BANK OF FINLAND DISCUSSION PAPERS

7/97

Monica Ahlstedt

Research Department
26.5.1997

Exchange Rate, Interest Rate and Stock Market Price Volatility for Value-at-Risk Analysis

Suomen Pankki
Bank of Finland
P.O.Box 160, SF-00101 HELSINKI, Finland
☎ + 358 0 1831

BANK OF FINLAND DISCUSSION PAPERS **7/97**

Monica Ahlstedt

Research Department
26.5.1997

Exchange Rate, Interest Rate and Stock Market Price Volatility for Value-at-Risk Analysis

ISBN 951-686-548-8
ISSN 0785-3572

Suomen Pankin monistuskeskus
Helsinki 1997

Exchange Rate, Interest Rate and Stock Market Price Volatility for Value-at-Risk Analysis

Bank of Finland Discussion Papers 7/97

Monica Ahlstedt
Research Department

Abstract

The study derives a theoretically and empirically founded procedure for volatility estimation and forecasting of daily financial return series for use in value-at-risk model frameworks. GARCH modelling is applied to account for time varying heteroskedastic conditional variances and covariances. Through univariate estimation, the historical conditional variance models are specified within a group of twelve markka-denominated exchange rates, a group of thirteen short-term interest rates, the long-term interest rate and Finland's general stock market index. Within these groups, the method of principal components is used to detect common short-term factors driving the high frequency stochastic processes. Spectral analysis is applied to identify the length and regularity in the cyclical behaviour of the estimated conditional variances and their principal components. Since there turned out to be a great similarity in the univariate estimation results within groups of rates, GARCH estimation on pooled data was performed to force the rates within groups into the same model. The estimated models on pooled data were found to be integrated in variance with closely similar parameter values for both exchange rates and interest rates.

Since a general multivariate framework is not possible to apply to the amount of series in this study due to the huge number of parameters to be identified, the covariances were calculated in two step-wise ways from the univariately estimated variances. First, assuming dependence between the autocorrelation structure of the conditional variances and covariances, univariately estimated parameters of the conditional variance models were used in identifying the pairs of conditional covariances. Second, assuming constant correlations, conditional covariances were estimated using joint information on the correlation coefficients of the GARCH standardized residuals and the univariate conditional variances. The first method is only applicable in estimating covariances within groups, the second is also applied in estimating the covariances between groups.

Although the magnitude or direction of the expected changes in rates cannot be forecast, the estimated GARCH structure makes it possible to forecast the expected future variances. By developing the parameter structure estimated on pooled data, a theoretically and empirically founded procedure is suggested to replace the usual ad hoc decision process of selecting the sample period and the weight structure for estimating variances and covariances.

Keywords: Time-dependent volatility, GARCH estimation, value-at-risk models

Tiivistelmä

Selvityksessä johdetaan teoreettisesti ja empiirisesti perusteltu järjestelmä taloudellisten tuottosarjojen päivähavaintojen volatiliteetin estimoimiseksi ja ennustamiseksi käytettäväksi value-at-risk malleissa. GARCH analyysia käytetään ajassa muuttuvien heteroskedastisten ehdollisten varianssien ja kovarianssien mallittamiseen. Yksiulotteisella estimoinnilla määritellään historialliset ehdolliset varianssimallit kaksitoista markkakurssia sisältävälle valuuttakurssiryhmälle, kolmentoista lyhyen koron ryhmälle, pitkälle korolle ja osakemarkkina indeksille. Ryhmien sisäisessä tarkastelussa käytetään pääkomponenttianalyysiä korkeafrekvenssisten stokastisten prosessien taustalla olevien yhteisten faktoreiden tunnistamiseksi. Spektrianalyysia käytetään estimoitujen ehdollisten varianssien ja niiden pääkomponenttien syklien pituuksien ja säännönmukaisuuksien arvioimisessa. Yksiulotteisissa ryhmien sisäisissä estimointituloksissa saavutetun korkean asteen yhdenmukaisuuden perusteella GARCH estimointi suoritettiin myös poolatulle aineistolle, jossa ryhmän sisäiset yksittäiset tuottosarjat pakotettiin noudattamaan samaa mallia. Poolatussa aineistossa identifioidut mallit osoittautuivat varianssi-integroiduiksi ja estimoinnin tuloksena saadut parametriarvot olivat liki pitäen samat sekä valuuttakursseille että koroille.

Koska yleistä moniuloitteista mallia ei suuren parametrimäärän vuoksi voitu soveltaa näin kattavaan muuttujamäärään, estimoitiin kovarianssit kahdella vaiheittaisella mentelmällä. Ensiksi, havainto varianssien ja kovarianssien autokorrelaatiorakenteen riippuvuudesta mahdollistaa yksiulotteisten ehdollisten varianssien estimointitulosten käytön parittaisten ehdollisten kovarianssien identifioinnissa. Toiseksi, periodien sisäisten korrelaatioiden vakioisuusoletus mahdollistaa ehdollisten kovarianssien määrittelemisen yhdistämällä standardisoitujen jäännöstermien estimoidut korrelatiokertoimet ja yksiulotteisen estimoinnin ehdolliset varianssit. Ensimmäinen menetelmä soveltui ryhmien sisäisten kovarianssien estimointiin, toista menetelmää sovellettiin myös ryhmien välisten kovarianssien estimointiin.

Vaikkakin tuottosarjojen odotettujen muutosten suuruutta tai merkkiä ei voi ennustaa, estimoitu GARCH rakenne mahdollistaa varianssin ennustamisen. Soveltamalla poolatussa aineistossa estimoidun mallin parametrirakennetta päädytään teoreettisesti ja empiirisesti perusteltuun menettelyyn, jolla voidaan korvata yleensä ad hoc perusteinen estimointiperiodin pituuden ja painorakenteen valinta odotettujen varianssien ja kovarianssien ennustamisessa.

Asiasanat: Aikariippuvainen volatiliteetti, GARCH estimointi, value-at-risk mallit

Contents

1 The objectives of the study

Evaluation of risk, measured by the variance of a given probability function, is a central issue in financial economics. Important areas where an appropriate estimation of variance is crucial are option pricing, hedging strategies and risk premium identification. Along with these and other financial applications, value-at-risk (VAR) models have recently become popular among financial institutions and supervisory bodies.

In this study theoretical and empirical models for the measurement and estimation of volatility in financial time series are presented. The estimated results can be utilized in the value-at-risk model developed at the Bank of Finland (Ahlstedt 1990).

The value-at-risk framework is a statistical procedure which measures, at a certain confidence interval, the amount of value that can be lost or gained in a portfolio due to changes in market prices of the underlying assets (Simons 1996). Although value-at-risk models can be used in assessing credit risk and liquidity risk the main area of application is evaluating market risk. Thus, estimates for expected changes in exchange rates, interest rates and stock prices are needed. The efficient market hypothesis states that the magnitude and direction of the expected changes in these rates cannot be forecast. This fact has also been confirmed in empirical work that shows the mean of the probability distributions to be zero. In the value-at-risk applications, therefore, the measure of the variance, rather than the mean, of probability distributions is estimated. Through this estimate, the historical variances of changes in market rates can be used as forecasts of the future behaviour of the rates.

As there is no generally accepted way to calculate the variance of the value of a portfolio, there are a number of value-at-risk models in use. Each yields results that mirror the underlying assumptions and methodological approach. There are three main approaches represented in the commercial packages distributed in the markets: the historical, the analytical and the simulation approach.[1] The internal applications developed within financial institutions are essentially variations on these main approaches. Each approach has its strengths and weaknesses, which thus have to be weighted against the purpose of the use of the model. These models are characterized by unrealistic or simplifying assumptions, mostly about the probability functions, which are sometimes in contradiction with the empirical realizations of the financial time series. In particular the normality assumption of the return series, the selection of sample time horizon and weight structure in variance estimation and forecasting are issues where no common agreement exists. In this study, generalized autoregressive heteroskedastic (GARCH) methodology is used to solve these problems. Through GARCH parametrization of financial rates, we end up with standardized stochastic processes, which, by definition, are normal or at least much closer to normal than the raw data on which random walk processes usually are applied. GARCH interpretation also makes it possible to forecast expected future variances. Based on the parameter structure of the estimated conditional variances, a theoretically and empirically founded formula

---

[1] J.P. Morgan's Riskmetrics, Bankers Trust's RAROC2020 and Chase's RISK$.

for selecting the sample period and the decay factors giving the weight vector for estimation of the future conditional variances and covariances can be derived.

Financial time series of high frequency data are known to have clustering as a typical feature. This feature can bee seen in the statistical properties of the unconditional frequency function as skewness, fatter tails and a higher peak around the mean (leptokurtosis) than in the stable normal distribution. In VAR model applications, the main assumption of the stochastic process in first differences of financial rates, is that of a random walk generating a normal distribution with a constant unconditional variance, although we know that the random walk model does not fit observed data. There is strong empirical evidence that the Autoregressive Conditional Heteroskedasticity models are adequate to capture the volatility clustering and the thicker tails in the unconditional distribution (Nerlove et al. 1988). The family of Autoregressive Conditional Heteroscedasticity, ARCH, models was introduced by Engle (1982) and later generalized by Bollerslev (1986) to GARCH. Further applications as the IGARCH and EGARCH have been developed to capture both linear and nonlinear dependencies in the second and higher moments.

The body of research on ARCH has grown extensively since the seminal paper of Engle (see Bollerslew et al. 1992 for an extensive survey). Although the implementation of ARCH means a huge methodological step forwards, there have been some doubts about its ability to capture all the nonlinearity within time series. Hsieh (1988), for example, concludes that time-varying means and variances are not sufficient to account fully for the leptokurtosis in exchange rates, but that a flexible stochastic GARCH model with time varying parameters explains the nonlinearity of the data (Hsieh 1991).

There is a possibility that even the ARCH modelling is too simple to capture the true nature of the stochastic process driving financial markets. More complicated nonlinearities could lead to the methodology of complexity and chaos, which has been applied in a number of scientific fields.

In particular, the abrupt huge changes in financial time series as the stock market crash of 19 October 1987 have fostered the idea of extending the methodology of explaining time-dependence in volatility in financial data to deterministic chaotic dynamics. In time-series models as Box-Jenkins and the family of GARCH, the economy has a stable momentary equilibrium but is constantly being perturbed by external shocks. The behaviour of economic time series comes about as a result of these external chocks. In chaotic models, the time series follow non-linear dynamics, which are self-generating and never die out. The fact that the fluctuation in financial time series can be internally generated is highly appealing, especially since it has been very difficult to find the theoretical framework in economics for explaining the GARCH approach in modelling non-linearity.

Chaos can be searched for using the method of correlation dimension proposed by Grassberger and Procaccia (1983). Unfortunately, this method requires large data sets, which are available in natural sciences but not in economics and finance. It also lacks a statistical theory for hypothesis testing (Hsieh 1991). Brock, Dechert and Scheinkman (1987) have developed a related method from the correlation dimension called the BDS statistic. It tests the null hypothesis that a time series is IID against an unspecified alternative using a nonparametric technique. This statistics has been shown to have good asymptotic and finite sample properties and good power against chaotic behaviour and most nonlinear structures. Thus, the BDS test is applied in this study to test the adequacy of fit of the estimated models.

The outline of the study is the following. Section 3 deals with twelve markka-denominated exchange rates, section 4 with thirteen short-term interest rates, section 5 with the long-term interest rate and section 6 with the general stock market index. The same estimation procedure is carried through for all rates. Pooling is used to force the individual rates within groups into the same process, which then is used in forecasting. In section 8 covariances within groups and between groups of rates are calculated in two ways: first by assuming dependence between the autocorrelation functions of conditional variances and covariances and second by assuming constant conditional correlation within periods. Based on the estimation results on pooled data, conditional forecast formulas are developed in section 9 for variances and covariances both within groups and between groups. The ad hoc based selection of the sample period and the weight structure for historical estimation of variance commonly used in value-at-risk model applications, is herewith replaced by a theoretically and empirically founded formula.

2 Methodology

The usual underlying assumption in VAR model applications is that financial return series follow a random walk data generating process with a normal frequency distribution and a constant variance. It is, however, known that changes in financial time series exhibit clustering, meaning that the unconditional frequency distribution differs from the normal in having fatter tails and a higher peak around the mean. These statistical features are interpreted as signs of a time-varying variance. In this study the stylized facts empirically found in the rates are modelled using the GARCH methodology of Engle (1982) and Bollerslev (1986). The model identification allows the construction of new time series in the form of GARCH standardized residuals, which should be normal, or at least much closer to normal than the random walk residuals. The normality of the transformed residuals also justifies and allows the making of probability statements on confidence intervals of expected future variances.

This study covers a group of twelve markka denominated exchange rates, a group of thirteen money market interest rates, the long-term interest rate and the general stock market index. The risks in a portfolio are not only measured by the variances of the individual rates, but also of covariances of pairs of variables. The variances of these twenty-seven rates should therefore be estimated in a multivariate framework. The multivariate GARCH model is developed in Bollerslev, Engle and Wooldridge (1988) and can, in principle, be estimated efficiently by maximum likelihood. However, the number of parameters in the general form may be very large. Although more or less plausible restrictions, such as assuming the parameter matrices to be diagonal, can be imposed to reduce the dimensionality of the parameter space, a complete multivariate GARCH for the amounts of rates in this study is too big to be elaborated. We, therefore, first apply the univaraite GARCH model to the individual rates, and thereafter estimate the covariances in two ways. First, by assuming equality between the autocorrelation structure of the conditional variances and covariances, the univariately estimated parameters of the conditional variance models are used in identifying the conditional covariances. Second, by assuming constant conditional correlations as proposed in Bollerslev (1990), the conditional covariances are estimated using the joint information on the correlation coefficients of the standardized GARCH residuals and the univariate conditional variances. The first method is applied in estimating the covariances within groups, while the second is also applied in estimating the covariances between groups.

In the univariate estimation the same methodology is applied to daily changes in all rates. The estimation period, 1 January 1987 – 31 December 1995 was divided into three non-overlapping subperiods to account for structural changes trigged by realignments in the Finnish markka. Pre-whitening of the data was applied when found necessary to remove linear dependence. Prior to model specification, unit root tests were applied to grant stationarity in mean. Next the mean equation identification was performed and the parsimonious GARCH(1,1) model was estimated for all rates. The goodness of fit is evaluated using the BDS statistics along with the usual statistical tests. Pooled data within periods is used to force individual rates within periods into the same process. The method of principal components is used to detect common factors driving the high-frequency stochastic processis. Spectral analysis is performed to identify the lenght and regularity in the cyclical behaviour of the estimated conditional variances and their principal components. Since there turned out to be a great likeness in the univariate estimation results within groups of rates, GARCH estimation on pooled data was applied to force the rates within groups into the same model.

3 Exchange rates

3.1 Markka-denominated rates

Most studies dealing with modelling the probability distribution of foreign exchange rates concentrate on the behaviour of dollar-denominated rates. This study deals with the markka-denominated exchange rates which measure the exchange rates as the domestic price of foreign currency. Results from rates denominated in other currencies are not necessary applicable to markka-denominated rates due to a different institutional structure affecting the rate generating process and also due to the Finnish market which is small in scale and scope. Three studies, to our knowledge, deal with markka-denominated rates (Ahlstedt 1990 and 1995, Sulamaa 1995). Some of these earlier results (Ahlstedt 1995), not repeated here, are referred to in the sections covering the empirical work.

3.2 Frequency

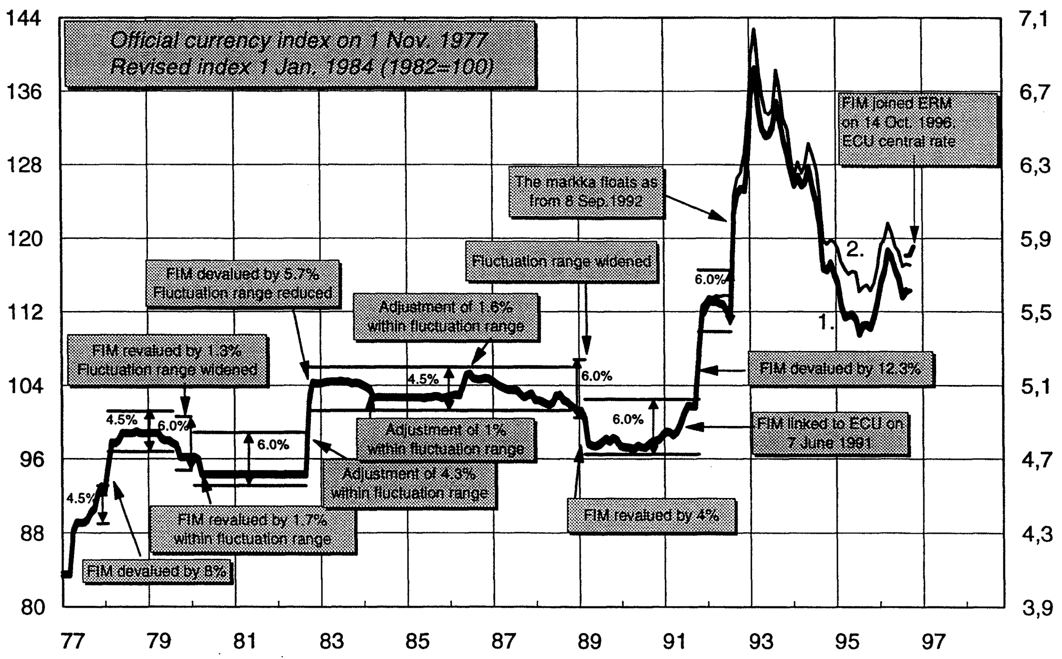
It is a well documented empirical result that certain distributional properties of financial time series, such as heteroscedasticity and leptokurtosis, decrease with frequency. Under temporal aggregation, convergence to unconditional normality occurs (Nerlove et al. 1988), so that one-month changes display less time-varying volatility and are closer to normality than one-week changes (on a monthly level), which, in turn, are closer to normality than daily observations. For exchange rates, even intra-day prices are quoted. In the intra-day quotations, the volumes and prices of exchange rates are determined at points where supply and demand are in balance. These momentary equilibrium points are reached at numerous discrete points in time during on-going trading. The quoted prices on the way towards the long-term equilibrium mirror the traders reaction to news coming into the markets. An attempt to explain these stylized facts in foreign exchange rate movements has been sought in common factors. For low frequency data, international economic variables have been tested and for high frequency data, the source of the pattern of variability has been sought in the news arrival process in the form of either meteor showers or heat waves (Engle, Ito and Lin 1990).

The purpose of this study, however, is to quantify, using time series techniques to model time-varying conditional variances, the inherent riskiness of short-term changes in the values of the banks' portfolios, which are marked to market on a daily bases, and therefore, the daily frequency is selected for the data. High frequency common factors will be tested for using principal components analysis on estimated daily variances.

3.3 Structural changes

Since this study also deals with the interaction between exchange rates and interest rates, the data used is extended backwards to cover the longest possible common interval for these rates in the data base at the Bank of Finland. This period is 1 Jan. 1987 - 31 Dec. 1995. The period (Figure 1) includes a 4 % revaluation of the Finnish markka basket 17 Mar. 1989, a 12.30 % devaluation on 15 Nov. 1991 and the transition into a floating regime from 8 Sep. 1992 onwards. At the beginning of the floating, there is a period of large followed by a period of depreciation strengthening of the Finnish markka. To account for possible structural shifts generated by the the sample period change in the exchange-rate regime, is divided into two main periods: one covering the pegged regime, 1 Jan. 1987 - 5 Sep. 1992, and the other covering the floating regime, 8 Sep. 1992 - 31 Dec. 1995 (Figure 1). As our goal is not to explain the effects or the transmission mechanism of structural shocks or to forecast turning points, pre and post data around shifts in exchange rate regimes are excluded.

Figure 1. **External value of the markka**



1 The Bank of Finland currency index (left scale)
2 Markka value of the ECU from 7 June 1991 (right scale)

The data consists of daily observations on log changes of closing rate bid-ask midpoint. Weekends and holidays were omitted. Monday is taken as the next day after Friday. Weekend or weekday effects have not been found (Ahlstedt 1990) in the exchange rates or interest rates. First differences are used referring to numerical studies on dollar rates, which show that higher order differencing is not necessary to reach stationarity (Chappell and Padmore 1995). For the markka's pegged period this is most certainly true, since the goal of the intervention mechanism is the stability of the currency.

Exchange rates included in this study are the twelve major currencies USD, GBP, SEK, NOK, DKK, DEM, NLG, BEF, CHF, FRF, ITL and JPY, whose markka-values display the following special features during the period under consideration which may well have affected the final variance-covariance estimates to be presented in this study.

Rates	Special volatility features
ALL	All series have huge peaks at 15 Nov. 1991 devaluation and 8 Sep. 1992 shift to the floating regime
USD	the magnitude of the changes in the USD exchange rate is bigger compared to the changes in the ERM currencies
GPB	peaks at joining the ERM 8 Oct. 1990 and turbulence and excess volatility at exit from the system 16 Sep. 1992
SEK	increased volatility at the time preceding transition to the floating regime in September 1992
NOK	increased volatility at the time preceding transition to the floating regime in September 1992
DKK	high volatility in connection with the turbulence among the crises in the ERM and the other Nordic currencies in September 1992 but staying in the ERM band
DEM	3 % revaluation 12 Jan. 1987
NLG	3 % revaluation 12 Jan. 1987, high volatility in connection with the ERM crises in September 1992
BEF	2 % revaluation 12 Jan. 1987; increased volatility in September 1992
CHF	turbulence in September 1992
FRF	turbulence in September 1992 as for the other ERM currencies DKK, BEF, NLG and CHF
ITL	the lira's band was narrowed from 6 % to 2.25 % 5 Jan. 1990, the effects of which can clearly bee seen in reduced volatility; 3.5 % devaluation 14 Sep. 1992 and excit from the system 16 Sep. 1992 and left floating
JPY	increased volatility at the time of unstability in the Nordic currencies and the ERM system early autumn 1992 but for a longer period

As expected, currencies within the ERM system tend to have lower variances than USD and JPY, which float freely.

Preliminary statistical analysis to test the hypothesis of a normal distribution with zero mean and a constant variance for the twelve exchange rates covering was conducted both main periods. Since the test values are extremely sensitive to even a single outlier, the observations of the revaluation and devaluation trading days within this period strongly affect the descriptive statistics.

For the first main period, 1 Jan. 1987 – 5 Sep. 1992, the hypothesis of zero mean could not be rejected on a 95 % confidence level. The skewness and leptokurtosis measures are high. To control for the possible effects of structural breaks, the pegged period is further divided into two subperiods where the dividing date is the revaluation date 17 Mar. 1989.

The full series were accordingly split into three non-overlapping sub-series

the pegged period	1Jan. 1987 - 16 Mar. 1989
	21 Mar. 1989 - 5 Sep. 1992
the floating period	8 Sep. 1992 - 31 Dec. 1995

The realignments are treated in two alternative ways, they are either used to divide the data into three periods or their effects are captured by dummy variables, that is, within the second pegged period. The skewness figures are neglible in all periods except the second pegged period, which includes the realignment.

3.4 The pegged period

3.4.1 First subperiod 1 Jan. 1987 - 16 Mar. 1989

Summary statistics for the two subperiods 1 Jan. 1987 - 16 Mar. 1989 and 21 Mar. 1989 - 5 Sep. 1992 show that the data for the first period is much closer to a normal distribution than the data for the latter. In most cases, the skewness measure does not significantly differ from that of the theoretical distribution. Mean percentage change of spot exchange rates is significantly different from zero only for SEK, NOK and ITL.

While the magnitude of the excess kurtosis for the first subperiod is only a fraction of the measures for that latter subperiod, it is nevertheless significant for all currencies. Kurtosis in the unconditional distribution may be seen as indication of conditionality in the second and higher moments.

The later subperiod for the first main period 21 Mar. 1989 - 5 Sep. 1992 includes the devaluation of 15 Nov. 1991. Although the actual day and nearby devaluation days are excluded from the data, spillover effects from the devaluation remain. The subperiod ends with the volatile markets preceding the switch from a pegged basket regime to the floating regime for FIM, SEK and ITL. It also covers the GBP joining the ERM system and the period preceding its exit from the system. This turbulence can be seen in the higher figures for variances in all currencies, except for the FRF and ITL. The skewness measures differ significantly from zero for all currencies but the JPY. This means that during this period extreme values have occurred more often than they do in the theoretical distribution. The figures for excess kurtosis are, as a rule, very high in this set of the data. USD and JPY display less kurtosis, although it remains statistically significant.

For floating period, 8 Sep. 1992 - 31 Dec. 1995, the hypothesis of a zero mean rate of depreciation is rejected only for the ITL. The skewness measures differ significantly from zero for all currencies except DKK, NLG and BEF. All excess kurtosis measures differ significantly from zero. Thus, most of the empirical unconditional distributions appear to display assymetries and have fat tails relative to the normal.

The magnitude of the variances are, as expected, largest for the floating regime period.

Next we proceed to the modelling the conditional variance. Estimation of the ARCH process depends on the specification of the conditional mean equation. Since first differencing produces stationarity, log changes of the exchange rates could be initially expressed as

$$R_t - R_{t-1} = \alpha + \varepsilon_t \tag{13}$$

where $R_t = \ln(X_t)$ denotes the natural log of the original series, X_t, α is a constant and ε_t is a zero mean error term. Under a serially uncorrelated and homoscedastic error process, R_t follows a random walk, possible with a drift. The results for each series R_t reveal the constant to be insignificantly different from zero, confirming the absence of a deterministic trend or drift. There is no evidence of serial correlation in the residuals with the exception of the ITL. The ITL Ljung-Box test statistics of linear serial correlation for lags up to five are highly significant. The Jarque-Bera normality test statistics is significant for all currencies except CHF, which lends support to earlier results showing deviation from normality in the form of leptokurtosis and skewness.

