet Transform (MODWT) to obtain the time-frequency domain cyclical decomposition of quarterly U.S. GDP component data, and simulated optimal fiscal policy. Crowley and Hudgins (2017b,c) then expanded the wavelet-based control model to simulate jointly optimal fiscal and monetary policy within a closed economy framework. This research has its origins in work done by Kendrick and Shoukry (2014), who employed an optimal control accelerator model to analyze optimal fiscal policy with an interest rate component for monetary policy.

The aim of this paper is to expand the theoretical framework used in Crowley and Hudgins (2017a) to derive a generalized comprehensive MODWT wavelet-based optimal control open-economy accelerator model that can be applied across countries that utilize flexible exchange rates. Once this is accomplished, the model is used to simulate the effects of varying exchange targets and restrictions. The simulation analysis example utilizes the Crowley and Hudgins (2017a) MODWT wavelet decomposition of South African post-apartheid data over the period 1970 – 2016. Section 2 develops the theoretical framework of the accelerator model. Section 3 derives the optimal tracking control framework, and Section 4 utilizes a MATLAB software program to simulate the optimal control policies within a large state-space system under varying policymaker preferences for short-term real exchange rate stability.

2. Wavelet-Based Optimal Control Model Derivation

2.1 Discrete Wavelet Analysis

Wavelet analysis has now become more accepted in economics, with numerous papers appearing in economics journals exploring a variety of existing issues in the time-frequency domain. Examples include Aguiar-Conraria and Soares (2011), Dar, Samantaraya, and Shah (2014), Tiwari et al. (2015), Chen (2016), Kumar et al. (2016), Verona (2016) and Power et al. (2017). Other examples of macroeconomic analysis using the MODWT can be found in Gallegati et al. (2011), Crowley and Hughes Hallett (2016), Faria and Verona (2016).

Time-frequency domain methods permit the extraction of cyclical information from a time series. Following the discrete wavelet analysis in Crowley and Hudgins (2015, 2017b,c), the value of a variable x at time instant k, x_k, can be expressed using Mallat's pyramid algorithm and multiresolutional analysis, as

$$x_k \approx S_{J,k} + d_{J,k} + d_{J-1,k} + ... + d_{1,k} \tag{1}$$

In wavelet analysis, the $d_{j,k}$ terms are wavelet detail crystals, $j = 1,\ldots, J$; $S_{J,k}$, is the wavelet smooth (which is a trend component), and J represents the number of scales (frequency bands). Although there are many different wavelet filter functions that are used in discrete wavelet analysis, this paper employs the asymmetric Daubechies 4-tap (D4) wavelet function with periodic boundary conditions. Table 1 defines the time-frequency ranges for all of the wavelet decompositions.

Table 1
The time intervals associated with each of the frequencies

J	*Time interval in quarters*	*Time interval in years*
1	2 to 4 quarters	6 months to 1 year
2	4 – 8 quarters	1 – 2 years
3	8 – 16 quarters	2 – 4 years
4	16 – 32 quarters	4 – 8 years
5	32 – 64 quarters	8 – 16 years

2.2 Open Economy Partial Accelerator Model

This section extends the closed-economy partial accelerator models of Crowley and Hudgins (2015, 2017b,c) to include the MODWT-decomposed series for exports, imports, the real exchange rate, and the foreign interest rate. The GDP components of domestic output (Y) are nested in the following blocks: personal consumption (C_j); private domestic investment (I_j); government expenditure (G_j); exports (EX_j); and imports (IM_j). At each frequency range, the wavelet-based GDP components remove the effects at all other four frequency ranges, so that each component only includes the crystal (d) at that frequency range and the modified smooth base-level trend (S)[1]. The wavelet-based components for any variable are therefore defined in equation (2) as follows:

$$X_{j,k} = d_{X,j,k} + S_{X,j,k} \qquad j = 1, \ldots, 5; \quad k = 1, \ldots, K \tag{2}$$

Equation (2) provides for a cyclical analysis of the level series over different frequency ranges by allowing economic cycle fluctuations to be superimposed onto the wavelet smooth. Thus, the level series can be analyzed by incorporating the separate cycles inherent within the pre-determined frequency ranges from discrete wavelet analysis.

The reduced form model in equations (3) through (15) expands the linear accelerator reduced-form block component matrix system of Crowley and Hudgins (2015, 2017c) for each frequency range, $j = 1, \ldots, 5$, as defined in table 1, where the $\beta_{j;0}$ coefficients are constants and the number of lags for any given variable is denoted by $L_{(\cdot)}$. The ir_j^{dom} and ir_j^{for} blocks represent the wavelet decomposition of the short-term domestic and foreign interest rates, respectively. Block RER_j is the wavelet decomposed real exchange rate (index of foreign currency unit per domestic currency unit), and the $\omega_{(.),j}$ terms represent blocks of random disturbance errors. Equation (3) specifies the consumption block as linearized functions of lag structures of consumption, investment, government spending, and the real exchange rate. Equation (4) specifies the investment block as linearized functions of the domestic GDP (Y^{dom}) and the domestic interest rate.

[1] The components of MODWT decomposed series are not orthogonal and thus the components only approximately sum to the original series. As in Crowley and Hudgins (2017a,b,c), the modified smooth is calculated as the residual of the original series and the sum of the component crystals, so that the sum of the crystals and the modified smooth equals the original data for each point in the time series.

$$C_{j,k} = \beta_{C,j,0} + f_{C,j}(C_{j,k-1},\dots, C_{j,k-L_C}, I_{j,k-1},\dots, I_{j,k-L_I},$$
$$G_{j,k-1},\dots, G_{j,k-L_G}, RER_{j,k-1},\dots, RER_{j,k-L_{RER}}) + \omega_{C,j,k-1} \tag{3}$$

$$I_{j,k} = \beta_{I,j,0} + f_{I,j}(Y^{dom}{}_{j,k-1},\dots, Y^{dom}{}_{j,k-L_{Ydom}},$$
$$ir^{dom}{}_{j,k-1},\dots, ir^{dom}{}_{j,k-L_{irdom}}) + \omega_{I,j,k-1} \tag{4}$$

$$G_{j,k} = \rho_j G_{j,k-1} + \omega_{G,j,k-1} \tag{5}$$

Equation (5) extracts the current trend in the government spending block (G_j). Future government spending will be determined by an optimal control system in section 4, which simulates the optimal policy forecasts. Equation (6) gives the export equation block, where exports are a function of the lag structures of exports, foreign national income (Y^{for}), and the real exchange rate at each frequency range. Similarly, equation (7) specifies the import equation block, where imports are a function of the lag structures of imports, domestic income (Y^{dom}), and the real exchange rate.

$$EX_{j,k} = \beta_{EX,j,0} + f_{EX,j}(EX_{j,k-1},\dots, EX_{j,k-L_{EX}}, Y^{for}{}_{j,k-1},\dots, Y^{for}{}_{j,k-L_{Yfor}},$$
$$RER_{j,k-1},\dots, RER_{j,k-L_{RER}}) + \omega_{EX,j,k-1} \tag{6}$$
$$IM_{j,k} = \beta_{IM,j,0} + f_{IM,j}(IM_{j,k-1},\dots, IM_{j,k-L_{IM}}, Y^{dom}{}_{j,k-1},\dots, Y^{dom}{}_{j,k-L_{Ydom}},$$
$$RER_{j,k-1},\dots, RER_{j,k-L_{RER}}) + \omega_{IM,j,k-1} \tag{7}$$

The real exchange rate equation block is given by equation (8), where the real exchange rate is determined by the lagged structures of the domestic interest rate, the foreign interest rate, the real exchange rate, and the country default risk. Equation (8) therefore also captures interest rate parity influences on domestic and foreign interest rates.

$$RER_{j,k} = \beta_{RER,j,0} + f_{RER,j}(ir^{dom}{}_{j,k-1},\dots, ir^{dom}{}_{j,k-L_{irdom}},$$
$$ir^{for}{}_{j,k-1},\dots, ir^{for}{}_{j,k-L_{irfor}}, RER_{j,k-1},\dots, RER_{j,k-L_{RER}},$$
$$Risk_{k-1},\dots, Risk_{k-L_{Risk}}) + \omega_{RER,j,k-1} \tag{8}$$

Equation (9) models the modified smooth trend processes for national output, consumption, investment, government spending, exports, imports, the interest rate, and the real exchange rate as first-order difference equations, where the ω_S terms represent random disturbances in each equation, thereby satisfying the standard assumptions.

$$S_k = s_1 S_{k-1} + s_2 X_{k-1} + \omega_{S,k-1} \tag{9}$$

In equation (9), the coefficients on the lagged modified smooth trend variables (S), and the coefficients on the lagged component variables (X), produce a weighted average growth contribution toward the current trend values of each component series.

Following Crowley and Hudgins (2015, 2017a,b,c), the current national debt level influences consumption and investment through changes in expected national output, due to rational expectations. Define the following variables:

$DEBT_k$ = the total stock of government debt in quarter k

$\hat{G}^{d}_{j,k}$ = the trend government obligations at frequency range j in quarter k

$G^{e}_{j,k}$ = expected contribution of government spending to national output

Equation (10) defines the trend process for government spending at each frequency range, where the current trend value depends on the lagged value of the actual level of government spending, where ρ_j is the growth coefficient, estimated by equation (5).

$$\hat{G}^{d}_{j,k} = \rho_j G_{j,k-1} + \omega_{G,j,k-1} \qquad j = 1, \ldots, 5 \qquad (10)$$

In equation (11), the expected value of government spending in any period k is determined based on a weighted average of the lagged actual spending and the lagged trend value within the frequency range.

$$G^{e}_{j,k} = \phi_{j,k}\left[G_{j,k-1} - \pi_{j,k}(DEBT_{k-1} - DEBT_0)\right] + (1-\phi_{j,k})\hat{G}^{d}_{j,k-1} \qquad (11)$$

$$0 < \phi_j < 1; \; j = 1,\ldots,5$$

Government spending primarily affects the GDP components through expected national output, so that all government spending changes have a limited impact. The effectiveness of fiscal policy at any given frequency range increases with the value of ϕ_j. This formulation permits rational expectations behavior. Any new fiscal initiative pulses the current cycle at each frequency range, but the current contribution of government spending toward national output production is crowded-out by any national debt stock that exceeds its initial value.

The model is closed by equations (12) through (15). Equation (12) contains the national income identity.

$$Y_k = C_k + I_k + G_k + EX_k - IM_k \qquad (12)$$

Equation (13) defines net taxes (T_k), as the total government tax and income minus total government transfer payments in quarter k, which are to be generated as a constant percentage (τ) of national output.

$$T_k = \tau Y_k \qquad (13)$$

Following Kendrick and Shoukry (2014) and Crowley and Hudgins (2015, 2017a,c), we limit active fiscal policy to government spending at each frequency range, and compute government tax income and transfer payments as passively determined variables. This is consistent with Kliem and Kriwoluzky (2014), which finds little empirical evidence for the typical simple fiscal policy rules where tax rates respond to output that derived in Dynamic Stochastic General Equilibrium (DSGE) models.

Equation (14) calculates the resulting government budget deficit (or surplus, when the value is negative) in quarter k, which is given by DEF_k :

$$DEF_k = G_k - T_k \tag{14}$$

$$DEBT_k = 0.25 DEF_k + (1 + i_k) DEBT_{k-1} \tag{15}$$

The national debt ($DEBT_k$) in equation (15) is the sum of the current budget deficit (converted from annualized rates to quarterly levels) and the previous period debt stock, which grows at the quarterly interest rate of i_k.

This model in equations (3) through (15) can be specified with either constant coefficients, as in this paper, or with time varying coefficients. The model derives from the widely accepted macroeconomic accelerator framework that has been employed by Kendrick (1981), Kendrick and Shoukry (2014), Crowley and Hudgins (2015, 2017b,c), Hudgins and Na (2016), and Hudgins and Crowley (2017). It includes both interest rate and real exchange rate outlets for the transmission of monetary policy, and also adjusts fiscal policy effectiveness for rational expectations component based on government debt stock.

3. Optimal Tracking Control

The objective of the LQ tracking problem is as follows: given the linear state equations specified by (3) – (15), the fiscal policymakers choose the level of government spending at each of the five frequency ranges, while the monetary authorities choose the short-term interest rate at each frequency range in order to minimize the expected value of a quadratic performance index consisting of the weighted tracking errors for the variables in the model. A discussion of the benefits and drawbacks of symmetric quadratic performance indices can be found in Kendrick (1981)

Using the methods of Crowley and Hudgins (2016, 2017b,c), let a (*) represent the target for any given variable. The control vector elements are the tracking errors between the actual and targeted level of the fiscal and monetary variables at each frequency:

$$u_k = \left[u_{1,k} \ ; \ u_{2,k} \ ; \ \dots \ ; \ u_{10,k} \right]^T \tag{16}$$

$$u_{m,k} = G_{m,k} - G^*_{m,k} \text{ for } m = 1, \dots, 5;$$

$$u_{m,k} = ir^{dom}{}_{m-5,k} - ir^*{}_{m-5,k}\,; m = 6, \dots, 10$$

The ten control variables ($u_{m,k}$) contain the negative of the targeted levels of government spending and the domestic interest rate at each frequency; thus, these target variables are added to state equations for consumption, investment, exports, imports, and the real exchange rate, thereby converting the specification into a standard LQ-regulator format. The target level of government spending and the interest rate are added to $u_{m,k}$ over each frequency range in order to retrieve the component values for the simulations. These values for government spending and the interest rate at each frequency are also automatically recorded after a one period lag. The 126-equation matrix state-space equation system given by (17) includes: a state vector (x) which embeds the constants, wavelet-decomposed variables, aggregate variables, target variables; the control vector (u); and the disturbance vector (ω) that includes all of the disturbance terms.

$$x_{k+1} = A_k x_k + B_k u_k + D_k \omega_k \quad ; \qquad x(1) = x_1 \tag{17}$$

$dim\, x$ = (126, 1) $dim\, u$ = (10, 1) $dim\, \omega$ = (37, 1)
$dim\, A$ = (126, 126) $dim\, B$ = (126, 10) $dim\, D$ = (126, 37)

Given equation (17), the objective is to minimize the following performance index:

$$\min_{u} \; E[J(u)] = x_{K+1}^T Q_f x_{K+1} + \sum_{k=1}^{K} \left[x_k^T Q_k x_k + u_k^T R_k u_k \right] \tag{18}$$

where the penalty weighting matrices have sizes as follows:

$dim\, Q_f$ = (126, 126) $dim\, Q_k$ = (126, 126) $dim\, R_k$ = (10, 10)

This paper only considers the deterministic LQ-regulator problem, which sets the disturbance vector to be the null vector ($\omega_k = 0$), or alternatively, it defines the disturbance coefficient matrix to be zero ($D_k = 0$). The solution is computed by solving the recursive equations (19) and (20) offline in retrograde time, as in Crowley and Hudgins (2017c).

$$F_k = \left[B_k^T \; P_{k+1} \; B_k + R_k \right]^{-1} B_k^T \; P_{k+1} \; A_k \tag{19}$$

$$P_k = Q_k + A_k^T \; P_{k+1} \left[A_k - B_k F_k \right] ; \qquad P_{k+1} = Q_f \tag{20}$$

Equation (21) generates the unique optimal feedback control policy in forward time, by using the matrices from equations (19) and (20).

$$u_k^{Optimal} = - F_k \, x_k \tag{21}$$

When the disturbance terms in equation (17) fluctuate, then the model can be simulated as a stochastic LQG design, as in Chow (1975), Kendrick (1981), and Kendrick and Shoukry (2014), or as a robust design as in Basar and Bernhard (1991) and Hudgins and Na (2016), or as a mixed robust/stochastic LQG design (Hudgins and Na, 2016; Hudgins and Crowley, 2017).

