

Juuso Vanhala

# Labour taxation and shock propagation in a New Keynesian model with search frictions



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Bank of Finland Research  
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12 • 2006

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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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This research was conducted while the author was visiting the Bank of Finland Monetary Policy and Research Department. The author wishes to thank Juha Kilponen, Erkki Koskela, Pekka Sinko, Jouko Vilmunen and participants of the Bank of Finland Monetary Policy and Research Department Workshop and the XXVIII Annual Meeting of Finnish Economists for comments and discussions. The author thanks Giuseppe Bertola, Peter Fredriksson, Seppo Honkapohja, Lars Ljungqvist and participants of the RUESG Labour Workshop for comments on an earlier version of this paper.

<http://www.bof.fi>

ISBN 952-462-282-3  
ISSN 0785-3572  
(print)

ISBN 952-462-283-1  
ISSN 1456-6184  
(online)

Helsinki 2006

# Labour taxation and shock propagation in a New Keynesian model with search frictions

Bank of Finland Research  
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## Abstract

This paper studies the implications of labour taxation in determining the sensitivity of an economy to macroeconomic shocks. We construct a New Keynesian business cycle model with matching frictions of the labour market, where sluggish employment adjustment implies a key role for labour markets in determining shock propagation. We consider three policy instruments to analyze the steady state and dynamic effects of tax reforms: the marginal tax rate and replacement ratio amplify shock responses whereas employment subsidies weaken them. The tax instruments affect the degree to which the wage absorbs shocks. We show that the relative effects of the tax instruments and thus the effects of tax progression are sensitive to the initial degree of tax progression in the economy. Increasing tax progression when taxation is initially progressive is harmful for steady state employment and output, and amplifies the sensitivity of macroeconomic variables to shocks. When taxation is initially proportional, increasing progression is beneficial for output and employment and dampens shock responses of macroeconomic variables.

Key words: matching, income taxation, business cycles

JEL classification numbers: J64, E24, E32

# Työn verotuksen ja työmarkkinakitkan vaikutukset makrotaloudellisten sokkien välittymiseen uuskeyneiläisessä mallissa

Suomen Pankin tutkimus  
Keskustelualoitteita 12/2006

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Rahapolitiikka- ja tutkimusosasto

## Tiivistelmä

Tässä tutkimuksessa tarkastellaan työn verotuksen ja työmarkkinoiden etsintäkitkan merkitystä makrotaloudellisten sokkien välittymisessä. Mallissa liitetään etsintäkitkaa sisältävät työmarkkinat uuskeynesiläiseen suhdannemalliin, mikä johtaa työllisyyden verkkaiseen sopeutumiseen mallissa ja antaa työmarkkinoille keskeisen merkityksen suhdannesokkien välittymisessä. Tutkimuksessa tarkastellaan kolmen veroinstrumentin vaikutuksia makrotalouden tasapainoon ja dynamiikkaan: tulovero ja työttömyyskorvaus voimistavat sokkien impulssivasteita ja työntekijöille kohdistettu työllisyystuki heikentää niitä. Veroinstrumentit vaikuttavat siihen, kuinka suuri osa sokeista välittyy palkkoihin. Tutkimuksessa osoitetaan veroinstrumenttien muutosten suhteellisten vaikutusten ja veroprogression vaikutusten riippuvan talouden lähtökohtaisesta veroprogression asteesta. Kun verotus on alun perin suhteellista, progression kasvattamisella on myönteiset tasapainovaikutukset työllisyyteen ja kokonaistuotantoon. Makrotaloudellisten muuttujien herkkyys sokeille vaimenee. Kun verotus on alun perin progressiivista, progression kiristäminen on haitallista tasapainotyöllisyyden ja -tuotannon kannalta ja se voimistaa makrotaloudellisten muuttujien herkkyyttä sokeille.

Avainsanat: dynaamiset yleisen tasapainon mallit, etsintäkitka, työn verotus, suhdannesykli

JEL-luokittelu: J64, E24, E32

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# 1 Introduction

The design of tax and labor market policy may be motivated by a number of objectives, such as collecting tax revenue, promoting income equality, guaranteeing minimum income to relatively disadvantaged workers or reducing unemployment. The trade-offs faced by policy makers to achieve the desired goals may differ according to the institutional setup of the economy as well as the equilibrium levels of macroeconomic variables such as output, unemployment or job turnover. As economies are not isolated in their steady state, events such as technology or monetary policy shocks shake the economy from time to time out of the steady state equilibrium. The way the economy responds to these shocks depends on the steady state of the economy, which is shaped by tax policies. Accordingly, another concern of policy makers when designing the structure of taxation are the implied economic dynamics and sensitivity of the economy to exogenous shocks.

The purpose of this paper is to study the effects of labor taxation on shock propagation in a monetary business cycle model. We construct a New Keynesian business cycle model which incorporates matching frictions of the labor market à la Mortensen-Pissarides (eg Pissarides 2000, Mortensen and Pissarides 1999a) to introduce real rigidity into the monetary model. The model incorporates three labor market policy instruments: a marginal income tax, a tax subsidy for employed workers and a replacement ratio for unemployed workers. The marginal income tax and employment subsidy jointly determine the degree of progression in income taxation and the replacement ratio determines the income when unemployed (in addition to the value of home production or leisure). With these tax policy tools, we study how income taxation affects the steady state equilibrium of the economy and how taxation transmits to the sensitivity of the economy to macroeconomic shocks.

A recent body of literature has explored the role of real rigidities of the labor market in business cycle models by combining the search-matching framework of the labor market to real business cycle models (Merz 1995, Andolfatto 1996, den Haan et al 2000) and the New Keynesian monetary model (Walsh 2003, 2005, Trigari 2004, Krause and Lubik 2005). These studies have been successful in improving the performance of business cycle models in generating shock persistence in macroeconomic variables observed in the data. A key feature of these models is that they introduce employment adjustment in business cycle models through changes in the number of employed workers (the extensive margin) instead of in the number of hours (the intensive margin). This, combined with search frictions of the labor market, generates involuntary unemployment and sluggish employment adjustment into the business cycle models. The rigidity in the adjustment of labor has proved to be of essence in generating persistence into the business cycle models.

In a search labor market, the equilibrium labor market variables depend on the incentives for firms to create vacancies, on workers valuation of employment relative to unemployment and the decisions of firms and workers to separate when outside opportunities are more attractive. These depend on how well the matching market works, but also on the institutional features of the

economy that determine the relative values of different labor market states. Indeed, the search-matching literature (eg Pissarides 1998, 2000, Mortensen and Pissarides 2003) has demonstrated how labor market policy, eg taxes, influences equilibrium labor market variables: unemployment, wages, labor market tightness, job creation rate and job destruction. A natural extension of this work is to ask whether the effects of the tax structure on the labor market equilibrium is of relevance in determining the sensitivity of the economy to exogenous shocks.

We show that individual tax policy instruments have well-defined comparative static and dynamic effects. In steady state, the marginal tax and replacement ratio dampen economic activity whereas the tax subsidy stimulates it. Higher marginal tax rates and replacement ratios amplify shock responses both in terms of peak effects and persistence whereas higher tax subsidies dampen the impulse responses. These clear cut results abstract from any tax revenue questions, so we proceed to study the effects of tax revenue neutral changes in tax progression with alternative assumptions on the initial tax scheme of the economy.

Although the effects of tax progression are by no means a novel area of research, the literature is all but conclusive on the subject. Koskela and Vilmunen (1996) refer to the ‘*widely held popular belief that the more progressive the tax system is, the greater the disincentive to work effort*’. Their analysis shows that under plausible assumptions increased tax progression lowers wages and unemployment in three trade union models of the labor market; the monopoly union, the ‘right-to-manage’ and the efficient bargain model. They conclude that the effects of taxation appear to be very sensitive to the structure of labor markets. Indeed, Pissarides (1998) studies the effects of employment tax cuts on unemployment and wages in four different equilibrium models of the labor market: competitive, union bargaining, search and efficiency wages. He points out that there is no definitive model of the European labor market and shows that effects of changes in the structure and level of taxation sometimes depends on the underlying model of the labor market. He finds that when wages are determined by bargaining, a revenue neutral increase in tax progression reduces unemployment in steady state. In a more general setting with endogenous job destruction Sinko (2005) obtains qualitatively similar results. Mortensen and Pissarides (2003) consider various tax and subsidy effects on wages and unemployment. They study policies that drive the labor market closer to ‘efficiency’ in terms of search frictions but they do not explicitly address tax progression schemes. Their calibrations show that the tightness to which the labor market is calibrated matters for the steady state outcomes. The interaction between shocks and institutions in a matching model is studied in Mortensen and Pissarides (1999b), but their focus is in unemployment compensation and employment protection policies.

We take new steps by analyzing the importance of the initial tax scheme on tax progression effects in a general equilibrium framework and taking a look at the dynamics of the model. We show that the effects of tax revenue neutral changes in tax progression depend crucially on the initial degree of tax progression in the labor market. When taxation is set to be progressive in the initial state (our benchmark case), we show that the effect of the marginal tax

on labor market variables dominates the tax subsidy effect. In steady state this implies that a ‘lower activity economy’ ie lower output and employment. The dynamic responses to exogenous shocks are amplified by tax progression. When we set taxation to be proportional in the initial state, we obtain qualitatively similar results to Pissarides (1998) and Sinko (2005). When taxation is initially proportional, increasing progression is beneficial for output and employment and dampens shock responses of macroeconomic variables. This is so because the relative strengths of the two tax effects are reversed when the tax subsidy is sufficiently small. Thus we find that a government tax revenue neutral change in tax progression has opposite effects on the steady state and shock responses depending on the degree of tax progression in the initial steady state.

The structure of this study is as follows. In section 2, we construct a New Keynesian model which incorporates matching frictions of the labor market and the tax policy instruments. Section 3 characterizes and solves the steady state of the model and presents the linearized system of equations. The model calibration is discussed in section 4. In section 5 we first analyze the effects of labor market policy on the steady state of the model at some length, as this reveals intuition and the mechanisms that drive the dynamics of the model. Thereafter consider the dynamic responses to shocks for various tax policy regimes. Section 6 summarizes and discusses paths for further research.

## 2 Model

The model economy follows the structure of Trigari (2004) and Walsh (2003, 2005) by incorporating a Mortensen-Pissarides type of labor market with matching frictions into a New Keynesian monetary model. The two main driving forces of the model’s dynamics are nominal rigidities in price setting and matching frictions. A characteristic feature of the model is the separation of firms into two types, each type taking account of one type of rigidity. This separation is made to separate the nominal rigidities from the real rigidities, thus making the model more tractable. The economy consists of the following:

*Households* – Households supply labor, purchase goods for consumption and hold bonds. Labor is supplied at the extensive margin, so adjustment in the labor market takes place through additional employed workers rather than varying the hours of work. We consider the households as extended families who pool consumption. This assumption is conventional and is made to avoid distributional issues. Households own the firms in the economy.

