

Juha Kilponen

**Euler consumption equation with
non-separable preferences over
consumption and leisure and
collateral constraints**



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The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Euler consumption equation with non-separable preferences over consumption and leisure and collateral constraints

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Abstract

This paper derives and estimates an aggregate Euler consumption equation which allows one to compare the importance of collateral constraints and non-separability of consumption and leisure as alternative sources of excess sensitivity of consumption to current income. Estimation results suggest that during a severe financial distress both non-separability and collateral constraints are needed to capture excess sensitivity of consumption to current economic conditions. During more tranquil times, evidence on collateral effects is more limited and non-separability is sufficient to make the Euler consumption equation agree well with the data.

Keywords: housing, financial distress, excess sensitivity of consumption

JEL classification numbers: E21, E32, E44

Kulutuksen Euler yhtälö, ei-separoituvuus ja vakuusrajoitteet

Suomen Pankin keskustelualoitteita 9/2009

Juha Kilponen
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä tutkimuksessa johdetaan ja estimoidaan kulutuksen Euler-yhtälö, joka sallii sen, että sekä kulutuksen ja työn tarjonnan välinen vuorovaikutus että vakuusrajoitteet saattavat selittää kulutuksen yliherkkyyttä käytävissä olevien tulojen suhteen. Kulutuksen Euler-yhtälö perustuu kotitalouden preferensseihin, jotka ovat myös sopusoinnussa pitkän aikavälin työntarjontafaktojen kanssa. Estimointitulokset osoittavat, että selitettäessä 1990-luvun alun rajua notkahdusta yksityisen kulutuksen kasvussa Suomessa oletukset niin vakuusrajoitteista kuin kulutuksen ja vapaa-ajan välisestä vuorovaikutuksesta ovat välttämättömiä. Periodilla 1995/I–2008/II, jolloin rahoitusmarkkinat ovat toimineet käytännössä häiriöttömästi, rahoitusrajoitteiden merkitys on ollut selkeästi pienempi. Tällöin oletus kulutuksen ja vapaa-ajan välisestä vuorovaikutuksesta hyötyfunktiossa, eli ns. ei-separoituvuus riittää.

Avainsanat: asuntomarkkinat, rahoitusmarkkinahäiriöt, kulutus

JEL-luokittelu: E21, E32, E44

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1 Introduction

It is widely known and agreed that the simplest form of consumption Euler equation is only weakly consistent with aggregate consumption data. Empirical failure is commonly associated to excess sensitivity of consumption to current income. Possible explanations in the large literature include financial market imperfections in the form of interest rate differentials, credit rationing and collateral constraints (Flavin, 1985, Hubbard and Judd, 1986, Hayashi, 1987, Jappelli and Pagano 1989, Iacoviello, 2004), as well as non-separable preferences and durability of goods and habits (Browning, 1991, Attanasio, 1995, Basu and Kimball, 2002, Kiley, 2007).

In this paper I derive and estimate an aggregate consumption Euler equation which features both non-separability and financial market imperfections in the form of binding collateral constraints. The contribution to the theoretical literature is that the form of the consumption Euler equation allows one to compare collateral constraints to non-separability of consumption and leisure as an explanation of excess sensitivity of consumption to current income. I thus circumvent the problem pointed out by Attanasio (1995) concerning Campbell and Mankiw's (1989) regression: that a coefficient of income growth can be interpreted as the fraction of individuals subject to liquidity constraints only if consumption and leisure are separable in the utility function. At the same time, the form of utility function used in this paper is consistent with the long-run labour supply facts, namely that there is no pronounced trend in hours worked per person but there is a strong trend in real wage. In other words, income and substitution effects should roughly offset each other.

In order to test the empirical fit of the consumption Euler equation developed in this paper, I estimate the resulting (linearized) Euler equation using aggregate data from Finland for 1987Q1–2008Q2 period and also for the subsample 1995Q1–2008Q2. The Finnish data should be highly informative on the importance of collateral effects, since the sample includes a period of a dramatic drop and recovery of private consumption in the aftermath of the house price bubble and economic recession in the early 1990s. The recession years were characterised by heightened financial distress and deteriorating financial conditions, as major banks failed. The sample also includes a period of more tranquil times without a major financial distress (mid-1990s to early 2008) allowing me to assess the relative importance of collateral constraints under widely different financial market conditions.¹ Finally, I compare the estimation results to the more simple consumption Euler equations estimated in Hall (1988), Campbell and Mankiw (1989), Basu and Kimball (2002) and Iacoviello (2004) for the US data.

Estimation results show that both non-separability of consumption and leisure and collateral effects are necessary for capturing the dramatic drop and recovery of consumption growth in Finland in the early 1990s. During

¹Honkapohja and Koskela (1999) have earlier argued that direct financial restraints in general played an important role in cutting aggregate demand during the recession years in Finland, yet they did not find strong direct evidence of the impact of house prices on consumption.

more tranquil times since the mid-1990s, evidence on collateral constraints is more limited and non-separability is sufficient to make the consumption Euler equation agree well with the consumption growth data. Furthermore, I find no support for the rule-of-thumb consumption behaviour, once the non-separability is accounted for.² Estimated values of the intertemporal elasticity of substitution (IES) and consumption share of collateral constrained households also seem reasonable and accurately determined. Both IES and consumption share of collateral constrained households are around 0.6 in the whole-sample estimation, including the period of financial distress. In the subsample after the mid-1990s, collateral constraints become less important, the consumption share of collateral constrained households drops to less than half and the intertemporal elasticity of substitution increases to almost 0.7.

The rest of the paper is organised as follows. Section two develops the model and derives a linearized aggregate Euler equation. Section three presents the empirical results and section four concludes. The appendix provides detailed derivations.

2 The model

The economy consists of two types of households, un-constrained and constrained. Both households have preferences over consumption, leisure and housing. Housing is separable in consumption and leisure, and all agents can trade houses, consumption goods and riskless real bonds. As for leisure and consumption, I impose cancellation between non-zero income and substitution effects, by writing the utility function so that the real wage is proportional to consumption times some function of labour quantity. A convenient form of utility function which delivers this is the King-Plosser-Rebelo (1988) version also used in Basu and Kimball (2002). Otherwise, the model can be seen as an extension of Iacoviello (2004).

2.1 Un-constrained household

An un-constrained household maximises standard lifetime-utility. The problem reads as

$$\max_{\{C_t^u, H_t^u, N_t^u\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^u)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t^u)} + \kappa f(H_t^u) \right)$$

²Iacoviello (2004) finds strong evidence of collateral effects in the US data, while Basu and Kimball (2002) find that for the last two decades, the Euler equation based on non-separable preferences explains the aggregate consumption growth data very well in the US. Also Kiley (2007) has found evidence of non-separability in the US data. Basu and Kimball (2002) also argue that after accounting for the effects of predictable movements in labour implied by non-separability, the evidence of excess sensitivity of consumption to predictable changes in income is substantially reduced, if not eliminated. Also Ham and Reilly (2002) provide evidence on non-separability of the consumption- leisure choice.

s.t.

$$B_t^u + Y_t^u + W_t N_t^u = R_{t-1} B_{t-1}^u + C_t^u + Q_t (H_t^u - H_{t-1}^u)$$

where C_t^u is consumption of un-constrained households, Q_t is real price of real estate, H_t^u is housing, R_t is the gross real interest rate, B_t^u is real bonds ($-B_t^u$ is lending), W_t is the real wage rate, N_t^u is employment and Y_t^u is random endowment. I assume that $f'(H_t^u) > 0$, $f''(H_t^u) < 0$ and κ is a positive constant. $v(N_t)$ denotes disutility from labour, with the usual properties: $v'(N_t^u) > 0$, $v''(N_t^u) < 0$. γ is the risk aversion parameter. Housing is treated as durable consumption that never depreciates. Straightforward maximization reduces to Euler consumption equation

$$(C_t^u)^{-\gamma} e^{(\gamma-1)v(N_t)} = \beta R_t E_t [(C_{t+1}^u)^{-\gamma} e^{(\gamma-1)v(N_{t+1})}] \quad (2.1)$$

Linearizing (2.1) yields

$$-\gamma \hat{c}_t^u + (\gamma - 1) \tau^u \hat{n}_t^u = -\gamma \hat{c}_{t+1}^u + \hat{r}_t + (\gamma - 1) \tau^u \hat{n}_{t+1}^u + \mathcal{O}_t \quad (2.2)$$

where $\hat{\cdot}$ denotes percentage deviation of a variable from the steady state. \mathcal{O}_t denotes higher order terms due to first order linearization. When linearizing (2.1), I used the fact that

$$v'(N^u) N^u = \left(\frac{W N^u}{C^u} \right) \equiv \tau^u. \quad (2.3)$$

where N^u denotes steady state (optimal) labour supply. Furthermore, using the approximation that

$$\Delta v(N_t^u) \approx \tau^u (n_t^u - n_{t-1}^u) = \tau^u \Delta n_t^u \quad (2.4)$$

equation (2.2) reduces to

$$\hat{c}_t^u = E_t \hat{c}_{t+1}^u - s \hat{r}_t - (1 - s) \tau^u E_t \Delta n_{t+1}^u \quad (2.5)$$

where $s \equiv 1/\gamma$ is the intertemporal elasticity of substitution. Linearization is accurate as long as there is no strong trend in labour supply, and we are not too far from the steady state, since the approximation to $v(N_t^u)$ is applied around some constant (optimal) value of N^u .