In equation (18), the first term penalizes the tracking errors for the aggregate and wavelet decomposed variables in the final period at the end of the planning horizon. The second term penalizes the tracking errors for the state and control variables in each period. Following Crowley and Hudgins (2015, 2017c), there is an embedded penalty for large changes in government spending between periods at each frequency, which incorporates the fact that most new budgets are constructed through traditional incremental budgeting, rather than by zero-based budgeting. The index also assigns a penalty for large changes in the aggregate interest rate and wavelet decomposed interest rates between periods, thus effectively penalizing the monetary authorities for an unstable market interest rate.

The model allows the optimal targets for the state and control variables to grow at distinct quarterly target rates of $g(.)$, that are specified by the policymakers, which results in annual growth rates of $\{[1 + g(.)]^4 - 1\}$ per year. Equation (22) defines the target variable equation for quarterly growth for each of these respective series.

$$X^*_{k+1} = [1 + g(.)_k] X^*_k \tag{22}$$

The state vector also includes the aggregate and wavelet decomposed target variables of consumption, investment, government expenditure, and the interest rate. It also includes aggregate output, net exports, net taxes, and the current and target levels of the government budget deficit and government stock of debt. In order to compute the penalty for excessive control variable changes between periods, and to recover the government spending and interest rate variables from the tracking error specification of the control vector in (16), the state vector also includes the lags and lagged differences of government spending and interest rates.

4. Simulation Analysis Example: South Africa

To illustrate the usage of this approach for a small open economy, in this section we apply this framework to the economy of post-apartheid South Africa. Our objective is to use the optimal control model to simulate the changes in optimal fiscal and monetary policy and the forecasted trajectories of the other macroeconomic variables under different scenarios for the relative importance that policymakers assign to real exchange rate (*RER*) stability. Although the framework for economic policy and the behavior of the South African economy changed considerably after 1994, apart from the early 2000s and during the great recession, changes in both exports and imports have largely been positive. The South African real exchange rate (*RER*)[2], however, has been quite volatile and has been on a depreciating trend against a basket of currencies. Since the early 1980s there has been a persistent interest rate spread between South African and U.S. short term rates. Due to this depreciating *RER* trend, we simulate the case where policymakers employ a constant target for *RER*, and compare this to the case where policymakers employ a depreciating target for the *RER*. The simulations thus consider three cases:

(1) small tracking weight on *RER* with constant *RER* target;

(2) large tracking weight on *RER* with constant *RER* target;

(3) large tracking weight on *RER* with depreciating *RER* target.

Following Crowley and Hudgins (2015, 2017a,c), all simulations specify political cycle targeting, where frequency ranges 3 and 4 get the most weight. This is

[2] The real exchange rate is sourced from the Reserve Bank of South Africa (see http://wwwrs.resbank.co.za/webindicators/EconFinDataForSA.aspx). It is described as a real effective exchange rate: average for the month against 20 trading partners - Trade in manufactured goods. The monthly index was aggregated up to a quarterly index using a simple average of the monthly values.

consistent with the primary South African political cycle of 5 years, and it captures the interaction between the political and economic motivations of the policymakers, with primary emphasis being on the cycles between 2 and 8 years[3].

The simulations consider a 4-year (16-quarter) planning horizon. The state variables are assigned their initial values at period $k = 1$. The fiscal authorities choose the optimal level of government spending, and the monetary authorities determine the optimal market interest rate at each frequency range, $j = 1, \ldots, 5$, starting in period $k = 1$. At the end of the planning horizon, the optimal government spending and interest rate in quarter $K = 16$ determines the levels of consumption, investment, and the other state variables in period $K + 1 = 17$.

Following Crowley and Hudgins (2017a), the target growth rates for all GDP components for all frequency ranges and aggregates are set at 1%, which is consistent with the data for the post-apartheid period of 1994 – 2016. The target short-term quarterly interest rate is set at .0195 (about 8% annually). The initial values for the state variables in period 1 correspond to the South African and foreign data in 2016, quarter 2. The initial stock of government debt is set at $DEBT_0 = 0$, since only the discrepancy between the current value and initial value has an impact on the state equations and the tracking errors.[4] The net tax rate as a percentage of national income is fixed at $\tau_0 = 0.20$.[5] The quarterly interest rate on the government debt is set at $i_0 = .02$, which is 8.24% per year. The expectation formation equation (11) sets the weight for the current level of government spending at $\phi = 0.6$, and the parameter weight for the adjustment for the national debt differential in the expectation equation at $\pi = 0.005$ for all frequency ranges in all periods. A good fit is obtained for the empirical estimations for equations (3) – (10), and further details are given in the appendix. We developed a MATLAB program that will simulate the large-scale optimal tracking control system, where the user can select any number of quarters for the horizon, after entering the performance index parameter weights and all system coefficients.

4.1 Small Tracking Weight on RER with Constant Target [Case 1]

In case (1), both fiscal and monetary policymakers actively track consumption and investment, and each policymaker also assigns a sizeable weight to tracking its respective policy growth target. The policymakers desire real exchange rate stability, and thus the target growth rate in the real exchange rate is 0. The relative weight on the deviations from the exchange target are given relative small weight, however, so that the policymakers are relatively more concerned with achieving their targets in the other macroeconomic variables.[6] Figure 1, panels (a) through (h), shows the forecast trajectories for South African government spending, the short-term interest rate, consumption, investment, exports, imports, net exports, and the real exchange rate, respectively.

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[3] The specifics of South African fiscal policy and monetary policy are discussed in Crowley and Hudgins (2017a).

[4] Government debt to GDP was 50.1% in 2015. It averaged 37.9% between 2000 and 2015, and has been on an upward trajectory since 2008.

[5] The tax rate is calculated using SARS data for 2016 by dividing the total nominal tax revenue of 1.0699 trillion rand by the nominal GDP value of 4.086812 trillion rand.

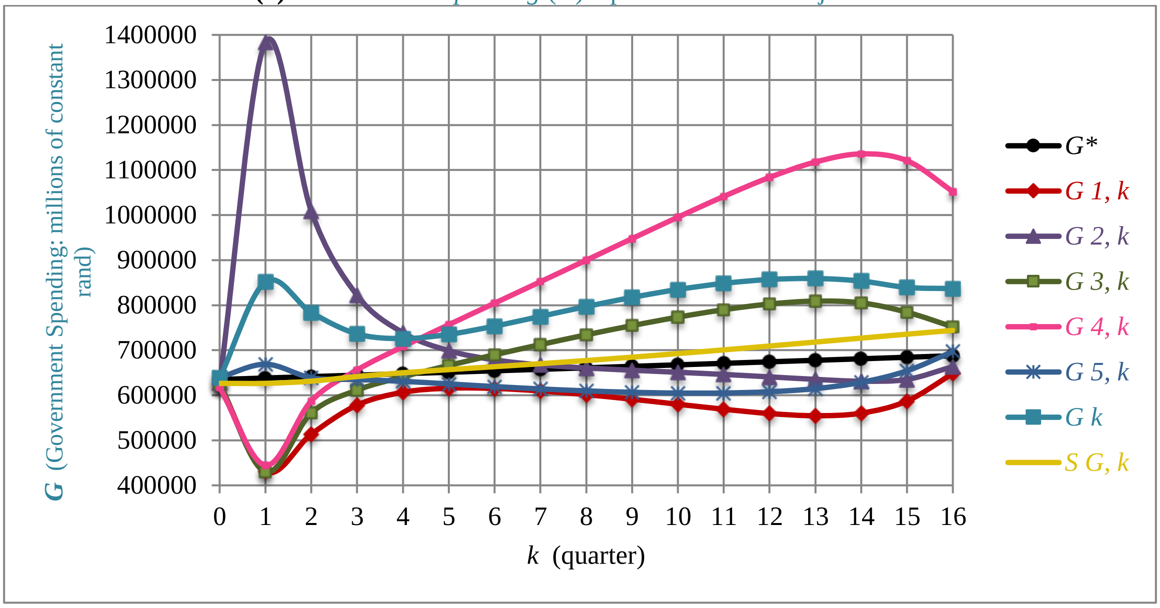
[6] We have also simulated the model where the target *RER* depreciates annually by 3%, with all else the same as in case (1). Since the relative penalty weight on the exchange rate tracking errors is relatively small, the results for all control and state variables are very similar to those in case (1).

Figure 1

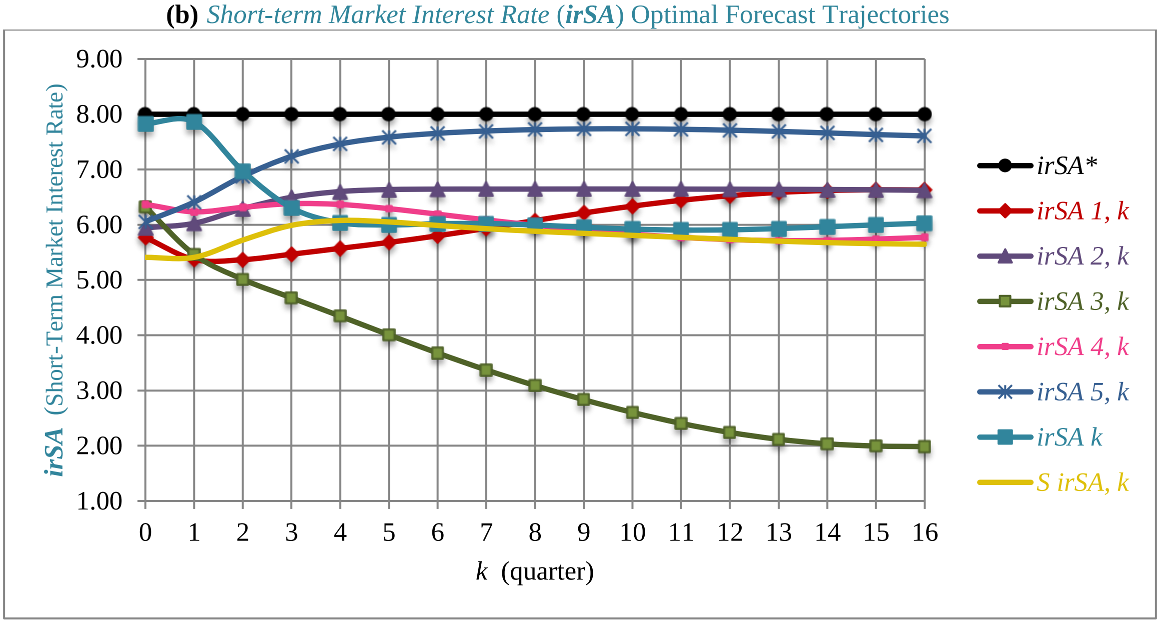
Small Weight on *RER* Tracking Error

RER Growth target of 0% per quarter

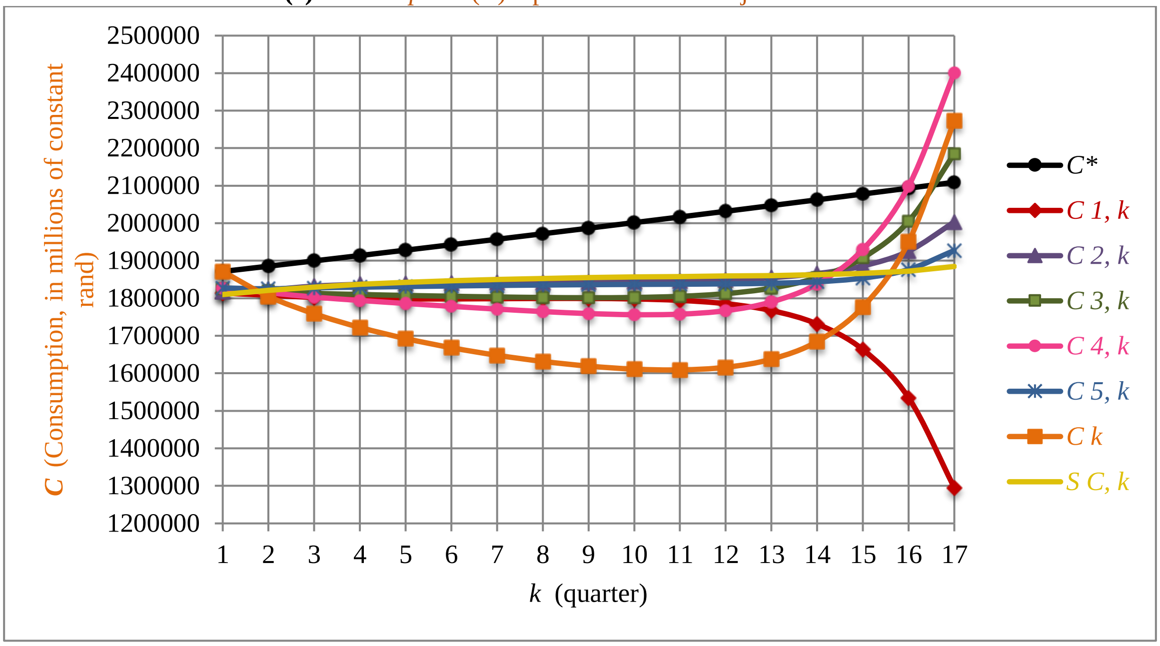
(a) *Government Spending* (***G***) Optimal Forecast Trajectories



