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Central bank liquidity auction mechanism design and the interbank market*

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Abstract

We study whether the mechanism design in the central bank liquidity auctions matters for the interbank money market interest rate levels and volatility. Furthermore, we compare different mechanisms to sell liquidity in terms of revenue, efficiency and auction stage interest rate levels and volatility. Most importantly, we ask which mechanism is the best at implementing the target policy interest rates to the interbank market and what are the trade-offs involved. We construct a relatively general model of strategic bidding with interdependent valuations, and combine it with a stylized model of the interbank market. The novel feature of the model is that the expectations of the interbank market outcomes determine the valuations in the liquidity auctions. The model captures the relevant features of how the European Central Bank sells liquidity. We use simulations to compare discriminatory price, uniform price and Vickrey auctions to a posted price mechanism with full allotment. In order to analyze interactions between the primary and the secondary market under four different mechanisms, we need to make a lot of assumptions and simplifications. Given this caveat, we find that posted prices with full allotment is clearly the superior alternative in terms of implementing the policy interest rate to the interbank markets. This comes at the cost of less revenue compared to the revenue maximizing discriminatory price auction, but surprisingly, will not result in efficiency losses compared even to the Vickrey auction.

Keywords: ECB liquidity auctions, Interbank markets, Mechanism design, Multi-unit auctions, Monetary policy, Posted-Prices.

JEL: C63, C72, D02, D44, D47, D53, E43, E44, E52, E58, G21.

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1 Introduction

Interbank markets are probably the single most important institution in financial markets. They are used to transfer liquidity from banks with surplus to banks with deficit. Interbank interest rates either directly determine or are strongly involved in the determination of all the interest rates used in the economy, from mortgages to corporate loans. Interbank markets are the focus of central banks' implementation of monetary policy. The central bank determines target policy interest rates and tries to steer the interbank interest rates to that target rate. In some countries, such as the US, the central bank implements the policy rates using open market operations in which it buys or sells some secure long maturity asset, such as government securities. In some other countries or regions, such as the EU, monetary policy is implemented by main refinancing operations (MROs). In the European Cenral Bank (ECB) MROs, the central bank sells liquidity (money) from central bank reserves to banks in weekly tenders. The policy rate is the reservation price in these multi-unit auctions.

In general, the macroeconomic literature linking central bank policies to interbank markets is scarce. The connection between the interbank markets and central bank open market operations has been previously studied by Allen et al. (2009). Välimäki (2008) has studied the relationship between MROs and the interbank market. He explains how the spread between the interbank interest rate and the policy interest rate and the volatility of interbank rates can be explained by the timing of the auctions and the reserve maintenance period, and uncertainty over allocations in the MROs. Our aim is to study whether the actual auctions mechanism used in the MROs influences the spread between the interbank interest rate and the policy interest rate, and the volatility of the interbank rates. Given that the applied mechanism makes a difference to the allocation in the MROs and the uncertainty over the allocation, and given Välimäki's (2008) result, the used mechanism should have a direct effect on the behavior of the interbank market.

Multi-unit auction mechanisms have previously been compared mainly in the light of revenue and efficiency (e.g. Hortacsu and McAdams 2010, Kastl 2011). These comparisons make sense since revenue is the main policy goal in most auctions, for example in the government treasury auctions. However, a central bank's policy goal is different. Although the ECB finances its operations using the revenue from MROs and probably also cares to some extent for the efficiency of the allocation of liquidity in the auctions, these questions are of second order to their main task, which is the implementation of monetary policy. Therefore, it makes sense to ask the question of first order importance: Is the auction mechanism relevant for the successful implementation of monetary policy and which mechanism is the best in this respect? Answering this question is the main contribution of our paper. Previously, similar studies that answer important macroeconomic questions using microeconomic theory on auctions have been presented by Cassola et al. (2013), who analyze the subprime financial crisis with ECB auction bid data, and Balat (2013), who uses procurement auction bid data to analyze the impacts of a stimulus package.

The main theoretical contribution of our work is to allow bidders to account for the interbank market equilibrium when formulating equilibrium bidding strategies in the liquidity auctions. The equilibrium in the interbank market is driven by interbank trading frictions. First, the interbank market is bilateral in nature and thus finding the right number and type of partners to trade with may bring about search costs and other transaction costs (see e.g. Välimäki 2008). Moreover, bilateral trading can be inefficient in general (Myerson and Satterthwaite 1983). Second, lenders require collateral, which may have varying opportunity costs (e.g. Ewerhart et al. 2010). Third, lending in these markets might be risky and thus undesirable for potentially risk averse banks (e.g. Välimäki 2008). On the one hand, these frictions, together with a given auction stage allocation, explain the variation in the interbank market equilibrium prices. On the other hand, expectations over the interbank market outcomes determine the marginal valuations used to derive the equilibrium bidding strategies. Admittedly, we model the interbank market in a very simplistic way and capture the aforementioned sources of frictions only with a single parameter in order to maintain the tractability of the auction model.

We also provide some new insights into the analysis of multi-unit auctions as such. Even when agents are symmetric and their valuations are private, analytical equilibrium characterizations are not simple and in many cases impossible (e.g. Hortacsu, 2011). Moreover, especially in the uniform price auction the number of equilibria is potentially infinite (e.g. Klemperer and Meyer, 1989; Wang and Zender, 2002). Furthermore, since different mechanisms cannot be compared analytically, only empirical comparison is meaningful (Ausubel and Cramton, 2002; Ausubel et al. 2011). Therefore, in order to compare the different mechanisms theoretically, we need to make some simplifying assumptions concerning marginal valuations, the distribution of uncertainty, and both the strategic set and the equilibrium selection. However, we show that our maintained assumptions do not remove the known characteristic differences of the different mechanisms. Under our assumptions, it is possible to characterize the equilibrium solution in all the standard multi-unit auction mechanisms and to compare the mechanisms using simulations. The interesting novel contribution is that we analyze how different assumptions concerning the information structure influence the mechanism comparisons.

Ewerhart et al. (2010) study questions most directly related to our goal. They construct a bidding model that incorporates the institutional features of ECB MROs and are able to explain some puzzling empirical regularities in the MROs auction and interbank market data. Thus their question is of a positive nature, whereas we ask a more normative question. They show that the discount observed in MROs (spread between the interbank and auction interest rates) can be explained by strategic bidding. Their auction model also explains why uncertainty leads to higher bids and why bid schedules are very flat. Our modeling assumptions are somewhat similar to theirs, especially in terms of capturing the essential features of the ECB institution, but there are important differences. In their model, bidders' valuations are influenced by the availability of collateral. In our model, bidder valuations are determined by the expectations over the interbank market equilibrium. Moreover, we provide a strict generalization by introducing interdependent valuations. Thus, our results are potentially of more interest in other multi-unit auction contexts and apply to a wider range of ECB operations. The available empirical evidence suggests that uncertainty about the common value of liquidity is not a central feature of the MROs (e.g. Bindseil et al. 2005), but in the ECB longer-term refinancing operations, common values may play an important role (e.g. Linzert et al. 2007). Our model applies to both settings. Moreover, changes in macroeconomic conditions, for example a financial crisis, may possibly increase the importance of common uncertainty.

Allowing interdependent valuations comes with some costs. First, we do not allow uncertainty to the total amount to be auctioned. In our model, the central bank sets the quantity sold in MROs to equal the expected aggregate demand. The total quantity is fixed before bids are compared and this quantity is announced to the bidders prior to submitting the bids. To our understanding, this assumption reflects reasonably well how the ECB behaves in practice, with the expectation that in reality, a small amount of uncertainty over the total allotment may be present. However, there will still be uncertainty about the individual allocations, and due to the interplay of correlated valuations and mechanism rules, the final total allocation may be lower than the amount that the central bank is auctioning. Moreover, we have to make some restrictive assumptions about the bidding strategies and cannot derive analytical comparisons between the different mechanisms. Fortunately, we can simulate the results. Other novel features of our model are that interbank trading is modeled explicitly, albeit in a very simple way. To our knowledge, we are the first to explicitly compare the different mechanisms in terms of how they influence the interbank market. Otherwise we follow and combine contributions by Ewerhart et al. (2010), Vives (2010, 2011), Rostek et al. (2010) and Holmberg (2008, 2009b). See also Table 1 for how our assumptions follow the existing literature. Our main analysis and policy conclusions are based on simulations of the different mechanisms under otherwise the same modeling assumptions.

Table 1: Some literature on theoretical models of multi-unit auctions

Reference	Values	Mechanisms	Marginal values	Strategies
Ausubel et al. (2011)[a]	Interdependent	UPA, DPA	Constant	General
Back and Zender (1993)	Common	UPA, DPA	Constant	General
Ewerhart et al. (2010)	IPV	UPA, DPA	Linear	Linear
Holmberg (2008)	IPV	UPA	General	General
Holmberg (2009b)	IPV	DPA	General	General
Klemperer and Meyer (1989)	IPV	UPA	General	General
Rostek et al. (2010)	IPV	UPA, DPA, VA	Linear	Linear
Wang and Zender (2002)	Common	UPA, DPA	Constant	General
Wilson (1979)	Common	UPA, VA	Constant	General
Vives (2010)	Interdependent	UPA	Linear	Linear
This paper (Ollikka and Tukiainen 2013)	Interdependent	UPA, DPA, VA, PP-FA	General	General

[a] Ausubel et al. (2011) combines the results of Ausubel and Cramton (2002) and Rostek et al. (2002). These notes refer to Section 4 of Ausubel et al. (2011). Notes on Rostek et al. (2010) in this Table refer to Section 5 of Ausubel et al. (2011).

Previous studies (e.g. Ewerhart et al. 2010) make comparisons only between discriminatory price auction (DPA) and the uniform price auction (UPA), whereas we compare four different mechanisms: Discriminatory price auction (called variable rate tenders by ECB), uniform price auction (called fixed rate tenders by ECB), the Vickrey auction (VA) and the posted price selling mechanism with full allotment (PP-FA).[1] Therefore, we have three different auction mechanisms, of which two have been used by the ECB in practice at some point, and one non-auction mechanism that has been used by the ECB since the recent financial crisis. Our comparison is based on theoretical modeling and simulations. We do not provide actual empirical evidence, although our models and simulation results are broadly consistent with the observed bidding behavior in the ECB auctions (e.g. Cassola et al. 2013) and the observed interbank interest rate data (e.g. Ayoso and Repullo 2003).

The stated policy goal of the ECB is to implement the target policy interest in the interbank market. This means minimizing the spread between the policy rate and the interbank interest rate. We find that the posted price mechanism with full allotment is the superior mechanism in this respect. It is able to steer the interbank rate to the policy rate correctly on average and with high accuracy, whereas the other mechanisms are off target on average and have larger variance.

The posted price mechanism with full allotment is also the most efficient mechanism. We measure efficiency by how much the banks need to rely on the ECB standing facilities to meet their reserve requirements after the auction and interbank trading stages have been played. Heuristically, this measures the amount of liquidity that the market mechanisms, i.e. the auction and the interbank market, fail to allocate. More formally, this measures the amount of liquidity traded away from the equilibrium prices. More traditional efficiency measures are not interesting because all banks have to meet the reserve requirements. The efficiency result is surprising because the standard result is that auctions are more efficient than posted prices, because in auctions, bidders can condition their bidding over the expectations of all the bidders' signals, given the price, whereas with posted prices they can only condition over their own signal. The presence of fixed total allotment and reserve prices changes this standard efficiency comparison. In the case of low average signals on demand, the reserve price limits the possibility of price delivering information about the other bidders' signals. In the case of high average signals on demand, fixed total allotment limits the supply in the auction mechanisms, but not in the posted prices with full allotment. Therefore, even if the price is in this case more informative of the competitors' signals in an auction, posted prices with full allotment is more efficient due to the gains from supply adjusting to higher demand.

The only trade-off for posting prices that comes out from our simulations is that the posted price with full allotment is not the optimal (i.e. revenue maximizing in the typical language of auction theory) mechanism. Under our modeling and simulation assumptions, discriminatory price auction is optimal. Moreover, the central bank may also prefer any auction mechanism over the posted price mechanism, because auctions may provide more information about the demand for liquidity in the market than the posted price mechanism, since in auctions, the bids are entire demand functions. All our results are robust to different parameter settings in the simulations. Furthermore, it is interesting to note that the relative advantage of posted prices compared to auction mechanisms gets smaller when we increase the interdependency of the signals, i.e. common values become more important, and when signals become less informative. Regarding the revenue and efficiency comparison between the uniform auction and the discriminatory auction, our results are in line with findings by Ewerhart et al. (2010).

In Section 3, we construct models of strategic bidding under the five different mechanisms of interest that are broadly consistent with the institutional features of the ECB liquidity auctions outlined in Section 2. In Section 4, we present the results from simulating the models and Section 5 concludes.

[1] Although posted prices with limited allotment have been used in practice by some central banks, we do not analyze that mechanism, because it is clearly inferior to posted prices with full allotment in all relevant dimensions.

2 Institutional setup of the ECB

Although our analysis may apply to many central bank or to some other multi-unit auction settings, we built our model to mimic the ECB tenders and the Eurosystem monetary policy. The ECB sets the monetary policy by determining the level of three official interest rates. The marginal lending rate and the deposit rate determine the interest rates for central bank lending and deposits to the standing facility. Any bank may at any time lend any amount of money from or deposit any amount of money to the central bank. Thus these rates set the lower and upper limits for the interbank rate. The rates are adjusted symmetrically around the key policy rate, which is the minimum bid rate. This minimum bid rate is the reservation price in the MROs. Although not explicitly announced, the key target of the ECB is to stabilize the short-term interbank rates to a level close to the key policy rate (Välimäki 2008).

The main institutional feature that influences the liquidity needs is the reserve maintenance period and the minimum reserve requirements. At the end of this period the banks need to fulfill their reserve requirements. The ECB requires the credit institutions (banks) to hold deposits on accounts with their national central bank (NCB). Minimum reserve requirements are determined as fractions of banks' demand deposits as cash reserves: 1% from the beginning of 2012. Reserve requirements are averaged over the reserve maintenance period, which lasts approximately a month. The required reserves are remunerated with the marginal MRO rate. In March 2004, the ECB synchronized the timing of the end of the reserve maintenance period and when the policy rate could be adjusted. This removed the effect of expectations of the policy rate on the interbank interest rate, thus making the spread between the policy rate and the market rate smaller (Välimäki 2008). Therefore, we do not explicitly model the effect of the reserve maintenance period on bidding behavior. However, there is a period of one week between the last auction of a given reserve maintenance period and the end of the period. Due to this difference, bidders are uncertain about their reserve needs at the time of bidding. This is the main source of uncertainty in our auction model.

Another important determinant of demand in these auctions is the availability of collateral. Both the ECB and the interbank market requires collateral for the liquidity. The range of eligible collateral is wider in the ECB operation than in the interbank markets. For this reason, banks may prefer ECB money over interbank money.[2] In the model of Ewerhart et al. (2010), the availability of collateral of different quality means the banks' demand curves are decreasing. In our model, we derive the marginal value of money in the ECB MROs for each bank from the expected interbank market outcomes. The expectations over the aggregate and individual demand imply decreasing demand curves in the auctions. The marginal value of money in ECB MROs is the avoided costs of lending (or borrowing) money in the secondary market. Hence, in contrast to Ewerhart et al. (2010), it is not (only) the opportunity cost of collateral that defines the banks' value of money in ECB MROs. We assume that the opportunity costs are (mainly) based on the probability of finding a trading partner in the interbank market and on the frictions (which may include quality of collateral) of the interbank market.

Another important institutional feature that has to be accounted for in our model is how the ECB decides the amount of liquidity sold in the auction and what information is revealed and when concerning this volume. Eisenschmidt et al. (2009) explain the procedure. Prior to auction, the ECB estimates the liquidity needs of the Eurosystem based on the sum of the expected outstanding autonomous factors (such as banknotes, government deposits and net foreign assets) and banks' reserve requirements. The allotment volume that satisfies these liquidity needs of the banking sector exactly is called the "benchmark allotment". Prior to bidding, either on the previous day, or the same morning, the ECB publishes a forecast of these autonomous factors on which the "benchmark

[2] Välimäki (2008) also mentions risk aversion, credit line capacity constraints and capital adequacy for reasons why ECB money could be preferred over interbank money.

allocation" calculation is based. Therefore, the main uncertainty over the total allocation arises from possible changes in autonomous factors between their publication and the actual auction. An important thing to note is that the aim of the central bank is to provide this "benchmark allocation". To our understanding, they will not change the allocation as a reaction to the submitted bids. This is important, since the bidding strategies do not have to account for a strategic reaction from the central bank to the bids. However, there is some uncertainty over the sold volume because only a forecast of the inputs of the calculation are published, but not the actual amount itself. On the other hand, given the short time lag, this uncertainty is likely to be relatively small. We will model this setup in following way. The total quantity sold in the auction corresponds to the expected demand at the marginal rate. We simplify the analysis by assuming that there is no uncertainty over this total quantity at the time of submitting bids.

In ECB tenders, the banks submit demand functions in terms of volume and price. The auction rules constrain these bids to be step functions with a maximum of 10 steps. Kastl (2011) shows that these steps influence bidding behavior. Nonetheless, in our theoretical model we abstract away from this detail to keep the analysis tractable. One important feature is that bids may be rationed if the demand exceeds supply at the marginal rate. A pro rata amount is allocated to all marginal bidders of the remaining liquidity (Eisenschmidt et al. 2009).

Figure 1 shows the histories of the ECB official interest rates and the EONIA rate (Euro Overnight Index Average), which is the index rate of the interbank market. From January 1999 to June 2000 the ECB used the uniform price auction mechanism in MROs. From June 2000 to July 2008 it used the discriminatory pricing rule. Due to the financial crisis, after October 2008, the ECB reverted back to the uniform price rule but with full allotment at the policy rate, effectively making it a posted price mechanism instead of an auction. We compare all these three rules and in addition make comparisons with the Vickrey auction, since the Vickrey auction guarantees efficient allocation in many situations.

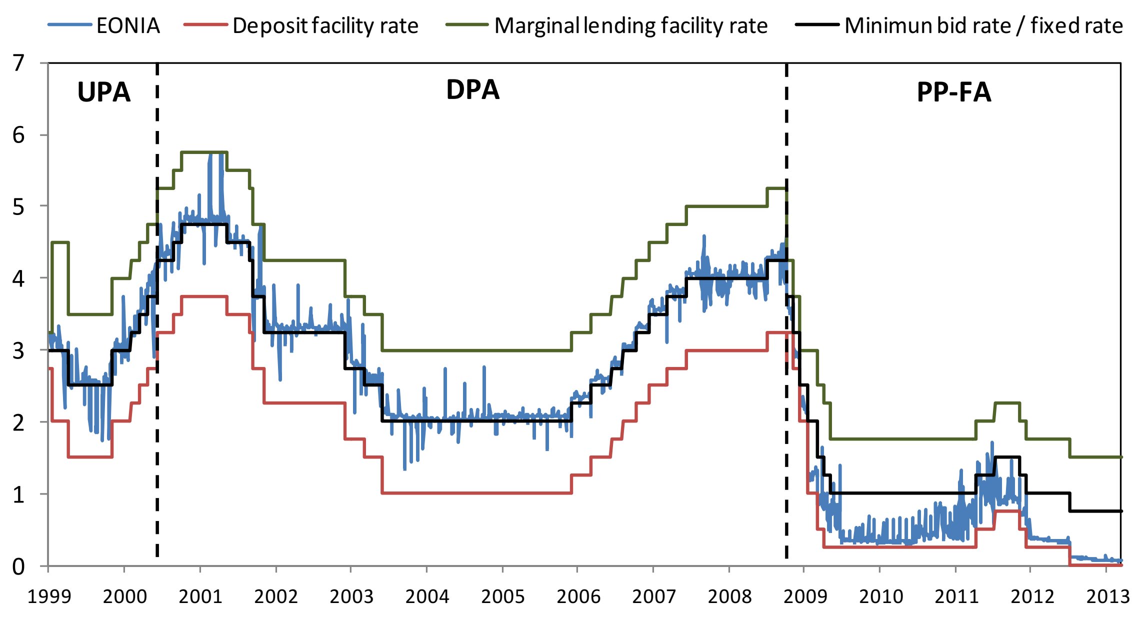

Figure 1: Policy and interbank rates

In a typical ECB auction, there are several hundred bidders. Due to computational reasons, we are not able to simulate a realistic number of bidders for the uniform auction. For now, we have calculated the results only for one hundred bidders. From this perspective, our simulation is more in line with the fine tuning operations of the ECB, to which typically only tens rather than hundreds of banks participate. When the number of bidders grows very large, the differences between the auction mechanisms in our model should vanish in all other respects except the revenue. In the model of Ewerhart et al. (2010), the difference in the price outcomes between uniform and discriminatory price auctions does not vanish due to uncertainty in the total allotment. Since we find notable differences between the mechanisms with 100 bidders, it is likely that even several hundred cannot be considered a large number in this respect. Thus, our results for one hundred bidders probably reveal relevant comparisons for ECB auctions with a more realistic amount of bidders.