*Firms* – There are two types of profit maximizing firms: wholesale and retail firms. Production takes place in the wholesale firms who use labor as the sole factor of production. Matching workers and wholesale firms is a time consuming and costly process which generates real rigidity into the economy. Wholesale firms sell all their output to the retail firms at a competitive price. Retail firms transform the intermediate goods purchased from the wholesale firms into differentiated final goods and sell them in a monopolistically competitive market with staggered pricing which generates the nominal rigidity of the model.

*Central bank* – The central bank does not behave optimally and it controls the nominal interest rate according to a policy rule.

*Government* – The government raises tax revenue by levying an income tax from employed workers. The tax revenue is used to finance unemployment benefits, tax subsidies paid to workers and other government expenditures.

## 2.1 Households

There is a continuum of households on the unit interval in a discrete-time economy. The representative household maximizes the expected present discounted utility

$$E_t \sum_{i=0}^{\infty} \beta^i u(\mathcal{C}_{t+i}, \mathcal{C}_{t+i-1}) \quad (2.1)$$

where  $\mathcal{C}_t = C_t + \psi h$ , and  $C_t$  is the consumption of a market purchased composite good. The composite good consists of the differentiated goods produced by the retail firms.<sup>1</sup>  $h$  is nontradable home production and  $\psi$  is an indicator function taking the value of zero when an individual is employed and one otherwise. The utility function allows for habit persistence. As monetary policy is represented by an interest rate rule and our focus is not on the stock of money, we consider a limit economy where the weight of the utility of the household's holdings of real money balances approaches zero in the utility function.<sup>2</sup>

The households budget constraint is

$$P_t D_t + (1 + i_{t-1}) B_{t-1} = P_t C_t + B_t \quad (2.2)$$

where  $D_t$  is the family income which consists of wage income, unemployment income and family share of firms profits.  $B_t$  is the household's nominal holdings of bonds and  $P_t$  is the retail price index. Using (2.1) and (2.2) we can derive the first-order condition

$$\lambda_t = \beta (1 + i_t) E \left( \frac{P_t}{P_{t+1}} \right) \lambda_{t+1} \quad (2.3)$$

which is the household's Euler condition, where

$$\lambda_t \equiv u_1(\mathcal{C}_t, \mathcal{C}_{t-1}) + \beta E_t u_2(\mathcal{C}_{t+1}, \mathcal{C}_t).$$

---

<sup>1</sup>The composite good will be defined below.

<sup>2</sup>The household's objective function with utility of holdings of real money balances  $\phi(m_{t+i})$  would be

$$E_t \sum_{i=0}^{\infty} \beta^i [u(\mathcal{C}_{t+i}, \mathcal{C}_{t+i-1}) + \phi(m_{t+i})].$$

## 2.2 Wholesale firms and labor market search

In the wholesale (intermediate product) market production takes place in firms that use labor as the sole input. Adjustments in the labor input are sluggish as matching firms and workers is time consuming. Due to the search frictions in the labor market a firm-worker match generates surplus ie in addition to productivity the match itself has a positive value because separation of the firm and worker leads to new search. Consequently the wage in the intermediate sector does not equal the marginal productivity of a worker. In addition to match productivity, the wage depends on the value of being idle for the firm and worker and the ease with which each side can find an alternative match. Unemployed workers receive an unemployment benefit and enjoy a value of nontradable home production (or leisure). The match surplus and labor market tightness influence the wage rate and govern job creation and destruction.

### 2.2.1 Match productivity and job flows

To keep the model simple we assume that labor is the only input in the production of intermediate goods. Match productivity is given by

$$y_{it} = a_{it}z_t$$

where  $a_{it}$  is match specific productivity and  $z_t$  is a common aggregate productivity measure. Each period  $a_{it}$  is drawn from a time-invariant distribution with c.d.f.  $F(a)$  and density  $f(a)$ . Denote the price at which wholesale firms sell output to competitive retail firms by  $P_t^w$ , the retail price index is  $P_t$  and  $\mu_t = \frac{P_t}{P_t^w}$  is the markup of retail over wholesale prices. The real value of output in terms of time  $t$  consumption is  $\mu_t^{-1}a_{it}z_t$ .

Production takes place once a firm and worker are matched. Matching of firms and workers in the intermediate sector is characterized by a constant returns to scale matching function

$$m(u_t, v_t) = Au_t^\alpha v_t^{1-\alpha}$$

where  $u_t$  and  $v_t$  are unemployed workers and open vacancies at time  $t$  respectively,  $0 < \alpha < 1$  and  $A > 0$  is a shift parameter.<sup>3</sup> The hazard rates for a firm of meeting a worker and a worker of meeting a firm are respectively

$$q_t^f = \frac{m(u_t, v_t)}{v_t} = A\theta_t^{-\alpha} \quad (2.4)$$

$$q_t^w = \frac{m(u_t, v_t)}{u_t} = A\theta_t^{1-\alpha} \quad (2.5)$$

where  $\theta_t = \frac{v_t}{u_t}$  is labor market tightness. The tighter the labor market, the easier it is for the worker to find a partner and harder for a firm to find a partner. Thus  $q_t^f$  is decreasing and  $q_t^w$  is increasing in  $\theta_t$ .

---

<sup>3</sup>The Cobb-Douglas matching function is supported by a number of empirical studies. For a survey on the matching function see Petrongolo and Pissarides (2001).

Jobs are destroyed due to exogenous shocks and endogenous separation decisions of firms and workers. Exogenous shocks arrive at rate  $\rho^x$  at the beginning of each period. For the matches that survive, the firm and worker jointly observe the realization of match productivity and decide whether to continue or destroy the match. Jobs with a productivity realization that is below a reservation productivity  $\tilde{a}_t$  are destroyed. Endogenous job destruction is then

$$\rho_t^n = \Pr[a_t \leq \tilde{a}_t] = F(\tilde{a}_t) \quad (2.6)$$

and the aggregate separation rate is

$$\rho_t = \rho^x + (1 - \rho^x) \rho_t^n. \quad (2.7)$$

With job creation and destruction characterized as above, the number of matches (employment) that enter period  $t$  is

$$n_t = (1 - \rho_{t-1}) n_{t-1} + m(u_{t-1}, v_{t-1}) \quad (2.8)$$

where  $n_t$  is period  $t$  employment. The measure of searching workers is

$$u_t = 1 - n_t + \rho_t n_t = 1 - (1 - \rho_t) n_t. \quad (2.9)$$

The number of searching workers in period  $t$  differs from the number of unemployed workers,  $1 - n$ , in the beginning of period  $t$  as some of the employed workers separate from their matches and start searching for a new job within the same period.

Furthermore, we determine the net job creation rate. Each period  $q_t^f v_t$  vacancies are filled. Of these vacancies a fraction  $\rho^x$  is immediately destroyed exogenously. The rate of turnover is then  $q_t^f \rho^x n_t$  and the net job creation rate can be expressed as

$$j c_t = \frac{q_t^f v_t}{n_t} - q_t^f \rho^x. \quad (2.10)$$

### 2.2.2 Employment taxes and unemployment income

From the variety of possible tax policy schemes we will focus on income taxation and unemployment benefits.<sup>4</sup> Taxes on labor income and unemployment earnings are modeled in a simple manner by using three policy instruments: a marginal tax on total labor earnings, a tax subsidy for employed workers and unemployment compensation. We assume that wage taxes are linear and smooth functions of income. In our benchmark case employed workers receive a tax subsidy  $v$  and are subsequently taxed for their total earnings, the subsidy included, at proportional rate  $\tau$  (*s.t.*  $0 \leq \tau \leq 1$ ).<sup>5</sup> The net income of a worker with match specific productivity

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<sup>4</sup>We abstract from other policy aspects such as employment protection or promotion through firing costs and hiring subsidies respectively, or the role of payroll taxes.

<sup>5</sup>The benchmark labor market policy setup follows Pissarides (2000), but in the analysis that follows we will consider departures from these assumptions.

$a_{it}$  is then  $(1 - \tau)[w_{it}(a_{it}z_t) + v]$ , where  $w_{it}(a_{it}z_t)$  is the wage of a worker with match-specific productivity  $a_{it}$ . The transfer from the worker to the tax authorities is

$$T_{it}(w_{it}(a_{it}z_t)) = \tau w_t(a_{it}z_t) - (1 - \tau)v \quad (2.11)$$

When the tax subsidy  $v$  is positive, taxation is progressive s.t. the average tax rate increases with the wage. When  $v = 0$  taxation is proportional.

Unemployment compensation is modeled to be a policy determined replacement ratio of net income.<sup>6</sup> As there is a distribution of wages, one possibility would be to set the unemployment compensation proportional to the average net wage. To simplify the model we use instead average productivity and assume that the unemployment compensation is proportional to the sum of the average productivity and the tax subsidy. The unemployment compensation is then

$$b_t = \rho^\tau (1 - \tau) (H(\tilde{a}_t) z_t + v). \quad (2.12)$$

where  $\rho^\tau$  is the replacement rate and  $H(\tilde{a}_t)$  is the conditional expectation  $E[a | a \geq \tilde{a}_t]$ . This setup effectively implies that the unemployment benefit is subject to the marginal tax rate.

### 2.2.3 Match surplus and value functions

Match surplus is a key element in determining job creation and destruction. The surplus is the difference of the values of being matched and the outside values and is given by

$$S_t(a_{it}) = J_t(a_{it}) + W_t(a_{it}) - V_t - U_t \quad (2.13)$$

where  $J_t(a_{it})$  and  $W_t(a_{it})$  are the values for a firm and worker respectively of being matched and  $V_t$  and  $U_t$  are the values of idleness for the worker and firm, that is having an open vacancy for the firm and being unemployed for the worker.

#### *Firm*

The value for a firm of a filled job  $J_t(a_{it})$  and a vacancy  $V_t$  are given by

$$J_t(a_{it}) = \frac{a_{it}z_t}{\mu_t} - w_t(a_{it}z_t) + \max E_t \beta_{t+1} (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}) \quad (2.14)$$

$$V_t = -\kappa + E_t \beta_{t+1} \left[ q_t^f (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}) + (1 - q_t^f) V_{t+1} \right]$$

---

<sup>6</sup>This is not the case in all European countries. Therefore a replacement ratio that is proportional to the gross wage will also be considered below.

