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## Abstract

This paper deals with the interaction of fiscal and monetary policy when the central bank is pursuing a price stability-oriented monetary policy. In particular, we study the durability of the price stability regime when public debt accumulates as a result of ultimately unsustainable deficits. The growth of indebtedness causes the collapse of the price stability regime after a period of rising deficits. The budget deficit is endogenously determined in the model, as a result of government's decisions on how to finance its expenditure. The alternative methods of finance are taxes, debt, and seigniorage. Under the price stability regime, only the first two methods are available, but in the long run taxes and seigniorage are the only alternatives. The price stability regime collapses when the public debt reaches an edogenously determined threshold, which makes reneging on price stability as attractive as accumulating more tax burden for the future. We are able to solve for the critical level of debt, the timing of the collapse, and the reaction of taxes to the collapse of the price stability regime. The critical level of debt depends, inter alia, on the level of government consumption, the real interest rate, the velocity of money, and the efficiency-effects of taxation. The results are illustrated by several numerical simulations.

Key words: inflation, fiscal policy, fiscal theory of inflation, Stability and Growth Pact

# Finanssipolitiikan vaikutukset inflaationvastaisen rahapolitiikan kestävyyteen

## Suomen Pankin keskustelualoitteita 14/2002

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## Tiivistelmä

Tutkimuksessa tarkastellaan finanssi- ja rahapolitiikan vuorovaikutusta, kun keskuspankki harjoittaa hintavakauteen tähtäävää rahapolitiikkaa. Erityisesti tutkitaan hintavakauspolitiikan kestävyyttä tilanteessa, jossa julkinen velka kumuloituu liiallisten budjettialijäämien vuoksi. Velan kasvu aiheuttaa tietyn ajan kuluessa inflaation vastaisen rahapolitiikan murtumisen. Budjettivajaus määräytyy tutkimuksessa käytetyssä mallissa endogeenisesti valtion rahoituspäätösten perusteella. Julkisten menojen rahoitusvaihtoehdot ovat verot, velka tai keskuspankin tuloksesta saatava rahoitustulo (senjoraasi). Hintavakauden oloissa vain kaksi ensimmäistä tulevat kysymykseen, mutta pitkällä aikavälillä vain verot tai rahoitustulo ovat mahdollisia. Hintavakauspolitiikka romahtaa, kun julkinen velka kasvaa niin paljon, että hintavakauden pettäminen tulee yhtä houkuttelevaksi kuin lisävelan ja siis tulevan verotaakan edelleen kasvattaminen. Käytetyssä mallissa on mahdollista laskea sekä velan kriittinen taso että hintavakauspolitiikan romahduksen ajankohta ja myös verojen reaktio vakauspolitiikan murtumiseen. Velan kriittinen taso riippuu mm. julkisista kulutusmenoista, reaalikorosta, rahan kiertonopeudesta ja verotuksen tehokkuusvaikutuksista. Tuloksia havainnollistetaan muutamilla numeerisilla esimerkeillä.

Asiasanat: inflaatio, finanssipolitiikka, fiskaalinen inflaatioteoria, vakaus- ja kasvusopimus

# Contents

# 1    Introduction

Central banks are usually thought to be responsible for maintaining price stability. However, the interaction of fiscal and monetary policies may lessen the power of the central bank to control the price level. If the fiscal policy authority can rely on seigniorage revenues when considering different ways to balance the budget, central bank's ability to fight for price stability is limited. Therefore, budget deficits and high debt levels may threaten low inflation policy targets because the government has always a temptation to use seigniorage to reduce its financial burden. This is especially the case if the central bank is not very independent. Questions considering independence of european central banks and especially the interaction of fiscal and monetary policies are important issues in this study as well as in public debate nowadays. All European countries are committed to low inflation, which could be threatened, at least to some extent, by loose fiscal policies. For this reason, the need for fiscal discipline has quite often been stressed by euroarea monetary policy authorities. Fiscal discipline in all euroarea countries is thought to be necesasry, if not sufficient to secure the ability of the European Central Bank to fight for price stability. Deficit and debt requirements ensure that no country with fiscal discipline is hurt by some other country that might use loose fiscal policy for its own purposes. Question concerning how the sustainability of low inflation policy regime depends on fiscal policy is very important when deciding what kind of norms fiscal policy should follow.

In this study we analyse how a zero inflation policy regime can be maintained when public expenditures are exogenous byt deficits are endogenous. Although we do not model any direct interaction between fiscal and monetary policy authorities, we can find out the (maximal) effects of fiscal constraints on the sustainability of monetary policy by assuming simply that there exists only one policy authority responsible for both fiscal and monetary policies. The policy authority's loss function is quadratic  in output gap and inflation, and the policy authority always minimises present value of its losses. The policy authority finances exogenous public expenditures and interest payments due to existing debt with new debt, taxes and seigniorage revenues. As in Obstfeld (1991), the seigniorage revenue is the sum of the change in real money balances and the inflation tax on previous period's real money holdings.[1] When the policy authority is committed to a zero inflation policy target, taxes and debt are the only ways to

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[1] Seigniorage revenues could be determined also as central bank transfers as in Canzoneri, Cumby and Diba (1998a and 1998b). Higher nominal money balances imply that the central bank has a bigger proportion of public debt. This helps the government to finance public expenditures because interest payments to central bank increase central bank's profit, which is paid back to the government. Because according to this determination seigniorage is interest payments which the government is able to avoid, seigniorage revenues are zero if the nominal interest rate is zero.

balance the budget. In the long run, however, maximum level of debt is restricted by the intertemporal budget constraint, ie fiscal solvency requires that the policy authority is able to pay back the existing debt with future surpluses (taxes + seigniorage revenues – public expenditures – interest payments). In the zero inflation policy regime, the policy authority is forced to use tax increases to finance deficits if indebtedness reaches its maximum level. When losses due to high taxation become unbearable, the policy authority is forced to give up defending the zero inflation policy target. After the regime switch the policy authority is able to lower taxes because budget is partly balanced with seigniorage due to inflation.

When the policy authority gives up defending the zero inflation policy target, in which he has been committed to, he bears a fixed (political) cost. This cost reflects reputational losses of the policy authority, or the cost of being forced to leave the office. Because the fixed cost increases the perceived costs of inflation, debt and taxes become more tempting relative to inflation. We assume that the possible regime switch is a once-for-all choice, ie all policy solutions are discretionary after the regime switch. Hence, in our model the policy authority gives up defending the low inflation regime only in an extreme situation because costs of doing so are so high.

This study is related to the fiscal theory of the price level as well as to the basic models of balance-of-payments crises. Also ideas presented in the study by Sargent and Wallace (1981) have benefited us in great deal. Our main theoretical contribution is that we are able to study both short run and long run effects of fiscal policy on monetary policy. As in the fiscal theory of the price level determination, we solve inflation from the intertemporal budget constraint. However, the standard literature on fiscal theory of the price level determination excludes questions concerning durability of different policy regimes. In this study the policy authority is first committed to a zero inflation policy regime. The core of this study is to reveal some aspects concerning the mechanism, which triggers the regime switch. We are able to derive conditions for the sustainability of the zero inflation regime which are valuable also when we consider other low inflation regimes. Another contribution is that, contrary to Sargent and Wallace (1981), we endogenise public deficit by allowing the policy authority to use taxes as a source of financing public expenditures and interest payments. Hence, government expenditures is the only exogenous policy variable in our study. We show how the policy authority uses taxes to postpone the possible regime switch.

