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Estimating Intertemporal Elasticity of Substitution in a Sticky Price Model

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Abstract

Cancellation of income and substitution effect implied by King-Plosser-Rebelo (1988) preferences breaks tight coefficient restriction between the slope of the Phillips curve and the elasticity of consumption with respect to real interest rate in a sticky price macro model. This facilitates the estimation of intertemporal elasticity of substitution using full information Bayesian Maximum Likelihood techniques within a structural model. The US data from the period 1984–2007 supports low intertemporal elasticity of substitution and strongly rejects a logarithmic and an additively separable utility specification commonly applied in the New Keynesian literature.

Keywords: Monetary policy, Bayesian estimation, Non-separable utility.

JEL: E32, E52,E21

1 Introduction

There are two important conditions that the preferences must fulfill in order for the balanced growth path to exist in the neoclassical growth model. First, the intertemporal elasticity of substitution must be invariant to the scale of consumption and the income and substitution effects associated to sustained growth in the labour productivity must not change the labour supply (cf. King-Plosser-Rebelo, 1988). The latter condition states that, in the long-run, the income and substitution effects must cancel the each other[1].

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[1] Consensus from a large number of empirical work on labour supply elasticities also suggests that the income elasticity cannot be much larger than the substitution elasticity. Hence, the preferred estimate of the uncompensated wage elasticity is *weakly* positive.

In order to fulfill these restrictions one possibility is to assume additively separable preferences, where, in addition, consumption enters logaritmically so that the cross-elasticity between consumption and hours worked is zero and the intertemporal elasticity of substitution is one. In this case, the household utility function typically takes a form $\ln(C_t) - v(N_t)$ where C_t is consumption and $v(N_t)$ is some strictly increasing function of the quantity of labour N_t, representing disutility from work. However, both of these assumptions can be challenged on empirical grounds.

First, under this specification of preferences the elasticity of consumption growth with respect to the real interest rate should be one. However, estimates based on the consumption Euler equation yields consistently much lower values (see e.g. Hall, 1988, Barsky et al., 1995, Cambell and Mankiw, 1989; Basu and Kimball, 2002; Fuhrer and Rudebusch, 2004, Yogo, 2002). second, the zero cross-elasticity between consumption and hours is generally rejected by the empirical research based on micro-data based literature. In particular, the level of consumption tends to fall after a retirement or after a person becomes unemployed[2] The latter evidence is consistent with the complementarity between consumption and work: households like to consume more when they work more.

Furthermore, the additively separable logarithmic in consumption preferences[3] also imply that the Frisch elasticity of labour supply and the consumption-constant elasticity of labour supply coincide. The Frisch elasticity of labour supply[4] primarily tells how a transitory change in the real wage impacts on the labour supply while the consumption-constant elasticity gives the impact of a permanent change in the real wage on the labour supply. When these two elasticities coincide, the same model can not say why a permanent increase in the marginal tax rate leads to a substantial decline in hours, but a transitory movement in the real wage does not cause as big a change in hours worked at the business cycle frequency. Much of the large literature on Frisch elasticity in the general equilibrium macroeconomic models hovers around this tension.

In this paper, we apply the class of preferences similar to King-Plosser-Rebelo (1988, henceforth KPR) and estimate the intertemporal elasticity of substitution within the structural New Keynesian model. KPR preferences allow for a non-zero cross-elasticity between consumption and labour and this elasticity is tightly linked to intertemporal elasticity of substitution. We show analytically that with KPR preferences the relationship between inflation and output gap i.e. the slope of the Phillips curve depends only weakly on the intertemporal elasticity of substitution in contrast to more usual separable preference specification. This means that estimating the IES together with the New Keynesian Phillips curve, does not constrain the estimate of IEs, which is still

[2]See e.g. Banks et al., (1998), Hammermesh (1984), Bernheim, (2001), Browning and Crossley (2001), Ameriks et al. (2007), Hurst (2008).

[3]In what follows we shall refer to these preferences with somewhat less precise terminology of 'additively separable log(arithmic) preference (utility)'.

[4]As argued in Kimball and Shapiro (2008) the Frisch elasticity governs, in a frictionless world, the intertemporal substitution in labour supply and is tightly linked to the effects of real interest rate on labour supply.

primarily identified from the relationship between output and real interest rate. At the same time, we can make use of Full Information Bayesian Maximun Likelihood methods, instead of relying on GMM estimation as is wilely done in the consumption Euler equation literature. As shown e.g. in Yogo (2002), Kiley (2010) and Kilponen (2012), weak instrument problem makes it difficult to identify IES using GMM techniques from macro data.

Using Bayesian Maximum Likelihood Methods and the U.s. data from the period 1984Q1-2007Q4, we find that the real interest rate elasticity of output is $0.22\ (0.09, 0.36)$. This value is in stark contrast to unitary real interest elasticity of output implied by the logarithmic and additively separable utility, but better in line with Euler consumption equation based estimates. At the same time, such low IES implies a strong complementarity between consumption and labour and seems to be difficult to reconcile with micro evidence of labour supply. The respective curvature of the utility with respect to consumption is far higher than in Smets and Wouters (2007), who estimate a fully fledged DsGE model with KPR preferences We show that differing results are driven by the choice of priors, the data prefering a high curvature and strong complementarity.

The remaining paper is organized as follows. Section 2 describes the model. Section 3 discusses the properties of the model with KPR preferences. section 4 provide the estimation results including robustness analysis and section 5 concludes.

2 The model

This section develops a stylized sticky price monetary policy model featuring King-Plosser-Rebelo preferences. We follow closely a text book type derivation of the sticky price monetary policy model (see e.g. Goodfriend and King, 1997; Walsh, 2010; Woodford, 2003; Gali, 2008).

2.1 Households

The economy is populated by identical infinitely-lived households who solve the following problem

$$
\begin{aligned}
& \max_{C_t(i),N_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\
& s.t. \\
\int_0^1 P_t(i) C_t(i) di + Q_t B_t &= B_{t-1} + W_t N_t + T_t \\
C_t &\equiv \left(\int C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \\
\lim_{T\to\infty} \mathbb{E}_t(B_T) &\geq 0.
\end{aligned}
$$

where $C_t(i)$ is the quantity of good i consumed by the representative household in period t; $P_t(i)$ is the price of good i; N_t is quantity of labour; W_t is nominal wage, B_t represents purchases of one period bonds of which price is Q_t; T_t is lump sum component of income and finally ϵ is the elasticity of substitution between the differentiated goods.[5] Following King-Plosser-Rebelo (1988), Kimball (1995), Basu and Kimball (2002), we assume that the additively time-separable felicity function $U(C_t, N_t)$ takes a form

$$U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t)}, \tag{1}$$

where $\gamma \neq 1$ controls the concavity of the utility function.[6] We shall in the following focus on the case where $\gamma > 1$. In this formulation s$\equiv 1/\gamma$ denotes the labour-held-constant intertemporal elasticity of consumption. It is important to notice that γ is (up to scaling) equal to the usual risk aversion measure only in the special case of exogenously fixed labour, as shown by Swanson (2012, corollary 1, p. 1671). That is, the usual measure of risk aversion ignores the household's ability to offset income shocks by adjustment of labour. As discussed further by Swansson (2012), high values of γ (or low values of s) are not ruled out by empirical micro estimates of risk aversion when labour margin is taken into account. $v(N_t)$ is some strictly increasing function of quantity of labour, representing the disutility from work.[7] Note that in the limiting case where $s \equiv \gamma^{-1} = 1$, the function $U(C_t, N_t) - \frac{1}{1-\gamma}$ converges, by l'Hopital's rule, to $\ln(C_t) - v(N_t)$.