This is in principle the same Euler equation as the one derived in Basu and Kimball (2002). As discussed by Basu and Kimball, there is a non-trivial linear restriction between the coefficient of the real interest rate \hat{r}_t and the coefficient of employment growth. Restriction comes from the facts about long-run labour supply.³ A low value of the elasticity of intertemporal substitution means that the marginal utility of consumption falls rapidly along with the higher level of consumption. Without any interaction between consumption and labour in the utility function, ie with separable preferences, a decline in the marginal utility of consumption would lead households to want more leisure unless the real wage increased markedly. When consumption and labour are complements, as here, an increased level of consumption, and thus a decline in the marginal utility of consumption, makes labour more pleasant. This renders the association between consumption and real wage stronger compared to that implied by separable preferences.

³Note that the parameter restriction does not depend on an exact value of the labour supply elasticity indicating the size of the income and substitution effects.

2.2 Constrained households

There is a fraction ζ of households who are borrowing constrained. At each point of time, the amount they can agree to repay in the following period cannot exceed a fraction $m \leq 1$ of next period's expected value of real estate holdings ($Q_{t+1}H_t^c$).⁴ One may think of $m \leq 1$ as representing liquidation costs in case of default. Formally, constrained households' real obligations $R_t B_t^c$ are limited by

$$R_t B_t^c \leq m E_t[Q_{t+1}H_t^c] \quad (2.6)$$

This type of collateral constraint can be rationalized by limited enforcement, the idea being that the creditors protect themselves from the threat of repudiation by collateralizing part of the household's real estate holdings. An important feature of (2.6) is that expected movements in the price of collateralised asset (real estate) affect the borrowing. Potentially, the prices of collateralized assets could also be affected by the size of the credit limits as emphasised by Kiyotaki and Moore (1997).

Following Iacoviello (2004) I assume that constrained households do not discount the future. Otherwise, the constrained households share the preferences with the un-constrained households. Their optimization problem is then

$$\begin{aligned} \max_{\{C_t^u, H_t^u, N_t^u\}} & \left(\frac{(C_t^c)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t^c)} + \kappa f(H_t) \right) \\ & \text{s.t.} \\ C_t^c + Q_t(H_t^c - H_{t-1}^c) + R_{t-1}B_{t-1}^c &= B_t^c + W_t^c N_t^c + Y_t^c \\ R_t B_t^c &\leq m E_t(Q_{t+1})H_t^c \end{aligned} \quad (2.7)$$

The first order conditions for B_t^c and H_t^c and N_t^c yield

$$(C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = R_t \phi_t \quad (2.8)$$

$$-Q_t (C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = \kappa f'(H_t^c) + m E_t(\phi_t Q_{t+1}) \quad (2.9)$$

$$W_t^c = C_t^c v'(N_t^c) \quad (2.10)$$

As can be seen from 2.8, current marginal utility of consumption is affected by the shadow value of the borrowing constraint, ϕ_t . There is a distortion in housing demand, since housing can be used as collateral. Furthermore, also the intratemporal decision is affected indirectly by the shadow value of borrowing constraint, since current marginal utility of consumption enters into the intratemporal condition (see 2.10). In the steady state, the constrained households borrow up to the limit, and it is assumed that the constraint holds also in the neighbourhood of the steady state. An increase in real estate prices relaxes the borrowing constraint, leads to higher borrowing, and thus increases consumption of constrained households. Due to non-separability, increasing consumption makes labour more pleasant and thus non-separability

⁴This is a form of collateral constraint used by Kiyotaki and Moore (1997).

has a tendency to further amplify the impact of house prices on consumption. The opposite is of course true when prices fall. When prices fall, the non-separability enforces the decline in consumption.

Linearizing and combining the first order conditions appropriately (see mathematical appendix for details) delivers the following consumption equation for the constrained households

$$\hat{c}_t^c = s[\theta\hat{h}_t^c + \omega(\hat{r}_t - E_t\hat{q}_{t+1}) + (1 + \omega)\hat{q}_t] + (1 - s)\tau^c\hat{n}_t^c \quad (2.11)$$

where $1 + \omega \equiv \frac{1}{1 - m\beta}$ and $\theta \equiv -\frac{f''(H^c)H^c}{f'(H^c)}$. $1 + \omega$ is the inverse of the downpayment needed to purchase one unit of housing, while θ is related to long-run demand elasticity of housing services. \hat{h}^c and \hat{q} denote housing demand and real house price in percentage deviation from the steady state.

I have linearized $v(N_t^c)$ around the optimal (trend) level of labour supply, similarly to the case of un-constrained households. Thus τ^c is defined correspondingly as above in equation (2.3). The borrowers' consumption is a positive function of house prices, with a coefficient equal to the inverse of the downpayment times the intertemporal elasticity of substitution, s . Consumption depends positively also on the measure of labour supply, due to the non-separability. With $s = 1$ and $\tau = 0$, this equation is in principle the same as the one derived in Iacoviello (2004).

2.3 Derivation of the aggregate Euler equation

Having derived the consumption Euler equations for the un-constrained and constrained households, the final step is to obtain the aggregate Euler equation, which can then be estimated. Recall for convenience the following Euler equations for un-constrained and constraint agents

$$\begin{aligned} \hat{c}_t^u &= E_t\hat{c}_{t+1}^u - s\hat{r}_t - (1 - s)\tau^u E_t\Delta n_{t+1}^u \\ \hat{c}_t^c &= s\theta\hat{h}_t^c + s\omega(\hat{r}_t - E_t\hat{q}_{t+1}) + s\omega\hat{q}_t + (1 - s)\tau^c\hat{n}_t^c \end{aligned}$$

Making the rational expectations assumption explicit, *ie* that

$$\begin{aligned} E_t\hat{c}_{t+1}^u &= c_{t+1}^u + \epsilon_t^u \\ E_t\Delta n_{t+1}^u &= \Delta n_{t+1}^u + \epsilon_t^{\Delta n} \\ E_t\hat{q}_{t+1} &= \hat{q}_{t+1} + \epsilon_t^q \\ E_t\hat{n}_{t+1} &= \hat{n}_{t+1} + \epsilon_t^n \end{aligned}$$

and substituting rational expectations into the corresponding Euler equations yields

$$\Delta c_{t+1}^u = s\hat{r}_t + \tau^u(1 - s)\Delta n_{t+1}^u + \epsilon_t^u + \alpha_u\epsilon_{t-1}^u$$

$$\hat{c}_t^c = s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1 - s)\tau^c\hat{n}_t^c + \epsilon_t^c + \alpha_c^c\epsilon_{t-1}^c$$

The ε_t^i , $i = u, c$, are forecast error terms which contain forecast errors related to future consumption, labour and real housing prices. Let λ denote consumption share of constrained households and note that aggregate consumption can be expressed in log first differenced form

$$\Delta c_t = \lambda \Delta c_t^c + (1 - \lambda) \Delta c_t^u \quad (2.12)$$

Substituting then Δc_{t+1}^u and \hat{c}_t^c in (2.12), and manipulating appropriately the resulting equations (see mathematical appendix for details) yields

$$\Delta c_t = \lambda s \theta \Delta h_t^c + \lambda s \omega [\Delta r_t - \Delta \Delta q_{t+1}] + (1 - s) \tau \Delta n_t + (1 - \lambda) s \hat{r}_t + \varepsilon_t \quad (2.13)$$

where ε_t is a linear combination of forecast errors ε_t^i . Re-organising slightly and using the fact that

$$\Delta r_t - \Delta \Delta q_{t+1} = r_t^h - r_{t-1}^h$$

where r_t^h denotes housing real interest rate, I arrive at the Euler consumption equation

$$\Delta c_t - \tau \Delta n_t = \lambda s [\theta \Delta h_t^c + \omega \Delta r_t^h - \hat{r}_t] + s (\hat{r}_t - \tau \Delta n_t) + \varepsilon_t \quad (2.14)$$

On the one hand, compared to Basu and Kimball (2002), there is a new term $\lambda s [\theta \Delta h_t^c + \omega \Delta r_t^h - \hat{r}_t]$. This comes from the presence of collateral constrained households, and it captures the sensitivity of consumption growth to price fluctuations in collateral, ie collateral effects in short. On the other hand, in comparison to Iacoviello (2004), there is also the term $\tau \Delta n_t$ on both sides of equation (2.14).⁵ This captures the effects of non-separability. Equation (2.14) thus nests both the Basu-Kimball (2002) and Iacoviello (2004) specifications. Naturally, it also nests the standard Euler equation with $\lambda = 0$, $\tau = 0$. In that case, only the real interest rate appears on the right hand side of equation (2.14). Finally, note that (2.14) allows one to compare collateral constraints to non-separability, since both λ and s can be identified separately.