These deviations from normal errors may be evidence that the ε_t's are not independently distributed across time, although they as such these non-normalities do not run counter to the assumption of a martingale process for exchange rates. The graphs of the logarithmic differences show clustering, which, on balance of the evidence is typical for high frequency dollar-denominated exchange rate data. Thus, there is a tendency for daily exchange rate changes to be followed by large residuals and small changes by small ones, but of unpredictable sign. This type of behaviour, as well as various other sources of heteroscedastic behaviour, can be modelled using ARCH(q) and GARCH(p,q) processes developed by Engle (1982) and Bollerslev (1986, 1987), which explicitly allow for this type of temporal dependence by parameterizing the conditional variance as a function of the past squared residuals and the past conditional variances themselves.

Bollerslev et al. (1992) suggests that the inclusion of one period lag for the squared innovations ϵ_t^2 and conditional variance h_t respectively, in the variance functions, ie GARCH(1,1) model, is usually sufficient to capture most of the conditional heteroscedasticity in financial market returns data. This is also confirmed by previous results for the markka-denominated exchange rates and interest rates (Ahlstedt 1990, 1995). Consequently a GARCH(1,1) structure is assumed

$$\begin{aligned} &R_t - R_{t-1} = \gamma_0 + \varepsilon_t \\ &\varepsilon_t \sim N(0,h_t) \\ &h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned} \tag{14}$$

If the value of the parameter β_1 is insignificantly different from zero then the process is an ARCH-model. If α_1 is zero we have a process that only depends on its past history. If both α_1 and β_1 are zero, ϵ_i is simply white noise.

The GARCH model was estimated by the method of maximum likelihood assuming conditional normality. The Jarque-Bera normality test statistics, however, strongly rejects the null hypothesis of normal errors. Conditional normality is not, however, necessary for the consistency and asymptotic normality of the estimators (West and Cho, 1995). Most usefully, then, the MLE based on normal density in equation (14) may be given a quasi-likelihood interpretation.

The results of the GARCH-estimation are shown in Table 1. The models were hard to iterate to convergence. A large amount of iterations were necessary. Based on the Ljung-Box test statistics for the ITL, the lagged endogenous variable is included in the mean equation for this currency. The drift parameter in the variance equation α_0 is statistically significant for all currencies. Both the ARCH-parameter α_1 and the GARCH parameter β_1 are significant in all equations.

Table 1. **GARCH-estimation of the volatility of foreign exchange rates 1 Jan. 1987 – 16 Mar. 1989 (t-statistics in parenthesis)**

	α_0	α_1	β_1	ARCH(1,1) test	Ljung-Box test statistics				
					LAG(1)	LAG(2)	LAG(3)	LAG(4)	LAG(5)
USD	0.1341·E−5 (2.47)	0.0883 (3.29)	0.8661 (23.37)	2.20	1.72	1.83	1.83	2.83	3.98
GBP	0.1764·E−6 (2.04)	0.0305 (2.97)	0.9473 (52.78)	5.46	1.01	2.45	4.24	4.33	4.68
SEK	0.3315·E−7 (2.23)	0.0733 (4.35)	0.9011 (43.49)	8.75	6.97	7.70	8.83	9.03	12.80
NOK	0.1363·E−6 (2.64)	0.0757 (3.51)	0.8868 (27.81)	2.30	2.87	6.86	8.63	8.69	8.71
DKK	0.5654·E−6 (3.17)	0.1271 (3.11)	0.7070 (9.89)	11.32	0.10	2.79	4.35	4.40	4.41
DEM	0.2490·E−6 (4.04)	0.1468 (4.67)	0.7896 (23.86)	30.83	0.92	2.96	3.37	4.19	5.27
NLG	0.2207·E−6 (3.16)	0.1479 (4.50)	0.7967 (21.78)	25.77	0.57	1.64	1.64	2.12	3.89
BEF	0.1489·E−6 (2.62)	0.0914 (3.84)	0.8601 (26.25)	32.51	0.48	4.57	4.99	6.63	9.75
CHF	0.3854·E−6 (1.62)	0.0756 (2.85)	0.8659 (17.33)	21.96	0.41	4.61	4.90	5.41	5.45
FRF	0.1630·E−6 (2.62)	0.1054 (3.99)	0.8355 (20.95)	18.30	0.41	0.54	0.87	4.29	5.34
ITL	0.4655·E−5 (9.82)	0.2284 (4.30)	0.3007 (4.69)	47.95	57.44	59.95	64.02	64.41	66.54
JPY	0.1208·E−5 (2.62)	0.0925 (3.18)	0.8094 (14.12)	2.62	0.51	2.81	3.44	3.56	3.72

The effect of the squared surprises, or shocks, on the variance is measured by the parameter α_1. The magnitude of the impact is very similar for the freely floating currencies USD and JPY and the European currencies besides the ITL, which has a pattern of its own. The sum of the parameters α_1 and β_1 is close to one, thus indicating a GARCH process integrated in variance or a GARCH process with persistance in the sense of Engle and Bollerslev (1986).[1] In such an persistent variance model, the current information remains important for the forecasts of the conditional variance for all horizons.

An extension of the GARCH model to the regression framework is the GARCH-in-Mean (GARCH-M) model proposed by Engle, Lilien and Robbins (1987). Applications in finance of the GARCH-M model is employed to capture a linear relationship between return and variance, ie risk according to the intertemporal capital asset pricing model of Merton (1973) (Mills 1993, p. 137)

$$\begin{aligned} R_t - R_{t-1} &= \gamma_0 + \gamma_1 \sqrt{h_t} + \varepsilon_t \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned} \qquad (15)$$

The conditional standard deviation (or variance) is included as an explanatory variable in the mean equation. The impact of the standard deviation on returns is interpreted as a time-varying risk premium.

To test for the existence of time-varying risk premia in the foreign exchange market and to ensure the selection of the right model, a GARCH(1,1), a GARCH(1,1)-M model was also tested for comparison. The results showed that the parameter values γ_1 for the risk-premium term are not statistically significant. These results coincide with the outcome of other studies dealing with other than markka-denominated exchange rates (for example Chappel and Padmore 1995) where no risk premium was found when modelling the return over riskless yield.

According to portfolio theory, or the Capital Asset Pricing Theory, the risks in the portfolio are not only captured by the variance of the individual currencies but also by their covariances. One way to estimate the covariances would be to move from the univariate framework to multivariate modelling. Theoretically, the multivariate case is a direct generalization of the univariate model with the exception that an entire variance-covariance matrix is modelled. The problem is that a the number of parameters in the general form may be too large for the approach to be practically feasible. Although various restrictions can be imposed to reduce the dimensionality of the parameter space, a multivariate GARCH for a system of twelve exchange rates is considered too large to be elaborated.

An alternative method to the multivariate GARCH for investigating the simultaneous dependence between rates is to use principal component techniques to test for common factors driving the individual exchange rate variances h_{it}. The underlying assumption is that exchange rate movements depend on a common set of international variables observable only at certain frequencies (Bollerslev 1990). If the common factors are macroeconomic variables, they are relevant only at high frequencies. If the common factors are to be found in the news arrival process,

---

[1]See Nelson (1990a) for a general analysis of persistance and convergence in GARCH(1,1) models.

they are relevant only on high frequencies. Through the GARCH model, then, we can predict how the exchange rates react to shocks or news, with the principal component method we try to identify the shocks.

The principal components were calculated for the conditional variances $h_{i,t}$ for the twelve rates. The eigenvalues and the cumulative fractions of variance explained are shown in Table 2. When the variables are highly correlated and form a homogenous group, the first principal component explains more than 90 % of the total variation. This is usually the case for a set of macroeconomic variables. The results presented in Table 5 indicate that the variances within the group of exchange rates are more heterogenous, and the total variance cannot be concentrated into a few common factors as for macroeconomic variables. The fraction of explained variance for the exchange rates starts from 50 % for the first principal component and grows then approximately 10 % for every additional component. The factor loading values of the individual variances show that the variance of USD dominates the first principal component with a value of 0.815. The GARCH estimation results of the exceptional behaviour of the ITL are confirmed also in the principal component calculations. The factor loading values of ITL are only 0.152 and 0.036, thus showing practically no correlation with any of the two first principal components. The removal of this currency would increase the fraction explaned by the first components.

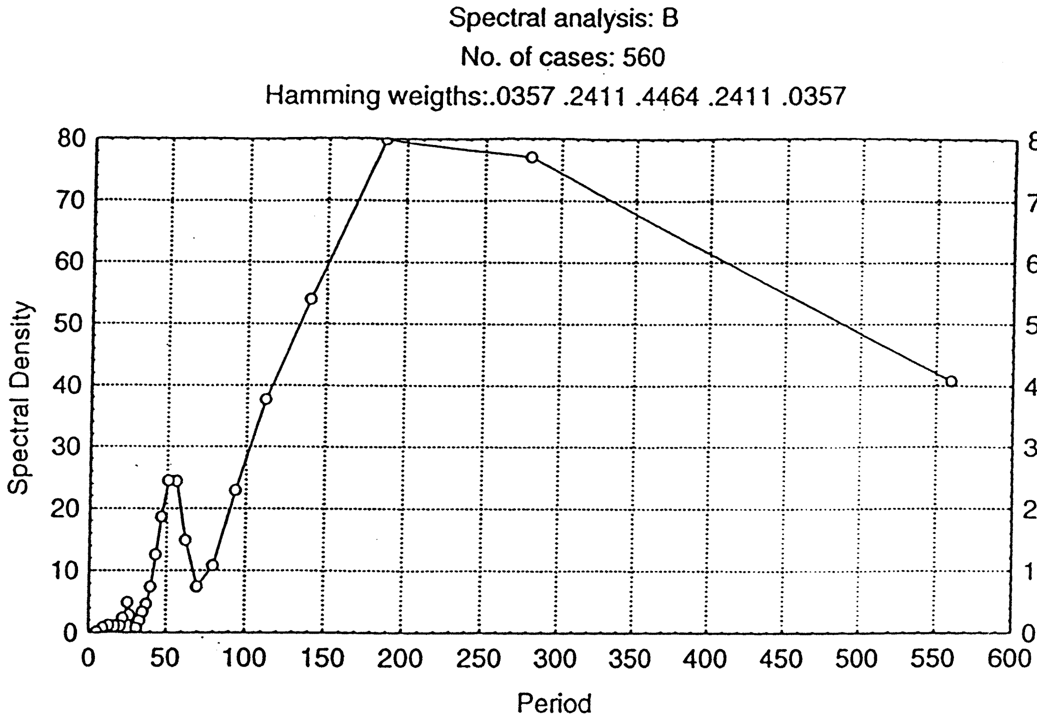
Table 2. **Principal components of conditional variances. Eigenvalues and cumulative fraction explained. Foreign exchange rates 1 Jan. 1987 – 16 Mar. 1989.**

Component	Eigenvalue	Cumulative R-Squared
1	5.5140	0.4595
2	1.5249	0.5865
3	1.3212	0.6966
4	0.9697	0.7775
5	0.8193	0.8457
6	0.5164	0.8888
7	0.4753	0.9284
8	0.0118	0.9294
9	0.0767	0.9358
10	0.1658	0.9496
11	0.2974	0.9744
12	0.3068	1.0000

Using spectral analysis on both the individual conditional variances and the principal components, we can decompose the observed time-variability of the conditional variances or of the principal components into contributions from periodic cycles at different angular frequencies (and, hence, of different cycle lengths). Furthermore, visual inspection of the power spectra provides us with potentially a powerful tool for identifying the autocorrelation structure of the underlying process generating the observed time variability of the conditional variances, and ultimately the process itself. Finally, spectral analysis may prove useful in constructing optimal filters to remove specific cycles of a given length from the data.

Now, the overall shape of the power spectrum of the first principal component (Figure 2) of the conditional variances gives us evidence of persistence in the component, ie the general shape of the spectrum resembles that of a positively autocorrelated process. Furthermore, additional contributions to the time-variability of the conditional variances come from cycles with frequencies in the range 0,0224–0,0561 radians or 0,0036–0,0089 cycles per day (corresponding to wavelengths between 112 and 280 days). Given the shape of the spectrum, however, cycles within this range need not be all that regular.

Figure 2. **Spectral density function of the first principal component of conditional variances of foreign exchange rates 1 Jan. 1987 – 16 Mar. 1989**



3.4.2 Second subperiod 21 Mar. 1989 – 5 Sep. 1992

The results of the GARCH(1,1) estimation for the second subperiod, 21 Mar. 1989 – 5 Sep. 1992, of the pegged regime are shown in Table 3. This period includes a 12.3 % devaluation of the Finnish markka on 15 Nov. 1991. The effects of this realignment of the markka are modelled by three dummy variables, which take the value one for the actual devaluation day and the two supsequent days, respectively. The estimated coefficients for the dummy variables show a devaluation effect of 13 % for the actual devaluation day, a strengthening of 4 % on the following day and a weakening of 1 % on the third day. The cumulative effects of the three days amount to a 10 % strengthening of the other currencies against the markka.

The ARCH coefficient is significant for all currencies. The GARCH parameter is not significant for GBP and CHF. The change in the pattern of the

Table 3. **GARCH-estimates of the volatility of foreign exchange rates 21 Mar. 1989 – 5 Sep. 1992 (t-statistics in parenthesis) Estimation with 3 dummy variables**

	α_0	α_1	β_1	D_1	D_2	D_3	ARCH(1,1) test	Ljung-Box test statistics				
								LAG(1)	LAG(2)	LAG(3)	LAG(4)	LAG(5)
USD	0.7865·E−6 (2.21)	0.0679 (5.02)	0.9187 (59.2)	0.1299 (18.93)	−0.0495 (7.21)	0.9870·E−2 (1.44)	0.66	2.48	3.45	7.02	7.09	7.80
GBP	0.9137·E−5	0.2004 (5.02)	0	0.1280 (38.31)	−0.0364 (10.91)	0.8440·E−2 (2.52)	12.77	0.18	3.99	4.39	10.57	12.38
SEK	0.5207·E−7 (5.16)	0.0506 (6.53)	0.9280 (100.68)	0.1285 (83.22)	−0.0346 (22.39)	0.0131 (8.48)	6.88	6.07	6.07	8.69	9.49	9.57
NOK	0.2631·E−7	0.0619 (6.78)	0.9286 (120.12)	0.1291 (86.40)	−0.0357 (23.86)	0.0113 (7.57)	9.66	8.36	8.40	14.55	29.14	30.77
DKK	0.1645·E−6 (4.40)	0.0590 (5.83)	0.8975 (57.18)	0.1302 (66.06)	−0.0361 (18.34)	0.0116 (5.90)	7.52	3.64	4.64	6.23	6.23	6.24
DEM	0.6303·E−7 (3.00)	0.0615 (7.34)	0.9229 (91.51)	0.1303 (66.16)	−0.0348 (17.67)	0.0102 (5.18)	4.23	0.89	1.20	2.95	6.19	8.42
NLG	0.8201·E−7 (3.71)	0.0701 (7.22)	0.9095 (81.73)	0.1303 (65.95)	−0.0348 (17.63)	0.0103 (5.22)	10.65	2.13	3.12	4.10	9.60	12.84
BEF	0.1212·E−5 (5.07)	0.1023 (5.40)	0.5968 (8.64)	0.1300 (63.63)	−0.035 (17.30)	0.0114 (5.59)	68.48	8.02	9.64	10.10	11.24	12.11
CHF	0.1113·E−4 (24.61)	0.1061 (5.33)	0	0.1310 (37.16)	−0.0342 (9.71)	0.9660·E−2 (2.74)	2.55	0.21	4.57	8.10	8.56	9.40
FRF	0.1139·E−6 (4.52)	0.0585 (5.94)	0.9050 (66.03)	0.1299 (72.98)	−0.0352 (19.76)	0.1173 (6.59)	3.03	1.26	2.79	4.20	5.04	5.38
ITL	0.9295·E−6 (2.95)	0.1101 (4.26)	0.7120 (9.18)	0.1297 (56.39)	−0.0358 (15.55)	0.0111 (4.84)	7.48	33.03	33.73	33.93	34.51	35.12
JPY	0.7420·E−6 (2.49)	0.0790 (3.96)	0.9009 (39.09)	0.1288 (23.10)	−0.0428 (7.68)	0.5930·E−2 (1.06)	0.03	0.74	0.74	0.80	1.66	1.67

variance of the GBP is explained by the fact that the period includes the entry and exit from the ERM of the GBP. The sum of α_1 and β_1 is, strictly speaking, less than one suggesting that the underlying variance processes are weakly stationary. In most cases, however, the sum of the parameters is very close to one.

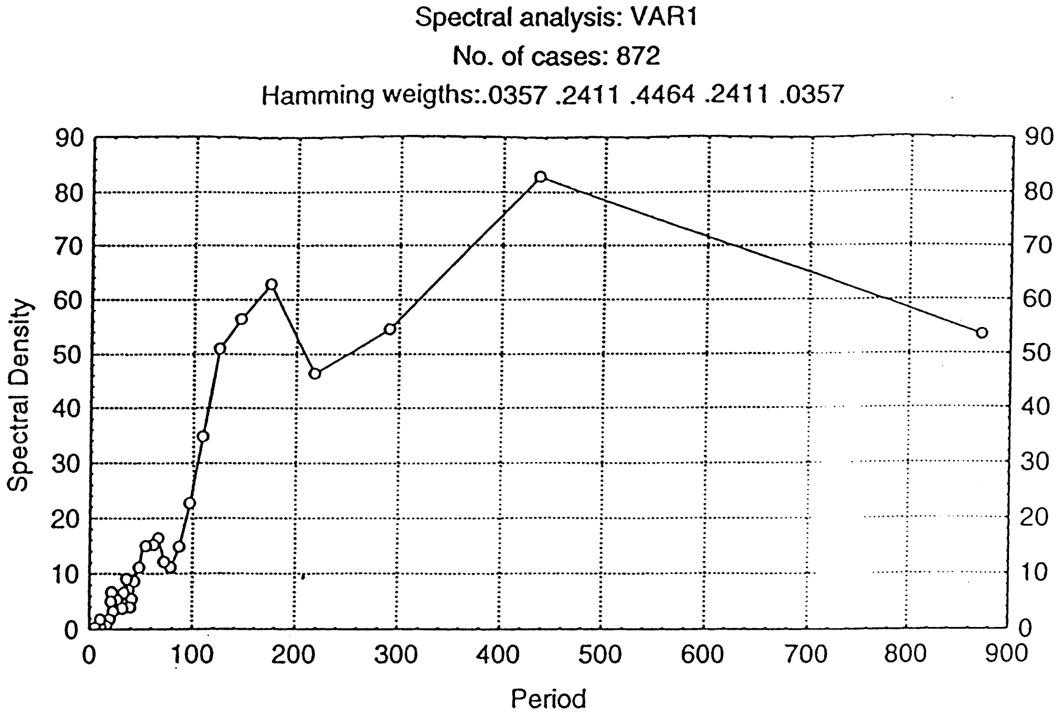
The values of the estimated principal components appear in Table 4. The fractions explained are almost identical to the previous subperiod of the pegged regime. The dominant currency is, however, not USD but DEM (and DEK and NLG because of their strong correlation with DEM).

The spectral density functions of the individual conditional variances peak at 180, 430 and 860 days. This is also confirmed in the spectral density function of the first principal component (Figure 3). At this period, the second and third peak frequency seem to be harmonics of the first one.

Table 4. **Principal components of conditional variances. Eigenvalues and cumulative fraction explained. Foreign exchange rates 21 Mar. 1989 – 5 Sep. 1992.**

Component	Eigenvalue	Cumulative R-Squared
1	5.5139	0.4594
2	1.6188	0.5944
3	1.2500	0.6985
4	0.9555	0.7781
5	0.8184	0.8464
6	0.5995	0.8963
7	0.4758	0.9360
8	0.5208	0.9794
9	0.1007	0.9878
10	0.0200	0.9894
11	0.0717	0.9954
12	0.0543	1.0000

Figure 3. **Spectral density function of the first principal component of conditional variances of foreign exchange rates 21 Mar. 1989 – 5 Sep. 1992**



3.5 The floating period

The floating regime of the markka is analyzed here more thoroughly, because forecasting will be based on the estimates of the conditional variances for this period. The estimates for the pegged period are used for comparing the volatility estimates across regimes. These comparisons may prove useful, since formally markka's free float come to an end on 14 Oct. 1996, when it entered the ERM. The institutional circumstances and obligations of the membership of ERM are presently closer to those of the floating period than to the earlier pegged periods, however.

Statistical stationarity tests were performed for the floating period 8 Sep. 1992 – 31 Dec. 1995. The Weighted Symmetric τ test, the augmented Dickey-Fuller τ test and the Phillips-Perron Z-test were employed both for the logs of exchange rates and the log differences. The estimated test statistics for the levels imply that the hypothesis of a unit root cannot be rejected, not even at a 1 % level of significance. The only value close to the 1 % critical value is the Dickey- Fuller τ test for the USD; the other two statistics for this currency do not sustain rejection. Pantula (1985) has shown that the asymptotic distribution of the Dickey-Fuller statistics is invariant to ARCH, meaning that the test is asymptotically robust to autoregressive conditional heteroskedasticity. The Phillips-Perron test, on the other hand, was good finite sample properties and may thus be more reliable here. Based on all the test statistics for the first differences, the hypothesis of a unit root can thus be rejected. The presence of a trend, which is detected for the levels, cannot be found in the differences any longer. The tests support the presence of one, and only one, unit root in the levels of the series. Thus, each series is appropriately made stationary by taking first differences.

The results of the GARCH(1,1) estimation are presented in Table 5. The first turbulent days of the floating regime are omitted and the estimation period begins from 14 Sep. 1992. The ARCH-parameter α_1 is zero for DEM and JPY and 1 for BEF. These values are set in the iteration process when the estimated values are reaching the boundaries of 0 and +1 for the parameters. The constant α_0 is significant for all currencies, but very small in magnitude. The sum $\alpha_1 + \beta_1$ is close to one for most currencies; through the forcing of the α_1 parameter to its boundary value in the iteration, the sum of the coefficients is much higher than one for the BEF. The parameter values for ITL indicate nonstationarity in variance.

Table 5. **GARCH-estimation of the volatility of foreign exchange rates 14 Sep. 1992 – 31 Dec. 1995 (t-statistics in parenthesis)**

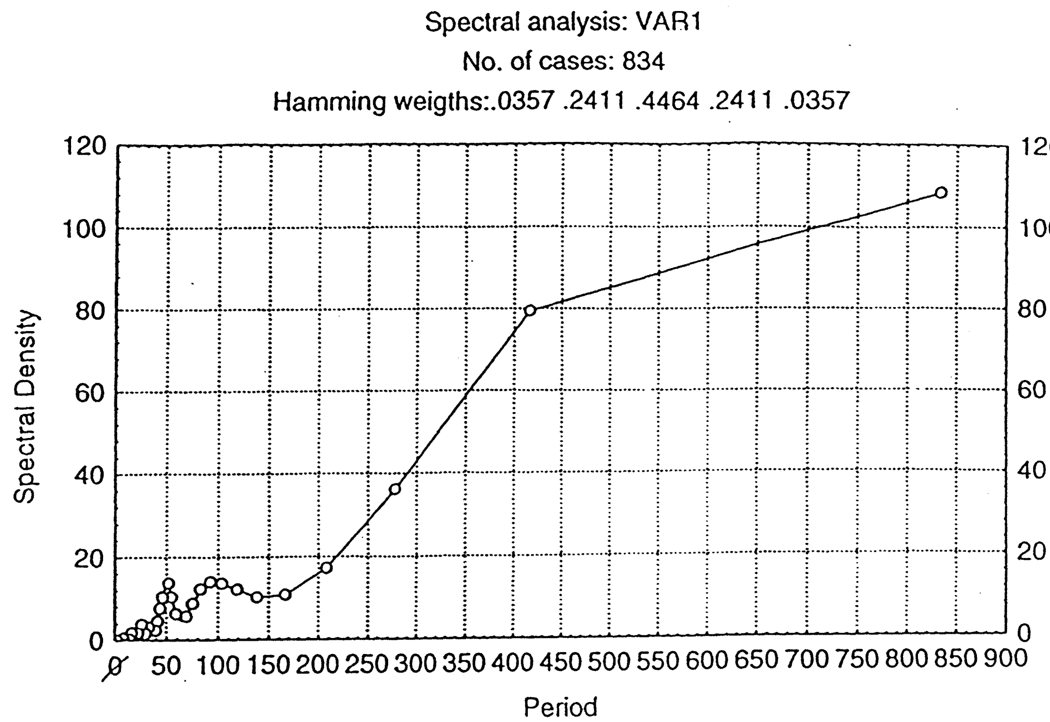
	α_0	α_1	β_1	ARCH(1,1)	Ljung-Box test statistics				
					LAG(1)	LAG(2)	LAG(3)	LAG(4)	LAG(5)
USD	0.3189·E−5 (4.57)	0.0792 (4.87)	0.8608 (39.79)	63.71	0.84	1.12	1.61	2.03	2.03
GBP	0.1134·E−5 (3.53)	0.0393 (4.20)	0.9225 (59.51)	64.60	0.51	2.20	18.04	20.16	22.65
SEK	0.4929·E−8 (4.27)	0.0549 (5.03)	0.9196 (58.09)	22.55	22.45	23.32	36.55	41.28	47.71
NOK	0.2793·E−8 (4.43)	0.0670 (7.42)	0.9292 (139.94)	30.06	8.12	8.40	9.35	11.54	22.94
DEK	0.3186·E−5 (4.03)	0.4586 (14.77)	0.5686 (13.24)	0.02	0.93	5.50	12.05	14.17	14.20
DEM	0.1653·E−6 (7.78)	0	0.9859 (699.75)	26.20	4.85	4.87	14.68	14.93	14.94
NLG	0.6442·E−7	0.1025 (7.59)	0.8623 (77.52)	18.76	8.53	11.93	12.41	15.64	66.44
BEF	0.1568·E−6 (6.20)	1	0.4677 (36.17)	6.93	10.79	26.02	43.04	44.87	52.81
CHF	0.4272·E−5 (7.72)	0.5814 (24.93)	0.3996 (12.87)	70.94	4.29	4.62	6.71	7.21	7.33
FRF	0.2884·E−5 (5.61)	0.2109 (6.91)	0.6612 (14.86)	74.82	0.14	3.54	7.18	7.35	7.60
ITL	0.8532·E−7 (17.41)	0.4615 (17.60)	0.6914 (55.29)	41.70	17.26	85.97	86.29	97.85	98.02
JPY	0.9024·E−9 (11.98)	0	0.9898 (1563.21)	26.51	1.32	4.54	9.57	9.58	14.10

The principal components are presented in Table 6. Compared to the pegged period, the fraction explained is 10 % higher for the three first components. This indicates a greater homogeneity in the variance structures. The dominant currencies seem to be GBP and DEM. Visual inspection of the spectrum of the first principal component of the conditional variances (Figure 4) once again strongly suggests the variance processes are persistent. The spectrum decreases almost monotonically from its value at the lowest frequence of 0.00754 (radians per day, or 0.0012 cycles per day corresponding to a wavelength of 834 days) to its value at the Nyqvist frequency (π radians or ½ cycles per day, wavelength 2 days).

