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Research Department
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The Term Structure of Real Interest Rates: Theory and Evidence from UK Index-Linked Bonds

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The views expressed are those of the author and do not necessarily correspond to the views of the Bank of Finland

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Abstract

This paper studies the behavior of the default-risk-free real term structure and term premia in two general equilibrium endowment economies with complete markets but without money. In the first economy there are no frictions as in Lucas (1978) and in the second risk-sharing is limited by the risk of default as in Alvarez and Jermann (2000ab). Both models are solved numerically, calibrated to UK aggregate and household data, and the predictions are compared to data on real interest rates constructed from the UK index-linked data. While both models produce time-varying risk or term premia, only the model with limited risk-sharing can generate enough variation in the term premia to account for the rejections of expectations hypothesis.

Key words: term structure of interest rates, general equilibrium, default risk, term premia, index-linked bonds.

JEL classification: E43, E44, G12

Reaalikorkojen aikarakenne: teoriaa ja soveltaminen brittiläisiin indeksiobligaatioihin

Suomen Pankin keskustelualoitteita 22/2000

Juha Seppälä
Tutkimusosasto

Tiivistelmä

Tutkimuksessa tarkastellaan reaalikorkojen aikarakenteen ja siihen liittyvien aikapreemioiden vaihtelua kahdessa erilaisessa yleisen tasapainon mallissa. Ensimmäisessä malliversiossa markkinat ovat kitkattomat, kuten Lucasin (1978) esittämässä mallissa. Toisessa versiossa luottotappioriski sen sijaan estää riskien täydellisen jakamisen sijoittajien kesken samalla tavalla kuin Alvarezin ja Jermanin (2000) esittämässä mallissa. Kumpikin malliversio kalibroidaan Ison-Britannian taloutta kuvaavan kokonaistaloudellisen ja kotitalouksia koskevan aineiston mukaan ja mallien tuottamia ennusteita verrataan brittiläisiä indeksiobligaatioiden markkinoita kuvaavaan aineistoon. Kumpikin malli tuottaa ajassa vaihtelevia aikapreemioita, mutta vain se, jossa riskin jako on rajoitettua, johtaa niin suuriin aikapreemion vaihteluihin, että sen avulla voidaan selittää, miksi korkojen aikarakenteen odotushypoteesi ei päde.

Asiasanat: korkojen aikarakenne, yleinen tasapaino, luottotappioriski, aikapreemiot, indeksiobligaatiot.

JEL-luokitus: E43, E44, G12

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1 Introduction

One of the oldest problems in economic theory is the interpretation of the term structure of interest rates. It has been long recognized that the term structure of interest rates conveys information about economic agents' expectations about future interest rates, inflation rates, and exchange rates. In fact, it is widely agreed that the term structure is *the best* source of information about economic agents' inflation expectations for one to four years ahead.¹

Since it is generally recognized that monetary policy can only have effect with “long and variable lags” as Friedman (1968) put it, the term structure is an invaluable source of information for monetary authorities.² Moreover, empirical studies indicate that the slope of the term structure predicts consumption growth better than vector autoregressions or leading commercial econometric models.³

Empirical research on the term structure of interest rates has concentrated on the (pure) expectations hypothesis. That is, the question has been whether forward rates are unbiased predictors of future spot rates. The most common way to test the hypothesis has been to run a linear regression (error term omitted):

$$r_{t+1} - r_t = a + b(f_t - r_t),$$

where r_t is the one-period spot rate at time t and f_t is the one-period-ahead forward rate at time t . The pure expectations hypothesis implies that $a = 0$ and $b = 1$. Rejection of the first restriction $a = 0$ is consistent with the expectations hypothesis with a term premium that is nonzero but constant.

By and large the literature rejects both restrictions.⁴ Rejection of the second restriction, $b = 1$, requires, under the alternative, a risk or term premium that varies through time and is correlated with the forward premium, $f_t - r_t$. Many studies—e.g., Fama and Bliss (1987) and Fama and French (1989)—take this to indicate the existence of time-varying risk or term premium.⁵ In order for policy makers to extract information about market expectations from the term structure, they need to have a general idea about the sign and magnitude of the term premium.⁶ Therefore, it is interesting to ask if there are models that are capable of generating term premia that are similar to the ones observed in actual time series. Unfortunately, as Söderlind and Svensson (1997) note in their review,

“We have no direct measurement of this (potentially) time-varying covariance [term premium], and even ex post data is of limited use since

¹See, e.g., Fama (1975, 1990) and Mishkin (1981, 1990a, 1992) for studies on inflation expectations and the term structure of interest rates using U.S. data. Mishkin (1991) and Jorion and Mishkin (1991) use international data. Abken (1993) and Blough (1994) provide surveys of the literature.

²Svensson (1994ab) and Söderlind and Svensson (1997) discuss monetary policy and the role of the term structure of interest rates as a source of information.

³See, e.g., Harvey (1988), Chen (1991), and Estrella and Hardouvelis (1991).

⁴The literature is huge. Useful surveys are provided by Melino (1988), Shiller (1990), Mishkin (1990b), and Campbell, Lo, and MacKinlay (1997). Shiller (1979), Shiller, Campbell, and Schoenholtz (1983), Fama (1984, 1990), Fama and Bliss (1987), Froot (1989), Campbell and Shiller (1991), and Campbell (1995) are important contributions to this literature.

⁵The literature is somewhat inconsistent in the definitions of risk and term premium. In the discussion below, both terms are used interchangeably. In Section 3.3, I will define the risk premium as the term that accounts for the rejections of expectations hypothesis in prices and term premium as the term that accounts for the rejections of expectations hypothesis in rates.

⁶Obviously, the sign and the magnitude of the term premium are also interesting for market professionals. See Cochrane (1999).

the stochastic discount factor is not observable. It has unfortunately proved to be very hard to explain (U.S. ex post) term premia by either utility based asset pricing models or various proxies for risk.”

The question whether utility-based asset pricing models are capable of generating risk premia similar to the ones observed in actual time series was originally posed in Backus, Gregory, and Zin (1989). They use a complete markets model first presented by Lucas (1978), and their answer is that the model can account for neither sign nor magnitude of average risk premia in forward prices and holding-period returns.⁷ In addition, they show that one cannot reject the expectations hypothesis with data generated via the Lucas model. Donaldson, Johnsen, and Mehra (1990) obtain the same result for a general equilibrium production economy, and den Haan (1995) investigates the issue further.

The problem is that given the variability in U.S. aggregate consumption series, the stochastic discount factor derived from frictionless utility-based asset pricing models is not sufficiently volatile. This is closely related to the equity premium puzzle first posed by Mehra and Prescott (1985) and the risk-free rate puzzle posed by Weil (1989).

Mehra and Prescott (1985) conjecture that the most promising way to resolve the equity premium puzzle is to introduce features that make certain types of intertemporal trades among agents infeasible. Usually this has meant that markets are *exogenously incomplete*. Economic agents are allowed to trade only in certain types of assets, and the set of available assets is exogenously predetermined.

Heaton and Lucas (1992) use a three-period incomplete markets model to address the term premium puzzle. Their answer is that “uninsurable income shocks may help explain one of the more persistent term structure puzzles” but “the question remains whether the prediction of a relatively large forward premium will obtain in a long horizon model.”

Alvarez and Jermann (2000ab) study the asset pricing implications of an endowment economy when agents can default on contracts. They show how endogenously determined solvency constraints that prevent the agent from defaulting on his own contracts help explain the equity premium and the risk-free rate puzzles. For the purpose of dynamic asset pricing, this framework has three advantages over the standard incomplete markets approach described above.⁸

First, allocations do not depend on a particular arbitrary set available securities. Second, the markets are complete and hence any security can be priced. This is particularly important in addressing questions related to the term structure of interest rates. Finally, finding the solution to an incomplete markets problem involves solving a very difficult fixed point problem, whereas solving the model in Alvarez and Jermann can be very fast and easy to implement.

However, it should be noted that, while empirical research has concentrated on the *nominal* term structure, both Lucas (1978) and Alvarez and Jermann (2000ab) are models of endowment economies without money. This paper studies the implications of the Lucas and Alvarez-Jermann models on the behavior of the default-risk-free *ex-ante real* term structure and term premia. Both models are solved numerically and calibrated to UK aggregate and household data, and the predictions are

⁷LeRoy (1973), Rubinstein (1976), and Breeden (1979) were important early contributions to the literature on dynamic general equilibrium asset pricing models.

⁸The work by Alvarez and Jermann builds on contributions by Kehoe and Levine (1993), Kocherlakota (1996), and Luttmer (1996). See also Aiyagari (1994) and Zhang (1997) for studies on endogenous solvency constraints.

compared to data on *ex-ante real* interest rates constructed from UK index-linked data. While both models produce time-varying risk premia, only the model with limited risk-sharing can generate enough variation in the term premia to account for the rejections of the expectations hypothesis.⁹

It should also be emphasized that in both the Lucas and Alvarez-Jermann economies the object of the study is the term structure of *default-risk-free* real interest rates. Even though in the Alvarez-Jermann economy risk-sharing is limited by the risk of default, the instruments that are traded in state-contingent markets are default-risk-free. Therefore, it is natural to compare the Lucas and Alvarez-Jermann term structures with the term structure of real bonds issued by the UK government. At least in principle, these bonds are default-risk free. Hence, the thesis of this paper is not that default risk directly explains the term premium in the real term structure, but rather that default risk limits risk-sharing in such a way as to produce a “realistic” term premium.

The rest of the paper is organized as follows. Section 2 presents the data on nominal and index-linked bonds. Section 3 presents the basic features of the Lucas and Alvarez-Jermann models. Section 4 calibrates the models given the data on the risk-free rate, aggregate consumption, business cycles, and individual incomes in the UK. Section 5 presents the numerical results for both models and compares the models’ behavior to the behavior of UK nominal and real term structures. Section 6 concludes. Appendix A explains how the models are numerically solved, and Appendix B presents sensitivity analysis of the Alvarez-Jermann model.

⁹There are several other interesting models that offer at least partial resolution of the equity premium and/or risk-free rate puzzles. A few examples are Constantinides (1990), Constantinides and Duffie (1996), Bansal and Coleman (1996), Campbell and Cochrane (1999), and Abel (1999). In principle, any of these models could be (but have not been) used to study the Backus-Gregory-Zin “term premium puzzle” in UK data.

2 Data

2.1 UK Index-Linked Bonds

The main complication in analyzing ex-ante real interest rates is that in most economies they simply are unobservable. The most important exception is the UK market for index-linked debt. It constitutes a significant proportion of marketable government debt, and its daily turnover is by far the highest in the world. The UK market for index-linked debt was started in 1981, and by March 1994 it accounted approximately 15% of outstanding issues by market value.¹⁰

Unfortunately, UK index-linked bonds do not provide a correct measure of ex-ante real interest rates. The reason is that the nominal amounts paid by index-linked bonds do not fully compensate the holders for inflation; the indexation operates with a lag. Both coupon and principal payments are linked to the level of the RPI (Retail Price Index) published in the month seven months prior to the payment date. In addition, the RPI number relates to a specific day in the previous month, so that the effective lag is approximately eight months. The motivation behind this procedure is that it always allows the nominal value of the next coupon payment to be known, and nominal accrued interest can always be calculated.

Different authors have made different assumptions in order to overcome the “indexation lag problem.” Woodard (1990), Deacon and Derry (1994) and Brown and Schaefer (1994) impose the Fisher hypothesis: nominal yields move one for one with changes in inflation, which means that there is no “inflation risk premia.” On the other hand, Kandel, Ofer, and Sarig (1996) and Barr and Campbell (1997) assume that different versions of the expectations hypothesis hold. Finally, using the properties of stochastic discount factors, Evans (1998) isolates the “indexation lag problem” to a conditional covariance term between the future (maturity less the inflation lag) inflation and nominal bond prices. He then estimates this term using a VAR model and derives the real interest rates.

Evans also tests for the versions of expectations hypothesis used by Kandel, Ofer, and Sarig (1996) and Barr and Campbell (1997) and rejects both versions at the 1 percent significance level.¹¹ Since this paper is mainly concerned on the expectations hypothesis, the methodology of Evans seemed best suited for my purposes. The data presented in the next section was provided by him. However, before I move to the data, I explain his methodology in more detail.

Following Evans, let Q_t , Q_t^+ , and Q_t^* denote the prices of nominal, index-linked, and real bonds. Throughout the analysis, it is assumed that the price index for month t , P_t , is known at the end of month t . If the economy admits no pure arbitrage opportunities, there exists a stochastic discount factor, M_{t+1} , that can be used to price one-period nominal returns on any asset i , as follows

$$E_t[M_{t+1}R_{t+1}^i] = 1, \tag{1}$$

where R_{t+1}^i is the gross return on asset i between t and $t + 1$, and E_t denotes the expectation conditioned on the time t information set. In an economy where there is a complete set of markets for state-contingent claims, there is a unique stochastic discount factor, $M_{t+1} > 0$, satisfying (1).

¹⁰See Brown and Schaefer (1996) for more details.

¹¹He also rejects the Fisher hypothesis. He finds that expected inflation is negatively correlated with real yields.

In the case of a nominal bond with h periods to maturity, the one-period nominal return is $Q_{t+1}(h-1)/Q_t(h)$. Hence,

$$Q_t(h) = E_t[M_{t+1}Q_{t+1}(h-1)], \quad (2)$$

where $Q_t(0) = 1$. Similarly,

$$Q_t^*(h) = E_t[M_{t+1}^*Q_{t+1}^*(h-1)], \quad (3)$$

where $M_{t+1}^* = M_{t+1}P_{t+1}/P_t$. Next, let l be the indexation lag. Obviously, $Q_t^+(l) = Q_t(l)$, and when $h > l$,

$$Q_t^+(h) = E_t[M_{t+1}^*Q_{t+1}^+(h-1)]. \quad (4)$$

Log-linearizing equations (2)–(4), one obtains

$$q_t(h) = E_t \left[\sum_{i=1}^h m_{t+i} \right] + \frac{1}{2} \text{var}_t \left[\sum_{i=1}^h m_{t+i} \right] \quad (5)$$

$$q_t^*(h) = E_t \left[\sum_{i=1}^h m_{t+i}^* \right] + \frac{1}{2} \text{var}_t \left[\sum_{i=1}^h m_{t+i}^* \right] \quad (6)$$

$$q_t^+(h) = E_t \left[\sum_{i=1}^{\tau} m_{t+i}^* \right] + \frac{1}{2} \text{var}_t \left[\sum_{i=1}^{\tau} m_{t+i}^* \right] \quad (7)$$

$$+ E_t[q_{t+\tau}(l)] + \frac{1}{2} \text{var}_t[q_{t+\tau}(l)] + \text{cov}_t \left[\sum_{i=1}^{\tau} m_{t+i}^*, q_{t+\tau}(l) \right],$$

where $\tau \equiv h - l$. These equations are exact when the (period t) conditional joint distribution for $\{M_{t+j}, P_{t+i+1}/P_{t+i}\}_{j>0, i>0}$ is log normal. Otherwise they contain approximation errors.

Since the log nominal and real stochastic discount factors are related via $m_{t+1}^* \equiv m_{t+1} + \Delta p_{t+1}$, using equations (5)–(7) one obtains

$$q_t^+(h) = q_t^*(\tau) + [q_t(h) - q_t(\tau)] + \gamma_t(\tau),$$

where

$$\gamma_t(\tau) \equiv \text{cov}_t[q_{t+\tau}(l), \Delta^\tau p_{t+\tau}].$$

In other words, as long as $l > 0$, index-linked bonds are an imperfect measure of ex-ante real bonds. However, Evans estimates $\gamma_t(\tau)$ using different VAR specifications. He finds that the estimates show high uniformity and imply that $\gamma_t(\tau)$ contributes approximately 1.5 basis points to the annualized yields.

2.2 The Term Structures of Nominal and Real Interest Rates

Figures 1 and 2 present end-of-month observations on term structures of nominal and real interest rates in the UK from January 1984 until August 1995, as estimated by Evans (1998). Several conclusions can be drawn from the figures. First, while the nominal term structure is generally upward-sloping, the real term structure contains both upward and downward sloping patterns, with neither shape clearly dominating. However, at first sight it seems that at the beginning of observation period the real structure was mostly downward sloping. Since Brown and Schaefer (1994, 1996)

Term Structure of Nominal Interest Rates

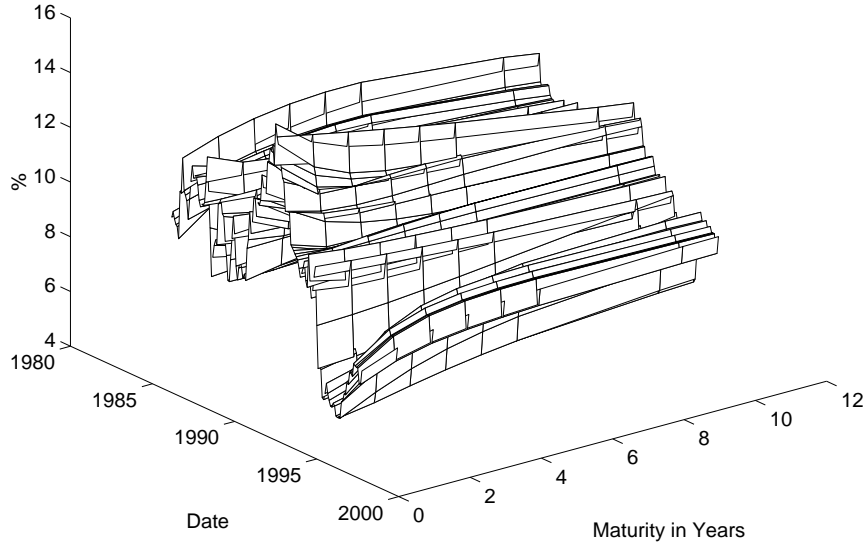


Figure 1: Term structure of nominal interest rates in the UK, 1984:1–1995:8.

Term Structure of Real Interest Rates

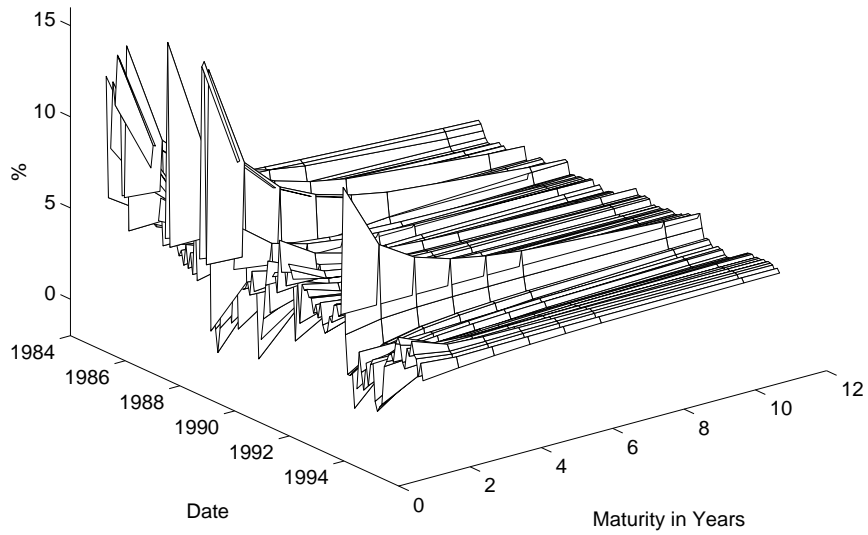


Figure 2: Term structure of real interest rates in the UK, 1984:1–1995:8.