⁵Iacoviello (2004) derives the aggregate Euler equation in a slightly different way. He replaces the conditional expectation of unconstrained household's consumption by the long term interest rate, which would yield to

$$\hat{c}_t = -s(1 - \lambda) [\hat{l}_t + \hat{r}_t] + \omega \lambda s [\hat{r}_t + \hat{q}_t - E_t \hat{q}_{t+1}] + \lambda s \hat{q}_t + \lambda s \theta \hat{h}_t^c + (1 - s) \lambda \tau \hat{n}_t$$

where \hat{l}_t is long-term interest rate.

3 Evidence

3.1 Data

I estimate the variants of (2.14) with Finnish data, using the sample since 1987.⁶ Basu and Kimball (2002) estimated (2.14) (with $\lambda = 0$) using instrumental variable estimation and assuming that ϵ_t has MA(1) structure. I follow Iacoviello (2004), and also Kiley (2007), by using Hansen's (1982) Generalised Methods of Moments (GMM).

As a dependent variable and as a measure of consumption, I use log change in total private consumption per capita (Δc). The real short-term interest rate is the difference between the quarterly 3-month money market rate and quarter-on-quarter change in the log private consumption deflator (r). The housing real interest rate is the difference between the quarterly 3-month money market rate and quarter-on-quarter change in the log house price index (r^h). Real house price is the log house price index (whole economy) deflated by the private consumption deflator (q). As a proxy for housing demand (for constrained agents), I use the detrended log total residential investment per capita (h). Implicitly, I assume that most of the variation in housing demand is due to variation in housing demand of constrained households. As a measure of labour supply growth, I use log difference of hours (total economy) per capita (Δn). I have smoothed consumption deflator, house price index and hours worked slightly using HP-filter, in order to re-move extra noise from the quarterly series.

In order to calibrate τ , I compute τ as an average of $\tau_t = \frac{(1-\tau_{t,l})W_tN_t}{C_t}$, following equation (2.3). The average value of τ for the period of 1987–2008 is roughly 0.5, which is the calibrated constant used in the estimation of the whole sample. $\tau_{t,l}$ is the average effective tax rate on labour. The method of computing the average effective tax rate on labour from aggregate National accounts data is described in OECD (2000) and Mendoza et al (1994). Given that there is a reasonable degree of uncertainty about the actual value of τ , I check the results for different values of τ , ranging from 0.4 to 0.6.

As instruments (\mathbf{z}) in the GMM estimation, I use three or four lags of each right hand side variable of (2.14) in levels. In order to take into account the first order moving average term in the errors, I use lags greater than 2 for these variables. As additional instruments, I use log disposable income (y), consumption income ratio ($c - y$), world GDP (y_w), and the household's debt-to-disposable income ratio ($d - y$). Disposable income is calculated according to the National Accounts definition. I report the results also using different instruments subsets in order to check the robustness of the results. All regressions include a constant term (not reported). The constant in a regression captures the higher order terms due to the precautionary savings motives of the consumers and approximation error due to linearisation. In all regressions I used rational expectations hypothesis and replaced expected

⁶This is motivated by the relatively late process of financial deregulation in Finland. Although financial market liberalisation started in the early 1980s, it intensified during the second half of the decade. For instance, regulation of lending rates was abolished as late as 1986 and market interest rate *helibor* was introduced in 1987.

inflation (both house price and consumer price inflation) with its ex post realizations. This is a strong assumption, since the real-ex post interest rate could be an inaccurate measure of the real interest rate perceived by households. A lack of quarterly survey data prevents me from checking the importance of this assumption to the results.

3.2 Results

3.2.1 Sample 1987Q1–2008Q2

I start by first estimating the following three consumption Euler equations with the following sets of orthogonality conditions

$$(I) : E_t\{(\Delta c_t - sr_t)\mathbf{z}_t\} = 0 \quad (3.1)$$

$$(II) : E_t\{(\Delta c_t - \tau\Delta n_t - s[r_t - \tau\Delta n_t])\mathbf{z}_t\} = 0 \quad (3.2)$$

$$(III) : E_t\{(\Delta c_t - \tau\Delta n_t - \lambda s[\theta\Delta h_t^c + \omega\Delta r_t^h - r_t] - s(r_t - \tau\Delta n_t))\mathbf{z}_t\} \\ = 0 \quad (3.3)$$

The first is the standard Euler equation, the second is the specification with non-separable labour only corresponding to Basu and Kimball (2004), and the last one is the specification with collateral constrained households and non-separable preferences, I refer to the third equation often as the encompassing model. Results are provided in Table 1, together with the preferred set of instruments used.

The estimates of the intertemporal elasticity of substitution (IES) are somewhat sensitive to whether the collateral constraint and/or non-separability of consumption and leisure is assumed. Estimates vary roughly from 0.3 to 0.6. Apart from the standard Euler equation, however, IES is always significantly greater than zero at the 5% significance level. The consumption share of collateral constraint households (λ) is roughly 60%. The 95% confidence interval for λ includes values as low (high) as 0.18(1.08). This seems reasonable, although the upper bound is somewhat above one. The mean estimate is somewhat high in comparison to other international studies. A high number, however, could reflect the fact that in the early 1990s financial markets in Finland tightened due to a severe economic recession and banking crisis. Moreover, the financial markets were liberalised relatively late in Finland.⁷

⁷Financial deregulation started in the early 1980s, but major changes, such as abolition of regulation of lending rates took place during the second half of the decade. For details, see for instance Honkapohja and Koskela (1999) and Honkapohja (2009).

The estimated parameter related to demand elasticity of housing θ and the parameter related to liquidation costs (ω) have high standard errors. ω is estimated to at value which is perhaps unrealistically low, given that $1 + \omega = \frac{1}{1-m\beta}$ is inverse of the downpayment needed to purchase one unit of housing services.⁸ Given that $\theta \equiv -\frac{f''(H^c)H^c}{f'(H^c)}$, a small estimated value of θ implies that preferences are roughly linear in housing services.⁹

The results are not particularly sensitive to calibrated values of τ . Using $\tau = 0.4$ changes the estimates of IES to 0.29 and to 0.51 in columns II and III of Table 1 respectively. Using $\tau = 0.6$ leads to estimates of IES of 0.40 and 0.67 in columns II and III. The estimate for the consumption share of collateral constrained households changes to 0.56 and 0.67 respectively, while the estimates for θ and ω change only marginally. The higher value of intertemporal substitution in column III suggests that non-separability is somewhat less important once collateral constrained households are included in the model. However, it is still possible to reject the hypothesis that $s = 1$ in column III at the 95% level.

Judging the goodness-of-fit on the basis of correlation between actual and fitted series, the results suggest that the encompassing model in column III, which combines collateral constraints and non-separable utility, adds predictive power with respect to other models. The correlation between actual consumption growth and dynamic forecasts resulting from the three Euler equation estimations shows that correlation is clearly highest in the model with collateral constrained households included. Correlation ranges from -0.46 to 0.57 in columns I, II and III respectively. Also the root mean squared error (RMSE) of one-step ahead predictions is the lowest in the specification which includes collateral constrained households. This could be due to the fact that the model with collateral constraint households captures better the consumption growth pattern around the 1990s recession. Visual inspection of Figure (1) confirms this. Figure (1) compares the dynamic forecasts of the models resulting from columns II and III in Table 1 to actual consumption growth. Clearly, the encompassing model captures a dramatic drop and recovery of consumption growth during the aftermath of the housing market collapse in early 1990 much better than the model of non-separable preferences alone. This is precisely the reason for the better fit.

