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Testing Nonlinear Dynamics, Long Memory and Chaotic Behaviour with Macroeconomic Data

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Abstract

This paper contains a set of tests for nonlinearities in economic time series. The tests correspond both to standard diagnostic tests for revealing nonlinearities and some new developments in modelling nonlinearities. The latter test procedures make use of models in chaos theory, so-called long-memory models and some asymmetric adjustment models. Empirical tests are carried out with Finnish monthly data for ten macroeconomic time series covering the period 1920–1994. Test results support unambiguously the notion that there are strong nonlinearities in the data. The evidence for chaos, however, is weak. Nonlinearities are detected not only in a univariate setting but also in some preliminary investigations dealing with a multivariate case. Certain differences seem to exist between nominal and real variables in nonlinear behaviour. Also in terms of short and long-term behaviour some differences can be detected.

Tiivistelmä

Tässä tutkimuksessa testataan taloudellisiin aikasarjoihin liittyviä epälineaarisuuksia. Testit koostuvat sekä tavanomaista diagnostisista testeistä että eräistä uusista epälineaarisuuksien olemassaoloa selvittävistä testimenetelmistä. Jälkimmäiset testit liittyvät kaaosteorian sovellutuksiin, ns. pitkän muistin malleihin ja epäsymmetrisen sopeutumisen malleihin. Empiiriset analyysit tehdään kymmenellä Suomea koskevalla kuukausisarjalla, jotka kattavat ajanjakson 1920–1994. Testit tulevat kiistatta sitä oletusta, että aikasarjoissa on epälineaarisuuksia. Epälineaarisuudet eivät kuitenkaan välttämättä heijasta determinististä kaaosta. Näitä ominaisuuksia ilmenee sekä yksittäisten muuttujien suhteen, mutta myös tutkittaessa muuttujien välisiä riippuvuuksia. Nimellisten ja reaalisten aikasarjojen välillä näyttää olevan jonkin verran eroja epälineaarisuuksien määrässä ja luonteessa. Myös lyhyen ja pitkän aikavälin käyttäytymisen suhteen ilmenee eroavaisuuksia.

Contents

1 Introduction

Even though economic relationships are thought to be fundamentally nonlinear, most modelling practices start with linear tests and modelling. The most apperent reason for this has been the difficulty to choose from numerous nonlinear alternatives. Economic theory rarely helps the researcher with anything else than maybe giving the assumed sign the between two variables. Given the amount of tests and statistical theory based on linear spaces, it has been almost too easy to attention to linear models. However, the poor performance of these models in forecasting eg. business cycles has alerted that maybe things are not that simple.

Apart from some almost self-evident nonlinear functions like the production function or the utility function, nonlinearities have rerely been treated satisfactory in economics. Although the state of art in nonlinear economics has started to get more attention, the biggest problem is that we do not have any clear cut procedure to approach these nonlinearities. So far we do not have any better advise than just to begin with linear testing and try to limit the nature of nonlinarities to some well specified class of models.

This paper examines several long Finnish macroeconomic time series. The purpose of the examination is to find out whether indeed there are any signs of nonlinearities in these series. Thus, we carry out a set of tests analogously to Lee, White and Granger (1993). At this stage, most of these tests are applied to univariate models although a multivariate application would obviously be more interesting. When scrutinizing the series we pay special attention to the distinction between nominal and real series. This can be motivated by the fact that nonlinearities are presumably quite different with nominal and real variables. (For an extensive survey to the literature, see Mullineux and Peng (1993).) Thus, it is of some interest of compare a typical real series, say industrial production, and a nominal series, say stock prices, in this respect.

Most monetary series – like relative prices, changes in price level and money aggregates – show some form of nonlinear behaviour. Prices are often more volatile than the real series, since they have a role of clearing device in the market. Monetary phenomenon are based upon valuations that could be adjusted without any relevant cost. In the market clearing situation it is often – but not necessarily always – easier to change the price than the quantity. Although prices could easily move into both directions, crises in the market produce excessively large negative (positive) changes. Nominal price rigidities would also have similar effects. Therefore it may be no surprise that real exchange rate, stock prices or inflation seem to adjust asymmetrically to shocks.

This affects the volatility of these series. Another major observation about the origin of "price shocks" relates to their unstable variance in time. It has been verified that in many cases price changes – e.g. in the stock market – cluster significantly. Forecasting price changes is therefore a harder task for economic agents than forecasting smoother real variables.

Nowadays, a general response to situations of changing volatility (heteroskedasticity) is to use an ARCH model specification. It may well be, however, that the ARCH model is not the proper framework. It could be the case that prices have the so-called long memory property, thus containing permanent components. In particular, the long memory property shows up in high and persistent serial correlation over long lags between absolute values of the (linearly filtered) series. It also shows up in so-called rescaled range analysis which provides estimates of the persistence of time series. Obviously, this kind of long memory phenomenom is at variance with a linear structure and therefore it may be useful to consider it also here.

However, in many cases also real economic variables vary in a nonlinear way. Obvious evidence of nonlinear adjustment could be seen e.g. from the apparent and persistent tendency to cycles in most important production variables (see, e.g., Pfann and Palm (1993) for details). Whether these nonlinearities in real series araise from the generating process of a series itself or random shocks is largely an empirical question. So far no agreement has emerged on the subject whether real or monetary phenomenon are responsible about business cycles. We hope that our estimates about the nonlinearity of these series could shed some light on this issue as well.

One general class of explanations for nolinearities is chaotic behavior. Quite recently, there have bee numerous theoretical and empirical applications of "chaos theory". In particular, the behavior of financial variables have been analyzed from this point of view (see, e.g., the books by DeGrauwe, et al (1993), Greedy and Martin (1994), Peters (1993) and Vaga (1994)). The analyses have concentrated on testing the existence of chaos; theoretical analyses have mainly been presented as examples of various possibilities in which (determinisitic) chaos might arise. Here, we leave the theoretical developments aside and concentrate solely on empirical testing. It is easy not derive a theoretical model which would easily apply to all macroeconomic series which are in our disposal.

Although the analysis mainly deals with univariate models, some preliminary work is done to identify nonlinear relationships between variables. In this context, we do not follow any specific hypothesis concerning the relationships between variables. By contrast, we simply make use of a cross-correlation analysis with respect different moments of our variables. Thus, the analyses represent some sort of first step towards a generalized Granger tests for nonlinear relationships. This analysis gives us a general idea of the magnitude and nature of these relationships. An obvious next step is to go back to theory and think about how the findings coincide with different theoretical approaches.

The structure of the paper is very straightforward. First, we have a look at the data in section 2, then we briefly present the test statistics and illustrate their properties with some simulated data in section 3 and in section 4 we go through the test results for univariate models. The results deal with various diagnostic tests procedures and with a set of analyses with correlation dimension, rescaled range, time irreversibility, nonlinear adjustment, parameter stability and long memory. In section 4, we scrutinize the results from a cross-correlation analyses between different moments of these series and, finally, in section 5 we present some concluding remarks.

2 The data

The data are monthly Finnish data covering the period 1920M1–1994M10. After data transformations the period 1992M5–1994M10 is covered. Thus, there are 893 observations in each series. The following ten series are analyzed in this connection.

Industrial production (ip)
Bankruptcies (bank)
Terms of trade (tt)
The real exchange rate index (fx)
Yield on long-terms government bonds (r)
The consumer price index (cpi)
The wholesale price index (wpi)
Banks' total credit supply (credit)
Narrow money (M1)
The UNITAS (Helsinki) stock exchange index (sx)

The first four series are real and the subsequent six nominal. The data are presented in Figure 1. For presentational convenience, most of the series have been presented in logs. To get some idea of the timing of changes in these variables the recession periods are marked by shaded areas.

Otherwise, the details of the data are presented in Virén (1992). We only point out that ip, bank, credit and M1 series are seasonally adjusted. This is simply because of data reasons – only seasonally adjusted data were available for the prewar period 1920–1938. As for the World War II (1939–1945), the data are treated in the same way as for the peace years.

The overall quality of the time series is rather good. Only money and interest rate series are somewhat deficient which is apparent on the basis of also Figure 1. The money, M1, series for 1922M1–1948M12 is based on rather crude assumptions on banks' cash holdings and hence the series is "too smooth" for this period. The interest rate series, in turn, suffers from the fact that banks' borrowing and lending rates were administratively fixed from mid 1930s to early 1980s and, therefore, the bond yields were not genuinely market-based but they were indirectly rationed, too. Because of these frictions with M1 and r, we leave them out from more sophisticated econometric analyses.

3 The test statistics

Data transformations

Testing nonlinearities is preferred to be started by estimating linear model and analysing the respective residuals. Although economic relationships are most likely to be nonlinear, there is also danger of unnecessary complication, if the difference to a linear model is small.

The need for nonlinear model depends also on the purpose of the model. For short-run forecasting linear models may do the thing, but for long-run forecasts or explanation of apparent nonlinear features a more proper modelling is needed. Since testing linearity is widely covered in Granger and Teräsvirta (1993), we give here only few basic standpoints. The linearity tests could be divided into two groups, depending on whether a specific nonlinear alternative exists or not. Since our data does not refer to any specific nonlinear formulation, we concentrate on testing against the general nonlinear alternative.

As it was mentioned above, we analyze only univariate models. A some sort of basic specification is a linear AR(4) which turned out to a reasonably good approximation for all time series. In specifying the order of the autoregressive models, we used model selection criterions (SC, HQ, AIC). In order to study the dynamic dependencies between variables, we though that in the first place it would be best to filter the original series with the linear autoregressive model of the same order. Thus, the residuals are not severely (linearly) autocorrelated. A few exceptions do exist, however, for higher order autocorrelation (for the lag 12, for instance). Anyway, we prefer the parsimonious AR(4) model to more sophisticated specifications.[1] In fact, we also used first log differences for all relevant variables instead of AR(4) residuals. The residuals were qualitatively very similar suggesting that the AR(4) transformation is not that crucial. For space reasons, the results with the first difference data are not reported here.

Log transformations was applied to most of the series. Thus, only the terms of trade, the real exchange rate and the interest rate series were left untransformed. To assess the validity of this transformation we made use of the Box–Cox transformation. The results of this procedure generally supported to the above mentioned choice. Only in the case of consumer prices and the terms of trade, one could not sure whether or not to make the log transformation.

Dealing with nonlinearities is often easier after the linear dependencies in a time series have already taken care of. Therefore nonlinear adjustment can be found from a series property filtered with autoregressive (linear) model. However, empirical problems do emerge at this point. It often happens, especially in multivariate analysis, that filtering is almost too effective, since all the significant relationships between variables are removed. Therefore too long autoregressive lag models that also affect the asymmetricity in the series should be avoided.

Standard diagnostic tests

Given the autoregressive model, we compute the following sets of tests: First some basic statistics on residuals of this linear AR(4) model (see Table 1). These statistics include the coefficients of skewness and kurtosis in addition to the median. Quite obviously, we intend to discover possible asymmetries with these data. The second set of tests consist of traditional specification tests for functional misspecification/nonlinearity. The tests (reported in Table 2) consists of Engle' s (1982) ARCH test in terms of lagged squared residuals, Ramsey's

[1] We are well aware that the remaining higher-order autocorrelation might invalidate the subsequent test statistics which are related to the measure of correlation dimension (see Ramsey (1990) for details).

(1969) RESET test in terms of higher-order powers of the forecast value of x_t, White's (1980) heteroskedasticity/functional form misspecification test in terms of all squares and cross products of the original regressors, The Jarque and Bera (1980) test for normality of residuals and Tsay's (1986) nonlinearity test in terms of squared and cross-products of lagged values x_t.[2] Finally, the Hsieh (1991) third order moment coefficients are computed. They should detect models which are nonlinear in mean and hybrid models which are nonlinear in both mean and variance but not models which are nonlinear in variance only.

BDS-test for chaotic process

In addition to these "traditional" test statistics we also computed the BDS (Brock, Dechert and Scheinkman) test statistic (see Table 4) and Ramsey's (1990) irreversibility $G_{1,2}$ test. BDS tests is designed to evaluate hidden patterns of systematic forecastable nonstationary in time series. The test was originally constructed to have high power against deterministic chaos, but is was find out that it can be used to test other forms of nonlinearities as well (see, e.g., Brock, Scheinkman and LeBaron (1991) Frank and Stengos (1988) and Medio (1992) for details).

BDS test could be applied also as a test for adequacy of a specified forecasting model. This could be accomplished by calculating the BDS test for the standardized forecast errors. Then BDS test is used as a specification test. If no forecastable structure exists among forecast errors, the BDS test should not alarm. BDS test has been found useful as a general test for detecting forecastable volatility. The key concept here is the correlation dimension, which could be applied in finding the topological properties of series. For purely random variable, the correlation dimension increases monotonically with the dimension of the space and the correlation dimension remains small even when the topological dimension of the space (embedding dimension) increases (Brock, Hsieh and LeBaron (1991)).

For a single series x_t for which $x_{t,m}$ is the set of m adjacent values of this time series x_{t+j}, j=0, ..., m−1 the m-correlation integral $C_m(\varepsilon)$ is defined as

$$C_m(\varepsilon) = \lim_{T\to\infty} T^{-2}[\text{pairs (i,j) for which} |x_i - x_j| < \varepsilon, ..., |x_{i+m-1} - x_{j+m-1}| < \varepsilon],$$

where T = N − m + 1, N is the length of the series.

The idea is that for chaotic series, the subsequent values of x_i and x_j will be very close. If the time series is a stochastic sequence, this does not happen. Now defining the correlation dimension d(m) as

[2] As for the properties of these test statistics see e.g. Petruccelli (1990) and Lee, White and Granger (1993).

$$d(m) = \lim_{\varepsilon \to \infty} \frac{\partial \log C_m(\varepsilon)}{\partial \log \varepsilon}$$

it will be seen, that for truly chaotic process $C_m(\varepsilon) \approx \varepsilon^d$, if ε is small. This means that correlation dimension is independent of m if the process is chaotic. Otherwise, if the process is truly stochastic the correlation dimension will increase linearly with m.

The purpose of the correlation measure is to describe the complexity of the true series and measure the nonlinear dimension (degrees of freedom) of the process. Tests of chaos concentrate on low-dimensional deterministic chaos prosesses, since there is no efficient way to tell the difference between high-dimensional chaos and randomness.

Although the correlation dimension can be calculated and interpreted rather easily, there are some major problems with the estimation of this measure, mainly due to fact that economic data are relatively noisy and there too few observations available (see Ramsey (1990) and Ramsey, Rothman and Sayers (1991) for more details). It can be shown that when the dimension of the data set is based on this Grossberger-Procaccia measure, the estimate of it is necessarily biased because of the following small sample problem: With finite data set the value of ε cannot be too small because otherwise $C_m(\varepsilon)$ will be zero and thus d(m) is not defined. By contrast, with large values of ε, $C_m(\varepsilon)$ saturates at unity so that the regression of $\log(C_m)$ on $\log(\varepsilon)$ is simply zero. Thus, the smaller the number of observations, the larger ε has to be, and the more biased the estimate of the dimension will be.

Although theory concerns the properties of $C_m(\varepsilon)$ as $\varepsilon \to 0$, the reality is that the range of ε used in estimating d(m) is far from zero and inevitably increases away from zero as the embedding dimension is increased. Smaller values of ε require substantial increases in sample size in order to determine a linear relationship between $\log(C_m(\varepsilon))$ and $\log(\varepsilon)$. In fact, the relationship is linear only for a narrow range of values for ε. Thus, one should be very careful in evaluating single point estimates of d(m). By scrutinizing the entire path of d(m) with respect to ε one may obtain a more reliable estimate of the true dimension. Alternatively, one may use the test procedure suggested by Brock, Hsieh and LeBaron (1991) in calculating the following BDS test statistic:

$$BDS(m,\varepsilon) = \sqrt{T}(\hat{C}_m(\varepsilon) - [\hat{C}_1(\varepsilon)]^m)/\sigma(m,\varepsilon),$$

Where $\sigma(m,\varepsilon)$ is an estimate of the standard deviation. BDS tests whether $C_m(\varepsilon)$ is significantly greater than $C_1(\varepsilon)^m$, and when this happens nonlinearity is present. Under the null hypothesis of x_t following i.i.d., and for fixed m and ε, $C_{m,T}(\varepsilon) \to C(\varepsilon)^m$, as $T \to \infty$, and SDB(m,ε) has the standard normal distribution. (Notice, however, that $C_m(\varepsilon) = C(\varepsilon)^m$ does not imply i.i.d..) The power of the test will depend critically on the choice of ε.

BDS test statistic is complicated since it depends on the embedding dimension (m) and the chosen distance (ε) related to standard deviation of the data. The selection of m is important in small samples especially when m is large, since increasing m means that the number of nonoverlapping sequences will become smaller. And when sample is less than 500 the asymptotic distribution may be different than the sampling distribution of the BDS statistic. The selection of ε is even more crucial and a failure to detect non-normality in calculating BDS with small ε is a consequence of too few observations. Brock, Hsich and LeBaron (1991, p. 52) suggests that for 500 or more observations, the embedding dimension m should be smaller or equal to 5, whereas ε should be 0.5–2 times the standard deviation of the data. In the empirical application, some alternative values of the dimension parameter m and the distance parameter ε are used.