3 Theoretical model

3.1 Interbank market

In this section, we first provide a stylized model of the interbank market. Then we discuss how the interbank model interacts with bidding strategies in five different mechanisms to allocate liquidity. For clarity, we begin the analysis by describing the interbank market as a market of only two representative banks. Let bank 1 be a representative bank on the demand side and bank 2 on the supply side of the interbank market. Consider that bank 1 is q^{ID} short of liquidity and bank 2 has money to the amount of q^{IS} over its reserve requirements after the auction stage. Let $\overline{p}$ denote the marginal lending facility (LF) rate and $\underline{p}$ the deposit facility (DF) rate. The ECB target policy rate is the mean of the standing facilities rates, $p^0 = \frac{1}{2}\left(\overline{p} + \underline{p}\right)$. In a frictionless interbank market, the inverse demand for money is defined by bank 1's willingness to pay for liquidity:

$$P_D^{IB}\left(\Delta q\right) = \begin{cases} \overline{p}, & \text{for } \Delta q \leq q^{ID} \\ \underline{p}, & \text{for } \Delta q > q^{ID}. \end{cases}$$

Now Δq denotes the amount of trading in the interbank market. Bank 2 is willing to sell liquidity according to the inverse supply

$$P_S^{IB}\left(\Delta q\right) = \begin{cases} \underline{p}, & \text{for } \Delta q \leq q^{IS} \\ \overline{p}, & \text{for } \Delta q > q^{IS}. \end{cases}$$

In the equilibrium, banks trade money at a maximum amount. Therefore,

$$\Delta q = \min\left(q^{ID}, q^{IS}\right).$$

If there is demand or excess liquidity left, banks have to turn to the standing facilities. The use of the standing facilities is given as

$$q^{LF} = \max(0, q^{ID} - q^{IS}), \qquad q^{DF} = \max(0, q^{IS} - q^{ID}).$$

Depending on the bargaining power of the two banks, the interbank rate may be any rate between standing facility rates. We assume that the interbank rate is defined by the point where the demand and supply cross, such that

$$p^{IB} = \begin{cases} \overline{p}, & \text{if } q^{ID} > q^{IS} \\ p^0, & \text{if } q^{ID} = q^{IS} \\ \underline{p}, & \text{if } q^{ID} < q^{IS}. \end{cases}$$

Thus, we make a following assumption:

Assumption 1. Banks have equal bargaining power in the interbank market.

On average over many auctions this is natural, because in our auction model, it is ex ante random on which side of the markets banks end up in the interbank market. However, given the auction outcome, the assumption may be less realistic. Nonetheless, we maintain it for the sake of tractability.

Next, suppose instead that trading in the interbank market is not costless and the symmetric marginal cost of trading on both sides of the market is given as

$$MTC\left(\Delta q\right)=\eta\Delta q,$$

where η denotes the trading cost coefficient. η can capture various sources of frictions. First, the interbank market is bilateral in nature and thus finding the right number and type of partners to trade with may induce search costs and other transaction costs (see e.g. Välimäki 2008). Moreover, bilateral trading can be inefficient in general (Myerson and Satterthwaite 1983). Second, lenders require collateral which may have varying opportunity costs (e.g. Ewerhart et al. 2010). Third, lending in these markets might be risky and thus undesirable for potentially risk averse banks (e.g. Välimäki 2008).

Hence, η reduces bank 1's incentives to pay for liquidity and increases bank 2's requirements for selling rates. The effective inverse demand for liquidity in the interbank market becomes thus

$$P_D^{IB}\left(\Delta q\right)=\begin{cases}\overline{p}-\eta\Delta q, & \text{for } \Delta q\leq q^{ID}\\ \underline{p}-\eta\Delta q, & \text{for } \Delta q>q^{ID}.\end{cases}$$

Respectively, the effective inverse supply is given as

$$P_S^{IB}\left(\Delta q\right)=\begin{cases}\underline{p}+\eta\Delta q, & \text{for } \Delta q\leq q^{IS}\\ \overline{p}+\eta\Delta q, & \text{for } \Delta q>q^{IS}.\end{cases}$$

In the equilibrium, supply and demand are equal. Therefore, four different cases arise. These are presented in Figure 2. Firstly, suppose that the market is symmetric and the amount of trading equals the total demand and supply, $\Delta q=q^{ID}=q^{IS}$ (Figure 2a). Then it must hold that $P_D^{IB}\left(\Delta q\right)\geq P_S^{IB}\left(\Delta q\right)$, and the equilibrium rate is any price between $P_D^{IB}\left(\Delta q\right)$ and $P_S^{IB}\left(\Delta q\right)$. Again, we define the interbank rate as the average of these two. With symmetric trading costs it is thus equal to the policy target rate

$$p^{IB}=\frac{1}{2}\left(\overline{p}-\eta\Delta q+\underline{p}+\eta\Delta q\right)=p^0.$$

In this case, the ECB target is reached and banks do not need the standing facilities in order to fulfill their reserve requirements.

The second case (Figure 2b) is otherwise similar, but the effective supply and demand curves cross before the total demand is fulfilled. Now the excess supply is not absorbed by the interbank market. Hence, we have $q^{ID}>\Delta q$ and $q^{IS}>\Delta q$. The amount of interbank trading is given as

$$\Delta q=\frac{1}{2\eta}\left(\overline{p}-\underline{p}\right),$$

and the interbank rate is, again, equal to the policy target rate

$$p^{IB} = P_D^{IB}(\Delta q) = P_S^{IB}(\Delta q) = p^0.$$

However, both banks must turn to the standing facilities. Bank 1 lends the amount of $q^{LF} = q^{ID} - \Delta q$ and pays $\overline{p}$ for this money. Respectively, bank 2 deposits $q^{DF} = q^{IS} - \Delta q$ to the deposit facility with the remuneration rate $\underline{p}$. Furthermore, the higher the η, the less banks trade money in the interbank market and the more banks have to use standing facilities.

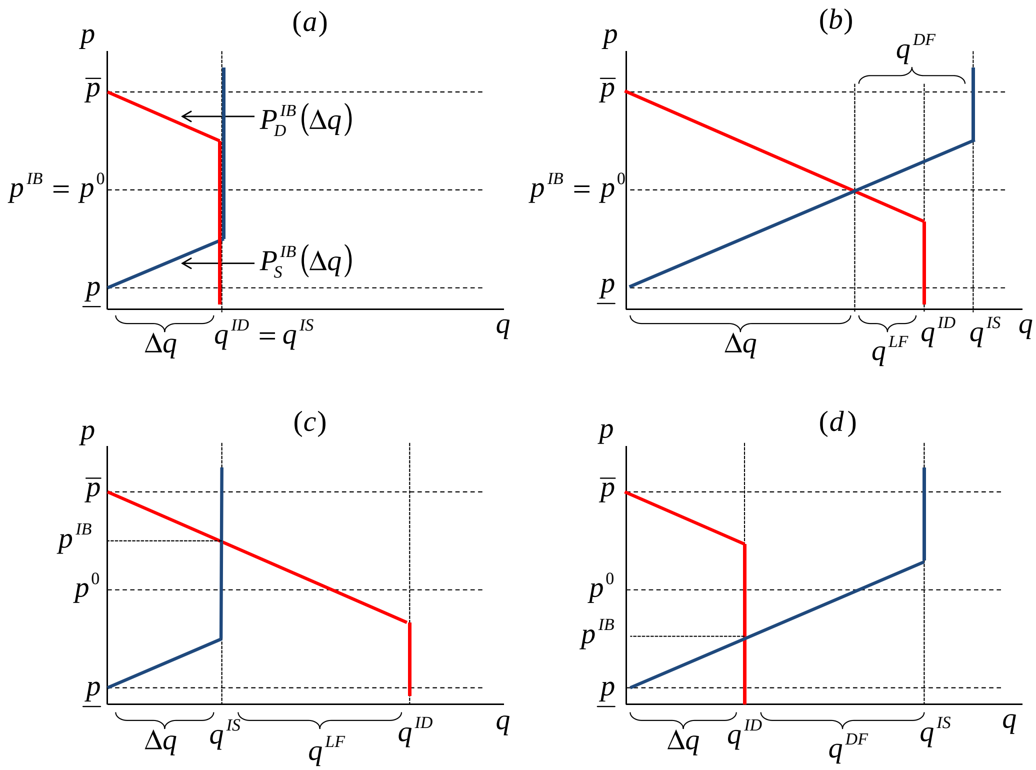

Figure 2: Stylized model of the interbank market - two representative banks.

In the third case (Figure 2c) the interbank market cannot satiate all the demand, $\Delta q = q^{IS} < q^{ID}$. Hence the market clearing rate is higher than the policy rate, $p^{IB} = \overline{p} - \eta q^{IS} > p^0$, and bank 1 has to lend money from the marginal lending facility, $q^{LF} = q^{ID} - q^{IS}$. The fourth case (Figure 2d) is symmetric with the third case, but now there is excess supply in the interbank equilibrium, $\Delta q = q^{ID} < q^{IS}$, and the interbank rate is lower than the policy rate, $p^{IB} = \underline{p} + \eta q^{ID} < p^0$. Deposits to the deposit facility total $q^{DF} = q^{IS} - q^{ID}$.

Clearly, the best case with regards to ECB policy targets is the first case of Figure 2. If there are frictions in the interbank market, it is in the interest of the ECB to get the allocation of liquidity in the MROs as close to banks' reserve requirements as possible and also to make the demand and supply of the interbank market symmetric. The higher the frictions, the more important this is. In this paper we ask: Do different mechanisms used by the ECB to sell liquidity in MROs reflect differently on these targets?

Suppose next that the number of banks increases to n. Let n^{IS} denote the number of banks on the supply side and n^{ID} on the demand side of the interbank market. Let Q^{IS} and Q^{ID} denote the total interbank supply and demand.

Suppose also that the model is symmetric, i.e. $Q^{IS} \approx Q^{ID} \Leftrightarrow n^{IS} \approx n^{ID}$ and $Q^{IS} \left\{ \begin{array}{c} \gg \\ \ll \end{array} \right\} Q^{ID} \Leftrightarrow n^{IS} \left\{ \begin{array}{c} \gg \\ \ll \end{array} \right\} n^{ID}$. Then the interbank rate is close to $\overline{p}$ if $Q^{ID} > Q^{IS}$ and close to $\underline{p}$ if $Q^{IS} > Q^{ID}$, given that η is low enough. We may write the interbank rate as

$$p^{IB} = \begin{cases} \max\left\{p^0, \overline{p} - \eta \frac{Q^{IS}}{n^{ID}}\right\}, & \text{if } Q^{IS} < Q^{ID} \\ p^0, & \text{if } Q^{IS} = Q^{ID} \\ \min\left\{p^0, \underline{p} + \eta \frac{Q^{ID}}{n^{IS}}\right\}, & \text{if } Q^{IS} > Q^{ID}. \end{cases} \tag{1}$$

Let $F_D\left(Q^{ID}\right)$ denote the cumulative distribution function of the interbank demand and $F_S\left(Q^{IS}\right)$ c.d.f. of the interbank supply. In the symmetric case these are identical functions. Furthermore, the expected interbank rate may then be written as:

$$\begin{aligned} E\left[p^{IB}\middle|\,\eta\right] \equiv p^{IB}(\eta) &= \int_0^\infty \left\{ \int_0^{Q^{IS}} \min\left\{p^0, \underline{p} + \eta \frac{Q^{ID}}{n^{IS}}\right\} f_D\left(Q^{ID}\middle|\,Q^{IS}\right) dQ^{ID} \right\} f_S\left(Q^{IS}\right) dQ^{IS} \\ &+ \int_0^\infty \left\{ \int_{Q^{IS}}^\infty \max\left\{p^0, \overline{p} - \eta \frac{Q^{IS}}{n^{ID}}\right\} f_D\left(Q^{ID}\middle|\,Q^{IS}\right) dQ^{ID} \right\} f_S\left(Q^{IS}\right) dQ^{IS}. \end{aligned} \tag{2}$$

The terms inside curly brackets define the expected interbank rate for a given Q^{IS}. The first term is the expected rate given that $Q^{IS} \geq Q^{ID}$ and the second term the expected interbank rate, given that $Q^{ID} \geq Q^{IS}$. These are multiplied by the density of Q^{IS} and integrated over all possible Q^{IS}. Equation (2) defines the expected price of money in the interbank market. However, this is not the same as the marginal valuation in the ECB MROs for each bank.

We assume that reserve requirements are uncertain to banks before ECB operation, but these requirements are drawn from the same commonly known distribution. However, before the MROs, banks receive noisy signals about their reserve requirements. In the following section, we describe the information structure of the model in more detail. Now, we derive the marginal value functions of banks in ECB operations while they form expectations over the reserve requirement r_i. The expected marginal valuation for an additional unit of money at total quantity q_i is

$$\begin{aligned} v_i(q_i|r_i,\eta) &= Pr_i(r_i \leq q_i)\left[Pr_i^{IB}\left(q_i|\,r_i \leq q_i, \eta, Q^{ID}, Q^{IS}\right) p^{IB}(\eta) + \left(1 - Pr_i^{IB}\left(q_i|\,r_i \leq q_i, \eta, Q^{ID}, Q^{IS}\right)\right) \underline{p}\right] \\ &+ Pr_i(r_i > q_i)\left[Pr_i^{IB}\left(q_i|\,r_i > q_i, \eta, Q^{ID}, Q^{IS}\right) p^{IB}(\eta) + \left(1 - Pr_i^{IB}\left(q_i|\,r_i > q_i, \eta, Q^{ID}, Q^{IS}\right)\right) \overline{p}\right]. \end{aligned} \tag{3}$$

We denote the probabilities that bank i is successful in selling or buying the marginal unit q_i in the interbank market respectively by $Pr_i^{IB}\left(q_i|\,r_i \leq q_i, \eta, Q^{ID}, Q^{IS}\right)$ and $Pr_i^{IB}\left(q_i|\,r_i > q_i, \eta, Q^{ID}, Q^{IS}\right)$. Therefore, marginal valuation depends on the expectations over whether the bank is short or long of liquidity, expectation over the interbank prices and whether the bank is successful at trading. If the bank cannot trade the q_i^{th} unit, it can turn to the standing facilities. In our simulation exercise, (3) is computationally a very challenging program because the probabilities of trade need to be calculated separately for each level of total interbank supply and demand. Therefore, we resort to an approximation where $v_i(q_i|r_i,\eta)$ is generated by a convex combination of the extreme cases of no frictions and very large frictions, because in these cases the problematic probabilities are canceled out.

Without any trading costs, coefficient $\eta = \eta_0 = 0$, and the marginal valuation reduces to

$$v_i\left(q_i|r_i,\eta_0\right) = \overline{p} - \underbrace{\int_0^{\infty} F_D\left(Q^{IS}\middle| Q^{IS}\right) f_S\left(Q^{IS}\right) dQ^{IS}}_{Pr\left(Q^{IS}>Q^{ID}\right)}\left(\overline{p}-\underline{p}\right), \tag{4}$$

where $Pr\left(Q^{IS} > Q^{ID}\right)$ is the probability of the interbank supply being greater than the interbank demand. When $\eta = 0$, the interbank rate is either $\overline{p}$ or $\underline{p}$ (or p^0, but then Pr_i^{IB}'s are 1 and we get the same result). In both cases, it does not make any difference whether the bank i is short or long of liquidity after MROs. Suppose, for instance, that the resulting interbank rate is $\overline{p}$ and bank i is long of liquidity. It sells excess liquidity at a price $\overline{p}$ and it finds a counterpart with the probability of one, while the total interbank demand is higher than the supply and trading is costless. In contrast, if bank i is short of money, it is indifferent whether it purchases the needed liquidity from the interbank market or from the lending facility. It is then only the aggregate probability which defines the marginal value of money in the auction when trading costs are low. More formal proof of (4) is presented in Appendix A.

What if the interbank market does not function properly and the trading costs are high for one reason or another? To some extent banks are willing to trade liquidity, but almost all banks must turn to standing facilities in order to fulfill their reserve requirements (see Figure 2b). We define η_H such that $\eta \to \infty$. At the limit, the interbank market is totally collapsed. In this case, $Pr_i^{IB}\left(q_i\middle| r_i < q_i, \eta, Q^{ID}, Q^{IS}\right) = Pr_i^{IB}\left(q_i\middle| r_i \geq q_i, \eta, Q^{ID}, Q^{IS}\right) \to 0$. However, for an individual bank, the expected rate it has to pay for money after the ECB MRO is effectively $\overline{p}$. Respectively, if a bank has excess liquidity after the ECB MRO, the marginal remuneration rate is $\underline{p}$. Thus, when $\eta = \eta_H$, the marginal value of money in the ECB operation is defined by

$$v_i\left(q_i|r_i,\eta_H\right) = \overline{p} - Pr_i\left(r_i \leq q_i\right)\left(\overline{p}-\underline{p}\right), \tag{5}$$

where $Pr_i\left(r_i \leq q_i\right)$ is the probability of bank i being long of money after the ECB MRO. Therefore, in the case of a frictionless interbank market, the marginal valuation is based on the probability that the market is on aggregate short or long. In the case of a collapsed interbank market, the marginal valuation is based on the probability that the given individual bank is short or long. In the intermediate case, the marginal valuation is a combination of these two probabilities. Before showing how we generate the intermediate case from the two extremes, we need to define the information structure of the auction game. We turn to this in the next sections.

3.2 Affine information structure

In ECB operations, bank i's expected marginal value of money is denoted by a function $v_i\left(q_i|r_i,\eta\right) = v_i\left(q_i|s\right)$, where q_i is the volume of liquidity received by bank i and $s = (s_1, \ldots, s_n)$ is the signal vector of the banks' reserve requirements $r = (r_1, \ldots, r_n)$. η is omitted for notational purposes. The information structure of the model builds on Vives (2010, 2011).

Definition 1. The affine information structure is defined by equations (6) - (12), where distribution functions of uncertain variables and frictions of the interbank market are common knowledge.

Assumption 2. The information about banks' reserve requirements is based on the affine information structure.

Firstly, reserve requirements are normally distributed around $\bar{r}$ and they are correlated between banks, i.e. $r_i \sim N\left(\bar{r}, \sigma_r^2\right)$ and $cov\left[r_i, r_j\right] = \rho\sigma_r^2$. However, banks do not observe their reserve requirements exactly before ECB MROs. Instead, each bank receives a signal $s_i = r_i + \varepsilon_i$, where noise terms ε_i are independent and identically distributed with normal distribution around zero, $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$. The average signal is denoted by $\tilde{s} = \frac{1}{n}\sum_{i=1}^{n} s_i$ and the average reserve requirement by $\tilde{r} = \frac{1}{n}\sum_{i=1}^{n} r_i$. The average signal and the average reserve requirement have an expected value $E[\tilde{s}] = E[\tilde{r}] = \bar{r}$ and variances $var[\tilde{s}] = \frac{1}{n}\left(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_r^2\right)$ and $var[\tilde{r}] = \frac{1}{n}(1+(n-1)\rho)\sigma_r^2$, respectively.[3] Moreover, banks may update their beliefs about the reserve requirements given the information they have. For example, the expected value of r_i conditional on signal s_i is

$$\begin{aligned} E[r_i|\, s_i] &= E[r_i] + \frac{cov[r_i, s_i]}{var[s_i]}(s_i - E[s_i]) \\ &= \bar{r} + \left(\frac{\sigma_r^2}{\sigma_\varepsilon^2 + \sigma_r^2}\right)(s_i - \bar{r}). \end{aligned} \tag{6}$$

Indeed, this is the information banks use in the posted price mechanism.

If bank i knew both the signal s_i and the average signal $\tilde{s}$, then it would further update its beliefs and the conditional expected value of r_i would be (see Appendix B, and e.g. DeGroot, 1970; Vives 2011)

$$E[r_i|\, s_i, \tilde{s}] = A\bar{r} + Bs_i + Cn\tilde{s}, \tag{7}$$

where

$$A = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_r^2} \tag{8}$$

$$B = \frac{(1-\rho)\sigma_r^2}{\sigma_\varepsilon^2 + (1-\rho)\sigma_r^2} \tag{9}$$

$$C = \frac{\rho\sigma_r^2\sigma_\varepsilon^2}{(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_r^2)(\sigma_\varepsilon^2 + (1-\rho)\sigma_r^2)}. \tag{10}$$

We denote the remaining uncertainty by $\varepsilon_i^s = r_i - E[r_i|s_i, \tilde{s}]$. Its cumulative distribution function $G_\varepsilon(\varepsilon_i^s)$ has parameters $\varepsilon_i^s \sim N\left(0, (B+C)\sigma_\varepsilon^2\right)$. Moreover, let $\tilde{G}_\varepsilon(n\tilde{\varepsilon}^s)$ denote the c.d.f. of the remaining aggregate uncertainty, that is $n\tilde{\varepsilon}^s = \sum_{i=1}^n \varepsilon_i^s$. It has normal distribution with parameters $n\tilde{\varepsilon}^s \sim N\left(0, (1-A)n\sigma_\varepsilon^2\right)$. We need distribution functions $G_\varepsilon(\varepsilon_i^s)$ and $\tilde{G}_\varepsilon(n\tilde{\varepsilon}^s)$ below, when we derive the marginal value functions of banks in ECB MROs.