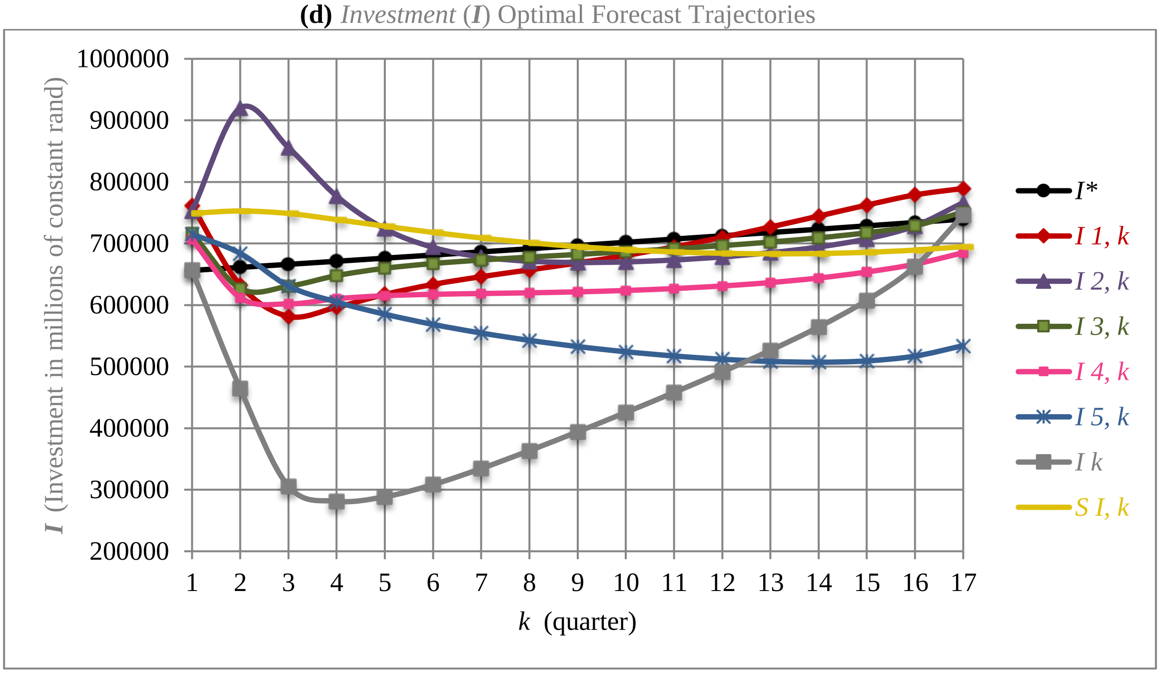
(b) *Short-term Market Interest Rate* (***irSA***) Optimal Forecast Trajectories



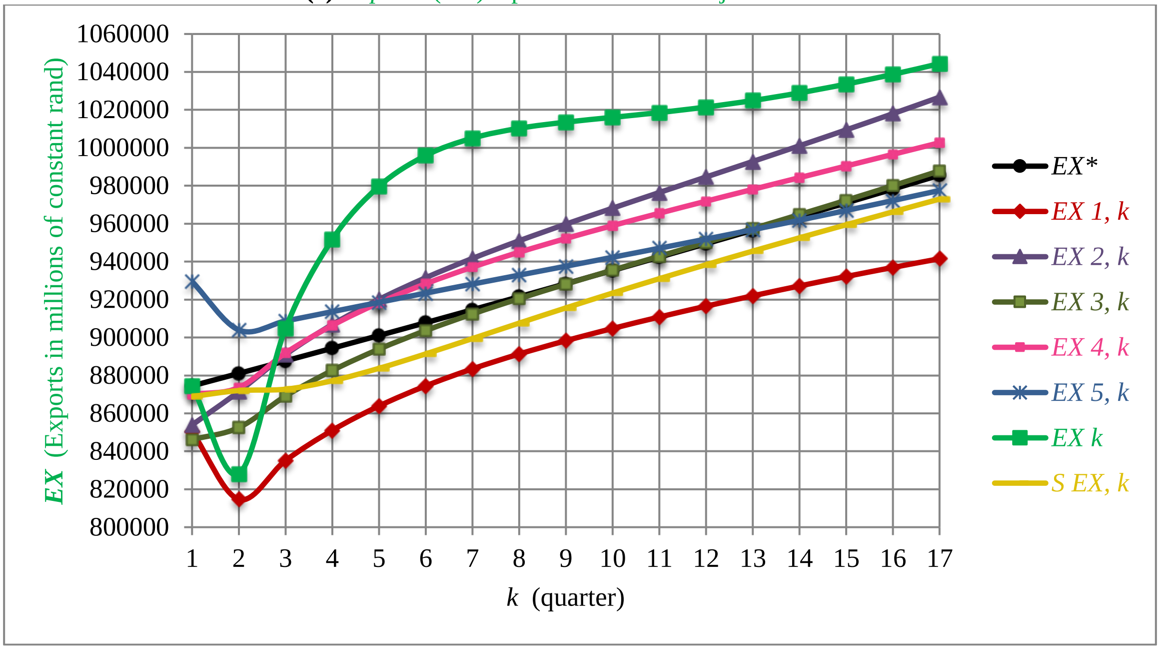
(c) *Consumption* (***C***) Optimal Forecast Trajectories



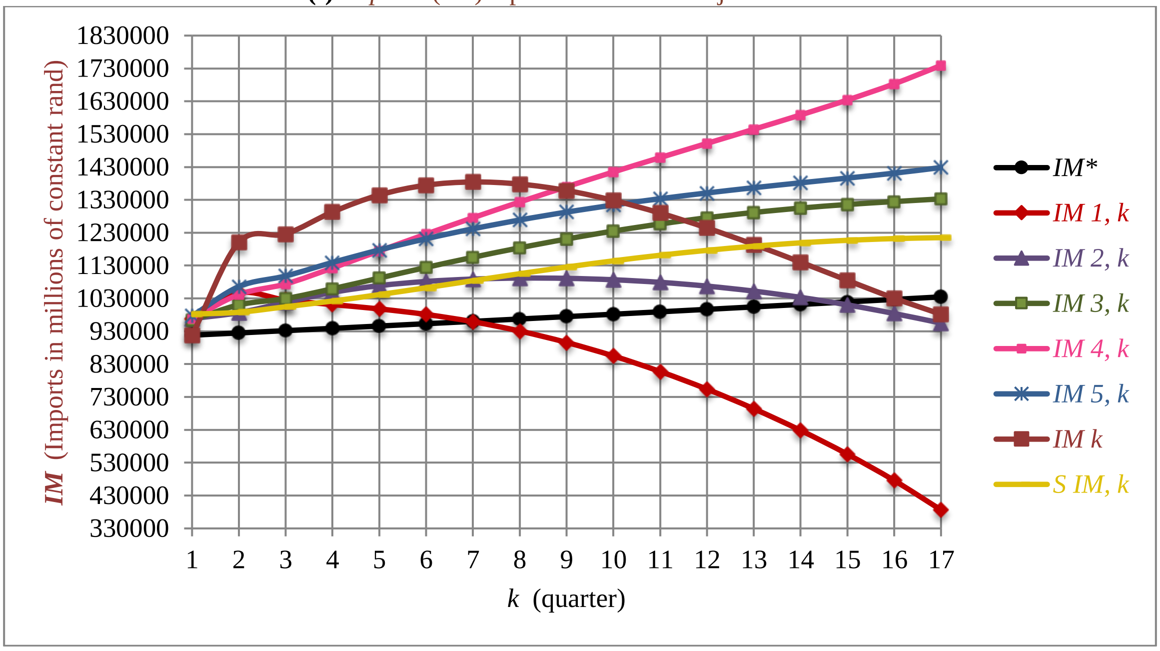
(d) *Investment* (***I***) Optimal Forecast Trajectories



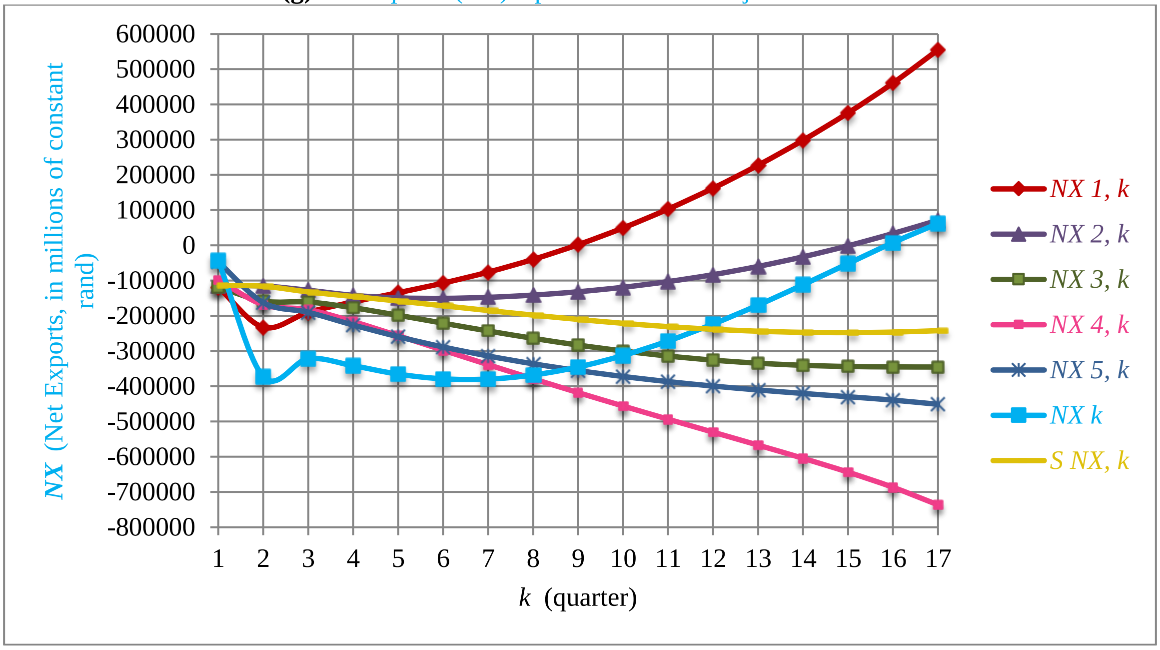
(e) *Exports* (***EX***) Optimal Forecast Trajectories



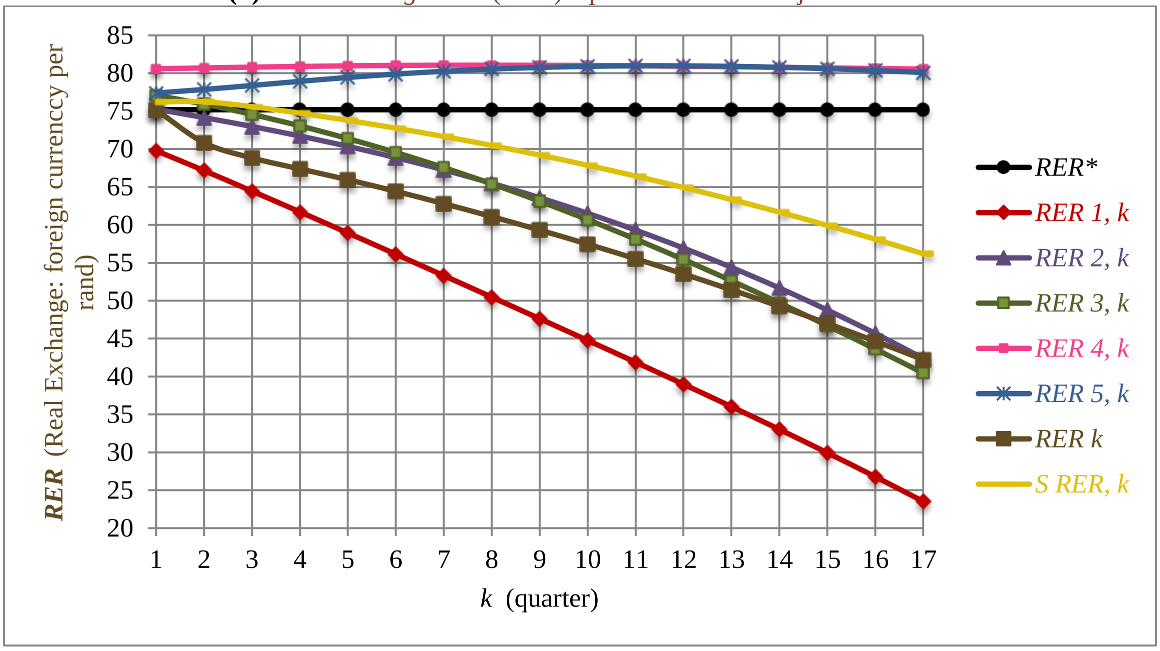
(f) *Imports* (***IM***) Optimal Forecast Trajectories



(g) *Net Exports* (***NX***) Optimal Forecast Trajectories



(h) *Real Exchange Rate* (***RER***) Optimal Forecast Trajectories



Panel (a) shows that government spending at all frequency ranges, and in the aggregate, initially decreases, except for a large increase at frequency 2 and a small increase at frequency 5. Over the horizon, the optimal spending mix shifts toward the political frequencies (3 and 4), which are the most heavily weighted frequency ranges. Spending at frequency 1 is consistently below the target, as is spending at frequency 5 (with the exception of period 1). The fiscal stance is expansionary, since aggregate government spending is consistently above the target throughout the entire horizon. After a large initial increase in quarter 1, aggregate government spending decreases for the next three periods. It then begins increasing until its maximum in periods 12 and 13, and then slightly descends thereafter.

