# Information and credit cycles

## Causes and consequences
## of financial instability

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# Information and credit cycles
## Causes and consequences of financial instability

by

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# Abstract

This thesis consists of an introductory chapter and three self-contained essays that apply insights from the microeconomic theory of corporate finance in a macroeconomic setting in order to explain and understand various market failures that were at the roots of the global financial crisis of 2007–2009. In particular, I study various forms of incomplete information in the credit market, and their implications on financial stability and on business cycles in the aggregate economy. I also seek to understand how monetary and macroprudential policies can be used to maintain financial stability, and how these two policies interact.

In the first essay, I set up and analyse a New Keynesian dynamic stochastic general equilibrium (DSGE) model where the financing of investments is affected by a double moral hazard problem. I then solve for jointly Ramsey-optimal monetary and macroprudential policies. In the main contribution of this essay, I find that the optimal policy can achieve an efficient allocation if the social planner can conduct both monetary and macroprudential policy. Using monetary policy alone is not enough: in this case a short-run policy trade-off between stabilising inflation and output gap arises. The second contribution is a systematic assessment of the performance of simple policy rules in comparison with the optimal policy. When policy follows simple rules instead of the Ramsey-optimal policy, the source of fluctuations is relevant for the choice of the appropriate policy mix. I find that following a monetary policy rule that reacts to asset prices performs well in stabilising cyclical fluctuations. A separate macroprudential policy rule, in the form of countercyclical bank capital regulation, is useful in counteracting shocks that arise from the financial sector itself. However, it can be counterproductive in mitigating the impacts of shocks that arise from the real sector but affect the financial sector.

In the second essay, I study the link between house prices, lending standards, and aggregate over-investment in housing. I develop an overlapping ge-

nerations model of the housing market. In the model, the market for housing loans is affected by asymmetric information: borrowers have private information on their idiosyncratic income risk. Selection is towards less creditworthy borrowers. I show that this adverse selection together with strategic complementarity in housing demand and deadweight costs of default can create endogenous boom-bust cycles in house prices. I also show that lending standards are loose and the incentives for less-than-creditworthy borrowers to apply for loans are particularly strong, first, when future house values are expected to be high, which leads to high leverage of borrowers; and second, when safe interest rates are low, which implies low costs of borrowing. However, there are strong non-linearities in the relationship between borrowing incentives and economic fundamentals. The results shed light on incentive mechanisms that can help explain the developments in the U.S. housing market in the early 2000s that led to the subprime crisis. They also imply that monetary policy has a direct impact on the stability of the housing market and on default incentives through the cost of borrowing and the opportunity cost of housing investment.

A prominent explanation for the subprime crisis of 2007–2009 in the U.S. states that it was fuelled by a loosening of lending standards and an expansion of credit supply, as financiers believed that house prices would keep rising and that investment in the housing market was safe. In the final essay, I estimate a conditional capital asset pricing model (CCAPM) with Bayesian learning on returns in the residential housing market in the U.S. in 1987–2016. In the model, I assume that the true systematic risk of the housing portfolio is unobservable, and the investors update their beliefs on the true risk based on observed returns. I show evidence in the data that financiers' perceptions about systematic risk in the housing market were relatively low in the 1990's and the early 2000's, in the period where house prices were increasing. At the onset of the subprime crisis, the beliefs about this risk were updated upward most swiftly in regions that had experienced the strongest house price appreciation. These empirical results lend support to the above-mentioned narrative on the causes of the crisis.

# Tiivistelmä

Tämä väitöskirja koostuu johdantoluvusta sekä kolmesta itsenäisestä esseestä, joissa tarkastellaan vuonna 2007 Yhdysvalloissa puhjenneen finanssikriisin taustalla vaikuttaneita markkinahäiriöitä. Tutkimuksessani selvitän, miten erilaiset informaatio-ongelmat markkinaosapuolten välillä voivat johtaa rahoitusmarkkinoiden epävakauteen ja siten talouden suhdannevaihteluihin. Tutkin myös, miten näiden häiriöiden aiheuttamia luottosyklejä voidaan ehkäistä tai hillitä. Tällaista koko rahoitusmarkkinoiden vakauttamiseen tähtäävää sääntelyä kutsutaan *makrovakauspolitiikaksi*. Lisäksi tarkastelen rahapolitiikan ja makrovakauspolitiikan välisiä kytköksiä sekä sitä, miten korkopolitiikka osaltaan vaikuttaa rahoitusmarkkinoiden vakauteen.

Menetelmällisesti tämän väitöskirjan kaksi ensimmäistä esseetä ovat teoreettisia tutkielmia, joissa sovellan vakiintuneita mikrotalousteorian informaatiomalleja rahoitusmarkkinoiden tasapainon ja makrotaloudellisten suhdanteiden tarkastelussa. Kolmannessa esseessä tutkin tilastollisin menetelmin sijoittajien odotuksia Yhdysvaltain asuntomarkkinoilla.

