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Research Department
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# Declining labour share – Evidence of a change in the underlying production technology?

Suomen Pankin keskustelualoitteita
Finlands Banks diskussionsunderlag

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# Declining labour share – Evidence of a change in underlying production technology?

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## Abstract

The study demonstrates that the decline in the labour share in Finland can not be explained by the Cobb-Douglas production function. Instead, we propose an approach based on the constant-elasticity-of-substitution (CES) production function with labour- and capital-augmenting technical progress. The model is augmented by imperfect competition in the output market. According to the empirical results based on estimation of the first-order-conditions, the technical elasticity of substitution is significantly less than unity (0.6) and hence the Cobb-Douglas production function is rejected. The growth rate of the estimated labour-augmenting technical progress has decreased in recent years, which is not consistent with the 'new-economy' hypothesis. Capital-augmenting technical trend has exploded during the same period, which provides a possible explanation for the rapid growth of the Solow residual. The main contributing factor behind the declining labour share is, however, the increasing mark-up.

Keywords: production function, elasticity of technical substitution, input-augmenting technical progress, new economy

# Heijastaako palkkatulojen laskeva kansantulo-osuus tuotantotekniikan muutosta?

## Suomen Pankin keskustelualoitteita 10/2001

Antti Ripatti – Jouko Vilmunen
Tutkimusosasto

## Tiivistelmä

Palkkojen kansantulo-osuuden laskua ei voida selittää tavanomaisella Cobb-Douglasin tuotantofunktiolla (CD), joka nimenomaan perustuu vakioiseen funktionaaliseen tulonjakoon. Tarkasteltaessa syitä palkkojen kansantulo-osuuden laskuun tutkimuksessa nojataan vakioiseen substituutiojoustoon perustuvaan tuotantofunktioon (CES), jota täydennetään termeillä, jotka kuvaavat pääoman ja työvoiman tuottavuutta lisäävää teknistä kehitystä. Tämän lisäksi oletetaan, että kilpailu hyödykemarkkinoilla on epätäydellistä. Tuotantofunktion parametrien, hintamarginaalin ja teknisen kehityksen estimointi perustuu yrityksen voitonmaksimointiongelman ensimmäisen kertaluvun ehtoihin. Saadut estimaatit viittaavat siihen, ettei CD sovellu kuvaamaan Suomen taloutta aggregaattitasolla ja että tuotantopanokset ovat toisiaan täydentäviä (estimoitu substituutiojousto on 0.6). Estimoidut teknisen kehityksen indikaattorit viittaavat siihen, että työn tuottavuutta lisäävän teknisen kehityksen kasvuvauhti on hidastunut merkittävästi 1990-luvun loppupuolella ja että samaan aikaan esiintyneen kokonaistuottavuuden kasvun kiihtymiseen on vaikuttanut pääoman tuottavuutta lisäävän teknisen kehityksen poikkeuksellisen voimakas kasvu. Palkkojen aiempaa alhaista kansantulo-osuutta on pitänyt yllä hintamarginaalien kasvu – erityisesti sähköteknisessä vientiteollisuudessa.

Asiasanat: tuotantofunktio, tuotantopanosten substituutiojousto, tekninen kehitys, uusi talous

# Contents

# 1 Introduction

According to the so called Kaldor facts, a set of empirical regularities — stylized facts — seem to characterize observed growth processes in several countries despite considerable cross country heterogeneity (Kongsamut, Rebelo and Xie 2000):

- Per capita output grows at a rate that is roughly constant
- The capital-output ratio is roughly constant
- The real rate of return to capital is roughly constant
- The shares of labour and capital in national income are roughly constant

These stylized facts suggest that several aggregate "great ratios" evolve smoothly over time and appear to provide a set of assumptions that can be exploited when constructing models of economic growth. In fact, as Kongsamut et al. (2000) argue, they have had an enormous impact on the construction of growth models. Also, since the *Cobb-Douglas functional form* for the production function is so widely used in the literature, many economists have come to believe that capital accumulation, technical progress and labour force expansion have no lasting effect on unemployment (Rowthorn 1999b).

Although it is true that 25 years of data maybe too short to serve as a basis for the evaluation of growth related proposition, Figures 1–3 plot data on the key macroeconomic great ratios for Finland over the period from 1975 onwards. From these figures we can try to take a stand as to whether the Kaldor facts are too stylized to be facts. First of all, apart from the recession in the 1990's, the growth rate of per capita output has been fairly stable over the last 25 years (figure 1). A qualification can be added that the post 1993 growth rate of per capita output may be somewhat higher than the pre-1990 average growth rate. The capital-output ratio, on the other hand, remained fairly stable, perhaps slightly falling, up till 1991 in the Finnish economy. Thereafter it has taken a sharp swing, initially rising due to the sharp fall in the aggregate output, and then, from 1993-94 onwards, it has fallen so that it is currently below the level it was prior to the 1990's (figure 3). Also, the real rental price of capital has remained remarkably stable over the sample period, although its short run fluctuations can be nontrivial (figure 3). Finally, the data indeed seems to suggest (figure 2) that the share of labour in national income also fluctuated around a constant prior to the recession in early 1990's; after peaking in 1990-91 labour's share has, however, collapsed.

Clearly then, the data indicate that during the 1990's there have been nontrivial changes in the great ratios in the Finnish economy. The observed changes have taken place after the onset of the deep recession in the Finnish economy in early 1990's. Most likely, this recession was not a typical growth slowdown at the business cycle frequencies, but also, and perhaps mostly, a response of the economy to a structural shock generated by the sharp fall in the trade with the former Soviet Union, when the countries switched to convertible currencies in the beginning of 1991. It is thus possible, and certainly conceivable, that this type of a shock triggered, or even necessitated, deep changes in the Finnish economy which, subsequently, have shown up as changes in eg the observed great ratios.

Since the post recession period in the Finnish economy is particularly a period of the rise and high growth of the IT sector and, more controversially, of increased diffusion of the implied new technologies in the economy, it its clearly conceivable that the underlying changes in the economy has been predominantly technological in nature. However, this being said, one should not forget that the end-1980's boom coincided with a period of financial market deregulation, while the deep recession in the early 1990's involved a banking crisis. Hence, the post-recession period is also characterized by the restructuring of the banking sector, and more generally, of the financial markets amid the process of increasing international integration of financial markets. A plausible conclusion from these developments appears to be that the financial market pressure on the Finnish economy has increased considerably in the 1990's; in particular even a considerable increase in the required return on capital may have resulted from these financial market developments.

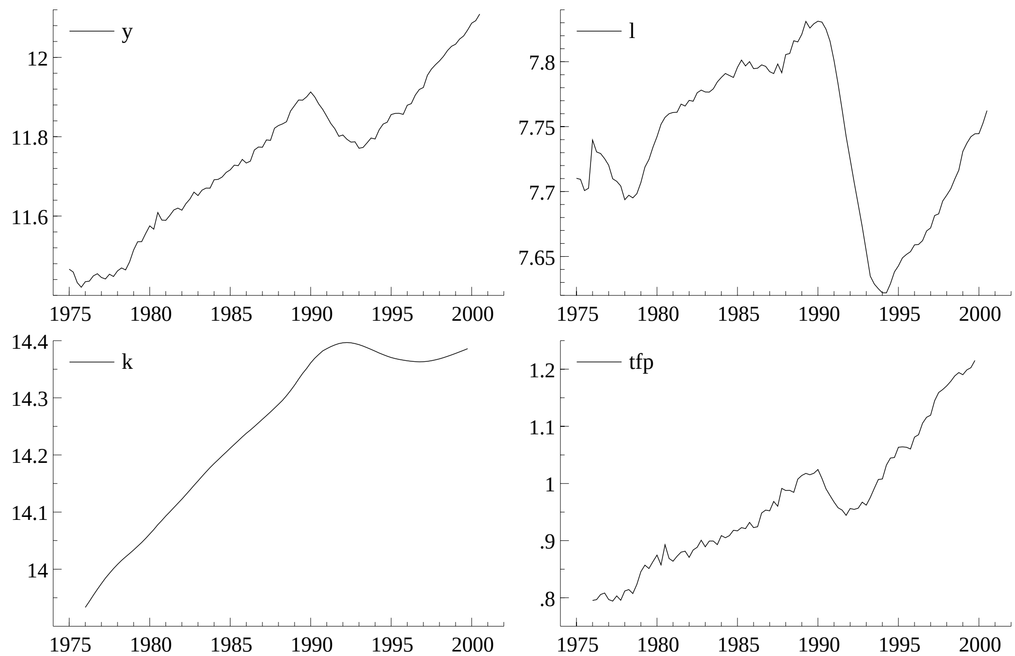
Therefore, the interesting question seems to be whether the deep recession in early 1990's marked a difference in the Finnish economy as to the structure of firms' technology or whether the observed changes in the great ratios reflect other factors, like changes in the composition of aggregate production, changes in the pricing behaviour of Finnish firms (mark-ups etc.) or, more generally, fundamental changes in the market structure. If there has been a technological change, then this could have resulted in a move away from the benchmark Cobb-Douglas aggregate production function which, on balance, was often supported by the pre-1990 data on aggregate output and which was frequently used in simulations incorporating aggregate supply behaviour. If indeed the elasticity of technical substitution between capital and labour is different from one (Cobb-Douglas case), then capital augmentation in technical progress could be an important source of shifts in the labour share.[1] However, other factors, such as locally increasing returns to scale in, for example, the IT sector coupled with an upward (local) trend in the (aggregate) mark-up, could also be present to explain the observed change in the labour share.[2]

In this paper, we seek a perspective on these issues. More specifically, we analyze the basic properties of the CES production function after which we provide estimates of the parameters of the assumed specification using quarterly data on aggregate capital stock, employment, output, real wages and rental price of capital in Finland over the post 1975 period. The first issue addressed in the estimation is the size of the elasticity of technical substitution. The data strongly suggest that labour and capital are gross complements, ie that the elasticity is less than one. Secondly, our econometric approach provide us with a framework for deriving estimates of the processes driving capital and labour augmenting technical progress as well as of the process underlying the dynamics of the (aggregate) mark-up. Perhaps surprisingly, our estimates indicate relatively strong capital-augmentation in technical change as

[1] Acemoglu (2000) has shown that in a standard model of endogenous growth, where firms invest in input augmenting technical change, all technical progress will be labour-augmenting along the balanced growth path. Hence, under the standard assumptions for endogenous growth, the result that technical change will be purely labour-augmenting follows from profit maximizing incentives. However, along the transition path the economy will often experience capital-augmenting technical change, and, interestingly, as long as capital and labour are gross complements — the elasticity of substitution is less than one — it will converge to the balanced growth path.

[2] It should, perhaps, also be emphasized that increasing returns and imperfect competition could be reflected in the (cyclical) movement of total factor productivity (TFP), which is often thought to reflect only innovations to technology (Bils and Chang n.d., Hall 1988).

Figure 1 **Output, capital stock, employment and total factor productivity**

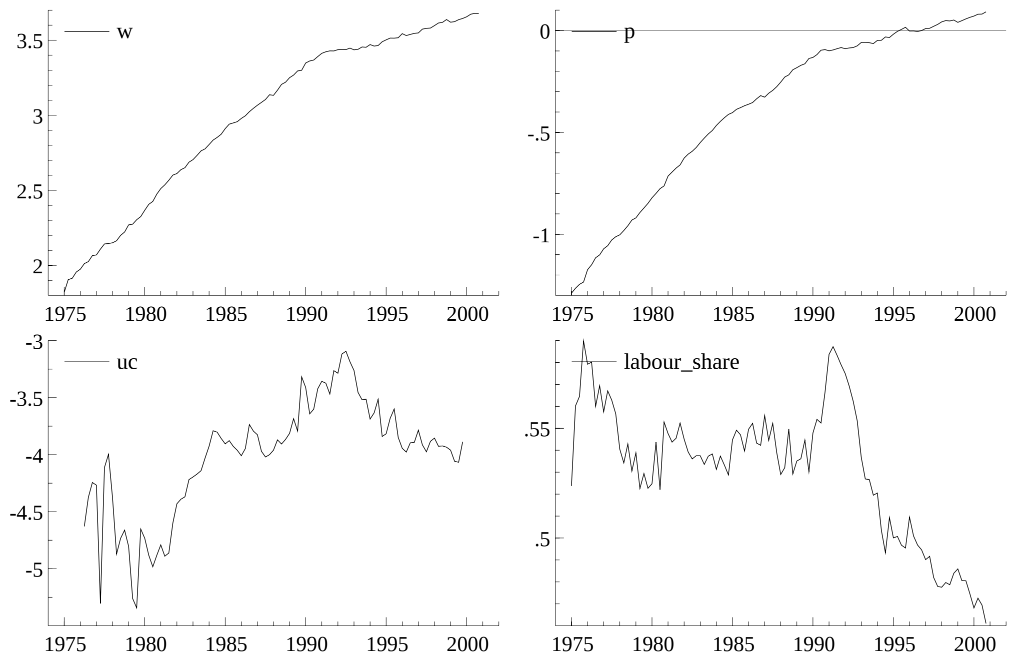



Note that the data are in logs. Output $y$ is measured as the GDP at 1995 prices; Employment is $l$; Capital stock $k$ is the capital stock of the whole economy; Total factor productivity is measured as a Solow residual computed from the standard Cobb-Douglas production function, where the parameter is estimated as the average labour share of the sample.

well as a clear upward trend in mark-up. Consequently, contrary to public conceptions, the growth rate of labour-augmenting technical progress has been modest during the post 1995 period. We argue that this is consistent with observing strong labour productivity during the recession due to destruction of low productivity jobs. Also, the hypothesis of capital-augmentation goes nicely with the fact the measures of aggregate capital stock indicate essentially no increase in it during the post 1993 period. Hence, we think that our decompositions are plausible.

The paper has the following structure. The next section reviews the basic properties of the Cobb-Douglas functional form. Admittedly, the Cobb-Douglas functional form is special, but, at the same time, possesses local properties that give a surprising sense of generality. It is also argued that the Cobb-Douglas aggregator is not much favoured by data. Section 3 introduces the CES production function and focuses on the analysis of the determinants of the labour share as implied by firms' (short-run) profit maximization. Naturally, the relationship between the capital-output ratio and the labour share is emphasized. The nature of this relationship is largely determined by the elasticity of substitution and factors that make the relationship shift are reviewed. Section 4 briefly reviews the profit maximizing input structure corresponding to the CES technology. Sections 5 and 6 take the CES to the data and review the estimation results. The last two sections discuss and conclude.

Figure 2 **Nominal rental prices of labour and capital, prices and labour share**




Note that the data are in logs. Nominal wage is measured as employee's quarterly wage. Price level is GDP deflator (1995=1).