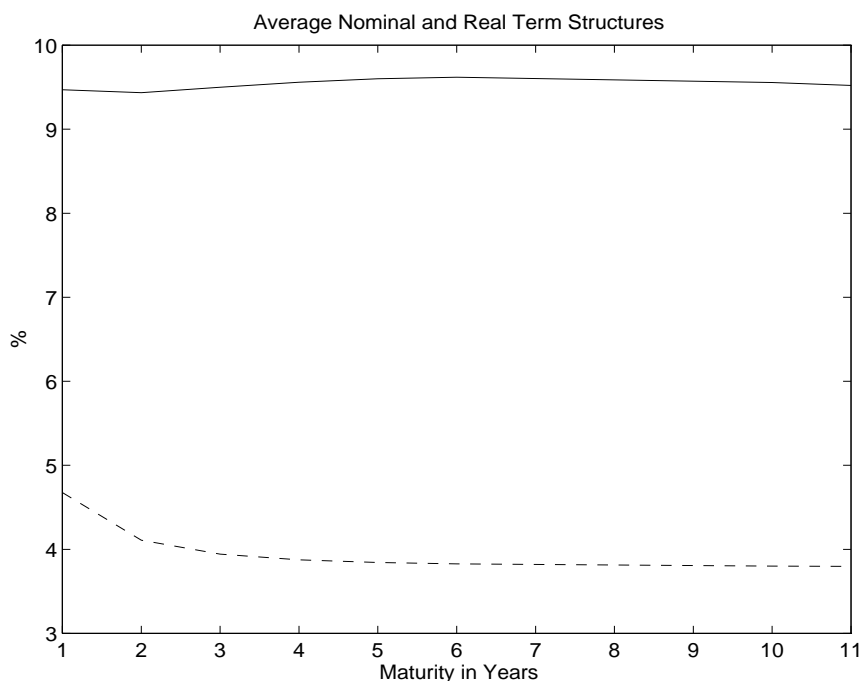


Figure 3: Average term structure of nominal (solid line) and real interest rates in the UK 1984:1–1995:8.

estimated the term structure of real interest rates to be upward-sloping on average, this issue warrants more attention.

Recall that the UK market for index-linked debt was started in 1981. The first bonds matured on March 30, 1988. This means that estimates for the short end of the real term structure were not available before 1987. For this reason, I present in Figures 3 and 4 the average term structures for nominal and real interest rates for the periods January 1984 to August 1995 and January 1987 to August 1995, respectively. The results are very similar; the average nominal term structure is flat while the average real term structure is downward sloping.¹²

Second, the long-term of the nominal term structure is quite volatile whereas the long-term of the real term structure appears to be highly stable. This observation is very interesting since Dybvig, Roll, and Ross (1996) showed that under very general conditions the limiting forward rate, if it exists, can never fall. In affine-yield models, such as Cox, Ingersoll, and Ross (1985), this means that the long-term of the term structure should converge, and these models have been criticized on the grounds that the long-term of the nominal term structure does not appear to be stable. Third, the short-term of the nominal term structure is less volatile than the short-term of the real term structure. As the matter of fact, the high volatility of short-term of the real term structure is probably the most striking feature of the real term structure data. These observations are confirmed in Figures 5 and 6 that present the standard deviations of both nominal and real term structures for both time periods.

Fourth, both the short-term and the long-term of the nominal term structure seem to be more autocorrelated than corresponding maturities in the real term structure. This is confirmed in Figures 7 and 8, which show annual autocorrelations for both the nominal and real term structures for both time periods.

¹²According to Evans, pricing errors at the beginning of the sample period give no evidence of a poor fit.

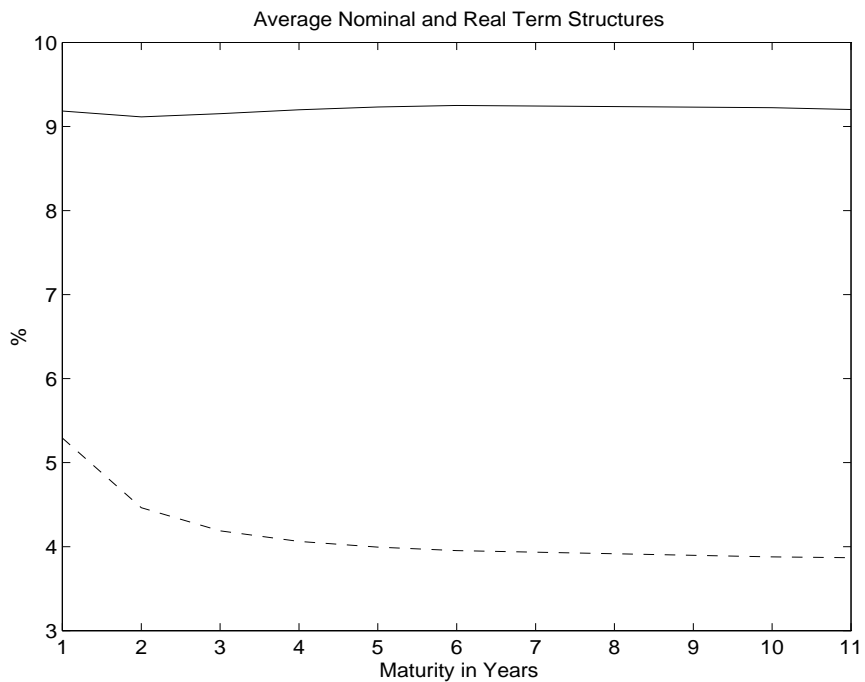


Figure 4: Average term structure of nominal (solid line) and real interest rates in the UK 1987:1–1995:8.

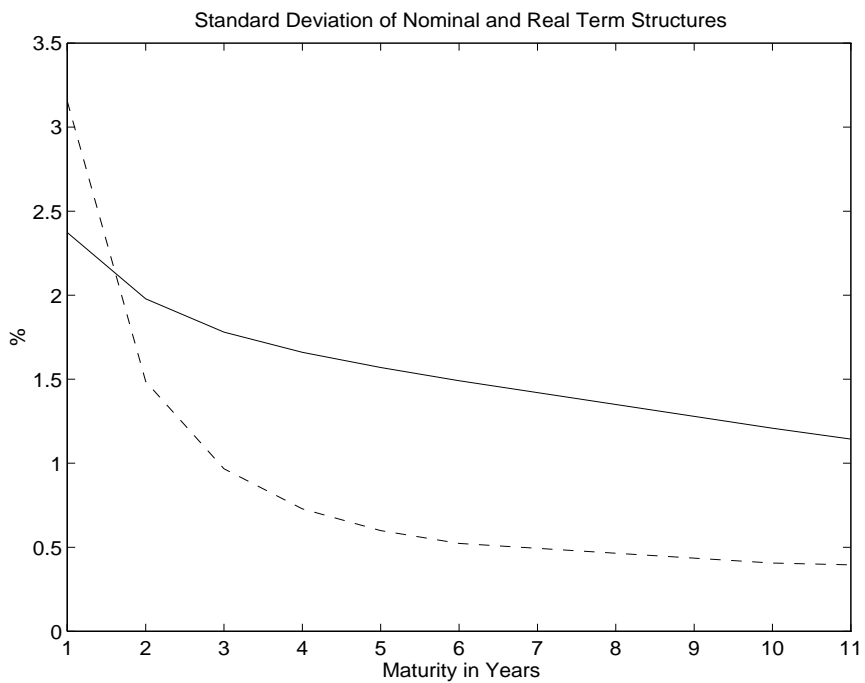


Figure 5: Standard deviation of the term structure of nominal (solid line) and real interest rates in the UK, 1984:1–1995:8.

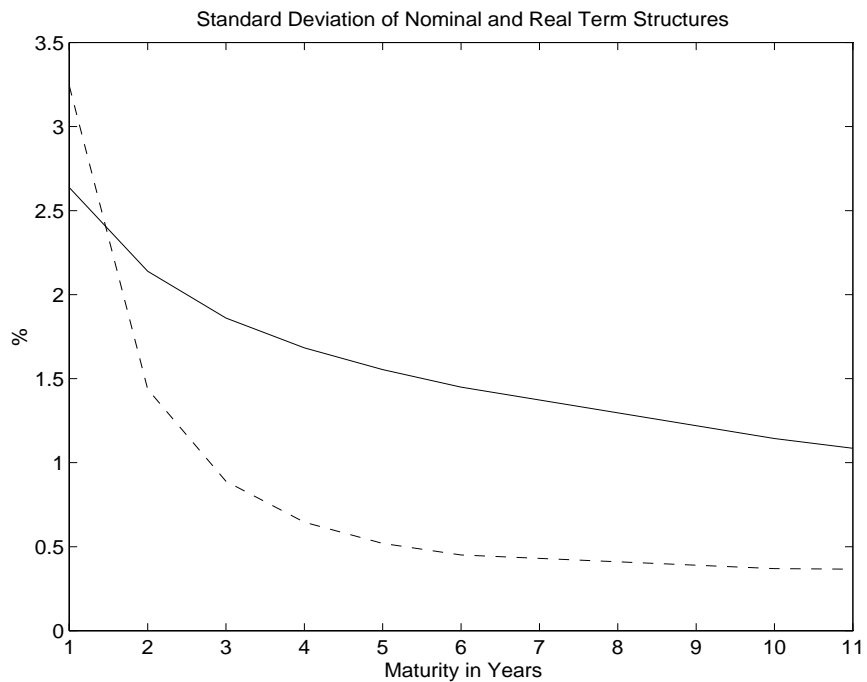


Figure 6: Standard deviation of the term structure of nominal (solid line) and real interest rates in the UK, 1987:1–1995:8.

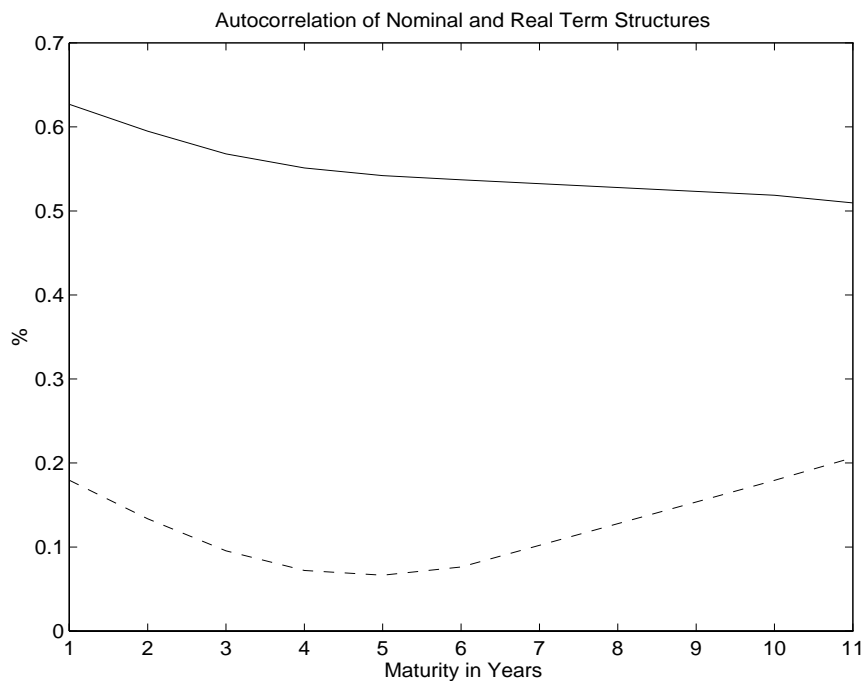


Figure 7: Autocorrelation of the term structure of nominal (solid line) and real interest rates in the UK 1984:1–1995:8.

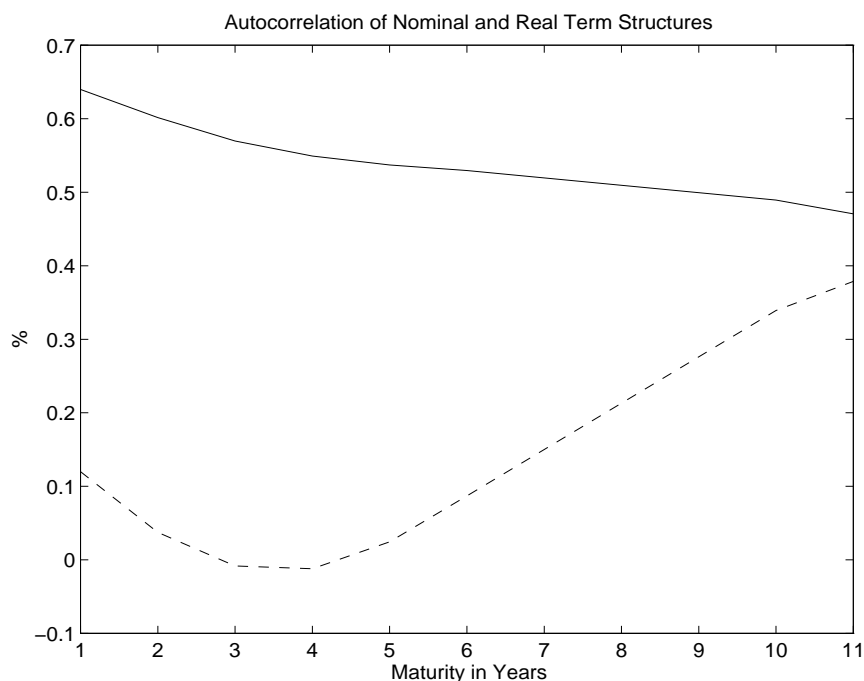


Figure 8: Autocorrelation of the term structure of nominal (solid line) and real interest rates in the UK 1987:1–1995:8.

Finally, the shapes of the real term structure are relatively simple compared to the nominal term structure. This means that single-factor term structure models such as Cox, Ingersoll, and Ross (1985) can be used successfully to estimate the term structure of real interest rates, as was shown by Brown and Schaefer (1994, 1996).

Similar observations have been obtained by Woodward (1990) and Brown and Schaefer (1994, 1996) who made different assumptions to overcome the “indexation lag problem” and estimated the yield curve using different methods than Evans used. With the exception of the average shape of real term structure, the results in Brown and Schaefer are consistent with results presented here. The issue of the average shape of the real term structure clearly warrants further investigation.

3 The Lucas and Alvarez-Jermann Models

3.1 The Environment

Alvarez and Jermann (1996, 2000ab) consider a pure exchange economy with two agents.¹³ Agents have identical preferences represented by time-separable expected discounted utility. Their endowments follow a finite-state first-order Markov process. The difference between the Alvarez-Jermann economy and the Lucas economy is that in the former agents cannot commit to their contracts. The agents have an incentive to default on their contracts if honoring their contracts would leave them worse off than they would be in autarky. Since everyone knows this and there is no private information, nobody is willing to lend more than the debtor is willing to pay back. Therefore, in equilibrium nobody defaults but risk-sharing is limited by the risk of default.

In the planning problem, the possibility of default is prevented by participation constraints. The economy can be decentralized through complete asset markets where the positions that the agents can take are endogenously restricted by person, state, and time-dependent solvency constraints. From now on, the discussion concentrates on the Alvarez-Jermann economy, the Lucas economy being a special case of the Alvarez-Jermann economy where participation or solvency constraints are never binding.

Let $i = 1, 2$ denote each agent and $\{z_t\}$ denote a finite-state Markov process $z \in Z = \{z_1, \dots, z_N\}$ with transition matrix Π . z_t determines the aggregate endowment e_t , and the individual endowments, $e_t^i, i = 1, 2$ as follows:

$$e_{t+1} = \lambda(z_{t+1})e_t \quad \text{and} \quad e_t^i = \alpha^i(z_t)e_t \quad \text{for } i = 1, 2,$$

where $\lambda(z_{t+1})$ is the growth rate of aggregate endowment between t and $t+1$ when the state in period $t+1$ is z_{t+1} and $\alpha^i(z_t)$ determines agent i 's share of the aggregate endowment when the state in period t is z_t .

Let $z^t = (z_1, \dots, z_t)$ denote the history of z up to time t . The matrix Π determines the conditional probabilities for all histories $\pi(z^t|z_0)$. Households care only about their consumption streams, $\{c\} = \{c_t(z^t) : \forall t \geq 0, z^t \in Z^t\}$, and rank them by discounted expected utility,

$$U(c)(z^t) = \sum_{j=0}^{\infty} \sum_{z^{t+j} \in Z^{t+j}} \beta^j u(c_{t+j}(z^{t+j})) \pi(z^{t+j}|z^t),$$

where $\beta \in (0, 1)$ is a constant discount factor and the one-period utility function is of the constant relative risk aversion type

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad 1 < \gamma < \infty,$$

where γ is the agents' constant coefficient of relative risk-aversion.

In the planning problem, the participation constraints force the allocations to be such that under no history will the expected utility be lower than that in autarky

$$U(c^i)(z^t) \geq U(e^i)(z^t) \quad \forall t \geq 0, z^t \in Z^t, i = 1, 2. \quad (8)$$

In other words, in no time period and in no state of the world can either agent's expected discounted utility be less than what the agent would obtain in autarky. If

¹³ Alvarez and Jermann (2000a) consider the more general case with $I \geq 2$ agents.

this were the case, the agent would not commit to his contract and would choose to default. The punishment for default would be permanent exclusion from the risk-sharing arrangement. The motivation for autarky constraints of the form (8) is quite natural. In the real world, it is sometimes difficult to make a debtor pay. People do not always keep their promises and debt collection can be costly and possibly useless.

One can argue that the model is unrealistic in two respects. On the one hand, the punishment is too severe; typically an agent who declares bankruptcy can return to the risk-sharing arrangement after a finite number of periods. On the other hand, the agent who reverts to autarky can keep his endowment stream; there is no confiscation of assets as punishment for default. Trying to relax both of these unrealistic features of the model probably would not change the asset pricing implications too much. Relaxing the first assumption would reduce risk-sharing, since punishment for default would not be so serious. Relaxing the second assumption would increase risk-sharing, since punishment from default would be more serious. Hopefully, the two effects would offset each other.

Note that the model abstracts completely from game-theoretic issues related to bargaining and renegotiation. Punishing one agent by forever excluding him from the risk-sharing arrangement makes the punishing agent much worse off than with finite punishment. The justification is that as in Alvarez and Jermann (2000a), there are actually many more agents than just one and each agent has a very small weight in the collective bargaining problem. The model is easiest to solve when there are only two agents. In Section 3.3, it is shown that adding more agents need not change the asset pricing implications of the model.

3.2 Constrained Optimal Allocations

The constrained optimal allocations are defined as processes, $\{c^i\}$, $i = 1, 2$, that maximize agent 1's expected utility subject to feasibility and participation constraints at every date and every history, given agent 2's initial promised expected utility. The recursive formulation of the constrained optimal allocations is given by the functional equation

$$TV(w, z, e) = \max_{c^1, c^2, \{w_{z'}\}} u(c_1) + \beta \sum_{z' \in Z} V(w_{z'}, z', e') \pi(z'|z)$$

subject to

$$c^1 + c^2 \leq e$$

$$u(c^2) + \beta \sum_{z' \in Z} w_{z'} \pi(z'|z) \geq w$$

$$V(w_{z'}, z', e') \geq U^1(z', e') \quad \forall z' \in Z \quad (9)$$

$$w_{z'} \geq U^2(z', e') \quad \forall z' \in Z, \quad (10)$$

where primes denote next-period values, $V(w, z, e)$ is agent 1's value function, w is agent 2's promised utility this period, $w_{z'}$ is agent 2's promised utility when the next-period state of the world is z' , and (9) and (10) are the participation constraints for agent 1 and agent 2, respectively. The second welfare theorem holds for the economy.¹⁴ Hence, one can solve for the allocations by solving the planning problem and read the prices off the first-order conditions of the competitive equilibrium.