Table 6. **Principal components of conditional variances. Eigenvalues and cumulative fraction explained. Foreign exchange rates 14 Sep. 1992 – 31 Dec. 1995.**

Component	Eigenvalue	Cumulative R-Squared
1	6.5713	0.5476
2	1.5919	0.6802
3	1.3205	0.7903
4	0.9224	0.8671
5	0.5081	0.9095
6	0.4612	0.9479
7	0.2909	0.9722
8	0.1673	0.9861
9	0.0773	0.9926
10	0.0463	0.9964
11	0.0381	0.9996
12	0.0043	1.0000

Figure 4. **Spectral density function of the first principal component of conditional variances of foreign exchange rates 14 Sep. 1992 – 31 Dec. 1995**

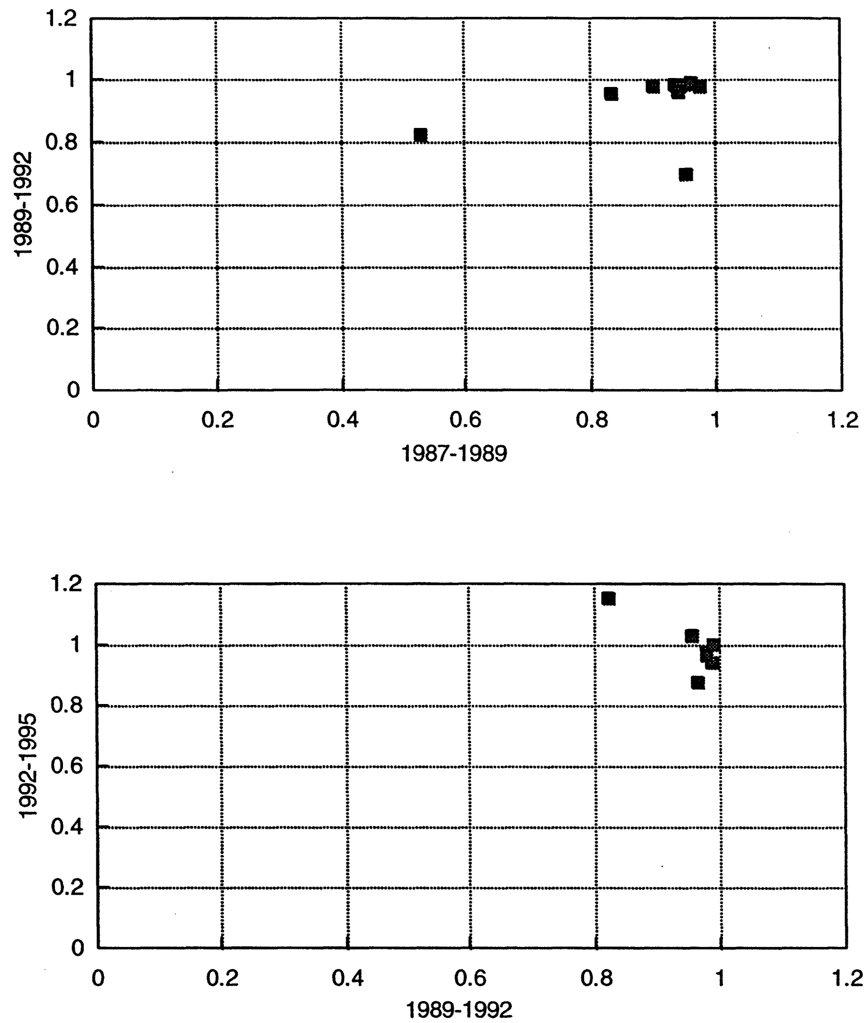


3.6 Pooled data

The main purpose of this study is to find a formula for the variance as a risk measure, which could be applied to all currencies. The results of the GARCH estimation for the individual currencies during both the pegged and the floating period show that there is a great similarity in the estimated parameter values of the variance process within periods. To evaluate the similarity between the individual conditional variance models, the sum $\alpha_1 + \beta_1$ for the variances in two periods were plotted against each other. In the first Figure in 5, the sum for the first floating period is plotted against the second pegged period. In the lower figure, the second pegged period is plotted against the floating period. The figures show clear clustering, which is interpreted as similarity between the individual parameter structures thus justifying pooling of the data.

Figure 5. **The sum $\alpha_1 + \beta_1$ for different periods plotted against each other**

Exchange rates



Next step then was to force the conditional variances for all the currencies into the same model by identifying a GARCH model on pooled data. In the estimation, the log differences for the individual twelve currencies were pooled separately for each period and a GARCH(1,1) model was estimated on this data. The pooled data within periods was constructed by simply connecting the data on the individual currencies together. While this implies incorrectness in the data at the connecting points, given the huge amount of data, the impact of so few data points is considered to be negligible.

GARCH(1,1) estimates the first pegged period 1 Jan. 1987 - 16 Mar. 1989 are

$$h_t = \underset{(7.69)}{0.2692*E\text{-}7} + \underset{(37.73)}{0.0567\varepsilon^2_{t-1}} + \underset{(953.01)}{0.9406h_{t-1}} \qquad (16)$$

To compare the goodness of fit of the pooled model to the individual models maximum values of the likelihood functions were calculated. The sum of the individual maximum likelihood functions is 31794, whereas the value for the pooled model is 31582 and the corresponding test statistics $\chi^2_{(44)}$ statistics for the null of the same GARCH(1,1) model is 420. This is highly significant thus confirming the expectation of forcing leading to an inferior model.

The impact of news given by the parameter $\alpha_1 = 0.0567$ is not very strong. The persistence parameter, however, is $\beta_1 = 0.9406$. The estimated mean lag of the variance expression, $1/(1-\beta_1)$ equals 16,7 meaning that it takes more than 3 weeks for the shocks to come through in the model. The sum $\alpha_1 + \beta_1 = 0.9973$ indicates an integrated process. One way to measure how long the impact of the shocks stays in the process, that is the persistence, is to use the half-life figure λ, which gives the number of days over which a shock to volatility diminishes to half its original size (Lamoureux and Lastrapes 1990). The half-life figure depends only on the sum of $\alpha_1 + \beta_1$ and is given by

$$\lambda = 1 - \left(\frac{\log 2}{\log(\alpha_1 + \beta_1)} \right) \tag{17}$$

For an integrated process, $\log(\alpha_1 + \beta_1)$ approaches zero from below and the λ value will be ∞. This is an other way to express the typical feature in an integrated process that the impacts of the shocks into the variance will never die out but remain for ever. For the pooled data the sum $\alpha_1 + \beta_1$ gives a half-life value $\lambda = 257$ days.

For the second subperiod 21 Mar. 1989 – 5 Sep. 1992 of the pegged regime the GARCH(1,1) estimates on pooled data were

$$\underset{}{h_t} = \underset{(13.41)}{0.3813 * E\text{-}7} + \underset{(71.22)}{0.1295 D_1} - \underset{(32.37)}{0.0361 D_2} + \underset{(9.17)}{0.0109 D_3} + \underset{(32.19)}{0.0621 \varepsilon^2_{t-1}} + \underset{(532.95)}{0.9353 h_{t-1}} \tag{18}$$

The estimated values of the ARCH and GARCH parameters are almost the same as for the previous subperiod. The models indicate a rather weak reaction of the conditional variance to shocks but a strong persistence. Even the values of the variance drift parameters are very close to one another. It is then reasonable to conclude that the behaviour of the exchange rates is homogenous all through the pegged period when the effects of the realignments are eliminated.

The sum of the values of the individual maximum likelihood functions was 48089 and the value for the pooled data model was 47969. The $\chi^2_{(79)}$ was 240. Although this test statistics is also highly significant it is clear that the violence made to the data by forcing the same model to the individual exchange rates is much less during this period than during the others.

The GARCH(1,1) estimation on the pooled data for the floating period 14 Sep. 1992 – 31 Dec. 1995 gave the following results

$$h_t = \underset{(24.79)}{0.2642 * E\text{-}5} + \underset{(71.53)}{0.1883 \varepsilon^2_{t-1}} + \underset{(169.51)}{0.7556 h_{t-1}} \tag{19}$$

The sum of the maximum likelihood functions for the individually estimated currencies is 47249 and for the pooled data 38013. The test statistic $\chi^2_{(44)}$ 18471 is highly significant, which rates the forced model estimated from the pooled data more inferior to the freely estimated individual models for this period than for the pegged period.

The value of the α_1, 0.1883, shows that the impact of news on the variance is much greater than during the pegged period. The impact of the lagged conditional variance dies considerably faster in this period than during the pegged period. The estimated mean lag of the variance expression, $1/(1-\beta_1)$, equals 4.17 or about four days. The sum $\alpha_1 + \beta_1$ is 0.9439, which means that the model is highly persistent but strictly speaking not integrated. The half-life figure λ equals 13 days. The value of the estimated parameter of the drift in variance, α_0, is much higher for the floating period than for the pegged period.

If we look at the figure of the currency index (Figure 1) there is a clear turning point in the middle of March 1993. From the beginning of the floating regime, 8 September 1992, there is a strong positive trend in the level of the index up to 10 March 1993. From that date on a similarly strong negative trend can be seen. This kind of changes in the trend may have implications on the estimation results worth to be considered. Perron (1989) has suggested that the widespread evidence of unit root in the univariate representation of time series may be due to the presence of important structural changes in the trend function. The changes can occur in the intercept, in the slope or in both. Similarly, ARCH effects may occur due to misspecification of the mean of the process, or, to be more precise, of the markka's trend during floating. The trend reversal itself may be an indication of the markka overshooting its long-term value or of a shift in the intervention policy pursued by the central bank. In any case, the observed point of the trend reversal is taken as exogenously given, and to account for its possible effects on estimated volatility, the sample is split into two subsamples around this observed point.

In the case of the currency index the hypothesis of an exogenously chosen break point is preferable especially when the turning of the slope occurred not slowly but after reaching a certain probably "overshooting" level, which may have trigged the intervention activity at the central bank (see Hung 1995 for a clearance of the effects of intervention strategies on exchange rate volatilities in US). There is a clear break point in the data found ex post, that can be interpreted as a sign of nonstationarity eg an unpredictable regime change. To account for this change in the regime the floating period was divided into two and new pooled estimations were made: one covering the upwards sloping period of the currency index and the other covering the downwards sloping period.

GARCH(1,1) estimation for pooled data covering the period of the markka's trend depreciation, 14 September 1992 – 10 March 1993 gave the following results

$$h_t = \underset{(8.54)}{0.2958 * E\text{-}2} + \underset{(7.88)}{0.3176}\varepsilon^2_{t-1} + \underset{(7.30)}{0.4455}h_{t-1} \qquad (20)$$

The results of the GARCH(1,1) estimation on pooled data for the period starting with the break date 10 March 1993 and ending at 31 December 1995 are

$$h_t = \underset{(14.87)}{0.9189*E\text{-}6} + \underset{(18.16)}{0.0809}\varepsilon_{t-1}^2 + \underset{(152.92)}{0.8847}h_{t-1} \tag{21}$$

The maximum value of the log-likelihood function for the whole floating period based on pooled data is 38013. The sum of the maximum values of the log-likelihood function for the subsamples is 38303. The value of the test statistics χ^2, which is strongly significant, indicates that the splitting of the floating period results in a superior model. The nonstationarity within the original full floating period is embedded in α_0. When accounting for the trend beak by allowing the constant to be freely estimated in the subperiods, we get considerably different values for α_0. Also the values of α_1 and β_1 differ between subsets. The identified model for the upwards sloping period is far from integrated with $\alpha_1 + \beta_1 = 0.7731$. For the downwards sloping period the sum is 0.9656 and the half-life figure $\lambda = 21$ days.

The pooled model for the first pegged period 1 Jan. 1987 - 16 Mar. 1989

$$h_t = \underset{(7.69)}{0.2692*E\text{-}7} + \underset{(37.73)}{0.0567}\varepsilon_{t-1}^2 + \underset{(953.01)}{0.9406}h_{t-1} \tag{22}$$

and for the second pegged period 21 Mar. 1989 - 5 Sep. 1992

$$h_t = \underset{(13.41)}{0.3813*E\text{-}7} + \underset{(71.22)}{0.1295}D_1 - \underset{(32.37)}{0.0361}D_2 + \underset{(9.17)}{0.0109}D_3 + \underset{(32.19)}{0.0621}\varepsilon_{t-1}^2 + \underset{(532.95)}{0.9353}h_{t-1} \tag{23}$$

have very similar α_1 and β_1 parameter values and we therefore conclude that the same model is applicable for the whole pegged period. The estimated model for the downwards sloping floating period

$$h_t = \underset{(14.87)}{0.9189*E\text{-}6} + \underset{(18.16)}{0.0809}\varepsilon_{t-1}^2 + \underset{(152.92)}{0.8847}h_{t-1} \tag{24}$$

is also very close to the model identified for the pegged period. The F-test of equality of the coefficients estimated for different periods was calculated and turned out be highly significant thus rejecting the null hypothesis. Given the large number of observations in the pooled data, however, this formal rejection of the null is perhaps not surprising. We therefore have to lean on pure common sense judgement to justify the conclusion that the conditional volatility of exchange rates can be modelled as the same integrated process regardless of the exchange rate regime. The assumption of equality simplyfies the multivariate analysis considerably.

3.7 BDS-statistics

The abrupt huge changes in financial time series especially in the stock market prices has fostered the idea that even GARCH modelling is too simple to capture the dynamics of the stochastic process driving the financial markets. This has led to attempt to apply the method of complexity and chaos to financial market data.

Most applied studies on chaotic behaviour of financial time series deal with stock returns. The results are mixed. Chaos is found in some papers in US stock returns, while others dispute the claim. Chaos as a general model of German stock returns is also rejected (Booth et al. 1992). In an extensive study, Hsieh (1991) rejects the hypothesis that the weekly stock returns are IID. He tests various explanations for the rejection: linear dependence, nonstationarity, chaos and nonlinear stochastic processes. The cause can not be found either in regime changes or chaotic dynamics but rather to be conditional heteroscedasticity. Similar results are reported in a study by Booth et al. (1992) on Finnish stock returns. The paper concludes that the stock returns exhibit nonlinear dependence but that the form of dependence is not chaotic. The nonlinear behaviour in their data is best explained by a GARCH model.

Although evidence for presence of deterministic chaotic generators in economic and financial time series has found not so far been very strong, the search for such generators has led to the development of new statistical tests (Brock et al. 1991) of which the most used one is the Brock, Dechart and Scheinkman BDS-test (Brock et al. 1987).

The BDS statistics is a general test for model misspecification. It is a diagnostic test where a rejection of the null hypothesis of IID innovations is consistent with some type of dependence in the data. They may result from a linear stochastic system, a non-linear stochastic system, or a non-linear deterministic system, ie. chaos. Additional diagnostic tests are therefore needed to determine the source of the rejection (Mills 1993, p. 125).

The asymptotic distribution of the BDS statistics, N(0,1), can approximate the finite sample distribution for 500 or more observations. The approximation appears uneffected by skewness or heavy tails. Simulations made by Hsieh (1991) confirm that neither the asymptotic nor the finite sample distribution of the BDS test is altered by using residuals instead of raw data linear models. This is not the case, however, when the test is applied to residuals from GARCH and EGARCH models. For these conditional variance models, the BDS test may reject too infrequently. Hsieh (1991) gives simulated critical values of the BDS statistic, to be used at 2,5 % and 97.5 % confidence levels for GARCH and EGARCH residuals.

BDS-statistics figures for the standardized residuals of the mean equation of the log differences are reported in Table 7 for the entire floating period. For this data, the N(0,1) assumption of the distribution of the test statistics is applicable. There is strong evidence against the null hypothesis of IID for all series. Simulations made by Hsieh (1991) show that the BDS test has good power to detect at least four types of non-IID features: linear dependence, nonstationarity, nonlinear stochastic processis and low dimensional chaos. In our case, prefiltering of the data rules out linear dependence. Nonstationarity caused by structural changes are accounted for by division of the estimation period into three intervals.

What is left then is nonlinearity in mean and variance. To capture nonlinearity in mean, the GARCH-M(1,1) model was tested. The results showed that the MEAN parameter is not statistically significant for any currency. The GARCH(1,1) was postulated to capture nonlinearity in variance. If the GARCH model is correctly specified, the standardized residuals should be IID in large samples. To determine whether any remaining non-linear structure is present in the model the BDS test was applied to the standardized GARCH(1,1) residuals (Table 7). For five currencies, the null of IID cannot be rejected when we use the simulated critical value of Hsieh, which is 2.11 for m=2 and $\epsilon/\sigma = 0.5$. For SEK, DKK, DEM, NLG, BEF, CHF and ITL the test finds evidence of remaining non-linearity or deterministic chaos. These findings are much in line with results reported in the literature for dollar-denominated exchange rates.

Table 7. **BDS-statistics for exchange rates**
9 Sep. 1992 – 31 Dec. 1995, m = 2, $\epsilon/\delta = 0.5$

	Standardized residuals	GARCH-residuals
USD	5.32	0.88
GBP	4.66	0.64
SEK	9.55	7.99
NOK	9.16	1.52
DKK	7.22	2.79
DEM	7.32	5.49
NLG	13.92	2.90
BEF	15.19	4.29
CHF	8.19	3.13
FRF	6.85	1.23
JPY	16.26	1.00
ITL	7.79	6.34

3.8 Summing up for exchange rates

So far we have shown that the stylized facts found in the FIM bilateral exchange rates can be modelled with a GARCH(1,1) process. Log-changes in the spot exchange rates are martingales, since conditional means are zero and there is no serial correlation.

The ARCH and GARCH parameters are significant for all exchange rates. The sum of the estimated parameters in the conditional equation for the individual currencies is close to one thus indicating an integrated variance process. This is also seen in the model estimated on pooled data, which turned out to be integrated for all periods. The principal component analysis applied to the estimated conditional variances was used as a method to detect a common set of variables generating exchange rate movements. Spectral analysis was performed on the estimated principal components to assess and measure common cyclical behaviour for the variances. There is a peak in the spectral density functions of the individual variances $h_{i,t}$ and the first principal components at 180 days for both pegged periods. The spectral density function of the first principal component for the floating period shows a peak at 420 days, but the overall interpretation of the density function is that of an at least persistent conditional variance process, perhaps even an integrated one.

In VAR model applications, the most used assumption of the stochastic process in first differences of financial rates is that of a random walk generating a normal distribution with a constant unconditional variance. Although we know that the random walk model does not fit observed data as well as autoregressive conditional variance models, it does not necessarily mean that its average performance is inferior to the time varying models. The estimated integrated conditional model for exchange rates derived in this study indicates that a constant variance forecast may be a good approximation of the time varying model. The random walk model can be considered as a benchmark against which the more sophisticated changing volatility models can be compared (Heynen and Kat 1994).

The alternative measures of the conditional, unconditional and sample variance of movements in the individual exchange rates can be summed up in a performance evaluation. The alternatives are

- GARCH(1,1) model conditional variance $h_{i,t}$. (KUH1 in Figure 6)
- GARCH(1,1) unconditional variance $\alpha_0/(1-(\alpha_1 + \beta_1))$, which also is the convergence limit for the conditional variance h_t. (KU837 in Figure 6)
- sample variance constant for the peak frequency evaluated on the cyclical behaviour of the individual conditional variances h_t and their principal components; 180 days for the pegged periods and 420 days for the floating period. (KU420 in Figure 6)
- sample variance calculated on quarterly data; the frequency selection is based on previous results (Ahlstedt 1990) where a subsample of 70 observations was found to be large enough to yield reasonable statistical efficiency, but still small enough to make it likely that the sample variance remains constant. (KU70 in Figure 6)

Figure 6. **Conditional, unconditional and sample variance comparison: USD floating period**

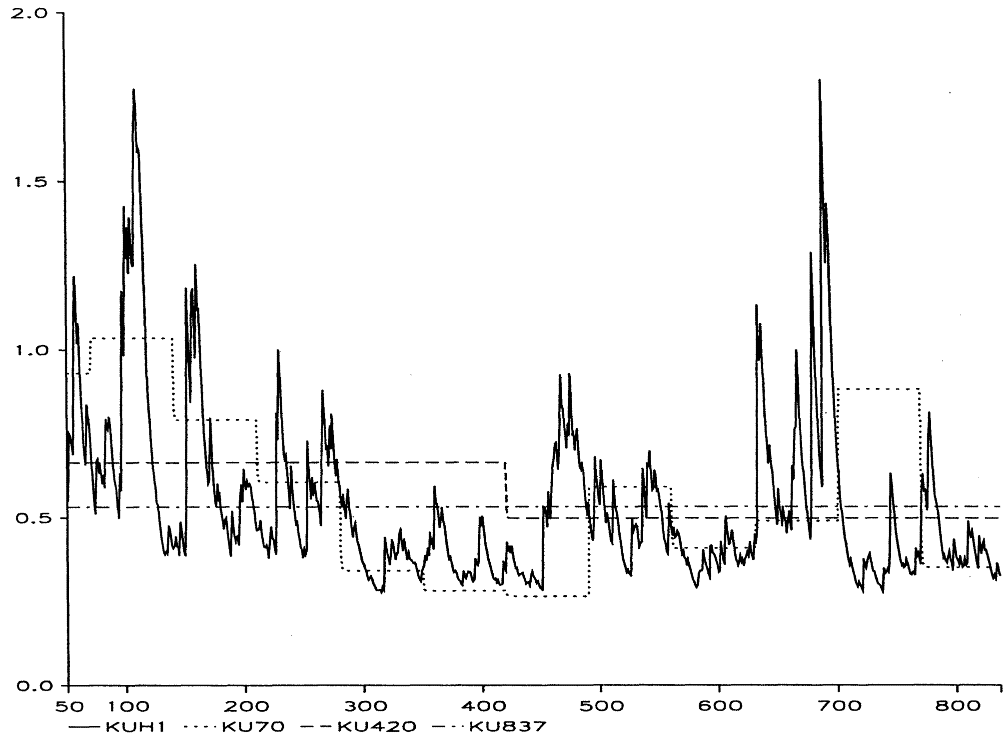
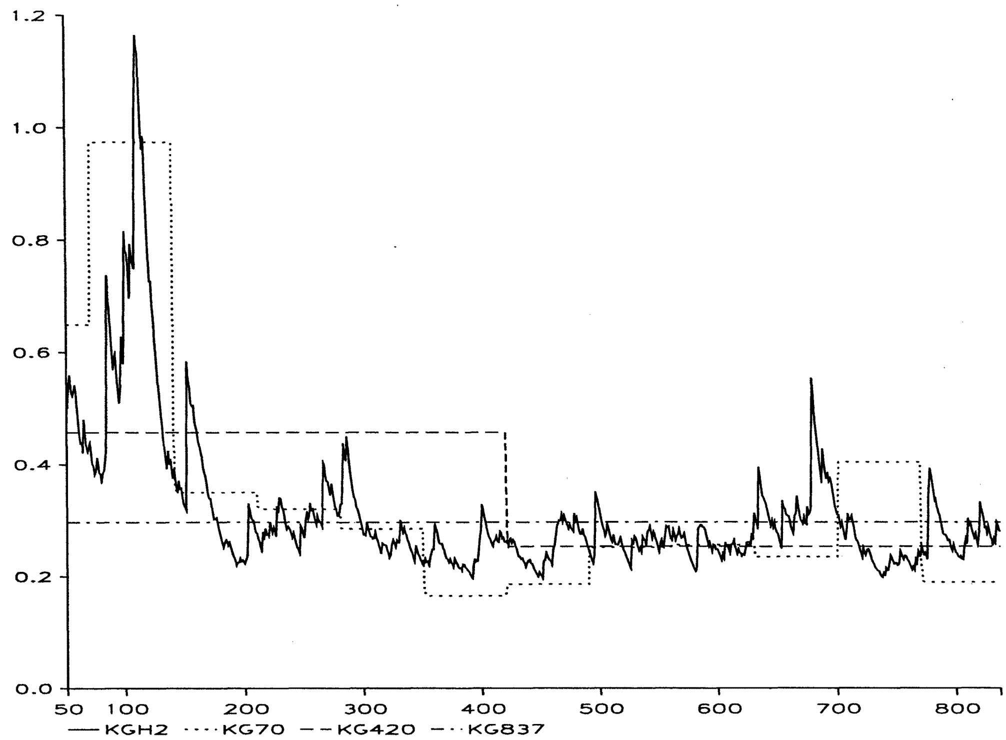


Figure 6 shows the comparison between these four methods for the USD and the floating period. The GARCH unconditional variance seems to be a good mean approximation of the conditional variance. The dominant frequency for the floating period, 420 days, appears twice in the sample size. This two-step function also gives a good visual approximation of the mean of the conditional variance. The step-function formed by the 70-day sample period, ie. quarterly frequency, smooths out the huge swings in the conditional variance and seems to capture the basic pattern in the fluctuation of the variance.