In the first step, the household makes a decision on consumption and labour supply. The optimal choice of consumption and labour supply yields the following consumption Euler equation and the labour supply equation

$$Q_t = \beta \mathbb{E}_t \left\{ \frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)} \frac{P_t}{P_{t+1}} \right\} \tag{2}$$

$$\frac{W_t}{P_t} = -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} \tag{3}$$

where $U_C(C_{t+j}, N_{t+j}) = C_{t+j}^{-\gamma} e^{(\gamma-1)v(N_{t+j})}$; $U_N(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t)}(\gamma - 1)v'(N_t)$. $\mathbb{E}_t$ is the usual conditional expectation operator. As usual, the optimal labour supply condition states that the intratemporal marginal rate of substitution between labour and consumption is equal to the real wage. The representative household must also decide on the allocation of her consumption expenditure among the differentiated goods. This gives rise to the familiar

[5] ϵ also denotes the absolute value of the own price elasticity of the demand for a good.

[6] Jaimovich and Rebelo (2009) further extend this class of preferences by considering also a time-non-separability. The original KPR preferences arise as a special case of their preferences. Note that Greenwood, Hercowitz and Huffman (1998) (GHH) preferences are not consistent with the balanced growth path due to lack of income effect. Hence, a permanent change in productivity would lead into a permanent change of the labour supply.

[7] Smets and Wouters (2007) use the similar utility function with the additional assumption of a particular functional form for $v(N_t)$, namely $v(N_t) = (\sigma - 1)/(1 + \xi_l)N_t^{(1+\xi_l)}$, where ξ_l is the labour supply elasticity.

demand equations:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t. \tag{4}$$

where $P_t \equiv \left(\int P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the aggregate price index.

Focusing on the first-order terms in the Taylor expansion and assuming homoscedasticity of the stochastic processes for $c_t \equiv \ln(C_t)$, $p_t = \ln(P_t)$ and $n_t = \ln N_t$, the optimal consumption and labour supply dynamics can be reparameterized as

$$\begin{aligned} c_t &= \mathbb{E}_t c_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - \rho) - (1-s)\tau \mathbb{E}_t \Delta n_{t+1} && (5) \\ w_t - p_t &= c_t + \varphi n_t + \iota, && (6) \end{aligned}$$

where $\pi_{t+1} \equiv \ln P_{t+1} - \ln P_t$, $i_t \equiv -q_t$, s$\equiv 1/\gamma$, $\rho \equiv -\ln\beta$, $\varphi \equiv \frac{v''(N)/N}{v'(N)}$ and $\iota \equiv \ln\tau - (1+\varphi)n$ and $\tau = WN/PC = v'(N)N$.

Equation (6) shows that KPR preferences imply that there is a unitary elasticity between the real wage and consumption. The unitary elasticity is important, since by the definition of the balanced growth path, the real wage and the consumption must grow at the same rate in the long-run. At the same time, the elasticity between the real wage and labour depends on the term $\varphi \equiv \frac{v''(N)}{v'(N)}N$. This can be interpreted as an inverse of the consumption-constant elasticity of labour supply, not the Frisch elasticity of labour supply.[8] However, it is possible to relate this term to Frisch labour supply elasticity ξ i.e. to the labour supply elasticity which keeps the marginal utility of consumption constant (see appendix A for detailed derivation). specifically, we show in the Appendix that

$$\xi = \frac{1}{\varphi + \tau(1-s)}. \tag{7}$$

From (7) it is easy to see that the Frisch elasticity of labour supply ξ is in general lower than the consumption-constant elasticity of labour supply φ^{-1}. The difference between these two elasticities depends directly on the cross-elasticity of consumption and labour (and thus on the long-run-labour share, τ, and the IES, s). It can be shown that $(1-s)\tau = -\frac{U_{CN}}{U_{CC}}\frac{N}{C} = -\frac{dC}{dN}\frac{N}{C}$ is the cross-elasticity of consumption and labour. Hence, $(1-s)\tau$ parameterizes elasticity of consumption w.r.t. labour supply. Equation (7) also reveals that the intertemporal aspects of consumption and the labour supply elasticity are tightly linked within this class of preferences.[9]

[8] Kimball (1995) argues that inverse of the consumption-constant labour supply elasticity φ can be calibrated on the basis of marginal expenditure share of leisure being equal to the ratio of marginal expenditure share of consumption to leisure times the wage income consumption share. His preferred value for φ^{-1} is one.

[9] As discussed by Kimball and Shapiro (2008), the consumption-constant labour supply elasticity is most useful for understanding how a permanent change in the real wage impacts on labour supply. The Frisch elasticity gives the impact of a temporary change in the real wage on labour supply. This means that the Frisch elasticity is a more useful concept at the business cycle frequency. In accordance with this interpretation, it is also natural to find

Finally, notice that letting $s \to 1$, the optimal consumption and labour supply equations given in equations (5)-(6) collapse to

$$\begin{aligned} c_t &= \mathbb{E}_t c_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1} - \rho), & (8) \\ w_t - p_t &= c_t + \varphi n_t + \iota, & (9) \end{aligned}$$

and where φ can now be intrepreted directly as inverse of the Frisch elasticity of labour supply. These equations are also consistent with the balanced growth path but with two important differences. First, employment is no longer part of the dynamic Is equation. second, the elasticity of consumption with respect to real interest rate is restricted to unity. As is also well known, in this case the intertemporal elasticity of consumption is equal to unity.

2.2 Firms, optimal price setting and inflation equation

Specification of the supply side of the model follows the standard setup. We assume that there is a continum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good using homogenous technology. Firms' production possibilities are given by the production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}. \tag{10}$$

A_t represents the common stochastic level of technology. All firms face identical isoelastic demand schedule (4) and they take aggregate price and quantities as given. In this model, the absence of (nominal) rigidities would imply that movements in technology, A_t, would not induce any movements in hours worked: output would move hand-in-hand with the technology. Hours worked would not be affected, because substitution and income effect cancels each other, the key property of the preferences which we have discussed above. Consequently, price rigidity is the sole reason why variations in technology induce movements in hours.

In order to introduce price rigidity into the model, we make the typical assumption that each firm may re-set its price only with probability $1 - \theta$. Thus a measure of $1-\theta$ producers reset their prices in each period. The average duration of price is given by $1/(1-\theta)$. In this framework, (log linearized) optimal price setting rule of the firms can be characterized as

$$p_t^* - p_{t-1} = (1 - \theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \{\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\} \tag{11}$$

where $\mu \equiv \ln \mathcal{M} = \ln \frac{\epsilon}{\epsilon - 1}$, and $\widehat{mc}_{t+k|t}$ denotes the log deviation of real marginal cost from its steady state value in period $t+k$ for a firm whose price was last set in period t. Combining the optimal price setting rule of the firms with the goods

that the Frisch elasticity of labour supply is lower than the consumption-constant elasticity. Finally, note that when $s = 1$, these two elasticities coincide.

market and the labour market clearing conditions as well as with the dynamic Is curve in equation (5) delivers a inflation equation:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \widehat{mc}_t, \tag{12}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$ and $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. Importantly, the slope of the marginal cost term λ is independent on the parameters of the utility and, hence, up to a first order approximation, the relationship between inflation and marginal costs is independent on the choice of the utility functional and intertemporal elasticity of substitution[10]