Further, let $I = \sum_{i=1}^n s_i = n\tilde{s}$ denote the aggregate market information. Note that given signal s_i, bank i may derive the conditional distribution of I before the ECB MROs. It is given as above cumulative distribution function $G(I|\, s_i)$ with the expected value and variance, respectively,

$$E[I|\, s_i] = nE[\tilde{s}] + \frac{n \cdot cov[\tilde{s}, s_i]}{var[s_i]}(s_i - E[s_i]), \tag{11}$$

$$var[I|\, s_i] = n^2 \cdot var[\tilde{s}] - \frac{(n \cdot cov[\tilde{s}, s_i])^2}{var[s_i]}. \tag{12}$$

Hence, each bank has to solve the decision problem with a different conditional expectation about the aggregate market information, but they share the same conditional variance. This difference is relevant in the discriminatory price auction, whereas in other auction mechanisms the distribution $G(I|\, s_i)$ has no role in determining the equilibrium strategies.

[3] Note also that $var[\tilde{s}] = cov[s_i, \tilde{s}]$.

Using normal distributions causes problems of unconstrained values of random parameters. We assume the number of winning banks is n in every auction with high probability. Hence, we assume throughout this paper that there is a positive constant $\underline{r}$ for which the probability to get a signal $s_i < \underline{r}$ is extremely low and banks ignore that possibility.

3.3 Marginal valuation of money in the ECB MRO

Next we derive the expected interbank rate and the marginal value of money in the ECB auctions given the affine information structure. Consider first a frictionless interbank market ($\eta = \eta_0$). Now, denote with $\tilde{G}_\varepsilon \left(n\bar{r} - nE\left[\tilde{r}|\, \tilde{s}\right]\right)$ the probability that the interbank market is short of money given the average signal $\tilde{s}$. Hence, the marginal value of money conditional on $\eta = \eta_0$, defined in (4) is constant for a given $\tilde{s}$, that is

$$v_i\left(q_i|s, \eta_0\right) = \overline{p} - \tilde{G}_\varepsilon\left(n\bar{r} - nE\left[\tilde{r}|\, \tilde{s}\right]\right)\left(\overline{p} - \underline{p}\right). \tag{13}$$

In contrast, suppose that the interbank market suffers from high frictions ($\eta = \eta_H$). Then the marginal value of money of bank i can be defined by (5). Now, denote with $G_\varepsilon\left(q_i - E\left[r_i|\, s_i, \tilde{s}\right]\right)$ the probability that the individual bank i is short of money given the signal s_i and the average signal $\tilde{s}$. Hence, the opportunity cost of the interbank money conditional on s_i, $\tilde{s}$ and $\eta = \eta_H$ is

$$v_i\left(q_i|s, \eta_H\right) = \overline{p} - G_\varepsilon\left(q_i - E\left[r_i|\, s_i, \tilde{s}\right]\right)\left(\overline{p} - \underline{p}\right). \tag{14}$$

For the sake of simplicity, we make the following assumption

Assumption 3. For $\eta_0 < \eta < \eta_H$, the marginal value lies between the equations (13) and (14). The marginal value function is given as

$$\begin{aligned} v_i\left(q_i|s\right) &= v_i\left(q_i|s, \eta_H\right) + \left[v_i\left(q_i|s, \eta_0\right) - v_i\left(q_i|s, \eta_H\right)\right]e^{-b\eta} \qquad (15)\\ &= \overline{p} - e^{-b\eta}\tilde{G}_\varepsilon\left(n\bar{r} - nE\left[\tilde{r}|\, \tilde{s}\right]\right)\left(\overline{p} - \underline{p}\right) - \left(1 - e^{-b\eta}\right)G_\varepsilon\left(q_i - E\left[r_i|\, s_i, \tilde{s}\right]\right)\left(\overline{p} - \underline{p}\right). \end{aligned}$$

Assumption 3 allows us to compute the intermediate cases as a convex combination of the extreme cases as discussed in section 3.1. In Figure 3 in Appendix F, we present examples of how marginal valuations and interbank rates behave under assumption 3 when scale parameter $b = \frac{1}{10}$. We use this value also in all the simulations. It also very important to point out that since under our assumptions the marginal value function does not depend on how the different mechanisms allocate liquidity between the bidders, we avoid the complication of having to construct a different value function for the different mechanisms.

3.4 Posted price mechanism with full allotment (PP-FA)

In the posted price mechanism, the ECB sets the policy target rate p^0 at which the banks may purchase as much liquidity as they wish. In this mechanism, the price does not reveal the signals of the other banks. Moreover, the expected interbank rate is p^0. After receiving signals, banks condition their reserve requirements only on their own signal and demand according to

$$q_i^{pp}\left(s_i\right) = E\left[r_i|\, s_i\right] = \bar{r} + \left(\frac{\sigma_r^2}{\sigma_\varepsilon^2 + \sigma_r^2}\right)\left(s_i - \bar{r}\right). \tag{16}$$

3.5 Auction mechanisms

In an auction, banks must submit monotonic and non-increasing demand schedules, $D_i\left(p; s_i\right)$. The auctioneer (ECB) uses bids to determine an aggregate bid function, $D\left(p; s\right) = \sum_{i=1}^{n} D_i\left(p; s_i\right)$, and then equalizes the demand and supply $Q = D\left(p; s\right)$. The supply is given as the benchmark allotment $Q = n\bar{r}$. However, if the total demand at policy rate p^0 is less than the benchmark allotment, then only $D\left(p^0; s\right)$ is allotted. All demand above or equal to the clearing price p, given $p \geq p^0$, will be accepted. Each bank submits a bid curve to maximize its expected profits:

$$\max_{D_i(p;s_i)} E\left[\pi_i^k \middle| s_i\right] = \int_{p^0}^{\overline{p}} \left\{ \int_0^{D_i(p;s_i)} v_i\left(x;s\right)dx - T_i^k \right\} dH\left(p, D_i\left(p;s_i\right)\right),$$

where T_i^k is the payment schedule in an auction $k = upa,\ dpa,\ va$ (for simplicity we omit the superscript k from $D_i\left(p;s_i\right)$). $H\left(p, D_i\left(p;s_i\right)\right)$ is the probability distribution of the clearing price. Hence, it is the probability that the clearing price p is not higher than the bid for unit q_i:

$$\begin{aligned} H\left(p,q_i\right) &= Pr\left(P_i\left(q_i\right) \geq p\right) \\ &= Pr\left(q_i \leq Q - D_{-i}\left(P_i\left(q_i;s_i\right);s_{-i}\right)\right). \end{aligned}$$

Further, $D_{-i}\left(p;s_{-i}\right) = \sum_{j\neq i}^{n} D_j\left(p;s_j\right)$ is the aggregate demand of every other bank but bank i. Hence, residual supply for bank i writes $RS_i\left(p\right) = Q - D_{-i}\left(p;s_{-i}\right)$. The inverse bid function is denoted by $P_i\left(q_i;s_i\right) \equiv D_i^{-1}\left(q_i;s_i\right)$.

Payments differ depending on the auction design. In the uniform price auction, banks pay the clearing price for all the liquidity they win and the payment schedule of bank i is thus

$$T_i^{upa} = pD_i\left(p;s_i\right). \tag{17}$$

In the discriminatory price auction, banks pay the amounts of their bids for all accepted demand bids, thus

$$T_i^{dpa} = \int_0^{D_i(p;s_i)} P_i\left(x;s_i\right)dx. \tag{18}$$

In both of these mechanisms, strategic banks shade their bids. Therefore, the allocation is not efficient ex ante and the clearing price does not necessarily reveal the marginal values of banks in the equilibrium (Ausubel and Cramton, 2002).

In the Vickrey auction, in addition to the clearing price and the allocation of units, the regulator determines paybacks for each bank.[4] The final payment banks have to pay for the accepted bids is not the clearing price. The share of the paybacks is given as

$$\alpha_i = 1 - \frac{\int_0^{q_i} RS_i^{-1}(x)dx}{RS_i^{-1}(q_i)q_i}. \tag{19}$$

Hence, the payment rule in the Vickrey auction[5] is defined by the residual supply for bank i as

$$T_i^{va} = pq_i\left(1-\alpha_i\right) = \int_0^{D_i(p;s_i)} RS_i^{-1}(x)dx. \tag{20}$$

[4] Montero (2008) uses a similar kind of Vickrey-Clarke-Groves payback function in the full information environment in a pollution problem setup (Vickrey, 1961; Clarke 1971; Groves 1973).

[5] In Ausubel and Cramton (2004) the same Vickrey auction payment rule is defined by,

$$T_i^{va} = \int_0^{q_i^e(s_i,s_{-i})} v_i\left(x;\hat{s}_i\left(s_{-i},x\right),s_{-i}\right)dx.$$

where signal $\hat{s}_i$ is the lowest possible signal for which firm i would have won the unit x given other bidders' signals s_{-i},

$$\hat{s}_i\left(s_{-i},x\right) = \inf_{s_i}\left\{ s_i \middle| q_i^e\left(s_i,s_{-i}\right) \geq x\right\}.$$

$q_i^e\left(s_i,s_{-i}\right)$ is the efficient allocation rule. Due to the symmetric correlation ($\rho_{ij} = \rho,\ \forall i,j$) this is equivalent to (20).

The payback mechanism induces banks to bid with their marginal valuations, conditional on the aggregate information. Note that the marginal payback function for bank i, that is $RS_i^{-1}(q_i)$, is determined by the strategies of all other banks but bank i.

Payment rules from (17) - (20) determine the optimal strategies of the banks in different auction designs. First order conditions (Euler equations) of the maximization problems are, respectively (see e.g. De Castro and Riascos, 2009; Hortaçsu 2011; Wang and Zender, 2002; Wilson, 1979),

$$VA: \quad v_i(q_i; s) = p, \tag{21}$$

$$UPA: \quad v_i(q_i; s) = p - q_i \frac{H_q}{H_p}, \tag{22}$$

$$DPA: \quad v_i(q_i; s) = p + \frac{H}{H_p}. \tag{23}$$

Furthermore, using the affine information structure and, in particular, the distribution of aggregate information I, the first order condition of the uniform price auction (22) is written as (see the derivation in Appendix C)

$$v_i(q_i; s) = p - q_i \frac{1}{D'_{-i}(p; s_{-i})}. \tag{24}$$

Respectively, the first order condition of the discriminatory price auction (23) becomes

$$v_i(q_i; s) = p + \frac{1}{D'_{-i}(p; s_{-i})} \frac{G(I|\, s_i)}{g(I|\, s_i)} \frac{dq_i}{dI}, \tag{25}$$

where $\frac{G(I|s_i)}{g(I|s_i)} = \lambda(I|\, s_i)$ is the inverse hazard rate of aggregate information conditional on the signal s_i.

As noted by e.g. Hortaçsu (2011), in most cases it is not possible to analytically evaluate the probability distribution H, which is needed in the uniform price and discriminatory price auction equilibrium characterizations. Moreover, it might be difficult even numerically if the number of bidders is large. In this paper, we approximate the probability distribution H and equilibrium strategies by utilizing the following assumptions. First, we derive a unique equilibrium strategy for each bank by fixing the end points of the bid functions. The end points are defined by the minimum bid rate (reservation price) and the expected reserve requirements (this is similar to capacity constraints in Holmberg 2008) of the banks. Banks bid only for those units that have a positive expected value over the minimum bid rate. The bidding function is thus constrained by the point at which the marginal value of money falls into the level of the minimum bid rate (see Assumption 4 below). Second, we reduce banks' optimization problems given by a system of n differential equations to a system of only two equations by taking an average over banks' strategies other than i, conditional on the information that bank i has. In this, we assume that the price derivative of the bid function, $D'_i(p; s_i)$, is linear in signal s_i (Assumption 5). By these assumptions the clearing price reveals the average signal of banks, with a sufficient accuracy. Third, in order to show analytical results and to reduce the computational burden in simulations, we make some further approximations: we assume that the difference between the slopes of the average bank's bid function and the individual bank's bid function is not relevant when calculating the slopes of the residual supply functions (Assumption 6). Furthermore, when we approximate equilibrium strategies of the uniform price auction and the discriminatory price auction, we linearize the marginal value functions around the competitive equilibrium (Definition 2 and Assumption 7). Next, these assumptions are described formally.

Assumption 4. The unique equilibrium candidate of all auction models is the one for which each bank's demand at p^0 (reservation price) is equal to the amount where its marginal value of money conditional on $\tilde{s} = \bar{r}$

falls to the level of the reservation price p^0. Hence, $P_i(\hat{q}_i; s_i) = p^0$ where,

$$\hat{q}_i = E[r_i|\, s_i, \bar{r}]\,. \tag{26}$$

Assumption 5. At every given price, the aggregate demand $D(p;s)$ is approximately equal to the expected demand of an average bank multiplied by the number of all participating banks. Hence, when the subscript m denotes the bank receiving the average signal $s_m = \tilde{s}$, we assume that,

$$D(p;s) \approx nD_m(p;s_m) \;\Rightarrow\; D'(p;s) \approx nD'_m(p;s_m)\,. \tag{27}$$

For prices $p^0 \leq p \leq \overline{p}$, we may thus write $D_m(p;s_m) = q_m = \frac{Q}{n} = \bar{r}$.[6] From (27) we further get

$$D'_{-m}(p;s_{-m}) \approx (n-1)\,D'_m(p;s_m) = \frac{n-1}{P'_m(q_m;s_m)}. \tag{28}$$

Assumption 6. The difference $D'_m(p;s_m) - D'_i(p;s_i)$ is not significant when calculating $D'_{-i}(p;s_{-i})$ and thus $D'_{-i}(p;s_{-i}) \approx (n-1)\,D'_m(p;s_m)$. This approximation is close at least for relatively large n and for relatively low σ_r^2 and, most importantly, when s_i is not far from $\tilde{s}$.

We will use a benchmark case of competitive equilibrium to help define the equilibrium stragegies also in the non-competitive mechanisms.

Definition 2. When all banks act as price takers in the ECB auction, the auction equilibrium is said to be competitive. The competitive auction equilibrium is symmetric, such that each bank must have equal probability of being short of money after the auction.

In the competitive auction equilibrium, banks are indifferent to whether they purchase the last unit in the auction or in the interbank market. Let $q_i^c(s_i,\tilde{s})$ denote the allocation of the competitive auction equilibrium. Then by Definition 2,

$$G_\varepsilon\left(q_i^c(s_i,\tilde{s}) - E[r_i|\, s_i,\tilde{s}]\right) = G_\varepsilon\left(q_j^c(s_j,\tilde{s}) - E[r_j|\, s_j,\tilde{s}]\right), \quad \forall i,j$$

where $q_i^c(s_i,\tilde{s}) - E[r_i|\, s_i,\tilde{s}]$ is the expected demand of liquidity of an individual bank after the competitive auction. Furthermore, while ECB allocates only $Q = n\bar{r}$ in MROs (when $\tilde{s} \geq \bar{r}$), on average banks receive $\bar{r}$. With symmetric distribution functions G_ε, when comparing bank i and the average bank, we get the the competitive equilibrium allocation of bank i as

$$\begin{aligned} q_i^c(s_i,\tilde{s}) &= \bar{r} + E[r_i|\, s_i,\tilde{s}] - E[\tilde{r}|\,\tilde{s}] \\ &= \bar{r} + B(s_i - \tilde{s})\,. \end{aligned} \tag{29}$$

[6] This holds if the price derivatives of bid functions are approximately linear in signals. If the price derivative of the demand function $D'_i(p;s_i)$ is not linear in s_i, the clearing price and the average signal have no easily solvable connection. In that case, the expected reserve requirement $E[r_i|\, s_i, I]$ is not informationally equivalent to $E[r_i|\, s_i, p]$. However, when deriving equilibrium strategies, we assume that banks are able to track the aggregate market information I from the clearing price p accurately enough. This follows from (27). Consider, on the contrary, that $\frac{\partial^2 |D'_i(p;s_i)|}{\partial s_i^2} > 0$. Then with normally distributed signals $|D'_m(p)| < \frac{1}{n}|D'(p)|$, which implies $|D'_{-m}(p)| > (n-1)|D'_m(p)|$ and from the first order condition we see that a bank which receives the average signal gets an amount $D_m(p) > \frac{Q}{n}$ in the auction. Instead, throughout the analysis of this paper, we assume that banks approximate the opponents' bid functions by assuming that $\frac{\partial D'_j(p;s_j)}{\partial s_j}$ is roughly constant for all $i \neq j$. Then our assumption (27) holds and $D_m(p) = q_m \approx \frac{Q}{n}$.

The competitive auction equilibrium price can now be written from (15) as

$$p^c(\tilde{s}) = v_i(q_i^c(s_i,\tilde{s})|s,\eta_H) + [v_i(q_i|s,\eta_0) - v_i(q_i^c(s_i,\tilde{s})|s,\eta_H)]\,e^{-b\eta}. \tag{30}$$

Assumption 7. When deriving equilibrium strategies of the uniform price auction and the discriminatory price auction, we use a first order linear approximation of the marginal value function of bank i around the competitive auction equilibrium $(p^c(\tilde{s}), q_i^c(s_i,\tilde{s}))$. This approximation is defined in (31) and (32).

The first order linear approximation of $v_i(q_i|s)$ around the competitive auction equilibrium $(p^c(\tilde{s}), q_i^c(s_i,\tilde{s}))$ is written as

$$\hat{v}_i(q_i|s) = p^c(\tilde{s}) + \beta(\tilde{s})(\bar{r} + B(s_i - \tilde{s}) - q_i) \tag{31}$$

where $\beta(\tilde{s})$ is the slope of the marginal value function evaluated at $q_i^c(s_i,\tilde{s})$,

$$\beta(\tilde{s}) = -\left.\frac{dv_i(q_i|s)}{dq_i}\right|_{q_i^c} = \left(1 - e^{-b\eta}\right) g_\varepsilon(\bar{r} - E[\tilde{r}|\,\tilde{s}])\left(\overline{p} - \underline{p}\right). \tag{32}$$

Assumptions 4-7 are by no means realistic nor innocuous. However, they are necessary for us to be able to conduct the analysis of interest, which is to compare various mechanisms in terms of their impact on the interbank market under different information regimes. Furthermore, the focus of this paper is not the correct analysis of bidding behavior in auctions, but rather we are satisfied with approximations that do not affect the qualitative mechanism comparisons from the interbank perspective. One comforting observation is that these assumptions do not change the known characteristic differences of auction designs: i) the generalized Vickrey auction is producing efficient allocation with a correct price signal, ii) due to the bid shading, the clearing prices of the uniform and discriminatory price auctions are lower than the "correct" price and iii) in the uniform (discriminatory) price auction bidders with "low" signals get more (fewer) units in the equilibrium than in the competitive case and vice versa for bidders with "high" signals (Ausubel et al., 2011; Vives, 2010; Jackson and Kremer, 2007).

3.5.1 Vickrey auction (VA) - competitive bidding strategy

The first step is to ask whether in our model, the Vickrey auction is competitive, i.e. bidders submit sincere bids. Ausubel and Cramton (2004) prove (their Theorem 1) that for any value function satisfying continuity, value monotonicity and the single-crossing property, Vickrey auction with reserve pricing has truthful bidding as an ex post equilibrium for any monotonic aggregate quantity rule and associated monotonic efficient assignment rule. In addition, Ausubel and Cramton (2004) also show that if the Vickrey auction with a reserve pricing is followed by any resale process that is coalitionally-rational against individual bidders, the truthful bidding remains to be an ex post equilibrium (their Theorem 2). In our model, the private information is one-dimensional and the marginal value of money $v_i(q_i;s)$ from (15) satisfies the continuity, value monotonicity and single-crossing properties.[7] Furthermore,

[7]Note that $B(s_i - \tilde{s}) = \frac{B}{n}\left((n-1)s_i - \sum_{j\neq i}^n s_j\right)$. It is then easy to see from (31), that all the three required assumptions from Ausubel and Cramton (2004) are satisfied:

1. Continuity: $v_i(q_i;s)$ is jointly continuous in (s, q_i).
2. Value monotonicity: $v_i(q_i;s)$ is non-negative, and $\frac{\partial v_i(q_i;s)}{\partial s_i} > 0$ and $\frac{\partial v_i(q_i;s)}{\partial q_i} \leq 0$.
3. Single-crossing: Let s' denote a signal vector $s' = \left(s_i', s_{-i}\right)$ and $s = (s_i, s_{-i})$. Then $v_i(q_i;s)$ has a single-crossing property, if for all i, $j \neq i$, q_i, q_j, s_{-i} and $s_i' > s_i$,
$$v_i(q_i;s) > v_j(q_j;s) \Rightarrow v_i\left(q_i;s'\right) > v_j\left(q_j;s'\right)$$
and
$$v_i\left(q_i;s'\right) < v_j\left(q_j;s'\right) \Rightarrow v_i(q_i;s) < v_j(q_j;s).$$

the fixed ECB allotment rule we impose $Q = n\bar{r}$ is monotonic in $\tilde{s}$. Therefore, in our model, $q_i^c(s_i, \tilde{s})$ is the efficient assignment rule $q_i^e(s)$ and is monotonic in s_i and $\tilde{s}$.[8] Moreover, our assumption 1 guarantees that the resale process is coalitionally-rational against individual bidders. Hence to conclude, Vickrey auction is an ex post efficient mechanism in our model whenever the clearing price is larger than the reservation price. Each price is associated with one possible $\tilde{s}$.[9] Thus, the clearing price reveals the average signal and banks are able to derive their $v_i(q_i; s)$ functions. Given the efficient allocation rule and the payment mechanism, it is optimal for each bank to bid sincerely if every other bank bids sincerely. The quantity bid at price p is thus equal to the efficient allocation rule. When the clearing price equals the reservation price, price no longer reveals information about the average signal. It should be noted that the concept of the ex post efficient auction equilibrium refers to the revelation of signals $s = (s_1, \ldots, s_n)$ and not to the revelation of true reserve requirements $r = (r_1, \ldots, r_n)$.

