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Research Department
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Short-Selling Restrictions, Strategic Stock Holdings and Index Futures Markets in Finland

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Abstract

The goal of the paper is to rationalize the observed persistent underpricing in the Finnish stock index futures market. It is shown that under a binding short-selling restriction on stocks the observed futures "underpricing" can be a result of strategic motives of the Finnish industrial and financial groups to hold large amounts of stocks, which implies a net futures demand for hedging part of the financial risk brought in by these strategic holdings. "Underpricing" can also emerge under short-selling restrictions, if strategic investors are better informed than other traders. Two main empirical implications of the model – a negative relationship between the futures basis and stock index volatility and a positive relationship between the basis and private information signals received by informed investors – are supported by the Finnish data from May 1988 to December 1990.

Tiivistelmä

Tutkimuksen tavoitteena on tarjota rationaalinen selitys sille, miksi suomalaiset FOX-osakeindeksitermiinit ovat olleet pitkiä aikoja alihinnoiteltuja ns. cost-of-carry –malliin nähden. Osoittautuu, että yksinkertaisessa yleisen tasapainon mallissa, jossa osakkeiden lyhyeksi myyminen ei ole mahdollista ja jossa osalla sijoittajista on strategisia sijoitusmotiiveja, futuurit voivat olla "alihinnoiteltuja". "Alihinnoittelua" voi lyhyeksimyyntirajoitusten vallitessa aiheuttaa myös se, että strategiset sijoittajat ovat muita paremmin informoituja. Testit Suomen aineistolla toukokuusta 1988 joulukuuhun 1990 tukevat mallin tärkeimpiä empiirisiä implikaatioita: havaitun ja cost-of-carry –mallin mukaisen termiinihinnan erotus, eli *basis*, on keskimäärin negatiivinen, basis on vähenevä funktio osakeindeksin odotetusta volatiliteetista ja kasvava funktio informoitujen sijoittajien saamasta yksityisestä informaatiosignaalista.

Contents

1 Introduction[1]

In frictionless markets with unlimited short-selling, stock index futures prices obey the well-known cost-of-carry formula to allow no arbitrage opportunities. When there are constraints on short-selling, the pure arbitrage approach does not necessarily give the answer to the futures pricing problem any more. This is because in order to do riskless arbitrage with an undervalued futures contract the arbitrageur should be able to short the stock index while buying the futures. One attempt to maintain the no-arbitrage relationship even under short-selling restrictions is to argue that investors, who can sell stocks out of their current portfolios, can engage in profitable arbitrage activity should the futures price decline relative to its cost-of-carry relationship with the index. This argument may not always hold, however. Selling stocks out of one's current portfolio clearly has its limit. Secondly, current stock holders may have other reasons not to sell their stocks. For example, they may care for the corporate control granted by their stock positions. Another limitation of the pure arbitrage approach to pricing futures contracts (or any derivative securities) is its implicit assumption that futures are redundant assets. Pricing derivative securities may not, though, always be possible without taking into account their explicit supply and demand conditions. Hence, general equilibrium analysis as opposed to partial equilibrium no-arbitrage analysis is an obvious way to study the pricing of derivative securities under market restrictions such as a short sales constraint. In such a setting these securities can have an active role in risk sharing between different market participants and are, thus, non-redundant.

In this paper we analyze the effect of a short-selling restriction on the prices of stock index futures in a simple general equilibrium model. In particular, we argue that the observed index futures pricing behaviour in the Finnish market, open since May 1988, may not be fully understood without such an approach. Unlike in the U.S. market for example, the arbitrage based cost-of-carry model has hardly been a fair description of the futures prices in Finland. From May 1988 till December 1990 the futures contracts in Finland have been underpriced relative to the cost-of-carry relationship most of the time. The pattern of this underpricing is very persistent. Puttonen & Martikainen (1991), who study the same period, have shown that the documented futures underpricing would have offered profitable arbitrage opportunities to the low-cost traders but not to the high-cost traders of the market. However, they argue that since the low-cost traders – brokers and market-makers – do not own large stock positions, the arbitrage opportunities have not been real to them, given the restriction on short-selling in Finland.[2] In our understanding this argument is not necessarily true. Indeed, there are low-cost

[1] The authors would like to thank Seppo Honkapohja, Markku Lanne, Lars Tyge Nielsen, Vesa Puttonen, Pedro Santa Clara and the seminar participants at the Bank of Finland research department and the University of Illinois for comments. Esa Jokivuolle and Yrjö Koskinen would like to thank Osuuspankkiryhmän tutkimussäätiö, Suomen arvo-paperimarkkinoiden edistämis-säätiö and Yrjö Jahnsson Foundation for financial support. Yrjö Koskinen would also like to thank the research department at the Bank of Finland for providing an excellent research environment during the summer 1994.

[2] There is no institutionalized short-selling of stocks in the Finnish market.

traders in Finland who have large stock holdings: commercial banks,[3] most of whom also act as brokers in the index derivatives markets. Hence, in principal it should have been possible for the Finnish commercial banks to arbitrage away at least part of the observed futures underpricing. However, banks may have other reasons why they are not willing to sell stocks out of their portfolios for index arbitrage purposes. First of all, banks may prefer control in corporations in order to monitor their business activities and, hence, protect the value of their own debt assets in these companies. We refer to this as the agency cost of debt motive for holding corporate shares. Secondly, bank executives may get private benefits from power in other companies. This may take the form of direct perks consumption or extra financial compensation from board membership and the like. Exercising economic power can also be rewarding in itself. We call these the managerial motives. Together we call the above motives, in the lack of a better expression, the strategic motives for holding corporate shares. To summarize, in our view a potential explanation for the existence of persistent seeming arbitrage opportunities (for low-cost traders) in the Finnish market is based on the short-selling restriction on one hand and the strategic stock ownership motives on the other. The strategic motives offer an explanation for why Finnish banks have not exploited the seeming arbitrage opportunities available to them in the index futures market.[4] The remaining task is to try to understand how the observed futures pricing behavior in Finland could have emerged in equilibrium in the first place.

In what follows we build a simple two-period general equilibrium model of index futures markets with three separate investor groups, which differ with respect to their demands for holding stocks and futures. In the second step different investor groups are also allowed to be asymmetrically informed about the future spot price. Various assumptions are motivated by the institutional characteristics of the Finnish capital markets. We assume that the first group of traders are strategic stock owners, who do not sell their stocks but sell futures instead for hedging reasons. In the Finnish context, these investors may be identified as industrial groups centered around commercial banks and strengthened by cross ownership. There are clear examples of such institutions in Finland and their stock holdings are known to be large. Above we discussed the strategic stock ownership motives of banks especially, but similar motives can characterize a whole industrial and financial group – both their managers and stock holders. Importantly, cross ownership between companies may be motivated by strategic concerns, since it can eliminate the threat of corporate take overs. Secondly, there are arbitrageurs who trade both in the stock and in the futures market and do not have any strategic stock ownership motives. These traders' initial stock endowment is assumed to be limited. This is characteristic of the

[3] The law in Finland allows commercial banks to hold at most 10% of an individual corporation's voting power granted by its shares. In practise it seems to be fairly easy for the banks to increase their effective power beyond the 10% through coalitions and ownership arrangements.

[4] In a more recent work Puttonen (1993c, p. 27) also concludes that a current owner of index stocks (with low enough transaction costs) would have been able to exploit arbitrage opportunities in the Finnish futures market, but may not have done so, for futures may not be perfect substitutes for stocks (Puttonen, 1993b).

market makers and brokers of the Finnish index derivatives market other than commercial banks, since they are typically relatively small companies with limited capacity to hold large amounts of stocks. Thirdly, there are investors who invest in the stock market only. It can be thought that they have prohibitively high costs of entering the futures market for informational reasons. Although somewhat *ad hoc* this assumption may not be so unrealistic. It is supported by the views of Miller et al. (1994) that a large part of investors invest only in one market. Asset trade in the market described above is supposed to go as follows. Strategic stock owners are not willing to sell any of their stocks but instead want to hedge part of their financial risk by selling futures contracts. Arbitrageurs buy these futures and wish to sell stocks in turn to hedge their own risk. Investors who only invest in stocks buy shares from arbitrageurs to optimize their portfolio risk exposure.We show that if short-selling is not possible and if the initial stock endowment of the strategic investors is large enough relative to that of the arbitrageurs, equilibrium futures price can be below its cost-of-carry relationship with the spot price. The reason for this is that under a short-selling restriction arbitrageurs cannot hedge all their futures purchases from the strategic stock holders by selling enough stocks and, hence, require compensation in the form of a lower futures price. It turns out that the futures "underpricing" is an increasing function of the expected stock index volatility. As we allow the strategic stock holders to be better informed about the future stock price development than the other traders, the "underpricing" also becomes a decreasing function of the private information signal received by the strategic investors. In other words, private bad news increase "underpricing" whereas private good news decrease it. These results are supported by empirical evidence from the Finnish market. Our theoretical model can be seen as a simplification of Fremault (1991). However, our treatment of the effects of a short-selling restriction is more explicit, and our results concerning asymmetric information are different. We think that the theoretical part of our paper is general enough to be applied in futures markets other than the Finnish one provided that they share the same essential characteristics: concentrated stock ownership and costly (or no) short-selling of stocks.