(2.15)

The value of a filled job is determined by the real value of match output  $\frac{a_{it}z_t}{\mu_t}$  (in terms of time  $t$  consumption goods) minus the wage  $w_t(a_{it}z_t)$  paid to the worker, and the expected future value of the job, which is discounted according to the discount factor  $\beta_{t+i} = \frac{\beta^i \lambda_{t+i}}{\lambda_t}$ . The wage paid by the firm includes the taxes the worker pays to the government. The expected value of the job takes into account that the job may be destroyed due to an exogenous shock with probability  $\rho^x$  and that jobs with a productivity realization  $a_{it+1} < \tilde{a}_{t+1}$  will be destroyed endogenously.

The value of having an open vacancy consists of the periodical cost  $\kappa$  of having an open vacancy and the expected surplus of a filled job. The latter depends on the probability  $q_t^f$  of finding an appropriate worker, and that the job is not destroyed due to an exogenous shock or endogenously due to a low realization of match specific productivity. We assume free-entry of firms to the market so firms enter until  $V_t = 0$ . Substituting the free-entry condition into (2.15)

$$\frac{\kappa}{q_t^f} = (1 - \rho^x) E_t \beta_{t+1} \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}). \quad (2.16)$$

The job creation equation states that the expected surplus for the firm must equal the cost of posting a vacancy. The right hand side of the equation gives the expected surplus that accrues to the firm from a filled job. The left hand side is the expected cost of filling the vacancy, where  $q_t^f$  is the probability of the firm finding a worker so  $\frac{1}{q_t^f}$  is the expected duration of search.

### Worker

The values for the worker of employment  $W_t(a_{it})$  and unemployment  $U_t$  are respectively

$$\begin{aligned} W_t(a_{it}) &= w_t(a_{it}z_t) - T(w_t(a_{it}z_t)) \\ &\quad + E_t \beta_{t+1} \left[ (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} W_{t+1}(a_{it+1}) dF(a_{it+1}) + \rho^x U_{t+1} \right] \end{aligned} \quad (2.17)$$

$$\begin{aligned} U_t &= h + b_t(H(\tilde{a}_t)z_t) \\ &\quad + E_t \beta_{t+1} \left[ q_t^w (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} W_{t+1}(a_{it+1}) dF(a_{it+1}) + (1 - q_t^w (1 - \rho^x)) U_{t+1} \right] \end{aligned} \quad (2.18)$$

An employed worker earns a wage of  $w_t(a_{it}z_t)$  and makes a transfer  $T(w_t(a_{it}z_t))$  to the tax authorities. The expected value of employment depends on the probability of not being destroyed by an exogenous shock and that the match specific productivity realization satisfies  $a_{it+1} \geq \tilde{a}_t$ . In the case of destruction the worker enjoys the value of unemployment  $U_{t+1}$ . An unemployed worker enjoys the value of leisure (or home production)  $h$  and an unemployment compensation  $b_t(w_t^e(H(\tilde{a}_t)z_t))$ , which was defined above. The probabilities and values of being employed or unemployed next period affect the value of unemployment in the current period.



## 2.2.4 Bargaining and the wage

The wage is determined by Nash bargaining as is conventional in the matching literature. The match surplus is shared between the firm and the worker according to the parameter  $\eta$  which represents the workers share (bargaining power) of the match surplus. The wage rate satisfies<sup>7</sup>

$$w_t(a_{it}z_t) = \arg \max [J_t(a_{it}) - V_t]^\eta [W_t(a_{it}) - U_t]^{1-\eta} \quad (2.19)$$

The first order condition is

$$\eta [1 - T'(w_t(a_{it}z_t))] J_t(a_{it}) = (1 - \eta) [W_t(a_{it}) - U_t] \quad (2.20)$$

and implies the following relations

$$J_t(a_{it}) - V_t = \frac{1 - \eta}{[1 - \eta T'((a_{it}z_t))]} S_t(a_{it}) \quad (2.21)$$

$$W_t(a_{it}) - U_t = \frac{\eta [1 - T'((a_{it}z_t))]}{[1 - \eta T'((a_{it}z_t))]} S_t(a_{it}). \quad (2.22)$$

These relations show how the share parameter  $\eta$  increases the relative share of match surplus going to the worker. From these relations we also see that increasing the marginal tax rate  $T'(w_t(a_{it}z_t)) = \tau$  has similar effects to the division of surplus as a decrease in the share parameter. The higher is the marginal tax rate, the lower is the worker's share of surplus relative to the firm's. Substituting the value equations into the first order condition (2.20) and rearranging produces the wage equation<sup>8</sup>

$$w_t(a_{it}z_t) = \eta \left( \frac{a_{it}z_t}{\mu_t} + \kappa\theta_t \right) + (1 - \eta) \left( \frac{h}{(1 - \tau)} + \rho^\tau H(\tilde{a}_t) z_t - (1 - \rho^\tau) v \right). \quad (2.23)$$

In addition to the real value of the marginal product  $\frac{a_{it}z_t}{\mu_t}$  of the match, the wage depends on the cost related to search in the case of separation as well as the outside value of the worker. The wage is increasing in labor market tightness  $\theta_t$  which reflects the ease with which a worker can find an alternative employer in the case of separation. The higher the value of home production  $h$ , the higher is the required wage for the worker to agree to work. The wage is increasing in the bargaining share  $\eta$  of the worker.

The partial comparative statics of the wage wrt. the policy parameters are

$$\frac{\partial w_t}{\partial \rho^\tau} > 0, \quad \frac{\partial w_t}{\partial v} < 0, \quad \frac{\partial w_t}{\partial \tau} > 0.$$

A higher replacement rate  $\rho^\tau$  raises the worker's unemployment income and threat point in the wage bargain, thus raising the wage. The tax subsidy  $v$  paid

<sup>7</sup>See appendix for detailed derivation of the wage.

<sup>8</sup>Note that this is the gross wage that the firm pays to a worker while the worker's after tax net wage is  $w_t(a_{it}z_t) - T(w_t(a_{it}z_t)) = (1 - \tau)[w_t(a_{it}z_t) + v]$ .

to an employed worker reduces the negotiated wage. This is because the cost of labor to the firm is reduced as the worker's employment is partly compensated by the tax subsidy. As the wage is bargained for, the firm and worker share the subsidy in the same way as they share the surplus of the job. The net gain from the subsidy received upon job formation is  $(1 - \rho^\tau)v$ : employed workers receive the full subsidy  $v$ , but as unemployment is proportional to net income (including the subsidy), they already received a fraction  $\rho^\tau$  of it in their unemployment benefit. The marginal tax  $\tau$  reduces the worker's share of match surplus. From any increase in the wage conceded by the firm, the worker receives only a fraction  $1 - \tau$ , so there is a joint loss to the firm and worker from the marginal tax. As the value of unemployment includes the value of home production which is not taxed, the marginal tax increases this value relative to the value of working. Thus the marginal tax increases the gross wage.

A government tax revenue neutral increase in tax revenue may be implemented by increasing the marginal tax and making the necessary increase in the tax subsidy to exhaust the change in tax revenue. The effect on the wage is a priori ambiguous as the two tax policy instruments affect the wage in opposite directions. The parameters of the second term in the wage equation (2.23) determine the relative effects of an increase in the two tax instruments. The size of the relative effects will be of key importance to the general equilibrium effects of the model.

For the purposes of the present study it may be intuitive to see the wage as consisting of market and nonmarket components. The first term in (2.23) consists of variables that reflect market conditions, match productivity and labor market tightness. The wage responds to changes and volatility in the labor market through this term. The second term consists of non market or fixed parameters. The larger is this part of the wage relative to the market part, the more rigid is the wage. The relative importance of these two terms determines how much of exogenous shocks are absorbed by the wage. The more rigid the wage is, the more the shocks are transferred on to the profitability of jobs and thus on job creation and destruction.

To illustrate this, consider two extreme cases of the wage negotiation outcome, namely approaching solutions where one of the partners has all of the bargaining power. When the worker's bargaining power approaches unity ( $\eta \rightarrow 1$ ) the second term in the wage equation approaches zero and the wage equation becomes

$$w_t(a_{it}z_t) = \frac{a_{it}z_t}{\mu_t} + \kappa\theta_t.$$

Now there are no fixed components of the wage and it consists only of 'market terms' making it more sensitive to market disturbances. The whole of the real value of the marginal product  $\frac{a_{it}z_t}{\mu_t}$  of the match accrues to the worker and the value of unemployment becomes irrelevant. The worker can appropriate all of the match surplus. The policy parameters have no influence in this extreme case.

In the other extreme the firm has all bargaining power ( $\eta \rightarrow 0$ ) and the wage equation reduces to

$$w = \frac{h}{(1 - \rho^\tau)(1 - \tau)} - v$$

where the match product and the ease at which new partners are found have no relevance. The wage is now immune to market disturbances but the policy parameters have a key influence on the wage. In this case the match surplus goes entirely to the firm and the wage paid to workers will be only as high as the value of leisure and unemployment compensation. Here the policy parameters have qualitatively similar, but more important, effects on the wage as in the basic case.

### 2.2.5 Job creation and destruction

To derive (relatively) explicit expressions for job creation and destruction we first manipulate the value equation for a filled job following Pissarides (2000, ch. 2). Substitute the wage equation into the value equation for a filled job (2.14) to get

$$\begin{aligned} J_t(a_{it}) = & (1 - \eta) \left( \frac{a_{it}z_t}{\mu_t} - \frac{h}{(1 - \tau)} - \rho^\tau H(\tilde{a}_t) z_t + v(1 - \rho^\tau) \right) - \eta \kappa \\ & + E_t \beta_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}). \end{aligned} \quad (2.24)$$

Evaluate this expression at  $a_{it} = \tilde{a}_t$  and subtract the resulting equation from (2.24) after noting that  $J_t(\tilde{a}_t) = 0$  by the definition of reservation productivity (jobs are destroyed when match surplus goes to zero).<sup>9</sup> We obtain

$$J_t(a_{it}) = (1 - \eta) \frac{z_t}{\mu_t} (a_{it} - \tilde{a}_t). \quad (2.25)$$

Substituting this into the job creation condition (2.16) we get

$$E_t \beta_{t+1} (1 - \rho^x) (1 - \eta) \frac{z_{t+1}}{\mu_{t+1}} \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} (a_{it+1} - \tilde{a}_{t+1}) dF(a_{it+1}) = \frac{\kappa}{q_t^f}. \quad (2.26)$$

This condition restates the condition that the firm's share of expected surplus must equal the job creation cost. From the partial comparative statics we see that the job creation condition of intermediate good firms depends negatively on labor market tightness  $\theta_t$  (through  $q_t^f$ ), positively on the reservation value  $\tilde{a}$  for match specific productivity, positively on general productivity  $z_t$  and negatively on the price markup  $\mu_t$  of retail firms.