Interestingly, after the crisis, from 1995 onwards, the two models from column II and column III have some difficulties to capture the volatility of consumption, albeit the model with collateral constraint does somewhat better. One reason for this could be related directly to the King-Plosser-Rebelo (1988) utility function, which has typically been argued to be unsuccessful in generating business cycle movements consistent with the data of small open economies in particular. The difficulty arises from the fact that these

⁸Since ω is typically calibrated in the DSGE models with collateral constrained households, I have experimented by fixing the value of ω at a more reasonable level. Higher values of ω tend to result in negative values for the consumption share of collateral constraint households, and smaller IES.

⁹To see this, assume for instance that $f(H) = \frac{1}{\delta}H^\delta$. Then, $\theta = (\delta - 1)$. A small value of θ means that δ is very close to unity, and thus preferences are roughly linear in housing services.

preferences typically yield much-too-low standard deviations of consumption in general equilibrium models (and a counterfactual procyclical trade balance, as shown by Correia, Neven and Rebelo, 1995, and Schmitt-Grohe and Uribe, 2003). The fact that the encompassing model seems to do somewhat better shows that inclusion of collateral effects improves the fit in this dimension too.

Table 1. Estimation results for the whole sample

Parameter	I	II	III
λ			0.63*** (0.23)
s	0.29* (0.15)	0.35*** (0.07)	0.61** (0.21)
θ			0.003 (0.01)
ω			0.11 (0.13)
{	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$
z	$y_{t-1}^w..y_{t-4}^w$	$n_{t-2}..n_{t-4}$ $(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$	$n_{t-2}..n_{t-4}$ $(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$ $r_{t-2}^h..r_{t-4}^h$ $h_{t-2}..h_{t-4}$ $(d-y)_{t-1}..(d-y)_{t-4}$
}			
j-stat (p-value)	0.07 (0.32)	0.07 (0.43)	0.14 (0.40)
s.e.	0.010	0.009	0.009
RMSE	0.0108	0.009	0.0074
Corr	-0.46	0.19	0.57
HAC	Bartlett, NW	Bartlett,NW	Bartlett,NW
τ	0 (rest.)	0.5	0.5

This Table reports GMM estimates of the structural parameters ($\lambda, s, \theta, \omega$) in equations (3.1)–(3.3) together with the list of instruments (z). Dependent variable is Δc_t . Heteroskedasticity and autocorrelation corrected standard errors are reported in parentheses. The j-stat gives the minimised value of an objective function and p-value associated with the Hansen (1982) test for overidentifying restrictions is in parentheses. RMSE is the one-step-ahead root mean squared prediction error. Corr denotes the ordinary correlation coefficient between dynamic forecast and actual consumption growth. HAC reports options used for computing the weighting matrix of the objective function and the last row reports τ used in the estimation. The estimation period is 1987Q1–2008Q2.

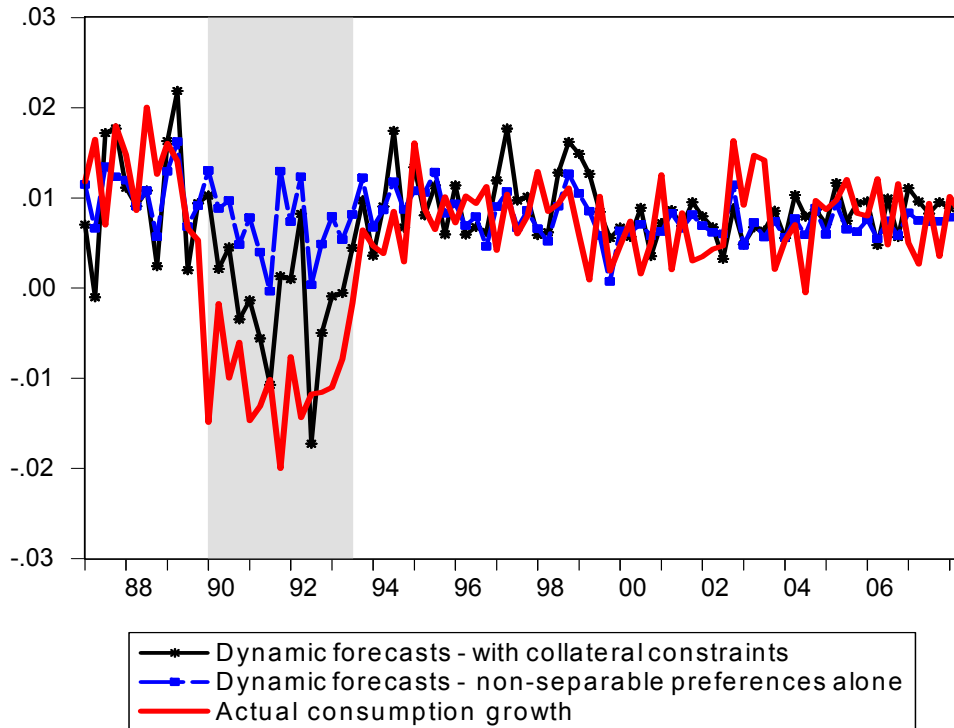


Figure 1: Dynamic forecasts and actual consumption growth – comparisons

3.3 Subsample 1995Q1–2008Q2

I next estimate the three consumption Euler equations for the sample 1995Q1–2008Q2. Results are provided in Table 2. First, there is some evidence that intertemporal elasticity of substitution increased in the later sample compared to the whole sample. Columns II and III suggest that the IES is 0.58 in the Basu and Kimball specification, and about 0.67 in the specification including collateral constrained households. IES is highly significant, except in the standard formulation (see column I). The share of collateral constrained households is now 0.47, slightly lower than for the whole period. This seems reasonable given that the Finnish economy from 1995 to early 2008 was not subject to any major turmoil and financial conditions remained rather stable. The parameters directly related to housing are not significant, although ω is now somewhat more reasonable. All in all, it seems that the intertemporal elasticity of substitution does not change widely across the estimation periods and specifications once non-separability is accounted for. Moreover, estimating the model with $\tau = (0.4, 0.6)$ does not markedly change the results. Higher values of τ imply somewhat higher values of IES.

The difference between the Basu and Kimball specification and the encompassing model is now rather small in terms of empirical fit. The correlations between actual and dynamic forecasts are 0.28 and 0.23 respectively. Moreover, the difference between RMSEs is negligible. Consequently, it seems that during a more tranquil period collateral effects are less important and the consumption equation derived from the model

with non-separability alone captures the fluctuations in consumption growth reasonably well.

Table 2. Estimation results for the subsample 1995Q1–2008Q2

Parameter	I	II	III
λ			0.47*** (0.23)
s	0.22 (0.15)	0.58*** (0.09)	0.67*** (0.18)
θ	–	–	0.01 (0.02)
ω	–	–	0.41 (0.39)
{	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$
z	$y_{t-1}^w..y_{t-4}^w$	$n_{t-2}..n_{t-4}$ $(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$	$n_{t-2}..n_{t-4}$ $(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$
}			$h_{t-2}..h_{t-4}$ $r_{t-2}^h..r_{t-4}^h$ $(d-y)_{t-1}..(d-y)_{t-4}$
j-stat (p-value)	0.13 (0.46)	0.15 (0.22)	0.21 (0.86)
s.e.	0.004	0.004	0.004
RMSE	0.0056	0.0043	0.0044
Corr	-0.30	0.28	0.23
HAC	Bartlett, NW	Bartlett,NW	Bartlett,NW
τ	0 (rest)	0.43	0.43

This Table reports GMM estimates of the structural parameters ($\lambda, s, \theta, \omega$) in equations (3.1)–(3.3) together with the list of instruments (z). The dependent variable is ΔC_t . Heteroskedasticity and autocorrelation corrected standard errors are reported in parentheses. The j-stat gives the minimised value of an objective function and p-value associated with Hansen (1982) test for overidentifying restrictions is in parentheses. RMSE is the one-step-ahead root mean squared prediction error. Corr denotes the ordinary correlation coefficient between dynamic forecast and actual consumption growth. HAC reports options used for computing the weighting matrix of the objective function and the last row reports τ used in the estimation. The estimation period is 1995Q2–2008Q2.

Tables 3–4 show the results from the estimation of the Basu and Kimball specification and the encompassing model using alternative instrument sets. Overall, the results from the Basu and Kimball specification are quite robust to different instruments sets. IES varies from the lowest value of 0.32 to the highest of 0.40 in the whole sample, and from 0.32 to 0.57 in the subsample 1995Q2–2008Q2. As for the encompassing model, the variability is greater.