The corresponding variance measures are displayed in Figure 7 for GBP and the same floating period.

Figure 7. **Conditional, unconditional and sample variance comparison: GBP, floating period**



4 Conditional variance modelling of interest rate data

4.1 The statistical distribution of interest rates

ARCH has mainly been applied to interest rate data to explain relationships between long and short-term interest rates and to model the time-varying risk premium in future interest rates. The time-series variable to be modelled in these studies has been a measure of excess return of long-term yields over short-term yields or yields on corporate bonds over yields on credit risk-free Treasury bonds.

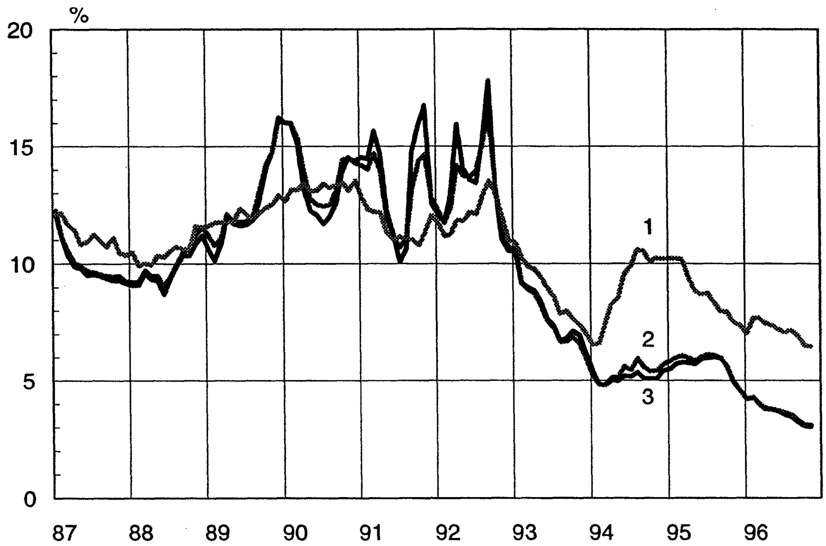
Especially ARCH-M and GARCH-M models have been applied, where a function of the conditional variance is included as a regressor in the mean equation to measure the risk premium. These models have not, however, been very successful. The inclusion of a MEAN term usually makes variables which have previously been found significant, no longer so. As a result, the usefulness of the model has also been challenged both on theoretical grounds by Backus, Gregory and Zin (1989) and on empirical grounds by Mehra and Prescott (1985), who showed that ARCH effects are more closely related to forecast errors than to risk premium.

Since most studies involving interest rates have, nevertheless, adopted GARCH(p,q) or GARCH-M(p,q) specifications, these models are selected also here. Usually the studies concentrate on yields, which are measured separately for individual bonds. The aim of this study, however, is to find a measure for interest rate risk in banks' portfolios without knowing the individual bond holdings. Thus, the statistical data for interest rates are used instead of yields.

To include the entire term structure of interest rates for all currencies in the study is not feasible. One way of diminishing the number of variables to be considered, but at the same time allow, however, the inclusion of the behaviour of the entire term structure, would be to use the method of principal components. Through this method, the variances of all interest rates for one currency are transformed into three main variables describing changes, respectively, in the general level of the term structure and changes in the slope and curvature of the term structure (Kärki and Reyes 1994). If the principal component method is used, then forecasting should accordingly concentrate on these changes in the behaviour of the curve. Since the objective of this study is to construct an estimate for the future behaviour of the rates themselves, we decided not to use the principal component method in this context. Instead, the solution to the problem with term structure coverage is sought by selecting one rate to represent all short rates up to one year. The correlation matrix for 1-month, 3-month, 6-month and 12-month rates was therefore calculated. The 3-month rate was tested as to have the highest correlation with the other short-term rates and was consequently selected to represent the term structure of the interest rates up to one year. The domestic 3-year rate was selected to represent the longer rates.

The same main time periods as for the exchange rates were chosen. Although the structural changes on which the division is based is not as clear as for the exchange rates but it is however defendable (Figure 8). The daily changes in interest rates are expressed as differences proportional to the levels. The order of differencing is dictated by the requirement of stationarity. To this end Weighted Symmetric τ test, Phillips-Perron Z-test and the augmented Dickey-Fuller test were employed to levels and differences both for the pegged period 1 Jan. 1987 - 5 Sep. 1992 and the floating period 9 Sep. 1992 - 31 Dec. 1995. The hypothesis of a unit root in levels was not rejected, but was strongly rejected in first differences by all three tests for both periods and for all interest rates. The estimated p-values in differences for Type I error is zero for the pegged period. The largest p-value for the floating period is 0.005 % for ERGDP. Based on the results of the augmented Dickey-Fuller test we conclude that there is no trend or constant in the unit root process generating observed of interest rates.

Figure 8. **Key interest rates**



1 Long-term bond rate (close to ten year)
2 3-month Helibor
3 1-month Helibor

The interest rate differentials reveal, unlike the exchange rates, strong linear serial correlation measured by the Ljung-Box test statistics. The ARCH(1) test statistics, calculated from a regression of the squared residuals on the lagged squared residuals, were also significant for most series for all three periods.

Prior to specifying GARCH-models for the interest rate series they had to be filtered from linear dependence. AR(p) models, $p \leq 5$, were identified. The selection of the order p (Table 16) is based on the 1 % probability level for the Jung-Box test statistics. It would have been very convenient to use the same order of AR-filtering for all series. It was, however, found that over-filtering for some interest rates removed the significant GARCH effects in the data under lower order filtered data. To avoid the harmful effects on the data of over-filtering, therefore, the order of the linear autoregressive filtering models were chosen individually for all thirteen series. The test values for the residuals of these pre-filtered models show that the filtering process produced linearly independent data for all interest rates with the exception of ERGBP. For ERGBP, even using 12 lags is insufficient to remove serial correlation during the first pegged period.

Table 16. **Selected order of pre-filtering. 3-month interest rates.**

	Lags up to order AR(p)		
	1.1.87–16.3.89	21.3.89–5.9.92	8.9.92–31.12.95
ERUSD	AR(1)	AR(1)	AR(3)
ERGBP	AR(12)*	AR(2)	AR(3)
ERSEK	AR(2)	AR(2)	AR(5)*
ERNOK	AR(1)	AR(3)	AR(5)
ERDKK	AR(2)	AR(1)	AR(3)
ERDEM	AR(1)	AR(2)	AR(3)
ERNLG	AR(1)	AR(1)	AR(1)
ERBEF	AR(4)	AR(1)	AR(1)
ERCHF	AR(1)	AR(2)	AR(3)
ERFRF	AR(5)	–	AR(4)
ERITL	AR(2)	AR(2)	AR(5)*
ERJPY	AR(1)	AR(1)	AR(1)
ERFIM	AR(1)	AR(2)	AR(4)

* Linear dependence remaining in the pre-filtered data.

Next step was to estimate A GARCH-M(1,1) model for both the pegged and the floating period. Based on the test results no constant term was included in the mean equation. For the floating period, the MEAN variable for only ERUSD was statistically significant. The inclusion of the MEAN, however, makes the GARCH-parameter insignificant, thus confirming the results from other studies.

4.2 The pegged period

4.2.1 First subperiod 1 Jan. 1987 – 16 Mar. 1989

Table 17 shows the results of GARCH(1,1) estimation for the first pegged period, 1 Jan. 1987 – 16 Mar. 1989, on the prefiltered interest rate differentials of ERUSD, ERGBP, ERSEK, ERNOK, ERDKK, ERDEM, ERNLG, ERBEF, ERCHF, ERFRF, ERITL, ERJPY and ERFIM. In the iterative estimation, the ARCH parameter was set to its lower boundary value zero for ERDKK, which means that there is no impact of news on the variance process. The GARCH parameter for ERGBP and ERCHF were also set to the lower boundary value zero. For these two interest rates, then, past conditional variance does not help forecasting future conditional variances.

Table 17. **GARCH (1,1) estimation of the volatility of 3-month interest rates 1 Jan. 1987 – 16 Mar. 1989 (t-statistics in parenthesis) (data multiplied by 100)**

	α_0	α_1	β
ERUSD	0.4374E-2 (6.12)	0.2885 (4.53)	0.1442 (1.27)
ERGBP	0.0294 (73.42)	0.1105 (2.55)	0
ERSEK	0.1371E-2 (4.37)	0.2400 (6.47)	0.7390 (25.79)
ERNOK	0.1177E-2 (2.49)	0.1253 (8.25)	0.8481 (41.37)
ERDKK	0.8462E-5 (0.31)	0	0.9957 (1200.28)
ERDEM	0.1516E-3 (2.70)	0.0927 (3.85)	0.8871 (31.55)
ERNLG	0.7243E-4 (2.92)	0.0947 (4.68)	0.8888 (40.72)
ERBEF	0.4927E-3 (4.99)	0.0930 (5.60)	0.8359 (41.33)
ERCHF	0.8482E-2 (19.93)	0.0892 (1.59)	0
ERFRF	0.1237E-2 (3.77)	0.1766 (4.39)	0.7299 (15.32)
ERITL	0.5298E-2 (5.07)	0.2976 (6.42)	0.5605 (9.56)
ERJPY	0.1143E-3 (2.14)	0.0583 (3.20)	0.9080 (38.19)
ERFIM	0.6582E-4 (4.83)	0.2949 (10.33)	0.7559 (38.97)

The estimated models are (weakly) stationary in variance with the exception of ERFIM for which the sum $\alpha_1 + \beta_1$ is 1.0508. This value is probably not significantly different from one, but the Finnish interest rate was anyhow excluded from the pooled data. In forecasting experiment, the domestic interest rate will be forced to follow the model which is estimated on the pooled data.

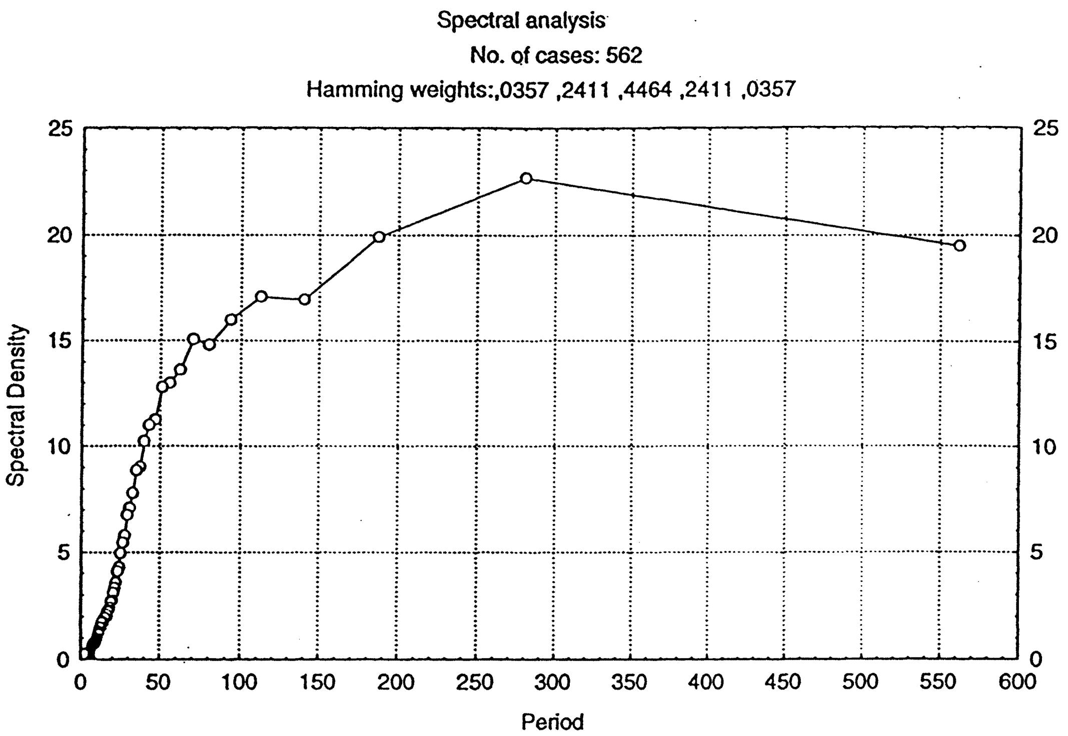
In order to detect common factors driving the conditional variances of interest rates, principal components were estimated for the pegged periods. The eigenvalues and cumulative fraction explained by the components for the first pegged period are shown in Table 18. The fractions explained by the first components is overall, relatively small compared to macroeconomic data. The conditional variance of ERBEF has the strongest factor loading on the first principal component followed by ERITL and ERJPY. The factor loading of US interest rate is practically neglible. Graphical analysisis of the principal components indicate, however, that it is not possible through this method to identify strong common factors, which could have been used as substitutes for latent factors found through multivariate GARCH estimation (Diebold and Nerlove 1989).

Table 18. **Principal components of conditional variances. Eigenvalues and cumulative fraction explained. 3-month interest rates 1 Jan. 1987 – 16 Mar. 1989.**

Component	Eigenvalue	Cumulative R-Squared
1	4.2484	0.3268
2	1.9312	0.4753
3	1.4134	0.5840
4	0.9874	0.6600
5	1.0417	0.7401
6	0.6315	0.7887
7	0.8260	0.8523
8	0.7940	0.9133
9	0.4775	0.9501
10	0.3746	0.9789
11	0.1327	0.9891
12	0.1082	0.9974
13	0.0328	1.0000

Spectral analysis of the individual variances, h_t, and of the first principal component (Figure 10) once again strongly suggest persistence in the underlying factors affecting the time variability of the conditional variances. Moreover, a cycle corresponding to a period of 281 days (second harmonic and 0.0224 radians or 0.0036 cycles per day).

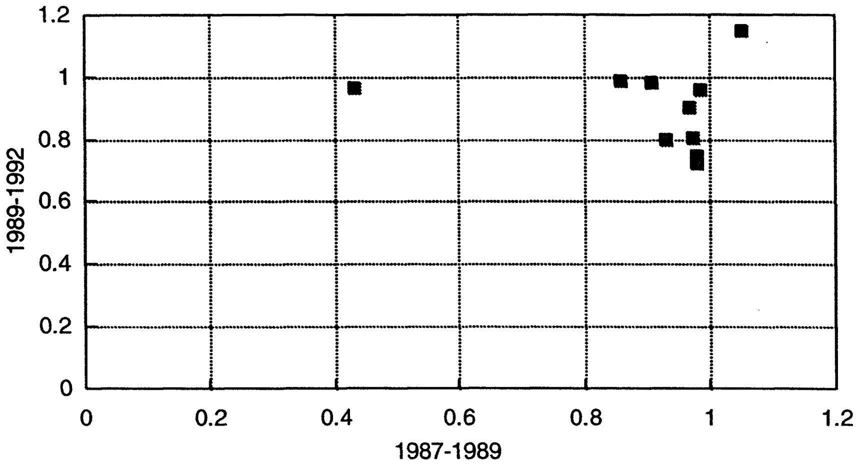
Figure 10. **Spectral density function of the first principal component of conditional variances of 3-month interest rates 1 Jan. 1987 - 16 Mar. 1989**

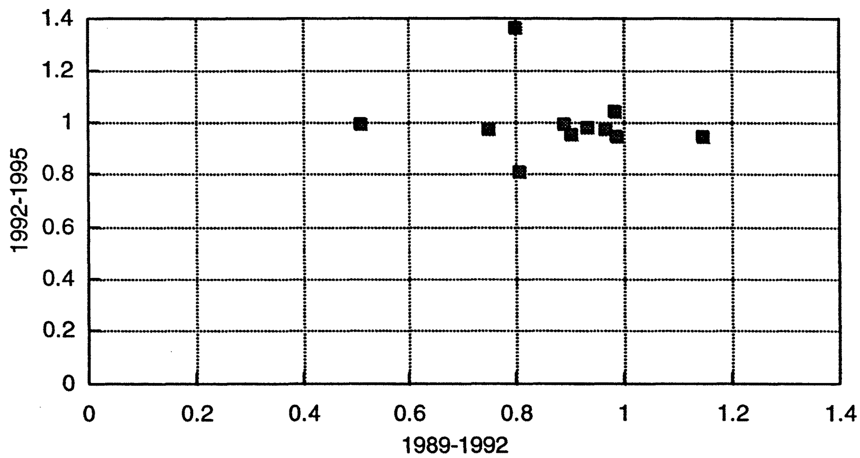


As for exchange rates the use of pooled data would impose the same structure on all the interest rates. In the same way as for exchange rates, the similarity between the estimated individual interest rate models was graphically tested by plotting the sum of the ARCH and GARCH coeffients for the three main periods against each other. Figure 13 displays strong clustering and in this sense sustain analysis on pooled data.

Figure 11. **The sum $\alpha_1 + \beta_1$ for different periods plotted against each other**

Interest rates





The following GARCH(1,1) model was estimated on the pooled data for the first subset of the pegged period

$$h_t = \underset{(6.36)}{0.3770*E\text{-}6} + \underset{(49.08)}{0.0669\varepsilon_{t-1}^2} + \underset{(1526.19)}{0.9418h_{t-1}} \tag{25}$$

The sum of the estimated ARCH and GARCH parameters is 1.0087, which makes conditional variance process of the interest rates integrates. The mean lag $1/(1-\beta_1)$ equals 17 days. The half-life frequency λ for an integrated process is infinite.

Prior to GARCH estimation the data for the second pegged period 21 Mar. 1989 - 5 Sep. 1992 was pre-filtered. The selected order p based on Box-Ljung statistics are presented in Table 16.

4.2.2 Second subperiod 21 Mar. 1989 - 5 Sep. 1992

Table 19 presents the results from GARCH(1,1) estimation for the second pegged period 21 Mar. 1989 - 5 Sep. 1992. Both ARCH and GARCH parameters are significant for all interest rates. With exception of ERFIM, all interest rates are stationary in variance. For the first pegged period, the sum $\alpha_1 + \beta_1$ was 1.0508 for the ERFIM and 1.0476 for this second period. Both sums do not significantly differ from one, and we can conclude that there is a unit root in the conditional variance process for both pegged periods for the Finnish interest rate. This second part of the pegged period includes a 12.3 % devaluation of the Finnish currency on 15 Nov. 1991. This realignment is accounted for in estimation of GARCH models for the foreign exchange rates for the corresponding period by using dummy variables. In the Finnish interest rate data, there is a huge peak at the devaluation date. An alternative model was tested for ERFIM including a dummy variable for the crucial date. There was no change in the estimated ARCH and GARCH parameter values compared to the model estimated without the dummy variable.

Results for principal components analysis for finding common factors in the second pegged period are shown in Table 20. The cumulative fraction explained by the components grows very slowly with the number of components included. There is an even stronger heterogenity in this the group of conditional variances than in the previous period. The same dominant interest rates in the factor loadings of the first principal component as in the first pegged period are found also for this period. Whereas the spectral density function of the first principal component also in this subperiod clearly gives evidence of persistent factors underlying the conditional variance processes, contributions from higher frequencies, most notably from those corresponding to cycle lengths of 62-174 days (0.0362-0.1014 radians or 0.0058-0.0161 cycles per day, ie 5.-14. harmonics), are visible in the figure.

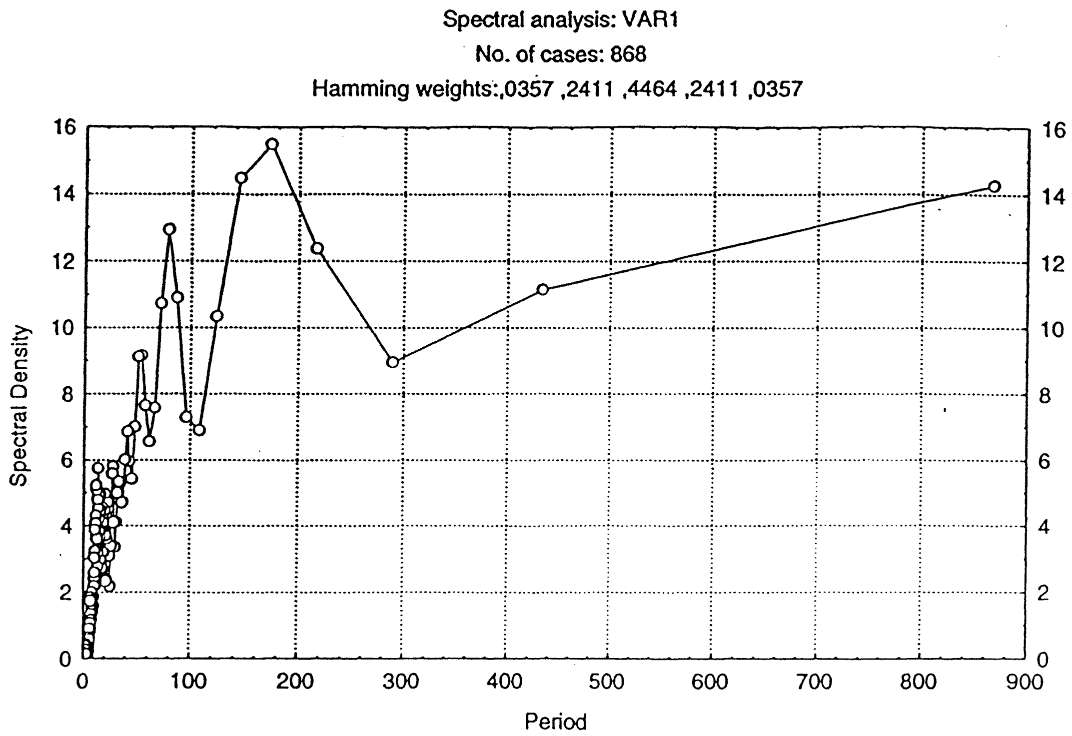
Table 19. **GARCH(1,1) estimation of the volatility of 3-month interest rates 21 Mar. 1989 – 5 Sep. 1992. (t-statistics in parenthesis) (data multiplied by 100)**

	α_0	α_1	β_1
ERUSD	0.2160E-3	0.0878	0.8772
	(4.39)	(5.80)	(44.77)
ERGBP	0.6923E-2	0.4098	0.0983
	(15.50)	(6.40)	(1.81)
ERSEK	0.8382E-2	0.2768	0.4710
	(14.39)	(11.03)	(13.26)
ERNOK	0.3710E-2	0.2058	0.6001
	(10.97)	(5.94)	(17.60)
ERDKK	0.1520E-2	0.2788	0.6092
	(9.23)	(8.77)	(21.71)
ERDEM	0.1387E-2	0.1862	0.5381
	(4.68)	(5.74)	(6.67)
ERNLG	0.1504E-3	0.1037	0.8551
	(3.52)	(5.68)	(33.57)
ERBEF	0.9879E-3	0.0743	0.7244
	(3.24)	(3.51)	(9.81)
ERCHF	0.8691E-3	0.1359	0.7958
	(2.98)	(7.02)	(22.55)
ERFRF	0.8490E-4	0.0565	0.9258
	(3.44)	(5.26)	(80.28)
ERITL	0.7565E-3	0.2001	0.7874
	(6.32)	(10.72)	(45.44)
ERJPY	0.4072E-3	0.1881	0.7140
	(10.37)	(7.36)	(37.19)
ERFIM	0.1788E-2	0.5929	0.5547
	(10.05)	(19.10)	(45.05)

Table 20. **Principal components of conditional variances. Eigenvalues and cumulative fraction explained. 3-month interest rates 21 Mar. 1989 – 5 Sep. 1992.**

Component	Eigenvalue	Cumulative R-Squared
1	2.8366	0.2182
2	2.0931	0.3792
3	0.2627	0.3994
4	0.3257	0.4244
5	0.4213	0.4568
6	1.1914	0.5485
7	0.6008	0.5947
8	0.6849	0.6474
9	0.7879	0.7080
10	0.8417	0.7728
11	0.9601	0.8466
12	1.0010	0.9236
13	0.9923	1.0000

Figure 12. **Spectral density function of the first principal component of conditional variances of 3-month interest rates 21 Mar. 1989 – 5 Sep. 1992**



The GARCH(1,1) model for the pooled data for the second pegged period is

$$h_t = \underset{(58.15)}{0.1497*E\text{-}5} + \underset{(61.76)}{0.0958}\varepsilon^2_{t-1} + \underset{(1444.69)}{0.9005}h_{t-1} \tag{26}$$

The sum of the ARCH and GARCH coefficients is 0.9963 which indicates an integrated variance process even for this second pegged period. Although the sum of the coefficients is the same for both parts of the pegged period, the estimated values differ between the individual coefficients. The impact of news, α_1, is bigger for the second period and, consequently, that of the past conditional variance smaller. Since the value of β_1 determines the mean lag of shocks, this lag measure will also differ between the two periods. The mean lag for the first period is 17 days and for the second period 10 days. The half-life statistics λ is 188 days for the second period.

