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23/95

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Research Department
3.8.1995

Fiat Exchange in Finite Economies

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Abstract

The state of the art of rendering fiat money valuable is either to impose a boundary condition, or to make the boundary condition unimportant by using infinities concerning the sequence of markets and/or the number of agents, so as to circumvent backward induction. We present two models of fiat exchange in deliberately finite economies in which the usage is not imposed. In the first approach agents have incomplete information about their relative position in the trade cycle. The second approach relies on the possibility that multiple non-monetary equilibria of the one-shot game can support monetary equilibria in the repeated game.

Tiivistelmä

On olemassa kaksi tapaa, jolla rahalle saadaan jokin arvo. Voidaan joko asettaa jokin rajaehto rahan määrälle tai voidaan tehdä rajaehto merkityksettömäksi käyttämällä äärettömää määrää markkinoita ja/tai talousyksiköitä. Siten voidaan kiertää takaisinpäin tapahtuva induktio. Esitämme kaksi rahan vaihtoa koskevaa mallia, jotka koskevat selkeästi äärellisiä talouksia, joissa tätä ehtoa ei ole asetettu. Ensimmäisessä lähestymistavassa talousyksiköillä on epätäydellinen informaatio suhteellisesta asemasta suhdannesyklin suhteen. Toisessa lähestymistavassa tukeudutaan siihen mahdollisuuteen, että yhden askeleen peliin liittyvät useat ei-monetaariset tasapainot voivat tukea monetaarista tasapainoa toistetussa pelissä.

Contents

1 Modelling Fiat Money

The eloquent review by Hellwig (1993) on the foundations of monetary theory is centered around the Hahn (1965) problem. In the words of Hellwig: "Why does fiat money have a positive value in exchange against goods and services even though it is not intrinsically useful?" Hahn observed that in the Walrasian type general-equilibrium models there is no role for fiat currency. Because these models are static, they cannot adequately capture the dynamic store-of-value function that money serves between transactions. The first requirement of any theory of money is therefore to be explicit about the sequential nature of monetary exchange. The literature on the topic is almost unanimous on this point.

Granting the necessity of a sequence economy, the currently known solutions to the Hahn problem can be broadly classified along two lines. In a dynamic economy fiat money has value in a transaction because it is expected to be valuable in future exchange. Borrowing terminology from the theory of differential equations, fiat money can be rendered valuable either by imposing a boundary condition, or by pushing away the boundary condition to infinity. Both are devices to circumvent the unravelling of the monetary equilibrium through backward induction. Consider for example the following quote from Cass and Shell (1980, p. 252) who, in their defense of an indefinite future as a prerequisite for modelling money argue:[1]

> "It is obvious (and well-known) that money cannot have a positive price – that is, cannot be a store of value- in the conventional finite-horizon model in which the 'end of the world' is known with certainty. The reason is simple. At the end of the last period, money is worthless. Therefore, in the next-to-last period, all individuals desire to dispose of money holdings in order to avoid capital losses. This drives the price of money to zero at the end of the next-to-last period. And so on. Individuals with foresight, not wanting to be stuck with the monetary 'hot potato', thus drive the price of money to zero in each period."

We now review these two solutions in some detail. The first response is axiomatic and resembles the way in which the budget constraint is handled in micro theory. The resemblance is closest with the Clower (1967) or cash-in-advance constraint, which is a specific form of a transaction technology such as in Hahn (1971) and Niehans (1971). The Clower constraint 'requires' that transactions are settled in terms of money. Thus cash balances need to be sufficient to cover the expenditures. Similarly, the budget constraint 'requires' that expenditures do not exceed income. The so called legal restrictions theory, see Wallace (1983), imputes essentially the same requirement.

[1] Note that, counter to some popular folklore, the backward unravelling also occurs if the end of time is uncertain, but bounded from above by some finite number. This is exploited by the play of langugage in the riddle of the prisoner who has read her verdict. The prisoner is told that she will be released during the next week at a day which is chosen at random. Her face brightens. But then the verdict adds the Kafkaesque statement that she will not know the eve before that she will be released the next day. Her face darkens.

A somewhat different tack is based on Lerner's (1947) idea of an external agency that is able to impose balanced budgets at the end of time. This is made explicit in Shubik (1981), who imposes a bankruptcy penalty. Obviously, the bankruptcy penalty serves as a boundary condition on the differential equation for the dynamic behavior of the agents. Another way of doing this is by sticking money directly into the utility function, or indirectly on the grounds of some transactions technology, see e.g. McCallum (1983). Albeit less obvious, this is formally equivalent to explicitly postulating a boundary condition like the bankruptcy penalty (if the sequence of markets is finite).

An effective boundary condition not further rationalized by the existence of an outside agency or transactions technology, but supported by appealing to the bounded rationality of agents was introduced by Grandmont (1982), see also the exposition by Radner (1982). Grandmont assumes that agents always attach a positive probablity to money having a positive price in the next trade, even in the last market.

A number of authors felt the boundary condition approach involved too much adhocery. To make explicit what is left implicit in Grandmont's temporary equilibrium cum bounded rationality approach, one might remove the finite upper bound on the sequence of trades. Thus, with positive probability the sequence extends beyond any given finite number of trades. This renders the Grandmont boundary condition compatible with rational expectations. Because, if the sequence is endless, it never becomes binding and needs not to be imposed.

Today's most popular model, in the class of models validating the use of money by pushing the boundary condition to infinity, is probably still Samuelson's (1958) overlapping generations (OLG for short) model, see e.g. the comprehensive treatment in Balasko and Shell (1981). Even though each agent only lives for two periods, the infinity of periods and generations may support a monetary equilibrium.[2] The monetary OLG model has been explored initially by Bryant (1980), Wallace (1980), Townsend (1980), Starr (1980), Gale (1982) and Hahn (1982). But it is still used by many authors as the starting point of their analysis, see e.g. Sargent (1987), Azariadis (1993) and Farmer (1993).

