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Unconventional government debt purchases as a supplement to conventional monetary policy*

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Abstract

In response to the Great Financial Crisis, the Federal Reserve and the Bank of England have adopted unconventional monetary policy instruments. We investigate if one of these, purchases of long-term government debt, could be a valuable addition to conventional short-term interest rate policy even if the main policy rate is not constrained by the zero lower bound. To do so we add a stylised financial sector and central bank asset purchases to an otherwise standard New Keynesian DSGE model. Asset quantities matter for interest rates through a preferred habitat channel. If conventional and unconventional monetary policy instruments are coordinated appropriately then the central bank is better able to stabilise both output and inflation.

Keywords: Quantitative Easing, Large-Scale Asset Purchases, Preferred Habitat, Optimal Monetary Policy
JEL-Classification: E40, E43, E52, E58

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1 Introduction

The Great Financial Crisis has seen the emergence of monetary policy instruments that are often described as "unconventional". The events of 2007-2008 forced monetary policy authorities to adopt new tools, even though they had little previous experience with them and there was considerable uncertainty about their likely impact. The general belief was that unconventional policy was an emergency response that would be phased out once the crisis was over. However, if it is designed carefully it may help a central bank reach its objectives even in non-crisis times.

This paper investigates whether the unconventional policy of central banks purchasing long-term government debt could be useful even after the Great Financial Crisis has passed. We obtain our results in a New Keynesian DSGE model with a stylised financial sector and a Taylor-type policy rule for central bank asset purchases. Asset quantities matter as a result of a friction in the interaction between households and financial institutions. Households can transfer income between periods only with the help of banks, who invest the deposits they receive into government bonds of different maturities because they perceive savers as being heterogeneous with respect to their preferred investment horizons. Central bank purchases of long-term government bonds reduce the supply of long-term debt available to the private sector, which increases the marginal willingness of banks to pay for it. This reduces yields on long-term debt, discourages saving, and hence increases output and inflation. When the policy parameters are chosen optimally, a combination of conventional and unconventional policies leads to significantly lower losses compared to when the central bank uses only conventional policies.

Central bank purchases of government bonds in our model have an effect through a "preferred habitat" channel of the type identified by Modigliano and Sutch (1966), and later developed by Vayanos and Vila (2009). The idea is that investors see government bonds of different maturities as imperfect substitutes and so are willing to pay a premium on bonds of their preferred maturity. The quantities of assets available then matter for prices and returns; if the central bank purchases government debt of a particular maturity then supply of that asset to the private sector is reduced, its price rises and its return falls. The preferred habitat channel operates in our model as banks hold government debt of different maturities in response to a perception that savers have heterogeneous investment horizons. The closest models to ours are Andrés et al. (2004) and Chen et al. (2011), although they consider different mechanisms and have less emphasis on policy implications.

The main focus of the current unconventional monetary policy literature is on credit policy, i.e. central bank purchases of private financial assets.[1] An important exception is Eggertsson and Woodford (2003), who examine central bank purchases of government debt. They argue that unconventional monetary policy works by acting as a signal for the future path of short-term interest rates, so will be especially useful when the main policy rate is constrained by the zero

[1] See Cúrdia and Woodford (2011), Del Negro et al. (2011), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Gertler et al. (2011) and Kiyotaki and Moore (2012).

lower bound. In their model, though, the risk-premium component of long-term interest rates is unaffected by any reallocation of assets between the central bank and the private sector. This is because risks are ultimately born by the private sector, even if government debt is purchased by the central bank. If the central bank makes losses on government debt then government revenue falls and taxes on the private sector have to rise to satisfy the government budget constraint.[2] This view is not supported by the empirical evidence in Bernanke et al. (2004), D'Amico and King (2010), D'Amico et al. (2011), Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011) and Neely (2012). Instead, these studies find strong evidence that central bank purchases of government debt have small but significant effects via the term premium component of long-term interest rates. The evidence supporting a preferred habitat mechanism comes from the "scarcity channel" in D'Amico et al. (2011) and the "safety channel" in Krishnamurthy and Vissing-Jorgensen (2011).

The paper is organised as follows. Section 2 introduces the model. Section 3 discusses the transmission mechanism of central bank asset purchases, and characterises the optimal mix of conventional and unconventional monetary policies. A final section concludes.

2 Model

The model economy consists of households, monopolistically competitive firms, banks, a treasury and a central bank. There is price stickiness, wages are assumed to be fully flexible, and firms use labour as the only input in the production of consumption goods.[3]

2.1 Households

The preferences of the representative household are given by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\chi_t^C \frac{C_t^{1-\delta}}{1-\delta} - \chi_t^L \frac{L_t^{1+\psi}}{1+\psi} \right)$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ is a CES consumption index composed to minimise cost and L_t is the time devoted to market employment. χ_t^C and χ_t^L are exogenous preference shock processes that evolve according to

$$\ln(\chi_t^C) = \rho_C \ln(\chi_{t-1}^C) + \varepsilon_t^C \quad \text{with } \varepsilon_t^C \sim N(0, \sigma_C^2) \tag{1}$$

$$\ln(\chi_t^L) = \rho_L \ln(\chi_{t-1}^L) + \varepsilon_t^L \quad \text{with } \varepsilon_t^L \sim N(0, \sigma_L^2) \tag{2}$$

[2]This intuition is given on page 5 of Cúrdia and Woodford (2010).

[3]The model incorporates elements from Benigno and Woodford (2005), Gali (2008) and Woodford (2003).

The household maximises expected utility subject to the budget constraint:

$$P_t C_t + T_t + P_t^S S_{t,t+1} = S_{t-1,t} + W_t L_t \tag{3}$$

where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ is the price of the composite consumption good and T_t is a lump-sum tax paid to the government. $S_{t,t+1}$ is the quantity of a savings device purchased from perfectly competitive banks at unit price $P_t^S < 1$ in period t. One unit of the savings device yields a certain payoff of one in period $t+1$, so the household can secure a payment of $S_{t,t+1}$ in period $t+1$ by saving $P_t^S S_{t,t+1}$ in period t. W_t is the nominal market wage.[4]

The first-order conditions of the household's optimisation problem are

$$1 = \beta E_t \left[\frac{\chi_{t+1}^C}{\chi_t^C} \left(\frac{C_{t+1}}{C_t} \right)^{-\delta} \frac{1}{\Pi_{t+1}} \right] \frac{1}{P_t^S} \tag{4}$$

$$\frac{W_t}{P_t} = \frac{\chi_t^L}{\chi_t^C} L_t^{\psi} C_t^{\delta} \tag{5}$$

with $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$. Equation (4) is the intertemporal Euler equation that characterises the optimal consumption-savings decision. Equation (5) is the intratemporal condition describing optimality between labour supply and consumption.

2.2 Firms

There is a continuum of monopolistically competitive firms indexed by $i \in [0,1]$. Firm i's production function is $Y_t(i) = A_t L_t(i)^{\frac{1}{\phi}}$, where $L_t(i)$ is labour employed and A_t is an exogenous technology shock process that evolves according to

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \varepsilon_t^A \quad \text{with } |\rho_A| < 1 \text{ and } \varepsilon_t^A \sim N(0, \sigma_A^2) \tag{6}$$

As in Calvo (1983), only a fraction $1-\alpha$ of firms can adjust the price of their respective good in any given period. Let $P_t^*(i)$ be the price chosen by a firm that is able to reset its price in period t. The evolution of the aggregate price level is then described by

$$P_t = \left[(1-\alpha) P_t^*(i)^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

All firms that can change their price in period t choose $P_t(i)$ to maximise the expected discounted stream of their future profits

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left[P_t(i) Y_T(i) - W_T L_T(i) \right]$$

[4] All prices are measured in units of the numeraire good "money", which is outside the model.

subject to the relevant demand constraints. $M_{t,T} \equiv \beta^{T-t}\frac{\chi_T^C}{\chi_t^C}\frac{C_T^{-\delta}}{C_t^{-\delta}}\frac{P_t}{P_T}$ is the firm's stochastic discount factor, derived from the consumption Euler equation (4).

