BANK OF FINLAND
ECONOMICS DEPARTMENT
WORKING PAPERS

20.1.2003 1/2003

Heli Virta

The Bank of Finland's short-term density forecast for inflation

Sisäiseen käyttöön

The Bank of Finland's short-term density forecast for inflation[1]

Economics department working papers x/2002

Heli Virta

Abstract

The paper discusses density forecasting in general and the Bank of Finland's short-term density forecast for inflation in particular. An additional topic of interest is incorporating subjective judgement into the forecast distribution. In general, estimating an entire distribution for the future values of a variable should be aimed at. Another goal should be a systematic method for taking subjective assessments into account. As to projecting future inflation, the Bank of England and Sveriges Riksbank have been forerunners in making and publishing subjectively adjusted density forecasts. Particularly the presentation in the form of a fan chart has attracted attention. Better documentation and the ease of use imply that the Riksbank method is preferable for the Bank of Finland use, while the need for some modifications arises from differences in the forecasting exercises. The discussion on the properties of the NIPE forecast, for which the density distribution and the fan chart are compiled, implicates in addition that the Riksbank model is suitable.

Key words: inflation forecast, density forecasting, subjective judgement, fan chart

---

[1] The primary goal of the project has been compiling a distribution for the Bank of Finland's NIPE forecast in a systematic manner that also incorporates subjective judgement. Special thanks are directed to Mårten Blix and Stefan Laséen from Sveriges Riksbank for the helpful presentation of their density forecasting procedure. Moreover, I am grateful to them for letting the Bank of Finland use (a modified version of) their program for making density forecasts.

The usual disclaimer applies. The opinions expressed in the paper are mine and need not reflect those of the Bank of Finland.

Contents

1 Introduction

The introduction of inflation targeting has laid emphasis on predicting inflation, particularly because international studies suggest that monetary policy affects inflation with a lag of 1–2 years. The quality and information content of forecasts have therefore become increasingly important. Since economic development is inherently uncertain, an outlook for inflation should comprise a range of possible outcomes and some probability estimates for different realisations besides the traditional point estimate.

Point forecasts have historically played a central role even if they do not reveal anything about the uncertainty concerning a prediction. Also somewhat more informative interval and probability forecasts, which specify the probability that the outcome will fall within a stated interval, have therefore been introduced. It is nevertheless only recently that density forecasts depicting an estimate of the complete probability distribution of the possible future values of the variable in question have gained in popularity, and an even more recent issue is incorporating subjective judgement into the forecast distribution. The development has mainly resulted from general improvement in numerical and simulation techniques, greater availability of relevant data and also from central banks' willingness to communicate more detailed views on future development to the public.

Although point forecasts still dominate presentation, other approaches have become increasingly popular. For example Blix and Sellin (1999) find compiling uncertainty intervals around central forecasts advantageous based on the following arguments. First, such bounds emphasise that the forecast is uncertain[2]. Without the bounds, it would be misleadingly easy to concentrate on the central figure presented. Second, the intervals communicate the central bank's view on the balance of risks to the public. The bank can thus announce whether inflation is more likely to be above or below the main scenario forecast. Third, constructing the bands requires intense internal discussion on the expected economic development, the main risks to the forecast and their most likely quantitative effects. The process therefore directs the discussion to crucial issues, which is an important benefit of the approach. Fourth, the assessment of uncertainty may affect the way in which monetary policy is conducted, as significant uncertainty might require a cautious policy stance so that unnecessarily large and sudden movements in interest rates can be avoided. The points are in line with the general view on the subject.

The public status of density forecasting rose in 1996, as the Bank of England introduced a density distribution for its inflation forecast. The top-down production of the distribution relies heavily on subjective judgement and the Bank's internal processes. Moreover, the approach and particularly the presentation of the forecast contain some problematic aspects. Sveriges Riksbank (the Riksbank) has also initiated its own density forecast for inflation. The process of deriving this forecast is more transparent and has been documented in more

[2] Forecast uncertainty derives from various factors besides the inherently uncertain economic development. Ericsson (2002) divides the sources of uncertainty to predictable ("what we know that we don't know") and unpredictable ("what we don't know that we don't know"). The former comprises cumulation of future errors to the economy and inaccuracies in estimates of the forecast model's parameters while the latter is composed of currently unknown future changes in the economy's structure, misspecification of the forecast model and mismeasurement of the base-period data.

detail, because of which this paper concentrates on the latter approach. The method adopted at the Bank of Finland relies likewise more heavily on the Riksbank's way of incorporating subjective judgement into the inflation forecast distribution. However, since the Bank of Finland's narrow inflation projection exercise (NIPE), to which the approach is applied, differs from the procedure followed at the Swedish central bank, some adjustments are both useful and necessary.

The paper proceeds as follows. To begin with, section 2 discusses density forecasts more closely. The section presents the traditional approaches to compiling the forecast distribution and the earliest density forecasts for inflation. Section 3 shows at a fairly general level how subjective judgement is incorporated into the forecast distribution at the Bank of England, while also criticism against the method is discussed. Section 4 then presents in detail the approach used at the Riksbank since this is, as already mentioned, the method that is closely replicated at the Bank of Finland. The central features and the implementation of the Bank of Finland approach are discussed in section 5 before concluding. Appendix 1 compares the methods of the Bank of England and the Riksbank, while appendix 2 concentrates on the variables the Riksbank and Norges Bank use to derive their forecast distributions for inflation. Appendix 3 is dedicated to the compositions of the subgroups of the Bank of Finland's NIPE model, and appendices 4–6 explore the properties of the past NIPE forecasts for the harmonised index of consumer prices and its sub-components.

2 An introduction to density forecasts

A density forecast of the realisation of a random variable at some future time is an estimate of the probability distribution of the possible future values of that variable (Tay and Wallis 2002). The definition stands in a clear contrast to that of the point forecast, which does not convey any information about uncertainty and is thus less informative. An interval forecast is the intermediate form between the two extremes, but only a probability density function or its equivalent fully characterises a random variable, like inflation, to be forecast. Such a function is conditional on the information set[3] used (Granger and Newbold 1986).

Compiling density forecasts has gained in popularity in recent years. The trend started in finance, particularly in risk management, but has then expanded to macroeconomic forecasting. Several factors explain the long neglect of density forecasting. First, only recent improvements in numerical and simulation techniques as well as in computer technology have made compiling density forecasts a fairly straightforward procedure[4]. Also the availability of relevant data has increased. Third, point and interval forecasts have historically been found adequate for most users' needs (Diebold, Gunther and Tay 1997). Today it is however thought that policy decisions should be based on a more informative outlook of the future. Moreover, the demands of increasing openness in the actions of central banks have led to the requirement of communicating a more detailed outlook of the future to the public.

---

[3] Information set consists of the data, knowledge, theories and assumptions about the process on which the variable is based available at the time the forecast is made.

[4] As late as in 1986 Granger and Newbold stated that deriving a complete density function is generally a too ambitious goal.

Various options exist for presenting a density forecast. The first one is a graph of the probability distribution depicting the main scenario forecast and for example the 90% uncertainty interval (see figure 2 in section 4.2 for an example). In accordance with the first approach, the table approach shows a probability distribution for a single point of time, this time in the form of a table containing the probabilities that inflation will lie within a specified range (table 1, section 2.1 and table 4, section 5.2). Third, a fan chart can be used to clarify how the probability distribution evolves over time. Such a chart presents the path of the main scenario forecast together with the chosen uncertainty intervals, so that the width of the fans measures the degree of uncertainty while the skew of the distribution is depicted by how far the bands stretch out on the sides of the central projection. Figures 1 (section 3.2) and 3 (section 4.6) present two different fan chart versions. The approach is particularly useful in emphasising that forecast uncertainty increases, as the prediction horizon becomes longer. The bands then become wider, ie fan out (hence the name of the chart). The fourth option of depicting the forecast in three dimensions results in a probability hill, in which the ridge represents the mode (as defined by the Bank of England (2002, 48)). Through time, as uncertainty increases, the ridge of the hill declines in height and the slopes of the hill become less steep (see also Britton et all 1998).

2.1 The evolution of density forecasts for inflation

The longest-running series of macroeconomic density forecasts was originated in the United States in 1968, when the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) initiated the ASA-NBER survey. In 1990, the Federal Reserve Bank of Philadelphia took over the withering survey and changed its name to the Survey of Professional Forecasters (the SPF). A special feature of the survey is that the respondents, most of whom represent the private sector, particularly the business world and the Wall Street, are asked to give density forecasts for aggregate output and inflation in addition to point forecasts of various variables. In fact, the SPF has asked the respondents to provide a complete probability distribution for future inflation since the beginning of the survey: each respondent is requested to announce the probability that inflation will be within a certain preassigned interval. The individual forecasts for the current and the next year are averaged over respondents to form a picture of the overall forecast uncertainty, and the SPF finally reports for each interval the mean probability that inflation will fall in that interval. This mean probability distribution, which can be presented as a histogram, can be viewed as a representative forecaster. The interest in the SPF density distributions for inflation has increased over time, because of which they nowadays feature in the public release of survey results. An example distribution is depicted in table 1. (Croushore 1993, Diebold, Tay and Wallis 1997)

Table 1. A density forecast of the Survey of Professional Forecasters for US inflation: mean probabilities (%) of 32 forecasters attached to possible percent changes in GDP price index

Inflation rate (%)	2001-2002	2002-2003
8.0 or more	0.03	0.07
7.0 to 7.9	0.04	0.16
6.0 to 6.9	0.07	0.23
5.0 to 5.9	0.17	0.51
4.0 to 4.9	0.94	3.16
3.0 to 3.9	4.59	12.33
2.0 to 2.9	19.22	40.22
1.0 to 1.9	55.78	33.23
0.0 to 0.9	18.03	9.28
Will decline	1.13	0.81

Source: Federal Reserve Bank of Philadelphia, Survey of Professional Forecasters, May 21, 2002. (http://www.phil.frb.org/files/spf/survq202.html)

Compared to that of the United States, the UK history of density forecasts for inflation is short, as the first ones were published only in the 1990s. Some of these reports have been relatively short-lived, but particularly the Bank of England fan charts launched in 1996 have attracted attention. The method of compiling the charts is discussed in section 3. Yet another way of publishing density forecasts was in use in the November 1996 Report of the Panel of Independent Forecasters, the forecast distributions of whom were published separately[5]. (Tay and Wallis 2002)

The National Institute Economic Review has also comprised a density forecast for inflation since 1996. The forecast distribution discussed by Poulizac et al (1996) is based on a macroeconometric model subject to the forecaster's residual adjustments so that a normal density forecast is centred on a point forecast and the variance equals that of the past forecast errors at the same horizon. The method is suitable if the forecast errors are normally distributed[6], as the assumption of a normal distribution can then be used to provide the probability ranges for the forecast. Moreover, if the forecasts seem unbiased, the variances of the errors can be used instead of mean square errors to calculate the probability ranges. As Melliss and Whittaker (1998) state, the approach reflects both the underlying model and the subjective judgement of the forecasters but is not reliable if a structural break or a regime change has occurred. Furthermore, the method does not allow for asymmetry in the forecast distribution.

As to the current state of density forecasting, the Bank of England and the Riksbank as well as the Norwegian central bank all publish density forecasts for inflation in the form of fan charts. In comparison to the traditional approach, the banks have taken a further step by incorporating subjective judgement into their

[5] Here it could be easily observed that reporting probabilities with a strong subjective element results often in rounding the shares.

[6] For example Tay and Wallis (2002) find that the conventional null hypothesis of normality would hardly be rejected in practice, because of which the case is relevant.

forecast distributions. The next section lays a foundation for discussing these methods by presenting an overview of the standard approaches for compiling density forecasts.

2.2 The general statistical approach: the framework and implementation

A density forecasting problem can be seen as a direct generalisation of a point forecasting problem of a decision situation (Tay and Wallis 2002). As the literature concerning explicit decision problems in the context of density distributions is still scarce, this section discusses point forecasts before generalising the approach to apply to forecast distributions.

2.2.1 Loss functions

Evaluating the goodness of point forecasts requires a criterion against which the alternatives can be judged. For example Granger and Newbold (1996, 121–122) introduce the cost (or loss) function in this context. The idea is that forecast errors cause costs and that the best forecast is the one that minimises the expected cost, the expectation being conditional on the information set used. Wallis (1999) specifies in addition that as to forecasting, the loss is usually set zero for a correct prediction and positive for a wrong prediction. The most common form for the loss function is the quadratic one, in which the loss is proportional to the square of the forecast error. The most popular alternative is the linear loss function. When the linear loss function is symmetric, the loss is proportional to the absolute value of the forecast error. In the case of the former type, the mean of the forecast distribution minimises the expected loss while in the case of the latter, the loss-minimising parameter is the median of the conditional distribution. Tay and Wallis (2002) emphasise that in practice, the loss function is always present even if it was not explicitly specified.

Tay and Wallis formalise the approach in a decision theoretic environment and, which is of interest here, in the context of density forecasts. They first assume that a forecast user has a loss function $L(a(p(y)),y)$, which depends on the action $a(\cdot)$ and the realisation of the forecast variable y. The choice of the action $a(\cdot)$ is based on the density forecast $p(y)$. An optimal action $a*$ minimises the expected loss so that

$$(2.1)\qquad a*(p(y)) = \arg\min_{a\in A} \int L(a,y)p(y)dy\,,$$

where A stands for all the different actions the forecast user might take. The standard approach to point forecasting defines the action as the choice of a point forecast $\hat{y}$ with the simple nonnegative loss function of the error $(y-\hat{y})$ taking the value zero for a zero error. Equation (2.1) is based on the assumption of a correct density forecast $p(y)$. The expected value for the loss $L(a^*,y)$ with respect to the true density $f(y)$ is then

$$(2.2)\qquad E\left[L(a^*,y)\right] = \int L(a^*,y)f(y)dy\,.$$

The choice of the loss function is particularly important when $f(y)$ is asymmetric, since different functions may result in different optimal forecasts.

