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Research Department 18.4.2000

Markets, Reserves and Lenders of Last Resort as Sources of Bank Liquidity

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The views expressed are those of the author and do not necessarily correspond to the views of the Bank of Finland

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Abstract

We study the long standing issue of whether markets can supply banks with sufficient liquidity or whether markets should be complemented with a lender of last resort (LOLR). For this purpose, we develop an extended version of the recent model of Holmström and Tirole (1998) on the supply of liquidity to firms.

H&Ts original model analyses liquidity supply to firms that are facing solvency shocks. We apply their framework to banking and extend the framework to admit the analysis of problems associated with transitory liquidity outflows, even absent any change in a bank's value. Our premise is that the scope for moral hazard may increase in connection with liquidity outflows. Moral hazard, which we interpret as the possibility of laxity in banks' monitoring of firms, may increase with liquidity outflows because banks need to increase their monitoring efforts in order to safeguard their own interests.

The model illustrates many key aspects the classical LOLR debate. The model shows how moral hazard limits of banks' ability to borrow from markets to cover liquidity outflows. It also predicts banks' demand for liquid reserves and the economies associated with centralization of reserves in a liquidity pool when the holding of liquid reserves entails opportunity costs. Finally, the model enables discussion of viable lending policies for the LOLR and contrasts these with the 'Bagehotian principles', which are still widely used as benchmark criteria in evaluating LOLR operations.

Key words: liquidity, lender of last resort, banking, central banking

JEL classification numbers: E 58, G 21.

Markkinat, reservit ja hätärahoittajat pankkien likviditeetin lähteinä

Suomen Pankin keskustelualoitteita 3/2000

Risto Herrala Tutkimusosasto

Tiivistelmä

Työssä tarkastellaan klassista kysymystä, tarvitseeko pankkisektori hätärahoittajan (lender of last resort, LOLR). Kysymystä lähestytään teoreettisesti yritysten likviditeettipolitiikkaa käsitelleen Holmströmin ja Tirolen (1998) mallin laajennuksen avulla.

Mallissa pankit saattavat ajautua epälikvidiin tilaan paitsi kokiessaan kannattavuussokkeja myös pyrkiessään löytämään rahoittajaa väliaikaisille likviditeettitarpeilleen. Lähtökohtanamme on, että moraalikato-ongelmat vaikeutuvat likviditeettivajausten sattuessa. Pankkien tulisi tiivistää luottoasiakkaidensa valvontaa, kun nämä kokevat likviditeettisokkeja ja hakevat lisärahoitusta pankeilta. Pankin asiakasvalvonnan laatua on ulkopuolisten vaikeata todentaa. Kun tallettajien usko pankin harjoittamaan valvontaan horjuu, pankki ajautuu epälikvidiin tilaan.

Malli selventää monia hätärahoitustoimintaan liittyviä keskeisiä kysymyksiä. Se selittää, miksi pankki saattaa olla kyvytön hankimaan riittävästi rahoitusta tallettajilta. Se auttaa ymmärtämään reservien keräämisen motiiveja sekä niitä hyötyjä, joita reservien keskittämisellä (hätärahoittajan luomisella) on saavutettavissa. Mallin avulla voidaan keskustella hätärahoituksen klassisten kriteereiden, ns. 'Bagehotin periaatteiden', asianmukaisuudesta.

Asiasanat: likviditeetti, hätärahoitus, pankki, keskuspankki

JEL luokittelu: E 58, G 21.

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1 Introduction

We study the long-standing issue of the need for a lender of last resort for the banking sector. The issue revolves around the following key questions:

- 1. Will free exchange of liquidity in the markets rescue banks when they are worth rescuing?
- 2. Can the establishment of a central liquidity pool improve upon the decentralised solution, in which each bank holds sufficient reserves in its balance sheet?
- 3. What are the optimal lending policies of a liquidity pool?
- 4. Should banks be aware, ex ante, of the policy of the pool?
- 5. Is there a role for public intervention in operating the liquidity pool?

In this paper we will analyse questions 1–4. Question 5, which is obviously crucial for the design of central banks' reserve and discount policies, is analysed in a forthcoming paper.

We approach the problem via an extended version of the model of Holmström and Tirole (H&T 1998) on the supply of liquidity to firms. In our model, banks have three alternative ways to deal with the possibility of liquidity shocks. First, they can rely on borrowing from the markets secondly they can hold liquid reserves; and third they can rely on a liquidity pool (such as a central bank). The latter alternative requires making a reserve deposit to the pool (or the central bank) in return for liquidity insurance.

Our model uses moral hazard as the basis for its explanation of the limits of banks' ability to borrow from markets to cover liquidity outflows. This gives rise to the demand for liquid reserves by banks, and the economies of centralising the reserves to a liquidity pool when the holding of liquid reserves entails opportunity costs. The model also illustrates the potential benefits of the right to issue legal tender. Finally, the model enables us to discuss the viable lending policies of the central liquidity pool.

The debate on the need for an LOLR for the banking sector started in the 1800s in England in connection with several situations in which the Bank of England was allowed to deviate from its banknote cover regulations in order to help the banking sector overcome liquidity drains. In a contemporary analysis, Walter Bagehot¹ spelled out the classical principles for LOLR assistance:

- lend freely during a crisis
- assist any and all sound borrowers
- lend against all acceptable collateral
- apply penalty rates for assistance
- assure the markets in advance

Bagehot felt that to lend freely to all against good collateral and to assure the markets of this in advance was necessary in order to maintain public confidence in the financial system. Collateral should always be accepted at its value obtaining in 'normal times'. The reasoning behind imposition of penalty rates is to discourage

¹ Original edition Bagehot (1873); our version Bagehot (1910, pp 175, 198–200).

risk taking, and unnecessary use of the system and to provide a 'carrot' for the central bank to supply LOLR assistance.

Bordo (1990) identifies four main schools of thought in the on-going debate on LOLR. <u>The classical position</u>, in line with Bagehot, supports the principles presented above. The 'financial stability' school (often identified with Charles Goodhart, eg 1995) maintains that it may be optimal to provide LOLR loans to both solvent and insolvent banks. <u>The free banking school</u> does not see a need for a public LOLR but does not rule out private LOLR arrangements (Selgin 1996). <u>The 'monetary policy school'</u> (eg Goodfried and King 1988) does not admit of a need for LOLR assistance to individual institutions. Goodfried and King specifically argue against central banks as LOLRs because they are not likely to have an information advantage over other market participants concerning a bank's financial condition.