This independence of inflation equation from the utility function in the first order approximation breaks down once the real marginal cost term is related to the measure of economic activity. Eliminating real wage from the definition of log real marginal costs $mc_t \equiv w_t - p_t - mpn_t$, (where mpn refers to marginal productivity of labour) and imposing market clearing conditions, we obtain

$$mc_t = \frac{(1+\varphi)}{1-\alpha}(y_t - a_t) - (\ln(1-\alpha) - \iota). \tag{13}$$

Note then that under flexible prices, the real marginal cost is constant and is given by $mc = -\mu$. Defining the natural level of output as the equilibrium level of output under flexible prices, y_t^n, it follows from (13) that

$$y_t^n = a_t + \vartheta_y^n, \tag{14}$$

where $\vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha)-\iota-\mu)}{(1+\varphi)}$. Furthermore, we have that

$$\widehat{mc}_t = \frac{(1+\varphi)}{1-\alpha}(y_t - y_t^n). \tag{15}$$

Finally, by combining (15) with (12), and defining $\tilde{y}_t \equiv (y_t - y_t^n)$, we obtain the New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \frac{(1+\varphi)}{1-\alpha}\tilde{y}_t. \tag{16}$$

Equation (14) says that the movements in the flexible price equilibrium output are solely due to movements in the productivity shifter. As discussed above, this is what we should expect at the flexible price equilibrium due to the KPR preferences. Furthermore, the marginal cost term (15) now depends on labour supply elasticity φ. However, note that the relationship between the marginal cost term and the output wedge is still independent of s, and hence on curvature of utility, if φ is taken as a parameter.[11] With additively separable

[10] Eichenbaum and Fisher (2007) derive a model in which the elasticity of demand facing firms is variable, capital is firm-specific and costly to adjust. This leads to specification of the inflation equation in which the firm specific capital reduces the response of inflation to marginal cost i.e. leads to a smaller slope of the marginal cost term in equation (12).

[11] The independence of slope of the Phillips curve from IES under KPR preferences is not itself a new result, but is not emphasised in the literature (see e.g. Smets and Wouters, 2007).

log preferences where $U(C_t, N_t) = \ln C_t - v(N_t)$, this also holds true. If we write φ following equation (7) then it is the case that the slope of the Phillips curve does depend on s. This dependence is however rather weak (see section 3)

An important distinction, however, is that the additively separable preferences that are consistent with the balanced growth path, constrains the IES to equal unity. In a more general case of additively separable constant relative risk aversion utility function the relationship between the real marginal costs and output wedge can be written as $\widehat{mc}_t = \left(\gamma + \frac{\xi+\alpha}{1-\alpha}\right)\tilde{y}_t$. This implies that IES and the slope of the Phillips curve are much more tightly linked. A lower intertemporal elasticity of substitution (a higher γ), given other parameter values, implies that inflation is more responsive to fluctuations in output wedge (see e.g. Gali, 2008).

2.3 IS equation

The final step in the derivations is to express the IS curve in terms of the output wedge $\tilde{y}_t$ and to define the natural rate of interest. Using the approximate log linear production function $y_t = (1-\alpha)n_t + a_t$ and substituting Δn_{t+1} away from the IS curve yields:

$$y_t = \mathbb{E}_t y_{t+1} + \frac{(1-s)\tau}{1-\alpha-(1-s)\tau}\mathbb{E}_t \Delta a_{t+1} - \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}(i_t - \mathbb{E}_t \pi_{t+1} - \rho) \quad (17)$$

Re-writing above in terms of the output wedge $\tilde{y}_t \equiv (y_t - y_t^n)$ by subtracting y_t^n from both sides, and using the fact that $y_t^n = a_t + \vartheta_y^n$ we arrive to:

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \psi(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \quad (18)$$

where r_t^n denotes the natural (real) rate of interest:

$$r_t^n = \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}. \quad (19)$$

and $\psi \equiv \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}$. The natural rate of interest given in equation (19) is the equilibrium real rate of return in the flexible price economy.

Equation (18) takes exactly the same form as in the model with additively separable log preferences, but with the following important difference: The elasticity of output wedge with respect to the real interest rate ψ is different from unity. In this set up ψ can be interpreted as (equilibrium) intertemporal elasticity of substitution. At given $\tau < (1-\alpha)$, the relationship between ψ and the labour-held-constant-intertemporal elasticity of substitution s is concave. In particular, as the curvature of the utility function increases (s declines), ψ decreases less than proportionately. This implies that a high curvature of utility can be supported by empirically reasonable values of ψ. However, this comes at the cost of introducing a stronger complementarity between consumption and leisure.

2.4 Alternative formulations of the IS curve

Another way of formulating the IS equation is to express it in terms of expected growth in labour. This alternative formulation can be achieved by using $y_t = (1-\alpha)n_t + a_t$ to substitute for output in equation (17). This yields

$$\mathbb{E}_t \Delta n_{t+1} = \psi'(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{20}$$

where $r_t^n \equiv \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}$, as defined earlier and $\psi' = \frac{s}{[(1-\alpha)-(1-s)\tau]}$. ψ' now gives the elasticity of labour w.r.t. the real interest rate. It is equal to ψ in the special case where $\alpha = 0$. Otherwise, due to concavity of the production function, $\psi' > \psi$. Equation (20) shows that labour can be used as an observable in the estimation instead of the output wedge $y_t - y_t^n$, which requires a proxy for the unobservable natural rate of output y_t^n. A clear benefit of using labour is that there is a much less controversy on how to measure labour than how to measure y_t^n, or how to treat the growth component of output in the estimation. Yet another way of writing the IS curve is not to substitute for labour, but simply subtract y_t^n from both sides of (17) and use the fact that $y_t^n = a_t + \vartheta_y^n$. This yields

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) - (1-s)\tau \mathbb{E}_t \Delta n_{t+1} \tag{21}$$

where $r_t^n \equiv \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}$.

3 Discussion

We highlight the impact of different values of s on the elasticity of output with respect to the real interest rate ψ, and on the slope of the Phillips curve κ in the model with additively separable preferences (the standard model) and in the model which allows non-zero cross elasticity between consumption and labour (KPR preferences). The first model is referred to as the standard NK model, where $\psi = s$ and $\kappa \equiv \lambda[s^{-1} + \frac{(\xi+\alpha)}{1-\alpha}]$ and where ξ is Frisch elasticity of labour supply. They key equations of the standard model are re-produced in the Appendix. The key equations and parameter definitions of the model with KPR preferences are given below:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \tag{22}$$

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \psi(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{23}$$

where $r_t^n \equiv \rho + \frac{1}{s}\mathbb{E}_t \Delta a_{t+1}$ and $\psi \equiv \frac{(1-\alpha)s}{1-\alpha-(1-s)\tau}$, $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, $\kappa = \lambda\frac{(1+\varphi)}{1-\alpha}$, $\tilde{y}_t \equiv (y_t - y_t^n)$, $y_t^n = a_t + \vartheta_y^n$, $\vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha)-\iota-\mu)}{(1+\varphi)}$, $\iota \equiv \tau - (1+\varphi)n$.

Baseline calibration is shown in Table 1 and the results from comparisons are reported in Table 2. The main result is that the standard NK model with additively separable preferences yields (in an empirical sense) implausibly steep Phillips curve at low values of s (and of course, is also inconsistent with the

Table 1: Parameterization

ξ	α	θ	β	ϵ	τ
1	0.33	0.67	0.99	6	$0.5^{(a}$

Note: Except for τ, these calibrated parameters are taken from Gali (2008, Ch. 3, p. 52). a) This value is chosen to reflect roughly the narrow measure of (after tax) labour share in the Us.[13]

Table 2: Key Tensions

	Additively separable Preferences				KPR preferences			
s	1	1/2	1/5	1/10	1	1/2	1/5	1/10
ψ	1	0.50	0.20	0.1	1	0.80	0.50	0.30
κ	0.13	0.19	0.38	0.69	0.13	0.11	0.10	0.09
φ^{-1}	1	1	1	1	1	1.33	1.67	1.82

Note: This table shows the key tensions in the standard model with additively separable preferences and in the model with KPR preferences when s varies from unity to 1/10, and otherwise the parameter values are chosen according to Table 1.

balanced growth path requirement). On the contrary, the slope of the Phillips curve is practically invariant to different values of s[12] in the model with KPR preferences.