IES substitution varies from 0.40 to 0.97 in the whole sample and from 0.81 to 1.67 in the subsample 1995Q2–2008Q2. The consumption share of collateral constrained households varies almost equally much, from 0.34 to 0.80 and from 0.41 to 0.92 in the whole sample and subsample, respectively. As for ω and θ , the standard errors are in general large and the magnitudes are typically comparable to the preferred specification in Table 2. While the results from encompassing model are quite sensitive to instrument sets, the general conclusions from Tables 1 and 2 remain valid.

Table 3. Alternative instruments in Basu-Kimball specification

Instrument set	S	
	87Q1-08Q1	95Q1-08Q2
$r_{t-2}..r_{t-4}$	0.40***	0.56**
$n_{t-2}..n_{t-4}$	(0.14)	(0.29)
$r_{t-2}..r_{t-4}$	0.33***	0.32*
$n_{t-2}..n_{t-4};$	(0.10)	(0.19)
$c_{t-2}..c_{t-4}$		
$c_{t-2}..c_{t-4}; r_{t-2}..r_{t-4}$	0.32***	0.57**
$n_{t-2}..n_{t-4}$	(0.08)	(0.23)
$y_{t-2}..y_{t-4}$		
$c_{t-2}..c_{t-4}; r_{t-2}..r_{t-4}$	0.36***	0.57**
$y_{t-2}..y_{t-4}; n_{t-2}..n_{t-4}$	(0.07)	(0.24)
$(c-y)_{t-2}..(c-y)_{t-4}$		

This Table reports GMM estimates of the IES (S) for the two sample periods based on (3.2) using different sets of instruments (z). Heteroskedasticity and autocorrelation corrected standard errors are reported in parentheses.

Table 4 Alternative instrument sets in encompassing model

Instrument set	s			λ			θ			ω		
	87Q1-08Q1	95Q1-08Q2	87Q1-08Q2	87Q1-08Q1	95Q1-08Q2	87Q1-08Q2	87Q1-08Q1	95Q1-08Q1	87Q1-08Q1	95Q1-08Q1	87Q1-08Q1	95Q1-08Q2
$\Gamma_{t-2} \cdot \Gamma_{t-4}; \mathbf{n}_{t-2} \cdot \mathbf{n}_{t-4}$	0.95** (0.47)	0.81*** (0.21)	0.34 (0.37)	0.41** (0.21)	0.08 (0.14)	-0.04 (0.02)	-0.38 (0.79)	0.35 (0.23)				
$\mathbf{h}_{t-2} \cdot \mathbf{h}_{t-4}; \Gamma_{t-2}^h \cdot \Gamma_{t-4}^h$												
$Y_{t-2} \cdot Y_{t-4}$												
$\Gamma_{t-2} \cdot \Gamma_{t-4}; \mathbf{n}_{t-2} \cdot \mathbf{n}_{t-4}$	0.97** (0.38)	1.64*** (0.39)	0.85*** (0.15)	0.41 (0.26)	-0.004 (0.006)	-0.04 (0.03)	0.14 (0.11)	0.58 (0.53)				
$\mathbf{h}_{t-2} \cdot \mathbf{h}_{t-4}; \Gamma_{t-2}^h \cdot \Gamma_{t-4}^h$												
$\mathbf{c}_{t-2} \cdot \mathbf{c}_{t-4}; Y_{t-2} \cdot Y_{t-4}$												
$\mathbf{dsy}_{t-1} \cdot \mathbf{dsy}_{t-4}$												
$\Gamma_{t-2} \cdot \Gamma_{t-4}; \mathbf{n}_{t-2} \cdot \mathbf{n}_{t-4}$	0.40* (0.21)	1.65*** (0.15)	0.78*** (0.25)	0.80*** (0.04)	0.002 (0.011)	0.001 (0.002)	-0.27 (0.24)	-0.07 (0.04)				
$\mathbf{h}_{t-2} \cdot \mathbf{h}_{t-4}; \Gamma_{t-2}^h \cdot \Gamma_{t-4}^h$												
$(d-y)_{t-1} \cdot (d-y)_{t-4}$												
$Y_{t-1}^w \cdot Y_{t-4}^w; \mathbf{c}_{t-2} \cdot \mathbf{c}_{t-4}$												
$\mathbf{c}_{t-2} \cdot \mathbf{c}_{t-4}; \Gamma_{t-2} \cdot \Gamma_{t-4}$												
$\mathbf{h}_{t-2} \cdot \mathbf{h}_{t-4}; Y_{t-2} \cdot Y_{t-4}$	0.97** (0.38)	1.47*** (0.36)	0.80*** (0.18)	0.92*** (0.10)	-0.00 (0.01)	0.03*** (0.008)	0.14 (0.12)	-0.54*** (0.13)				
$\Gamma_{t-2}^h \cdot \Gamma_{t-4}^h; \mathbf{n}_{t-2} \cdot \mathbf{n}_{t-4}$												
$(c-y)_{t-2} \cdot (c-y)_{t-4}$												

This Table reports GMM estimates of the structural parameters ($\lambda, s, \theta, \omega$) based on equation (3.3) using different sets of instruments (z). Heteroskedasticity and autocorrelation corrected standard errors are reported in parentheses.

3.4 Further robustness checks and comparisons

I now proceed to compare the results to the Campbell and Mankiw (1989) model with rule-of-thumb consumers. The Campbell and Mankiw (1989) specification is achieved by imposing the restrictions $\tau = 0$ and $\lambda = 0$, but augmenting the standard Euler equation with a measure of disposable income (Δy_t). Furthermore, I test whether, after accounting for non-separable consumption-leisure choice, consumption is still sensitive to changes in disposable income. Finally, I also report the estimates of Iacoviello's specification with separable preferences in column III of Table 5. Iacoviello's specification is obtained from (3.3) by imposing restrictions $\tau = 0$ and $s = 1$.¹⁰ This corresponds to separable logarithmic utility. The results are provided in Table 5. For convenience, I state the regression specifications here. As earlier, estimations are done using GMM.

$$\begin{aligned}
 \text{(I)} & : \quad \Delta c_t - \tau \Delta n_t = s(\hat{r}_t - \tau \Delta n_t) + \alpha \Delta y_t + \epsilon_t^1 \\
 \text{(II)} & : \quad \Delta c_t = s \hat{r}_t + \alpha \Delta y_t + \epsilon_t^2 \\
 \text{(III)} & : \quad \Delta c_t = \lambda [\theta \Delta h_t^c + \omega \Delta r_t^h] + (1 - \lambda) \hat{r}_t + \epsilon_t
 \end{aligned} \tag{3.4}$$

Column I reports the estimates of the Basu and Kimball regression with current disposable income included, while column II reports the estimates of the Campbell and Mankiw (1989) specification. Column III reports the results from the Iacoviello (2004) specification with the restrictions $s = 1$ and $\tau = 0$. The first observation as to columns I and II is that real disposable income is either insignificant or it enters with incorrect sign. Second, IES is small and insignificant in the Campbell-Mankiw (1989) specification (see Column II) and disposable income is marginally significant only in the whole sample. Finally, in Iacoviello's specification the share of collateral constraints λ as well as θ and ω are comparable with the results obtained in Tables 1 and 2. However, the predictive power of Iacoviello's (2004) specification, in comparison to the non-separable case, is weaker. These results thus provide further support that collateral effects are important, yet non-separability combined with collateral constraints delivers a better performance. This is especially true for the whole sample, which includes a period of financial distress in the early 1990s.

¹⁰Iacoviello (2004) arrived at the aggregate consumption Euler equation in a slightly different way, allowing him also to estimate intertemporal elasticity of substitution even with separable labour. The specification in this paper does not allow to identify separately the intertemporal substitution and consumption share of constrained households. Therefore, I imposed the additional restriction that $s = 1$. This corresponds to logarithmic utility with respect to consumption.

Table 5 Robustness checks – alternative specifications

Parameter	I		II		III	
	87Q1–08Q2	95Q1–08Q2	87Q1–08Q2	95Q1–08Q2	87Q1–08Q2	95Q1–08Q2
s	0.21** (0.09)	0.46*** (0.09)	0.08 (0.17)	-0.06 (0.18)	I (rest.)	I (rest.)
α	-0.05 (0.06)	-0.15*** (0.02)	0.23* (0.06)	-0.10 (0.06)	-	-
λ	-	-	-	-	0.87*** (0.11)	0.58*** (0.09)
θ	-	-	-	-	-0.004 (0.005)	-0.02*** (0.01)
ω	-	-	-	-	0.15** (0.07)	0.31 (0.08)
j-stat (p-value)	0.09 (0.42)	0.17 (0.11)	0.07 (0.75)	0.12 (0.11)	0.12 (0.16)	0.14 (0.25)
s.e.	0.008	0.005	0.009	0.005	0.01	0.004
RMSE	0.008	0.004	0.009	0.005	0.01	0.004
Corr	0.42	0.17	0.31	-0.24	-0.19	0.31
HAC	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW
τ	0.5	0.43	0 (rest.)	0 (rest.)	0 (rest.)	0 (rest.)