# 2 Cobb-Douglas aggregator

## 2.1 Special, but has some surprising features ...

The constant returns to scale (CRS) Cobb-Douglas functional form[3]

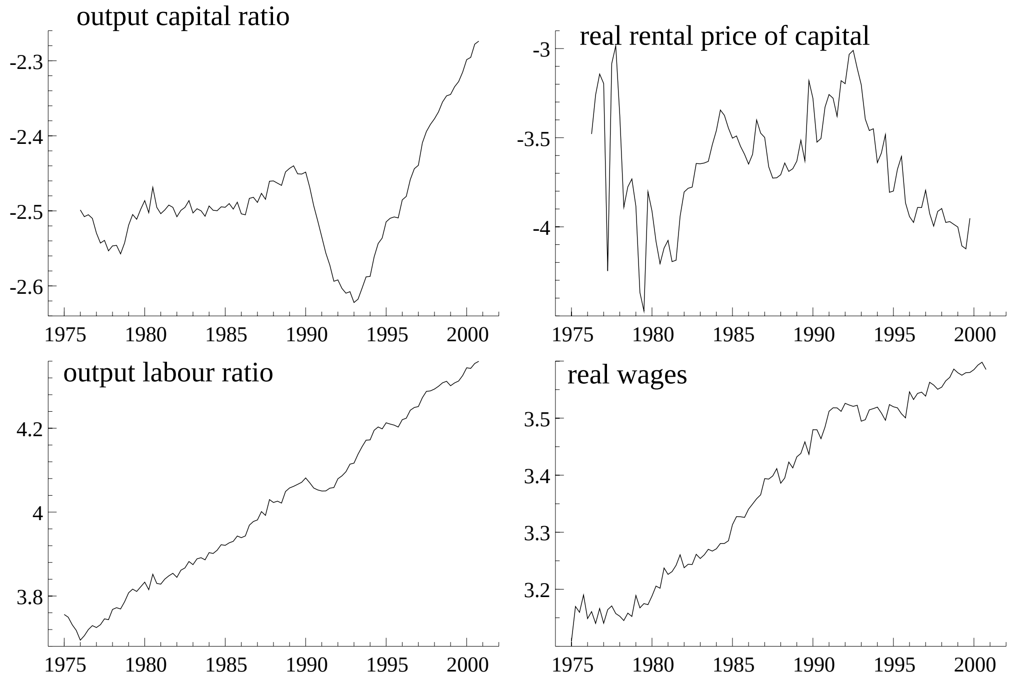
$$Y = AK^{\alpha}L^{1-\alpha}$$

is very special, but at the same time has, as demonstrated eg by Mitchell (1990) in the context of approximating underlying cost functions, some peculiar and surprising features. As is well known, it embeds an assumption of constant shares of labour and capital in national income, equal to their respective output elasticities $\alpha$ and $1-\alpha$ (ie $1-\alpha = WL/pY = s_L$ and $\alpha = RK/pY$). This is a particularly strong property, since it implies that in this type of a Cobb-Douglas economy, different types of shocks do not affect the relevant income shares, so that the technology, coupled with the (competitive) price system has a built in risk-sharing mechanism. Thus, from the point of view of hedging and risk-sharing, the existence of highly sophisticated asset markets may actually not be necessary.[4]

[3]We shall present the functional form in terms of the production function, where $Y$ is the (aggregate) output, $K$ and $L$ refer to (the rental services of) capital and labour, respectively and $A$ denotes total factor productivity (TFP).

[4]In a different context, similar redundancy of markets for (internationally traded) securities under Cobb-Douglas preferences (over domestic and foreign consumption indices) is emphasized by Obstfeld and Rogoff (1998, 1999). See also Corsetti and Pesenti (2001).

Figure 3 **Capital and labour productivities and the corresponding rental prices**




Note that the data are in logs. The three months money market rate is used as a short term interest rate in the computation of the real rental price of capital. The inflation expectations are based on the filtered stochastic slope of the log of investment deflator. The depreciation is computed from the capital accumulation equation.

On the other hand, Mitchell (1990) demonstrates that a continuous cost function has a first-order Taylor series approximation interpretation[5] if and only if it is the dual of a homogenous Cobb-Douglas production function. That is, *all* Taylor series expansions of cost functions, when expanded jointly in prices and output, will collapse to the Cobb-Douglas form when terms of second- and higher-order are dropped (Mitchell 1990, p. 513). This result is really surprising, since it not only implies that the cost function is homogenous of degree one in prices, as dictated by economic theory, but that in "order to have a first-order Taylor series approximation interpretation it must be *homogenous in output as well*" (Mitchell 1990, p. 513, italics added). Because of unit substitution elasticities (as well as constant output elasticity), the Cobb-Douglas functional form is so restrictive so that probably the most important conclusion from Mitchell's analysis is to use, whenever needed, a higher than first-order approximation to the (unknown) underlying cost function. That is, take at least a second-order approximation to the cost function and work

[5]Even though the exact definition of a first-order Taylor series approximation interpretation — or, more generally, of generalized quasilinear functions — is certainly more general, the basic idea can be nicely captured by starting from a n$^{th}$-order ($n = 2$ typically) Taylor series approximation of a (continuously differentiable) function at a given point. This approximation should be interpreted generally in the sense that it can be done in units other that the natural units of the variables. Anyway, the question now is what is the outcome when we delete all the second- and higher-order terms from the approximation.

out, if possible, how the restrictions imposed by economic theory on cost functions, impinge upon the approximate structure.[6]

The analysis by de La Grandville and Klump (1999)[7] on the relationship between elasticity of (input) substitution and (Solow type) economic growth contributes to demonstrating the special nature of the Cobb-Douglas functional form.[8] The authors work with a class of functional forms called the normalized constant elasticity of substitution (CES) functions; these are CES functional forms indexed by the elasticity of substitution parameter $\sigma$, ie two such functions differ only in the value of the substitution parameter $\sigma$.[9] They demonstrate that, for two economies with a common capital-output ratio, equal population growth rates and equal investment (-saving) rates, the one with higher elasticity of (input) substitution will, over time, have a higher per capita income. Also, if the model admits a finite steady state, that the under these same conditions, an economy with higher elasticity of substitution will have a higher capital intensity as well as higher per capita income in the steady state.[10] An additional corollary of these two findings is that the growth rate of per capita income is higher in an economy where technical input substitutability is higher.[11]

In a number of contributions, Rowthorn[12] has provided a critical analysis of the restrictions implied by the Cobb-Douglas functional form particularly on (long-run) unemployment. He notes that most of the literature on the causes of the dramatic rise in the European unemployment over the last 20 years has focused on labour market issues, such as wage fixing institutions, the role of welfare benefits as well as the quality and motivation of the workforce, whereas eg capital formation has played at best a secondary role (Rowthorn 1999b, p. 413). Commenting critically on the influential econometric work of Layard and Nickell,[13] on British unemployment, where, according to the cross equation restrictions, investment has no permanent effect on

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[6]Of course, the point raised by Blackorby, Primont and Russel (1978) in the context of applying flexible functional forms is relevant here. That is, applying flexible functional forms to cost functions is more difficult than applying them to eg utility or production functions, since for the former, the regularity conditions are much stronger.

[7]See also de La Grandville (1997), Klump (1997), Klump and Preissler (2000).

[8]According to Solow (1956) the form of the CES function he used in his comparative growth analysis, $Y = \left(aK^{1/2} + L^{1/2}\right)^2$, "offers a bit of variety" to the more usual ones of "Harrod-Domar" and "Cobb-Douglas".

[9]A particular class of normalized CRS-CES functions is determined by fixing three critical ratios; the output-labour (ie labour productivity) and capital-labour ratios, $y = Y/L$ and $k = K/L$, as well as the marginal rate of technical substitution $F_L/F_K = [f(k) - kf'(k)]/f'(k)$. The ratios serve to determine a common point of tangency of the CES function within a particular family.

[10]In the original Solow (1956) growth model, the critical threshold for the savings rate $s$ to generate investment large enough for perpetual income growth is $s = \frac{n}{a^2}$, where $n$ denotes population growth and $a$ is a parameter intimately related to capital augmentation and income distribution in Solow's specification of the production function. de La Grandville (1989) shows how this critical value of the savings rate depends on the elasticity of substitution $\sigma$; $s = n\beta(\sigma)^{\sigma/(1-\sigma)}$, where $\beta(\sigma)$ is capital's coefficient in the corresponding normalized CES production function $y = Y/L = A(\sigma)\left[\beta(\sigma)k^{(\sigma-1)/\sigma} + (1-\beta(\sigma))\right]^{\sigma/(\sigma-1)}$ (for more details, see de La Grandville and Klump (1999, p. 284)).

[11]See Klump and Preissler (2000) for a more detailed analysis of how the elasticity of substitution affects different facets of growth (ie the nature of the steady state, growth rate as well as the speed of adjustment).

[12]See Rowthorn (1995, 1999a, 1999b).

[13]See Bean, Layard and Nickell (1986).

unemployment, as well as on their subsequent book on European unemployment[14] Rowthorn argues that these analyses share a common weakness, the postulated high substitutability between labour and capital. This implies, in particular, that variations in real wages have a large effect on employment, so that investment in new capital stock actually leads to no net job creation in their model. More precisely, since the authors assume a Cobb-Douglas production function, the implied labour demand function is so elastic, that wage increases generated by investment in new capital stock leads to a loss of employment on existing equipment which is enough to offset entirely the extra jobs created on new equipment (Rowthorn 1999b, p. 414). Rowthorn argues that assuming a Cobb-Douglas form is unrealistic,[15] and provides a thorough analysis of the case whether elasticity of substitution between labour and capital is different from, and in particular below unity.

Rowthorn builds his model on the CES production function

$$Y = \left[\alpha\left(\Lambda_L N\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\Lambda_K K\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

ie technical process involves input augmentation $(\Lambda_L$ and $\Lambda_K)$[16]. Otherwise, in a model with imperfectly competitive labour and goods markets, where wages are determined by noncooperative Nash bargaining between industry unions and firms under the constraint imposed by firms' labour demand, he is able to show that, in a symmetric equilibrium, long-term unemployment in the model is determined by an equation of the form

$$u = 1 - \frac{N}{L} = f(\upsilon)$$

where

$$f(\upsilon) = \frac{1-\kappa}{(1-b)\,\phi\left\{\epsilon_{SN}\left[1 - \frac{(\kappa+\rho)(1-\alpha)}{(1-\kappa)\upsilon^{\rho}(\kappa+\rho)(1-\alpha)}\right] + \frac{\kappa}{\beta}\left[1 - \frac{(1-\alpha)}{(1-\kappa)\upsilon^{\rho}+\kappa(1-\alpha)}\right]\right\}}$$

where $\rho = (1-\sigma)/\sigma > 0$, $\kappa = 1 - 1/\theta$, ie inverse of the mark-up, $\theta$ is the price elasticity of demand faced by firms, $\beta$ is the relative bargaining power of a union, $\epsilon_{SN} = \frac{N}{S}\frac{\partial S}{\partial N}$ is the elasticity of the share of union members, $S$, who will keep their job after the wage settlement w.r.t. employment $N$, $\phi$ is a constant and where $b = \frac{B}{W}$ is the replacement ratio. The "technological" variable $\upsilon = \frac{\Lambda_K K}{Y}$ is implicitly determined by

$$\left[\upsilon^{\rho} - (1-\alpha)\right]\left[1 - f(\upsilon)\right]^{\rho} = \alpha\left[\frac{\Lambda_K K}{\Lambda_L L}\right]^{\rho}$$

Hence, $\upsilon$, which has the interpretation of a capital-output ratio measured in efficiency units, incorporates information about the production technology and affects long-term unemployment, or NAIRU for that matter, only to the extent that $\rho$ is different from zero. From the definition of $\rho$ we can immediately see that it is zero

[14]See Layard, Nickell and Jackman (1991, LNJ).

[15]See also his critical comments on Blanchard (1997, 1998), who assumes the elasticity of substitution is at least 1, in Rowthorn (1999a).

[16]Here $N$ denotes employment and $L$ labour force.

only when $\sigma$, the elasticity of substitution between capital and labour, equals one, only when $\sigma = 1$. But this is the Cobb-Douglas case, under which the equation determining long-term unemployment reduces to

$$f(\upsilon) = \frac{1 - \alpha\kappa}{(1-b)\,\phi\left[\epsilon_{SN} + \frac{\alpha\kappa}{\beta}\right]}$$

Hence, long-term unemployment is fundamentally determined by the degree of competition in labour and goods markets, $\beta$ and $\eta$, as well as labour market institutions, $b$ and $\phi$.[17] Comparatic statics are in this case well known, and can be found in eg Layard et al. (1991).

## 2.2 . . . and is largely data incongruent

On balance, estimates of the elasticity of substitution parameter $\sigma$ quite strongly suggest that it is significantly different from one. However there seems to be a considerable amount of uncertainty as to whether $\sigma$ exceeds or fall short of unity. In the former case labour and capital are technical substitutes, while technical complementarity results from $\sigma$ being less than one. The approach taken to estimate the substitution parameter appears to bear on its estimated size.

More specifically, Rowthorn (1999b) presents evidence on cross-country estimates of the substitution parameter, based on three sets of reported estimates of the elasticity of labour demand w.r.t. the real wage.[18] The number of countries varies from 16 to 19, so altogether 52 estimates of the substitution parameter are available. At best ten out of the 52 estimates exceed 0.5, of which only three are above unity and three in the vicinity of unity.[19]

Rowthorn's (1999b) estimates are derived from estimates of the elasticity of labour demand w.r.t. the real wage coupled with given values for the (short-term[20]) share of profits in national income, $s_\pi$, and elasticity of output demand w.r.t. to the price, $\eta$. This approach to obtaining estimates of the substitution parameter from estimates of labour demand elasticity is based on the fact, as shown by Rowthorn (1999b, Appendix), that under the CRS-CES production function and under the assumption that labour is rewarded according to marginal productivity, the relationship between elasticity of technical substitution $\sigma$ and elasticity of labour demand (w.r.t. wage) $\epsilon$ is

$$\epsilon = \frac{\kappa\sigma}{\kappa - (1 - s_\pi)} = \frac{\kappa\sigma}{\kappa - s_L}$$

where $\kappa = 1 - \frac{1}{\theta}$ $(= \frac{1}{\mu})$. Hence, given an estimate of $\kappa$ and $s_\pi$, we can obtain an estimate of the substitution parameter by solving this equation for $\sigma$

$$\sigma = \frac{\epsilon(\kappa - s_L)}{\kappa}$$

[17]Note that $u \to 0$ as $\beta \to 0$, ie unemployment vanishes under competitive labour market conditions.