Furthermore, in the model with KPR preferences, the relationship between ψ and s is concave. As the labour-held-constant intertemporal elasticity of substitution falls from unity to 1/10, the interest rate elasticity of output ψ only falls from unity to roughly 0.30. Therefore, the model permits a low value s, yet to achieve empirically plausible degree of real interest rate elasticity of output. Very low values of s, in turn generates a strong complementarity between consumption and leisure.

Because the slope of the Phillips curve is not very sensitive to different values of s, this allows us to identify the it primarily from the relationship between ex-ante real interest rate and output, just like it is done in the consumption Euler equation estimations by GMM methods. However, the advantage is that we have a structural model for inflation and interest rates, permitting us to use full information maximum likelihood based methods, instead of GMM methods. GMM methods applied to estimate s from aggregate data typically suffer from weak instrument problems.

[12] The variability of κ is due to the fact that we have fixed Frisch elasticity of labour supply equal to unity, and instead let φ vary in accordance with the equation (7).

4 Estimation

Relaxing the assumption of non-zero cross elasticity between consumption and labour allows permits a wide range of values for the curvature of utility with respect to consumption without distorting the relationship between output wedge and inflation in the sticky price monetary policy model, and yet keep the model consistent with the long-run labour supply facts. Many empirical studies that rely on the estimation of Euler consumption equation with aggregate data, infer the value of IES directly from the elasticity of consumption or output growth on the real interest rate.[14] This is correct only when the preferences are additively separable between consumption and labour and there is a direct correspondence between the curvature of utility and IES. If preferences are non-separable, like here, then the coefficient is biased due to the omitted variable bias (see equation, 5). The problem with structural interpretation of this elasticity is also illustrated in Table 2, where we found a concave relationship between s and ψ, where ψ measures the equilibrium IES.

Typical values found in the empirical macro literature for the respective elasticity are closer to zero than one. At the same time, one of the key weaknesses of estimating IES directly from Consumption Euler equation is that some form of instrumental variable estimation need to be employed. As shown for instance in Yogo (2002), Kiley (2010) and Kilponen (2012), the weak instrument problem makes it difficult to identify IES. In order to rest on more reliable inference, the weak instrument problem is addressed by using weak instrument robust confidence intervals, which are typically much wider than the classical ones (see e.g. Yogo, 2002, Stock and Yogo, 2005). The Bayesian Maximum Likelihood method applied to structural model does not suffer from a similar problem. As can be seen later on, credible sets around the point estimates of the IES are rather tight even when un-informative priors are used.

In this section, we estimate s together with the other key parameters of the model using the structural equilibrium relations given in (22)-(23), and the respective definitions given underneath these equations. For comparison, we also estimate the model with additively separable logarithmic utility. We label these models as $\mathcal{M}1$ and $\mathcal{M}2$ in what follows.[15] In order to make our estimation exercise more comparable to many other studies, we re-write the policy rule by allowing interest rate smoothing and assuming that the shocks to interest rate rule are i.i.d. over time:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t] + v_t,\ \ v_t \sim N(0, \sigma_v^2). \tag{24}$$

We also allow AR(1) shocks to the inflation equation (mark-up shocks) and to

[14] See e.g. Hall (1988), Cambell and Mankiw, (1989), Fuhrer and Rudebusch, 2004 and Yogo (2002).

[15] While it is well known that the standard model falls short of capturing features such as hump-shaped responses of output and inflation to technology and monetary policy shocks, our primary interest is to demonstrate the plausibility of the alternative formulations of utility in the simple model.

productivity shifter a_t as is standard in the literature:

$$\epsilon_t^\pi = \rho_\pi \epsilon_{t-1}^\pi + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \quad (25)$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a,\ \epsilon_t^a \sim N(0, \sigma_{\epsilon^a}) \quad (26)$$

4.1 The Data

As observable variables, we use hours worked, interest rate and inflation. Interest rate is quarterly federal funds rate and inflation is measured as quarterly log difference of the consumer price index. In contrast to many others, our observable vector does not contain the output gap. A clear benefit of using labour as observable is that there is much less controversy on how to measure labour than how to measure y_t^n, or how to treat the deterministic growth component of output in the estimation.[16] Hours worked are calculated following Hall (2009). specifically, we average over monthly series of hours (H=LNU02033120) and unemployment (U=LNS14000000) from the Bureau of Labour statistics and compute total hours as N=H*(1-U/100), where (1-U/100) is the employment rate. Our measure of quarterly hours then represents (seasonally adjusted) hours worked at non-agricultural industries in the US.[17] The corresponding observable variables are shown in Figure 1. Parameters α, τ and ϵ are fixed according to Table 1. Estimation sample is 1984Q1-2007Q4.

[Figure 1 about here]

4.2 Choice of priors

We rely primarily on the evidence summarized in Hall (2009) when choosing the priors for the key labour market parameters and s. Hall's "priors" for the Frisch elasticity and the interest rate elasticity are as follows: $\xi = 0.7$ and $\psi = 0.5$. These priors translate to following prior values for the IES and the (inverse) consumption-constant labour supply elasticity, $s = 0.20$ and $\varphi = 1.03$ (at given $\tau = 0.5$, $\alpha = 0.33$), respectively. The implied prior for the cross-elasticity between consumption and hours is $\frac{dC}{dN}\frac{N}{C} = (1-s)\tau = 0.4$. Direct empirical evidence on this cross-elasticity is clearly more scarce than on the Frisch elasticity. Hall (2009) provides a brief summary of the empirical literature which attempts to identify this elasticity by looking at what happens to the level of consumption when a person stops working. This means that the cross elasticity is identified from the correlation of consumption and the exogenous movements in the labour supply (due to e.g. unemployment, disability or retirement). Based on this literature, Hall's (2009) preferred value of this cross-elasticity is 0.3.Kimball and shapiro (2008) use a specific survey evidence on

[16] See e.g. Canova (1998)

[17] Smets and Wouters (2007) use a different measure of labour supply i.e. they compute $n = \ln((H/L)*(E/100))$, where H=average weekly hours in non-farm business (PRS85006023), E = Employment of 16 years of age and older (CE16OV) and L = population of 16 years and older (LNS10000000). We have estimated our model also with this measure of labour. The results are qualitatively similar i.e. we obtain small intertemporal elasticity of subsitution.

the response of hours to a large wealth shock to estimate different labour supply elasticities. Unfortunately, they are not able to uncover the cross-elasticity discussed herein. However, their baseline value needed to infer the other labour supply elasticities is also 0.3. Chetty (2006) argues that upper bound of this elasticity is 0.15, considerably lower than the values preferred by Hall (2009) and Kimball and Shapiro (2008).

In comparison to Smets and Wouters (2007), who also use KPR preferences to estimate a more fully specified DSGE model, our prior mean of the curvature of the utility function is quite a bit higher. Otherwise, our priors are rather standard (for comparison, see for instance Del Negro and Schorfheide, 2008 and Smets and Wouters, 2007). Prior densities and estimation results are summarized in Table 2.