2.2.2 Implementation

A density forecast can be based on a formal statistical or econometric model, be that either an ARCH model for a single time series or a more comprehensive macroeconomic model for aggregate macroeconomic variables (Diebold, Tay and Wallis 1997). At all events, the starting point of the standard statistical approach is the econometric model used to predict the variable in question. In the case of a linear multivariate model, the shocks are generally assumed to follow the normal distribution, because of which also endogenous variables are normally distributed and the generation of error bands is a well-known procedure.

The non-linear case is slightly more complicated because there are no analytic methods available. The current technology allows however using stochastic simulation to derive the uncertainty intervals. For example Tay and Wallis (2002) describe the procedure, which is frequently based on generating the underlying pseudo-random error terms assuming a normal distribution. Model equations are often log-linear or include some complex forms while definitions and accounting identities are linear. Resulting predictive densities are therefore nonnormal and of unknown form, because of which they are usually reported either graphically or numerically instead of an analytical presentation of the density distribution. Tay and Wallis state furthermore that stochastic simulation may be advantageous even with linear models, as the method can account for other sources of uncertainty, such as coefficient estimation error and errors in projecting exogenous variables, besides model's random error terms.

A more detailed description of using stochastic simulation to derive error bounds specifically for an inflation forecast is given by Blake (1996), who views the subject in the context of the National Institute model for the UK economy. The model is first solved without taking unexpected disturbances into account, which results in a deterministic forecast. Representative shocks are then added into the solution[7] so that the shocks are consistent with historical experience and have the same contemporaneous covariance structure. The shocks must likewise be consistent with the residuals of the model equations. The procedure of solving the model depends on the properties of the model, but irrespective of whether the model comprises forward expectations or not, a large number of simulations with different sets of shocks will result in an estimated range for the target variable. These replications make calculating standard error bounds possible although further assumptions are necessary to justify the use of standard error bounds as the confidence limits for the forecast. However, as Tay and Wallis (2002) find, it is generally not too problematic to assume that the deviations from the deterministic forecast are normally distributed. The per period standard errors can be calculated

[7] National Institute utilises historical shocks instead of generating new ones. The order in which the shocks are used is randomised, and the shocks of a certain historical period are applied across all the equations. On the one hand, the approach requires that the shocks are serially uncorrelated. On the other hand, it makes truncating too large shocks or considering their variance-covariance properties unnecessary. All in all, the approach reduces the amount of intervention required. (Blake 1996)

as the square root of the sum of the squares of the forecast errors, which is divided by the number of replications. A figure depicting the deterministic forecast and the lines of one and two standard deviations on both sides of the deterministic forecast can then be drawn. In the case of an approximately normal distribution around the mean, the two standard deviations lines approximate the 95% and the one standard deviation lines the 70% confidence interval.

2.2.3 Operational difficulties of the model-based approach

Despite the advantage of the fairly simple estimation of the forecast distribution, the standard model-based approach lacks some important features as to predicting short-term inflation. First, it is based on an existing (single) model for making the forecast. Blix and Sellin (2000) state that no single model is used at the Riksbank, which is why another method is necessary. Likewise, the standard approach does not incorporate any simple mechanism for taking period-specific information into account. The same operational difficulties apply to incorporating subjective judgement that has proven to be important when making short-term forecasts, which are relevant here.

To overcome the shortcomings mentioned, subjective judgement can be incorporated into inflation outlook uncertainty for example by adjusting the coefficients of the model used to generate the forecast or by additive shifts in the forecasts (Svensson 1999). The Bank of England and the Riksbank have in addition developed more analytic methods for incorporating subjective judgement into the forecast distribution. The following sections discuss both approaches, and a comparison of the methods is presented in appendix 1. The models have various similarities especially as to central assumptions, but also significant differences can be found.

3 The Bank of England approach

The introduction of the 2.5% inflation target for UK monetary policy in 1993 stressed the role of inflation forecasts, and the Bank of England (henceforth the BoE) started publishing its Inflation Report after the adoption of the new regime[8]. Each of these reports presents projections for inflation and GDP growth for the following two years besides a detailed account of recent economic developments. Already the early reports comprised in addition estimated uncertainty intervals for the inflation forecast based on the forecast errors from the previous ten years, while the current method of publishing a density forecast of inflation for one to nine quarters ahead was adopted in 1996. The forecast is based on an assumption of unchanged UK short-term interest rates during the forecast period, although a fan chart based on market expectations on interest rates is also depicted[9].

---

8 The minutes of the monthly Monetary Policy Committee are the other main vehicle of transparency.

9 The assumption of a constant short-term policy-determined interest rate originates from the time when the BoE was responsible for the inflation forecast but the Chancellor made interest rate decisions and it was not appropriate for the Bank to publicly express its views on the Chancellor's future actions. Vickers (1998) notes in addition that since the Bank cannot sensibly postulate its

The final density forecast presents the view of the Monetary Policy Committee (the MPC)[10], which has been the Bank's highest decision-making body since 1997. The Committee, aided by the Bank staff, forms its opinion about the risks affecting the inflation forecast by discussing the major economic issues. The necessary information originates from different models, official data, financial market intelligence and business surveys, in addition to which the MPC relies on its subjective judgement of current and future economic development (Bank of England 2000). The BoE's density forecast for inflation is therefore based on the Committee's view on the main scenario inflation forecast and the risks surrounding it. The introduction to Inflation Reports states this as follows: "Although not every member will agree with every assumption on which our projections are based, the fan charts represent the MPC's best collective judgement about the most likely paths for inflation and output, and the uncertainties surrounding those central projections."

3.1 The method

Before the final assessment, the MPC evaluates various scenarios utilising the BoE's multiple models approach to forecasting. It is after all expected to be able to explain the background of the presented distribution either with statistical or judgmental factors. Britton et al (1998), on whom this section is based if not stated otherwise, emphasise that in an ideal situation, all different scenarios would be evaluated. In practice, however, the forecast distribution is expected to follow the two-piece normal distribution, which is discussed more accurately below in the context of the Riksbank. However, choosing a different distribution when suitable is not ruled out and it is likewise possible to change the distribution between Reports. In the case of the two-piece normal distribution, determining the exact distribution requires estimates of only three parameters: a measure of central tendency, the balance of risks (ie skewness) and the degree of uncertainty.

The BoE defines the mode, ie the most frequent observation in a distribution, as the measure of central tendency. The definition follows from the Bank's willingness to concentrate on the most likely outcome, which is then generated by an economic model based on the central assumptions. During the process, the MPC considers several different models and the model used can thus vary between the forecasts. The choice of presenting the mode as the central projection has been criticised because of the properties of the parameter and the form of the loss function that justifies its use (see section 3.3).

The balance of risks is defined so that the difference between the mean and the mode measures skewness. The MPC first simulates the effects of alternative assumptions in the cases where risks are estimated to be unbalanced and attaches then probability weights to the different scenarios. The Bank of England (2000) notes that there are two kinds of events that may be incorporated into the inflation skewness. The first category comprises events that have already taken place or are about to do so, while the second category is composed of events that have not occurred yet and are therefore not incorporated into the central projection. The overall balance of risks expressed in terms of an effect on the

---

own future reaction function, the number of practicable alternatives as to expressing the distribution is quite limited.

[10] See Bean and Jenkinson (2001) for a detailed description of the internal processes of the MPC.

inflation forecast is in any case the aggregate derived by adding up the separate simulations weighted with the attached probabilities.

As to the degree of uncertainty, the initial assessment of the variance is based on the inflation forecast errors from the previous ten years. For example Tay and Wallis (2002, 49) note that historical forecast errors are an appropriate starting point, as they incorporate various possible error sources including the model error. However, the approach of projecting the variance forward is problematic in the presence of structural breaks such as the introduction of inflation targeting[11]. Because the inflation variance relates to the variances of the different shocks considered, the BoE uses simulations to estimate the contributions of the underlying variances on the inflation variance itself. The MPC's view on the degree of inflation uncertainty is then formed by adjusting the past inflation forecast error variance in accordance with the estimated effects of the underlying variances.

The MPC finally discusses the inflation distribution derived in the process described above. If it feels that the distribution is not consistent with its discussions, the assumptions and the estimated probabilities can be modified to calculate a new distribution. The forecast distribution is therefore adjusted until the variance and the skewness reflect the views of the Committee. In addition, the probability area of the probability density function must equal one as required by statistics. The process is thus of iterative nature and arriving to the final forecast requires a series of meetings between the MPC and Bank staff[12].

3.2 The fan chart

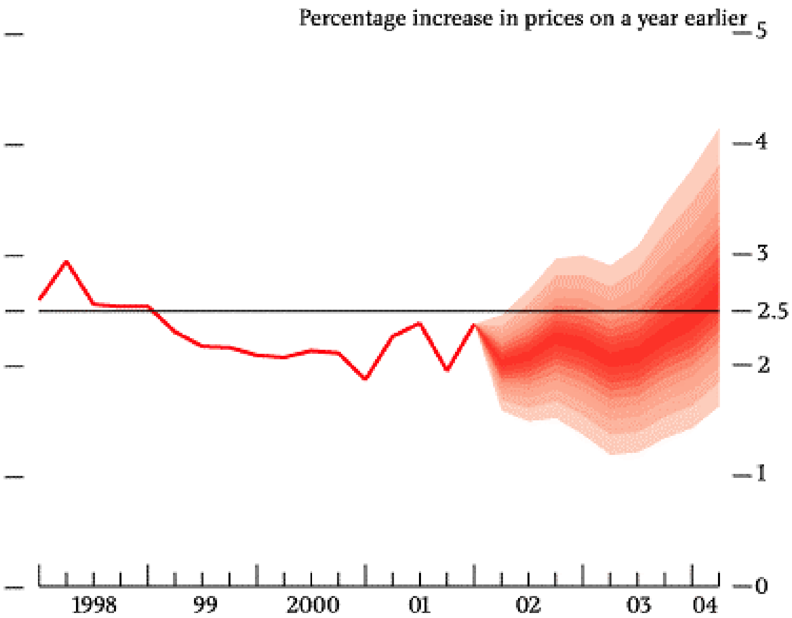
The parameters are estimated for one- and two-year projections, after which they are interpolated to achieve a quarterly fan chart such as the May 2002 chart depicted in figure 1. Each of the nine quarters of the forecast could naturally be presented as the standard two-piece normal probability distribution such as the one depicted in figure 2 in section 4.2.

---

[11] Wallis (2001) finds that the standard deviation used to estimate the fan chart has generally been too high. Since it seems likely that the recent forecast errors have been smaller than the past ones (see Bank of England (2000, 63)), the overestimation might result from the approach based on the past forecast errors.

[12] Britton et al (1998) discuss the timing of these meetings.

Figure 1. The Bank of England's fan chart in May 2002



Source: Bank of England Inflation Report May 2002 (http://www.bankofengland.co.uk)

The shading of the BoE's fan chart can be seen as a kind of contour map. At each point of time, the darkest band is centred at the mode projection and covers 10% of the total probability. The bands become lighter as the paths they depict become increasingly unlikely, and each shade of red covers additional 10% of the probability. It should however be noted that the bands do not coincide with the standard percentile-based intervals and that the tail probabilities at a certain point of time do not necessarily coincide with each other. The BoE has not found it necessary to report the tail probabilities.

3.3 Criticism

The goals of presenting inflation prospects in the form of a density forecast are generally accepted and agreed on. The criticism against the method adopted by the BoE has thus been mainly concerned with parameter and presentation choices. Above all, emphasising the role of the mode at the expense of the mean (and the median) and compiling the fan chart from as narrow bands as possible have been criticised.

Wallis (1999) first approaches the issue of a suitable central measure for inflation by discussing the form of a loss function of a central bank. He concludes that the loss function associated with the use of the mode as the measure of central tendency is the so-called step loss function, in which the loss is zero for a correct prediction and a constant positive value otherwise. Although this conclusion is generally accepted[13], a situation in which the size of the deviation from the realised value is insignificant seems quite unrealistic. Particularly, Wallis does not find it probable that the BoE's loss function is of this form. As to the statistical properties of the central measures, he reminds furthermore that the mode is problematic also in the sense that for random variables X and Y with asymmetric

[13] See for example Vickers (1998).

distributions mode[X+Y] does not generally equal mode[X] + mode[Y] even in the linear case.

Wallis claims in addition that the use of the mode as the central projection in a fan chart gives a biased impression of the average inflation over a number of years: with positive skewness (as in the BoE fan chart of August 1997), the mode is biased downwards. The problems thus relate to the asymmetry of the two-piece normal distribution. Also Svensson (1999) believes that it might be better to plot the mean (or at least the median in the case of distribution targeting) instead of the mode, since he finds the former in many cases a more appropriate focus than the latter.

Moreover, the prediction (or confidence) intervals of the BoE's fan charts do not follow the standard practice of expressing percentile-based intervals. Wallis (1999) emphasises that in statistical literature, the prediction intervals are usually based on the percentiles of the forecast distribution. He also notes that prediction intervals for asymmetric distributions generally used in literature are central prediction intervals defined in accordance with confidence intervals as follows. “A central 100p % prediction interval covers the stated proportion in the centre of the distribution, with equal probability in each of the tails.” The problem with the BoE's fan chart is therefore that it is not constructed of central prediction intervals, as the customary procedure would require. Instead, the BoE sets the bands as narrowly as possible for the assigned probabilities, which centres the bands on the mode. In the case of an asymmetric distribution, these intervals do not coincide with the central intervals and the tail probabilities are unequal[14]. Wallis shows for example that “for the Bank's forecasts [of August 1997], the probability that inflation will lie above the interval exceeds the probability that inflation will lie below the interval”. This, in turn, makes interpreting the fan chart correctly more difficult.