The paper is organized as follows. Section 2 presents the theoretical model with separate cases for symmetric and asymmetric information. Section 3 reviews the empirical results, and Section 4 concludes the paper.

2 The Model

We will analyze the equilibrium price behavior of stocks and futures contracts in a simple general equilibrium model with two time periods. The aim is to study market equilibria with and without a constraint on short-selling both under symmetric and asymmetric information in order to understand the observed futures pricing behavior in Finland. There are three assets in the economy: a stock, S, which can be understood as a market index, a futures contract, F, written on the stock and a riskless bond, B. The stock and the bonds have a positive net supply whereas the futures are in zero net supply. We next specify three groups of agents. The first group is hedgers (denoted by "h"), who hold a portfolio of the stock they want to hedge in the futures markets. In this simple setup hedgers don't trade at all in the stock market. They can be thought of as being strategic investors who get private benefits by exercising control in individual companies. In the context of Finland hedgers may naturally be identified as industrial and financial groups centered around large commercial banks. The second group of agents is arbitrageurs (denoted by "a"), who trade in both futures and stock markets and try to profit from any possible price differences between the stock and futures contracts. Lastly, there are investors who trade only in the stock. We call this group speculators (denoted by "s"). Speculators can be thought of as individual investors that have high entry costs to the futures markets. The number of investors in each group is normalized to be one.

With respect to investor groups our model is a simplification of Fremault (1991). In her model two other investor groups are specified: futures markets speculators and stock market hedgers. We could easily add these groups to our model, but the increased complexity wouldn't add any new insights to questions we are addressing. For the same reason we don't include explicit trading costs that Fremault imposes on stock market trades.

The three assets we postulated are traded on competitive markets. Trading takes place in the first period (labeled "0"). Assets pay off in the consumption good of the economy in the second period (labeled "1"), when traders consume their wealth. The price of the bond is normalized to one in both periods, so the interest rate is zero. The period 0 price of the stock is P_s and its period 1 price is a random variable $\tilde{v}$, whose realization is unknown in period 0. All agents have an initial belief that $\tilde{v}$ is normally distributed with mean $\bar{v}$ and variance σ_v^2.[5] The period 0 price of the futures contract is P_f. The future's price equals the spot price at period 1. We define the basis to be the period 0 price of futures minus the period 0 price of the stock, i.e. basis = $P_f - P_s$.

Hedgers and arbitrageurs have initial endowments of the stock, $\tilde{e}^h$ respectively. Endowmets are normally distributed, $\tilde{e}^h \sim N(\bar{e}^h, \sigma_h^2)$ and $\tilde{e}^a \sim N(0,\sigma_a^2)$ and the mean of the hedgers' endowment is strictly greater than zero.[6] The distributions of endowments are common knowledge, but each trader knows only his or her exact endowment. The endowments are uncorrelated with the second period price and thus give no information about the realization of $\tilde{v}$. In the case of symmetric information we could assume constant endowments as well without affecting the results. However, in the case of asymmetric information the stochasticity of endowments is a crucial (but standard) assumption for obtaining meaningful results. All the traders are assumed to have endowment of bonds (denoted by B and the relevant superscript) to make sure that they have the funds necessary to execute the trades they wish.

All agents maximize the expected utility of their terminal wealth. The expected utility function is a constant absolute risk aversion function

[5] We later add asymmetric information so that agents have different beliefs about the second period price.

[6] For simplicity we set the endowment of the speculators to zero.

$$U(\tilde{W}_1^k) = E[-\exp(-\tilde{W}_1^k)], \qquad (1)$$

where $\tilde{W}_1^k$ is agent k's end wealth.

It is well known that when agents have a CARA-utility function and uncertainty is normally distributed the objective function can be written in a mean-variance form. Combining with budget constraints the maximization problem for arbitrageurs, hedgers and stock market investors can be written in the following form:[7]

$$\begin{aligned}
&\max_{x^a, f^a} (\bar{v} - P_s)x^a + \bar{v}e^a + (\bar{v} - P_f)f^a + \bar{B}^a - \left(\frac{1}{2}\right)\sigma_v^2 (x^a + f^a + e^a)^2 \\
&\max_{f^h} \bar{v}e^h + (\bar{v} - P_f) f^h + \bar{B}^h - \left(\frac{1}{2}\right)\sigma_v^2 (f^h + e^h)^2 \qquad (2) \\
&\max_{x^s} (\bar{v} - P_s)x^s + \bar{B}^h - \left(\frac{1}{2}\right)\sigma_v^2 (x^s)^2
\end{aligned}$$

where x^a and x^S are the net demands of arbitrageurs and speculators in the stock market and f^a and f^h are the net demands of arbitageurs and hedgers in the futures markets, respectively. We now start solving the different cases of equilibria. Throughout the paper it is important to keep in mind that since the model parameters are constant by definition, all the comparative statics that we do have to be interpreted so that we are comparing different economies with different parameter values and not one economy with changing parameter values.

2.1 Equilibrium under symmetric information

The case when all investors have the same beliefs about the second period prices is useful in highlighting the simple mechanism that drive the results in our model. The symmetric information case also serves as a benchmark for the pricing relations under asymmetric information. The crucial thing is the short sales restiction. Without that the stock and the futures contract would be perfect substitutes and the basis would always be zero. As will be seen later on, this result holds even with asymmetric information.

[7] See appendix A for details. Note that now the endowment terms are realized endowments. Alternatively, in this case of symmetric information they could be simply interpreted as constant endowments.

A. Short sales are allowed

Using the four demand equations derived from the mean-variance maximization problem and the market clearing conditions, prices for the stock and futures are found to be in the following equilibrium:

$$P_s = P_f = \bar{v} - \frac{e^a + e^h}{3} \sigma_v^2. \tag{3}$$

So there is no difference between the stock and the futures price. Consistent with standard results the stock and the futures price is an increasing function of the future expected stock value and decreasing in stock variance.

B. No short sales

Next we assume that short sales of stocks are not allowed and that the short sales constraint is binding for arbitrageurs. This means that in equilibrium arbitrageurs will sell their whole stock endowment to stock speculators. Formally, $-x^a = e^a = x^s$. Now we get the following equilibrium prices (see Appendix A):

$$P_s = \bar{v} - e^a \sigma_v^2$$

and (4)

$$P_f = \bar{v} - \frac{e^h}{2} \sigma_v^2.$$

Proposition 1: In the absence of informational reasons for trading, the basis is negative if $e^h > 2e^a$. Furthermore, it is a decreasing function of volatility.