4.3 The floating period

Prefiltering of order p shown in Table 16 was performed. Despite prefiltering, ERSEK and ERITL still showed linear dependence measured by the Ljung-Box test statistics.

The results for the GARCH(1,1) estimation for the floating period, 9 Sep. 1992 - 31 Dec. 1995, are shown in Table 21. The GARCH coefficient is significant for all interest rates. For ERDEM and ERNLG, the ARCH parameter was set to its lower boundary value zero. For these interest rates, there is no impact of news on the interest rates. The sum $\alpha_1 + \beta_1$ is less than one for most interest rates. The sum is exactly one for ERGBP, thus indicating an integrated model. The models for ERNOK, ERDKK, ERBEF and ERFRF are, however, non-stationary in variance.

The graphs of the individual pre-filtered interest rate data display increasing volatility during the turbulen times at the beginning of the floating period; interest rates react strongly to the perceived uncertainty in the currencies while they are approaching equilibrium after the fierce attacks against the currencies. This period, however, must be regarded as exceptional on which forecasts should not be based. Thus, the turbulent period can be left out from the estimation period for those currencies where the sum of $\alpha_1 + \beta_1$, exceeds one. For these currencies, the estimation period was chosen by the requirement that the conditional variance process be at most integrated. In the case of ERNOK, this was constructed by dropping the first 100 data points; for DKK, by dropping the first 250 observations. The non-stationarity in ERBEF could not be eliminated by selection of a subperiod, since the non-stationary features in the variance are distributed over the entire period. ERFRF shows clear non-stationarity at the beginning and at the end of the period. The middle period is too short to be used for identification of the model. While several subperiods were tested, stationarity was not achieved.

The GARCH-estimation results for stationary periods for ERNOK and ERDKK appear in Table 22.

Table 21. **Garch (1,1) estimation of the volatility of the 3-month interest rates 8 Sep. 1992 – 31 Dec. 1995 (t-statistics in parenthesis) (data multiplied by 100)**

	α_0	α_1	β_1
ERUSD	0.4320·E–4	0.0460	0.9298
	(4.10)	(5.01)	(66.52)
ERGBP	0.2766·E–4	0.0663	0.9289
	(4.52)	(7.51)	(144.17)
ERSEK	0.4519·E–3	0.0610	0.9098
	(8.74)	(7.48)	(114.59)
ERNOK	0.2272·E–2	0.9826	0.4215
	(6.06)	(59.82)	(17.69)
ERDKK	0.8906·E–3	0.4743	0.6849
	(18.12)	(18.75)	(57.47)
ERDEM	0.8772·E–5	0	0.9902
	(11.35)		(1550.77)
ERNLG	0.3544·E–5	0	0.9941
	(5.57)		(1731.46)
ERBEF	0.1576·E–2	0.8218	0.5428
	(14.36)	(16.34)	(40.95)
ERCHF	0.5007·E–4	0.0393	0.9420
	(3.70)	(5.17)	(93.02)
ERFRF	0.2938·E–4	0.1705	0.8687
	(6.90)	(27.10)	(361.04)
ERITL	0.1029·E–6	0.0944	0.8501
	(8.24)	(6.08)	(52.53)
ERJPY	0.1038·E–7	0.1447	0.8113
	(6.72)	(10.30)	(45.28)
ERFIM	0.2387·E–3	0.0711	0.8768
	(14.87)	(14.99)	(131.87)

Table 22. **GARCH (1,1) volatility estimation, interest rates; SUB-periods of 9 Sep. 1992 – 31 Dec. 1995**

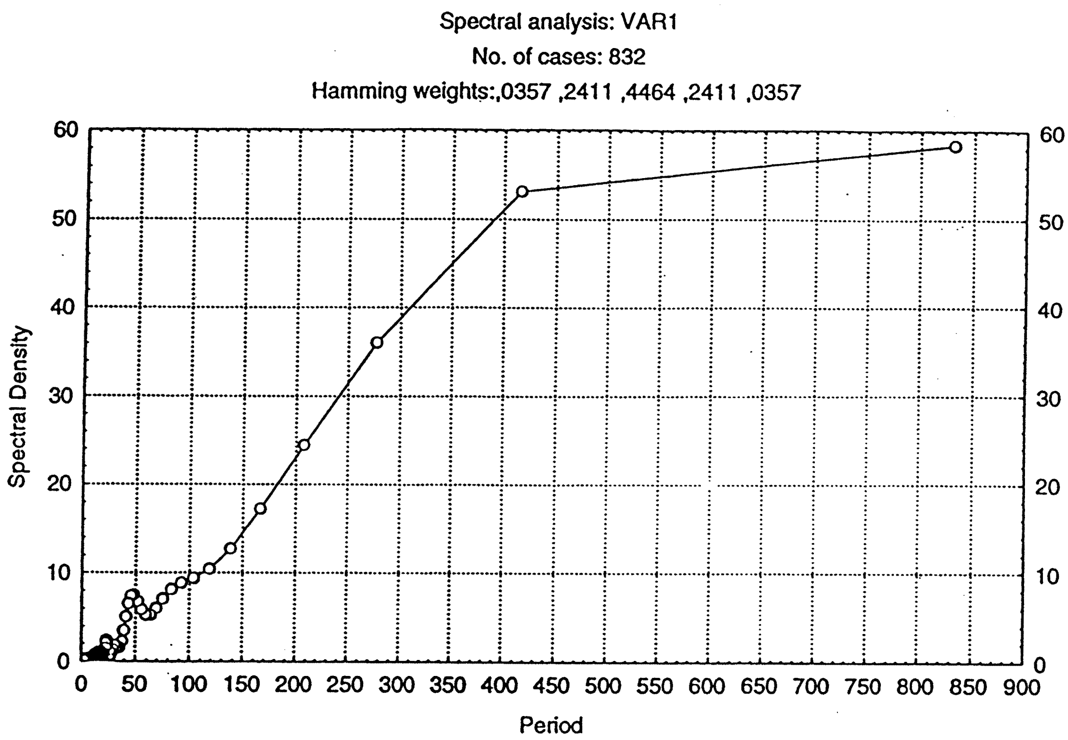
	α_0	α_1	β_1	
ERNOK	0.2520E–2	0.1319	0.6772	(–100)
	(3.49)	(4.41)	(8.83)	
ERDKK	0.1380E–2	0.4133	0.5829	(–250)
	(12.92)	(11.29)	(22.77)	

Principal components are presented in Table 23. The figures for the cumulative fraction explained by the principal components show a much higher degree of homogeneity for this period than for the pegged period. The same pattern is also present in the factor loadings of the first components. This time the power spectrum of the first principal component in Figure 13 nicely conforms to the spectrum of a highly persistant component process, although there is a peak at 420 days and its harmonics 840 days. The overall interpretation of the power spectrum for interest rates during the floating regime is the same as for the exchange rates during the corresponding regime, ie the spectrum is typical for an integrated stochastic process.

Table 23. **Principal components of conditional variances. Eigenvalues and cumulative fraction explained. 3-month interest rates 8 Sep. 1992 – 31 Dec. 1995.**

Component	Name	Eigenvalue	Cumulative R-Squared
1	P1	7.3626	0.5663
2	P2	1.4651	0.6790
3	P3	1.0793	0.7620
4	P4	0.8284	0.8258
5	P5	0.9202	0.8966
6	P6	0.5711	0.9405
7	P7	0.3500	0.9674
8	P8	0.1778	0.9811
9	P9	0.1012	0.9889
10	P10	0.0674	0.9941
11	P11	0.0516	0.9981
12	P12	0.0205	0.9996
13	P13	0.0041	1.0000

Figure 13. **Spectral density function of the first principal component of conditional variances of 3-month interest rates 9 Sep. 1992 – 31 Dec. 1995**



A pooled series was formed from the stationary estimation period for the individual interest rates which was the full period for ERUSD, ERGBP, ERSEK, ERDEM, ERNLG, ERCHF, ERITL, ERJPY and ERFIM. Sub-periods was used for ERNOK and ERDKK. Due to the non-stationarity of its conditional variance process over the entire period, ERBEF is excluded from the pooled data. Since no stationary sub-period was found for ERFRF, this interest rate is also excluded from the pooled series. In forecasting, ERBEF and ERFRF will, along with the other interest rates, be forced to follow the process estimated from the pooled data.

The estimated GARCH(1,1) model for the pooled data for this period is

$$h_t = \underset{(173.49)}{0.4223*E\text{-}8} + \underset{(112.32)}{0.0881\varepsilon^2_{t-1}} + \underset{(2683.54)}{0.9367h_{t-1}} \qquad (27)$$

The sum $\alpha_1 + \beta_1$ is 1.0247 which can be interpreted as non-stationarity in variance. The null of an integrated variance model, $\alpha_1 + \beta_1 = 1$, should pass statistical testing.

At the starting point of the empirical part of this study, the data was divided into three separate subperiods to account for exogenously identified structural changes in the exchange rate regime. Within the third period, there is a clear change in the trend of the levels of the exchange rates. Consequently this subperiod was divided into two parts: one covering the upwards sloping trend, the other covering the downwards sloping trend. The same partition was now made for the interest rates and a GARCH(1,1) model was estimated on pooled data for the second half of the third period, 11 Mar. 1993 - 31 Dec. 1995, to detect the possible effects of the trend break. The results are very similar to the model covering the full period

$$h_t = \underset{(136.96)}{0.1981*E\text{-}9} + \underset{(95.07)}{0.0793\varepsilon^2_{t-1}} + \underset{(2699.19)}{0.9399h_{t-1}} \tag{28}$$

As the sum of $\alpha_0 + \beta_1 = 1.0192$ does not significantly differ from 1, we can conclude that the inclusion of a structural change has neglible impact on the estimated parameter values and that the resulting model is approximately IGARCH(1,1).[1] The estimated mean lag for the floating period is about 17 days, the same lag as was estimated for the first pegged period.

The similarity between the estimated models on pooled data for the three main periods is not as strong as for the exchange rates. The estimated GARCH(1,1) model on the pooled data for the first subperiod of the pegged period is

$$h_t = \underset{(6.36)}{0.3770*E\text{-}6} + \underset{(49.08)}{0.0669\varepsilon^2_{t-1}} + \underset{(1526.19)}{0.9418h_{t-1}} \tag{29}$$

The GARCH(1,1) model for the pooled data for the second pegged period is

$$h_t = \underset{(58.15)}{0.1497*E\text{-}5} + \underset{(61.76)}{0.0958\varepsilon^2_{t-1}} + \underset{(1444.69)}{0.9005h_{t-1}} \tag{30}$$

and for the second part of the floating period

$$h_t = \underset{(136.96)}{0.1981*E\text{-}9} + \underset{(95.07)}{0.0793\varepsilon^2_{t-1}} + \underset{(2699.19)}{0.9399h_{t-1}} \tag{31}$$

Due to the large number of observations, the equality of the coefficients in a formal F-test is rejected; differences in the estimated parameter vectors are, however, fairly small so that the conditional variance of the interest rate process is assumed to be the same across exchange rate regime.

---

[1]See Lamourex and Lastrades (1990) on the effects of structural changes on the persistance parameters.

4.4 BDS-statistics

The BDS-statistics figures for the standardized residuals of the prefiltered raw data are reported in Table 24 for the full period and for shorter periods. There is strong evidence against the null hypothesis of IID for all series. The value of the test statistics for ERNOK and ERDKK is reduced through shortening of the period, but remain significant. After controlling for linear dependence, nonstationarity due to possible structural changes, deviations from the null of IID residuals could be either due to nonlinearity in mean or in variance. To capture nonlinearity in mean the GARCH-M(1,1) model was tested. The results showed that the MEAN-parameter is not statistically significant for most interest rates. In a single case where it is significant, it makes the GARCH parameter insignificant. The GARCH(1,1) was postulated to capture nonlinearity in variance. If the GARCH model is correctly specified, the standardized residuals should be IID in large samples. To determine whether any remaining non-linear structure is present in the model, the BDS test was applied to the standardized GARCH(1,1) residuals (Table 24). Although the figures are smaller than those of the prefiltered raw data, small amounts of nonlinearity still appear in most of the residual processes.

Table 24. **BDS-statistics for filtered interest rates**
9 Sep. 1992 – 31 Dec. 1995, m = 2, $\epsilon/\delta = 0.5$

	Long period	Shortened period	GARCH-residuals
ERUSD	9.97		7.84
ERGBP	7.71		3.74
ERSEK	11.94		8.39
ERNOK	14.90	1.96	-0.98
ERDKK	17.89	11.62	2.66
ERDEM	8.32		7.66
ERNLG	4.17		2.70
ERBEF	12.66		2.99
ERCHF	4.74		
ERFRF	18.07		
ERJPY	8.40		7.46
ERITL	10.75		6.07
ERFIM	11.55		10.10

4.5 Summing up for short interest rates

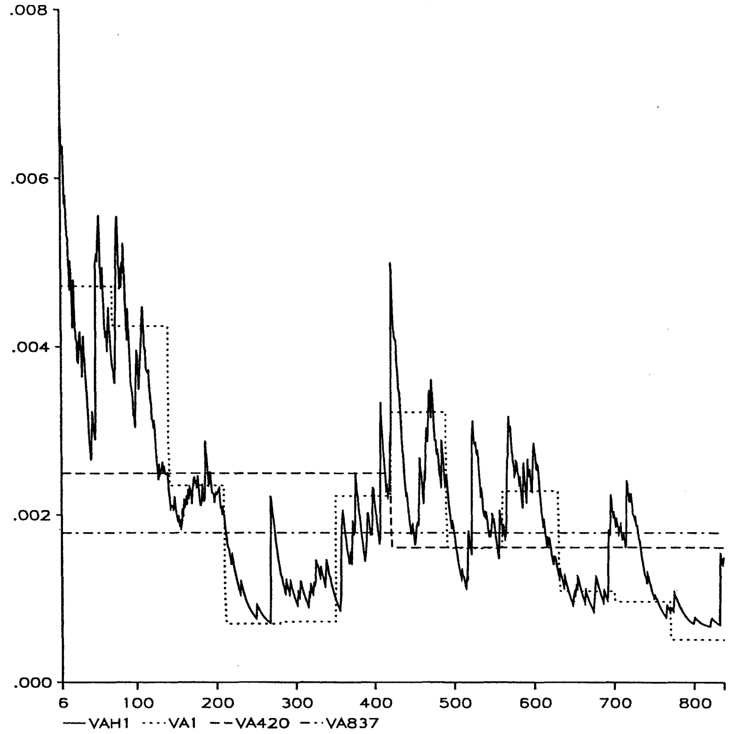
For the thirteen three-month interest rates, GARCH(1,1) models were estimated for three intervals selected to account for possible structural changes trigged by realignments of the domestic currency. The interest rates differed from the foreign exchange rates in that they reveal strong linear dependence in the raw data, which called for pre-whitening of the data. Also, non-stationarity conditional variance tend to be more typical for the interest rates than for exchange rates. We were not able to identify the whole model for some interest rates since either the ARCH parameter or the GARCH parameter was set to its lower boundary value in the iteration process regardless of the selection of initial values. The models estimated on pooled interest rate data turned out to be integrated in variance. The same parameter values are valid for all periods regardless of the current exchange rate regime. The same result was found for the exchange rates.

Spectral analysis of the individual conditional variances and the first principal components also suggest that the conditional variance processes are highly persistent; large contributions to the time variability of the conditional variances or their first principal component also come from shorter cycles, especially during the later pegged period. A common cyclical period of 180 days for both pegged periods and 420 days for the floating period is traceable. Even in this feature the results coincide with the corresponding results for the exchange rates.

Next we compare the alternative measures of interest rate volatility derived from the estimation results. The expressions for the conditional, unconditional and sample variance measures are the same as for the foreign exchange rates

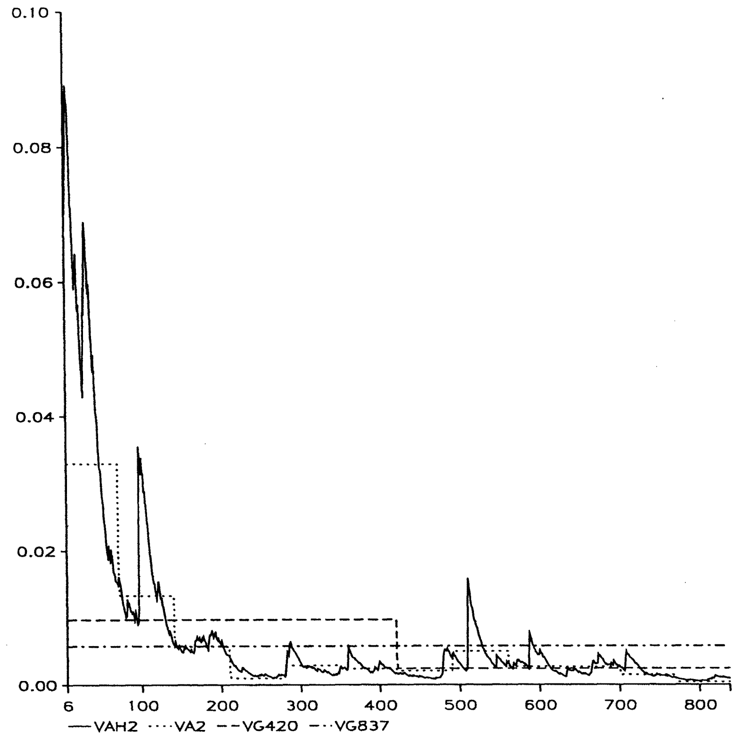
- GARCH(1,1) model conditional variance $h_{i,t}$. (VAH1 in Figure 14)
- GARCH(1,1) unconditional variance $\alpha_0/(1-(\alpha_1+\beta_1))$, the convergence limit for the conditional variance h_t. (VA837 in Figure 14)
- sample variance constant for the peak frequency evaluated on the cyclical behaviour of the individual conditional variances and their principal components; 180 days for the pegged period and 420 days for the floating period. (VA420 in Figure 14)
- sample variance calculated on quarterly data; the frequency selection is based on previous results (Ahlstedt 1990, 1995). A subsample of about 70 observations was found to be large enough to yield reasonable statistical efficiency, yet small enough to make it likely that the sample variance remains constant. (VA1 in Figure 14)

Figure 14. **Conditional, unconditional and sample variance measure comparison ERUSD floating period**



The four alternative volatility measures are plotted in Figure 14 for the ERUSD and in Figure 15 for ERGBP for the floating period. The sample variances work as smooth mean values for the rough fluctuations in the daily conditional volatility $h_{i,t}$.

Figure 15. **Conditional, unconditional and sample variance measure comparison ERGBP floating period**



5 Long-term bond rate

To capture the interest rate risk inherent in the bonds in the investment portfolio of the banks, we need an estimate of the variance of the long rate. The stock of bonds in that portfolio mainly consists of domestic currency nominated bonds, so we therefore concentrate on the three-year markka bond rate to be used as a proxy for the average interest rate term of the entire bond portfolio. Selection of the three-year term is based on the historical data on the average duration of the bonds in the trading portfolios of the individual Finnish banks.

Figure 8 shows the long-term bond rate for the period under consideration. The data was once again divided into two periods: the pegged regime of the currency 1 Jan. 1990 – 5 Sep. 1992 and the floating regime 8 Sep. 1995 – 31 Dec. 1995.

Next we look at the stationarity of the long rate. The unit root test statistics of the Weighted symmetric τ test, the Dickey-Fuller τ test and the Phillips-Peron Z test on the differentiated series are all statistically significant, which means that the hypothesis of a unit root is rejected. Differencing once produces a mean stationary series.

To detect linear dependency, Ljung-Box test was performed on the differences. The test statistics reviel strong autocorrelation. AR-filtering of order one for the first period and of order three for the second period is sufficient to remove linear dependence in mean. ARCH effects are detected for both periods.

The GARCH(1,1) model was estimated for both periods. The results are shown in Table 25. All estimated parameter values are statistically significant. For both periods, the process is not integrated: the sum $\alpha_1 + \beta_1$ is 0.8870 for the pegged period and the mean lag 1.1 days and for the floating period the sum is 0.6183 and the mean lag 1.2 days. Low persistence is also measured by the half-life statistics λ, which is 7 days for the pegged period and 2.5 days for the floating period. The low persistence also means a strong mean reverting process in the time path of the conditional variance, ie the effects of shocks to the current conditional variance of the forecast of the future variance dies out relatively quickly.

Table 25. **Long Rate, Differences**

GARCH(1,1) estimation of the volatility (t-values in parenthesis)

		α_0	α_1	β_1
1 Jan. 90 – 5 Sep. 92	AR(1)	0.9206E–7 (19.50)	0.7800 (10.50)	0.1070 (3.84)
8 Sep. 92 – 31 Dec. 95	AR(3)	0.2030E–6 (14.93)	0.4420 (7.02)	0.1763 (3.19)

BDS statistics to detect remaining nonlinearity are presented in Table 26. Conditional variance modelling reduced the values of the test statistics to half of the value for the filtered raw data but they were still high enough to reject the null of IID.

Table 26. **BDS-statistics of the standardized residuals for the 3-year interest rate and the general stock market index**

	Residuals from AR-filtered data	GARCH(1,1) residuals
Pegged period		
Long rate	8.71	4.17
Stock index	1.74	1.14
Floating period		
Long rate	8.57	4.22
Stock index	4.97	1.71

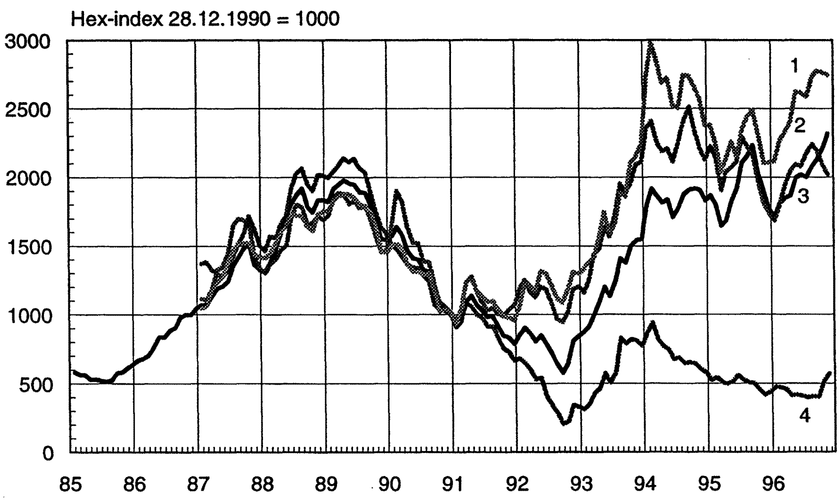
To test the hypothesis of a time-varying risk premium in the long rate, the GARCH-M(1,1) model was estimated for both periods. For the pegged period, the inclusion of the standard deviation with a coefficient of γ_1 in the mean equation resulted in a statistically significant γ_1 parameter but at the same time β_1 lost its significance. For the floating period, the estimated γ_1 parameter was set to its boundary value zero in the iteration process.

6 Statistical measures of general stock market index

Market risk includes also uncertainty about the future values of traded shares in the portfolios. To measure this risk, we need an estimate of volatility in stock prices. Assuming that banks behave as enlightened traders, diversifying their portfolios to remove non-systematic risk, the variance of the general stock market index can be used as an estimate of the remaining systemic risk in the equity portfolio.

The data employed are daily log changes of the general HEX index of the Helsinki Stock Exchange's. This index transformation is used to measure compounded yield since both dividends and capital gains are included. Figure 16 displays the general stock market index in levels.

Figure 16. **Helsinki stock exchange. Share prices by sector**



1 Metal industries
2 Forest industries
3 All-share index
4 Banks and finance

Latest observation October 1996

Empirical studies have shown that index-series have one unit root, ie stationarity is achieved by transformation into first differences (Malkamäki 1993). To check if first differencing is enough to produce stationarity, unit root tests were performed on log differences of the stock market index for the pegged period, 1 Jan. 1987 – 5 Sep. 1992, and the floating period, 14 Sep. 1992 – 31 Dec. 1995, of the Finnish currency. Based on the results of the Weighted Symmetric τ test, Dickey-Fuller τ test and Phillips-Perron Z test, the hypothesis of a unit root was rejected. Thus, the logarithmic transformation of the general stock market index is integrated of order one.

Descriptive statistics of the stock market index were then calculated. The null hypothesis of zero mean cannot be rejected for the pegged period, but is rejected for the floating period. While skewness for the pegged period is huge, removal of the "black Monday" observation substantially reduces it. Again, we see that a single outlier can considerably affect the value of the test statistics.

In empirical studies, skewness has been found to be a much stronger feature in stock prices than in exchange rates and interest rates. This is not, however, the case with the Finnish general stock market index.