Wallis notes further that the best prediction interval for an all-or-nothing loss function is the shortest, while the central prediction interval is more suitable in the case of a linear loss function. The complaint is in line with Svensson (1999, 22), who claims that “under a loss function assigning a constant loss outside a tolerance interval and a zero loss inside, the forecast should maximise the probability of being inside the interval, which implies that the upper and lower bounds should have the same probability density”.

Wallis (1999) constructs the BoE's August 1997 Inflation Report fan chart again and finds that his central interval lies above the interval presented by the Bank. Furthermore, the alternative chart highlights the median instead of the mode. Wallis emphasises that both charts represent the same distribution but that the alternative chart nevertheless gives an impression of higher future inflation because the distribution is positively skewed[15]. Also Tay and Wallis (2002)

---

[14] Wallis (2001) complains likewise that the tail probabilities are not reported.

[15] Wallis (2001) evaluates the BoE's forecast performance after the MPC's inauguration. As to the one-year-ahead forecasts, he finds that the null hypothesis of unbiasedness would be rejected against the one-sided alternative of an upward bias at the 5% significance level. In contrast to the one-year-ahead forecasts, the current-quarter forecasts seem to perform well: they appear unbiased and the standard deviations seem correct. The results indicate likewise that the standard deviation used in preparing the fan charts has generally been too high. One-year-ahead inflation forecasts have also frequently been too high, and the test implying that a null hypothesis of a correct unconditional coverage should not be rejected in favour of the view that the bands are too wide does not take into account the fact that all the misses are in the lower tail. Wallis therefore

emphasise that it is important to pay attention to the presentation of a density forecast, as inappropriate presentation might be misleading and thus misinterpreted. The authors find graphical presentations useful particularly in eliciting asymmetries, in addition to which it is often infeasible to present the distribution in the analytic form.

The criticism has not made the BoE to modify its approach. Instead, Vickers (1998) states that although the fan chart draws attention to the mode this does not mean that the mode determines monetary policy. He instead emphasises that the whole distribution and its other moments are taken fully into account. Also the Bank of England (1999) highlights that although there are different ways of illustrating the in principle same probability distribution, the methods do not alter what is being illustrated. The BoE claims furthermore that policy is unaffected by how a fan chart is drawn[16]. Moreover, Vickers reminds that the conditional mean is not necessarily the optimal intermediate target even with strict inflation targeting and a quadratic loss function unless uncertainty is additive.

4 The Riksbank approach

The Riksbank adopted an explicit inflation target of 2% annual rise in the consumer price index with a tolerance interval of ± 1 percentage points in 1993. The Bank has since then been developing its approach to predicting inflation contingent on the prevailing repo rate. Although the assumption of a constant interest rate restricts comparisons with other forecasts that assume some response from monetary policy, it provides a simple rule of thumb for conducting monetary policy: an inflation forecast above (below) the target rate indicates too expansionary (restrictive) monetary stance and the repo rate should therefore be raised (lowered) in the near future (Berg 1999).

Blix and Sellin (1998, 1999, 2000) describe the procedure for producing the Riksbank's density forecast for inflation, and this section follows them if not stated otherwise. In addition, the author has benefited from discussions with Mårten Blix and Stefan Laséen from the Riksbank. In short, the Bank's approach relies on evaluating the development of the macro variables deemed to affect future inflation and gives a major role to subjective judgement, which has been found particularly important in making short-term forecasts. In contrast to the BoE, the final uncertainty estimate and the balance of risks for the inflation forecast derive from the chosen macro variables. Blix and Sellin (1998, 3) emphasise that “since it is the inflation forecast and the inflation uncertainty that matter for policy decisions, it is desirable to have inflation uncertainty *endogenously* determined from underlying assumptions”. The outlook is presented to the public quarterly in the Riksbank's Inflation Report, which has comprised an inflation forecast with uncertainty intervals since December 1997. The section concerning the density forecast for inflation discusses both upside and downside risks besides presenting a fan chart and interval forecasts for one and two year predictions.

---

concludes that this implies that the MPC has exaggerated the upside risks and that the Bank's density forecasts place too much probability in the upper ranges of the inflation forecast.

[16] Svensson (1999) agrees that the illustration need not have effect on how the policy is conducted if the policy is based on the entire distribution.

4.1 Justifying the mode as the measure of central inflation

Despite the criticism presented in section 3.3, also the Riksbank uses the mode (denoted here somewhat exceptionally by μ) as the main scenario point forecast. As the primary reason for the use of the mode instead of the mean Blix and Sellin (2000) present the claim that it is intuitively more natural to view a point forecast as the most likely outcome[17]. In the case of the two-piece normal function used in the Riksbank method, the parameter that maximises the probability density function is naturally the mode. The authors base their statement on two factors. First, they state that point forecasts are predominantly judgmental since the element of subjective judgement is always present in generating them – even despite the use of econometric models. Second, they find that judgmental forecasts are closer to the mode than the mean since forecasters in general concentrate on the most likely scenario instead of weighting all scenarios, including the most exceptional ones, into the forecast by taking their probabilities into account. Furthermore, Blix and Sellin find that the practical implementation of the method at the Riksbank supports the conjecture: the approach has helped to display the balance of risks correctly. Moreover, Blix and Sellin (1999) emphasise that the discussion should not concentrate on the measure of central tendency instead of the more relevant forecast distribution.

In general, the use of the mode has both advantages and disadvantages. On the one hand, the mode uses less information about the distribution than the mean and might thus be misleading in some cases. On the other hand, the parameter is less sensitive to exceptional values such as observations in the tails of the distribution. Moreover, the mode does not have the same asymptotic justification as the mean as measure of central tendency. However, this is not an issue when the finite-sample distribution is assumed to be known, as is the case here.

Despite the weight given to the mode, the mean $\tilde{\mu}$ is not ignored in the analysis. In accordance with the BoE, the difference between the mean and the mode, that is $\gamma \equiv \tilde{\mu} - \mu$, plays an important role by measuring the skewness of the distribution. When $\gamma = 0$, the distribution is symmetric but when γ is negative, the distribution is skewed to the left and downside risks dominate upside risks. And vice versa. The parameter γ therefore expresses the balance of risks. Significant skewness could indicate a need to revise the forecast and the assumptions behind it.

4.2 Two-piece normal distribution

Blix and Sellin assume that macro variables $X_j(t)$, j = 1,…,n, deemed to affect future inflation follow the two-piece normal distribution (4.1) defined by parameters μ, σ_1 and σ_2 so that

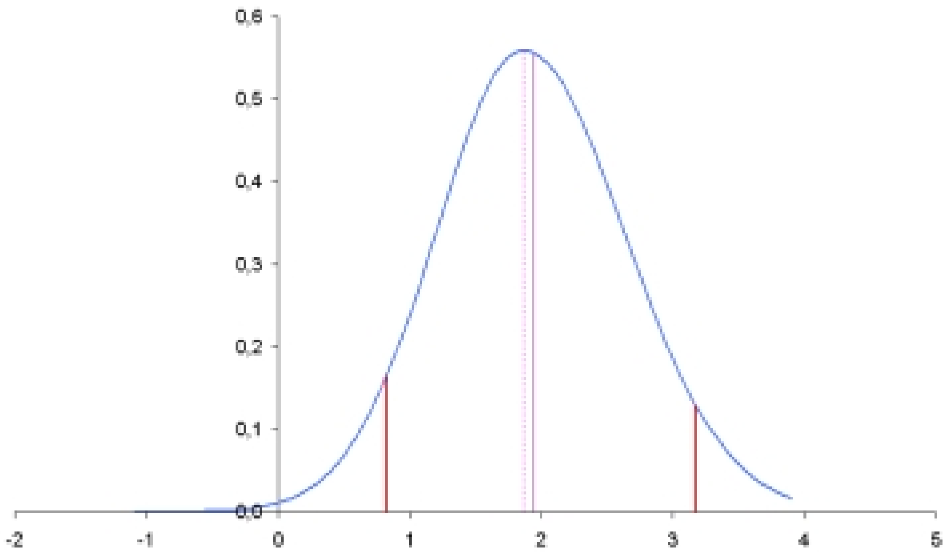
[17] The argument relates to that of the BoE, see section 3.1.

$$(4.1) \quad f(x;\mu,\sigma_1,\sigma_2) = \begin{cases} C\exp\left[-\dfrac{(x-\mu)^2}{2\sigma_1^2}\right], x \le \mu \\ C\exp\left[-\dfrac{(x-\mu)^2}{2\sigma_2^2}\right], x > \mu \end{cases}$$

where $C = \sqrt{2/\pi}(\sigma_1 + \sigma_2)^{-1}$. The underlying macro variables the Riksbank uses are discussed in appendix 2.

The distribution to the left of the mode is proportional to the standard normal distribution (ie the Gaussian) with a standard deviation σ_1, as is the distribution to the right of the mode but with a different standard deviation σ_2. If $\sigma_1 > \sigma_2$, the distribution is skewed to the left and conversely when $\sigma_1 < \sigma_2$. Moreover, the distribution coincides with the standard Gaussian if σ_1 equals σ_2. Wallis (1999) specifies the derivation of the two-piece distribution further by stating that the halves of the two original normal distributions are scaled to give the common value $f(\mu)$ so that the scaling factor applied to the normal probability density function on the left is $2\sigma_1/(\sigma_1+\sigma_2)$ while the factor applied on the right is $2\sigma_2/(\sigma_1+\sigma_2)$. Tay and Wallis (2002) state in addition that the coefficient of kurtosis exceeds 3 whenever $\sigma_1 \ne \sigma_2$, because of which the distribution is leptokurtic. Figure 2 presents a positively skewed two-piece normal distribution. In such a case, the mode depicted by the weaker dotted line is the smallest of the three measures of central tendency. The figure also shows the 90% confidence interval.

Figure 2. Two-piece normal distribution



The choice of the two-piece normal function derives mainly from three factors. First, the distribution is easy to work with. Second, the distribution treats both upside and downside risks in the same way unlike many other asymmetric distributions. Third, the distribution has the Gaussian as a special case. As mentioned, the distribution collapses to the standard normal distribution if no special information is available, which is often the implicit assumption among forecasters. Also, because the distribution is closely related to the normal distribution, central limit theorems can be used, while special circumstances can in most events justify departures from the Gaussian distribution. Tay and Wallis (2002) note however that two-piece normal distribution does not have any convenient multivariate generalisation.

As discussed above, Blix and Sellin measure skewness with[18]

$$(4.2) \quad \gamma(t) \equiv \tilde{\mu}(t) - \mu(t) = \sqrt{2/\pi}(\sigma_2(t) - \sigma_1(t))$$

The difference between the mean and the mode is proportional to the third central moment, which is generally used as a measure of skewness (Berg 1999). It is now easy to obtain the mean of the distribution from (4.2).

4.3 The role of subjective judgement

At the Riksbank, the economics department evaluates future prospects, including the degree of uncertainty, for the macro variables deemed to affect inflation (see appendix 2 for a discussion of the variables). The approach described here is claimed to substitute for the common estimation of a system of equations.

The economists of the department specialise in different factors and the individual assessments are then discussed together to achieve consistency in the overall picture for 12 and 24 months ahead. To focus the discussion, each subjective judgement must be accompanied by a verbal explanation particularly if the future development is expected to differ from the past. The process has been made easier in advance by showing all the participants how their evaluations of the variables they are responsible for affect the other variables. However, general knowledge and understanding of the entire process is not required from everyone. The forecasting performance of each forecaster is followed and the forecasts of the previous round are sent to the respective economists at the beginning of a new round. According to the prevailing practice, it is more recommendable to be sluggish than to change the future prediction and its direction constantly.

The economics department finally delivers the forecast including the uncertainty assessments to the Executive Board, which may require revisions to them. In any case, the Board uses the assessment as the basis for the uncertainty analysis finally presented in the Inflation Report. The published outlook reflects the majority view of the Board, and one of the benefits of the approach is that a single differing opinion can be taken into account in the asymmetry of the distribution without completely modifying the forecast.

Incorporating subjective assessments of economic development into inflation forecast uncertainty is based on two questions posed for each of the X_j's. The questions aim at defining the balance of risks and the degree of uncertainty. The former is concerned with the possible asymmetry of the distribution while the latter measures the uncertainty of the future development in comparison to historical forecast errors. Blix and Sellin find that deriving these

[18] The derivation of (4.2) based on John (1982) is presented in Blix and Sellin (1998).

two figures helps to focus internal discussion at the Bank on the factors causing uncertainty and further on the underlying assumptions.

The first question concerning the balance of risks is stated explicitly as "what is the probability that the realised value of the variable X_j falls below the mode forecast?" The question can also be formulated more formally as "what is $P_j =$ pr[$X_j \le \mu_j$]?", which gives the definition for the parameter P. The reference value for P used with no specific information available is 50%.

The second question concerns the degree of uncertainty and relates uncertainty of the current forecast to historical uncertainty measured by the standard deviation of the variable. The reference value for this parameter depicted by h_j is 1. If current uncertainty is estimated to exceed historical uncertainty, h_j>1 and vice versa. If the future is estimated 30% more uncertain than the past, the value given for h_j is 1.3. A belief that the economy is approaching a turning point might be reflected in a high value of h, while a low h could be justified in a situation in which all indicators point to the same direction. According to the Riksbank, the values of h have usually ranged between 0.7 and 1.3.