The first-order condition for price setting and the assumed price adjustment process imply that equilibrium inflation is given by[5]

$$\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{\theta(\phi-1)+1}} \tag{7}$$

where

$$F_t = \chi_t^C C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta-1} F_{t+1} \tag{8}$$

$$K_t = \frac{\theta\phi}{\theta-1}\chi_t^L L_t^{\psi}\left(\frac{Y_t}{A_t}\right)^{\phi}\mu_t + \alpha\beta E_t \Pi_{t+1}^{\theta\phi} K_{t+1} \tag{9}$$

F_t and K_t are auxiliary variables and μ_t is a mark-up shock that follows an AR(1) process:

$$\ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu \quad \text{with } \varepsilon_t^\mu \sim N(0,\sigma_\mu^2) \tag{10}$$

Firms do not pay dividends. An assumption commonly made to explain this simplification is that all profits are taken up by investments in an exogenous production factor, for example additions to physical capital needed to offset depreciation.

2.3 Banks

The role of the representative bank is to determine the maturity composition of the aggregate savings device offered to households. Perfect competition in the banking sector ensures that all bank revenues are returned to households, but it is the bank that decides how deposits are invested into short-term and long-term government bonds. We assume when doing so that banks perceive households as heterogeneous with regards to their desired investment horizons, an assumption consistent with the preferred habitat literature where investors value characteristics of assets other than just their payoff. In the context of our model, this means that investors are perceived as having a preference for assets with maturities that match their preferred investment horizon. Assets of other maturities are viewed as only imperfect substitutes. As a result, the price of an asset with a given maturity is influenced by supply and demand effects local to that maturity, see Vayanos and Vila (2009), and the central bank can use purchases of government bonds to influence prices and yields at different maturities.[6]

Formally, in every period t the representative bank collects nominal deposits $P_t^S S_{t,t+1}$ from households, which it uses to purchase $B_{t,t+1}$ units of a short-term government bond and $Q_{t,t+\tau}$

[5]See section A.1 in the appendix for the derivation.

[6]The assumption that households are *perceived* as having heterogeneous preferences over different investment horizons is made so the model remains within a standard representative agent framework. If households differ with respect to their *actual* preferences then a full heterogeneous agent model would be needed.

units of a long-term government bond. The budget constraint of the representative bank is

$$P_t^S S_{t,t+1} = P_t^B B_{t,t+1} + P_t^Q Q_{t,t+\tau}$$

A unit of the short-term bond can be purchased at price P_t^B in t and has a payoff of one in the next period $t+1$. The long-term bond is traded at the price P_t^Q in t; a unit of this bond yields a payoff of $\frac{1}{\tau}$ in every period between $t+1$ and $t+\tau$. It is assumed that both types of bond have to be held until maturity.[7]

The bank constructs the savings device S offered to households on the basis of a function V that reflects its perception of the different investment horizons preferred by heterogenous households. The allocation problem of the representative bank is

$$\max_{B_{t,t+1}, Q_{t,t+\tau}} \quad V\left(\frac{B_{t,t+1}}{P_t}, \frac{Q_{t,t+\tau}}{P_t}\right)$$

subject to the budget constraint.[8] The asset demand schedules that solve the allocation problem depend on the functional form chosen for V. We prefer not to impose restrictive properties on the demand curves, so borrow from the empirical literature on demand estimation and adopt the flexible functional form of the Generalised Translog (GTL) model introduced by Pollak and Wales (1980). Rather than specifying V directly, the GTL model specifies the indirect utility function $V^* \equiv V(\frac{B^*}{P}, \frac{Q^*}{P})$:

$$\begin{aligned}\log(V_t^*) &= a_0 + \sum_k a_1^k \log\left(\frac{P_t^k}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right) + \\ &\frac{1}{2}\sum_k \sum_l a_2^{kl} \log\left(\frac{P_t^k}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right) \log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)\end{aligned}$$

with $k, l \in \{B, Q\}$, $a_2^{kl} = a_2^{lk}$ and $s_t \equiv \frac{S_{t,t+1}}{P_t}$. The asset shares $as^B \equiv \frac{P^B B}{P^S S}$ and $as^Q \equiv \frac{P^Q Q}{P^S S} = 1 - as^B$ can be derived using the logarithmic form of Roy's identity, see Barnett and Serletis (2008):

$$as_t^k = \frac{P_t^k g^k}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right) \frac{a_1^k + \sum_l a_2^{kl} \log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)}{\sum_l a_1^l + \sum_k \sum_l a_2^{kl} \log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)}$$

To ensure that the above asset shares are the result of the bank's optimisation problem, the parameter space has to be restricted such that four "integrability conditions" hold. Under these

[7]This simplifying assumption is frequently made in the literature, see Andrés et al. (2004) for a discussion.

[8]The optimisation problem is equivalent to that of a bank in a model where V aggregates the heterogeneous preferences of households over bonds of different maturities.

conditions, quasi-homotheticity, $a_1 \equiv a_1^B$ and $a_2 \equiv a_2^{BB}$, the asset demands are:[9]

$$\frac{B_{t,t+1}}{P_t} = g^B + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^B}\left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \tag{11}$$

$$\frac{Q_{t,t+\tau}}{P_t} = g^Q + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^Q}\left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \tag{12}$$

The income of the representative bank from their holdings of long-term bonds in period t is

$$\frac{1}{\tau}\sum_{j=1}^{\tau} Q_{t-j,t+\tau-j}$$

Perfect competition in the banking sector requires that banks return all their revenues from government bonds to depositors, so the return to a household depositing $P_{t-1}^S S_{t-1,t}$ is

$$S_{t-1,t} = B_{t-1,t} + \frac{1}{\tau}\sum_{j=1}^{\tau} Q_{t-j,t+\tau-j} \tag{13}$$

The prices P_t^B and P_t^Q of the short and long-term government bonds are known with certainty in period t but not in advance. The gross nominal interest rate paid on the one-period bond is

$$1 + i_t = \frac{1}{P_t^B} \tag{14}$$

For comparison purposes, it is possible to calculate an interest rate associated with the τ-period bond traded in t. It is implicitly defined by

$$\begin{aligned} P_t^Q &= \frac{\frac{1}{\tau}}{1+i_t^Q} + \frac{\frac{1}{\tau}}{\left(1+i_t^Q\right)^2} + \frac{\frac{1}{\tau}}{\left(1+i_t^Q\right)^3} + \ldots + \frac{\frac{1}{\tau}}{\left(1+i_t^Q\right)^\tau} \\ &= \frac{1}{\tau}\frac{1}{1+i_t^Q}\frac{1-\left(\frac{1}{1+i_t^Q}\right)^\tau}{1-\frac{1}{1+i_t^Q}} \end{aligned} \tag{15}$$

where i_t^Q is the per-period interest rate that equates the unit price P_t^Q with the value of the discounted payoff stream from a unit investment in the long-term bond.