¹⁴See Alvarez and Jermann (2000a).

3.3 Asset Pricing

To analyze asset prices, the economy must be decentralized. The markets are complete, so that in every state z there are one-period Arrow securities to each next period state of nature, z' . However, the solvency constraints prevent agents from holding so much debt in any state that they would like to default on their debt contracts. The solvency constraints affect each state differently, since the relative value of autarky compared to honoring the contract is different in every state.

Let $q(z', z)$ be the price of an Arrow security that pays one unit of consumption good at the beginning of the next period when the next-period state is z' and the current-period state is z . Agent i 's holdings of this asset are denoted by $a_{z'}^i$. Finally, let $B^i(z', z)$ be the minimum position agent i can take in the asset that pays off when the next-period state is z' and the current-period state is z . Hence,

Definition 1. *The household's problem given the current state (a, z) is to maximize the expected utility*

$$H^i(a, z) = \max_{c, \{a_{z'}\}_{z' \in Z}} u(c) + \beta \sum_{z' \in Z} H^i(a_{z'}, z') \pi(z'|z)$$

subject to solvency and budget constraints

$$\begin{aligned} a_{z'} &\geq B^i(z', z) \quad \forall z' \in Z \\ \sum_{z' \in Z} q(z', z) a_{z'} + c &\leq a + e^i(z). \end{aligned}$$

The equilibrium can now be defined.

Definition 2. *The equilibrium is a set of solvency constraints $\{B_t^i\}$, prices $\{q_t\}$, and allocations $\{c_t^i, a_{t+1}^i\}$ such that*

1. *Taking the constraints and prices as given, the allocations solve both households' optimization problems.*
2. *Markets clear:*

$$a^1(z^{t+1}) + a^2(z^{t+1}) = 0 \quad \forall t \geq 0, \forall z^{t+1} \in Z^{t+1}.$$

3. *When the solvency constraints are binding, the continuation utility equals autarky utility:*

$$H^i(B_{t+1}^i(z^{t+1}), z^{t+1}) = U^i(e^i)(z^{t+1}) \quad \forall t \geq 0, \forall z^{t+1} \in Z^{t+1}. \quad (11)$$

Condition (11) means that solvency constraints are endogenously generated. This ensures that the solvency constraints prevent default and it allows as much insurance as possible. A binding solvency constraint means that the agent is indifferent between defaulting and staying in the risk-sharing regime. A slack solvency constraint means that the agent's expected discounted utility is strictly higher than what he would obtain in autarky. Hence, the model provides endogenous justification for debt, solvency, short-selling, and other exogenous constraints that are commonly used in the literature on incomplete markets.

As far as the asset pricing is concerned, the crucial point of the model is that the prices of Arrow securities are given by the maximum of the marginal rates of

substitution for agents 1 and 2.¹⁵ That is, if the current state is z^t , the price of an Arrow security that pays one unit of consumption good at the beginning of the next period if the next-period state is z^{t+1} is given by

$$q(z^{t+1}, z^t) = \max_{i=1,2} \beta \frac{u'(c_{t+1}^i)}{u'(c_t^i)} \pi(z^{t+1}|z^t). \quad (12)$$

The economic intuition is that the unconstrained agent in the economy does the pricing. Since $B(z^{t+1})$ gives the minimum amount of an asset one can buy, the constrained agent would like sell that asset and hence his marginal valuation of the asset is lower. In other words, the constrained agent has an internal interest rate that is higher than the market rate. Therefore, he would like to borrow more than is feasible, to keep the autarky constraints satisfied. In the full risk-sharing regime (Lucas economy), the two marginal rates of substitution are equalized.

Notice that one can introduce as many new agents into the economy as desired without changing the asset pricing implications, provided that the new agents' marginal valuations are always less than or equal to market valuations. A corollary of this is that each new agent whose income process is perfectly correlated with aggregate income has no effect on asset prices.

In addition, let $q(z^{t+j}, z^t)$ be the price of an Arrow security from state z^t to state z^{t+j} , which is given by

$$q(z^{t+j}, z^t) = \prod_{k=t}^{t+j-1} q(z^{k+1}, z^k), \quad (13)$$

and let $q(z^{k+1}, z^k)$ be given by (12).

Let m_{t+1} denote the real stochastic discount factor

$$m_{t+1} \equiv \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})}.$$

The price of an n -period zero-coupon bond is given by

$$p_{n,t}^b = \sum_{z^{t+n} \in Z^{t+n}} q(z^{t+n}, z^t) = E_t \left[\prod_{j=1}^n m_{t+j} \right]. \quad (14)$$

Using (12), (13), and the equation above, note that

$$p_{n,t}^b = \sum_{z^{t+1} \in Z^{t+1}} \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})} p_{n-1,t+1}^b \pi(z^{t+1}|z^t) = E_t[m_{t+1} p_{n-1,t+1}^b]. \quad (15)$$

The bond prices are invariant with respect to time, and hence equation (15) gives a recursive formula for pricing zero-coupon bonds of any maturity.

Forward prices are defined by

$$p_{n,t}^f = \frac{p_{n+1,t}^b}{p_{n,t}^b},$$

and the above prices are related to interest rates (or yields) by

$$f_{n,t} = -\log(p_{n,t}^f) \quad \text{and} \quad r_{n,t} = -(1/n) \log(p_{n,t}^b). \quad (16)$$

¹⁵This result was also derived by Cochrane and Hansen (1992) and Luttmer (1996).

To define the risk premium as in Sargent (1987), write (15) for a two-period bond using the conditional expectation operator and its properties:

$$\begin{aligned}
p_{2,t}^b &= E_t[m_{t+1}p_{1,t+1}^b] \\
&= E_t[m_{t+1}]E_t[p_{1,t+1}^b] + \text{cov}_t[m_{t+1}, p_{1,t+1}^b] \\
&= p_{1,t}^b E_t[p_{1,t+1}^b] + \text{cov}_t[m_{t+1}, p_{1,t+1}^b],
\end{aligned}$$

which implies that

$$p_{1,t}^f = \frac{p_{2,t}^b}{p_{1,t}^b} = E_t[p_{1,t+1}^b] + \text{cov}_t\left[m_{t+1}, \frac{p_{1,t+1}^b}{p_{1,t}^b}\right]. \quad (17)$$

Since the conditional covariance term is zero for risk-neutral investors, I will call it the *risk premium* for the one-period forward contract, $rp_{1,t}$, given by

$$rp_{1,t} \equiv \text{cov}_t\left[m_{t+1}, \frac{p_{1,t+1}^b}{p_{1,t}^b}\right] = p_{1,t}^f - E_t[p_{1,t+1}^b],$$

and similarly $rp_{n,t}$ is the risk premium for the n -period forward contract:

$$rp_{n,t} \equiv \text{cov}_t\left[\prod_{j=1}^n m_{t+j}, \frac{p_{1,t+n}^b}{p_{1,t}^b}\right] = p_{n,t}^f - E_t[p_{1,t+n}^b].$$

In addition, I will call a difference between the one-period forward rate and the expected value of one-period interest rate next period the *term premium* for the one-period forward contract, $tp_{1,t}$:

$$tp_{1,t} \equiv f_{1,t} - E_t[r_{1,t+1}]$$

and similarly $tp_{n,t}$ is the term premium for the n -period forward contract:

$$tp_{n,t} \equiv f_{n,t} - E_t[r_{1,t+n}].$$

4 Calibration

4.1 Free Parameters

In the spirit of Backus, Gregory, and Zin (1989), I will solve the model for the simplest possible case that produces nonconstant interest rates and has income heterogeneity.¹⁶ This is obtained by introducing uncertainty in the growth rate of aggregate endowment while treating the agents in a symmetric fashion. Similar techniques can be applied for more complicated cases.

In particular, there will be three exogenous states: z_{hl} , z_e , and z_{lh} . The states z_{hl} and z_{lh} are associated with a recession, and following Mankiw (1986) and Constantinides and Duffie (1996), the recessions are associated with a widening of inequality in earnings.¹⁷ During the expansion both agents have the same endowment z_e . The states are ordered so that

$$\alpha^1(z_{hl}) = \alpha^2(z_{lh}) \geq \alpha^1(z_e) = \alpha^2(z_e) \geq \alpha^1(z_{lh}) = \alpha^2(z_{hl})$$

and the transition matrix Π preserves symmetry between the agents.

Thus, there are five free parameters associated with aggregate and individual endowment. These are α_{hl}^1 for individual incomes,

$$\alpha^1 = \begin{bmatrix} \alpha_{hl}^1 \\ 0.5 \\ 1 - \alpha_{hl}^1 \end{bmatrix} \quad \text{and} \quad \alpha^2 = \begin{bmatrix} 1 - \alpha_{hl}^1 \\ 0.5 \\ \alpha_{hl}^1 \end{bmatrix},$$

λ_e and λ_r for the growth rates of aggregate endowment,

$$\lambda = \begin{bmatrix} \lambda_r \\ \lambda_e \\ \lambda_r \end{bmatrix},$$

and π_r and π_e for the transition matrix

$$\Pi = \begin{bmatrix} \pi_r & 1 - \pi_r & 0 \\ (1 - \pi_e)/2 & \pi_e & (1 - \pi_e)/2 \\ 0 & 1 - \pi_r & \pi_r \end{bmatrix}.$$

The order of calibration is as follows. First, the transition matrix for the aggregate states can be expressed as a function of the fraction of time spent in the expansion state, π , and the first-order autocorrelation of aggregate consumption, θ :¹⁸

$$\Pi = \begin{bmatrix} (1 - \theta)\pi + \theta & (1 - \theta)(1 - \pi) \\ (1 - \theta)\pi & (1 - \theta)(1 - \pi) + \theta \end{bmatrix}.$$

Clearly,

$$\pi = \frac{1 - \pi_r}{2 - (\pi_e + \pi_r)}$$

$$\theta = \pi_e + \pi_r - 1.$$

Next, given the transition matrix for the aggregate states, λ_e and λ_r determine the average growth rate of aggregate consumption and its the standard deviation.

¹⁶Without income heterogeneity, the Alvarez-Jermann and Lucas models would be identical.

¹⁷Storesletten, Telmer, and Yaron (1999) document this for U.S. data.

¹⁸See Barton, David, and Fix (1962).

Finally, α_{hl}^1 determines the standard deviation of individual income. Note that with this parameterization one cannot pin down the persistence of individual income. An alternative would be to allow individual incomes to take different values also during expansions. I discuss in Section 4.2.2 why I chose not to do so.

In addition, there are two free parameters, β and γ , associated with preferences. Section 4.2 explains the five equations that determine endowment parameters, and Section 4.3 shows how the preference parameters are pinned down to match the first and second moments of the risk-free rate in the UK Appendix A gives details on how the model is solved numerically, and Appendix B shows that the main results are highly robust with respect to measurement error in moment conditions.

4.2 Aggregate and Individual Endowment

4.2.1 Aggregate Consumption and Business Cycles

The first step is to calibrate the law of motion for the aggregate endowment so that it matches a few facts about aggregate consumption (in the model, aggregate endowment equals aggregate consumption) and business cycles. Campbell (1998) reports that in the annual UK data for 1891–1995, the average growth rate of aggregate consumption is 1.443%, the standard deviation of the growth rate of aggregate consumption is 2.898%, and the first-order autocorrelation is 0.281.

Since the UK does not publish official definitions of expansions and recessions, I followed Chapman (1997), who used the following definitions in his study of the cyclical properties of U.S. real term structure. Business cycle expansions are defined as at least two consecutive quarters of positive growth¹⁹ in a three quarter equally-weighted, centered moving average of the real GDP/capita (“output”). Cycle contractions are at least two consecutive quarters of negative growth in the moving average of output. A peak is the last quarter prior to the beginning of a contraction, and a trough is the last quarter prior to the beginning of a expansion. Business cycles thusly defined are presented in Figure 9 for the UK from the first quarter of 1957 until the last quarter of 1997. The quarterly observations on the GDP at 1990 prices and annual observations on the population were obtained from the CD-ROM July 1999 version of the International Monetary Fund’s International Financial Statistics. The quarterly population series were constructed by assuming that population grows at constant rate within the year and that the original annual data are as at December 31 of each year. According to the definitions, expansions are 3.8824 times more likely to occur than recessions in the UK from the first quarter of 1957 until the last quarter of 1997.

4.2.2 Individual Endowment

The next step is to calibrate the process for the individual endowments. As mentioned above, the current specification only allows one to match the standard deviation of individual income. An alternative would be to allow individual incomes to take different values also during expansions. Unfortunately, in the UK there are only two data sets that provide information on household income. The first is the Family Expenditure Survey (FES), which covers the years 1968–1992 and the second is the British Household Panel Survey (BHPS), which covers the period September 1990 to January 1998. Since I am interested in household income heterogeneity for particular households and the FES is a survey, I chose to use the BHPS. This approach

¹⁹The growth (rate) is the difference in the logarithm of the series.

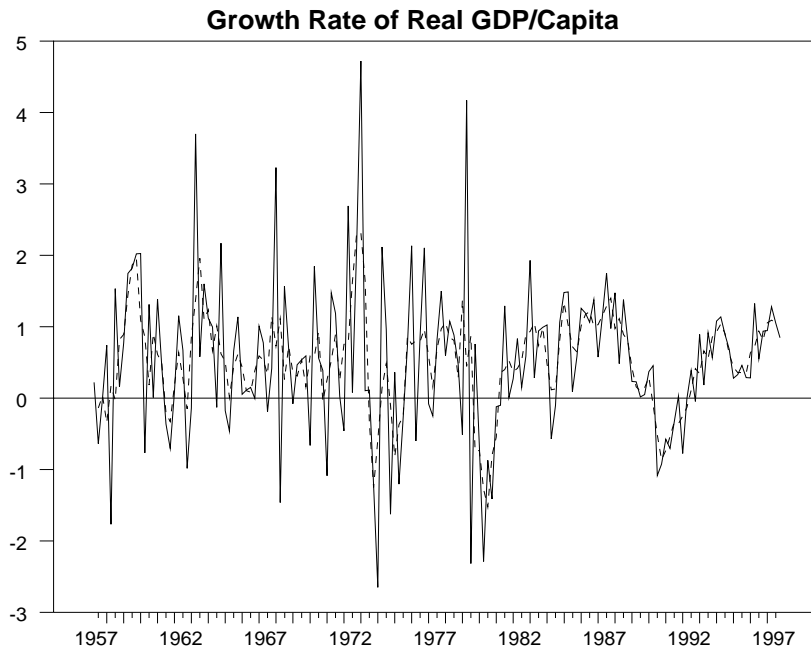


Figure 9: The growth rate of real GDP/capita in the UK 1957:1–1997:4 and its moving average (quarterly observations).

is similar to that taken in previous studies, such as Heaton and Lucas (1996) and Storesletten, Telmer, and Yaron (1999), which calibrate asset pricing models to the U.S. data on households.

Heaton and Lucas (1996) and Storesletten, Telmer, and Yaron (1999) also provide examples on how to estimate the standard deviation of individual income. However, in order to implement the Storesletten, Telmer, and Yaron method, one needs to know how the cross-sectional variance in household income varies over business cycles. The BHPS was started only in September 1990 when the UK was in depression, and the depression ended in the first quarter of 1992. Thus, the data does not cover a full cycle and it has only one data point for the recession. Therefore, I decided to concentrate on the simplest possible specification (the least free parameters) that would provide different results from the Lucas model.

The BHPS provides a panel of monthly observations of individual and household income and other variables from September 1990 until January 1998 for more than 5,000 households, giving a total of approximately 10,000 individuals.²⁰ I use the subset of the panel that has reported positive income in every year since 1991. This gives a total of 2,391 individuals in the panel. For households with more than one member with reported income, I constructed the individual income as the total household income divided by the number of the members of the household with reported income. Following Heaton and Lucas (1996), the individual annual income dynamics are assumed to follow an AR(1)-process:

$$\log(\eta_t^i) = \bar{\eta}^i + \rho^i \log(\eta_{t-1}^i) + \epsilon_t^i, \quad (18)$$

where $\eta_t^i = e_t^i / \sum_{i=1}^n e_t^i$. This provides a close connection for the standard deviation of ϵ_t^i in the data with the standard deviation of α 's in the model. Note that it

²⁰For more details on the BHPS, see Taylor (1998).

Table 1: Cross-sectional means and standard deviations of the coefficient estimates in the regression (18). Data: British Household Panel Survey.

Coefficient	Cross-Sectional Mean	Cross-Sectional Standard Deviation
$\bar{\eta}^i$	-6.4331	5.2376
ρ^i	0.2035	0.6315
σ^i	0.2830	0.2853

Table 2: Cross-sectional means and standard deviations of the coefficient estimates in the regression (18) obtained by Heaton and Lucas (1996). Data: Panel Study of Income Dynamics

Coefficient	Cross-Sectional Mean	Cross-Sectional Standard Deviation
$\bar{\eta}^i$	-3.354	2.413
ρ^i	0.529	0.332
σ^i	0.251	0.131

also imposes a cointegration relationship between aggregate and individual income. Unfortunately, given the length of data it seems unlikely that one could test whether this relationship is reasonable.

Table 1 reports cross-sectional means and standard deviations of the coefficient estimates in the regression (18). The relevant numbers are the first-order autocorrelation coefficient, ρ^i , and the standard deviation of the error term, $\sigma^i = \sqrt{E[(\epsilon_t^i)^2]}$. Heaton and Lucas (1996) estimate the same coefficients using a sample of 860 U.S. households in the Panel Study of Income Dynamics (PSID) that have annual incomes for 1969 to 1984. Table 2 reports cross-sectional means and standard deviations of the coefficient estimates obtained by Heaton and Lucas. The estimated autocorrelation coefficient in the BHPS, 0.2035, is significantly smaller than that in the PSID, 0.529. On the other hand, the standard deviation of the error term in the BHPS, 0.2830, is roughly the same as that in the PSID, 0.251. Storesletten, Telmer, and Yaron (1999) obtain a higher number for the standard deviation of individual income.²¹

The question of the persistence of individual income is, unfortunately, an unresolved issue. Heaton and Lucas (1996) estimate a relatively low number while Storesletten, Telmer, and Yaron (1999) obtain estimates much closer to one. Finally, Blundell and Preston (1998) impose the presence of a random walk component. Baker (1997) provides analysis that questions the assumption of a unit root. In order to reasonably pin down the persistence of individual income one would need to have very long panels of data, which are currently unavailable.

²¹For previous estimates of the standard deviation of individual income using the UK data, see Meghir and Whitehouse (1996) and Blundell and Preston (1998). Both studies use the FES data. Blundell and Preston (1998), which is closer to exercise here, decompose the variance in income into permanent and transitory components, and their assumption is that the process is composed of a pure transitory component and a random walk. They show strong growth in transitory inequality toward the end of this period, while young cohorts are shown to face significantly higher levels of permanent inequality. At the beginning of the sample, their estimates are close to mine, but at the end their estimates are higher, as in Storesletten, Telmer, and Yaron (1999).