The parameters P and h received as the answers to the questions can be used to derive the parameters that specify the two-piece normal distribution (4.1). The assessment of uncertainty has its starting point in historical data, and the first step is scaling the variance of X_j with the uncertainty parameter h_j. The parameter ω defined in (4.3) combines subjective judgement with the historical standard deviation of X_j (denoted with $\sigma_j(t)$) as follows:

$$(4.3) \quad \omega_{j,j}(t) = \left[h_j(t)\sigma_j(t)\right]^2 .$$

Blix and Sellin (1998) show that the two missing parameters needed to specify the two-piece normal distribution can be expressed as

$$(4.4) \quad \sigma^2_{1,j}(t;\omega_{j,j},P_j) = \omega_{j,j}(t)\left[(1-2/\pi)\left(\frac{1-2P_j(t)}{P_j(t)}\right)^2 + \left(\frac{1-P_j(t)}{P_j(t)}\right)\right]^{-1}$$

and

$$(4.5) \quad \sigma^2_{2,j}(t;\omega_{j,j},P_j) = \omega_{j,j}(t)\left[(1-2/\pi)\left(\frac{1-2P_j(t)}{1-P_j(t)}\right)^2 + \left(\frac{P_j(t)}{1-P_j(t)}\right)\right]^{-1} .$$

Equations (4.4) and (4.5) are proportional to $\sigma_1^2 \cong h^2\sigma^2 P/(1-P)$ and $\sigma_2^2 \cong h^2\sigma^2(1-P)/P$. Therefore, the higher the h is (ie the higher the degree of future uncertainty), the higher the standard deviations are. The effect of the factor P can likewise be analysed: when the value of the parameter increases (ie there is more downward risk), σ_1 increases while the effect on σ_2 is to the opposite direction. A larger P means thus that the proportion of the probability mass to the left of the mode increases, which in turn means that downside risks are larger. This is compatible with the assumptions.

4.4 The distribution for the inflation forecast

After having established a framework for incorporating subjective judgement at the level of underlying macro variables, Blix and Sellin move forward to aggregating the effects to concern inflation. They emphasise again that the Riksbank's approach does not require assessing the degree of uncertainty and the balance of risks for inflation as such. Instead, the inflation forecast is assumed to follow the two-piece normal distribution of (4.1) in accordance with the factors affecting inflation[19]. Since the inflation forecast $\mu_{\Delta p}(t)$ and its uncertainty measured by the historical average of the past forecasting errors (see Blix and Sellin 2000, 9) are for now taken as given, the remaining task is to derive the estimates for $\sigma_{1,\Delta p}$ and $\sigma_{2,\Delta p}$ by connecting the uncertainty assessments of the underlying variables. Here Blix and Sellin resort to a statistical approximation, which they first show to be qualitatively sensible[20]. The approach is similar to that of the Norwegian central bank[21] and relies on combining the skews of the individual macro variables into inflation skewness through equation

$$(4.6) \quad \gamma_{\Delta p}(t) = \sum_{j=1}^{n} \beta_j(t) \gamma_j(t),$$

where γ with the lower case symbol Δp is the skewness of inflation and the skewness parameter on the right hand side relates to the X_j's. According to the Riksbank's experiences, the skewness of inflation is usually below 0.2.

As can be seen from equation (4.6), the skewness of a single macro variable X_j affects the total skewness of inflation with the weight β_j, which reflects the importance of the variable in question. The weights are the elasticities with respect to inflation and are derived by shocking each respective factor in isolation and by calculating its impact on inflation one and two years ahead in a macroeconometric model. In July 2002, the weights were based on five models, of which the one found most suitable was used to calculate the final weight parameters. Other considerations comprised choosing the weights close to the Riksbank's rules of thumb and monitoring that the weights chosen do not differ too drastically from the weights derived from the other macroeconometric models under consideration. Moreover, the weights between the 12 and 24 months

---

19 Assuming a linear relationship between the macro variables and inflation is not found helpful since aggregating several two-piece normal variables does not result in any known distribution (Blix and Sellin 1999). Preliminary results of research conducted at the central bank of Portugal indicate however that the two-piece normal distribution is an appropriate approximation.

20 This can be shown by evaluating equation (4.6). First, if there is no skewness in the macro variables, there will not be any in inflation. Second, Blix and Sellin note that negative skewness in for example consumption will result in negative skewness in inflation since, in the case of consumption, the weight is typically non-negative. Third, when underlying macro variables are skewed to opposite directions, the weights describing the relative importance of the variables for future inflation will determine whether inflation is skewed to the left or to the right.

21 Norges Bank approximates the skewness of inflation forecast as a weighted sum of the skews of the underlying factors. The weights express the effects of each individual factor to inflation 1–2 years ahead in time, while the skewness of a factor is based on the situation in the market on the one hand and on a subjective assessment of risks on the other. (Norges Bank 2001)

forecasts differ also in that some spill-over from the first period is incorporated into the second period weights.

The skewness parameters on the right hand side of (4.6) can be obtained by substituting the square roots of (4.4) and (4.5) into (4.2), which results in

$$(4.7) \quad \gamma_j(t) \equiv \tilde{\mu}_j(t) - \mu_j(t) = \sqrt{2/\pi}\,(\sigma_{2,j}(t) - \sigma_{1,j}(t)) .$$

John (1982) shows that the variance of a variable following the two-piece normal distribution can be calculated as (here the variance is calculated for inflation Δp)

$$(4.8) \quad \sigma^2_{\Delta p}(t) = (1 - 2/\pi)\left(\sigma_{2,\Delta p}(t) - \sigma_{1,\Delta p}(t)\right)^2 + \sigma_{1,\Delta p}(t)\sigma_{2,\Delta p}(t) ,$$

because of which the values for $\sigma_{1,\Delta p}$ and $\sigma_{2,\Delta p}$ can be calculated from (4.8) and

$$(4.9) \quad \gamma_{\Delta p}(t) = \sqrt{2/\pi}\,(\sigma_{2,\Delta p}(t) - \sigma_{1,\Delta p}(t)) .$$

These can be reduced to

$$(4.10) \quad \sigma^2_{1,\Delta p}(t) + b\sigma_{1,\Delta p}(t) + c = 0 ,$$

where $b = \dfrac{\gamma_{\Delta p}}{\sqrt{2/\pi}}$ and $c = -\left[(1 - \pi/2)\gamma^2_{\Delta p} + \sigma^2_{\Delta p}\right]$.

Here and for now the variance $\sigma^2_{\Delta p}$ is based on the Riksbank's historical inflation forecast errors adjusted to account for the assumption of a constant repo rate (Blix and Sellin 1999). Of the two solutions obtained for (4.10) only one is typically relevant. This solution can be substituted into (4.9) to obtain $\sigma_{2,\Delta p}(t)$. The standard deviation parameters can then be substituted further into (4.1), which defines the entire two-piece normal distribution for the inflation forecast with the given mode forecast.

4.5 Incorporating judgement of the underlying macro variables into the inflation forecast variance

Thus far, the inflation forecast variance was solely based on past forecast errors. It is however sometimes desirable to base the variance estimate on the uncertainties concerning the underlying macro variables[22]. The conventional approach to relating unconditional variances would rely on a (typically linear) relation of the form

[22] Despite the possible advantages, the Riksbank does not follow the approach presented in this section. Instead, the Bank's procedure for deriving a forecast distribution allows using a parameter similar to the parameter *h* in the context of the underlying macro variables to modify historical uncertainty based on subjective assessments.

$$(4.11)\quad \begin{aligned} \Delta p_t &= \theta' X_t + \eta_t \\ \sigma^2_{\Delta p} &= \theta' \Omega_X \theta + \sigma^2_\eta , \end{aligned}$$

where Ω_X is the covariance matrix of X_t and $\sigma^2_\eta = \mathrm{var}[\eta_t]$. Blix (1999) finds however that although this method is a mathematically correct way of relating variances with standard OLS assumptions, the standard regression assumptions are probably violated in the case relevant here. Blix and Sellin (1998) therefore present a different method for incorporating subjective judgement concerning the underlying macro variables into the inflation variance. The method is based on the assumption that

$$(4.12)\quad \sigma^2_{\Delta \mathrm{p}}(t;\sigma^2_\varepsilon) = \beta'(t)\Sigma(t;\sigma^2_\varepsilon)\beta(t)\,,\ t = 1,\ldots,T,$$

where $\beta' = \begin{bmatrix} 1 & \beta_1 & \cdots & \beta_n \end{bmatrix}$,

$$\Sigma(t;\sigma^2_\varepsilon) = \begin{bmatrix} \sigma^2_\varepsilon & 0 & \cdots & 0 \\ 0 & \omega_{1,1} & \cdots & \omega_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \omega_{n,1} & \cdots & \omega_{n,n} \end{bmatrix} \text{ and}$$

$$\omega_{i,j} = \begin{cases} \mathrm{var}[X_j(t)] & \text{for } i = j \\ \mathrm{cov}[X_i(t), X_j(t)] & \text{for } i \neq j \end{cases}.$$

The variance was defined in (4.3), while the covariance is calculated as

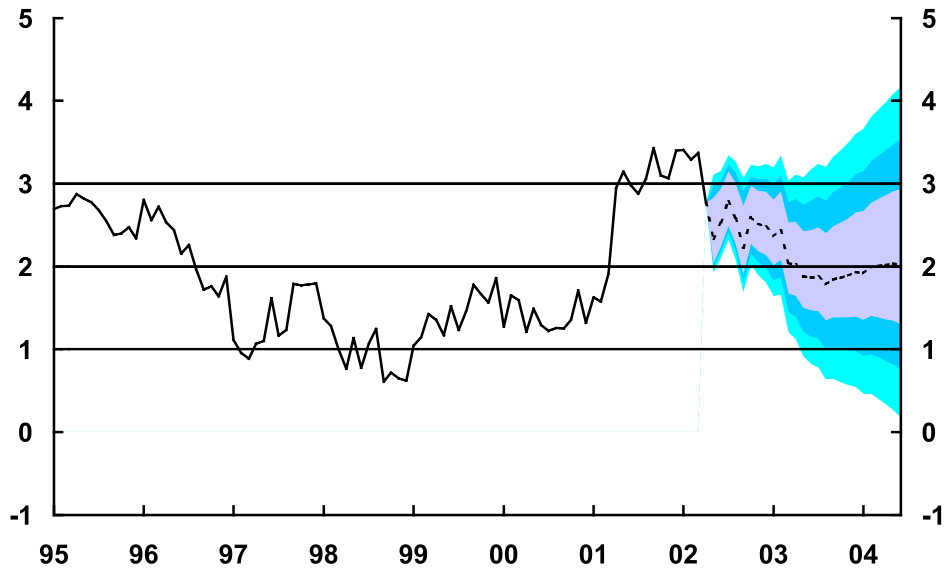
$$(4.13)\quad \mathrm{cov}[X_i(t), X_j(t)] = \rho_{i,j}\sqrt{\omega_{i,j}(t)\omega_{j,j}(t)}\ .$$

The parameter $\rho_{i,j}$ is the estimated time-independent data correlation between X_i and X_j, and the covariance matrix Σ can be viewed as the empirical covariance matrix scaled to account for subjective judgement. Differences between the two therefore result from the subjective assessments of the variable development. Furthermore, the term $\sigma^2_\varepsilon(t)$ is defined as the variance of an inflation shock independent of the X_j's. It thus represents that part of uncertainty that increases, as the forecast horizon becomes longer. The parameter can be calibrated for the required time periods (for one and two years in the case of the Riksbank) by setting all h's at 1 and choosing then $\sigma^2_\varepsilon(t)$ so that (4.12) equals the standard deviation of the historical forecast errors t years ahead. The derived $\sigma^2_\varepsilon(t)$'s can then be used together with the subjective variances to yield an adjusted $\sigma^2_{\Delta p}(t)$, which can be substituted into (4.10) to replace the historical variance used in the previous section. Compared to the use of mere historical uncertainty, the approach leads to an increasing variance when the outlook of an underlying macro variable is conceived as increasingly uncertain, and conversely.

4.6 Presentation as a fan chart

At the Riksbank, the time interval between the probability distributions for one and two year projections is interpolated to produce a fan chart. The Bank accomplishes the task by adjusting the intervals upwards monthly by a given factor, which is chosen so that the adjustment lies within the interval limits given by the one and two years distributions. As a response to the criticism Wallis (1999) presents against the BoE's fan chart, Blix and Sellin (2000) emphasise that in contrast to the BoE's charts and particularly in the case of an asymmetric distribution, the Riksbank's confidence intervals are not centred at the mode. Instead, they are symmetric around the median and have therefore the standard interpretation. This can be seen from figure 3, which depicts the Riksbank's fan chart[23]. The fan chart is composed of the mode forecast and the confidence intervals for 50, 75 and 90% probabilities.

Figure 3. The Riksbank's fan chart



Source: The Riksbank's Excel workbook

Future possibilities include publishing a bivariate distribution for output and inflation[24]. Although the method in itself is implementable, it has not yet been found necessary at the Executive Board level.

5 The Bank of Finland's short-term density forecast for inflation

At the Bank of Finland (BoF), incorporating subjective judgement into the inflation forecast distribution follows closely the Riksbank's approach. Some modifications relating particularly to underlying macro variables are nevertheless necessary and also useful due to differences in forecasting procedures. The

[23] The figure is not from the Bank's latest Inflation report (2/2002) although it follows closely the UND1X inflation figure presented there.
[24] See Blix and Sellin (2000).

implementation of the forecast distribution at the BoF is based on an inflation prediction model called NIPE, which is in contrast to the Riksbank's "no single model" approach. Since the NIPE model relies on predicting the future development of the sub-components of the harmonised index of consumer prices (HICP), the BoF utilises these subgroups as underlying variables instead of macro variables such as international economic growth or public demand[25] to compile the distribution for the inflation projection. This main difference between the approaches leads to further modifications in other parts of the process. Since the practical implementation is discussed in detail in Virta (2002), only general principles are covered here. A discussion on the implications of the past NIPE forecasts and their accuracy precedes presenting the actual implementation.