Proof:

$$P_f - P_s < 0 \Leftrightarrow \left(e^a - \frac{e^h}{2} \right) \sigma_v^2 < 0. \tag{4}$$

So if the realization of hedgers' stock endowment is at least twice as high as that of arbitrageurs', then the futures is "underpriced" relative to the price implied by the cost-of -carry formula.[8] Considering the large stock ownership concentration of the leading industrial and financial groups in Finland as well as the relatively small number and size of market makers and brokers other than commercial banks in the index derivatives market, it is not hard to believe that this condition has often been fulfilled in the Finnish market. The intuition for the "underpricing" result is that given the relative sizes of initial stock endowments and the constraint

[8] In this model where the interest rate is normalized to be zero and there are no interim dividends, the cost-of-carry formula is simply $P_f = P_s$.

on short-selling, arbitrageurs will not be able to hedge all their futures purchases from the hedgers by selling stocks to stock speculators. Hence, in order to share part of the hedgers' risk they require compensation in the form of a lower futures price. The result that the basis is a decreasing function of volatility offers an interesting testable prediction, which will be addressed in the empirical part of the paper. Finally, in solving the equilibrium without short sales we assumed that the Kuhn-Tucker inequality constraint on arbitrageurs' stock sales was binding, i.e. that the constraint held as an equality. It can be shown that the condition in Proposition 1 which results in a negative basis also guarantees that a binding constraint will give the optimal solution.[9]

2.2 Equilibrium under asymmetric information

In this section we assume that hedgers and arbitrageurs are better informed than speculators.[10] They receive a noisy signal $\tilde{\theta} = \tilde{v} + \tilde{\varepsilon}$ about the second period stock price, where $\tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2)$. This enables them to achieve a higher precision of the period 1 stock price expectation than before, where precision is defined as the inverse of expected stock price variance. The new precision for informed traders is $h_i = h + h_\varepsilon$, where $h = \frac{1}{\sigma_v^2}$ and $h_\varepsilon = \frac{1}{\sigma_\varepsilon^2}$. Using the received signal informed traders update their beliefs and the conditional expectation about the stock price becomes[11]

$$v_i = \bar{v} + \frac{h_\varepsilon(\theta^* - \bar{v})}{h + h_\varepsilon} \tag{5}$$

where θ^* is the observed signal.

Speculators don't observe the signal, but they are able to learn some of the hedgers' and arbitrageurs' information by observing prices. The informed agents don't learn anything from trading with the speculators. In order to determine how much the uninformed agents learn and what are the resulting prices for the stock and the futures we first assume that the uninformed agents have linear conjectures

[9] The proof is available from the authors on request.

[10] For simplicity all the hedgers and all the arbitrageurs are assumed to share the same information. In the context of the model it is natural to think that hedgers, given their access to corporate board rooms, are better informed than other trader groups. Moreover, it would make no difference to assume that arbitrageurs are uninformed, since there would exist two sources of uncertainty for them (hedgers' endowment and the signal) and two markets open for them. Hence they would recover all the available information from the prices.

[11] The precision h_i and the conditional expectation v_i follow easily from the fact that the second period price and the signal are bivariate normally distributed random variables.

about what the prices should be.[12] These price conjectures are linear functions of the expected second period price, the realized signal, the expected endowments and realized endowments with unknown parameters. Using these price conjectures and the properties of multivariate normal distributions we calculate the conditional expectation and precision that the speculators have. At this stage the expressions contain the unknown parameters from the price conjectures. The unknown parameters are solved in equilibrium equating the conjectured price with the price which is calculated using the conditional expectation and precision. Finally the solved parameters are put back into the expression of conditional expectation and precision and the equilibrium prices in terms of the known model parameters are obtained.

A. Short sales allowed

Since the stock and the futures contract are perfect substitutes, when short-selling is allowed, both prices have exactly the same information content. As a result the uninformed traders don't gain anything by observing two prices. Thus, we may simply assume that speculators observe the stock price only and then infer some of the information that hedgers and arbitrageurs posess. The resulting equilibrium would be exactly the same, whether speculators observed the futures price only or both prices.

Observing the stock price allows the uninformed traders to achieve a precision

$$h_u = h + \frac{h_\varepsilon}{1 + \frac{\sigma_a^2 + \sigma_h^2}{4h_\varepsilon}}, \tag{6}$$

which is higher than the unconditional precision h, but lower than the full information precision h_i.

It is easy to see that $h_u = h_i$, if there were no uncertainty concerning the endowments, i.e. $\sigma_a^2 = \sigma_h^2 = 0$, and thus the speculators would learn all the information that the hedgers and arbitrageurs have.

The conditional expectation of the second period price for speculators is now

$$v_u = \bar{v} + \frac{h_\varepsilon(\theta^* - \bar{v}) - ½(e^a + e^h)}{h + h_\varepsilon + \frac{h_\varepsilon(\sigma_a^2 + \sigma_h^2)}{4h_\varepsilon}} \tag{7}$$

In equilibrium the price of the stock is the same as the price of futures (see Appendix B):

[12] The solution concept is the same as in Grossman (1976) or Diamond and Verrechia (1981). See appendix B for details.

$$P_s = P_f = \bar{v} + \frac{2h_\varepsilon(\theta^* - \bar{v}) - (e^a + e^h) + \dfrac{2h_\varepsilon \bar{e}^h}{6h_\varepsilon + \sigma_a^2 + \sigma_h^2}}{2h_\varepsilon + 2h + \dfrac{h(\sigma_a^2 + \sigma_h^2)}{6h_\varepsilon + \sigma_a^2 + \sigma_h^2}} . \tag{8}$$

So when short sales are unrestricted or the short sales constraint is not binding stock and futures prices react in a same way to new information. For example, if hedgers receive negative information, it gets reflected in the futures price but the stock price declines by the same amount so that the basis stays always at zero.

It can be easily seen that

$$\lim_{h_\varepsilon \to 0} P_s = P_f = \bar{v} - \frac{e^a + e^h}{3h} , \tag{9}$$

so when the information content of the signal converges to zero, the prices converge to those that prevail in the symmetric information case with short sales allowed.

When the precision of the signal approaches infinity, then $\lim_{h_\varepsilon \to \infty} P_s = P_f = \theta^*$, so the signal becomes dominant in determining the equilibrium prices. Next we start analyzing the case of asymmetric information under a short sales contraint, which again results in the possibility of a negative basis. Along with stock volatility the basis is also shown to be a function of the private information signal received by the informed traders.

B. Equilibrium without short sales

When the short sales constraint is binding the stock and the futures are not perfect substitutes anymore. This means that the respective prices contain different information. So in the absence of any informational costs it would be suboptimal for the uninformed investors (speculators) to pay attention only to the stock price and not at all to the futures price when making investment decisions. Nevertheless, we will start by first analyzing the case when the uninformed investors only use the stock price in making inferences about the second period price. First, this case serves the purpose of highlighting the importance of the informational role the futures price can play when there are short selling constraints. Secondly, we will argue later on that analyzing such a boundedly rational equilibrium may be used to bring some dynamic aspects to our framework of analysis. The case of a fully rational equilibrium where uninformed investors take into account information from both markets will be analyzed right after.

When uninformed investors only take into account information from the stock market, we get the following prices in equilibrium (see Appendices A and B):

$$P_s = \bar{v} - \frac{e^a}{h},$$

$$P_f = \bar{v} + \frac{h_\varepsilon(\theta^* - \bar{v})}{h + h_\varepsilon} - \frac{e^h}{2(h + h_\varepsilon)}. \tag{10}$$

The price of the stock is found to be the same as under symmetric information without short sales. This means that uninformed investors don't learn anything from observing the stock price, i.e. the precision remains h and the conditional expectation of second period price v_u is the same as the unconditional expectation v.

Proposition 2: The basis is negative if $e^h > 2h_\varepsilon(\theta^* - \bar{v}) + 2\left(\frac{h + h_\varepsilon}{h}\right)e^a$.

Proof:

$$\text{basis} = \frac{2hh_\varepsilon(\theta^* - v) - he^h + 2(h + h_\varepsilon)e^a}{2h(h + h_\varepsilon)}, \tag{11}$$

from which the proposition follows directly.[13]

From Proposition 2 we see that if the private information signal is neutral, i.e. it equals its expected value, the hedgers' endowment has to be $2\left(\frac{h + h_\varepsilon}{h}\right)$ times bigger than arbitrageurs' endowment in order to induce a negative basis. This is a stricter condition than in the case of symmetric information. This is simply because $(h + h_\varepsilon) > h$ whenever the signal has information content. If the signal has no informational value, i.e. $h_\varepsilon = 0$, then we have the same situation that prevails with symmetric information. Then the basis is solely determined by relative endowments. Note that in contrast to the case of symmetric information the basis can now be negative also for informational reasons, if $h_\varepsilon > 0$. Having the relative endowments fixed, there are always some values of the signal for which the basis becomes negative.

Proposition 3: When the short-selling constraint is binding, i.e. when the basis is negative, then the basis is an increasing function of the signal.

[13] Although we have not worked out the optimality of the Kuhn-Tucker conditions, appealing to the corresponding result in the symmetric information case we conjecture that the negativity condition for the basis in Proposition 2 also guarantees optimality of the solution using the binding short-selling constraint. We will make a corresponding conjecture concerning Proposition 6 below.