The bid function of bank i in the Vickrey auction is equal to the competitive case when $\tilde{s} \geq \bar{r}$. On the other hand, when $\tilde{s} < \bar{r}$ the bid follows from Assumption 4. Hence we get

$$P_i^{va}\left(q_i^{va}(s_i, \tilde{s}); s_i\right) = p^{va}(\tilde{s}) \quad = \quad \begin{cases} p^c(\tilde{s}), & \text{if } \tilde{s} \geq \bar{r} \\ p^0, & \text{if } \tilde{s} < \bar{r} \end{cases} \tag{34}$$

where the equilibrium quantity $q_i^{va}(s_i, \tilde{s}) = D_i^{va}\left(p^{va}(\tilde{s}); s_i\right)$ is given as

$$q_i^{va}(s_i, \tilde{s}) = \begin{cases} q_i^c(s_i, \tilde{s}), & \text{if } \tilde{s} \geq \bar{r} \\ \hat{q}_i, & \text{if } \tilde{s} < \bar{r} \end{cases}. \tag{35}$$

Suppose next that $\tilde{s} < \bar{r}$. Then the auction clears with the policy rate $p = p^0$ and bank i receives $q_i^{va}(s_i, \tilde{s}) = \bar{r} + B(s_i - \bar{r})$. In that case, the total demand is less than the benchmark allotment,

$$\begin{aligned} D^{va}\left(p^0\right) &= \sum_i^n \{\bar{r} + B(s_i - \bar{r})\} \\ &= n\left(\bar{r} + B(\tilde{s} - \bar{r})\right) \\ &= n\tilde{q}^0 < n\bar{r}, \end{aligned}$$

where $\tilde{q}^0$ is the average allotment conditional on $\tilde{s} < \bar{r}$. It is important to note that if $\rho, \sigma_\varepsilon^2, \sigma_r^2 > 0$, the clearing price does not contain all the information and the allotment of bank i at p^0 is more than the expected reserve requirements conditional on s_i and $\tilde{s} < \bar{r}$. This follows from,

$$\begin{aligned} D_i^{va}\left(p^0; s_i\right) &> E\left[r_i | s_i, \tilde{s}\right] \\ \bar{r} + B(s_i - \bar{r}) &> \bar{r} + (1 - A)(\tilde{s} - \bar{r}) + B(s_i - \tilde{s}) \\ B(\tilde{s} - \bar{r}) &> (1 - A)(\tilde{s} - \bar{r}) \\ B &< B + nC. \end{aligned}$$

[8] The efficient allocation rule $q_i^e(s)$ is defined by (see Ausubel and Cramton, 2004)

$$v_i\left(q_i^e(s); s\right) \begin{cases} \leq v_{-i}\left(q_{-i}^e(s); s\right), & \text{for } i \text{ such that } q_i^e(s) = 0 \\ = v_{-i}\left(q_{-i}^e(s); s\right), & \text{for } i \text{ such that } 0 < q_i^e(s) < Q \\ \geq v_{-i}\left(q_{-i}^e(s); s\right), & \text{for } i \text{ such that } q_i^e(s) = Q \end{cases} \tag{33}$$

where $v_{-i}(q_{-i}; s)$ is the aggregate marginal value of money of every other bidder but bidder i.

[9] This holds while the correlation is symmetric, i.e. $\rho_{ij} = \rho$, $\forall i, j$.

Moreover, the greater the value of the parameter B, the more banks rely on their own signal. However, as the relative noise of the signal, i.e. $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$, or the correlation coefficient ρ increases, the more weight banks put on the aggregate information and B decreases. At p^0 the demand of bank i is equal to its expected reserve requirements conditional on signal s_i and as if the average signal was $\bar{r}$. If $\rho = 0$, this equals the demand in posted price mechanisms. However, when $\rho > 0$, banks with a signal lower (higher) than the expected reserve requirement demand more (less) than in the posted price mechanisms, while $\frac{\sigma_r^2}{\sigma_\varepsilon^2+\sigma_r^2} > \frac{(1-\rho)\sigma_r^2}{\sigma_\varepsilon^2+(1-\rho)\sigma_r^2} = B$. Thus we get,

$$D_i^{va}\left(p^0; s_i\right) = \bar{r} + B\left(s_i - \bar{r}\right) \begin{cases} > q_i^{pp}\left(s_i\right), & \text{if } s_i < \bar{r} \\ = q_i^{pp}\left(s_i\right), & \text{if } s_i = \bar{r} \\ < q_i^{pp}\left(s_i\right), & \text{if } s_i > \bar{r}. \end{cases}$$

Hence, there is expected to be excess liquidity after the ECB operation whenever $\tilde{s} < \bar{r}$ and also more excess liquidity than in the posted price mechanisms. Respectively, when the clearing price is greater than p^0 and $\tilde{s} > \bar{r}$, in expected terms, the interbank market is short of money. However, this is not due to the insufficient information aggregation but to limiting the supply to $Q = n\bar{r}$. This implies that when (the Vickrey) auction mechanism is restricted by price and the total quantity limits, the information mechanism, which is the comparative advantage of auctions when compared to direct price mechanism, is not working properly. When $\tilde{s} > \bar{r}$ the posted price mechanism adjusts better in aggregate positive demand shocks. When $\tilde{s} < \bar{r}$, on the other hand, the information mechanism of the auction is limited, while the clearing price does not provide accurate information to bidders.

Finally, note that the auctioneer has to pay information rent to banks in order to get them to reveal their expected marginal values. The revenues are thus reduced by the share of paybacks and the total revenues left to the auctioneer in the Vickrey auction is,

$$R^{va} = \sum_{i=1}^{n} \int_{0}^{q_i^{va}(s_i,\tilde{s})} RS_i^{-1}(x)\, dx.$$

In simulations, we approximate revenues by taking the revenue from the average bank and multiply it by n. Hence the total revenue in the Vickrey auction (if $\tilde{s} \geq \bar{r}$) is calculated by, $R^{va} = n\int_0^{\bar{r}} RS_m^{-1}(x)\, dx$. Note that whenever $\tilde{s} < \bar{r}$, the revenue is $\left(R^{va}\middle|\, \tilde{s} < \bar{r}\right) = n\tilde{q}^0 p^0$ while $RS_i^{-1}\left(q_i\middle|\, \tilde{s} < \bar{r}\right) = p^0$ for $q_i \in [0, E\left[r_i\middle|\, s_i, \tilde{s}\right]]$.

3.5.2 Uniform price auction (UPA)

In the case of uniform price auction, no tractable solutions are generally available when values are uncertain and interdependent, and banks use non-linear strategies. The general problem is the infinite number of equilibria. However, Holmberg (2008) shows that by using a supply function model in the symmetric private values case, a unique supply function equilibrium can be derived if the auctioneer sets a price cap and bidders have capacity constraints, which bind with positive probability. Moreover, if agents are asymmetric and if strategies are not constrained to being linear, an unique solution can be derived only if both start and end points of the bid functions are fixed or the equilibrium quantity has unbounded support.

In our case, banks with higher than the average signal have steeper bid functions (and less steep inverse bid functions $P_i(q_i; s_i)$) than the average bank and vice versa for banks receiving a lower than the average signal. Thus, in order to solve the problem, we must simultaneously solve a number of n differential equations given by the first order condition (24) with fixed end points. This is a demanding task analytically, and also numerically, even in the absence of value uncertainty.[10] However, using Assumptions 4 - 7, and writing $p^{upa}(\tilde{s}) = P_m(q_m)$, the approximative two equation model of the uniform price auction is given as

$$v_m(q_m; s) = P_m(q_m) - \left(\frac{1}{n-1}\right) P'_m(q_m)\, q_m \tag{36}$$

$$v_i(q_i; s) = P_m(q_m) - \left(\frac{1}{n-1}\right) P'_m(q_m)\, q_i. \tag{37}$$

Utilizing linearized marginal value functions, the solution for this problem is derived in Appendix D. Solving (36) gives the expected price with the given I. In this we apply the models of Holmberg (2008), Rudkevich et al. (1998), and Anderson and Philpott (2002). Furthermore, the price equation (36) can be used when deriving the solution for (37). Thus, the equilibrium strategy in the uniform price auction may be defined by

$$P_i^{upa}\left(q_i^{upa}(s_i, \tilde{s}); s_i\right) = p^{upa}(\tilde{s}) = \begin{cases} p^c(\tilde{s}) - W^{upa}(\tilde{s}), & \text{if } \tilde{s} \geq \bar{r} \\ p^0, & \text{if } \tilde{s} < \bar{r} \end{cases}, \tag{38}$$

where the equilibrium quantity $q_i^{upa}(s_i, \tilde{s}) = D_i^{upa}(p^{upa}(\tilde{s}); s_i)$ is written as

$$q_i^{upa}(s_i, \tilde{s}) = \begin{cases} q_i^c(s_i, \tilde{s}) - \left(\frac{W^{upa}(\tilde{s})}{\bar{r}\beta(\tilde{s}) + W^{upa}(\tilde{s})}\right) B(s_i - \tilde{s}), & \text{if } \tilde{s} \geq \bar{r} \\ \hat{q}_i, & \text{if } \tilde{s} < \bar{r}. \end{cases} \tag{39}$$

The bid shading function of the average bank in the uniform price auction is the last term of the RHS in (38),

$$W^{upa}(\tilde{s}) = \bar{r}^{n-1} \frac{B}{n} \int_{n\bar{r}}^{n\tilde{s}} \left(\frac{\beta\left(\frac{I}{n}\right)}{\left(\bar{r} + B\left(\tilde{s} - \frac{I}{n}\right)\right)^{n-1}} \right) dI. \tag{40}$$

When $\tilde{s} > \bar{r}$, the equilibrium price of the UPA is lower than the competitive price and allocation is not ex post efficient. Suppose that bank i gets some signal which is lower than the average signal, $s'_i < \tilde{s}$. This would give bank i more quantity in the uniform price auction than in the competitive case because $W^{upa}(\tilde{s}) > 0$ and $\beta(\tilde{s}) > 0$, for all $\tilde{s} > \bar{r}$, and from (39) it is clear that $q_i^{upa}(s'_i, \tilde{s}) > q_i^c(s'_i, \tilde{s})$. A similar argument applies for $s''_i > \tilde{s}$, which gives $q_i^{upa}(s''_i, \tilde{s}) < q_i^c(s''_i, \tilde{s})$. Banks receiving higher than the average signal get fewer units in the uniform price auction when compared to the competitive case, whenever $\tilde{s} > \bar{r}$. The effective response to increased signal for bank i is lower than in the competitive case, $\frac{\partial q_i^{upa}(s_i, \tilde{s})}{\partial s_i} \leq \frac{\partial q_i^c(s_i, \tilde{s})}{\partial s_i} = B$. However, the closer we get to p^0, the closer we get to the competitive case and thus the ex post efficient allotment.

Our equilibrium selection is based on the end condition of the bid functions. Note that when $\tilde{s} > \bar{r}$ and $p \geq p^0$, we could choose any equilibrium strategy for the average bidder, which satisfies the first order condition and goes through the point $(p_{\bar{r}}^{upa}, \bar{r})$ where $p_{\bar{r}}^{upa} \in \left[p^0, p^c(\tilde{s})\right]$. However, our equilibrium selection is consistent with the idea that bidders will bid with their expected reserve requirements at policy target rate p^0. Moreover, equilibrium strategies, associated with the end conditions $\left(p^0, \hat{q}_i\right)$ for all i, would be unique if there would exist even a slight exogenous uncertainty over the volume of total allotment and hence over the residual supply for bank i, with the support overlapping $q_i^{upa}(s_i, \tilde{s})$ and $\hat{q}_i$ (see Holmberg 2008). Even though we do not explicitly model this

[10] Holmberg (2009a) shows a numerical method with capacity constraints in the case of supply function equilibrium model with asymmetric agents and no uncertainty in marginal cost functions. Armantier et al. (2008), on the other hand, define a concept of constrained strategic equilibrium for Bayesian games and they use the concept to approximate equilibrium strategies in a multi-unit discriminatory price private values auction model. The solution concept in this paper is loosely related to model of Armantier et al. (2008).

uncertainty, we discussed in Section 2 that in practice there is some uncertainty over the sold volume. Also, for an individual bank the uncertainty can arise from the uncertain number of participating banks in ECB MROs or any deviation of other banks out of their equilibrium strategies. Note also that our equilibrium strategies are based on approximations and thus suffer from such deviations by assumption. Hence, we argue that our equilibrium selection is reasonable. Total revenues of the uniform price auction are simply,

$$R^{upa} = p^{upa}\left(\tilde{s}\right)\sum_{i=1}^{n} q_i^{upa}\left(s_i, \tilde{s}\right).$$

3.5.3 Discriminatory price auction (DPA)

In the discriminatory price auction, banks have to pay exactly the bid they have submitted for all accepted demand units. Banks try to guess the equilibrium price and bid for that price with all infra-marginal units. However, due to the uncertainty, bidders use different discounts in the bid function for different units. Moreover, all banks share the same variance of the conditional distribution of the average signal $\tilde{s}$. In other words, each bank faces a similar decision problem only with a different expected value of the average signal. Hence, for every I separately, each bank needs to solve the following system of two differential equations from equation (25),

$$v_m\left(q_m; s\right) = P_m\left(q_m\right) + \left(\frac{1}{n-1}\right) P_m'\left(q_m\right)\frac{G\left(I|\,\tilde{s}\right)}{g\left(I|\,\tilde{s}\right)}\frac{dq_m}{dI} \tag{41}$$

$$v_i\left(q_i; s\right) = P_m\left(q_m\right) + \left(\frac{1}{n-1}\right) P_m'\left(q_m\right)\frac{G\left(I|\,s_i\right)}{g\left(I|\,s_i\right)}\frac{dq_i}{dI}, \tag{42}$$

where $G\left(I|\,\tilde{s}\right)$ is the probability distribution of the aggregate information of the average bank receiving a signal $\tilde{s} = \frac{I}{n}$. We follow Holmberg (2009) and derive first the equilibrium price equation $p^d\left(\tilde{s}\right) = P_m\left(q_m\right)$ from (41). Using this and (42) we get the approximative equilibrium strategy for bank i as the price-quantity pair (see Appendix E),

$$P_i^{dpa}\left(q_i^{dpa}\left(s_i, \tilde{s}\right); s_i\right) = p^{dpa}\left(\tilde{s}\right) = \begin{cases} p^c\left(\tilde{s}\right) - W^{dpa}\left(\tilde{s}\right), & \text{if } \tilde{s} \geq \bar{r} \\ p^0, & \text{if } \tilde{s} < \bar{r} \end{cases} \tag{43}$$

$$q_i^{dpa}\left(s_i, \tilde{s}\right) = \begin{cases} q_i^c\left(s_i, \tilde{s}\right) + \frac{W^{dpa}\left(\tilde{s}\right)}{\beta\left(\tilde{s}\right)}\left[1 - \frac{\lambda\left(n\tilde{s}|s_i\right)}{\lambda\left(n\tilde{s}|\tilde{s}\right)}\right], & \text{if } \tilde{s} \geq \bar{r} \\ \hat{q}_i, & \text{if } \tilde{s} < \bar{r}. \end{cases} \tag{44}$$

The bid shading function of the average bank is written as

$$W^{dpa}\left(\tilde{s}\right) = \frac{B}{n}\frac{\int_{n\bar{r}}^{n\tilde{s}} \beta\left(\frac{I}{n}\right)\left[G\left(I|\,\tilde{s}\right)\right]^{n-1} dI}{\left[G\left(n\tilde{s}|\,\tilde{s}\right)\right]^{n-1}}. \tag{45}$$

Note that for each $s_i \geq \bar{r}$ there is one I for which $I = ns_i$. This event has a conditional probability $g\left(ns_i|\,s_i\right)$ and bank i thinks it is the average bank, which gives the competitive amount $\frac{Q}{n} = \bar{r}$ to the bank i in the equilibrium. The bid for that quantity is lower than in the competitive case and it is defined by (43). If bank i, on the contrary, receives some lower signal $s_i' < \frac{I}{n}\left(\geq \bar{r}\right)$, then due to the decreasing inverse hazard rate of normal distribution $\lambda\left(I|\,s_i'\right) > \lambda\left(I|\,\tilde{s}\right)$, the equilibrium allotment is lower than in the competitive case, $q_i^{dpa}\left(s_i', \tilde{s}\right) < q_i^c\left(s_i', \tilde{s}\right)$. Similarly, for a higher signal, $s_i'' > \frac{I}{n}$, we get $q_i^{dpa}\left(s_i'', \tilde{s}\right) > q_i^c\left(s_i'', \tilde{s}\right)$, while $\lambda\left(I|\,s_i''\right) < \lambda\left(I|\,\tilde{s}\right)$. This makes $\frac{\partial q_i^{dpa}\left(s_i, \tilde{s}\right)}{\partial s_i} \geq \frac{\partial q_i^c\left(s_i, \tilde{s}\right)}{\partial s_i} = B$ and banks with higher than the average signal receive in expected terms more units in the discriminatory price auction than in the competitive case (or in the Vickrey auction) and the allocation is inefficient ex post. When s_i increases, it affects the conditional distribution of aggregate information $(I|\, s_i)$. With a high signal, the bank assumes that the average signal is also higher and thus the competition for liquidity is tighter. This induces the bank to increase its bid in order to guarantee enough liquidity. However, the difference between the discriminatory auction and the competitive mechanism diminishes when closing to p^0.

Similarly with the Vickrey auction, we approximate revenues of the discriminatory price auction in simulations by taking the revenue from the average bank and multiplying it by n. Hence, denoting $I(q_m, s_m) \equiv \left(I : q_m = q_i^{dpa}\left(s_m, \frac{I}{n}\right)\right)$ the total revenue of the discriminatory price auction is given as

$$R^{dpa} \approx n \int_0^{\bar{r}} p^{dpa}\left(\frac{I(x, s_m)}{n}\right) dx.$$

4 Simulations and mechanisms comparisons

In simulations we examine the money market and ECB MROs with respect to two dimensions. First, we examine cases where we vary the performance of the interbank market. We simulate two cases where the market suffers from different levels of friction and one case where the interbank market is totally collapsed ($\eta = \eta_H$). In these cases, the banks' marginal value of money in ECB MROs is defined by (15). Secondly, we examine differences in the information structure. On one hand, we consider a case where the banks are fairly certain about their reserve requirements and the correlation between banks' reserve requirements is low (the almost private values case). On the other hand, we consider three cases where the banks are either highly uncertain about their own reserve requirements before the ECB operation, or the reserve requirements are correlated between the banks, or both (interdependent values).

We run these simulations in order to examine how the information structure and the market performance of the interbank market affect the auction and interbank outcomes. The fixed parameters of the model are presented in Table 2. We normalize the ECB rates such that the policy rate is zero and standing facility rates are 100 units above and below the policy target rate. When we calculate auction revenues and revenues from standing facilities we instead use the "true" policy rate $p_T^0 = 100$. In other simulations the normalized value $p^0 = 0$ is used.

Table 2: Fixed parameter values.

Variable		Value
b	Scale parameter of the marginal value function	0.1
$\bar{r}$	Expected reserve requirement	100
σ_r	Standard deviation of the reserve requirement	20
n	Number of banks	100
$\overline{p}$	Normalized marginal lending facility rate	100
$\underline{p}$	Normalized deposit facility rate	-100
p^0	Normalized minimum bid rate	0
p_T^0	True minimum bid rate	100

Information structures are presented in Table 3. Information structure I is closest to the private values case, because correlation between valuations is low and signals provide fairly accurate estimates of banks' reserve requirements.[11] In information structure II, the correlation between uncertain reserve requirements is relatively high and in information structure III signals are noisier when compared to the first structure. We also adjust the signals to be both more interdependent and noisier to account for the possible case where both common uncertainty and higher private uncertainty arise simultaneously, possibly due to a financial crisis (information structure IV). Market performance of the interbank market is defined by the trading cost coefficient (see Table 4). It varies in simulations from the relatively modest value $\eta_1 = 5$ to the case where the interbank market is collapsed, $\eta_3 \to \infty$.