Most empirical implementations of GARCH(p,q) models to stock market indices have adopted low orders for the lag lengths p and q. Typically, GARCH(1,1), GARCH(1,2) or GARCH(2,1) models have been selected. A limitation in GARCH models is, however, the assumption that only the magnitude, and not the sign, of unanticipated returns determines volatility (Mills 1992, p. 140). Nelson (1990) presented an alternative to the GARCH model, the exponential GARCH labelled EGARCH, which encompasses the observed feature that changes in stock return volatility are negatively correlated with the returns themselves, ie volatility tends to rise in response to "bad news" and fall in response to "good news". Hsieh (1990) has shown by applying the BDS test to the residuals from a EGARCH(1,1) model for stock market indices and portfolios that the EGARCH model typically cannot completely account for all deviation from IID in stock returns.

In this study we hope to be able to use a GARCH model with the same order of p and q for all the market risks and therefore the GARCH(1,1) process, which was selected for exchange rates and interest rates, is selected for the Finnish stock market index as well.

Prior to the GARCH identification, the data was pre-filtered to remove linear dependence. An AR(3) process was selected for the pegged period and AR(1) for the floating period. The selection was based on the Ljung-Box test statistics.

The estimated parameter values of the GARCH(1,1) model for both periods are presented in Table 27. The values of the parameters α_1 and β_1 differ between periods, while the estimated α_0 parameters are very similar indeed. For the pegged period, the sum is 0.8584, the mean lag 7 days and the half-life frequency 6 days. For the floating period, the sum of the two parameters is 0.9475, the mean lag 7 days and the half-life measure 14 days.

Table 27. **General stock market index, compounded yield, log differences**

GARCH(1,1) estimation of the volatility (t-values in parenthesis)

		α_0	α_1	β_1
1 Jan. 87 – 5 Sep. 92	AR(3)	0.1208E–4	0.3275	0.5309
		(11.43)	(10.99)	(15.96)
8 Sep. 92 – 31 Dec. 95	AR(1)	0.9196E–5	0.0966	0.8509
(15 Nov. 92 included)		(2.37)	(4.31)	(21.97)

BDS statistics for the standardized residuals prior to and aftger GARCH estimation are displayed in Table 26. The test statistics are significantly reduced after accounting for GARCH(1,1) effects in the conditional variance and the hypothesis of IID residuals cannot be rejected for the resulting standardized returns to the general stock market index.

The GARCH-M(1,1) was also estimated for the stock market index to detect a possible time-varying risk-return relationship in the mean equation. The γ_1 parameter was not significant in the two data periods.

Prior to selecting the maintained volatility models, the odds for the general idea of imposing the same GARCH structure on all the rates affecting banks' portfolio returns were perceived least favourable in the case of stock market returns, since evidence from other sources strongly favoured an EGARCH model these returns. Yet, the empirical estimation revealed that GARCH(1,1) was best-suited to capture heteroscedasticity in the stock market returns. For exchange rates and interest rates, the BDS test statistics still detects deviations from IID in the standardized GARCH residuals.

7 Summing up for all rates

The objective of the study has been to model the time varying variances in twelve exchange rates, thirteen short-term interest rates, one long-term rate and the general stock market index. A GARCH structure was entertained to account for the observed heteroscedasticity in the rates, and GARCH(1,1) turned out to perform reasonably well for all the rates. In the end, evidence of significant GARCH-M effects remains inconclusive and the study argues against them. Hence, it is concluded that no significant time variability can be observed in the risk-return relationship in the selected data set.

One of the strongest conclusion of the present study is that the conditional variance model of the individual exchange rates and short-term interest rates is at least approximately the same across exchange rate regimes. The model for long-term interest rate volatility, on the other hand, displays less persistence under floating than under fixed exchange rates, although the estimated conditional variance process appears (weakly) stationary under both regimes.

Furthermore, results from the pooled data suggest that the changes in markka exchange rates and short interest rates have a time-varying conditional variance which can be modelled as an identical IGARCH process. Perhaps suprisingly, observed volatility of the general stock market index also seems to follow the same IGARCH(1,1) process, while the long term rate exhibits strong mean reverting behaviour. The finding of an IGARCH process is consistent with the common results that when the GARCH model is applied to high-frequency data, shocks to variance are strongly peristant; that is, the sum of the ARCH and GARCH parameters is very close to one. One possible explanation for integration in the conditional variance can be found in Nelson (1990b), who derives the stationary distribution of the GARCH conditional variance process in continuous time. This underlying diffusion model, which is close to IGARCH, provides accurate approximations to high frequency data. Furthermore, the distribution of the diffusion limit, and hence of the approximating process in high frequency data, displays some interesting properties; the GARCH innovation process is conditionally normal (ie given the conditional variance), but unconditionally its distribution is approximately Student t. Also, in the special case of the diffusion limit of the IGARCH(1,1) model, the Student t has an infinite variance. Lamoureux and Lastrapes (1990) give an other explanation suggesting that the persistence is overstated when the estimation is based on long series. The resulting IGARCH could as well be due to the existence and failure to take into account, of deterministic structural shifts in the model or to time-varying parameters. Structural shifts may result in instability of the drift parameter α_0 over the sample period, ie nonstationarity of the conditional variance and high persistance in α_1 and β_1. The reason for the division of the full data into subsets in this study was to account for the possibility of such structural shifts, due to changes in the exchange rate regime. In the GARCH estimation on the pooled data, the same model was forced on the individual rates and the individual drift parameters α_0 were also imposed into a single constant in the estimation for each period. The drift parameters in the individual models are very small in magnitude, but differ between rates and then can be interpreted as structural shifts. This feature might have had the effect on the estimation results of the model identified on the pooled data leading to the appearance of extremely strong persistence in variance. The average sum of the ARCH and GARCH parameters of the individual models, where the structural changes were accounted for, is, however, close to one, and thus supports the hypothesis of an integrated variance process.

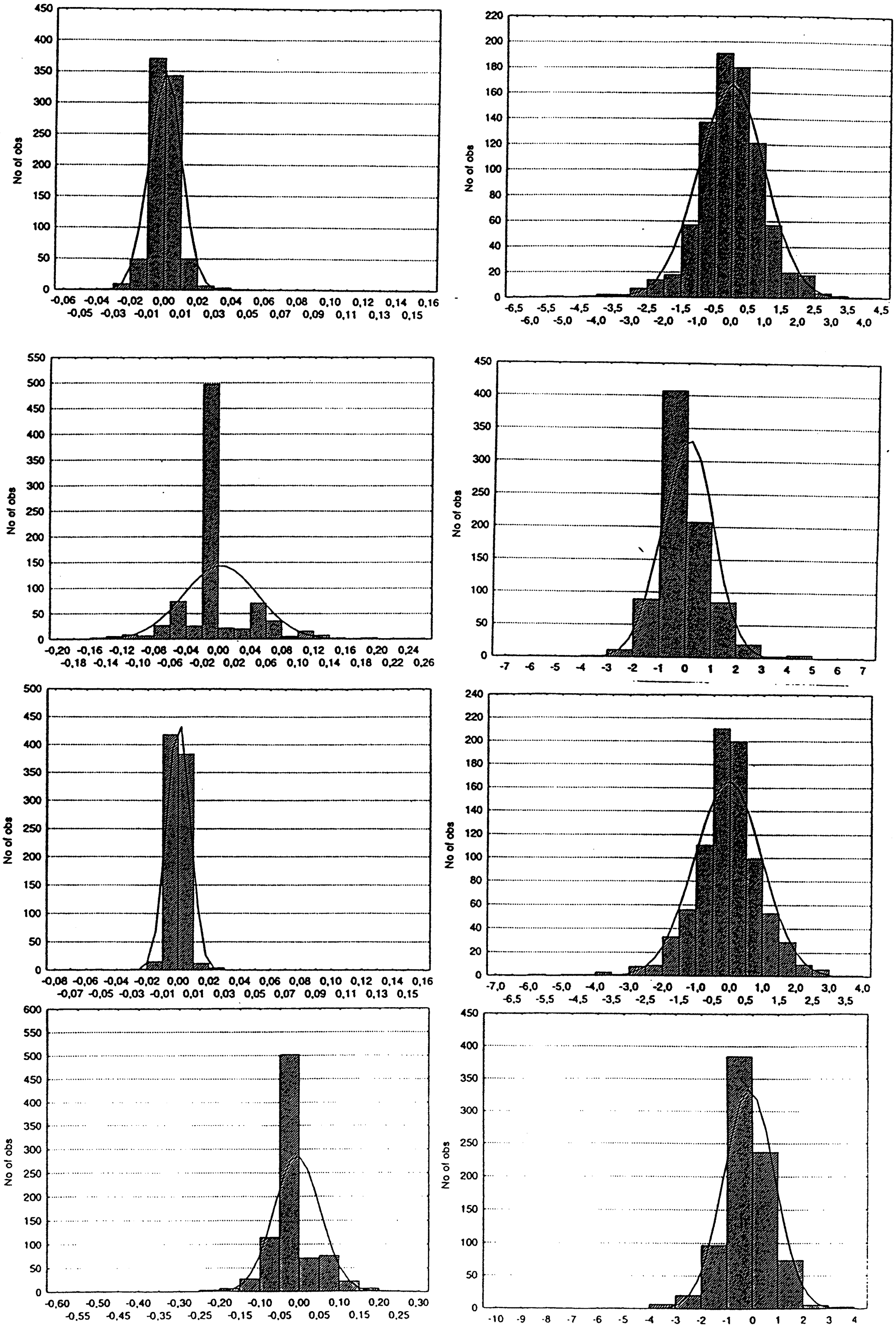
As a result of the GARCH estimation, it was possible to construct new variables by standardizing the raw data with the estimated standard deviations. Through this procedure we should, theoretically, end up with series which are normal or at least closer to normal than the raw data. Figure 17 and Table 28 compare the skewness and kurtosis figures between the raw data and the GARCH residuals for USD, ERUSD, DEM, ERDEM, ERFIM, FIM long-term rate and the general stock market index HEX. Skewness is found in the raw data only in ERFIM. The kurtosis figures are also in most cases substantially reduced through the conditional variance modelling, with the ERLONG being the exception, whereas for the ERFIM substantial amount of kurtosis still remains after filtering with the estimated GARCH model. Hence, while the GARCH(1,1) model is able to track the own temporal dependencies, the assumption of conditionally normally distributed innovations may need further considerations in the present data. As a reference, under the null of IID normally distributed standardized residuals, the sample skewness should be the realization of a normal distribution with a mean of 0 and a variance of $6/831 = 0.085^2$, while the sample kurtosis is asymptotically normally distributed with a mean of 3 and a variance of $24/831 = 0.17^2$.

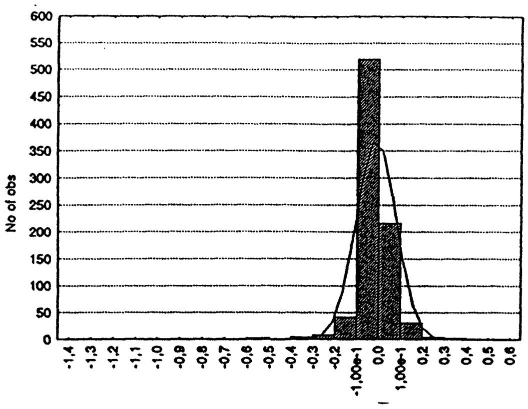
Table 28. **Skewness and Kurtosis statistics for raw data and GARCH(1,1) residuals for USD, ERUSD, DEM, ERDEM, ERFIM, FIM long-term rate and HEX stock market index for the floating period 14 Sep. 1992 – 31 Dec. 1995**

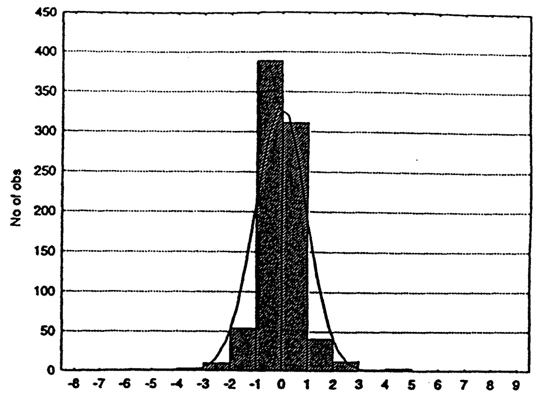
	Skewness figures	
	Raw data	GARCH(1,1) residuals
USD	0.34	−0.23
ERUSD	0.60	0.75
DEM	0.11	−0.52
ERDEM	−1.33	−0.78
ERFIM	−3.27	0.59
ERLONG	−0.45	−1.55
HEX	−0.07	0.04

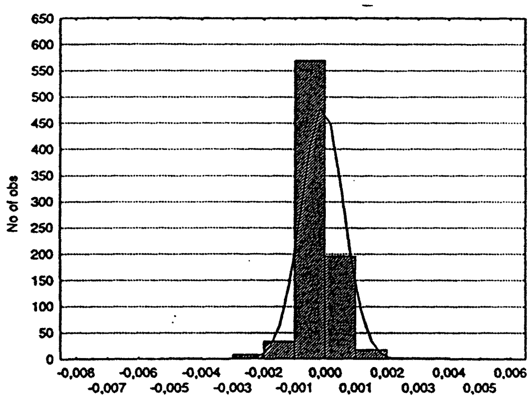
	Kurtosis figures	
	Raw data	GARCH(1,1) residuals
USD	78.11	2.40
ERUSD	3.42	5.73
DEM	240.28	3.17
ERDEM	14.24	6.74
ERFIM	42.74	16.02
ERLONG	23.49	27.35
HEX	2.36	0.71

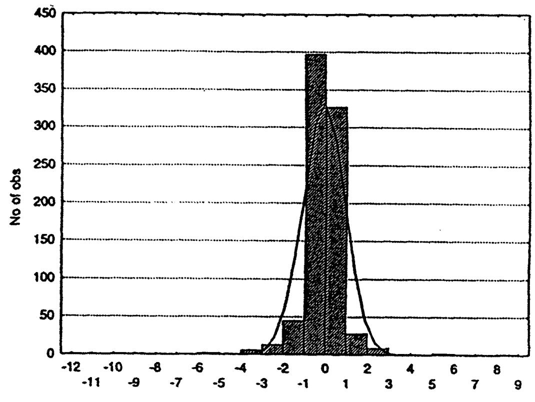
Figure 17. **Raw data figures in the left column of USD, ERUSD, DEM, ERDEM, ERFIM, FIMlog and HEX. The figures for corresponding GARCH residuals are in the right column.**

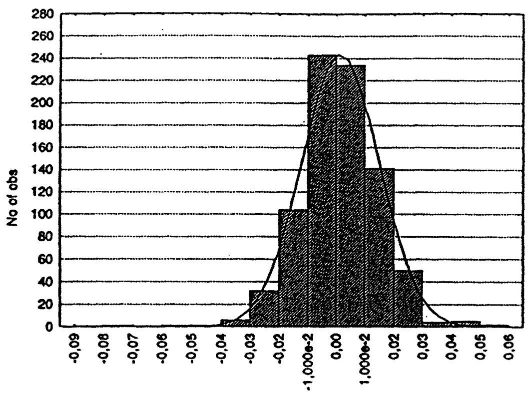


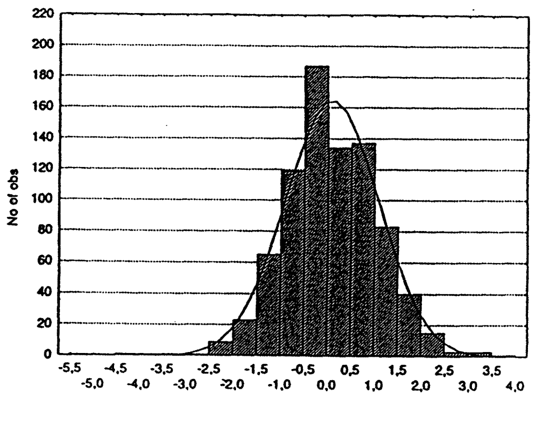

No of obs
0 50 100 150 200 250 300 350 400 450 500 550 600
-1,4 -1,3 -1,2 -1,1 -1,0 -0,9 -0,8 -0,7 -0,6 -0,5 -0,4 -0,3 -0,2 -1,00e-1 0,0 1,00e-1 0,2 0,3 0,4 0,5 0,6


No of obs
0 50 100 150 200 250 300 350 400 450
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9


No of obs
0 50 100 150 200 250 300 350 400 450 500 550 600 650
-0,008 -0,006 -0,004 -0,002 0,000 0,002 0,004 0,006
-0,007 -0,005 -0,003 -0,001 0,001 0,003 0,005


No of obs
0 50 100 150 200 250 300 350 400 450
-12 -10 -8 -6 -4 -2 0 2 4 6 8
-11 -9 -7 -5 -3 -1 1 3 5 7 9


No of obs
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280
-0,09 -0,08 -0,07 -0,06 -0,05 -0,04 -0,03 -0,02 -1,000e-2 0,00 1,000e-2 0,02 0,03 0,04 0,05 0,06


No of obs
0 20 40 60 80 100 120 140 160 180 200 220
-5,5 -4,5 -3,5 -2,5 -1,5 -0,5 0,5 1,5 2,5 3,5
-5,0 -4,0 -3,0 -2,0 -1,0 0,0 1,0 2,0 3,0 4,0

8 Covariances

In this study, two applications will be suggested and used to measure the covariances. The first is based on the assumption of identical autocorrelation structures for variances and covariances between rates. The assumption allows us to extend the univariate estimation results the conditional variances to obtained for the conditional covariances. The other method follows Bollerslev (1990) and is based on the assumption of constant correlation between rates, which simplifies the estimation procedure by using the results for the individual conditional variances in calculation of the conditional covariances. The first method can be applied only for covariances within groups of rates. The other can be applied also for covariances between groups of rates.

8.1 Conditional covariances: identical autocorrelation structure of variances and covariances

In the first method for covariance estimation, we test for dependence between the autocorrelation structure of the variances and covariances. If dependence is found to exist, then the conditional covariances can be modelled with the same parameter structure as their conditional variances.

By analogy to the conditional variance formula

$$h_t=\sigma_t^2=\alpha_0+\alpha_1\varepsilon_{t-1}^2+\beta_{t-1}h_{t-1} \tag{32}$$

the conditional covariances can be expressed as

$$\sigma_{ij,t}=\alpha_{0,ij}+\alpha_{1,ij}(\varepsilon_{i,t-1}\varepsilon_{j,t-1})+\beta_{1,ij}\sigma_{ij,t-1} \tag{33}$$

Since the series σ_{ijt-1} are not observable, the covariances cannot be estimated using an univariate GARCH method.

The null hypothesis of independence is tested using the non-parametric Kendall coefficient of concordance W (Siegel 1956), which expresses the degree of association among a set of ranked variables. The variables to be ranked in this case are the autocorrelation functions of variances and covariances within the two groups of rates.

In case of dependence between the autocorrelation structure in the univariate conditional variance

$$h_{i,t}=\alpha_{0,i}+\alpha_{1,i}\varepsilon_{i,t-1}^2+\beta_{1,i}h_{t-1} \tag{34}$$

and the conditional covariance

$$\sigma_{ij,t}=\alpha_{0,ij}+\alpha_{1,ij}(\varepsilon_{i,t-1}\varepsilon_{j,t-1})+\beta_{1,ij}\sigma_{ij,t-1} \tag{35}$$

the parameter values for the β_1 univariately estimated for the variances can also be used for covariances. Most conditional variances in interest rates in this study were estimated to follow an integrated process, whereby the ARCH parameter α_1 and the autocorrelation or GARCH parameter β_1 sum up to one. If we can show that the autocorrelation structure measured by β_1 in not independent between variances and covariances, it then follows from the unit root proposition that there is dependence between α_1 parameters for variances and covariances.

Through repeated substitutions, the conditional variance formula in (34) can be developed into the following expression

$$h_t = \frac{\alpha_0}{(1-\beta_1)} + \alpha_1 \sum_{j=1}^{\infty} \beta_1^{j-1} \varepsilon_{t-j}^2 \tag{36}$$

where the conditional variance is expressed in the form of a geometrically weighted average of past squared residuals so that the parameter β_1 gives the decay rate.

For the IGARCH process, formula (36) gets the following form

$$h_t = \frac{\alpha_0}{\alpha_1} + \alpha_1 \sum_{j=1}^{\infty} (1-\alpha_1)^{j-1} \varepsilon_{t-j}^2 \tag{37}$$

The expression for the covariances corresponding to formula (37) for the variances is then

$$\sigma_{ij,t} = \frac{\alpha_0}{\alpha_1} + \alpha_1 \sum_{s=1}^{\infty} (1-\alpha_1)^{s-1} \varepsilon_{i,t-s} \varepsilon_{j,t-s} \tag{38}$$

In the empirical implementation of the derived formula for the conditional covariances, we consequently use the parameter estimates of α_i's and β_i's from the pooled data within groups and periods and for ϵ_i's and ϵ_j's the observations on the individual returns.

8.2 Conditional covariances: constant correlation

In the second method for covariance estimation, the assumption of time varying variances and covariances but constant conditional correlation between the N stochastic processes made in Bollerslev (1990) allows the univariate GARCH estimation to be extended into a multivariate framework through a simplified estimation and inference procedure. The GARCH(1,1) structure for the conditional variances and covariances is expressed as

$$\varepsilon_{i,t} = z_{i,t} h_t^{1/2}$$

$$h_{ii,t} = \alpha_i + \alpha_{i1} \varepsilon_{i,t-1}^2 + \beta_{i1} h_{ii,t-1} \tag{39}$$

$$h_{ij,t} = \varrho_{ij} (h_{ii,t} h_{jj,t})^{1/2}$$

In the original application, the correlations coefficients ρ_{ij} of the standardized residuals are estimated simultaneously with the conditional moments. In our application we use a two-step method: in the first stage we calculate the correlation coefficients on the univariately estimated standardized GARCH(1,1) residuals, in the second stage we calculate the covariances from the joint information on the correlation coefficients and the estimated conditional univariate variances. In our application we assume constant correlation within periods, but allow for time-variation between periods.

8.3 Covariances between exchange rates

Non-trivial covariation of the exchange rates and interest rates is most likely, not only because of new information coming into the markets affecting all the rates, but also because of the intervention policy of the central banks.

In the first method of measuring the covariances between exchange rates, we test for the possibility to encompass the coherence between rates into the analysis by extending the estimated parameter structure from the conditional variances to the conditional covariances. In order to do so, we have to test for dependence between these conditional moments.

The null hypothesis of independence was tested using the Kendall coefficient of concordance W (Siegel 1956), which expresses the degree of association among sets of ranked variables. The variables to be ranked is the sample autocorrelation functions of variances and covariances. The test was performed separately for the group of twelve exchange rates and the group of thirteen interest rates.

Autocorrelations in variances and covariances up to the fifth order were calculated from the exchange rate data separately for the pegged and floating period. The numerical autocorrelation values were then ranked. The test statistics W was calculated to test the null hypothesis that the rankings are unrelated. The numerical value of the coefficient of concordance W is 0.691 for the pegged period and 0.469 for the floating period. The coefficient W is in this case approximately distributed as $\chi^2_{(4)}$ and the corresponding test statistics are 215.59 and 146.33. These test statistics are highly significant, which means that the null of independence can be rejected for both periods. Based on the outcome of the Kendall W test procedure showing that the variances and covariances are not independent, we thus apply the method of modelling the conditional covariances between exchange rates with the same parameter structure, ie the values of $\alpha_{1,i}$ and $\beta_{1,i}$, as their conditional variances.

The Kendall W test was performed on ranked autocorrelation values of variances and covariances. The numerical values of the autocorrelation function can also be used to approximate the similarity between the variances and covariances. In Table 29, the mean of the numerical values of autocorrelation functions up to order five for variances and covariances are presented. For the exchange rates we can conclude that the structures are very close to each other.

Table 29. **Autocorrelation mean values of variances and covariances**

Exchange rates

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
Pegged period					
Variances	0.1279	0.0202	0.0322	0.0214	-0.0050
Covariances	0.0908	0.0421	0.0248	0.0057	0.0144
Floating period					
Variances	0.1964	0.0830	0.0851	0.0301	0.0612
Covariances	0.2190	0.1247	0.1165	0.0383	0.0687

Interest rates

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
Pegged period					
Variances	0.1863	0.0619	0.0874	0.0422	0.0273
Covariances	0.0143	0.0534	0.0105	0.0013	0.0123
Floating period					
Variances	0.1509	0.0987	0.1083	0.0926	0.1001
Covariances	0.0049	0.0160	0.0009	0.0031	0.0185

A third method of evaluating the dependence between conditional variances and covariances is based on principal components analysis. Principal components were calculated separately for the sample variances and covariances for the second pegged period, which in the estimation were found to be identical to the first pegged period and the floating period. Then the correlation coefficient R was then determined by regressing the first principal component of variances on the first principal component of covariances. The correlation coefficient is 0.54 for the pegged period and 0.87 for the floating period. A strong dependence can therefore be found in this way between variances and covariances.

The outcome of the Kendall W test, the visual interpretation of the mean values of the autocorrelation functions and the high degree of correlation between principal components, all support the use of the same parameter structure for variances and covariances of exchange rates.

The estimated conditional variance model of the pooled data can then be used as the basic model for the conditional covariances between exchange rates. The estimated pooled model for the pegged period is

$$h_t=0.3813*E\text{-}7+0.0621\varepsilon_{t-1}^2+0.9353h_{t-1} \qquad (40)$$

and for the floating period

$$h_t=0.9189*E\text{-}6+0.0809\varepsilon_{t-1}^2+0.8847h_{t-1} \qquad (41)$$

The sum $\alpha_1 + \beta_1$ does not significantly differ from one, so we may conclude that the conditional variance of exchange rates follows a GARCH process integrated in variance and that appears to apply across exchange rate regimes.