A more recent approach is the one followed by Kiyotaki and Wright (1989). They extend the work of Jones (1976) which showed the endogeneous appearance of a commodity money. Within the setup of a random matching model and specialization of agents Kiyotaki and Wright derive the endogeneous appearance of a fiat currency. In order to sustain the fiat money equilibrium it is assumed that there is a continuum of agents who are infinitely lived. Other work in this search theoretic vein includes Kiyotaki and Wright (1993) and Ritter (1995). Williamson and Wright (1994) use the search model to study the informational role of money, i.e. in the sense that the quality of money is more easily verified than the quality of other commodities, as was initially discussed by Brunner and Meltzer (1971). Banjerjee and Maskin (1991) also focus on the informational role, albeit in a Walrasian setup. The endogenous formation of prices through bilateral bargaining, as was first discussed in De Vries (1986), is combined with the search literature in Trejos and Wright (1995).

[2] Some care has to be taken that the (Pareto optimal) monetary equilibrium is within the core; see Kovenock (1984) and Fisher (1994).

In De Vries (1986) and Faust (1989) the infinity of periods is replaced by the assumption of finite but continuous time. In this way one 'maps' the countable infinity of periods into the uncountable infinity of instances over a bounded interval, where there is no penultimate instance of time. In the setup of Faust, agents receive their instaneously perishable endowment before part of their consumption needs are realized.[3] Because different agents are paid at different instances, fiat exchange can achieve the desired intertemporal consumption smoothing. Even towards the end of time, the value of money does not drop to zero because marginal utility is unbounded at zero.[4]

The successes of the above analyses not withstanding, there are, however, several motives for investigating the possibilities for monetary exchange in a finite economy. One motive is that infinite economies usually permit a rich multiplicity, often a continuum of equilibria (folk theorem). This indeterminacy does not give these theories high predictive power. Another motive is that the (atemporal) continuum of agents assumption is easily falsified, and that the possibility of an unbounded time axis or a continuum of trades (quantum mechanics) is questionable on the basis of physics. And even if an endless life of the universe or continuous trading were a possibility, then the theory is still untestable on the crucial assumptions that drive it. In this respect it is of interest to mention the experimental work on bubbles, as reported in Smith et al. (1988) and Plott (1991), and (monetary) overlapping generations model like in Marimon and Sunder (1990) and Aliprantis and Plott (1992). These experiments all invoke a boundary condition by necessity, but the theories to be tested rely on the possibility of an infinite sequence of markets. Thus while these experiments are of interest, and the models of finite economies which are actually being tested are interesting as well, the experiments do not provide a test of the theory. Empirically, the boundary condition approaches and the infinity approaches appear to be equivalent. It is sometimes suggested that one should take the models that rely on infinities as approximations to reality. But the experimental work tries just the opposite. This yields yet another motive for studying finite monetary economies. If the infinite economy is an approximation to the finite economy, one may do better by explicitly studying the finite economy.

In game theory a similar development has taken place. The folk theorem of infinitely repeated games was regarded unsatisfactory for similar reasons. Two responses to the backward induction unravelling have been developed. The first approach based on adding a small amount of incomplete information was pioneered by Kreps and Wilson (1982) and Milgrom and Roberts (1982). The presentation in Kreps (1990, pp. 536–544) of the resolution in terms of the centipede game is of particular relevance for our case. The backwards unravelling of the centipede game can be circumvented if, with some very small probability, there is another payoff structure such that the first mover has an incentive to continue playing if nature selects this state of affairs. The second

[3] We note in Faust's model that the uncertainty about the time h when the future consumption needs realize is not driving the result. Letting g denote the time the endowment is received and letting T denote the end of time, then h = (g+T)/2 suffices.

[4] This assumption appears to be crucial to prevent unravelling of the monetary equilibrium. Alternatively, some positive storage capability would also hamper monetary exchange.

player being aware of this possibility, but not being informed about the true state of nature, may now deem it profitable to continue playing if the first agent does so. It follows that each player may choose to continue playing even if nature has not selected the alternative payoff structure for the first mover, because the second player is uninformed. Just the mere possibility is enough to get 'cooperation' started.

The other game theoretic resolution relies on the presence of multiple equilibria in the stage game. Benoit and Krishna (1985) show that if there are multiple equilibria in the single shot version, then the finitely repeated analogue can have other equilibria that are perfect. The idea is that if the multiple equilibria of the single shot game are payoff nonequivalent, then it may be possible to support a multistage equilibrium that does not consist of a sequence of single shot equilibria, through the threat of playing an equilibrium with a lower payoff for a deviating player in the last round. In the last round only the single shot equilibria can be played, and hence playing the less-desirable equilibrium is a credible threat.

In the third section we show how a monetary equilibrium can evolve in a finite economy by using Benoit and Krishna's theory. This is of some interest because of the following remark in the conclusion of Faust (1989): "It is difficult to imagine what second equilibrium could exist in the final period of the monetary model besides one in which the value of money is zero." Faust recognizes the Benoit and Krishna analysis as the: "only one other relevant solution to the terminal problem", but does not think that it can be used to validate fiat money because there cannot be multiple and payoff nonequivalent non-monetary equilibria. Our positive result using the Benoit-Krishna framework builds on the presence of an autarkic equilibrium that is dominated by a barter equilibrium in the single shot version of the game. Both are genuine non-monetary equilibria, as the barter equilibrium relies on the double coincidence of wants.

The second section exploits the idea of agents being incompletely informed. But the theory we develop is quite different in spirit from the Kreps-Wilson-Milgrom-Roberts resolution. In these models there is a small probability that some agent has a different nature, more conducive to cooperation; or has a different payoff structure such that under these circumstances the agent has an incentive not to defect immediately. The possible presence of such an alternative payoff structure may make sense in many circumstances, but does not seem plausible for the problem at hand. It is hard to imagine the possibility that there exists an agent in the last market of the economy who is willing to accept intrinsically worthless paper money in return for giving up valuable goods and services, unless one is willing to entertain the possibility that agents are boundedly rational as in Radner's (1982, sect. 3.6) interpretation of Grandmont (1982). In that case agents, even in the last market, attach a positive probability of money having value in the future, and are therefore willing to accept fiat money at all times.

