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Monetary policy, expected inflation and inflation risk premia

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Suomen Pankki
Bank of Finland
P.O.Box 160
FI-00101 HELSINKI
Finland
☎ + 358 10 8311

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Federico Ravenna* – Juha Seppälä**

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* University of California, Department of Economics.
E-mail: fravenna@ucsc.edu
** University of Illinois, Department of Economics.
E-mail: seppala@uiuc.edu

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Federico Ravenna – Juha Seppälä
Monetary Policy and Research Department

Abstract

Within a New Keynesian business cycle model, we study variables that are normally unobservable but are very important for the conduct of monetary policy, namely expected inflation and inflation risk premia. We solve the model using a third-order approximation that allows us to study time-varying risk premia. Our model is consistent with rejection of the expectations hypothesis and the business-cycle behaviour of nominal interest rates in US data. We find that inflation risk premia are very small and display little volatility. Hence, monetary policy authorities can use the difference between nominal and real interest rates from index-linked bonds as a proxy for inflation expectations. Moreover, for short maturities current inflation is a good predictor of inflation risk premia. We also find that short-term real interest rates and expected inflation are significantly negatively correlated and that short-term real interest rates display greater volatility than expected inflation. These results are consistent with empirical studies that use survey data and index-linked bonds to obtain measures of expected inflation and real interest rates. Finally, we show that our economy is consistent with the Mundell-Tobin effect: increases in inflation are associated with higher nominal interest rates, but lower real interest rates.

Keywords: term structure of interest rates, monetary policy, expected inflation, inflation risk premia, Mundell-Tobin effect

JEL classification numbers: E5, E43, E44, G12

Rahapolitiikka, inflaatio-odotukset ja inflaatioriskin korkokustannukset

Suomen Pankin keskustelualoitteita 18/2007

Federico Ravenna – Juha Seppälä
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tutkimuksessa tarkastellaan rahapolitiikan harjoittamisen kannalta tärkeiden, mutta tavanomaisesti havaitsemattomien muuttujien, odotetun inflaation ja inflaatioepävarmuudesta aiheutuvan riskipreemion määräytymistä modernissa uuskeynesiläisessä makromallissa. Riskipreemion mahdollisesti ajassa vaihtelevia ominaisuuksia voidaan analysoida mallin ratkaisemisessa käytetyn kolmannen asteen approksimointimenetelmän ansiosta. Korkojen aikarakenteen ratkaisu mallissa ei edellytä odotushypoteesin voimassaoloa. Lisäksi mallin korkojen suhdannedynamiikka on sopusoinnussa Yhdysvalloista kerättyjen vastaavien korkohavaintojen kanssa. Tutkimuksen tulosten perusteella inflaation riskipreemio on hyvin pieni ja ajan suhteen melko vakaa, minkä ansiosta keskuspankki voi käyttää nimellisten ja inflaatiosuojatuista valtion joukkovelkakirjoista laskettujen reaalikorkojen erotusta inflaatio-odotusten mittarina. Lyhyissä maturiteeteissa vallitseva inflaatiovauhti lisäksi ennustaa hyvin inflaation riskipreemion. Myös reaalikorkojen ja odotetun inflaation välinen negatiivinen korrelaatio lyhyissä maturiteeteissa on tilastollisesti vahva. Reaalikorot lisäksi vaihtelevat voimakkaammin kuin odotettu inflaatio. Mallin implikaatiot ovat näiltä osin sopusoinnussa sellaisten empiiristen tutkimusten kanssa, joissa käytetään kyselytutkimuksia inflaatio-odotusten ja inflaatiosuojattujen valtion joukkovelkakirjojen tuottoja reaalikorkojen mittaamiseen. Mallin ratkaisu on myös sopusoinnussa ns. Mundellin – Tobinin vaikutuksen kanssa. Tämän vaikutuksen mukaan talouden nimelliskorko nousee, mutta reaalikorko alenee inflaation kiihtyessä.

Avainsanat: korkojen aikarakenne, rahapolitiikka, inflaatio-odotukset, inflaation riskipreemio, Mundellin – Tobinin vaikutus

JEL-luokittelu: E5, E43, E44, G12

Contents

1 Introduction

Several variables that are very important for the conduct of the monetary policy are unobservable. These include (ex-ante) real interest rates, expected inflation, and inflation risk premia. New Keynesian models assume that monetary policy responds to inflationary shocks by raising short-term nominal interest rates sufficiently to increase short-term real interest rates, given expected inflation.[1] Assuming that the government issues both nominal and index-linked bonds, a measure for both nominal and real interest rates is available. However, to obtain expected inflation as the difference between the two interest rates, the sign and magnitude of the inflation risk or term premium must be known.[2]

In this paper we study the behavior of inflation risk premia using a New Keynesian model. The New Keynesian framework has become the workhorse model for monetary policy analysis, but its predictions for the term structure of interest rates have only recently attracted attention.[3] We provide a model solution accurate to the third order, and use a parameterization which ensures the model is consistent with important stylized facts about the behavior of the US nominal term structure.

There is a large empirical literature on the dynamic behavior of real interest rates and expected inflation, but the literature has not identified generally accepted empirical regularities. As Ang, Bekaert, and Wei (2007) note:

> 'For example, whereas theoretical research often assumes that the real interest rate is constant, empirical estimates for the real interest rate process vary between constancy as in Fama (1975), mean-reverting behavior (Hamilton, 1985), or a unit-root process (Rose, 1988). There seems to be more consensus on the fact that real rate variation, if it exists at all, should only affect the short end of the term structure but that the variation in the long-term interest rates is primarily affected by shocks to expected inflation (see, among others, Mishkin, 1990, and Fama, 1990, but this is disputed by Pennacchi, 1991). Another phenomenon that has received wide attention is the Mundell (1963) and Tobin (1965) effect: the correlation between real rates and (expected) inflation appears to be negative.'

Similarly, views about inflation risk premia vary considerably in the literature. Buraschi and Jiltsov (2005) study the inflation risk premium in a continuous-time general equilibrium model in which the monetary authority sets the money supply based on targets for long-term growth of the nominal money supply, inflation, and economic growth. They identify

[1] See, eg, Clarida, Gali, and Gertler (1999) or Woodford (2003).

[2] Inflation risk premia and term premia refer to the same concept (compensation required by the nominal bond-holder to bear the risk of changes in currency value) expressed in terms of bond price spread or bond return spread. The premia are defined formally in Section 4.

[3] See Bekaert, Cho, and Moreno (2005), Gallmeyer, Hollifield, and Zin (2005), Hördahl, Tristani, and Vestin (2005), Hördahl and Tristani (2007), Ravenna and Seppälä (2007), Rudebusch and Wu (2004).

the time-variation of the inflation risk premium as an important explanatory variable for deviations from the expectations hypothesis. In contrast, in Ravenna and Seppälä (2007) the monetary policy authority follows an interest rate rule – a more accurate description of the actual conduct of monetary policy in most countries. Contrary to results in Buraschi and Jiltsov, Ravenna and Seppälä find that in the New Keynesian model monetary policy shocks and inflation risk premia do not explain rejections of the expectations hypothesis.