The problem with BDS test is however, that it does not have a simple interpretation. Nonlinearity based on BDS test could be a result from chaos or nonlinear stochastic process. However, BDS test was originally designed to test whether data generating process of a series is deterministic (chaotic) or not (Granger & Teräsvirta (1993), p. 63). Since the BDS test is based on the null hypothesis that the observations (here AR(4) residuals) are i.i.d., a rejection merely reveals that this is not the case. The specific form of nonlinearity is therefore an open question.

As for the practical implementation of the test, it is here done by using the residuals of the AR(4) model as inputs. The use of the autoregressive filter is based on the invariance property of chaotic equations shown by Brock (1986). Brock showed that if one carried out a linear transformation of chaotic data, then both the original and the transformed data should have the same correlation dimension and the same Lyapunov exponents. Some alternative values for the dimension parameter m and distance parameter ε are applied.

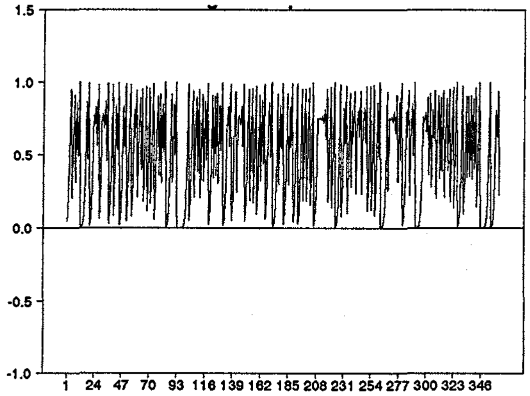
In order to get some idea of implications of deterministic chaos we illustrate the case by comparing a truly deterministic chaos series to a random N(0,1) series. A logistic map model which takes the form $x_t = 4*x_{t-1}(1-x_{t-1})$ is used to generate the chaotic series. Both series contain 2000 observations; the initial value of the logistic map series is 0.3.[3] The figure on the following pages illustrates the time paths of these two series (only the first 200 observations are graphed), the respective autocorrelations for 60 lags, two dimensional plots in terms of the current and lagged value of the variable, correlation dimension estimates with an embedding dimension 2–5 and the BDS test statistics with the the embedding dimension 2 over the ε values 0.5–3.0.

[3] The first value of the series is .300. The series are very sensitive with respect to this initial value. If the initial value is changed to .30001, the new series diverges from the original series after 14 observations and never converges. In addition to the logistic map specification we used the Henon map (which is equally often used a benchmark example). Here, the Henon map takes the following parametrization: $x_{t+1} = 1 + y_t + 1.4*x_t^2$ and $y_{t+1} = .3x_t$ with x(0) = .1 and y(0) = .1. Needless to say, also these series depend very much on the initial values.

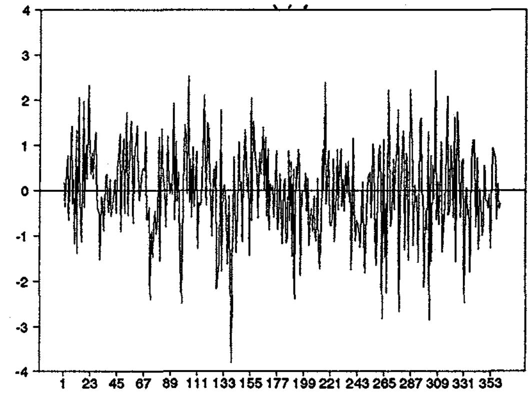
Figure 1. **Comparison of logistic map and random series**

First 200 observations of the time series

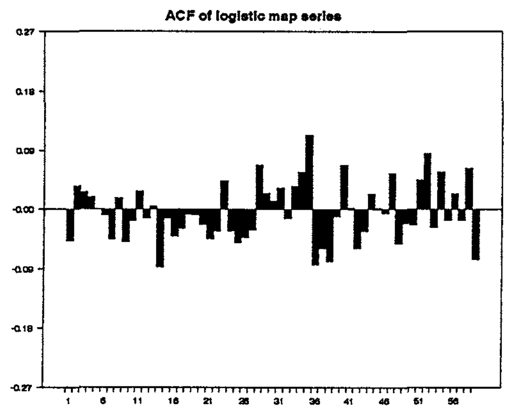
Logistic map series

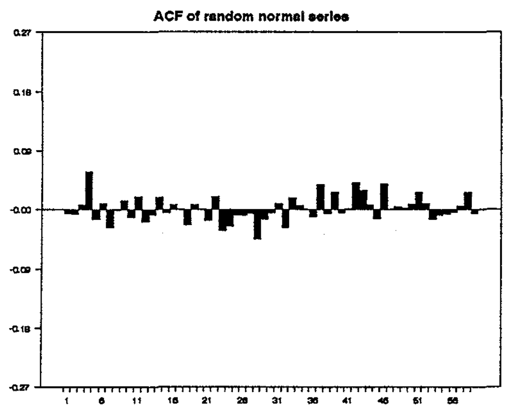


Random N(0,1) series



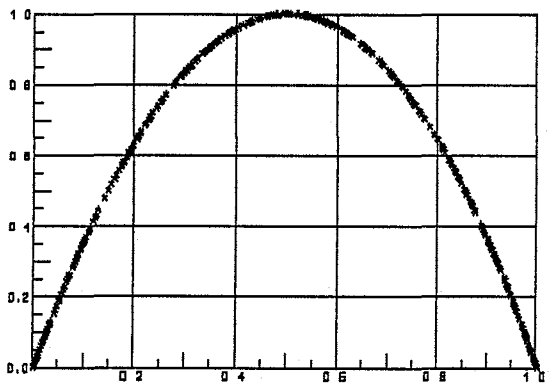
Autocorrelations



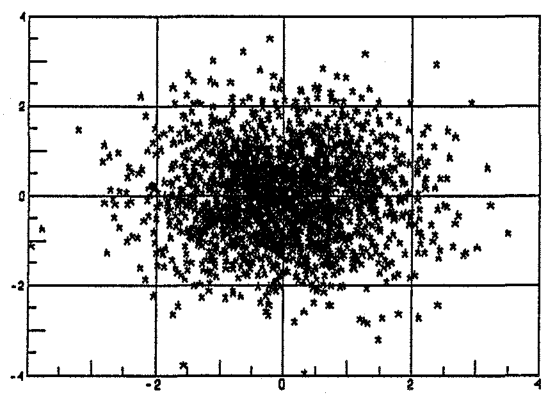


Two-dimensional plots

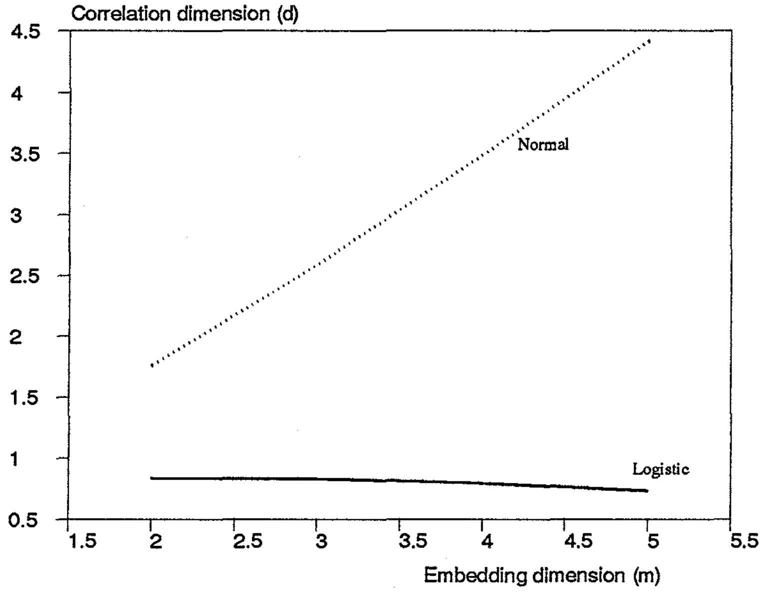
Logistic map series



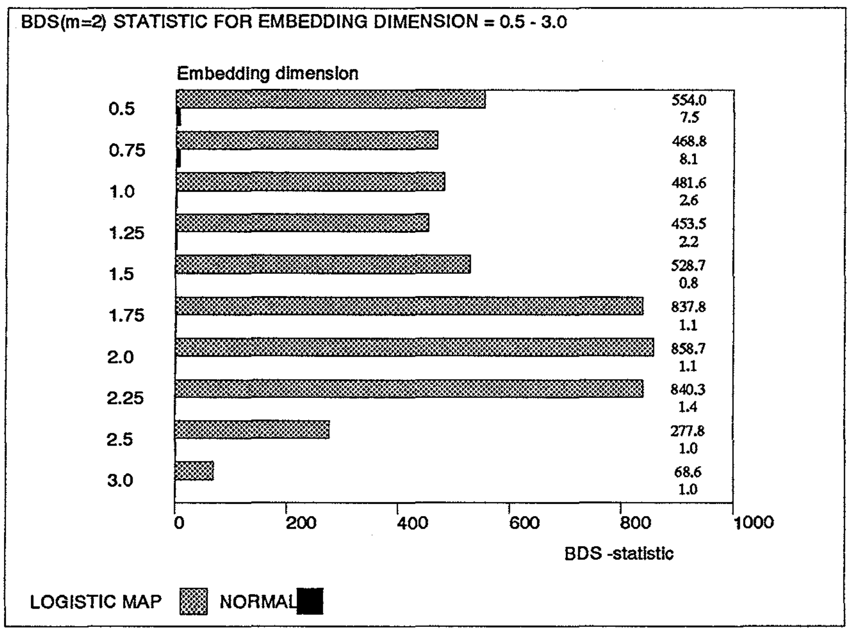
Random N(0,1) series



Correlation dimensions of logistic map and random normal processes

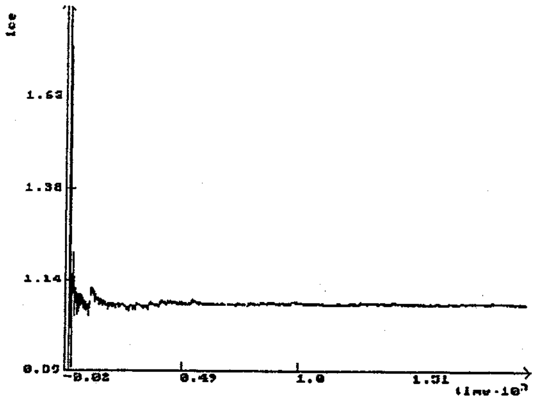


BDS(2) statistics for ε = 0.5–3.0

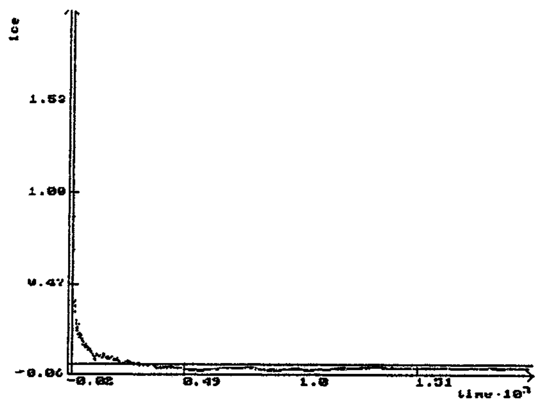


Lyapunov exponents

Logistic map, $L_1 = 1.07$

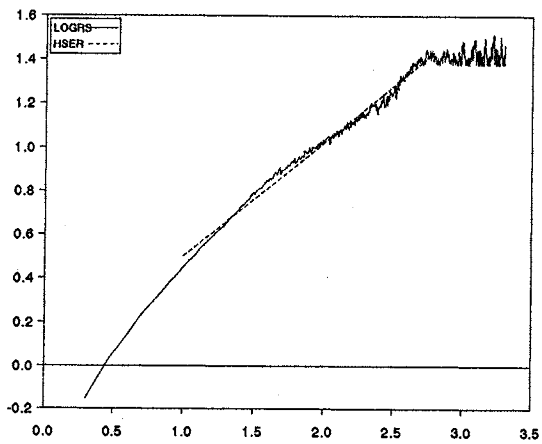


Random N(0,1), $L_1 = -0.02$

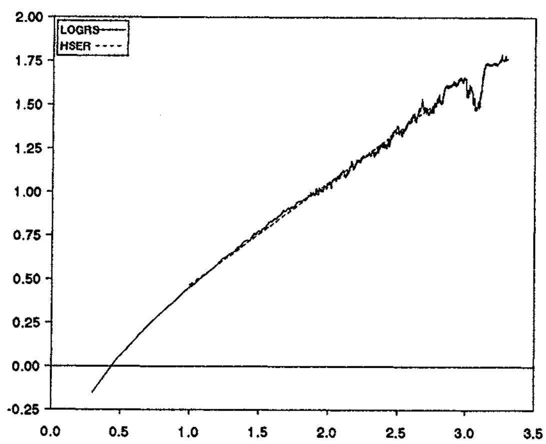


Hurst exponents

Logistic map, H = .426 (.520/.024)



Random N(0,1), H = .588



The purpose of Figure 1 is to show that the time series and the autocorrelations are quite similar. In fact, one might at first glance consider the logistic map series to be random walk series. The dimension plots show, however, that there is a fundamental difference between these two series. The random N(0,1) series is spread quite evenly over the plane while the logistic map series do not fill enough space at a sufficiently high embedding dimension which is a generic property of chaotic processes. The clustering of two-dimensional plots also shows up in the dimension estimates (and in the BDS test statistics). The estimate for the logistic map series is about one irrespective of the embedding dimension (it can be shown that the correlation dimension for the logistic map is 1.00 ±0.02, see, e.g., Hsieh (1991)). Finally, the BDS test statistics clearly discriminates these two series. Thus, the statistic for random normal series typically fails to exceed the critical value while the test statistic for the logistic map exceeds the critical value by many hundreds.

Lyapunov exponents

The Lyapunov exponents measure the average stability properties of the system on the attractor. Frequently the presence of at least one positive Lyapunov exponent is taken to be the definition of chaos. For a fixed point attractor, the Lyapunov exponents are the absolute numbers of the eigenvalues of the Jacobian matrix evaluated at the fixed point. Thus, the Lyapunov exponents can be considered as generalizations of eigenvalues (see, e.g. Medio (1992) and Frank and Stengos (1988a) for further details).

To define the Lyapunov exponents consider the following dynamical system:

$$\frac{dx}{dt} = F(x),$$

where x is a vector with N components. Consider a trajectory x*(t) that satisfies this equation and an arbitrarily small positive initial displacement from the start of x*(t) denoted by D(t). Now, it can be shown under fairly general conditions that, for given D(0), the following limit exists:

$$L_i = \lim_{t\to\infty}(t^{-1}\ln|D_i(t)|),\ \ t = 1,2,...N.$$

Notice that the Lyapunov exponents are not local properties as one might think. Thus, the values if L_i are independent of the choice of D(0). In fact, one may interpret that the exponent(s) measures the average rate of separation over the entire strange attractor.

A positive Lyapunov exponent measures how rapidly nearby points diverge from each other. A negative Lyapunov exponent, in turn, measures how long it takes for a system to reestablish itself after it has been perturbated. Basically, this is the reason why the Lyapunov exponents offer a way to classify attractors.

The problem is that it is not easy to estimate Lyapunov exponents from experimental data. Wolf, et al (1985) have developed a FORTRAN program which estimates the largest exponent L_1 from these kinds of data but it has been shown (see, e.g., Brock (1986) and Brock and Sayers (1988)) that the estimates are very sensitive with respect to the nuisance parameters used in the context of the program. Thus, for instance, large positive Lyapunov estimates may be obtained for pure noise data. Our own experience points to the same direction. Therefore we are reluctant to use the Wolf, et al (1985) estimates to characterize our real data.

Quite recently, McCafferty, et al (1991) and Dechert and Genacay (1993) have proposed an alternative algorithm using the so called multilayer feedforward networks which appears to have superior properties with respect to the Wolf, et al (1985) algorithm. This will allow us to rescrutinize the values of the Lyapunov exponents in a more affirmative way. Although we do not go through the analysis of Lyapunov exponents with the real data we may refer to Figure 1 in text where the largest Lyapunov exponent is presented for random normal and logistic map time series.[4] In the case of logistic map series, the exponent is large and positive while with random noise series the exponent converges to a (small) negative value.