Proof:

$$\frac{\partial \text{basis}}{\partial \theta} = \frac{h_\varepsilon}{h + h_\varepsilon} > 0. \tag{12}$$

This is a direct consequence of the fact that information is reflected in the futures price but not in the stock price. So negative private information shocks decrease the basis and positive shocks increase it, when the short-selling constraint is binding. We further emphasize that given the relative initial stock endowments of different investor groups large enough positive signals can make the short-selling constraint not binding (i.e. the basis negativity condition in Proposition 2 would not hold any longer) and, hence, cause a zero basis.

Proposition 4: When the short-selling constraint is binding, the basis is an increasing function of h, if $e^h > 2h_\varepsilon(\theta^* - \bar{v}) + 2\left(\frac{h + h_\varepsilon}{h}\right)^2 e^a$.

Proof:

$$\frac{\partial \text{basis}}{\partial h} = \frac{2h^2 h_\varepsilon(\bar{v} - \theta^*) + h^2 e^h - 2(h + h_\varepsilon)^2 e^a}{2h^2(h + h_\varepsilon)^2}, \tag{13}$$

from which the proposition follows.

Note that the basis negativity condition in Proposition 2 is a necessary condition for Proposition 4, since this guarantees that the short-selling restriction is binding (see footnote 12). However, it is easily seen that Proposition 2 holds whenever Proposition 4 holds so that the condition in Proposition 4 is actually a sufficient condition for the basis being an increasing function of the precision h. Proposition 4 compares to the volatility result of Proposition 1 in the symmetric information case, since it implies that the basis is a decreasing function of the volatility. Compared to the case of symmetric information (Proposition 1) the hedgers' endowment has to be $2\left(\frac{h + h_\varepsilon}{h}\right)^2$ times bigger in order the basis to be a decreasing function of volatility, given that the signal is neutral. Note that the basis can also be an increasing function of volatility, if the short-selling restriction is binding (Proposition 2 holds) but Proposition 4 does not hold. However, we conjecture that this would not be very likely given reasonable parameter values.

Proposition 5: When the short-selling constraint is binding, the basis is an increasing function of h_ε, *if* $e^h > 2h(\bar{v} - \theta^*)$.

Proof:

$$\frac{\partial \text{basis}}{\partial h_\varepsilon} = \frac{2h(\theta^* - \bar{v}) + eh}{2(h + h_\varepsilon)^2}, \tag{14}$$

from which the proposition follows.

According to both Propositions 4 and 5 the basis is an increasing function of the respective precision parameters, if hedgers' endowment is big enough. However, there could be situations where an increase in h increases the basis, whereas an increase in h_ε decreases the basis. This could happen for example when the basis is negative because of a strong negative signal, but the relative size of the hedgers' endowment is small.

We now turn to the case where the uninformed investors use also the futures price in assessing the second period stock price, i.e. the resulting equilibrium will be fully rational. After introducing the fully rational equilibrium stock and futures prices, we will also consider the question how the boundedly rational equilibrium analyzed above and the fully rational equilibrium may be used in addressing certain dynamic aspects.

When the uninformed traders use information both from the stock and the futures market, their precision for the second period expected stock price becomes

$$h_u = h + \frac{h_\varepsilon}{1 + \frac{\sigma_h^2}{4h_\varepsilon}}, \tag{15}$$

which is lower than the precision for informed traders, but higher than the precision which prevails when there are no restrictions on short sales (see Appendix B). This is because the futures price has information content, since under a binding short-selling restriction the stock and the futures contract are not perfect substitutes. As a result of the short sales restrictions the informativeness of markets has improved.

The conditional expectation of the second period stock price for the uninformed traders becomes now

$$v_u = \bar{v} + \frac{h_\varepsilon(\theta^* - \bar{v}) - \frac{1}{2}(e^h - \bar{e}^h)}{h + h_\varepsilon + \frac{h_\varepsilon \sigma_h^2}{4h_\varepsilon}}. \tag{16}$$

The equilibrium prices are given by

$$P_s = \bar{v} + \frac{h_\varepsilon(\theta^* - \bar{v})}{h + h_\varepsilon + \frac{h\sigma_h^2}{4h_\varepsilon}} - \frac{2e^a + e^h - \bar{e}^h + \frac{e^a\sigma_h^2}{2h_\varepsilon}}{2(h + h_\varepsilon) + \frac{h\sigma_h^2}{2h_\varepsilon}}, \qquad (17)$$

$$P_f = \bar{v} + \frac{h_\varepsilon(\theta^* - \bar{v})}{h + h_\varepsilon} - \frac{e^h}{2(h + h_\varepsilon)}.$$

The futures price is the same as in the case when uninformed traders use only the stock price in making their decisions. The difference is that the realized signal is now also reflected in the stock price. However, the signal is given less weight in the stock price than in the futures price. It can be shown that, for example, when the hedgers' initial stock endowment equals its expected value and the arbitrageurs' initial endowment is close to zero, for negative signals (less than the expected value) the stock price is higher in the boundedly rational equilibrium than in the fully rational equilibrium, and vice versa if the signal is positive (more than its expected value). We use this result to sketch some price dynamics in our framework. We may think that the boundedly rational equilibrium, where the uninformed investors use information only from the stock market, is a temporary equilibrium on the way to the fully rational equilibrium. This is admittedly ad hoc, but it could be seen as learning behavior, where the uninformed investors do not immediately learn from the futures price. Hence, the fully rational equilibrium would be achieved first with a lag. Suppose also that it would take two periods from the hedgers and the arbitrageurs to execute all the trades needed for their portfolio optimization. Now the futures price would immediately reflect all available information already in the first period as a result of futures trading between the informed traders, but the stock price would adjust first in the second period, when the uninformed traders start using information from the futures market. This would induce the futures price to lead the stock. Moreover, the lead-lag relationship could be asymmetric in the sense that a high enough positive signal would break the binding short-selling restriction, result in a zero basis and, hence, induce no lead-lag relationship at all. For negative signals the lead-lag relationship would always be there (given that other parameter values would not break the binding short-selling restriction). It can be further added that if the asymmetry of information gradually vanishes in the course of time, the stock price moves closer and closer to the futures price so that eventually any remaining negative basis is due only to differences in relative endowments. We are aware that the above sketched scenario is rather daring, since it combines elements from two completely separate static equilibria and does not address the question in a truly dynamic model. Still, we offer it as a possible explanation for the empirical lead-lag results obtained by Puttonen (1993a) with the Finnish index futures data (see the discussion in section 3). We will finish this section by working out the comparative statics of the fully rational equilibrium and state the negativity condition for the basis which is also presumed to guarantee the binding short-selling restriction.

Proposition 6: The basis is negative if[14]

$$e^{h} > \frac{2hh_{\varepsilon}\sigma_{h}^{2}}{(4hh_{\varepsilon}+4h_{\varepsilon}^{2}+h\sigma_{h}^{2})}(\theta^{*}-\bar{v})+\frac{2h_{i}}{h_{u}}e^{a}. \tag{18}$$

Proof:

$$\text{basis}=\frac{2hh_{\varepsilon}\sigma_{h}^{2}(\theta^{*}-\bar{v})-e^{h}(4hh_{\varepsilon}+4h_{\varepsilon}^{2}+h\sigma_{h}^{2})+2e^{a}(4hh_{\varepsilon}+4h_{\varepsilon}^{2}+h\sigma_{h}^{2}+h_{\varepsilon}\sigma_{h}^{2})}{2(h+h_{\varepsilon})(4hh_{\varepsilon}+4h_{\varepsilon}^{2}+h\sigma_{h}^{2})} \tag{19}$$

Dividing the denominator and numerator by

$$4h_{\varepsilon}+\sigma_{h}^{2} \tag{20}$$

and noting that

$$h_{u}=\frac{4hh_{\varepsilon}+4h_{\varepsilon}^{2}+h\sigma_{h}^{2}}{4h_{\varepsilon}+\sigma_{h}^{2}} \tag{21}$$

we get that

$$\text{basis}=\frac{\frac{2hh_{\varepsilon}\sigma_{h}^{2}}{4h_{\varepsilon}+\sigma_{h}^{2}}(\theta^{*}-\bar{v})-e^{h}h_{u}+2e^{a}(h+h_{\varepsilon})}{2(h+h_{\varepsilon})(h_{u})}, \tag{22}$$

from which the proposition follows.