Table 3: Simulations - information structures

Variating variables		I	II	III	IV
σ_ε	Standard deviation of the signal noise	5	5	20	20
ρ	Correlation coefficient	0.2	0.5	0.2	0.5

Table 4: Simulations - market performance

Variating variable		η_1	η_2	η_3
η	trading cost coefficient	5	20	∞

For a given information structure $(\sigma_\varepsilon, \rho)$, we generate a signal space (s) and a reserve requirement space (r) of 1000 draws.[12] Furthermore, for each draw l, we calculate the equilibrium quantities $q_{i,l}^k\left(s_{i,l}, \tilde{s}_l\right)$ of every bank for posted price and auction mechanisms according to (16), (35), (39), and (44). For the auction mechanisms, we derive the equilibrium prices from (34), (38), and (43) and calculate the auction revenues using (17) - (20).[13]

Using allocations of ECB MROs we further derive the interbank demand or supply for each bank i, of mechanism k, and draw l (in what follows, subscript l is omitted for simplicity), that is $q_i^{ID,k} = \left(r_i - q_i^k\left(s_i, \tilde{s}\right)\right)\mathbf{1}_{\left\{q_i^k\left(s_i,\tilde{s}\right)<r_i\right\}}$ or respectively $q_i^{IS,k} = \left(q_i^k\left(s_i, \tilde{s}\right) - r_i\right)\mathbf{1}_{\left\{q_i^k\left(s_i,\tilde{s}\right)>r_i\right\}}$. Total demand and supply of the interbank market are thus $Q^{ID,k} = \sum_{i=1}^n q_i^{ID,k}$ and $Q^{IS,k} = \sum_{i=1}^n q_i^{IS,k}$. We also derive the number of banks on the demand and in the supply side of the market. Furthermore, given η, it is then possible to derive the resulting interbank rates $p^{IB,k}$ according to (1). If bank i is on the demand side of the interbank market, the amount of trading is given as

$$\Delta q_i^k = \begin{cases} q_i^{ID,k}, & \text{if } q_i^{ID,k} \leq \frac{1}{\eta}\left(\overline{p} - p^{IB,k}\right) \\ \frac{1}{\eta}\left(\overline{p} - p^{IB,k}\right), & \text{if } q_i^{ID,k} > \frac{1}{\eta}\left(\overline{p} - p^{IB,k}\right) \end{cases}$$

and the use of the lending facility is $q_i^{LF,k} = q_i^{ID,k} - \Delta q_i^k$. If bank i is on the supply side of the market, we get respectively,

$$\Delta q_i^k = \begin{cases} q_i^{IS,k}, & \text{if } q_i^{IS,k} \leq \frac{1}{\eta}\left(p^{IB,k} - \underline{p}\right) \\ \frac{1}{\eta}\left(p^{IB,k} - \underline{p}\right), & \text{if } q_i^{IS,k} > \frac{1}{\eta}\left(p^{IB,k} - \underline{p}\right) \end{cases}$$

and bank i deposits to the deposit facility $q_i^{DF,k} = q_i^{IS,k} - \Delta q_i^k$. Total use of standing facilities is thus $Q^{F,k} = \sum_{i=1}^n q_i^{LF,k} + q_i^{DF,k}$.

The revenues that we report here are the joint revenues to the ECB from both the MROs and the standing facilities. In addition, the required reserves are remunerated with the marginal MRO rate. In practice, the remuneration

[11] Of cource, values are not private while $\sigma_\varepsilon, \rho \neq 0$.

[12] The parameter spaces are generated in R using the mvtnorm package (Multivariate Normal and t Distributions).

[13] To ease the computational burden, for Vickrey and discriminatory price auctions we calculate the auction payments of the average bank $(s_i = \tilde{s})$ and multiply it by n.

is the average MRO rate over the maintenance period. We denote the average clearing price of the mechanism k over all the draws by $\tilde{p}^k = \sum_{l=1}^{1000} p^k(\tilde{s}_l)$. Total revenues are thus

$$TR^k = R^k + \overline{p}Q^{LF,k} - \underline{p}Q^{DF,k} - \tilde{p}^k \sum_{i=1}^{n} r_i,$$

where R^k denotes the revenues from ECB MROs, $\overline{p}Q^{LF,k}$ are the revenues from the lending facility, $\underline{p}Q^{DF,k}$ is the total remuneration of liquidity deposited in the deposit facility, and $\tilde{p}^k \sum_{i=1}^{n} r_i$ is the remuneration of the required reserves. In the total revenue results, "true" rates are used for $\tilde{p}^k$, $\overline{p}$ and $\underline{p}$.

We present three sets of results in tables 5, 6 and 7, where we have calculated mean values and standard deviations of the simulated outcomes under all the information structures and the interbank market conditions. Table 5 shows results of simulated interbank rates, Table 6 the total use of standing facilities and Table 7 the total revenues from ECB MROs and standing facilities. See also Appendix F where we present the densities of the simulation outcomes.

In Table 5, we report the means and standard deviations of interbank rates. When $\eta \to \infty$ the interbank rate is p^0 by assumption and thus, we omit that case from Table 5. Recall that with perfect interbank market ($\eta = 0$), the resulting interbank rates would be either $\overline{p}$ or $\underline{p}$ and all mechanisms would give relatively similar results with regard to the interbank rates. Hence, when trade frictions are low ($\eta_1 = 5$) the simulated densities of interbank rates are fairly similar in all mechanisms (see the left panel of Figure 5 in Appendix F). The higher interdependency tends to increase and noisier signals reduce the variance of interbank rates. Secondly, when signals are noisy ($\sigma_\varepsilon = 20$), the average interbank rate after auctions is closer to the target rate. The third notable difference is that after auction mechanisms the interbank rate tends to be larger on average than after the full allotment posted price mechanism. This is due to the limited aggregate supply in ECB MROs.

Table 5: Interbank rates. Means and standard deviations of simulation results.

		$\eta_1 = 5$		$\eta_2 = 20$	
		Mean	S.D.	Mean	S.D.
Information structure I	VA	23.14	88.16	29.56	62.67
$\sigma_\varepsilon = 5,\ \rho = 0.2$	DPA	22.91	88.07	29.23	62.50
	UPA	23.14	88.15	29.56	62.66
	PP FA	1.22	84.85	0.68	41.06
Information structure II	VA	12.82	92.51	21.88	73.73
$\sigma_\varepsilon = 5,\ \rho = 0.5$	DPA	12.33	92.28	21.25	73.47
	UPA	12.82	92.51	21.87	73.73
	PP FA	2.35	86.10	0.99	46.33
Information structure III	VA	7.95	74.42	9.24	34.73
$\sigma_\varepsilon = 20,\ \rho = 0.2$	DPA	7.95	74.42	9.23	34.71
	UPA	7.96	74.43	9.24	34.74
	PP FA	0.46	68.73	-0.12	20.09
Information structure IV	VA	4.40	86.07	7.46	62.17
$\sigma_\varepsilon = 20,\ \rho = 0.5$	DPA	4.41	86.07	7.47	62.17
	UPA	4.40	86.07	7.46	62.17
	PP FA	0.71	78.38	-0.27	41.63

Further, when trade frictions become larger ($\eta_2 = 20$), interbank rates after the posted price mechanism with full allotment are relatively closer to the ECB target rate than after other mechanisms. With auction mechanisms the average interbank rates are greater than in PP-FA, and average values are even increased from the case $\eta_1 = 5$, due to the high right tail of interbank rate distributions (see the right panel of Figure 5 in Appendix F). Again, when signals become noisier the interbank rate distributions become more concentrated around the target rate. Increased correlation of reserve requirements has the opposite effect. In terms of policy targets, is high private uncertainty of banks and high frictions of interbank markets something the ECB should wish for? However, a more concentrated interbank rate distribution due to these conditions comes at a cost.

the use of standing facilities stands for the efficiency metric of the money market. Simulation results of these are presented in Table 6. The posted price is also the best mechanism with regard to banks needs of the standing facilities, while PP-FA adjusts not only to low demand conditions but also when money demand is high. Mean values and standard deviations of Q^F are lowest in PP-FA with all simulations (Figure 6). All auction mechanisms provide fairly similar results. This is intuitively clear. For low signal values ($\tilde{s} < \bar{r}$) the allocation is equivalent in all auctions. Respectively, when $\tilde{s}$ is higher than but relatively close to $\bar{r}$, the bid functions of each bank do not distinguish that much from each other in VA, UPA, and DPA. Besides, when the demand for money is high ($\tilde{s} \gg \bar{r}$) and when auction mechanisms have different efficiency rates, it is, however, more probable that more or less all banks are short of liquidity, due to the limited aggregate supply in ECB tenders. The trading in the interbank market is reduced and most of the banks have to turn to standing facilities. Thus the efficiency of the Vickrey auction, for instance, is of no use, even with relatively high interbank market frictions. Moreover, when the auction demand is low ($\tilde{s} < \bar{r}$), banks bid too much in auctions, while the bids are conditioned, in addition to individual signal, to the minimum bid rate, whereas in posted price mechanism the bids are conditioned only on their own signals. Moreover, the use of standing facilities increases in banks' uncertainty (σ_ε) and frictions of the interbank market (η). Furthermore, when these factors are high, the banks' need for using standing facilities are fairly similar in all the mechanisms. The higher interdependency of banks (ρ) increases the right tails of Q^F simulation distributions. This shows in higher average values and standard deviations when the correlation coefficient increases.

Table 6: Total use of standing facilities. Means and standard deviations of simulation results.

		$\eta_1 = 5$		$\eta_2 = 20$		$\eta_3 \to \infty$	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Information structure I	VA	422.3	503.1	429.4	498.6	608.6	402.6
$\sigma_\varepsilon = 5,\ \rho = 0.2$	DPA	431.0	576.7	441.0	609.0	622.1	547.8
	UPA	422.3	503.1	429.4	498.6	608.7	402.6
	PP-FA	96.2	33.2	109.5	29.5	387.3	29.4
Information structure II	VA	654.6	787.2	658.7	784.4	802.2	698.7
$\sigma_\varepsilon = 5,\ \rho = 0.5$	DPA	663.5	845.7	674.2	888.7	821.0	833.2
	UPA	654.7	787.2	658.7	784.4	802.3	698.6
	PP-FA	111.5	46.5	122.3	42.2	387.5	30.1
Information structure III	VA	630.9	441.2	845.1	332.0	1212.9	248.8
$\sigma_\varepsilon = 20,\ \rho = 0.2$	DPA	631.0	441.3	845.1	332.1	1213.1	249.3
	UPA	630.9	441.2	845.1	332.0	1212.9	248.7
	PP-FA	449.7	245.9	723.3	146.8	1130.8	119.4
Information structure IV	VA	979.9	731.6	1078.0	659.0	1333.0	537.8
$\sigma_\varepsilon = 20,\ \rho = 0.5$	DPA	979.9	731.6	1078.0	659.0	1333.0	537.9
	UPA	979.9	731.6	1078.0	659.0	1333.0	537.8
	PP-FA	623.1	402.0	788.2	305.3	1134.3	221.6

Table 7: Total net revenues from MROs and standing facilities ($\times 10^3$). Means and standard deviations of simulation results.

		$\eta_1 = 5$		$\eta_2 = 20$		$\eta_3 \to \infty$	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Information structure I	VA	18.3	449.8	21.1	423.1	40.0	408.4
$\sigma_\varepsilon = 5,\ \rho = 0.2$	DPA	566.7	57.3	625.8	71.9	661.0	67.5
	UPA	41.7	442.2	42.6	414.0	60.9	401.6
	PP-FA	9.7	5.2	11.0	4.0	38.7	2.9
Information structure II	VA	49.6	470.8	50.4	460.9	65.3	450.2
$\sigma_\varepsilon = 5,\ \rho = 0.5$	DPA	549.0	74.8	585.8	91.6	611.2	89.5
	UPA	65.0	466.0	65.6	457.1	80.3	448.5
	PP-FA	11.3	5.8	12.3	4.8	38.7	3.0
Information structure III	VA	24.2	364.1	56.2	265.8	95.7	243.2
$\sigma_\varepsilon = 20,\ \rho = 0.2$	DPA	675.1	37.3	778.4	37.0	838.3	36.6
	UPA	62.8	367.0	84.7	264.6	121.3	243.3
	PP-FA	45.0	25.9	72.3	17.0	113.1	11.9
Information structure IV	VA	61.1	406.3	75.6	358.5	101.9	342.7
$\sigma_\varepsilon = 20,\ \rho = 0.5$	DPA	678.9	58.2	763.1	53.3	809.4	42.4
	UPA	98.4	405.5	108.4	358.3	133.4	344.8
	PP-FA	62.6	41.2	78.7	31.8	113.4	22.2

Even though PP-FA is the best mechanism with respect to interbank rates and the use of standing facilities, this is not the case with the net revenues from the MROs and standing facilities. In expected terms, the discriminatory price auction provides the largest revenues. This is mainly due to greatest auction revenues (see the simulation results of auction clearing prices and revenues in Appendix F). Besides, the spread of net revenues is much higher in VA and UPA than in other mechanisms.

5 Conclusions

We have presented a model of strategic bidding in multi-unit auctions that incorporates the main feature of the ECB liquidity auctions: The presence of secondary market. The novelty of our model is that the secondary market equilibrium generates the marginal valuation functions upon which bidders base their equilibrium bidding strategies in these multi-unit auctions. We have also used the auction allocations to model the outcomes in the interbank market and the need for the banks to turn to standing facilities of the ECB. We have compared four different mechanism to sell liquidity: The discriminatory price, the uniform price and the Vickrey auction, and the posted price mechanism with full allotment. All except the Vickrey auction have been used by the ECB in practice at one point or the other.

Our main objective was to compare which of these mechanisms is the best at achieving the stated goal of the ECB: The implementation of the target interest rate (reservation price) to the interbank market. We find that the current mechanism of posted price with full allotment is by far the most superior mechanism in this respect. Moreover, mechanism selection involves only limited trade-offs, since the posted price with full allotment is more efficient than even the Vickrey auction in our model. The only trade-off that emerges from our simulations is that the discriminatory price auction is optimal and thus generates more revenue than the posted price mechanism.

Nonetheless, optimality is probably of second order considering the role of ECB and the role of liquidity auctions. However, if the central bank values the information about the market that the bids provide, they should adapt an auction mechanism over a posted price mechanism, because in auctions, the central bank learns the entire demand function, whereas in posted prices the bid is only a single point in the price-quantity plane. Despite these consideration, the auction mechanisms seem inferior to the posted with full allotment in the relevant policy dimensions.

At the end of the day, we think that the mechanism design in the ECB liquidity auctions should be decided by the main purpose of the ECB: The implementation of monetary policy. Thus, we conclude that the ECB should continue using posted prices with full allotment, even after the current crisis. This conclusion is very intuitive. If the goal of the regulator is to regulate price, it is best achieved by regulating the price directly and letting the quantity adjust, instead of regulating the quantity and hoping that the price will adjust to the desired level.

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Appendix

A. Marginal valuation in an auction when the interbank market is frictionless

Suppose that in (3), $\eta = \eta_0 \approx 0$. Then

$$\begin{aligned} v_i\left(q_i|r_i,\eta_0\right) &= \underbrace{Pr_i\left(r_i \le q_i\right)\left[Pr_i^{IB}\left(q_i|\, r_i \le q_i, \eta_0, Q^{ID}, Q^{IS}\right)p^{IB}\left(\eta_0\right) + \left(1 - Pr_i^{IB}\left(q_i|\, r_i \le q_i, \eta_0, Q^{ID}, Q^{IS}\right)\right)\underline{p}\right]}_{V_1} \\ &+ \underbrace{Pr_i\left(r_i > q_i\right)\left[Pr_i^{IB}\left(q_i|\, r_i > q_i, \eta_0, Q^{ID}, Q^{IS}\right)p^{IB}\left(\eta_0\right) + \left(1 - Pr_i^{IB}\left(q_i|\, r_i > q_i, \eta_0, Q^{ID}, Q^{IS}\right)\right)\overline{p}\right]}_{V_2}. \end{aligned}$$

The second term can be written (omitting η_0) as

$$\begin{aligned} V_2 &= Pr_i\left(r_i > q_i\right)Pr\left(Q^{ID} > Q^{IS}\right)\left[Pr_i^{IB}\left(q_i|\, Q^{ID} > Q^{IS}, r_i > q_i\right)\overline{p} + \left(1 - Pr_i^{IB}\left(q_i|\, Q^{ID} > Q^{IS}, r_i > q_i\right)\right)\overline{p}\right] \\ &+ Pr_i\left(r_i > q_i\right)Pr\left(Q^{ID} = Q^{IS}\right)\left[Pr_i^{IB}\left(q_i|\, Q^{ID} = Q^{IS}, r_i > q_i\right)p^0 + \left(1 - Pr_i^{IB}\left(q_i|\, Q^{ID} = Q^{IS}, r_i > q_i\right)\right)\overline{p}\right] \\ &+ Pr_i\left(r_i > q_i\right)Pr\left(Q^{ID} < Q^{IS}\right)\left[Pr_i^{IB}\left(q_i|\, Q^{ID} < Q^{IS}, r_i > q_i\right)\underline{p} + \left(1 - Pr_i^{IB}\left(q_i|\, Q^{ID} < Q^{IS}, r_i > q_i\right)\right)\overline{p}\right]. \end{aligned}$$

Note that $Pr_i^{IB}\left(q_i|\, Q^{ID} = Q^{IS}, r_i > q_i\right) = Pr_i^{IB}\left(q_i|\, Q^{ID} < Q^{IS}, r_i > q_i\right) = 1$, when $\eta = \eta_0$. Also, whenever the total interbank demand is higher than the total supply, the resulting interbank rate is $\overline{p}$. Respectively, when the supply is higher than demand (or equal to demand) the interbank rate is $\underline{p}$ (or p^0), and thus

$$V_2 = Pr_i\left(r_i > q_i\right)\left[Pr\left(Q^{ID} > Q^{IS}\right)\overline{p} + Pr\left(Q^{ID} = Q^{IS}\right)p^0 + Pr\left(Q^{ID} < Q^{IS}\right)\underline{p}\right].$$

Similarly for V_1 we get

$$\begin{aligned} V_1 &= Pr_i\left(r_i \le q_i\right)Pr\left(Q^{ID} > Q^{IS}\right)\left[Pr_i^{IB}\left(q_i|\, Q^{ID} > Q^{IS}, r_i \le q_i\right)\overline{p} + \left(1 - Pr_i^{IB}\left(q_i|\, Q^{ID} > Q^{IS}, r_i \le q_i\right)\right)\underline{p}\right] \\ &+ Pr_i\left(r_i \le q_i\right)Pr\left(Q^{ID} = Q^{IS}\right)\left[Pr_i^{IB}\left(q_i|\, Q^{ID} = Q^{IS}, r_i \le q_i\right)p^0 + \left(1 - Pr_i^{IB}\left(q_i|\, Q^{ID} = Q^{IS}, r_i \le q_i\right)\right)\underline{p}\right] \\ &+ Pr_i\left(r_i \le q_i\right)Pr\left(Q^{ID} < Q^{IS}\right)\left[Pr_i^{IB}\left(q_i|\, Q^{ID} < Q^{IS}, r_i \le q_i\right)\underline{p} + \left(1 - Pr_i^{IB}\left(q_i|\, Q^{ID} < Q^{IS}, r_i \le q_i\right)\right)\underline{p}\right]. \end{aligned}$$

But now $Pr_i^{IB}\left(q_i|\, Q^{ID} = Q^{IS}, r_i \le q_i\right) = Pr_i^{IB}\left(q_i|\, Q^{ID} > Q^{IS}, r_i \le q_i\right) = 1$, and hence

$$V_1 = Pr_i\left(r_i \le q_i\right)\left[Pr\left(Q^{ID} > Q^{IS}\right)\overline{p} + Pr\left(Q^{ID} = Q^{IS}\right)p^0 + Pr\left(Q^{ID} < Q^{IS}\right)\underline{p}\right].$$

Combining V_1 and V_2 yields

$$\begin{aligned} v_i\left(q_i|r_i,\eta_0\right) &= Pr\left(Q^{ID} = Q^{IS}\right)p^0 + Pr\left(Q^{ID} > Q^{IS}\right)\overline{p} + Pr\left(Q^{ID} < Q^{IS}\right)\underline{p} \\ &= Pr\left(Q^{ID} = Q^{IS}\right)\left(\overline{p} + \underline{p}\right)\frac{1}{2} + (1 - Pr\left(Q^{ID} \le Q^{IS}\right))\overline{p} + Pr\left(Q^{ID} < Q^{IS}\right)\underline{p} \\ &= \overline{p} - Pr\left(Q^{ID} < Q\right)\left(\overline{p} - \underline{p}\right) - Pr\left(Q^{ID} = Q^{IS}\right)\frac{1}{2}\left(\overline{p} - \underline{p}\right) \\ &\approx p^{IB}\left(\eta_0\right). \end{aligned}$$

Since we use a continuous distribution of r_i in our simulations, $Pr\left(Q^{ID} = Q^{IS}\right) = 0$, and we can use the given approximation in the simulation.