[18]Estimates of the elasticity of labour demand are obtained from Layard et al. (1991), Newell and Symons (1985) and Bean et al. (1986).

[19]See Rowthorn (1999b, Table 2, p. 417).

[20]ie holding the capital stock fixed.

which reduces to

$$\sigma = \epsilon (1 - s_L) = \epsilon s_\pi$$

under perfectly competitive goods market ($\eta \to \infty$). But, as Rowthorn ironically notes, if this were the case, then, given a plausible value of $0.3$ for the capital's share $s_\pi$ and Cobb-Douglas technology, $\sigma = 1$, the implied value of the labour demand elasticity would be $3.3$. Such a value is totally implausible and much larger than typical estimates from econometric studies. These values would imply, interestingly, that a reduction in the real wage of only 2 – 3 would be enough to eliminate the whole of European unemployment using the existing amount of capital and existing technology (Rowthorn 1999b, p. 415)! Reducing the demand elasticity $\eta$ to a reasonable level of eg $5$, would, under the specified conditions, produce an estimate of the labour demand elasticity, $\epsilon$, of $8$, while reducing the substitution parameter to levels that we can observed in the data, ie to around $0.3$ would result in an labour demand elasticity of $1.1$ and $2.4$, far more reasonable figures in the light of the data[21].

Duffy and Papageorgiou (2000) provide further cross-country evidence concerning the aggregate production function specification. They use a panel of 82 countries over a 28-year period from 1960 to 1987 to estimate a general CES production function specification, and find, using different estimation methods as well as two alternative measures of labour input, that for the entire sample of countries they can reject the Cobb-Douglas specification. To be more specific, the estimated value of CES substitution parameter $\rho$ spans the range from $-0.2$ to $-0.7$, implying that the estimated rate of technical substitution between labour $\sigma$ is (statistically significantly) above one. However, their result also indicates that the growth rate of exogenous Hicks-neutral technological progress — or mean growth rate of TFP — is systematically negative, approximately $-1.5$% p.a.[22] This suggests that the period covered in the study can be characterized as one of technological regress in the countries included in the sample.

The authors note (p. 100) this feature of the estimation results, ie that ”the log of real GDP has, on average, declined over the period 1960 to 1987”, and experiment with the negative time trend by estimating a broken time trend function over the periods 1960 to 1973 and 1974 to 1987. The estimated coefficient for these time trends are still (slightly) negative, while rest of the parameters roughly retain their original estimated values. Later, in the context of linear estimation results, the authors report the results from an experiment that may at least partly explain this apparently puzzling estimation result. By allowing for country specific time trends, the authors find (p. 105), first, that 78% of the estimated time trend coefficients are significantly different from zero. Also, there is a mixture of negative and positive coefficients. 86% of the statistically significant coefficients are negative. *The countries with significantly positive time trend estimates tend to be among the more developed countries*. Hence, it is this form of heterogeneity among the countries that helps to explain the authors' result of negative coefficients on time trends in model specifications with a single common time trend (or a broken time trend). Also, this finding accords much better with one's preconception of a positive contribution to output from technical

[21] See eg Rowthorn (1999b, Table 1).

[22] See Table 1 (p. 99) in Duffy and Papageorgiou (2000).

progress as well as with many empirical studies showing positive technical progress at least among the more developed economies.[23]

A simple (regression) procedure to obtain an estimate of the substitution parameter builds directly on the idea that dates back to the birth, in the contribution of Arrow, Chenery, Minhas and Solow (1961), of the CES function. These authors observed that a power function of the form $y = cw^{\sigma}$ could fit the empirical relationship between national income per head ($y$) and the (product) real wage $w$.[24] They successfully tested this functional form with the data concluding that $\sigma$ is significantly less than one. Here we repeat this exercise on Finnish quarterly data over the period $1981(1) - 2000(3)$. A restricted cointegration analysis of the vector $(\ln Y_t, \ln L_t, \ln (W/P)_t, t)$ ($t$ a proxy for technical progress) indicates that the stationarity of the linear combination

$$\ln Y_t - \ln L_t - 0.309 \ln (W/P)_t - 0.005t$$

is not rejected by the data.[25] Hence, the estimated elasticity of substitution is $\sigma = 0.309$, which is considerably less than one. Now, given the estimate of $\sigma$, we can trace back the underlying CES technology by solving the ordinary differential equation

$$y = c \left[ f(k) - kf'(k) \right]^{\sigma}$$

where $y = Y/L$ and $k = K/L$, which results from using the first order condition for profit maximizing employment under CRS production technology.[26]

The fact that the elasticity of substitution is less than one has the important implication that the share of labour in national income tends to increase as the latter increases. Indeed, since $s_L = WL/PY = w/y$ and since $\sigma = \frac{dy/y}{dw/w} < 1$, we have $dw/w > dy/y$ so that $w/y$ increases. So real wages increase faster than labour productivity. Also, in the context of Solow or optimal growth model, positive endogenous growth cannot occur in the long run, when the elasticity of factor

[23] The authors estimate the linear regression specification also using alternative subsamples of countries. More precisely, they classify the countries into four groups according to the size of the capital-labour ratio $k$: high-$k$, middle-$k$, low middle-$k$ and low-$k$ (see Table 3, p. 109). Since per capita income and $k$ are highly positively correlated, the classification corresponds to wealth ranking of the countries. Now, the substitution parameter $\rho$ is significantly negative at $1\%$ or $5\%$ level only for the middle-$k$ countries ($\widehat{\rho} = -0.08992$). At $10\%$ it is positive ($\widehat{\rho} = 0.21204$) for the low-$k$ countries using the human capital based measure of labour input. Otherwise, the authors cannot reject the null of Cobb-Douglas (ie the hypothesis of $\rho = 0$).

[24] As the notation suggests, $\sigma$ signifies elasticity of technical substitution. $c$, on the other hand, is a constant that is equal to $(1/(1-\alpha))^{\sigma}$ under the CRS-CES $y = \alpha k^{(\sigma-1)/\sigma} + (1-\alpha)$.

[25] The hypothesis of CRS cannot be rejected over the sample period $1981(1) - 2000(3)$. Also, the dimension of the cointegration space is one (ie one cointegrating vector). The restricted cointegrating vector alluded to in the text involves restricting the coefficient of the employment variable to equal $-1$, given that the cointegrating vector is normalized by the coefficient of the output variable. For a more thorough analysis, see section 5.

[26] Technically, the ODE in the text corresponds to the so called Bernoulli equation:

$$y = f(k) = \left[ f(k) - kf'(k) \right]^{\sigma} \Leftrightarrow$$

$$\frac{d}{dk} \left[ f(k) \right]^{\frac{\sigma-1}{\sigma}} - \left( \frac{\sigma-1}{\sigma k} \right) \left[ f(k) \right]^{\frac{\sigma-1}{\sigma}} = \frac{\sigma-1}{c\sigma k}$$

which can be solved by standard methods, once the equation is transformed via a change of variable $z(k) = \left[ f(k) \right]^{(\sigma-1)/\sigma}$ into an ordinary linear differential equation.

substitution $\sigma$ is less than one. The principal reason is that, given $\sigma < 1$, marginal productivity falls down to zero asymptotically; if $\sigma < 1$, then $MP_K = f'(k) = \alpha\left[\alpha + (1-\alpha)k^{(1-\sigma)/\sigma}\right]^{1/(\sigma-1)} \to 0$, as $k \to \infty$. [27] On the other hand, the case of low technical factor substitutability, $\sigma < 1$, is also interesting in the context of the Solow growth model, since it is a necessary condition for the existence of multiple (ie two) locally stable steady states, as demonstrated by Galor (1996). All in all, then, the role of factor substitution is critically important in the growth context, since sufficiently high substitutability, more precisely $\sigma > 1$, is a necessary condition for long-run endogenous growth, while sufficiently low substitutability introduces the possibility of multiple long-run equilibria.

# 3 Factor substitution and the labour share; the SK schedule

The basic question we ask in this paper is whether the aggregate production technology in the Finnish economy should be taken from the class of CES technologies, where the elasticity of substitution between factors is different from one (Cobb-Douglas). In particular, is the observed decline in labour's income share an indication of a change in (aggregate) production technology or should it be attributed to some specific aspects of agents' behaviour, like pricing behaviour (changes in market structure and mark-ups) or to underlying changes in the composition of aggregate output due, most notably, to the much higher than average growth rate of the IT sector.

Since the CRS Cobb-Douglas production function is a special case of the CRS CES form, the subsequent discussion can be framed entirely in terms of deriving specific implications of assuming that the aggregate production technology is given by the CES production function

$$Y = F(K, L) = \left[\delta (XK)^{-\rho} + (1-\delta)(BL)^{-\rho}\right]^{-\frac{1}{\rho}}$$

where the meaning of all the variables and parameters are well known: $Y$, $K$ and $L$ stand for aggregate output, capital stock and labour, respectively; $\rho$ is the substitution parameter (ie $1/(1+\rho) = \sigma$ is the elasticity of technical substitution between capital and labour), while $\delta$ is the "distribution" parameter.[28] The specification above allows for input augmentation in technical progress, the exact nature of which is capture by the parameters $X$ and $B$; within this specification, the common factor driving $X$ and $B$ can be regarded as the total factor productivity (TFP) process. For many purposes it is better to write the CRS production function in its intensive form; defining $y$ as labour productivity in effective units, $y = \frac{Y}{BL}$, we can

[27] Labour productivity — both marginal and average — remains bounded away from zero; $\lim_{k\to\infty} y = (1-\alpha)^{\frac{\sigma}{\sigma-1}} = \lim_{k\to\infty}\left[f(k) - kf'(k)\right] = \lim_{k\to\infty} MP_L$.

[28] $\delta$ can be regarded as a distribution parameter in the sense that $\lim_{\rho\to 0} F(K, L) = \Lambda K^{\delta} L^{1-\delta}$ and we know that the Cobb-Douglas parameter $\delta$ measures the capital's share in income.

rewrite the CES function as

$$y = f(k) = F\left(\frac{K}{L}, 1\right) = \left[\delta k_L^{-\rho} + (1-\delta)\right]^{-\frac{1}{\rho}} \tag{1}$$

where $k_L = XK/BL$ is the capital-labour ratio in efficiency units.

Profit maximization implies that

$$1 - \delta\left(\frac{k_L}{y}\right)^{-\rho} = \mu\frac{WL}{pY} \equiv \mu s_L \tag{2}$$

where $\mu$ denotes the mark-up.[29] Defining capital-output ratio as $k_Y = K/Y$, the efficiency condition (2) for the use of labour input reduces to

$$1 - \delta\left(Xk_Y\right)^{-\rho} = \mu s_L$$

which, of course, further reduces to the familiar constant labour share condition, $1 - \delta = s_L$, under the Cobb-Douglas production technology $\rho = 0$. Hence, we can immediately deduce that, for a fixed mark-up and stable capital-augmenting technical progress, the effects of movements in the capital-output ratio, or movements in capital productivity $1/k_Y$, on labour's share $s_L$ depends critically on the nature of input substitutability as measured by the elasticity parameter $\rho = (1-\sigma)/\sigma$.[30] If labour and capital are technical complements, ie when $\rho > 0$ or $\sigma < 1$, labour's share will increase as the capital-output ratio increases (ie under falling capital productivity). The opposite is true, if labour and capital are substitutes, $\rho < 0$ or $\sigma > 1$. Anyway, writing, in the spirit of Bentolila and Saint-Paul (1999),

$$s_L = \mu^{-1} g\left(k_Y; X\right) \tag{3}$$

for the implied relationship between the labour's share and the capital-output ratio, clearly gives us a convenient framework to think about the factors that could possibly affect the evolution of the labour share. To reiterate, for a stable mark-up and capital augmenting technical progress, we should be able to pick up, from the data, the unique relationship between $s_L$ and $k_Y$. Also, we should observe shifts in the $g$-schedule, if there are shifts in the mark-up or if, at any given level of the capital stock, capital productivity shifts due to technical progress.

[29] $\mu = \frac{p}{mc} = \frac{\eta}{\eta-1}$, where $mc$ means marginal costs and $\eta$ denotes own price elasticity of output demand.

[30] Bentolila and Saint-Paul (1999) built their analysis of factors explaining movements in the labour share on what they call the *share-capital*, SK, schedule $s_L = g(k_Y)$ implied by the condition for profit maximizing use of labour input under the (implicit) assumption of a fixed mark-up. As they emphasize, this relationship is unique under the assumptions made, and is unaltered by changes in factor prices or quantities or in input augmenting technical progress. Changes in these factors will thus trigger changes in the labour share along the schedule, and cannot explain any deviation from the SK relationship, ie any residual in the equation $s_L = g(k_Y)$. As for accounting for these deviations, Bentolila and Saint-Paul (1999) experiment with factors that drive a wedge between the marginal product of labour and the real wage, with capital-augmenting technical progress and with factors that drive a wedge between the capital-output ratio and the employment elasticity of output $\eta = \left[f(k_L) - k_L f'(k_L)\right]/Lf(k_L)$. Thus imperfect competition in product or labour markets — mark-ups, unions etc. — or labour adjustment costs belong to the first category, while shifts in the production function due, for example, effects from imported materials as well as heterogeneity in the composition of the workforce are example of factors in the third category.

It should be noted that the approach taken here to the determinants of the labour's share strongly emphasizes the structure of firms' profits maximization. Hence, the relationship in (3) survives, as demonstrated by Bentolila and Saint-Paul (1999), a number of alternative ways to complete the model. For example, a model of wage determination could be incorporated into the set up to see how it possibly affects the behaviour of the labour's share and, in particular, how it impinges, if at all, on the relationship between the capital-output ratio and labour's share. To be more specific, think of a right-to-manage model of wage determination, where (utility maximizing) unions determine, within the limits of their bargaining power, the wage while firms, through profit maximization, determine employment. Since, in terms of employment, we are still on the labour demand curve, the preceding analysis is valid; in particular, the relationship between the labour's share and capital-output ratio remains intact.