4.3 The results

To begin with, the main result for $\mathcal{M}1$ is that the data supports a low value for the s. Posterior mean of s is as low as 0.07 with a relatively tight 90% credible set, ranging from 0.02 to 0.12. This implies together with the other estimated and calibrated parameters of the model that the posterior mean estimate for the elasticity of output with respect to real interest rate is $\hat{\psi} = 0.21$ $(0.09, 0.36)$. This accords well with the consumption Euler equation based literature. such a low value of s implies a very strong complementarity between consumption and labour and hence a rather large difference between Frisch and consumption-constant elasticity of labour supply. The posterior mean estimate for the Frisch labour supply elasticity is $\hat{\xi} = 0.96$ $(0.50, 1.73)$ while the posterior mean estimate for the consumption-constant elasticity of labour supply $\hat{\varphi}^{-1} = 1.32$ $(0.68, 2.38)$.[18] Finally, the posterior mean estimate for the slope of the Phillips curve $\hat{\kappa}$ is 0.014 $(0.003, 0.033)$. The estimated parameters of the policy rule are $\rho_i = 0.90$, $\hat{\phi}_x = 0.32$, $\hat{\phi}_\pi = 2.10$. These are relatively close to e.g. Erceg, Guerrieri and Gust (2006). Taylor (1993) coefficients are within 90% credible set. In comparison to Smets and Wouters (2007), our estimated value for the intertemporal elasticity of substitution is far much lower. In section 4.5, we show that this is primarily due to a choice of priors.

As for the shocks, the monetary policy shock has an (annualized) standard deviation of 48 basis points, while the cost-push shock has an (annualized) standard deviation of 88 basis points and a low persistence. The technology shock is strongly serially correlated and the standard error of innovations is equal to 37 basis points in quarterly terms. This is somewhat smaller than given by most of the estimates based on Solow residuals.

[Figure 2 about here]

Now compare these results to the model where the intertemporal elasticity of substitution is restricted to unity ($\mathcal{M}2$). Figure 2 compares the posterior densities against the common prior densities in the two models. The main

[18] The numbers in the brackets provide 90% probability sets.

Table 3: Priors and Summary of Posteriors

	Prior			Posterior					
				$\mathcal{M}1$			$\mathcal{M}2$		
Parameter	*Density*	*Mean*	*P*	*Mean*	*90% CI*		*Mean*	*90% CI*	
				Firms and Households					
θ	$\mathcal{B}$	0.60	0.15	0.88	0.82	0.95	0.84	0.77	0.94
$s^{(*}$	$\mathcal{G}$	0.20	0.10	0.07	0.02	0.12	–	–	–
ξ^{-1}	$\mathcal{G}$	1.40	0.50	1.22	0.50	1.90	1.27	0.52	1.97
β	$\mathcal{B}$	0.99	0.001	0.987	0.985	0.989	0.987	0.986	0.989
				Interest rate rule					
ρ_i	$\mathcal{B}$	0.50	0.15	0.89	0.86	0.92	0.84	0.80	0.88
ϕ_x	$\mathcal{G}$	0.20	0.10	0.32	0.15	0.59	0.32	0.10	0.53
ϕ_π	$\mathcal{G}$	1.50	0.40	2.10	1.32	2.83	2.53	1.66	3.33
				Exogenous shock processes					
ρ_a	$\mathcal{B}$	0.40	0.15	0.83	0.75	0.92	0.89	0.84	0.94
ρ_π	$\mathcal{B}$	0.40	0.15	0.48	0.32	0.63	0.42	0.27	0.58
σ_a	$\mathcal{G}^{-1}$	0.80	0.50	0.32	0.21	0.43	2.16	1.49	2.38
σ_π	$\mathcal{G}^{-1}$	0.30	0.50	0.20	0.14	0.25	0.23	0.18	0.28
σ_v	$\mathcal{G}^{-1}$	0.20	0.20	0.13	0.12	0.14	0.14	0.12	0.16
$\mathcal{LMD}$					1173.07			1157.97	

Note: $\mathcal{B}, \mathcal{G}$ and $\mathcal{G}^{-1}$ correspond to Beta, Gamma and inverse Gamma distributions. Mean corresponds to mean and P is the standard deviation of the respective prior distribution. Fixed parameters are $\alpha = 0.33$, $\tau = 0.5$, $\epsilon = 6$ as in Table 1. Prior and posterior moments for the standard errors ($\sigma_.$) are in percentage form. *) In $\mathcal{M}2$, s is fixed to 1. Hence, prior and posterior moments are irrelevant. $\mathcal{LMD}$ is log marginal density. Estimation sample is 1984Q1-2007Q4 and estimations were done using Dynare version 4.3.1. Posterior distibution was obtained by Metropolis–Hastings algorithm.

difference between the two models is that the standard error of innovations to technology shocks in $\mathcal{M}2$ (at posterior mean) is almost 6(!) times larger than the respective standard error in $\mathcal{M}1$. Furthermore, the data prefers the model $\mathcal{M}1$. The ratio of marginal likelihood values ($\mathcal{LMD}$s) between the two models, in favour to $\mathcal{M}1$, is equal to 1.013.(see Table 2). As for the reduced form parameters, the slope of the Phillips curve in $\mathcal{M}2$ is quite much larger. The posterior mean estimate for the slope is 0.05 $(0.014, 0.11)$.

4.4 Equilibrium responses to technology and monetary policy shocks

Figures 3-4 show the equilibrium responses (at posterior mean) to one standard deviation technology and monetary policy shock in the two models. As discussed above, the standard deviations of technological innovations are 0.32% and 2.16% in $\mathcal{M}1$ and $\mathcal{M}2$ respectively. Note also that due to cancelling out of the income and substitution effect the response of the natural output (output under flexible prices) tracks the exogenous response of technology to its innovation (not shown in the Figure) in the both models exactly.

The most important difference between the two models is the response of output to technology shocks. Technology shock does open a negative output wedge in the both models and leads to a fall in employment (hours worked). However, the sign of the response of output in $\mathcal{M}1$ is the same as that of the output wedge. That is, the productivity shock generates a negative response to output in the short-run. Output will eventually be pushed to a positive territory (after 10 quarters or so) as the negative employment response fades away and technology shock persists. Negative short-run output reaction in $\mathcal{M}1$ is explained by the strong degree of complementarity between output and employment. Our results suggest that the cross-elasticity between consumption and hours is $0.47(0.43, 0.49)$, which is a rather high number. This is manifested by much stronger relative response of employment to technology shock in $\mathcal{M}1$ when compared to $\mathcal{M}2$. In $\mathcal{M}1$ employment falls on impact roughly 1% given a one standard deviation shock (0.37%) to technology. In $\mathcal{M}2$, the size of the technology shock is 2.16%, but employment falls on impact only 1%.

Note furthermore that the natural rate of interest falls far more in $\mathcal{M}1$ than in $\mathcal{M}2$. since the real rate, due to slow reaction of the nominal rate, follows the fall in the natural rate with a considerable delay, technology shock opens up a *positive* interest rate gap between the real rate and the natural rate of interest. This contributes negatively to output due to usual interest rate channel. However, this contribution is undermined by low interest rate sensitivity of output (due to the low intertemporal substitution) in $\mathcal{M}1$.

[**Figures 3-4 about here**]

In $\mathcal{M}2$ both employment and the natural rate reacts relatively little such that the output response is clearly positive. Technology shocks opens up a small positive interest rate gap also in $\mathcal{M}2$. On the contrary to $\mathcal{M}1$, this has relatively strong negative contribution to output since the intertemporal substitution is high.