B. Affine information structure

Consider the following multivariate normal random variable $X_i = (X_1, X_{2.1}, X_{2.2}) = (r_i, s_i, I)$ where $s_i = r_i + \varepsilon_i$ and $I = n\tilde{s} = n(\tilde{r} + \tilde{\varepsilon})$. This has a mean vector

$$\mu = E[X_i] = \begin{bmatrix} \mu_1 \\ \mu_{2.1} \\ \mu_{2.2} \end{bmatrix} = \begin{bmatrix} E[r_i] \\ E[s_i] \\ E[I] \end{bmatrix} = \begin{bmatrix} \bar{r} \\ \bar{r} \\ n\bar{r} \end{bmatrix}$$

and a covariance matrix

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix},$$

where

$$\begin{aligned}
\Sigma_{11} &= var[r_i] = \sigma_r^2 \\
\Sigma_{12} &= \Sigma_{21}^T = \begin{bmatrix} cov[r_i, s_i] \\ cov[r_i, I] \end{bmatrix}^T = \begin{bmatrix} \sigma_r^2 \\ (1 + (n-1)\rho)\sigma_r^2 \end{bmatrix}^T \equiv \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}^T \\
\Sigma_{22} &= \begin{bmatrix} var[s_i] & cov[s_i, I] \\ cov[s_i, I] & var[I] \end{bmatrix} = \begin{bmatrix} \sigma_r^2 + \sigma_\varepsilon^2 & \sigma_\varepsilon^2 + (1 + (n-1)\rho)\sigma_r^2 \\ \sigma_\varepsilon^2 + (1 + (n-1)\rho)\sigma_r^2 & n\left[\sigma_\varepsilon^2 + (1 + (n-1)\rho)\sigma_r^2\right] \end{bmatrix} \equiv \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix}.
\end{aligned}$$

Inverse of Σ_{22} is written as

$$\Sigma_{22}^{-1} = \frac{1}{\det(\Sigma_{22})} \begin{bmatrix} \det(\Delta_{22}) & -\det(\Delta_{21}) \\ -\det(\Delta_{12}) & \det(\Delta_{11}) \end{bmatrix} = \frac{1}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} \begin{bmatrix} \Delta_{22} & -\Delta_{21} \\ -\Delta_{12} & \Delta_{11} \end{bmatrix}.$$

The conditional distribution of the random variable $(r_i|s_i, I)$ has an expected value (DeGroot, 1970) of

$$\begin{aligned}
& E[r_i|s_i, I] \\
= \ & \mu_1 + \Sigma_{12}\Sigma_{22}^{-1} \begin{bmatrix} s_i - \mu_{2.1} \\ I - \mu_{2.2} \end{bmatrix} \\
= \ & \bar{r} + \frac{1}{\det(\Sigma_{22})} \begin{bmatrix} \delta_1\Delta_{22} - \delta_2\Delta_{12} \\ -\delta_1\Delta_{21} + \delta_2\Delta_{11} \end{bmatrix}^T \begin{bmatrix} s_i - \bar{r} \\ I - n\bar{r} \end{bmatrix} \\
= \ & \left(1 - \frac{\delta_1\Delta_{22} - \delta_2\Delta_{12} + n(\delta_2\Delta_{11} - \delta_1\Delta_{21})}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}}\right)\bar{r} \\
& + \frac{\delta_1\Delta_{22} - \delta_2\Delta_{12}}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} s_i \\
& + \frac{\delta_2\Delta_{11} - \delta_1\Delta_{21}}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} I.
\end{aligned}$$

Using the equations defined above, and after some calculations, the expected value of r_i can be written as

$$E[r_i|s_i, I] = A\bar{r} + Bs_i + CI, \tag{46}$$

where

$$
\begin{aligned}
A &= \frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + \left(1 + (n-1)\,\rho\right)\sigma_r^2\right)} && (47)\\
B &= \frac{(1-\rho)\,\sigma_r^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\,\sigma_r^2\right)} && (48)\\
C &= \frac{\rho\sigma_r^2\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\,\sigma_r^2\right)\left(\sigma_\varepsilon^2 + \left(1 + (n-1)\,\rho\right)\sigma_r^2\right)}. && (49)
\end{aligned}
$$

Moreover, the conditional variance is

$$
\begin{aligned}
var\left[r_i|s_i, I\right] &= \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}^T && (50)\\
&= \Sigma_{11} - \delta_1 \frac{\delta_1\Delta_{22} - \delta_2\Delta_{21}}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}} - \delta_2 \frac{\delta_2\Delta_{11} - \delta_1\Delta_{12}}{\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}}\\
&= \sigma_r^2 - \sigma_r^2 B - \left(1 + (n-1)\,\rho\right)\sigma_r^2 C\\
&= \sigma_r^2\,(1-B)\,(1-\rho+\rho A)\\
&= \sigma_r^2\,(1-B)\,(1-\rho) + \sigma_r^2\,(1-B)\,\rho A\\
&= (B+C)\,\sigma_\varepsilon^2.
\end{aligned}
$$

We denote the remaining uncertainty by $\varepsilon_i^s = r_i - E\left[r_i|s_i, I\right]$. It is a normal random variable with zero expected value and a variance $var\left[\varepsilon_i^s\right] = var\left[r_i|s_i, I\right]$. Further, a covariance between remaining uncertainties may be written as

$$
\begin{aligned}
cov\left[\varepsilon_i^s, \varepsilon_j^s\right] &= E\left[\left\{(r_i - \bar{r}) - B\,(s_i - \bar{r}) - Cn\,(\tilde{s} - \bar{r})\right\}\left\{(r_j - \bar{r}) - B\,(s_j - \bar{r}) - Cn\,(\tilde{s} - \bar{r})\right\}\right]\\
&= E\left[(r_i - \bar{r})\,(r_j - \bar{r}) + B^2\,(s_i - \bar{r})\,(s_j - \bar{r}) + C^2 n^2\,(\tilde{s} - \bar{r})\right]\\
&\quad -2E\left[B\,(r_i - \bar{r})\,(s_j - \bar{r}) + Cn\,(r_i - \bar{r})\,(\tilde{s} - \bar{r}) - BCn\,(s_i - \bar{r})\,(\tilde{s} - \bar{r})\right]\\
&= \rho\sigma_r^2 + B^2\rho\sigma_r^2 + C^2n^2 \cdot var\left[\tilde{s}\right] - 2B\rho\sigma_r^2 - 2C \cdot var\left[\tilde{r}\right] + 2BCn \cdot var\left[\tilde{s}\right]\\
&= var\left[\varepsilon_i^s\right] - (1-B)^2\,\sigma_r^2\,(1-\rho) - B^2\sigma_\varepsilon^2\\
&= var\left[\varepsilon_i^s\right] - B\sigma_\varepsilon^2\\
&= C\sigma_\varepsilon^2.
\end{aligned}
$$

The aggregate uncertainty is given as (note that $B + nC = 1 - A = \frac{var[\tilde{r}]}{var[\tilde{s}]}$)

$$
n\tilde{\varepsilon}^s = \sum_{i=1}^{n} \varepsilon_i^s = (\tilde{r} - \bar{r}) - (1 - A)\,(\tilde{s} - \bar{r})\,.
$$

It has a normal distribution with zero expected value and a variance

$$\begin{aligned}
var\left[n\tilde{\varepsilon}^{s}\right] &= E\left[\left(\left(\tilde{r}-\bar{r}\right)-\left(1-A\right)\left(\tilde{s}-\bar{r}\right)\right)^{2}\right] \\
&= E\left[\left(\tilde{r}-\bar{r}\right)^{2}+\left(1-A\right)^{2}\left(\tilde{s}-\bar{r}\right)^{2}-2\left(1-A\right)\left(\tilde{r}-\bar{r}\right)\left(\tilde{s}-\bar{r}\right)\right] \\
&= var\left[\tilde{r}\right]+\left(1-A\right)^{2}var\left[\tilde{s}\right]-2\left(1-A\right)cov\left[\tilde{r},\tilde{s}\right] \\
&= A^{2}var\left[\tilde{r}\right]+\left(1-A\right)^{2}var\left[\tilde{\varepsilon}\right] \\
&= \left(1-A\right)n\sigma_{\varepsilon}^{2} \\
&= n^{2}\left(\frac{B}{n}+C\right)\sigma_{\varepsilon}^{2}.
\end{aligned}$$

C. First order conditions of VA, UPA and DPA

Maximization problems of bidder i in the Vickrey, uniform price and discriminatory price auction are the following, respectively,

$$\max_{D_i(p;s_i)} E\left[\pi_i^{va}\middle| s_i\right]=\int_{p^0}^{\bar{p}}\left\{\int_0^{D_i(p;s_i)} v_i\left(x;s\right)-RS_i^{-1}\left(x\right)dx\right\}dH\left(p,D_i\left(p;s_i\right)\right), \tag{51}$$

$$\max_{D_i(p;s_i)} E\left[\pi_i^{upa}\middle| s_i\right]=\int_{p^0}^{\bar{p}}\left\{\int_0^{D_i(p;s_i)} v_i\left(x;s\right)dx\right\}-pD_i\left(p;s_i\right)dH\left(p,D_i\left(p;s_i\right)\right), \tag{52}$$

$$\max_{D_i(p;s_i)} E\left[\pi_i^{dpa}\middle| s_i\right]=\int_{p^0}^{\bar{p}}\left\{\int_0^{D_i(p;s_i)} v_i\left(x;s\right)-P_i\left(x;s_i\right)dx\right\}dH\left(p,D_i\left(p;s_i\right)\right), \tag{53}$$

where $H\left(p,D_i\left(p;s_i\right)\right)$ is the probability distribution of the market clearing price, i.e. the probability that the market clearing price p is not higher than the bid for unit q_i. Hence,

$$\begin{aligned}
H\left(p,q_i\right) &= Pr\left[P_i\left(q_i;s_i\right)\geq p\right] \\
&= Pr\left[q_i\leq Q-D_{-i}\left(P_i\left(q_i;s_i\right);s_{-i}\right)\right].
\end{aligned}$$

First order conditions (Euler equations) of the maximization problems are, respectively (see e.g. De Castro and Riascos, 2009; Hortaçsu 2011; Wang and Zender, 2002; Wilson, 1979),

$$VA: \qquad \left\{v_i\left(q_i;s\right)-p\right\}H_p=0, \tag{54}$$

$$UPA: \qquad v_i\left(q_i;s\right)=p-q_i\frac{H_q}{H_p}, \tag{55}$$

$$DPA: \qquad v_i\left(q_i;s\right)=p+\frac{H}{H_p}, \tag{56}$$

where we have used $p=RS_i^{-1}\left(D_i\left(p;s_i\right)\right)$ in the first order condition of VA.

Next we derive the first order conditions of UPA and DPA using the affine information structure and, in particular, the distribution of aggregate information I. Suppose that the clearing price of the auction is such that $p^0<p<\bar{p}$. Consider also that bank i utilizes the optimal bidding strategy in the equilibrium $P_i\left(q_i;s_i\right)=p$ and all other banks use optimal strategies. This results in a residual supply $RS_i(p) = Q - D_{-i}(p; s_{-i}) = q_i(s_i, I)$ given $I = n\tilde{s}$. In other words, let I be the aggregate information, which clears the auction with the clearing price p and gives an allocation $q_i(s_i, I)$ to bank i. Hence, the probability of the clearing price $H(p, q_i)$ is equal to $G(I|\, s_i)$ in the equilibrium. Given optimal strategies, bank i receives at least q_i units with a probability $G(I|\, s_i)$, when bidding with a non-increasing function $P_i(q_i; s_i)$ with a value higher or equal to p for q_i. Consider further that bank i increases its bid for the marginal unit q_i by an infinitesimal amount dp (and bids for all infra-marginal units are at least $p + dp$). With a given I, the residual supply increases by $dRS_i = -D'_{-i}(p; s_{-i})\, dp$. Hence in order to get exactly q_i units with a clearing price $P_i(q_i; s_i) = p + dp$, the aggregate information I should increase such that

$$\begin{aligned} 0 &= \frac{\partial RS_i(p)}{\partial p} dp + \frac{\partial RS_i(p)}{\partial I} dI \\ &\Rightarrow \frac{dI}{dp} = \frac{D'_{-i}(p; s_{-i})}{\frac{\partial RS_i(p)}{\partial I}}. \end{aligned}$$

Thus the partial derivative of H with respect to the clearing price evaluated at p and $q_i(s_i, I)$ is

$$\frac{\partial H(p, q_i)}{\partial p} = \frac{dG(I|\, s_i)}{dI}\frac{dI}{dp} = g(I|\, s_i)\frac{D'_{-i}(p; s_{-i})}{\frac{\partial RS_i(p)}{\partial I}} > 0.$$

Similarly, if bank i bids for a unit $q'_i = q_i + dq_i$ with a bid defined by $P_i(q'_i; s_i) = p$, in order to receive this unit the aggregate information should decrease by $dI = \frac{1}{\frac{\partial RS_i(p)}{\partial I}} dq_i$ and the partial derivative of H with respect to q_i is

$$\frac{\partial H(p, q_i)}{\partial q_i} = g_i(I|\, s_i)\frac{1}{\frac{\partial RS_i(p)}{\partial I}} < 0.$$

Hence, the first order conditions of the uniform price auction and the discriminatory price auction is written as

$$UPA: \qquad v_i(q_i; s) = p - \frac{q_i(s_i, I)}{D'_{-i}(p; s_{-i})}, \tag{57}$$

$$DPA: \qquad v_i(q_i; s) = p + \frac{1}{D'_{-i}(p; s_{-i})}\frac{G(I|\, s_i)}{g(I|\, s_i)}\frac{dq_i}{dI}, \tag{58}$$

where $\frac{\partial RS_i(p)}{\partial I} \equiv \frac{dq_i}{dI}$ and $\frac{G(I|s_i)}{g(I|s_i)} = \lambda(I|\, s_i)$ is the inverse hazard rate of aggregate information conditional on the signal s_i.

D. Equilibrium strategy of UPA

We can derive an analytical solution for the uniform price auction from the first order condition of the average bank (36) by using $(1/q_m)^{n-1}$ as an integrating factor (see e.g. Holmberg 2008; Rudkevich et al. 1998; and Anderson and Philpott, 2002). Thus, multiplying (36) by q_m^{-n}, we get

$$\frac{1}{q_m^{n-1}} P'_m(q_m) - \left(\frac{n-1}{q_m^n}\right) P_m(q_m) = -\left(\frac{n-1}{q_m^n}\right) v_m(q_m; s),$$

or equivalently

$$\frac{d}{dq_m}\left(\frac{1}{q_m^{n-1}} P_m(q_m)\right) = -\left(\frac{n-1}{q_m^n}\right) v_m(q_m; s).$$

Furthermore, integrating both sides gives

$$\frac{1}{q_m^{n-1}} P_m(q_m) = \Gamma_m^{upa} - (n-1) \int \frac{v_m(q_m; s)}{q_m^n} dq_m,$$

where Γ_m^{upa} is the constant of integration. Using the end condition $P_m(\hat{q}_m; s_m) = p^0$, the constant of integration Γ_m^{upa} can be defined as

$$\Gamma_m^{upa} = \frac{1}{(\hat{q}_m)^{n-1}} p^0 + (n-1) \left\{ \int \frac{v_m(q_m; s)}{q_m^n} dq_m \bigg|_{\hat{q}_m} \right\}.$$

Hence, the bid of the average bank for the quantity $q_m = \bar{r}$ and thus the expected equilibrium price $p^{upa}(\tilde{s})$ conditional on the market information $ns_m = n\tilde{s} \geq n\bar{r}$ may be written as

$$P_m(q_m) = \frac{\bar{r}^{n-1}}{(\hat{q}_m)^{n-1}} p^0 + \bar{r}^{n-1}(n-1) \int_{\bar{r}}^{\hat{q}_m} \frac{v_m(q_m; s)}{q_m^n} dq_m. \tag{59}$$

While $P_m(q_m) = p^{upa}(\tilde{s})$ when $s_m = \tilde{s}$, we next describe (59) in a form of $p^{upa}(\tilde{s}) = p^c(\tilde{s}) - W^{upa}(\tilde{s})$ where the last term is the bid shading function of the average bidder. When deriving the equilibrium strategy for the average bank indexed with m and receiving a signal s_m (which is equal to $\tilde{s} > \bar{r}$ in the equilibrium), we assume that the quantity of the average bank q_m is not constant. Instead, we change the variable of integration from q_m to I, where I gets values from $n\bar{r}$ to $n\tilde{s}$. We will show that the bank which gets a signal s_m will receive fewer units in the equilibrium of UPA when compared to the competitive case, whenever the aggregate information has values $n\bar{r} < I < n\tilde{s}$. However, we know by assumption that the equilibrium quantities of the average bank are equal to the competitive allocation at the end points of the region $I \in (n\bar{r}, n\tilde{s})$. Thus we average out the quantity q_m when we integrate the equilibrium price from the first order condition of the average bank. Hence, we use an approximation from the competitive case $q_m(s_m, I) = \bar{r} + B\left(s_m - \frac{I}{n}\right)$. Thus, q_m increases from $q_m(s_m, n\tilde{s}) = \bar{r}$ to $q_m(s_m, n\bar{r}) = \hat{q}_m$. Furthermore, in order to integrate (59), we need to define the derivative of the marginal value function with respect to the aggregate information in the region $I \in (n\bar{r}, n\tilde{s})$, that is

$$\frac{dv_m(q_m; s)}{dI} = \frac{dv_m(q_m; s)}{dq_m} \frac{dq_m}{dI}.$$

We use the linearized model and thus the approximation $\frac{dv_i(q_i; s)}{dq_i} \approx -\beta(\tilde{s})$. Moreover, we average out the derivative $\frac{dq_m}{dI}$ in the region $I \in (n\bar{r}, n\tilde{s})$ by

$$\frac{dq_m}{dI} \approx \frac{\Delta q_m}{\Delta I} = \frac{\bar{r} - \bar{r} - B(s_m - \bar{r})}{n\tilde{s} - n\bar{r}} = -\frac{B}{n}.$$

These give an approximation $\frac{dv_m(q_m;s)}{dI} \approx \frac{B}{n}\beta\left(\frac{I}{n}\right)$. Thus, we may write (59) again by,

$$\begin{aligned} p^{upa}(\tilde{s}) &= \left(\frac{q_m(s_m,n\tilde{s})}{q_m(s_m,n\bar{r})}\right)^{n-1} p^0 - (q_m(s_m,n\tilde{s}))^{n-1}(n-1)\int_{n\bar{r}}^{n\tilde{s}} \frac{v_m(q_m;s)}{\left(q_m\left(s_m,\frac{I}{n}\right)\right)^n}\frac{dq_m}{dI}dI \qquad (60) \\ &= \left(\frac{q_m(s_m,n\tilde{s})}{q_m(s_m,n\bar{r})}\right)^{n-1} p^0 - (q_m(s_m,n\tilde{s}))^{n-1}(n-1)\int_{n\bar{r}}^{n\tilde{s}} v_m(q_m;s)\frac{d}{dI}\left(-\frac{1}{(n-1)}\frac{1}{\left(q_m\left(s_m,\frac{I}{n}\right)\right)^{n-1}}\right)dI \\ &= p^c(\tilde{s}) - (q_m(s_m,n\tilde{s}))^{n-1}\frac{B}{n}\int_{n\bar{r}}^{n\tilde{s}}\left(\frac{\beta\left(\frac{I}{n}\right)}{\left(q_m\left(s_m,\frac{I}{n}\right)\right)^{n-1}}\right)dI. \end{aligned}$$

In the last step we have integrated by parts. The bid shading function of the average bidder can be written as

$$W^{upa}(s_m,n\tilde{s}) = (q_m(s_m,n\tilde{s}))^{n-1}\frac{B}{n}\int_{n\bar{r}}^{n\tilde{s}}\left(\frac{\beta\left(\frac{I}{n}\right)}{\left(q_m\left(s_m,\frac{I}{n}\right)\right)^{n-1}}\right)dI. \qquad (61)$$

It has a derivative

$$\frac{dW^{upa}(s_m,n\tilde{s})}{d\tilde{s}} = B\beta(\tilde{s}) - B(n-1)(q_m(s_m,n\tilde{s}))^{n-2}\frac{B}{n}\int_{n\bar{r}}^{n\tilde{s}}\left(\frac{\beta\left(\frac{I}{n}\right)}{\left(q_m\left(\frac{I}{n}\right)\right)^{n-1}}\right)dI.$$

In order to derive the quantity bid for bank i, we need to determine the derivative of the average bank's bid function $P'_m(q_m)$ at the equilibrium point. We may write $\frac{dP_m(q_m)}{dq_m} = \frac{dp^{upa}(\tilde{s})}{d\tilde{s}}\frac{d\tilde{s}}{dq_m}$, and hence

$$\begin{aligned} \frac{dP_m(q_m)}{dq_m} &= -\frac{1}{B}\left(\frac{dp^c(\tilde{s})}{d\tilde{s}} - \frac{dW^{dpa}(s_m,n\tilde{s})}{d\tilde{s}}\right) \qquad (62) \\ &= -(n-1)(q_m(s_m,n\tilde{s}))^{n-2}\frac{B}{n}\int_{n\bar{r}}^{n\tilde{s}}\left(\frac{\beta\left(\frac{I}{n}\right)}{\left(q_m\left(\frac{I}{n}\right)\right)^{n-1}}\right)dI \\ &= -\frac{(n-1)}{q_m(s_m,n\tilde{s})}W^{upa}(s_m,n\tilde{s}). \end{aligned}$$

While $s_m = \tilde{s}$, this yields $\frac{dP_m(q_m)}{dq_m} = -\frac{(n-1)}{\bar{r}}W^{upa}(\tilde{s})$. Finally, using linearized marginal value functions (around the competitive equilibrium), equations (60) and (62), and first order conditions, we can easily derive the approximative equilibrium quantity for bank i,

$$\begin{aligned} D_i^{upa}(p^{upa}(\tilde{s});s_i) &= q_i^{upa}(s_i,\tilde{s}) \qquad (63) \\ &= \bar{r} + \left[\frac{\bar{r}\beta(\tilde{s})}{\bar{r}\beta(\tilde{s}) + W^u(\tilde{s})}\right]B(s_i - \tilde{s}) \\ &= \bar{r} + B(s_i-\tilde{s}) - \left[\frac{W^{upa}(\tilde{s})}{\bar{r}\beta(\tilde{s}) + W^{upa}(\tilde{s})}\right]B(s_i-\tilde{s}). \end{aligned}$$

While $\frac{W^{upa}(\tilde{s})}{\bar{r}\beta(\tilde{s})+W^{upa}(\tilde{s})} \geq 0$, bidders with high signals $s_i > \tilde{s}$ will receive fewer units in the uniform price auction than in the competitive case.