Standard Deviation of (Individual Consumption/Aggregate Consumption)

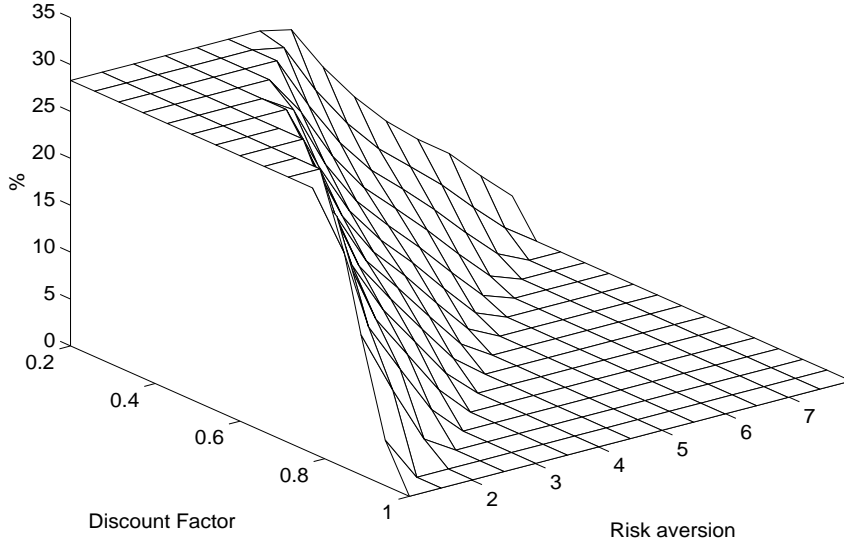


Figure 10: Standard deviation of (individual consumption/aggregate consumption) as a function of the discount factor and coefficient of relative risk aversion.

4.2.3 Calibrated Values

Solving the system of five unknowns in five equations leads to the endowment vectors and growth rates²²

$$\alpha^1 = \begin{bmatrix} 0.7706 \\ 0.5 \\ 0.2294 \end{bmatrix}, \quad \alpha^2 = \begin{bmatrix} 0.2294 \\ 0.5 \\ 0.7706 \end{bmatrix}, \quad \text{and} \quad \lambda = \begin{bmatrix} 0.9573 \\ 1.0291 \\ 0.9573 \end{bmatrix},$$

and the transition matrix

$$\Pi = \begin{bmatrix} 0.4283 & 0.5717 & 0 \\ 0.0736 & 0.8527 & 0.0736 \\ 0 & 0.5717 & 0.4283 \end{bmatrix}.$$

4.3 Preferences

The next step is to match agents' preference parameters, the discount factor β , and the coefficient of relative risk aversion, γ , to the key statistics of the asset market data. To illustrate the features of the model, Figure 10 plots the standard deviation of individual consumption, $\text{std}(\log(\frac{c^i}{\sum_j c^j}))$, as a function of β and γ .

The flat segment in the upper-left corner of the Figure corresponds to autarky and the flat segment in the lower-right corner to perfect risk-sharing. The parameter values which are interesting for the purpose of asset pricing are those that generate allocations between these two extremes. To accomplish this, one has to choose either relatively low risk aversion and a relatively high discount factor or relatively high risk aversion and a relatively low discount factor. In Section 5, I report numerical

²²Due to highly nonlinear nature of the model, the results are quite sensitive to the used parameter values. Therefore, I will report all the parameter values with four decimal precision.

Expected Riskfree Rate

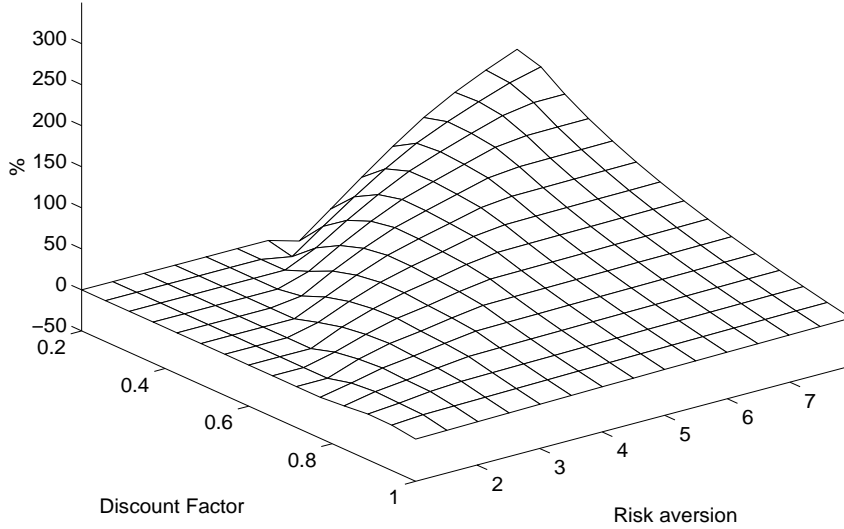


Figure 11: Average risk-free rate as a function of the discount rate and risk aversion in an Alvarez-Jermann economy.

results only for one pair of coefficients of risk aversion and discount factor, and in Appendix B I show that these results are very robust to measurement error in risk-free rates, aggregate consumption, business cycles, and individual income. In addition, I show how increasing the coefficient of risk-aversion, discount factor, or the standard deviation of individual income moves the results to the direction of more risk-sharing (the Lucas model).

Figures 11–14 present the average and standard deviations of the risk-free rate in the Alvarez-Jermann and Lucas economies. Note that in the Alvarez-Jermann economy in autarky no trade is allowed because one of the agents would default on his contract, and hence all the statistics have been set to zero. The shapes in the Lucas economy are relative easy to understand using log-linear approximations, as was done in Section 2.

Recall that the price of an n -period zero-coupon bond in a Lucas economy is given by

$$p_{n,t}^b = \beta^n E_t \left[\left(\frac{e_{t+n}}{e_t} \right)^{-\gamma} \right].$$

Taking a log-linear approximation of the n -period interest rate,²³ one obtains

$$r_{n,t} = -\log(\beta) + \frac{\gamma}{n} E_t \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right] - \frac{\gamma^2}{2n} \text{var}_t \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right], \quad (19)$$

$$E[r_{n,t}] = -\log(\beta) + \frac{\gamma}{n} E \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right] - \frac{\gamma^2}{2n} \text{var} \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right]. \quad (20)$$

²³Again, the approximation is exact only if consumption growth has a log-normal distribution. See Campbell (1986).

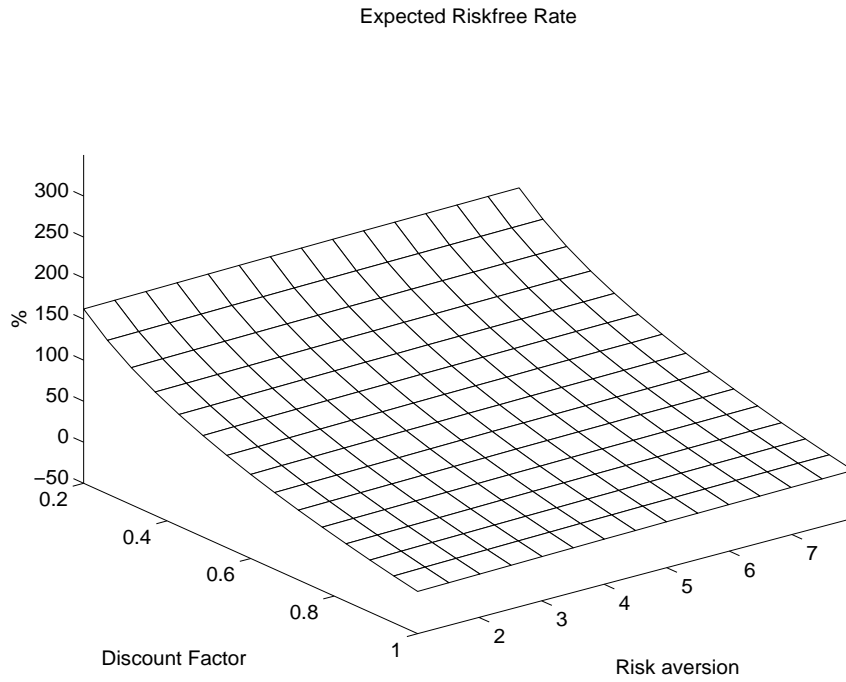


Figure 12: Average risk-free rate as a function of the discount rate and risk aversion in a Lucas economy.

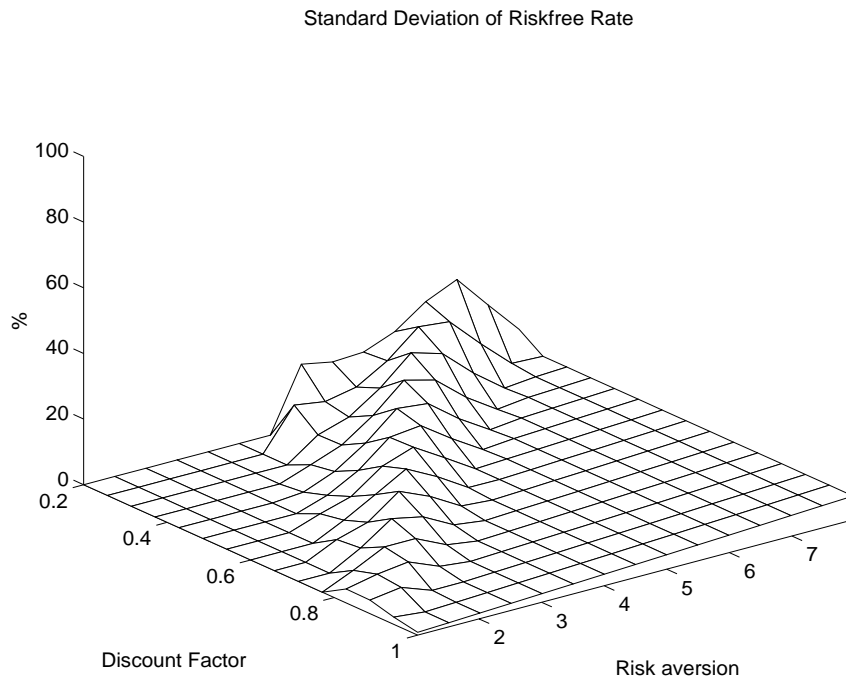


Figure 13: Standard deviation of the risk-free rate as a function of the discount rate and risk aversion in an Alvarez-Jermann economy.

Standard Deviation of Riskfree Rate

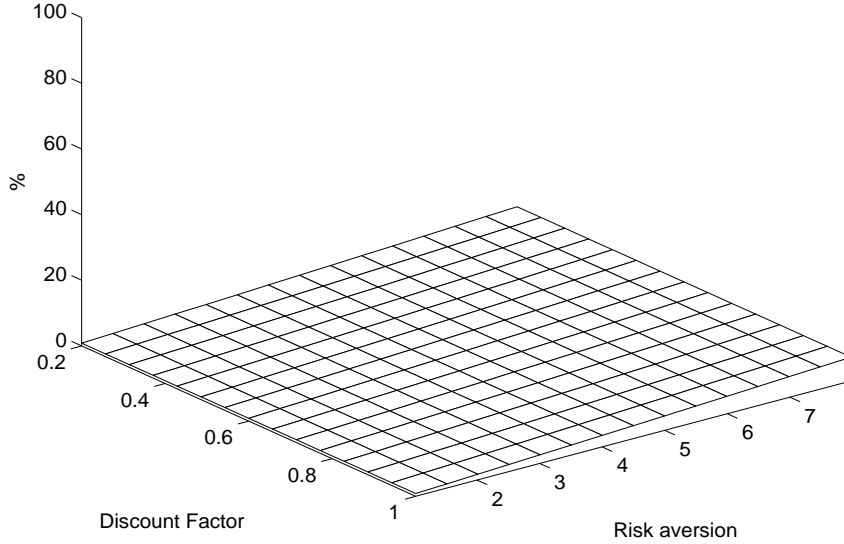


Figure 14: Standard deviation of the risk-free rate as a function of the discount rate and risk aversion in a Lucas economy.

Therefore, the interest rate decreases exponentially in the discount factor and increases slowly in the risk-aversion coefficient.

In the Alvarez-Jermann model, the relationship between parameter values and interest rates is more complicated because the interest rates depend not only on aggregate endowment but also on the consumption share of the unconstrained agent:

$$-\exp(r_t) = p_t^b = \beta E_t \left[\left(\frac{e_{t+1}}{e_t} \right)^{-\gamma} \max_{i=1,2} \left\{ \left(\frac{\hat{c}_{t+1}^i}{\hat{c}_t^i} \right)^{-\gamma} \right\} \right], \quad (21)$$

where \hat{c}_t^i is the agent i 's consumption share in period t . When the discount factor is lowered, the incentive to participate in the risk-sharing arrangement is reduced. This leads to an increase in variability in consumption shares and hence an increase in the “max” operator, thereby increasing bond prices and reducing one-period interest rates.

Campbell (1998) reports that in annual UK data for 1891–1995, the average real risk-free rate was 1.198 and its standard deviation was 5.446. In the Alvarez-Jermann model, matching these values leads to $\beta = 0.3378$ and $\gamma = 3.514$. $\beta = 0.3378$ is considerably less than what either complete markets or incomplete markets literature typically use. The reason is that, in order to be able to match asset market data, one has to reduce risk-sharing. In the Alvarez-Jermann endowment economy, an incentive to participate in risk-sharing is very high so that only by lowering the discount factor can autarky become tempting. In addition, note that the persistence of individual income is lower than the values estimated by either Heaton and Lucas (1996) or Storesletten, Telmer, and Yaron (1999). The quantitative results in Alvarez and Jermann (2000b) indicate that increasing the persistence does not change the asset pricing implications, provided that one is allowed to increase the discount factor. That is, the more persistent the individual income, the more tempting is default

Table 3: Selected statistics for the Lucas model when $\beta = 0.99$ and $\gamma = 6.1149$, for the Alvarez-Jermann model when $\beta = 0.3378$ and $\gamma = 3.514$, and for the data.

	Lucas Model	Alvarez-Jermann Model	Data
$E[r]$ (%)	7.93	1.198	1.198
$\text{std}[r]$ (%)	5.446	5.446	5.446
$E(\Delta c)$ (%)	1.443	1.443	1.443
$\text{std}[\Delta c]$ (%)	2.898	2.898	2.898
$\text{corr}[\Delta c]$	0.281	0.281	0.281
$\text{Pr}(\text{exp.})/\text{Pr}(\text{rec.})$	3.882	3.882	3.882
$\text{std}(\log(c^i / \sum_j c^j))$ (%)	0.0	26.71	—
$\text{corr}(\log(c^i / \sum_j c^j))$	1.0	0.4336	—
$\text{std}(\log(e^i / \sum_j e^j))$ (%)	28.3	28.3	28.3
$\text{corr}(\log(e^i / \sum_j e^j))$	0.4193	0.4193	0.2035

(the agents would like to accumulate state-contingent debt during bad times and default during good times) and hence the risk-sharing is reduced for more patient agents.

Obviously, in the Lucas economy risk-sharing is never limited, so it is not possible to match both the average risk-free rate and the standard deviation simultaneously. Since the standard deviation is more important for explaining the behavior of the term premium, I chose the discount factor and risk-aversion coefficients in the Lucas economy

$$(0.99, 6.1149) = \arg \min_{\beta \in (0, 0.99], \gamma \in (1, 100)} \{ |E[r(\beta, \gamma)] - 1.198| \text{ s.t. } \text{std}[r(\beta, \gamma)] = 5.446 \}.$$

Table 3 summarizes the main statistics for the models, given the above discount factor and risk aversion values, and in the data. Notice that, in the Alvarez-Jermann economy, the standard deviation of individual consumption is close to the standard deviation of individual income. In other words, the allocations are close to autarky allocations. In Appendix B, I show that this result is also very robust. In order to be able to match the basic asset pricing data, one has to reduce risk-sharing considerably from the full risk-sharing benchmark. It is difficult to say how reasonable this result is: In a recent paper, Brav, Constantinides, and Geczy (1999) conclude that the observation error in the consumption data makes it impossible to test the complete consumption insurance assumption against the assumption of incomplete consumption insurance.

The mechanism for limited risk-sharing works as follows. Both agents are off the solvency constraint when the current state is the same as the previous state. The average duration of expansion is about six years and the average duration of depression is about two years. Only when expansion changes to recession or vice versa is somebody constrained. When expansion changes to recession, the agent who got the lower share during the recession has the higher consumption growth rate. Hence, he would like to default. When recession changes to expansion, the agent who got the lower share during the recession has the higher consumption growth and would like to default. The next section shows how this mechanism translates to the behavior of the term structure of interest rates.²⁴

²⁴Clearly, with only two agents, one cannot interpret the agents literally, but rather how the

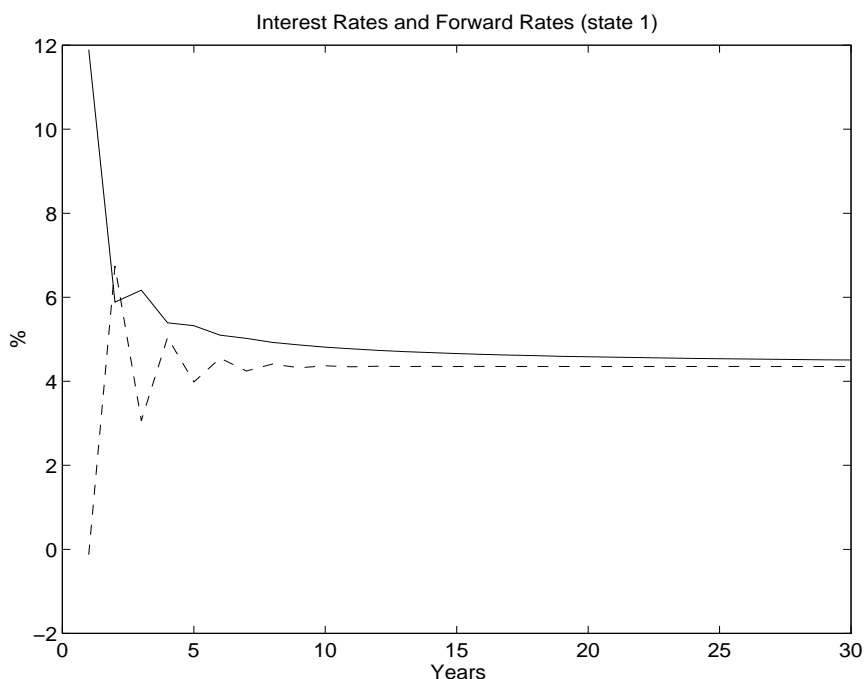


Figure 15: Interest rates (solid line) and forward rates in the Alvarez-Jermann economy during recessions.

5 Results

5.1 The Term Structure of Interest Rates

Figures 15–18 present the interest rates for maturities of 1 to 30 years and the forward rates of 1 to 30-year forward contracts during expansions and recessions in the Alvarez-Jermann and Lucas economies. A few things are worth noting from the Figures. First, both models produce both upward and downward-sloping term structures. However, the models’ cyclical behavior is exactly the opposite. In the Alvarez-Jermann model, the term structure of interest rates is downward-sloping in recessions and upward-sloping during expansions. In the Lucas model, the term structure of interest rates is upward-sloping in recessions and downward-sloping during expansions. Also, in both models upward-sloping term structures are always uniformly below downward-sloping term structures.²⁵ The cyclical behavior of the term structure is of particular interest since empirical and theoretical results from previous studies have been contradictory.