If, on the other hand, wages and employment are determined by efficient bargaining or contracting, where firms and unions enter into a cooperative game to determine labour market outcomes, then it is well known that the efficient employment-wage pair should satisfy the conditions

$$w = \frac{F(K,L)}{L} - \left(\frac{1-\theta}{\theta}\right)\left(\frac{V-\overline{V}}{V_w}\right)$$
$$w = \beta\left[\frac{F(K,L)}{L}\right] + (1-\beta)\, F'_L\,(K,L)$$

(for a fixed capital stock $K$), where $V = V\,(w, L; K)$ denotes the (reduced form of the) unions' utility, and $\overline{V}$ is the fall-out utility. $\theta$ measures unions' bargaining power and $\beta = \beta_V$ denotes the weight attached to average productivity and it depends, in general, on $V$ (and, hence, on $w$, $L$ and $K$).[31] These two equations imply that the marginal product of labour satisfies

$$F'_L\,(K,L) = w - \left[\frac{\beta\,(1-\theta)}{\theta\,(1-\beta)}\right]\left(\frac{V-\overline{V}}{V_w}\right) \equiv \widetilde{w} \quad (< w)$$

where $\widetilde{w}$ denotes the real opportunity cost of labour.[32] Efficient bargaining thus entails — along the contract curve — choosing a wage-employment pair, where the wage rate is equal to the weighted average of average and marginal productivities.[33]

[31]It can be shown that $\beta = \left[1 + \left(\frac{1-\theta}{\theta}\right)\left(\frac{V-\overline{V}}{V_L L}\right)\right]^{-1}$. In the case of a utilitarian union we can choose $V - \overline{V} = [v\,(w) - v\,(b)]\, L$ ($v\,(w)$ is the (indirect) utility function of a representative employed worker, $v(b)$ that of an unemployed worker and $b$ represents unemployment benefits), so that in this widely used case, $\beta = \theta$.

[32]Note that since $\widetilde{w} < w$, a solution to the efficient bargaining problem displays excessively high employment relative to the case where the bargaining solution lies on the labour demand schedule. In particular, excess employment results relative to the competitive case, where $\theta = 0$ ($V = \overline{V}$).

[33]Two major implications follow from this particular feature of efficient bargaining. Firstly, there could be excess employment (relative to the competitive level). Secondly, efficient bargains are hard to implement, since firms have, ex post, all the incentives to renege on the contract and choose employment on the labour demand curve. This is so, because on the labour demand curve, labour is, from the point of view of firms, efficiently used, ie profits are maximized. There is a wage-employment pair that is sustained by an efficient bargain which gives firms the same amount of profits that the profit maximizing wage-employment pair generates. This is because an efficient bargain occurs at a point in the employment-wage plane, which is tangent to the unions' indifference curve and firms' isoprofit curve.

Furthermore, when the bargaining power of unions increases, $\beta$ also increases, so that under stronger union dominance, the wage rate more strongly reflects average productivity developments. In an economy where the labour market outcome is dominated by firms, on the other hand, the wage rate is more strongly pinned down by the marginal productivity of labour.

To derive more quantitative effects of efficient bargaining on the SK relationship, assume a utilitarian union the bargaining power of which is $\theta$ and a CRS firm with the production function $Y = XKf(\ell)$, $\ell = BL/XK$. Recalling that labour's share is given by $s_L = \theta + (1-\theta)\,\ell f'(\ell)/f(\ell)$, we finally have

$$s_L = \theta + (1-\theta)\, g(k; X)$$

which implies that in an efficient bargaining context, the bargaining power of unions will act on the SK relationship as a shift factor. In the CRS CES case, we can further show that

$$s_L = \theta + (1-\theta)\left[1 - \delta (Xk_Y)^{-\rho}\right] = 1 - (1-\theta)\,\delta (Xk_Y)^{-\rho}$$

Hence, the higher bargaining power of unions tends to make the SK schedule flatter in the $(k_Y, s_L)$ plane.[34]

The way we have specified aggregate production technology implies that changes or shifts in the production function come through input augmentation only. This, however, need not be entirely satisfactory, since intermediate inputs in the form of eg imported materials or heterogeneity in the labour force can be important sources of shifts in the SK schedule, as also argued by Bentolila and Saint-Paul (1999). Now, to account for imported materials, assume that the CRS production function is of the form $Y = F(XK, BL, M)$, where $M$ denotes imported materials. This can be rewritten as $Y = XKf(\ell, m)$, $\ell = BL/XK$ and $m = M/XK$. Define the value added as $\widetilde{Y} = Y - qM$, where $q$ is the relative (or real) price of imported materials. The SK schedule is now defined in terms of the value added: denote by $\widetilde{s}_L$ the share of labour in value added, $\widetilde{s}_L = WL/p\widetilde{Y}$ and by $\widetilde{k}_{\widetilde{Y}}$ the capital-valued added ratio

$$\widetilde{k}_{\widetilde{Y}} = \frac{1}{f(\ell, m) - qm}$$

so that the labour share in value added is

$$\widetilde{s}_L = \frac{wL}{\widetilde{Y}} = \frac{BLf_1(\ell, m)}{XK\left(1/X\widetilde{k}_{\widetilde{Y}}\right)} = \frac{X\ell f_1(\ell, m)}{1/\widetilde{k}_{\widetilde{Y}}}$$

The comparative statics effect of an increase in the relative price of imported materials, holding $\widetilde{k}_{\widetilde{Y}}$ constant, on the labour's share in valued added is, in general, ambiguous. Three effects are at work here:[35] first, in order to maintain a constant capital value added ratio as materials prices increase, the labour-capital ratio must

[34] It can be shown that after allowing for firms' mark-up, the SK schedule can be written as $s_L = 1 - (1-\theta)\left[1 - \delta (Xk_Y)^{-\rho}/\mu\right]$, which is slightly more complicated, but does not change the conclusion as to how unions' bargaining power impinges upon the SK schedule.

[35] See Bentolila and Saint-Paul (1999, p. 9).

rise, which pushes pushes the labour share up. Secondly, imports fall as their relative price increase. Consequently, if $f_{12}(\ell, m) > 0$, which perhaps is a realistic assumption, labour's marginal product falls, the wage rate falls as does the labour share. Thirdly, a negative effect comes from the fall in the wage rate induced by the required increase in the labour-capital ratio (taking into account of the induced (indirect) effect on $m$).

Extending the CRS CES production function to account for the presence of imported materials, ie assuming that the production function is

$$\begin{aligned} Y &= \left[\delta_K (XK)^{-\rho} + \delta_L (BL)^{-\rho} + (1 - \delta_K - \delta_L)(CM)^{-\rho}\right]^{-\frac{1}{\rho}} \\ &= XK\left[\delta_K + \delta_L \ell^{-\rho} + (1 - \delta_K - \delta_L) m^{-\rho}\right]^{-\frac{1}{\rho}} \end{aligned}$$

the labour share in the value added can be shown to be

$$\widetilde{s}_L = 1 - \left[1 - (1 - \delta_K - \delta_L)^{\frac{1}{1+\rho}} q^{\frac{\rho}{1+\rho}}\right]^{-(1+\rho)} \left(X\widetilde{k}_{\widetilde{Y}}\right)^{-\rho}$$

From this we can immediately see that the SK schedule will shift upwards as the relative price of imported materials increases if and only if $\rho < 0$, ie if and only if labour and capital are technical substitutes. This is so because, to maintain the capital-output ratio constant, higher substitutability implies that a lower wage fall is required to increase the labour-capital ratio when imported materials fall as their relative price increases. This contributes to increase the labour share.[36]

Recently, a fair amount of discussion has focused on the changes in returns to skills that have been observed in various countries since the mid 1970's. These development may have affected the evolution of the labour share, and, conceivably, changes in skill premia in general affect the SK schedule. Hence it would be desirable if the previous framework could be extended to account for differences in workers skills. However, the way heterogeneity is incorporated in the production function, ie the restrictions imposed on the production technology, turns out to be critical for the one-to-one relationship between the labour share and capital-output ratio. If the CRS production function takes the form

$$Y = H\left[XK, G(B_1 L_1, B_2 L_2)\right]$$

where the subaggregator $G$ is homogenous of degree one in the two types of labour, $L_1$ and $L_2$, then one can show[37] there exists a one-to-one relationship between the labour share and capital-output ratio, $s_L = g(k_Y)$, where $g$ depends only on $H$. If, on the other hand, there is more complementarity between skilled labour and capital than between unskilled labour and capital, it turns out that the wage ratio of the two types of labour enter the SK relationship. To be more specific, assume that the CRS CES production function takes the form

$$Y = \left[\delta (XK + B_1 L_1)^{-\rho} + (1 - \delta) B_2 L_2^{-\rho}\right]^{-\frac{1}{\rho}}$$

which[38] gives rise to the following expression for the labour share

$$s_L = 1 - \delta \left[\frac{1}{1 + \psi(\omega)^{\rho}}\right]^{\frac{1+\rho}{\rho}} Xk_Y$$

[36]Bentolila and Saint-Paul (1999, p. 10).

[37]Bentolila and Saint-Paul (1999, p. 11).

[38]The intuition behind this specification is that tasks can be done either by unskilled labour or capital and skilled labour is needed to monitor tasks Bentolila and Saint-Paul (1999, Appendix A).

where $\psi(\omega) = (B_1\omega/B_2)^{1/(1+\rho)} = \left(\frac{B_1 w_2}{B_2 w_1}\right)^{1/(1+\rho)}$, ie $\psi(\omega)$ is an increasing function of the wage premium $\omega = w_2/w_1$. So, an increase in the wage premium, *ceteris paribus*, will shift the SK schedule up in the $(k_Y, s_L)$ plane.

Clearly, then, if labour heterogeneity enters the production function through an aggregator which is homogenous of degree one in the two types of labour, the SK schedule will be unaffected by relative factor prices as well as relative factor supplies. Moreover, it is also unaffected by any change in the relative demand for skilled labour induced by technology, as long as this change shows up in the subaggregator $G$, but not in $H$.

Finally, a reformulation of the efficiency condition (2), which explicitly brings in the (long-run or structural) unemployment may also be useful, particularly in the context of how features of the "new economy", by making the problem of mismatch in the labour market more severe, could impinge on the labour's share. Defining $k_N = \frac{XK}{BN}$ as the capital-labour *force* ratio ($N$ for labour force), we have

$$1 - \frac{\delta\left(\frac{k_N}{1-u}\right)^{-\rho}}{1-\delta+\delta\left(\frac{k_N}{1-u}\right)^{-\rho}} = \mu s_L$$

where the unemployment rate is given by $u = \frac{N-L}{N}$. Hence, at any given level of the capital-labour force ratio $k_N$ and mark-up $\mu$, labour's share tends to increase, as the (long run) unemployment rate $u$ increases, if labour and capital are technical complements ($\rho > 0$). In the case of technical substitutes ($\rho < 0$), the opposite is true. All this comforms well with one's intuition, since low factor substitutability means that the production technology effectively constrains a firm from (profitably) reshuffling its input mix, once one observes changes in relative factor productivities. Or, when relative factor prices change, firms have only limited technical capabilities.

# 4 Profit maximizing input demands under the CES technology

Allowing for decreasing returns to scale the profit maximizing input structure $(L^\pi, K^\pi)$ corresponds to the solution

$$(K^\pi, L^\pi) = \arg\max_{(K,L)} \{pF(K,L) - WL - RK\}$$

where $W$ is the nominal wage rate and $R$ the nominal rental price of capital. The production technology is now given by

$$Y = \left[\delta(XK)^{-\rho} + (1-\delta)(BL)^{-\rho}\right]^{-\frac{\eta}{\rho}} \tag{4}$$

so that the CES form is homogenous of degree $\eta < 1$. The necessary F.O.C. for maximum profits are

$$\begin{aligned} \frac{\eta}{\mu}\left[\delta(XK^\pi)^{-\rho} + (1-\delta)(BL^\pi)^{-\rho}\right]^{-\frac{\eta+\rho}{\rho}}(1-\delta)B(BL^\pi)^{-(1+\rho)} = w \\ \frac{\eta}{\mu}\left[\delta(XK^\pi)^{-\rho} + (1-\delta)(BL^\pi)^{-\rho}\right]^{-\frac{\eta+\rho}{\rho}}\delta X(XK^\pi)^{-(1+\rho)} = r \end{aligned} \tag{5}$$

where the real input prices are denoted by small case letters. Consequently, the profit maximizing input structure is given by

$$L^{\pi} = (1-\delta)^{\sigma} B^{-1} \left[\frac{\mu}{\eta} c(w,r)\right]^{\frac{1}{(\eta-1)}} \left[\frac{w/B}{c(w,r)}\right]^{-\sigma}$$
$$K^{\pi} = \delta^{\sigma} X^{-1} \left[\frac{\mu}{\eta} c(w,r)\right]^{\frac{1}{(\eta-1)}} \left[\frac{r/X}{c(w,r)}\right]^{-\sigma}$$

where the real unit production cost or minimum unit cost function — the dual of the CES production technology at unit output level — is of the form

$$c(w,r) = \left[\delta^{\sigma}\left(\frac{r}{X}\right)^{\sigma\rho} + (1-\delta)^{\sigma}\left(\frac{w}{B}\right)^{\sigma\rho}\right]^{\frac{1}{\sigma\rho}}$$

Here, as previously, the elasticity of technical factor substitution is denoted by $\sigma = \frac{1}{1+\rho}$. The interpretation of the unit cost function $c(w,r)$ is that it corresponds to the minimum cost of obtaining the unit output level given that (real) unit input prices are $w$ and $r$.[39] It is a homogenous of degree one function, which implies that the nominal unit cost function $C(W,R)$, say, is given by $p \cdot c(w,r)$. Of course, under competitive goods market conditions, $\mu \to 1$, the profit maximizing labour and capital demand functions above are not well defined, if the production technology displays constant or increasing returns to scale, $\eta \geq 1$. To analyze these cases one usually resorts to cost minimization to derive the corresponding compensated (Hicksian) input demand functions

$$\left(K^{C}, L^{C}\right) = \arg\min_{(K,L)} \left\{WL + RK;\ F(K,L) \geq y\right\}$$

where $y$ is a fixed output level. This cost minimizing input structure is in the case of the CES production technology

$$K^{C} = \delta^{\sigma} X^{-1} \left[\frac{R/X}{C(W,R)}\right]^{-\sigma} y^{\frac{1}{\eta}}$$
$$L^{C} = (1-\delta)^{\sigma} B^{-1} \left[\frac{W/B}{C(W,R)}\right]^{-\sigma} y^{\frac{1}{\eta}}$$

An alternative approach starts from a "short run" perspective and assume that the capital stock is fixed and this is the reason for the production function to display decreasing returns to scale. Optimal use of labour in production is based on the restricted profit maxization problem, where only the amount of labour is derived from profit maximizing behaviour. The relevant optimality condition is given by the first equation in (5), but with the capital input fixed exogenously at level $K$. The implied restricted or short run labour demand function is given by

$$L^{\pi_{SR}} = \left[g(w_t;\mu)\right]^{\frac{1}{\rho}} \left(\frac{XK}{B}\right)$$

[39] Hence, $c(w,r)$ is the solution to the problem

$$\min\left\{wL + rK;\ F(K,L) \geq 1\right\}$$

Given that the CES function is homogenous, the cost function can be written as $C(W,R)\, y^{\frac{1}{\eta}}$, where $y$ denotes a fixed output level.

where the "normalized" labour demand function — ie equilibrium labour-capital ratio measured in efficiency units of capital, $L^*/XK$ — is given by $g(w_t;\mu) = [((1-\delta)B/\mu\omega)^{\sigma\rho} - (1-\delta)]/\delta$. Note that (effective) capital accumulation tends to make the labour demand schedule shift "up and to the right" over time, thus generating, at any given level of $w$ and $X$, dynamic increases in firms' labour demand.