When indebtedness and deficits are constrained by stability pact type requirements, and if these requirements are credible, it is easier for central banks to control the price level. However, even if these requirements are met, fiscal solvency is also needed. The stability pact prevents a situation where existing debt can not be repaid with expected future surpluses. Instead of explicit stability type constraints on debt and deficits, we use the fiscal theory of the price level

determination to bind debt to expected future surpluses. Therefore, fiscal solvency is not an issue in this study. It is always required and it also determines the maximum possible level of indebtedness of the policy authority. Because a single policy authority decides on issuing new debt, taxes and inflation, the problem of possible indeterminacy or multiplicity of the equilibrium price level does not exist. In this way, implications of this study differ from implications of the fiscal theory of the price level. Among other things, our model determines how long the zero inflation policy regime is maintained, how much taxes are cut in the regime switch, what is the inflation rate after the regime switch, and what is the time path of debt. Actually, it turns out that the debt path determines whether the policy authority is able to maintain the zero inflation regime or is the regime switch inevitable. In short, the zero inflation regime is sustainable only if long run fiscal policy is in line with long run monetary policy.

This paper has the following structure. Section 2 reviews the literature related to this study. Section 3 presents the simple model we use in this study. In section 4 we solve equilibrium paths of taxes, inflation and debt in the case when the intertemporal budget constraint is binding, ie in the post-collapse discretionary policy regime. We show that zero inflation is not a solution in this regime. This implies that the zero inflation policy regime inevitably collapses when debt reaches its maximum value, determined by the post-collapse paths of public expenditures, inflation and taxes. Section 5 deals with solutions of taxes and inflation when the intertemporal budget constraint is not binding. Instead, we assume that the policy authority is committed to a zero inflation policy regime. Hence, the policy authority uses taxes and debt to finance government expenditures and interest payments. The policy solution also determines whether the policy authority is able to maintain the zero inflation regime or is collapse of the zero inflation regime inevitable. In section 6 we determine two conditions for the regime switch and we solve how long the zero inflation regime is maintained. In section 7 we combine results derived in sections 4, 5 and 6 and we solve the whole model numerically. We also calculate some elasticities to show effects of changes in parameter values on endogenous variables. Section 8 concludes and discusses some issues for further research.

# 2    Short review of the literature related to this study

In this section we review briefly the literature which is used in this study. We have extended the fiscal theory of the price level to include a temporary zero inflation target. The mechanism which is incorporated in the breakdown of the zero inflation regime in this study is basically the same as in basic balance-of-payments crises models (see for example Krugman, 1979). Also theories of

optimal taxation (about seigniorage revenues, see for example Mankiw, 1987) is an essential element of this study when considering different ways to finance public expenditures and interest payments.

The fiscal theory of the price level (Leeper, 1991; Sims, 1994 and Woodford, 1994, 1995, 1996) provides a theoretical framework for our analysis. The fiscal theory of price level in its simplest form states that the equilibrium price level is whatever makes the real value of nominally denominated government liabilities equal to the present value of expected future real government budget surpluses. Hence, if fiscal policy does not ensure the above equality, the price level must do so.

Leeper (1991) studies monetary and fiscal policy interaction in a stochastic maximising model. Monetary and fiscal policy authorities are called active and passive depending on the constraints a policy authority faces. An active policy authority does not face any constraints, and is thus free to set its control variable. On contrary, a passive policy authority takes the active authority's policy decision as given. Hence, he is forced to balance the budget constraint. From now on, if the monetary authority is the active one, we call the policy setting as money dominant (MD) regime. In this regime, the central bank sets money supply, and fiscal policy has to adjust to satisfy the government's present value budget constraint for any given real value of current government liabilities. On the other hand, if the fiscal authority is the active one, we call the policy setting as fiscal dominant (FD) regime.[2] In this regime, the fiscal policy authority sets surpluses via expenditures and taxes, and the money supply adjusts to satisfy the government present value budget constraint. In a FD regime, monetary authority is not able to control the price level.

Leeper analyses the existence and uniqueness of equilibria produced by a class of monetary and fiscal policy rules. If one of the policy authorities is active and the other is passive, there exist a unique saddle-path equilibrium. Policy behaviour of the active authority completely specifies policy and uniquely determines the equilibrium pricing function. Passiveness of the other policy authority ensures that the path of government debt is not explosive. On the other hand, if both policy authorities are passive, the price level is indeterminate. Finally, if both policy authorities are active, there exist an equilibrium only by accident.

Sims (1994) and Woodford (1994) extend the analysis by Leeper. Woodford considers the determinacy of the equilibrium price level in the cash-in-advance monetary economy. He analyses two types of monetary policy regimes. Firstly, the money supply grows at a given exogenous rate, and secondly, the nominal interest rate on one-period government debt is pegged at a given non-negative

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[2] The MD/FD terminology is based on Canzoneri and Diba (1996). Policy regimes could also be called "Ricardian" (MD) and "non Ricardian" (FD) as in Woodford (1995, 1996).

level. Sims uses same policy regimes as Woodford, allthough the model is different. They both consider the possible multiplicity of perfect foresight equilibria, and also "sunspot" equilibria are possible. In the case of a constant money growth regime the price level can be indeterminant. However, according to Woodford, high growth rates of money guarantee uniqueness of equilibrium. This can result in a contradiction of government's objectives. On the one hand the government maximises the welfare in the steady state equilibrium. On the other hand uniqueness of the steady state equilibrium is also preferred. Hence, in order to ensure uniqueness of the equilibrium, the government is forced to accept a higher level of money growth than it would be optimal to maximise welfare. In the case of interest rate pegging regime, both Sims (1994) and Woodford (1994) argue that a monetary policy with fixed nominal interest rate support uniqueness of equilibrium.

Woodford (1995) compares the fiscal theory of the price level to the quantity theory of money, which is the more conventional way to determine the general price level. He shows that equilibrium conditions of the quantity theory are often insufficient to determine the equilibrium price level. Changes in the price level affect private sector wealth and hence private sector demand for goods and services. Equality of aggregate demand and supply usually imply unique price level. Therefore fiscal policy affects price level in two different ways. Firstly, changes in aggregate demand depend on the size of the debt.[3] Higher debt (higher private sector wealth) induces larger wealth effects and thus larger changes in the price level. Secondly, expectations about future fiscal policies have also wealth effects.

Contrary to Leeper (1991) Cochrane (1998b) extends the fiscal theory of the price level to include long-term debt.[4] This implies that the real value of nominally denominated debt changes not only due to price level changes, but also due to changes in the nominal value itself. Changes of expected future price levels affect nominal bond prices and thus the nominal value of debt. The central equality of the fiscal theory of the price level no more induce simple relationships. For example, expectations of smaller future surpluses can result in current or future inflation.

In Sargent and Wallace (1981) fiscal policy is taken as given and it does not depend on current or future monetary policies. Another crucial assumption is that the real interest rate exceeds the growth rate of the economy. If the monetary authority finances interest payments and the exogenous deficits by issuing more debt, the second assumption implies that the real debt would grow faster than the size of the economy. Because this is limited by the demand for bonds, seigniorage

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[3] Real value of government liabilities (debt and the monetary base) equals real value of net assets of the private sector.