The Hurst exponent (rescaled range analysis)

The Hurst exponent is a new measure which can classify time series in terms of persistence (or, "antipersistence"), stability of the data generating mechanism and the importance of outlier-type observations. Thus, it can distinguish between a random series from a non-random series, even if the random series is non-Gaussian. The Hurst exponent was first applied to natural systems (first, in analyzing water reservoir control within the Nile River Dam project in early 1900) but recently there have numerous applications to financial data (see, e.g., DeGrauwe, et al (1993) and Peters (1993)).
Computing the Hurst exponent (H) requires the following steps:
First, compute the cumulative deviation $X_{t,\tau}$ over τ periods:

$$X_{t,\tau} = \sum_{i=1}^{t}(e_i - M_\tau)$$

where e_i is the inflow in year i and M_τ is the recursive average of e_i over N periods. Second, compute the recursive range between the maximum and minimum of X so that:

[4] Lyapunov exponents have been estimated in several empirical studies, see, e.g., Frank and Stengos (1988c), Frank, et al (1988) and Peters (1993). The results have been somewhat mixed, partly depending on the algorithm (thus, for instance, Frank and Stengos (1988) do not find support to the existence chaos while Peters's results point to the opposite direction). There are, however, a lot of ambiguity with the results because of convergence problems and computational sensitivity.

$$R_\tau = \max(X_{t,\tau}) - \min(X_{t,\tau})$$

Finally, compute the exponent estimating the parameter H from the following model for the "rescaled range" R/S:

$$\frac{R}{S} = (\alpha \cdot N)^H,$$

where S is the (recursive) standard deviation of the original observations and α a scaling constant (for further details, see., e.g., DeGrauwe, et al (1993) and Peters (1993)).

According to the statistical mechanics, H should equal to 0.5 if the series is a random walk. In other words, the range of cumulative deviations should increase with the square root of time. With many (most?) time series from natural system, the value of H has turned out be much higher than 0.5. In surprisingly in many cases the value of 0.73 is obtained (see DeGrauwe et al).

When H is different from 0.5, the observations are no more independent in sense that they carry a memory of all preceding events. This memory can be characterized as "long-term memory". Theoretically, it lasts forever. Thus, the current data reflects everything which has been in the past. Notice that this is something which cannot be taken into account in standard econometrics in which time invariance is assumed.[5]

Now, consider the case in which $H < ½$ and $H > ½$. In the former case, the system is antipersistent or "mean reverting". Thus if the system has been up in the previous period, it is more likely to be down in the next period. By contrast, when $0.5 < H < 1$, the system is persistent or "trend-enforcing". If the series has been down in the last period, then the changes are that is it will continue to be down in the next period.

A R/S plot for random N(0,1) and a logistic map series is presented in Figure 1 in text. Notice, the estimated slope (i.e., the Hurst exponent) is 0.59 which is quite close to the theoretical value of 0.5. The estimated slope of the logistic map series is instead 0.43 (as for the Henon map, an estimate of .38 is obtained) which says (in statistical terms) that this series have no population mean and the distribution of variance is undefined. Obviously, there is nothing we can forecast with these series.

[5] The values of Hurst exponent can to related to a correlation (C) measure in the following way: $C = 2^{(2H-1)} - 1$. Thus, when $H = ½$, $C = 0$, and we are dealing with random series. Its probability density function can be the normal curve but is does not have to be. By contrast, if H is different from ½, the distribution is not normal.

The Ramsey irreversibility test

The irreversibility test, which has been derived by Ramsey and Rothman (1988) and Rothman (1993), deals with the concept of time reversibility.[6] Time irreversibility is concept which is useful in analyzing possible asymmetries (nonlinearities) in economic time series, for instance, in output series. According to conventional Mitchell–Keynes business cycle hypothesis cyclical upturns are longer, but less steep, than downturns (see also the "plucking model" of Friedman (1993)) If one traces out the behaviour of cycles in reverse time it can be seen that the symmetric cycle is time reversible and the asymmetric cycle is time irreversible.

Ramsey and Rothman (1988) propose that the presence of time irreversibility is checked by estimating a symmetric bicovariance function in terms of x_t. The test statistic which is obtained from this bicovariance function is of the following type:

$$G_{i,j}^{k} = T^{-1}\sum_{t=1}^{T}\left[(x_{t-1})^{i}(x_{t-k})^{j} - (x_t)^{j}(x_{t-k})^{i}\right], \qquad k=1,2,...,K.$$

If the time series is time reversible, $G_{i,j}^{k}=0$ for all k. As for the choice of exponents, i and j, we assume here that i = 2 and j = 1 (here we just follow Ramsey (1990)). In addition, we experiment with the pair i = 3 and j = 1. The maximum lag length K is set at 120. To ensure stationarity, we use also here AR(4) residuals instead of the original time series. The significance of the G statistic is tested by computing the confidence limits according to the following formula for the variance of $G_{1,2}^{k}$:

$$\mathrm{Var}\left[G_{1,2}^{k}\right] = \left(\frac{2}{(T-k)}\right)\left[\mu_4\mu_2 - \mu_2^3\right],$$

where $\mu_2 = E\left[x_t^2\right]$ and $\mu_4 = E\left[x_t^4\right]$. Assuming that the data are independent and identically distributed $N(0,\sigma^2)$, the right hand side of the above formula can be simplified to be $\left(\frac{4}{(T-1)}\right)\left[\mu_2^3\right]$. This is clearly a crude approximation because the normality assumption does not hold, nor are the variables uncorrelated. However, it is not all clear how the variance terms should be computed when x_t is not i.i.d. but follows e.g. some general ARMA(p,q) model (see Ramsey and Rothman (1988) for various experiments). Here the test statistics and the respective confidence limits are displayed in Figure 6.

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[6] A stationary time series $\{x_t\}$ is time reversible if for any positive integer n, and for every t_1, t_2, ..., t_n, $\in$ z, where z is the set of integers, the vectors $(x_{t1}, x_{t2}, ..., x_{tn})$ and $(x_{-t1}, x_{-t2}, ..., x_{-tn})$ have the same joint probability distributions. A stationary time series which is not time reversible is said to be irreversible. Notice, that by definition, a non-stationary series is time irreversible. See e.g. Tong (1983) for further details.

A nonlinear adjustment equation

Instead of just computing test statistics for nonlinearity, it would be tempting to estimate a general nonlinear time series model and compare its properties with a linear model. Unfortunately, such general nonlinear model does not exist nor is there any agreement of a reasonable approximation which could be used to capture the possible nonlinear elements of the data. Still, the situation is not completely hopeless. There some interesting candidates for a nonlinear specification. The first which deserves to be mentioned is the threshold model specification introduced by Tong (see e.g. Tong (1983)). Another specification which is clearly worth mentioning is the nonlinear employment (output) equation introduced by Pfann (1992). This (estimating) equation takes the following form:

$$x_t = a_0 + a_1 t + a_2 x_{t-1} + a_3 x_{t-2} + a_4(x_{t-1}x_{t-2}) + a_5(x_{t-1}^3 x_{t-2}) + a_6(x_{t-1} - x_{t-2})^3 + \mu_t,$$

where μ is the random term. According to Pfann (1992) and Pfann and Palm (1993), the parameter of the nonlinear terms can be unambiguously signed in the case employment equations. Thus, a_4 should be positive (if hiring costs are larger than firing costs, or in general, if the cycle spends more time rising to a peak than time falling to a trough). Moreover, parameter a_5 is expected to be negative if the asymmetry (skewness) of magnitude (i.e. the magnitude of troughs exceeds the magnitude of peaks) is negative and parameter a_6 also negative if the asymmetry (skewness) of duration is negative (i.e., it takes longer for a series to rise from a trough to a peak than to fall from a peak to a trough).

Although this model may make more sense with (productive) input and output series we also apply it to all ten (here, in fact, thirteen) Finnish series partly to see whether the real and nominal series can be discriminated on the basis of this equation. The results are reported in Table 4. This table also includes a comparison of this model with a linear alternative.[7]

4 Test results with univariate models

4.1 Results from diagnostic tests

The message of the empirical analyses is quite clear and systematic: the data do not give much support to linear models. Thus, all tests statistics in reported in

[7] Here, we merely replicate the experiments by Pfann (1992). Thus, we take the same detrending procedure (see the second term on the right hand side) and the same lag structure. Obviously, extending the lag length beyond 2 would enormously complicate the model.

Table 2 and 3 indicate that at least a linear AR(4) model is trouble.[8] According to Table 2, the residuals from the AR(4) model suffer from heteroskedasticity and non-normality. The ARCH(7) statistic significant for all variables (perhaps excluding the interest rate). Thus, even with real series like industrial output an autoregressive conditional heteroskedasticity effect can be discerned. This is something new. Nobody is surely surprised to find an ARCH effect in stock prices but here a similar result applies to other variables as well.

Nonnormality is clearly a severe problem. It is quite obvious that normality is violated because of outlier observations. Clearly, some observations can classified as outliers and it might well be that these observations contribute to the rejection of linearity. This can be seen from Figures 2 and 3 which contain the time series and frequency distributions for the AR(4) residuals. In accordance with Table 1, the main problem seems to be excess kurtosis, not so much excess skewness. Although the normality assumption is rejected, the graphs suggest that the distributional problems are not, after all, so severe as the Jarque–Bera normality test statistic suggests.

Unfortunately, there is no obvious remedy to non-normality and outlier observations. One alternative is, of course, to use robust estimators and examine whether the results (e.g., the properties of residuals) change importantly due to the change in estimators. In fact, we did do this but it turned out that the results with the least absolute deviations estimator were qualitatively very similar to the OLS results. Another possibility is to reconsider the relevant sampling distributions of the nonlinearity tests statistics in the light of observed behaviour of OLS residuals. Here, we have not yet worked out this alternative.

After these considerations, some comments on the RESET and TSAY nonlinearity test statistics merit note. Both tests do suggest that the (linear) functional form is misspecified for most of the variables. The results are, however, very systematic. Thus, for instance, industrial production and bankruptcies, on the one hand, and narrow money and credit supply, on the other hand, behave in a different way in these tests. Moreover, the test results do not allow from drawing a clear line between real and nominal variables. As far as the Hsieh's (1991) third order moment coefficients are concerned, one can see that with some variables the coefficients are very high. Some of the highest coefficients are in fact quite similar to those of the logistic map series! High coefficient values are obtained for the real exchange rate, consumer and wholesale prices, money and – somewhat surprisingly – stock prices. By contrast, the values for industrial production, bankruptcies and terms and trade are somewhat lower although all of them are not "clean". Thus, nonlinearities do exist and nonlinearities are not only a problem for real variables. Because the third order moment coefficients are not intended to test models which are nonlinear in variance one may conclude that the high coefficient values for the nominal series do not (only) reflect some ARCH effects but other sorts of nonlinearities (say GARCH-in-Mean effects or long memory behaviour).

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[8] In addition of the test statistics reported in Table 2 we also computed the Keenan (1985) and McLeod-Li (1983) test statistics. Both of these turned out to be highly significant. Thus the marginal significance levels were in all cases well below 5 per cent. The test statistics were also computed for the post Second Word War period. Results were quite similar to those reported in Table 2. Thus the war itself cannot explain why the results are favourable to nonlinearities.

4.2 Results from analyses of correlation dimension

Next, turn to results from the analysis of the correlation dimension. Those results are presented in the following way: First, the two-dimensional plots of the AR(4) residuals are presented in Figure 5, then the correlation dimension estimates are presented in Figure 6 (Figure 6 consists of two plots showing the correlation integral and the derivative of $C(\varepsilon)$ in terms of ε; the respective numerical values are reported in Table 3) and, finally the BDS test statistics are reported in Table 4.

Unfortunately, the results from these exercises are somewhat different. First, the dimension plots are not consistent with the existence of low-dimensional chaotic behavior (notice, however, that we just look at thing very informally in two dimensions). Although there are some differences between variables none of variables behaves in a chaotic manner. Stock prices may best correspond to a random variable (observations are evenly distributed over the x_t, x_{t-1} plane) while some clustering takes place in consumption and wholesale prices.

As one might expect on the basis of the dimension plots, the estimates of the correlation dimension (the embedding dimension running from 2 to 5) lend very little support to model of chaotic behavior. The estimate of d(m) increases almost linearly with the embedding dimension m. Only wholesale prices represent an opposite result. The estimate of d(m) remains in the neighbourhood of one even if the embedding dimension is increased to 5. Figure 2 may explain why this result emerges. The behaviour of prices in the 1920s and 1930s was completely different from the rest of the sample period (i.e. the price level was practically stationary for the pre-war period while after the outbreak the Second World War the rate of inflation turned out to be stationary). If the 1920s and 1930s are dropped from the sample the correlation dimension estimates behave well in accordance with the other variables.[9]

Somewhat contrary to these results, the BDS statistics turn out to be very high and suggesting that the data generating mechanism is not linear. The null hypothesis that the series are random i.i.d variates is rejected in all cases with standard significance levels. If the series are shuffled, i.e. the observations are arranged in a random order, the null hypotheses of independent observations is typically not rejected which suggests that the distributional assumptions are not very critical in terms of the outcome of the BDS statistics. By contrast, the time-series structure is the important thing which produces the very high values of the BDS statistics.

But how should we interpret this conflicting evidence? Should stress more the correlation dimension estimates or the BDS test statistics. The answer is not easy. Perhaps, the best way to summarize this evidence is to conclude that there are definitely some signs of nonlinearity but not necessarily of deterministic chaos.

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[9] For the period 1939M9–1993M8 the following set of dimension estimates were obtained: m = 2: 1.901 (1.02); m = 3: 2.709 (1.30); m = 4: 3.617 (1.94) and m = 5: 4.226 (1.01). These values are clearly in accordance with the other values in Table 3 and hardly consistent with the existence of deterministic chaos.

4.3 Results from the rescaled range analysis

Turn next to the analysis of Hurst exponents. The estimates are presented in Table 5 and the corresponding graphs in Figure 7. The results are derived for the AR(4) residuals. In addition to these residuals we have also used shuffled residuals where the order of observations is based on random drawing. By shuffling the order of observations we destroy the memory of the observations and effectively make the series random series (obviously, the distribution function stays invariant).

The OLS estimates or H are all well above 0.5 suggesting that the long-memory property indeed holds for the data. It can be argued that, for instance, economic crises have some systematic patterns. This may induce correlations that seem to repeat themselves. With shuffled residuals, the estimates are, however, quite close to benchmark value 0.5. Thus, the critical feature of the data is just the time-dependency of observations.

An interesting question is how long is the long-memory phenomenon. Is there certain time span – say one year – in which observations are not independent. One could for instance argue that for various reasons (see, e.g., Peters (1993)) the stock market is not efficient in the short run but efficient in the long run. In other words, we have some cycles which just reflect these inefficiencies (or, more generally market imperfections). This might show up in a change of the R/S slope. In fact, this kind of reasoning seems to apply to the Finnish stock price series. There is quite clear change in the slope after 200 data points corresponding the period of about 16 years. In the short run, stock prices are very persistent (H equals to .70) while in the long run stock prices can be characterized as independent or even antipersistent. Thus, an estimate of .11 is obtained for the data points exceeding 200 (see Figure 7).

After these comment we may turn to the analysis of shuffled series, i.e. series in which the observations are arranged in a random order. As point out above, the values of the Hurst exponent should now converge towards 0.5. The results in Table 5 show that this is indeed the case. In all cases, the values of the Hurst exponent decrease. They do not necessarily go to 0.5 but to some value between 0.5 and 0.6. Thus, it is clear that the order in which the observations originally are is crucial for the analysis. This order must be maintained in order to preserve the memory of the data, by destroying the order one destroy also the memory and the data look like genuine random series generated with nonbiased random walk. After all, when dealing with time series the time dimension is important. It is possibly also unidirectional, as we see in the next section.

4.4 Results from the time irreversibility analysis

A similar result emerges with Ramsey's (1990) irreversibility tests statistics reported in Figure 8.1. Although, the confidence limits are only indicative some signs of nonlinearities can be discerned with all series. Somewhat surprisingly, stock prices do not seem to be the most striking example of this sort of nonlinearities. Thus, for instance, the test results for industrial production tell more about nonlinearities than the results for the stock index (see Figure 8.2).

Also bankruptcies and banks' total credit supply seem to be more obvious candidates. Perhaps, this is something which is in accordance with the observed nature of indebtedness and the relationship between indebtedness, credit supply and bankruptcies (see, for instance, Stiglitz and Weiss (1981) and Bernanke (1983)).

Recently, Rothman (1993) has shown that the Ramsey–Rothman irreversibility tests is relatively powerful against the threshold model. Thus, our findings could also be interpreted from this point of view. In other words, there are nonlinearities but not of deterministic chaos type but rather resulting from nonlinear model structure or parameter instability. In the subsequent sections, we deal with these alternatives.

4.5 Estimates of adjustment equations

Can anything else be said about the nature of nonlinearities? Tables 2 and 6 suggest that this is the case.[10] Table 1 indicates that the real series and the nominal series behave in a very different way. The nominal series do not show up any signs of negative skewness. Moreover, the nonlinear adjustment equations (reported in Table 6) behave very badly, for instance, in terms of stationarity.[11] It is particularly interesting to compare the behaviour of industrial production and stock prices. Industrial output is characterized by clear negative skewness (in magnitude) while there is no apparent skewness in stock prices. With industrial production, positive residuals are much smaller and obviously more numerous than negative residuals. Intuitively, this makes sense since capacity constraints limit increasing production while a decrease in orders or bankruptcies may lower production more rapidly. With stock prices, there is no difference between positive and negative residuals. Thus, adjustment of stock prices does not contain significant asymmetries. See Figure 9 for details; notice that positive and (absolute values of) negative AR(4) residuals are presented here in an ascending order.

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[10] Here, we have introduced three additional real variables: the real interest rate and the (inverses of) money and credit velocities.