In panel (b), the interest rate is aggressively expansionary at the political cycle frequency ranges 3 and 4. The interest rate consistently remains close to the target at the longest cycle (frequency 5), partially due to fiscal crowding-out effects. The interest rate at the shortest cycles (frequencies 1 and 2) is mildly expansionary. The aggregate interest rate steadily decreases until quarter 4, and then remains close to 6% for the remainder of the horizon. The monetary authorities thus pursue an interest rate policy that cushions the fall in investment by allowing the interest rate to consistently remain below the target.

Figure 1(c) shows that consumption remains below the target in order to facilitate higher consumption in the future and larger current investment. Towards the end of the horizon, consumption increases rapidly to exceed its target level, mostly due to the upturn in the detail crystals in frequency ranges 3 and 4. Even with the higher government spending levels in the wavelet-based model, aggregate consumption lags behind the individual frequency ranges during the first few periods before beginning to close the gap at the end of the planning horizon. The fall in aggregate consumption is due to the negative detail crystals in frequency ranges 1, 2, and 5, which causes consumption over those frequency ranges to fall below the smoothed consumption trend. Consumption at frequency ranges 3 and 4 is the largest, due to the emphasis given to the tracking errors at the political cycle ranges.

Panel (d) shows that investment tracks the target closely at the frequency cycle 3. Aggregate investment shows a large initial decrease over the first three quarters, but then encounters a steady increase until it reaches the target at the end of the horizon. Investment immediately rises in the 1-2 years cycle, but in other cycle frequencies, it immediately falls, and then at all cycle frequencies we see an increase over time until it is larger than the target level at the shortest 3 frequencies.

In panel (e), exports mostly dip and then recover to exceed the target rate of growth, in a J-curve type effect which implies a real depreciation of the South African rand (SAR) and therefore monetary expansion. Particularly at the 1-2 year frequency (d_2) and 4 to 8 year frequency (d_4), the detail crystals are above target levels for almost the entire simulation period.

In panel (f), imports start above target at all frequencies (once again, in a J-curve type effect) but then as quantities fall particularly at higher frequencies, this starts to depress aggregated imports. Particularly at the 2-4 quarter frequency (d_1), imports start to rapidly fall off after 2 quarters, and this short-term effect continues to drag down the aggregate, despite the fact that at lower frequencies (likely growth effects) import growth remains robust. The level of aggregate imports initially increases, but then falls toward the target starting in the middle of the horizon. Imports are the largest at the longer cycles (frequency ranges 3, 4, 5), and fall below the growth target for the shorter cycles. When we combine *EX* and *IM* to get net exports (*NX*) in panel (g), we can see that the net effect is a deterioration for half of the planning period, followed by a steady improvement that causes net exports to finish the horizon with a positive real trade balance.

The target for the real exchange rate (*RER*) is no movement, but immediately as the planning period begins, the exchange rate is on a depreciating trend, with higher frequency crystals all below the smooth, but longer term (4 years and above) frequency crystals pointing to a real exchange rate appreciation. The net effect at all frequencies is a substantial real depreciation of the SAR. This depreciation is most extreme at the shortest cycle.

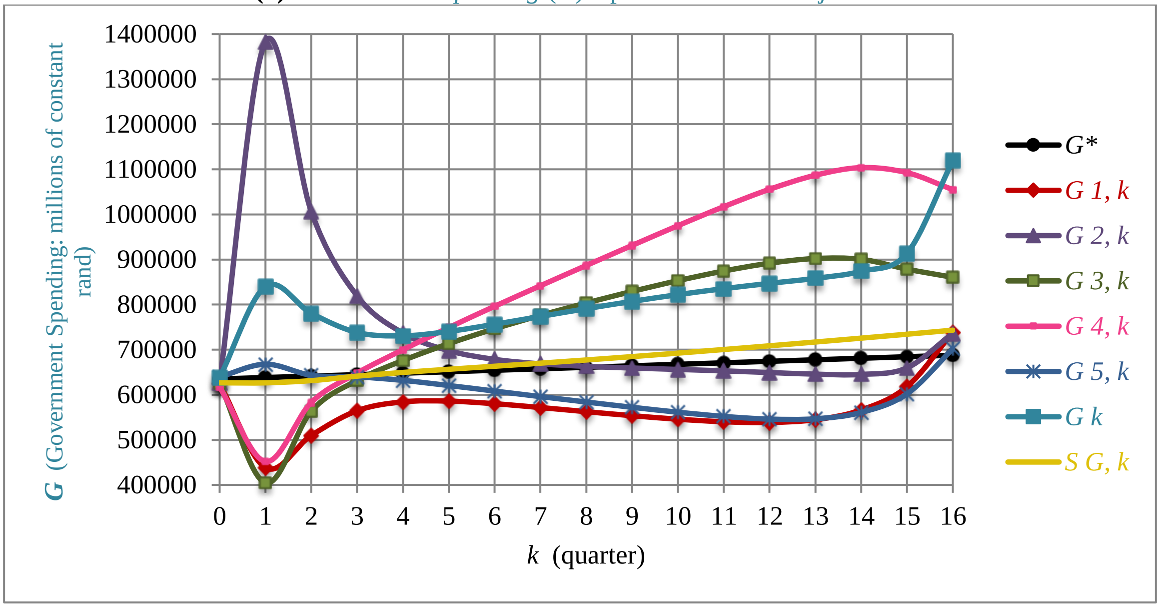
4.2 Large Tracking Weight on RER with Constant Target [Case 2]

Case (2) explores the scenario where policymakers place a relatively high importance on exchange rate stability. Here, fiscal policymakers are more strongly subject to political concerns than are monetary authorities. Whereas case (1) simulated the situation where both fiscal and monetary policy were aligned to a political targeting emphasis, the central bank is likely to be less concerned with the political cycle, and more concerned with short-term stabilization policy, as well as with conducting policies that promote a stable real exchange rate. In this scenario, the monetary authorities place a heavy emphasis on stabilizing the real exchange rate, but also place some emphasis on achieving consumption and investment targets. Previous models, such as Crowley and Hudgins (2017b,c), do not permit this scenario, since they do not model the open economy.

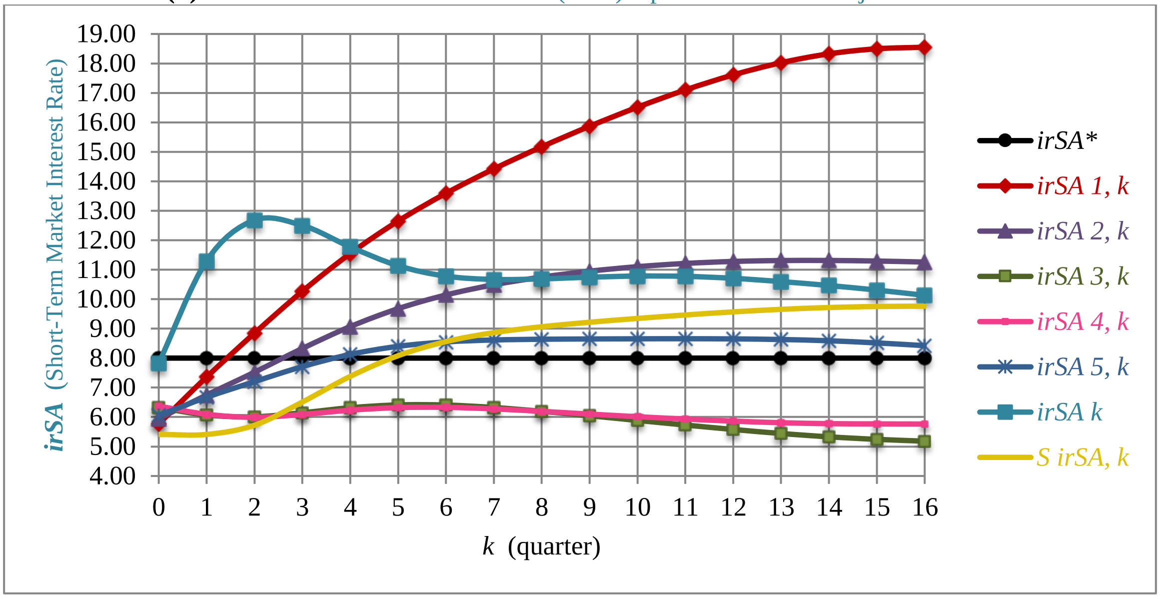
To simulate case (2), the relative weights in the performance index are the same as in case (1), except that the tracking error on the aggregate real exchange rate is much more heavily weighted. Government spending is initially the largest at frequency range 2, but then transfers to longer cycle spending. The fiscal spectrum in figure 2(a) shows the dominance of thrust placed upon activity at the longer 4 to 8 year political cycle and the shorter 2 to 4 year cycle. Unlike case (1), aggregate spending increases in the last two quarters in case (2). Fiscal policy is more aggressive in case (2), and cumulative aggregate government spending across all periods in case (2) is 2.5% larger compared with the simulation in case (1).

Figure 2
Large Weight on *RER* Tracking Error
RER Growth target of 0% per quarter

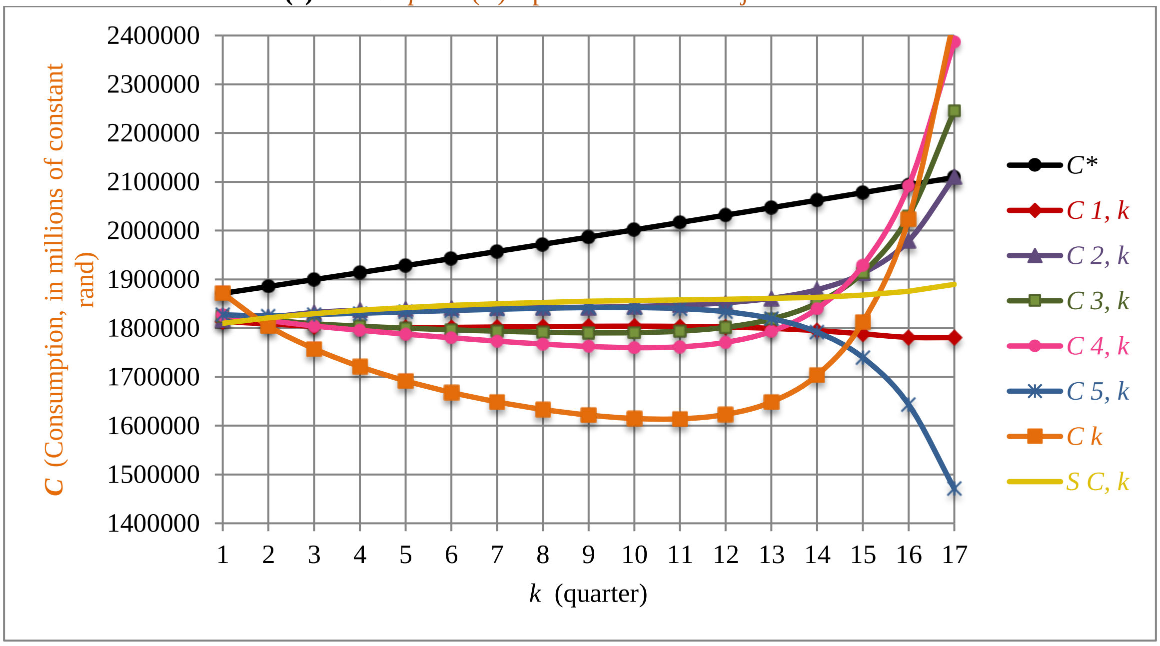
(a) *Government Spending* (***G***) Optimal Forecast Trajectories