[4] Leeper (1991), Sims (1994) and Woodford (1994, 1995) use one-period debt.

is inevitable sooner or later. Hence, the monetary authority can make money tighter now only by making it looser later. If the demand for money depends on the expected rate of inflation, tighter money now can mean higher inflation now. This happens because now the current price level depends on both current and anticipated future money supplies. If the monetary authority cuts the current money supply, this must be compensated by higher money supplies in the future. These in turn raise current inflation via expectations.

In a way, our analysis is closely related to literature on balance-of-payments crises. In both cases the policy authority has access to some reserve that enables it to defend its policy goal. In the models of balance-of-payments crises the policy authority defends the exchange rate with foreign reserves, and in our model the low inflation regime can be defended with debt. Contrary to the simplest model of balance-of-payments crises, the breakdown of the zero inflation regime is not inevitable in our model if losses due to tax increases are bearable.

# 3 The model

We study the effects of fiscal policy on monetary policy by incorporating a policy authority's budget constraint in the model with a quadratic loss function and expectations and tax augmented Phillips curve. Via debt the intertemporal budget constraint links up the policy choices of different time periods.

The policy authority's nominal flow budget constraint is

$$B_{t+1} + T_t + M_t - M_{t-1} = (1 + i_t)B_t + G_t, \tag{3.1}$$

where $B_t$ is beginning-of-period nominal debt, $T_t$ is nominal taxes, $M_t$ is the end-of-period nominal money balances, i is the nominal interest rate and $G_t$ is nominal public expenditures. Hence, the policy authority finances its liabilities with new debt, taxes and growth in the money stock. Dividing equation (3.1) by $P_t$ gives

$$b_{t+1} + \tau_t + \frac{M_t - M_{t-1}}{P_t} = (1 + i_t)b_t \frac{P_{t-1}}{P_t} + g_t, \tag{3.2}$$

where $P_t$ denotes the price level during period t and all lower case letters are variables determined in real terms. Like in Woodford (1996), the value of nominal debt is deflated by the previous period's price level. This implies that the real debt $(b_t)$, like the nominal debt $(B_t)$, is a predetermined state variable at date t. In order to get intuitively easy real budget constraint, we define inflation rate between periods t and t–1 as

$$\pi_t = \frac{P_t - P_{t-1}}{P_t}. \tag{3.3}$$

Equation (3.3) implies that $1-\pi_t = P_{t-1}/P_t$. Equation (3.2) simplifies when we approximate the real interest rate $(1+r)$ by $1+r = (1+i_t)(1-\pi_t)$.[5] The change in desired real money balances can be divided into two parts when inflation is determined by equation (3.3) (see Obstfeld, 1991), namely inflation tax on previous period's real money holdings, and the change of real money holdings. Hence, the policy authority's real flow budget constraint is

$$b_{t+1} + \tau_t + \pi_t m_{t-1} + m_t - m_{t-1} = (1 + r)b_t + g_t. \tag{3.4}$$

In equation (3.4), real public expenditures $(g_t)$ and costs (including interest payments) due to previous period's debt are exogenous. The policy authority balances its budget with new debt $(b_{t+1})$, conventional taxes $(\tau_t)$ and seigniorage revenue $(\pi_t m_{t-1} + m_t - m_{t-1})$. The seigniorage revenue is a sum of the inflation tax on real money balances carried over from previous period and the desired increase in real money balances.

The budget constraints for each t can be summed to get the intertemporal budget constraint

$$(1 + r)b_t \le \sum_{i=0}^{\infty} \left( \frac{1}{1+r} \right)^i (\pi_{t+i} m_{t-1+i} + m_{t+i} - m_{t-1+i} + \tau_{t+i} - g_{t+i}). \tag{3.5}$$

According to equation (3.5), the maximum level of outstanding debt including interest payments is determined by the discounted sum of seigniorage revenues and surpluses $(\tau_{t+i} - g_{t+i})$. Hence, if the intertemporal budget constraint is not binding, the policy authority is able to generate, at the relevant policy optimum, tax and seigniorage revenues in excess of its current commitments, incorporated in the debt service costs and the present discounted value of expected future expenditure. Instead, if the intertemporal budget constraint is binding, higher debt levels are feasible only through a credible commitment to larger surpluses and seigniorage in the future. The timing and amount of these extra seigniorage revenues and surpluses can differ a lot. If the real interest rate is very low, timing is not very important. On the other hand, if the real interest rate is high, future increases of inflation and taxes are not powerful in financing current debt. Hence the policy authority finances the debt increase during a shorter period but with higher inflation and taxes. However, to solve for the paths of the policy variables,

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[5] Usually, the real interest rate is defined as $(1+r) = (1+i)/(1+\pi)$. The error we make here is of second order, because in this study $i-r-\pi-i\pi = 0$, instead of the normal $i-r-\pi-r\pi = 0$.

we have to use both the intertemporal budget constraint and the intertemporal loss function.

In this study, we assume that monetary policy always takes into account constraints of fiscal policy, ie debt and deficits. On the other hand, fiscal policy can always to some extent rely on seigniorage revenues, but not totally. This implies that the policy mix is somewhere between fiscal dominant and monetary dominant policy regimes. If the policy mix is predetermined, we can assume that there exist only one policy authority and he is responsible for both fiscal and monetary policies. The policy authority's one-period quadratic loss function is

$$l_t = (y_t - y^*)^2 + \alpha \pi_t^2, \tag{3.6}$$

where $y_t$ is output, $y^*$ is the desired level of output, $\pi_t$ is inflation and $\alpha$ captures the preferences of the policy authority. According to equation (3.8), the policy authority chooses a combination of inflation and output gap. The bigger is the weight put on inflation, the more reluctant is the policy authority to suffer losses due to inflation. Later when we study the problem of regime switches, we add a fixed cost into the loss function. The fixed cost captures effects of reputational costs due to a regime switch.

The output is given by the expectations and tax augmented Phillips curve

$$y_t = y + \delta(\pi_t - \pi_t^e) - \gamma \tau_t, \tag{3.7}$$

where $\pi_t^e$ is the expected inflation based on the information available in period $t–1$, $\tau_t$ is taxes and $\delta$ and $\gamma$ are positive constants. $y$ determines the constant level of output around which the actual output fluctuates. Equation (3.7) implies that unanticipated inflation and lower taxes have a positive effect on output.

We assume that money market equilibrium always holds. In our model the central bank supplies the amount of money that is demanded. The interest rate has no role in equilibrating money markets. The money demand, and hence the amount of money in the economy, is

$$m_t^D = ky_t = k\left[y + \delta\left(\pi_t - \pi_t^e\right) - \gamma \tau_t\right] = m_t^S = m_t, \tag{3.8}$$

ie output is multiplied by a constant k, which can be interpreted to be inverse of the velocity of money.

After substituting equation (3.8) into equation (3.5) it is easy to see how taxes and inflation affect the intertemporal budget constraint. Because the demand for real money balances depends (negatively) on taxes, the policy authority is able to affect seigniorage revenue with both taxes and inflation. Hence, taxes have both

14

direct (positive) and indirect (negative) effect on budget constraint, but inflation has only a direct (positive) effect on budget constraint.

# 4 Policy solutions after the possible regime switch

The policy authority minimises the sum of the discounted losses from the current period to infinity. However, to avoid problems concerning time consistency, we divide the minimisation problem in two parts. In this section we solve the policy authority's policy problem after the regime switch, ie in the discretionary policy regime. In the discretionary policy regime the policy authority has given up defending the zero inflation regime. Therefore, the policy authority is able to choose both inflation and taxes freely. When we have solved the policy problem in the discretionary policy regime, we are able to solve the minimisation problem in the zero inflation policy regime, and tie up these two solutions to get solution of the whole model.