[11] With consumer and wholesale prices there seems to be positive skewness indicating that prices tend to increase faster than to decrease, which obviously makes sense. The behaviour of long-term interest rate may only reflect this same fact. The real exchange rate, in turn, is characterized by gradual deterioration of competitiveness and once-for-all devaluations of the currency. Money and credit seem to behave in the same way as stock prices in terms of skewness although the estimations results are somewhat different. With bankruptcies, the results represent some sort of puzzle. Industrial output and bankruptcies do not seem to be just mirror images – quite the contrary. Thus, there are some (although not very significant) signs of negative skewness indicating that peaks in bankruptcies are smaller than the corresponding troughs. This clearly indicates that bankruptcies are perhaps more related to financial and institutional variables than just to demand and output.

4.6 Results from stability analysis

The adjustment properties could, of course, be scrutinized in a straightforward way by looking at the parameter stability over depressions and booms. Table 7 contains some indicators of parameter stability for the univariate AR(4) which is used as some sort of point of departure in this study. Thus, we have computed the average lag length for depression (the shaded areas in Figure 1) and non-depression periods, the Chow stability test statistic in terms of the sample split and a F-test statistic for the significance of multiplicative (x_{t-i}*depression dummy) terms. It turns out that the stability property is at variance with the data. Moreover, there is some, although not very strong, evidence of asymmetric adjustment in the sense that the average lag length is shorter in depressions than in "normal years".

The stability measures are to some extent consistent with the evidence from the nonlinear adjustment model but also some clear inconsistencies arise. For instance, somewhat conflicting results are obtained for bankruptcies and stock prices. It should be noticed, however, that the classification of observations is based on output behaviour and the cyclical behaviour of other variables, such as stock prices, do no coincide with output movements and, therefore, the results cannot be identical.

Thus, if anything can be learned from this exercise, it is the fact that nonlinearities do seem to exist with the long Finnish times but there are clear differences between nominal and real variables. Thus, it is perhaps futile to analyze all sorts of nonlinearities using a single model as a frame of reference.

4.7 Evidence on long-memory properties

In time series, a long-term memory property is said to be present if absolute values of a stationary variable r_t has significant autocorrelations for long lags i.e. $\rho(|r_{t-k}|, |r_t|) \neq 0$, when k is large. This property was first noted for speculative price series by Taylor (1986) and called thereafter also the Taylor effect (see Granger and Ding (1993)). In practice, this property implies that the simple random walk model does not hold for stock prices, even if the price changes are serially uncorrelated. This phenomenon also shows up in the rescaled range analysis with the Hurst exponents. As pointed out earlier, high ($H > 0.5$) values of the Hurst exponent imply strong persistence – the fact that the observations (residuals) are independent but they have a memory. Thus, the data are not generated with random walk but, instead, with biased random walk or, with other words, with fractional brownian motion.

For instance if we consider stock price changes, it seems intuitively appealing to observe that they are uncorrelated, but this does not explain anything about the heteroskedasticity found in them. Statistically stock prices could be martingales with non-constant innovation variance (see e.g. Spanos (1986)). However, from the economic point of view the problem is to find out whether residual variance from linear model follow conditional heteroscedasticity (ARCH), generalized version of it (GARCH), asymmetric power ARCH (A-PARCH as defined in Ding, Granger and Engle (1993)) or some other form of heteroskedasticity appropriate for the particular time series.

However, univariate models could be helpful in identification and prediction of the type of heteroskedasticity, but likely insufficient for understanding these prosesses.[12]

Heteroskedasticity in residuals shows already that stronger forms of rational expectations rationality, which imply efficient use of all information, does not hold for higher moments of the process. In fact expectation error are not white noise, but rather innovation processes with non-constant variance. The long-memory phenomenon puts emphasis also to the long-term cyclical swings often accounted in economic time series. These cyclical swings could relate to business cycles or even Kutznets and Kontrajev cycles or tendency to generate serious financial crises as those withnessed in 1930's and 1980's. However, as Granger and Ding (1993) emphasize, that caution in interpretation should be maintained, since it is not the series themselves but their absolute values, that have the long-memory property.

If the efficient market hypothesis would hold strictly, the random walk property implies that r_t is an i.i.d process. In addition any transformation of r_t, like $|r_t|$ or r_t^2 should also be i.i.d process (Ding, Granger, Engle (1993), s. 87). The sample autocorrelations of i.i.d process will have finite variance $1/\sqrt{(T)}$ and larger correlations for $|r_t|$ will indicate long-memory property. Ding, Granger and Engle (1993) show that, if $|r_t|^d$ is taken for yardstick in measuring the strengthness of autocorrelation for long lags, the long-memory property is strongest around $d = 1$.

In the same way as Ding, Granger and Engle (1993), we found out that all variables in our data set showed clear evidence of long-memory, thus the sample autocorrelations for absolute values of residuals were greater than the autocorrelations of squared residuals. This resemblance could indicate that economic time series have characteristics of models, not fully described and understood so far.

Series, which had $|r_t|$ well above r_t^2 were industrial production, bankruptcies, bank loans and both price price indexes. (Clearly, this is consistent with results from the rescaled range analysis, i.e. with the estimates of the Hurst exponents). A little bit different were series like terms of trade and real exchange rate, money supply and stock prices, which mostly shared the same characteristics. This could due to rare, but large discrete changes in these series e.g. like the effects of devaluations. The results from these long-memory tests performed to AR(4)-residuals of our time series are presented in Table 8 below. Figures of sample autocorrelation functions for the absolute values of the AR(4) residuals are shown in Figures 8.

In this connection we also applied the fractional differencing model approach proposed by Geweke and Porter–Hudak (1983). Thus, we estimated the differencing parameter d for the AR(4) residual series. The results are presented in Table 8 (see the last column). Except for two cases, the estimates are positive suggesting the time series processes do indeed exhibit long memory (in the case of negative estimates, the processed could be said to have intermediate memory; see Chung and Ballie (1993)). Summing up, one can say

[12] Granger and Teräsvirta (1993, p. 51–53) note that a series may have short-memory in mean, and long-memory in variance, but not so likely the opposite i.e. long-memory in mean with short-memory in variance. Short-memory in mean is often found in stationary series, whereas long-memory is present in integrated "level" series.

that all the test we have applied thus far clearly suggest that a dominant feature of the data is just this long memory property.

Among other things these results indicate that linear filtering with AR(4) model is not sufficient to remove dependence on faraway past in these series, even though model selection criteria would suggest in most times 4th order autoregressive polynomial should be long enough. Despite the fact that these series have dominant long-run features like unit roots and trends, parsimonous linear models seem unable to account for this task. Observations refer therefore to conclusion that trends in economic time series are most likely stochastic rather than deterministic. Nonlinearities are hereby faced again.

The main message is however, that long-memory property is very persistently present in all of the real and monetary series. In addition there seems to be no difference between real and monetary variables about how fast autocorrelations would die out for long lags.

5 Testing dependencies between residual moments

The purpose of applying first an autoregressive model to the series is to remove the potential trend component from series. Removing deterministic or stochastic long term trend could be done by other means as well e.g. differencing or modelling by structural time series models and thereafter eliminating the trend component. We proceed by calculating dependency measures of different transformations of these AR(4) residuals.[13] Different moments of residual series and absolute values of residuals are considered as transformations. Therefore we calculate dependence tests from cross-autocorrelations between these univariate residuals as a first step in searching for dynamic relationships.

As could be seen this procedure looks like an extension of the Granger causality test. However, we start by calculating Portmanteau test statistics without conditioning on past observations of the transformed residuals of the series itself. Portmanteau tests give us potential evidence about the direction and strengthness of the dynamic dependencies between variables. If relationship is one-sided it simplifies greatly the identification of the sources of shocks in these series.

To test whether residuals of the autoregressive model satisfies properties of independent white noise series could be accomplished with calculating Portmanteau (Q) statistic. This test is designed to pick up departures from randomness among the k first auto- or crosscorrelations. Test has the following form

$$Q = T(T+2)\sum_{k=1}^{M}(T-k)^{-1}r_k^2,$$

where r_k^2 are the squared correlation of the residuals.

[13] We also computed the same measures with respect to the ARCH-model residuals. The results turned out to be so close to the results with squared OLS residuals that we do not report them.

This modification of the basic Box-Pierce statistic was first presented in Ljung and Box (1978). The test statistic is asymptotically $\chi^2(M)$ distributed when the original residuals are independent. There is no clear solution in choosing M, but in our case a too small values could result in a failure to detect dependencies between important higher order lags. As could be guessed, increasing M will on the other hand lead to lower power of the test (Harvey (1981), p. 211).

The Portmanteau statistic could be applied also to the higher moments or absolute values of stationary series as a general test against non-randomness. McLeod and Li (1983) have shown that for squared residuals have the same standard asymptotic variance (1/T) as the original series if the residuals are random. In the following tests we assumed lag order to be 24 (2 years) to be large enough to pick up long term dependencies between different moments of residuals. In our application economic theory has rather little to say about the lags between shocks leading to variation in other variables.

Table 9 presents a summary of the estimated Q test statistics. Only the number of significant cases is reported here. The test statistics have been computed both for leads and lags to get some idea of causality. A more detailed report of the results from cross-correlation analysis is available upon request from the authors.

With reference to the table we point out that in general the number of significant values is very high. Almost two third of the coefficients are significant at the 5 per cent level of significance. Particularly in the case of absolute values of the AR(4) residuals, the dependencies are very strong. In accordance with the results from univariate long-memory tests, the results in Table 9 suggest that the long-memory phenomenon also applies to co-movements of different variables – and not only within real and nominal variables but between all macroeconomic variables.

As for the role of different variables, one may note that the bankruptcy variable is very important in terms of the correlation structure. In fact, the number of significant correlations for bankruptcies is bigger than with all other variables. By contrast, the interest rate and M1 series are only moderately correlated with other variables.

The test results do not tell very much of causation. In general, the cross-correlation coefficients are of the same magnitude with respect to leads and lags. Therefore, it is very hard to draw any far-reaching conclusions on this matter. Perhaps one may still point out that industrial production looks like an exogenous variable while money supply is rather an endogenous than an exogenous variable.

Calculating the contemporaneous correlations between variables does not have any dynamic causal interpretation as it indicates only instantaneous linear co-movement (positive or negative) within a month. As could be seen from table 9, about one third of the off-diagonal correlations are significant at 5 per cent level. The interpretation of (significant) correlations is in most cases rather straightforward. Thus, for instance, consumer prices correlate in an expected way with, wholesale prices, monetary variables like credit, money aggregate, stock prices and the real exchange rate but not with other real variables.[14]

Altogether, the correlations between higher moments of the AR(4) residuals – in the same way as between the absolute values – are so strikingly high that further analysis in a multivariate nonlinear set-up is clearly required. The first step is simply to find out why volatility changes are so much related. In addition, one has to think about a possible explanation to the observed strong co-skewness between variables. Finally, one has also to take into account the fact that the long-memory property seems to apply also to the co-movements of different series – both nominal and real. It seems at least that a (multivariate) ARCH model is not a sufficient or a proper specification to account for these features of the data.

6 Concluding remarks

The empirical analyses which are presented in this paper have given strong and unambiguous support to the existence of nonlinearities in Finnish historical time series. The univariate case is very clear but it seems that nonlinearities may be even stronger and more important in the multivariate set-up. Obviously this calls for further research in this area.

It is surely not surprising that the exact nature of non-linearities cannot be identified. We are inclined to conclude that deterministic chaos is not the probable explanation. It is noticeable that Brock and Potter (1993) arrive at similar conclusion when they review some recent evidence from macroeconomic and financial data. Another explanation which is often mentioned in this context concerns ARCH and GARCH effects. It typically found that after these effects are accounted for the evidence for nonlinearity and chaos is weakened (see, e.g., Hsieh (1991)). In this study, we found the ARCH effects of minor importance. Thus, the explanations for nonlinearities must be looked for elsewhere. Nonlinearities may, for instance, reflect neglected nonstationarities but in this connection we would rather argue in favour of the specific (asymmetric) properties of short-run (cyclical) adjustment process. There can well be various institutional arrangements and constraints, informational deficiencies, capacity constraints and so on which prevent immediate and symmetric adjustment and which, in turn, explain the empirical findings. Finally, various stability tests clearly indicate that the behaviour of macroeconomic variables is quite different in recession and expansion periods.

It seems well possible that nonlinearities may change some widely accepted assumptions or results. Thus, for instance, the neutrality of money may not be so good approximation as is looks like in the context of linear models. It may also be that the conventional symmetric adjustment mechanisms represent a very poor framework for dynamic specification. Also the short and long run

[14] On the other hand it is interesting to note that wholesale prices do correlate with both real and monetary variables. Industrial production correlates only with wholesale prices and bankruptcies, but in both cases the sign of the correlation seems to be the opposite than expected. It is also hard to interpret why interest rate correlates positively with stock prices. According to present value formulae, the relation should be just opposite.

properties of different time series and the way in which the corresponding markets function need to carefully rethought in the light of, for instance, the long-memory results which have been obtained in this study. Finally, it may be that the importance of certain variables (and unimportance of the other variables) in the propagation mechanism of nominal and real shocks in the economy will change a lot if nonlinearities are taken into account. The Finnish data suggest that, for instance, bankruptcies is such a neglected variable.

Table 1. **Descriptive statistics for the residuals of a linear AR(4) model**

	skewness	kurtosis	median	med(–)	med(+)	stand.dev.
ip	–0.65	5.16	.274	–.008	.587	.056
bank	–0.59	4.57	.237	–1.154	2.424	.309
tt	0.67	27.13	.034	–.081	.146	2.261
fx	4.15	68.34	–.139	–.214	–.081	3.044
r	0.44	17.73	–.000	–.157	.157	.272
cpi	3.31	29.67	–.123	–.178	–.092	.013
wpi	3.01	20.92	–.117	–.181	–.069	.013
credit	0.06(1)	8.34	.016	–.033	.062	.011
M1	0.89	17.02	.040	–.129	.129	.025
sx	–0.12(1)	5.40	.063	–.262	.290	.049

Skewness and kurtosis denote the coefficients of skewness and kurtosis, respectively. Median denotes the sample median, med(–) and med(+) denote the endpoints of the confidence interval for the median. In the case of log transformation, the values of the median, med(–) and med(+) have been multiplied by 100. ip denotes (log) industrial production, bank (log) bankruptcies, tt terms of trade, fx the real exchange rate index, r yield on long-terms government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money and sx the (log) UNITAS stock price index. The sample period is 1922M5–1994M10. (1) Not significant at the 5 per cent level.

Table 2. **Diagnostic test statistics for a linear AR(4) model 1920M5–1993M3**

	ARCH	RESET1	RESET2	Func. form	WHITE	J–B	TSAY
ip	19.15	0.42	2.18	13.11	10.88	272.1	7.53
bank	17.61	4.59	7.34	14.98	24.17	237.1	33.41
tt	10.25	8.77	8.62	3.90	5.21	2645.6	30.07
fx	7.98	4.07	8.68	4.55	2.81	1066.4	83.10
r	2.79	0.57	0.79	2.10	1.00	1698.0	16.55
cpi	40.26	2.78	3.72	22.42	13.17	461.0	101.21
wpi	7.78	3.84	5.15	8.84	9.13	550.4	100.50
credit	21.99	13.99	7.02	13.70	6.90	722.3	33.16
M1	27.67	7.70	6.60	34.28	14.36	1362.5	163.31
sx	46.19	0.04	0.23	43.56	7.08	403.9	44.76
5 %	2.02	3.85	1.70	2.61	1.65	3.8	18.31
1 %	2.66	6.66	2.10	3.80	2.01	6.0	22.21

ARCH denotes the Engle's ARCH test statistic (with 7 lags), RESET1 test statistic adds the second power of the fitted value as an additional regressor RESET2 includes both the second and third powers of y. Func. form is the F-test of the second power of the explanatory variables and their cross-terms included into the regression. White denotes White' heteroskedasticity/ functional form test statistic, J–B the Jarque-Bera test statistic for residual normality and TSAY Tsay's nonlinearity test statistic for 4 lags. 1 % and 5 % denote the critical values of the respective test statistics.

Table 2. **continued**

	r(1,1)	r(1,2)	r(1,3)	r(1,4)	r(2,2)	r(2,3)	r(2,4)	r(3,3)	r(3,4)	r(4,4)
ip	−.142	.112	−.011	−.119	.114	−.006	.031	−.144	.019	.086
bank	.194	.005	−.101	.021	.115	−.115	−.049	.206	−.019	.192
tt	−.237	−.002	−.123	−.041	.106	.032	.018	−.262	−.083	−.027
fx	−.494	−.370	−.404	.152	−.560	.345	−.634	−.547	.193	−.351
fr	−.237	−.049	.013	.038	−.039	−.056	−.046	−.142	−.121	−.418
cpi	.619	.393	−.498	−.598	−.042	−.019	.353	.007	.796	.796
wpi	−.353	.113	−.118	.001	.137	.052	.302	−.378	.124	.044
credit	.124	−.147	−.212	.055	.112	−.113	.148	.069	.198	.009
M1	−.495	−.089	.313	.134	−.837	−.040	.266	−.638	.035	−.297
sx	.298	.188	−.038	.015	−.429	−.133	.148	.058	.115	−.031
logistic map	.669	.536	.556	.558	.848	.544	.561	.833	.669	.536
random N(0,1)	−.040	−.015	−.011	−.016	−.005	.020	−.050	−.055	−.039	−.015

r_{ij}'s are Hsieh's (1991) third order moment coefficients $[\Sigma x_t x_{t-i} x_{t-j}/T]/[\Sigma x_t^2/T]^{1.5}$.