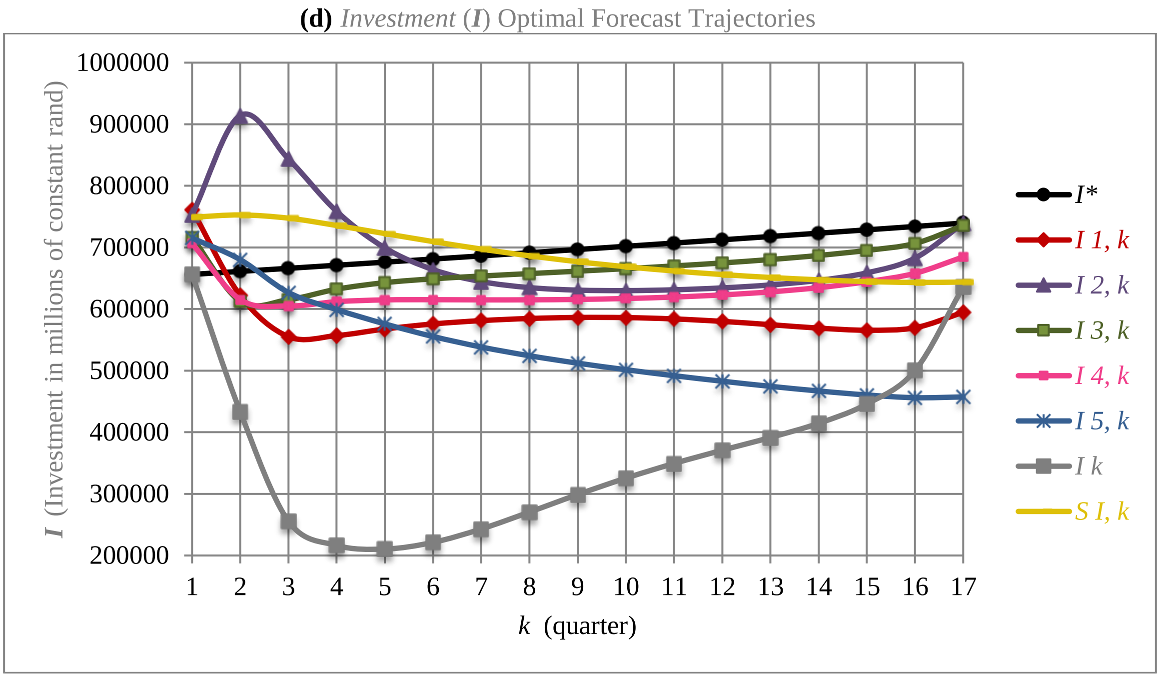


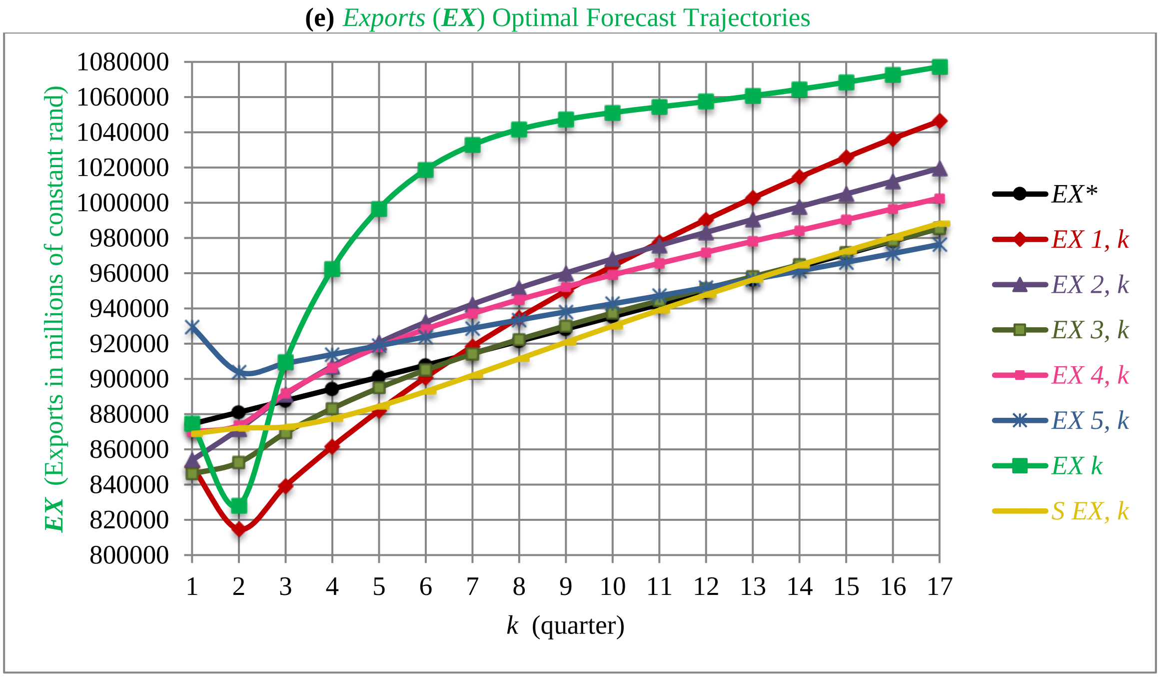
(b) *Short-term Market Interest Rate* (***irSA***) Optimal Forecast Trajectories



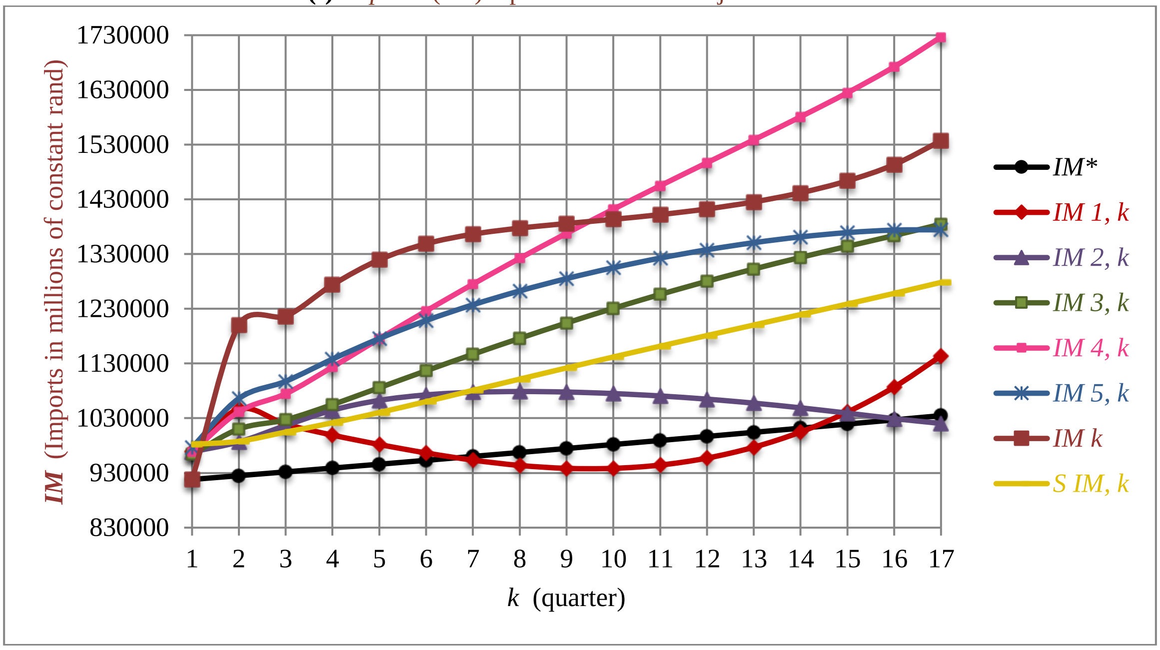
(c) *Consumption* (***C***) Optimal Forecast Trajectories



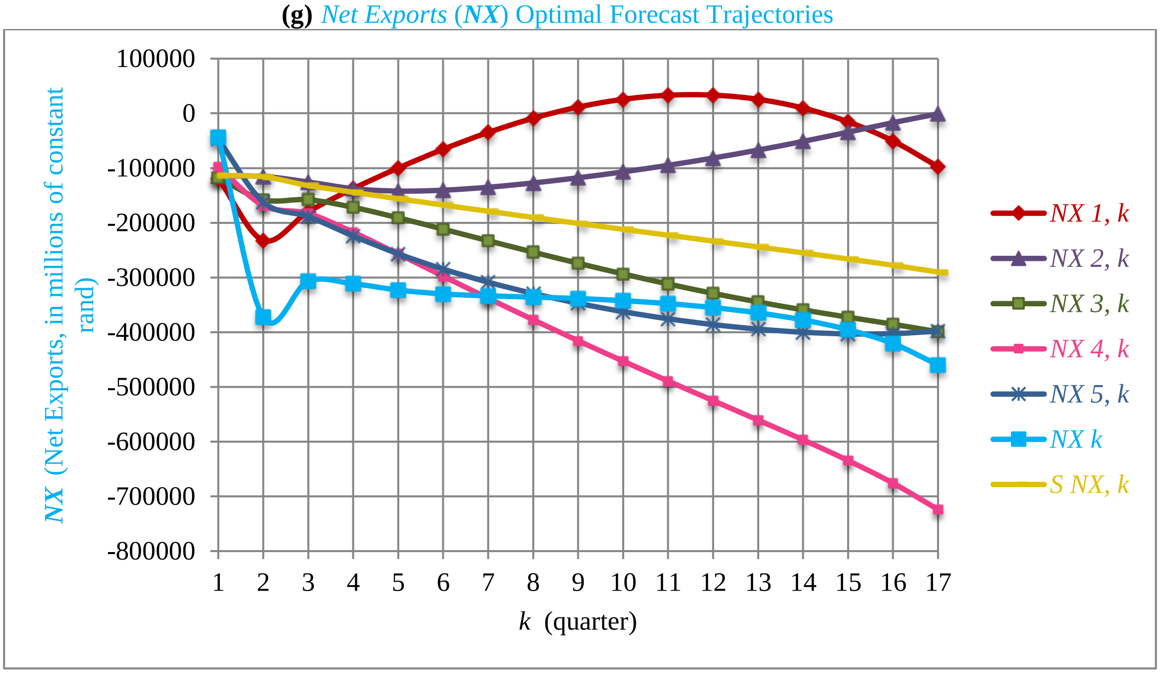

(d) Investment (I) Optimal Forecast Trajectories
I (Investment in millions of constant rand)
1000000
900000
800000
700000
600000
500000
400000
300000
200000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
k (quarter)
I*
I 1, k
I 2, k
I 3, k
I 4, k
I 5, k
I k
S I, k


(e) Exports (EX) Optimal Forecast Trajectories
EX (Exports in millions of constant rand)
1080000
1060000
1040000
1020000
1000000
980000
960000
940000
920000
900000
880000
860000
840000
820000
800000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
k (quarter)
EX*
EX 1, k
EX 2, k
EX 3, k
EX 4, k
EX 5, k
EX k
S EX, k

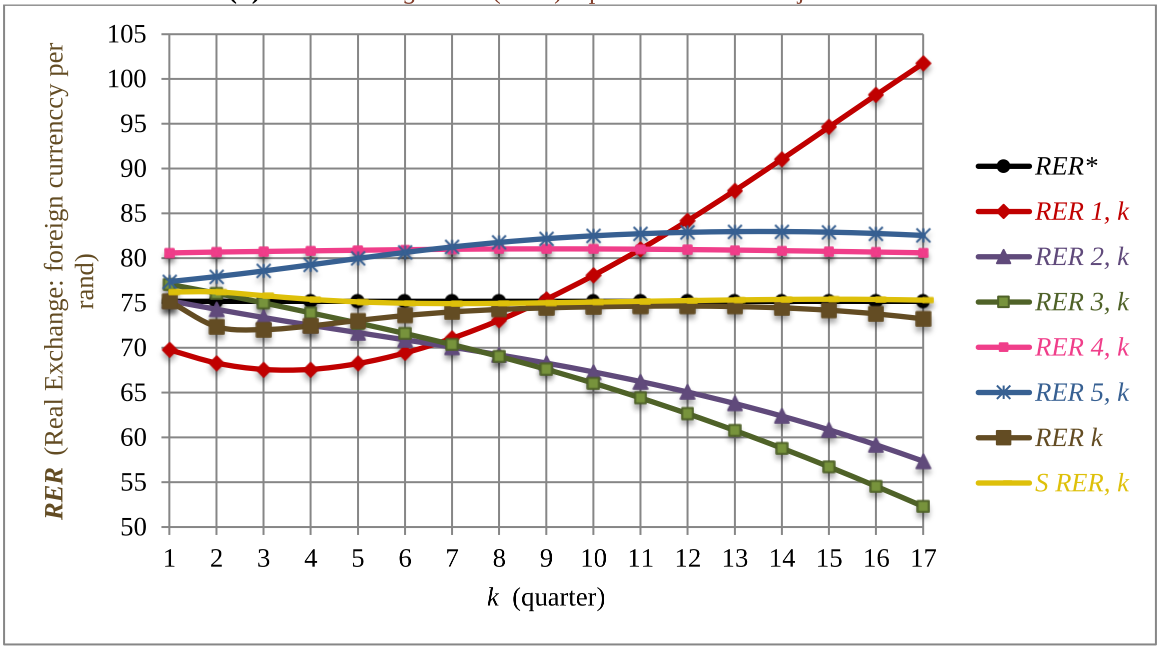
(f) *Imports* (***IM***) Optimal Forecast Trajectories



(g) *Net Exports* (***NX***) Optimal Forecast Trajectories



(h) *Real Exchange Rate* (***RER***) Optimal Forecast Trajectories



The aggregate interest rate trajectory is higher when the real exchange rate is restricted, since the real appreciation has a positive influence on the domestic interest rate. Whereas the interest rate trajectories in case (1) are consistently below the target at all frequency ranges, all trajectories are consistently above the target in case (2). In figure 2(b), the interest rate rises immediately and reaches almost 13% in quarter 2, and remains above the target and smooth trend for the entire forecast horizon in order to support the real exchange rate. Thus, it maintains a minimal depreciation that is consistent with the interest parity adjustments in equation (8). Towards the end of the forecast period, interest rates fall slightly, to about 10%.

Consumption has a similar pattern to case (1), except that the higher real exchange rate causes a slightly higher aggregate trajectory compared with case (1). The cumulative aggregate consumption is about 1% larger in case (2) than in case (1). Aggregate consumption only exceeds the target in the last quarter, where there is a substantial increase.

In case (2), the higher interest rate clearly depresses investment, with cumulative aggregate investment falling below that obtained with dual emphasis by 20%. Longer-term investment (d_5) is clearly hurt by the consistently tighter monetary policy required to keep the real exchange rate relatively constant throughout the forecast period, as is investment at the shortest frequency range (d_1). The negative influence on investment associated with the appreciated *RER* trajectory is consistent with findings of Ng and Souare (2014).

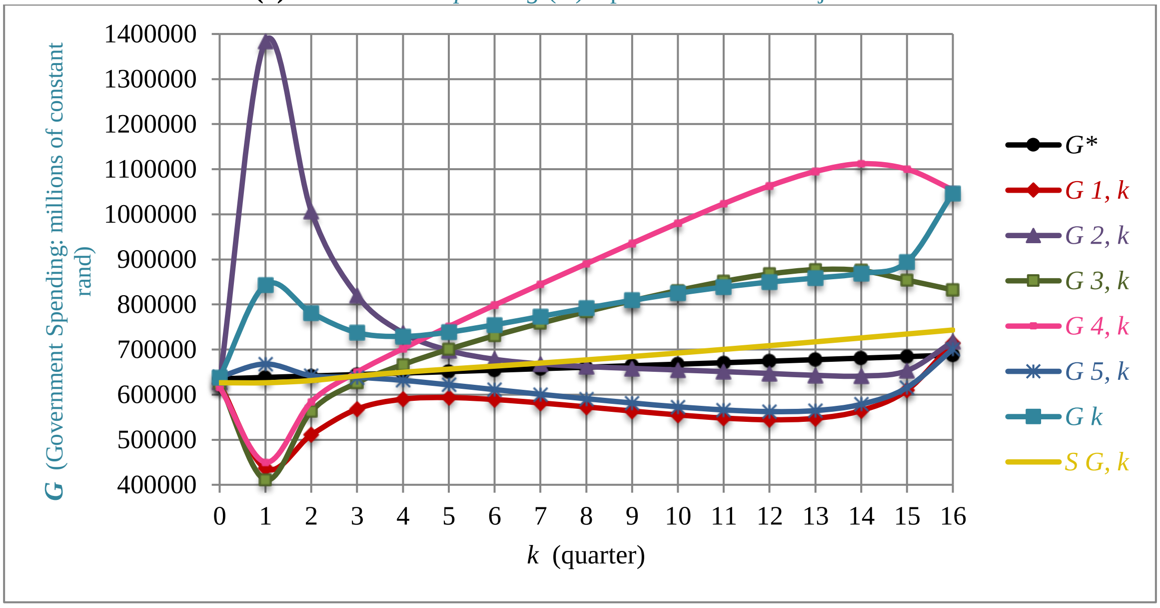
In case (2), the relatively larger (appreciated) real exchange rate results in cumulative increases in aggregate exports and imports of 2.5%, and 10%, respectively, compared to case (1). After a small initial real depreciation, the aggregate real exchange rate index stays constant for around 8 quarters before starting to depreciate again, where it eventually ends the horizon at 31 points higher than in case (1). Although the real exchange rate at frequency ranges 2 and 3 appear to still suggest depreciation, the real exchange rate is actually appreciating at the longer frequency ranges (4 and 5). The real exchange rate at the shortest frequency range (d_1) depreciates during the first 5 quarters, and then consistently appreciates thereafter. Aggregate net exports initially deteriorate and then remain roughly flat for the next 10 quarters, and then slightly decline in the last 4 quarters. The much lower trajectory of net exports in case (2) clearly reflects the real exchange rate appreciation in case (2) compared to case (1). The cumulative value of net exports across the entire horizon is 45% smaller in case (2) than in case (1).