We therefore propose an incomplete information structure which is amenable to the case of monetary economies. As was noted in fn. 1, uncertainty about when the world comes to an end is not enough to prevent backward unravelling. One simply applies the argument to each branch of the probability tree. But with incomplete information the information sets are no longer singletons. The probabilities, reflecting beliefs about types, are jointly determined with the strategies, i.e. are not exogenously imputed. And hence backward induction cannot be used. In effect, we will assume that agents have incomplete information about exactly where they are located in the chain of (monetary) exchanges. They know who they are trading with, but they cannot monitor who started the chain and where it ends. This uncertainty prevents one from invoking the backward induction argument. The uncertainty we introduce has the effect of turning the game into a lottery over who will end up with the 'hot potato'.[5] The penultimate agent, who does not know that he is next to last because he does not know how many went before him, may therefore decide to accept money from the agent that comes before him because he attaches a high enough probability to not being the penultimate agent. (Note that this is different from the case where one does know how many went before, and one just faces the uncertainty over whether there will be one other stage or not, cf. fn. 1).

Interestingly, and in contradistinction with the Kreps and Wilson (1982) resolution of the chain store paradox, we do not add a branch to the game along which players have an incentive which is opposite from the incentive along the other branch. In fact, if agents knew along which branch they were playing, then the monetary equilibrium would always unravel. Thus we do not rely on the possible existence, albeit with slightest positive probability, of a Santa Claus who picks up the final bill.

The next section starts with a finite and even very small economy of three agents who buy and sell only once. Because the economy is so small, the uncertainty over one's position in the chain needs to be quite large. This requirement, however, may rapidly be relaxed as the chain of exchanges becomes longer and longer. At the same time, this effect renders plausible the assumption that agents have imperfect information over where exactly their exchanges are located in the chain of exchanges. Given the incredible number of transactions that take place every day, there are not many people who have an idea where their money goes one or two transactions after they have spent it.

The only attempt to model fiat money in a finite economy that we are aware of is the recent and interesting work by Kultti (1994). In essence, Kultti considers the atemporal OLG model as developed by Cass and Yaari (1966) with a finite number of agents. The agents attach zero utility to the good they are endowed with, but desire the good of their left neighbor. There are two equilibria, one is autarky, and the other involves trade, whereby one sells to the right hand neighbor and buys from the left hand neighbor. Because there is no double coincidence of wants, this might be designated monetary exchange. But note that the first agent along the ring, who initiated the monetary exchange, only accepts the monetary bills from the last agent along the ring because the goods he gives up in return for the money yield him no utility. As soon as these endowments do yield some utility, the first agent has an incentive to renege and the monetary exchange breaks down; cf. the discussion of Faust above. In

[5] We note that many people appear to be willing to play the state lottery, which is commonly known to be a negative sum game. In our case, like in the centipede game, the monetary equilibrium can easily yield a positive sum to every one. A well known hot potato game is the children's card game Old Maid, or 'Zwarte Pieten' as it is called in Dutch and German. In this game all cards come in pairs, except for one card which is the hot potato.

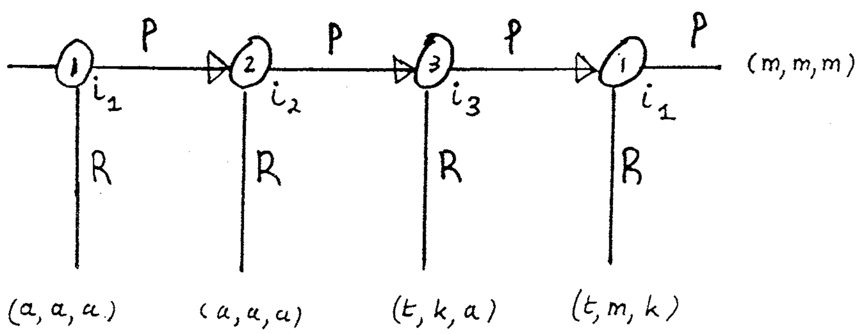
comparison, the equilibrium we discuss breaks down with complete information over positions in the chain of exchanges, but appears otherwise robust.

The paper does not deal with the velocity, price formation, dominance and search issues. The dominance issue can, in our opinion, be satisfactorily treated by recognizing the different marketability properties of bonds versus currency, see e.g. Banerjee and Maskin (1991). Moreover, with the advances in technology, interest paying currency may well replace the non interest paying fiat money we know today. The experience with the French interest bearing war time currency, and the usage of credit cards or emoney through internet, points in this direction. The determination of velocity is search technology dependent and is therefore better treated in a more elaborate model that recognizes richer specialization patterns as in Kiyotaki and Wright (1989). The discussion in Hellwig (1993) is also illuminating in this respect. Treating all these intricate issues in a single paper seems impossible, and we therefore focus on the Hahn problem.

2 Helicopter Money

Consider a very simple economy made up of 3 agents, labelled 1, 2, and 3. Agents always meet in the following order 1, 2, 3, 1, 2, 3, 1, ... By this we mean that agent 1 first encounters agent 2, then 2 meets 3, and 3 meets 1, etc. The chain of meetings does not necessarily start with agent 1, i.e. 2, 3, 1, 2 ... is admissible, but the order of meetings is the same. To begin with, we only consider a single round of meetings, e.g. 1, 2, 3, 1. During a meeting agents can decide to trade, but there is an absence of a double-coincidence-of-wants. For simplicity we assume agents are specialized so that the lower number agent has (net) nothing to offer to the higher labelled agent; except agent 1 whose goods are desired by 3. Thus one can view agents being grouped around a triangle. Each agent values his endowment positively so that autarky is a possible outcome. The agent's payoff to autarky is labelled as "a". In the aggregate, though, there is quid-pro-quo. Therefore, if all agents could meet together the Walrasian trade equilibrium would yield a payoff vector (m, m, m), where m > a. The friction imposed by the sequential encounters, however, necessitates sequential trade imbalances. How are these imbalances settled?