Theoretical models rationalize LOLR activity via incomplete information considerations. An LOLR is one potential tool for eliminating bank runs due to coordination failure by depositors. The fear that collapse of the financial system, eg via bank-to-bank contagion of problems, may lead to significant welfare loss has been used to rationalize LOLR activity by central banks. Imperfections in the functioning of markets may give rise to a need for an LOLR.²

The very important recent contribution of Holmström and Tirole has shown how to analyze the question from the market imperfection perspective. H&T argue that adverse shocks, which reduce the value of firms, may jeopardize a firm's ability to obtain external financing on demand even though the firm may still have positive continuation value. The markets for liquid funds do not work perfectly because moral hazard reduces a firm's pledgeable value. Outside investors should optimally commit liquid funds to the firm ex ante, which the firm can use as a reserve buffer (reserves) for liquidity shocks.

After demonstrating the inability of markets to supply liquidity in sufficient quantities,³ Holmström and Tirole demonstrate the existence of economies in pooling reserves compared with a situation in which each firm holds liquidity in its own balance sheet. Economies in reserve pooling can be used as rationale for an institution (such as a bank) to provide liquidity insurance to firms.⁴ We argue analogously that one can rationalize an LOLR as a liquidity insurance scheme for the banking sector.

In H&Ts original model, the shock is a loss of value to the firm (a solvency shock). However, a theory of LOLR should, in line with the classical debate, also analyse the potential role of the LOLR in financing transitory liquidity outflows. Is there a role for a central bank in ensuring that transitory liquidity problems do not force banks to liquidate valuable investment projects?

A hypothesis that adverse shocks to bank value are a potential source of liquidity problems in banking and that LOLR arrangements serve as buffers against such value shocks is not contrary to empirical observation: during banking crises solvency and liquidity problems generally coexist.⁵ However, common

² See Bank of England (1999) for an extensive review.

³ See Bhattacharya and Gale (1987) for an alternative approach. They base the incomplete markets hypothesis on a free riding argument.

⁴ As H&T note, the model is not specific about the type of institutional arrangement required for governance of the pool.

⁵ See eg Kindleberger (1978), Schwartz (1986), Bordo (1986), Lindgren, Garcia, Saal (1998), Herrala (1999) for descriptions of banking crisis episodes.

wisdom in banking concerning potential causes of illiquidity seems to hold that illiquidity may have broader origins than merely adverse developments in bank value. Traditionally, bankers and bank supervisors have categorized liquidity crises on the basis of whether they were caused by a bank's solvency problems or were 'pure liquidity crises'. Nowadays, this distinction is considered misleading, It is recognized that any liquidity crisis – whether stemming from suspicions concerning a bank's financial condition or other factors – could, in the end, to jeopardize banks' solvency.⁶

We extend the H&T model to allow banks to experience problems in raising liquidity in connection with transitory liquidity outflows, absent any change in the value of a bank. Our premise is that the scope for moral hazard may be increased in connection with liquidity outflows. Moral hazard, which we interpret as the possibility of laxity in banks monitoring of firms, may increase with outflows of liquidity because in that case banks would need to increase their monitoring efforts in order to safeguard their own interests.

Our analysis starts with a description of the basic setup of the model. We then proceed to analyse the properties of optimal policy by the bank, as determined by the contract between the bank and outside investors. We show that the optimal policy entails pre-hoarding of reserves. We then investigate the implementation of the policy and discuss the potential benefits of pooling reserves and the right to issue legal tender. Finally we contrast the model with the classical LOLR model and discuss some areas for future research.

2 The model

In the model, a single good can be used for both consumption and investment. Two groups of agents are the main focus of interest, banks and depositors. Both types of agents are risk neutral and have additive and separable utility functions over non-discounted consumption streams, ie they are unbiased as regards the timing of consumption. An unlimited number of firms have projects with positive net present value but with insufficient own-funds to complete the projects. The banks (but not depositors) have access to a constant returns to scale lending technology, which they can use to provide financing for the projects of the firms. There are three periods (t = 0,1,2).

Chart 1.

Ordering of events in the model



⁶ Materials on banks' and supervisors' views on this issue were kindly made available to the author by the Finnish Financial Supervision Authority.

In period zero, a bank and some depositors negotiate a contract on a banking project. The contract specifies:

- the amount to be invested in loans (L) in period zero,
- a continuation policy (λ) which determines whether the banking project will continue in period 2 or end in period 1 with zero payoff, and
- a distribution of period 2 returns from the banking project, with the bank's share being R_b and depositors' share R_d .

After this policy choice is made, the bank proceeds to grant loans to firms from its own funds and from the outside funds provided by depositors. Lending to firms can be made at any scale. For period 0 lending, the bank has at its disposal own funds A. We denote by D the amount collected by the bank from depositors. The return requirement for depositors' funds is normalized to 0. During this initial period the bank can also invest in liquid reserves (z) if the continuation policy requires that the bank should be able to withstand liquidity shocks in period 1 that exceed the amount the bank can raise by leveraging itself up in period 1.

In period 1, firms in need of extra liquidity to finish their investment projects, turn to the bank for extra liquidity. Subsequently, the bank faces a liquidity shock, which is assumed to be proportional to the amount initially invested in loans (ρ L). The liquidity shock parameter ρ is drawn from a continuous distribution f[ρ], assumed nonzero in the positive domain. Banks can potentially raise liquidity for firms by either leveraging themselves up with depositor funds or by selling liquid reserves.

We will investigate two cases as regards the feasible region of continuation policy. A binary continuation policy (λ =0 means default in period 1, and λ =1 means continue to period 2) refers to a situation in which the bank needs to cover the whole liquidity shock or else it will default. Default yields zero payoffs for both the bank and the depositors. A continuous $\lambda \in [0,1]$ refers to a situation where the banking project can 'continue partially', ie the bank can reject part of the liquidity shock (discontinue part of its loan portfolio in period 1, with zero payoff), while keeping some loans until maturity. One may interpret this as meaning that the case that obtains depends on whether or not the bank has precommitted to supply firms with liquidity. Since both types of arrangement empirically exist between banks and firms, we will investigate both cases.