5.1 The realised forecast errors of the Narrow Inflation Projection Exercise

The BoF's NIPE model bases the estimated future HICP on the five sub-components of energy, non-energy industrial goods, unprocessed food, processed food, and services. Appendix 3 presents the June 2002 compositions of the NIPE subgroups in a fairly general level without announcing the possible lags. The adjustment factors mean that subjective judgement can be used in addition to the actual model to generate the final forecast path.

The top row of figure 4 depicts the frequency distributions for 4- and 10-month HICP forecast errors since the beginning of 1999[26]. The forecast horizons mentioned have been chosen for closer inspection because they play the largest roles in the August 2002 NIPE forecast (ie the complete density distributions are compiled for December 2002 and June 2003). As expected, the pictures show that the range is longer for 10-month forecasts and that 4-month forecasts usually hit closer to the actual inflation rate. On the other hand, the 10-month frequency is also at its highest at near zero errors. The majority of observations are positive in both cases, which means that the forecasts have generally underestimated the realised inflation rate. The pictures in the middle row graph the estimated density and the distribution for HICP. The 4-month distribution seems to be skewed to the left while the 10-month distribution is closer to the normal distribution. The pictures at the bottom row depict normality even more explicitly: the closer the curve is to the straight line, the closer the distribution of the errors is to the Gaussian. The 4-month forecast distribution is somewhat heavy-tailed, but the 10-month forecasts seem to be quite close to the normal distribution.

[25] Appendix 2 discusses the underlying macro variables the Riksbank and Norges Bank use.
[26] See appendix 4 for the frequency distributions and estimated densities for the subgroups.

Figure 4. Historical NIPE forecast errors of HICP

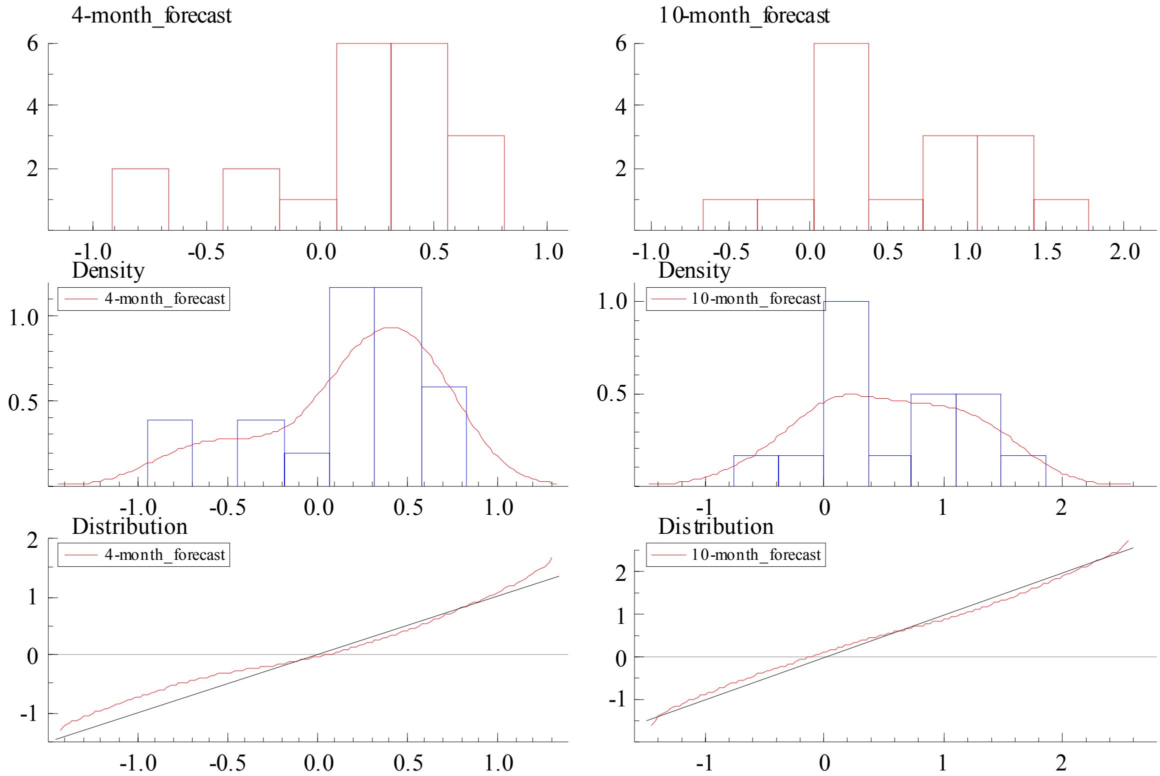


Table 2 presents some descriptive statistics for the 4- and 10-month HICP forecasts[27]. The parameters confirm the conclusions made on the basis of figure 4: the mean is lower for the 4-month forecast as is the standard deviation. Furthermore, the assumption of normality is rejected at 5% risk level for 4-month forecast errors, while the same assumption at the same risk level is not rejected for 10-month forecast errors. The analysis implies that the approach directly based on the normality of the forecast errors is not suitable for the entire NIPE path although it could be used to generate the forecast distribution at the end of the projection horizon. Moreover, the Riksbank's method facilitates incorporating subjective judgement into the forecast distribution, which is a further justification for the use of the approach.

[27] Appendix 5 comprises the similar statistics for the subgroups.

Table 2. Descriptive statistics

	4-month forecast	10-month forecast
Observations	20	16
Mean	0.19437	0.54964
Std.Devn.	0.43635	0.61394
Skewness	-0.95546	-0.013719
Excess Kurtosis	-0.21345	-1.0355
Minimum	-0.75384	-0.60941
Maximum	0.72841	1.5458
Asymptotic test:	Chi^2(2) = 3.0810 [0.2143]	Chi^2(2) = 0.71539 [0.6993]
Normality test:	Chi^2(2) = 7.5934 [0.0224]*	Chi^2(2) = 0.32284 [0.8509]

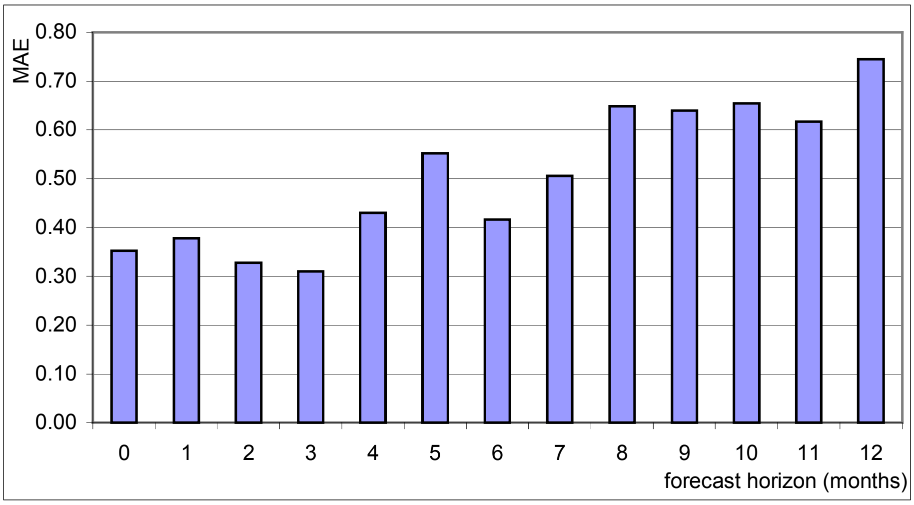
In addition, the one-sample t-test of table 3 reveals that the mean of the 4-month forecast errors does not differ statistically significantly from zero at 5% risk level, although only barely. The mean of the 10-month forecast errors does however differ from zero at the same risk level.

Table 3. One-sample t-test

	4-month forecast	10-month forecast
T_0	1.992	3.581
Critical value	2.093	2.131
P	0.061	0.003

The mean absolute errors of the past HICP forecasts are depicted in figure 5. The figure shows how uncertainty increases as the forecast horizon becomes longer. The forecast errors have therefore been larger for longer-term predictions. The similar figures for the subgroups are presented in appendix 6. They can be used to deduce suitable values for the parameter h. The figures differ greatly so that historical uncertainty has been at its largest in the subgroups of energy and unprocessed food. It is somewhat surprising that uncertainty has not increased in every subgroup as a result of a longer horizon. Such behaviour of the forecast errors might after all be expected.

Figure 5. The historical uncertainty of short-term HICP forecasts at different horizons



5.2 Implementation at the Bank of Finland

As to implementation, one important issue relates to complete forecast distributions, which the Riksbank calculates for 12- and 24-month horizons. Although the horizon of the NIPE forecast is shorter, complete probability distributions can be calculated for two distinct points of time in accordance with Sweden's central bank. It is natural to choose the end of the respective projection horizon as the latter point, as deriving an interval forecast for that time is one of the main objectives of the whole procedure. The other point can then be chosen as wanted between the present and the end of the horizon. In August 2002, the complete distributions were calculated for December 2002 and June 2003. In line with the Riksbank procedure, the final fan chart is interpolated on the basis of these two distributions.

In the most practical level, the parameters required for the two-piece normal distribution are calculated in Excel, which is also used to draw the fan chart. For this purpose, the program received from the Riksbank[28] was applied to the BoF use. Deriving the forecast distribution begins by inserting P's, h's and historical standard deviations for all the subgroups into the Excel workbook. In accordance with the Riksbank, the mean absolute error of the forecasts of the corresponding length is used to measure historical uncertainty. Sub-component predictions are then fused into an inflation forecast by annually changing weights, which the BoF utilises also to combine the skews of the subgroups into the skew of the inflation forecast. The use of predefined weights instead of simulated ones is one of the differences between the approaches of the BoF and the Riksbank.

The historical standard deviation of inflation forecasts defined as in the context of the sub-components is also necessary. This standard deviation can be adjusted to account for subjective views on future development by using a parameter similar to the h's of the sub-components. To make updating the calculations as quick as possible, a macro has been compiled. Using the macro requires only pressing a button, after which the update is complete.

[28] We gratefully acknowledge Sveriges Riksbank for allowing us to use their program.

The most important outputs of the process of introducing subjective judgement into NIPE's density forecast are presented in table 4 and figure 6. Table 4 first depicts the complete interval forecast at the end of the projection horizon in June 2003 according to August 2002 forecast. The column on the left has been compiled without subjective judgements of the future development of the HICP subgroups. The column on the right reflects then the estimated upside risks mainly in services but also in processed food.

Table 4. The August 2002 density forecast for June 2003: without and with subjective assessments

%	without subjective assessments	with subjective assessments
[6.5;7.0[	0	0
[6.0;6.5[	0	0
[5.5;6.0[	0	0
[5.0;5.5[	0	0
[4.5;5.0[	0	0
[4.0;4.5[	0	0
[3.5;4.0[	0	0
[3.0;3.5[	1	3
[2.5;3.0[	5	9
[2.0;2.5[	16	19
[1.5;2.0[	28	29
[1.0;1.5[	28	26
[0.5;1.0[	16	11
[0.0;0.5[	5	2
[-0.5;0.0[	1	0
[-1.0;-0.5[	0	0
Total	100	100

Figure 7 shows the complete fan chart according to the August 2002 NIPE forecast, while the same chart is depicted in figure 6 without the subjective judgements. It is easy to observe that the subjective distribution at the end of the forecast horizon in June 2003 is skewed to the right as is the December 2002 distribution but in a lesser scale. A comparison between the figures shows how subjective assessments have shifted the 90% confidence interval upwards.