Figure 1. Backward Unravelling



t > m > a > k

Suppose the chain of exchanges is: 1, 2, 3, 1. And suppose agent 1 is handed some fiat money that can be used to settle temporal trade imbalances. Then fiat currency performs the role of a tally, and the Walrasian outcome (m, m, m) might be observed in the decentralized economy. The situation is depicted in Figure 1; capital P along a branch that eminates from the node or information set i_j, j = 1,2,3, indicates that the agent j decides to trade and has accepted currency from the agent on his left side. Note, however, that at the final node when agent 3 offers currency to 1 in return for goods, agent 1 has an incentive to renege and not honour the notes. At the second node agent 1 was a net receiver of goods with a payoff of "t", say, such that $t > m$. By accepting the fiat currency at the last node, he would give up something for nothing. Suppose that the payoff to agent 3 in the case that 1 refuses his money is $k < a$. Due to the fact that agent 3 accepted money from 2, and is unable to spend it on agent's 1 commodities, he is worse off than if he had remained autarkic. Realizing this, agent 3 does not engage into monetary exchange with agent 2. This is depicted by the 'R' branch in Figure 1. Continuing this backward induction, it follows that the monetary equilibrium collapses at the outset. The autarkic equilibria (PR, R, R) and (RR, R, R) with payoff (a, a, a) are the only subgame perfect equilibria.[6]

The collapse of the monetary equilibrium clearly also occurs if agent 2 or 3 is handed the money instead of agent 1. It also does not help to randomly initiate the trade cycle so long as it is observed who gets the money first. We now introduce some incomplete information which we believe is plausible within economies with large and highly interwoven chains of transactions. Suppose a helicopter randomly, i.e. with probability $\pi = 1/3$, drops the fiat money at one of the agent's doorsteps. Each agent knows whether or not he has received the money. Thus the agent who receives the money also knows he will be the first and the last in the chain of transactions. But the other two agents remain in the dark about which of the other agents has received the money.

The new situation is depicted in Figure 2 (with the 'helicopter' situated in the center). The dashed curves indicate the uncertainty of an agent about the node at which he is located. Information set i_j^1 j = 1,2,3 represents the node at which the money has been dropped on the doorstep of agent j. If agent j decides to trade the money for goods, he will meet with agent (j+1)(mod 3); and (j+1)(mod 3) must decide whether to accept or reject the money. Since (j+1)(mod 3) does not know which of the other agents received the money first, he must choose an action at an information set i_{j+1}^2 which has two nodes. If agent j+1 chooses to accept the money, he then attempts to pass the money to agent (j+2)(mod 3) who, in turn, cannot distinguish which of the nodes in i_{j+2}^2 is relevant. If agent j+2 accepts the money, he in turn will try to pass it to agent j. At information set i_j^3 agent j finds it optimal to reject the money.

It is evident from the information structure illustrated in Figure 2 that a strategy for player j, j = 1,2,3 in the game, is a triple (s_j^1, s_j^2, s_j^3) specifying whether to accept or reject money in return for goods at information sets i_j^1, i_j^2 and i_j^3. It is always a conditionally strictly dominant strategy for j to reject

[6] The first entry of the strategy vector denotes the actions chosen by player 1 at his two information sets i_1 and i_4. The second entry gives player 2's action at the information set i_2, and the third entry is player 3's action at i_3.

money at i_j^3 and a weakly dominant strategy to offer monetary exchange at i_j^1. Behavior at i_j^2, j = 1,2,3 determine the nature of the equilibrium.

Proposition 1. Let money be randomly dropped to one of the agents; i.e. π = 1/3. Suppose payoffs are such that k + m > 2a. Then there exists a symmetric sequential (monetary) equilibrium in which each player j uses a strategy (P, P, R) and attaches probability 1/2 to each node in the information set i_j^2, j = 1,2,3. Along the branch of the game tree in which player j must receive money, j receives a payoff t, (j+1)(mod 3) receives m and (j+2)(mod 3) receives k. Each player's expected payoff in the game is (t + m + k)/3.

Proof. We need only verify the sequential rationality and consistency of the assesment specified by the equilibrium. The random action of the central bank is completely revealing to the agent who is issued the money. Thus the agent knows he is the first and the last agent in the chain, and that the other two agents are uninformed about his identity. Clearly, because it is conditionally strictly dominant for j to reject money at i_j^3 and weakly dominant to accept money at i_j^1, the actions specified at these information sets are sequentially rational. Any agent j who must make a choice at i_j^2 updates as follows: Given the knowledge that he was not issued the money first, agent j allocates probability ½ to be along each one of the two remaining branches. Since j is offered monetary exchange by agent (j−1)(mod 3), j calculates ½.1/(½.1 + ½.1) = ½ as the probability of being on either of the branches, since in equilibrium along either branch he is offered monetary exchange with probability 1. Given the strategies chosen by his rivals, accepting money is a best response, since (k + m) > 2a. Consistency follows immediately because in equilibrium every decision made is reached with positive probability. □

Remark 1. Along the branch that starts with 1 suppose that for some reason 1 plays R. Or suppose 2 does not accept the money. Does 2 have an incentive to print his own money? If 2's money is distinct from the helicopter money, then 3 infers along which branch he is and rejects to trade with 2. If perfect counterfeiting is possible, no monetary exchange would develop either. Thus in both cases printing one's own money does not pay.

Corollary 1. The other two symmetric perfect Bayesian equilibria are the autarkic equilibrium (R, R, R) or (P, R, R), and a mixture between the monetary and autarkic equilibrium whereby the uninformed players accept money at i_j^2 with probability

$$q = (a - k)/(m - a).$$

Proof. First, consider the autarkic equilibrium. Note that the Bayesian updating process is irrelevant for the decision as to whether to accept the money or not. The Nash property is verified as follows. At any node of the game agents are expected to reject trading in the continuation of the game. Therefore it never pays for the uninformed to accept money, even if an out of equilibrium offer of monetary exchange occurs.