In the final period (2), the bank chooses the level at which it will monitor the firms to which it has granted loans. There are two alternatives: good banking with adequate monitoring, and bad banking with no monitoring. The good strategy results in a probability p_H of successful completion of the banking project and the bad strategy a success probability $p_L < p_H$. Unmonitored firms tend to engage in riskier projects than monitored firms.

After this choice of monitoring level, returns from the loan portfolio are realized. Verifiable returns from lending are RL (R = (r + $\alpha_1 \rho$), r > 1) in the good state of the world (project is successful) and zero in the bad state of the world. The 'recovery parameter' α_1 , $0 < \alpha_1 < 1/p_H$ denotes the expected recovery of the liquidity outlay in period 2 in the good state of the world. A private benefit BL (B = (1 + $\alpha_2 \rho$)b), which is the savings from monitoring, accrues to the bank if it chooses the bad strategy. The 'incentive parameter' α_2 , $0 < \alpha_2 < 1$ captures the

idea that a bank needs to increase its loan-portfolio monitoring when firms' liquidity problems increase.⁷

Finally, costs related to implementation of continuation policy effect the viability of the banking project. Banks will be able to leverage themselves up with funds from the depositors on period one to the level, where the continuation value of the bank for the depositors is greater than the default value (zero). We call this level the 'depositor threshold', and denote it by ρ_D . The depositor threshold ρ_D satisfies:

$$p_{H}\left(r - \frac{b}{\Delta p}\right) - \left(1 - p_{H}\alpha_{1} + p_{H}\alpha_{2}\frac{b}{\Delta p}\right)p_{D} = 0$$

from which one can solve:

$$\rho_{\rm D} = \frac{p_{\rm H} \left(r - \frac{b}{\Delta p} \right)}{1 - p_{\rm H} \alpha_1 + p_{\rm H} \alpha_2 \frac{b}{\Delta p}}$$
(2.1)

A bank may implement a continuation policy, which requires it to supply liquidity to firms in excess of the depositor threshold, by hoarding reserves (z L) on period zero. We denote by $q \ge 1$ the price of a liquid asset of unit size. Then, (q - 1)z is the liquidity cost of a continuation policy, which requires a bank to be able to clear shocks up to the level $\rho_L = \rho_D + z$ per unit of lending.

To sum up, the banking project in our model has the following return matrix⁸:

	Good banking strategy	Bad banking strategy
Total outlays	$(1+\rho)L+(q-1)zL$	$(1+\rho)L+(q-1)zL$
Success probability	р _н	p_L
Returns if success	$(r+\alpha_1\rho)L$	$(r+\alpha_1\rho)L + (1+\alpha_2\rho)bL$
Returns if fail	0	$(1+\alpha_2\rho)bL$
Expected net returns	$p_{\rm H}(r{+}\alpha_{\rm l}\rho)L-(1{+}\rho)L-$	$p_L(r+\alpha_1\rho)L+(1+\alpha_2\rho)bL-$
	(q-1)zL	$(1+\rho)L - (q-1)zL$

We will assume throughout the analysis, that monitored lending to firms according to the good banking strategy carries positive net present value⁹ for the bank, but unmonitored lending according to the bad banking strategy does not:

⁷ That $\alpha_2 > 0$ can be rationalised from the premise that banks need to increase monitoring of firms with the leverage of the firm to ensure that entrepreneurs increase their (private) effort in line with the demands of the situation (see for example Holmström and Tirole 1997). Such a view could also be based on the empirical observation that monitoring of loan clients becomes more costly as the amount lent to the clients approaches the pledgeable value of the client.

⁸ The model of H&T is given by the special case $\alpha_1 = 0$ and $\alpha_2 = 0$.

⁹ This return cannot be competed away by the banks because under moral hazard a positive rate of return for the banker is a prerequisite for the feasibility of the banking project (see below).

$$\int \max \{ p_{H}(r + \alpha_{I}\rho) - \rho, 0 \} f[\rho] d\rho - 1$$

$$> 0 >$$

$$\int \max \{ p_{L}(r + \alpha_{I}\rho) + (1 + \alpha_{2}\rho)b - \rho, 0 \} d\rho - 1$$
(2.2)

(2.2) implies that:

$$\int_{0}^{p_{1}} \left\{ \left(r - \frac{b}{\Delta p} \right) + \left(\alpha_{1} - \alpha_{2} \frac{b}{\Delta p} \right) p \right\} f[\rho] d\rho > 0$$
(2.3)

For further reference, (2.3) implicitly defines the feasible range of values for r, b and Δp .

Under (2.2), the banker needs to be given incentives to choose the good banking strategy. The banker will choose the good banking strategy, if:

$$R_{b}[\rho] \ge \frac{(1+\alpha_{2}\rho)b}{\Delta p} \forall \rho$$
(2.4)

In what way does our model of the banking project relate to the theory of banking? In our model, a bank is a liquidity provider to firms and a monitor. The role of a bank as a liquidity provider to firms can be rationalised from first principles as in H&T. We use (in chapter 4.2) analogous reasoning to rationalise a LOLR as a liquidity pooling arrangement to banks. The ability of a bank to economise on monitoring firms has been illustrated by Diamond (1984). These two roles of a bank combined give a banking project a similar temporal structure that the projects of firms have in H&Ts model (initial investment at t = 0, the liquidity shock at t = 1 and, returns at t = 2). We do abstract from the widely studied issue of bank runs: a liquidity shock in our model is an increase in the demand for liquidity by the loan clients of the bank.¹⁰

3 Optimal policy

3.1 Policy at bank level when partial continuation is not possible

In this chapter our aim is to show that a bank cannot rely on depositors to leverage itself up on period 1 to sufficient degree. It is optimal for the bank to collect liquid reserves at the start of the game. Liquid reserves function as insurance against the possibility for permanent and or transitory liquidity outflows on the middle period, which may endanger the ability of the bank to continue to the final period.

We will solve for the optimal policy of the bank by maximising the expected return from the project to the banker, with respect to the return requirement for depositors, and the incentive compatibility constraint of the banker.