Fama (1990) reports that

“A stylized fact about the term structure is that interest rates are pro-cyclical. (...) [I]n every business cycle of the 1952–1988 period the one-year spot rate is lower at the business trough than at the preceding or following peak. (...) Another stylized fact is that long rates rise less than short rates during business expansions and fall less during contractions. Thus spreads of long-term over short-term yields are counter-cyclical. (...) [I]n every business cycle of the 1952–1988 period the five-year yield

fraction of population is affected by solvency constraints over the business cycle.

²⁵See Proposition 1 below for the explanation.

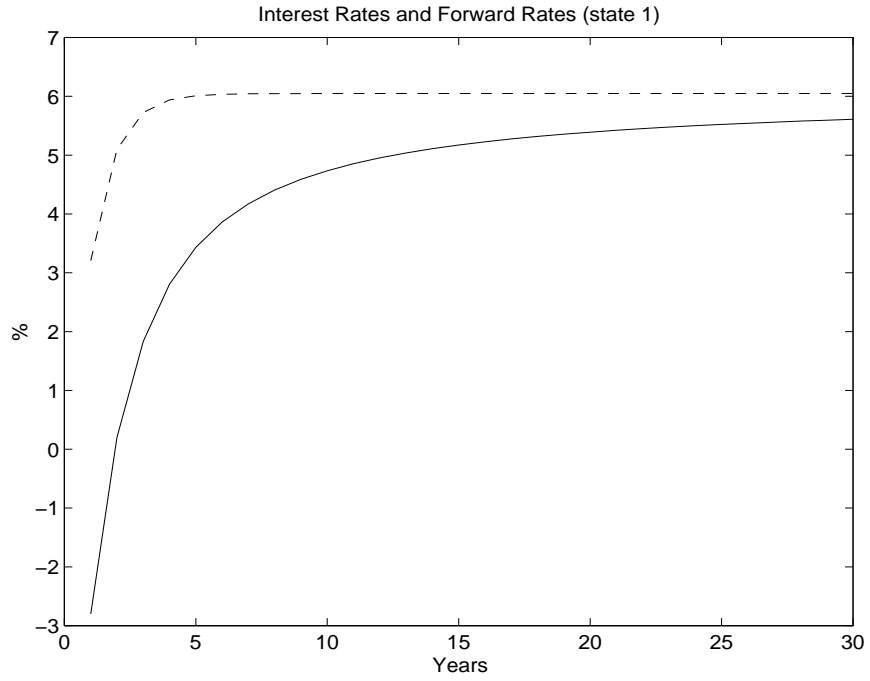


Figure 16: Interest rates (solid line) and forward rates in the Lucas economy during recessions.

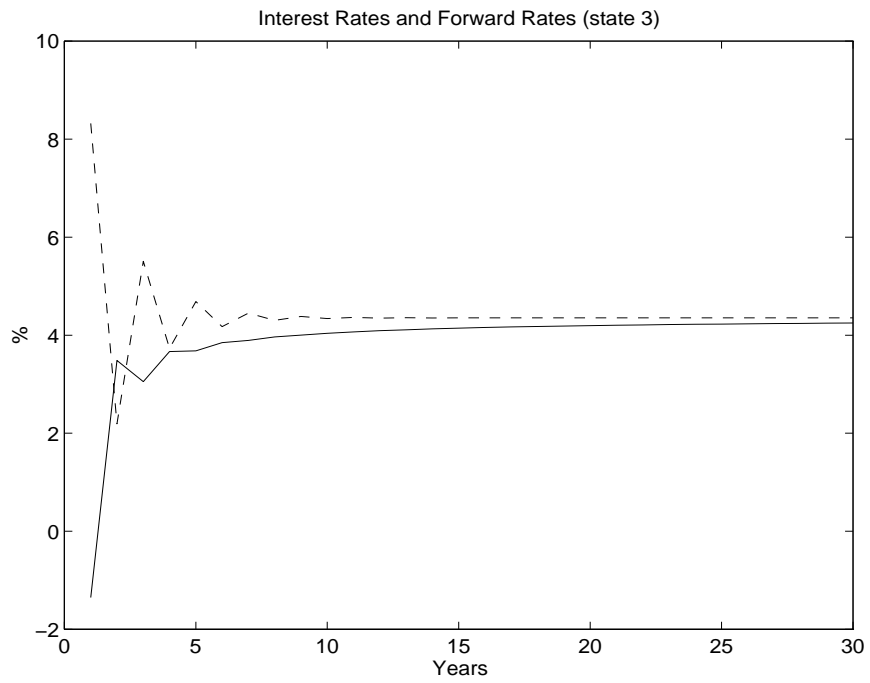


Figure 17: Interest rates (solid line) and forward rates in the Alvarez-Jermann economy during expansions.

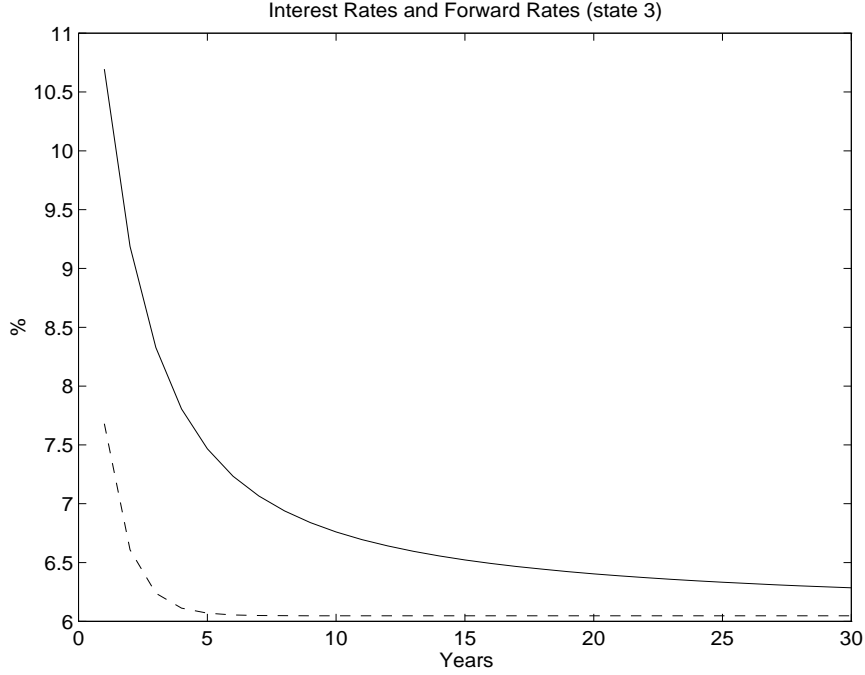


Figure 18: Interest rates (solid line) and forward rates in the Lucas economy during expansions.

spread (the five-year yield less the one-year spot rate) is higher at the business trough than at the preceding or following peak.”

Notice that this statement applies to the term structure of nominal interest rates. On the other hand, Donaldson, Johnsen, and Mehra (1990) report that in a stochastic growth model with full depreciation the term structure of (ex-ante) real interest rates is rising at the top of the cycle and falling at the bottom of the cycle. In addition, at the top of the cycle the term structure lies uniformly below the term structure at the bottom of the cycle.

In the Lucas economy, the cyclical behavior of the term structure will depend on the autocorrelation of consumption growth. Recall equation (19):

$$r_{n,t} = -\log(\beta) + \frac{\gamma}{n} E_t \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right] - \frac{\gamma^2}{2n} \text{var}_t \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right].$$

It implies that if consumption growth is positively autocorrelated, then a good shock today will forecast good shocks in the future and consequently high interest rates for the near future. As the maturity increases, however, the autocorrelation decreases and the maturity term in the denominator starts to kick in reducing the interest rates. The interest rates move one-for-one with the business cycle, exactly as Donaldson, Johnsen, and Mehra (1990) report.²⁶

In the Alvarez-Jermann economy, the prices of multiple-period bonds are determined by

$$-n \exp(r_{n,t}) = p_{n,t}^b = \beta^n E_t \left[\left(\frac{e_{t+n}}{e_t} \right)^{-\gamma} \times \max_{i=1,2} \left\{ \left(\frac{\hat{c}_{t+1}^i}{\hat{c}_t^i} \right)^{-\gamma} \right\} \times \cdots \times \max_{i=1,2} \left\{ \left(\frac{\hat{c}_{t+n}^i}{\hat{c}_{t+n-1}^i} \right)^{-\gamma} \right\} \right].$$

²⁶However, the question of the cyclical behavior of the term structure is more complicated for production economies. See den Haan (1995) and Vigneron (1999).

However, to study the slope of the term structure it is sufficient to know whether interest rates are procyclical or countercyclical. To see this, note that the following version Dybvig, Roll, and Ross (1996) holds in the Alvarez-Jermann economy.

Proposition 1. *If the transition matrix for the economy is ergodic, then as n approaches infinity, the forward price, $p_{n,t}^f$, converges to a constant.*

Proof. Let m_{ij} denote the pricing kernel between states i and j . In state i , the price of a one-period bond is $\sum_j \pi_{ij} m_{ij}$. This can be expressed as $\sum_j b_{ij}$, where b_{ij} defines a matrix B . Similarly, the price of a two-period bond is

$$\sum_j \sum_k \pi_{ij} m_{ij} \pi_{jk} m_{jk}$$

or $\sum_j b_{ij}^{(2)}$, where $b_{ij}^{(2)}$ denotes the (i, j) element of B^2 . In general, the price of an n -period bond is given by $\sum_j b_{ij}^{(n)}$. Since the transition matrix is ergodic, the Perron-Frobenius theorem guarantees that the dominant eigenvalue of B is positive and that any positive vector operated on by powers of B will eventually approach the associated eigenvector and grow at the rate of this eigenvalue. Recall that n -period forward price is the ratio of the price of an $(n + 1)$ -period bond to the price of an n -period bond. As n gets large, the ratio converges to the dominant eigenvalue of B regardless of the current state. ■

An immediate corollary of Proposition 1 is that the risk premia will converge. Since the transition matrix is ergodic, the expected spot price, $E_t[p_{1,t+n}^b]$, will converge and the limiting risk premium is the difference between the limiting forward price and the limiting expected spot price.

Therefore, it is enough to study one-period bonds that are determined by equation (21)

$$-\exp(r_t) = p_t^b = \beta E_t \left[\left(\frac{e_{t+1}}{e_t} \right)^{-\gamma} \max_{i=1,2} \left\{ \left(\frac{\hat{c}_{t+1}^i}{\hat{c}_t^i} \right)^{-\gamma} \right\} \right].$$

Recall that the variability of consumption shares increases in recessions and that aggregate income growth is positively autocorrelated. Hence, the two terms inside conditional work opposite to each other. In the current parameterization, the dominant term is consumption heterogeneity. The greatest variability inside the “max” operator occurs when one moves from the expansion state to the recession state. This means that bond prices increases during expansions or that interest rates decline. Interest rates are countercyclical even though consumption growth is positively autocorrelated.²⁷

Next, Figures 19–22 present the risk and term premia for maturities of 1 to 30 years during expansions and recessions in the Alvarez-Jermann and Lucas economies.

²⁷In the British data presented in Section 2, the correlation between one-year real interest rate and the cyclical component of real GDP/capita (obtained using a Hodrick-Prescott filter with a smoothing parameter of 1,600) is -0.17 . The correlation between yield spread (five-year yield minus one-year yield) and the cyclical component is 0.43 . The correlations between nominal data and the cyclical component are 0.19 and -0.48 , respectively. In other words, the Alvarez-Jermann model seems to be consistent with the real data and the Lucas model with the nominal data. These estimates, unfortunately, are not very reliable, as the Britain had time to go through only one business cycle in the sample. It is interesting to note that King and Watson (1996) obtained the same result for the cyclical behavior of nominal and real interest rates in U.S. data. They obtained real interest rates by estimating expected inflation using VAR.

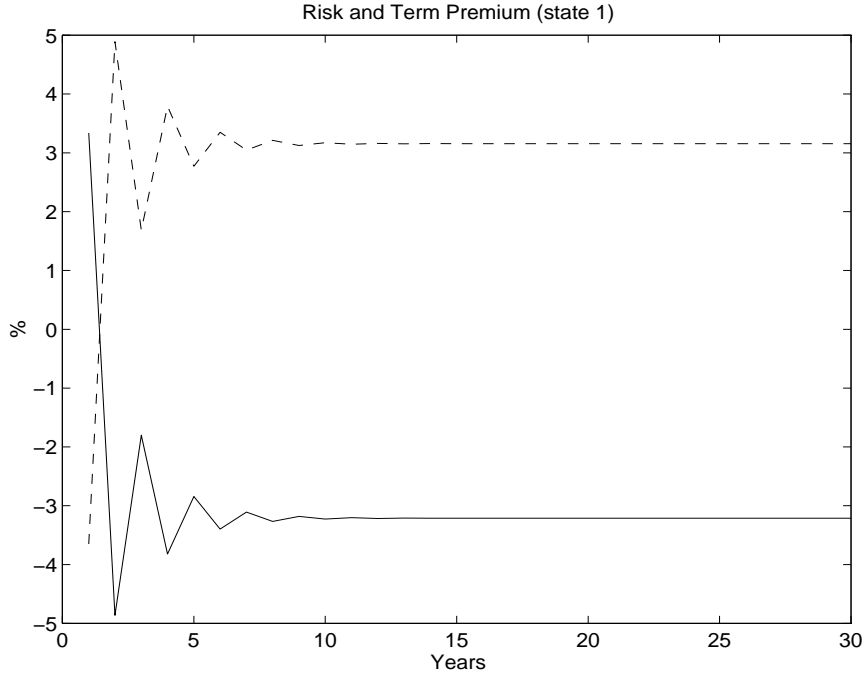


Figure 19: Risk premium (solid line) and term premium in the Alvarez-Jermann economy during recessions.

Note how the signs of risk and term premia are opposite in the Lucas vs. Alvarez-Jermann economies. A positive sign on the term premium means that forward rates tend to overpredict future interest rates and a negative sign means that the term premium underpredicts. In addition, in the Lucas model the term premium is very stable, and in the Alvarez-Jermann model, although the term premium is also stable over the cycle for long maturities, it varies considerable and negatively with the level of interest rates for short maturities. This result indicates that the Alvarez-Jermann model may be useful in accounting for rejections of the expectations hypothesis.

Figures 23–28 present the mean and the standard deviation of the interest rates, forward rates, risk, and term premia. The average term structure is upward-sloping in the Alvarez-Jermann economy and downward-sloping in the Lucas economy. In both economies, the standard deviations decrease with the maturity as in the UK nominal and real data. In the Lucas economy, the average shape of the term structure is easy to explain. Recall equation (20):

$$E[r_{n,t}] = -\log(\beta) + \frac{\gamma}{n} E \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right] - \frac{\gamma^2}{2n} \text{var} \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right].$$

If the $E \left[\log \left(\frac{e_{t+n}}{e_t} \right) \right] \approx n\bar{e}$, then the average shape of the term structure is determined by the ratio of the variance term to maturity. If consumption growth is positively autocorrelated, the variance term grows faster than the maturity since shocks in the growth rate are persistent.²⁸ In the Alvarez-Jermann economy, unconditional expectations are more difficult to obtain, but it is sufficient to note that the term structure is upward-sloping during the expansions and the economy is growing most of the time.

²⁸See den Haan (1995).

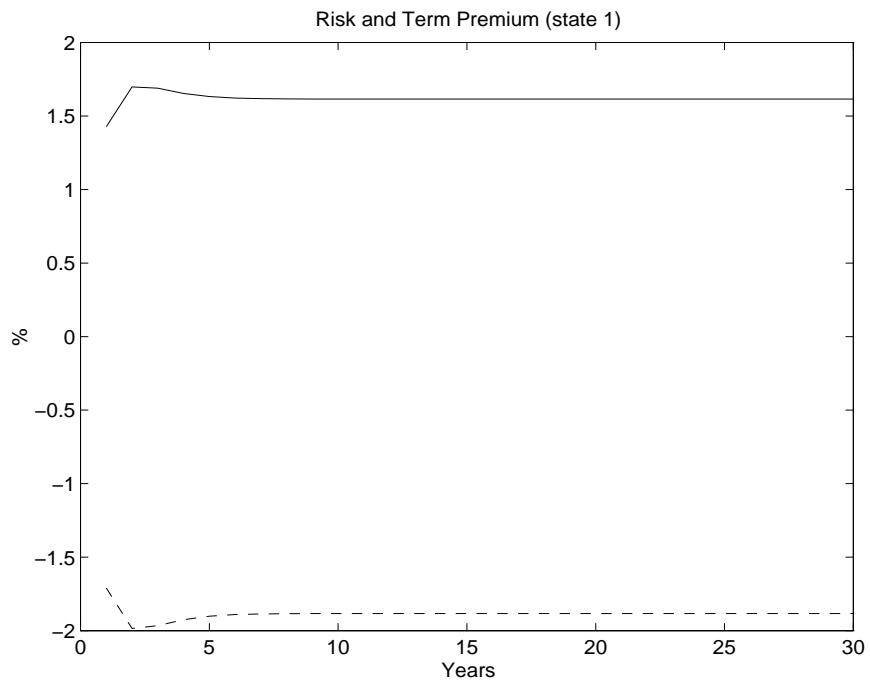


Figure 20: Risk premium (solid line) and term premium in the Lucas economy during recessions.

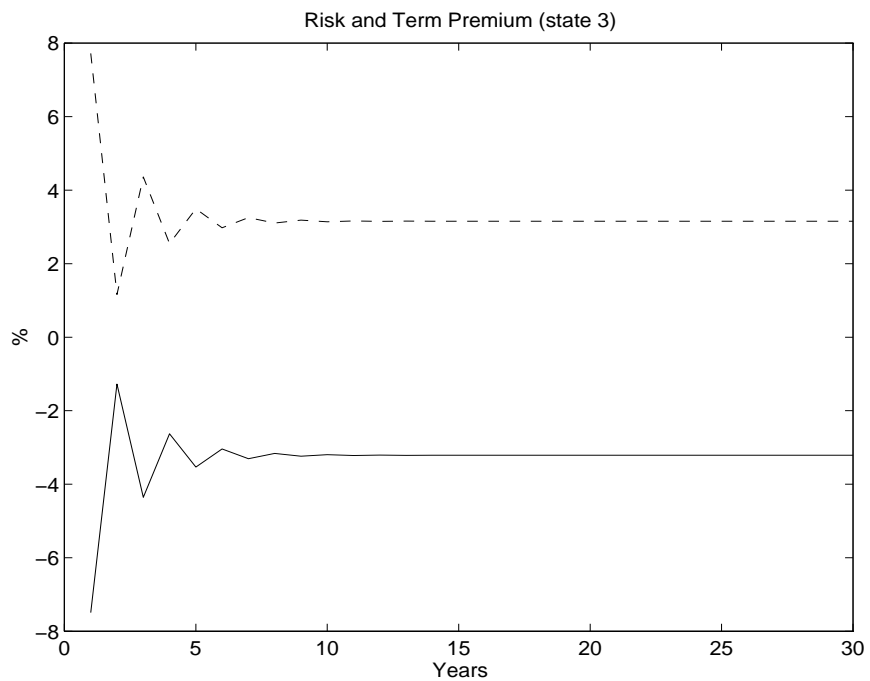


Figure 21: Risk premium (solid line) and term premium in the Alvarez-Jermann economy during expansions.