The policy authority's minimisation problem in the discretionary policy regime is solved as follows. We substitute output equation (3.7) into the period loss function (3.6) and money market equilibrium (3.8) into the policy authority's flow budget constraint (3.4). After these substitutions the constrained minimisation problem facing the policy authority can be solved using the Lagrangean expression

$$
\begin{aligned}
\text{Lagr} = E_T \sum_{t=T}^{\infty} \beta^{t-T} & \left[ \left( y - y* + \delta\left(\pi_t - \pi_t^e\right) - \gamma\tau_t^a \right)^2 + \alpha\pi_t^2 \right] \\
& + E_T \sum_{t=T}^{\infty} \beta^{t-T} \lambda_t \left[ b_{t+1} + \tau_t^a + k\left(y + \delta\left(\pi_{t-1} - \pi_{t-1}^e\right) - \gamma\tau_{t-1}^a\right)\pi_t \right. \\
& + k\left(y + \delta\left(\pi_t - \pi_t^e\right) - \gamma\tau_t^a\right) - k\left(y + \delta\left(\pi_{t-1} - \pi_{t-1}^e\right) - \gamma\tau_{t-1}^a\right) \\
& \left. - (1+r)b_t - g_t \right]
\end{aligned}
\tag{4.1}
$$

where T is the period when the regime switch from the zero inflation policy regime to the discretionary regime occurs. $\tau_t^a$ denotes taxes after the regime switch and $\lambda_t$ denotes the Lagrange multiplier. Hence, we solve here how the policy authority optimally finances its expenditures with inflation, taxes and debt.

From now on we assume rational expectations. Because there is no uncertainty in our model, the model is solved with perfect foresight. The first partial derivatives of the Lagrangean with respect to $\pi_t$, $\tau_t^a$ and $b_{t+1}$, $t > T$, give us thefollowing first order conditions

$$2\delta(y - y* + \delta(\pi_t - \pi_t^e) - \gamma\tau_t^a) + 2\alpha\pi_t + \lambda_t k(y + \delta(\pi_{t-1} - \pi_{t-1}^e) - \gamma\tau_{t-1}^a)$$
$$+ \beta\delta kE_t[\lambda_{t+1}(\pi_{t+1} - 1)] = 0 \tag{4.2}$$
$$- 2\gamma[y - y* + \delta(\pi_t - \pi_t^e) - \gamma\tau_t^a] + \lambda_t(1 - \gamma k) + \beta\gamma kE_t[\lambda_{t+1}(1 - \pi_{t+1})] = 0$$

and

$$\lambda_t - \beta(1 + r)E_t\lambda_{t+1} = 0. \tag{4.3}$$

If we assume that $\beta(1+r)$ is equal to one, then $\lambda$ is constant. Constancy of $\lambda$ and the above assumptions imply that $\tau_t^a$, solved from the second equation of (4.2), is

$$\tau_t^a = \frac{1}{\gamma}(y - y*) + \frac{1}{2\gamma^2}[\beta\gamma k\pi_{t+1} + (1 - \beta)\gamma k - 1]\lambda, \tag{4.4}$$

where we have used the fact that $\pi_t = \pi_t^e$ for all t. The bigger is the gap between the desired level of output (y*) and the constant level of output (y), the more the policy authority lowers period t taxes. Tax cut implies higher output, and thus the actual period t output is closer to the desired output. In order to have positive taxes, the expression in paranthesis has to be negative. When we choose a reasonable set of parameters, we see that the expression in paranthesis is negative. Therefore, the effect of period t+1 inflation on period t taxes is negative. This is quite obvious result because higher inflation in the future induces higher seigniorage revenues, and hence the need for tax financing is lower.

Substituting $\tau_t^a$ and $\tau_{t-1}^a$ from equation (4.4) into equation (4.2) and rearranging, we obtain

$$\pi_t = \frac{k[(1 - \beta)\gamma k - 1]}{\gamma(4\alpha - \beta k^2\lambda^2)}\lambda^2 - 2\frac{\delta + \gamma ky*}{\gamma(4\alpha - \beta k^2\lambda^2)}\lambda. \tag{4.5}$$

Equation (4.5) implies that $\pi_t$ is constant. Hence, according to equation (4.4), also $\tau_t^a$ is constant. If we substitute the constant solutions of $\pi$ and $\tau^a$ into the intertemporal budget constraint (3.5), we get a fifth order equation for $\lambda$. It is impossible to get closed form solutions for all the endogenous variables. Instead, we use the first order conditions in a different way to solve this problem. However, as we will see below, the above constancy result proves to be very useful. From the second equation in (4.2) we have

$$\lambda = 2\gamma\frac{y - y* - \gamma\tau_t^a}{1 - (1 - \beta)\gamma k - \beta\gamma kE_t\pi_{t+1}}. \tag{4.6}$$

Hence, $\lambda$ can be solved explicitly from equation (4.6). It follows that substituting $\lambda$ into the first equation in (4.2) and using the above constancy result of $\pi$ and $\tau^a$, yields

$$\gamma^3 k(\tau^a)^2 - \gamma(\delta + 2\gamma ky - \gamma ky^*)\tau^a - \alpha\beta\gamma k\pi^2 + \alpha[1 - (1-\beta)\gamma k]\pi$$
$$+ (\delta + \gamma ky)(y - y^*) = 0. \tag{4.7}$$

In order to get solutions of $\pi$ or $\tau^a$ with respect to exogenous variables, we use the intertemporal budget constraint. The intertemporal budget constraint implies that when $\lambda$, $\pi$, $\tau^a$ and g are constants, and thus m is constant, also the maximum level of debt is constant. Utilising the well known properties of infinite geometric sum, the intertemporal budget constraint reduces to

$$ky\pi + (1 - \gamma k\pi)\tau^a = rb^{max} + g. \tag{4.8}$$

Equation (4.8) determines the maximum level of debt that the policy authority is able to issue without losing its fiscal solvency. The maximum level of debt is constant and indebtedness is always equal to this level after the possible regime switch. Each period constant public expenditures and interest payments are financed with constant seigniorage revenue and taxes. Hence, the policy solution is identical in all periods. Equation (4.8) is forward looking, therefore implying for example a negative relationship between public expenditures and debt. If public expenditures are cut, the maximum possible level of debt increases. However, if there is no reason to increase debt, the policy authority can cut taxes and inflation. Solving $\tau^a$ from equation (4.8) and substituting it into equation (4.7) implies

$$\alpha\beta\gamma^3 k^3\pi^4 - \alpha\gamma^2 k^2[2\beta + 1 - (1-\beta)\gamma k]\pi^3 + \gamma k[\delta\gamma ky * + \alpha\beta + 2\alpha(1 - (1-\beta)\gamma k)]\pi^2$$
$$+ [(\gamma^3 k^2 y * - \delta\gamma^2 k)(rb + g) + \delta\gamma ky - 2\delta\gamma ky * - 2\gamma^2 k^2 yy * - \alpha(1 - (1-\beta)\gamma k)]\pi$$
$$- \gamma^3 k(rb + g)^2 + (\delta\gamma + 2\gamma^2 ky - \gamma^2 ky*)(rb + g) - \gamma ky^2 - \delta(y - y^*)$$
$$+ \gamma kyy* = 0. \tag{4.9}$$

where $\pi$ and $b^{max}$ are endogenous variables. Equation (4.9) implies that zero inflation cannot be sustained as an optimal solution when the intertemporal budget constraint is binding.[6] This implies that the zero inflation policy regime inevitably

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[6] From the first equation of (4.2) and equation (4.6) we can solve $\tau^a$ which would be required to sustain $\pi=0$ as a solution of equation (4.9). These solutions of $\tau^a$ are $(\delta+\gamma ky)/\gamma^2 k>y$ and $(y-y^*)/\gamma<0$ which can be ruled out, given out we require nonnegative taxes. The latter solution yields a negative level of indebtedness which the policy authority is able to maintain. Hence, also this solution can be ruled out because the policy authority has no initial savings.

collapses when debt reaches its maximum value, which determined by the post-collapse paths of inflation, taxes and public expenditures.