Table 3. **Estimates of correlation dimension with AR(4) residuals**

	Embedding dimension			
	2	3	4	5
ip	1.97	2.78	3.60	4.47
	(0.05)	(0.08)	(0.08)	(0.09)
bank	1.84	2.68	3.68	4.62
	(0.45)	(0.57)	(1.16)	(1.61)
tt	1.86	2.59	3.19	3.78
	(0.31)	(0.30)	(0.21)	(0.27)
fx	1.68	2.42	3.28	4.08
	(0.52)	(1.02)	(2.08)	(2.67)
cpi	1.87	2.67	3.32	3.53
	(0.43)	(0.57)	(0.38)	(2.29)
wpi	.84	1.04	1.19	1.24
	(1.59)	(4.50)	(9.08)	(11.36)
credit	1.77	2.54	3.35	4.12
	(0.33)	(0.46)	(0.69)	(0.80)
sx	1.81	2.66	3.49	4.19
	(0.43)	(0.60)	(0.59)	(0.43)
random N(0,1)	2.03	2.93	3.87	4.81
	(0.06)	(0.05)	(0.04)	(0.15)
henon map	1.28	1.29	1.27	1.30
	(3.55)	(2.69)	(0.36)	(0.92)
logistic map	0.91	0.98	1.01	1.03
	(0.10)	(0.37)	(0.88)	(1.67)

Numbers inside parentheses are chi-square test statistics for the goodness of fit.

Table 4. **BDS test statistics for the residuals of a linear AR(4) model**

	m=2 ε=0.5	m=3 ε=0.5	m=4 ε=0.5	m=10 ε=0.5	m=2 ε=1.0	m=5 ε=1.0
Original AR(4) residuals						
ip	12.3	17.7	22.3	29.5	10.7	20.6
bank	9.0	11.2	13.0	15.3	10.3	16.2
tt	11.5	14.4	17.7	26.3	8.7	15.0
fx	15.7	17.7	19.5	22.3	17.1	16.8
r	13.4	16.4	18.4	20.0	8.6	11.1
cpi	10.7	14.6	16.1	20.9	11.3	14.6
wpi	8.1	10.4	12.5	15.8	10.7	12.9
credit	10.7	14.4	18.3	23.5	11.4	18.5
M1	22.6	34.1	54.3	86.3	13.7	22.2
sx	7.8	8.5	9.7	10.9	9.1	13.3
random N(0,1)	0.6	0.2	–0.3	–0.4	0.7	0.1
Henon map	165.9	280.8	428.9	717.7	76.0	91.3
logistic map	669.1	881.1	1152.1	1570.0	282.0	250.3
ARCH(4) residuals of an AR(4) model						
ip	10.0	13.9	15.8	17.7	4.6	10.1
bank	11.5	14.2	16.3	18.1	9.9	12.0
tt	3.0	4.3	5.9	6.3	1.8	5.1
fx	6.1	9.1	9.2	9.2	1.7	4.6
r	5.5	5.5	5.5	6.0	3.8	4.3
cpi	13.7	14.5	14.3	14.3	8.8	9.3
wpi	10.2	10.4	10.2	9.6	10.8	9.7
credit	14.1	15.9	16.7	17.4	11.6	13.1
M1	12.8	13.4	14.0	14.1	9.3	9.6
sx	10.4	13.6	15.7	17.4	9.5	13.7
Shuffled AR(4) residuals						
ip	–2.2	–1.4	–1.0	0.4	–2.5	–1.3
bank	–0.8	–0.2	–0.3	0.4	–0.9	1.1
tt	1.6	2.1	2.0	1.9	1.9	1.7
fx	0.4	1.0	0.7	0.5	1.4	1.5
r	1.9	1.6	1.3	1.2	1.7	1.0
cpi	2.7	2.6	2.3	0.2	0.7	1.2
wpi	–1.0	–1.6	–1.5	–1.2	–1.3	–1.6
credit	0.4	–0.1	–0.6	–0.4	–0.6	–0.8
M1	1.9	1.1	0.5	0.2	2.5	1.1
sx	–0.5	0.0	0.5	0.5	–0.7	1.0
Henon map	0.6	0.3	0.2	0.3	0.1	–0.0
logistic map	1.0	1.3	2.2	0.2	–0.2	1.2

The test statistic is BDS = $T^{½}[C_m(\varepsilon)-C_1(\varepsilon)^m]/\sigma_m(\varepsilon)$, where T = N – m + 1 and N = the number of observations, $C_m(\varepsilon)$ = the correlation integral = T^{-2}*[number of pairs (i,j) such that $| y_i - y_j | < \varepsilon$, $| y_{i+1} - y_{j+1} | < \varepsilon, \ldots,$ $| y_{i+m-1} - y_{j+m-1} | < \varepsilon$] so that $y_i,\ldots,y_{i+m-1}$ and $y_j,\ldots,y_{j+m-1}$ are two segments of the series y_t of length m and $\sigma_m(\varepsilon)$ is the respective standard deviation. Under the null that the series is independently and identically distributed, BDS has a limiting standard normal distribution. Here, ε = 0.5 corresponds to ε = 0.5*{the standard deviation of the residual series}. ε = 1.0 is defined in the same way. ip denotes (log) industrial production, bank (log) bankruptcies, tt terms of trade, fx the real exchange rate index, r yield on long-terms government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money and sx the (log) UNITAS stock price index. The shuffled series are obtained by sampling randomly with replacement from the data until one obtains a shuffled series of the same length as the original. The sample period is 1922M5–1994M10.

Table 5. **Estimates of the Hurst exponent**

	AR(4) residuals		Shuffled AR(4) residuals
ip	.592		.593
bank	.586		.620
tt	.567		.607
fx	.659		.428
cpi	.765		.516
wpi	.768		.595
credit	.716		.651
sx	.698	(.105)	.614
random N(0,1)	.588		.519
Henon map	.381		.355
logistic map	.426		.565

Estimates correspond to the whole range of data. The graphs of the R/S series are presented in Figure 7. On the basis of this figure, one may conclude whether the slope (i.e., the estimate of the Hurst exponent) is constant over all data points. In the case of stock prices (sx) instability is obvious. Therefore, two estimates are presented, first for the time span 0–200 observations (months) and the second (inside parentheses) for the time span of 200 and more observations.

Table 6. **Estimation results of a nonlinear AR model**

	a_0	a_1	a_2	a_3	a_4	a_5	a_6	SEE	DW	F3
ip	.319	.098	.580	.157	.055	–.771	–.525	.056	2.09	2.68
	(3.21)	(2.89)	(10.33)	(2.42)	(2.74)	(2.63)	(0.59)			
bank	.926	.070	.271	.156	.097	–.744	–.013	.325	2.23	3.70
	(3.64)	(1.30)	(3.24)	(1.20)	(2.74)	(2.24)	(0.35)			
tt	.218	.689	1.103	–.619	.343	–.499	–.570	.023	2.06	10.31
	(2.71)	(1.26)	(13.57)	(4.88)	(2.76)	(3.06)	(4.08)			
fx	.211	–.966	1.132	–.598	.295	–.301	–.230	.038	1.89	15.28
	(2.97)	(1.51)	(16.46)	(5.38)	(3.19)	(3.91)	(4.77)			
r	.458	.062	.894	.274	–.016	.031	.029	.259	1.95	1.47
	(0.70)	(1.36)	(9.66)	(1.65)	(0.69)	(0.50)	(1.40)			
cpi	–.132	.025	1.408	–.406	–.003	.048	1.168	.014	2.13	3.80
	(0.81)	(2.57)	(34.48)	(9.88)	(2.66)	(2.12)	(0.44)			
wpi	–.161	.024	1.553	–.487	–.007	.020	–13.560	.015	2.13	10.83
	(3.09)	(2.33)	(37.24)	(10.63)	(3.13)	(2.83)	(3.54)			
credit	–.017	.020	1.460	–.454	–.001	.001	–102.35	.011	2.16	11.60
	(0.94)	(2.04)	(34.58)	(10.66)	(1.26)	(0.93)	(4.88)			
M1	–.030	.058	.738	.278	–.002	.006	6.256	.025	2.00	8.20
	(1.44)	(2.21)	(17.43)	(6.35)	(2.19)	(1.71)	(3.54)			
sx	.000	.001	1.284	–.309	.000	.000	.158	.049	1.97	0.53
	(0.05)	(4.08)	(32.04)	(7.70)	(0.32)	(0.07)	(0.17)			
rr	.009	–.000	1.193	–.194	–.001	.000	–.022	.484	2.069	1.15
	(0.17)	(0.14)	(33.80)	(5.47)	(1.07)	(1.42)	(1.43)			
c/y	.006	.000	.740	.147	.557	–.004	–15.005	.010	2.032	4.22
	(1.26)	(1.05)	(15.35)	(2.42)	(1.89)	(2.65)	(0.83)			
m/y	.001	.000	.805	.147	.659	–.041	–410.62	.003	1.960	1.35
	(0.82)	(0.64)	(15.22)	(2.47)	(0.78)	(1.11)	(1.35)			

The estimating equation is of the form: $x_t = a_0 + a_1 t + a_2 x_{t-1} + a_3 x_{t-2} + a_4(x_{t-1}x_{t-2}) + a_5(x_{t-1}^3 x_{t-2}) + a_6(x_{t-1} - x_{t-2})^3 + \mu_t$, where μ is the random term. If we restrict $a_4 = a_5 = a_6 = 0$, we end up with a standard linear model. F3 represents a F test statistic for this restriction. The corresponding 5 % (1 %) critical value(s) is 2.64 (3.86). ip denotes (log) industrial production, bank (log) bankruptcies, tt terms of trade, fx the real exchange rate index, r yield on long-terms government bonds, cpi the (log) consumer price index, wpi the (log) wholesale price index, credit the (log) banks' total credit supply, M1 the (log) narrow money and sx the (log) UNITAS stock price index. In this case, we also consider three additional real variables which are rr the five-year real interest rate, c/y banks' credit supply in relation to nominal output (i.e., ip*wpi) and m/y the corresponding measure in terms of money supply. The sample period is (with some exceptions) 1922M5–1994M10. Coefficient a_5 has been divided by 1000.

Table 7. **Some stability test results**

	Average lag length		Stability tests	
	I	II	Chow	Dummy test
ip	1.44	1.80	5.08	5.18
bank	2.12	2.12	0.68	0.30
tt	0.42	0.74	3.06	3.24
fx	0.88	0.89	3.05	3.03
r	0.79	1.01	1.32	1.55
cpi	0.30	0.83	8.14	10.10
wpi	0.33	0.46	3.64	3.77
credit	0.48	0.38	2.51	2.41
M1	0.68	1.52	8.92	11.75
sx	0.72	0.68	3.77	4.52
5 %	..	..	2.22	2.38
1 %	..	..	3.04	3.34

The average lag length is computed for the depression periods (I) and non-depression periods (II). Chow notes a Chow test statistic for the hypothesis that the coefficients of the AR(4) model are the same for these two subperiods. Dummy test denotes a F test for the multiplicative dummy*x_{t-1} -terms.

Table 8 **Long-memory tests for AR(4) residuals of the historical time series, Period: 1922/M1–1993/M6**

	Significance level of the Ljung-Box Q(60) statistic for residual transformation			First order autocorrelation coefficients for residual transformations			Geweke and Porter–Hudak differenting parameter estimates
Variable	r_t	$\|r_t\|$	r_t^2	r_t	$\|r_t\|$	r_t^2	$\hat{d}$
ip	.000	.000	.000	–.006	.289**	.137**	.249
bank	.000	.000	.000	–.000	.208**	.084*	.212
tt	.004	.000	.000	.016	.187**	.036	–.026
fx	.528	.000	.027	–.013	.392**	.095*	.134
r	.037	.000	.000	–.002	.247**	.058	.020
cpi	.000	.000	.000	–.013	.388**	.302**	.338
wpi	.003	.000	.000	–.007	.324**	.180**	.132
credit	.000	.000	.000	–.008	.351**	.317**	.386
M1	.000	.000	.000	–.004	.423**	.346**	.439
sx	.001	.000	.000	.000	.268**	.182**	–.244

* = significant at 5 per cent level ($\pm 2/\sqrt{T}$) = 0.068
** = significant at 1 per cent level ($2.58/\sqrt{T}$) = 0.088

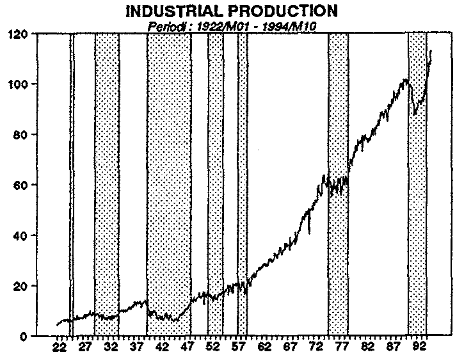
Table 9. **Number of significant Box–Ljung test statistics for the cross-correlation coefficients of different powers of AR(4) residuals**

	u	u^2	u^3	$\|u\|$	u_t
ip	3(7)	6(8)	1(6)	7(10)	0
bank	6(8)	8(8)	8(8)	10(9)	1
tt	7(2)	9(7)	5(4)	10(5)	3
fx	5(6)	5(6)	2(5)	8(9)	3
r	2(1)	5(4)	6(4)	6(5)	2
cpi	7(6)	7(9)	6(5)	8(9)	5
wpi	7(8)	8(8)	7(7)	9(9)	4
credit	8(7)	7(7)	6(6)	8(9)	3
M1	6(2)	4(4)	5(2)	8(9)	3
sx	6(6)	7(6)	6(5)	8(8)	5

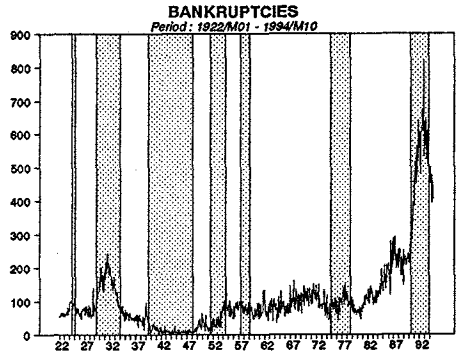
The first number indicates the number of significant Box–Ljung test statistics at the 5 per cent level of significance for the first 24 positive lags of the other variable. The second column (inside parentheses) indicates the corresponding number for the same number of leads. u indicates untransformed residuals, u^2 squared residuals, u^3 third power of residuals, $|u|$ absolute values of residuals and u_t contemporaneous values of residuals (these values are computed from simple correlation coefficients).