Second, to derive the mixed strategy equilibrium we first focus on the Bayesian updating process of player 2 when he is not the first receiver of money. Along the branch starting with 1 he is offered the money with probability 1. Along the branch starting with 3 he is offered to trade with 1 with probability $q < 1$. Note that this is different from the case of Proposition 1 where $q = 1$. The probability to be at either branch is 1/2. Therefore the probability to be along the branch which starts with 1 is given by Bayes' rule: $\frac{1}{2}.1/(\frac{1}{2}.1 + \frac{1}{2}.q) = 1/(1 + q) > 1/2$. In equilibrium the uniformed must be indifferent between refusing to trade for which he receives a payoff a, and accepting to trade:

$$a = \frac{q}{1+q}k + \frac{1}{1+q}[qm + (1-q)k].$$

If he accepts to trade he is either the penultimate agent and gets k, or he is second to last and the next agent in line determines to trade or not to trade through randomization. Solving this equation for q yields $(a-k)/(m-a)$. Furthermore sequential rationality is clearly satisfied. □

Corollary 2. With n agents and n branches, $n > 2$, and $\pi = 1/n$, then (P, ..., P, R) is a perfect Bayesian monetary equilibrium if

$$\frac{n-2}{n-1}m + \frac{1}{n-1}k > a.$$

Proof. Analogous to the proof of Proposition 1. □

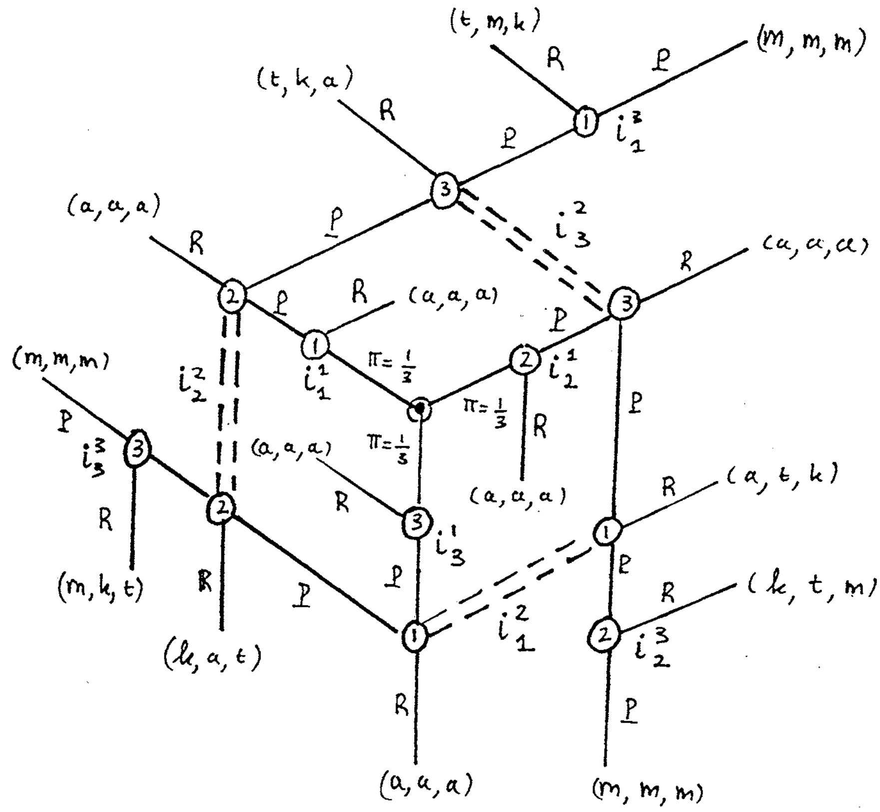
Remark 2. The possible loss of being the penultimate agent rapidly becomes a negligible factor as n increases. Thus the question of whether to trade or not to trade is almost completely determined by the differential $m-a$. In fact, for any triple $m > a > k$ and a number of agents $n \geq n^*$,

$$n^* = 1 + (m-k)/(m-a),$$

the above monetary equilibrium can be supported. Presumably a and k are not too far apart. Hence, n is much larger than n^* in our current economies with seemingly endless chains of transactions. This is why we believe that our assumption of asymmetric information is both plausible and supportive for the existence of a monetary equilibrium.

Instead of considering longer and longer chains, it is also of interest to consider repetitions of the basic chain, with money being issued anew in every round. Suppose that the simple 3-agent chain economy is replicated s times before it comes to an end. The monetary equilibrium (P, P, R) obtains under the same conditions as in Proposition 1. The reason is that only the last cycle counts. If a monetary equilibrium exists in the last round, then, by backward induction, it can exist all along. Another equilibrum is of course the autarkic solution (R, R, R). Alternations between the different equilibria of Proposition 1 and Corollary 1 are possible too.

Figure 2. Helicopter Money



Instead of considering s repetitions of the game with money being issued anew in every trading round, it is of interest to consider multiple trading rounds whereby the money is carried over from one round to another (the money is issued only once). Without proof we state the result for these latter type of trading rounds.

Corollary 3. With s trading rounds the monetary equilibrium (P, P, P, ..., P, P, R) yields along a specific branch the payoff vector

$$([s-1]m+t,\ sm,\ [s-1]m+k),$$

and an expected payoff of (s−1)m/3 + (t+m+k)/3. The autarkic equilibrium yields the payoff vector (sa, sa, sa). □

Remark 3. The single round monetary equilibrium yields an ex post loss to the penultimate agent in comparison with the autarkic equilibrium. But $s > 1$ trading rounds (money being issued only once) of the monetary equilibrium yield a gain to the ultimate loser as long as: (s−1)m − (a−k) > 0. Under the same conditions as before, i.e. a being close to k, m being relatively large compared to a, and s being very large, it would be hard to swallow a backward induction argument that renders money worthless to begin with, even though the ultimate loser stands to gain a lot from cooperating. With the above asymmetric information resolution, though, no such problems arise. And the size conditions on s, m, a and k seem reasonable for our economies.

Remark 4. We noted above that a perenial problem with the models that rely on infinities is, that these models are not directly testable. Only the implications, in so far as these are unambiguously related to these models, can be used in trying to falsify these models. Alternatively, the models can be altered by turning them into finite (time) economies by imposing a boundary condition that reflects the rational expectations solution if time were indefinite. This is the approach taken by e.g. Marimon and Sunder (1990), Aliaprantis and Plott (1992) in testing OLG models experimentally. But theorists might not regard this as a valid test, because the infinity is omitted. We note that the monetary models we introduced lend themselves easily to an experimental design and test, because they are finite in every respect.