Finally, note that when $\tilde{s}$ increases, the quantity from (63) for every s_i is closing to $\bar{r}$ while $\beta(\tilde{s})$ is closing to zero faster than $W^{upa}(\tilde{s})$ when $\tilde{s} \to \infty$ and thus $p^{upa}(\tilde{s}) \to \overline{p}$. This yields a non-monotonic bid function $P_i(q_i;s_i)$.

However, we assume that ECB accepts only non-increasing bid functions. In simulations, instead of (63), we thus use a quantity equation,

$$q_i^{upa}(s_i,\tilde{s}) = \begin{cases} 0, & \text{if } p^{upa}(\tilde{s}) = \overline{p} \\ q_i^{min}, & \text{if } \overline{p} > p^u(\tilde{s}) \geq p^{upa}\left(\tilde{s}^{i,high}\right) \\ q_i^c(s_i,\tilde{s}) - \left(\frac{W^{upa}(\tilde{s})}{\overline{r}\beta(\tilde{s})+W^{upa}(\tilde{s})}\right) B(s_i - \tilde{s}), & \text{if } p^{upa}\left(\tilde{s}^{i,high}\right) > p^{upa}(\tilde{s}) > p^0 \\ \hat{q}_i, & \text{if } p^{upa}(\tilde{s}) = p^0 \end{cases} \tag{64}$$

where $\tilde{s}^{i,high} \equiv (\tilde{s} : min\{D_i^{upa}(p^{upa}(\tilde{s});s_i)\})$ and $q_i^{min} = q_i^{upa}\left(s_i,\tilde{s}^{i,high}\right)$ from (39). This is, however, not quantitatively important in most of the cases.

E. Equilibrium strategy of DPA

When we derive the (approximative) closed form solution of the discriminatory price auction, we follow the model of Holmberg (2009). We start by solving the expected clearing price from (41) with the given $I = n\tilde{s}$ and using $[G(I|\,\tilde{s})]^{n-2}$ as an integrating factor. First, we denote the signal of the average bank by s_m. This is by assumption equal to $\tilde{s}$. However, in what follows, we treat I as a continuous variable moving from $n\overline{r}$ to $n\tilde{s}$. Moreover, we assume that the allotment $q_m(s_m,I) = \overline{r} + B\left(s_m - \frac{I}{n}\right)$ is not a constant. It increases from $\overline{r}$ to $\hat{q}_m = \overline{r} + B(s_m - \overline{r})$ as I decreases from $n\tilde{s}$ to $n\overline{r}$. Hence, multiplying (41) by $[G(I|\,s_m)]^{n-1}$ yields

$$\begin{aligned} & [G(I|\,s_m)]^{n-1}\frac{dP_m(q_m)}{dq_m}\frac{dq_m}{dI} + (n-1)[G(I|\,s_m)]^{n-2} g(I|\,s_m) P_m(q_m) \\ = \;& (n-1) v_m(q_m;s)[G(I|\,s_m)]^{n-2} g(I|\,s_m), \end{aligned}$$

or equivalently

$$\frac{d}{dI}\left([G(I|\,s_m)]^{n-1} P_m(q_m)\right) = (n-1) v_m(q_m;s)[G(I|\,s_m)]^{n-2} g(I|\,s_m).$$

Integrating both sides gives

$$[G(I|\,s_m)]^{n-1} P_m(q_m) = \Gamma_m^{dpa} + \int (n-1)[G(I|\,s_m)]^{n-2} v_m(q_m;s) g(I|\,s_m)\, dI. \tag{65}$$

As a result, we have multiple equilibria, each determined by a different value of the constant of integration Γ_m^{dpa}. However, with the end condition, $P_m(\hat{q}_m) = p^0$, we get the unique solution by solving Γ_m^{dpa} as

$$\Gamma_m^{dpa} = [G(n\overline{r}|\,s_m)]^{n-1} p^0 - (n-1)\left\{\int v_m(q_m;s)[G(I|\,s_m)]^{n-2} g(I|\,s_m)\, dI\right\}\Bigg|_{n\overline{r}}.$$

Hence, we can write the expected price $p^{dpa}(\tilde{s}) = P_m(q_m)$ as a function of $\frac{I}{n}$ evaluated at $\tilde{s}$:

$$P_m(q_m) = \frac{[G(n\overline{r}|\,s_m)]^{n-1}}{[G(n\tilde{s}|\,s_m)]^{n-1}} p^0 + \frac{(n-1)}{[G(n\tilde{s}|\,s_m)]^{n-1}} \int_{n\overline{r}}^{n\tilde{s}} v_m(q_m;s)[G(I|\,s_m)]^{n-2} g(I|\,s_m)\, dI. \tag{66}$$

Using the same approximation as in the uniform price auction, that is $\frac{dv_m(q_m;s)}{dI} \approx \frac{B}{n}\beta\left(\frac{I}{n}\right)$, we integrate (66) by parts and thus get the equilibrium price

$$\begin{aligned} p^{dpa}(\tilde{s}) &= v_m(\bar{r};s) - \frac{(n-1)}{\left[G(n\tilde{s}|\,s_m)\right]^{n-1}} \int_{n\bar{r}}^{n\tilde{s}} \left(\frac{dv_m(q_m;s)}{dI}\right)\left(\frac{1}{n-1}\right)\left[G(I|\,s_m)\right]^{n-1} dI \\ &= p^c(\tilde{s}) - \frac{B}{n}\frac{\int_{n\bar{r}}^{n\tilde{s}} \beta\left(\frac{I}{n}\right)\left[G(I|\,s_m)\right]^{n-1} dI}{\left[G(n\tilde{s}|\,s_m)\right]^{n-1}}. \end{aligned} \tag{67}$$

The bid shading function of the average bank is

$$W^{dpa}(s_m, n\tilde{s}) = \frac{B}{n}\frac{\int_{n\bar{r}}^{n\tilde{s}} \beta\left(\frac{I}{n}\right)\left[G(I|\,s_m)\right]^{n-1} dI}{\left[G(ns_m|\,s_m)\right]^{n-1}}.$$

When $s_m = \tilde{s}$ we may also write $W^{dpa}(s_m, n\tilde{s}) = W^{dpa}(\tilde{s})$. The shading function has a derivative

$$\begin{aligned} \frac{dW^{dpa}(s_m, n\tilde{s})}{d\tilde{s}} &= B\frac{d}{d(n\tilde{s})}\left(\frac{\int_{n\bar{r}}^{n\tilde{s}} \beta\left(\frac{I}{n}\right)\left[G(I|\,s_m)\right]^{n-1} dI}{\left[G(n\tilde{s}|\,s_m)\right]^{n-1}}\right) \\ &= \left\{\frac{B}{\left[G(n\tilde{s}|\,s_m)\right]^{n-1}}\right\}\frac{d}{d(n\tilde{s})}\left(\int_{n\bar{r}}^{n\tilde{s}} \beta\left(\frac{I}{n}\right)\left[G(I|\,s_m)\right]^{n-1} dI\right) \\ &\quad + \left\{B\int_{n\bar{r}}^{n\tilde{s}} \beta\left(\frac{I}{n}\right)\left[G(I|\,s_m)\right]^{n-1} dI\right\}\frac{d}{d(n\tilde{s})}\left(\left[G(n\tilde{s}|\,s_m)\right]^{-n+1}\right) \\ &= B\beta(\tilde{s}) - n(n-1)\frac{g(n\tilde{s}|\,s_m)}{G(n\tilde{s}|\,s_m)}W^{dpa}(s_m, n\tilde{s}). \end{aligned}$$

In the equilibrium, the bid function of the average bank has a slope

$$\begin{aligned} \frac{dP_m(q_m)}{dq_m} &= \frac{dp^{dpa}(\tilde{s})}{d\tilde{s}}\frac{d\tilde{s}}{dq_m} \\ &= -\frac{1}{B}\left(\frac{dp^c(\tilde{s})}{d\tilde{s}} - \frac{dW^{dpa}(s_m, n\tilde{s})}{d\tilde{s}}\right) \\ &= -(n-1)\frac{g(n\tilde{s}|\,s_m)}{G(n\tilde{s}|\,s_m)}W^{dpa}(s_m, n\tilde{s})\frac{n}{B}. \end{aligned} \tag{68}$$

The optimal bid of bank i is the quantity q_i that solves (42) using equations (67) and (68). Recall that the inverse hazard rate is $\lambda(I|\,s_i) = \frac{G(I|s_i)}{g(I|s_i)}$. Then, using linearized marginal value functions (around the competitive equilibrium) and after some simple calculations, the approximative bid function of bank i in the discriminatory price auction may be written as (when $s_m = \tilde{s}$)

$$\begin{aligned} D_i^{dpa}\left(p^{dpa}(\tilde{s}); s_i\right) &= q_i^{dpa}(s_i, \tilde{s}) \\ &= \bar{r} + B(s_i - \tilde{s}) + \frac{W^{dpa}(\tilde{s})}{\beta(\tilde{s})}\left[1 - \frac{\lambda(n\tilde{s}|\,s_i)}{\lambda(n\tilde{s}|\,\tilde{s})}\right], \end{aligned}$$

where $\bar{r}+B\left(s_i-\tilde{s}\right)$ is the equilibrium quantity in the competitive case. In other words, utilizing our approximative assumptions and due to monotonic functions, each price $p^{dpa}\left(\tilde{s}\right)$ is associated with one possible aggregate information $I=n\tilde{s}$ by (67) and this, on the other hand, defines the optimal quantity bid $q_i^{dpa}\left(s_i,\tilde{s}\right)$.

F. Simulation results

Figure 3 presents the simulated interbank rates for one of our simulation specifications (information structure IV), when the auction equilibrium is competitive. The purpose of this figure is twofold: First, to simply describe how interbank rates change with the trading frictions, and second, to show that resulting prices are in line with what we should expect. The blue curve in every panel of Figure 3 describes,

$$v_i\left(q_i^c\left(s_i,\tilde{s}\right)|s,\eta_H\right)=\overline{p}-G_\varepsilon\left(\bar{r}-E\left[\tilde{r}|\,\tilde{s}\right]\right)\left(\overline{p}-\underline{p}\right), \tag{69}$$

which is the competitive bid of bank i for quantity $q_i=q_i^c\left(s_i,\tilde{s}\right)$, when the interbank market is assumed to be collapsed. Respectively, the black curve is the expected interbank rate when interbank market is perfect. The red dotted curve is the marginal value of money of banks in a competitive equilibrium with different levels of interbank market frictions. In different panels of Figure 3, we plot the simulated interbank rates with different interbank market conditions. In the right panel, the interbank market is collapsed ($\eta_3\to\infty$) and the interbank rate is p^0 with every $\tilde{s}$. In the left panel, the interbank trade frictions are relatively modest ($\eta_1=5$) and in the middle the coefficient of the trading frictions is $\eta_2=20$. Resulting interbank rates are closing to $\underline{p}=-100$ when the money demand is low ($\tilde{s}<\bar{r}=100$) and to $\overline{p}=100$ when the money demand is high ($\tilde{s}>\bar{r}$), because the interbank market is unbalanced. With a balanced interbank market ($\tilde{s}\approx\bar{r}$) the interbank rate tends to be closer to p^0 the higher trading costs are. Recall that without any trading costs ($\eta=0$) the interbank rate would be either $\underline{p}$ or $\overline{p}$. According to Figure 3, our selection for $v_i\left(q_i;s\right)$, in particular $b=\frac{1}{10}$, seems to reflect quite accurately the changes of interbank rates due to different market conditions.

Figure 4 presents the bid functions of the bank with the signal $s_i=100$ in all three auction mechanisms (bold curves), when the interbank market is collapsed ($\eta\to\infty$). The horizontal lines describe the clearing prices and vertical lines the allocations to bank i when the demand conditions are high ($\tilde{s}=120$, left panel) and when the average signal is only slightly higher than expected ($\tilde{s}=102$, right panel). Due to the high uncertainty ($\sigma_\varepsilon=20$) the bid function in the discriminatory price auction (red curve) is relatively close to the bid schedule in the Vickrey auction (black curve). In the uniform price auction (blue dotted curve) the bid shading is stronger at prices higher than p^0. For prices close to $\overline{p}$ the price bids are again not shaded that heavily in the UPA. Note that the Vickrey auction bid function in Figure 4 is equal to the blue curve in Figure 3 only with different horizontal axes. Both of these curves plots prices $p=p^c\left(\tilde{s}|\,\eta\to\infty\right)$, but in Figure 4 the horizontal axis associated with these prices is given as $q_i=q_i^c\left(s_i,\tilde{s}\right)$ whereas in Figure 3 it is directly given as $\tilde{s}$. Figure 4 shows that the allocation is fairly similar after all the auction mechanisms when the average signal is close to the expected value $\bar{r}$ (right panel). However, when the average signal is high, the bank with a lower than the average signal ($s_i=100<120=\tilde{s}$) receives fewer units in DPA and more units in UPA than in the efficient allocation (VA).

Next, we present the densities of the simulation outcomes. Figure 5 shows the densities of the simulated interbank rates, Figure 6 the total use of the standing facilities and Figure 7 the total revenues from the ECB MROs and the standing facilities. In all the figures, information structure I is presented at the top and information structure IV at the bottom. The left panel of figures presents the case when the interbank market suffers only from mild frictions $\eta_1=5$, in the middle, the trading cost coefficient is $\eta_2=20$ and in the right panel, the interbank market is collapsed $\eta_3\to\infty$. When $\eta\to\infty$, the interbank rate is p^0 by assumption and we omit that case from Figure 5.

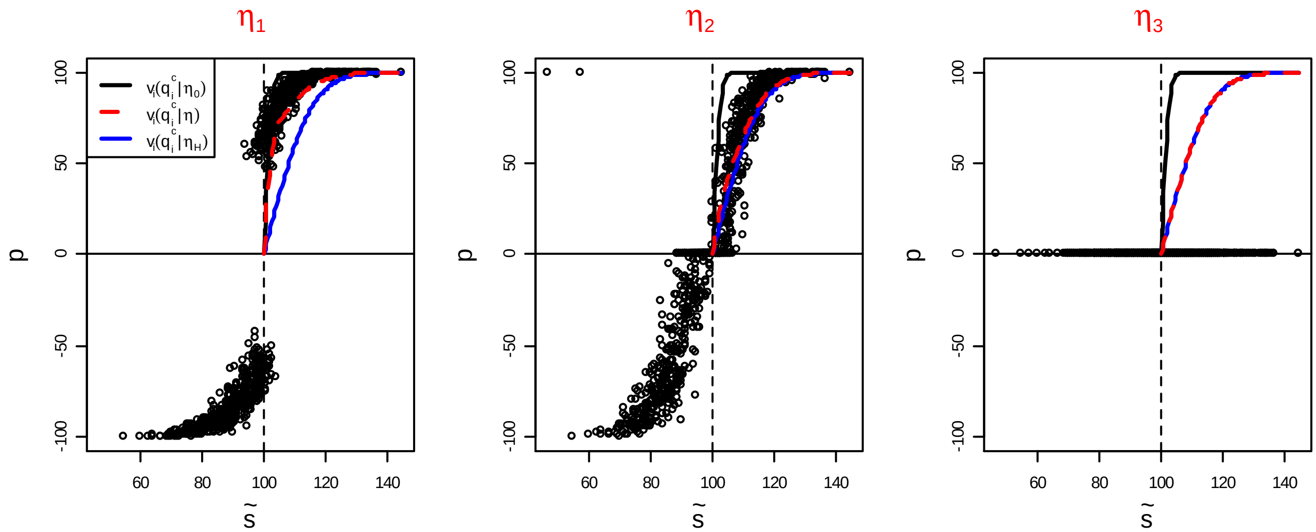

Figure 3: The marginal valuations of bank i in the ECB MROs for the competitive allocation $q_i^c(s_i, \tilde{s})$ and the simulated interbank rates after the competitive auction equilibrium with different trading cost coefficients: $\eta_1 = 5$ (left), $\eta_2 = 20$ (middle) and $\eta_3 \to \infty$ (right). The information structure: $\sigma_\varepsilon = 20$, $\rho = 0.5$. The other parameter values are presented in Table 2.

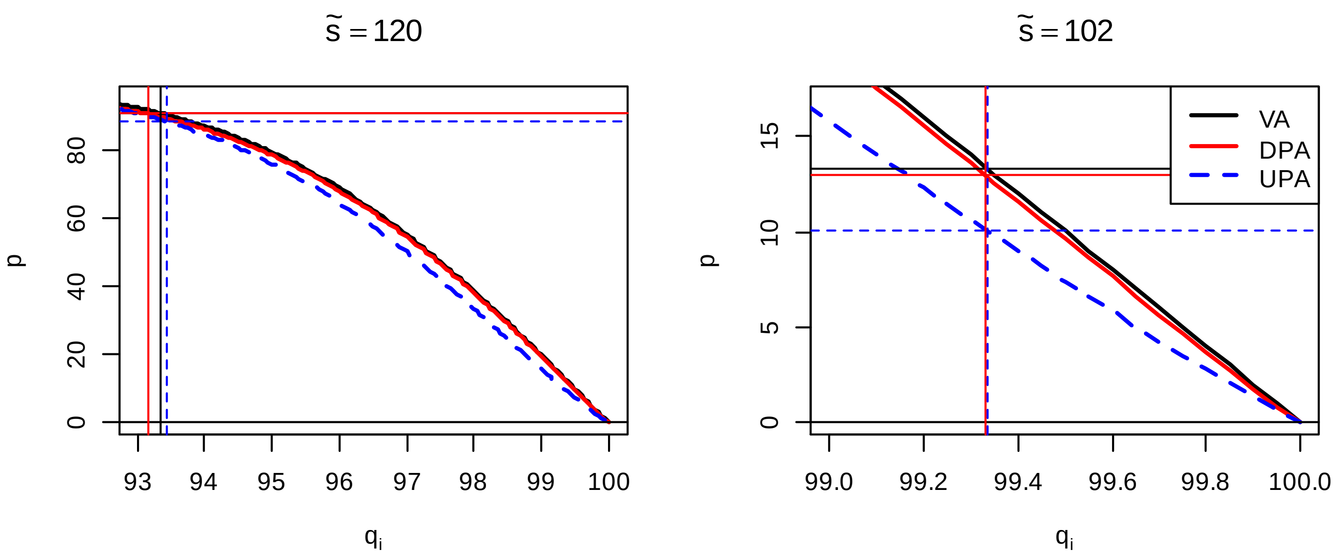

Figure 4: The bid functions $P_i(q_i; s_i)$ for the bank with the signal $s_i = 100$ and equilibrium outcomes of the Vickrey (VA), discriminatory price (DPA) and uniform price (UPA) auction, when the average signal is $\tilde{s} = 120$ (left) and $\tilde{s} = 102$ (right). The trading cost coefficient is $\eta \to \infty$ and the information structure: $\sigma_\varepsilon = 20$, $\rho = 0.5$. The other parameter values are presented in Table 2.

In addition, Table 8 and figures 8 and 9 present the densities of auction clearing prices and auction revenues.