4.3 Large Tracking Weight on RER with Depreciating Target [Case 3]

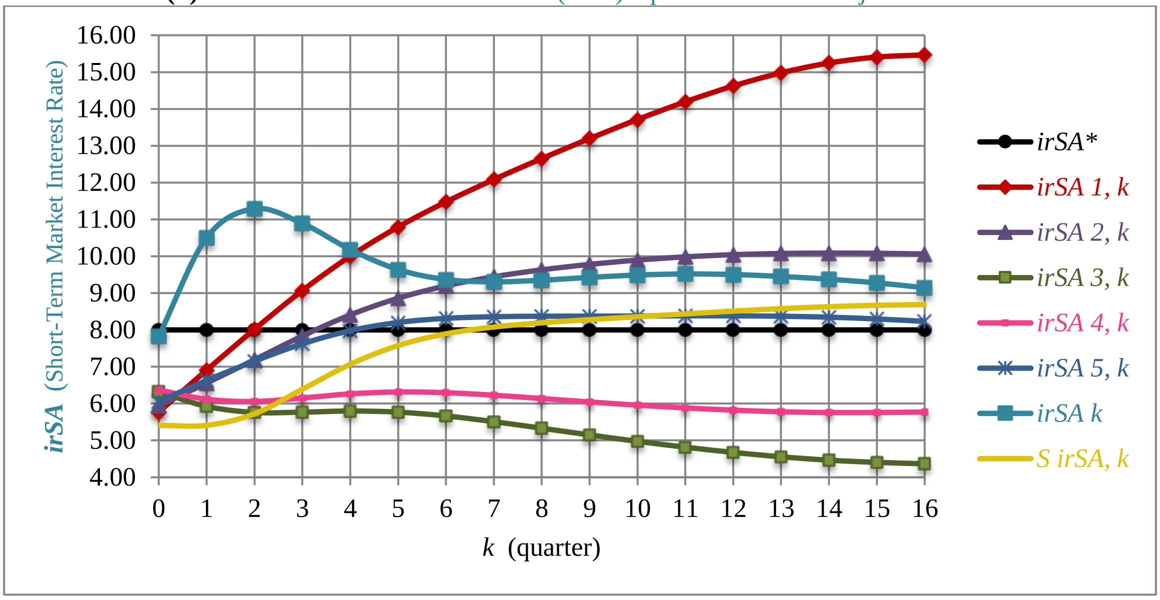
In case (3), the real exchange rate target depreciates annually by 3% (0.75% per quarter). The government spending trajectories in figure 3(a) are very similar to those in case (2). One notable difference is that aggregate spending in case (3) exhibits a much smaller increase in the last two quarters of the horizon than in case (2). Aggregate spending in case (2) is also about 0.66% less in case (3) than in case (2), which allows for fiscal savings that are still significant. However, the spending is not less over the entire horizon at all frequency ranges. Cumulative spending across the horizon in case (3) is less at frequency ranges 2, 3, and 5, but is greater at frequencies 1 and 4 when compared to the constant *RER* target in case (2).

Figure 3
Large Weight on *RER* Tracking Error
RER Growth target of – 0.75% per quarter

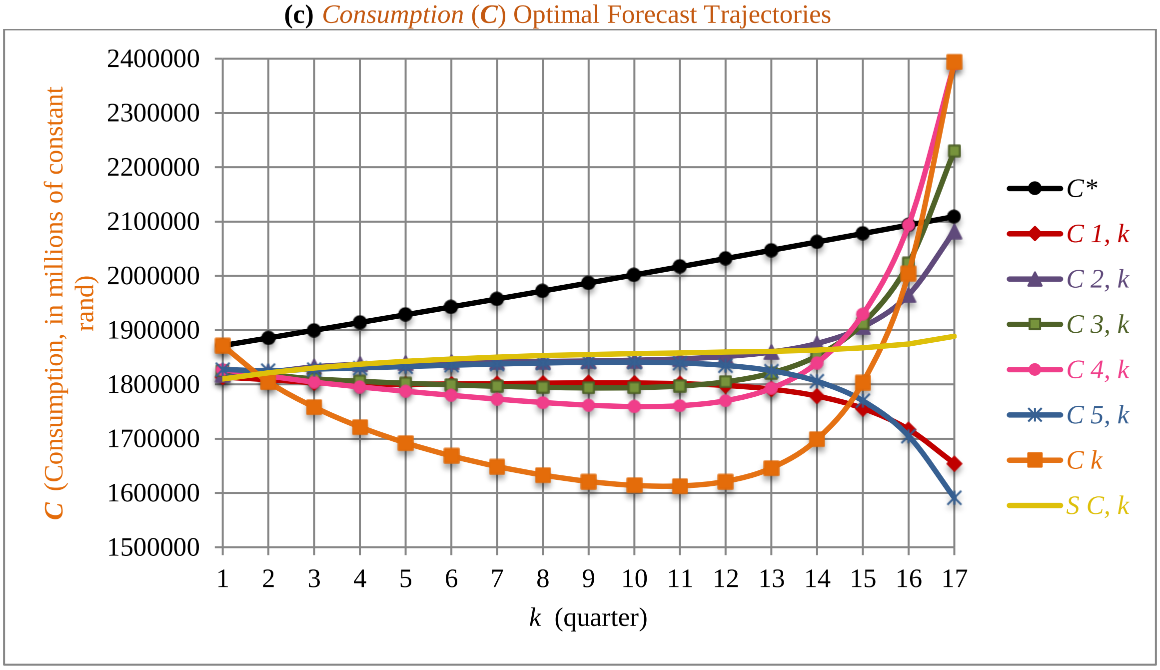
(a) *Government Spending* (***G***) Optimal Forecast Trajectories



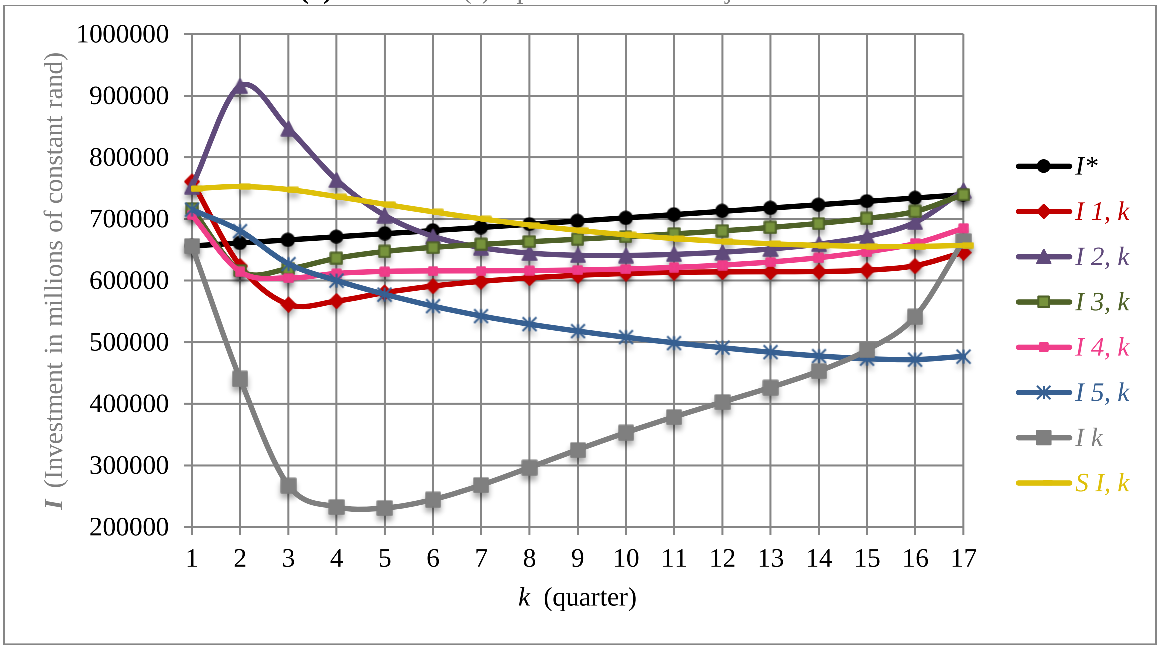
(b) *Short-term Market Interest Rate* (***irSA***) Optimal Forecast Trajectories



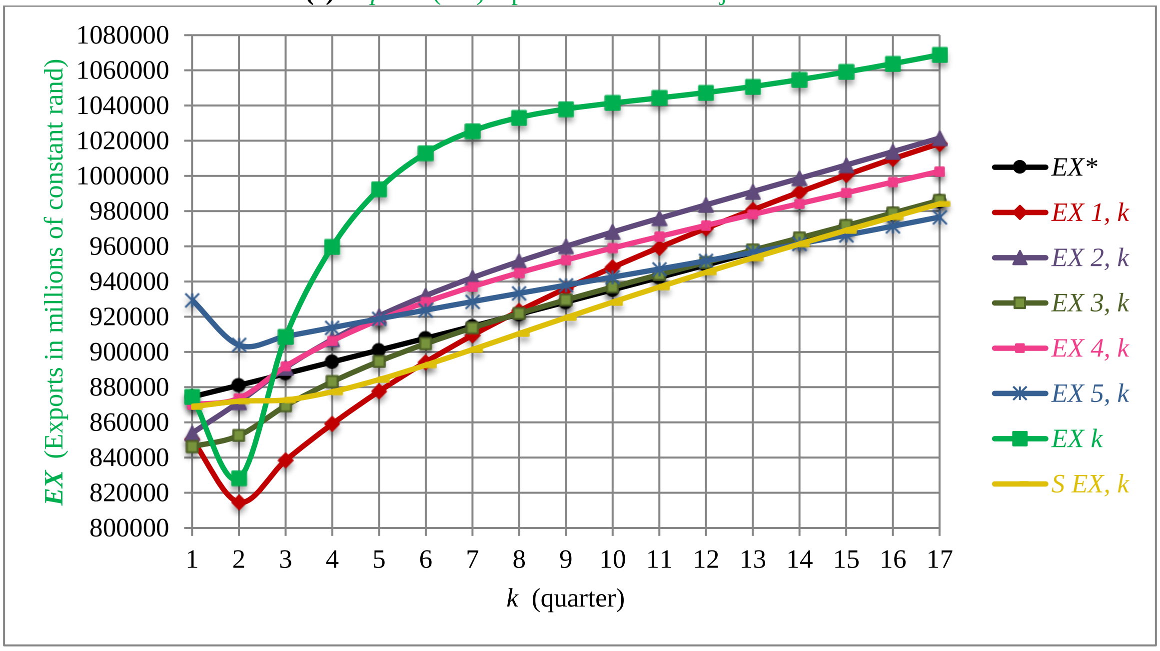
(c) *Consumption* (***C***) Optimal Forecast Trajectories



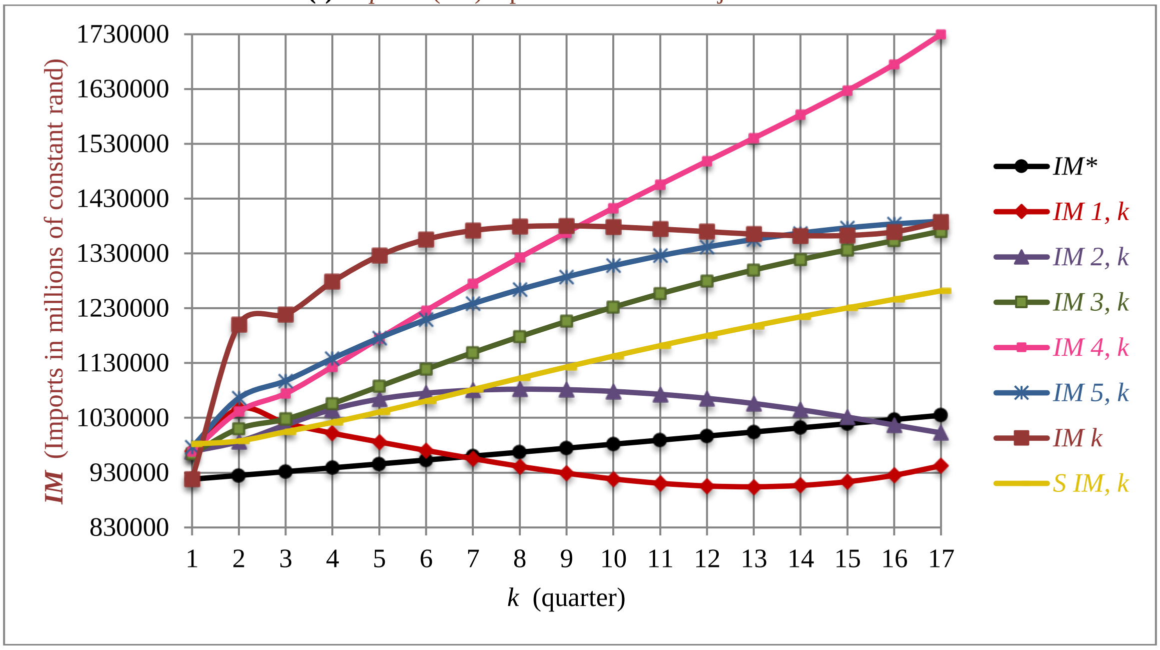
(d) *Investment* (***I***) Optimal Forecast Trajectories