Figure 6. The complete August 2002 fan chart with subjective judgement

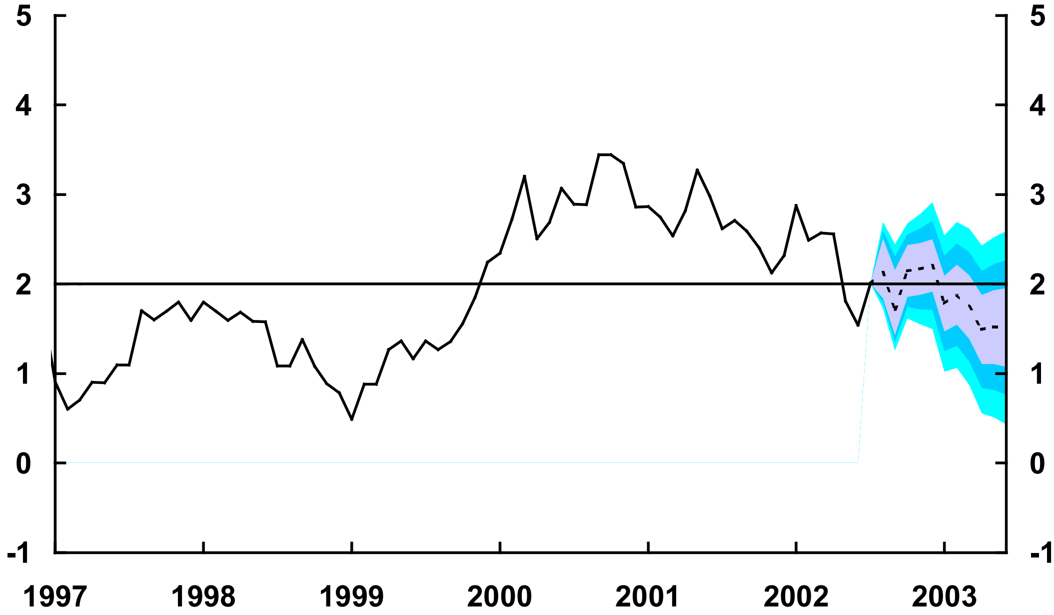
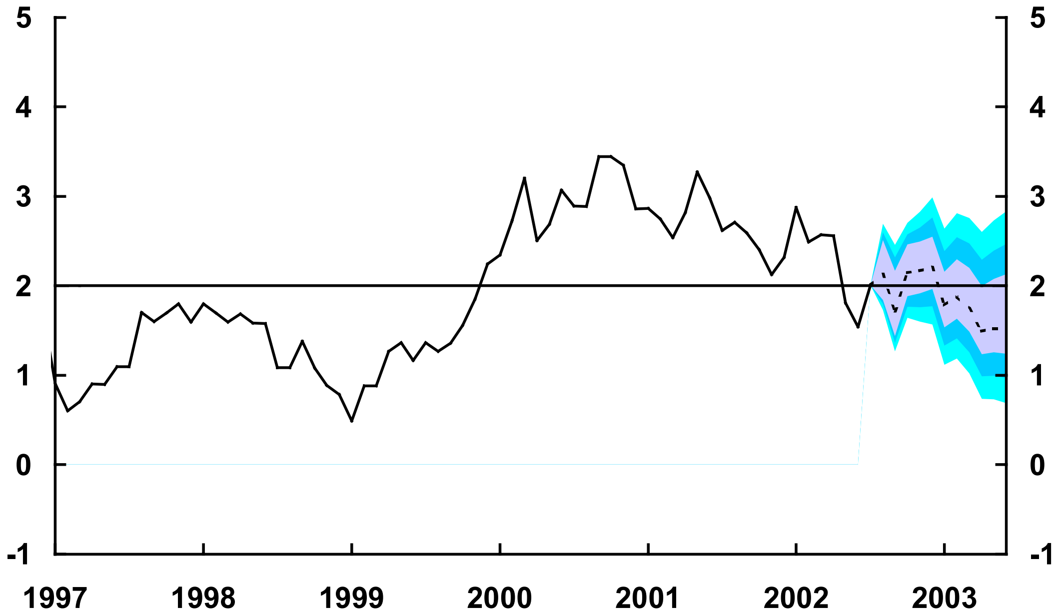


Figure 7. The complete August 2002 fan chart with subjective judgement



The role of the central projection warrants further discussion, as the NIPE forecast depicted by the dashed line is the sum of the weighted (mode) forecasts of the subgroups. Therefore, it cannot be viewed as a mode forecast as such. However, the figures can also be presented without the central projection.

6 Conclusions

Lately, density forecasts for inflation and other macro variables have become increasingly popular for two reasons. First, compiling the complete forecast distribution used to be a too complicated task before the general improvement in numerical and simulation techniques and the availability of data. Second, communicating a more detailed picture of the estimated future development has gained in popularity. The latter has also resulted in developing more systematic methods for incorporating subjective judgement into forecast distributions. The development has various benefits although the methodology has until recently been somewhat inadequate.

The introduction of inflation targeting has made central banks to allocate more resources for developing inflation forecasts. Both the BoE and the Riksbank have introduced procedures for incorporating subjective judgement into their density forecasts for inflation. The approach of the Riksbank has been reported in more detail and escapes likewise some criticism that has been directed against the BoE procedure, because of which the method the BoF uses is very close to the Riksbank's approach. Moreover, the Excel workbook the Riksbank utilises for compiling the forecast distribution and the fan chart has been an enormous help in implementing the approach at the BoF. Some modifications have however been made due to the different natures of the inflation forecasts in question.

The main benefits of the resulting procedure are the quickness and the ease of implementation. Moreover, the framework for incorporating subjective judgement is very serviceable. And although the approach developed by the Riksbank is not perfect, it is clearly more systematic and therefore better than the earlier method of compiling the distribution for the NIPE forecast at the end of the forecast horizon. After all, no systematic method was in use. Furthermore, the procedure can be applied almost as such to other variables to be forecast, in addition to which it can be developed further to be of use in longer-term forecasts although the role of subjective judgement might get smaller, as the forecast horizon lengthens.

References

Bank of England (1999) **Inflation Report – February 1999.** 52. (Accessed at: http://www.bankofengland.co.uk)

Bank of England (2000) **Inflation Report – August 2000.** 49, 63. (Accessed at: http://www.bankofengland.co.uk)

Bank of England (2002) **Inflation Report – May 2002.** 38–52. (Accessed at: http://www.bankofengland.co.uk)

Bean, C. – Jenkinson, N. (2001) **The formulation of monetary policy at the Bank of England.** Bank of England Quarterly Bulletin, Winter, 434–441.

Berg, C. (1999) **Inflation Forecast Targeting: the Swedish Experience.** Penning- och valutapolitik, Vol. 3.

Blake, A.P. (1996) **Forecast Error Bounds by Stochastic Simulation.** National Institute Economic Review, May, 72–79.

Blix, M. (1999) **Forecasting Swedish Inflation with a Markov Switching VAR.** Sveriges Riksbank Working Paper No. 76.

Blix, M. – Sellin P. (1998) **Uncertainty Bands for Inflation Forecasts.** Sveriges Riksbank Working Paper No. 65.

Blix, M. – Sellin P. (1999) **Inflation Forecasts with Uncertainty Intervals.** Sveriges Riksbank Quarterly Review No. 2, 12–28.

Blix, M. – Sellin P. (2000) **A Bivariate Distribution for Inflation and Output Forecasts.** Sveriges Riksbank Working Paper No. 102.

Britton, E. – Fisher, P. – Whitley, J. (1998) **The Inflation Report projections: understanding the fan chart.** Bank of England Quarterly Bulletin, February, 30–37.

Croushore, D. (1993) **Introducing: The Survey of Professional Forecasters.** Business Review, Federal Reserve Bank of Philadelphia, November/December, 3–15.

Diebold, F.X. – Gunther T.A – Tay, A.S. (1997) **Evaluating Density Forecasts.** Federal Reserve Bank of Philadelphia Working Paper No. 97-6.

Diebold, F.X. – Tay, A.S – Wallis, K.F. (1997) **Evaluating Density Forecasts of Inflation: The Survey of Professional Forecasters.** NBER Working Paper No. 6228.

Ericsson, N.R. (2002) **Predictable Uncertainty.** In Clements, M.P. – Hendry, D.F. (ed.) A Companion to Economic Forecasting, Blackwell Publishers, 19–44.

Federal Reserve Bank of Philadelphia (2002) **Survey of Professional Forecasters.** May 21. (Accessed at: http://www.phil.frb.org/files/spf/survq202.html)

Granger, C.W.J. – Newbold, P. (1986). **Forecasting Economic Time Series.** Second Edition. Academic Press, San Diego.

John, S. (1982) **The three-parameter two-piece normal family of distributions and its fitting.** Communications in Statistical Theory and Methods, 11(8), 879-885.

Melliss, C. – Whittaker, R. (1998) **The Treasury Forecasting Record: Some New Results.** National Institute Economic Review, April, 65–79.

Norges Bank (2001) **Inflation Report 2/2001.** 16–20. (Accessed at: http://www.norges-bank.no)

Poulizac, D. – Weale, M. – Young, G. (1996) **The Performance of National Institute Economic Forecasts.** National Institute Economic Review, May, 55–62.

Svensson, L.E.O (1999) **Price Stability as a Target for Monetary Policy: Defining and Maintaining Price Stability.** CEPR Discussion Paper No. 2196.

Sveriges Riksbank (1998) **Inflation Report 1998:4.** (Accessed at: http://www.riksbank.se)

Sveriges Riksbank (2002) **Inflation Report 2002:2.** (Accessed at: http://www.riksbank.se)

Tay, A.S. – Wallis, K.F. (2002) **Density Forecasting: A Survey.** In Clements, M.P. – Hendry, D.F. (ed.) A Companion to Economic Forecasting, Blackwell Publishers, 45–68.

Vickers, J. (1998) **Inflation targeting in practice: the UK experience.** Bank of England Quarterly Bulletin, November, 368–375.

Virta, H. (2002) **Jakauma ja viuhkadiagrammi NIPE-ennusteelle: ohje.** Bank of Finland mimeo. (In Finnish)

Wallis, K.F. (1999) **Asymmetric density forecasts of inflation and the Bank of England's fan chart.** National Institute Economic Review, January, 106–112.

Wallis, K.F. (2001) **Chi-squared tests of interval and density forecasts, and the Bank of England's fan charts.** European Central Bank Working Paper No. 83.

Appendix 1. A comparison between the Bank of England's and the Riksbank's approaches to constructing density forecasts for inflation

The Riksbank's goals of presenting the inflation forecast in the form of a fan chart coincide with those of the BoE, as can be seen from table A.1. Moreover, the distribution and therefore the crucial parameters are the same, in addition to which both approaches emphasise the value of subjective judgement. Furthermore, both processes are of iterative nature: at the BoE, the final fan chart is achieved only after several steps while at the Riksbank, the estimates for P and h are discussed together. The most significant difference is that the Riksbank uses its estimates of the underlying macro variables to deduce the distribution for inflation forecast while the BoE bases its forecast more directly on the expected development of inflation itself. Some differences are also observable in the chosen method of presenting the forecast to the public.

Table A.1. Density forecasts for inflation at the Bank of England and at the Riksbank

	The Bank of England	**The Riksbank**
Preparation	Monetary Policy Committee (Bank staff assists)	Economics department (individual economists)
Interest rate assumption	Constant UK short-term rates	Constant repo rates
Forecast distribution	Two-piece normal	Two-piece normal
The parameter of **central tendency**	The mode	The mode
Questions	1) What is the balance of risks for inflation based on different scenarios? 2) How large is the degree of uncertainty (the variance) based on past forecasts and the current situation?	1) How large is the downside risk for each underlying macro variable? What is P_j = pr[X_j<μ_j]? 2) How large is forecast uncertainty compared to historical uncertainty (measured by h_j)?
The balance of risks (skewness)		
- the parameter	The difference between the mean and the mode	The difference between the mean and the mode
- the method of estimation	Simulating the effects of different risk scenarios and attaching probability weights to these to derive the aggregate balance of risks	Based on the underlying macro variables, aggregated into inflation forecast skewness by using appropriate weight parameters
The degree of uncertainty		

- the parameter	The variance	The variance
- the method of estimation	Initial assessment based on the past inflation forecast errors, adjusted according to expected shocks and their estimated (simulated) impacts	Based on the underlying macro variables, aggregated into inflation forecast uncertainty through a mathematical model
The nature of the process	Iterative: the fan chart	Iterative: the P_j's and the h_j's
Presentation	Fan chart, quarterly forecasts, centred at the mode, not based on standard percentiles	Fan chart, monthly forecasts, centred at the median, based on standard percentiles
Published	Quarterly	Quarterly

Appendix 2. Underlying macro variables at the Riksbank and Norges Bank

The Riksbank does not use any single model to generate the inflation forecast, because of which it relies on assessments of chosen macro variables to derive an inflation forecast and its distribution. The choice of the underlying variables is thus a crucial phase of the approach. On the one hand, the variables must be such that they affect future inflation. On the other hand, the Bank must have well-grounded opinions on the future development of the chosen variables.

In general, the Riksbank divides the factors affecting inflation into five groups of international economic activity and inflation, financial asset prices, domestic demand and supply, inflation expectations, and other cost shocks and transitory effects. The Bank finds these factors particularly essential to inflation development within the relatively short-term forecast horizon in use. At a more detailed level, international factors comprise economic activity in the rest of the world, external inflation, exchange rate movements and import prices mirroring external inflation as well as exchange rate movements. Also the oil price is considered. Second, the Riksbank discusses financial asset prices and financial market development in general. Third, domestic demand and supply are treated through a discussion on the subcomponents of final domestic demand. Demand relative to supply is also found important, because demand exceeding long-term productive capacity is likely to generate inflationary pressure. The Riksbank uses capacity utilisation, labour market situation and output gap to measure the factor. Fourth, inflation expectations are discussed since price and wage increases can also stem from high inflation expectations as such. Fifth, other shocks and transitory effects are considered. These include price movements resulting from supply side shocks and fiscal policy impulses. (Riksbank 1998, 2002)

In July 2002, the variables featuring in the Riksbank's underlying macro variables comprised international export prices, international economic growth, exchange rate, price of raw oil, potential GDP, private consumption, public consumption, exports, gross investments and wages per hour. The current choice of the variables relies heavily on the special knowledge of the Bank economists so that every underlying variable has an expert who is concentrated on the variable in question.

Correspondingly, Norges Bank (2001) determines global growth prospects, oil prices and the exchange rate as important factors as to imported price inflation while fiscal policy, domestic demand and wage formation are deemed to affect domestic price inflation. The picture of risks is therefore compiled of six components. Norges Bank gives also more detailed information of the factors it uses to measure the variables. This information is presented in table A.2.