Again, on the grounds of the corresponding result in the symmetric information case, we presume that the condition in Proposition 6 also guarantees the optimality of a binding short-selling constraint (see footnote 12). From Proposition 6 we see that if the received signal is neutral, then the hedger's endowment has to be $2(h_i/h_u)$ times the arbitrageurs endowment. So in this case the hedger's endowment can be smaller relative to arbitrageur's than in the case when speculators only followed the stock price. The higher the speculator's precision, the closer we are to the situation that prevails in the symmetric information case.

Like in the previous case the basis can be negative for informational reasons. Now the signal has to be more negative in order to make the basis negative,

[14] For simplicity in propositions 6 to 9 we assume that $e^{h}=\bar{e}^{h}$.

because the stock price is more informative. If we assume that $e^h = 2e^a$ (so the basis would be zero in the symmetric information case), then the condition for negative basis is that $-h(\theta^* - \bar{v}) > e^a$. On the other hand, a big enough positive signal can induce a zero basis even if the hedgers' relative stock endowment would be large.

Proposition 7: When the short-selling restriction is binding, the basis is an increasing function of the signal.

Proof:

$$\frac{\partial basis}{\partial \theta} = \frac{hh_\varepsilon \sigma_h^2}{(h+h_\varepsilon)(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)} > 0. \tag{23}$$

Both prices are increasing functions of the signal, but the price of the futures reacts more to the signal – thus the result in Proposition 7.

Proposition 8: When the short-selling restriction is binding, the basis is an increasing function of h, if

$$e^h > \frac{2h_\varepsilon \sigma_h^2 (4h^2 h_\varepsilon + h^2\sigma_h^2 - 4h_\varepsilon^3)}{(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2}(\theta^* - \bar{v}) + \frac{2h_i^2}{h_u^2} e^a. \tag{24}$$

Proof:

$$\frac{\partial basis}{\partial h} = \frac{2h_\varepsilon \sigma_h^2(\bar{v} - \theta^*) + e^h(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2 - 2e^a(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2 + h_\varepsilon\sigma_h^2)^2}{2(h+h_\varepsilon)^2(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2} \tag{25}$$

Dividing the denominator and numerator by

$$(4h_\varepsilon + \sigma_h^2)^2$$

and using the definition of h_i and h_u we get that the desired result.

Proposition 9: When the short-selling restriction is binding, the basis is an increasing function of h_ε, if

$$e^h > \frac{2h_\varepsilon \sigma_h^2 (8hh_\varepsilon^2 + 8h_\varepsilon^3 - h^2\sigma_h^2)}{(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2}(\theta^* - \bar{v}) + 2h_i^2 e^a \left(\frac{1}{h_u^2} - \frac{\sigma_h^2}{(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2} \right) \tag{27}$$

Proof:

$$\frac{\partial basis}{\partial h_\varepsilon} = \frac{2h\sigma_h^2(\bar{v} - \theta^*)(8hh_\varepsilon^2 + 8h_\varepsilon^3 - h^2\sigma_h^2) + e^h(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2}{2(h + h_\varepsilon)^2(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2}$$

$$\frac{-2e^a(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2 + h_\varepsilon\sigma_h^2)^2 + 2e^a\sigma_h^4(h + h_\varepsilon)^2}{2(h + h_\varepsilon)^2(4hh_\varepsilon + 4h_\varepsilon^2 + h\sigma_h^2)^2}. \tag{28}$$

Divide the denominator and numerator by

$$(4h_\varepsilon + \sigma_h^2)^2$$

and use the expressions for h_i and h_u.

The result in Proposition 8 compares closest to the result concerning volatility and the basis in the symmetric information case in Proposition 1. Although it is possible for certain parameter values in Proposition 8 that the basis is a decreasing function of h (an increasing function of volatility), it can be shown that this is not very likely. We conclude that like in the case of symmetric information, the basis is (likely to be) decreasing in the stock volatility.

In our model we have analyzed the futures basis behavior in a static general equilibrium setting. We have considered four different cases: equilibria with and without short sales when all the investors are symmetrically informed, and equilibria with and without short sales in the presence of an information asymmetry. The last case – no short sales in an asymmetric information environment – was further divided into two separate cases: a boundedly rational equilibrium where the uninformed traders use information only from the stock market and a fully rational equilibrium where they use information both from the stock and the futures market. The general result from the model is that the futures basis can be negative only if short sales are not allowed and furthermore the short sales constraint has to be binding. This means that in equilibrium arbitrageurs will be selling their whole initial stock endowment. Otherwise the basis will always be zero. It was shown in the symmetric information case that conditions on the negativity of the basis and the short-selling constraint to be binding coincide, and it was presumed that this holds also for the corresponding conditions in the asymmetric information case. With symmetric information the basis is negative if the initial stock endowment of the hedgers is large enough relative to that of the arbitrageurs. With asymmetric information the condition depends also on the other model parameters. The symmetric and asymmetric information cases share the implication that when the short-selling constraint is binding, the basis is a decreasing function of stock volatility (to be precise, this is only the most likely case when information is asymmetric). When information is asymmetric, a binding short-selling constraint also implies that the basis is an increasing function of the private information signal received by the informed investors. In the next section we attempt to test these predictions empirically with data from the Finnish market.

3 Empirical Evidence

In this section we attempt to test the predictions of our theoretical model empirically. Some caution with interpreting the regression results will be in order, since the model is essentially a static one, but we plan to test it with time series data. We start by describing the data.

Data

We use the basis calculated from the nearby index futures contract in the Finnish Options Market (FOM) to test our model implications. The data spans the period 88/05/02 – 90/12/21, the same as used by Puttonen (1993a). Of the two futures contracts simultaneously available in the market the nearby contract is the one with at most two months to expiration. The other contract always has a maturity of two months plus the maturity of the nearby contract. We switch to a new contract one week before the expiration day of the current contract. The main reason for using the nearby contract for analysis is its higher liquidity. Other variables needed are the FOX index, the implied volatility from the nearby FOX index options, the 3 month Helibor rate and the realized dividend payments in 88/05/02 – 90/12/21.[15,16]

The FOX index, which the futures and options are based on, is computed as a value-weighted index from the 25 most actively traded stocks in the Helsinki Stock Exchange (HeSe). Its correlation with the larger market indices of HeSe is quite high. Hence, it should be well suited for hedging against market wide risks. There is a potential problem of nontrading of stocks in the FOX index, since the index is updated on the basis of the latest transaction prices. In the presence of nontrading the observed index value does not necessarily reflect the true market value of the index portfolio and, hence, would bias the futures basis measurement as well. Nontrading could be the cause behind the high positive first-order serial correlation (more than 30 per cent) encountered in the daily logdifferences of the FOX index. A model of an equally-weighted index presented in Lo & MacKinley (1988) suggests that the probability of nontrading in a given period for a stock, common to each stock in the index, implies a theoretical first-order index return autocorrelation coefficient of the same size. In other words, a probability of 30 per cent of each constituent index stock not trading during a day would induce a first-order daily serial correlation coefficient of .30. Puttonen (1993b, Fig. 2) provides a crude estimate of the nontrading probability of the FOX index stocks, which

[15] The Helibor rates for different maturities are calculated daily by the Bank of Finland as the average of the offered rates at the Helsinki interbank money market.

[16] The data were kindly provided by The Finnish Options Market. We thank Vesa Puttonen for providing the data adjusted for expected dividends needed in the cost-of-carry formula; see Puttonen (1993a) for details.

ranges between 1 and 14 per cent depending on the month considered.[17] The fact that the FOX index is a value-weighted index and the largest capitalization stocks in it probably have a lower nontrading probability than the smallest capitalization stocks further implies that the spurious serial correlation induced into the index returns by nontrading is likely to be less than what the model of Lo & MacKinley (1988) and the figures of Puttonen (1993b) suggest. Hence, the major part of the FOX index short term serial correlation cannot be attributed to nontrading effects. Berglund & Liljeblom (1988) draw attention to the specific trading system used in the HeSe as a source of spurious index serial correlation, but are still forced to conclude that a large part of the documented serial correlation in the Finnish stock returns remains unexplained. Besides, the trading systems in HeSe have undergone important changes during 1987–1989, which supposedly have reduced spurious autocorrelation effects. In essence, these changes should have reduced the problem of non-synchronous trading, i.e. emergence of non-trading periods before closing inside one trading day. To conclude, certain biases may enter the futures basis measurement via the use of the observed FOX index level, which exhibits significant serial correlation in its daily logdifferences. However, in the lack of a comprehensive explanation for the serial correlation we are so far hesitant to do any specific corrections to adjust the observed index level.[18]

Regression results

The futures basis (see Figure 1), measured as the percentual difference between the observed futures value and the theoretical futures value according to the cost-of-carry formula, is regressed on the implied volatility and a "signal" variable. Theoretical futures prices according to the cost-of-carry formula correspond to the case in our theoretical model, where the stock index price equals the futures price but, unlike our model with a zero risk-free rate, adjust for the real world discounting and dividend payments.[19] The implied volatility is computed as a weighted average of the implied volatilities of all available nearby FOX index option contracts, where the weights are based on the Black-Scholes derivatives

[17] Puttonen's (1993b) measure, interpreted here as a nontrading probability, is the monthly average percentage of nontrading observations. This figure exceeds the value of 35 per cent in February, 1990, when there was a bank strike in Finland. For 1989 the measure never exceeds 5 per cent, whereas for 1990, excluding February, the average measure is in the 5 – 10 per cent range.