¹⁰ See Diamond and Dybvig (1983) for an analysis of the deposit contract.

$$\max_{I,\lambda,R_{f}} U = L \int_{0}^{\infty} p_{H}R_{b}[\rho]\lambda[\rho]f[\rho]d\rho - A$$

s.t (i) $L \int_{0}^{\infty} \{p_{H}(r + \alpha_{I}\rho - R_{b}[\rho]) - \rho\}\lambda[\rho]f[\rho]d\rho - L(q - 1)z \ge L - A$ (3.1)
(ii) $R_{b}[\rho] \ge \frac{(1 + \alpha_{2}\rho)b}{\Delta p} \forall \rho$

Function U in (3.1) is the expected value of the banking project for the banker, net of the banker's initial outlays to the banking project. Condition (i) is the return requirement of depositors. It states that expected project returns, diluted by the share of the banker and the costs of implementing the continuation policy must not be smaller than depositor outlays in the bank. (ii) is the incentive compatibility constraint of the banker. Choice variables in the problem are the size of lending $(L \ge 0)$, cut-off policy (here binary: $\lambda = 1$ for continue and or $\lambda = 0$ for discontinue), and the banker's share of total returns ($R_b[\rho]$).

To simplify (3.1) note that, as U is linearly increasing in L, condition (i) must be binding or the problem will have a solution where the bank's loan portfolio is infinite in size. We proceed with the assumption that the depositor return requirement holds with equality. A necessary condition for this is, that the left hand side of (i) (value of the bank for depositors) grows more slowly than the right hand side (deposits) as the size of the bank gets larger. It is sufficient, that the expected marginal unit value of the bank is less than one for depositors at it's maximum. To achieve this, we make the following assumption:

$$\int_{0}^{\rho_{\rm D}} \left\{ p_{\rm H} \left(r + \alpha_{\rm I} \rho - \frac{(1 + \alpha_{\rm 2} \rho) b}{\Delta p} \right) - \rho \right\} f[\rho] d\rho < 1$$
(3.2)

The upper bound of the integral is the depositor threshold (see (2.1)). (3.3) implies, that the marginal returns from banking must not be sufficient to finance both the incentive compatible share of the banker, and the return requirement of depositors, else optimal bank size grows towards infinity. This condition rules out self-financing of the banking project.

Inserting (i) (as equality) into U one notices, that banker utility is maximised with minimum incentive compatible payoffs so that we can also regard (ii) to hold with equality.

We also need to establish a link between continuation policy and the level of reserves carried by a bank. Note that any z enables continuation up to $\rho_L = z + \rho_D$. It can be verified (as a solution to (3.1)), that the value of continuation is decreasing in ρ , so that continuation will optimally be up to some threshold liquidity shock. We can, therefore, replace λ with an upper bound ρ_L in the integral terms, and set $z = (\rho_L - \rho_D)$.

After these steps, and after some manipulation, we can express problem (3.1) in the following form:

$$\max_{\rho_{\rm L},I} U = A\left(-1 + \frac{\int_{0}^{\rho_{\rm L}} p_{\rm H} \frac{(1+\alpha_2\rho)b}{\Delta p} f[\rho]d\rho}{1 - \int_{0}^{\rho_{\rm L}} \left(p_{\rm H} \left(r+\alpha_1\rho - \frac{(1+\alpha_2\rho)b}{\Delta p}\right) - \rho\right) f[\rho]d\rho + \max[(q-1)(\rho_{\rm L}-\rho_{\rm D}),0]}\right) (3.3)$$

We see that the problem, in fact, entails maximising the expected share of the banker in project returns (denominator of second term in parenthesis in (3.3)) in proportion to the share of the banker in initial outlays.

From (3.3) we can deduce that the discontinuation threshold will optimally be at or above the depositor threshold. Recall that for the pair $(\alpha_1, \alpha_2) = (1/p_H, 0)$ we get an infinite depositor threshold so that (3.3) is not well defined. For other cases, at and below ρ_D both the numerator and the denominator (in the right term in parenthesis) are positive by assumption (3.2), the numerator is increasing and the denominator is non-decreasing. U must, therefore, be increasing up to and at ρ_D . We can also derive an upper bound for the discontinuation threshold from the observation that, in optimum, the denominator cannot be growing faster than the numerator.

Formally, the boundaries for the feasible region of the optimal discontinuation threshold (ρ^*) can be expressed as:

$$\rho_{\rm D} \le \rho^* < \rho_1 - \frac{(q-1)}{f[\rho^*](1-p_{\rm H}\alpha_1)}$$
(3.4)

where $\rho_1 = p_H r/(1 + p_H \alpha_1)$ maximises problem (3.1) absent the incentive compatibility constraint.

Solving for the first order condition for maximum of the discontinuation threshold we get:

$$(p_{H}\alpha_{1} - 1 - p_{H}\alpha_{2}r)\int_{0}^{\rho^{*}} F[\rho]d\rho + (1 + \alpha_{2}\rho^{*}) + (q - 1)(1 + \alpha_{2}\rho^{*})max[(\rho^{*} - \rho_{D}), 0] = 0$$
(3.5)

(3.5) equates marginal unit costs from continuation with average unit costs from continuation. It can be verified from a second order condition that (3.5) yields a maximum whenever liquidity costs are not too high. Formally, (3.5) yields a maximum whenever:

$$(q-1) \left\{ (\rho^* - \rho_D) + \frac{f'[\rho^*]}{f[\rho^*]^2} \int_0^{\rho^*} (1 + \alpha_2 \rho) f[\rho] d\rho \right\}$$

$$< (1 - p_H \alpha_1) F[\rho^*] + \alpha_2 (p_H r F[\rho^*] - 1)$$

$$(3.6)$$

The optimal discontinuation threshold cannot be solved explicitly from the first order condition at this level of generality so that we cannot improve much on the accuracy afforded by (3.4). (3.5) and (3.6) do, however, illustrate, that the

optimum will be strictly greater than the depositor threshold whenever the liquidity premium is not too large. This is because the right hand side of (3.5) is positive at and below the depositor threshold. Only when the liquidity premium is so large that marginal liquidity costs overstep average unit costs from continuation after ρ_D , will the optimal discontinuation threshold be ρ_D .

All in all, the analysis shows that, under the assumption of binary continuation and absent extreme values of the parameters, it will be optimal for the bank to collect reserves. Reserves shield the bank against liquidity shocks which decrease the value of the bank, and/or increase the moral hazard of the banker. The bank will implement a continuation policy, which enables it to continue beyond the depositor threshold because the utility of the banker is still increasing at the margin at that point.