Hördahl and Tristani (2007) estimate inflation risk premia in the euro area using a three-equation log-linear New Keynesian model to describe macroeconomic variables and an essentially affine term structure model for bond pricing. They find that on average inflation risk premia were negligible and that fluctuations around the mean are relatively small but occasionally statistically significant. Veronesi and Yared's (2000) estimates imply that for US bonds inflation risk premia have been extremely small since the 1990s. In contrast, Ang, Bekaert, and Wei (2007) use an essentially affine term structure model with regime switching and find that inflation risk premia are positive and increasing in maturity in the US data. The literature on inflation risk premia is vast, but as noted in Hördahl and Tristani (2007) there is no robust consensus on the results for the sign, magnitude and volatility of the premia.

Ravenna and Seppälä (2007) show that a New Keynesian model can explain the behavior of the US term structure over the business cycle. The model produces procyclical interest rates and countercyclical term spreads, and the term spread has predictive power for future economic activity. Most importantly, the model is able to account for rejections of the expectations hypothesis. Our aim in this paper is to explore the implications of this same model concerning the dynamic behavior of real interest rates, expected inflation, and inflation risk premia.

We find that in the model (i) inflation risk premia are very small on average and have low volatility, (ii) the behavior of the short-maturity inflation risk premia can be well approximated by a linear function of current inflation, (iii) short-term real interest rates and expected inflation are significantly negatively correlated, and (iv) short-term real interest rates display greater volatility than expected inflation. Finally, we show that our economy is consistent with the Mundell-Tobin effect, that increases in inflation are associated with higher nominal interest rates but lower real interest rates. Result (i) is consistent with Hördahl and Tristani (2007). Results (i) and (ii) imply not only that the difference between nominal and index-linked yields is an accurate measure of expected inflation also but that current inflation can explain most of the residual. Result (iii) is consistent with empirical evidence in Pennacchi (1991), Woodward (1992), and Barr and Campbell (1997). Result (iv) is consistent with empirical evidence in Pennacchi (1991). The Mundell-Tobin effect has been confirmed in cross-sectional empirical studies by Monnet and Weber (2001) and Rapach (2003) and in long-term US data by Ahmed and Rogers (2000).

The rest of the paper is organized as follows. Section 2 explains our version of the New Keynesian model. Section 3 reports the results concerning the nominal term structure. Section 4 reports the results for the term structures of real interest rates, expected inflation, and inflation risk premia. Section 5 concludes. Appendix A presents the model's equilibrium conditions, Appendix B the parameterization, and Appendix C our algorithm to solve the model numerically.

2 The Model

The term structure of interest rates and risk premia are derived from a money-in-utility-function model where nominal rigidities allow monetary policy to affect real variables. As in Calvo (1983) and the New Keynesian literature on the business cycle, we assume output that is produced by monopolistically competitive firms that face an exogenous and constant probability of being able to reset prices in any period t.

Each household owns shares in all firms, and profits from the output sector are distributed to the household sector. Savings can be accumulated in money balances or in a range of riskless nominal and real bonds of several maturities. The government runs a balanced budget in every period, and rebates to consumers any seigniorage revenue from issuing the monetary asset. Output is produced with undifferentiated labor supplied by households.

Overall, the model follows closely the baseline New Keynesian framework that has become the workhorse for analyzing of monetary policy. To match the behavior of the US nominal term structure over the business cycle in the postwar period, we rely on habit-persistent preferences, and three exogenous shocks affecting total factor productivity, households' preferences and monetary policy. The model is described in detail in Ravenna and Seppälä (2007). First order conditions and parameterization are contained in Appendices A and B.

Households

There is a continuum of infinitely-lived households, indexed by $j \in [0,1]$. Households obtain utility from a consumption index C_t which is a Dixit-Stiglitz aggregate defined over a continuum of differentiated goods $z \in [0,1]$ with elasticity of substitution θ. The utility of household j is given by

$$V_t = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{\left(C^j_{t+i} - bC^j_{t+i-1}\right)^{1-\gamma}}{1-\gamma} D_{t+i} - \frac{\ell {N^j_{t+i}}^{1+\eta}}{1+\eta} + \frac{\xi}{1-\gamma_m} \left(\frac{M^j_{t+i}}{P_{t+i}}\right)^{1-\gamma_m} \right\} \tag{2.1}$$

where N_t denotes labor supply, M_t nominal money balances, P_t the aggregate price level, and B_t bond holdings. D_t is an aggregate stochastic preference shock that distorts the labor-leisure decision.[4] The household's budget constraint is

$$\int_0^1 C_t^j(z)P_t(z)\,dz = W_t N_t^j + \Pi_t^j - (M_t^j - M_{t-1}^j) - (\overrightarrow{p}_t \overrightarrow{B}_t^j - B_{t-1}^j) - \tau_t^j \quad (2.2)$$

Each element of the row vector $\overrightarrow{p}_t$ represents the price of an asset with maturity k that will pay one unit of currency in period $t+k$. The corresponding element of $\overrightarrow{B}_t$ represents the quantity of such claims purchased by the household. B_{t-1}^j indicates the value of the household portfolio of claims maturing at time t. Bonds are in zero-net supply, since the government does not issue bonds. Therefore in equilibrium it must hold that $B_{t,f} = 0$ for any component f of the vector $\overrightarrow{B}_t$. W_t is the nominal wage rate, τ is a lump-sum tax imposed by the government, and Π_t is the profit received from firms.

Firms

The firm producing good z employs a linear technology $Y_t(z) = A_t N_t(z)$ where A_t is an aggregate productivity shock. Cost minimization implies that the real marginal cost MC_t of producing one unit of output is

$$MC_t(z)MPL_t(z) = W_t/P_t$$

where $MPL$is the marginal product of labor.

In each period there is a constant probability $(1-\theta_p)$ that the firm will be able to adjust its price regardless of past history. This implies that the fraction of firms setting prices at t is $(1-\theta_p)$ and that the expected waiting time for the next price adjustment is $\frac{1}{1-\theta_p}$. The problem for the firm setting the price at time t is to choose $P_t(z)$ so as to maximize the expected discounted stream of profits

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \frac{MUC_{t+i}}{MUC_t} \left[\frac{P_t(z)}{P_{t+i}} Y_{t,t+i}(z) - \frac{MC_{t+i}^N}{P_{t+i}} Y_{t,t+i}(z) \right] \quad (2.3)$$

where MUC is the marginal utility of consumption and MC^N is the nominal marginal cost, subject to

$$Y_{t,t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\theta} Y_{t+i} \quad (2.4)$$

In (2.4), $Y_{t,t+i}(z)$ is the firm's demand function for its output at time $t+i$, conditional on the price set at time t, $P_t(z)$. Market clearing insures that $Y_{t,t+i}(z) = C_{t,t+i}(z)$ and $Y_{t+i} = C_{t+i}$.