An important feature of the interaction between banks and households is that the closed-form asset demand schedules are simple and intuitive, in contrast with previous contributions to the literature on unconventional monetary policy. Both demand curves are downward sloping for small enough values of a_2. Low asset quantities are associated with high asset prices, which allows unconventional monetary policy instruments to work through a scarcity channel. For example,

[9] See Section A.2 in the appendix for the details.

central bank purchases of long-term bonds reduce the quantity of those bonds available to the banks and thus increase their price.

2.4 Policy

The government's economic policy is implemented by the actions of a treasury and a central bank. We model the operations of the treasury in a simplified way to maintain the focus of our paper on monetary policy.

The treasury issues both short and long-term government debt. Short-term bonds are supplied in a quantity consistent with the central bank's setting of the short-term nominal interest rate. The quantity of long-term bonds issued in period t is $\bar{Q}_{t,t+\tau}$, to be purchased by the banks and the central bank. Long-term bonds can only be purchased from the treasury in the period they are issued and then have to be held until maturity, i.e. there is no secondary market for long-term government debt. The supply of the long-term bond is determined by the simple rule

$$\frac{\bar{Q}_{t,t+\tau}}{P_t} = fY \tag{16}$$

where $f > 0$ and Y is steady-state output.

The treasury also uses lump-sum taxes to finance government spending. The government consumption good G_t has exactly the same same CES aggregator of firms' production goods as the composite household consumption good. It is given exogenously by

$$G_t = \rho_G G_{t-1} + \varepsilon_t^G \quad \text{with } \varepsilon_t^G \sim N(0, \sigma_G^2) \tag{17}$$

Government consumption is financed by lump-sum taxes $T_t = P_t G_t$.

The central bank sets the short-term nominal interest rate according to a Taylor rule:

$$\frac{1+i_t}{1+i} = \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi} \left(\frac{Y_t}{Y}\right)^{\gamma_Y} \nu_t \tag{18}$$

Variables without time subscript denote steady-state values and ν_t is an interest rate shock term that evolves according to

$$\ln(\nu_t) = \rho_\nu \ln(\nu_{t-1}) + \varepsilon_t^\nu \quad \text{with } \varepsilon_t^\nu \sim N(0, \sigma_\nu^2) \tag{19}$$

Asset purchases are carried out according to a Taylor-type rule:

$$\frac{\bar{Q}_{t,t+\tau} - Q_{t,t+\tau}^{CB}}{\bar{Q}_{t,t+\tau}} = \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi^{QE}} \left(\frac{Y_t}{Y}\right)^{\gamma_Y^{QE}} \xi_t \tag{20}$$

where $Q_{t,t+\tau}^{CB}$ is the quantity of long-term bonds purchased by the central bank in period t. The logarithm of the shock ξ_t follows an AR(1) process.

$$\ln(\xi_t) = \rho_\xi \ln(\xi_{t-1}) + \varepsilon_t^\xi \quad \text{with } \varepsilon_t^\xi \sim N(0, \sigma_\xi^2) \tag{21}$$

The central bank is a net buyer of long-term bonds of the type issued in t if $Q_{t,t+\tau}^{CB} > 0$ and is a net supplier of those to the market if $Q_{t,t+\tau}^{CB} < 0$.[10] For example, if inflation and output are below their respective steady-state values then with $\gamma_\Pi^{QE} > 0$, $\gamma_Y^{QE} > 0$ and $\xi_t = \xi = 1$ the central bank stimulates the economy by purchasing long-term bonds. Conversely, if inflation and output lie above their steady-state values then asset sales are used to depress prices and productive activity.

2.5 Market Clearing

The model is completed by conditions for clearing in bond, goods and labour markets.

Demand and supply in the market for long-term bonds are equated for

$$\bar{Q}_{t,t+\tau} = Q_{t,t+\tau} + Q_{t,t+\tau}^{CB} \tag{22}$$

The market for short-term bonds clears by assumption. In goods markets the aggregate resource constraint is

$$Y_t = C_t + G_t \tag{23}$$

Labour market clearing requires that hours supplied by the representative household L_t are equal to aggregate hours demanded by the firms, $\int_0^1 L_t(i)di$, implying that aggregate production is

$$Y_t = A_t \left(\frac{L_t}{D_t} \right)^{\frac{1}{\phi}} \tag{24}$$

where $D_t \equiv \int_0^1 \left[\frac{P_t(i)}{P_t} \right]^{-\theta\phi} di$ is a measure of price dispersion. The dynamics of the price dispersion term are given by[11]

$$D_t = (1-\alpha) \left(\frac{1 - \alpha \Pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{\theta\phi}{\theta-1}} + \alpha \Pi_t^{\theta\phi} D_{t-1} \tag{25}$$

Price dispersion is a source of inefficiency in the model. It acts in addition to the mark-up distortions created by the market power of firms.

[10] Since long-term bonds are only traded in the period in which they are issued, unconventional policy in our model should not be viewed as changing the existing stock of long-term bonds held by the central bank. It should instead be seen as affecting the supply of an asset available for purchase at a particular point in time.

[11] See Section A.3 in the appendix for derivations.

2.6 Model Summary and Calibration

Equations (1)-(25) are sufficient to describe the behaviour of the endogenous variables in rational expectations equilibrium. Real variables are stationary, but for $\Pi > 1$ there is a positive trend in all nominal variables. Section A.4 of the appendix shows how we take a first order numerical approximation of a trend-stationary version of the model and simulate it around the steady state described in Section A.5 of the Appendix. In what follows we define $s_t \equiv \frac{S_{t,t+1}}{P_t}$, $b_t \equiv \frac{B_{t,t+1}}{P_t}$ and $q_t \equiv \frac{Q_{t,t+\tau}}{P_t}$.

Parameter	Value	Description
β	0.99	Households discount factor
δ	2	Inverse elasticity of intertemporal substitution in consumption
ψ	0.5	Inverse Frisch elasticity of labour supply
θ	6	Intratemporal elasticity of substitution between consumption goods
ϕ	1.1	Inverse of returns to scale in production
α	0.85	Degree of price rigidity
τ	20	Horizon of long-term bond
a_1	0.95	Asset demand
a_2	0	Asset demand
g^B	20	Asset demand (subsistence level of B)
g^Q	0.625	Asset demand (subsistence level of Q)
Π	1.005	Steady-state inflation
f	0.7587	Parameter in long-term bond supply rule
ρ_ν	0.1	Persistence of shock to Taylor rule
ρ_ξ	0.1	Persistence of shock to asset purchase rule
ρ_G	0.1	Persistence of government spending
ρ_C	0.1	Persistence of consumption preference shock
ρ_L	0.7	Persistence of labour supply preference shock
ρ_A	0.7	Persistence of technology shock
ρ_μ	0.95	Persistence of price mark-up shock
σ_ν	0.0025	Standard deviation of shock to Taylor rule
σ_ξ	0.0025	Standard deviation of shock to asset purchase rule
σ_G	0.0025	Standard deviation of government spending shock
σ_C	0.0025	Standard deviation of consumption preference shock
σ_L	0.0025	Standard deviation of labour supply preference shock
σ_A	0.01	Standard deviation of technology shock
σ_μ	0.012	Standard deviation of price mark-up shock

Table 1: Calibration

Table 1 gives an overview of the model calibration. The calibration of the parameters in the household and firm problems are standard and in line with Gali (2008) and Smets and Wouters (2003, 2007). The households discount factor is 0.99. The inverse of the intertemporal substitution elasticities in consumption and labour supply are 2 and 0.5 respectively. The elasticity of consumption is calibrated at a relatively low value to ensure that the effects of long-term asset purchases are not over-stated. The intratemporal elasticity of substitution between consumption goods equals 6, which implies a steady-state mark-up of 20 percent. The production function exhibits decreasing returns to scale. Price rigidity is calibrated as being relatively high because wages are fully flexible in the model. Steady-state inflation corresponds to an annual inflation rate of close to 2 percent. τ is 20 so the long-term bond has a maturity of 5 years.