We cannot solve for the levels of the four endogenous variables because we have only three equations. However, solving for the tax and debt policies in the zero inflation policy regime as well as solving the regime switching problem between the two regimes provides us with one extra equation which enables us to solve the whole model.

# 5      Policy solutions in a zero inflation policy regime, and the problem of regime switching

The general conclusion from the above analysis is that at the policy optimum the policy authority's commitments are financed by seigniorage revenues and taxes. Now we assume that the policy authority follows a zero inflation policy. Hence, fiscal commitments have to be financed by taxes and debt. However, debt has an upper bound, which is determined by post-collapse paths of inflation, taxes and public expenditures. The zero inflation policy regime collapses when the debt reaches its maximum value, and the policy authority is reluctant to increase taxes further.

The policy authority's minimisation problem can be solved by using the following Lagrangean expression

$$
\begin{aligned}
\text{Lagr} = {} & E_0 \sum_{t=0}^{T-1} \beta^t \left[ y - y * - \delta \pi_t^e - \gamma \tau_t^b )^2 \right] \\
& + E_0 \sum_{t=0}^{T-1} \beta^t \mu_t \Big[ b_{t+1} + \tau_t^b + k(y - \delta \pi_t^e - \gamma \tau_t^b) \\
& \quad - k(y - \delta \pi_{t-1}^e - \gamma \tau_{t-1}^b) - (1+r)b_t - g_t \Big] \\
& + \beta^T \left[ \frac{1}{1-\beta} \left[ (y - y * - \gamma \tau^a)^2 + \alpha \pi^2 \right] + C \right],
\end{aligned}
\tag{5.1}
$$

where the post-collapse part of losses, ie the term in the square brackets, is independent of pre-collapse paths of taxes and debt. The first partial derivatives of the Lagrangean with respect to $\tau_t^b$ and $b_t$ give us the first order conditions (t<T)

$$
-2\gamma \left[ y - y * - \delta \pi_t^e - \gamma \tau_t^b \right] + \mu_t (1 - \gamma k) + \beta \gamma k E_t \mu_{t+1} = 0
\tag{5.2}
$$

and

18

$$\mu_t - \beta(1 + r)E_t\mu_{t+1} = 0. \tag{5.3}$$

When expectations are rational and $\beta(1+r)$ is equal to one, equation (5.3) implies that $\mu$ is constant. Hence, according to equation (5.2), also $\tau^b$ is constant.

As we have shown, Lagrange multipliers are constant both in the zero inflation policy regime ($\mu$) and in the discretionary policy regime ($\lambda$). In order to solve how $\mu$ and $\lambda$ are related to each other, we can take the first partial derivative of the whole minimisation problem with respect to debt. The first partial derivative implies (the proof is provided in appendix 1) that $\mu$ is equal to $\lambda$. For now on we use $\lambda$ to denote the constant Lagrange multiplyer in both policy regimes.

As in the discretionary policy regime, the first order conditions (5.3) and (5.2) and our assumptions imply that we get a constant solution for taxes. When we iterate the flow budget constraint backwards, we get

$$b_t = (1 + r)^t b_0 + \frac{(1 + r)^t - 1}{r}(g - \tau^b), \tag{5.4}$$

where $t<T$, ie the policy authority is maintaining the zero inflation policy regime. Constancy of $\tau^b$ and g and the assumption that $\tau^b - g < rb_0$ imply that debt starts to increase. This in turn imply that interest payments increase. Hence, debt path is non-linear. It is worth noting that this is the case even if $\tau^b = g$. Interest payments which are financed by issuing debt yield a non-linear growing debt path.[7] If $\tau^b - g = rb_0$, debt stays at its initial constant level $b_0$.

We are able to solve for the number of periods the zero inflation policy regime can be maintained. The policy authority has two instruments to postpone the regime switch. Firstly, it can increase its indebtedness to the level determined by post-collapse paths of taxes, inflation and exogenous expenditures. Secondly, it can use taxes. The difference between these two instruments is that taxes decrease output and therefore affect directly pre-collapse losses. However, postponement of the regime switch resulting from higher pre-collapse taxes decrease discounted post-collapse losses. The zero inflation policy regime collapses when losses due to one extra period in zero inflation policy regime plus the fixed cost discounted by $\beta$ is equal to losses due to one period in discretionary policy regime plus the fixed cost, ie

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[7] This has some implications already mentioned in the introduction. The higher is the interest rate, the faster is debt growth. If we had a model with endogenous interest rate, the policy authority could improve its ability to maintain the zero inflation policy regime by lowering interest rates. Hence, if the level of indebtedness is high, the policy authority may be forced to lower interest rates even if it is not in line with the target of price stability.

$$\gamma^2(\tau^b)^2 + \beta C = \gamma^2(\tau^a)^2 + \alpha\pi^2 + C. \tag{5.5}$$

Where we have assumed that the output target and the steady state output are equal, $y = y^*$. Another condition associated with the regime switch is the debt constraint. The debt has to reach its maximum value, determined by post-collapse taxes, inflation and public expenditures, in the last period of the zero inflation policy regime. Hence,

$$(1+r)b_{T-1} + g - \tau^b = b_T^{max}, \tag{5.6}$$

where T is the period when the regime switch occurs. When we iterate equation (5.6) backwards we get

$$b_0 = \frac{b_T^{max}}{(1+r)^T} + \sum_{i=0}^{T-1}\left(\frac{1}{1+r}\right)^{i+1}(\tau^b - g). \tag{5.7}$$

Using the definition for finite geometric sum, T is determined by equation (5.7) as follows

$$T = \frac{\ln\left[\dfrac{b^{max} - \left(\dfrac{\tau^b - g}{r}\right)}{b_0 - \left(\dfrac{\tau^b - g}{r}\right)}\right]}{\ln(1+r)}. \tag{5.8}$$

Equation (5.8) determines how long the policy authority maintains the zero inflation regime. The lower is the initial debt ($b_0$), the longer the policy authority is able to postpone the collapse via debt financing. If $b_0$ equals the maximum level of debt, determined by the post-collapse paths of inflation and taxes, the zero inflation policy regime collapses immediately. On the other hand, depending on the preferences, the policy authority may be able to set higher pre-collapse taxes. Again, this postpones the regime switch because it reduces debt growth. It is worth noting, that also higher post-collapse taxes and inflation postpone the collapse of the zero inflation policy regime because then the policy authority is able to issue more debt.

Public expenditure, g, has two effect on timing of the regime switch. Firstly, public expenditure affects surpluses after the regime switch. Hence, higher public expenditures lower the maximum level of debt, and therefore shorten the time period when the zero inflation policy regime is maintained. Secondly, higher public expenditure lowers pre-collapse surpluses, and therefore makes the regime switch happen earlier. Because both these effects work in the same direction,

higher public expenditure has a strong adverse effect on sustainability of the zero inflation policy regime.