Figure 2. **Historical Finnish time series**

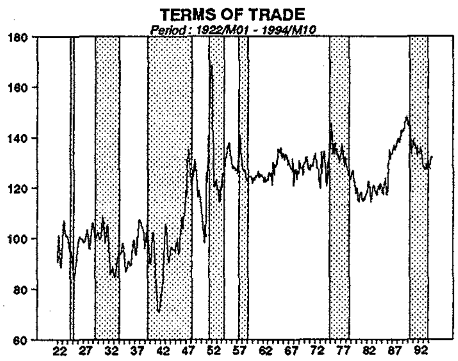
Industrial production



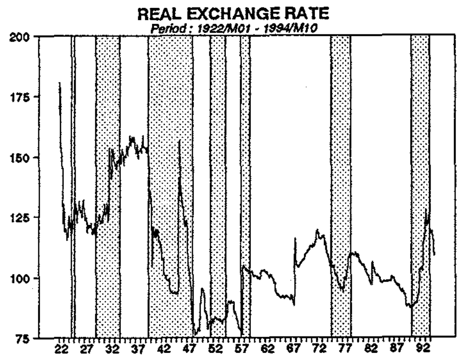
Bankruptcies



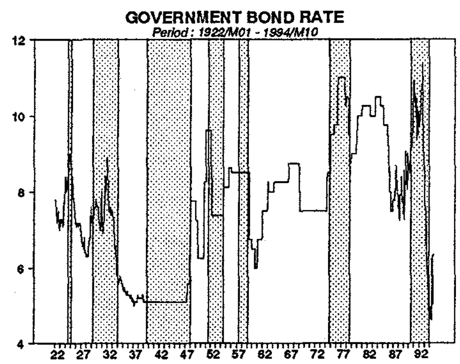
Terms of trade



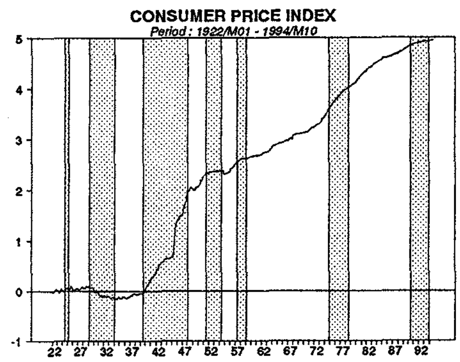
The real exchange rate index



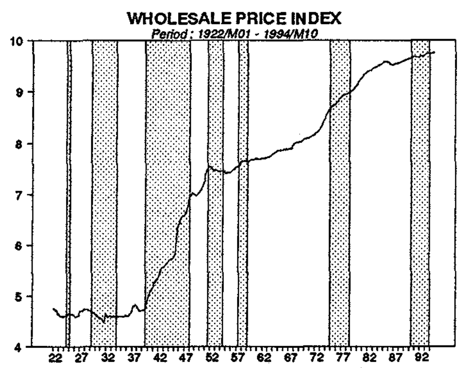
Yield on long-terms government bonds



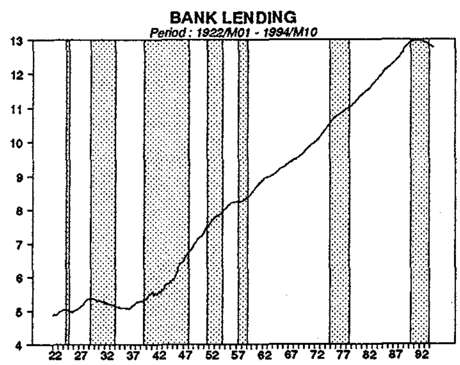
The consumer price index



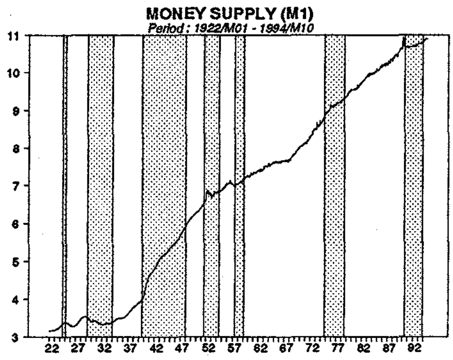
The wholesale price index



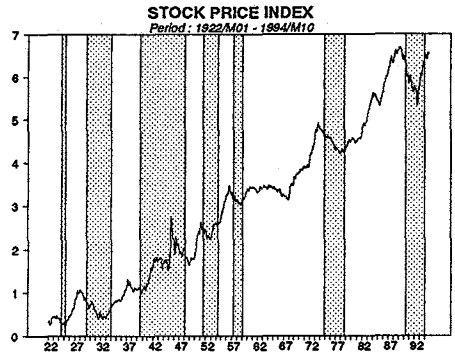
Bank's total credit supply



Narrow money (M1)



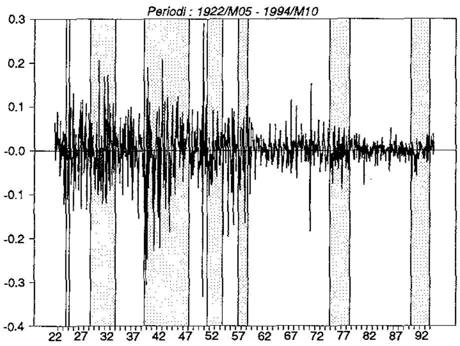
The Unitas stock exchange index



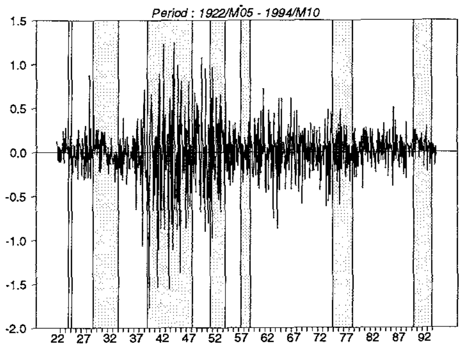
Nominal variables (except for the interest rate) are expressed in logs.

Figure 3. **Time series of AR(4) residuals**

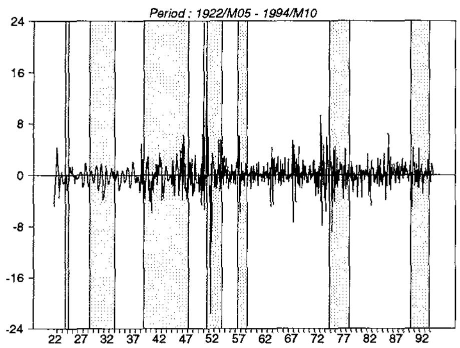
Industrial production



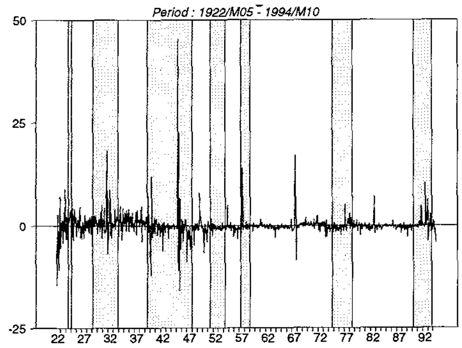
Bankruptcies



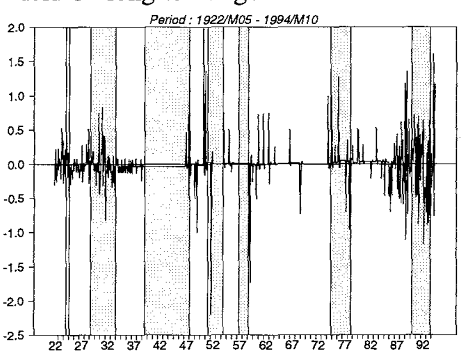
Terms of trade



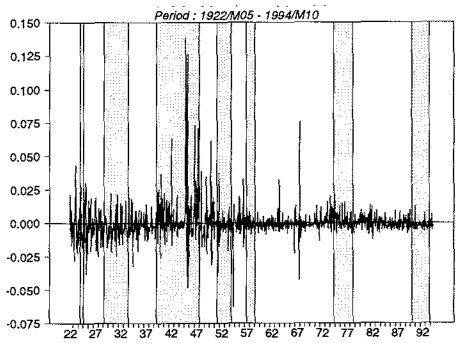
The real exchange rate index



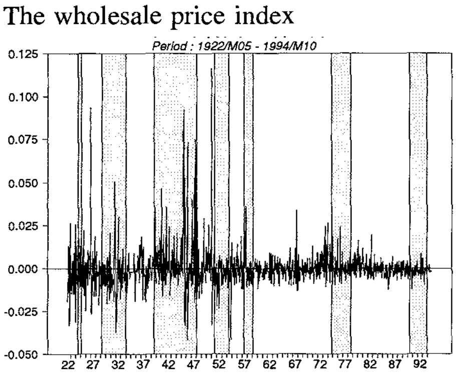
Yield on long-terms government bonds



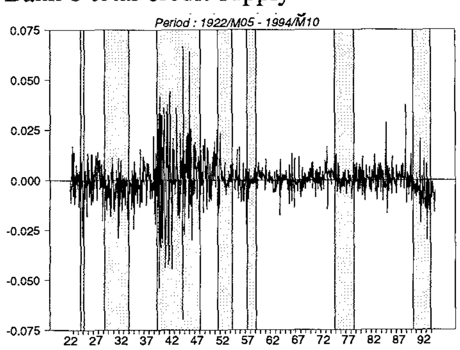
The consumer price index



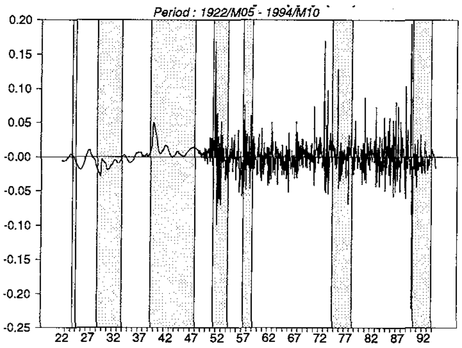
The wholesale price index



Bank's total credit supply



Narrow money (M1)



The Unitas stock exchange index

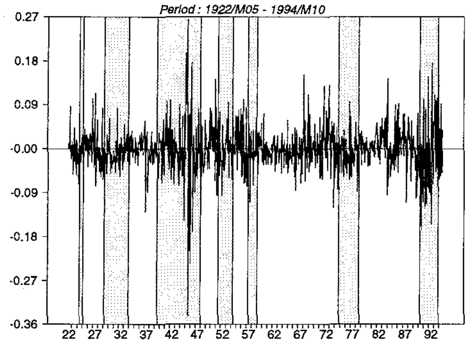


Figure 4. **Frequency distribution of AR(4) residuals**

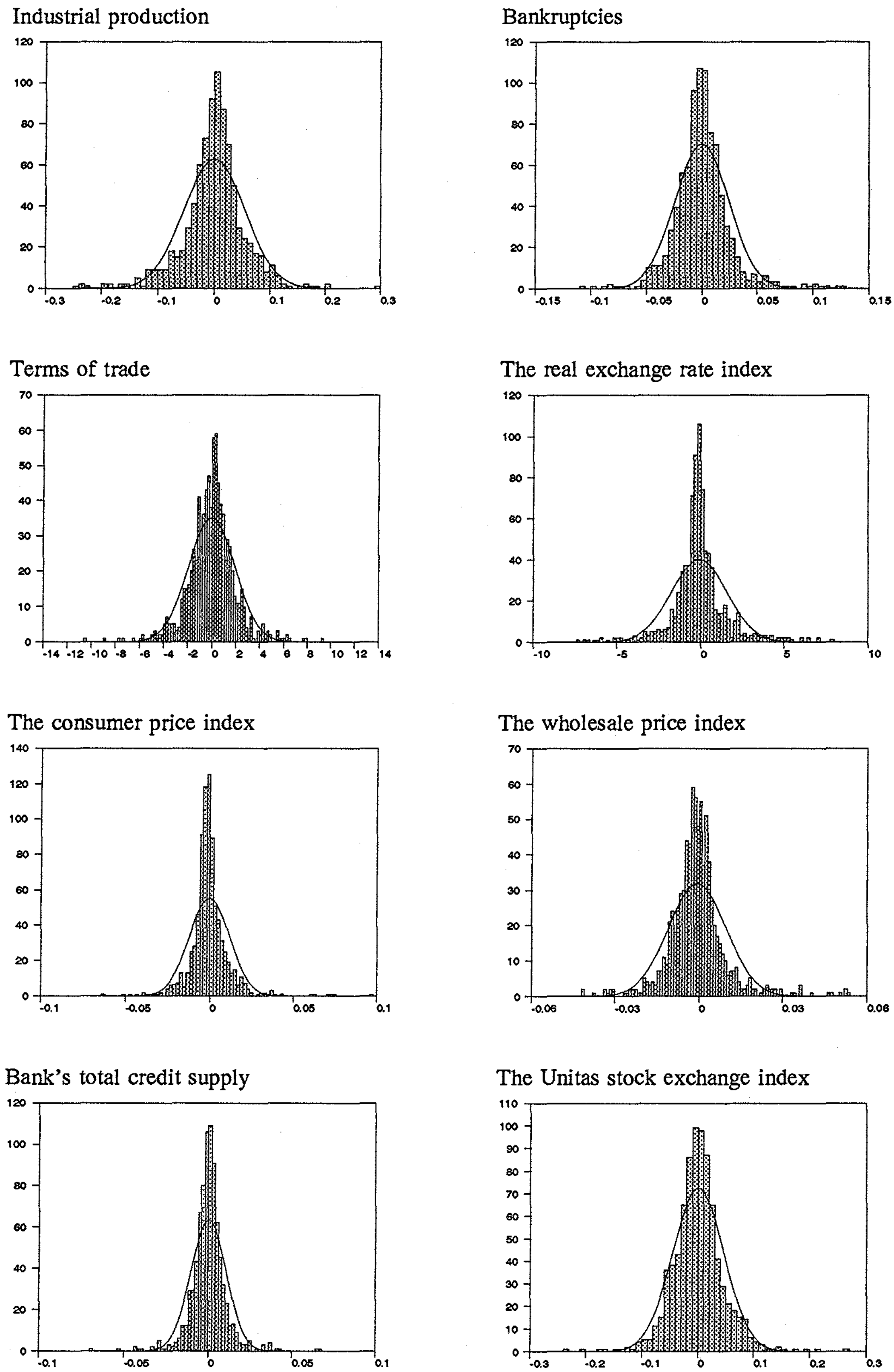
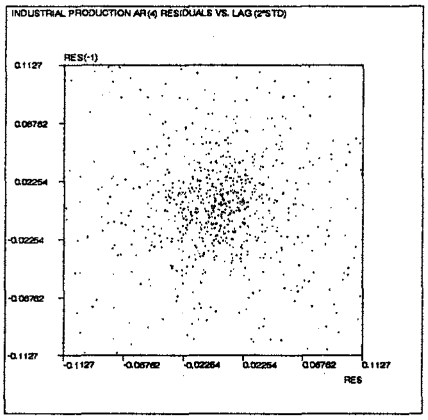
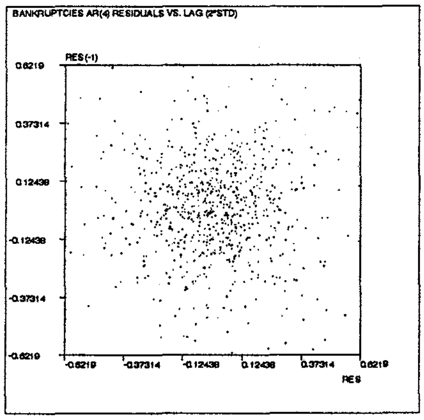


Figure 5. **Two-dimensional plots of AR(4) residuals**

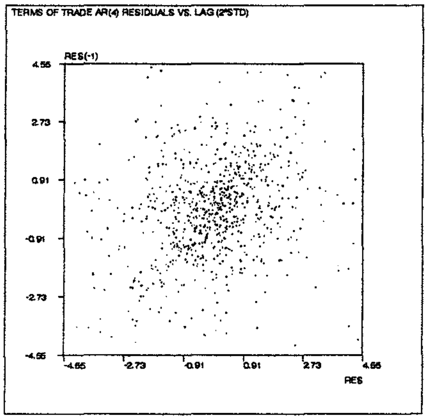
Industrial production



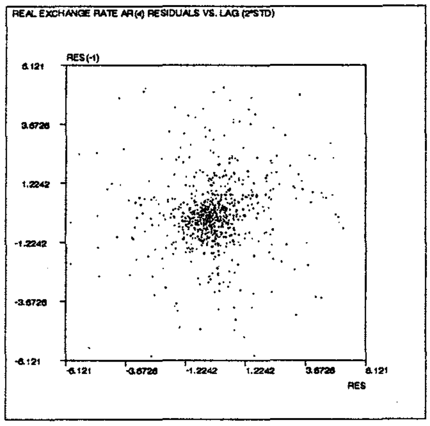
Bankruptcies



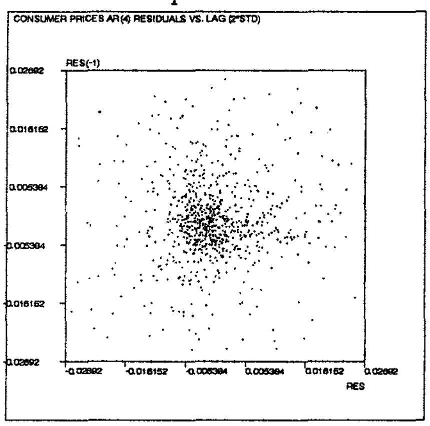
Terms of trade



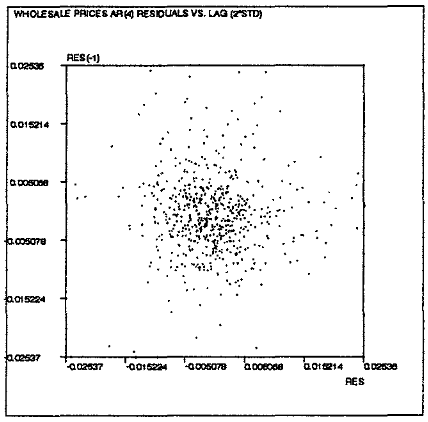
The real exchange rate



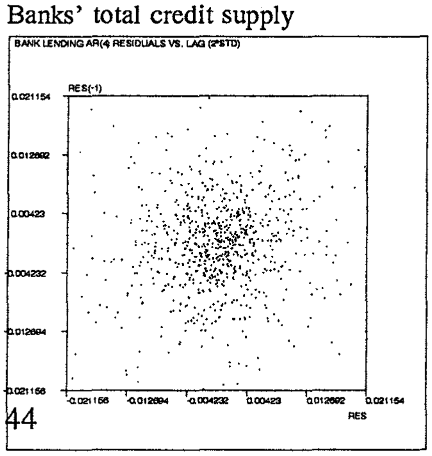
The consumer prices index



The wholesale price index



Banks' total credit supply



The Unitas stock exchange index

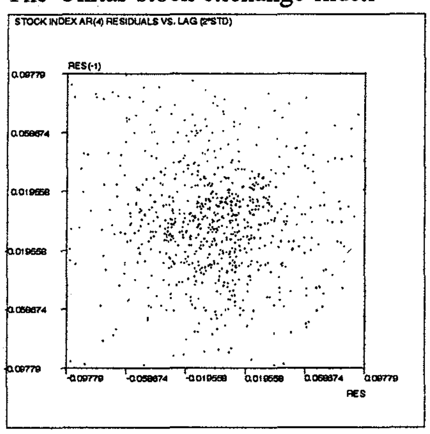
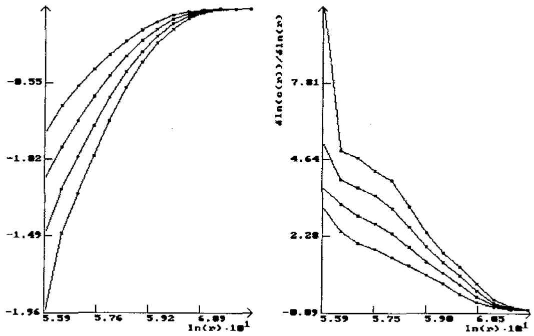
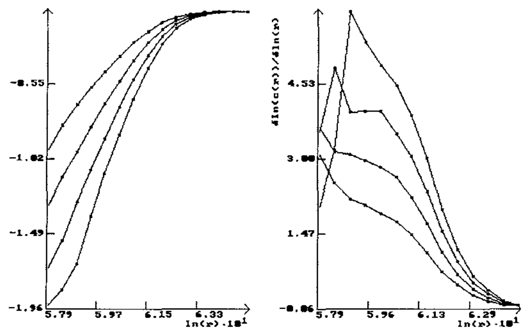