# 5 Econometric setup

The econometric approach is based on the specification of the CES production function as reported in (4) in the previous section. The specification departs from the standard CES by allowing for non-constant returns to scale. In deriving the first order conditions of the profit maximization we rely on the assumption of the monopolistic competition. Due to this fact the mark-up, $\mu$, enters to the first order conditions. In order to exploit the possible changes in the competitive structure of the Finnish economy we allow the mark-up to vary over time. The same holds for the labour and capital augmenting technical changes $B_t$ and $X_t$.

Let small letters denote the logarithm of variables denoted by the respective capital letters and $e_t^y \equiv \log(1/\mu)$. The log-linearized version of the first order conditions are given by:

$$\log\eta(1-\delta) + e_t^y - \rho b_t + (1+\rho/\eta)y_t - (\rho+1)l_t - (w_t - p_t) = 0 \tag{6}$$
$$\log\eta\delta + e_t^y - \rho x_t + (1+\rho/\eta)y_t - (\rho+1)k_t - (uc_t - p_t) = 0, \tag{7}$$

where $w_t - p_t$ is the real wage and $uc_t - p_t$ is the real user cost of capital. We have three unobservables in the first order conditions: $e_t^y$, $x_t$ and $b_t$. The unobserved parts of the first order conditions (6) and (7) are blocked together as follows:

$$\tau_t^l \equiv \log(\eta(1-\delta)) + e_t^y - \rho b_t \tag{8}$$
$$\tau_t^k \equiv \log(\eta\delta) + e_t^y - \rho x_t. \tag{9}$$

Hence, the first order conditions can be abbreviated by

$$\tau_t^l + (1+\rho/\eta)y_t - (\rho+1)l_t - (w_t - p_t) = 0 \tag{10}$$
$$\tau_t^k + (1+\rho/\eta)y_t - (\rho+1)k_t - (uc_t - p_t) = 0. \tag{11}$$

We need further assumptions to identify these unobservables.

Our estimation problem is a multivariate nonlinear filtering problem, where the unobserved state variables are $e_t^y$, $x_t$ and $b_t$ and the measurement equations are given by (4), (7) and (6). In order to identify the three unobservables, one need to make assumption about their stochastic specification and combine this with further restrictions. We proceed by assuming that all unobservables are driven by stochastic trends and by analyzing the cointegrating features of the various restrictions.

## 5.1 Stochastic specification of unobservables

In order to be able to nest some interesting special cases we parameterize the unobservables as follows

$$b_t = \gamma_0^b + \gamma_1^b c_{1t} + \gamma_2^b (t/T) + s_t^b \quad (12)$$
$$x_t = \gamma_0^x + \gamma_1^x c_{1t} + \gamma_2^x c_{2t} + \gamma_3^x g_{xt} + s_t^x \quad (13)$$
$$e_t^y = \gamma_0^y + \gamma_1^y c_{1t} + s_t^y, \quad (14)$$

where $c_{it}$ is an independent random walk process:

$$c_{it} - c_{it-1} = \varepsilon_t^i, \quad \varepsilon_t^i \sim \text{iid}(0, \sigma_i^2) \quad i = 1, 2.$$

The $s_t^j$ ($j = b, x, y$) represent unspecified stationary processes with zero mean, ie $s_t^j \sim I(0)$. The deterministic function $g_{xt}$ is assumed to be characterized by the following logistic function:

$$g_{xt} \equiv g_x(t/T, \mu) = \{1 + \exp\left[-\gamma_g (t/T - \tau)\right]\}^{-1},$$

where the parameter $\tau$ determines the inflection point and the parameter $\gamma_g$ the steepness of the logistic function. Note that this specification allows for estimation of the "break point", which is determined by the parameter $\tau$.

The specifications (12) – (14) imply that — without further restrictions — the unobservables are driven by nonstationary stochastic trends. The labour-augmenting technical progress, $b_t$, contains a deterministic linear trend. In addition to this the capital augmenting technical progress, $x_t$, does not contain a drift term but is allowed to contain a deterministic level change of the logistic form.[40] The demand elasticity might also contain a stochastic trend but no drift is allowed.

Combining equations (8)–(9) and (12)–(14) we obtain the following specification for $\tau_t^l$ and $\tau_t^k$:

$$\begin{aligned} \tau_t^l =& (\gamma_0^y - \rho\gamma_0^b + \log[\eta(1-\delta)]) + (\gamma_1^y - \rho\gamma_1^b)c_{1t} - \rho\gamma_2^b (t/T) + (s_t^y - \rho s_t^b) \\ \equiv& \lambda_l^0 + \lambda_l^1 c_{1t} - \rho\gamma_2^b (t/T) + s_t^l \quad (15) \\ \tau_t^k =& (\gamma_0^y - \rho\gamma_0^x + \log(\eta\delta)) + (\gamma_1^y - \rho\gamma_1^x)c_{1t} - \rho\gamma_2^x c_{2t} \\ & - \rho\gamma_3^x g_{xt} + (s_t^y - \rho s_t^x) \\ \equiv& \lambda_k^0 + \lambda_k^1 c_{1t} + \lambda_k^2 c_{2t} - \rho\gamma_3^x g_{xt} + s_t^k. \quad (16) \end{aligned}$$

## 5.2 Cointegration and common trends

We may approximate the law-of-motion of the system with the vector error correction model as follows

$$\begin{aligned} \Delta z_t &= \nu + \sum_{j=1}^{p-1} \Gamma_j \Delta z_{t-j} + \alpha\beta^{*\prime} z_{t-1}^* + \varepsilon_t \quad \varepsilon_t \sim NID(0, \Omega) \\ &= \nu + \Gamma q_t + \alpha\beta^{*\prime} z_{t-1}^* + \varepsilon_t, \quad (17) \end{aligned}$$

[40] The discussion in the previous section relates to the choice of this function, which is assumed to be exogenous with respect to other variables.

where $z_t = [y_t \; l_t \; k_t \; (w-p)_t \; (uc-p)_t]'$ and $z_t^{*\prime} = [z_t' \; t/T \; g_{xt}]$, $q_t = [\Delta z_{t-1}' \cdots \Delta z_{t-p}']'$ and $\Gamma = [\Gamma_1 \cdots \Gamma_{p-1}]$.

Assume that $z_t \sim I(1)$ and some interesting hypotheses of the cointegrating rank emerge. They involve restrictions on the nonstationary components in (15) and (16). Given the first order conditions and the specifications of unobservables, the number of cointegrating vectors may vary from zero to two. We classify them according to table 1.

Table 1 **The hypothesis concerning the cointegration rank**

| Rank | | $\tau_t^l$: $\sim I(0)$, ie $\lambda_l^1 = 0$ | $\tau_t^l$: $\sim I(1)$, ie $\lambda_l^1 \neq 0$ |
|---|---|---|---|
| $\tau_t^k$ | $\sim I(0)$, ie $\lambda_k^1 = \lambda_k^2 = 0$ | 2 | 1 |
| | $\sim I(1)$, ie $\lambda_k^1 \neq 0$ and/or $\lambda_k^2 \neq 0$ | 1 | 1/0 |

Since we have two first order conditions, the number cointegrating vectors in (17) is at most two.[41] Two cointegrating vectors are obtained when the non-stationary parts of $\tau_t^l$ and $\tau_t^k$ are zero and the cointegration between other variables hold as is predicted by the economic theory. The parameter restriction is then $\lambda_l^1 = \lambda_k^1 = \lambda_k^2 = 0$. Another extreme, cointegrating rank zero, is obtained when $\tau_t^l$ and $\tau_t^k$ contain independent stochastic trends. This is implied by the restriction $\lambda_l^1 \neq 0$ and $\lambda_k^2 \neq 0$. The intermediate case can be obtained in three ways. It might be possible that one of the first order conditions is stationary — when conditioned on the deterministic variables. An interesting special case is obtained when a linear combination of the first order conditions is stationary. This is achieved with the parameter restriction $\lambda_k^2 = 0$.

These hypothesis are valid in the case when all the observable variables are $I(1)$ processes. This assumption might not hold for the real user cost, $(uc - p)_t$. If this is true then there will be an extra cointegrating vector in the system (17) and the above discussion may easily be generalized to such a case.

The cointegrating vectors, $\beta^*$, look very different in the cases discussed above. We study two special cases: the first relies on the first order condition with respect to labour (7) in the estimation of the parameters and the second one on the linear combination of the first order conditions.

It is typical, see eg Bolt and van Els (2000), to base the parameter estimation of the CES production on the first order condition with respect to labour. This means that $\tau_t^k$ is assumed to be nonstationary and $\tau_t^l$ stationary and, consequently, $\lambda_k^2 \neq 0$ and $\lambda_l^1 = 0$. It is also typical to assume a constant[42] mark-up process, $e_t^y$. We allow it to vary, possibly, in a nonstationary manner. The parameter restriction $\lambda_l^1 = 0$ implies that $\gamma_1^y = \rho\gamma_1^b$, ie the loadings of the common trend in $b_t$ and $e_t^y$ processes

[41] Assuming that there is no extra (linear) economic theory that relates these variables in the long-run.

[42] or stationary

are linearly dependent. The implied cointegrating vector,[43] $\beta^*$, is given by

$$\beta^{*\prime} = \begin{bmatrix}(1+\rho/\eta) & -(\rho+1) & 0 & -1 & 0 & -\rho\gamma_2^b & 0\end{bmatrix}. \tag{18}$$

The cointegrating space is identified and three (linear) over-identification restrictions are given. Estimation may be performed using the restricted linear estimation techniques and the deep parameters $\rho$, $\eta$ and $\gamma_2^b$ may be recovered from the normalized estimates of the cointegration vector. To obtain asymptotic standard errors one may use the delta method or the approach given in the next section.

Another, interesting, special case is the one where the unobservable processes share a common stochastic trend $c_{1t}$. This is obtained when the random walk component $c_{2t}$ is zero in (13), or, equivalently, $\lambda_k^2 = 0$. The implied cointegrating vector is the following:

$$\beta^{*\prime} = \begin{bmatrix}(\lambda_k^1 - \lambda_l^1)(1+\rho/\eta) & -\lambda_k^1(1+\rho) & \lambda_l^1(1+\rho) & -\lambda_k^1 & \lambda_l^1 & -\lambda_k^1\rho\gamma_2^b & \lambda_l^1\rho\gamma_3^x\end{bmatrix}.$$

It is easy to see that the linear combination $\begin{bmatrix}\lambda_k^1 & -\lambda_l^1\end{bmatrix}'$ of $\tau_t^l$ and $\tau_t^k$ cancels the common trend in (15) and (16). Note, however, that the cointegrating space spanned by $\beta^*$ in above formula is not identified. This means, in particular, that $\lambda_l^1$ and $\lambda_k^1$ cannot be identified. If we multiply the cointegrating space by $1/\lambda_l^1$, and denote the ratio as $\lambda \equiv \lambda_k^1/\lambda_l^1$, the ratio is identified and the cointegrating vector, which contains one nonlinear overidentifying restriction is given by

$$\beta^{*\prime} = \begin{bmatrix}(\lambda-1)(1+\rho/\eta) & -\lambda(1+\rho) & (1+\rho) & -\lambda & 1 & -\lambda\rho\gamma_2^b & \rho\gamma_3^x\end{bmatrix}. \tag{19}$$

Given that the cointegrating rank equals one, the above two hypotheses are nested to the unrestricted case and can, therefore, be tested using standard methods. This means that we can let the data determine the stochastic specification of the unobservables. As discussed in previous sections, there are some interesting hypotheses concerning the parameters $\rho$ and $\eta$. If $\eta = 1$ we obtain constant returns to scale. Another interesting hypothesis corresponds the Cobb-Douglas production function, ie $\rho = 0$. This is a problematic hypothesis since the parameters $\eta$, $\gamma_2^b$ and $\gamma_3^x$ are not identified under the null hypothesis. We need to make further assumption concerning these parameters in order to test the CD production function hypothesis.

## 5.3 Maximum likelihood estimation and hypothesis testing

In this section we spell out the conditions for the consistency and the identification of the likelihood function and the limiting distribution of the parameters. The underlying statistical theory is developed in Saikkonen (2001a) and Saikkonen (2001b) and applied in Ripatti and Saikkonen (2001) (see also Pesaran and Shin 1999).

The cointegrating space need to be normalized for the hypothesis testing. In the following, $z_{1,t}$ denotes the normalized variable. We write the statistical model (17)

[43] Remember that $z^{*\prime} = [y_t\ l_t\ k_t\ (w-p)_t\ (uc-p)_t\ (t/T)\ g_{xt}]$.