In this section we have solved how the policy authority uses debt and taxes to finance its expenditures in the zero inflation policy regime. We have also constructed an equation, (5.5) in which pre-collapse taxes are related to post-collapse taxes, inflation and fixed cost of the regime switch. In section 6 we solve the whole model by combining our results concerning the zero inflation and discretionary policy regimes.

# 6    Numerical solution of the model

In the policy optimum of section 4, we found constant solutions for inflation and post-collapse taxes, which in turn implied that the maximum level of debt is also constant. In section 5 we assumed that the policy authority is committed to a zero inflation policy regime. The regime collapses when debt reaches its maximum possible value, and the policy authority is reluctant to increase taxes. In terms of losses this means that the regime switch occurs when losses due to one period in zero inflation policy regime plus the fixed cost discounted by β is equal to losses due to one period in discretionary policy regime plus the fixed cost. However, we could not provide an analytical solution that combines the two parts of the model.

Here we collect all the relevant equations to solve the model numerically. We continue to assume that the desired level of output equals the constant steady state level of gross output, ie y* = y. This assumption removes one incentive to lower taxes and create surprise inflation shocks. However, it does not have crucial effects on how we can study the sustainability of the zero inflation policy regime when maximum level of debt is determined by post-collapse paths of inflation, taxes and public expenditures, which is the core of our study. The simplified model is

$$2\gamma^2\tau^b + \lambda - (1-\beta)\gamma k\lambda = 0, \tag{6.1}$$

$$2\delta\gamma\tau^a + 2\alpha\pi + k\lambda(y - \gamma\tau^a + \delta) + \beta\delta k\lambda(\pi - 1) = 0, \tag{6.2}$$

$$2\gamma^2\tau^a + \lambda - (1-\beta)\gamma k\lambda - \beta\gamma k\lambda\pi = 0, \tag{6.3}$$

$$ky\pi + (1 - \gamma k\pi)\tau^a = rb^{max} + g, \tag{6.4}$$

$$\gamma^2(\tau^b)^2 + \beta C - \left[\gamma^2(\tau^a)^2 + \alpha\pi^2\right] - C = 0. \tag{6.5}$$

and

$$T = \frac{\ln\left[\dfrac{b^{max} - \left(\dfrac{\tau^b - g}{r}\right)}{b_0 - \left(\dfrac{\tau^b - g}{r}\right)}\right]}{\ln(1 + r)}.$$  (6.6)

Equation (6.1) is the first order condition with respect to taxes in the zero inflation policy regime and equations (6.2) and (6.3) are the first order conditions with respect to inflation and taxes in the post-collapse policy regime. Equation (6.4) is the intertemporal budget constraint, which implies the maximum possible level of debt determined by post-collapse paths of inflation, taxes and exogenous expenditures. Equations (6.5) and (6.6) are the conditions of the regime switch. In equation (6.5) losses due to one period in zero inflation policy regime plus the fixed cost discounted by β is equal to losses due to one period in discretionary policy regime plus the fixed cost. Equation (6.6) determines how many periods the zero inflation policy regime is maintained.

In the model (equations (6.1)–(6.6)) we have six endogenous variables and eight parameters and exogenous variables.

Endogenous variables:

$\tau^b$      taxes bebore the regime switch
$\tau^a$      taxes after the regime switch
$\pi$      inflation after the regime switch
$b^{max}$      maximum level of debt determined by post-collapse inflation, taxes and public expenditures
$\lambda$      Lagrange multiplyer
T      number of period when the regime switch occurs

Parameters and exogenous variables:

$\alpha$      relative weight placed on inflation in the loss function
$\gamma$      tax elasticity of output
$\delta$      elasticity of output w.r.t. unanticipated inflation
k      inverse of the velocity of money
g      public expenditure (constant)
$b_0$      initial level of debt
C      fixed cost of a regime switch
r      real interest rate

The model is non-linear and hence quite sensitive to changes in values for parameters and exogenous variables. When considering the set of values for parameters and exogenous variables, we had to kept couple of things in mind. Firstly, the parameters and exogenous variables should have economically reasonable values. Secondly, the model with chosen set of parameters and exogenous variables should converge with high accuracy in order to enable us to calculate elasticies with sufficient accuracy. Finally, in order to emphasize characteristics of our model, we wanted to choose a set of parameters and exogenous variables that results in a solution in which the zero inflation policy regime does not collapse immediately. This requires that the relative weight placed on inflation in the loss function is small compared with the weight placed on taxes in the loss function. Another requirement is that the inverse of velocity of money has to be high enough. High k strengthens the effect of inflation on seigniorage revenues, and hence makes inflation more tempting. Also the fixed cost due to a regime switch is important. If C is low enough, the discretionary policy regime is always preferred to the zero inflation policy regime, hence causing an immediate breakdown of the the zero inflation policy regime.

The model is solved as follows. First we solve the five equations (6.1)–(6.5) system in which the discretionary policy regime and the problem of the regime switch are tied together. We get solutions for the discretionary policy variables and for pre-collapse taxes. When we have solutions for post-collapse taxes and inflation, the maximum possible level of debt is determined. Hence, when the initial level of debt is known, we can calculate from equation (6.6) how many periods the zero inflation policy regime is maintained. After trying different values for parameters and exogenous variables, we decided to use the set of values presented in table 1. Also the numerical solution of the model is presented in table 1.

Table 1. **Numerical solution of the model with a given set of values for parameters and exogenous variables**

$\alpha=0.05$, $\gamma=0.45$, $\delta=0.2$, $k=0.6$, $g=0.42$, $C=0.05$, $r=0.03$ and $b_0=0$

| $\tau^b$ | $\tau^a$ | $\pi$ | $b^{max}$ | $\lambda$ | T |
|---|---|---|---|---|---|
| 0.404 | 0.388 | 0.155 | 1.490 | –0.165 | 45.795 |

In the model the constant steady state level of gross output (y) is assumed to be equal to one. Hence, public expenditure, taxes and debt can be interpreted to be percentage shares of the steady state or full employment GDP. As we can see from table 1, taxes decrease after the zero inflation policy regime collapses.

Hence, output increses, but only 45 per cent of the 1.2 percentage cut in taxes. Inflation in the discretionary regime is 15.5 percentage points annually. The zero inflation policy regime is maintained 45.5 periods after which the policy regime is discre-tionary. The debt (to GDP ratio) is 1.49 when the regime switch occurs and it remains at this level also in the discretionary policy regime.

It is also worth stressing what is the magnitude of the fixed cost due to the regime switch compared with other losses. From the one period loss function we can calculate the one period loss in the discretionary policy regime using results of the basic model. The one period loss is 0.032,[8] which means that the fixed cost is roughly 1.58 times the one period loss. The discounted infinite sum of one period losses is 1.088, ie the fixed cost is about 4.6 percent of discounted losses due to the discre-tionary policy. One might argue that the fixed cost is too low, but this criticism can be omitted when we compare the fixed cost to the difference of one period losses due to the two policy regimes. It turns out that the difference of the one period loss due to the zero inflation policy regime and the one period loss due to the discretionary policy regime is only 2.7 per cent of the fixed cost. This comparison implies that the fixed cost is surely high enough. Therefore the policy authority wants, when minimising discounted losses, to postpone the regime switch as long as 45.5 periods.