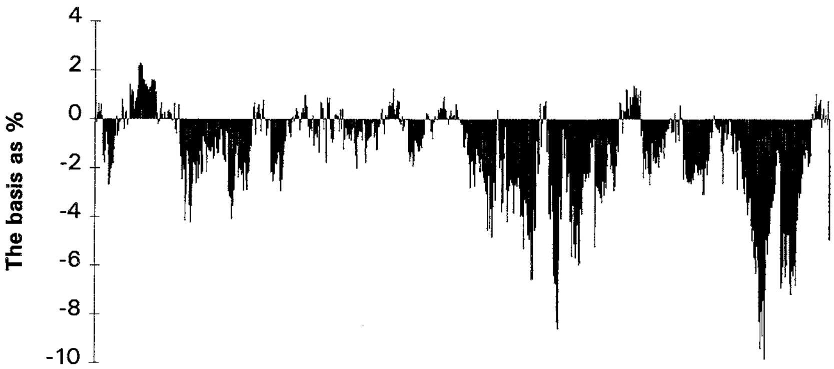
[18] For example, when the index serial correlation is all due to nontrading of individual stocks, Jokivuolle (1994) has suggested a theoretical correction for measuring the true unobservable market value of the index portfolio. This procedure could also be used to account for the effect of nonsynchrorous trading. We might consider this in future work.

[19] Choosing the way of estimating expected dividends is not a major concern with the Finnish futures data, since in Finland companies pay dividends only once a year during a few spring months. Hence, it cannot be the case that for example the large negative basis observations during the fall months could result from biased expected dividend estimates.

with respect to volatility of each contract. Single implied volatility figures exceeding 50 percentage points at annual level are excluded from the computation, for they are considered unreliable.[20]

The signal received by the informed traders between the trading periods t–1 and t is not directly observable to an econometrician, but will be realized in the current futures return from t–1 to t. According to our model, the private signal will be most efficiently reflected in the futures price, since the informed investors will trade in futures. Hence, we measure the signal by the current index return implied by the futures prices, computed from the cost-of-carry formula, minus the required rate of return on the index.[21] In other words, the idea is to use positive and negative "excess" returns as a measure of positive and negative signals, respectively.

Figure 1. **The basis during 88/05/02–90/12/21**



Our model predicts that a negative basis will occur only if the short-selling restriction is binding. Further, a negative dependence between the basis and volatility and a positive dependence between the basis and signal will be present only for a binding short-selling restriction. On the other hand, in our theoretical model no positive basis can occur. In order to test for all these predictions, we form two dummy variables such that the first one assumes the value one whenever the basis is negative and the second one displays ones in the opposite case.[22,23]

[20] We thank Jouni Torasvirta for describing the procedure of computation of the implied volatility series in the FOM data files.

[21] During the sample period the realized average FOX index return was negative and so does not give a plausible estimate for the required rate of return. The risk-free rate was used as a crude proxy instead.

[22] Small positive basis observations in the data are compatible with our model, since we do not consider trading costs explicitly.

[23] There is a potential problem of simultaneity in this setup, since the explanatory dummy variables are constructed using the dependent variable. Hence, our OLS estimates will be potentially inconsistent and biased. However, we conjecture this problem not to be very serious, since the dependent variable enters as an explanatory variable only after first being transformed into a dummy variable.

We then decompose volatility and signal into two separate variables each by multiplying them by the two dummies. Volatility and signal multiplied by the first dummy should show negative and positive dependence with the basis, respectively, but the variables constructed with the second dummy should show no relationship with the basis whatsoever.[24] Although our model is a two-period one and, hence, the impact of time on discounting and uncertainty is not explicitly modelled in it, following Chen, Cuny & Haugen (1994) one can argue that the relevant volatility variable should be the implied volatility times the remaining time to futures maturity. In order to account for this we include time to maturity in the regression equation.

Since the basis is heavily autocorrelated, almost nonstationary, its first lag is included as an explanatory variable. This strong persistence in the level of basis suggests that efficient arbitrage activity, which would tend to pull the basis back towards zero level, is not characteristic of the Finnish index futures market in our sample period. Rather, the basis behaviour – persistent deviations from the traditional arbitrage-free relationship – leaves room for a general equilibrium type of explanation pursued in this study (see also Chen, Cuny & Haugen, 1994).

Due to futures hedging demand of the strategic stock holders and a binding short-selling restriction, we expect the average basis to be negative. This is, indeed, the case, since the average basis in the sample period is about –1.5 per cent. Table 1 also provides some direct evidence on the relative sizes of stock holdings of different investor groups in Finland. The figures reveal the importance of financial and non-financial corporations as stock holders. Moreover, so far investment funds have been next to non-existent in Finland. We include a dummy for the period 90/01/18–90/03/02, when Finland experienced a bank strike. Puttonen (1993) shows figures of decreased trading activity in that period, which are consistent with the notion that executing trades during the strike became more difficult. This may have increased uncertainty in the market and affected the average basis.

[24] Some notes are in order about this test setting. We are implicitly assuming that the short-selling restriction is always binding whenever the basis is negative, but we do not test that hypothesis directly. On the other hand, if the short-selling restriction were not binding when the basis is negative, it means that someone could do arbitrage by selling stocks short and buying futures. Hence, assuming that there are no true arbitrage opportunities in the market strengthens our argument for using negative basis as an indicator of a binding short-selling restriction. These arguments may appear cyclical at the first glance, but one has to bear in mind that we are, indeed, attempting to explain why the seeming arbitrage opportunities in the market, i.e. the negative basis observations, have not been real.

Table 1. **The ownership division of the Finnish public companies**

	households	non-profit	companies	bank	insurance	others
1991	21.6	9.7	32.6	13.6	9.7	12.8
1990	24.8	9.3	26.5	15.1	10.1	14.2
1989	27.1	10.0	23.8	12.6	14.9	11.6
1988	31.3	12.0	19.6	16.4	12.3	8.4
1987	35.1	13.1	15.6	16.0	12.8	7.4

Source: Kansallis Bank: *Listed companies 1992.*

Table 2 presents the OLS regression results. The independent variables in Panel A are the following: the first lag of the basis, the dummy for the bank strike, volatility when the basis is negative and positive, respectively, time to the futures maturity, signal when the basis is negative and positive, respectively, the first and the second lag of the futures implied index return, respectively, and a constant term. We include the first two lags of the futures implied index in the regression in order to take care of a mild residual autocorrelation problem that showed up in a regression without the lags.[25] Their coefficients can be anticipated to obtain significant negative signs, since they essentially capture the empirical lead-lag effect found in Puttonen (1993a). In general, the results are supportive of our model. Both volatility and signal have highly significant coefficients, when the basis is negative. These relationships are graphically illustrated in Figures 2 and 3, respectively. When the basis is positive, the coefficient of volatility is not even close being significant. For positive basis the coefficient of signal is highly significant, but the magnitude of the coefficient is only about one over seventy of the magnitude of the corresponding coefficient when the basis is negative, and its sign is opposite. Without any potential explanation for this latter result, it might be safer to attribute it to a random effect. Hence, also the result concerning the signal variable can be interpreted in favor of our model.