The optimal discontinuation threshold does never reach the first best discontinuation level where the value of the bank is zero (under the good banking strategy), because the banker requires a positive marginal return to choose the good banking strategy. The bank should be discontinued before it has lost all value.

3.2 Optimal policy at bank level with partial continuation

Partial continuation refers to a situation, where a bank may discontinue part of it's loan portfolio with zero payoffs, while keeping some until maturity. We interpret this as referring to a case where a bank has not, a priori, committed to credit lines for it's loan clients.

We present the problem in a form, where the incentive compatibility constraint is already included in the maximised function and the return requirement of outside investors. We also show the border conditions for the now continuous continuation function λ . The continuation function can obtain values in between zero (which denotes discontinuation of all banks) and one (all continue). The upper bound must be smaller than one (part of banks continue), whenever the amount of liquidity z is not sufficient to cover the shortfall of liquidity $\rho - \rho_D$.

The problem to be maximised thus becomes:

$$\begin{aligned} \max_{I,\lambda,z} U &= L \int_{0}^{\infty} \lambda[\rho, z] \left\{ p_{H} \frac{(1+\alpha_{2}\rho)b}{\Delta p} \right\} f[\rho] d\rho \\ \text{s.t} \quad (i) \quad L \int_{0}^{\infty} \lambda[\rho, z] \left\{ p_{H} \left(r + \alpha_{1}\rho - \frac{(1+\alpha_{2}\rho)b}{\Delta p} \right) - \rho \right\} f[\rho] d\rho - L(q-1)z \ge L - A \quad (3.7) \\ (ii) \quad 0 \le \lambda[\rho, z] \le \min \left\{ 1, \frac{z}{\rho - \rho_{D}} \right\} \end{aligned}$$

We again make the assumption, that (i) holds with equality to get finite investment size. To solve the other endogenous variables, we use the Lagrangian method, and denote by δ the Lagrangian coefficient. It represents the marginal value of wealth within the bank, and it's value must therefore be above unity.

As in the previous chapter with binary continuation, for the special case $\alpha_1 = 1/p_H$ and $\alpha_2 = 0$ the solution for problem (3.7) becomes trivial. For other parameter values, the solutions of the problem for the optimal values of λ and z satisfies

$$\lambda[\rho, z] = \begin{cases} \min\left\{1, \frac{z}{\rho - \rho_{D}}\right\} \text{ if } \rho_{D} < \rho \leq \overline{\rho} \equiv \frac{p_{H}\left(\delta r - (\delta - 1)\frac{b}{\Delta p}\right)}{\delta - \delta p_{H}\alpha_{1} + (\delta - 1)p_{H}\frac{b}{\Delta p}\alpha_{2}} \\ 0 \text{ if } \rho > \overline{\rho} \end{cases}$$
(3.8)
$$\int_{\rho_{D}+z}^{\overline{\rho}} \left\{\frac{\partial\lambda[\rho, z]}{\partial z}\right\} \left\{p_{H}\frac{(1 + \alpha_{2}\rho)b}{\Delta p} + \delta\left(p_{H}\left(r + \alpha_{1}\rho - \frac{(1 + \alpha_{2}\rho)b}{\Delta p}\right) - \rho\right)\right\} f[\rho]d\rho \\ -\delta(q - 1) = 0 \end{cases}$$
(3.9)

The second order derivative is always zero for this linear problem, but closer inspection helps to verify that we do, again, have a maximum whenever liquidity costs are not too large.¹¹ For q sufficiently large, the right hand side of (3.9) is always negative, in which case we have z = 0 in optimum.

In other cases, (3.9) indicates that the bank allows at least some firms to continue whenever the liquidity shock falls below the threshold value $\overline{\rho}$. The bank utilises partial continuation for some liquidity shocks, else (3.9) cannot hold (the continuation function must be increasing for (3.9) to hold).

We observe that the discontinuation threshold varies in the interval

 $\rho_{\rm D} < \overline{\rho} < \rho_1$

The lower bound is approached when the marginal value of wealth approaches infinity (moral hazard increases), and the upper bound is approached when the marginal value of wealth approaches one (its lower bound).

The bank will choose:

 $0 < z < \overline{\rho} - \rho_D$

All in all, then, the bank will choose to collect reserves even when partial continuation is possible, unless liquidity costs are too high. Reserves enable continuation fully up to some value above the depositor threshold. From this point on there is a finite interval where the bank rations liquidity outlays to firms (it resorts to partial continuation by liquidating a part of it's portfolio). The bank is discontinued (whole portfolio is liquidated) at some level below the first best.

¹¹ The target function is graphically the area under a downward sloping line. The optimal discontinuation threshold cuts the line at the interception with the x-axis. For maximum q is not so large as to make the slope of the line positive.

4 On the benefits of an LOLR

4.1 Reserves at bank level

The analysis in section 3 indicates that reserves are not needed, when the liquidity shock is fully recovered and the liquidity shock does not affect the bankers' incentives ($\alpha_1 = 1/p_H$, $\alpha_2 = 0$). In this case, the depositor threshold is infinite irrespective of whether partial continuation is possible or not. A bank will always be able to get enough funds from depositors on period 1 to continue. The optimal policy can, then, be implemented with an arrangement, where depositors give the bank the amount L on period zero against a claim on the bank's returns on period 2. No reserves are needed for implementation of the policy.

In all other cases the depositor threshold is below the optimal discontinuation threshold ($\rho^* - \rho_D > 0$, $\overline{\rho} - \rho_D > 0$), given that liquidity costs are not too high. Banks need to invest in reserve assets on period zero to implement the optimal discontinuation threshold on period 1. In the following we investigate the issue, whether banks can use inter bank assets as reserve assets alone, or whether (costly) outside assets are needed as a reserve asset. We present a proof¹² that inter bank assets cannot guarantee implementation for all parameter values of the problem.

Inter bank assets are claims, issued on period t = 0 on the value of banks. Banks can acquire such assets on period zero and sell them to obtain finance from the outside investors on period 1. As inter bank assets are also sold to depositors which have a zero return requirement, holding these assets cannot be costly. Inter bank assets, therefore, are a potentially premium free reserve asset for the banking sector.