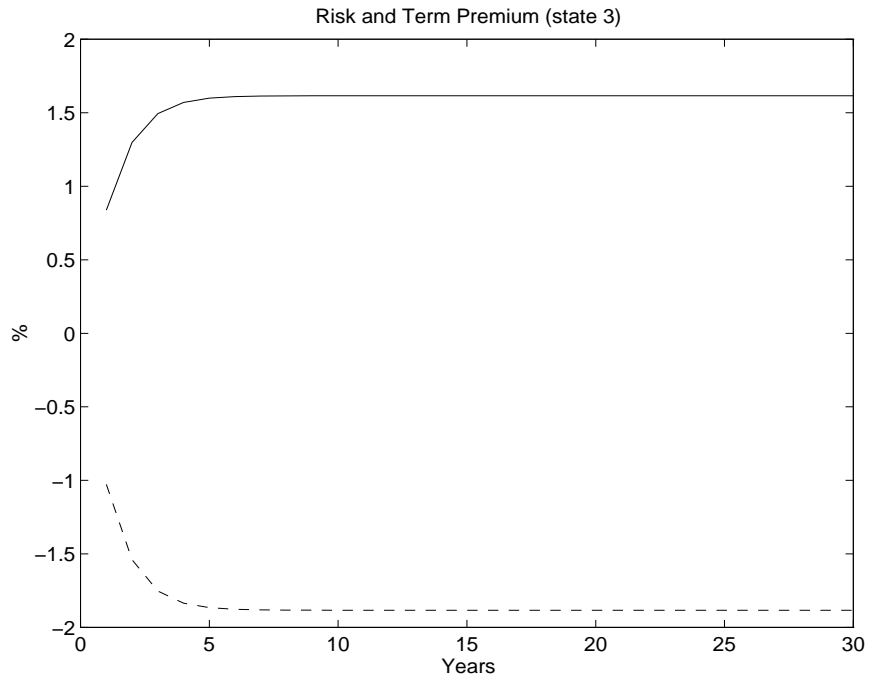


Figure 22: Risk premium (solid line) and term premium in the Lucas economy during expansions.

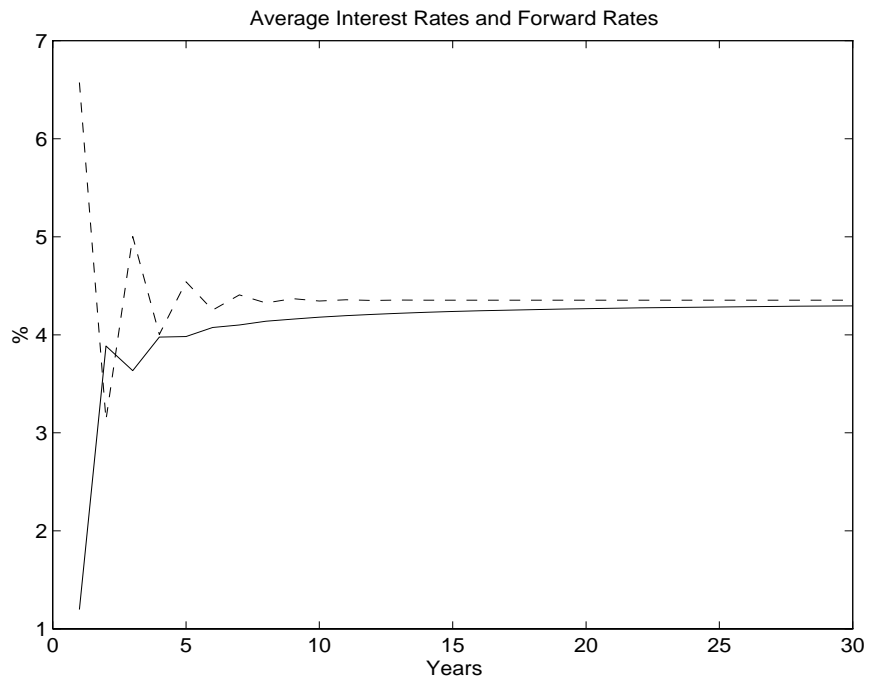


Figure 23: Average interest rates (solid line) and forward rates in the Alvarez-Jermann economy.

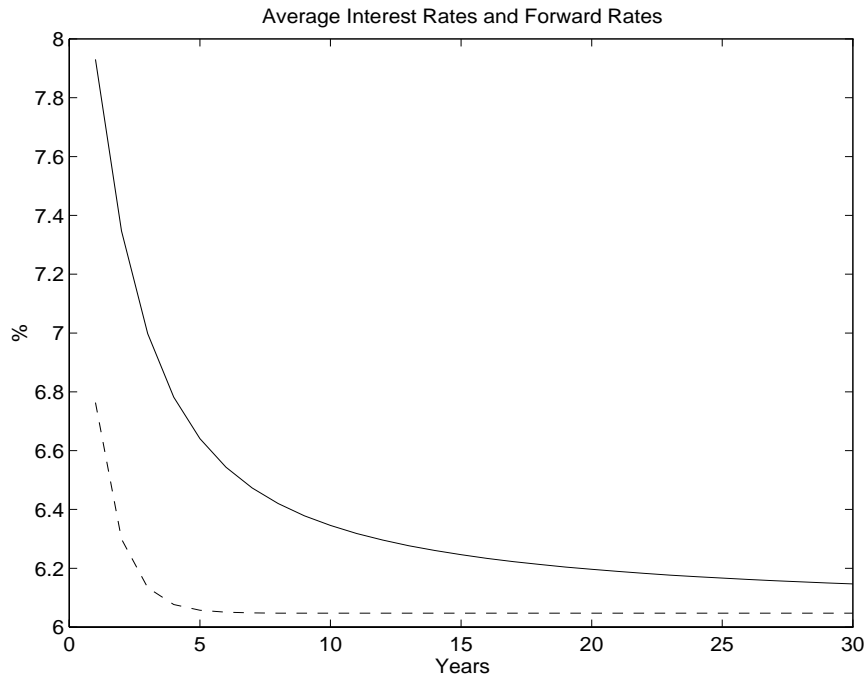


Figure 24: Average interest rates (solid line) and forward rates in the Lucas economy.

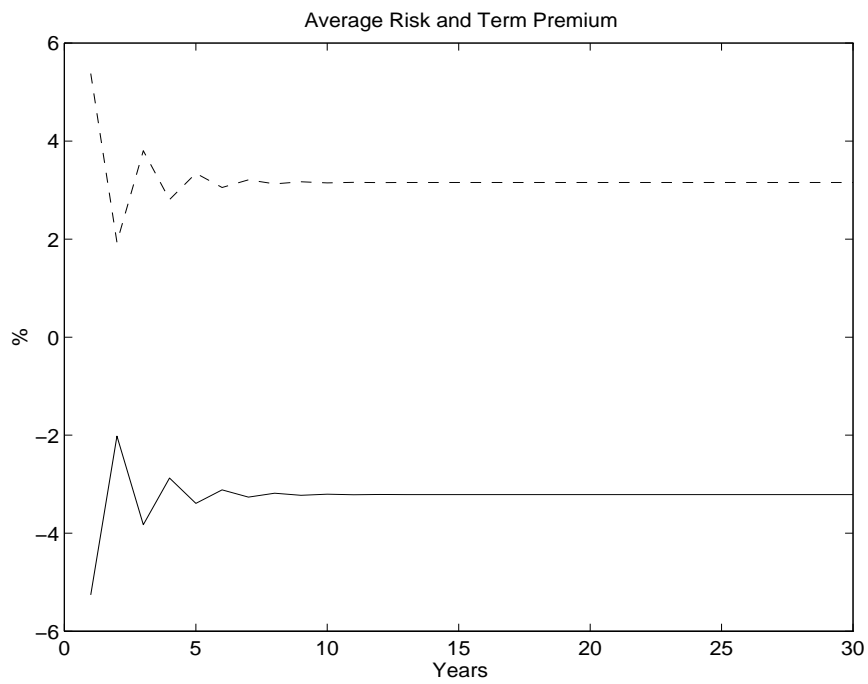


Figure 25: Average risk premium (solid line) and term premium in the Alvarez-Jermann economy.

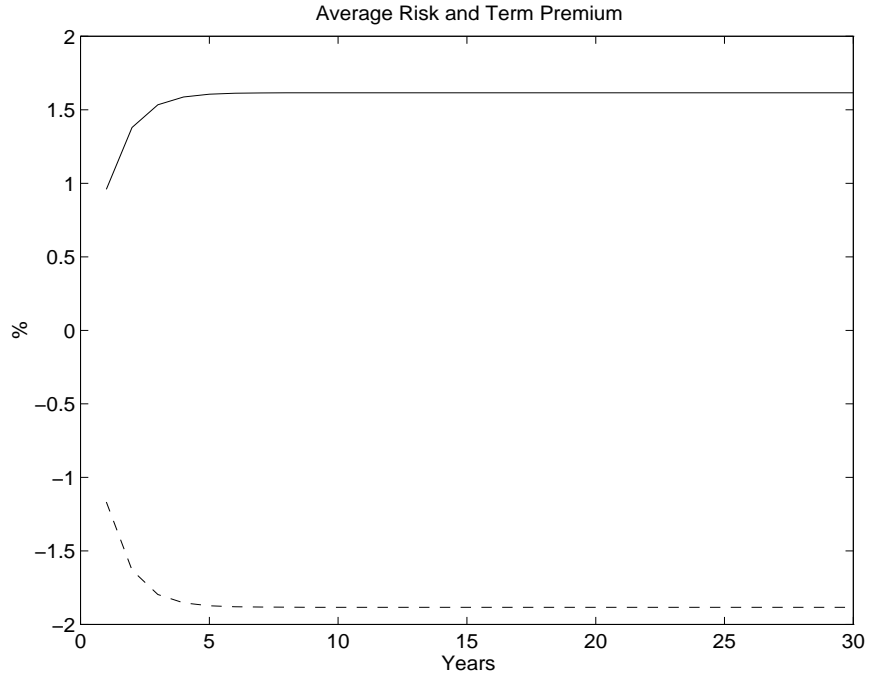


Figure 26: Average risk premium (solid line) and term premium in the Lucas economy.

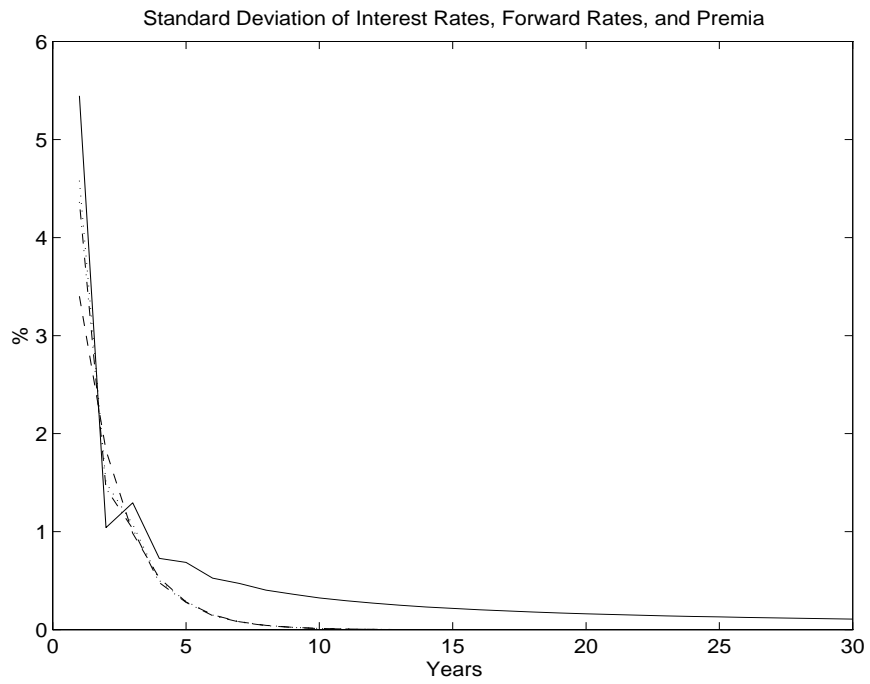


Figure 27: Standard deviation of interest rates (solid line), forward rates (dashed line), risk premium (dash-dot line), and term premium (dotted line) in the Alvarez-Jermann economy.

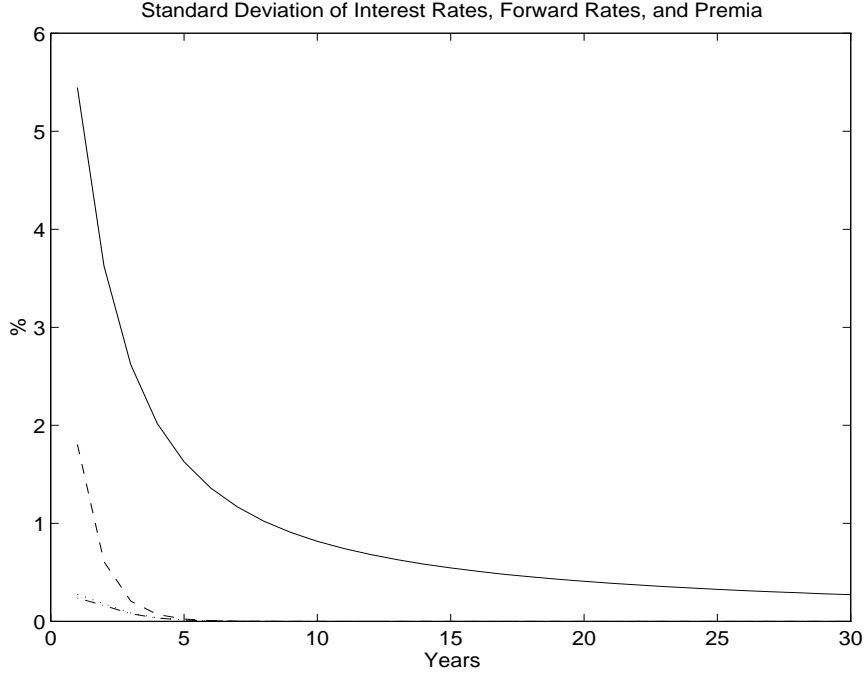


Figure 28: Standard deviation of interest rates (solid line), forward rates (dashed line), risk premium (dash-dot line), and term premium (dotted line) in the Lucas economy.

The relationship between the term premium and the shape of the term structure is as follows. Yields can be expressed as averages of forward rates:

$$r_{n,t} = \frac{1}{n} \sum_{j=0}^{n-1} f_{j,t}.$$

Therefore, the average term structure can be expressed as

$$E[r_{n,t}] - E[r_{1,t}] = \frac{1}{n} \sum_{j=0}^{n-1} E[f_{j,t} - E_t[r_{1,t+j}]] = \frac{1}{n} \sum_{j=0}^{n-1} tp_{j,t}.$$

Using the log-linear approximation and assuming homoscedastic errors, the term premium can be expressed as

$$tp_{n,t} = -\frac{1}{2} \text{var}_t[\log(p_{1,t+n}^b)] - \text{cov}_t[\log(p_{1,t+n}^b), \log(m_{t+n})].$$

In the Lucas economy, this reduces into

$$tp_{n,t} = -\frac{\gamma^2}{2} \text{var}_t \left[E_{t+n} \log \left(\frac{e_{t+n+1}}{e_{t+n}} \right) \right] - \gamma^2 \text{cov}_t \left[E_{t+n} \log \left(\frac{e_{t+n+1}}{e_{t+n}} \right), \log(e_{t+n}) \right].$$

Suppose that $\Delta \log(e_t) = \rho \Delta \log(e_{t-1}) + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. Then

$$tp_{1,t} = -\frac{\gamma^2}{2} \rho^2 \sigma_\epsilon^2 - \gamma^2 \rho \sigma_\epsilon^2 = -\gamma^2 \sigma_\epsilon^2 \rho^2 \left[\frac{1}{2} + \frac{1}{\rho} \right],$$

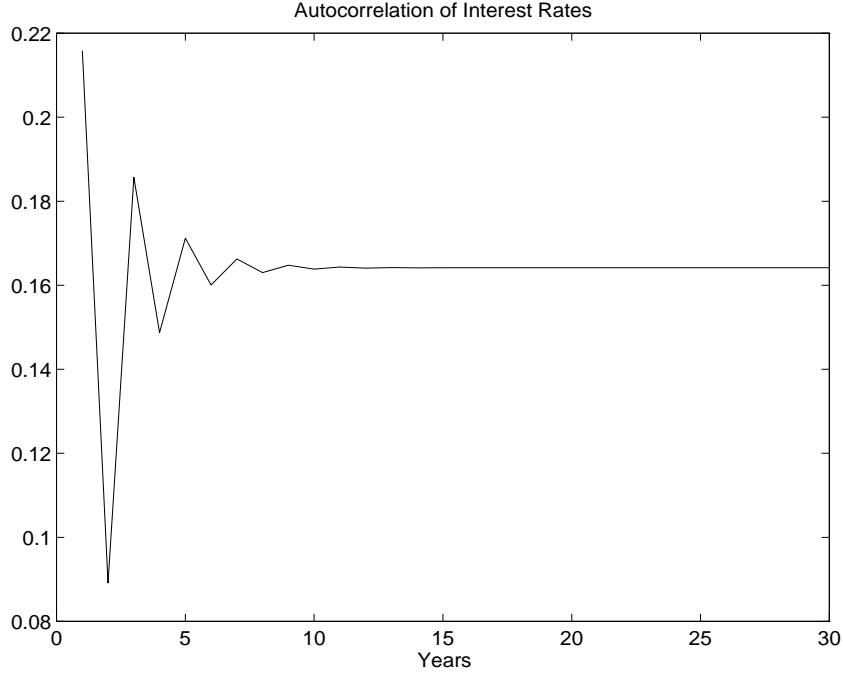


Figure 29: Autocorrelation of interest rates in the Alvarez-Jermann Economy.

which is less than zero, if $\rho > 0$. For example, in the British data

$$tp_{1,t} = -(6.1149)^2 (0.02898)^2 (0.281)^2 \left[\frac{1}{2} + \frac{1}{0.281} \right] = -1\%.$$

Alvarez and Jermann (2000b) show that in the Alvarez-Jermann economy the sign of the term premium depends on one-period ahead individual income variance, conditional on current aggregate state or “heteroscedasticity ex-ante”:

$$\frac{\sigma_r}{\sigma_e} \equiv \frac{\text{std}[\log(\alpha^i(z_{t+1})) | \lambda_t = \lambda_r]}{\text{std}[\log(\alpha^i(z_{t+1})) | \lambda_t = \lambda_e]}.$$

When $\frac{\sigma_r}{\sigma_e} > 1$, the term premium is negative and the average term structure is downward-sloping; when $\frac{\sigma_r}{\sigma_e} < 1$, the term premium is positive and the average term structure is upward-sloping. $\frac{\sigma_r}{\sigma_e} < 1$ means that in expansions one expects more idiosyncratic risk in the future. From (21) it follows that the max-term in the stochastic discount factor becomes more volatile and hence bond prices are higher (interest rates lower) in expansions. Therefore, a positive term premium is required to compensate bond holders. The next section studies whether this term premium is volatile enough to account for rejections of the expectations hypothesis.