The asset demand parameters and f are calibrated so the quarterly interest rate on short and long-term debt is 1.3 percent in the non-stochastic steady state and long-term debt makes up close to one quarter of all outstanding debt.[12] A quarterly rate of 1.3 percent translates to an annual rate of 5.3 percent, which is representative of bond markets before the outbreak of the Global Financial Crisis.

The calibration of the shock processes emphasises the significance of productivity and mark-up shocks and generates a plausible degree of volatility. Monetary policy and preference shocks make a smaller but equal contribution to overall volatility.

3 Conventional and Unconventional Monetary Policy

The possibility of purchasing long-term government debt presents the central bank with an additional tool to help stabilise the economy. To understand the implications of this it is useful to begin with a discussion of how conventional and unconventional monetary policy actions are transmitted in the model. It is then possible to understand the optimal coordination of conventional and unconventional policies. The final part of this section addresses the implications of policy coordination for equilibrium determinacy.

3.1 Transmission Mechanisms

The response of the economy to a contractionary shock in the Taylor rule for the short-term nominal interest (18) rate is depicted in Figure 1; the response to an expansionary shock in the rule for long-term bond purchases is shown in Figure 2. Both figures are obtained under the calibration $\gamma_{\Pi} = 1.01$, $\gamma_Y = 0.3$ and $\gamma_{\Pi}^{QE} = 0$, $\gamma_Y^{QE} = 60$ for conventional and unconventional monetary policies. A value of 60 for γ_Y^{QE} implies in steady state that a one percent decrease in output leads to the central bank buying approximately 45 percent of the new long-term bonds issued in that period. This amounts to less than five percent of all outstanding long-term debt.

[12] In the absence of exogenous shocks, the present discounted value of outstanding payment obligations from long-term debt at the beginning of any given period is $PW^Q = (1/\tau)\tau q + (1/\tau)\frac{(\tau-1)q}{1+i} + (1/\tau)\frac{(\tau-2)q}{(1+i)^2} + \ldots + (1/\tau)\frac{q}{(1+i)^{\tau-1}}$. The share of outstanding long-term debt in our calibrated economy, calculated as $\frac{PW^Q}{b+PQ^Q}$, is then 24.

The shock to the Taylor rule immediately increases the short-term nominal interest rate and equivalently reduces the price of the short-term bond. Demand for the short-term bond therefore rises. The lowering of P^B also reduces the price of the composite savings device available to households, leading to an increase in savings $P^S s$ and a fall in both output and consumption. The fall in output is accompanied by inflation below its steady-state level.

The effects on the market for long-term bonds work through two channels. An increase in savings shifts out the demand curve for the long-term bond, which puts upward pressure on its price P^Q. In addition, output and inflation fall below their respective steady-state levels so the central bank uses unconventional monetary policy to purchase long-term government bonds. This reduces the quantity of long-term bonds available to the private sector, further increasing their price.

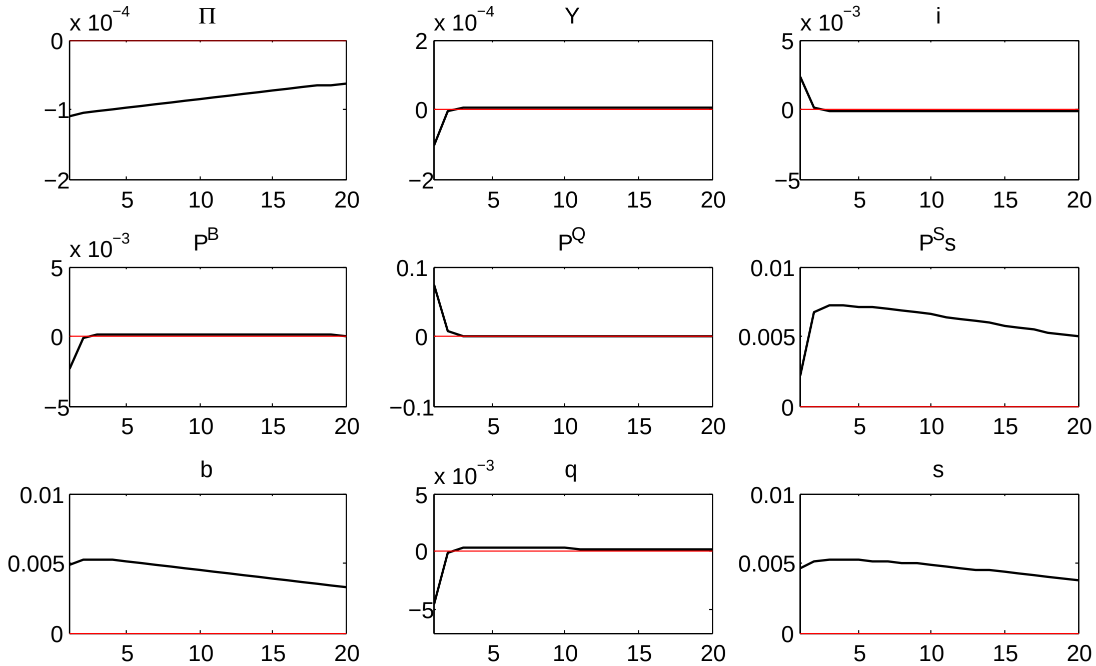

Figure 1: Response to a short-term nominal interest rate shock

Prices quickly return to their steady-state values once the shock to the short-term nominal interest rate has passed. The temporary boost to the yield of the short-term bond has a small positive effect on the wealth of households, allowing them to maintain higher levels of savings, consumption and leisure for a number of periods following the shock. A more elaborate description of public finances would eliminate this effect, since taxes should rise endogenously to finance the higher interest payments on short-run debt. However, the wealth effect on consumption is very small so our assumption that taxes are exogenous does not significantly weaken our optimal policy exercises.

Figure 2 shows the response of the economy to an expansionary shock in the central bank's long-term bond purchase rule (20). The unanticipated rise in central bank purchases of long-term bonds q^{CB} reduces the supply of this asset to the private sector and pushes up its price. The increase in P^Q similarly increases the price of the composite savings device P^S, causing households to save less, consume more and supply more labour to firms. Output then increases and prices grow at a faster rate. The rise in the price P^Q of the long-term bond is associated with a decline in the long-term interest rate, as can be seen from equation (15) defining i_t^Q. This corresponds to a "flattening of the yield curve" effect generally attributed to purchases of long-term government securities.