Table 2 **Definitions**

| symbol | definition<br>the first case | <br>the second case |
|---|---|---|
| $z_{1,t}$ | $w_t - p_t$ | $uc_t - p_t$ |
| $z_{2,t}$ | $[y_t\ l_t\ k_t\ (uc_t - p_t)]'$ | $[y_t\ l_t\ k_t\ (w_t - p_t)]'$ |
| $\beta(\theta_{11})'$ | $[-(1+\rho/\eta)\ \rho+1\ 0\ 0]'$ | $[(1-\lambda)(1+\rho/\eta)\ \ -\lambda(\rho+1)\ (\rho+1)\ \ -\lambda]'$ |
| $\mu_1$ | $\rho\gamma_2^b$ | $\lambda\rho\gamma_2^b$ |
| $\mu_2$ | $0$ | $\rho\gamma_3^x$ |
| $\theta_{11}$ | $[\rho\ \eta]'$ | $[\lambda\ \rho\ \eta]'$ |
| $\theta_{12}$ | $\gamma_2^b$ | $[\gamma_2^b\ \gamma_3^x]'$ |
| $\theta_{13}$ | $0$ | $[\gamma_g\ \tau]'$ |

in the following normalized form.

$$\begin{aligned}\Delta z_t =&\nu + \sum_{j=1}^{p-1}\Gamma_j\Delta z_{t-j} + \alpha\left[z_{1,t-1} + \beta(\theta_{11})'z_{2,t-1} - \mu_1(t/T) + \mu_2 g_{xt}\right] + \varepsilon_t \\ =&\nu + \Gamma q_t + \alpha\left[z_{1,t-1} + \beta(\theta_{11})'z_{2,t-1} - \mu_1(t/T) + \mu_2 g_{xt}\right] + \varepsilon_t, \qquad (20)\end{aligned}$$

where $\varepsilon_t \sim \text{NID}(\mathbf{0}, \Omega)$. Due to the difference choice of normalization, our special cases involve definitions that are given in table 2. Other definition involves $z_t = [z_{1,t}\ z_{2,t}']'$, $q_t = [\Delta z_{t-1}'\ \cdots\ \Delta z_{t-p+1}']'$, $\Gamma = [\Gamma_1\ \cdots\ \Gamma_{p-1}]$. $g_{xt}$ is given by

$$g_{xt} \equiv g(t/T, \theta_{13}) = \{1 + \exp\left[-\gamma_g(t/T - \tau)\right]\}^{-1}$$

Note that $\beta(\theta_{11})$ is a nonlinear function of the underlying parameters, which, in both cases, satisfies the standard rank condition, ie identifies the parameters. We partition the parameter vector $\theta$ as $\theta = [\theta_1'\ \ \theta_2']'$, where $\theta_1 = [\theta_{11}'\ \ \theta_{12}'\ \ \theta_{13}']'$ and $\theta_2 = \text{vec}[\nu\ \ \Gamma\ \ \alpha]$. The operator vec denotes the standard columnwise vectorization operator. Note, that $\theta_1$ denotes the parameters that determine the cointegrating space and $\theta_2$ the rest of the parameters, ie short-run dynamics, constants etc. The CES production function and the stochastic specification of the system imply the following parameter restrictions: $\rho > -1$ and $\rho \neq 0$, $\eta > 0$

Conditioning on the initial values $z_{-p+1}, \ldots, z_0$ we can write the log-likelihood function of the data as

$$l_T(\theta, \Omega) = -\frac{T}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t(\theta)'\Omega^{-1}\varepsilon(\theta), \qquad (21)$$

where

$$\varepsilon_t(\theta) = \Delta z_t - \nu - \Gamma q_t - \alpha\left[z_{1,t-1} + \beta(\theta_{11})'z_{2,t-1} - \mu_1(t/T) + \mu_2 g_{xt}\right]. \qquad (22)$$

The maximum likelihood estimators of $\theta$ and $\Omega$, denoted by $\hat{\theta} = [\hat{\theta}_1'\ \ \hat{\theta}_2']'$ and $\hat{\Omega}$, are obtained by maximizing the function $l_T(\theta, \Omega)$. This maximization problem is highly nonlinear and the analytical gradients of the likelihood function, as will be derived below, are recommendable in the numerical optimization. Saikkonen (2001a) shows, that under suitable regularity conditions, the ML estimators $\hat{\theta}$ and $\hat{\Omega}$ exist with probability approaching one and are consistent. The limiting distribution of $\hat{\theta}$ is derived in Saikkonen (2001b). The estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ are asymptotically independent and also asymptotically independent of the estimator $\hat{\Omega}$. The limiting distribution of $\hat{\theta}_1$ is mixed normal and that of $\hat{\theta}_2$ is normal.

To study the covariance matrices of the estimator $\hat{\theta}$, we define the following matrices

$$G_{1,t}(\theta) = \begin{bmatrix} -\frac{\partial \operatorname{vec}[\beta(\theta_{11})']}{\partial \theta'_{11}}(z_{2,t-1} \otimes \alpha') - \frac{\partial \mu_1(t/T)}{\partial \theta'_{11}}\alpha' + \frac{\mu_2 g_{xt}}{\partial \theta'_{11}}\alpha' \\ \begin{bmatrix} \frac{\partial \mu_1}{\partial \theta'_{12}}(t/T) & -\frac{\partial \mu_2}{\partial \theta'_{12}} g_{xt} \end{bmatrix}' \alpha' \\ -\mu_2 g_{xt}^2 \left[ (t/T - \tau)\exp[-\gamma_g(t/T-\tau)] \quad -\gamma_g \exp[-\gamma_g(t/T-\tau)] \right]' \alpha' \end{bmatrix}$$

$$G_{2,t}(\theta_1) = \begin{bmatrix} -I_5 & -(q'_t \otimes I_5) & -(u'_t \otimes I_5) \end{bmatrix}'.$$

where the partial derivatives are given in the appendix. Under regularity conditions we may conclude (see Saikkonen 2001b) that,

$$\hat{M}_{1\cdot 2}^{1/2}(\hat{\theta}_1 - \theta_1) \xrightarrow{d} \mathrm{N}(0, I), \tag{23}$$

where

$$\hat{M}_{1\cdot 2}^{1/2} = \sum_{t=1}^{T} \hat{G}_{1,t}\hat{\Omega}^{-1}\hat{G}'_{1,t} - \sum_{t=1}^{T} \hat{G}_{1,t}\hat{\Omega}^{-1}\hat{G}'_{2,t} \left( \sum_{t=1}^{T} \hat{G}_{2,t}\hat{\Omega}^{-1}\hat{G}'_{2,t} \right)^{-1} \sum_{t=1}^{T} \hat{G}_{2,t}\hat{\Omega}^{-1}\hat{G}'_{1,t}$$

with $\hat{G}_{1,t} = G_{1,t}(\hat{\theta})$ and $\hat{G}_{2,t} = G_{2,t}(\hat{\theta}_1)$. For $\hat{\theta}_2$ we have

$$\hat{M}_{2\cdot 1}^{1/2}(\hat{\theta}_2 - \theta_2) \xrightarrow{d} \mathrm{N}(0, I), \tag{24}$$

where $\hat{M}_{2\cdot 1}^{1/2}$ is defined in the same way as $\hat{M}_{1\cdot 2}^{1/2}$ except that the roles of the subscripts 1 and 2 are interchanged. The explanation for that the limiting distribution of $\hat{\theta}_1$ is mixed normal and that of $\hat{\theta}_2$ is normal relies on the fact the $\hat{G}_{1,t}$ contains integrated processes and $\hat{G}_{2,t}$ asymptotically stationary processes and that (23) and (24) hold. Approximate standard errors can be obtained for the components of $\hat{\theta}_1$ and $\hat{\theta}_2$ by taking square roots of the diagonal elements of the matrices $\hat{M}_{1\cdot 2}$ and $\hat{M}_{2\cdot 1}$, respectively.

Saikkonen (2001b) shows that in the case when the parameters of the model are identified one is able to construct the LR, LM and Wald tests as usual and that their asymptotic distribution is chi-square under the null hypothesis.

## 5.4 Identification

In section 5.2 we showed how to estimate some parameters of interest using the assumptions related to common trends in the unobservables. What remains is the identification issue. The information needed to identify the random walk component of unobservables relies on the production function (4) and the estimated counterparts of $\tau_t^l$ and $\tau_t^k$.

The $\tau_t^l$ and $\tau_t^k$ processes are defined as follows

$$\tau_t^l = e_t^y - \rho b_t + \log[\eta(1-\delta)] \tag{25}$$

$$\tau_t^k = e_t^y - \rho x_t + \log(\eta\delta). \tag{26}$$

Given the estimates of the parameters $\rho$ and $\eta$, the estimates of $\hat{\tau}_t^l$ and $\hat{\tau}_t^k$ may be recovered from the first order conditions (10) and (11). We solve $B_t$ and $X_t$ from equations (25) and (26) as follows

$$B_t = \left[\frac{\exp(e_t^y)\eta(1-\delta)}{\hat{\mathfrak{T}}_t^l}\right]^{1/\hat{\rho}}$$
$$X_t = \left[\frac{\exp(e_t^y)\eta\delta}{\hat{\mathfrak{T}}_t^k}\right]^{1/\hat{\rho}},$$

where $\mathfrak{T}_t \equiv \exp(\tau_t)$. The unobservables $B_t$ and $X_t$ in the production function (4) are substituted by the above measures to obtain the following equation

$$y_t = \frac{\hat{\eta}}{\hat{\rho}}\log\hat{\eta} + \frac{\hat{\eta}}{\hat{\rho}}e_t^y - \frac{\hat{\eta}}{\hat{\rho}}\log[\hat{\mathfrak{T}}_t^k K_t^{-\hat{\rho}} + \hat{\mathfrak{T}}_t^l L_t^{-\hat{\rho}}]. \tag{27}$$

Note that the parameter $\delta$ cancels and the only unobservable is the mark-up process, $e_t^y$. This is the final piece of information that is needed to identify all the unobservables — or their common random walk component.

In the case of stationary first order condition wrt. labour, ie our first example, the identification is simple: the mark-up process, $e_t^y$, can be solved from the production function (27) and the technology processes, $b_t$ and $x_t$, as residuals from equations (25) and (26). Note, however, that we may not be able to identify the intercept terms of the unobservables.

If the cointegrating vector is based on the linear combination of the first order conditions, as in our second example, the identification scheme is more complicated. The common trend may be based on the fact that $\hat{\tau}_t^l$ and $\tau_t^k$ are cointegrated with cointegration vector $[\lambda \;\; -1]'$. According to, for example, Johansen (1995) a candidate for common trend is the orthogonal complement of the cointegrating vector. Here it is $[1 \;\lambda]'$. In our case it produces a common trend which has a drift term. Another candidate is the orthogonal complement of the loadings matrix of the vector error correction model for $\hat{\tau}_t = [\hat{\tau}_t^l \; \hat{\tau}_t^k]'$, ie $\alpha_\perp$ in our notation, multiplied by the cumulative sum of the residuals, $\sum_{i=0}^t \varepsilon_i$, of such a model. See Johansen (1995) for further discussion of the common trends.

It is important to note, however, that the common trend obtained as above is not identified. It may be multiplied by any arbitrary non-zero scalar. Given that we have an estimate of the common trend $\hat{c}_t$, we may parameterize the mark-up process as follows.

$$\begin{aligned} e_t^y &= \gamma_0^y + \gamma_1^y c_{1t} + s_t^y \\ &= \gamma_0^y + \gamma_1^y(\lambda_c^0 + \lambda_c^1\hat{c}_t) + s_t^y \\ &= \gamma_0^y + \gamma_1^y\lambda_c^0 + \gamma_1^y\lambda_c^1\hat{c}_t + s_t^y \\ &\equiv \lambda_0^y + \lambda_1^y\hat{c}_t + s_t^y. \end{aligned} \tag{28}$$

The unknown parameters $\lambda_0^y$ and $\lambda_1^y$ may be estimated from the production function (27). Since the production function contains now a stationary component $s_t^y$, the estimation should be performed using cointegration techniques. Due to nonlinearities, we may not apply the FIML methods proposed by Johansen (1988). Another type of complication is faced due to the fact that the approach relies on the pre-estimates of $\rho$, $\eta$, $\tau_t^l$ and $\tau_t^k$.

# 6 Estimation results

The need for the inclusion of the nonlinear deterministic term of the form $g_{xt}$ in cointegrating relationships, ie the structural change, may be tested by the approach derived in Saikkonen (2001a), (2001b) and applied in Ripatti and Saikkonen (2001). Heuristically, the deterministic structural change is approximated by the polynomials of the time trend and the null of zero coefficient is tested.

The trace test of cointegrating rank indicate two cointegrating vectors. The first cointegrating vector seems to correspond the one related to the first order condition with respect to labour. When testing nonlinearity in the deterministic part of the cointegrating vectors, it suggests that the nonlinearity might be present in the cointegrating relationship relating to stationary real user cost. The $p$-value of exclusion of second and third order time polynomials in the first cointegrating vector is 0.72 and first to third order time polynomials in the second — stationary real user cost — cointegrating vector is 0.02. This is not surprising since according to visual inspection (see figure 3), there seems to be a level shift in the real user cost of capital. Altogether, this suggest that the first order condition with respect to labour might be (trend) stationary whereas the one with respect to capital is nonstationary. Consequently, efficiency might be gained in analysing a smaller system, where the vector $z_t$ in (17) is replaced by $z_t' = [y_t \;\, l_t \;\, (w-p)_t]$ and in (20) by $z_{2,t} = [y_t \;\, l_t]'$. The deterministic part should include $t/T$ but exclude $g_{xt}$.

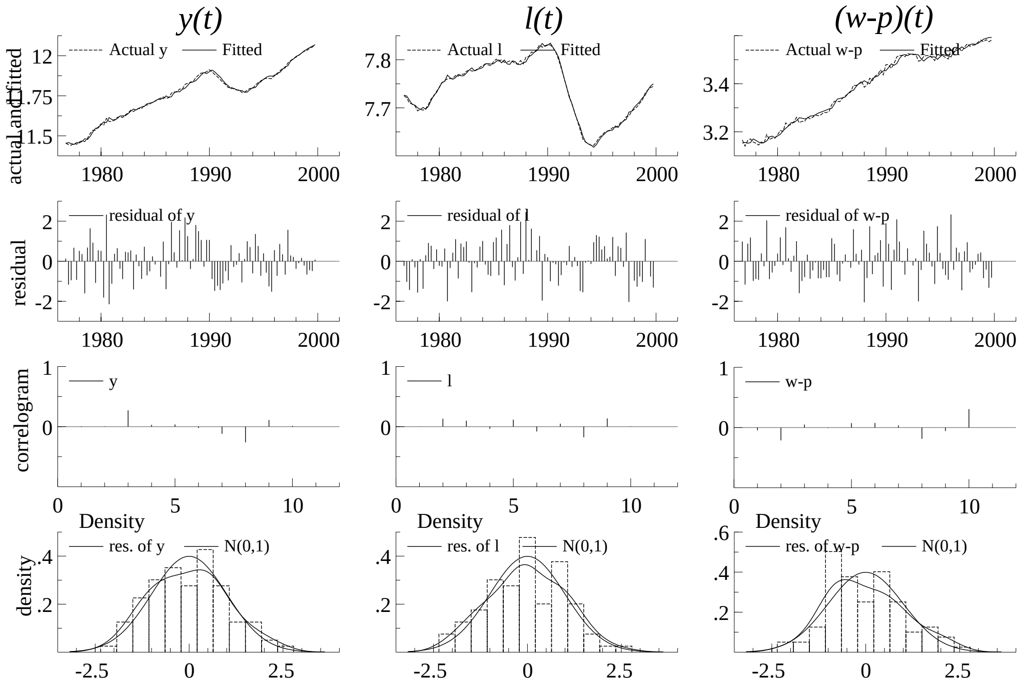
We end up analysing the first case studied in sections 5.2 and 5.4. We estimate VAR model (17) with two lags. The residuals obey the standard assumptions (see figure 4). There is clear evidence that the cointegrating rank equals one. The trace test for the hypothesis that the rank equals zero is 81.01 (95% fractile 42.4) and for the hypothesis that the rank equals one is 21.51 (95% fractile 25.3). The cointegrating residual is depicted in figure 5. The values of the deep parameters are reported in table 3. Parameter $\rho$ deviates from zero implying that the elasticity of substitution is 1.47. Note, however, that we may not test the hypothesis that $\rho = 0$ due to the fact that parameters $\eta$ and $\gamma_2^b$ are not identified in such a case. The constant-returns-to-scale $\eta = 1$ hypothesis is also clearly rejected ($p$-value 0.01). Consequently, the aggregate Finnish economy has decreasing returns to scale.