The models $\mathcal{M}1$ and $\mathcal{M}2$ also show a clear difference with respect to strength of the response to a monetary policy shock. Note that, in contrast to technology shocks, monetary policy shocks in the two models have almost exactly equal standard deviations. Thus the differences in the responses in Figure 4 can be read directly as the differences in the strength of the equilibrium responses. It is clear that in $\mathcal{M}2$ responses of all the variables are much stronger. This is explained simply by the restrictive assumption that the real-interest rate elasticity of output is unity in $\mathcal{M}2$. In $\mathcal{M}1$ the real interest rate elasticity of output is roughly 0.21. In summary, $\mathcal{M}1$ emphasizes the labour market responses, while $\mathcal{M}2$ puts emphasis on the nominal side, and the reaction of the monetary policy.

4.5 Sensitivity to priors and habit persistence

Our results imply a considerably higher curvature of utility function, due to low values of s, than those obtained by Smets and Wouters (2007). Although the results are not directly comparable due to various different modelling assumption, we demonstrate that the key reason for the differing results is the choice of priors. Smets and Wouters (2007) impose a relatively informative prior to the curvature parameter γ such that high values of labour-held-constant risk aversion are practically ruled out in their estimation. In order to demonstate this, we have extended the model by introducing external habit persistence into consumption and re-estimated the complete model using priors for γ and habit persistence similar to Smets and Wouters.[19] We have introduced habit formation in the model, since it is present in Smets and Wouters (2007). Habit formation alters the parametrisation of the dynamic IS and AS curves and it further complicates the relationship between equilibrium IES and curvature of utility. IES and γ reflect distinct characteristics of preferences when the utility function is not time-separable, as is the case with habits.

With habit persistence in consumption, the parameter which governs the sensitivity of consumption to the real interest rate can be expressed as

$$\psi \equiv \frac{(1-b)(1-\alpha)s}{(1+b)(1-\alpha)-(1-s)\tau} \qquad (27)$$

and where b measures the intensity of external habit persistence. A given real interest rate sensitivity of output is consistent with different combinations of s and habit intensity parameter b. Furthermore, it is clear from equation (27) that ψ is decreasing in b and increasing in s. This suggest then that setting a high (and informative) prior for the degree of habit persistence makes it more likely that the estimation also produces relatively high values for s.As for the New Keynesian Phillips curve, the introduction of habits introduces a current period output gap difference to the right hand side of the equation. Furtheremore, the

[19] See Appendix for description of the model with external habits.

relationship between the Frisch elasticity of labour supply and the consumption constant elasticity of labour supply now depends on the intensity of habits. When the intensity of habits increases, a difference between the Frisch elasticity of labour supply and the consumption constant elasticity of labour supply increases. Similarly, more intense habits increases the dependence of inflation on the output gap difference (see.36).

Smets and Wouters (2007) set the prior mean for habit persistence parameter to 0.7 with a standard error of 0.1, while their prior for γ is 1.5 with standard error of 0.37. Using (27) these priors imply a prior mean for ψ approximately equal to 0.138, given $\tau = 0.5$ and $\alpha = 0.33$ in our setup.

Table 4 shows the estimation results from the extended model with habit persistence[20]. Column I reproduces the benchmark results from table 2, while columns II-III shows the results using the priors comparable to Smets and Wouters (2007). In column II, habit persistence is fixed to zero, while in column III the prior mean for the habit persistence parameter has been set equal to 0.6. This is close to the value used by Smets and Wouters (2007)[21] while the prior for s have been translated from original prior for γ in Smets and Wouters (2007). Column IV shows the estimation results by using our own prior on s (from our benchmark model) and using a lower prior for habit persistence, where we rule out the high values of habit persistence.

The results show that estimated value for s is sensitive to the choice of priors. This also translates into different values of posterior mean of ψ, ranging from 0.06 to 0.28 in different specifications. Allowing for habit persistence leads in general to a lower value of ψ, but this lower value can be obtained with strikingly different values of s. As expected, with the priors from Smets and Wouters, the estimated values of s tend to be higher and with habit persistence the posterior estimates of both s and b are close to their prior contributions (see column III). However, based on log marginal likelihood, the data weakly supports the combination of low s and moderate degree of habit persistence (see column IV). At the same time, the estimate of the implied real interest rate elasticity of output is very similar in both cases. Note further that the credible sets in columns I and IV are remarkably tight relative to those in columns II and III. Consequently, using our own priors the posterior estimates of the key parameters tend to be in much tighter range than under the priors from Smets and Wouters.

[20] Appendix shows the key equations of the model with habit persistence.

[21] We have chosen this lower value due to the fact that the model does not permit much higher initial values of b. Otherwise, the model becomes unstable. We have also added into interest rate rule equation the term $\phi_{\Delta y}\Delta\tilde{y}_t$, which appears in the original contribution by Smets and Wouters (2007). We estimate the parameter $\phi_{\Delta y}$ alongisde with the other parameters. This helps to reconcile the stable equilibria in the model, even with relatively high value of IES and habit persistence.

Table 4: Sensitivity of Intertemporal Elasticity of Substitution to Priors

	I	II	III	IV
Prior, s	$\mathcal{B}(0.2, 0.15)$	$\mathcal{N}(0.67, 0.15)$	$\mathcal{N}(0.67, 0.15)$	$\mathcal{B}(0.2, 0.15)$
Prior b	fixed to zero	fixed to zero	$\mathcal{N}[0.6, 0.1]$	$\mathcal{N}[0.45, 0.05]$
Parameter		**Posterior Distribution**		
s	0.07 (0.02,0.12)	0.10 (0.02,0.23)	0.56 (0.23,0.89)	0.09 (0.02,0.15)
b			0.79 (0.58,0.95)	0.47 (0.41,0.52)
ψ	0.21 (0.09, 0.36)	0.28 (0.03, 0.64)	0.08 (0.01 0.21)	0.06 (0.02, 0.11)
$\mathcal{LMD}$	1173.07	1166.15	1181.68	1183.75

Note: $\mathcal{B}$ and $\mathcal{N}$ correspond to Beta and Normal distributions. Fixed parameters are $\alpha = 0.33$, $\tau = 0.5$, $\epsilon = 6$ as in Table 1. $\mathcal{LMD}$ is log marginal density. Estimation sample is 1984Q1-2007Q4 and estimations were done using Dynare version 4.3.1. Posterior distibution was obtained by Metropolis–Hastings algorithm.

5 Conclusions

One of the most common assumptions in sticky price monetary policy models is the additively separable utility in consumption and labour. In order to make this particular class of utility functions consistent with balanced growth path, consumption enters the utility function in a logarithmic form, the implications of which are not well supported by empirical evidence. Allowing for non-zero cross-elasticity between consumption and labour, for example along the lines of KPR type preferences employed in this paper, leads to empirically more plausible results and allows to estimate IES with full information maximum likelihood based methods. Key to this result is that KPR preferences break the tight coefficient restriction between the slope of the Phillips curve and the elasticity of consumption (output) with respect to real interest rate in the model economy studied here.The Bayesian estimation results suggest that the real interest rate elasticity of output is in the range $0.1 - 0.4$ in the US during period 1984-2007. In the model with habit persistence, the data weakly supports the combination of moderate degree of habit persistence and high curvature of utility as opposed to relatively low curvature of utility and high habit persistence as in Smets and Wouters (2007). At the same time, however, our estimates suggest a strong complementarity between consumption and labour, that is difficult to reconcile with micro evidence on labour supply. Hence, further work on testing alternative models of aggregate consumption and labour supply behaviour is needed, with the ultimate target of finding specifications that would be better in line with lower consumption-labour complementarity, a moderate responsiveness of consumption to real interest rates and the long-run labour supply facts.