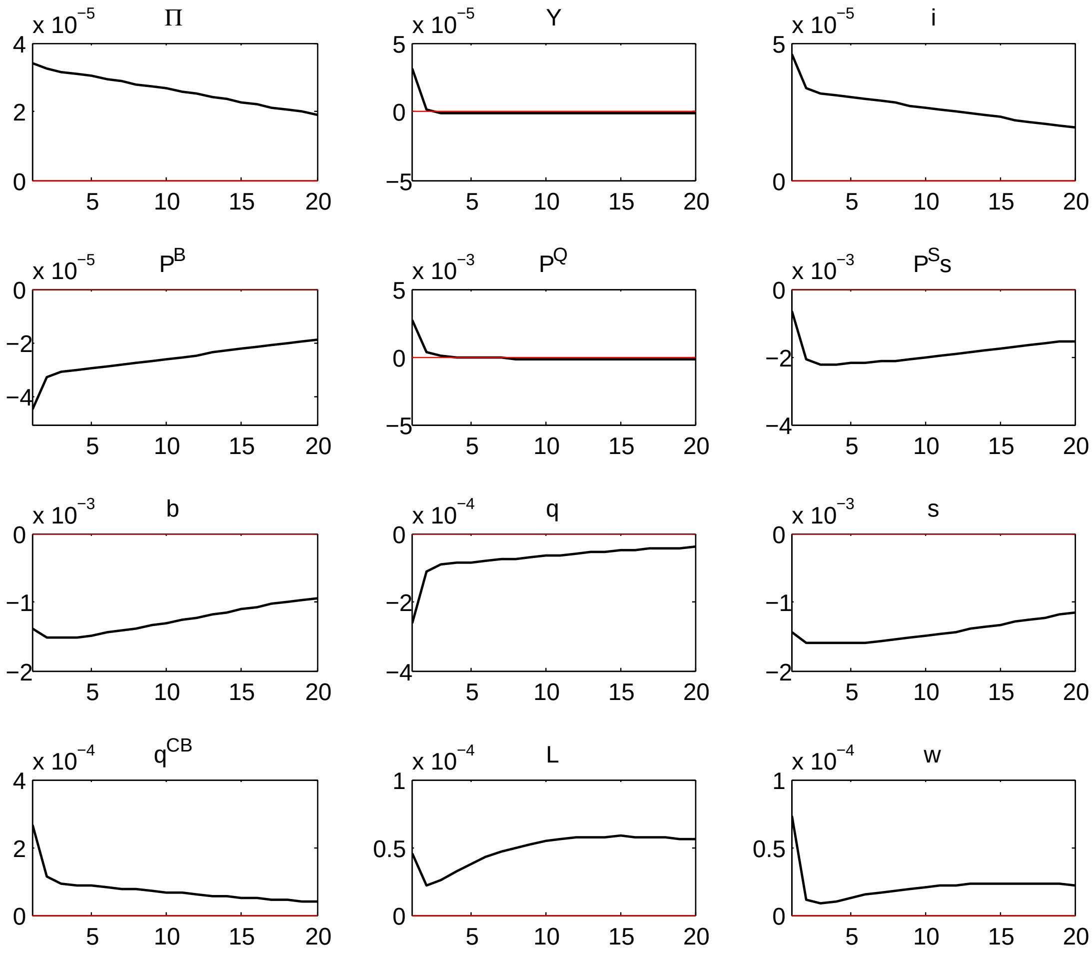

Figure 2: Response to a shock to purchases of the long-term bond

The decline in savings shifts the demand curves for both types of bonds inwards, leading to a fall in the quantity of the short-term bond held by the private sector. The fall in holdings is though limited because the central bank subsequently follows its Taylor rule and increases the short-term nominal interest rate, thereby reducing the price of the short-term bond and restoring some of its demand.

Savings $P^S s$ only gradually return to their steady-state level after the unexpected purchases of long-term bonds, where again we abstract from the small effect that changes in wealth would imply for taxes. Labour supply similarly remains elevated for a number of periods, which helps bring savings back to their steady-state level and ameliorates the potential impact of inflation on price dispersion.

The impulse response functions suggest that both conventional and unconventional monetary policies may be effective at stabilising the economy. In what follows, the focus is on the benefits of allowing the central bank to purchase long-run government bonds as an unconventional policy that acts as a supplement to conventional short-term interest rate policy.

3.2 Optimised Policy Rules

Let the central bank loss function be

$$\Omega_0 = E_0 \left(1 - \beta^{CB}\right) \sum_{t=0}^{\infty} \left(\beta^{CB}\right)^t \left[\omega_\Pi \left(\Pi_t - \Pi\right)^2 + \omega_Y \left(Y_t - Y\right)^2 + \omega_i \left(i_t - i\right)^2 + \omega_Q \left(i_t^Q - i^Q\right)^2\right]$$

where β^{CB} is a discount factor and ω_Π, ω_Y, ω_i and ω_Q are weights. The losses depend on the deviations of inflation, output and interest rates from their steady-state values. This specification allows for the volatility of both short-term and long-term interest rates to enter the central bank's considerations, although in the baseline case we set ω_i and ω_Q to zero. The discount factor of the central bank is assumed arbitrarily close to one, in which case minimising the loss function is equivalent to minimising a weighted sum of the variances of inflation, output and the interest rates.

The central bank sets Taylor rule parameters γ_Π and γ_Y together with asset purchase rule parameters γ_Π^{QE} and γ_Y^{QE}, subject to the constraints imposed by the model and assumed feasibility constraints $\gamma_\Pi, \gamma_Y \in [0, 6]$ and $\gamma_\Pi^{QE}, \gamma_Y^{QE} \in [0, 75]$. We allow for a high upper bound on the set of feasible Taylor rule parameters so that the benefits of unconventional monetary model do not arise solely when the feasibility constraints for γ_Π and γ_Y are binding. A lower upper bound on the set of Taylor rule parameters would likely make unconventional policy even more effective, as the central bank would then find it more difficult to stabilise the economy through conventional policy.

Table 2 shows optimised policy rules for different weights on inflation and output stabilisation in the central bank's loss function, with ω_i and ω_Q set to zero so the central bank is not concerned about stabilising interest rates. For each set of weights, the first row presents results when the central bank uses only conventional monetary policy. In this baseline case, coefficients on the asset purchasing rule are set to zero and only the coefficients on the interest rate rule are optimised. For the next case, the central bank fixes the coefficients of the interest rate rule at their baseline values, but optimises over the coefficients in the asset purchasing rule. This experiment shows the improvements attainable if the central bank starts purchasing long-term government bonds without internalising the potential effect this has on optimal interest rate policy. In the final case, the central bank acknowledges the complementarity of conventional and unconventional policies by jointly optimising over the parameters in the interest rate and asset purchasing rules.

The benefits of unconventional monetary policy are small if the central bank's interest rate rule is not adjusted to account for purchases of long-term government bonds. However, when both policies are jointly optimised then the ability of the central bank to stabilise inflation and output is greatly improved. The use of unconventional monetary policy reduces losses by more than seven percent compared to the baseline case, provided conventional monetary policy is adjusted accordingly.