Figure 6. **Correlation dimension estimates**

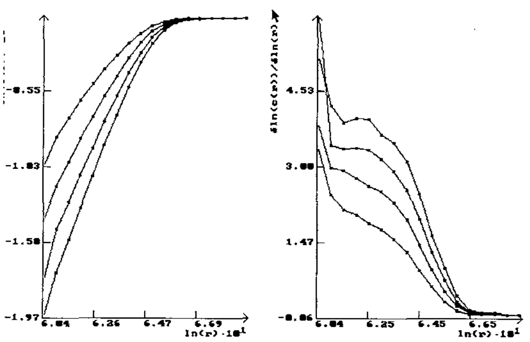
Industrial production



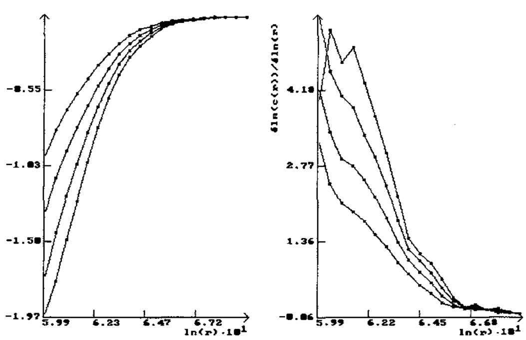
Bankruptcies



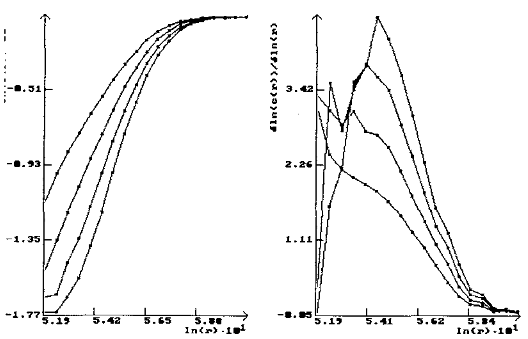
Terms of trade



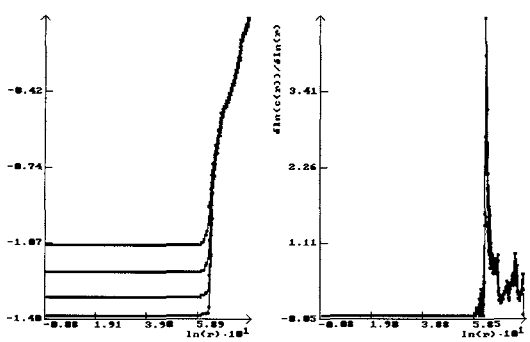
The real exchange rate



The consumer price index

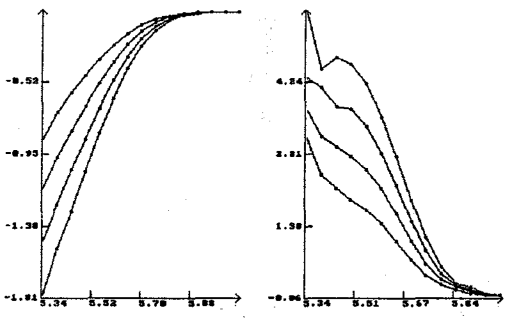


The wholesale price index



The x-axis in all figures is $10*\log(\varepsilon)$, the y-axis in the left-hand- side figure is $10*\log(C_m(\varepsilon))$ and in the right-hand-side figure $\partial\log(C_m(\varepsilon))/\partial\log(\varepsilon)$.

Banks' total credit supply



The Unitas stock exchange index

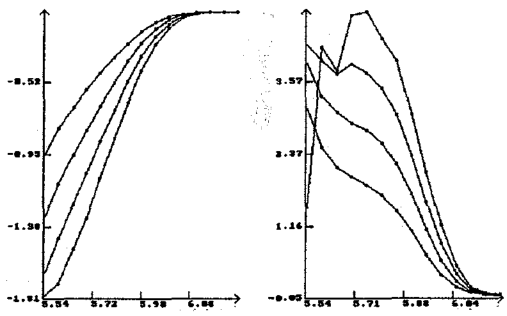
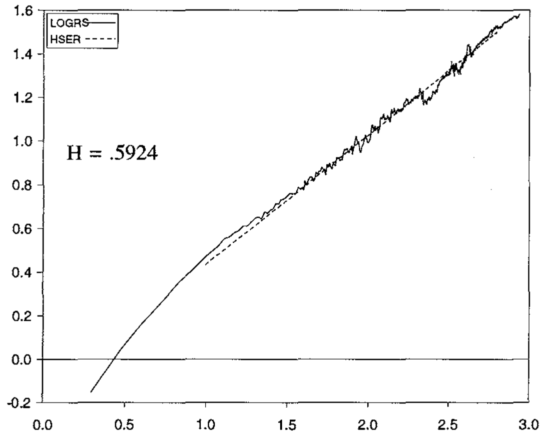
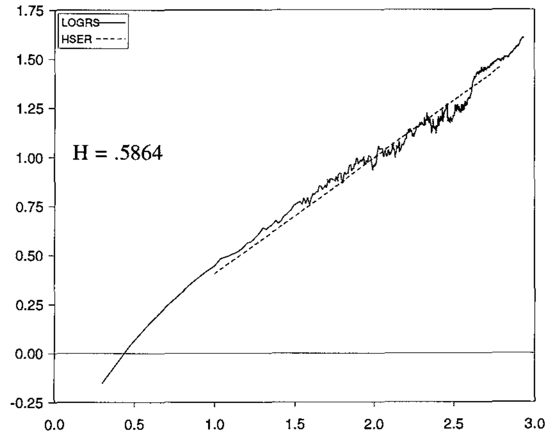


Figure 7. **Hurst exponents calculated for range 1.0–2.8**

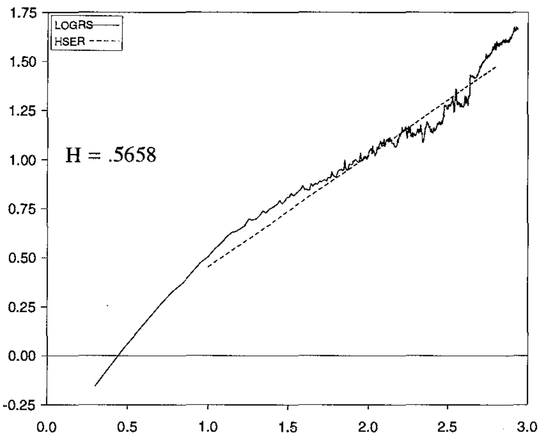
Industrial production



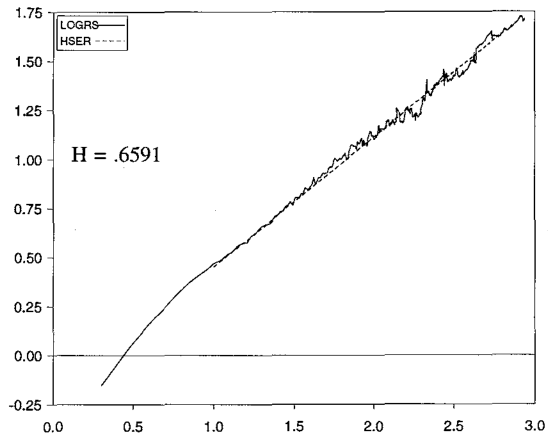
Bankruptcies



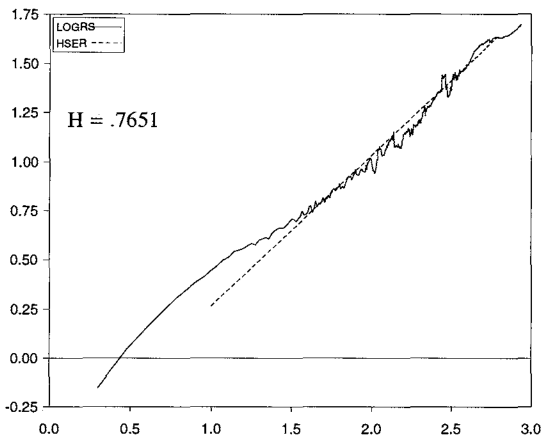
Terms of trade



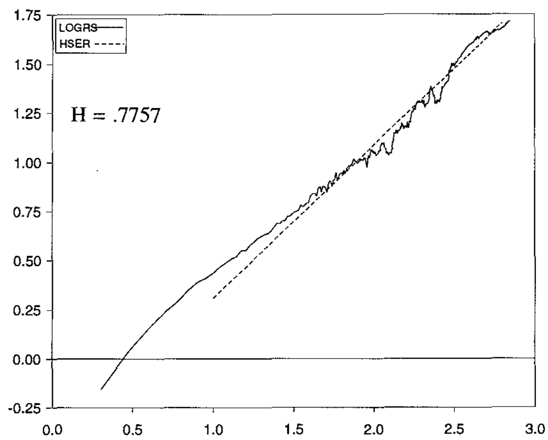
The real exchange rate index



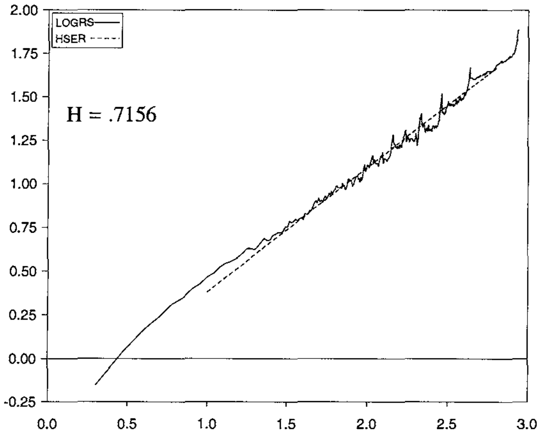
The consumer price index



The wholesale price index



Bank's total credit supply



The Unitas stock exchange index

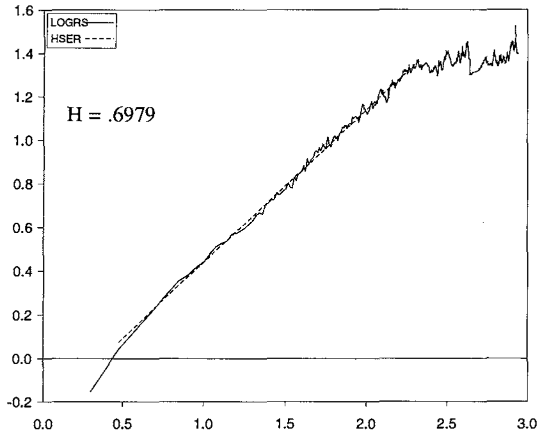
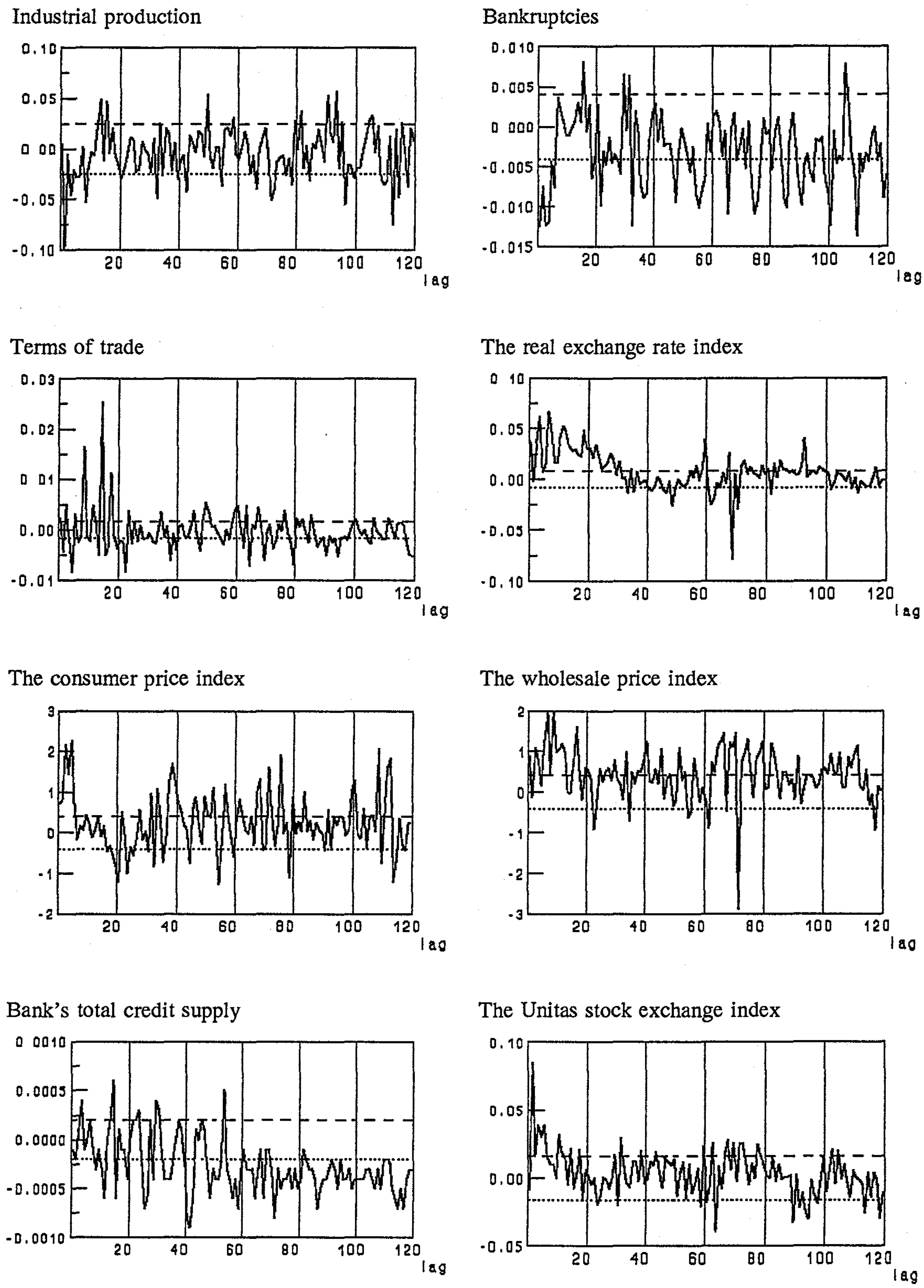


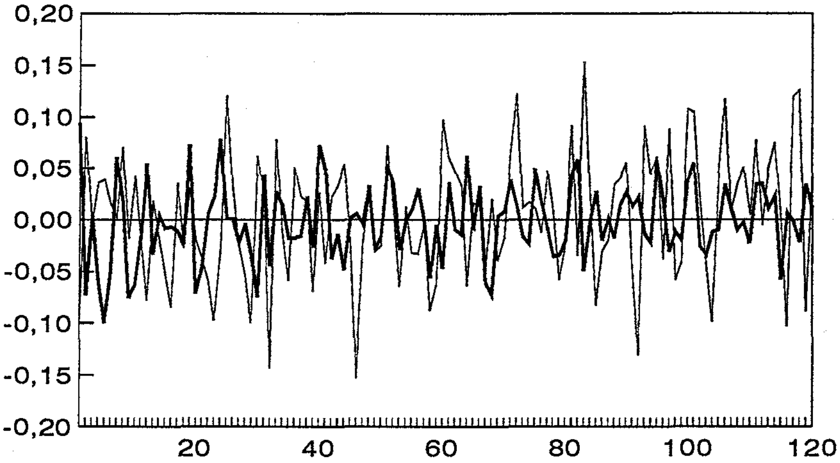
Figure 8.1 **Ramsey irreversibility test statistics**



The solid line denotes the $G^{k}_{2,1}$ test statistic, above and below it are the corresponding 5 per cent confidence limits.