[4] Hall (1997) defines this shock as a shift in "households' choice between work in the market and time spent in non-market activities" and shows that most of the volatility in US labor hours can be explained by this shock. This empirical evidence is consistent with Eichenbaum, Hansen, and Singleton (1988) results on comovements of US real wages, consumption and work effort.

Monetary policy

The monetary authority adjusts the interest rate in response to deviations of target variables from the steady state according to the forward-looking instrument feedback rule

$$\frac{(1+\overline{R}_{1,t})}{(1+R^{SS})} = E_t \left(\frac{1+\pi_{t+1}}{1+\pi_{SS}}\right)^{\omega_\pi} \left(\frac{Y_t}{Y_{SS}}\right)^{\omega_y} \tag{2.5}$$

where $\overline{R}$ is the target rate for the short-term nominal interest rate, R^{SS} is the steady state nominal interest rate, $\pi_{t+1} = \log(P_{t+1}) - \log(P_t)$, π_{SS} is the steady state inflation rate, Y_{SS} is the steady state level of output, and ω_π, $\omega_y \geq 0$ are the feedback coefficients for CPI inflation and output. We assume the central bank assigns positive weight to an interest rate smoothing objective, so that the domestic short-term interest rate at time t is set according to

$$(1+R_{1,t}) = \left[(1+\overline{R}_{1,t})\right]^{(1-\chi)} \left[(1+R_{1,t-1})\right]^{\chi} \varepsilon_t^{mp} \tag{2.6}$$

where $\chi \in [0,1)$ is the degree of smoothing and ε_t^{mp} is an unanticipated exogenous shock to monetary policy.

A large number of variants of the monetary policy instrument rule (2.6) Interest rate smoothing have been estimated with US data. We choose an inflation feedback coefficient ω_π equal to 1.5. This value is substantially lower than the one estimated by Clarida, Gali and Gertler (2000) for the Volker-Greenspan tenure, but closer to the estimate in Rabanal and Rubio-Ramirez (2005) for the longer 1960–2001 period and averages across different monetary regimes in post-war US data.

The choice of a value for ω_y is more controversial. Rabanal and Rubio-Ramirez (2005) estimate a value of 0.1, while Taylor's (1993) estimate is equal to 0.125. In our benchmark parameterization we chose a value of $\omega_y = 0$. Estimates of instrument rules across a large number of OECD countries consistently find very inertial behavior for the policy interest rate. Following the empirical evidence, we assume a smoothing parameter χ equal to 0.9. Quarterly steady state inflation is set at 0.75%, roughly the average US value over the period 1994–2004. The effects of alternative assumptions for monetary policy on the term structure results are discussed in Ravenna and Seppälä (2007).

3 Term structure of nominal interest rates

Let q_{t+1} denote the real stochastic discount factor

$$q_{t+1} \equiv \beta \frac{MUC_{t+1}}{MUC_t} \tag{3.1}$$

and let Q_{t+1} denote the nominal stochastic discount factor

$$Q_{t+1} \equiv \beta \frac{MUC_{t+1}}{MUC_t} \frac{P_t}{P_{t+1}} \tag{3.2}$$

The price of an n-period zero-coupon real bond is given by

$$\begin{aligned} p^b_{n,t} &= E_t \left[\prod_{j=1}^{n} q_{t+j} \right] \\ &= E_t \left[q_{t+1} p^b_{n-1,t+1} \right] \end{aligned} \tag{3.3}$$

and similarly the price of an n-period zero-coupon nominal bond is given by

$$\begin{aligned} p^B_{n,t} &= E_t \left[\prod_{j=1}^{n} Q_{t+j} \right] \\ &= E_t \left[Q_{t+1} p^B_{n-1,t+1} \right] \end{aligned} \tag{3.4}$$

The bond prices are invariant with respect to time, hence equations (3.3) and (3.4) yield a recursive formula for pricing zero-coupon real and nominal bonds of any maturity.

Prices are related to rates (or yields) by

$$R_{n,t} = -(1/n)\log(p^B_{n,t}) \quad \text{and} \quad r_{n,t} = -(1/n)\log(p^b_{n,t}). \tag{3.5}$$

Table 3.1 presents means, standard deviations, and correlations with output for selected maturities in the term structure in the model, and for US nominal data as estimated by Global Financial Data from the first quarter of 1952 to the first quarter of 2006. Output is filtered using the Hodrick-Prescott (1980) filter with a smoothing parameter of 1600, in both the model and the data.

Table 3.1 **Main term structure statistics. Data: 1952–2006**

	Mean	Standard deviation	Correlation with output
$R_{1,t}$ (model)	5.01198	1.84042	0.16560
$R_{4,t}$ (model)	6.22698	1.40906	0.25269
$R_{40,t}$ (model)	6.59399	0.56985	0.40510
$R_{80,t}$ (model)	6.59911	0.33910	0.40896
$R_{1,t}$ (data)	5.03359	2.81717	0.17491
$R_{4,t}$ (data)	5.60977	3.05969	0.14690
$R_{40,t}$ (data)	6.42456	2.76373	–0.01473
$R_{80,t}$ (data)	6.60014	2.71775	–0.04062
$R_{40,t} - R_{1,t}$ (model)	1.58202	1.50547	–0.04911
$R_{80,t} - R_{1,t}$ (model)	1.58713	1.63395	–0.10166
$R_{40,t} - R_{4,t}$ (model)	0.36702	0.98454	–0.12717
$R_{80,t} - R_{4,t}$ (model)	0.37213	1.14983	–0.18905
$R_{40,t} - R_{1,t}$ (data)	1.39097	1.13509	–0.46998
$R_{80,t} - R_{1,t}$ (data)	1.56654	1.32127	–0.45650
$R_{40,t} - R_{4,t}$ (data)	0.81479	1.01890	–0.48109
$R_{80,t} - R_{4,t}$ (data)	0.99037	1.24971	–0.44800

The average term structure is upward-sloping in both the model and the data. Means match quite well: the model produces nominal yields varying from 5.01% to 6.60% for three months to 20 years maturity, while the corresponding US yields varied from 5.03% to 6.60%.

Table 3.1 shows that the model generates procyclical nominal interest rates and countercyclical term spreads. This matches the positive correlation between yields and the cyclical component of output observed in US data at maturities up to one year. The nominal term spreads are countercyclical in both the US data and the model at all maturities. Ravenna and Seppälä (2007) perform extensive sensitivity analysis for the model's terms structure implications and discuss the dimensions in which the New Keynesian model succeeds or fails to explain the US term structure behavior.