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Appendix

A Frisch elasticity and consumption constant elasticity of labour supply

Frisch elasticity of labour supply is defined as the elasticity of labour supply where the marginal utility of consumption is held fixed. Hence, we must have that

$$\begin{aligned} dU_C(C,N) &= U_{CC}dC + U_{CN}dN = 0 \\ &\Leftrightarrow \qquad (28) \\ \frac{dc}{dn} &\equiv \frac{d\log C}{d\log N} = -\frac{U_{CN}}{U_{CC}}\frac{N}{C} = (1-s)\tau. \qquad (29) \end{aligned}$$

Furthermore, along constant marginal utility of consumption paths

$$c_\omega \equiv \frac{dc}{d\omega} = (1-s)\tau\frac{dn}{d\omega} = (1-s)\tau n_\omega, \qquad (30)$$

where ω denotes the log of the real wage. From intratemporal condition for labour, we know that

$$n_\omega = \varphi^{-1}(1-c_\omega) \qquad (31)$$

where $\varphi \equiv \frac{v''(N)N}{v'(N)}$. Hence, substituting (30) into (31) and solving for n_ω gives:

$$\begin{aligned} n_\omega &= \varphi^{-1}(1-(1-s)\tau n_\omega) \\ n_\omega &= \frac{1}{\varphi + (1-s)\tau} \qquad (32) \end{aligned}$$

and where $n_\omega \equiv \xi$ is Frisch elasticity of labour supply.

B Standard model

This appendix replicates the key equations and the parameter definitions of the standard sticky price monetary policy model (adapted from Gali, 2008, ch. 3).

$$\begin{aligned}
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t && (33)\\
\tilde{y}_t &= \mathbb{E}_t \tilde{y}_{t+1} - s(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) && (34)\\
i_t &= \rho + \phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + v_t && (35)
\end{aligned}$$

where $r_t^n \equiv \rho + \frac{1}{s}\psi_{ya}^n \mathbb{E}_t \Delta a_{t+1}$ and

$$\begin{aligned}
\lambda &\equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta, \ \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}\\
\kappa &\equiv \lambda\left(\frac{1}{s} + \frac{\xi+\alpha}{1-\alpha}\right), \ \tilde{y}_t \equiv (y_t - y_t^n),\\
y_t^n &= a_t + \vartheta_y^n, \ \vartheta_y^n \equiv \frac{(1-\alpha)(\ln(1-\alpha)-\mu)}{\frac{1}{s}(1-\alpha)+\xi+\alpha}\\
\psi_{ya}^n &\equiv \frac{1+\xi}{\frac{1}{s}(1-\alpha)+\xi+\alpha}
\end{aligned}$$

C Model with habit persistence

This appendix shows the key log linearised equations and the parameter definitions of the sticky price monetary policy model with external habit persistence and King-Plosser-Rebelo preferences. Detailed derivation of the model is available by request from the authors.

$$\begin{aligned}
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t + \lambda \frac{b}{1-b} \Delta \widetilde{y}_t && (36)\\
\tilde{y}_t &= \omega_1 \tilde{y}_{t-1} + \omega_2 \mathbb{E}_t \tilde{y}_{t+1} - \psi(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) && (37)\\
r_t^n &= \rho + \frac{1}{s}\begin{pmatrix} \frac{(1-s)\tau}{(1-b)(1-\alpha)} E_t\left(\Delta a_{t+1} - \Delta y_{t+1}^n\right) \\ -\frac{b}{(1-b)} y_{t+1}^n + \frac{b}{(1-b)} y_{t-1}^n + \Delta y_{t+1}^n \end{pmatrix} && (38)\\
y_t^n &= \frac{(1-\alpha)b}{(1+\varphi)(1-b)+(1-\alpha)b} y_{t-1}^n + \frac{(1+\varphi)(1-b)}{(1+\varphi)(1-b)+(1-\alpha)b} a_t && (39)\\
i_t &= \rho + \phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + v_t && (40)
\end{aligned}$$

where

$$\begin{aligned}
\lambda &\equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta, \ \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}\\
\kappa &\equiv \lambda\frac{(1+\varphi)}{1-\alpha}, \ \omega_1 = \frac{b(1-\alpha)}{(1+b)(1-\alpha)-(1-s)\tau}\\
\omega_2 &= \frac{(1-\alpha)-(1-s)\tau}{(1+b)(1-\alpha)-(1-s)\tau}, \ \psi = \frac{(1-b)(1-\alpha)}{(1+b)(1-\alpha)-(1-s)\tau}\\
\tilde{y}_t &\equiv (y_t - y_t^n)
\end{aligned}$$

Furthermore, it can easily be shown that the relationship between Frisch elasticity of labour supply, ξ_h, and consumption constant elasticity of labour supply in the presence of external habits is given by $\xi_h = 1/(\varphi + \frac{1-s}{1-b}\tau)$.

D Figures

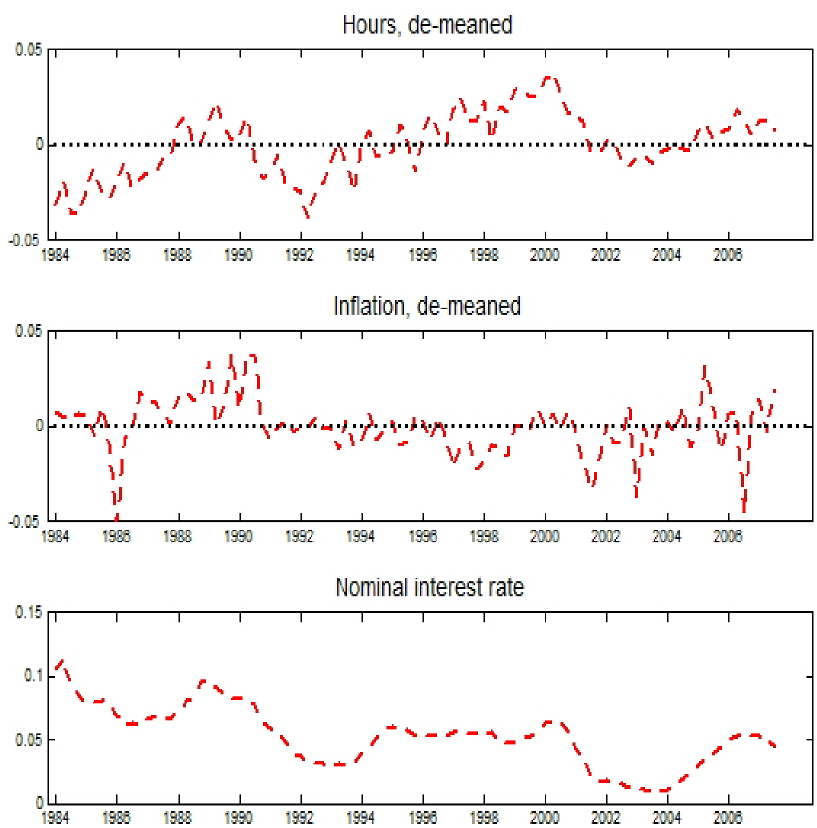

Figure 1: The Observable Variables Used in the Estimation
Note: This figure shows the quarterly data from the U.S. Hours is seasonally adjusted and de-meaned hours worked at non-agricultural industries. Inflation is annualized quarterly difference of log consumer price index. Nominal interest rate is annualized Federal Funds Rate. See section 3.1 for more details of the data.