Table 3 **Some Key Parameters of the First Order Condition Estimation**

| Parameter | Value | Standard Error |
|---|---|---|
| $\rho$ | 0.681 | 0.019 |
| $\eta$ | 0.695 | 0.050 |
| $\gamma_2^b$ | 0.984 | 0.059 |

We identify the mark-up process, $b_t$ and $x_t$ following the procedure that is described in section 5.4. These measures are depicted in figure 5. The mark-up was fairly stable during the 1980s before the collapse during the Gulf-war (and the collapse of the Soviet trade) hitting its low in 1992. There has been a constant rise since that period. The labour augmenting technical progress, $b_t$, portrays an interesting shape. Up to 1992 it had fairly steady and rapid growth rate following a decline

Figure 4 **Residual diagnostics**




The first column from the left contains diagnostic graphs for $y_t$ equation, the second for $l_t$ equation and the third for $w_t - p_t$. The top row depicts actual and fitted values, the second residual, the third correlogram and the lowest graphs density function (histogram, kernel estimate and normal). The $p$-values of vector tests for autocorrelation and normality are 0.19 and 0.58 respectively.

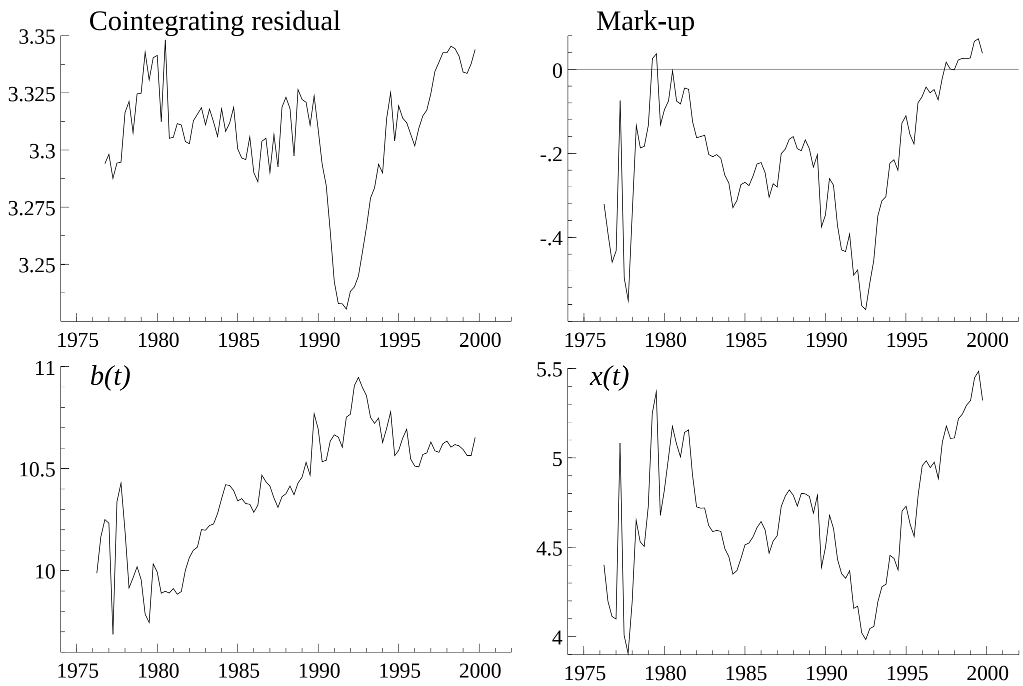
during 1993-95. Since then it has remained stable. The shape contradicts that of the total factor productivity depicted in figure 1. This is also surprising since one expects the ‘new-economy’ features to be present in the data particularly during the post 1995 period. Finally, an interest feature in the recent history of the Finnish economy is the substantial rise in capital augmenting technical progress. Its climb has compensated the slow growth rate of labour augmenting technical progress in the second half of 1990s.

Figure 6 depicts the surprising stability of the parameters of the CES production function when the sample size is expanded at the end. The same does not hold for the inverse case: Decreasing the sample from the start will increase the estimate of the return to scale parameter. Starting the estimation period from 1981 would yield the $p$-value of 0.11 for the null hypothesis that $\eta = 1$. The estimate 0.5 of the elasticity of substitution is obtained under the null. Shifting the starting period further would lead to higher $p$-values.

# 7 Discussion and the robustness of the results

Due to the identification scheme, which exhausts all the information regarding the first order conditions and the production function, the labour share can be perfectly modelled. To emphasize the role of mark-up, we compute the following mark-up-

Figure 5 **Cointegrating residual, mark-up, $b_t$ and $x_t$**




The upper left graph depicts the cointegrating residual and the upper right the mark-up process computed as the residual of the production function. The lower figures graph $b_t$ and $x_t$ processes.

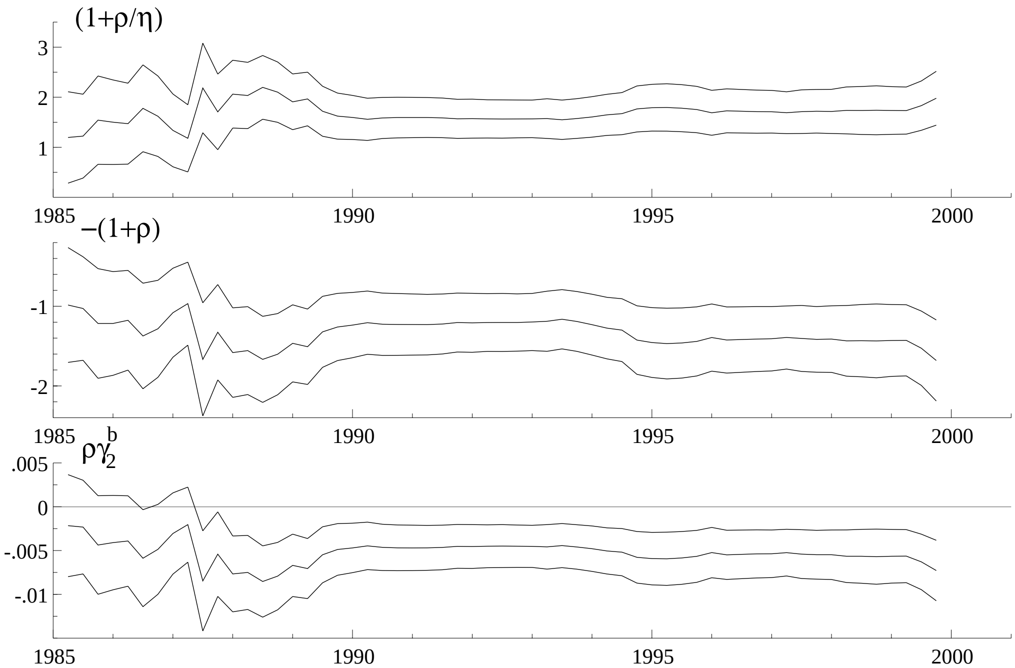
adjusted labour share

$$s_L^m = \log[\eta(1-\delta)] - \rho b_t + \frac{\rho}{\eta} y_t - \rho l_t.$$

The labour share and the mark-up-adjusted labour share are depicted in figure 7. Their difference in levels shows the deviation of the mark-up from the unity (no mark-up case). The mark-up-adjusted labour share declines much faster in the first years of great recession but recovers in the latter half of 1990s. The initial decline is due to the rapid increase in labour-augmenting technical progress. This was counteracted by the decrease in the mark-up. The opposite is true since 1995, when the mark-up returned to the original level and started climb over that and when the growth rate of labour-augmenting technical progress slackened. This figure clearly demonstrates that the mark-up has an important role in explaining the continuing lower level in the labour share during the latter half of 1990's.

Mark-up has interesting influences on the marginal productivities of labour and capital. These measures are depicted in upper graphs of figure 7.The marginal product of labour — when adjusted to the mark-up — contains no drift during the 1980s. It declined during the early 1990s and has been growing rapidly since then. These trends deviate substantially from those of the real wage. This graph repeats the story of the mark-up-adjusted labour share. The growth rate of real wages would be much higher in the competitive aggregate economy. The real rental price of capital and the marginal product of capital — when adjusted for the mark-up — display suprisingly high *positive* correlation as predicted by the economic model. Finally, the lower right graph of figure 7 highlights the differences in average and marginal

Figure 6 **Recursive estimates of $1 + \rho/\eta$, $-(1 + \delta)$ and $\rho\gamma_2^b$**




productivities of labour (with and without effect of the mark-up). These differences are due to the choice of CES production function and the time-varying mark-up.

To check the robustness of our results we use another identification scheme, which is based on the assumption that the capital augmenting technical progress is constant and on the Kalman filter estimation of the labour augmenting technical progress. Assume that the capital augmenting technical progress is unity, $X_t = 1$ for all $t \geq 0$ and that the parameters $\rho$ and $\eta$ are known. Then the estimation problem of the parameter $\delta$ and $B_t$ process will be linear

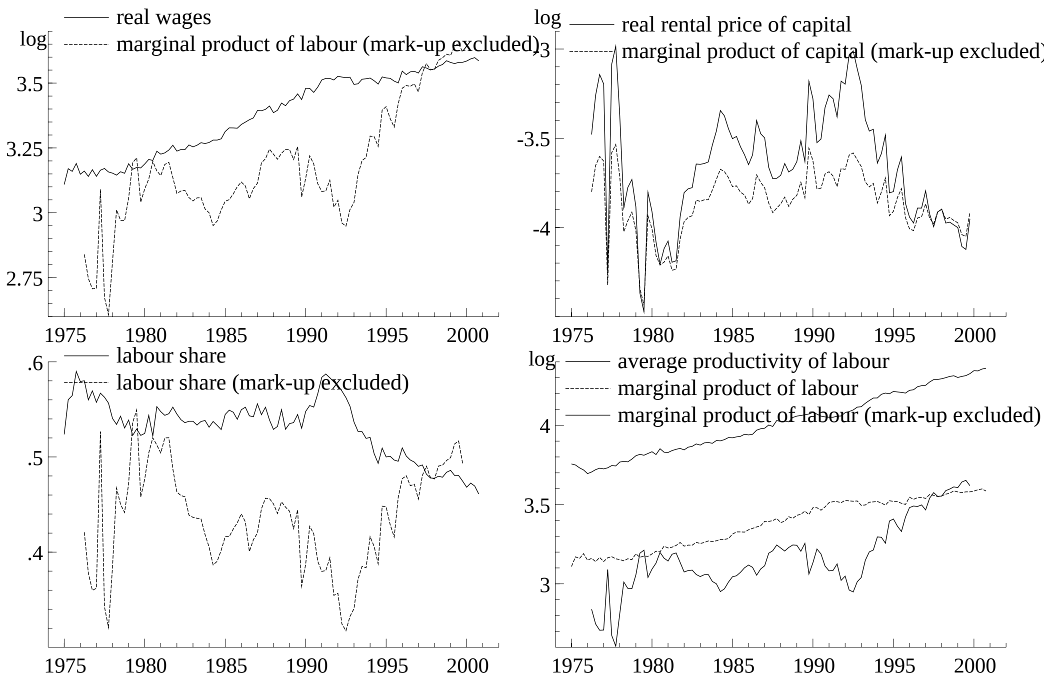
$$Y_t^{-\hat{\rho}/\hat{\eta}} = \delta K_t^{-\hat{\rho}} + (1 - \delta) B_t^{-\hat{\rho}} L_t^{-\hat{\rho}}. \tag{29}$$

Naturally, one needs to assume the source of uncertainty in the model. Equation (29) can be considered as a measurement equation in the state space setting. We approximate $B_t$ process with the random walk with drift process and estimate the system by maximum likelihood with the Kalman filter. The technical details are given in the appendix.

The estimate of $b_t$ is depicted in figure 8. The upper left graph demonstrates how much these two measures of $b_t$ deviate each other. The differences are substantial. Labour-augmenting technical trend, $b_t$, based on our original identification depicts much more rapid growth during the recession than the measure based on the Kalman filter approach. On the other hand, the growth rate of $b_t$ declines to zero during the latter half of 1990s according to our measure but continues growing at significant pace according to the KF measure. Our findings are in line with the study[44] by Maliranta (2001) which relies on plant-level data. The corresponding

[44]Maliranta (2001) demonstrates how the aggregate labour productivity in manufacturing rose substantially in the first half of 1990s due to the massive restructuring and has improved only marginally since then.