Table A.2. Underlying macro variables at Norges Bank

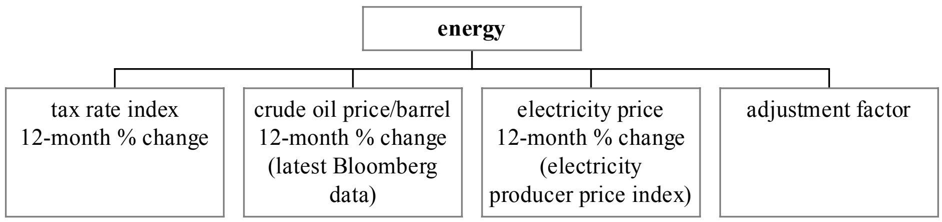
Variable	Measure
Oil price	The prices in the options market for delivery one year ahead
International prospects	Emphasis on the inflationary impulses to the Norwegian economy, based on an average analysis of developments this year and the next year
Exchange rate	The prices in the options market one month ahead

Public demand	The fiscal policy programme for the next year, also current year shifts in the fiscal stance
Private demand	Household and business saving and investment behaviour
Wage formation	The outcome of the wage settlement the following year

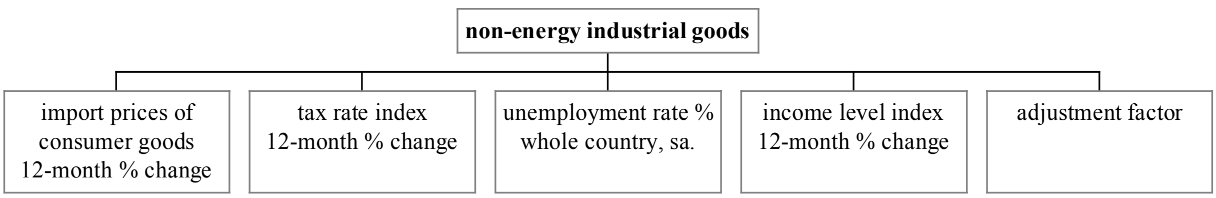
Appendix 3. The sub-component compositions of the NIPE forecast.

(1) Energy
(2) Non-energy industrial goods
(3) Unprocessed food
(4) Processed food
(5) Services

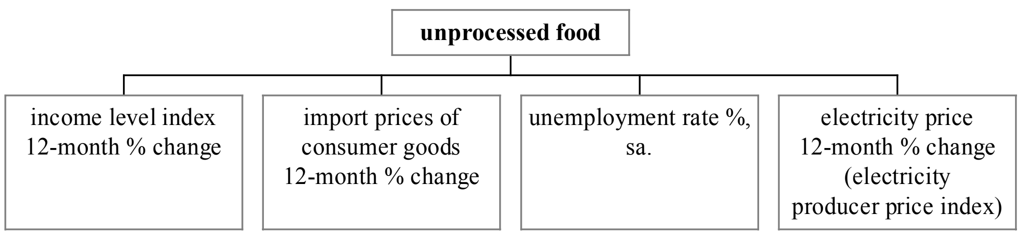
Subgroup composition (1)



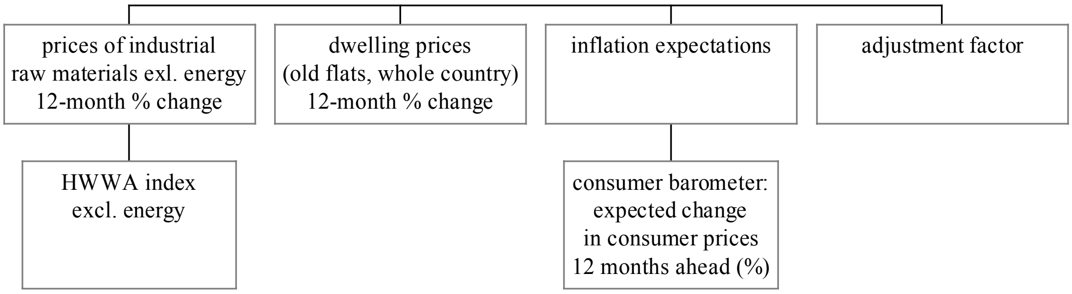
Subgroup composition (2)



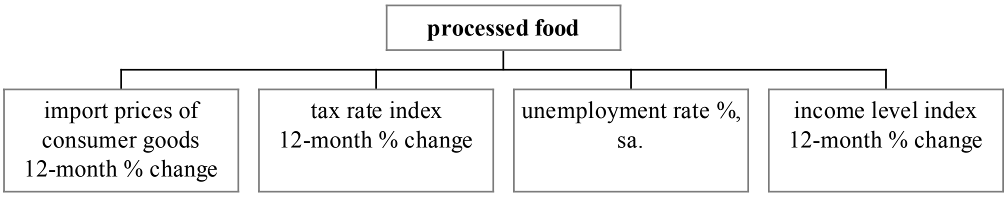
Subgroup composition (3a)



Subgroup composition (3b)

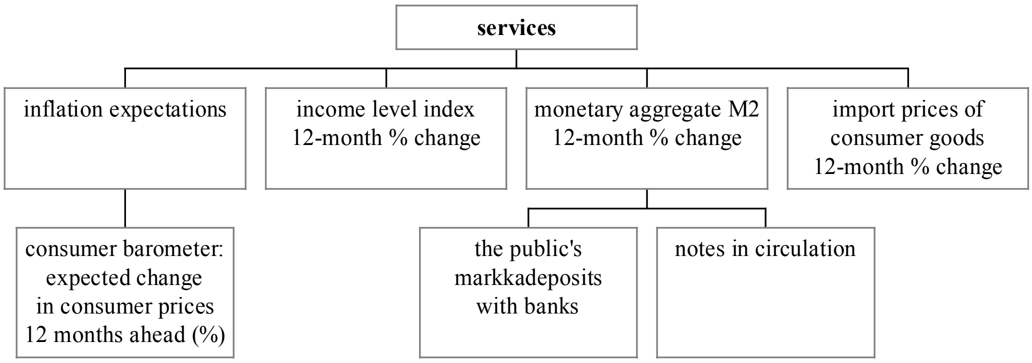


Subgroup composition (4a)

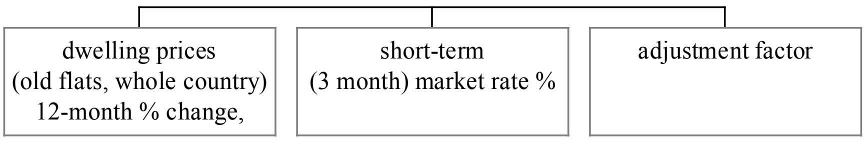


Subgroup composition (4b)

Subgroup composition (5a)



Subgroup composition (5b)



Appendix 4. Frequency distributions and estimated densities for the NIPE subgroups