Finally, Figures 29 and 30 present autocorrelations for interest rates in both economies. In the Alvarez-Jermann economy, autocorrelations are U-shaped between 0.08 and 0.2, as in the empirical term structure of real interest rates. However, in the Lucas economy the autocorrelation is constant and determined by the autocorrelation of consumption growth (0.281).

5.2 The Expectations Hypothesis

There are two main versions of the expectations hypothesis. While most of the empirical literature has concentrated on the expectations hypothesis in rates, it is

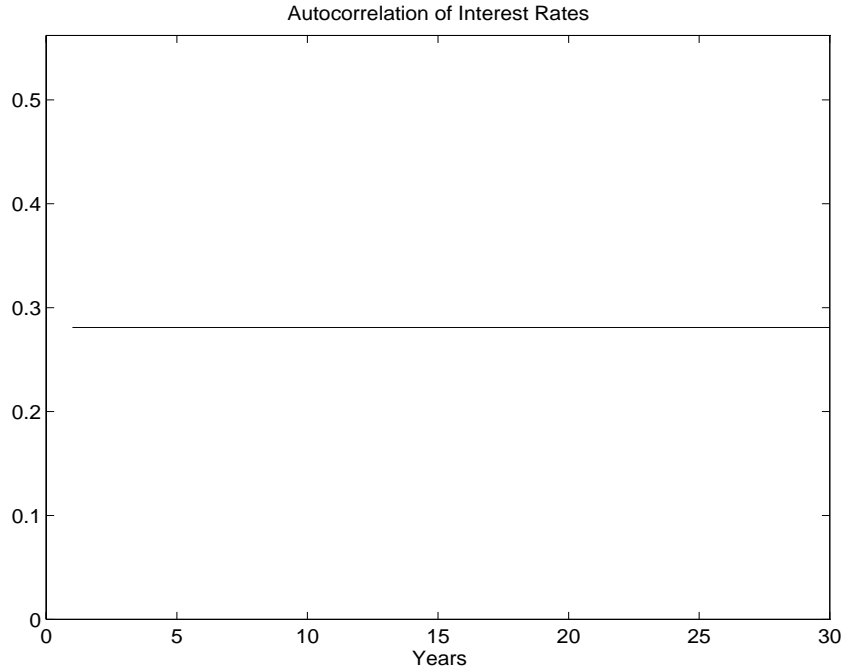


Figure 30: Autocorrelation of interest rates in the Lucas Economy.

pedagogical to start with the expectations hypothesis in prices. Recall equation (17)

$$p_{1,t}^f = E_t[p_{1,t+1}^b] + \text{cov}_t \left[m_{t+1}, \frac{p_{1,t+1}^b}{p_{1,t}^b} \right].$$

Backus, Gregory, and Zin (1989) tested the expectations model in the Lucas economy by starting with (17), assuming that the risk premium was constant, i.e.,

$$E_t[p_{1,t+1}^b] - p_{1,t}^f = a,$$

and regressing

$$p_{1,t+1}^b - p_{1,t}^f = a + b(p_{1,t}^f - p_{1,t}^b) \quad (22)$$

to see if $b = 0$. They generated 200 observations 1000 times and used the Wald test with White (1980) standard errors to check if $b = 0$ at the 5% significance level. They could reject the hypothesis only roughly 50 times out of 1000 regressions, which is what one would expect from chance alone. On the other hand, for all values of b except -1 , the forward premium is still useful in forecasting changes in spot prices. The hypothesis $b = -1$ was rejected every time.

Table 4 presents the number of rejections of different Wald tests in the regressions

$$y_{t+1} = a + bx_t$$

in the Lucas economy calibrated to UK data. Table 5 presents the same tests for the Alvarez-Jermann model. Unlike in the Lucas model, the results with the Alvarez-Jermann model are consistent with empirical evidence on the expectations hypothesis. The model can generate enough variation in the risk premia to account for rejections of the expectations hypothesis. When the risk premium is subtracted from $p_{1,t+1}^b - p_{1,t}^f$, b is equal to zero with 5% significance level.

Table 4: The number of rejects with regressions in the Lucas economy.

y_{t+1}	$p_{1,t+1}^b - p_{1,t}^f$	$p_{1,t+1}^b - p_{1,t}^f - rp_{1,t}$	$p_{1,t+1}^b - p_{1,t}^f$	$p_{1,t+1}^b - p_{1,t}^f - rp_{1,t}$
x_t	$p_{1,t}^f - p_{1,t}^b$	$p_{1,t}^f - p_{1,t}^b$	$p_{1,t}^b - p_{1,t}^f$	$p_{1,t}^b - p_{1,t}^f$
<i>Wald</i> ($a = b = 0$)	648	71	651	68
<i>Wald</i> ($b = 0$)	109	67	105	62
<i>Wald</i> ($b = -1$)	1000	1000	1000	1000

Table 5: The number of rejects with regressions in the Alvarez-Jermann economy.

y_{t+1}	$p_{1,t+1}^b - p_{1,t}^f$	$p_{1,t+1}^b - p_{1,t}^f - rp_{1,t}$	$p_{1,t+1}^b - p_{1,t}^f$	$p_{1,t+1}^b - p_{1,t}^f - rp_{1,t}$
x_t	$p_{1,t}^f - p_{1,t}^b$	$p_{1,t}^f - p_{1,t}^b$	$p_{1,t}^b - p_{1,t}^f$	$p_{1,t}^b - p_{1,t}^f$
<i>Wald</i> ($a = b = 0$)	1000	54	1000	62
<i>Wald</i> ($b = 0$)	1000	65	1000	62
<i>Wald</i> ($b = -1$)	1000	1000	1000	1000

In Table 6 the results of the regression (22) are presented for one realization of 200 observations for the Lucas model and for the Alvarez-Jermann model, and for UK real and nominal interest rate data. In Table 6, *Wald* rows refer to the marginal significance level of the corresponding Wald test. It is worth noting how close the values of the regression coefficients are for the Alvarez-Jermann model and the real UK data. On the other hand, in the nominal term structure, the forward premium has very little power in forecasting changes in spot prices.

Recent empirical literature has concentrated on the Log Pure Expectations Hypothesis. According to the hypothesis, the n -period forward rate should equal the expected one-period interest rate n periods ahead:

$$f_{n,t} = E_t[r_{1,t+n}].$$

To test the hypothesis, one can run the regression

$$(n - 1) * (r_{n-1,t+1} - r_{n,t}) = a + b(r_{n,t} - r_{1,t}) \quad \text{for } n = 2, 3, 4, 5, 6, 11. \quad (23)$$

According to the Log Pure Expectations Hypothesis, one should find that $b = 1$.²⁹ Table 7 summarizes the results from this regression for the models and for real and nominal data. The expectations hypothesis is clearly rejected in all cases except for the Lucas model.³⁰

²⁹See, e.g., Campbell, Lo, and McKinley (1997).

³⁰The negative a coefficients for the Alvarez-Jermann model and for nominal data follow from the upward-sloping average shape of the term structure. Recall from Section 2.2 that the average shape of the nominal term structure is upward-sloping, and it is downward-sloping for the real term structure estimated by Evans (1998). Since Brown and Schaefer (1994, 1996) estimated the term structure of real interest rates to be upward-sloping on average, this issue warrants more attention.

Table 6: Tests of the expectations hypothesis with a single regression.

Variable/Test	Lucas	Alvarez-Jermann	Real Data	Nominal Data
a	-0.0118	0.0279	-0.0047	0.0001
$se(a)$	0.0041	0.0032	0.002	0.0005
b	0.0070	-0.4970	-0.4982	-0.9033
$se(b)$	0.2711	0.0481	0.1046	0.0416
R^2	0.0067	0.4036	0.3457	0.7218
$Wald(a = b = 0)$	0.0013	2e-104	3e-7	0.0
$Wald(b = 0)$	0.9569	4e-19	2e-6	0.0
$Wald(b = -1)$	5e-15	1e-19	2e-6	0.02

Table 7: Expectations hypothesis regressions in rates.

Regression	a	$se(a)$	b	$se(b)$	R^2
Lucas ($n = 2$)	1.3841	0.0007	1.1830	0.0003	0.1443
Lucas ($n = 3$)	1.9277	0.0009	1.1609	0.0002	0.1808
Lucas ($n = 4$)	2.1219	0.0010	1.1449	0.0002	0.2120
Lucas ($n = 5$)	2.1896	0.0010	1.1339	0.0001	0.2370
Lucas ($n = 6$)	2.2131	0.0010	1.1264	0.0001	0.2564
Lucas ($n = 11$)	2.2265	0.0010	1.1103	0.0001	0.3056
Alvarez-Jermann ($n = 2$)	-2.5150	0.0009	-0.0586	0.0001	0.0024
Alvarez-Jermann ($n = 3$)	-0.8688	0.0004	0.5663	0.0001	0.5629
Alvarez-Jermann ($n = 4$)	-1.8223	0.0007	0.2910	0.0001	0.1152
Alvarez-Jermann ($n = 5$)	-1.3232	0.0005	0.4714	0.0001	0.3786
Alvarez-Jermann ($n = 6$)	-1.6003	0.0006	0.3983	0.0001	0.2522
Alvarez-Jermann ($n = 11$)	-1.5077	0.0006	0.4567	0.0001	0.3517
Real Data ($n = 2$)	0.2752	0.2336	0.2931	0.1448	0.0365
Real Data ($n = 3$)	0.2316	0.2543	0.4033	0.1113	0.1
Real Data ($n = 4$)	0.2055	0.2820	0.4199	0.1029	0.1121
Real Data ($n = 5$)	0.2056	0.3135	0.3135	0.4168	0.1060
Real Data ($n = 6$)	0.2225	0.3470	0.4059	0.1034	0.0942
Real Data ($n = 11$)	0.3891	0.5198	0.3891	0.1336	0.0292
Nominal Data ($n = 2$)	-0.1473	0.1679	0.3570	0.2623	0.0174
Nominal Data ($n = 3$)	-0.5544	0.3039	0.5518	0.3064	0.0435
Nominal Data ($n = 4$)	-0.8968	0.4275	0.6941	0.3745	0.0526
Nominal Data ($n = 5$)	-1.1834	0.5394	0.7673	0.4462	0.0539
Nominal Data ($n = 6$)	-1.4215	0.6383	0.7696	0.5111	0.0518
Nominal Data ($n = 11$)	-2.1516	0.9721	-0.1958	0.7253	0.0367

6 Conclusions and Further Research

With risk-averse agents, the term structure contains expectations plus term premia. In order for policy makers to extract information about market expectations from the term structure, they need to have a general idea about the sign and magnitude of the term premium. But as Söderlind and Svensson (1997) note in their review

“We have no direct measurement of this (potentially) time-varying covariance [term premium], and even ex post data is of limited use since the stochastic discount factor is not observable. It has unfortunately proved to be very hard to explain (U.S. ex post) term premia by either utility based asset pricing models or various proxies for risk.”

This paper studied the behavior of the default-risk free real term structure and term premia in two general equilibrium endowment economies with complete markets but without money. In the first economy there were no frictions, as in Lucas (1978) and in the second the risk-sharing was limited by the risk of default, as in Alvarez and Jermann (2000ab). Both models were solved numerically, calibrated to UK aggregate and household data, and the predictions were compared to the data on real interest rate constructed from UK index-linked data. While both models produce time-varying term premia, only the model with limited risk-sharing can generate enough variation in the term premia to account for the rejections of expectations hypothesis.

I conclude that the Alvarez-Jermann model provides one plausible explanation for the Backus-Gregory-Zin term premium puzzle in real term structure data. What is needed now is a theory to explain the behavior of the term structure of nominal interest rates. Since it is usually recognized that monetary policy can only have effect with “long and variable lags” as Friedman (1968) put it, it is crucial to understand what are the inflation expectations that drive the market. Once we understand the behavior of both nominal and real interest rates, we can get correct estimates of these inflation expectations. An interesting topic for further research is whether a nominal version of the Alvarez-Jermann model is consistent with the nominal data.

Another interesting topic would be to analyze the cyclical behavior of nominal and real term structures, both in data and in theory. King and Watson (1996) provide an example of how to do this, but they had to use real interest rates that they constructed using a VAR framework. In my opinion, the British data would provide a better approximation for ex-ante real interest rates. However, since the British data are still relatively short and neither the Lucas model nor the Alvarez-Jermann model were built to confront this question, this topic is left for further research.

A Algorithm

This section explains how the Alvarez-Jermann model can be solved numerically. The aggregate endowment is growing over time, but the CRRA utility function implies that the value function, $V(\cdot)$, autarky values of utility, $U^i(\cdot)$, and the policies, $\{C^1(\cdot), C^2(\cdot), W(\cdot)\}$, satisfy the following homogeneity property.

Proposition 2. *For any $y > 0$ and any (w, z, e) ,*

$$\begin{aligned} V(y^{1-\gamma}w, z, ye) &= y^{1-\gamma}V(w, z, e) \\ U^i(z, ye) &= y^{1-\gamma}U^i(z, e) \quad \text{for } i = 1, 2 \\ C^i(y^{1-\gamma}w, z, ye) &= yC^i(w, z, e) \quad \text{for } i = 1, 2 \\ W(y^{1-\gamma}w, z, ye) &= y^{1-\gamma}W(w, z, e). \end{aligned}$$

Proof. See the proof of Proposition 3.9 in Alvarez and Jermann (1996). ■

Defining a new set of “hat” variables as

$$\begin{aligned} u(c) &= e^{1-\gamma}u\left(\frac{c}{e}\right) = e^{1-\gamma}u(\hat{c}) \\ U^i(z', e') &= (e')^{1-\gamma}U^i(z', 1) = \hat{U}^i(z', 1) \quad \text{for } i = 1, 2 \\ w &= \frac{e^{1-\gamma}}{e^{1-\gamma}}w = e^{1-\gamma}\hat{w} \\ w' &= \frac{(e')^{1-\gamma}}{(e')^{1-\gamma}}w' = (e')^{1-\gamma}\hat{w}', \end{aligned}$$

and using the above proposition in the following way:

$$\begin{aligned} V(w, z, e) &= V\left(\frac{e^{1-\gamma}}{e^{1-\gamma}}w, z, \frac{e}{e}\right) = e^{1-\gamma}V(\hat{w}, z, 1) \\ V(w', z', e') &= (e')^{1-\gamma}V\left(\frac{w'}{(e')^{1-\gamma}}, z', 1\right) = (\lambda(z')e)^{1-\gamma}V(\hat{w}, z, 1), \end{aligned}$$

the functional equation can be rewritten with stationary variables as

$$TV(\hat{w}, z, 1) = \max_{\hat{c}^1, \hat{c}^2, \{\hat{w}(z')\}} u(\hat{c}_1) + \beta \sum_{z' \in Z} V(\hat{w}(z'), z', 1) \lambda(z')^{1-\gamma} \pi(z'|z)$$

subject to

$$\hat{c}^1 + \hat{c}^2 \leq 1 \tag{24}$$

$$u(\hat{c}^2) + \beta \sum_{z' \in Z} \hat{w}(z') \lambda(z')^{1-\gamma} \pi(z'|z) \geq \hat{w} \tag{25}$$

$$\begin{aligned} V(\hat{w}(z'), z', 1) &\geq \hat{U}^1(z', 1) \quad \forall z' \in Z, \\ \hat{w}(z') &\geq \hat{U}^2(z', 1) \quad \forall z' \in Z. \end{aligned} \tag{26}$$

In order to guarantee the above maximization problem is well-defined, it is needed to assume that

$$\max_{z \in Z} \left\{ \beta \sum_{z' \in Z} \lambda(z')^{1-\gamma} \pi(z'|z) \right\} < 1.$$

The results in Alvarez and Jermann (1996) indicate that during recession the allocations do not depend on past history. However, during the expansion there are

two endogenous states: one where $z_t = z_e$ and $z_{t-1} = z_{hl}$, and another where $z_t = z_e$ and $z_{t-1} = z_{lh}$. From now on, the states are ordered as follows:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} (z_t = z_{hl}) \\ (z_t = z_e, z_{t-1} = z_{hl}) \\ (z_t = z_e, z_{t-1} = z_e) \\ (z_t = z_e, z_{t-1} = z_{lh}) \\ (z_t = z_{lh}) \end{bmatrix} .$$

Moreover, it is possible to solve for the allocations and prices simply by solving for at most two systems of nonlinear equations. The first system corresponds to the case in which the participation constraints are not binding, and the second system corresponds to the case in which the agents are constrained when entering the boom period. In addition to the nonlinear equation, there is a set of inequalities that determines which case is valid.

From feasibility (24) and non-satiation, it follows that agent 1's consumption is always the aggregate endowment less agent 2's consumption. Hence, both systems have 10 equations in 10 unknowns: agent 2's consumption and continuation utility in each of the five states. From now on, c_n will denote $\hat{c}^2(z_n)$ and "hats" will be dropped from other variables as well.

In both the unconstrained case and the constrained case, five equations are given by (25); during the expansion neither agent has a reason to trade: $c_3 = 0.5$; (24) and symmetry imply that $c_1 + c_5 = 1$; and the participation constraint (26) holds with equality when agent 2 receives the most favorable shock: $w(z_5) = U^2(z_5)$. The two missing equations depend on whether the agents are constrained when entering the expansion state. In the unconstrained case, $c_2 = c_1$ and $c_4 = c_5$, and, in the constrained case, $w(z_2) = U^2(z_2)$ and $c_2 + c_4 = 1$.

To solve for the allocations, one needs only to solve for the unconstrained system and check whether $w(z_2) \geq U^2(z_2)$. If this is not the case, the solution is given by the constrained case.

B Sensitivity Analysis for the Alvarez-Jermann Model

B.1 Preferences

Table 8 shows how sensitive the main results are to the calibrated value of the discount factor, β , while holding all other parameter values constant. In the table, N/A refers to autarky where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22). The table shows how the allocations move closer to full sharing as the discount factor is increased, so rejecting the expectations hypothesis becomes more and more difficult.

Table 8: Main statistics as a function of discount factor in the Alvarez-Jermann model.

β	0.3	0.3778	0.4556	0.5333	0.6111	0.6889	0.7667
$\text{std}(\log(c^i / \sum_j(c^j)))$ (%)	28.3	23.38	17.63	12.91	9.142	4.553	0
$\text{corr}(\log(c^i / \sum_j(c^j)))$	0.4193	0.4688	0.5514	0.6364	0.6358	0.6361	1
$E[r]$ (%)	N/A	13.67	29.67	37.17	37.42	34.29	30.95
$\text{std}[r]$ (%)	N/A	3.206	6.243	12.04	14.14	6.979	3.031
$r_{30} - r_1$ (exp.) (%)	N/A	0.4609	-5.775	-11.11	-4.995	-3.73	-1.984
$r_{30} - r_1$ (rec.) (%)	N/A	-5.056	-5.775	-6.123	-12.32	-2.784	5.161
$E[r_{30} - r_1]$ (%)	N/A	0.008	-3.089	-3.061	-1.778	-1.046	-0.5207
$E[\text{tp}_1]$ (%)	N/A	1.745	-0.4647	0.2831	0.7052	0.0648	-0.3535
$\text{std}[\text{tp}_1]$ (%)	N/A	2.118	1.111	1.358	0.3195	0.3577	0.09668
$Wald(b = 0)$	N/A	1000	308	69	113	123	85
b in (22)	N/A	-0.2486	-0.0257	-0.0758	-0.0814	0.0703	0.2045
b in (23) ($n = 2$)	N/A	0.3056	0.8255	0.9021	0.9988	1.0460	1.0622

Table 9 shows how sensitive the main results are to the calibrated value of the coefficient of risk aversion, γ , while holding all other parameter values constant. In the table, N/A refers to autarky, where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22). The table shows how the allocations move closer to full sharing as the coefficient of risk aversion is increased. In addition, the term premium changes sign and becomes less and less volatile, so that rejecting the expectations hypothesis becomes more and more difficult. However, when the risk aversion coefficient is very high, the interest rates fluctuate more and so the number of expectations hypothesis rejections increases slightly.