Loss Function (ω_Π, ω_Y, ω_i, ω_Q)	Interest Rate Rule (γ_Π, γ_Y)	Asset Purchase Rule (γ_Π^{QE}, γ_Y^{QE})	Loss	Gain (in %)
700, 300, 0, 0	1.70, 5.84	0(f), 0(f)	10.4449	-
	1.70(f), 5.84(f)	0.03, 1.15	10.4233	0.21
	1.80, 0.00	0.00, 16.60	9.6864	7.26
800, 200, 0, 0	1.77, 5.73	0(f), 0(f)	11.7051	-
	1.77(f), 5.73(f)	0.05, 1.13	11.6808	0.21
	1.87, 0.00	0.00, 16.29	10.8266	7.49
900, 100, 0, 0	1.83, 5.64	0(f), 0(f)	12.9420	-
	1.83(f), 5.64(f)	0.08, 1.12	12.9152	0.21
	1.92, 0.00	0.00, 16.05	11.9446	7.71

(f): fixed, i.e. not chosen optimally

Table 2: Optimised policy rules without interest rate stabilisation

The optimal policy mix is dichotomised, assigning conventional short-term interest rate policy to react to inflation and unconventional asset purchasing policy to react to output. An important prerequisite for this result is that conventional and unconventional policies differ in their impact on inflation and output, so interventions on one margin are not exactly offset at another margin. To see why unconventional policy is so effective in our model, suppose that a shock increases inflation and decreases output. If only conventional monetary policy is available then the central bank has a difficult policy problem. It can increase the short-term nominal interest rate to reduce inflation at the cost of further depressing output, or it can reduce the short-term nominal interest rate to stimulate output at the cost of yet higher inflation. If conventional and unconventional policies differ in their impact then unconventional policy can be used to offset the unintended consequences of conventional policy. In our example, the central bank can coordinate policies by increasing its purchases of the long-term government bond at the same time as it increases the short-term nominal interest rate. The rise in the short-term nominal interest rate counteracts inflation, whilst the extra purchases of government debt stimulate output.

Loss Function (ω_Π, ω_Y, ω_i, ω_Q)	Interest Rate Rule (γ_Π, γ_Y)	Asset Purchase Rule (γ_Π^{QE}, γ_Y^{QE})	Loss	Gain (in %)
800, 150, 50, 0	1.84, 6.00	0(f), 0(f)	12.7319	-
	1.84(f), 6.00(f)	3.35, 10.58	12.2471	3.81
	0.22, 0.01	4.40, 15.34	11.2214	11.86
800, 100, 50, 50	0.06, 4.72	0(f), 0(f)	30.7101	-
	0.06(f), 4.72(f)	0.00, 7.96	27.7127	9.76
	1.25, 0.45	0.33, 11.57	13.3974	56.37

(f): fixed, i.e. not chosen optimally

Table 3: Optimised policy rules with interest rate stabilisation

Table 3 suggests that unconventional monetary policy is even more effective when the central bank is also concerned about the volatility of interest rates. If only variability in the short-term interest rate is costly then the unconventional policy of purchasing long-term government bonds almost completely replaces conventional short-term interest rate policy. The optimal short-term interest rate hardly moves at all, with the asset purchase rule alone sufficient to capture a significant proportion of the available welfare gains. If short-term and long-term interest rate volatilities are both costly then unconventional policy is extremely potent at controlling losses. This is because purchases of long-term government bonds allow the central bank to directly affect the long-term interest rate.

3.3 Determinacy

The coefficients in the conventional and unconventional policy rules play an important role in ensuring determinacy of the rational expectations equilibrium in our model, a desirable feature of policy emphasised by Bernanke and Woodford (1997). Fortunately, allowing the central bank to operate in the long-term government bond market makes it easier for it to use the short-term interest rate to guarantee equilibrium determinacy. It is therefore less demanding for a central bank to achieve determinacy once it suitably coordinates conventional and unconventional monetary policies.

Figure 3 shows the determinacy (white) and indeterminacy (shaded) regions as functions of the parameters in the short-term interest rate rule, under varying intensities of central bank intervention in the long-term bond market. Moving from left to right, even small-scale purchases of long-term bonds lead to a significant expansion of the determinacy region. A central bank using unconventional monetary policy is therefore significantly less constrained in its choices for conventional monetary policy. In the most extreme case, even a constant interest rate rule is consistent with determinate equilibrium if purchases of long-term government bonds are conducted appropriately.

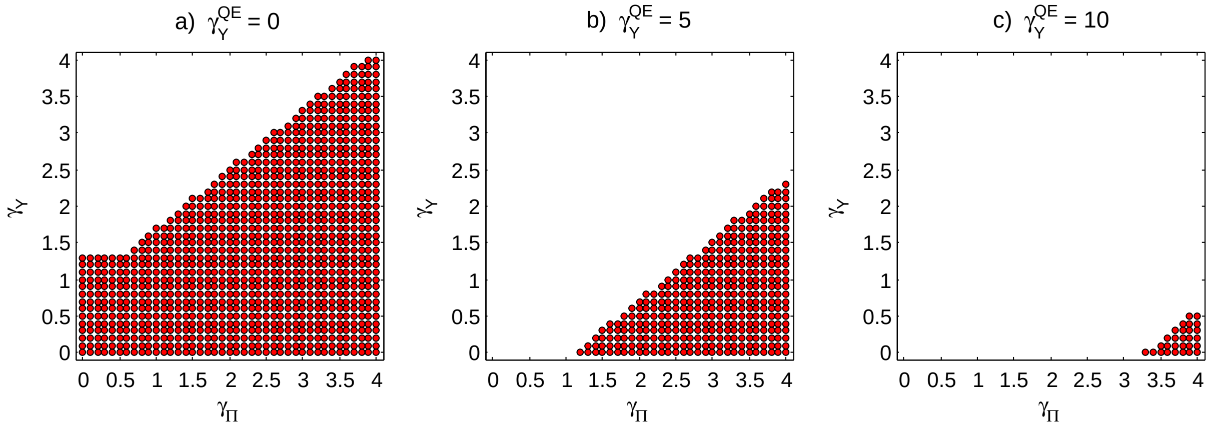

Figure 3: Determinacy (white) and indeterminacy (shaded) regions for different values of γ_Y^{QE} ($\gamma_\Pi^{QE} = 0$ in all three plots).

4 Conclusion

Our results call for central banks to consider purchasing long-term government bonds, even when the economy is not in financial crisis. Unconventional asset purchases of this type are a valuable addition to the tool kit of a central bank trying to stabilise inflation and output, whatever the state of the economy and not just when the short-term nominal interest rate is constrained by the zero lower bound. In order to reap the full benefit, though, it is important to coordinate unconventional and conventional monetary policy. In our model, this means that the short-term interest rate should respond to inflation while the central bank's purchases of long-term debt should respond to output. Unconventional monetary policy plays an even more important role if the central bank is additionally concerned about interest rate volatility.

The 'Maturity Extension Program and Reinvestment Policy" announced by the Federal Reserve in September 2011 is an example of the type of unconventional monetary policy measure we advocate.[13] Under this programme, the Federal Reserve lengthens the average maturity of its government bond portfolio by selling short-term treasuries and re-investing the proceeds in long-term treasuries. Our results suggest that such operations may be increasingly important

[13]In reference to a comparable programme from the 1960s, this policy is sometimes referred to as "Operation Twist" or "Operation Twist Again."

even when financial crisis has passed.

The benefit of unconventional monetary policy in our model is indicative of more general gains available from the coordination of monetary policy and the debt management of fiscal authorities. In our model, central bank purchases of long-term government bonds are equivalent to a reduced issuance of these bonds by the fiscal authority. Whilst our results abstract from active debt management by the treasury, Borio and Disyatat (2009) argues that fiscal authorities may be tempted to reduce debt financing costs by increasing the maturity of new debt at a time when long-term interest rates are relatively low. This is problematic, because it implies that any attempt by the central bank to use unconventional asset purchases to stimulate the economy may be unwound by the treasury issuing more long-term government debt.