This Table reports some robustness checks using the GMM estimation method. Column I reports the estimates for the Basu-Kimball specification with disposable income, while column II shows the results from Cambell and Mankiw (1989) specification. Column III reports the estimates of Iacoviello's specification without imposing non-separable labour. Dependent variable is Δc_t . Instrument sets in column I and III are the same as in Table 1, column II and III respectively. In column II instrument set is $Z = \{c_{t-2}, c_{t-4}, r_{t-2}, r_{t-4}, Y_{t-2}, Y_{t-4}, (c-y)_{t-2}, (c-y)_{t-4}\}$.

4 Conclusions

In this paper, I have derived an aggregate Euler consumption equation in which non-separability between consumption and leisure, and collateral constrained households, makes current consumption dependent on employment as well as on the development of the asset (house) prices. This form of Euler consumption equation makes it possible to compare collateral constraints with non-separability of consumption and leisure as an explanation of excess sensitivity of consumption to current income.

Estimation results clearly indicate that both complementarity of consumption and labour introduced by non-separability, and collateral effects are important features of aggregate consumption behaviour. The model that combines the two is able to explain a major share of the variation in consumption growth during 1987Q1–2008Q2 period. This is quite remarkable given that the period includes a dramatic drop and recovery of consumption during and after the 1990s recession in Finland. Estimates of intertemporal elasticity of substitution and consumption share of collateral constrained households also seem reasonable and accurately determined. Furthermore, I find no support for rule-of-thumb consumption behaviour, once non-separability is accounted for.

In more general, the results suggest that during financial market distress, like in Finland in the early 1990s, binding quantity constraints can become an important feature of aggregate behaviour. An interesting and useful extension would be to allow only occasionally binding constraints and then fully account for general equilibrium effects by completing the modelling of housing and production sides. This would provide a useful framework to assess, among other things, the transmission of monetary policy under different financial conditions.

References

- Attanasio, O P (1995) **The Intertemporal Allocation of Consumption: Theory and Evidence.** Carnegie-Rochester Conference Series on Public Policy 42, 39–89.
- Browning, M (1991) **A Simple Non-additive Preference Structure of Household Behavior over Time.** Journal of Political Economy 99, 607–637.
- Hayashi, F (1987) **Tests for Liquidity Constraints: A Critical Survey.** In Truman Bewley (ed.), *Advances in Econometrics, Fifth World Congress*, Vol. 2 (Cambridge: Cambridge University Press).
- Hall, R E (1988) **Intertemporal substitution in consumption.** Journal of Political Economy 96, 339–357.
- Hubbard, R – Judd, K (1986) **Liquidity Constraints, Fiscal Policy, and Consumption.** Brookings Papers on Economic Activity 1, 1–59.
- Basu, S – Kimball, M (2002) **Long-run labor supply and the elasticity of the intertemporal substitution for consumption.** Working Paper, University of Michigan.
- Campbell, J Y – Mankiw, N G (1989) **Consumption, income, and interest rates: reinterpreting the time series evidence.** NBER Macroeconomics Annual. MIT Press, Cambridge, MA.
- Correia, I – Neves, J C – Rebelo S T (1995) **Business Cycles in a Small Open Economy.** European Economic Review 39, 1089–1113.
- Flavin, M (1985) **Excess Sensitivity of Consumption to Current Income: Liquidity Constraints or Myopia?** Canadian Journal of Economics 18, 117–136.
- King, R – Plosser, C – Rebelo, S (1988) **Production, Growth and Business Cycles I: The Basic Neoclassical Model.** Journal of Monetary Economics 21, 195–232.
- Kiyotaki, N – Moore, J (1997) **Credit Cycles.** Journal of Political Economy 105, 211–248.
- Ham, J – Reilly, K (2002) **Testing Intertemporal Substitution, Implicit contracts, and Hours Restriction Models of the Labour Market Using Micro Data.** American Economic Review, Vol. 92, No. 4.
- Honkapohja, S – Koskela, E (1999) **The economic crisis of the 1990s in Finland.** Economic Policy 14(29), 399–436.
- Honkapohja, S (2009) **The 1990’s financial crises in Nordic countries.** Bank of Finland Discussion Paper 5/2009.

Iacoviello, M (2004) **Consumption, house prices, and collateral constraints: a structural econometric analysis.** Journal of Housing Economics 13(4), 304–320.

Iacoviello, M (2005) **House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle.** American Economic Review 95(3), 739–764.

Jappelli, T – Pagano, M (1989) **Consumption and Capital Market Imperfections: An International Comparison.** American Economic Review, 79, 1088–1105.

Kiley, M T (2007) **Habit persistence, Non-separability between consumption and leisure, or rule-of-thumb consumers: which accounts for the predictability of consumption growth.** Finance and Economics Discussion Series 2007-48, Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board, Washington.

Hansen, L P (1982) **Large Sample Properties of Generalized Method of Moments Estimators.** Econometrica 50(4), 1029–1054.

Schmitt-Grohe, S – Uribe, M (2003) **Small Open Economy Models.** Journal of International Economics 61, 163–185.

OECD (2000) **Taxing Wages 1999–2000.** Paris: OECD.

Mendoza, E G – Razin, A – Tesar, L L (1994) **Effective tax rates in macroeconomics: cross-country estimates of tax rates on factor incomes and consumption.** Journal of Monetary Economics, Vol. 34 (3), 297–323.

A Mathematical appendix – detailed derivation

A.1 Un-constrained household

The dynamic optimisation of un-constrained household is

$$\max_{\{C_t, H_t, N_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^u}{1-\gamma} e^{(\gamma-1)v(N_t)} + \kappa f(H_t^u) \right)$$

s.t.

$$C_t^u + Q_t(H_t^u - H_{t-1}^u) + R_{t-1}B_{t-1}^u = B_t^u + W_t^u N_t^u + Y_t^u$$

where C_t^u is consumption of un-constrained households, Q_t is relative price of real estate, H_t^u is housing, R_t is the gross real interest rate, B_t is real bonds ($-B_t$ is lending), W_t is the real wage rate, N_t is employment and Y_t is random endowment. I assume that $f'(H_t) > 0$, $f''(H_t) < 0$. $v(N_t)$ denotes disutility from labour with the usual properties, $v'(N_t) > 0$, $v''(N_t) > 0$ and $\gamma > 1$ is the risk aversion parameter. The first order conditions become

$$\begin{aligned} u_C(C_t^u, N_t^u, H_t^u) &= C_t^{u-\gamma} e^{(\gamma-1)v(N_t^u)} \\ u_H(C_t^u, N_t^u, H_t^u) &= Q_t C_t^{u-\gamma} e^{(\gamma-1)v(N_t^u)} \\ u_N(C_t^u, N_t^u, H_t^u) &= C_t^{u(1-\gamma)} e^{(\gamma-1)v(N_t^u)} v'(N_t^u) \end{aligned}$$

where $u_j(\cdot), j = C, H, N$, denotes marginal utility with respect to $j = c, H, N$. The intratemporal condition is

$$W_t = -\frac{u_N(C_t^u, N_t^u, H_t^u)}{u_C(C_t^u, N_t^u, H_t^u)} = C_t^u v'(N_t^u) \quad (\text{A.1})$$

Optimal choice of consumption implies the Euler consumption equation

$$\begin{aligned} u_C(C_t^u, N_t^u, H_t^u) &= \beta R_t E_t u_C(C_{t+1}^u, N_{t+1}^u, H_{t+1}^u) \\ C_t^{u-\gamma} e^{(\gamma-1)v(N_t)} &= \beta R_t E_t [C_{t+1}^{u-\gamma} e^{(\gamma-1)v(N_{t+1})}] \end{aligned} \quad (\text{A.2})$$

Linearizing (A.2) yields

$$-\gamma \hat{c}_t^u + (\gamma - 1) \tau \hat{n}_t^u = -\gamma \hat{c}_{t+1}^u + \hat{r}_t + (\gamma - 1) \tau \hat{n}_{t+1}^u \quad (\text{A.3})$$

\hat{c}_t denotes the percentage deviation of consumption from the steady state. In linearizing (A.2), I have used first order linear approximation of $v(N_t)$ around the constant level of optimal choice of labour N^* and the facts that

$$\tau \equiv v'(N)N = \left(\frac{WN}{C} \right)$$

and

$$\Delta v(N_t) \approx \tau (n_t - n_{t-1}) = \tau \Delta n_t. \quad (\text{A.4})$$

Substituting (A.4) into (A.3) yields finally

$$\hat{c}_t^u = E_t \hat{c}_{t+1}^u - s \hat{r}_t - (1 - s) \tau E_t \Delta n_{t+1}^u \quad (\text{A.5})$$

where $s \equiv 1/\gamma$ is the intertemporal elasticity of substitution.