4 Term structures of expected inflation, real interest rates, and inflation risk premia

The definitions of one-period zero-coupon nominal bond (3.4) and nominal stochastic discount factor (3.2) yield

$$p_t^B = E_t\left[Q_{t+1}\right] = E_t\left[\beta\frac{MUC_{t+1}P_t}{MUC_tP_{t+1}}\right] \tag{4.1}$$

To define the inflation risk premium, write (4.1) using the definition of conditional covariance and the definition of real bond price (3.3)

$$\begin{aligned} p_t^B &= E_t\left[\beta\frac{MUC_{t+1}P_t}{MUC_tP_{t+1}}\right] \\ &= E_t\left[\beta\frac{MUC_{t+1}}{MUC_t}\right]E_t\left[\frac{P_t}{P_{t+1}}\right] + cov_t\left[\beta\frac{MUC_{t+1}}{MUC_t},\frac{P_t}{P_{t+1}}\right] \\ &= p_t^bE_t\left[\frac{P_t}{P_{t+1}}\right] + cov_t\left[q_{t+1},\frac{P_t}{P_{t+1}}\right] \end{aligned}$$

Since the conditional covariance term is zero for risk-neutral investors and when the inflation process is deterministic, we call it the *inflation risk premium*, $irp_{1,t}$, given by

$$irp_{1,t} = cov_t\left[q_{t+1},\frac{P_t}{P_{t+1}}\right] = p_t^B - p_t^bE_t\left[\frac{P_t}{P_{t+1}}\right]$$

and similarly $irp_{n,t}$ is the n-period inflation risk premium

$$irp_{n,t} = cov_t\left[\prod_{j=1}^{n}q_{t+j},\frac{P_t}{P_{t+n}}\right] = p_{n,t}^B - p_{n,t}^bE_t\left[\frac{P_t}{P_{t+n}}\right]$$

Assuming that the inflation risk premium is zero, we obtain the Fisher hypothesis

$$p^B_{n,t} = p^b_{n,t} E_t \left[\frac{P_t}{P_{t+n}} \right]$$

or by taking logs and multiplying by $-(1/n)$

$$R_{n,t} \approx r_{n,t} + \frac{1}{n} E_t \left[\log \left(\frac{P_{t+n}}{P_t} \right) \right]$$

That is, the nominal interest rate is approximately the sum of the (ex-ante) real interest rate and the average expected inflation.

We define

$$epi_{n,t} \equiv \frac{1}{n} E_t \left[\log \left(\frac{P_{t+n}}{P_t} \right) \right]$$

and the inflation term premium as the difference between the nominal interest rate and the sum of real interest rate and average expected inflation

$$itp_{n,t} \equiv R_{n,t} - r_{n,t} - epi_{n,t}.$$

Table 4.1 presents the main statistics for the term structure of real interest rates, average expected inflation, inflation risk premia, and inflation term premia in the benchmark parameterization. Since average expected inflation is nearly constant as a function of maturity and the inflation term premia are very small, the average term structure of real interest rates follows exactly the same pattern as the average term structure of nominal interest rates in Table 3.1. Inflation risk and term premia are very small and have low volatility.[5]

[5] While the inflation risk and term premia are volatile relative to their average values, the variation is very small in absolute terms.

Table 4.1 **Main inflation statistics for real interest rates, average expected inflation, and inflation risk and term premia, benchmark case**

	Mean	Standard deviation	Correlation with output
$r_{1,t}$	2.44323	3.84144	–0.07976
$r_{2,t}$	3.24540	3.34813	–0.07399
$r_{4,t}$	3.64921	2.54032	–0.05532
$r_{8,t}$	3.83640	1.64610	–0.00476
$r_{12,t}$	3.89609	1.19887	0.04803
$r_{16,t}$	3.92604	0.94661	0.09410
$r_{20,t}$	3.94399	0.78679	0.13243
$epi_{1,t}$	2.70201	2.59401	0.23127
$epi_{2,t}$	2.70201	2.27734	0.24525
$epi_{3,t}$	2.70200	1.81323	0.26987
$epi_{8,t}$	2.70195	1.26079	0.30932
$epi_{12,t}$	2.70193	0.96434	0.33665
$epi_{16,t}$	2.70194	0.78670	0.35395
$epi_{20,t}$	2.70196	0.66892	0.36427
$irp_{1,t}$	0.03125	0.00792	–0.22185
$irp_{2,t}$	0.05827	0.01088	–0.23463
$irp_{4,t}$	0.09122	0.01148	–0.24944
$irp_{8,t}$	0.10012	0.01175	–0.19349
$irp_{12,t}$	0.07833	0.01620	–0.11236
$irp_{16,t}$	0.05105	0.01802	–0.09234
$irp_{20,t}$	0.02706	0.01955	–0.06929
$itp_{1,t}$	–0.13326	0.03596	0.21570
$itp_{2,t}$	–0.13343	0.02834	0.22101
$itp_{4,t}$	–0.12423	0.01934	0.22967
$itp_{8,t}$	–0.10589	0.01222	0.22076
$itp_{12,t}$	–0.09476	0.01074	0.17175
$itp_{16,t}$	–0.08887	0.01040	0.14064
$itp_{20,t}$	–0.08641	0.01033	0.12246

Table 4.2 shows a positive relationship between inflation term premia and expected inflation, and a negative relationship between inflation risk premia and expected inflation. The correlations are respectively 0.96792, and -0.96200 for $n = 1$. For longer maturities, they decrease in absolute value but are still very high. Notice also that nominal interest rates and average expected inflation are negatively correlated for short maturities but positively correlated for long maturities.

Table 4.2 Correlation between selected variables, benchmark case

	$n=1$	$n=2$	$n=4$	$n=8$	$n=12$	$n=16$	$n=20$
$\rho(r_{n,t}, R_{n,t})$	0.79443	0.77924	0.71054	0.64143	0.64166	0.66613	0.69209
$\rho(r_{n,t}, epi_{n,t})$	–0.90508	–0.89273	–0.84064	–0.69445	–0.52287	–0.36678	–0.24002
$\rho(R_{n,t}, epi_{n,t})$	–0.46075	–0.41324	–0.21622	0.10656	0.31825	0.44948	0.53452
$\rho(irp_{n,t}, epi_{n,t})$	–0.96200	–0.95162	–0.82097	–0.44215	–0.21947	–0.18939	–0.17546
$\rho(itp_{n,t}, epi_{n,t})$	0.96792	0.96517	0.90958	0.75572	0.53398	0.40181	0.35814

Since the model features persistent shocks, habit formation, sticky prices, and interest rate smoothing, one would expect a high degree of autocorrelation in inflation. Hence, current inflation should be a good predictor of expected future inflation. Given the correlation between inflation premia and expected inflation as shown in Table 4.2, current inflation should be a good predictor of movements in inflation premia. Table 4.3 presents the regression results for the equations

$$\begin{aligned} irp_{n,t} &= \beta_0 + \beta_1 \pi_t \\ itp_{n,t} &= \beta_0 + \beta_1 \pi_t \quad for \quad n = 1, 2, 4, 8, 12 \quad quarters \end{aligned}$$

for 200,000 model-generated observations. Up to the maturity of one year, the current inflation explains 80% of the variation in inflation term premia. For longer maturities, it does considerably worse. Overall the results in Tables 4.1–4.3 imply not only that the difference between nominal and index-linked yields is a good measure for expected inflation but also that current inflation can explain most of the residual.