Figure 8.2 **Ramsey irreversibility test statistics for ip and sx**

$G^k_{3,1}$-statistic for ip (thin line) and sx (bold line)



$G^k_{2,1}$-statistic for ip (thin line) and sx (bold line)

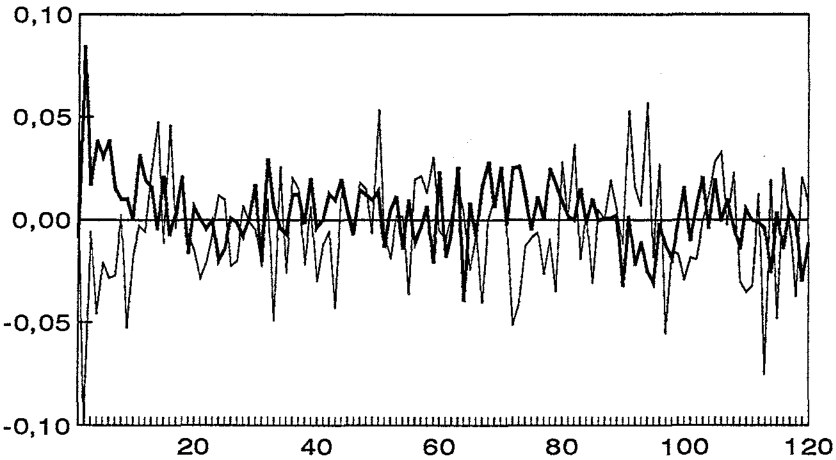
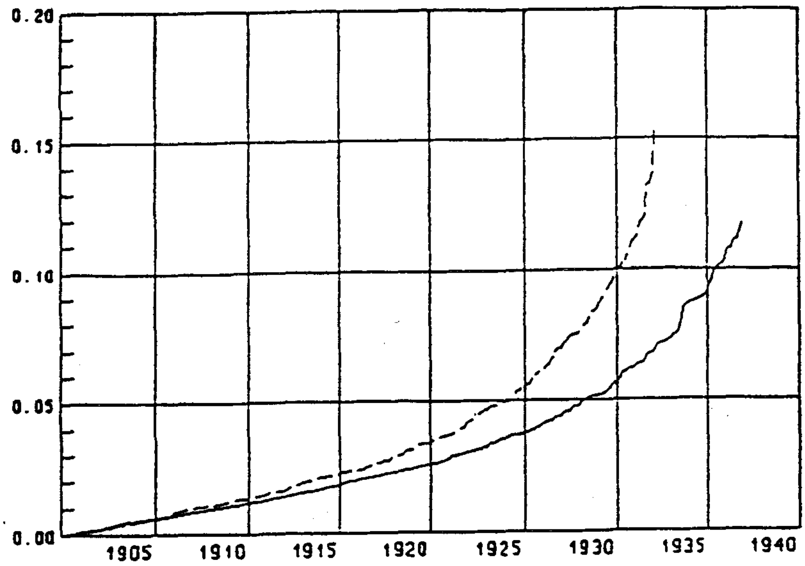


Figure 9. **Residuals for industrial production and stock prices**

Positive AR(4) residuals of industrial production ————
Absolute values of negative AR(4) residuals of industrial production — — — —



Positive AR(4) residuals of stock prices ————
Absolute values of negative AR(4) residuals of stock prices — — — —

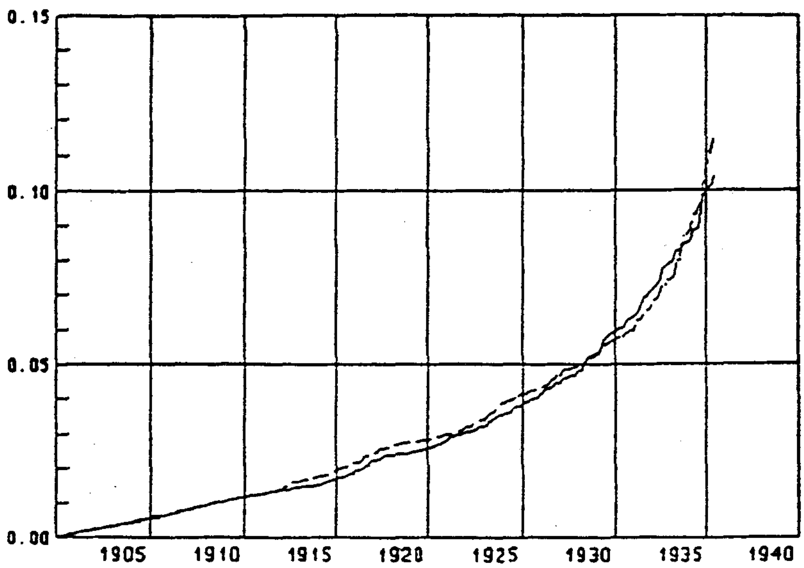
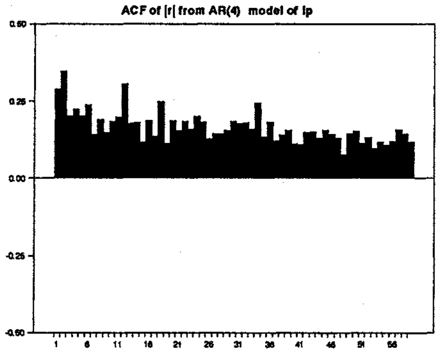
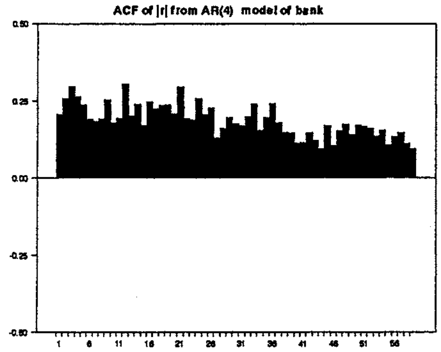


Figure 10. Autocorrelations of absolute values of AR(4) residuals

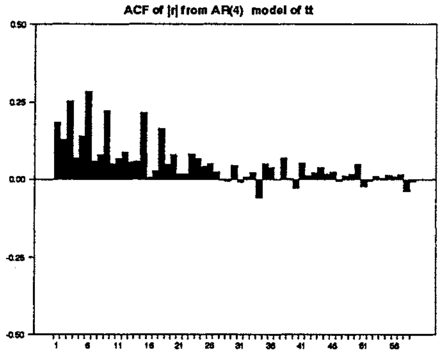
Industrial production



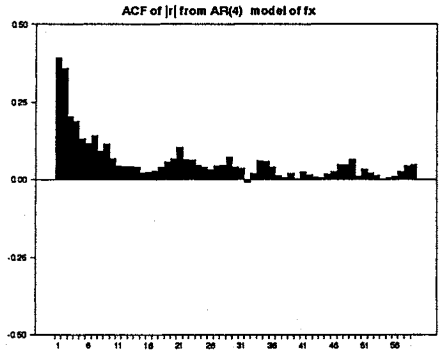
Bankruptcies



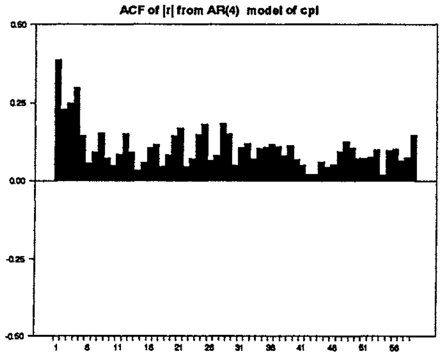
Terms of trade



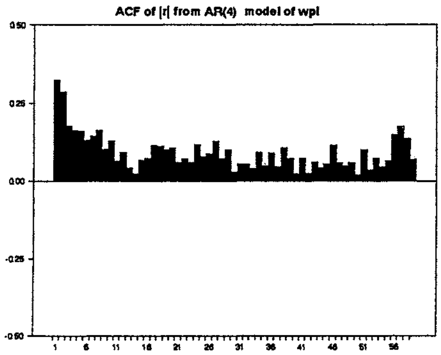
The real exchange rate index



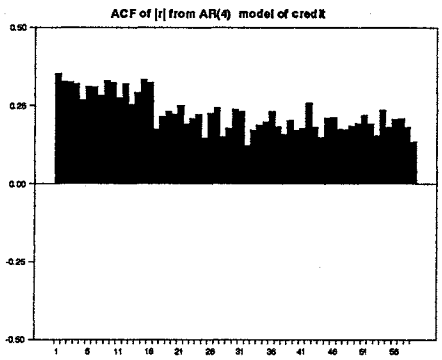
The consumer price index



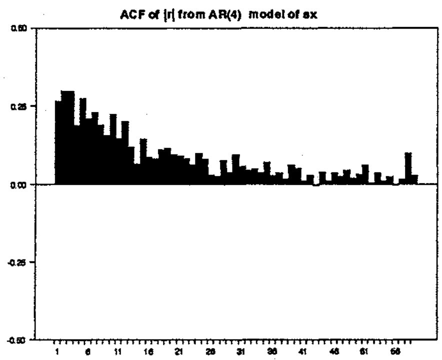
The wholesale price index



Bank's total credit supply



The Unitas stock exchange index



References

Bernanke, B. (1983) **Nonmonetary Effects of the Financial Crisis on the Propagation Mechanism of the Great Depression.** American Economic Review 73, 257–276.

Brock, W. (1986) **Distinguishing random and Deterministic Systems: Abridged Version.** Journal of Economic Theory 40, 168–195.

Brock, W.A., Hsieh, D.A. and LeBaron, B. (1991) **Nonlinear Dynamics: Chaos, and Instability.** The MIT Press, Cambridge MA.

Brock, W.A and Potter, S.M. (1993) **Nonlinear Time Series and Macroeconometrics,** in Maddala, G.S., Rao, C.R. and D.H. Vinod (eds.) Handbook of Statistics, Vol. 11, Elsevier Science Publishers B.V., 195–229.

Brock, W.A. and Sayers, C. (1988) **Is the Business Cycle Characterized by Deterministic Chaos?** Journal of Monetary Economics 22, 71–90.

Chung, C.–F. and Baillie, R.T. (1993) **Small Sample Bias in Conditional Sum-of-Squares Estimators of Fractionally Integrated ARMA Models.** Empirical Economics 18, 791–806.

Dechert, W.D. and Gencay, R. (1993) **Lyapunov Exponents as a Nonparametric Diagnostic for Stability Analysis,** in Nonlinear Dynamics, Chaos and Econometrics. Pesaran, M.H. and Potter, S.M. (editors), John Wiley, New York, 229-237.

DeGrauwe, P., Dewachter, H. and Embrechts, M. (1993) **Exchange Rate Theory: Chaotic Models of Foreign Exchange Markets.** Blackwell Publishers, Oxford.

Ding, Z., Granger, C.W.J. and Engle, R.F. (1993) **A Long Memory Property of Stock Market Returns and a New Model.** Journal of Empirical Finance, 1, 83–106.

Engle, R.F. (1982) **Autoregessive Conditional Heteroskedasticity, with Estimates of United Kingdom Inflations.** Econometrica 50, 987–1008.

Frank, M. and Stengos, T. (1988a) **Chaotic Dynamics in Economic Time-Series.** Journal of Economic Surveys 2, 103–133.

Frank, M.Z. and Stengos, T. (1988b) **Some Evidence Concerning Macroeconomic Chaos.** Journal of Monetary Economics 22, 423–438.

Frank, M.Z. and Stengos, T. (1988c) **The stability of Canadian Macroeconomic Data as Measured by the Largest Lyapunov Exponent.** Empirical Letters 27, 11–14.

Frank, M.Z. and Stengos, T. (1989) **Measuring the Strangeness of Gold and Silver Rates of Return.** Review of Economic Studies 56, 553–567.

Frank, M., Gencay, R. and Stengos, T. (1988) **International Chaos.** European Economic Review 32, 1569–1584.

Friedman, M. (1993) **The "Plucking Model" of Business Fluctuations Revisited.** Economic Inquiry 31, 171–177.

Geweke, J. and Porter–Hudak, S. (1983) **The Estimation and Application of Long Memory Time Series Models.** Journal of Time Series Analysis 4, 221–238.

Granger, C.W.J. (1993) **Modelling Non-Linear Relationships Between Long-Memory Variables.** Paper presented in Workshop on Recent Developments in Econometrics, Aarhus, Denmark 30.–31.8.1993.

Granger, C.W.J. and Ding, Z. (1993) **Some Properties of Absolute Return, An Alternative Measure of Risk.** Paper presented in Workshop on Recent Developments in Econometrics, Aarhus, Denmark 30.–31.8.1993.

Granger, C.W.J. and Teräsvirta, T. (1993) **Modelling Nonlinear Economic Relationships.** Oxford University Press.

Greedy, J. and Martin, V. editors (1994) **Chaos and Non-linear Models in Economics.** Edward Elgar, London.

Harvey, A.C. (1981) **The Econometric Analysis of Time Series.** Philip Allan.

Hsieh, D. (1991) **Chaos and Nonlinear Dynamics: Application to Financial Markets.** Journal of Finance 46, 1839–1877.

Jarque, C.M. and Bera, A.K. (1980) **Efficient Tests for Normality, Heteroskedasticity and Serial Independence of Regression Residuals.** Economics Letters 6, 255–259.

Keenan, D.M. (1985) **A Tukey Nonadditivity Type Test for Time Series Nonlinearity.** Biometrika 72, 39–44.

Lee, T.-H., White, H. and Granger, C.W.J. (1993) **Testing for Neglected Nonlinearity in Time Series Models.** Journal of Econometrics 56, 269–290.

Ljung, G.M. and Box, G.E.P. (1978) **On a Measure of Lack of Fit in Time Series Models.** Biometrica 65, 297–303.

McCafferty, D.F,. Ellner, S. Gallant, A.R. and Nychka, D.W. (1992) **Estimating the Lyapunov Exponent of a Chaotic System with Nonparametric Regression.** Journal of American Statistical Association 87, 682–695.

McLeod, A.J. and Li, W.K. (1983) **Diagnostic Checking ARMA Time Series Models Using Squared-Residual Correlations.** Journal of Time Series Analysis, 4, 269–273.

Medio, A. (1992) **Chaotic Dynamics, Theory and Applications to Economics.** Cambridge University Press.

Mullineux, A. and Peng, W.S. (1993) **Nonlinear Business Cycle Modelling.** Journal of Economic Surveys 7, 41–83.

Peters, E.E. (1993) **Chaos and Order in the Capital Market.** John Wiley, New York.

Petruccelli, J.D. (1990) **A Comparison of Tests for SETAR-type Non-linearly in Time Series.** Journal of Forecasting 9, 25–36.

Pfann, G.A. (1992) **Employment and Business Cycle Asymmetries.** Paper presented the ESEM Meeting in Brussels 1992.

Pfann, G.A. and Palm, F.C. (1993) **Asymmetric Adjustment Costs in Nonlinear Labour Demand Models.** Review of Economic Studies 60, 397–412.

Ramsey, J.B. (1969) **Tests for Specification Errors in Classical Least Squares Regression Analysis.** Journal of Royal Statistical Society, Series B, 31, 350–371.

Ramsey, J.B. (1990) **Economic and Financial Data as Nonlinear Processes,** in G.P. Dwyer and R.W. Hafer (eds) The Stock Market: Bubbles, Volatility, and Chaos, Kluwer Academic Publishers, Dordrecht.

Ramsey, J.B. and Rothman, P. (1988) **Characterization of the Time Irreversibility of Economic Time Series: Estimators and Test Statistics.** C.V. Starr Center for Applied Economics, New York University, Economic Research Reports #88–39.

Ramsey, J., Sayers, C. and Rothman, P. (1988) **The Statistical Properties of Dimension Calculations Using Small Data Sets: Some Economic Applications.** International Economic Review 31, 991–1020.

Rothman, P. (1993) **The Comparative Power of the TR Test Against Simple Threshold Models,** in Nonlinear Dynamics, Chaos and Econometrics, Pesaran, M.H. and Potter, S.M. (editors), John Wiley, New York, 229–237.

Spanos, A. (1986) **Statistical Foundations of Econometric Modelling.** Cambridge University Press.

Stiglitz, J. and Weiss, A. (1981) **Credit Rationing in Markets with Imperfect Information.** American Economic Review 71, 393–410.

Taylor, S. (1986) **Modelling Financial Time Series.** New York, John Wiley & Sons.

Tong, H. (1983) **Threshold Models in Non-linear Time Series Analysis.** Springer-Verlag, New York.

Tsay, R.S. (1986) **Non-linearity Tests for Time Series.** Biometrika 73, 461–466.

Vaga, T. (1994) **Profiting from Chaos.** McGraw–Hill, New Yorkcd.

Virén, M. (1992) **Some Long-Run Trends in Finnish Financial Variables.** University of Turku, Department of Economics, Discussion Paper No. 16.

White, H. (1980) **A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity.** Econometrica 48, 817–838.

Wolf, A., Swift, B., Swinney, J. and Vastano, J. (1985) **Determining Lyapunov exponents from a Time Series.** Physica D 16, 285–317.

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