In order to find out the effects of parameters and exogenous variables on endogenous variables, we calculate elasticities around the basic solution of the model. The calculated elasticities are presented in table 2.

Table 2.  **Elasticities of endogenous variables with respect to parameters and exogenous variables**

|  | $\tau^b$ | $\tau^a$ | $\pi$ | $b^{max}$ | $\lambda$ | T |
|---|---|---|---|---|---|---|
| $\alpha$ | 0.29 | 0.37 | −1.82 | −0.03 | 0.29 | 4.19 |
| $\gamma$ | −1.85 | −2.33 | 10.36 | −1.93 | 0.16 | −26.58 |
| $\delta$ | 0.30 | 0.62 | −7.59 | −7.88 | 0.30 | −0.10 |
| k | −0.85 | −1.33 | 10.36 | 8.46 | −0.84 | −7.31 |
| g | 0 | 0 | 0 | −9.40 | 0 | −19.67 |
| C | 0.27 | 0.28 | −0.05 | 2.22 | 0.27 | 5.14 |
| r | 0.27 | 0.27 | 0.06 | 1.37 | 0.28 | 4.20 |

Note: The elasticities are calculated in a standard way. First we solved the model numerically using the chosen set of exogenous parameters and variables (the basic solution presented in table 1). After that we solved the model numerically when the values of parameters and exogenous variables were changed 0.1 percentage points one after another. The resulting percentage changes of endogenous variables are divided by 0.001.

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[8] Although inflation is a substantial part of policy authority's revenues after the regime switch, it is only about 3.8 per cent of total one period losses due to a discretionary policy regime.

Most of the results presented in table 2 are very intuitive, but some of the elasticities seem to be at odds with what one would expect. However, it turns out that also these elasticities can be explained, usually by the maximum level of debt. We will next discuss these elasticities, concentrating, naturally, on cases in which intuition is not very straightforward.

The trade-off between inflation and pre- and post-collapse taxes is directly affected by the following three parameters. $\alpha$ denotes the relative weight placed on inflation in the loss function, $\gamma$ denotes the tax parameter in the output equation and k denotes inverse of the velocity of money. A rise in $\alpha$ lowers inflation and raises taxes. The effect of $\alpha$ on inflation is stronger than on post-collapse taxes, which results in a slight reduction in the maximum possible level of debt. On aggregate $\alpha$ has a positive effect on T, ie the time of the regime switch is postponed, because the reduction in $b^{max}$ is more than cancelled out by the increase in pre-collapse taxes. Contrary to $\alpha$, a rise in $\gamma$ or k results in lower taxes and higher inflation. The effect is stronger on post-collapse than on pre-collapse taxes. Because a rise in $\gamma$ decreases post-collapse taxes more than it raises inflation, the maximum possible level of debt decreases. On the contrary, a rise in k yields a strong positive effect on inflation and a smaller negative effect on post-collapse taxes, hence resulting in a higher maximum possible level of debt. Elasticities of $\tau^b$ and $b^{max}$ with respect to $\gamma$ are both negative, therefore resulting a strong negative effect of $\gamma$ on T. Although the maximum level of debt is reduced by the rise in $\gamma$, the policy authority cuts also pre-collapse taxes to avoid losses due to a bigger $\gamma$. However, elasticities of $\tau^b$ and $b^{max}$ may have opposite signs as well, as is the case with respect to k. Lower pre-collapse taxes dampen the positive effect of higher $b^{max}$ on T.

The fixed cost C and the real interest rate have quite similar effect on endogenous variables. However, the effect of C is slightly more powerful and also the transmission mechanisms differ. Higher C implies that the policy authority is more willing to postpone the abandonment of the zero inflation policy regime. Hence, the policy authority raises pre-collapse taxes ($\tau^b$) and allows higher debt level, which requires higher sum of post-collapse tax and seigniorage revenues. An interesting detail of the model is that although the maximum level of debt increases, inflation actually slightly decreases when C rises. The reason for this is as follows. Higher post-collapse taxes imply lower real money balances, which dampen the effect of inflation on seigniorage revenue. Hence, inflation looses

some of its power. When C rises, the policy authority finances its commitments with higher post-collapse taxes and lower inflation.[9]

In all, the policy authority equates the reduction in the loss due to postponement of the regime switch to the extra losses due to higher pre- and post-collapse taxes.

A rise in the real interest rate increases interest payments and decreases the discount factor. The policy authority is able to reduce the discounted sum of losses by postponing the regime switch. Hence, the policy authority increases pre-collapse taxes and post-collapse taxes and inflation, which result in a higher level of debt. This reveals the important role of the fixed cost. The advantage to postpone the realisation of the fixed cost exceeds the disadvantage of higher losses in the zero inflation policy regime.

Elasticities of endogenous variables with respect to public expenditure can be extracted from our model in a relatively straightforward manner. The level of public expenditure affects only the maximum level of debt and therefore timing of the collapse of the zero inflation policy regime. These effects are quite strong because they are not dampened by higher taxes or inflation in our model. This is an extreme feature of our model and potentially needs to be reconsidered in a more realistic setting. This would be the case if, for example, the level of public expenditure affects output.

The initial level of debt also has no impact on taxes and inflation. The initial debt level affects only timing of the regime switch and the debt path before the collapse. If $b_0$ is equal to the maximum level of debt, determined by post-collapse paths of taxes, inflation and public expenditure, the regime switch occurs immediately. Naturally, the smaller is $b_0$, the longer the zero inflation policy regime can be maintained. In table 2 we assumed that $b_0$ is zero, which implied that the zero inflation policy regime was maintained 45.8 periods. However, if $b_0$ is 0.3 the zero inflation policy regime can be maintained only 30.4 periods. If the initial debt level equals 0.6, the EMU-criterion, T is equal to 19.8. Although the initial debt level has no impact on policy variables, it directly affects sustainability of the zero inflation policy regime. Hence, lower debt levels help governments to commit to some policy targets because interest payments are then lower and the economy (public sector) is less vulnerable to the effects of adverse shocks hitting the economy. In short, lower debt levels provide governments better cushions for unfavourable circumstances.

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[9] It is worth noting that also a combination of higher inflation and lower post-collapse taxes would be sufficient to finance policy authority's commitments. In this case higher inflation would dampen the effect of $\tau^a$ on revenues. However, this would mean that the effects of higher C on taxes before and after the regime switch would have opposite signs, which would contradict the relevant necessary optimality conditions.

When we solved our model numerically, we chose a solution in which there existed a primary deficit, ie $\tau$–$g$ was negative. In this case all (positive) initial debt levels surely imply a breakdown of the zero inflation policy regime. The lower the debt level, the longer the zero inflation policy regime is maintained. Although the maximum possible level of debt is determined by post-collapse paths for inflation and taxes, it highlights the importance of fiscal solvency. Usually inflation is thought to ensure fiscal solvency in the long run. But if interest rates adjust to inflation expectations, and if we rule out explosive paths for inflation, monetization of debt is impossible in perfect foresight equilibria. However, high debt results in high interest rates if fiscal solvency can not be ensured by future budget surpluses. Thus both debt and deficit requirements matter. Neither of these requirements is sufficient alone, but together they provide fiscal discipline that is essential for central banks when they fight for price stability.