Clearly, a portfolio of inter bank assets cannot be valuable as liquidity if banks face synchronous shocks. This is because a bank needs to rely on reserve assets as a source of liquidity only in circumstances when it does not have positive outside value. If other banks are in an identical situation, the portfolio of inter bank assets is valueless. Inter bank assets can be valuable reserve assets if banks encounter idiosyncratic liquidity shocks. Then banks, which need liquidity, can sell claims of other banks, which are still valuable to outside investors, to obtain liquidity.

Nevertheless, we will show that, even if liquidity shocks are independently distributed across banks, the optimal policy still cannot, in general, be implemented in a decentralised setting utilising inter bank claims alone.

Consider the case with a binary continuation first.

Assume, that reserves of inter bank assets can provide banks with the means to reach ρ^* , when banks face idiosyncratic shocks. Consider a situation, where bank a owns a share $\beta < 1$ of other banks, which we denote by b. Then, the resale value S^a of bank a's portfolio on claims on b on period 1 is:

$$S^{a} = \beta L \int_{0}^{\rho_{*}} \left\{ p_{H} \left(r + \alpha_{I} \rho^{b} - \frac{\left(1 + \alpha_{2} \rho^{b} \right) b}{\Delta p} \right) - \rho^{b} \right\} f \left[\rho^{b} \right] d\rho^{b}$$

$$(4.1)$$

¹² This extends the proof given by Holmström and Tirole (1998).

Then, for bank a to reach ρ^* , the following inequality must hold

$$\rho * L < \rho_D L + S^a$$
,

which implies

$$\rho^{*} + \int_{0}^{\rho^{*}} \rho^{b} f\left[\rho^{b}\right] d\rho^{b} < \frac{p_{H}\left(r - \frac{b}{\Delta p}\right)}{1 + p_{H}\left(\alpha_{2}\frac{b}{\Delta p} - \alpha_{1}\right)} + p_{H}\int_{0}^{\rho^{*}} \left\{-r\frac{b}{\Delta p} + \left(-\alpha_{1} + \alpha_{2}\frac{b}{\Delta p}\right)\rho^{b}\right\} f\left[\rho^{b}\right] d\rho^{b}$$

$$(4.2)$$

Note, that the left-hand side of the inequality is always positive, and also increasing in b whenever $\alpha_2 > 0$. The right hand side, on the other hand, decreases as b increases. Within the boundaries determined by (2.3) it can be arbitrarily close to zero for large enough b. For large b, then, inter bank assets will not provide sufficient means to achieve the optimal discontinuation threshold for the banking sector even in the case when shocks are idiosyncratic. It is easily verified that an analogous proof can be utilised for the cases where a bank can utilise partial continuation.

Therefore, under autarky, the policy must be implemented with outside assets. Consider a situation, where implementation of the optimal policy (in line with (3.5) or (3.9)) calls for a discontinuation threshold $\rho_D + z^*$. Then, optimal policy can, in principle, be implemented so that investors invest $(1 + qz^*)L$ -A in the bank on period 0 with the covenant that the bank invests qz^* in liquid outside assets. It can sell these on period 1 to obtain z^* liquidity and implement the cutoff.

Prices of liquid outside assets should be expected to carry a liquidity premium (q > 1) at the markets if they provide a liquidity service.¹³ An additional drawback with hoarding of outside assets is that, as Holmström and Tirole (1998) argue, if liquid assets are costly to hold, implementing the above policy in a decentralised setting is complicated by the fact that liquidity is a public good. The equilibrium at the market of liquid assets breaks down in the case, when obtaining or holding these assets carries costs, as banks have an incentive not to invest directly in the costly liquid asset, but to invest in the assets of other banks which hold the liquid asset. This free rider problem could be overcome by the use of multiple securities to discriminate in between outside investors and banks.¹⁴

4.2 Pooling banks' reserve assets

In this chapter we discuss potential gains from pooling banks' reserve assets so that liquidity policy can be implemented on the banking sector level. These economies provide a rationale for a lender of last resort. We will demonstrate

¹³ See H&T 1998b for an analysis of the liquidity premium.

¹⁴ H&T's idea is to price-discriminate investors, who value liquidity services differently.

economies of reserve pooling with an example, where a group of (ex ante) identical banks i pool liquid reserves.

The optimal policy of the pool can be solved from:

$$\begin{split} & \underset{\lambda}{\text{Max}} U = L \int_{0}^{\infty} p_{H} \frac{(1 + \alpha_{2} \rho)b}{\Delta p} \lambda[\rho, z] f[\rho] d\rho \\ & \text{s.t.} \\ & (i) \quad L \int_{0}^{\infty} \lambda[\rho, z] \bigg\{ p_{H} \bigg(r + \alpha_{1} \rho - \frac{(1 + \alpha_{2} \rho)b}{\Delta p} \bigg) - \rho \bigg\} f[\rho] d\rho - L(q - 1) z \ge L - A \\ & (ii) \quad 0 \le \lambda[\rho, z] \le 1 \\ & (iii) \quad \int_{\rho_{D}}^{\infty} (\rho - \rho_{D}) \lambda[\rho, z] f_{t=1}[\rho] d\rho \le z \end{split}$$

The problem changes, in comparison to autarky (3.7), in the respect that we have (iii) as the aggregate constraint for realised shocks across the banking sector on period 1. We denote by $f_{t=1}$ the realised distribution of shocks across banks.