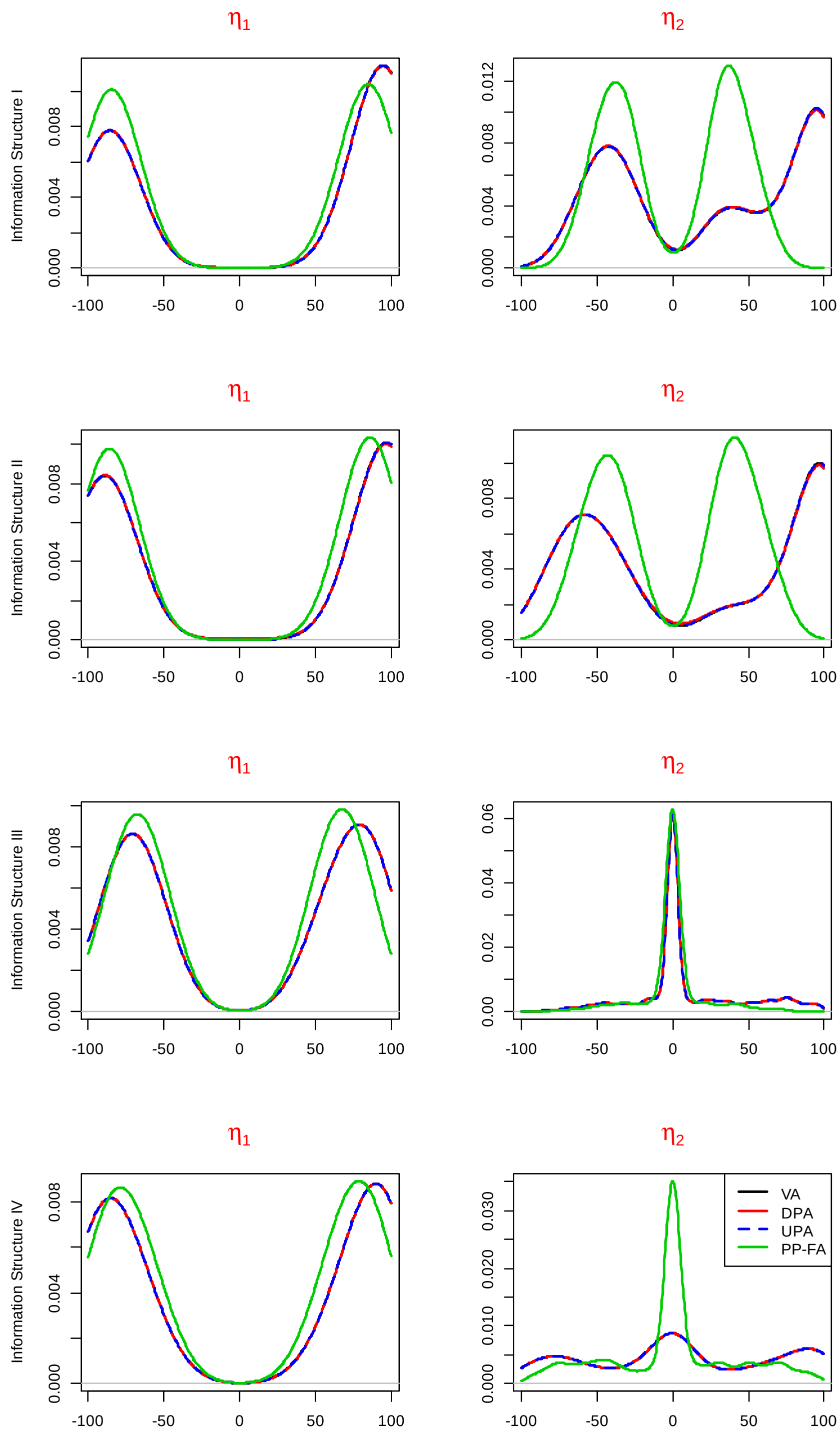

Figure 5: The interbank rates. The densities of the simulated values. The parameter values are presented in tables 2, 3, and 4.

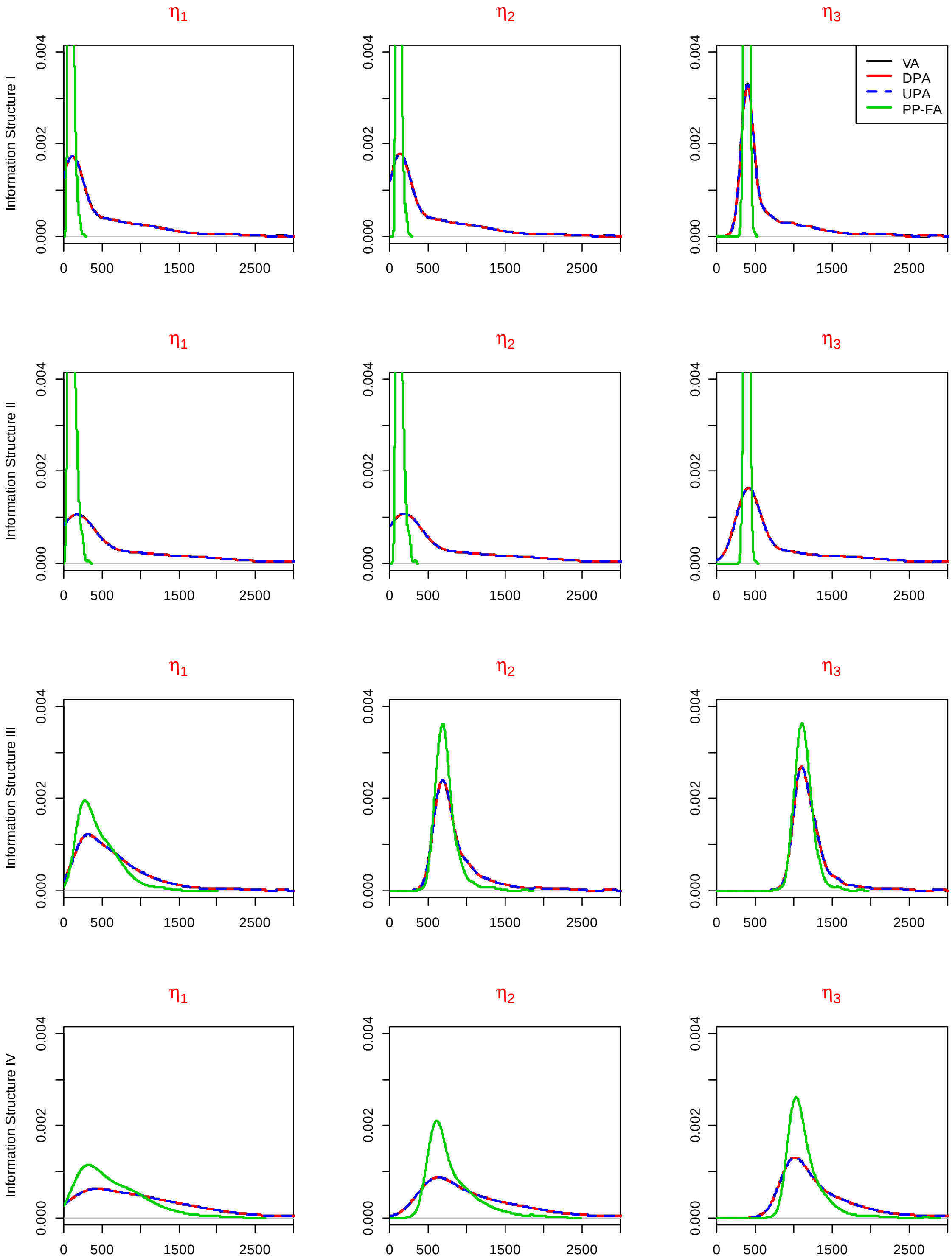

Figure 6: The total use of the standing facilities. The densities of the simulated values. The parameter values are presented in tables 2, 3, and 4.

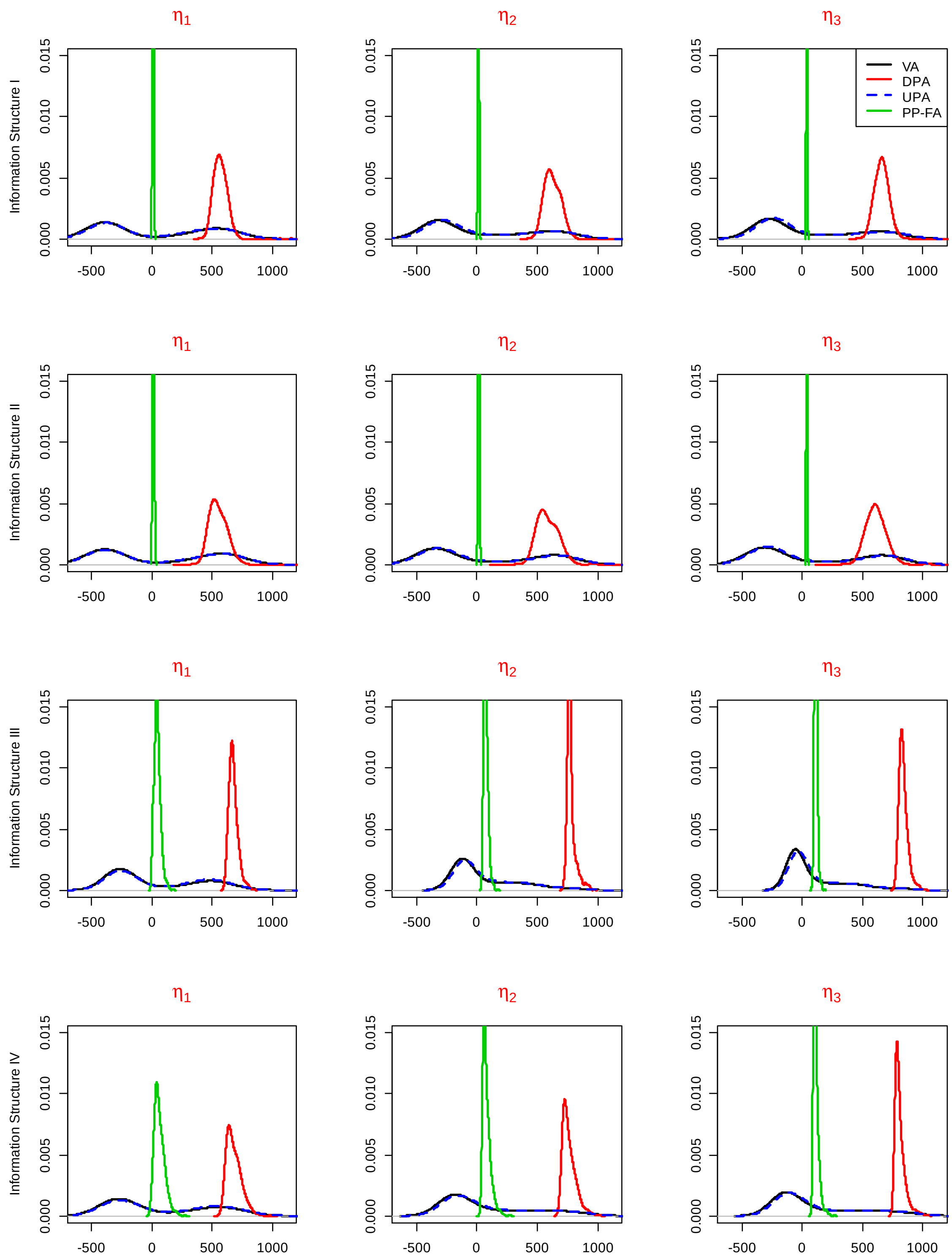

Figure 7: The total net revenues from the MROs and the standing facilities ($\times 10^3$). The densities of the simulated values. The parameter values are presented in tables 2, 3, and 4.

Table 8: Means and standard deviations of simulation results - auctions.

a) Information structure I: $\sigma_\varepsilon = 5$, $\rho = 0.2$

		$\eta_1 = 5$		$\eta_2 = 20$		$\eta_3 \to \infty$	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Clearing price	VA	43.29	44.71	36.74	41.38	34.86	40.92
	DPA	43.08	44.54	36.28	41.12	34.33	40.67
	UPA	41.81	43.47	33.49	39.57	31.10	39.20
Revenues	VA	1375.0	476.2	1311.5	439.1	1293.3	432.7
($\times 10^3$)	DPA	1920.7	106.2	1910.5	110.7	1907.5	112.0
	UPA	1383.7	469.4	1300.5	426.9	1276.6	421.5

b) Information structure II: $\sigma_\varepsilon = 5$, $\rho = 0.5$

		$\eta_1 = 5$		$\eta_2 = 20$		$\eta_3 \to \infty$	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Clearing price	VA	45.68	46.73	41.38	44.86	40.15	44.67
	DPA	45.54	46.62	41.07	44.74	39.78	44.57
	UPA	44.64	45.92	39.09	43.83	37.50	43.75
Revenues	VA	1390.0	516.6	1347.3	495.3	1335.1	492.1
($\times 10^3$)	DPA	1887.3	158.0	1878.1	163.7	1875.5	165.4
	UPA	1395.0	512.3	1339.5	487.9	1323.6	485.5

c) Information structure III: $\sigma_\varepsilon = 20$, $\rho = 0.2$

		$\eta_1 = 5$		$\eta_2 = 20$		$\eta_3 \to \infty$	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Clearing price	VA	33.76	37.22	22.11	27.51	18.77	25.34
	DPA	33.64	37.09	21.86	27.21	18.48	24.99
	UPA	32.92	36.37	20.28	25.92	16.65	23.69
Revenues	VA	1282.1	373.5	1175.8	274.1	1145.2	250.5
($\times 10^3$)	DPA	1931.8	55.8	1895.4	60.9	1884.9	62.4
	UPA	1312.4	379.5	1185.9	273.2	1149.6	249.7

d) Information structure IV: $\sigma_\varepsilon = 20$, $\rho = 0.5$

		$\eta_1 = 5$		$\eta_2 = 20$		$\eta_3 \to \infty$	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Clearing price	VA	38.94	41.83	30.59	36.31	28.20	35.22
	DPA	38.87	41.77	30.43	36.19	28.01	35.09
	UPA	38.17	41.10	28.89	35.02	26.23	33.89
Revenues	VA	1332.6	425.2	1253.7	365.7	1231.0	353.0
($\times 10^3$)	DPA	1949.7	62.1	1939.6	66.0	1936.7	67.2
	UPA	1362.2	429.7	1269.5	367.1	1242.9	354.8

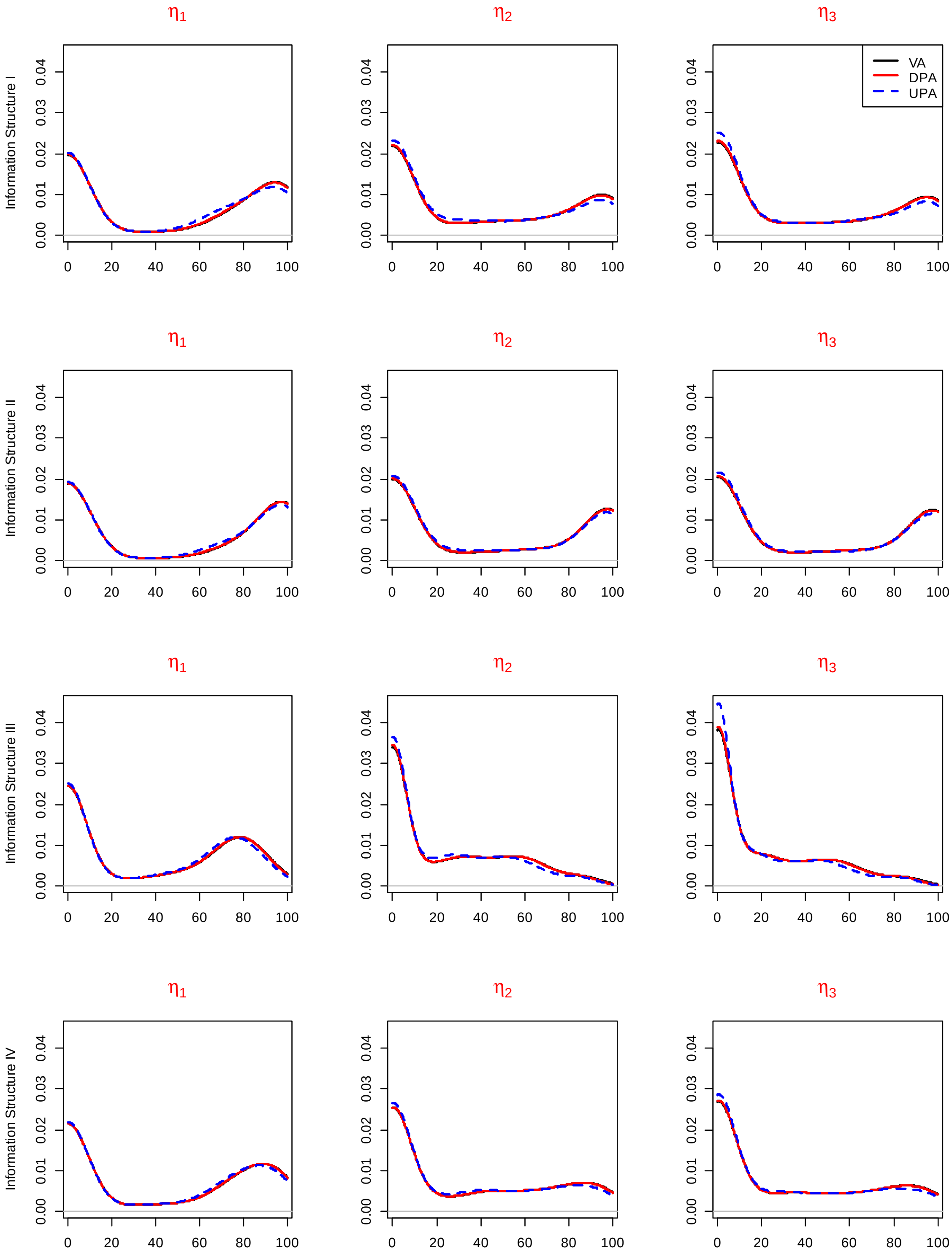

Figure 8: The auction clearing prices. The densities of the simulated values. The parameter values are presented in tables 2, 3, and 4.

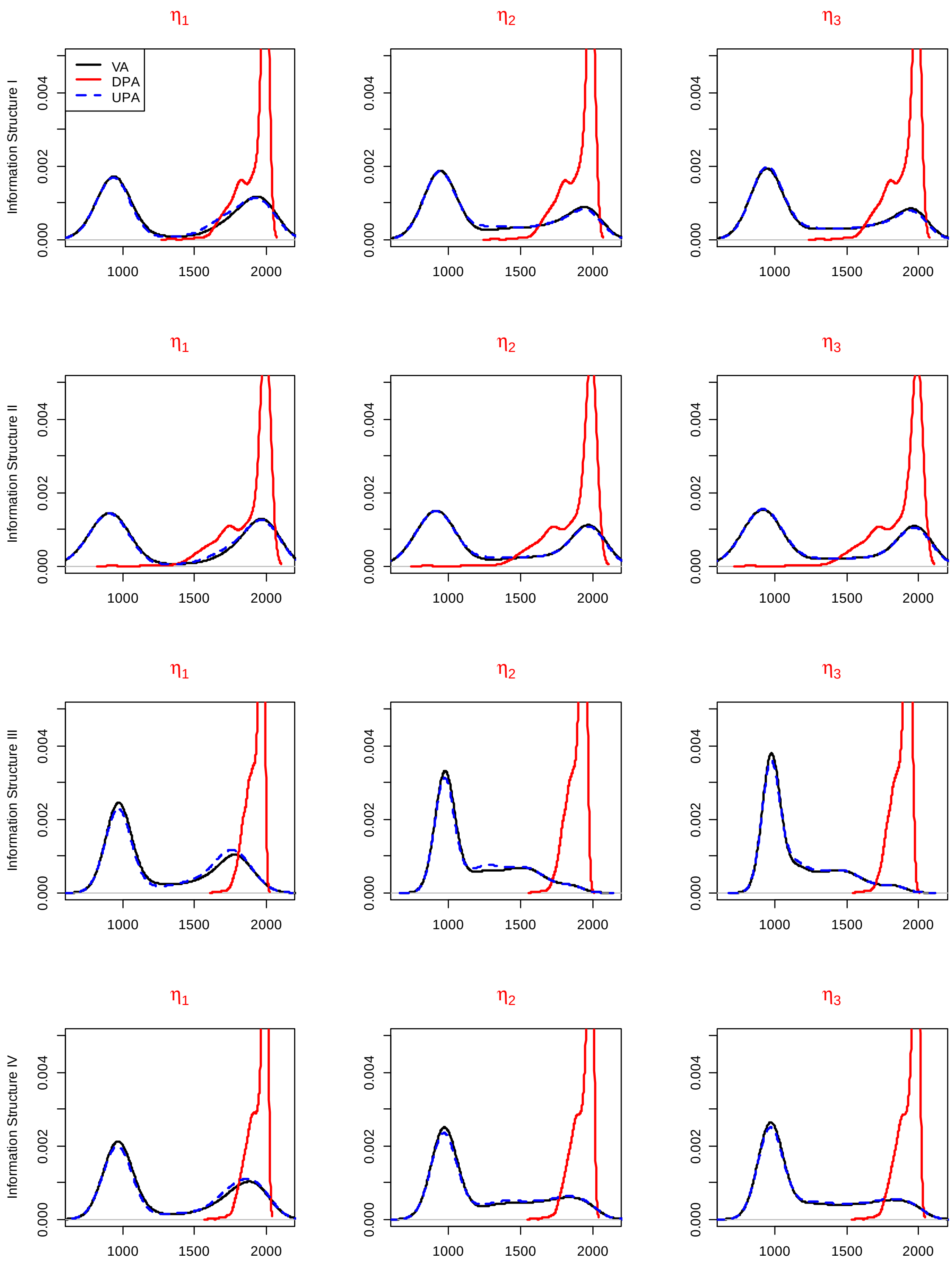

Figure 9: The Auction revenues ($\times 10^3$). The densities of the simulated values. The parameter values are presented in tables 2, 3, and 4.

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