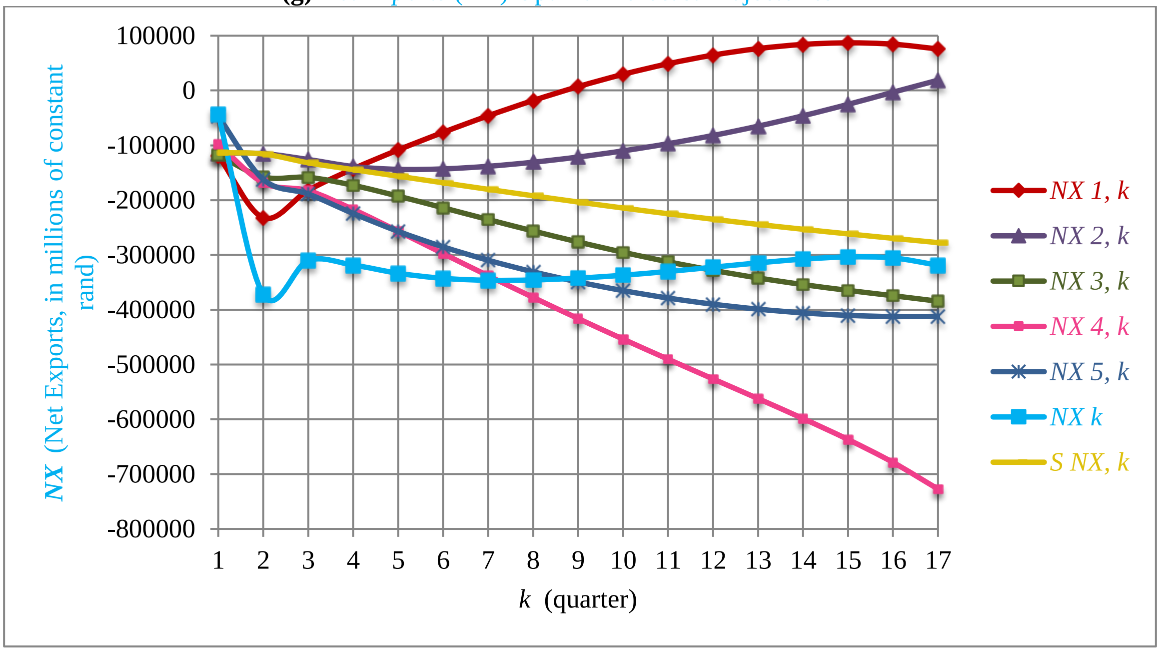
(e) *Exports* (***EX***) Optimal Forecast Trajectories



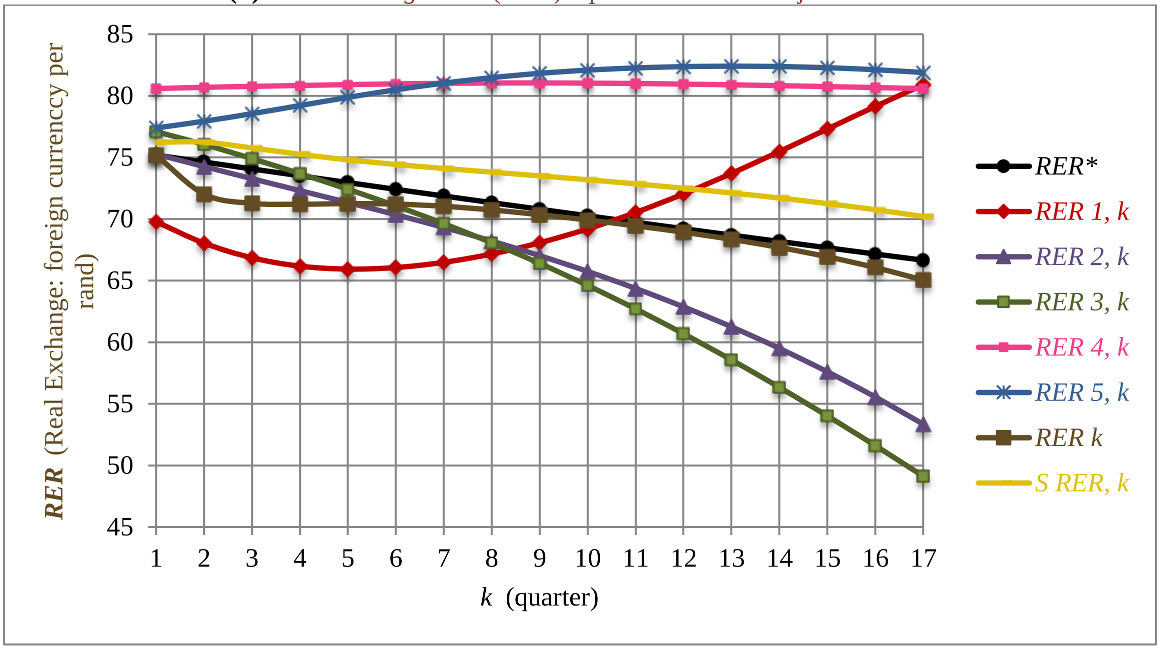
(f) *Imports* (***IM***) Optimal Forecast Trajectories



(g) *Net Exports* (***NX***) Optimal Forecast Trajectories



(h) *Real Exchange Rate* (***RER***) Optimal Forecast Trajectories



In case (3), the depreciating *RER* target allows the central bank to pursue a more aggressively expansionary interest rate policy relative to case (2), since the interest rate can sustain a lower trajectory and still maintain the interest rate parity adjustments. The interest rate trajectories all frequency ranges in case (3) have the same relative positions as in case (2), but all of the trajectories in case (3) are substantially lower. Whereas the aggregate interest rate reaches 12.7% in case (2) and eventually falls to about 10%, the aggregate rate in case (3) only reaches a maximum of 11.3% and falls to about 9% at the end of the horizon. The interest rate trajectory is almost identical for frequency range 4, but the trajectory is substantially lower at all other frequency ranges in case (3) when compared to case (2).

The consumption patterns are very similar in cases (2) and (3), but cumulative aggregate consumption is 0.27% smaller in case (3), which is mostly driven by the lower trajectory at frequency range 1 in figure 3(c). The difference in investment is much more substantial, however, when comparing cases (2) and (3). Cumulative investment is almost 7% higher in case (3) than in case (2), since the lower interest rates in case (3) are less stifling. In case (3), the investment trajectories are consistently higher at the highest and lowest frequency ranges (1 and 5) than in case (2). Thus, the central bank could avoid substantial losses in investment by targeting a small depreciation that aligns with the trend economic fundamentals, rather than pursuing an overly restrictive emphasis on exchange rate stability.

The general trajectory paths in panels (f) and (g) for exports and imports at all frequency ranges are similar in cases (2) and (3). In case (2), the cumulative value of exports is 0.67% larger than in case (3), but the cumulative value of imports is 2.58% larger in case (2) than in case (3). Thus, the difference in the trade balance is substantial. In case (3), the aggregate net export trajectory initially falls by a similar amount as that in case (2); the decline in net exports, however, is much more severe in case (2). Cumulative aggregate net exports are about 8.7% larger in case (3) than in case (2), which illustrates the significant improvement in the trade balance that occurs with a depreciating real exchange rate target.

As in case (2), the exchange rate in figure 3(h) again follows its target closely in the middle of the horizon, and depreciates more substantially below the target at the beginning and end of the horizon. The frequency range 1 trajectory is lower in case (3) than in case (2), showing a large effect in the shortest cycle. This simulation demonstrates one of the advantages of using the wavelet decomposition. When the policymakers emphasize exchange rate stability a primary objective, the exposure to short-term instability vastly increases at the shortest frequency range cycle (frequency 1), as shown in figure 2(h). This appreciated frequency 1 exchange rate is accompanied by an increased frequency 1 interest rate, as shown in figure 2(b).

Since the South African economy is attempting to promote faster growth to emerge from weak economic performance in the previous years, the *RER* emphasis with a constant target places a much greater restriction on the short-term interest rate policy cycles, leaving economic performance more susceptible to short-term trade balance and short-cycle exchange rate movements, as well as reduced investment. These short-term cycle stability problems are somewhat mitigated by allowing the *RER* target to depreciate. At the end of the horizon at frequency range 1, the interest rate is 3 percent lower, and the *RER* is 20 points lower, in case (3) than in case (2), while the frequency 1 net exports become positive in case (3). These variable changes demonstrate the short-term economic improvements of a depreciating *RER* target.

5. Conclusion

In this paper we construct a generalized open economy LQ tracking model for use with time-frequency methods, and in particular discrete wavelet analysis. We then apply the model to a small open economy, namely that of South Africa.

Once the model is estimated in the South African context, the simulations illustrate the results for three specific sets of policy assumptions and parameters. We simulate the model by incrementally increasing the *RER* stability emphasis for a constant target, and for different targeted rates of depreciation. The included cases represent the most relevant comparisons, since cases (2) and (3) provide a more heavily weighted targeting preference, but still leave possible some deviations from the target. Thus, the policymaker has the ability to utilize the model to ascertain the sensitivity of the simulations to incremental change in exchange rate restrictions. Although the results are not included in this paper, we have also simulated the model for changes in the other system parameters, such as replacing the political cycle emphasis with a short-term stabilization emphasis and a long-term growth emphasis, and adjusting the expectations parameter.

Although this paper only simulated the deterministic LQ tracking control design, the model can be extended to generate results from a stochastic LQG and robust worst-case controller designs, as in Hudgins and Crowley (2017). Further extensions could include explicit inclusion of price levels and inflation, and the modeling of different frequencies with different equation structures, rather than restricting the each equation block to follow the same structure. Although we have only simulated the large-scale open-economy model using South African data, the model we have developed can be implemented for other countries.

References

Aguiar-Conraria, L., Soares, M. (2011). Oil and the macroeconomy: using wavelets to analyze old issues. *Empirical Economics* 40(3), 645 – 655.

Basar, T. and Bernhard, P. (1991). *H^∞- optimal control and related minimax design problems*. Boston: Birkhauser.

Chen, Y-W. (2016). Health progress and economic growth in the USA: the continuous wavelet analysis. *Empirical Economics* 50(3), 831 – 55.

Chow, G. (1975). *Analysis and control of dynamic economic systems*. New York: John Wiley and Sons.

Crowley, P. (2007). A guide to wavelets for economists. *Journal of Economic Surveys* 21(2), 207 – 267.

Crowley, P. and Hudgins, D. (2017)a. Evaluating South African monetary and fiscal policy using a wavelet-based model. *Discussion Paper*, available at https://sites.google.com/site/soemacroeco/papers, forthcoming.

Crowley, P. and Hudgins, D. (2017)b. Wavelet-based monetary and fiscal policy in the Euro area under optimal tracking control. *Journal of Policy Modeling* 39(2), 206 – 231.

Crowley, P. and Hudgins, David. (2017)c. What is the right balance between U.S. monetary and fiscal policy? Explorations using simulated wavelet-based optimal tracking control. *Empirical Economics*, online first, DOI: 10.1007/s00181-017-1326-2.

Crowley, P. and Hudgins, D. (2015). Fiscal policy tracking design in the time–frequency domain using wavelet analysis. *Economic Modelling* 51, 501 – 514.

Crowley, P. and Hughes Hallett, A. (2016). Correlations between macroeconomic cycles in the US and UK: What can a frequency domain Analysis tell us*? Italian Economic Journal* 2(1), 5 – 29.

Dar, A., Samantaraya, A. & Shah, F. (2014). The predictive power of yield spread: evidence from wavelet analysis, *Empirical Economics* 46(3), 887 – 901.

Faria, G. and Verona, F. (2016). Forecasting the Equity Risk Premium with Frequency-Decomposed Predictors, Bank of Finland Research Discussion Papers 29/2016, Helsinki, Finland.

Gallegati, M., Gallegati, M., Ramsey, J., and Semmler, W. (2011). The US wage Phillips Curve across frequencies and over time. *Oxford Bulletin of Economics and Statistics* 73(4) 489 – 508.

Hudgins, D. and Crowley, P. (2017). Stress-Testing US Macroeconomic Policy: A Computational Approach Using Stochastic and Robust Designs in a Wavelet-Based Optimal Control Framework. *Working Paper*.

Hudgins, D. and Na, J. (2016). Entering H^∞-optimal control robustness into a macroeconomic LQ tracking model. *Computational Economics* 47(2) 121 – 155.

Kendrick, D. (1981). *Stochastic control for econometric models*. New York, NY, McGraw Hill.

Kendrick, D. and Shoukry, G. (2014). Quarterly fiscal policy experiments with a multiplier accelerator model. *Computational Economics* 44(3), 1 – 25.

Kliem, M. and A. Kriwoluzky (2014). Toward a Taylor Rule for fiscal policy. *Review of Economic Dynamics* 17(2), 294 – 302.

Kumar, S., Pathak, R., Tiwari, A., and Yoon, S. (2016). Are exchange rates interdependent? Evidence using wavelet analysis. *Applied Economics* 49(33), 3231 – 3245.

Ng, E. and Souare, M. (2014). On investment and exchange-rate movements. *Applied Economics* 46(19), 2301 – 2315.

Power, G. Eaves, J., Turvey, C., and Vedenov, D. (2017). Catching the curl: wavelet thresholding improves forward curve modelling. *Economic Modelling* 64, 312 – 321.

Tiwari, A., Bhanja, N., Dar, A., Islam, F. (2015). Time–frequency relationship between share prices and exchange rates in India: Evidence from continuous wavelets. *Empirical Economics* 48(2), 699 – 714.

Verona, F. (2016). Time–frequency characterization of the U.S. financial cycle. *Economics Letters* 144, 75 –79.

Appendix – Model Estimation for South Africa

The empirical estimates for the linearized control system follow Crowley and Hudgins (2017a). Each equation was estimated with variable lags up to two quarters. Tables A1 – A5 show the OLS regression coefficient estimates for with *p-values* (in parentheses) for the MODWT decomposition accelerator system in equations (3), (4), (6), (7), (8), respectively, using the data for the post-apartheid period 1994 quarter 1 – 2016 quarter 2. For all of the equations, R^2 is 0.99, suggesting that the estimated consumption equations over each frequency range have a good fit.