# 7    Conclusions

This paper considers the sustainability of a zero inflation policy regime from the perspective provided by the fiscal theory of the price level. The fiscal theory of the price level in its simplest form states that the equilibrium price level whatever makes the real value of nominally denominated government liabilities equal to the present value of expected future real government budget surpluses. Hence, if coventional fiscal instruments do not ensure the above equality, the path of the price level must do so. In our analysis discretionary solutions of the model provide us post-collapse paths of inflation and taxes that determine the maximum level of debt. The zero inflation policy regime collapses when two conditions are met. Firstly, the debt reaches its maximum level, and secondly, losses due to pre-collapse taxes equal gains due to postponement of losses which the policy authority faces if the zero inflation policy regime collapses.

Our main results are as follows. The timing of the collapse depends on the initial debt ($b_0$) and on pre-collapse taxes. The lower is the initial debt, the longer the policy authority is able to postpone the collapse via debt financing. If $b_0$ equals the maximum level of debt, determined by the post-collapse paths of inflation and taxes, the zero inflation policy regime collapses immediately. The timing also depends on the level of government expenditure. On the other hand, depending on the preferences, the policy authority may be able to set higher pre-collapse taxes. Again, this postpones the regime switch because it reduces debt growth. Also higher post-collapse taxes and inflation postpone the collapse of the zero inflation policy regime because then the policy authority is able to issue more debt.

The full elimination of debt by inflation is probably not an issue in industrialised countries nowadays. It would require an explosive path of inflation, because interest rates react to inflation expectations, thus dampening the effect of inflation on debt. Higher interest rates would also reduce economic growth. Hence, the costs of hyperinflation would be too high for it to constitute a policy option. In industrialised countries, inflation is one source of revenue, but usually very small and stable. However, when financial markets and economies as a whole are not very stable, inflation is a tempting alternative to erode debt. Therefore, for example Russia and some Latin American countries have sometimes resorted to high inflation. The temptation to use inflation in financing budget deficits depends also on access to credit markets. For governments with access to credit markets, the link between budget deficits and inflation is indirect because deficits can be temporarily financed by issuing government bonds. On the other hand, for governments with limited access to credit markets or that have strongly relied on money creation, deficit cuts usually have a direct beneficial effect on price stability. Therefore, the need for reduction in budget deficits in order to achieve price stability is most obvious in developing countries and economies in transition.

Nowadays almost all countries have committed to a low inflation target. Hence, seigniorage is not an important source of revenue. However, as we have shown, the ability to maintain the low inflation policy regime depends on the level of public debt. When a low debt country faces a positive expenditure shock, it can use debt to dampen the effects of the shock on taxes and inflation. Instead, when a high debt country faces a positive expenditure shock, its ability to use new debt is limited. This implies that price stability may be threatened by large enough expenditure shocks – such as caused by population ageing – if the initial debt level is high. Consequently, even a mere probability of this happening might conceivably weaken the credibility of monetary policy.

# References

Canzoneri, M.B. – Behzad, D. (1996) **Fiscal Constraints on Central Bank Independence and Price Stability.** CEPR Discussion Paper 1463.

Cochrane, J.H. (1998a) **A Frictionless View of U.S. Inflation.** NBER Working Paper 6646.

Cochrane, J.H. (1998b) **Long-term Debt and Optimal Policy in the Fiscal Theory of the Price Level.** NBER Working Paper 6771.

Dornbusch, R. (1996) **Debt and Monetary Policy: The Policy Issues.** NBER Working Paper 5573.

Leeper, E.M. (1991) **Equilibria Under "Active" and "Passive" Monetary Policies.** Journal of Monetary Economics, 27, 129–147.

Obstfeld, M. (1991) **Dynamic Seigniorage Theory: An Exploration.** CEPR Discussion Paper 519.

Sargent, T.J. – Wallace, N. (1981) **Some Unpleasant Monetarist Arithmetic.** Federal Reserve Bank of Minneapolis Quarterly Review, Fall, 1–17. (Reprinted in Rational Expectations and Inflation, New York: HarperCollins College Publishers, 1993)

Sims, C.A. (1994) **A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy.** Economic Theory, 4, 381–399.

Woodford, M. (1994) **Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy.** Economic Theory, 4, 345–380.

Woodford, M. (1995) **Price Level Determinacy Without Control of a Monetary Aggregate.** Carnegie-Rochester Conference Series on Public Policy, 43, 1–46.

Woodford, M. (1996) **Control of the Public Debt: A Requirement for Price Stability?** NBER Working Paper 5684.

# Appendix 1

## The proof of equality of Lagrange multipliers in zero inflation and discretionary policy regimes

The policy authority minimises the sum of discounted losses in the two policy regimes. This constrained minimisation problem is expressed by the Lagrangean

$$
\begin{aligned}
\text{Lagr} = E_0 \sum_{t=0}^{i+n-1} \beta^t \Big[ (y - y* - \delta\pi_t^e - \gamma\tau_t^b)^2 \Big] \\
+ E_0 \sum_{t=0}^{i+n-1} \beta^t \mu_t \Big[ b_{t+1} + \tau_t^b + k(y - \delta\pi_t^e - \gamma\tau_t^b) - k(y - \delta\pi_{t-1}^e - \gamma\tau_{t-1}^b) \\
- (1+r)b_t - g_t \Big] + E_0 \sum_{t=i+n}^{\infty} \beta^t \Big[ y - y* + \delta(\pi_t - \pi_t^e) - \gamma\tau_t^a)^2 + \alpha\pi_{t\,\text{sup}} 2 \Big] \\
+ \beta^{i+n} C + E_0 \sum_{t=i+n}^{\infty} \beta^t \lambda_t \Big[ b_{t+1} + \tau_t^a + k(y + \delta(\pi_{t-1} - \pi_{t-1}^e) - \gamma\tau_{t-1}^a)\pi_t \\
+ k(y + \delta(\pi_t - \pi_t^e) - \gamma\tau_t^a) - k(y + \delta(\pi_{t-1} - \pi_{t-1}^e) - \gamma\tau_{t-1}^a) \\
- (1+r)b_t - g_t \Big]
\end{aligned}
$$

$$\text{(A1.1)}$$

which is the sum of the zero inflation policy regime Lagrangean and the discretionary policy regime Lagrangean. In the text we showed that debt is constant starting from the period when the regime switch occurs. Hence, $b_{i+n}$, $b_{i+n+1}$, $b_{i+n+2}$, ... can be replaced by $b^{\text{max}}$. It is easy to see that $b^{\text{max}}$ ties together the two policy regimes. Taking the first partial derivative with respect to $b^{\text{max}}$ yields

$$
\begin{aligned}
\frac{\partial \text{LAGR}}{\partial b^{\text{max}}} = \beta^{i+n-1}\mu_{i+n-1} + \beta^{i+n}\big[1 - (1+r)\big]\lambda_{i+n} + \beta^{i+n+1}\big[1 - (1+r)\big]\lambda_{i+n+1} \\
+ \beta^{i+n+2}\big[1 - (1+r)\big]\lambda_{i+n+2} + \beta^{i+n+3}\big[1 - (1+r)\big]\lambda_{i+n+3} + ... = 0.
\end{aligned}
$$

$$\text{(A1.2)}$$

Using the constancy results of $\mu$ and $\lambda$, the definition of infinite geometric sum and dividing by $\beta^{i+n-1}$, the equation (A1.2) reduces to

$$\mu - \beta\frac{1}{1-\beta}r\lambda = 0, \qquad \text{(A1.3)}$$

which implies that $\lambda = \mu$.

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