Developing formula (37) we get the following weight structure for the floating period when $\alpha_1 = 0.08$ and $\beta_1 = 1 - \alpha_1$

$$h_t=\frac{\alpha_0}{0.08}+0.08\varepsilon_{t-1}^2+0.08(0.92)\varepsilon_{t-2}^2+0.08(0.92)^2\varepsilon_{t-3}^2+0.08(0.92)^3\varepsilon_{t-4}^2 + \ldots + 0.08(0.92)^{n-1}\varepsilon_{t-n}^2+\ldots \qquad (42)$$

The series of lagged squared residuals to be included in the formula is truncated at 28, as the weights of the observations there after have less than 10 % of the weight for the first observation.

Table 30 gives the numerical values of the weight series.

Table 30. **Numerical values for weights of the truncated sequence of lagged squared innovations**

lag number	weight
1	0.089
2	0.082
3	0.075
4	0.069
5	0.063
6	0.058
7	0.054
8	0.050
9	0.045
10	0.042
11	0.038
12	0.035
13	0.033
14	0.031
15	0.028
16	0.025
17	0.023
18	0.021
19	0.020
20	0.018
21	0.017
22	0.015
23	0.014
24	0.013
25	0.012
26	0.011
27	0.010
28	0.009

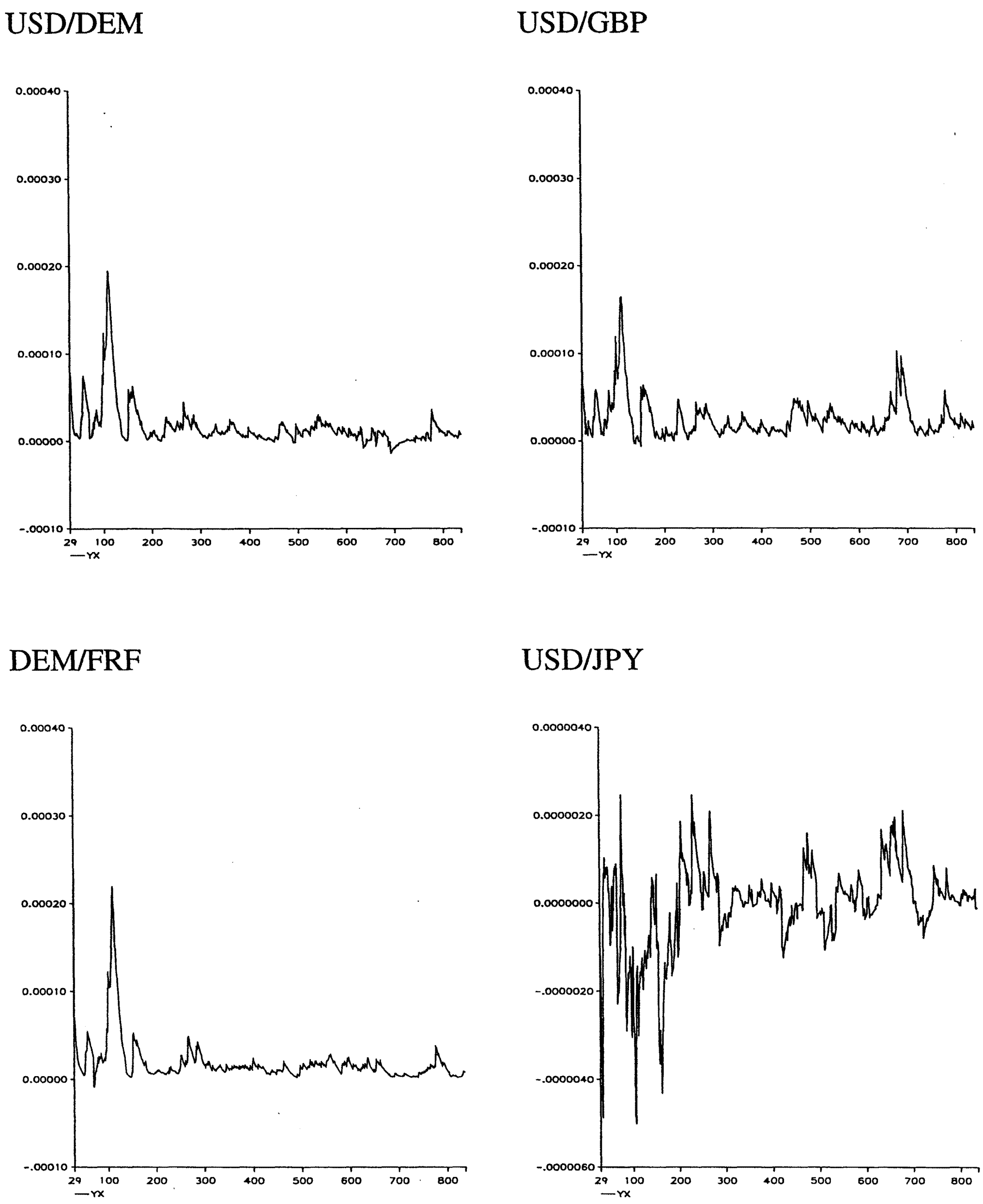
The expression for the covariances corresponding to formula (34) for the variances is then

$$\sigma_{ij,t}=\frac{\alpha_0}{0.08}+0.08\varepsilon_{i,t-1}\varepsilon_{j,t-1}+0.08(0.92)\varepsilon_{i,t-2}\varepsilon_{j,t-2}+0.08(0.92)^2\varepsilon_{i,t-3}\varepsilon_{j,t-3}$$
$$+0.08(0.92)^3\varepsilon_{i,t-4}\varepsilon_{j,t-4}+\ldots+0.08(0.92)^{n-1}\varepsilon_{i,t-n}\varepsilon_{j,t-n}+\ldots \quad (43)$$

In the empirical implementation of the derived formula for the conditional covariances, we consequently use the parameter estimates of α_1 and β_{1i} from the pooled data within groups and periods. For e_i's and e_j's we use observations on the individual exchange rates.

The plots of the conditional covariances calculated according to the formula (43) for USD/DEM, USD/GBP, DEM/FRF, and USD/JPY are displayed in Figure 18. Visually, the variation of the covariances is often very similar to that of the corresponding variances. The excessive time variability of the USD/JPY conditional covariance in the beginning of the period reflects increased volatility of the JPY during that time.

Figure 18. **Conditional covariances 9 Sep. 1992 – 31 Dec. 1995 USD/DEM, USD/GBP, DEM/FRF and USD/JPY (weighted series)**



The alternative approach to calculating covariances is the Bollerslev method of constant conditional correlations. In applying this method, we assume the correlations to be constant within the three main periods, but allow them to change between periods.

To evaluate the empirical correctness of the assumption of constant correlation in the Bollerslev method, a CUSUM test was applied to the standardized residuals to test the stability of the regression parameter in an OLS estimation where the exchange rates were regressed one at a time on one of the other exchange rates. The test values did not allow rejection of the null hypothesis that the regression parameter remains constant for the second pegged period and the floating period. For the first pegged period, however, the constant correlation hypothesis between DEM and NLG and the other rates is rejected.

The evidence during the second pegged period and the floating period in favour of the constant correlation assumption supports the use of the Bollerslev method.

In applying the method, the conditional correlation coefficients of pairs of GARCH standardized residuals of the individual exchange rates in (41) were first calculated using sample data from the three subperiods. Theoretically, these correlation coefficients are approximately normally distributed under either the null of small or no correlation, if in the latter case, one uses the normal to approximate the exact student t-distribution. The big sample sizes make even low correlations statistically significant. For the first pegged period, the sample size is 558 and the critical value at a 5 % confidence level for the correlation coefficient is 0.083. The corresponding figures for the second pegged period are 866 and 0.067, and for the floating period, 819 and 0.068.

The numerical values of within-group correlations are highest for the group of the exchange rates. This conforms with the theory that information coming into the market affects all markka rates instantaneously.

In the Bollerslev method, we assume constant conditional applying correlations within periods, but allow for changing correlations between periods reflecting different regimes. A significant difference in the level of the coefficients is also to be seen in the sample estimates.

During the first pegged period, 1 Jan. 1987 - 16 Mar. 1989, the highest covariances are found between the European currencies DKK, DEM, NLG, BEC, CHF and FRF. The ERM apparently underlies these correlations. For the USD, correlations are significant only with GBP, SEK, NOK and JPY.

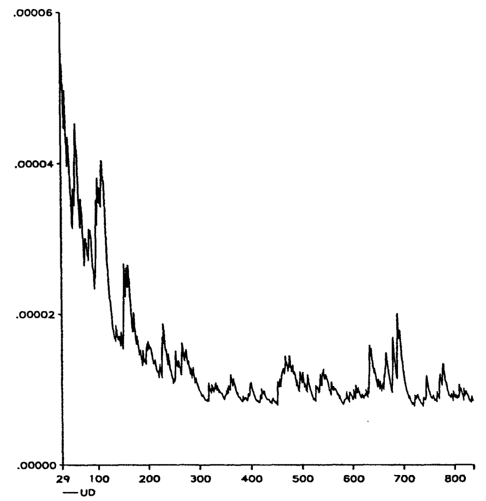
For the second pegged period, 21 Mar. 1989 - 5 Sep. 1992, the conditional correlations are significant for all pairs of exchange rates and much higher in value than during the first pegged period. An explanation of the phenomen could be intensified central bank intervention activity aiming at smoothing exchange rate movements during this turbulent period.

During the floating period, 8 Sep. 1992 - 31 Dec. 1995, the covariances between ERM currencies are again significant, although much lower than for the first pegged period. The correlation between the other currencies are generally insignificant. Correlation between USD and GBP and the Nordic countries is found in this period.

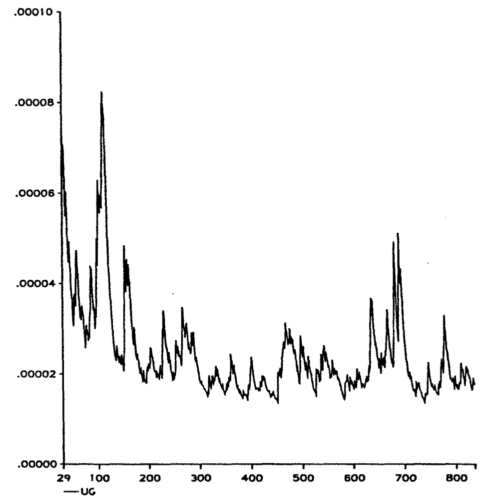
Figure 19 shows the conditional covariances for the floating period for USD/DEM, USD/GBP, DEM/FRF and USD/JPY using the constant correlation method. These figures can be compared with the figures in the previous Figure 18 showing the conditional covariances for the same pairs of exchange rates calculated by assuming an identical parameter structure for the variances and covariances.

Figure 19. **Conditional covariances 9 Sep. 1992 – 31 Dec. 1995 USD/DEM, USD/GBP, DEM/FRF and USD/JPY (constant correlation)**

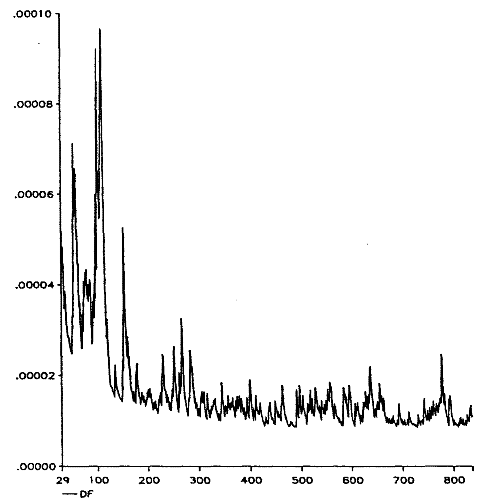
USD/DEM



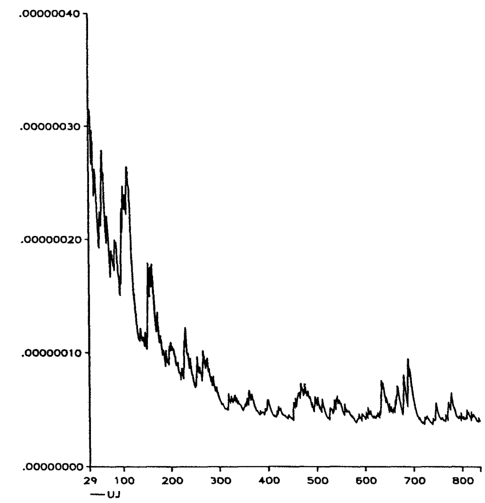
USD/GBP



DEM/FRF



USD/JPY



8.4 Covariances between interest rates

Similar calculations for determining covariances were then carried out for the group of thirteen interest rates as for the group of twelve exchange rates.

In the first method, we test the null hypotheses of liner independence of autocorrelation structure of the variances and covariances. If the null hypothesis is rejected, dependence between the $\alpha_{1,i}$ and $\alpha_{1,ij}$ parameters result from the assumption of IGARCH variance processes.

Ranking according to the numerical values of the autocorrelation functions of the sample variances and covariances of interest rates was performed. The value of the Kendall coefficient W was 0.1014 for the pegged period and 0.0264 for the floating period. The corresponding $\chi^2_{(4)}$ test statistics are 36.504 and 9.502. For the first period, the test statistics are significant even at a 0.1 % confidence level and for the second period at 5 % confidence level. The null hypothesis of independence can thus be rejected.

The test implies, both for interest rates and the exchange rates, that the parameter values estimated for the conditional variances can also be used to calculate the conditional covariances.

As for exchange rates the mean values of the numerical autocorrelation functions up to the fifth order were calculated separately for variances and covariances. The figures presented in Table 29 confirm the results of the Kendall W in revealing a clearly weaker, or even non-existing, dependence between the conditional second moments for interest rates in comparison to exchange rates. The assumption of the same parameter structure has therefore less empirical support for interest rates.

As a third test of dependence between variances and covariances, a principal component-based analysis was used as for the exchange rates. The correlation coefficient R between first principal components of variances and covariances for the first pegged period was estimated to be 0.88, for the second pegged period 0.83 and for the floating period 0.58. The outcome of this calculation supports the assumption of dependence between variances and covariances.

The estimated conditional variance of the pooled data was used as the basic expression for the conditional covariances between interest rates in the same way as for the exchange rates. The estimated pooled model for the pegged period for interest rates was

$$h_t = 0.1497 * E\text{-}7 + 0.0958\varepsilon^2_{t-1} + 0.9005h_{t-1} \tag{44}$$

and for the floating period

$$h_t = 0.1981 * E\text{-}9 + 0.0793\varepsilon^2_{t-1} + 0.9399h_{t-1} \tag{45}$$

For both periods, the sum $\alpha_1 + \beta_1$ of both the pooled exchange rate model and the pooled interest rate model does not significantly differ from one. Therefore, we conclude that the conditional variances of the rates follow a GARCH process integrated in variance.

For the floating period, we end up with the same weighted formula (37) for interest rates as for exchange rates

$$h_t = \frac{\alpha_0}{0.08} + 0.08\varepsilon^2_{t-1} + 0.08(0.92)\varepsilon^2_{t-2} + 0.08(0.92)^2\varepsilon^2_{t-3} + 0.08(0.92)^3\varepsilon^2_{t-4} + \ldots + 0.08(0.92)^{n-1}\varepsilon^2_{t-n} + \ldots \tag{46}$$

and consequently the same values of weights as in Table 30.

The expression for the covariances corresponding to formula (38) for the variances is then

$$\sigma_{ij,t}=\frac{\alpha_0}{0.08}+0.08\varepsilon_{i,t-1}\varepsilon_{j,t-1}+0.08(0.92)\varepsilon_{i,t-2}\varepsilon_{j,t-2}+0.08(0.92)^2\varepsilon_{i,t-3}\varepsilon_{j,t-3}$$
$$+0.08(0.92)^3\varepsilon_{i,t-4}\varepsilon_{j,t-4}+\ldots+0.08(0.92)^{n-1}\varepsilon_{i,t-n}\varepsilon_{j,t-n}+\ldots \qquad (47)$$

The conditional covariances of interest rates for the floating period are displayed in Figure 20 for ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ERFIM.

The CUSUM test statistics on constant correlation in standardized residuals within periods, which is the simplifying assumption in the second method of estimation of covariances, had no power to reject the null of constant correlation for any pair of interest rates except FIM and ITL for the two pegged periods. For the floating period, the hypothesis was rejected only for the correlation between ITL and some other rates. We, therefore, assume that it is possible to use the Bollerslev method.

In applying the Bollerslev method as a second method for covariance estimation, the conditional correlation coefficients on GARCH standardized residuals for all pairs of interest rates for the three periods into which the data was split to account for regime changes were calculated.

For all periods, the numerical values of the correlations are much smaller within the group of interest rates than within the group of exchange rates. For the first pegged period, there is significant contemporaneous correlation between ERUSD and almost all the other interest rates. Within the group of ERM currencies, only a few occasional significant coefficients are found.

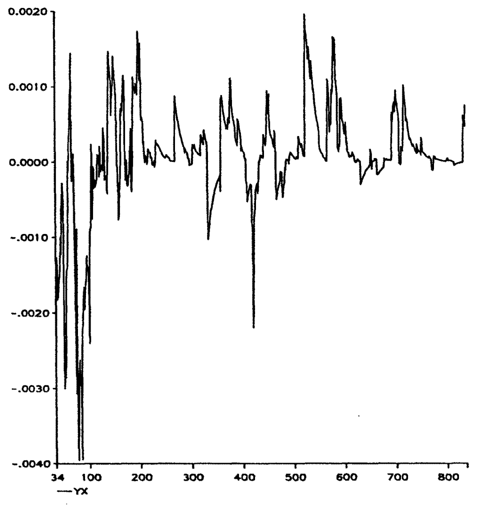
The pattern in the calculated coefficients is to a large extent the same for the second pegged period. The only difference is found in greater dependence between ERGBP and the other interest rates.

For the floating period, more of the correlations are significant compared to the pegged period, but they still remain low in comparison to the correlations within the group of exchange rates. Even ERJPY, which in the pegged periods appears to be completely uncorrelated with the other interest rates, shows under floating significant correlations with almost all the other interest rates. The change in the correlation between the interest rates may be attributed to a greater integration of financial markets both within and outside Europe.

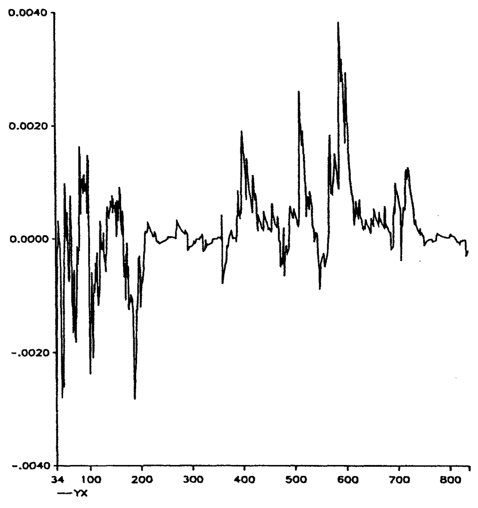
Covariances calculated according to the second method are displayed in Figure 21 for the floating period for the pairs of interest rates ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ERFIM. These figures are comparable to the outcome of the first method of covariance estimation based on identical parameter structure for variances and covariances for the same pairs of interest rates and the same period in Figure 20.

Figure 20. **Conditional covariances 9 Sep. 1992 - 31 Dec. 1995 ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ERFIM (weighted series)**

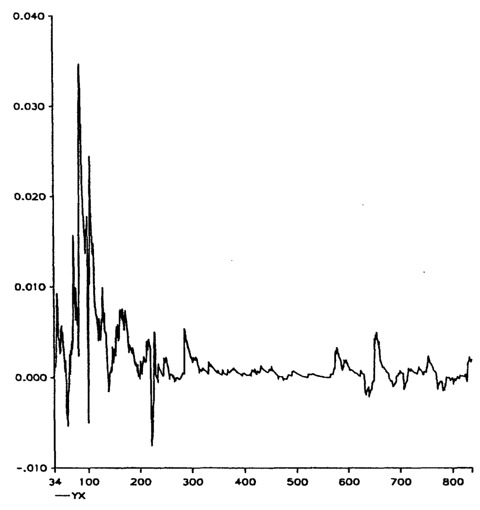
ERUSD/ERDEM



ERUSD/ERGBP



ERDEM/ERFRF



ERDEM/ERFIM

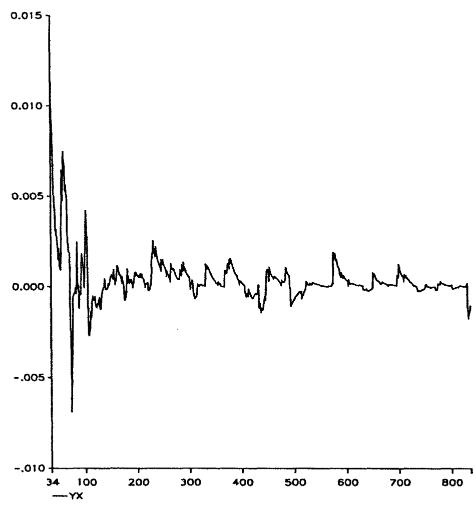
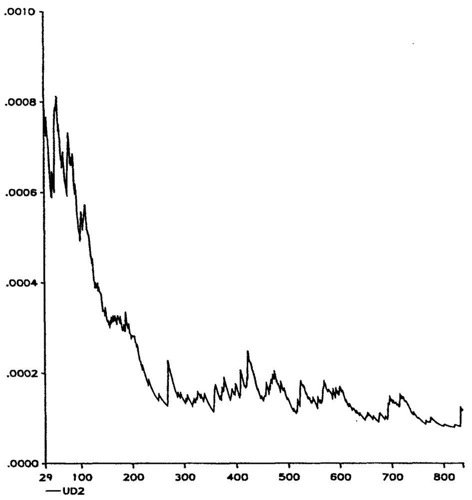
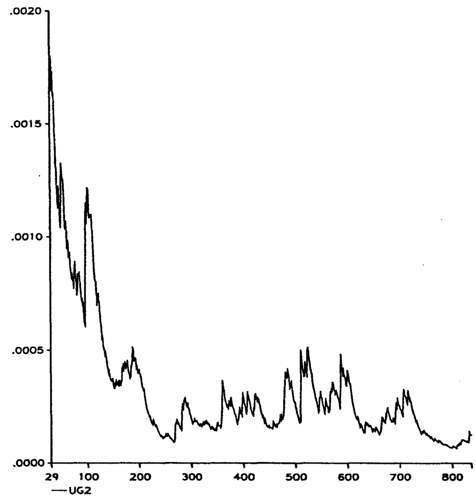


Figure 21. **Conditional covariances 9 Sep. 1992 – 31 Dec. 1995 ERUSD/ERDEM, ERUSD/ERGBP, ERDEM/ERFRF and ERDEM/ERFIM (constant correlation)**

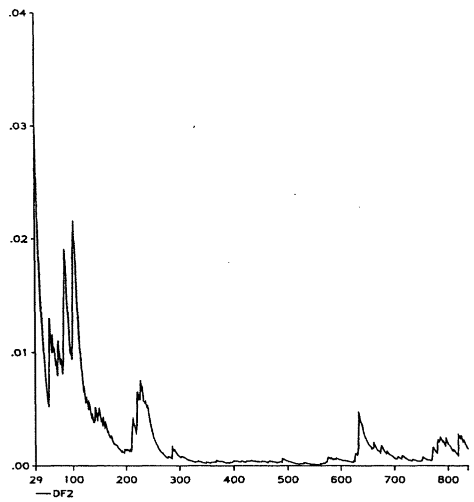
USD/DEM



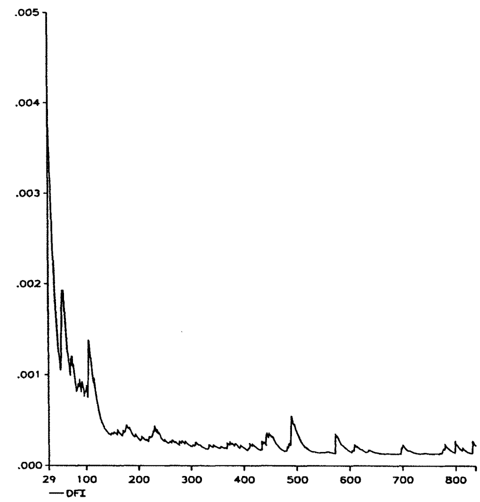
USD/GBP



DEM/FRF



ERDEM/ERFIM



8.5 Covariances between groups of rates

Measurement of the coherence between rates in efficient multivariate covariance estimation is not feasible due to the huge number of parameters to be estimated both within the groups of rates and even more so for all rates taken together. Within the groups of exchange rates and interest rates, the encompassing of the conditional covariances is therefore solved by showing that the autocovariance structure in variances and covariances is not independent. Within groups, the parameter estimates of the conditional variance processes in the pooled data are therefore used to model the conditional covariance processes. This method is not, however, applicable to covariances between groups. The other method developed by Bollerlev (1990) can be used also to measure coherence between groups. The validity of the working assumption of time-dependent conditional variances and covariances, but constant correlation, is, however, more disputable between groups than within groups.

To measure the covariation between groups in the covariance estimation method of Bollerslev, the correlation matrix was calculated for all twenty seven rates. The correlation coefficients were calculated for the GARCH residuals of the individual rates, which are assumed to be normal and IID. In this context, we assume the correlations to be constant within periods but allow them to change between periods with different regimes. The correlations were consequently calculated from sample data for each of the three main periods.