Jobs are destroyed when match surplus is zero,  $J_t(\tilde{a}_{it}) = 0$ . Setting (2.24) to equal zero and substituting the job creation condition for the second row

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<sup>9</sup>The firm and worker agree when to separate as  $J_t(a_{it}) = 0$  implies  $W_t(a_{it}) - U_t = 0$  by the Nash bargaining rule. Therefore we may consider job destruction from either the firm's or worker's perspective.

we obtain

$$\frac{\tilde{a}_{it}z_t}{\mu_t} - \frac{h}{(1-\tau)} - \rho^\tau H(\tilde{a}_t)z_t + v(1-\rho^\tau) - \frac{\eta}{1-\eta}\kappa\theta_t + \frac{1}{1-\eta}\frac{\kappa}{q_t^f} = 0 \quad (2.27)$$

We now see from (2.26) and (2.27) that the policy instruments are present only in the job destruction condition.

### 2.3 Aggregate output and consumption

The aggregate output of the economy produced by all firm-worker matches is given by

$$Q_t = (1-\rho_t)n_tz_t \int_{\tilde{a}_t}^{\bar{a}_t} \frac{a_{it}f(a_{it})da_{it}}{1-F(\tilde{a}_t)} = (1-\rho_t)n_tz_tH(\tilde{a}_t) \quad (2.28)$$

where  $H(\tilde{a}_t)$  as the conditional expectation  $E[a | a \geq \tilde{a}_t]$ . Finally, we also require that consumption  $C_t$  equals aggregate household income  $Y_t$  which equals production net of vacancy costs

$$C_t = Y_t = (1-\rho_t)n_tz_tH(\tilde{a}_t) - \kappa v_t. \quad (2.29)$$

### 2.4 Retail firms and price rigidity

There is a continuum of monopolistically competitive retail firms on the unit interval. Retail firms buy output of wholesale firms at price  $P_t^W$ , differentiate the good and sell it to households. No other inputs or costs are used in the production of final goods, thus retail firm's marginal cost is  $P_t^W$  and real marginal cost is  $\frac{P_t^W}{P_t}$ .

Output sold by retail firm  $j$  is  $y_{jt}$  at price  $p_{jt}$ . Final goods  $y_t$  are a composite of individual retail goods

$$y_t = \left[ \int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where  $\varepsilon > 1$  is the the elasticity of substitution across the differentiated retail goods. If resources are used efficiently output of good  $j$  equals the demand (consumption) of good  $j$ ,  $y_{jt} = c_{jt}$  so we have

$$C_t = \left[ \int_0^1 c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The demand for good  $j$  can be written as

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\varepsilon} C_t \quad (2.30)$$

where the price elasticity of good  $j$  is  $\varepsilon$ . As  $\varepsilon \rightarrow \infty$ , the goods become closer substitutes and firms have less market power.

Following Walsh (2005) and Christiano et al (2001) a fraction  $1 - \omega$  of randomly chosen firms adjusts its price optimally each period and a fraction  $\omega$  adjusts according to a rule of thumb.<sup>10</sup> Optimally adjusting firms set their price to maximize the expected discounted value of current and future profits and all adjusting firms choose the same price  $p^*$ . Profits at a future date  $t + i$  are affected by the price chosen at date  $t$  if the firm has not had the possibility to update its price optimally after  $t$ . The probability of this is  $\omega^i$ . Firms choose  $p_{jt}$  to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \frac{p_{jt}}{P_{t+i}} c_{jt+i} - \frac{P_{t+i}^w}{P_{t+i}} c_{jt+i} \right]. \quad (2.31)$$

where  $\beta_{t+i} = \frac{\beta^i \lambda_{t+i}}{\lambda_t}$ . Using the demand curve (2.30) faced by the firm to eliminate  $c_{jt}$  from the objective function and substituting  $\mu_{t+i}^{-1} = \frac{P_{t+i}^w}{P_{t+i}}$  we obtain

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\varepsilon} - \mu_{t+i}^{-1} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\varepsilon} \right] C_{t+i}. \quad (2.32)$$

The first order condition is after some manipulation<sup>11</sup>

$$\frac{p_t^*}{P_t} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \mu_{t+i}^{-1} \left( \frac{P_{t+i}}{P_t} \right)^{\varepsilon} C_{t+i} \right]}{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \left( \frac{P_{t+i}}{P_t} \right)^{\varepsilon-1} C_{t+i} \right]}. \quad (2.33)$$

This equation gives the price chosen by the firms that adjust their price optimally.

The aggregate price is given by

$$P_t^{1-\varepsilon} = (1 - \omega) (p_t^*)^{1-\varepsilon} + \omega p_{jt-1}^{1-\varepsilon} \quad (2.34)$$

where a fraction  $(1 - \omega)$  adjusts price optimally and a fraction  $\omega$  adjusts according to rule of thumb. We assume that firm  $j$  uses a rule of thumb based on the most recently observed rate of inflation and the most recently observed price level  $P_{t-1}$ ,

$$p_{jt} = \pi_{t-1} P_{t-1}. \quad (2.35)$$

To obtain an expression for aggregate inflation, equations (2.33) and (2.34) can be approximated around a zero average inflation steady state equilibrium. We obtain

$$\pi_t = \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \frac{1}{1 + \beta} \pi_{t-1} - \frac{\zeta}{1 + \beta} \hat{\mu}_t. \quad (2.36)$$

where  $\zeta = \frac{(1-\omega)(1-\omega\beta)}{\omega}$  and  $\hat{\mu}_t$  is the deviation of the price markup from the steady state value.

<sup>10</sup>This is a variant of Calvo (1983).

<sup>11</sup>See appendix for detailed derivation.

## 2.5 Monetary authority

The central bank controls the nominal rate of interest according to a policy rule that is a modified Taylor rule. The short-term nominal interest rate follows the process

$$R_t = R_{t-1}^{\rho_R} \left( \frac{P_t}{P_{t-1}} \right)^{\phi_\pi(1-\rho_R)} e^{\phi_t} \quad (2.37)$$

where  $\rho_R$  is the degree of interest rate smoothing,  $\phi_\pi > 1$  is the response coefficient for inflation and  $\phi_t$  is a serially uncorrelated, mean zero stochastic process. With this policy rule for the nominal rate of interest, the nominal quantity of money adjusts endogenously to satisfy the demand for money.

## 2.6 Government tax revenue

The government levies income taxes from workers to finance unemployment benefits and tax subsidies paid to workers. The government tax revenues are given by

$$TR_t = (1 - \rho^x) n_t T [w_t^e (H(\tilde{a}_t) z_t)] - (1 - n_t) b_t (H(\tilde{a}_t) z_t) \quad (2.38)$$

where  $T [w_t^e (H(\tilde{a}_t) z_t)]$  and  $b_t (H(\tilde{a}_t) z_t)$  are given by (2.11) and (2.12) respectively. The government receives tax payments (marginal tax on gross income net of the tax subsidy) from all employed workers whose jobs are not destroyed in the current period. The unemployed workers receive an unemployment compensation from the government.

## 3 Model solution and dynamics

In steady state we have  $\pi_t = 0$  and  $p_t^* = P_t = P$  and  $z_t = z = 1$ . This implies that the household's Euler condition reduces to  $R = \frac{1}{\beta}$  and the steady state values of  $n, \rho, u, q^f, q^w, jc, \theta, w, \tilde{a}, C$  and the policy variables  $TR, T$  and  $b$  are given by the steady state versions of equations (2.4), (2.5), (2.7), (2.8), (2.9), (2.10), (2.26), (2.23), (2.27), (2.29), (2.38), (2.11) and (2.12).<sup>12</sup> We proceed by first solving the non-stochastic zero-inflation steady state and then linearizing the model around this steady state to simulate the dynamics of the model. The variables are expressed in terms of percentage deviations around the steady state.

- The Euler condition from household's problem

$$0 = E_t \hat{y}_{t+1} - \hat{y}_t - \frac{1}{\sigma} (\hat{r}_t - E_t \hat{\pi}_{t+1}) \quad (3.1)$$

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<sup>12</sup>The steady state equations are listed in the appendix.

- Survival rate of matches  $\varphi_t = 1 - \rho_t$

$$\hat{\varphi}_t = - \left( \frac{\rho^n}{1 - \rho^n} \right) e_{F,a} \hat{a}_t \quad (3.2)$$

where  $e_{F,a} = \frac{\partial F(\bar{a})}{\partial \bar{a}} \frac{\bar{a}}{F(\bar{a})}$ .

- Employment (evolution of number of matches)  $n_{t+1}$

$$\hat{n}_{t+1} = \bar{\varphi} \hat{n}_t + \bar{\varphi} \hat{\varphi}_t + \left( \frac{\bar{v} \bar{q}^f}{\bar{n}} \right) \hat{v}_t + \left( \frac{\bar{v} \bar{q}^f}{\bar{n}} \right) \hat{q}^f \quad (3.3)$$

- Unemployment (number of unemployed job seekers  $u_t$ )

$$\hat{u}_t = - \frac{\bar{\varphi} \bar{n}}{\bar{u}} \hat{n}_t - \frac{\bar{n} \bar{\varphi}}{\bar{u}} \hat{\varphi}_t \quad (3.4)$$

- Probability of filling vacancy for firm  $q^f$

$$\hat{q}_t^f = \alpha (\hat{u}_t - \hat{v}_t) \quad (3.5)$$

- Equality of firms filling vacancies and workers finding jobs

$$\hat{v}_t + \hat{q}_t^f = \hat{u}_t + \hat{q}_t^w \quad (3.6)$$

- The nominal interest rate rule

$$\hat{r}_t = \rho_R \hat{r}_{t-1} + \phi_\pi (1 - \rho_R) \pi_t + \phi_t \quad (3.7)$$

- Inflation

$$\hat{\pi}_t = \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} + \frac{1}{1 + \beta} \hat{\pi}_{t-1} - \frac{\zeta}{1 + \beta} E_{t-1} \hat{\mu}_t \quad (3.8)$$

where  $\zeta = \frac{(1-\omega)(1-\omega\beta)}{\omega}$ .

- Output equation

$$\hat{y}_t = \frac{\bar{Q}}{\bar{y}} (\hat{z}_t + e_{H,a} \hat{a}_t + \hat{\varphi}_t + \hat{n}_t) - \frac{\kappa \bar{v}}{\bar{y}} \hat{v}_t \quad (3.9)$$

where  $e_{H,a} = \frac{\partial H(\bar{a})}{\partial \bar{a}} \frac{\bar{a}}{H(\bar{a})}$ .