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A Appendix

A.1 Firms

The profit maximisation problem of a firm i that can adjust its price in period t is

$$\max_{P_t(i)} \; E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left[P_t(i) Y_T(i) - W_T L_T(i) \right] \tag{26}$$

subject to

$$Y_T(i) = Y_T \left[\frac{P_t(i)}{P_T} \right]^{-\theta} \tag{27}$$

Equation (27) is the aggregate demand for firm i's product.[14] Firm i's production function implies that $L_T(i) = \left[\frac{Y_T(i)}{A_T} \right]^{\phi}$ which, together with (27), can be used to rewrite i's optimisation problem as

$$\max_{P_t(i)} \; E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left\{ P_t(i) Y_T \left[\frac{P_t(i)}{P_T} \right]^{-\theta} - W_T \left(\frac{Y_T}{A_T} \right)^{\phi} \left[\frac{P_t(i)}{P_T} \right]^{-\theta\phi} \right\} \tag{28}$$

The first-order condition is

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left\{ Y_T \left[\frac{P_t^*(i)}{P_T} \right]^{-\theta} - \theta Y_T \left[\frac{P_t^*(i)}{P_T} \right]^{-\theta} + \theta\phi \frac{W_T}{P_T} \left(\frac{Y_T}{A_T} \right)^{\phi} \left[\frac{P_t^*(i)}{P_T} \right]^{-\theta\phi-1} \right\} = 0 \tag{29}$$

By substituting in for the real wage from (5), and using the definition of the stochastic discount factor, the first-order condition can be expressed as

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \chi_T^C C_T^{-\delta} Y_T \left(\frac{P_T}{P_t} \right)^{\theta-1} = \left[\frac{P_t}{P_t^*(i)} \right]^{\theta\phi+1-\theta} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\theta\phi}{\theta-1} \chi_T^L L_T^{\psi} \left(\frac{Y_T}{A_T} \right)^{\phi} \left(\frac{P_T}{P_t} \right)^{\theta\phi}$$

Defining

$$F_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \chi_T^C C_T^{-\delta} Y_T \left(\frac{P_T}{P_t} \right)^{\theta-1} \tag{30}$$

$$K_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\theta\phi}{\theta-1} \chi_T^L L_T^{\psi} \left(\frac{Y_T}{A_T} \right)^{\phi} \left(\frac{P_T}{P_t} \right)^{\theta\phi} \tag{31}$$

followed by rearranging yields

$$\frac{P_t^*(i)}{P_t} = \left(\frac{K_t}{F_t} \right)^{\frac{1}{\theta(\phi-1)+1}} \tag{32}$$

[14] This demand schedule is standard in models with monopolistically competitive producers, see Woodford (2003) for a derivation.

Benigno and Woodford (2005) show in an analogous problem that (32) is both necessary and sufficient for an optimum. Note that F_t and K_t can be written recursively:

$$F_t = \chi_t^C C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta-1} F_{t+1} \tag{33}$$

$$K_t = \frac{\theta\phi}{\theta-1}\chi_t^L L_t^{\psi}\left(\frac{Y_t}{A_t}\right)^{\phi} + \alpha\beta E_t \Pi_{t+1}^{\theta\phi} K_{t+1} \tag{34}$$

The above equations are those in the paper when the price mark-up $\frac{\theta}{\theta-1}$ is multiplied by the exogenous shock μ_t.

The price level in the economy is

$$P_t = \left[(1-\alpha)P_t^*(i)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{35}$$

Rearranging yields

$$\frac{P_t^*(i)}{P_t} = \left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} \tag{36}$$

Combining the above equation with (32) gives

$$\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{\theta(\phi-1)+1}} \tag{37}$$

which is the same as equation (7) in Section 2.2.

A.2 Asset Demand Equations

In the GTL model, the asset shares $as^B \equiv \frac{P^B B}{P^S S}$ and $as^Q \equiv \frac{P^Q Q}{P^S S} = 1 - as^B$ are

$$as_t^k = \frac{P_t^k g^k}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right)\frac{a_1^k + \sum_l a_2^{kl}\log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)}{\sum_l a_1^l + \sum_k\sum_l a_2^{kl}\log\left(\frac{P_t^l}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)} \tag{38}$$

where $k, l \in \{B, Q\}$ and symmetry requires that $a_2^{kl} = a_2^{lk}$.

Demand curves are quasi-homothetic if they have linear Engel curves, i.e. the demand for an asset increases linearly with income. We impose quasi-homotheticity to limit wealth effects, which implies that $a_2^{BB} + a_2^{QB} = 0$ and $a_2^{BQ} + a_2^{QQ} = 0$. Together with symmetry this implies that $a_2^{BB} = a_2^{QQ} = -a_2^{BQ} = -a_2^{QB}$. Making the common normalisation $a_1^B + a_1^Q = 1$ and defining $a_1 \equiv a_1^B$ and $a_2 \equiv a_2^{BB}$, the asset shares are

$$as_t^B = \frac{P_t^B g^B}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right) \times$$

$$\left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right) - a_2 \log\left(\frac{P_t^Q}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)\right] \quad (39)$$

$$as_t^Q = \frac{P_t^Q g^Q}{P_t^S s_t} + \left(1 - \frac{P_t^B g^B + P_t^Q g^Q}{P_t^S s_t}\right) \times$$

$$\left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right) + a_2 \log\left(\frac{P_t^Q}{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}\right)\right] \quad (40)$$

which after rearranging is

$$\frac{B_{t,t+1}}{P_t} = g^B + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^B}\left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \quad (41)$$

$$\frac{Q_{t,t+\tau}}{P_t} = g^Q + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^Q}\left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] \quad (42)$$

Marshallian demands satisfy i) positivity, ii) adding up, iii) homogeneity of degree zero in prices and income and iv) symmetry and negative semidefiniteness of the matrix of substitution effects. Thus, the above demand system is only consistent with the firm's maximisation problem stated in section 2.3 if it satisfies these four "integrability conditions" (Barnett and Serletis, 2008). (41) and (42) satisfy ii) and iii), i) will be satisfied by an adequate calibration and iv) remains to be checked post simulation.

A.3 Price Dispersion

The labour market clearing condition is

$$L_t = \int_0^1 L_t(i) di \quad (43)$$

or, using the production function of firm i

$$L_t = \int_0^1 \left[\frac{Y_t(i)}{A_t}\right]^\phi di \quad (44)$$

Together with (27) this implies

$$L_t = \int_0^1 \left\{\frac{Y_t}{A_t}\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\right\}^\phi di = \left(\frac{Y_t}{A_t}\right)^\phi \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta\phi} di \quad (45)$$

Defining $D_t \equiv \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta\phi} di$ and rearranging yields

$$Y_t = A_t \left(\frac{L_t}{D_t}\right)^{\frac{1}{\phi}} \quad (46)$$

The law of motion of price dispersion (25) is derived as follows. Since only a share $1-\alpha$ of firms adjusts their price in every period, the dispersion of prices in t can be written

$$\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta\phi} di = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta\phi} + \alpha \int_0^\alpha \left[\frac{P_{t-1}(i)}{P_t}\right]^{-\theta\phi} di \tag{47}$$

where without loss of generality it is assumed that the firms that are not able to reset their price in t are those indexed by $i \in [0, \alpha]$. After simple manipulations one obtains

$$\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta\phi} di = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta\phi} + \alpha \Pi_t^{\theta\phi} \int_0^\alpha \left[\frac{P_{t-1}(i)}{P_{t-1}}\right]^{-\theta\phi} di \tag{48}$$