Correlations between exchange rates and interest rates are, as a rule, small compared to the correlations within groups. The statistically significant correlations between exchange rates and interest rates are found for ERFRF and ERITL. Although over half of the correlation coefficients between these two interest rates and the twelve exchange rates are significant, their numerical values are small compared to intra-group correlations. It is difficult to find a theoretical rationale for this particular pattern of correlation. Of cource, it could simply reflect data-specific features.

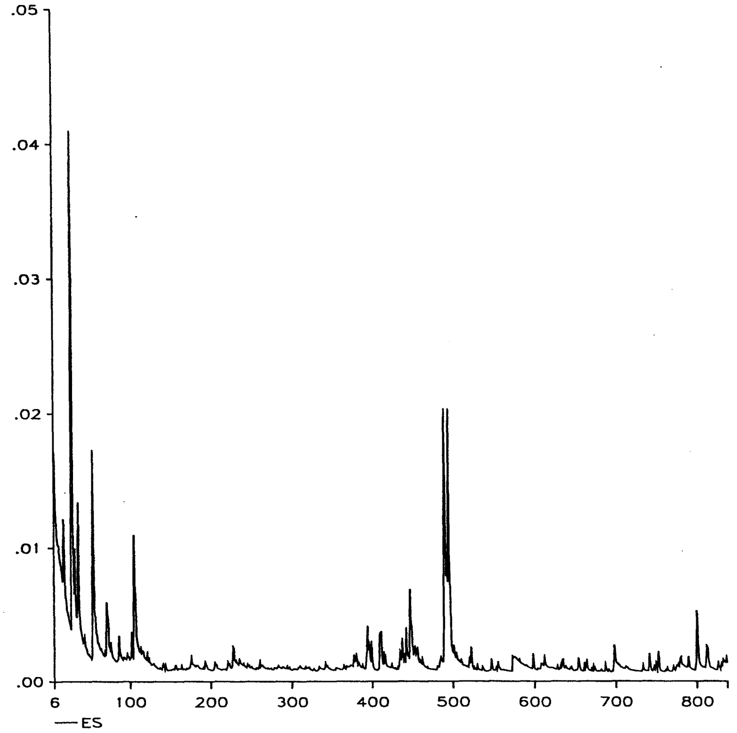
Strong correlation are found, as expected, between the long-term interest rate and the short-term rates. The movements in the general stock market index are totally independent of the contemporaneous movements in all other rates.

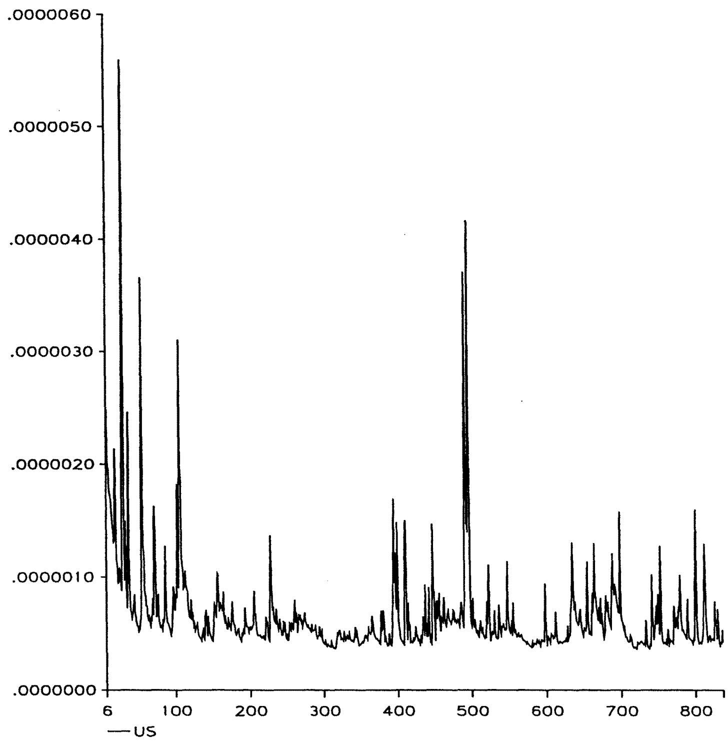
A strong correlation of $\rho = 0.330$ for the floating period is found between the Finnish long and short rate. The corresponding conditional covariance is displayed in Figure 22.

The hypothesis of an impact from the US exchange rate and interest rate on the European financial rates finds no support in the contemporaneous correlation coefficient values. To visualize the comovements between the USD and the Finnish long rate, the conditional covariance calculated with the value $\rho = 0.139$ is presented in Figure 22.

The negligible correlation values between exchange rates, short interest rates and the stock market index suggest that the covariances between these groups of rates do not have a significant contemporaneous impact on the cumulative risk of the portfolio. A stronger impact could be found if temporal adjustment processes are allowed for, or through temporal aggregation of the data.

Figure 22. **Conditional covariances, constant correlation 9 Sep. 1992 – 31 Dec. 1995. FIM long rate/FIM short rate and USD/FIM long rate**





9 Forecasting conditional variances and covariances

The importance of the identification of the GARCH-models on exchange rates, interest rates and stock market indices is, that eventhough the magnitudes and directions of changes in the rates cannot be predicted, since the expected conditional mean is zero, variance is predictable. In forecasting the time-dependent variance, we can also calculate the time dependent confidence with which one can forecast variation about the mean (Diebold and Nerlove 1986).

The aim of this study is to identify, for supervisory purposes, a measure of volatility for assessing the potential risks in bank portfolios using the value-at-risk model developed at the Bank of Finland (Ahlstedt 1990). Realized volatility should be measured on the reporting day but also a forecast of the expected future volatility is needed.

If log differences of exchange rates, interest rates and stock market indices are unpredictable and follow a homoscedastic process (ie random walk), we can write

$$R_t - R_{t-1} = \Delta R = \varepsilon_t \tag{48}$$

where ε is IID(μ,σ)

In this model, unconditional variance σ is constant and equals conditional variance. An unbiased estimator for the variance from a sample of size N is given by

$$\sigma^2 = \frac{1}{N-1} \sum_{t=1}^{N} (\Delta R_t)^2 \tag{49}$$

Based on the assumption of identically and independently distributed errors ε_t (IID), the volatility over a longer horizon can be estimated by multiplying the one-day volatility by the number of days as a scaling factor. The forecast of the volatility over a period of T days ahead is simple $\sigma^2 T$.

For the financial time series, the assumption of IID of the disturbances is typically violated. The interpretation of a GARCH model is that the disturbances are uncorrelated, but not independent. Current conditional variance is a function of past conditioning information. This means that the time-dependent conditional volatility can be forecast. For the GARCH(1,1) model, the one-step-ahead forecast of the conditional variance is

$$\sigma_t^2 = h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{50}$$

and forecast of the conditional variance s step ahead can be written as

$$E_t(h_{t+s}) = \alpha_0 + (\alpha_1 + \beta_1) E_t(h_{t+s-1}) \tag{51}$$

Under the assumption of stationarity of the foreign exchange rates, interest rates and stock market index in GARCH(1,1) parametrization ($\alpha_1 + \beta_1$ less than 1), the conditional variance will be near its unconditional mean at a sufficiently long horizon. This can be seen from the evaluation of (1) into the following form

$$E_t(h_{t+s}) = \sigma^2 + (\alpha_1 + \beta_1)^{s-1}\{h_{t+1} - \sigma^2\} \tag{52}$$

The forecast mean reverts to a constant volatility with a decay rate depending on $(\alpha_1 + \beta_1)$.

Through repeated substitutions in (51) we get an expression for the time t forecast of the variance over next s days expressed on a daily basis

$$E(h_{t,s}) = \frac{1}{s}\sum_{k=1}^{s} E_t(h_{t+k}) = \sigma^2 + (h_{t+1} - \sigma^2)\frac{1 - (\alpha_1 + \beta_1)^s}{s(1 - (\alpha_1 + \beta_1))} \tag{53}$$

where σ^2 is the unconditional constant variance, which can be shown to be $\alpha_0/(1-(\alpha_1 + \beta_1))$.

For the stationary GARCH(1,1) process, the current information continues to be important even for large s, while the relevant importance decreases with the horizon.

In the integrated process IGARCH α_1 and β_1 sum to one and the model can be expressed as follows, after imposing α_0=0

$$h_{t+1} = \alpha_1 \varepsilon_t^2 + (1 - \alpha_1)h_t \tag{54}$$

If $(\alpha_1 + \beta_1) = 1$ in (52) we can see from (53), by applying L'Hôspital's rule, that the s step forecast for this model is

$$E_t(h_{t+s}) = h_{t+1} \tag{55}$$

This means that the forecast for the conditional variance s steps in the future is the same as the conditional variance one step ahead for all horizons s ie the conditional variance follows a driftless random walk. Thus information today retains its importance in forecasting indefinitely into the future and the shocks to conditional variance are permanent. The forecast of the variance over the next s days is simply

$$E(h_{t,s}) = \Sigma E(h_{t+s}) = sh_{t+1} \tag{56}$$

The prediction error variance for the IGARCH process does not converge as the forecast horizon lengthens, but grows linearly with the length of the forecast horizon.

In the IGARCH(1,1) model with a trend

$$h_{t+1}=\alpha_0+\alpha\varepsilon_t^2+(1-\alpha_1)h_t \qquad (57)$$

the forecast for time t+s is

$$E_t(h_{t+s})=(s-1)\alpha_0+h_{t+1} \qquad (58)$$

and

$$E_t(h_{t,s})=s((s-1)\alpha_0+h_{t+1}) \qquad (59)$$

While the formula (53) can be used for ex post forecasts when the model estimation period ends at time t, we also need a formula for ex ante forecasting for periods starting from points in time where the h_t is not known. For this purpose, the GARCH(1,1) model can be rewritten as

$$h_t=\frac{\alpha_0}{(1-\beta_1)}+\alpha_1\sum_{j=1}^{\infty}\beta_1^{j-1}\varepsilon_{t-j}^2 \qquad (60)$$

where the conditional variance is expressed in the form of a geometric weighted average of past squared residuals so that the parameter β_1 gives the decay rate.

For the IGARCH process expression (60) reduces to

$$h_t=\frac{\alpha_0}{\alpha_1}+\alpha_1\sum_{j=1}^{\infty}(1-\alpha_1)^{j-1}\varepsilon_{t-j}^2 \qquad (61)$$

The estimated conditional variance of the pooled data for the floating period will be used as the forecasting formula. The estimated model for the exchange rates is

$$h_t=0.9189*E\text{-}6+0.0809\varepsilon_{t-1}^2+0.8847h_{t-1} \qquad (62)$$

and for the interest rates

$$h_t=0.1981*E\text{-}9+0.0793\varepsilon_{t-1}^2+0.9399h_{t-1} \qquad (63)$$

The sum $\alpha_1 + \beta_1$ does not significantly differ from one and we therefore conclude that the conditional variance of both exchange rates and 3-month interest rates can be modelled as a GARCH process integrated in variance. Based on the outcome of the Kendall W test procedure, we also conclude that the conditional covariances between exchange rates and conditional covariances between interest rates can be modelled with the same parameter structure as their conditional variances.

Expanding equation (61) we get the following weight structure for forecasting purposes when $\alpha_1 = 0.08$ and $\beta_1 = 1 - \alpha_1$

$$h_t = \frac{\alpha_0}{0.08} + 0.08\varepsilon_{t-1}^2 + 0.08(0.92)\varepsilon_{t-2}^2 + 0.08(0.92)^2\varepsilon_{t-3}^2 + 0.08(0.92)^3\varepsilon_{t-4}^2 + \ldots + 0.08(0.92)^{n-1}\varepsilon_{t-n}^2 + \ldots \tag{64}$$

and for the covariances

$$\sigma_{ij,t} = \frac{\alpha_0}{0.08} + 0.08\varepsilon_{i,t-1}\varepsilon_{j,t-1} + 0.08(0.92)\varepsilon_{i,t-2}\varepsilon_{j,t-2} + 0.08(0.92)^2\varepsilon_{i,t-3}\varepsilon_{j,t-3} + 0.08(0.92)^3\varepsilon_{i,t-4}\varepsilon_{j,t-4} + \ldots + 0.08(0.92)^{n-1}\varepsilon_{i,t-n}\varepsilon_{j,t-n} + \ldots \tag{65}$$

The series of lagged squared residuals to be included in actual calculations is truncated at 28 past observations. The weight of the observation there after are less than 10 % of the weight of the first observation. Table 30 gives the numerical values of the weights.

In VAR models, historical data on financial rates are used to estimate the expected variance to be implemented as a measure of risk in the portfolio. Applications differ from each other among other things in the ad hoc selection of the length of the sample period. In some applications a declining lag structure has been imposed on the historical observations in the sample without any well founded reason, although it is clear that different selected weight structures generate significant differences in resulting volatility estimates. J.P. Morgan's RiskMetrics uses a decay factor of 0.94 for all daily volatilities. Simons (1996) simulates the effects of a decay factor ranging between 0.94 and 0.97. These simulation, however, only give the sensitivity of the weight structure on the volatility measure, but not the in some sense correct solution. In the formula (64) for the variances and (65) for pairs of covariances, a theoretically and empirically derived solution to the problems of selecting the sample period and the weight structure for the estimation of the future conditional moments is presented. The sample period is truncated to 28 observations and the decay factor in the weight structure is $(1 - \alpha_1) = 0.92$. The short period of 28 observations means a rapid updating of the estimated volatility. It also means that in periods of growing volatility, the low weights on more distant observations give higher estimates on volatility compared to equally weighted observations and correspondingly lower values for periods of diminishing volatility.

In the quarterly ex post VAR model evaluation of the market risks in the supervised banks' portfolios developed at the Bank of Finland, formulas (64) and (65) will be used to calculate the individual conditional variances and the pairs of covariances for twelve exchange rates, thirteen short interest rates and the general stock market index on a daily basis. Since the variance model for the long rate is not integrated, we have to use a different structure for the forecast of this volatility. The point in time t is then the reporting day of the portfolio's composition. In forecasting, the inherent risk over the banks' planning horizon, which is assumed to be one year, forecast measures of variances and covariances for lower frequencies are needed. The monthly, quarterly, half-year and annual volatility forecasts are calculated using the formula (59) for the $h_{ij,t}$ derived from the expressions (64) and (65).

10 Summary and conclusions

The objective of this study has been to find ways to estimate the variances of the probability distribution of changes in financial time series which can be used as forecasts of the future behaviour of these series in a value-at-risk framework. The stylized facts found in markka bilateral exchange rates, short and long-term interest rates and stock market prices are modelled in a GARCH(1,1) process. The low parameter order specification was selected since it has proven to be an adequate representation for most financial time series. Prior to model identification, unit root tests for stationarity in mean were performed and also pre-whitening, where needed, to remove linear dependence. The full estimation period, 1 Jan. 1987 – 31 Dec. 1995, was divided into three subperiods to account for nonstationarity, ie structural changes trigged by realignments in the Finnish currency.

Univariate GARCH models for twelve exchange rates, thirteen short-term interest rates, one long-term interest rate and the general stock market index were estimated. Principal component analysis on the estimated conditional variances for each period for both exchange rates and interest rates was performed to detect common factors driving the rates. The analysis showed that, compared to macroeconomic variables, the groups of conditional variances revieled a high degree of heterogeneity, but could be used in concentrating the fluctuations in the individual variances into common factors. Spectral analysis was then performed in order to measure cyclical regularity in the estimated conditional variances. In the spectral density figures, the highest values were on average found for the period of 180 days and its harmonics for the pegged periods both for exchange rates and interest rates. The power spectrum for the floating period both for exchange rate and interest rates revealed an integrated process.

The results of the univariate GARCH estimation for exchange rates and interest rates during both the pegged and the floating period showed that there was a great likeness in the estimated parameter values within groups of rates. Therefore GARCH models were estimated with pooled data to force the conditional variances within the group of currencies and within the group of interest rates, respectively, into the same model. The estimated models for the pooled data were found to be integrated in variance both for exchange rates and interest rates. The striking results are that the parameter structure is independent of the exchange rate regime and that the almost same parameter values were found in models estimated on pooled data both for exchange rates and interest rates. Also the variance model of the general stock market index was estimated to have this same parameter structure.

BDS test statistics were applied to the standardized GARCH residuals to test for model misspecification. For all rates the applied GARCH(1,1) model produced decreasing test values compared to the raw data but as a rule evidence for some remaining nonlinearity was found. As a result of the GARCH estimation it was possible to construct standardized residuals as new variables. These variables are theoretically normal and empirically at least much closer to normal than the raw data. The standardization of the data thus makes the normality assumption on which VAR models typically are based more grounded.

The huge number of variables both within groups of rates and even more so for all rates taken together, did not allow the use of multivariate GARCH estimation to assess covariances in the system. The problem was handled in two ways. First, by assuming the same parameter structure for variances and covariances and second, by assuming constant correlation between standardized residuals. The dependence between of the autocorrelation structure of variances and covariances was tested using Kendalls W test. Based on the outcome of the test, the null of independence could be rejected both within the group of exchange rates and interest rates. The same estimated parameter values was therefore used in forecasting variances and covariances.

In the second method, the Bollerslev method, the comovements within and between groups were measured with a correlation matrix including all twenty seven rates under discussion. Correlation between exchange rates and interest rates were as a rule small compared to the correlations within the groups. Strong correlation was found as expected between the long interest rate and the short rate. The movements in the general stock market index were totally independent of the contemporaneous movements in all other rates. Based of the results of a CUSUM test on constant correlation within periods the Bollerslev method was applicable for covariance estimation both within groups and between groups for pair of rates with significant correlations.

Although the magnitude or the direction of expected changes in the rates cannot be forecasted the identification of a GARCH model means, however, that the conditional variances of the changes can be forecasted. The results derived in this study can be used to calculate the expected variance measures in VAR models. We end up with a forecasting formula for the conditional variances and covariances, which gives the solution to the problem of selecting the length of the sample period and the lagged weight structure in volatility forecasting.

References

Ahlstedt, M. (1990) **Measuring the Interest Rate and Exchange Rate Risk of a Commercial Bank's Portfolio.** Bank of Finland Discussion Paper 17/90.

Ahlstedt, M. (1995) **The Statistical Distribution of Exchange Rates and Interest Rates** (in Finnish). Bank of Finland Discussion Paper 37/95.

Backus, D. – Gregory, A.W. – Zin, S.E. (1989) **Theoretical Relations between Risk Premium and Conditional Variances.** Working paper, Queens University, Canada.

Bollerslev, T. (1986) **Generalized Autoregressive Conditional Heteroskedasticity.** Journal of Econometrics, Vol. 31.

Bollerslev, T. (1987) **A Conditional Heteroskedastic Time Series Model for Speculative Prices ans Rates of Return.** Review of Economics and Statistics, 69.

Bollerslev, T. (1990) **Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach.** Review of Economics and Statistics, 72.

Bollerslev, T. – Chou, R.Y. – Kroner, K.F. (1992) **ARCH Modelling in Finance.** Journal of Econometrics, Vol. 52.

Bollerslev, T.R. – Engle, R.F. – Wooldridge ,J.M. (1988) **A Capital Asset Pricing Model with Time Varying Covariances.** Journal of Political Economy, 96/1.

Booth, G.G. – Martikainen, T. – Sarkar, S.K. – Virtanen, I. – Yli-Olli ,P (1992): **Nonlinear Dependence in Finnish Stock Returns.** Preliminary paper, July.

Box, G.E.P. – Tiao, G.C. (1975) **International Analysis with Applications to Economic and Environmental Problems.** Journal of the American Statistical Association, 70.

Brock, W. – Deckert, W. – Scheinkman, J. (1987) **A Test for Independene Based on the Correlation Dimension.** Univeristy of Wisconsin, Madison, University of Huston and University of Chicago.

Brock, W.A. – Hsieh, D.A. – LeBaron B. (1991) **Nonlinear Dynamics, Chaos and Instability.** Massachusetts Institute of Technology.

Chappel, D. – Padmore, J. (1995) **Changes in Volatility of the Sterling-Deutchmark Exchange Rate: the Effect of ERM Membership.** Applied Economics Letters, 2.

Diebold, F. – Nerlove, M. (1989) **The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model.** Journal of Applied Econometrics, 4.

Engle, R.E. (1982) **Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation.** Econometrica, Vol 50.

Engle, R.E. – Ito, T. – Lin, W-L. (1990) **Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market.** Econometrica 58.

Engle, R.F. - Lilien, D.M. - Robins, P. (1987) **Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model.** Econometrica 55.

Grassberger, C. - Procaccia, I. (1983) **Measuring the Strangeness of Strange Attractors.** Physica 9D, 189-208.

Heynen, R.C. - Kat, H.M. (1994) **Volatility Prediciton: A Comparison of the Stochastic Volatility, GARCH(1,1), and EGARCH(1,1) Models.** The Journal of Derivatives. Winter.

Hsieh, D.A. (1988) **The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983.** Journal of International Economics, 24.

Hsieh, D.A. (1991) **Chaos and Nonlinear Dynamics. Application to Financial Markets.** The Journal of Finance. Vol XLVI, No. 5. December.

Hung, J.H. (1995) **Intervention Strategies and Exchange Rate Volatility: A Noise Trading Perspective.** Research Paper #9515. Federal Reserve Bank of New York.

Kärki, J. - Reyes, C. (1994) **Model Relationship.** Risk, Vol 7, Dec.

Lamoureux, C.G. - Lastrapes, W.D. (1990) **Persistance in Variance, Structural Change, and the GARCH Model.** Journal of Business & Economic Statistics, Vol 8.

Malkamäki, M. (1993) **Essays on Conditional Pricing of Finnish Stocks.** Bank of Finland B:48.

Mehra, R. - Precott, E. (1985) **The Equity Premium: A Puzzle.** Journal of Monetary Economics 15.

Merton, R.C. (1973) **An Intertemporal Capital Asset Pricing Model.** Econometrica, 41.

Mills, T.C. (1993) **The Econometric Modelling of Financial Time Series.** Cambridge University Press.

Nelson, D.B. (1990a) **Conditional Heteroscedasticity in Asset Returns: A New Approach.** Econometrica, 59.

Nelson, D.B. (1990b) **ARCH Models as Diffusion Approximations.** Journal of Econometrics, 45.

Nelson, C.R. - Plosser, C.I. (1982) **Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications.** Journal of Monetary Economics, 10.

Nerlove, M. - Diebold, F.X. - van Beeck, H. - Cheung, Y-W. (1988) **A Multivariate ARCH Model of Foreign Exchange Rate Determination.** Research Report.

Pantula, S.G. (1985) **Estimation of Autoregressive Models with ARCH Errors.** Working Paper, Department of Statistics, North Carolina State University.

Perron, P. (1989) **The Great Crash, the Oil Price Shocks and the Unit Root Hypothesis.** Econometrica, 57.

Siegel, S. (1956) **Nonparametric Statistics for the Behavioral Sciences.** McGraw-Hill Kogakusha Ltd.

Simons, K. (1996) **Value at Risk – New Approaches to Risk Management.** New England Economic Review. September/October.

Sulamaa, P. (1995) **Statistical Analysis of the Markka Bilateral Spot Exchange Rates.** University of Helsinki.

West, K.D. – Cho, D. (1995) **The Predictive Ability of Several Models of Exchange Rate Volatility.** Journal of Econometrics, 69.

List of variables

Foreign exchange rates		Three month interest rates
USD	US dollar	ERUSD
GBP	Great Britain pound	ERGBP
SEK	Swedish krona	ERSEK
NOK	Norwegian krona	ERNOK
DKK	Danish krona	ERDKK
DEM	Deutsch mark	ERDEM
NLG	Dutch guilder	ERNLG
BEF	Belgian/Luxenbourg franc	ERBEC
CHF	Swiss franc	ERCHF
FRF	French franc	ERFRF
ITL	Italian lira	ERITL
JPY	Japanese yen	ERJPY

Domestic rates	
ERFIM	three month interest rate
ERlong	three year long rate
HEX	general stock market index

BANK OF FINLAND DISCUSSION PAPERS

ISSN 0785-3572

1/97	Peter Nyberg **Macroenomic Aspects of Systemic Bank Restructuring**. 1997. 30 p. ISBN 951-686-541-0. (KASI)
2/97	Anne Brunila **Fiscal Policy and Private Consumption – Saving Decisions: Evidence from Nine EU Countries**. 1997. 36 p. ISBN 951-686-542-9. (TU)
3/97	Antti Ripatti **Limited and Full Information Estimation of the Rational Expectations Demand for Money Model: Application to Finnish M1**. 1997. 30 p. ISBN 951-686-543-7. (TU)
4/97	Mika Pösö – Mikko Spolander **Estimating Exchange Market Pressure and the Degree of Exchange Market Intervention for Finland during the Floating Exchange Rate Regime**. 1997. 24 p. ISBN 951-686-544-5. (RP)
5/97	Sinimaaria Ranki **Monetary Policy in the ERM: Internal Targets or External Constraints?** 1997. 62 p. ISBN 951-686-545-3. (TU)
6/97	Anne Brunila **Current Income and Private Consumption – Saving Decisions: Testing the Finite Horizon Model**. 1997. 70 p. ISBN 951-686-547-X. (TU)
7/97	Monica Ahlstedt **Exchange Rate, Interest Rate and Stock Market Price Volatility for Value-at-Risk Analysis**. 1997. 90 p. ISBN 951-686-548-8. (TU)