Table A1

Consumption coefficient estimates from equation (3), with *(p-values)*

$$C_{j;k} = \beta_{C,j,0} + \beta_{C,j,1} C_{j,k-1} + \beta_{C,j,2} I_{j,k-1} + \beta_{C,j,3} G_{j,k-1} + \beta_{C,j,4} C_{j,k-2} + \beta_{C,j,5} RER_{j,k-1} + \beta_{C,j,6} RER_{j,k-2} + \omega_{C,j,k-1}$$

j	*Quarters*	$\beta_{C,j,0}$	$\beta_{C,j,1}$	$\beta_{C,j,2}$	$\beta_{C,j,3}$	$\beta_{C,j,4}$	$\beta_{C,j,5}$	$\beta_{C,j,6}$
1	2 to 4	-10320.04 *(0.7154)*	1.9185 *(0.0000)*	0.0089 *(0.6804)*	0.1530 *(0.0000)*	-0.9691 *(0.0000)*	411.8688 *(0.4162)*	-370.9431 *(0.4766)*
2	4 to 8	-7453.99 *(0.7698)*	1.9344 *(0.0000)*	0.0069 *(0.7314)*	0.0657 *(0.0000)*	-0.9814 *(0.0000)*	715.8055 *(0.2485)*	-727.4892 *(0.2516)*
3	8 to 16	-33025.19 *(0.1694)*	1.8960 *(0.0000)*	-0.0149 *(0.4555)*	0.1357 *(0.0000)*	-0.9251 *(0.0000)*	205.3582 *(0.6852)*	-35.9853 *(0.9451)*
4	16 to 32	4966.38 *(0.8629)*	1.8398 *(0.0000)*	0.0212 *(0.4303)*	0.1699 *(0.0000)*	-0.8999 *(0.0000)*	452.0582 *(0.3751)*	-564.4508 *(0.2755)*
5	32 to 64	41024.70 *(0.1408)*	1.9401 *(0.0000)*	0.0278 *(0.3086)*	0.2013 *(0.0000)*	-1.0243 *(0.0000)*	581.5646 *(0.2716)*	-912.5597 *(0.0745)*

The consumption equation is given as table A1. The government spending coefficients ($\beta_{C,j,3}$) all have the expected positive signs, and are statistically significant. Investment has a crowding-in effect at all frequency ranges, except for frequency range 3. The one lag real exchange rate coefficients ($\beta_{C;j,5}$) are all positive, whereas the two lag real exchange rate coefficients ($\beta_{C;j,6}$) are all negative. The dominant exchange rate effect is negative at frequency ranges 2, 4, and 5, and is strongest at the lowest frequencies. This suggests that exchange rate appreciations have an initial positive influence on consumption, which includes imports, but that the substitution effect dominates over longer cycles.

Based on the estimates in table A1, the reduced form consumption equation employed in the state space system is obtained as follows. Substitute the variable G^e from equations (10) and (11) into equation (3) so that it replaces G, and then augment the equation (3) with the government debt and government trend spending to obtain the reduced form equation for determining consumption at each frequency range.

$$C_{j;k} = \delta_{j,0} + \delta_{j,1} C_{j,k-1} + \delta_{j,2} I_{j,k-1} + \delta_{j,3} G_{j,k-1} + \delta_{j,4} \hat{G}^{d}_{j,k} + \delta_{j,5} C_{j,k-2} + \delta_{j,6} RER_{j,k-1} + \delta_{j,7} RER_{j,k-2} + \delta_{j,8} DEBT_{k-1} + \omega_{C,j,k-1} \qquad \text{(A1)}$$

where

$\delta_{j;0} = \beta_{C,j,0} + \beta_{C,j,3}\, \phi_j \pi_j\, DEBT_0$; $\delta_{j;1} = \beta_{C,j,1}$; $\delta_{j;2} = \beta_{C,j,2}$; $\delta_{j;3} = \beta_{C,j,3}\, \phi_j$;
$\delta_{j;4} = \beta_{C,j,3}(1 - \phi_j)$; $\delta_{j;5} = \beta_{C,j,4}$; $\delta_{j;6} = \beta_{C,j,5}$; $\delta_{j;7} = \beta_{C,j,6}$; $\delta_{j;8} = -\beta_{C,j,3}\, \phi_j\, \pi_j$

The estimated investment equations are given in Table A2, where the R^2 values are all greater than 0.87. The national output coefficients ($\beta_{I,\,j,\,1}$) are positive for all frequency ranges, suggesting a crowding-in effect at all frequency ranges, and thus producing a crowding-in effect of income on investment. The interest rate coefficients ($\beta_{I,\,j,\,2}$) all have the expected negative signs at all frequencies, which means that larger domestic (South African) interest rate increases have a negative effect on investment.

Table A2

Investment coefficient estimates from equation (4), with *(p-values)*

$$I_{j;\,k} \;=\; \beta_{I,\,j,\,0} + \beta_{I,\,j,\,1}\,Y^{SA}{}_{j,\,k-1} + \beta_{I,\,j,\,2}\,d_{irSA,\;j,\,k-1} + \omega_{I,\;j,\,k-1}$$

j	*Quarters*	$\beta_{I,\,j,\,0}$	$\beta_{I,\,j,\,1}$	$\beta_{I,\,j,\,2}$	R^2
1	2 to 4	-273321.15 *(0.0000)*	0.3079 *(0.0000)*	-6411.1244 *(0.0047)*	0.87
2	4 to 8	-266955.22 *(0.0000)*	0.3051 *(0.0000)*	-7824.7925 *(0.3760)*	0.88
3	8 to 16	-244553.69 *(0.0000)*	0.2953 *(0.0000)*	-11314.5962 *(0.0434)*	0.90
4	16 to 32	-192207.47 *(0.0000)*	0.2732 *(0.0000)*	-9867.4535 *(0.0053)*	0.91
5	32 to 64	-165036.82 *(0.0000)*	0.2617 *(0.0000)*	-13051.7672 *(0.0047)*	0.92

Table A3

Export coefficient estimates from equation (6), with *(p-values)*

$$EX_{j;\,k} \;=\; \beta_{EX,\,j,\,0} + \beta_{EX,\,j,\,1}\,EX_{\,j,\,k-1} + \beta_{EX,\,j,\,2}\,Y^{OECD}{}_{j,\,k-1} + \beta_{EX,\,j,\,3}\,RER_{,\;j,\,k-1} + \beta_{EX,\,j,\,4}\,RER_{j;\,k-2} + \omega_{EX\;j,\,k-1}$$

j	*Quarters*	$\beta_{EX,\,j,\,0}$	$\beta_{EX,\,j,\,1}$	$\beta_{EX,\,j,\,2}$	$\beta_{EX,\,j,\,3}$	$\beta_{EX,\,j,\,4}$	R^2
1	2 to 4	-86738.65 *(0.3158)*	0.7299 *(0.0000)*	3028.40 *(0.0056)*	3667.48 *(0.0235)*	-3551.60 *(0.0265)*	0.95
2	4 to 8	-28036.73 *(0.7596)*	0.7091 *(0.0000)*	3049.31 *(0.0059)*	1119.09 *(0.4301)*	-1500.53 *(0.2900)*	0.95
3	8 to 16	-22228.14 *(0.7882)*	0.7166 *(0.0000)*	2803.42 *(0.0081)*	1710.06 *(0.2010)*	-1993.59 *(0.1256)*	0.95
4	16 to 32	-28036.73 *(0.7596)*	0.7091 *(0.0000)*	3049.31 *(0.0059)*	1119.09 *(0.4301)*	-1500.53 *(0.2900)*	0.95
5	32 to 64	-22228.14 *(0.7882)*	0.7166 *(0.0000)*	2803.42 *(0.0081)*	1710.06 *(0.2010)*	-1993.59 *(0.1256)*	0.95

Table A4

Import coefficient estimates from equation (7), with *(p-values)*

$$IM_{j;k} = \beta_{IM,j,0} + \beta_{IM,j,1} IM_{j,k-1} + \beta_{IM,j,2} Y^{SA}_{j,k-1} + \beta_{IM,j,3} RER_{j,k-1} + \beta_{IM,j,4} RER_{j;k-2} + \omega_{IM\ j,k-1}$$

j	Quarters	$\beta_{IM,j,0}$	$\beta_{IM,j,1}$	$\beta_{IM,j,2}$	$\beta_{IM,j,3}$	$\beta_{IM,j,4}$	R^2
1	2 to 4	-376443.52 *(0.0004)*	0.9351 *(0.0000)*	0.0492 *(0.1399)*	-4905.52 *(0.0472)*	8186.73 *(0.0007)*	0.96
2	4 to 8	-315210.38 *(0.0010)*	0.9435 *(0.0000)*	0.0376 *(0.1958)*	-8430.15 *(0.0038)*	11275.54 *(0.0001)*	0.97
3	8 to 16	-210607.76 *(0.0267)*	0.9409 *(0.0000)*	0.0401 *(0.1454)*	-6283.98 *(0.0141)*	7995.54 *(0.0015)*	0.97
4	16 to 32	-175969.30 *(0.1129)*	0.8719 *(0.0000)*	0.0700 *(0.0195)*	-5454.78 *(0.0185)*	6535.61 *(0.0040)*	0.95
5	32 to 64	-167362.51 *(0.1872)*	0.8593 *(0.0000)*	0.0694 *(0.0220)*	-7275.52 *(0.0064)*	8373.47 *(0.0008)*	0.95

Tables A3 and A4 show the estimates for the export and import equations. Exports at each frequency are positively related to foreign output (measured by the OECD production index), while imports vary positively with domestic output (Y^{SA}). Imports exhibit a type of real J-curve relationship with the real exchange rate, where the initial impact of a real exchange rate appreciation has a positive effect, while the lagged effect of an appreciation is negative and dominant.

Table A5

Real Exchange Rate coefficient estimates from equation (8), with *(p-values)*

$$RER_{j;k} = \beta_{RER,j,0} + \beta_{RER,j,1} ir^{SA}_{j,k-1} + \beta_{RER,j,2} ir^{US}_{j,k-1} + \beta_{RER,j,3} Risk_{k-1} + \beta_{RER,j,4} RER_{j;k-1} + \omega_{RER\ j,k-1}$$

j	Quarters	$\beta_{RER,j,0}$	$\beta_{RER,j,1}$	$\beta_{RER,j,2}$	$\beta_{RER,j,3}$	$\beta_{RER,j,4}$	R^2
1	2 to 4	2.9846 *(0.4447)*	0.5736 *(0.0060)*	-0.7452 *(0.0053)*	-0.1617 *(0.0138)*	0.9991 *(0.0000)*	0.87
2	4 to 8	1.3329 *(0.6024)*	0.2071 *(0.0003)*	-0.6383 *(0.0001)*	-0.1213 *(0.0016)*	1.0374 *(0.0000)*	0.94
3	8 to 16	1.8207 *(0.5501)*	0.2744 *(0.0664)*	-0.5110 *(0.0172)*	-0.0867 *(0.0432)*	1.0025 *(0.0000)*	0.92
4	16 to 32	7.0129 *(0.1034)*	0.1031 *(0.5619)*	-0.0745 *(0.7972)*	-0.0369 *(0.4596)*	0.9302 *(0.0000)*	0.85
5	32 to 64	9.1833 *(0.0366)*	0.3416 *(0.1172)*	-0.4703 *(0.1334)*	-0.0685 *(0.2104)*	0.9073 *(0.0000)*	0.85

Table A5 estimates the real exchange rate cycle at each frequency range based on the influence of interest rate parity. The positive coefficients ($\beta_{RER,\ j,\ 1}$) on the domestic interest rate (ir^{SA}) at each frequency range cause the exchange rate to appreciate as the domestic interest rate increases, while the negative coefficients ($\beta_{RER,\ j,\ 2}$) on the foreign interest rate (ir^{US}) cause the exchange rate to depreciate as the foreign interest rate increases. The negative value of the coefficient $\beta_{RER,\ j,\ 3}$ at each frequency range shows that the real exchange rate depreciates as the aggregate country default risk premium (measured by the ratio of domestic debt to domestic GDP) increases.

Table A6 gives the estimates for the coefficients of the modified smooth trends (with *p-values* in parentheses) for all of the series, as specified in equation (9). Summing the two coefficients in each of the equations produces a weighted average trend growth rate. In the consumption trend series equation, the coefficient on the lagged value of the series is $s_{C,1}$ = 0.9717, which is much larger than coefficient on the lagged value of aggregate consumption, given by $s_{C,2}$ = 0.0334. This pattern holds for all of the other modified smooth trend series, where the coefficients on the lagged value of each trend series exceeds 0.8, while the coefficients on the lagged aggregate variable of the series are less than 0.2. All four equations achieve a good fit, with statistically significant coefficients and R^2 > 0.98 in each equation.

Table A6

Empirical Estimation of the *Modified Smooth* Trend Series

	GDP		*Consumption*		*Investment*		*Government Spending*	
	$s_{YSA,1}$	$s_{YSA,2}$	$s_{C,1}$	$s_{C,2}$	$s_{I,1}$	$s_{I,2}$	$s_{G,1}$	$s_{G,2}$
Coefficient	0.9777	0.0293	0.9717	0.0334	0.9707	0.0390	0.9816	0.0257
(*p-value*)	*(0.0000)*	*(0.1073)*	*(0.0000)*	*(0.0905)*	*(0.0000)*	*(0.1047)*	*(0.0000)*	*(0.1275)*

	Exports		*Imports*		*RER*		*Interest rate* (ir^{SA})	
	$s_{EX,1}$	$s_{EX,2}$	$s_{IM,1}$	$s_{IM,2}$	$s_{RER,1}$	$s_{RER,2}$	$s_{irSA,1}$	$s_{irSA,2}$
Coefficient	0.9543	0.0490	0.9681	0.0403	0.8507	0.1515	0.8383	0.1516
(*p-value*)	*(0.0000)*	*(0.1325)*	*(0.0000)*	*(0.1125)*	*(0.0000)*	*(0.0000)*	*(0.0000)*	*(0.0000)*

The estimated government spending coefficients (ρ_j) are all about 1.007, which means that the average quarterly growth rate is about 0.007 per quarter (about 2.8% per year) at all frequency ranges.

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