Contrary to our expectations and the results of Chen, Cuny and Haugen (1994), time to the futures maturity does not obtain a significant coefficient. This may be due to the fact that we are only using the nearby futures contract and, hence, only relatively short futures maturities. Adding the longer futures contracts into the test sample might change the results in this respect. The dummy for the bank strike is significant at the 5 per cent level with a negative coefficient. In other words, the bank strike seems to have further depressed the average basis.

[25] The actual model parameters are not significantly affected by inclusion or exclusion of the lagged variables.

Table 2. **OLS regression for the basis as the dependent variable**

Panel A: Regression results

Indep.var.	*coeff.*	*t-value*	*p-value*	R^2
$Basis_{t-1}$	0.7856(**)	44.33	0.000	0.9285
$Dummy_1$ (bank strike)	-.2079(*)	-2.095	0.018	
Vol(-)	-81.43(**)	-8.011	0.000	
Vol(+)	11.63	0.4783	0.684	
Time	0.2865	0.5157	0.697	
Signal(-)	48.12(**)	25.69	0.000	
Signal(+)	-.7106(**)	-9.663	0.000	
$IFOX_{t-1}$	-14.12(**)	-7.845	0.000	
$IFOX_{t-2}$	-3.002(*)	-1.728	0.043	
Constant	0.3089(**)	3.676	0.000	

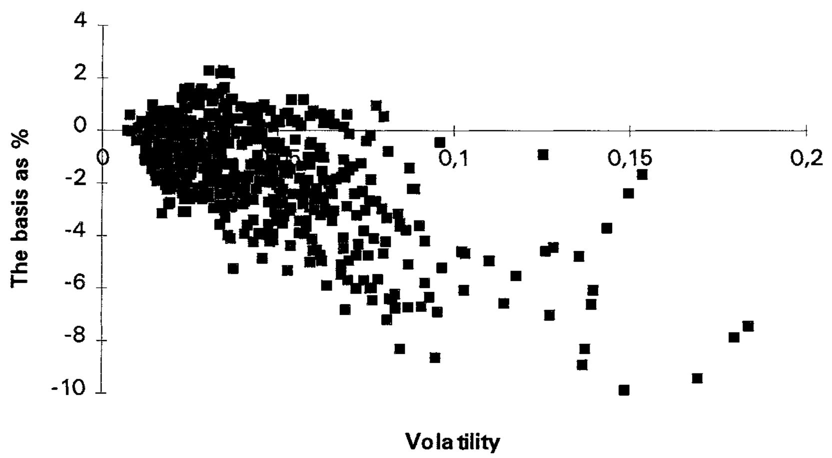
Panel B: Residual autocorrelations and Ljung-Box-Pierce Q-statistics

lag.	*coeff.*	*Q(lag)*	*p-value for Q*
1	0.03	0.54	0.462
2	-.07	3.69	0.158
3	0.06	6.45	0.092
4	0.12(**)	15.45	0.004
5	0.04	16.47	0.006
6	0.01	16.60	0.011
7	0.02	16.94	0.018
8	-.01	17.02	0.030
9	0.04	18.00	0.035
10	0.03	18.61	0.046

The first two lags of the futures implied index returns are both significantly negative, which is in line with the empirical results of a lead-lag relationship between the futures and the spot index in Finland found by Puttonen (1993a). Additionally, Puttonen found that the futures-spot lead-lag relation is asymmetric: the futures lead the index for one lag more on downtick than on uptick. Puttonen conjectures that this asymmetry is due to the short-selling restriction in the Finnish market. We argue that a formal explanation for an asymmetric lead-lag relationship based not only on a short-selling restriction but also on asymmetric information might be provided by our present analysis. As discussed before Proposition 6, interpreting the boundedly rational equilibrium analyzed in Section 2 as a temporary equilibrium on the way to the fully rational equilibrium could result in an asymmetric lead-lag relationship depending on the direction of the private information signal. Such a scenario could explain Puttonen's results.[26] Finally, the results in Panel B indicate that by and large the null hypothesis of white noise error terms need not be rejected, although the significant fourth-order sample autocorrelation coefficient makes the Q-statistics significant after the third lag.

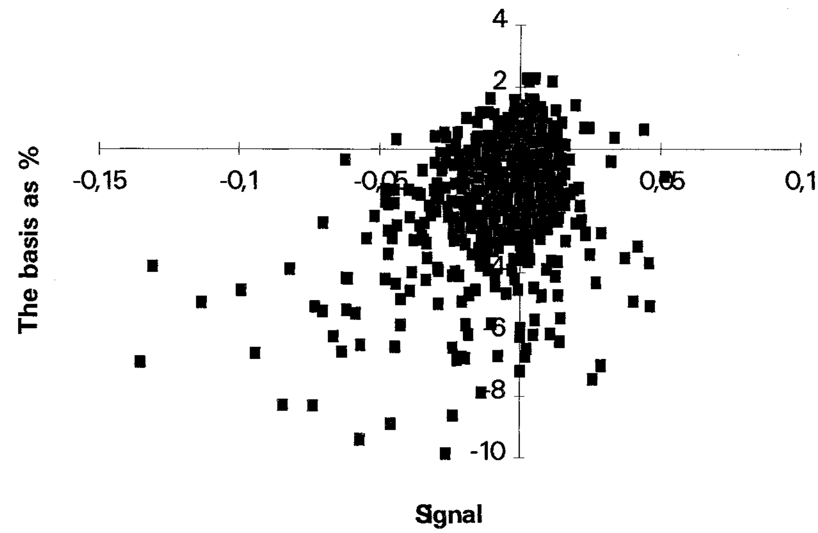
We conclude that there is clear evidence of the negative basis being negatively related to index volatility and positively related to the signal which the informed investors receive. These relationships do not carry over to the positive basis as expected. Not forgetting the strong persistence in the basis series, which indicates the lack of efficient arbitrage activity, we feel that the regression results give support to our general equilibrium model, suggesting one possible explanation for the behaviour of the Finnish index futures market.

Figure 2. **The basis vs. volatility**



[26] Chan (1992) argues that the model by Diamond & Verrecchia (1987) with private information and short sales restrictions would imply an asymmetric lead-lag relationship between the futures and the spot, and tests this hypothesis with the U.S. data. He finds no evidence of asymmetry in the lead-lag relation. This may be no wonder, though, since as a matter of fact Diamond & Verrecchia predict that trade in options (and futures), which offers a substitute for selling stocks short and, hence, lowers the selling short costs, should in itself decrease such an asymmetry. We find Chan's test hypothesis somewhat contradictory in this respect. Neither is Chan's finding of no asymmetry in the lead-lag relationship wonder from the view point of our model, for in the U.S. the short sales constraint is not binding in the same sense as in Finland and the information and ownership structures of the market may be quite different from those in Finland.

Figure 3. **The basis vs. the signal**



4 Conclusions

We have analyzed the behaviour of the Finnish stock index futures market in a simple rational expectations general equilibrium setting. The model is a simplification and in some ways an extension of the model in Fremault (1991). We also provide some empirical evidence supporting the model predictions in the period from May 1988 to December 1990 in the Finnish market. Our model is also related in spirit to a recent model by Chen, Cuny & Haugen (1994).[27]

The primary motive of the paper is to rationalize the observed seeming arbitrage opportunities in the form of persistent futures underpricing relative to the cost-of-carry model in the Finnish index futures market. The argument is built around the short-selling restriction and strategic stock ownership motives in the Finnish capital market in a general equilibrium framework. The effects of a certain form of asymmetric information that follows naturally our investor group specification are also studied. It is shown that under a binding short-selling restriction the observed futures "underpricing" can be a result of strategic motives of the Finnish commercial banks and industrial groups centered around them to hold large amounts of stocks, which in turn implies a net futures demand for hedging part of the financial risks brought in by these strategic holdings. In particular, this offers a potential answer to the question why the Finnish

[27] The volatility-basis relationship of our model is shared by the model of Chen, Cuny & Haugen (1994) with the exception that our model predicts this relationship only when the basis is negative (indicating a binding short-selling restriction). On the other hand, the Chen, Cuny & Haugen (1994) model does not require short-selling restrictions for explaining negative (or non-zero) basis. Their model received very strong empirical support in tests with the U.S. data (Chen, Cuny & Haugen, 1994) as well as with Dutch data (Berglund & Kabir, 1994).

commercial banks have not wiped out the arbitrage opportunities clearly available to them in the index futures market, since the violations from the futures arbitrage pricing relationship have been large enough to cover banks' low trading costs and since the banks with their relatively large share holdings have not been constrained by the short-selling restriction of the Finnish market. In the model allowing short-selling would bring the futures price back to its cost-of-carry level. Two main testable implications follow from the model: the negative futures basis is an increasing function of the expected stock index volatility (the basis becomes increasingly negative as volatility increases) and a decreasing function of the private information signal received by the informed investors (who are identified as the strategic stock holders in the model). Our empirical results support these predictions. Moreover, we sketch a scenario based on our analysis of a boundedly rational equilibrium along with a fully rational equilibrium in the case of asymmetric information, which could potentially explain the asymmetric lead-lag relation between the futures and the stock prices found by Puttonen (1993a) in Finland. In future work we plan to stretch our test sample period to cover the most recent years as well.