3 Money as a Sanctioned Exchange Intermediary

We now return to our claim in the introduction that the Benoit and Krishna (1985) construction, in which multiple payoff-nonequivalent equilibria in the single-shot game can support Pareto superior equilibria in the repeated game, is applicable to the problem of valuing fiat money. Consider again three agents who play the following dynamic game: At stage 1 the agents simultaneously decide whether to produce for autarky and not go to the market (action A) or to produce for trade and go to the market (action T). This pre-market "production stage" is intended to capture the idea that gains from specialization and scale may arise when producing for trade but that these gains may be lost when agents must provide a different set of goods at lower volumes in an autarkic solution. And the impossibility to communicate inhibits coordination on the Pareto superior production decisions, cf. Bryant (1983). All agents know the outcome of the first stage before the start of the second stage.

At the second, "market stage" several possibilities might arise. First, and trivially, there may be no exchange on any market because no more than one agent chose the action T (see Jehiel and Yanelle, 1995, for an extensive treatment of this outcome). In this case, we normalize the autarkic agents' single payoff to 7.5 and assume that a single agent producing for trade earns a lower payoff, 6, due to the loss in consumption resulting from production specialization (and possibly the cost of going to market).

If exactly two agents choose to produce for trade, monetary exchange and bilateral exchange are equivalent. We assume that the endowments of skills and other factors are such that agent (j+1)mod 3 produces the good most desired by agent j, but that the goods produced by j provide only moderate gains from trade for (j+1). We also assume that goods have an indivisibility of ½. Hence, when agents j and (j+1)mod 3 engage in bilateral trade there is a gain for both relative to the situation where they would be the sole agent that produced for exchange, but j gains more. For simplicity we assume that the payoff to j in such a two player coalition is 9 while the payoff to (j+1) is 6.5. All payoffs are recorded in Figure 3 below. In the Appendix these payoffs are backed up by a fully articulated, though simple, exchange economy with preferences and production possibility sets.

When all three agents produce for the market we must carefully specify the sequence of trade. We assume a Wicksellian triangular pattern of trade in which the agents are aligned in order 1,2,3. Agent 1 first meets agent 2 in market 1. Then 2 meets 3 in market II and, finally, 3 meets 1 in market III. Following the Samuelsonian "iceberg" model of trade, transportation costs and the limited production possibilities are such that it only pays to transport an item once. Hence, no commodity money can exist. On the other hand, it is assumed that the intrinsically worthless fiat money can be transfered costlessly. The production possibilities are such that an agent has a comparative advantage in producing the commodity which is desired most by one of his trade patterns. Hence, trade is necessary to reap the benefits from specialization vis à vis autarky. Also, production possibilities and preferences only partially overlap in the sense that one agent does not produce and value all the commodities produced and valued by his neighbor.

In the sequence of markets four types of trade patterns can emerge. A first trade pattern is a sequence of barter exchanges (TB, TB, TB). Monetary or fiat exchange can develop as follows. We assume monetary exchange must start in market I and always breaks down as soon as the chain of monetary exchanges is broken by a barter trade. Since in trade between j and (j+1)mod 3 player j is better off with such a transfer vis à vis barter exchange, he never opposes such a transaction. Whether bilateral exchange between j and (j+1)mod 3 is barter (B) or monetary (M) is decided by (j+1).

Hence, contingent upon all agents choosing T in the first stage, a second stage action vector such as (TB, TM, TM) has the following interpretation: In market I agent 2 proposes fiat exchange and agent 1 accepts. Similarly, in market II, the offer of fiat exchange by agent 3 is accepted by agent 2. However, in market III agent 1 proposes barter exchange and agent 3 must grudgingly accept, and is caught holding worthless fiat money.

Monetary exchange Pareto dominates barter because, given our assumption on the cost of resale of goods, barter requires bilateral coincidence of wants while monetary exchange allows the one way-flow of goods around the triangle (we elaborate in detail in the Appendix). Therefore, agents would benefit if all three engaged in fiat exchange. Figure 3 provides the agents' payoffs for each triple of player strategies. We have reduced each player's strategy set to three elements by equating strategies that are payoff equivalent. A player may choose stage 1 autarky (A), or stage 1 production for trade together with accepting only bater trade (TB) in the event that all three agents produce for trade, or accepting monetary trade as well (TM). As can be seen from the figure, there are three pure strategy equilibria of the game: (A, A, A), (TB, TB, TB) and (TM, TB, TB). The last two are payoff and observationally equivalent, and involve a sequence of bilateral barter trades between three agents, so we will refer to both as "the barter equilibrium". But only the equilibria (A, A, A) and (TB, TB, TB) are perfect equilibria in the normal for m of the game illustrated. The barter equilibrium strictly payoff dominates the autarkic equilibrium. Of course, agents would benefit if all three engage in fiat exchange (TM, TM, TM). Unfortunately, due to the boundary problem the first agent has incentive ro renege when he is asked to provide goods for worthless fiat money in the last market, see e.g. the payoffs to (TB, TM, TM). Hence, given that all three players choose initially to produce for the market, the only subgame equilibrium involves barter exchange. Fiat exchange is not an equilibrium of the game.

Figure 3. Payoffs

A

	A	TB	TM
A	7.5 7.5 7.5	7.5 6 7.5	7.5 6 7.5
TB	6 7.5 7.5	9 6.5 7.5	9 6.5 7.5
TM	6 7.5 7.5	9 6.5 7.5	9 6.5 7.5

TB

	A	TB	TM
A	7.5 7.5 6	7.5 9 6.5	7.5 9 6.5
TB	6.5 7.5 9	9.5 9.5 9.5	11.5 8 9.5
TM	6.5 7.5 9	9.5 9.5 9.5	11.5 8 9.5

TM

	A	TB	TM
A	7.5 7.5 6	7.5 9 6.5	7.5 9 6.5
TB	6.5 7.5 9	9.5 9.5 9.5	11.5 10 8
TM	6.5 7.5 9	9.5 9.5 9.5	10 10 10

Now, suppose the game in Figure 3 is repeated once. Playing each one-shot equilibrium twice, yielding each agent a payoff of twice 7.5 or 9.5 respectively, certainly represent equilibria in the repeated game. In addition, we have the following result.