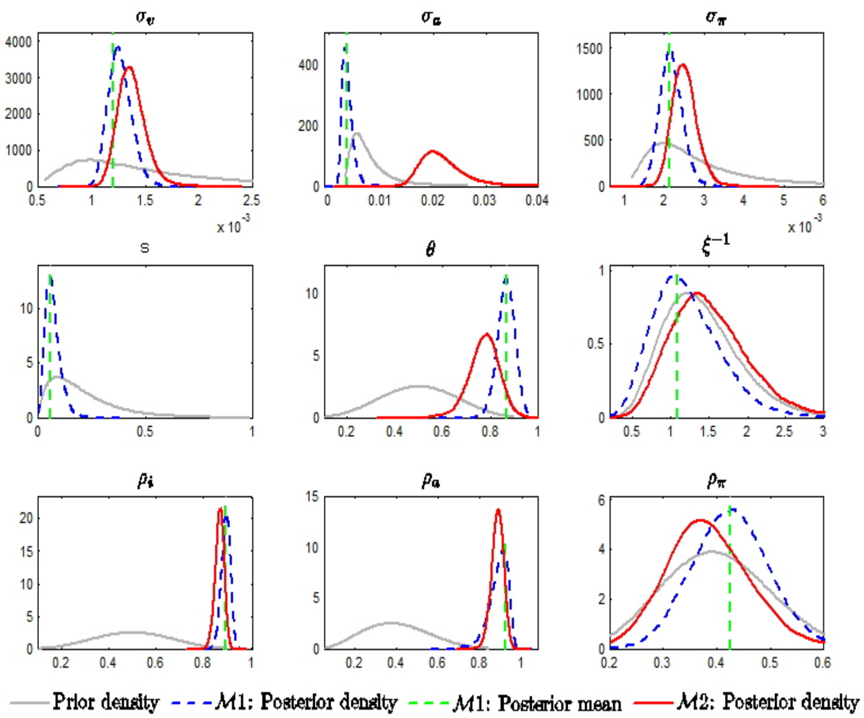

Figure 2: The Prior and Posterior Densities.

Note: This figure compares the prior and posterior densities after estimation of the model with KPR preferences ($M1$) and with logarithmic utility ($M2$).

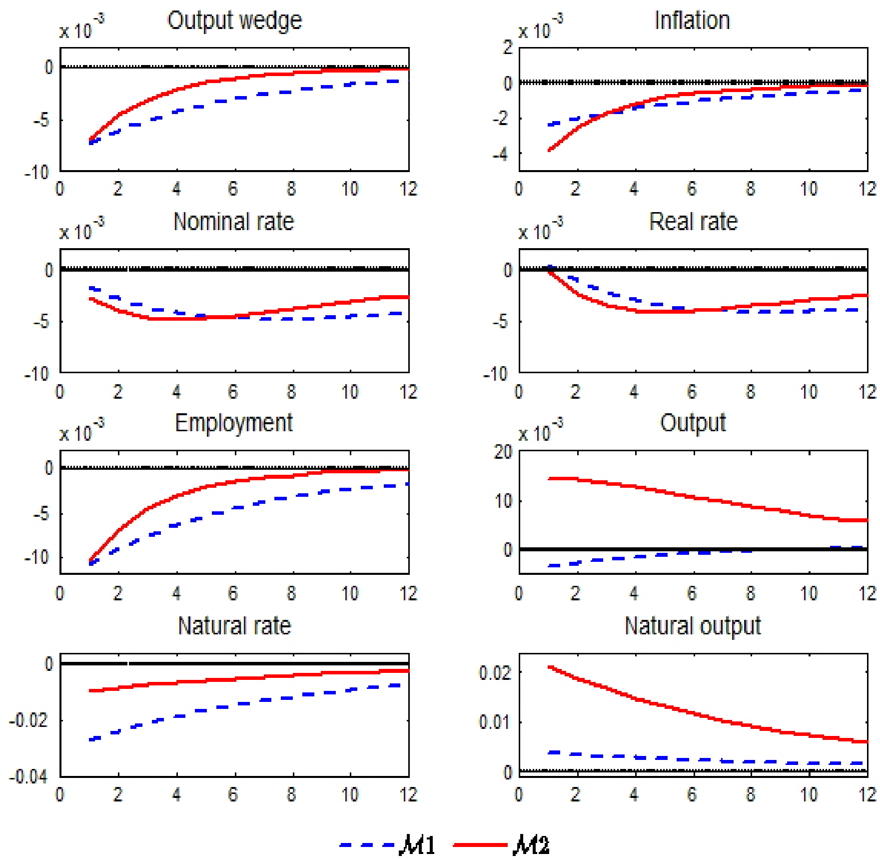

Figure 3: Equilibrium Responses to One Standard Deviation Technology Shock at Posterior Mean.

Note: This Figure shows the impulse responses of selected variables to technology shock in the model estimated under KPR preferences ($M1$) and under logarithmic utility ($M2$). Interest rates and inflation rates are annualised.

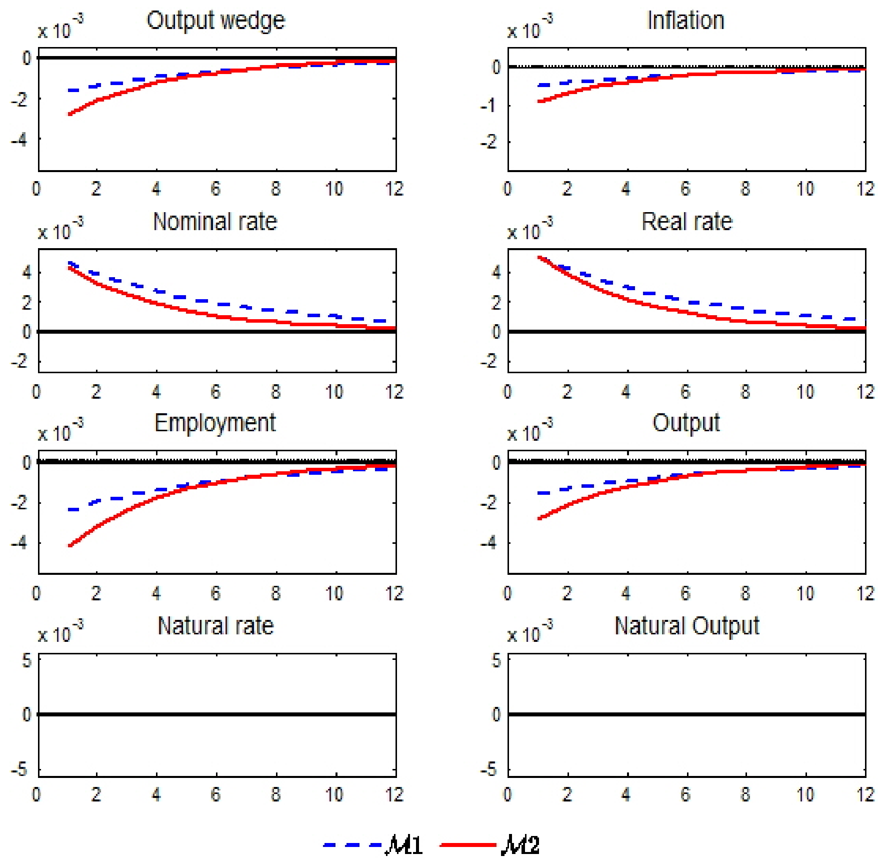

Figure 4: Equilibrium Responses to One Standard Deviation Monetary Policy Shock at Posterior Mean.

Note: This Figure shows the impulse responses of selected variables to monetary policy shock in the model estimated under KPR preferences ($M1$) and under logarithmic utility ($M2$). Interest rates and inflation rates are annualised rates.

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