- Endogenous job creation

$$\begin{aligned} -\hat{q}_t^f &= \hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\varphi}_{t+1} + \hat{z}_{t+1} - \hat{\mu}_{t+1} \\ &\quad + E_t \beta (1 - \eta) \bar{\varphi} \bar{q}^f \frac{H(\bar{a}_{t+1}) e_{H,a} - \bar{a}_{t+1}}{\bar{\mu} \kappa} e_{H,a} \hat{a}_{t+1} \end{aligned} \quad (3.10)$$

- Endogenous job destruction

$$\begin{aligned} &\frac{\bar{a}}{\bar{\mu}} \hat{a}_t + \left[ \frac{\bar{a}}{\bar{\mu}} - \rho^\tau \frac{H(\bar{a}_t)}{\bar{\mu}} \right] (\hat{z}_t - \hat{\mu}_t) - \rho^\tau \frac{H(\bar{a}_t) e_{H,a}}{\bar{\mu}} \hat{a}_t \\ &\quad - \frac{\eta \kappa \bar{q}^w}{(1 - \eta) \bar{q}^f} \hat{q}_t^w - \frac{(1 - \eta \bar{q}^w) \kappa}{(1 - \eta) \bar{q}^f} \hat{q}_t^f \\ &= 0 \end{aligned} \quad (3.11)$$

## 4 Calibration

The baseline parameter values are calibrated to a stylized U.S. economy and to be in line with previous literature.<sup>13</sup> As information on all parameters is not available, we calibrate these values indirectly as residual parameters from the steady state equations. The model's parameters can be separated into six groups: labor market parameters, labor market policy parameters, household preferences, parameters characterizing the degree of price rigidity, interest rate parameters and the parameters of exogenous shocks.

*Labor market* – Job flows are determined by the matching and separation probabilities of firms and workers. We set the time period to one quarter and the job finding rate of workers and the rate of filling vacancies at  $q^w = 0.6$  and  $q^f = 0.7$  respectively. The matching function parameters are set to  $\alpha = 0.4$  for the worker's elasticity parameter and  $1 - \alpha = 0.6$  for the firm's elasticity parameter. These are in accordance with empirical studies of the matching function.<sup>14</sup> The size of the labor force is normalized to one and the employment rate is set to  $n = 0.94$ , which implies an unemployment rate of 6 percent. The steady-state number of workers searching for a job is then  $u = 0.154$ , as  $u$  also includes the total  $\rho n$  of workers who move to the matching market because their matches dissolve before production is started. The total job destruction rate is set to  $\rho = 0.1$  which is roughly consistent with a large body of empirical studies.<sup>15</sup> These values and the matching function also imply  $v = 0.134$ . For the exogenous job destruction rate we use the value calibrated by den Haan et al (2000)  $\rho^x = 0.068$  implying the endogenous job destruction rate  $\rho^n = F(\tilde{a}) = 0.034$ . The reservation productivity  $\tilde{a}$  can be derived from  $\tilde{a} = F(\rho^n)^{-1}$ . Following eg. Mortensen and Pissarides (2003) we assume that  $F(a)$  is the uniform c.d.f. with support  $[\gamma, 1]$ . In the linearized model we need the elasticity of the c.d.f. at the reservation productivity level  $\tilde{a}$ , which is given by  $e_{F,a} = \frac{\partial F(\tilde{a})}{\partial \tilde{a}} \frac{\tilde{a}}{F(\tilde{a})} = \frac{\tilde{a} f(\tilde{a})}{F(\tilde{a})}$ . For the conditional expectation of  $a$  given the reservation productivity  $\tilde{a}$  we have  $H(\tilde{a}) = \int_{\tilde{a}}^{\bar{a}} a \frac{f(a)}{1-F(\tilde{a})} da$  and the elasticity  $e_{H,a} = \frac{\partial H(\tilde{a})}{\partial \tilde{a}} \frac{\tilde{a}}{H(\tilde{a})}$ . The worker and firm are assumed to get an equal share of the match surplus in the wage bargaining so we set  $\eta = 0.5$ . The value of leisure  $h$  and the lower support of the productivity distribution  $\gamma$  are calibrated s.t. the model is consistent with the values for  $\rho^n$  and  $n$  above. Finally  $q$  and  $\kappa$  are calibrated as residual parameters from the steady state equations.

*Labor market policy* – We calibrate the policy parameters together with the value of home production in such a way that we obtain steady state values that are roughly consistent with Walsh (2003, 2005) for reasonable tax parameter values. Our strategy is to first set the policy parameters to benchmark values s.t. taxation is initially progressive and the replacement ratio similar to examples of the U.S. used in the literature. We then reverse calibrate the value of home production  $h$  s.t. the model produces steady state values that are consistent with eg Walsh (2003, 2005) calibrations. For the

<sup>13</sup>See eg Walsh (2003, 2004), Trigari (2004), Krause and Lubik (2003) and den Haan et al (2000).

<sup>14</sup>See eg Petrongolo and Pissarides (2001) and Blanchard and Diamond (1989).

<sup>15</sup>See eg Davis et al (1998).



baseline calibration we set the marginal tax rate to  $\tau = 0.25$  and the tax subsidy  $\nu = 0.03$ . The positive tax subsidy implies that income taxation is progressive. Finally, the replacement rate is set to  $\rho^r = 0.2$  and reverse calibration of the value of home production produces  $h = 0.53$ .

*Household preferences* – We follow Walsh (2005) for the utility function  $u(C_{t+i}) = \frac{(C_{t+i} - \chi C_{t+i-1})^{1-\sigma}}{1-\sigma}$  where  $\chi$  is a parameter of habit persistence, and choose values for the parameters of household preferences that are standard in the literature. We set  $\chi = 0.5$ ,  $\beta = 0.989$  and the coefficient of relative risk aversion is chosen to be  $\sigma = 2$ . The steady state price markup for retail firms is set to equal  $\mu = 1.1$  which implies  $\varepsilon = 11$ , which is the parameter that determines the elasticity of demand of differentiated retail goods.

*Price rigidity* – The degree of price rigidity is determined by the share of firms who do not optimally adjust their price. We follow Walsh (2003) and set this fraction to equal  $\omega = 0.67$ .

*Monetary policy* – We set the parameters of the interest rate rule to equal  $\phi_\pi = 1.10$ , which gives a 110 basis points long-run nominal response to a 100 basis point increase in inflation, and  $\rho_R = 0.9$  which is roughly consistent with the empirical evidence on high inertia displayed by central bank policy rules (Walsh 2005).

*Shock processes* – We assume that the log aggregate productivity shock to follow an AR(1) process  $\log z_t = \rho_z \log z_{t-1} + \epsilon_t$  with  $\rho_z = 0.95$  and  $\rho_\epsilon = 0.01$ . The standard deviation of the policy shock is set to  $\phi_t = 0.002$ .

## 5 Model analysis

We proceed by first analyzing the steady state of the model and the comparative statics of the labor market policy parameters. Then we move to the impulse response analysis to study the effects of taxes on the dynamic behavior of the model.

### 5.1 Steady state labor market policy analysis

#### 5.1.1 Employment taxes and unemployment income

First we consider the effects of changes in policy parameters on the steady state of the economy independently of tax revenue considerations (figure 1). We then investigate compensating policy changes to study the impact of changes in the tax structure (figure 2). With tax revenue neutral changes we fix the government tax revenue and consequently the tax subsidy solves as an endogenous variable of the model which depends on the marginal tax rate. A general observation to make is that the effects of policy work through the wage on the job destruction condition, which jointly with the job creation condition determines the destruction productivity and labor market tightness.

	<i>Benchmark</i>	$d\tau$	$dv$	$d\rho^\tau$
$y$	0.66	0.62	0.69	0.62
$q$	0.67	0.63	0.70	0.63
$n$	0.94	0.91	0.96	0.90
$\theta$	0.88	0.76	1.02	0.74
$qf$	0.70	0.73	0.65	0.74
$qw$	0.60	0.56	0.67	0.55
$a$	0.58	0.59	0.56	0.59
$jc$	0.031	0.033	0.253	0.034

Figure 1: Percentage point changes in policy parameters.

*Marginal tax rate* – Consider a marginal increase in the income tax rate  $\tau$ . As home production (or leisure) is not taxed it's value relative to working increases making the latter less attractive. To restore the attractiveness of working the wage must be increased. Higher wages imply lower job creation and lower labor market tightness; less vacancies and more unemployed workers. Output falls as less people are employed and jobs are fewer.

*Tax subsidy* – Increasing the tax subsidy  $\lambda$  has opposite effects to the marginal tax rate. The tax subsidy paid to an employed worker reduces the negotiated wage as the worker's employment is partly compensated by the tax subsidy. Bargaining implies that the firm and worker share the subsidy. The reduction in the negotiated wage raises job creation, vacancies and labor market tightness. Unemployment falls as the job finding probability for workers increases. Output increases.

*Replacement ratio* – A higher replacement rate increases the worker's unemployment income and threat point in the wage bargain. The wage increases with effects similar to those of the marginal tax.

An alternative way to model taxes and the unemployment compensation scheme would be to follow eg Pissarides (1998) and Sinko (2005) by assuming that the net income of a worker with match specific productivity  $a_{it}$  is  $(1 - \tau) w_{it} (a_{it} z_t) + v$  ie the tax subsidy is not subject to the marginal tax (in the benchmark case we assumed that employed workers receive a tax subsidy  $v$  and are subsequently taxed for their total earnings, the subsidy included). The transfer from the worker to the tax authorities in the alternative setup is then.<sup>16</sup>

$$T_{it}(w_{it}(a_{it}z_t)) = \tau w_t(a_{it}z_t) - v. \quad (5.1)$$

Unemployment compensation can be assumed to be either fixed or proportional to the average productivity (without the tax subsidy)

$$b_t = \rho^\tau H(\tilde{a}_t) z_t. \quad (5.2)$$

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<sup>16</sup>Another possibility to model tax progression is to use a tax exemption and a marginal tax.

The results presented above are qualitatively unambiguous and general and are not sensitive to the calibration of the model or specific policy setup. However, the particular policy setup does influence the quantitative effects of the policy instruments. This is a feature to bear in mind when considering tax progression schemes.

### 5.1.2 Tax progression

We next examine the importance of the structure of taxes for the equilibrium values of the model. Keeping the government tax revenue fixed, we increase tax progression by increasing the marginal tax rate and then increase the tax subsidy so much that the change in tax revenue implied by the marginal tax raise is exhausted. Given the comparative statics of the marginal tax and tax subsidy described above, the effects of increasing tax progression are ambiguous *a priori*, and depend on the relative magnitude of the effects of the tax instruments.

*Progressive taxes in initial equilibrium* – Figure 2 shows how a revenue neutral increase in tax progression affects the steady state of the economy in the benchmark calibration. The wage rate increases, inducing more job destruction, reducing labor market tightness and thus raising unemployment. Output decreases. The effect of a tax revenue neutral increase in tax progression is qualitatively similar to an increase in the marginal tax. The effect on the steady state values of the change in the marginal tax dominates the effect of the tax subsidy. However the effect of the marginal tax is moderated by the opposite effect of the tax subsidy. A more progressive tax scheme thus shifts the economy to a lower output and higher unemployment equilibrium, so this tax structure involves a trade-off between income equality considerations and equilibrium unemployment and output.