From (4.3) it follows that optimal continuation policy satisfies:

$$\lambda[\rho, z] = \begin{cases} 1 & \text{if } \rho \le \rho_{L} = \min\{\overline{\rho}, \hat{\rho}\} \\ 0 & \text{if } \rho > \rho_{L} = \min\{\overline{\rho}, \hat{\rho}\} \end{cases}$$

where
$$\overline{\rho} = \frac{p_{H} \left(\delta r - (\delta - 1) \frac{b}{\Delta p}\right)}{\delta - \delta p_{H} \alpha_{1} + (\delta - 1) p_{H} \frac{b}{\Delta p} \alpha_{2}}$$

$$\hat{\rho} : L \int_{\rho_{D}}^{\hat{\rho}} (\rho - \rho_{D}) f_{t=1}[\rho] d\rho = z$$
(4.4)

Reserve policy in aggregate implementation satisfies

$$L\left\{p_{H}\frac{(1+\alpha_{2}\rho_{L})b}{\Delta p} + \delta\left(p_{H}\left(r+\alpha_{1}\rho_{L}-\frac{(1+\alpha_{2}\rho_{L})b}{\Delta p}\right) - \rho_{L}\right)\right\}f\left[\rho_{L}\right]\frac{\partial\rho_{L}}{\partial z}$$
(4.5)
$$-\delta(q-1) = 0$$

This, again, establishes a maximum, if q is not too high. (4.4) shows that the pool will implement a ceiling $\overline{\rho}$ above which no bank continues. When the aggregate liquidity constraint binds, the pool will adjust this discontinuation threshold downwards to $\hat{\rho}$ to equate aggregate demand of liquidity with aggregate supply. The pool achieves ex post efficient use of liquidity compared to autarky because, when liquidity shocks are asynchronous across banks, the pool can use excess liquidity of 'lucky' banks to support continuation of unlucky banks.

(4.5) shows that the pool will choose a level of reserves such that it expects the aggregate budget constraint to bind (else the first term in (4.5) is zero). This economises on liquidity costs. Note that reserve hoarding is based on the expected

aggregate demand for liquidity, and not on expected demand on bank level. This implies that the pool will achieve ex ante economies in hoarding reserves whenever the expected realisation of liquidity shocks is not singular across banks.

All in all, the previous analysis shows that a LOLR would utilise the possibility to implement 'soft budget constraints' for the banking sector on period one. This results in ex ante economies in reserve hoarding, and ex post economies in use of liquidity.

4.3 Legal tender

One might argue that a key characteristic of central banks, which makes them ideal as LOLRs is their ability to issue legal tender. When the LOLR can issue money which, by law, must be accepted as payment for debt, then the need to prehoard costly reserves vanishes. Then, banks can use all available funds on period zero for productive lending. The LOLR can inject legal tender to banks on demand in period 1, which the banks can use to settle their liquidity shocks.

In H&Ts terminology, the right to issue legal tender makes it possible for the LOLR to replicate a 'state contingent bond', the value of which follows the demand for liquidity by banks. Optimal continuation policy can in this case be solved from the following program

$$\begin{split} & \underset{\lambda}{\operatorname{Max}} U = L_{0}^{\infty} p_{H} \frac{(1 + \alpha_{2} \rho)b}{\Delta p} \lambda[\rho] f[\rho] d\rho \\ & \text{s.t.} \\ & (i) \quad L_{0}^{\infty} \lambda[\rho] \left\{ p_{H} \left(r + \alpha_{1} \rho - \frac{(1 + \alpha_{2} \rho)b}{\Delta p} \right) - \rho \right\} f[\rho] d\rho \geq L - A \\ & (ii) \quad 0 \leq \lambda[\rho] \leq 1 \end{split}$$

$$(4.6)$$

Problem (4.6) is similar to (4.3) except that the liquidity cost term is absent in (i). As (i) is binding by assumption, program (4.6) yields a higher level of utility than program (4.3). The solution to (4.6) satisfies

$$\lambda[\rho] = \begin{cases} 1 & \text{if } \rho \le \rho \# \\ 0 & \text{if } \rho > \rho \# \end{cases}$$
where
$$\rho \# = \frac{p_{H} \left(\delta r - (\delta - 1) \frac{b}{\Delta p} \right)}{\delta - p_{H} \delta \alpha_{1} + (\delta - 1) p_{h} \frac{b}{\Delta p} \alpha_{2}}.$$
(4.7)

To verify that we do, indeed, have a maximum we can use similar reasoning as in the section 3.2.

(4.7) illustrates the fact that the socially optimal cut-off will, again, be above the depositor threshold and below the first best. We observe that, in fact, the LOLR will implement the same discontinuation threshold $\overline{\rho}$ as in the previous case when reserve assets were needed (problem (4.3)). Recall that it was then optimal to adjust the discontinuation threshold downwards to $\hat{\rho}$ whenever liquidity was in short supply. This is never optimal here as the issuer of legal lender has the ability to create more liquidity.

We, therefore, see how the right to issue legal tender enables the LOLR to reduce the costs of hoarding reserves. Banks will be able to invest all available funds in lending and aggregate utility will be increased compared to the case when costly reserves are used. One might ask, whether this state contingent bond could be implemented at bank level. Banks traditionally had the right to issue notes and one might think that such a right would allow banks to solve their liquidity problems once and for all. We must, however, emphasize that the right to issue legal tender means here that depositors <u>must</u> accept the 'money' issued by the LOLR as payment for debt. Indeed, the right to enforcement is crucial here, since we observe that depositors would like to close down the bank before the optimal discontinuation threshold is reached (the optimal cut-off is above the depositor threshold). Therefore, to give banks the right to issue notes would not suffice to implement (4.7).

5 Discussion

To restate our main results about the sufficiency of markets as a source of liquidity, and the role of the LOLR:

- 1. Banks may find it hard to finance even transitory liquidity shocks from the markets if moral hazard problems are amplified by liquidity shocks.
- 2. There are economies in reserve pooling that arise from the ability of a pool to implement soft budget constraints for banks experiencing liquidity shocks. An ability to issue legal tender reduces liquidity costs further. The rationale for a LOLR depends on these economies of centralization.

The view offered by the model on the economies of LOLR is obviously at odds with the monetary-policy-school view, which holds that an LOLR cannot improve on the market outcome. Goodfriend and King argue against LOLR on the premise that central banks (as LOLRs) are not likely to have better information than market participants on potential recipients of LOLR assistance. While our model does not dispute this premise, it does indicate that an information advantage is not a prerequisite for the economies associated with an LOLR arrangement, which can be rationalized with its ability to distribute liquidity efficiently across banks. Due to this ability, an LOLR can economize on reserve holdings (compared to 'autarky') when market transactions are constrained by moral hazard.

To get a better grip on the reasons behind this difference in opinion, consider the situation in our model from the viewpoint of an outsider who observes only period 1, when banks experience liquidity shocks. The outsider would observe that some banks are in trouble because they need liquidity but cannot pay a competitive return to the depositors. There is no apparent reason why an outsider, eg a central bank, should support the bank so as to enable it to continue into period 2. The only one who would benefit from continuation is the banker, whose payoff depends on the ability of the bank to continue.