Since the distribution of prices among the firms that are unable to change their price in $t-1$ is the same as the distribution of all prices in $t-1$, the above equation becomes

$$\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta\phi} di = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta\phi} + \alpha \Pi_t^{\theta\phi} \int_0^1 \left[\frac{P_{t-1}(i)}{P_{t-1}}\right]^{-\theta\phi} di \tag{49}$$

and adequate substitutions yield the final result

$$D_t = (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}} + \alpha \Pi_t^{\theta\phi} D_{t-1} \tag{50}$$

A.4 Stationary Model

Definitions: $w_t \equiv \frac{W_t}{P_t}$, $s_t \equiv \frac{S_{t,t+1}}{P_t}$, $b_t \equiv \frac{B_{t,t+1}}{P_t}$, $q_t \equiv \frac{Q_{t,t+\tau}}{P_t}$, $\bar{q}_t \equiv \frac{\bar{Q}_{t,t+\tau}}{P_t}$, $q_t^{CB} \equiv \frac{Q^{CB}_{t,t+\tau}}{P_t}$

$$C_t + G_t + P_t^S s_t = \frac{s_{t-1}}{\Pi_t} + w_t L_t \tag{51}$$

$$1 = \beta E_t \left[\frac{\chi_{t+1}^C}{\chi_t^C}\left(\frac{C_{t+1}}{C_t}\right)^{-\delta} \frac{1}{\Pi_{t+1}}\right] \frac{1}{P_t^S} \tag{52}$$

$$w_t = \frac{\chi_t^L}{\chi_t^C} L_t^\psi C_t^\delta \tag{53}$$

$$\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\theta-1}{\theta(\phi-1)+1}} \tag{54}$$

$$F_t = \chi_t^C C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta-1} F_{t+1} \tag{55}$$

$$K_t = \frac{\theta\phi}{\theta-1} \chi_t^L L_t^\psi \left(\frac{Y_t}{A_t}\right)^\phi \mu_t + \alpha\beta E_t \Pi_{t+1}^{\theta\phi} K_{t+1} \tag{56}$$

$$s_t = b_t + \frac{1}{\tau}\left(q_t + \sum_{k=1}^{\tau-1} \frac{q_{t-k}}{\prod_{j=0}^{k-1} \Pi_{t-j}}\right) \tag{57}$$

$$1 + i_t = \frac{1}{P_t^B} \tag{58}$$

$$
\begin{aligned}
P_t^Q &= \frac{1}{\tau}\frac{1}{1+i_t^Q}\frac{1-\left(\frac{1}{1+i_t^Q}\right)^{\tau}}{1-\frac{1}{1+i_t^Q}} && (59)\\
b_t &= g^B + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^B}\left[a_1 + a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] && (60)\\
q_t &= g^Q + \frac{P_t^S s_t - P_t^B g^B - P_t^Q g^Q}{P_t^Q}\left[1 - a_1 - a_2 \log\left(\frac{P_t^B}{P_t^Q}\right)\right] && (61)\\
\bar{q}_t &= fY && (62)\\
\frac{1+i_t}{1+i} &= \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi}\left(\frac{Y_t}{Y}\right)^{\gamma_Y}\nu_t && (63)\\
\frac{\bar{q}_t - q_t^{CB}}{\bar{q}_t} &= \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi^{QE}}\left(\frac{Y_t}{Y}\right)^{\gamma_Y^{QE}}\xi_t && (64)\\
\bar{q}_t &= q_t + q_t^{CB} && (65)\\
Y_t &= C_t + G_t && (66)\\
Y_t &= A_t\left(\frac{L_t}{D_t}\right)^{\frac{1}{\phi}} && (67)\\
D_t &= (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}} + \alpha\Pi_t^{\theta\phi}D_{t-1} && (68)\\
\ln(\chi_t^C) &= \rho_C \ln(\chi_{t-1}^C) + \varepsilon_t^C && (69)\\
\ln(\chi_t^L) &= \rho_L \ln(\chi_{t-1}^L) + \varepsilon_t^L && (70)\\
\ln(A_t) &= \rho_A \ln(A_{t-1}) + \varepsilon_t^A && (71)\\
\ln(\mu_t) &= \rho_\mu \ln(\mu_{t-1}) + \varepsilon_t^\mu && (72)\\
G_t &= \rho_G G_{t-1} + \varepsilon_t^G && (73)\\
\ln(\nu_t) &= \rho_\nu \ln(\nu_{t-1}) + \varepsilon_t^\nu && (74)\\
\ln(\xi_t) &= \rho_\xi \ln(\xi_{t-1}) + \varepsilon_t^\xi && (75)
\end{aligned}
$$

A.5 Steady State

This section describes the steady state of the stationary model summarised in A.4. The equations are ordered recursively as far as possible; P^B, P^Q and i^Q have to be found numerically.

$$
\begin{aligned}
A &= \chi^C = \chi^L = \mu = \nu = \xi = 1 && (76)\\
G &= 0 && (77)\\
\Pi &= 1.005 && (78)\\
D &= \frac{1-\alpha}{1-\alpha\Pi^{\theta\phi}}\left(\frac{1-\alpha\Pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}} && (79)
\end{aligned}
$$

$$
\begin{aligned}
Y &= \left[\frac{1-\alpha\beta\Pi^{\theta-1}}{1-\alpha\beta\Pi^{\theta\phi}}\frac{\theta\phi}{\theta-1}D^{\psi}\left(\frac{1-\alpha\Pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(\phi-1)+1}{\theta-1}}\right]^{\frac{1}{1-\delta-\phi(\psi+1)}} && (80)\\
C &= Y && (81)\\
F &= \frac{Y^{1-\delta}}{1-\alpha\beta\Pi^{\theta-1}} && (82)\\
K &= \frac{1}{1-\alpha\beta\Pi^{\theta\phi}}\frac{\theta\phi}{\theta-1}D^{\psi}Y^{\phi(\psi+1)} && (83)\\
L &= DY^{\phi} && (84)\\
w &= L^{\psi}C^{\delta} && (85)\\
P^S &= \frac{\beta}{\Pi} && (86)\\
q^{CB} &= 0 && (87)\\
\bar{q} &= fY && (88)\\
q &= \bar{q} && (89)\\
s &= \frac{\Pi}{1-\beta}(C-wL) && (90)\\
b &= s-\frac{q}{\tau}\left(1-\frac{1}{\Pi^{\tau}}\right)\frac{\Pi}{\Pi-1} && (91)
\end{aligned}
$$

At this point, the asset demand equations (92) and (93) can be employed to solve numerically for P^B and P^Q

$$
\begin{aligned}
b &= g^B+\frac{P^S s-P^B g^B-P^Q g^Q}{P^B}\left[a_1+a_2\log\left(\frac{P^B}{P^Q}\right)\right] && (92)\\
q &= g^Q+\frac{P^S s-P^B g^B-P^Q g^Q}{P^Q}\left[1-a_1-a_2\log\left(\frac{P^B}{P^Q}\right)\right] && (93)
\end{aligned}
$$

Finally, i^Q is given implicitly for a particular value of P^Q by

$$
P^Q=\frac{1}{\tau}\frac{1}{1+i^Q}\frac{1-\left(\frac{1}{1+i^Q}\right)^{\tau}}{1-\frac{1}{1+i^Q}} \qquad (94)
$$

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