A.2 Constrained households

Constrained households solve the following problem:

$$\begin{aligned} & \max \left(\frac{(C_t^c)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N^c)} + \kappa f(H_t) \right) \\ & \text{s.t.} \\ & C_t^c + Q_t(H_t^c - H_{t-1}^c) + R_{t-1}B_{t-1}^c = B_t^c + W_t^c N_t^c + Y_t^c \\ & B_t^c \leq mE_t(Q_{t+1})H_t^c/R_t \end{aligned}$$

ϕ_t is the time t shadow value of the borrowing constraint. Assume that household's collateral constraint hold with equality. Forming a Lagrangian and substituting the budget constraint into the maximization problem yields

$$\begin{aligned} \mathcal{L}_t^c = & \frac{(B_t^c + W_t^c N_t^c + Y_t^c - Q_t(H_t^c - H_{t-1}^c) - R_{t-1}B_{t-1}^c)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N)} \\ & + k^c f(H_t^c) + \phi_t [mE_t(Q_{t+1})H_t^c - R_t B_t^c]. \end{aligned}$$

The first order conditions for B_t^c and H_t^c and N_t^c yields:

$$(C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = R_t \phi_t \quad (\text{A.6})$$

$$-Q_t (C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = \kappa f'(H_t^c) + mE_t(\phi_t Q_{t+1}) \quad (\text{A.7})$$

$$W_t^c = C_t^c v'(N^c) \quad (\text{A.8})$$

Linearizing (A.6) yields

$$\begin{aligned} -\gamma \hat{c}_t^c + (\gamma-1)\tau \hat{n}_t^c &= \hat{\phi}_t + \hat{r}_t. \\ \hat{c}_t^c &= \tau^c(1-s)\hat{n}_t^c - s\hat{\phi}_t - s\hat{r}_t, \quad s = \frac{1}{\gamma} \end{aligned} \quad (\text{A.9})$$

$v(N_t^c)$ is linearized around the optimal (trend) level of labour supply as in the case of un-constrained households. $\hat{\phi}_t$ is the Lagrange multiplier in percentage deviation from the steady state.

A.3 Linearizing asset demand equation for constrained households

Start with the equation (A.7)

$$Q_t (C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = k^c f'(H_t^c) + mE_t(\phi_t Q_{t+1}) \quad (\text{A.10})$$

Then, note that the steady state version of (A.6) gives

$$(C^c)^{-\gamma} e^{(\gamma-1)v(N^c)} = \bar{R}\phi \quad (\text{A.11})$$

where the steady state interest rate \bar{R} can be found from the steady state version of the consumption Euler equation for un-constrained agents (A.2)

$$\bar{R} = \frac{1}{\beta} \quad (\text{A.12})$$

Combining (A.12) and (A.11) yields

$$\beta (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} = \phi \quad (\text{A.13})$$

Linearizing $v(N_t^c)$ around optimal steady state level of labour supply N^{*c} , we find that LHS of (A.7) is

$$Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} (1 + \hat{q}_t) (1 - \gamma \hat{c}_t^c) (1 + (\gamma - 1) \tau^c \hat{n}_t^c)$$

where $\tau^c \equiv \frac{N^{*c} W}{C^c}$. Linearizing RHS of (A.7) yields

$$k^c f'(H^c) (1 - \theta \hat{h}_t^c) + mE(\phi Q) (1 + \hat{\phi}_t) (1 + \hat{q}_{t+1})$$

where $\theta \equiv -\frac{f''(H^c)H^c}{f'(H^c)}$. Combining linearized versions of LHS and RHS yields

$$\begin{aligned} & Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} (1 + \hat{q}_t) (1 - \gamma \hat{c}_t^c) (1 + (\gamma - 1) \tau^c \hat{n}_t^c) \\ &= k^c f'(H^c) (1 - \theta \hat{h}_t^c) + mE(\Phi Q) (1 + \hat{\phi}_t) (1 + \hat{q}_{t+1}) \\ & Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} [\hat{q}_t - \gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c] \\ &= -k^c f'(H^c) \theta \hat{h}_t^c + mE(\Phi Q) [\hat{\phi}_t + \hat{q}_{t+1}] \end{aligned} \quad (\text{A.14})$$

Next note that the steady state version of (A.10) implies that

$$\begin{aligned} (C_t^c)^{-\gamma} e^{(\gamma-1)v(N^c)} &= k^c f'(H^c) + m\Phi \\ (C_t^c)^{-\gamma} e^{(\gamma-1)v(N^c)} &= k^c f'(H^c) + m\beta [(C^c)^\gamma e^{(\gamma-1)v(N^c)}] \\ 1 - m\beta &= \frac{k^c f'(H^c)}{(C^c)^\gamma e^{(\gamma-1)v(N^c)}} \end{aligned}$$

Substituting this into (A.14), and using (A.13) yields

$$\begin{aligned} & Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^c)} [\hat{q}_t - \gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c] \\ &= -k^c f'(H^c) \theta \hat{h}_t^c + mE(\beta (C^c)^\gamma e^{(\gamma-1)v(N^c)} Q) [\hat{\phi}_t + \hat{q}_{t+1}] \\ & [\hat{q}_t - \gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c] \\ &= -\frac{k^c f'(H^c)}{Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})}} \theta \hat{h}_t^c + m\beta E_t [\hat{\phi}_t + \hat{q}_{t+1}] \\ & \hat{q}_t - \gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c \\ &= -(1 - m\beta) \theta \hat{h}_t^c + m\beta E_t [\hat{\phi}_t + \hat{q}_{t+1}], \quad Q = 1 \\ & \hat{q}_t - \gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c \\ &= -\theta (1 - m\beta) \hat{h}_t^c + m\beta \hat{\phi}_t + m\beta E_t \hat{q}_{t+1} \end{aligned} \quad (\text{A.15})$$

where I have normalised $Q = 1$. Furthermore, recall from the consumption Euler equation for constrained agents that

$$\begin{aligned} (C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} &= \phi_t R_t \\ &\approx \\ -\gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c &= \hat{\phi}_t + r_t \\ &\Rightarrow \\ \hat{\phi}_t &= -\gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c - r_t \end{aligned}$$

Using this to remove $\hat{\phi}_t$ from (A.15) yields

$$\begin{aligned}\hat{q}_t - \gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c &= - (1 - m\beta) \theta \hat{h}_t^c + m\beta [-\gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c - r_t] + m\beta E_t \hat{q}_{t+1} \\ \hat{q}_t - \gamma \hat{c}_t^c + m\beta \gamma \hat{c}_t^c + (\gamma - 1) \tau^c \hat{n}_t^c - m\beta [(\gamma - 1) \tau^c \hat{n}_t^c] &= - (1 - m\beta) \theta \hat{h}_t^c - m\beta r_t + m\beta E_t \hat{q}_{t+1}\end{aligned}$$

Multiplying both sides by $\frac{1}{\gamma}$ and using $s \equiv \frac{1}{\gamma}$ yields

$$s \hat{q}_t - \hat{c}_t^c + m\beta \hat{c}_t^c + (1 - s) \tau^c \hat{n}_t^c - m\beta (1 - s) \tau^c \hat{n}_t^c = -s (1 - m\beta) \theta \hat{h}_t^c - s m\beta r_t + s m\beta E_t \hat{q}_{t+1}$$

Solve for \hat{c}_t^c

$$\hat{c}_t^c = s \theta \hat{h}_t^c + s \frac{m\beta}{1 - m\beta} r_t - s \frac{m\beta}{1 - m\beta} E_t \hat{q}_{t+1} + \frac{s}{(1 - m\beta)} \hat{q}_t + (1 - s) \tau^c \hat{n}_t^c$$