The key to understanding the rationale for the LOLR scheme is to note that the banker is willing to pay for the benefit of continuation.¹⁵ There is scope for a deal between the banker and outsiders (depositors). Such a deal cannot be consummated when the trouble is at hand (in period 1) because the bank has nothing quid pro quo to offer. However, back in period 0, the banker would have been willing and able to trade some returns from the banking project for protection against liquidity shocks in period 1. Reserve hoarding in period 0 serves as a means by which outsiders can commit to such a policy. Reserve holding is costly and hence economies in reserve holding constitute a rationale for the LOLR. Thus, while there is no apparent reason why outsiders should help the bank when problems mount, it is optimal ex ante to establish an LOLR that is committed to providing support.

The model offers a view on the debate between the Bagehotian classical school and the financial stability school on whether LOLR assistance should be given only to solvent institutions or to certain others as well. Recall that in our model both banks and depositors have nonnegative expected returns from banking projects in period zero negotiations. In this sense, our model prescribes that banks participating in an LOLR scheme should be solvent ex ante.

In deciding whether a participating bank should receive LOLR assistance, the issue is not really whether the bank has any value per se but whether its continuation value is greater than its liquidation value. Another important insight is that a bank's continuation value differs for different interest groups (banker vs depositors) when there is moral hazard. Indeed, our extension of the H&T framework indicates that a bank's continuation value of the bank is unaffected. Optimally, a bank should have the ability to continue beyond the point at which continuation becomes unattractive for depositors but it should discontinue operations before its continuation value is totally exhausted.

As regards the collateral policy of the LOLR, the classical school advocates lending against all acceptable collateral, evaluated at normal-time value. The exact meaning of normal-time value is an interesting issue. A postulate of our model is that this is the bank's pledgeable value prior to the onset of crisis rather than during a crisis. The parties should commit to some continuation policy prior to the crisis.

This said, we would re-emphasize that the model does not support the postulate that collateral policy could help an LOLR to recover some of the injected funds from the receivers. LOLR assistance is needed only after the pledgeable (outside) value of a bank has been exhausted. A similar issue arises when one tries to rationalize the application of the Bagehotian principle regarding penalty rates in the context of our model. One is led to ask why a bank would need an emergency loan at all if it has collateral to offer and it can pay above-market rates for it.¹⁶ To rationalize such a facility, one may need to resort to the

¹⁵ Recall that in the model the incentive-compatible payoff of the bank is strictly positive. The bank has 'going concern' value but this cannot be pledged due to moral hazard. An alternative modelling strategy to explain non-pledgenbility would be via adverse selection.

¹⁶ Fischer (1999) proposes that the penalty interest rate should include a margin over the rate at 'normal times' but then the bank could just as well buy some liquidity from the market at market rates and receive some via an insurance payment from the LOLR.

assumption that the an LOLR has an information advantage vs the markets in respect to the recipient of LOLR assistance.

Finally, our model admits an interesting interpretation of the classical means of assuring the markets in advance concerning the LOLR scheme. The issue here is whether a central bank should make banks aware of the possibility that they may get LOLR assistance. According to the model, the participants should be aware of the LOLR's policy (they should know that the pool acts as an LOLR). Otherwise banks would keep their own reserves and economies of reserve pooling would be lost. However, there may be ambiguity as to whether any particular bank is allowed to continue given the shock that it has encountered. This is because it may be optimal for the LOLR to implement a continuation policy where the continuation of a bank depends on the aggregate demand for liquidity. This facilitates efficient use of liquidity at system level.

We wish to emphasize some of the issues that the model leaves open. The model abstracts from the incentive effects of LOLR schemes and is, likewise, silent on the issue of who should function as the LOLR, ie a private clearing house or a public institution such as the central bank. The model does, however, suggest that a right to issue legal tender is useful for a LOLR. In a related vein, the model is not rich enough to allow a proper treatment of monetary policy issues.

References

- Bagehot, W. (1910) Lombard Street A Description of the Money Market. Second edition, Smith, Elder & Co. London, England (First edition 1873).
- Bank of England (1999) Lender of Last Resort: a Review of the Literature. Financial Stability Review: November 1999.
- Bhattacharya, S. Gale, D. (1987) **Preference Shocks, Liquidity and Central Bank Policy.** In new approaches to monetary economics. Eds. Barnett, Singleton. Cambridge University Press.
- Bordo, M.D. (1990) The Lender of Last Resort: Alternative Views and Historical Evidence. Economic Review, January/February 1990.
- Diamond, D.W. Dybvig, P.H. (1983) Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy, Vol. 91, No. 3, 401–419.
- Diamond, D.W. (1984) Financial Intermediation and Delegated Monitoring. Review of Economic studies 51 (3): 393–414.
- Fischer, Stanley (1999) **IMF as a Lender of Last Resort**. Central Banking, Vol IX, Number 3.
- Goodfriend, M. King, R.G. (1988) Financial Derequlation, Monetary Policy and Central Banking. Economic Review, Federal Reserve Bank of Richmond (May/June 1988).
- Goodhart, C. (1995) The Central Bank and the Financial System. MacMillan Press, England.
- Herrala, R. (1999) Banking Crises vs. Depositor Crises: The Era of The Finnish Markka. Scandinavian Economic History Review 2/99.
- Holmström, B. Tirole, J. (1997) Financial Intermediation, Loanable Funds and the Real Sector Quarterly Journal of Economics 112, 663–982.
- Holmström, B. Tirole, J. (1998) Private and Public Supply of Liquidity. Journal of Political Economy, Vol. 106, No. 1, 1–40.
- Holmström, B. Tirole, J. (1998b) LAPM: a Liquidity-Based Asset Pricing Model. NBER Working Paper 6673, August 1998.
- Kindleberger, Charles P. (1978) Manias, Panics, and Crashes. A History of Financial Crises. Basic Books.
- Lindgren, C.-J. Garcia, G. Saal, M.I. (1996) Bank Soundness and Macroeconomic Policy. International Monetary Fund.
- Schwartz, Anna J. (1988) Financial Stability and the Federal Safety Net. In Restructuring Banking & Financial Services in America, Eds. Haraf, William F. and Kushmeider, Rose Marie, American Institute for Public Policy Research, 34–62.

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