Figure 7 **Role of mark-up**



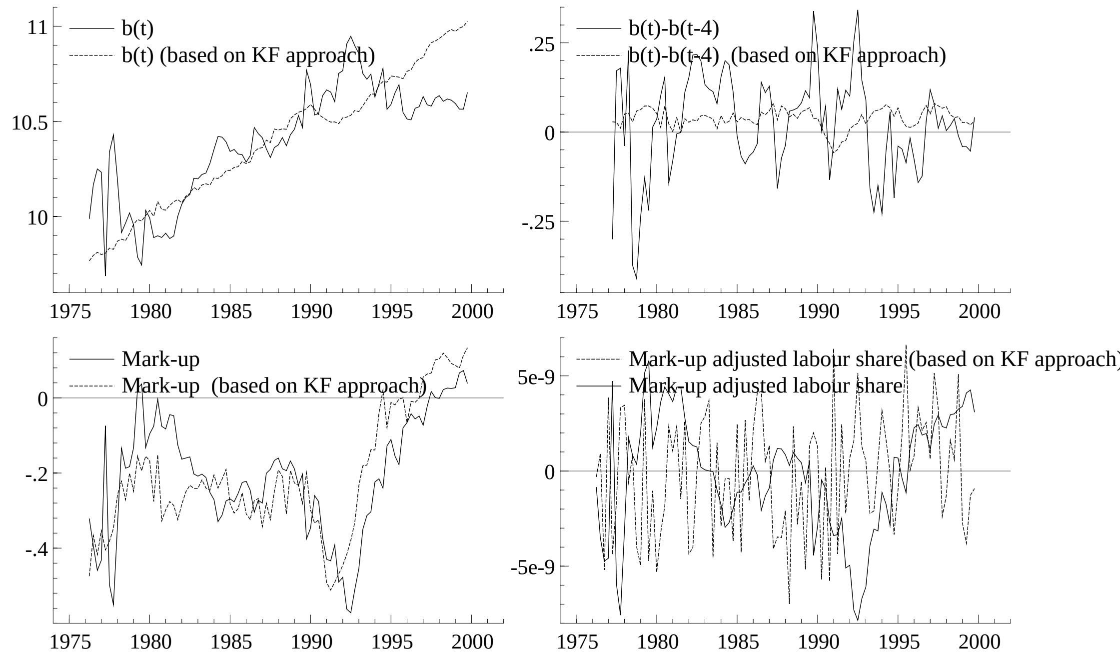

Note that the scales are adjusted in each graph. The upper left graph depicts the real wages and marginal product of labour without mark-up. The upper right graph depicts corresponding measure for capital. The lower left graph highlights the role of mark-up in the labour share and the lower right graph depicts the differences in marginal and average productivity of labour.

mark-up measures are, however, surprisingly alike. As in the labour-augmenting technical trend measures, the mark-up measures do not substantially deviate before 1990s. Also the shapes that demonstrate upward trend in mark-up during 1990s are alike. However, the timing of the slump and the subsequent recovery are different. The bottom and the recovery show up two years earlier in the Kalman filter based measure that in our preferable measure.

# 8 Concluding thoughts

We began this study by asking whether the Finnish data on aggregate wage, employment, capital stock and output provide us with sufficiently strong evidence to reject the hypothesis of a Cobb-Douglas aggregate production technology for the economy. We provided evidence, in particular from the 1990's that some of the famous Kaldor facts — the great aggregate ratios — do not seem to hold in the Finnish data. Most importantly, there has been a significant fall in the labour's national income share (or in the unit labour costs) during the last 10 years or so in Finland. This observation in itself suggests that the days of a stable aggregate Cobb-Douglas production function could be regarded as the good old days and that it could actually reflect an underlying process where the technical substitution possibilities between different factors of production in the economy may be changing.

Figure 8 **Comparison of $b_t$ measures**




The upper graph presents two estimates of $b_t$ and the lower part the corresponding mark-up-adjusted labour share figures. The scales are adjusted

To build appropriate testing ground for this idea, we postulated that in light of the latest data a CES production function with possibly a non unit substitution elasticity would provide a more appropriate description of the aggregate production technology of the Finnish economy. More specifically, given, in particular, the complementary observation that the capital-output ratio has been falling almost throughout the 1990's, we asked whether a CES production function with an elasticity of technical substitution between labour and capital of less than one could reasonably well explain the observed decline in the labour's income share. The critical underlying assumption here is that the (aggregate) mark-up has remained relatively constant, so that its potential variation over time cannot be the reason for the decline in labour's income share.

According to our estimation results, the elasticity of technical substitution between capital and labour indeed appears to be (even considerably) less than one in Finland (around $0.6$), implying that the two inputs are gross complements. Consequently, under the CES with complementarity among inputs, the observed fall in the capital-output ratio may have made a nontrivial contribution to the decline in the labour share. However, there are a number of qualifications that we need to consider before we can take a final stand as to whether the data are consistent with our initial hypothesis of a CES aggregate production function for the Finnish economy.

First of all, it is quite conceivable that the aggregate (or average) mark-up has not been constant during the latest growth swing in Finland. Actually, our estimates strongly suggest that, indeed, the rise in the aggregate mark-up may have been considerable. The fact that, in particular, the ICT sector has grown vigorously from early 1990's onwards, so that its share in the aggregate GDP has increased remarkably over the period, could conceivably be consistent with increasing aggregate mark-up. In a sense, however, this begs the question, since we would need convincing economic arguments to explain what is so different in the production/cost structure of the ICT sector and of the structure and behaviour of the markets for its goods that gives rise to extremely good profitability and high mark-ups? Continuous product differentiation and (locally) increasing returns may be the key factors here, but clearly further analysis is needed. Anyway, this argument obviously suggests that a more disaggregated approach may be preferable to the fully aggregate approach followed in this paper. For example, the production technologies of the ICT sector and the rest of the economy (excluding, possibly, the public sector) could be modelled and estimated separately, after which the aggregate could, in principle, be obtained through aggregation.

Secondly, we allowed for input augmentation in the underlying technical progresses. To this end, the results are interesting and highly intriguing. On the one hand, they indicate that the growth rate of capital augmenting technical progress has increased during the recent years quite significantly. Hence, measures have been taken, at the firm and plant level, to increase capital productivity. The (required) rate of return on capital may have gone up, providing all the necessary profit maximizing incentive to speed up capital augmentation.[45] Or higher prices commanded by new products provide the incentives favouring capital augmentation; being a scarce factor, this price effect implies that there will be more technological improvements favouring capital.[46] Anyway, in the present context one can only speculate what could be the exact role of financial markets in providing the appropriate (rate of return) incentives to these developments.

On the other hand, the results also suggest that labour productivity may have not increased as much as is usually thought of during the recent years. More specifically, our estimates indicate that labour productivity increased sharply during the years of the deep recession, but has considerably levelled off subsequently. Hence, there could be an element of 'creative destruction' in the Finnish growth story during the last ten years in the sense that low productivity jobs (along with unprofitable capital) were quickly destroyed during the first years of the recession, which shows up in a sharp increase in labour productivity. Of course, this is speculation, but there is additional evidence available that appears to support our interpretation.[47]

The following Table 4 provides a summary of the productivity development in the Finnish economy during the last 20 years. In the table we combine data on productivity trends with our estimates of the growth rate of labour and capital augmenting technical progres in three consecutive subsamples spanning the whole sample period of the last 20 years. Taken at face value the differences in the subsample estimates of the growth rate of labour and capital augmenting technical progresses are stark indeed. From their estimated base values of 7.63 and –1.42, respectively,

[45] As also emphasized by eg Acemoglu (2000), an increase in the growth rate of capital augmenting technical progress, driven by profit-maiximizing behaviour, is a transitory phenomenon.

[46] See Acemoglu (2001). He develops a framework to analyze the forces that shape the direction of technical change. In addition to the price effect alluded to in the main text, he identifies the market size effect as the second critical factor. The latter effect creates a force towards innovations complementing the abundant factor.

[47] See Maliranta (2001) and Pohjola (2001).

during the 1980's, the growth rate of labour augmenting technical progress dropped down to 3.55 per cent p.a., while that of the capital slightly increased to -0.23 per cent p.a. during the first half of the 1990's, ie during the recession years. Thereafter, the roles have been reversed in a surprisingly stark way with the growth rates showing vigorous capital augmentation (20.39 per cent p.a.) and actually declining labour (–1.25 per cent p.a.) in the underlying technical improvements.

Table 4 **Decompositions of productivities**

| Component of Productivity | Average growth rate, % p.a. 1980–89 | 1990–94 | 1995–99 | Estimated drift, % p.a. |
|---|---|---|---|---|
| GDP growth | 3.63 | -1.34 | 4.73 | — |
| Labour productivity (data) | 2.54 | 2.64 | 2.43 | — |
| $B_t$ (estimate) | 7.63 | 3.55 | -1.25 | 3.90 |
| Capital productivity (data) | 0.63 | -2.09 | 4.63 | — |
| $X_t$ (estimate) | -1.42 | -0.23 | 20.39 | 0 |
| Total factor productivity (data) | 1.64 | 0.39 | 3.46 | — |
| Technical progress, (estimate) weighted by elasticities | 1.84 | -1.02 | 3.32 | 1.92 |

Note, however, from the last column that the estimated changes in the input augmented technical progres are of transitory nature, the forecasts derived from the underlying unobserved components model for the technical progres indicate that, first, the growth rate of labour augmenting technical progres will return back to its level prior to 1990 and, second, that capital augmenting technical progress will cease to grow. While these forecasts reflect the built-in (convergence) properties of the underlying models of technical progres — along a 'balanced growth path' only labour augmentation occurs — the surprising feature about the estimates in table 4 is the wild variation in the growth rates of the two input augmenting technical progresses. Note also that the last row in Table for gives us an estimate of the growth rate of the TFP; it is the familiar Cobb-Douglas functional form that motivates the form of the estimate, since under the C-D form $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$, where $\alpha$ is the capital elasticity of output, $\alpha = \partial \ln Y / \partial \ln K$, the TFP factor $A$ is given by $A_t = X_t^\alpha B_t^{1-\alpha}$ so that $\ln Y_t - \alpha \ln K_t - (1-\alpha)\ln L_t = \ln A_t = \alpha \ln X_t + (1-\alpha)\ln B_t = [\partial \ln Y_t / \partial \ln K_t] \ln X_t + [\partial \ln Y_t / \partial \ln L_t] \ln B_t$. The path of the estimated TFP conforms qualitatively with the data, but the estimated drop in the growth rate of the TFP in the middle of the sample (1990-1994) as well as the subsequent (1995-2000) rise in it is (much) more pronounced than in the data.

That the direction of input augmentation in the technical progress has changed so much during the 1990's clearly invites thinking and speculation as to the root cause of it. In particular, it would be highly intriguing to perceive elements of the alleged 'new economy' in the estimated direction of input autmentation. If this is the case, then the argument is that the 'new economy' provides stronger incentives to develop capital augmenting technologies. Given profit maximizing behaviour the underlying reason could be that the goods produced with these technologies command higher prices. Note, however, that capital augmenting techonological progress does not necessarily imply that technological progress is capital-biased, ie the relative marginal product of capital increases under capital augmentation. In fact, the bias depends on the elasticity of substitution, $\sigma$, between labour and capital. More specifically, given our CES production function, we have

$$\frac{MP_K}{MP_L} = \frac{\gamma}{1-\gamma}\left(\frac{X}{B}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{K}{L}\right)^{-\frac{1}{\sigma}}$$

so that

$$\frac{\partial\left(\frac{MP_K}{MP_L}\right)}{\partial X}\Big|_{K/L} \lesseqgtr 0 \longleftrightarrow \sigma \lesseqgtr 1$$

These fundamental equations[48] imply that capital augmentation will be capital-biased if capital and labour are gross substitutes, ie $\sigma > 1$. Since $\sigma < 1$ according to our estimation results, we tend to conclude that capital augmenting technical progress in the aggregate Finnish economy is labour-biased. The intuition is that given gross complementarity, $\sigma < 1$, an increase in capital productivity increases the demand for labour by much more, effectively creating excess demand for labour. The result is that the marginal product of labour increases by more than the marginal product of capital.[49] Finally, note that the relative marginal product of capital is decreasing in the relative abundance of capital, $K/L$, or capital-labour ratio. This is related to the usual substitution effect, leading to a downward-sloping demand curve (for capital). Now, the measured capital-labour ratio has fallen during the latter half of the 1990's and this by itself has had a positive contribution to the relative marginal product of capital.

Overall, then, further work needs to be done in order to turn the hypothesis that the 'new economy' underlies the recent productivity developments in the Finnish economy into a compelling economic argument. If the 'new economy', or rise of the ICT sector, has been the driving force for improving capital productivity in the Finnish economy, our results suggest that the technological innovations that have been made to improve capital productivity do not have a permanent effect on the growth rate of capital productivity. What this implies eg for the stock market and for macroeconomic policy in particular, needs to be thought through carefully. We hope to be able to do this in a future work.

[48] See also Acemoglu (2001).
[49] See Acemoglu (2001, p. 9).

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# A Gradients

We present the formulas of certain partial derivatives that are used in the covariance matrix estimator (23) (see also table 2 for definitions). For the first special case they are as follows

$$\frac{\partial \operatorname{vec}[\beta(\theta_{11})']}{\partial \theta_{11}'} = \begin{bmatrix} -1/\eta & \rho/\eta^2 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \frac{\partial \mu_1}{\partial \theta_{11}'} = \gamma_2^b, \quad \frac{\partial \mu_1}{\partial \theta_{12}'} = -\rho, \quad \frac{\partial \mu_2}{\partial \theta_{11}'} = \frac{\partial \mu_2}{\partial \theta_{12}'} = 0.$$

For the second case they are

$$\frac{\partial \operatorname{vec}[\beta(\theta_{11})']}{\partial \theta_{11}'} = \begin{bmatrix} (1+\rho/\eta) & (\lambda-1)/\eta & -(\lambda-1)\rho/\eta^2 \\ -(1+\rho/\eta) & -\lambda & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix},$$

$$\frac{\partial \mu_1}{\partial \theta_{11}'} = \begin{bmatrix} \rho\gamma_2^b & \lambda\gamma_2^b & 0 \end{bmatrix},$$

$$\frac{\partial \mu_2}{\partial \theta_{11}'} = \begin{bmatrix} 0 & \gamma_3^x & 0 \end{bmatrix},$$

$$\frac{\partial \mu_1}{\partial \theta_{12}'} = \begin{bmatrix} \lambda\rho & 0 \end{bmatrix},$$

$$\frac{\partial \mu_2}{\partial \theta_{12}'} = \begin{bmatrix} 0 & \rho \end{bmatrix}.$$

# B State-Space Estimation of the System

Denote $\bar{y}_t \equiv Y_t^{-\hat{\rho}/\hat{\eta}}$, $\bar{k}_t \equiv K^{-\hat{\rho}}$, $\bar{b}_t \equiv B_t^{-\hat{\rho}}$ and $\bar{l}_t \equiv L^{-\hat{\rho}}$. Equation (29) can then be written as follows

$$\bar{y}_t = \delta\bar{k}_t + (1-\delta)\bar{b}_t l_t + \varepsilon_t^y, \quad \varepsilon_t^y \sim NID(0, \sigma_y^2). \tag{30}$$

This forms the *measurement equation* in our state-space setup.

Since $\bar{b}_t$ is unobservable, we need a transition equation for it. We assume that it follows a random walk process with drift:

$$\bar{b}_t = \bar{b}_{t-1} + \beta_0 + \varepsilon_t^b, \quad \varepsilon_t^b \sim NID(0, \sigma_b^2). \tag{31}$$

The system can be estimated using a standard Kalman filter in computing the likelihood function to be maximized. The initial condition for $\bar{b}_0$ is given by

$$\bar{b}_0 = \frac{1}{(1-\hat{\delta})\bar{l}_0}(\bar{y}_0 - \hat{\delta}\bar{k}_0),$$

where $\hat{\delta}$ is an estimate of $\delta$ in each iteration.

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