<u>Proposition 2.</u> In a once-repeated version of the game, playing (TM, TM, TM) in the first round and playing (TB, TB, TB) in the second round constitutes a subgame perfect equilibrium path. Each agent receives a total equilibrium payoff of 19.5.

<u>Proof.</u> Since second stage behavior represents an equilibrium of the one shot game, clearly it is consistent with subgame perfect equilibrium behavior. The first stage choice (TM, TM, TM) is supported by credibly punishing any deviation by reversion to the dominated equilibrium (A, A, A), which would yield only 19 to the deviator.

Remark 5. We do not consider the issue of renegotiation proofness as this seems to violate the spirit of decentralized markets where money plays a transactions role.

4 Conclusion

Main stream monetary economics bases the value of fiat money in exchange on infinities such as an indefinite future to circumvent backward induction. Due to the evident benefits from monetary exchange over barter, backward induction arguments against the use of fiat money in finite but nevertheless large economies seem implausible. And models that push the boundary condition to infinity present difficulties for positive economists who want to test these theories. One resolution to this problem is through incomplete information. In game theory this avenue has been followed in addressing the chain store paradox and the centipede game. This is accomplished by including an epsilon probability on playing against a type for which the backward unravelling does not occur. We believe this fortunate happenstance to be of lesser relevance in monetary economics, because fiat money is by definition intrisically worthless. Instead, we take the point of view that exchanges in present day economies are so numerous and intricate that people easily loose track of who is at the beginning and the end of a chain of exchanges. This is modelled explicitly by inserting uncertainty over who initiates the chain of monetary exchanges. Nevertheless, agents always know who they are exchanging with. Because agents do not know where their meetings in the chain take place relative to the beginning or end, i.e. at which node they are, the backward induction argument breaks down. We started with a relatively small economy, 3 agents and 4 exchanges, for which the non-observability of the initiator of monetary exchange may appear questionable. Nevertheless, this simple set up lends itself easily to experimental investigation. By repeating the chain, and making the chain longer and longer the non-observability assumption seems eminently plausible. Future work along these lines would probably benefit from more intricate trading patterns such as those discussed in the pairwise trade and random meeting models.

The other approach articulated in this paper does not rely on an incomplete information structure, but exploits the multiplicity of equilibria which are present in a single shot version of the game. A priori it appears quite natural to require that there exists an autarkic and a barter equilibrium. And the fact that the barter equilibrium Pareto dominates the autarkic equilibrium also seems plausible. In this respect, the setup of the model is reminiscent of the coordination failure literature. Given the cost of transporting commodities and the asymmetries in production possibilities and preferences, a unidirectional flow of goods coupled with fiat money (costlessly) flowing the other way, would constitute a more efficient outcome. Like in the finite horizon OLG model, however, this outcome cannot be sustained as an equilibrium in our decentralized market economy. The reason is that the agent who issues the notes is not willing to redeem. In a twice repeated setting, though, the fiat exchange can be supported in the first round by the threat of autarky in the second round. Obviously, this line of analysis can be extended by considering more agents, commodities and trading rounds. But the small model already captures the essence of the transactions facilitating role of money, even though money has no intrinsic value.

Finally, we like to point out that the above can also be interpeted as an analysis of bubbles in finite economies. The intimate connection between fiat money and bubbles has been analyzed in depth by Tirole (1985). The popular view which has emerged is that bubbles, defined as the difference between the market price and the 'fundamental' value, require a dynamic inefficiency and infinities with respect to the sequence of markets and agents.[7] In the literature on bubbles the assumption of complete information is usually adopted without stating so explicitly. A setting of incomplete information may, however, be quite natural and does not necessitate the infinities of the complete information setting. As was shown in the second section this can lead to the adoption of intrinsically worthless fiat money, i.e. a bubble. In a similar spirit, the mixed strategy equilibrium of Corollary 1 may be interpreted as a sunspot equilibrium in a finite economy, i.e. both agents receiving an independent random signal on which they base their action. The bubble 'bursts' once the penultimate agent finds out that his bills are without value. In a way the other setting which departs from multiple nonmonetary equilibria is even 'nicer'. When the bubble bursts in the last period, no one is caught holding the hot potato without having benefitted from it. In either setup the usage of fiat exchange removes an inefficiency, but in neither case do we rely on infinities. This setup may therefore also be more conducive to bubble tests, because it removes a crucial but untestable maintained hypothesis.

[7] See e.g. the statements in Azariadis, (1993, p. 474), Blanchard and Fischer (1989, Ch. 5) and Tirole (1985, p. 1521).

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Appendix

In this appendix we provide some economic detail behind the two stage game of the third section. Agents labeled, 1, 2 and 3 are aligned at the corners of a triangle, see Figure 4 below. The sequence of markets is as in Figure 1. Markets are indicated by roman numerals. As was explained in the main text, a bilateral market may take place or not take place, depending on whether both agents have decided to produce for the market or whether at least one of the agents adopts the autarky strategy.

There are six commodities labeled: a, b, c, d, e, and f respectively. The agents in our example have simple linear utility functions. Agents exhibit different preferences. The commodity weights for the utility functions of agents 1, 2 and 3 are as follows:

$U_1 = (10, 2, 0, 0, 3, 1).\ (a, b, c, d, e, f)^T,$

$U_2 = (1, 3, 10, 2, 0, 0).\ (a, b, c, d, e, f)^T,$

$U_3 = (0, 0, 1, 3, 2, 10).\ (a, b, c, d, e, f)^T,$

and where the superscript T indicates the transpose of a vector. Observe that agents do not care about all commodities, and that this differs across agents.