Table 4.3 **Regression of** $y_{n,t}$ on $\beta_0 + \beta_1 \pi_t$

$y_{n,t}$	β_0	β_1	R^2
$irp_{1,t}$	0.0370	–0.0022	0.9166
$irp_{2,t}$	0.0660	–0.0029	0.8713
$irp_{4,t}$	0.0980	–0.0026	0.6177
$irp_{8,t}$	0.1031	–0.0012	0.1222
$irp_{12,t}$	0.0795	–0.0005	0.0125
$itp_{1,t}$	–0.1601	0.0099	0.9339
$itp_{2,t}$	–0.1544	0.0078	0.9132
$itp_{4,t}$	–0.1376	0.0049	0.7979
$itp_{8,t}$	–0.1125	0.0024	0.4867
$itp_{12,t}$	–0.0984	0.0013	0.1911

Table 4.2 shows that real interest rates and expected inflation are significantly negatively correlated. This result is consistent with Pennacchi (1991) who estimated real interest rates and expected inflation as a state-space system using observations on the term structure of nominal interest rates and NBER-ASA survey forecasts of inflation. Pennacchi also found that

real interest rates display greater volatility than expected inflation. Table 4.1 confirms that the New Keynesian model is consistent with this empirical regularity as well.

The first result is not surprising. Monetary policy in our model, as in most New Keynesian models, operates through changes in real rates to reduce current and (expected) future inflation. Besides Pennacchi (1991), Woodward (1992) and Barr and Campbell (1997) also found the same negative correlation using data on UK nominal and index-linked bonds. The second result follows from the fact that in the model $(R_t, epi_t) < 0$. Table 4.6 below shows that the sign of this covariance is robust to most alternative parameterizations.

Sensitivity analysis

Tables 4.4–4.7 report selected correlations under different parameterizations. They reveal the robustness of our earlier results: (i) Inflation term premia are very small, have low volatility, and are on average negative; (ii) Short-term real interest rates and expected inflation are significantly negatively correlated; (iii) Short-term real interest rates display greater volatility than expected inflation. The monetary policy shock plays a key role in these results. Without policy shocks, the inflation term premium becomes positive, short-term real interest rates and expected inflation are positively correlated, and short-term real interest rates are less volatile than expected inflation. As the volatility of policy shocks increases, the magnitude of the inflation term premium increases. Interestingly, lower nominal rigidity in prices does not significantly change the expected inflation volatility – though it will change the inflation autocorrelation – but does significantly increase the magnitude of the inflation term premium.

Table 4.4 **Sensitivity of real interest rate and expected inflation statistics to different parameter values. BM = benchmark**

	$E[r_{1,t}]$	$\sigma(r_{1,t})$	$\rho(r_{1,t}, Y_t)$	$E[epi_{1,t}]$	$\sigma(epi_{1,t})$	$\rho(epi_{1,t}, Y_t)$
BM	2.44323	3.84144	–0.07976	2.70201	2.59401	0.23127
$b = 0$	3.91091	3.85959	–0.34255	2.70305	2.70596	0.48602
$\gamma = 1.5$	1.57240	4.01566	–0.09344	2.70586	2.98997	0.32506
$\chi = 0.7$	3.15546	1.88062	0.00681	2.93799	1.29973	0.36158
$\pi_{SS} = 1.0$	2.58478	3.98113	–0.07987	–0.31087	2.82735	0.20949
$\pi_{SS} = 1.01$	2.38704	3.80035	–0.08064	3.70419	2.53410	0.24241
$\omega_y = 0.1$	1.41685	4.43245	–0.05063	2.03919	5.10894	–0.27850
$\omega_\pi = 3.0$	3.00102	2.64209	–0.03573	2.94862	1.29731	0.17075
$\omega_\pi = 1.2$	2.15903	4.61458	–0.09958	2.18798	3.55571	0.25169
$\theta_p = 0.5$	3.01132	5.14716	–0.05187	2.77138	4.81021	0.13592
$\sigma_d=0$, $\sigma_a=0.01$	2.70905	3.91251	–0.53361	2.71190	2.60253	0.43339
$\sigma_a = 0$	2.46048	3.88249	–0.08605	2.71528	2.57105	0.23886
$\sigma_{mp} = 0$	3.62740	0.70511	–0.12057	2.98589	0.96212	0.45681
$\sigma_{mp} = 0.006$	–1.13281	9.31457	–0.12515	2.12654	5.34492	0.20446
$\rho_d=\rho_a=0.99$, $\sigma_{mp}=0, \chi=0.95$	3.26258	0.54433	–0.31659	2.96822	0.65423	0.36379

Table 4.5 **Sensitivity of inflation risk and term premia statistics to different parameter values. BM = benchmark**

	$E[irp_{1,t}]$	$\sigma(irp_{1,t})$	$\rho(irp_{40,t}, Y_t)$	$E[itp_{1,t}]$	$\sigma(itp_{1,t})$	$\rho(itp_{1,t}, Y_t)$
BM	0.03118	0.00791	–0.22202	–0.13326	0.03596	0.21570
$b = 0$	0.01183	0.00334	–0.46988	–0.05695	0.01882	0.46112
$\gamma = 1.5$	0.02474	0.00844	–0.32039	–0.10765	0.03811	0.31329
$\chi = 0.7$	0.00230	0.00154	–0.39511	–0.00980	0.00636	0.39513
$\pi_{SS} = 1.0$	0.04075	0.00848	–0.20000	–0.17466	0.03933	0.19330
$\pi_{SS} = 1.01$	0.02767	0.00772	–0.23420	–0.11831	0.03484	0.22804
$\omega_y = 0.1$	0.08749	0.04018	0.29107	–0.37114	0.17383	–0.28012
$\omega_\pi = 3.0$	0.01250	0.00197	–0.17014	–0.05267	0.00872	0.16492
$\omega_\pi = 1.2$	0.05052	0.01118	–0.24147	–0.21843	0.05263	0.23254
$\theta_p = 0.5$	0.10231	0.01577	–0.13378	–0.51707	0.09585	0.12053
$\sigma_d=0$, $\sigma_a=0.01$	0.02934	0.00710	–0.39130	–0.12698	0.03342	0.40043
$\sigma_a = 0$	0.03207	0.00773	–0.21817	–0.13670	0.03512	0.21186
$\sigma_{mp} = 0$	–0.00275	0.00030	0.00166	0.01095	0.00119	0.01463
$\sigma_{mp} = 0.006$	0.16828	0.04733	–0.17461	–0.71920	0.22571	0.16958
$\rho_d=\rho_a=0.99$, $\sigma_{mp}=0, \chi=0.95$	0.00013	0.00050	–0.22447	–0.00059	0.00203	0.22539