Denoting $1 + \omega = \frac{1}{1 - m\beta}$ so that $\omega = \frac{m\beta}{1 - m\beta}$ we finally arrive at the expression

$$\hat{c}_t^c = s \theta \hat{h}_t^c + s \omega (\hat{r}_t - E_t \hat{q}_{t+1}) + s (1 + \omega) \hat{q}_t + (1 - s) \tau^c \hat{n}_t^c \quad (\text{A.16})$$

Recalling furthermore that $\Delta \hat{q}_{t+1}$ is a change in the relative, or real price of housing, and that \hat{r}_t is the real ex-ante interest rate expressed in consumption price inflation, we obtain

$$\begin{aligned}\hat{r}_t - E_t (\hat{q}_{t+1} - \hat{q}_t) &= i_t - (p_{t+1}^c - p_t^c) - [(q_{t+1}^c - q_t^c) - (p_{t+1}^c - p_t^c)] \\ &= i_t - [(q_{t+1}^c - q_t^c)] \\ &\equiv r_t^h\end{aligned}$$

where r_t^h denotes the real ex ante housing interest rate. Consequently, we can express (A.16) as

$$\begin{aligned}\hat{c}_t^c &= s \theta \hat{h}_t^c + s \omega r_t^h + s \hat{q}_t + (1 - s) \tau^c \hat{n}_t^c \\ \hat{c}_t^c - \tau^c \hat{n}_t^c &= s [\theta \hat{h}_t^c + \omega r_t^h + \hat{q}_t - \tau^c \hat{n}_t^c]\end{aligned}$$

A.4 Derivation of aggregate Euler equation

In this section, I derive the aggregate Euler equation. Recall for convenience the following Euler equations for unconstrained and constrained agents

$$\begin{aligned}\hat{c}_t^u &= E_t \hat{c}_{t+1}^u - s \hat{r}_t - (1 - s) \tau E_t \Delta n_{t+1}^u \\ \hat{c}_t^c &= s \theta \hat{h}_t^c + s \omega (\hat{r}_t - E_t \hat{q}_{t+1}) + s \omega \hat{q}_t + (1 - s) \tau^c \hat{n}_t^c\end{aligned}$$

Make rational expectations (RE) assumption explicit *ie* that

$$\begin{aligned}E_t \hat{c}_{t+1}^u &= c_{t+1}^u + \epsilon_t^{cu} \\ E_t \Delta n_{t+1}^u &= \Delta n_{t+1}^u + \epsilon_t^{\Delta n} \\ E_t \hat{q}_{t+1} &= \hat{q}_{t+1} + \epsilon_t^q \\ E_t \hat{n}_{t+1} &= \hat{n}_{t+1} + \epsilon_t^n\end{aligned}$$

ϵ_t^i is the forecast error term. Substituting RE assumptions into the corresponding Euler equations yields

$$\Delta c_{t+1}^u = s\hat{r}_t + \tau^u(1-s)\Delta n_{t+1}^u + \epsilon_t + \alpha_u \epsilon_{t-1}$$

$$\hat{c}_t^c = s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c \epsilon_t$$

Note that aggregate consumption can be expressed in log first differenced form

$$\begin{aligned}\hat{c}_t &= \lambda\hat{c}_t^c + (1-\lambda)\hat{c}_t^u \\ \Rightarrow \\ \Delta c_t &= \lambda\Delta c_t^c + (1-\lambda)\Delta c_t^u\end{aligned}\tag{A.17}$$

Substituting Δc_{t+1}^u and \hat{c}_t^c in (A.17) yields

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c \epsilon_t - c_{t-1}^c] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_t + \alpha_u \epsilon_{t-1}]\end{aligned}$$

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c \epsilon_t - c_{t-1}^c] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_{t-1} + \alpha_u \epsilon_{t-2}]\end{aligned}$$

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c \epsilon_t - c_{t-1}^c] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_{t-1} + \alpha_u \epsilon_{t-2}]\end{aligned}$$

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\Delta\hat{h}_t^c + s\omega\Delta\hat{r}_t - s\omega\Delta\Delta q_{t+1} + (1-s)\tau^c\Delta\hat{n}_t^c + \Delta\epsilon_t + \alpha_c\Delta\epsilon_t] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_{t-1} + \alpha_u \epsilon_{t-2}]\end{aligned}$$

$$\Delta c_t = \lambda[s\theta\Delta\hat{h}_t^c + s\omega\Delta\hat{r}_t - s\omega\Delta\Delta q_{t+1}] + (1-s)[\lambda\tau^c\Delta n_t^c + \tau^u(1-\lambda)\Delta n_t^u] + (1-\lambda)s\hat{r}_t + \epsilon_t'\tag{A.18}$$

The problem with this expression is that there are several unobserved variables. In particular, we do not have observations on consumption and employment for unconstrained and constrained agents separately. However, there is a way to simplify the above equation. First, we make use of an aggregate constraint for labour

$$N_t = N_t^u + N_t^c$$

Linearizing this yields

$$\hat{n}_t = \lambda_n \hat{n}_t^c + (1-\lambda_n) \hat{n}_t^u$$

where λ_n is the average employment share of constrained households. Recall from the intratemporal condition for labour that for both households

$$W_t = C_t^u v'(N_t^u)\tag{A.19}$$

$$W_t = C_t^c v'(N_t^c)\tag{A.20}$$

Equalising (A.19) and (A.20), and evaluating these in the steady state delivers an expression for relative consumption shares of the households which depends on the relative marginal disutility of labour

$$\begin{aligned} C_t^c v'(N_t^c) &= C_t^u v'(N_t^u) \\ \frac{C^c}{C^u} &= \frac{v'(N^u)}{v'(N^c)} \end{aligned} \quad (\text{A.21})$$

We also know that $N^i v'(N^i) = \left(\frac{W N^{ci}}{C^{ci}}\right) \equiv \tau^i$. Using these to remove $v'(\cdot)$ from (A.21) yields

$$\frac{N^c}{N^u} = \frac{C^c \tau^c}{C^u \tau^u} \quad (\text{A.22})$$

Recall that λ is the consumption share of constrained agents and that $C = C^u + C^c$. Thus

$$\frac{\lambda_n}{1 - \lambda_n} = \frac{N^c}{N^u} = \frac{\tau^c C^c / C}{\tau^u C^u / C} = \frac{\tau^c \lambda}{\tau^u (1 - \lambda)} \quad (\text{A.23})$$

Consider the term

$$[\lambda \tau^c \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u]$$

in (A.18). Recalling from (A.23) that

$$\tau^c \lambda = \frac{\lambda_n}{1 - \lambda_n} \tau^u (1 - \lambda)$$

so that

$$\begin{aligned} [\lambda \tau^c \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u] &= \left[\frac{\lambda_n}{1 - \lambda_n} \tau^u (1 - \lambda) \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u \right] \\ &= (1 - \lambda) \tau^u \left[\frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c + \Delta n_t^u \right] \end{aligned}$$

Then, using the fact that

$$\Delta n_t^u = \frac{1}{1 - \lambda_n} \Delta n_t - \frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c$$

so that

$$\begin{aligned} [\lambda \tau^c \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u] &= (1 - \lambda) \tau^u \left[\frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c + \Delta n_t^u \right] \\ &= (1 - \lambda) \tau^u \left[\frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c + \left[\frac{1}{1 - \lambda_n} \Delta n_t - \frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c \right] \right] \\ &= \frac{(1 - \lambda) \tau^u}{1 - \lambda_n} \Delta n_t \end{aligned}$$

Finally recall that by definition $\frac{(1 - \lambda) \tau^u}{1 - \lambda_n} = \frac{\tau^c \lambda}{\lambda_n} = \tau$, and so we obtain

$$[\lambda \tau^c \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u] = \frac{(1 - \lambda) \tau^u}{1 - \lambda_n} \Delta n_t = \tau \Delta n_t \quad (\text{A.24})$$

Consequently, substituting (A.24) into (A.18) we arrive into following expression for linearized aggregate consumption Euler equation

$$\Delta c_t = \lambda[s\theta\Delta h_t^c + s\omega\Delta r_t - s\omega\Delta\Delta q_{t+1}] + (1-s)\tau\Delta n_t + (1-\lambda)sr_t + \epsilon_t \quad (\text{A.25})$$

Alternatively, replacing $\Delta r_t - \Delta\Delta q_{t+1} = r_t^h - r_{t-1}^h$, we finally arrive at

$$\Delta c_t - \tau\Delta n_t = \lambda s[\theta\Delta h_t^c + \omega\Delta r_t^h - \hat{r}_t] + s(\hat{r}_t - \tau\Delta n_t) + \epsilon_t \quad (\text{A.26})$$

where r_t^h denotes the housing real interest rate. This is equation (2.14) in the main text.

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