The set of production possibilies simply consists in two elements for each agent. The first element denoted by A gives production under autarky, the second element denoted by T yields the production for the market economies. The respective production possibility sets are as follows:

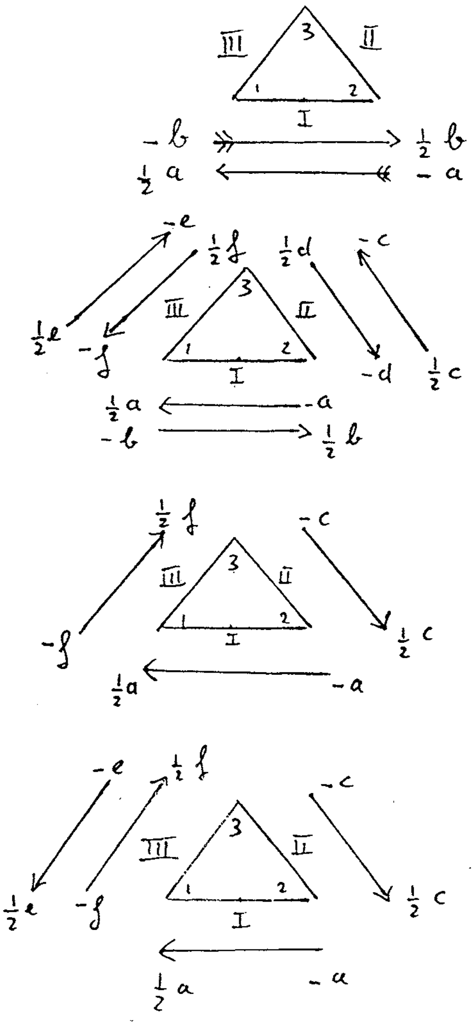
$$\begin{pmatrix} A_1 \\ T_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}, \frac{1}{2}, 0, 0, \frac{1}{2}, 0 \\ 0, \frac{3}{2}, 0, 0, \frac{1}{2}, \frac{3}{2} \end{pmatrix},$$

$$\begin{pmatrix} A_2 \\ T_2 \end{pmatrix} = \begin{pmatrix} 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0 \\ \frac{3}{2}, \frac{1}{2}, 0, \frac{3}{2}, 0, 0 \end{pmatrix},$$

$$\begin{pmatrix} A_3 \\ T_3 \end{pmatrix} = \begin{pmatrix} 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \\ 0, 0, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, 0, \end{pmatrix},$$

In autarky each agent can produce the commodity he desires most, but their comparative advantage lies in producing the commodities which are higher valued by the other agents. Thus the aggregate transformation schedule expands considerably if agents specialize and produce for trade. The gains from this specialization have to come from trade, because it is easily checked that $U_i(T_i) = 6 < U_i(A_i) = 7.5$, for i = 1,2,3.

Figure 4.



Strategy: TB, TB, A

bilateral exchange in market I

Strategy: TB, TB, TB

barter in all markets

Strategy: TM, TM, TM

monetary exchange

Strategy: TB, TM, TM

agent 1 'reneges' on monetary exchange with agent 3

The trade technology is as follows, To ship any of the commodities to another agent there are some semi-variable costs of transportation. These costs are modeled following the Samuelsonian iceberg model of trade. We assume that per unit shipment, one looses half the unit. In addition, it is assumed that there is an indivisibility of ½. Given that the maximum output of any commodity by a single agent is 3/2, we only have to consider shipment of 0, ½, 1, or 3/2 of the output. Shipping 0 amounts to nothing, but so does shipping ½ a unit because one foregoes the entire cargo as cost of transportation. Sending 1 unit gives the receiveing pary ½ a unit. Loading 3/2 costs a whole unit, and is equivalent to shipping only 1 unit from the receiving party's point of view. Hence, sending 3/2 units does not make sense. It follows that only 1 unit is exchanged, or nothing.

While it is assumed that exchanging commodities is costly, we also assume that transfering fiat money can be accomplished costlessly. This assumption appears plausible given that the defining characteristic of fiat money is its intrinsic worthlessness.

Typical trade patterns are depicted in Figure 4. The corresponding payoffs are given in Figure 3 in the main text. Under strategy (TB, TB, A) the third agent decides to remain autarkic and makes $(0, 0, 1, 3, 2, 10) . (0, 0, 0, ½, ½, ½)^T = 7.5$. Both other agents have produced for market exchange. Before trading they value their endowments as $U_{1,2} = 6$. The only mutually beneficial and feasible trade is for agent 1 to ship 1b to agent 2, while agent 2 forwards 1a. In the process only ½b and ½a arrive at the other side. This yields $U_1 = (10, 2, 0, 0, 3, 1) . (½, ½, 0, 0, ½, 3/2)^T = 9$, while $U_2 = (1, 3, 10, 2, 0, 0) . (½, 1, 0, 3/2, 0, 0) = 6.5$. Clearly, both agents gain from trading. Note that exchanging any of the other commodities would be pure waste because either the endowments are insufficient, i.e. less than 1, or because the commodity is not valued by the other party.

The second exchange pattern depicts the case of economy wide barter. It is easily shown that each agent has a strict incentive to trade in each market that he enters. Moreover, this multilateral trade program generates the highest possible payoff when all exchange takes place on a quid pro quo basis.

Under monetary exchange, i.e. TM, TM, TM, commodity flows are clockwise, while fiat money flows the other way. This exchange yields all agents $U_i = 10$. We note that this is not the only fiat exchange which is feasible. Consider e.g. the counter clockwise flow of commodities whereby agent 1 ships 1b to agent 2, agent 2 sends 1d to agent 3, and agent 3 forwards 1e to agent 1, while money flows the other way around. This yields each agent $U_i = 5.5$. Which leaves every agent worse off in comparison to the valuation of their endowment $U_i(T) = 6$. Because such trade patterns are not individually rational, they are not considered to be part of the game.

The last trade configuration in Figure 4 describes the case whereby agent 1 does not honor the fiat notes in his exchange with agent 3. Even though agent 1 has paid with these notes for his trade deficit with agent 2. The third agent is stuck with holding the useless paper money, and only partly recoups through the barter exchange with agent 1. Thus agent 3 makes significantly less than the other two agents.

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