Figure A4.1. Energy

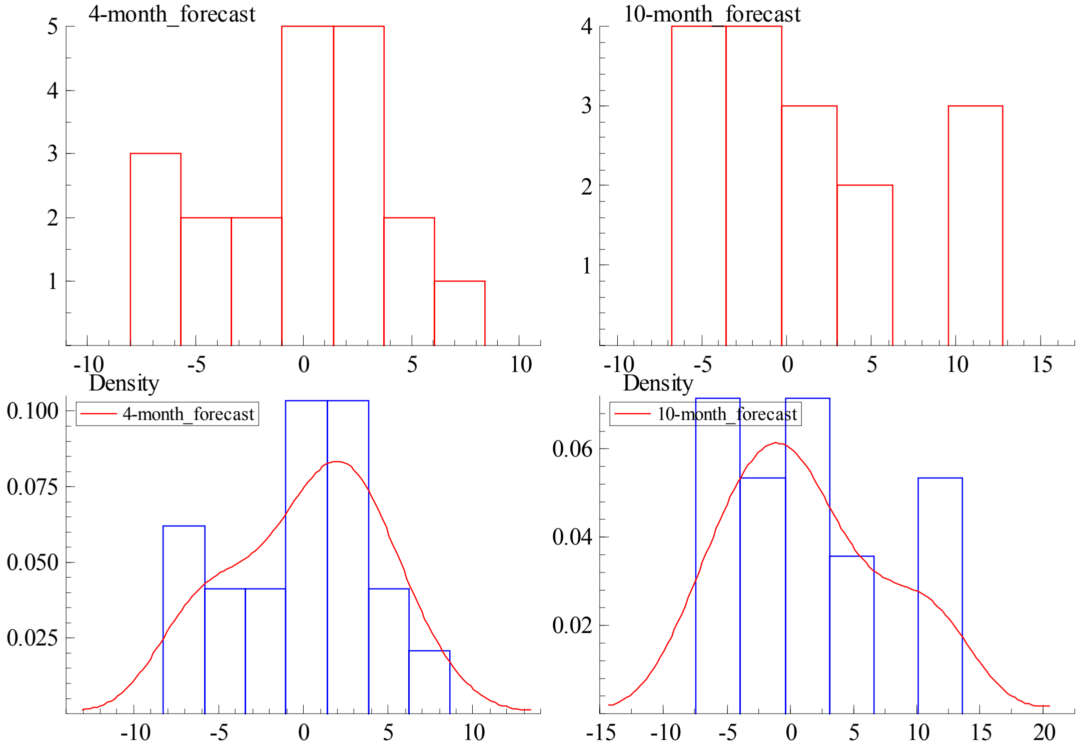


Figure A4.2. Non-energy industrial goods

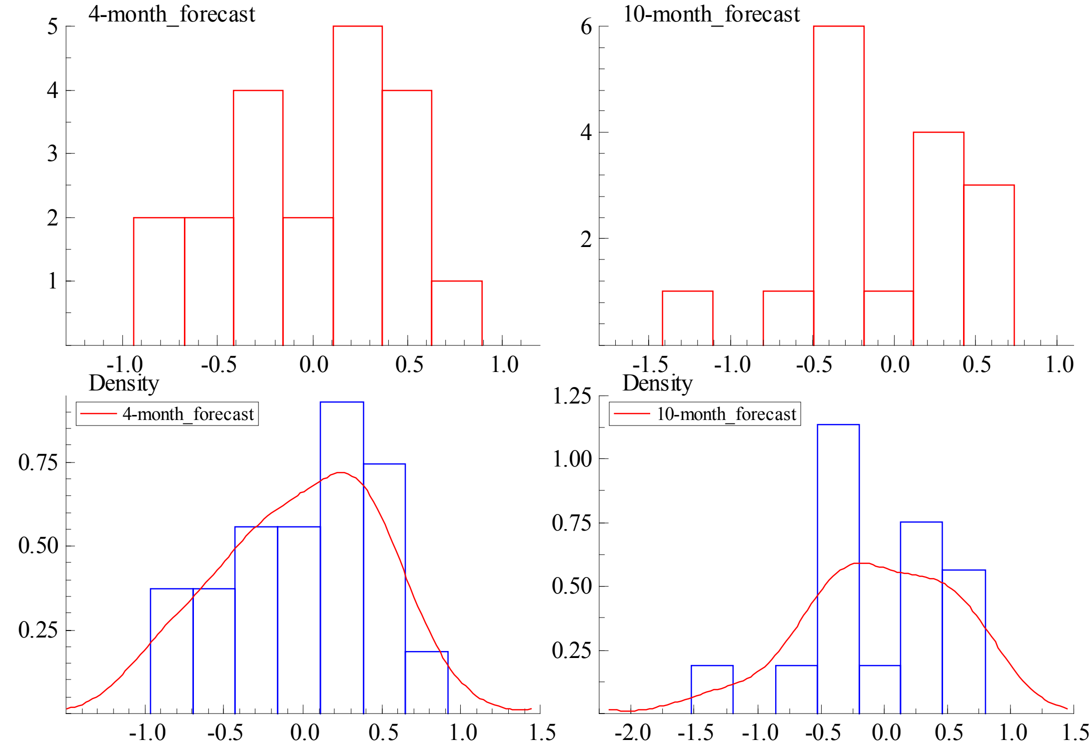


Figure A4.3. Unprocessed food

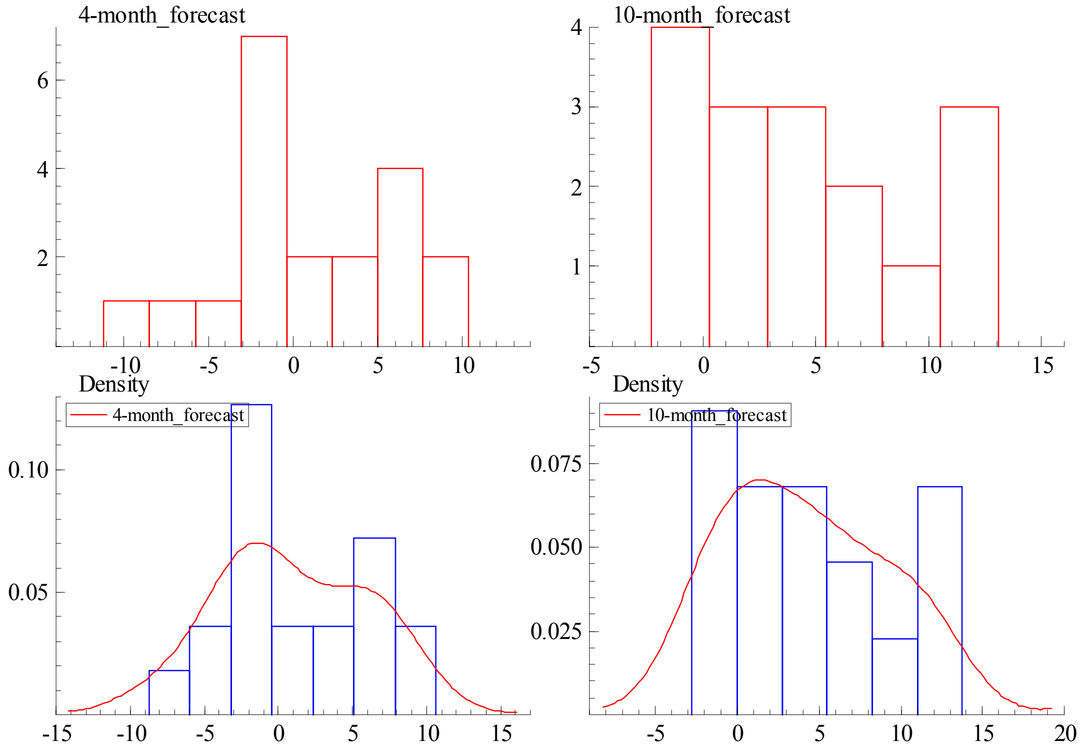


Figure A4.4. Processed food

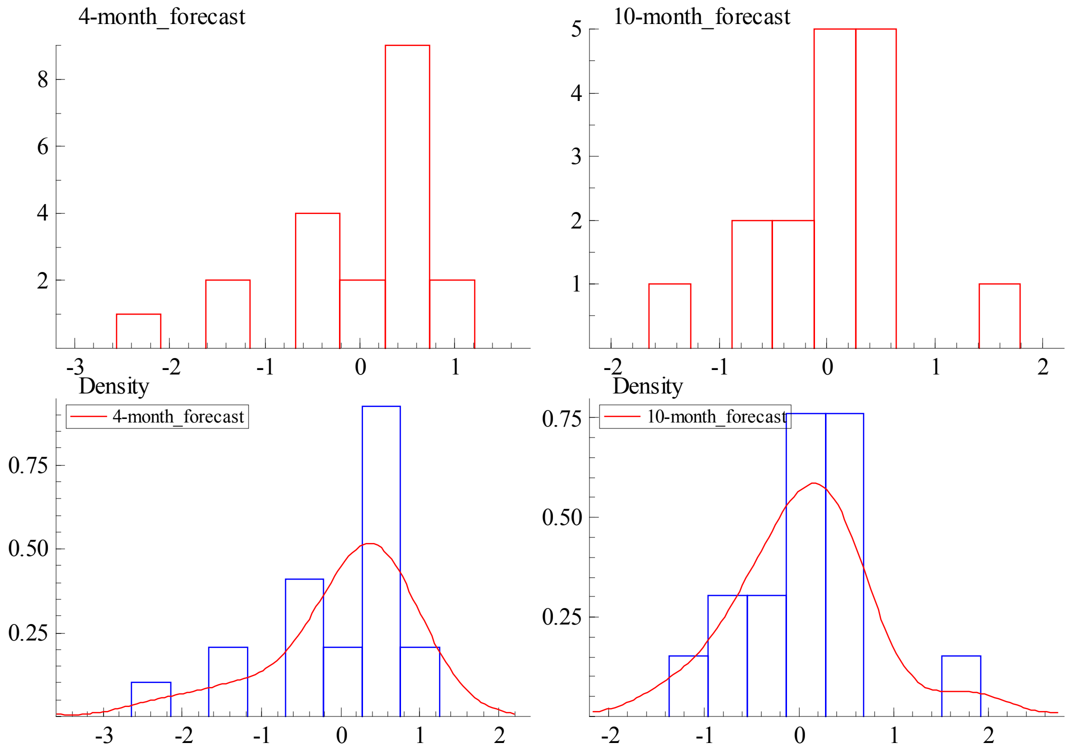
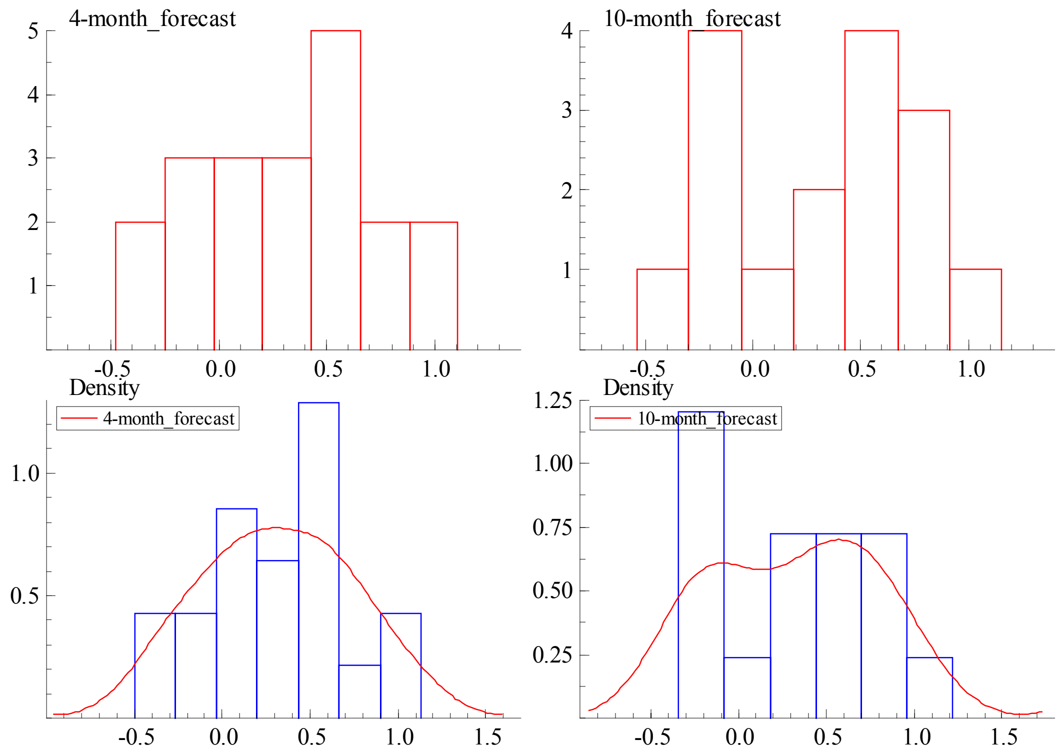


Figure A4.5. Services



Appendix 5. Descriptive statistics for the NIPE subgroups

Energy	4-month forecast	10-month forecast
Observations	20	16
Mean	0.198395	1.344091
Std.Devn.	4.156043	5.760533
Skewness	-0.272571	0.486197
Excess Kurtosis	-0.889264	-0.895712
Minimum	-6.895502	-6.453194
Maximum	7.721184	11.461847
Normality Chi^2(2)=	0.82781 [0.6611]	1.9716 [0.3731]

Non-energy industrial products	4-month forecast	10-month forecast
Observations	20	16
Mean	-0.024900	-0.032349
Std.Devn.	0.461911	0.543783
Skewness	-0.370448	-0.452704
Excess Kurtosis	-0.830987	-0.333581
Minimum	-0.897892	-1.294126
Maximum	0.755789	0.709808
Normality Chi^2(2)=	1.2397 [0.5380]	0.95008 [0.6219]

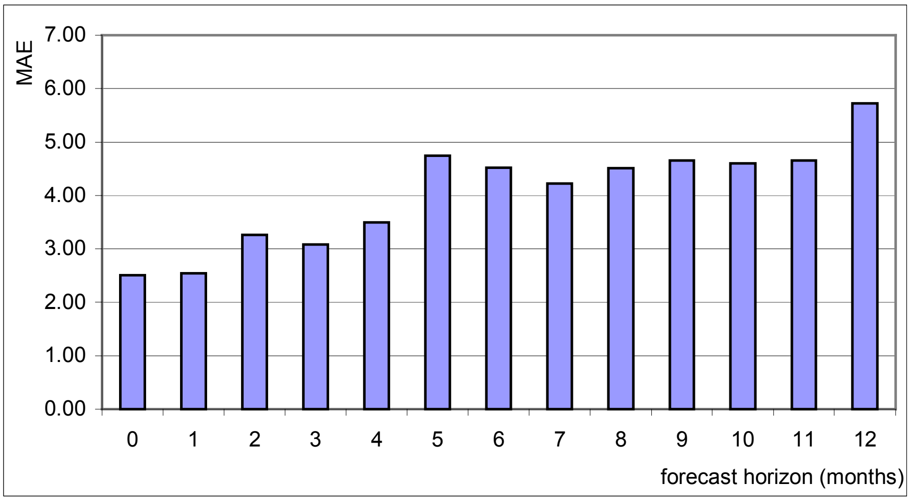
Unprocessed food	4-month forecast	10-month forecast
Observations	20	16
Mean	0.935951	4.128432
Std.Devn.	4.747596	4.525425
Skewness	0.003298	0.311421
Excess Kurtosis	-0.945912	-1.202486
Minimum	-8.589015	-2.228071
Maximum	8.818119	11.267871
Normality Chi^2(2)=	0.31336 [0.8550]	2.219 [0.3297]

Processed food	4-month forecast	10-month forecast
Observations	20	16
Mean	0.031358	0.072200
Std.Devn.	0.833767	0.674896
Skewness	-1.251170	0.328126
Excess Kurtosis	1.093029	1.037252
Minimum	-2.289020	-1.339545
Maximum	1.179067	1.782914
Normality Chi^2(2)=	7.1872 [0.0275] *	6.4788 [0.0392] *

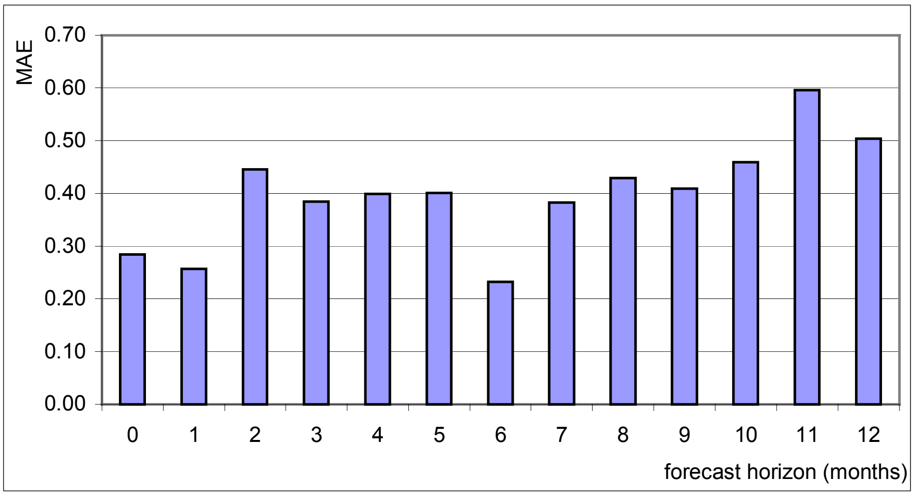
Services	4-month forecast	10-month forecast
Observations	20	16
Mean	0.315799	0.308918
Std.Devn.	0.400598	0.427354
Skewness	0.032378	-0.044630
Excess Kurtosis	-0.888577	-1.409971
Minimum	-0.384479	-0.326026
Maximum	1.034807	0.992572
Normality Chi^2(2)=	0.1928 [0.9081]	2.6226 [0.2695]

Appendix 6. Mean absolute errors for the HICP subgroups according to the length of the forecast horizon

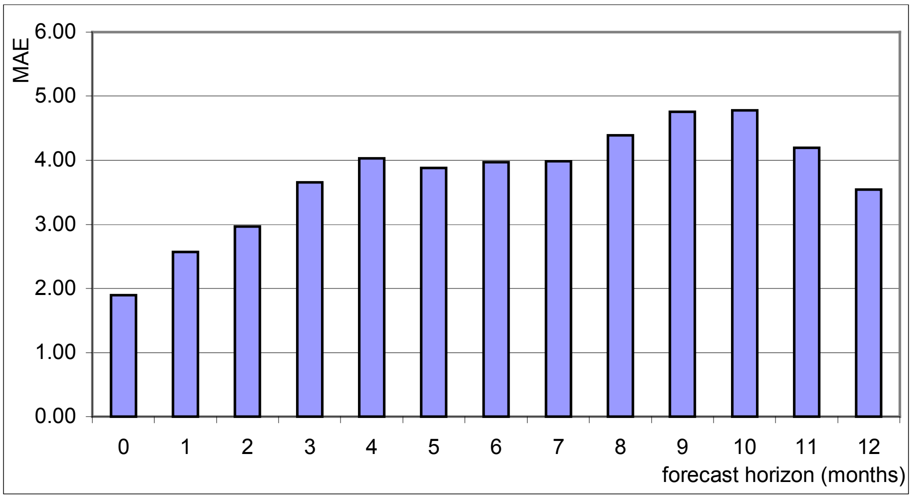
Energy



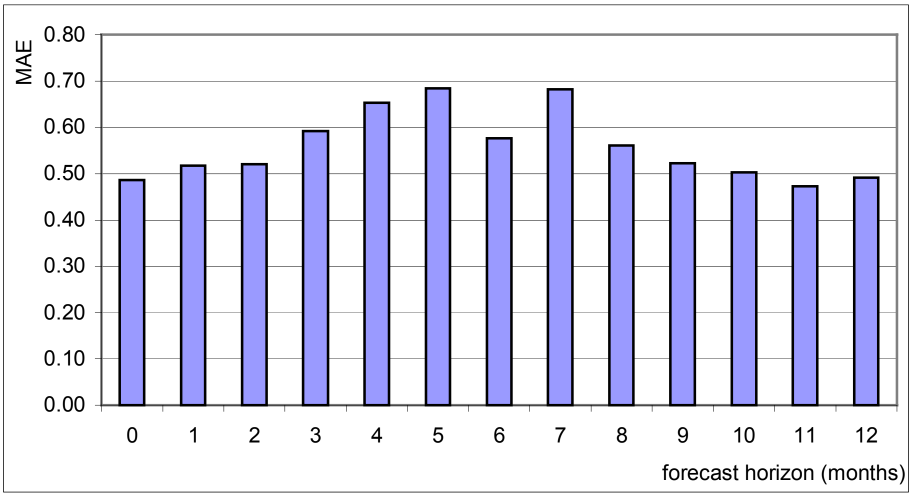
Non-energy industrial goods



Unprocessed food



Processed food



Services