*Proportional taxes in initial equilibrium* – The above result is in contrast with the results of Pissarides (1998) and Sinko (2005). In their studies increasing tax progression has a positive employment effect, whereas we find a negative one. The key issue between these opposite results is the initial degree of tax progression. Pissarides (1998) and Sinko (2005) consider the effects of a tax revenue neutral increase in tax progression when taxation is initially proportional (the tax subsidy is zero), whereas we start from an initially progressive tax scheme. Experimenting with the policy instruments reveals that our model also produces qualitatively similar results to the above studies when taxation is proportional in the initial state. The smaller is the tax subsidy in the initial state, the smaller is the negative effect of the marginal tax increase on employment relative to the positive effect of the tax subsidy increase. For a sufficiently small tax subsidy the relative effects are reversed and the employment effect turns positive. The wage rate decreases with tax progression, inducing more job creation and vacancies, higher labor market tightness and lower unemployment. Output increases. In this case promoting income equality is consistent with lower equilibrium unemployment and higher output.

	$\tau = 0.25, \nu = 0.03$	$\tau = 0.26, \nu = 0.038$	$\tau = 0.30, \nu = 0.071$
$y$	0.659	0.655	0.641
$q$	0.667	0.663	0.650
$n$	0.94	0.937	0.951
$\theta$	0.880	0.865	0.819
$qf$	0.70	0.70	0.71
$qw$	0.60	0.60	0.58
$a$	0.577	0.578	0.583
$jc$	0.030	0.031	0.032

Figure 2: Tax revenue neutral increase in tax progression.

Our opposite results to those of Pissarides (1998) and Sinko (2005) show that the effects of increasing tax progression depend on the initial degree of tax progression in the economy. Our results are not in conflict with these studies, but completes them by emphasizing the mechanism by which progression works. Our simulations show that, starting from a proportional tax scheme, the relative strength of the two tax policy instruments is reversed as progression increases. Initially the effect of the tax subsidy dominates, but once the initial tax scheme is sufficiently progressive, the effect of the marginal tax dominates. This implies that for economies with an initially low degree of tax progression, increasing it is beneficial in terms of employment and output. But for economies with a sufficiently progressive tax scheme initially, increasing progression further is harmful in terms of employment and output.

## 5.2 Tax reform and shock propagation

Now we investigate how changes in labor market policy instruments affect shock propagation. As in the previous section, our strategy is to first look at the effect of policy parameters separately without government tax revenue considerations and then examine tax revenue neutral changes in the tax structure.

### 5.2.1 Employment taxes and unemployment income

The effects of the individual policy instruments on the impulse response functions to productivity and interest rate shocks are plotted by the solid lines in figure 3 and 4 respectively. The dotted lines plot the impulse responses for a percentage point increase in the income tax rate. The impulse response functions for a percentage point increase in the tax subsidy are produced by the dashed lines. For the sake of clarity in the figure, the impulse response

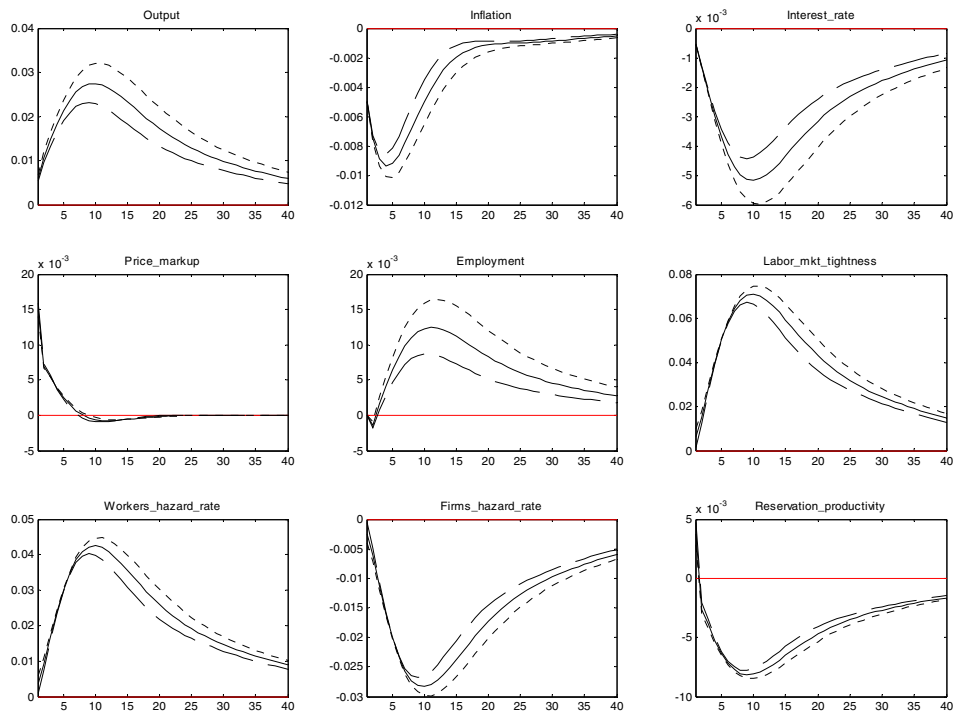


Figure 3: Impulse response functions to output shock. The baseline case is plotted by the solid lines, the dotted lines plot the impulse responses for  $\tau = 0.26$  and the dashed lines plot the impulse response functions for  $v = 0.04$ .

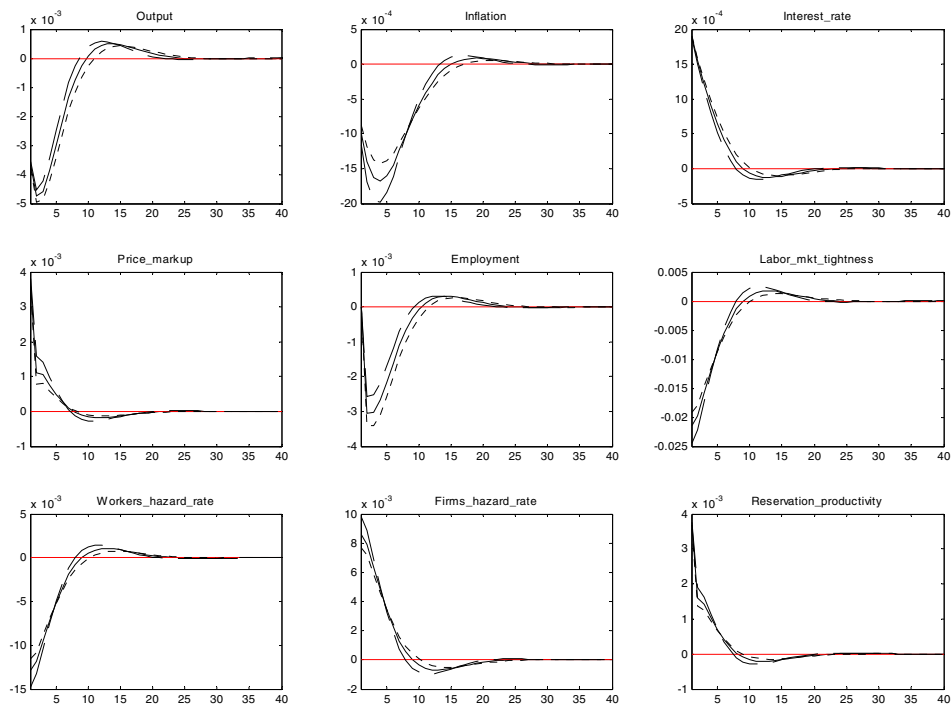


Figure 4: Impulse response functions to policy shock. The baseline case is plotted by the solid lines, the dotted lines plot the impulse responses for  $\tau = 0.26$  and the dashed lines plot the impulse response functions for  $v = 0.04$ .

functions for the replacement rate is not plotted as the plots overlap closely those of the marginal tax.

*Marginal tax rate* – The impulse response functions of a productivity shock are generally amplified by the marginal tax increase, but the shapes of the functions remain qualitatively the same. Both peak effects are larger and the shocks are more persistent. In fact, this effect is similar to the effect of reducing the bargaining power of workers described in Walsh (2003). This should not be surprising, considering the discussion in section 2.2.4 on the way the marginal tax affects the division of match surplus. A higher marginal tax increases the ‘non market’ component in the wage equation relative to the market sensitive part. This implies that the wage is more rigid and absorbs less of shocks, transmitting them on to the rest of the economy through job creation and destruction. The marginal tax affects the impulse responses to an interest rate shock in a more diverse way. The impulse responses of output and employment are amplified both in the peak effect and persistence. The peak effect of inflation is moderated but the impulse response is more persistent. This applies to the labor market tightness and to the firms and workers hazard rates as well.

*Tax subsidy* – The tax subsidy has the opposite effect to the marginal tax rate. The impulse responses to a productivity shock are smoothed: both peak effects and persistence are reduced by the tax subsidy. The tax subsidy increase has an opposite effect to the marginal tax in the wage equation. An increase in the tax subsidy increases relative size of the market sensitive component of the wage. This implies that the wage absorbs more of the shocks and less of them transmit to the rest of the economy. The tax subsidy smooths the impulse responses of output and employment wrt. an interest rate shock. The peak effects of inflation, labor market tightness and the hazard rates are amplified but the impulse responses are more persistent.

*Replacement ratio* – The replacement rate has qualitatively similar effects to the marginal tax for similar reasons.

### 5.2.2 Tax progression

We now proceed to investigate the importance of the structure of taxation for the dynamics of the economy wrt. shocks. As in the steady state analysis we consider increasing the marginal tax rate and making the necessary increase in the tax subsidy to keep government tax revenues neutral. A general remark to be made is that the same forces are at work here as in the steady state analysis: the policy setup of the labor market determines the relative effects of the tax parameters.