Table 4.6 **Sensitivity of selected correlations to different parameter values. BM = benchmark**

	$\rho(\pi_t, epi_{1,t})$	$\rho(r_{1,t}, R_{1,t})$	$\rho(r_{1,t}, epi_{1,t})$	$\rho(R_{1,t}, epi_{1,t})$
BM	0.99007	0.79443	–0.90508	–0.46075
$b = 0$	0.99016	0.79188	–0.92546	–0.50151
$\gamma = 1.5$	0.98668	0.65870	–0.78859	–0.05678
$\chi = 0.7$	0.98009	0.80360	–0.09781	0.51371
$\pi_{SS} = 1.0$	0.98805	0.77576	–0.92625	–0.48071
$\pi_{SS} = 1.01$	0.99065	0.79249	–0.89165	–0.43052
$\omega_y = 0.1$	0.98219	0.22357	–0.66846	0.57540
$\omega_\pi = 3.0$	0.99322	0.93120	–0.89752	–0.67503
$\omega_\pi = 1.2$	0.98857	0.66538	–0.90977	–0.29547
$\theta_p = 0.5$	0.97207	0.30724	–0.96350	–0.04133
$\sigma_d=0$ $\sigma_a=0.01$	0.99158	0.85386	–0.94422	–0.63483
$\sigma_a = 0$	0.99024	0.81989	–0.91598	–0.52131
$\sigma_{mp} = 0$	0.99320	0.48591	–0.18627	0.76821
$\sigma_{mp} = 0.006$	0.97400	0.88941	–0.96042	–0.72717
$\rho_d=\rho_a=0.99,$ $\sigma_{mp}=0,\chi=0.95$	0.97695	0.60954	–0.06265	0.75301

Table 4.7 **Sensitivity of selected correlations to different parameter values. BM = benchmark**

	$\rho(irp_{1,t}, epi_{1,t})$	$\rho(itp_{1,t}, epi_{1,t})$
BM	–0.96200	0.96792
$b = 0$	–0.98098	0.97798
$\gamma = 1.5$	–0.93459	0.95022
$\chi = 0.7$	–0.85067	0.85618
$\pi_{SS} = 1.0$	–0.95873	0.96381
$\pi_{SS} = 1.01$	–0.96235	0.96965
$\omega_y = 0.1$	0.57540	0.94390
$\omega_\pi = 3.0$	–0.91935	0.92941
$\omega_\pi = 1.2$	–0.94245	0.94969
$\theta_p = 0.5$	–0.95004	0.96228
$\sigma_d=0$ $\sigma_a=0.01$	–0.95724	0.96694
$\sigma_a = 0$	–0.97168	0.97521
$\sigma_{mp} = 0$	–0.13088	0.16868
$\sigma_{mp} = 0.006$	–0.86648	0.88168
$\rho_d=\rho_a=0.99,$ $\sigma_{mp}=0,\chi=0.95$	–0.76052	0.76218

Table 4.8 **Effect of changes in steady-state inflation rate on nominal and real interest rates and demand for money**

	$E\left[R_{1,t}\right]$	95% CI for $E\left[r_{1,t}\right]$	$E\left[M_t^d/Y_t\right]$
$\pi_{SS}=0\%$	2.09924	[2.5679,2.6016]	72.164%
$\pi_{SS}=3\%$	5.01198	[2.4264,2.4601]	67.823%
$\pi_{SS}=4\%$	5.97292	[2.3702,2.4039]	67.140%

The Mundell-Tobin effect

The Mundell-Tobin effect (Mundell, 1963, and Tobin, 1965) refers to the idea that higher inflation reduces demand for money and increases demand for interest-bearing assets. Therefore, the required return on bonds and/or marginal productivity of capital falls and the real interest rate declines. The Tobin (1965) effect implies that an increase in inflation also increases the capital stock and economic growth, and has generated much discussion in the literature.[6]

The Mundell-Tobin effect should be distinguished from the negative correlation between expected inflation and real interest rates in high frequency data, discussed earlier. The first is a statement about different steady states while the second applies to transition paths. The Mundell-Tobin effect has been confirmed in cross-sectional empirical studies by Monnet and Weber (2001) and Rapach (2003). Ahmed and Rogers (2000) also found support for the Tobin effect using long-term US data.

Typically standard general equilibrium models with long-lived agents will not generate the Mundell-Tobin effect.[7] Wang and Yip (1992) obtain the Mundell-Tobin effect with a utility function which is nonseparable in money and consumption. Ireland (1994) studies a model in which capital accumulation affects money's role as a medium of exchange, and finds that the effect of inflation on growth is small. Kam (2005) proposes a model in which the rate of time preference is an increasing function of real wealth. Finally, Bai (2005) obtains the Mundell-Tobin effect in a Bewley-type exchange economy with incomplete markets and a fixed cost of exchanging bonds for goods or money.

Our New Keynesian model will also generate the Mundell-Tobin effect, as shown in Table 4.8. An increase in the average level of inflation raises the average level of nominal interest rates but reduces the demand for money and the average level of real interest rates. The table reports the 95% confidence interval for real interest rates, to show that this effect is statistically significant.

[6] See, eg, Sidrauski (1967), Brock (1974), Stockman (1981), Drazen (1981), Ireland (1994), Jones and Manuelli (1995), Azariadis and Smith (1996), Ahmed and Rogers (2000), Kaas and Weinrich (2003), and Kam (2005). Orphanides and Solow (1990) provide a useful survey of the older literature.

[7] Drazen (1981), Chatterjee and Corbae (1992), Azariadis and Smith (1996), and Kaas and Weinrich (2003) obtain the result in models with short-lived agents. See also the survey by Orphanides and Solow (1990).

5 Conclusions

This paper explores the behavior of inflation risk premia in a New Keynesian model. Understanding the size and dynamics of inflation risk premia is essential for the monetary authority to measure expected inflation from bond prices. Assuming that the government issues both nominal and index-linked bonds, a measure of both nominal and real interest rates is available. However, to obtain from these rates a measure of expected inflation, the monetary authority needs to know the signs and magnitudes of inflation risk premia.

Our answer is that a benchmark New Keynesian model, able to explain important stylized facts about the behavior of the US nominal term structure, implies that inflation risk premia are very small on average and have very low volatility. As a consequence, the difference between nominal and index-linked yields is a good measure of expected inflation. In addition, we found that the correlation between expected inflation and inflation term premia is very close to one. Since in our parameterization, and in the data, the correlation between expected and current inflation is also very close to one, for short maturities current inflation explains a large fraction of the variation in inflation term premia, that is, of the difference between nominal yield and the sum of expected inflation and real yield. Taken together these results imply, contrary to the findings of Ang, Bekaert, and Wei (2007), that term premia between short and long rates predominantly reflect real risks rather than compensation for inflation uncertainty.