B.2 Individual Income

Table 10 shows how sensitive the main results are to the estimated value of the standard deviation of individual income relative to the aggregate income, $\text{std}(\log(e^i / \sum_j(e^j)))$, while holding all parameter values constant. In the table, N/A refers to autarky, where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22). The table shows how the allocations move closer to full sharing when the standard deviation of individual income is increased. In addition, the term premium changes sign and becomes less and less volatile, so that rejecting the expectations hypothesis becomes more and more difficult.

Table 11 shows how sensitive the main results are to the estimated value of the standard deviation of individual income relative to the aggregate income $\text{std}(\log(e^i / \sum_j(e^j)))$ when preference parameters are recalibrated to match the average risk-free rate and

Table 9: Main statistics as a function of the coefficient of risk aversion in the Alvarez-Jermann model.

γ	3.3	3.822	4.344	4.867	5.911	6.956	7.478
$\text{std}(\log(c^i / \sum_j(c^j)))$ (%)	28.3	22.87	17.58	13.5	7.065	0.3461	0
$\text{corr}(\log(c^i / \sum_j(c^j)))$	0.4193	0.4755	0.5533	0.6261	0.636	0.6362	1
$E[r]$ (%)	N/A	16.16	39.81	60.94	91.4	115	116.5
$\text{std}[r]$ (%)	N/A	5.226	8.903	15.98	18.32	5.764	6.76
$r_{30} - r_1$ (exp.) (%)	N/A	2.479	-1.915	-4.872	-6.829	-5.602	-6.08
$r_{30} - r_1$ (rec.) (%)	N/A	-8.163	-12.88	-20.25	-16.58	8.147	9.822
$E[r_{30} - r_1]$ (%)	N/A	1.079	-1.701	-3.194	-2.734	-2.413	-2.823
$E[\text{tp}_1]$ (%)	N/A	3.786	2.732	2.935	1.767	-1.411	-1.819
$\text{std}[\text{tp}_1]$ (%)	N/A	3.915	4.194	4.55	0.2946	0.0839	0.3929
$Wald(b = 0)$	N/A	1000	865	235	220	63	139
b in (22)	N/A	-0.4482	-0.1512	-0.0493	0.1054	-0.0248	0.0622
b in (23) ($n = 2$)	N/A	0.2128	0.5283	0.7062	1.0077	1.0703	1.1962

Table 10: Main statistics as a function of the standard deviation of individual income in the Alvarez-Jermann model.

$\text{std}(\log(e^i / \sum_j(e^j)))$ (%)	25	30	35	40	50	60	65
α_{hl}^1	0.7459	0.7826	0.8146	0.8423	0.8865	0.9186	0.9311
$\text{corr}(\log(e^i / \sum_j(e^j)))$	0.4193	0.4184	0.4158	0.4133	0.4084	0.404	0.402
$\text{std}(\log(c^i / \sum_j(c^j)))$ (%)	25	25.61	22.23	18.89	11.71	0.1176	0
$\text{corr}(\log(c^i / \sum_j(c^j)))$	0.4193	0.4588	0.5384	0.6174	0.6356	0.6362	1
$E[r]$ (%)	N/A	10.09	35.54	58.69	90.97	112.7	112.9
$\text{std}[r]$ (%)	N/A	6.226	9.702	15.67	18.15	2.948	3.031
$r_{30} - r_1$ (exp.) (%)	N/A	4.9323	1.375	-2.439	-5.263	-2.031	-1.984
$r_{30} - r_1$ (rec.) (%)	N/A	-9.188	-15.01	-21.52	-18.34	4.974	5.161
$E[r_{30} - r_1]$ (%)	N/A	2.545	0.1156	-1.994	-2.117	-0.529	-0.5207
$E[\text{tp}_1]$ (%)	N/A	5.402	4.648	3.693	1.353	-0.3422	-0.3535
$\text{std}[\text{tp}_1]$ (%)	N/A	4.978	5.594	5.476	1.238	0.0726	0.0967
$Wald(b = 0)$	N/A	1000	991	389	108	73	82
b in (22)	N/A	-0.3599	-0.2770	-0.0807	0.1169	0.2039	0.1358
b in (23) ($n = 2$)	N/A	0.0637	0.3735	0.6308	0.9402	1.0115	1.0802

its standard deviation. In the table, N/A refers to autarky where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22). The table shows that as the standard deviation of individual income is increased one can match the average risk-free rate and its standard deviation with a higher discount factor and lower coefficient of relative risk-aversion. Otherwise, all the results are robust with respect to the standard deviation of individual income, assuming that it is high enough. If the standard deviation of individual income is too low, the agents have no incentive to participate in a risk-sharing arrangement.

Table 11: Main statistics as a function of standard deviation of individual income in the Alvarez-Jermann model (model recalibrated).

std($\log(e^i / \sum_j(e^j))$) (%)	25	30	35	40	50	60	65
α_{hi}^1	0.7459	0.7826	0.8146	0.8423	0.8865	0.9186	0.9311
β	N/A	0.3589	0.4181	0.4708	0.5554	0.6164	0.6402
γ	N/A	3.207	2.505	2.016	1.402	1.047	0.9227
corr($\log(e^i / \sum_j(e^j))$)	0.4193	0.4184	0.4158	0.4133	0.4084	0.404	0.402
std($\log(c^i / \sum_j(c^j))$) (%)	25	28.23	32.63	36.93	45.27	53.33	57.28
corr($\log(c^i / \sum_j(c^j))$)	0.4193	0.4332	0.4316	0.4298	0.4255	0.4206	0.4181
$E[r]$ (%)	N/A	1.198	1.198	1.198	1.198	1.198	1.198
std $[r]$ (%)	N/A	5.446	5.446	5.446	5.446	5.446	5.446
$r_{30} - r_1$ (exp.) (%)	N/A	5.525	5.324	5.158	4.923	4.781	4.733
$r_{30} - r_1$ (rec.) (%)	N/A	-7.443	-7.596	-7.727	-7.931	-8.079	-8.138
$E[r_{30} - r_1]$ (%)	N/A	3.034	2.863	2.715	2.489	2.33	2.267
$E[tp_1]$ (%)	N/A	5.192	4.684	4.242	3.556	3.079	2.896
std $[tp_1]$ (%)	N/A	4.429	4.002	3.62	3.005	2.56	2.386
$Wald(b = 0)$	N/A	1000	1000	1000	1000	1000	998
b in (22)	N/A	-0.5190	-0.4487	-0.4691	-0.3841	-0.3379	-0.3374
b in (23) ($n = 3$)	N/A	0.5705	0.5457	0.5273	0.5063	0.5278	0.5306

B.3 Aggregate Consumption

Table 12 shows how sensitive the main results are to the estimated value of the fraction of time that the economy spends in expansion relative to time in recession. Again, preference parameters are recalibrated to match the average risk-free rate and its standard deviation. In Table, N/A refers to autarky where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22). The table shows that as the number of times the economy switches from the expansion state to the recession state is increased, one can match the average risk-free rate and its standard deviation with a higher discount factor and lower coefficient of relative risk-aversion.

Table 13 shows how sensitive the main results are to the estimated value of first-order autocorrelation of consumption growth. Again, preference parameters are recalibrated to match the average risk-free rate and its standard deviation. In the table, N/A refers to autarky, where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22). Note that, unlike in the Lucas model, the signs of the term premia and average term spread are unaffected by the sign of first-order autocorrelation of consumption growth.

Table 14 shows how sensitive the main results are to the estimated value of the standard deviation of consumption growth. Again, preference parameters are recal-

Table 12: Main statistics as a function of the fraction of time the economy spends in expansion relative to time in recession in the Alvarez-Jermann model (model recalibrated).

Pr(exp.)/ Pr(rec.)	4	3.5	3	2.5	2	1.5	1
α_{hl}^1	0.7730	0.7625	0.7510	0.7381	0.7238	0.7074	0.6885
λ_e	1.0289	1.0299	1.0312	1.0328	1.0349	1.0381	1.0434
λ_r	0.9565	0.9602	0.9642	0.9686	0.9734	0.9789	0.9855
π_e	0.8562	0.8402	0.8203	0.7946	0.7603	0.7124	0.6405
π_r	0.4248	0.4408	0.4607	0.4864	0.5207	0.5686	0.6405
β	0.3315	0.3615	0.4034	0.4636	0.5507	0.6687	0.7932
γ	3.528	3.451	3.311	3.074	2.685	2.101	1.422
$\text{corr}(\log(e^i / \sum_j(e^j)))$	0.4158	0.4318	0.4519	0.4778	0.5124	0.5609	0.6339
$\text{std}(\log(c^i / \sum_j(c^j)))$ (%)	26.7	26.77	26.84	26.9	26.99	27.16	27.84
$\text{corr}(\log(c^i / \sum_j(c^j)))$	0.4302	0.446	0.4659	0.4917	0.5263	0.5739	0.6396
$r_{30} - r_1$ (exp.) (%)	5.654	5.418	5.18	4.952	4.777	4.798	5.452
$r_{30} - r_1$ (rec.) (%)	-7.455	-7.159	-6.849	-6.517	-6.141	-5.661	-4.936
$E[r_{30} - r_1]$ (%)	3.188	2.79	2.354	1.873	1.348	0.8078	0.328
$E[\text{tp}_1]$ (%)	5.557	4.763	3.894	2.946	1.947	1.01	0.3417
$\text{std}[\text{tp}_1]$ (%)	4.623	4.422	4.139	3.729	3.13	2.313	1.456
$Wald(b = 0)$	1000	1000	1000	1000	1000	1000	964
b in (22)	-0.499	-0.5544	-0.5504	-0.4756	-0.3217	-0.4105	-0.3108
b in (23) ($n = 3$)	0.5936	0.5622	0.5353	0.5264	0.5130	0.5344	0.5884

Table 13: Main statistics as a function of the first-order autocorrelation of consumption growth in the Alvarez-Jermann model (model recalibrated).

$\text{corr}[\Delta c]$	0.45	0.3	0.15	0	-0.15
π_e	0.8873	0.8566	0.8259	0.7952	0.7645
π_r	0.5627	0.4434	0.3241	0.2048	0.0855
β	0.376	0.3416	0.3135	0.29	0.2669
γ	3.666	3.528	3.43	3.355	3.296
$\text{corr}(\log(e^i / \sum_j(e^j)))$	0.558	0.4346	0.3135	0.1923	0.0712
$\text{std}(\log(c^i / \sum_j(c^j)))$ (%)	26.61	26.7	26.77	26.83	26.87
$\text{corr}(\log(c^i / \sum_j(c^j)))$	0.5687	0.4488	0.3289	0.209	0.0889
$r_{30} - r_1$ (exp.) (%)	5.577	5.597	5.607	5.612	5.615
$r_{30} - r_1$ (rec.) (%)	-7.349	-7.383	-7.411	-7.436	-7.457
$E[r_{30} - r_1]$ (%)	3.102	3.098	3.09	3.079	3.069
$E[\text{tp}_1]$ (%)	5.113	5.35	5.543	5.704	5.842
$\text{std}[\text{tp}_1]$ (%)	4.893	4.622	4.267	3.846	3.374
$Wald(b = 0)$	1000	1000	1000	1000	1000
b in (22)	-0.4916	-0.5824	-0.4859	-0.4251	-0.3390
b in (23) ($n = 3$)	0.4239	0.5616	0.6761	0.7701	0.8468

ibrated to match the average risk-free rate and its standard deviation. In the table, N/A refers to autarky where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22).

Table 14: Main statistics as a function of the standard deviation of consumption growth in the Alvarez-Jermann model (model recalibrated).

std[Δc] (%)	1	2	3	4	5	6	7
λ_e	1.0195	1.0246	1.0297	1.0347	1.0398	1.0449	1.05
λ_r	0.9947	0.9750	0.9553	0.9356	0.9159	0.8962	0.8765
β	0.2886	0.3156	0.3402	0.3624	0.3824	0.4003	0.4163
γ	3.879	3.673	3.498	3.348	3.22	3.111	3.017
std($\log(c^i / \sum_j(c^j))$) (%)	27.27	26.99	26.68	26.33	25.94	25.51	25.03
corr($\log(c^i / \sum_j(c^j))$)	0.4282	0.4309	0.434	0.4377	0.442	0.447	0.4193
$r_{30} - r_1$ (exp.) (%)	5.814	5.704	5.587	5.462	5.326	5.176	5.009
$r_{30} - r_1$ (rec.) (%)	-7.283	-7.34	-7.392	-7.437	-7.475	-7.504	-7.521
$E[r_{30} - r_1]$ (%)	5.814	3.158	3.091	3.023	2.955	2.884	2.809
$E[tp_1]$ (%)	5.672	5.507	5.362	5.235	5.123	5.023	4.933
std[tp_1] (%)	5.001	4.778	4.559	4.344	4.13	3.915	3.693
$Wald(b = 0)$	1000	1000	1000	1000	1000	1000	1000
b in (22)	-0.6158	-0.6078	-0.6213	-0.5415	-0.4171	-0.5561	-0.3822
b in (23) ($n = 3$)	0.5944	0.5661	0.5921	0.5721	0.5786	0.5912	0.5956

Table 15 shows how sensitive the main results are to the estimated value of average consumption growth. Again, preference parameters are recalibrated to match the average risk-free rate and its standard deviation. In the table, N/A refers to autarky where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22).

Table 15: Main statistics as a function of average consumption growth in the Alvarez-Jermann model (model recalibrated).

$E[\Delta c]$ (%)	0.5	1	1.5	2	2.5	3	3.5
λ_e	1.0197	1.0247	1.0297	1.0347	1.0397	1.0447	1.0497
λ_r	0.9479	0.9529	0.9579	0.9629	0.9679	0.9729	0.9779
β	0.3295	0.3339	0.3383	0.3427	0.3471	0.3515	0.356
γ	3.535	3.524	3.513	3.501	3.49	3.478	3.466
std($\log(c^i / \sum_j(c^j))$) (%)	26.44	26.59	26.73	26.87	27.02	27.16	27.31
corr($\log(c^i / \sum_j(c^j))$)	0.4363	0.4349	0.4335	0.4321	0.4308	0.4294	0.4281
$r_{30} - r_1$ (exp.) (%)	5.465	5.537	5.607	5.675	5.742	5.807	5.87
$r_{30} - r_1$ (rec.) (%)	-7.466	-7.424	-7.382	-7.34	-7.299	-7.258	-7.218
$E[r_{30} - r_1]$ (%)	3.006	3.055	3.103	3.149	3.195	3.239	3.283
$E[tp_1]$ (%)	5.319	5.35	5.38	5.408	5.436	5.463	5.489
std[tp_1] (%)	4.566	4.574	4.582	4.589	4.596	4.602	4.608
$Wald(b = 0)$	1000	1000	1000	1000	1000	1000	1000
b in (22)	-0.5170	-0.5425	-0.5049	-0.4881	-0.5432	-0.5551	-0.4388
b in (23) ($n = 3$)	0.6003	0.5676	0.5657	0.5794	0.5624	0.5503	0.5481

B.4 Risk-Free Rate

Table 16 shows how sensitive the main results are to the estimated value of the standard deviation of the risk-free rate. Again, preference parameters are recalibrated to match the average risk-free rate and its standard deviation. In the table, N/A

refers to autarky where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22).

Table 16: Main statistics as a function of the standard deviation of the risk-free rate in the Alvarez-Jermann model (model recalibrated).

std[r] (%)	2	3	4	5	6	7	8
β	0.3994	0.3797	0.3616	0.3448	0.3293	0.3148	0.3012
γ	3.107	3.232	3.351	3.465	3.575	3.681	3.784
std($\log(c^i / \sum_j(c^j))$) (%)	27.26	27.09	26.93	26.78	26.64	26.51	26.39
corr($\log(c^i / \sum_j(c^j))$)	0.4284	0.43	0.4315	0.433	0.4344	0.4358	0.4371
$r_{30} - r_1$ (exp.) (%)	1.822	2.899	3.992	5.101	6.222	7.354	8.495
$r_{30} - r_1$ (rec.) (%)	-2.852	-4.187	-5.504	-6.809	-8.102	-9.385	-10.66
$E[r_{30} - r_1]$ (%)	0.9701	1.57	2.185	2.813	3.453	4.103	4.76
$E[tp_1]$ (%)	1.759	2.759	3.802	4.883	5.998	7.142	8.314
std[tp_1] (%)	1.548	2.387	3.261	4.167	5.103	6.065	7.051
$Wald(b = 0)$	1000	1000	1000	1000	1000	1000	1000
b in (22)	-0.4196	-0.4513	-0.5978	-0.4876	-0.4023	-0.5599	-0.5730
b in (23) ($n = 3$)	0.6174	0.5677	0.5730	0.5714	0.5742	0.5818	0.5827

Table 17 shows how sensitive the main results are to the estimated value of the average risk-free rate. Again, preference parameters are recalibrated to match the average risk-free rate and its standard deviation. In the table, N/A refers to autarky, where financial assets have no prices and $Wald(b = 0)$ refers to the number of rejections of the expectations hypothesis in the regression (22).

Table 17: Main statistics as a function of the average risk-free rate in the Alvarez-Jermann model (model recalibrated).

$E[r]$ (%)	0.5	1	1.5	2	2.5	3	3.5
β	0.3386	0.338	0.3374	0.3369	0.3364	0.336	0.3355
γ	3.495	3.509	3.523	3.536	3.55	3.563	3.576
std($\log(c^i / \sum_j(c^j))$) (%)	26.91	26.77	26.63	26.49	26.35	26.21	26.08
corr($\log(c^i / \sum_j(c^j))$)	0.4318	0.4331	0.4345	0.4358	0.4372	0.4385	0.4399
$r_{30} - r_1$ (exp.) (%)	5.694	5.626	5.557	5.487	5.415	5.342	5.268
$r_{30} - r_1$ (rec.) (%)	-7.329	-7.37	-7.412	-7.453	-7.495	-7.536	-7.578
$E[r_{30} - r_1]$ (%)	3.162	3.116	3.069	3.021	2.972	2.922	2.872
$E[tp_1]$ (%)	5.414	5.387	5.359	5.331	5.301	5.27	5.239
std[tp_1] (%)	4.587	4.583	4.578	4.573	4.568	4.562	4.555
$Wald(b = 0)$	1000	1000	1000	1000	1000	1000	1000
b in (22)	-0.5533	-0.5939	-0.4532	-0.4922	-0.4719	-0.5240	-0.5436
b in (23) ($n = 3$)	0.5706	0.5718	0.5649	0.5863	0.6068	0.6075	0.6024

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