*Progressive taxes in initial equilibrium* – Figures 5 and 6 plot the impulse responses of the benchmark setup to productivity and interest rate shocks respectively. With an increase in tax progression the impulse responses are amplified, both in terms of peak effects as well as persistence. The reasoning is analogous to that of the previous section where the steady state effects were analyzed. The amplifying effect on the impulse responses of the marginal tax dominates that of the tax subsidy. For interest rate shocks the impulse

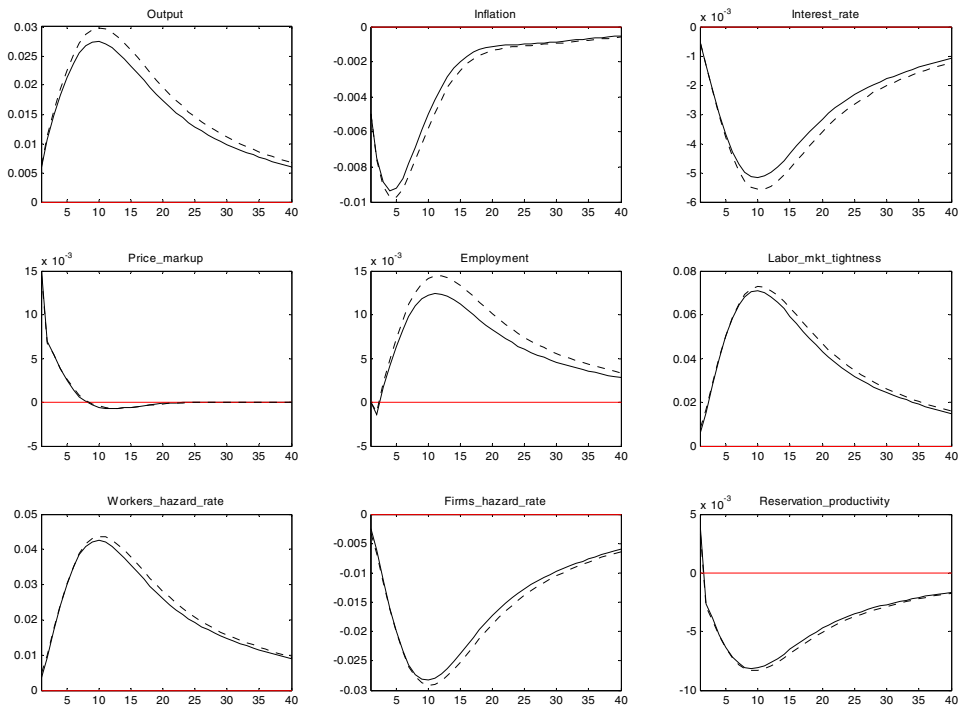


Figure 5: Impulse response functions to a productivity shock and tax progression. The baseline case is plotted by solid lines and increased progression is plotted by the dotted lines.

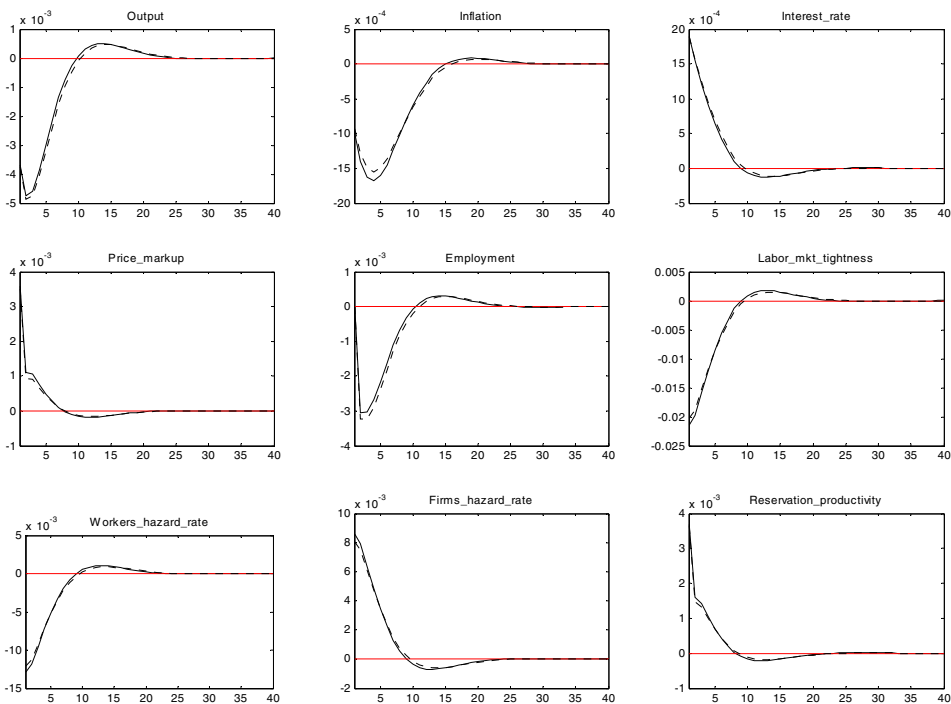


Figure 6: Impulse response functions to a policy shock and tax progression. The baseline case is plotted by solid lines and increased progression is plotted by the dotted lines.

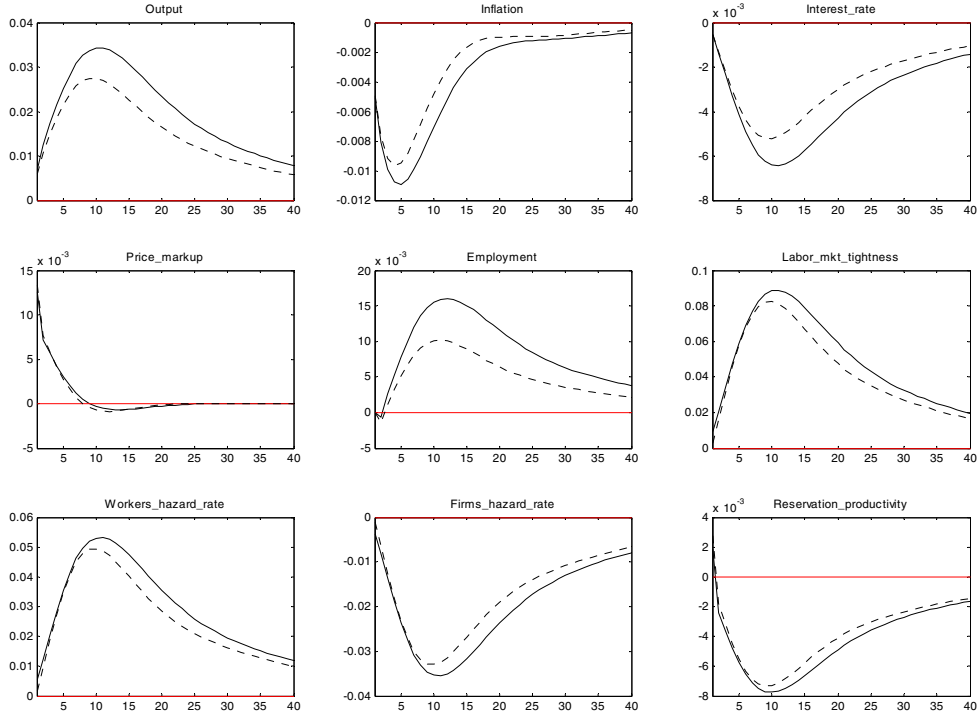


Figure 7: Impulse response functions to a productivity shock and tax progression. The initial value responses are plotted by solid lines and increased progression is plotted by the dotted lines.

responses are affected by tax progression qualitatively in the same way as by the marginal tax. Overall, tax progression implies a more volatile economy in the benchmark calibration.

*Proportional taxes in initial equilibrium* – The impulse responses to productivity and interest rate shocks for a calibration with the tax subsidy being zero in the initial state are plotted in figures 7 and 8. As in the steady state analysis the results of the alternative setup are opposite to the benchmark case. Now the impulse responses wrt. to a productivity shock are smoother and less persistent, both in terms of peak effects as well as persistence. In this alternative setup tax progression, or promoting income equality is consistent with a less volatile economy.

The implications of labor market policy depend crucially on the initial labor market policy scheme. As the marginal tax and the tax subsidy have opposite effects, their relative strengths depend on the initial degree of tax progression. To achieve any desired goals by using tax policies should bear in mind the specific context in to which the policies are implemented.



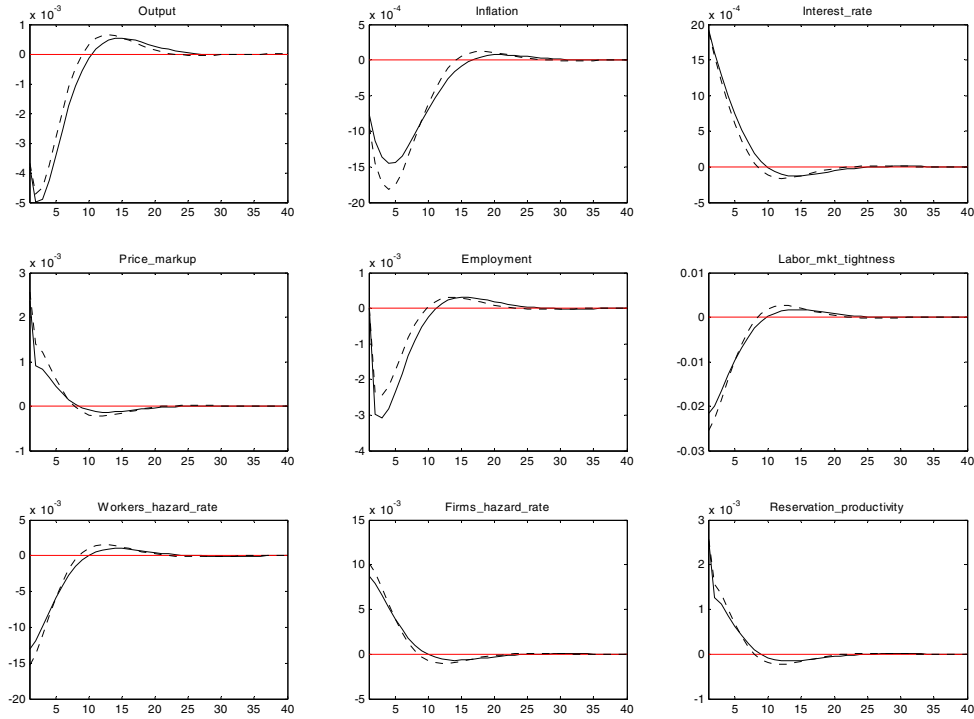


Figure 8: Impulse response functions to a policy shock and tax progression. The initial value responses are plotted by solid lines and increased progression is plotted by the dotted lines.

## 6 Concluding remarks

We have examined the effects labor taxation in a monetary business cycle model extended with search labor markets. The paper illustrates the importance of the initial state of the labor market in determining both the steady state and dynamic effects of labor taxation. The main conclusion is that the macroeconomic outcomes of tax reforms depend on the initial degree of tax progression which determines the relative effects of the tax instruments. In an economy with initially proportional labor taxation, increasing progression has desirable equilibrium employment and output effects and stabilizing dynamic effects. However, if the tax scheme is initially sufficiently progressive, increasing progression has opposite effects: the equilibrium employment and output effects are negative and the sensitivity to shocks is amplified.