Our model can also account for the empirical results in Pennacchi (1991): (i) real interest rates and expected inflation are significantly negatively correlated and (ii) short-term real interest rates display greater volatility than expected inflation. The first result has also been obtained by Woodward (1992) and Barr and Campbell (1997) using data on UK index-linked bonds. Finally, the New Keynesian model also generates the Mundell-Tobin effect, which has been confirmed in cross-sectional empirical studies by Monnet and Weber (2001) and Rapach (2003) and in long-term US data by Ahmed and Rogers (2000).

The main drawback of our approach is that to analyze time-varying risk premia we need a solution accurate at least to the third-order. Model estimation of a non-linear approximation to the rational expectations equilibrium requires econometric methodologies that have had limited application in macroeconomics (see, eg, Boragan Aruoba, Fernandez-Villaverde, and Rubio-Ramirez, 2006). Estimating the model is hence left for future research.

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Appendix

A. Equilibrium conditions

Household

The solution to the intratemporal expenditure allocation problem among the varieties of differentiated goods yields the demand function for individual good z

$$C_t^j(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\theta} C_t^j, \tag{A1.1}$$

and the consumption price index $P_t = \left[\int_1^0 P_t(z)^{1-\theta}\, dz\right]^{\frac{1}{1-\theta}}$.

The first order conditions with respect to labor and real money balances are

$$\begin{aligned} MUC_t &= E_t\left[\frac{D_t}{(C_t - bC_{t-1})^\gamma} - \beta b \frac{D_{t+1}}{(C_{t+1} - bC_t)^\gamma}\right] \\ \frac{W_t}{P_t} &= \frac{\ell N_t^\eta}{MUC_t} \end{aligned} \tag{A1.2}$$

$$MUC_t = \xi\left(\frac{M_t}{P_t}\right)^{-\gamma m} + E_t\left[\beta MUC_{t+1}\frac{P_t}{P_{t+1}}\right] \tag{A1.3}$$

where MUC is the marginal utility of consumption.

5.1 Firms

Minimizing the nominal cost $W_t N_t(z)$ of producing a given amount of output $\overline{Y}$ yields the labor demand schedule:

$$MC_t^N(z) MPL_t(z) = W_t, \tag{A1.4}$$

where MC^N is the nominal marginal cost, MPL is the marginal product of labor.

Substituting (2.4) into (2.3), firm z's objective function can be written as

$$E_t \sum_{i=0}^{\infty} (\theta_p \beta)^i \frac{MUC_{t+i}}{MUC_t}\left\{\left[\frac{P_t(z)}{P_{t+i}}\right]^{1-\theta} Y_{t+i} - \frac{MC_{t+i}^N}{P_{t+i}}\left[\frac{P_t(z)}{P_{t+i}}\right]^{-\theta} Y_{t+i}\right\}. \tag{A1.5}$$

Since $P_t(z)$ does not depend on i, the optimality condition is

$$\begin{aligned} & P_t(z) E_t \sum_{i=0}^{\infty} (\theta_p\beta)^i MUC_{t+i}\left[\frac{P_t(z)}{P_{t+i}}\right]^{1-\theta} Y_{t+i} \\ = {} & \mu E_t \sum_{i=0}^{\infty} (\theta_p\beta)^i MUC_{t+i} MC_{t+i}^N \left[\frac{P_t(z)}{P_{t+i}}\right]^{1-\theta} Y_{t+i}, \end{aligned} \tag{A1.6}$$

where $\mu = \frac{\theta}{\theta - 1}$ is the flexible-price level of the markup.

Market Clearing

Since the measure of the economy is unitary, in the symmetric equilibrium it holds that $M_t^j = M_t$, $C_t^j = C_t$, and the consumption shadow price is symmetric across households: $MUC_t^j = MUC_t$. The linear production technology ensures that MC is equal across firms – whether or not they are updating their price – regardless of the level of production, which will indeed be different. Firms are heterogeneous in that a fraction $(1 - \theta_p)$ of firms in the interval $[0, 1]$ can optimally choose the price charged at time t. In equilibrium each producer that chooses a new price $P_t(z)$ in period t will choose the same new price $P_t(z)$ and the same level of output. Thus the dynamics of the consumption-based price index will obey

$$P_t = \left[\theta_p P_{t-1}^{1-\theta} + (1 - \theta_p) P_t(z)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{A1.7}$$

The government rebates seigniorage revenues to the household in the form of lump-sum transfers, so that in any time t the government budget is balanced. Since we defined τ^j as the amount of tax levied by the government on household j, assuming $\tau_t^j = \tau_t^i \ \forall j, i \in [0, 1]$, at every date t the transfer will be equal to

$$-\int_0^1 \tau_t^j dj \ = -\tau_t \int_0^1 dj = = -\tau_t = M_t^s - M_{t-1}^s$$

Equilibrium in the money market requires that

$$M_t^s = M_t^{d^j} = M_t^d$$

Equilibrium in the goods market gives

$$Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\theta} C_t = A_t N_t(z)$$

Integrating over z

$$A_t \int_0^1 N_t(z)\, dz = \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\theta} dz C_t$$
$$A_t N_t = C_t s_t,$$

where $s_t \equiv \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\theta} dz$.

B. Model parameterization

Our specification for preference, technology and policy parameters follows the New Keynesian monetary business cycle literature.[8] When available, the deep parameters of the model are taken from Rabanal and Rubio-Ramirez's (2005) estimates of the New Keynesian model for the US over the 1960–2001 sample.

Household preferences are modeled within the internal habit-formation framework of Boldrin, Christiano, and Fisher (2001). The habit formation coefficient is parameterized to $b = 0.8$, a value that Constantinides (1990) finds can explain the equity premium puzzle. The value of γ is set at 2.5, to provide adequate curvature in the utility function and allow the model to generate sufficient risk-premia volatility. The labor supply elasticity $(1/\eta)$ is set equal to 2, a value in line with estimates in Rabanal and Rubio-Ramirez's (2005). The parameter ℓ is chosen to set steady state labor hours at about 30% of available time. The quarterly discount factor β is parameterized so that the steady state real interest rate is equal to 4% per year.

The parameterization of demand elasticity θ implies a flexible-price equilibrium producers' markup of $\mu = \theta/(\theta - 1) = 1.1$. The parameterization chosen for the Calvo (1983) pricing adjustment mechanism implies an average price duration of one year. This value is consistent with estimates for the US over the last forty years obtained from aggregate data (Gali and Gertler, 1999, Rabanal and Rubio-Ramirez, 2005).