References

Berglund, T. & Kabir, R. (1994): What explains the Difference Between the Futures' Price and its "Fair Value"? Evidence from the European Options Exchange, unpublished manuscript, Swedish School of Economics and Business Administration, Helsinki, Finland.

Berglund, T. & Liljeblom, E. (1988): Market Serial Correlation on a Thin Security Market: A Note, Journal of Finance, Vol. 43, No. 4.

Chan, K.A. (1992): Further Analysis of the Lead-Lag Relationship Between the Cash Market and Stock Index Futures Market, Review of Financial Studies 5:1, 123–152.

Chen, N.F., Cuny, C.J. & Haugen, R.A. (1994): Stock Volatility and the Levels of the Basis and Open Interest in Futures Contracts, forthcoming in Journal of Finance.

Diamond, D.W. and Verrechia, R.E. (1981): Information Aggregation in a Noisy Rational Expectations Economy, Journal of Financial Economics, vol. 9: 221–235.

Diamond, D.W. and Verrechia, R.E. (1987): Constraints on Short-Selling and Asset Price Adjustment to Private Information, Journal of Financial Economics, 18: 277–311.

Fremault, A. (1991): Stock Index Futures and Index Arbitrage in a Rational Expectations Model, Journal of Business, vol. 64, no. 41: 523–547.

Grossman, S.J. (1976): On the Efficiency of Competitive Stock Markets When Traders Have Diverse Information, Journal of Finance 31, 573–585.

Jokivuolle, E.M. (1994): A Note on Measuring the True Stock Index Value in the Presence of Infrequent Trading, unpublished manuscript, University of Helsinki.

Lo, A.W. & MacKinley, A.C. (1988): Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test, Review of Financial Studies, Vol. 1, No. 1, 41–66.

Miller, M., Muthuswamy, J. & Whaley, R. (1994): Predictability of S&P500 Index Basis Changes: Arbitrage-Induced or Statistical Illusion", Journal of Finance.

Puttonen, V. (1993a): Short Sales Restrictions and the Temporal Relationship Between Stock Index Cash and Derivatives Markets, The Journal of Futures Markets , vol. 13 (September): 645–664..

Puttonen, V. (1993b): The Ex Ante Profitability of Index Arbitrage in the New Finnish Markets, Scandinavian Journal of Management.

Puttonen, V. (1993c): The Efficiency of the Finnish Stock Index Derivatives Markets, a doctoral dissertation, University of Vaasa.

Puttonen, V. and Martikainen, T. (1991): Short sales restrictions, implications for stock index arbitrage, Economics Letters, vol. 37, no. 2:159–163.

Appendix A

Agents' mean-variance objective function is

$$E\tilde{W}_1^k - \frac{1}{2}\mathrm{var}(\tilde{W}_1^k), \tag{A1}$$

where $\tilde{W}_1^k$ is agent k's end wealth.

The budget constraint of the arbitrageurs for periods 0 and 1 are

$$W_0^a = P_s e^a + \bar{B}^a = P_s(e^a + x^a) + B^a,$$

and (A2)

$$W_1^a = \tilde{v}(e^a + x^a) + (\tilde{v} - P_f)f^a + B^a.$$

Solving for the end period wealth in (A2) and placing that expression into (A1) we get the following maximization problem for arbitrageurs

$$\max_{x^a, f^a} (\bar{v} - P_s)x^a + \bar{v}e^a + (\bar{v} - P_f)f^a + \bar{B}^a - (\frac{1}{2})\sigma_v^2(x^a + f^a + e^a)^2. \tag{A3}$$

Similarly we get the maximization problems for hedgers and speculators respectively

$$\max_{f^h} \bar{v}e^h + (\bar{v} - P_f)f^h + \bar{B}^h - (\frac{1}{2})\sigma_v^2(f^h + e^h)^2$$

and (A4)

$$\max_{x^s} (\bar{v} - P_s)x^s + \bar{B}^s - (\frac{1}{2})\sigma_v^2(x^2)^2$$

Solving (A3) and (A4) we get the demand functions

$$x^a = \frac{\bar{v} - P_s}{\sigma_v^2} - f^a - e^a,$$

$$f^a = \frac{\bar{v} - P_f}{\sigma_v^2} - x^a - e^a,$$

$$f^h = \frac{\bar{v} - P_f}{\sigma_v^2} - e^h$$

and (A5)

$$x^s = \frac{\bar{v} - P_s}{\sigma_v^2}.$$

Using (A5) in the case where there are no short selling restrictions or replacing the derived demand function for x^a with $x^a = -e^a$ when the short selling constraint is binding and applying the market clearing conditions

$$x^a + x^s = 0$$

and (A6)

$$f^a + f^h = 0$$

in both cases we get the equilibrium prices for stocks and futures contracts in the unrestricted case and when the short selling constraint is binding.

Appendix B

Let the speculators have the following price conjectures

$$P_s = x_0\bar{v} + x_1\tilde{\theta} + x_2\tilde{e}^a + x_3\tilde{e}^h + x_4\bar{e}_h$$

and (B1)

$$P_f = y_0\bar{v} + y_1\tilde{\theta} + y_2\tilde{e}^h + y_3\bar{e}^h,$$

where the x and y are unknown parameters (vectors) to be determined.

Denote the vector of price conjectures (B1) by X_2. Define the vector X_1 by

$$[\tilde{v} \;\; \tilde{e}^a \;\; \tilde{e}^h]$$

Let

$$m = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$

and

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

where

$$m_1 = [\bar{v} \;\; 0 \;\; \bar{e}^h]$$

$$m_2 = [\bar{v}(x_0 + x_1) \;\; \bar{e}^h(x_3 + x_4)]$$

$$\Sigma_{11} = \begin{bmatrix} 1/h & 0 & 0 \\ 0 & \sigma_a^2 & 0 \\ 0 & 0 & \sigma_h^2 \end{bmatrix}$$

$$\Sigma_{12} = \begin{bmatrix} x_1/h & y_1/h \\ \sigma_a^2 x_2 & 0 \\ \sigma_h^2 x_3 & \sigma_h^2 y_2 \end{bmatrix} = \Sigma_{21}$$

$$\Sigma_{22}=\begin{bmatrix} x_1^2(1/h+1/h_\varepsilon)+x_2^2\sigma_a^2+x_3^2\sigma_h^2 & x_1y_1(1/h+1/h_\varepsilon)+x_3y_2\sigma_h^2 \\ x_1y_1(1/h+1/h_\varepsilon)+x_3y_2\sigma_h^2 & y_1^2(1/h+1/h_\varepsilon)+y_2^2\sigma_h^2 \end{bmatrix}$$

Now

$$X_1 \sim N(m_1,\Sigma_{11})$$

and

$$X_2 \sim N(m_2,\Sigma_{22})$$

The conditional distribution of X_1 given X_2 is

$$X_1|X_2 \sim N(m_{12},\Sigma_{11.2})$$

where

$$m_{1.2}=m_1+\Sigma_{12}\Sigma_{22}^{-1}(X_2^{*}-m_2) \qquad \text{(B2)}$$

and

$$\Sigma_{11.2}=\Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}.$$

So using (B2) we calculate ν_u (the conditional expectation of the second period price for speculators) and h_u (the precision or the inverse of conditional variance). Then we use ν_u and h_u which still contain the unknown parameters from the price conjectures together with ν_i and h_i (the conditional moments for arbitrageurs and hedgers) in the maximization problem (see Appendix A) instead of the unconditional moments. We derive the new demand functions and apply market clearing conditions in order to determine the new equilibrium. Finally in the new equilibrium the unknown parameters x and y are determined by equating the conjectured prices with realized prices. The calculated values for x and y are then used in (B1) to determine the final equilibrium prices.

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