Our simulations show that interactions of policy tools differ depending on the state of the labor market. As very different policy schemes are implemented in European countries and these countries have large variation in labor market outcomes, it would be of interest to study the implications of tax reforms in these different setups. Also, as a large set of policy instruments is available to the policy maker, a more comprehensive study including tools such as payroll taxes, hiring subsidies and firing costs would offer more insight into the effects of tax reforms and the alternatives available and trade-offs involved when designing tax reforms.

There are several issues that deserve attention in future research. We have investigated the effects of taxation on macroeconomic outcomes in a framework which incorporates the search-matching model of the labor market to a New Keynesian business cycle model. A word of caution regarding the results may be in order. Pissarides (1998) points out that there is no definitive model of the European labor market and shows that effects of changes in the structure and level of taxation sometimes depends on the underlying model of the labor market. One avenue for future research would be to consider the implications of the choice of the labor market model nested in the New Keynesian framework.

Finally, an important issue in matching models is the inefficiency typically produced by matching frictions and decentralized bargaining. An alternative approach to labor market policy is to design taxation so as to internalize search externalities and improve the efficiency of resource allocation. This question is also left for future work.

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# Appendix

## A.1 Bargaining and wage

The match surplus is shared between the firm and the worker according to the parameter  $\eta$  which represents the workers share of the match surplus (bargaining power). The wage rate thus satisfies

$$w_t = \arg \max (W_t(a_{it}) - U_t)^\eta (J_t(a_{it}) - V_t)^{1-\eta}. \quad (\text{A1.1})$$

The first order condition is given by

$$\begin{aligned} & \eta \frac{\partial W_t(a_{it})}{\partial w_t} (W_t(a_{it}) - U_t)^{\eta-1} (J_t(a_{it}) - V_t)^\eta \\ & + (1 - \eta) \frac{\partial J_t(a_{it})}{\partial w_t} (W_t(a_{it}) - U_t)^\eta (J_t(a_{it}) - V_t)^{-\eta} \\ & = 0 \end{aligned}$$

Divide both sides by  $[J_t(a_{it}) - V_t]^{\eta-1} [W_t(a_{it}) - U_t]^{-\eta}$  to get

$$\eta \frac{\partial W_t(a_{it})}{\partial w_t} (J_t(a_{it}) - V_t) + (1 - \eta) \frac{\partial J_t(a_{it})}{\partial w_t} (W_t(a_{it}) - U_t) = 0$$

where  $\frac{\partial J_t(a_{it})}{\partial w_t} = -1$  and  $\frac{\partial W_t(a_{it})}{\partial w_t} = 1 - T'(w_t)$  so the first order condition becomes

$$\eta [1 - T'(w_t)] J_t(a_{it}) = (1 - \eta) (W_t(a_{it}) - U_t) \quad (\text{A1.2})$$

Substituting the value equations and  $V_{t+1} = 0$  into the first order condition and cancelling terms produces

$$\begin{aligned} & [1 - \eta T'(w_t(a_{it}z_t))] w_t(a_{it}z_t) \quad (\text{A1.3}) \\ & = \eta [1 - T'(w_t(a_{it}z_t))] \frac{a_{it}z_t}{\mu_t} \\ & + \eta [1 - T'(w_t(a_{it}z_t))] E_t \beta_{t+1} \left[ (1 - \rho^x) q_{t+1}^w \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}) \right] \\ & + (1 - \eta) [A + h + b_t + T(w_t(a_{it}z_t))] \end{aligned}$$

where we have used

$$\eta [1 - T'(w_t)] E_t \beta_{t+1} J_{t+1}(a_{it}) = (1 - \eta) E_t \beta_{t+1} [W_{t+1}(a_{it}) - U_{t+1}]$$

given by the first order condition. Substituting equations (2.11), (2.12) and

$$\begin{aligned} E_t \beta_{t+1} (1 - \rho^x) [W_{t+1}(a_{it}) - U_{t+1}] & = \frac{\eta [1 - T'(w_t)]}{(1 - \eta)} E_t \beta_{t+1} (1 - \rho^x) J_{t+1}(a_{it}) \\ & = \frac{\eta [1 - T'(w_t)]}{(1 - \eta)} \frac{\kappa}{q_t^f} \end{aligned}$$

into (A1.3) and dividing both sides of the resulting equation by  $(1 - \tau)$  produces

$$w_t(a_{it}z_t) = \eta \left( \frac{a_{it}z_t}{\mu_t} + \kappa\theta \right) + (1 - \eta) \left( \frac{A + h}{(1 - \tau)} + \rho^\tau a_t^e z_t - (1 - \rho^\tau)v \right) \quad (\text{A1.4})$$

where  $\mu_t = \frac{P_t}{P_t^w}$ .

## A.2 Price rigidity and Phillips curve

Firms choose  $p_{jt}$  to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \frac{p_{jt}}{P_{t+i}} c_{jt+i} - \frac{P_{t+i}^w}{P_{t+i}} c_{jt+i} \right]. \quad (\text{A2.1})$$

where  $\beta_{t+i} = \frac{\beta^i \lambda_{t+i}}{\lambda_t}$ . Using the demand curve (2.30) faced by the firm we can eliminate  $c_{jt}$  to get the objective function and substitute and  $\mu_{t+i}^{-1} = \frac{P_{t+i}^w}{P_{t+i}}$  to get

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\varepsilon} - \mu_{t+i}^{-1} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\varepsilon} \right] C_{t+i}. \quad (\text{A2.2})$$

The first order condition is

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ (1 - \varepsilon) \left( \frac{p_t^*}{P_{t+i}} \right)^{-\varepsilon} + \varepsilon \mu_{t+i}^{-1} \left( \frac{p_t^*}{P_{t+i}} \right)^{-\varepsilon-1} \right] \frac{1}{P_{t+i}} C_{t+i} = 0.$$

Re-express this as

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ (1 - \varepsilon) \left( \frac{1}{P_{t+i}} \right) + \varepsilon \mu_{t+i}^{-1} \frac{1}{p_t^*} \right] \left[ \left( \frac{p_t^*}{P_t} \right) \left( \frac{P_t}{P_{t+i}} \right) \right]^{-\varepsilon} C_{t+i} = 0$$

Divide by  $\frac{p_t^*}{P_t}$  and rearrange

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \left( \frac{1}{P_{t+i}} \right) \left( \frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \\ &= \frac{\varepsilon}{(\varepsilon - 1)} E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \mu_{t+i}^{-1} \left[ \frac{1}{p_t^*} \left( \frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \end{aligned}$$

Multiply and divide the left side by  $P_t$

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \frac{1}{P_t} \left( \frac{P_t}{P_{t+i}} \right) \left( \frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \\ &= \frac{\varepsilon}{(\varepsilon - 1)} E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \mu_{t+i}^{-1} \left[ \frac{1}{p_t^*} \left( \frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \end{aligned}$$

Then multiply both sides by  $p_t^*$  and rearrange to obtain

$$\frac{p_t^*}{P_t} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \mu_{t+i}^{-1} \left( \frac{P_{t+i}}{P_t} \right)^\varepsilon C_{t+i} \right]}{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[ \left( \frac{P_{t+i}}{P_t} \right)^{\varepsilon-1} C_{t+i} \right]} \quad (\text{A2.3})$$

The aggregate price is given by

$$P_t^{1-\varepsilon} = (1 - \omega) (p_t^*)^{1-\varepsilon} + \omega p_{jt-1}^{1-\varepsilon} \quad (\text{A2.4})$$

where a fraction  $(1 - \omega)$  adjusts price optimally and a fraction  $\omega$  adjusts according to rule of thumb. We assume that firm  $j$  uses a rule of thumb based on the most recently observed rate of inflation and the most recently observed price level  $P_{t-1}$ ,

$$p_{jt} = \pi_{t-1} P_{t-1}. \quad (\text{A2.5})$$

To obtain an expression for aggregate inflation, equations (A2.3) and (A2.4) can be approximated around a zero average inflation steady state equilibrium.

### A.3 Steady state equations

In steady state we have  $\pi_t = 0$  and  $p_t^* = P_t = P$  and  $z_t = z = 1$ . This implies that the household's Euler condition reduces to  $R = \frac{1}{\beta}$  and the steady state values of  $n, \rho, u, q^f, q^w, jc, \theta, w, \tilde{a}, C$  and the policy variables  $TR, T$  and  $b$  are given by the steady state versions of equations (2.4), (2.5), (2.7), (2.8), (2.9), (2.10), (2.26), (2.23), (2.27), (2.29), (2.38), (2.11) and (2.12)

- Firm's hazard rate

$$q^f = \frac{m(u, v)}{v} \quad (\text{A3.1})$$

- Worker's hazard rate

$$q^w = \frac{m(u, v)}{u} \quad (\text{A3.2})$$

- Destruction rate

$$\rho = \rho^x + (1 - \rho^x) F(\tilde{a}) \quad (\text{A3.3})$$

- Employment

$$\rho n = m(u, v) \quad (\text{A3.4})$$

- Unemployed job seekers

$$u = 1 - (1 - \rho) n \quad (\text{A3.5})$$

- Net job creation

$$jc = \frac{m(u, v)}{n} - q^f \rho^x \quad (\text{A3.6})$$

- Government tax revenue

$$TR = (1 - \rho^x) nT(w) - (1 - n) b \quad (\text{A3.7})$$

- Worker's tax transfer to government

$$T = \tau w - (1 - \tau) v \quad (\text{A3.8})$$

- Unemployment compensation

$$b = \rho^\tau (1 - \tau) (w + v). \quad (\text{A3.9})$$

- Free-entry

$$\frac{\kappa}{q^f} = \beta (1 - \rho^x) (1 - \eta) \frac{1}{\mu} \int_{\tilde{a}}^{\bar{a}} (a_i - \tilde{a}) dF(a_i) \quad (\text{A3.10})$$

- Wage

$$w = \eta \left( \frac{H(\tilde{a})}{\mu} + \kappa \theta \right) + 1 - \eta \left( \frac{h}{1 - \tau} + \rho^\tau H(\tilde{a}) - (1 - \rho^\tau) v \right) \quad (\text{A3.11})$$

- Job destruction treshold  $\tilde{a}$

$$\frac{\tilde{a}}{\mu} - \frac{h}{(1 - \tau)} - \rho^\tau H(\tilde{a}) + v (1 - \rho^\tau) - \frac{\eta}{1 - \eta} \kappa \frac{q^w}{q^f} + \frac{1}{1 - \eta} \frac{\kappa}{q^f} = 0 \quad (\text{A3.12})$$

- Aggregate income and consumption

$$Y = C = (1 - \rho) nH(\tilde{a}) - \kappa v \quad (\text{A3.12})$$

- The steady-state price markup

$$\mu = \frac{\varepsilon}{\varepsilon - 1}.$$



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