The preference and technology exogenous shocks follow an AR(1) process:

$$\log Z_t = (1 - \rho_Z) \log \overline{Z} + \rho_Z \log Z_{t-1} + \varepsilon_t^Z, \qquad \varepsilon_t^Z \sim \text{iid } N(0, \sigma_Z^2);$$

where $\overline{Z}$ is the steady state value of the variable. The policy shock ε_t^{mp} is a Gaussian i.i.d. stochastic process. The autocorrelation parameters for technology and preference shocks are equal to $\rho_a = 0.9$ and $\rho_d = 0.95$. The standard deviations of innovations ε_t^Z for technology, preference and policy shock are set at $\sigma_a = 0.0035$, $\sigma_d = 0.08$, $\sigma_{mp} = 0.003$. The low value for the policy shock volatility implies that the major part of the short term nominal interest rate dynamics is driven by the systematic monetary policy reaction to the state of the economy. The preference shock volatility is large but very close to the estimate of Rabanal and Rubio-Ramirez (2005) using US data. Compared to the business cycle literature, our assumption for the technology shock volatility is low. This parameterization is necessary to allow the model to generate a positive correlation between nominal interest rate and GDP, since technology shocks produce negative comovements between these variables.

[8]For references to estimated and calibrated staggered price-adjustment models, see Christiano, Eichenbaum and Evans (2005), Ireland (2001), Ravenna (2006), Rabanal and Rubio-Ramirez (2005), Woodford (2003).

An important concern in the parameterization of shocks has been to match the correlations between output and nominal and real rates with US data, to be able to evaluate whether the term structure generated by the model can predict output variation, as in many empirical studies with US data. Table A2.1 compares the model's second moments and correlations with output to the US post-war data sample.[9]

Table A2.1 **Selected variable volatilities and correlations.** The first line presents data and the second line the model valus. Sample: 1952–2006.

x_t	$\sigma(x_t)$	$\rho(x_{t-3}, Y_t)$	$\rho(x_{t-2}, Y_t)$	$\rho(x_{t-1}, Y_t)$	$\rho(x_t, Y_t)$	$\rho(x_{t+1}, Y_t)$	$\rho(x_{t+2}, Y_t)$	$\rho(x_{t+3}, Y_t)$
Y_t	1.59							
	2.01							
N_t	1.51	0.24	0.47	0.69	0.87	0.90	0.83	0.70
	2.28	0.57	0.75	0.89	0.95	0.89	0.76	0.59
π_t	3	–0.10	–0.01	0.09	0.19	0.28	0.31	0.31
	3.49	0.21	0.23	0.23	0.20	0.13	0.06	0.01
R_t	2.82	–0.10	–0.01	0.10	0.17	0.20	0.21	0.19
	1.84	0.12	0.14	0.15	0.17	0.17	0.18	0.17
r_t	2.32	–0.10	–0.13	–0.14	–0.13	–0.14	–0.15	–0.16
	3.84	–0.12	–0.12	–0.11	–0.08	–0.01	0.04	0.08

The model performs well in matching contemporaneous correlations with output. Output and labor hours show similars pattern of cross-correlations in the data and in the model. The real interest rate is countercyclical at all lags, while the lead cross-correlations become mildly positive in the model

[9] The output and labor hours series are logged and Hodrick-Prescott filtered. US data: Y_t is real GDP, N_t is average weekly hours for private industries multiplied by the ratio between total number of workers employed in the non-farm sector and the civilian non-institutional population, π_t is CPI inflation, R_t is 3-month T-bill rate, r_t is ex-post short term real interest rate. All rates are annual. Quarterly data sample is 1952:1–2006:1. The average weekly hours series starts at 1964:1. We chose to use the period following the Treasury-Federal Reserve Accord of 1951 in order to avoid having to contend with the constraint on interest rate movements imposed by the Federal Reserve's 'par pegging' of Government securities prices. Data sources: Bureau of Economic Analysis, Bureau of Labor Statistics, St. Louis Federal Reserve Bank.

and remain negative in the data. As in the data, the nominal interest rate generated by the model is a procyclical and lagging variable. Inflation lags output in the data, while it is a leading indicator in the model. This phase shift is explained by the forward-looking price-setting behavior. In the model economy demand shock explains a large part of the volatility, and the real marginal cost is correlated with output. Since inflation depends on future expected marginal costs, inflation will lead output in the model.

The model captures well the magnitudes of cross correlations for all variables. This result comes at the cost of overstating the volatilities of output, real interest rate and inflation, relative to the data. A less stylized model would allow enough degrees of freedom to better match the data moments.

An additional hurdle for the model fit of US macroeconomic volatility is the data sample, which for the period in Table A2.1 is heterogeneous with respect to US monetary policy goals and US Federal Reserve operating procedures, and includes the 1970s inflationary episode. Section 3 illustrates the model results conditional on alternative parameterizations for the policy rule, intended to capture varying degrees of inflation aversion for the monetary policymaker.

C. Solution algorithm

We solve the model using a third-order approximation around the non-stochastic steady state. The numerical solution is obtained using Dynare++ version 1.3.1. It is well known that taking a first-order approximation of bond prices will yield no risk premia and that a second-order approximation will yield only constant premia. The reason is simple: second-order approximation involves only squared prediction error terms with constant expectations.

The stochastic process describing the dynamics of bond prices is constructed in three steps. In the first step, we solve for the third-order numerical approximation to our model using Dynare++. The model has six state variables – $C_{t-1}, R_{t-1}, s_{t-1}, D_t, A_t, \varepsilon_t^{mp}$ – and seven control variables – $r_t, W_t/P_t, N_t, MUC_t, \pi_t, \tilde{G}_t, \tilde{H}_t$ – in 13 equations. In the second step, we generate 200,000 observations for state and control variables. In the final step, we build an approximation to the conditional expectations $E_t[\beta^n(MUC_{t+n}P_t)/(MUC_tP_{t+n})]$ and $E_t[\beta^n MUC_{t+n}/MUC_t]$ to obtain the prices of n-maturity zero-coupon bonds. Given that the model has a quarterly frequency and we are interested in building prices for both short and long-term bonds (where for the 20-year bond, $n = 80$), a Monte Carlo methodology proves computationally efficient. We regress the simulated values of $\beta^n(MUC_{t+n}P_t)/(MUC_tP_{t+n})$ and $\beta^n MUC_{t+n}/MUC_t$ on third-order complete polynomials of $[C_{t-1}, R_{t-1}, s_{t-1}, D_t, A_t, \varepsilon_t^{mp}]'$. The fitted regressions give the approximation for $E_t[\beta^n(MUC_{t+n}P_t)/(MUC_tP_{t+n})]$ and $E_t[\beta^n MUC_{t+n}/MUC_t]$. The approach is similar to that used in Evans and Marshall (1998).

Since we use the same third-order terms as the those given by a third-order Taylor approximation (that is, we use a complete polynomial basis), our approach is equivalent to taking a third-order approximation to the Euler equation for the bonds at each maturity. With third-order approximation, the current state variables multiply squared prediction error terms, and hence risk premia are time-varying.

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Suomen Pankki
Bank of Finland
P.O.Box 160
FI-00101 HELSINKI
Finland

. 2343