Ensimmäisessä esseessä tarkastelen ohjauskorkoon perustuvan rahapolitiikan sekä pankkisektorin vakauteen tähtäävän makrovakauspolitiikan vuorovaikutusta talouden suhdannevaihteluiden tasaamisessa. Käyttämässäni yleisen tasapainon mallissa pankkien pääoma on keskeisessä asemassa, sillä se määrää yritysrahoituksen ja edelleen investointien tason taloudessa. Koska pankit toimivat rahoituksen välittäjinä talouden eri sektorien — kotitalouksien ja yritysten — välillä, ne voivat levittää talouden sokkien vaikutuksia sektorilta toiselle.

Käyttämässäni mallissa yritysrahoitus ei allokoidu tehokkaasti rahoitusmarkkinaosapuolten kannustinongelmien vuoksi. Tästä syystä rahapolitiikka ei yksin pysty tasoittamaan talouden suhdanteita: vaikka sillä voidaankin vakauttaa hinnat, rahoituksen tehoton jakautuminen vaikuttaa investointien kokonaistasoon ja siten tuotantoon. Tämä tulos eroaa aiemmasta rahapolitiikan

tutkimuksesta, jossa rahoitusmarkkinahäiriöitä ei ole otettu huomioon ja jossa korkopolitiikka on sen seurauksena riittävä työkalu suhdanteiden vakauttamisessa. Toisaalta osoitan, että raha- ja makrovakauspolitiikka yhdessä mahdollistavat inflaation ja tuotannon vakauttamisen samanaikaisesti. Makrovakauspolitiikka näyttäytyy erityisen hyödyllisenä, kun sokkien lähde on pankkisektori itse, jolloin sopivilla vastasyklisillä pääomavaatimuksilla voidaan kokonaan ehkäistä rahoitussektorin häiriöiden leviäminen reaalitalouteen.

Väitöskirjan toisessa esseessä käsittelen asuntohintojen, luottoehtojen ja ylivelkaantumisen yhteyttä kokonaistalouden kannalta. Tulokset perustuvat teoreettiseen malliin, jossa luottomarkkinoihin vaikuttaa epäsymmetrinen informaatio lainanhakijoiden tuloriskin suhteen. Lainanantajat eivät mallissa havaitse täydellisesti yksittäisten lainanhakijoiden tuloriskiä ja siihen perustuvaa lainan takaisinmaksukykyä. Velallisiksi valikoituu siten myös kotitalouksia, jotka eivät ole luottokelpoisia suuren tuloriskin vuoksi.

Näytän, että tämä epäsymmetrinen informaatio yhdessä asuntojen kysyntää vahvistavan komplementaarisuuden sekä maksuhäiriöiden tehokkuustappioiden kanssa synnyttävät mallissa endogeenisia syklejä asuntojen hintoihin. Esseessä osoitan myös, että kun asuntojen tulevien hintojen odotetaan olevan korkeita ja korkotaso on matala, lainanantajien luottoehdot löystyvät ja suuririskisten lainanhakijoiden kannustimet hakea asuntolainoja ovat erityisen voimakkaat. Tämä johtaa kotitalouksien korkeaan velkaantumisasteeseen. Sen sijaan asuntojen hintojen maltillinen nousu sekä kohtuullinen korkotaso voivat päinvastoin johtaa asuntovelallisten keskimääräisen tuloriskin pienentymiseen ja velkaantumisasteen laskuun. Talouden perustekijöiden ja kannustinvaikutusten välinen yhteys on siten epälineaarinen. Nämä tulokset auttavat ymmärtämään velkaantumisen kannustimia, jotka johtivat osaltaan Yhdysvalloista alkunsa saaneeseen asunto- ja rahoitusmarkkinakriisiin 2000-luvun puolivälin jälkeen. Tulokset osoittavat myös, että rahapolitiikalla on suoria vaikutuksia asuntomarkkinoiden vakauteen lainojen korkomarginaalien ja velkaantumisen vaihtoehtoiskustannusten kautta.

Yleisen käsityksen mukaan Yhdysvaltain asuntomarkkinat ylikuumenivat 2000-luvulla, koska asuntohintojen uskottiin laajasti jatkavan nousuaan, ja asuntosijoitusten ajateltiin siksi olevan lähes riskittömiä. Hintojen nousu mahdollisti samanaikaisesti myös luottoehtojen löystymisen ja luottojen tarjonnan kasvun, mikä johti korkeariskisten subprime-luottojen määrän voimakkaaseen kasvuun 2000-luvun alkupuoliskon aikana. Hintojen käännyttyä laskuun ja talouden ajauduttua taantumaan näiden luottojen riskit realisoituivat.

Viimeisessä esseessä tutkin sijoittajien odotuksia asuntosijoitusten tuottojen tulevasta kehityksestä hinnoittelumallin avulla. Yleisesti sijoittajien tuotto-

odotukset asuntomarkkinoilla riippuvat sekä kokonaistaloudellisista että hajautettavissa olevista riskeistä, mutta käyttämässäni mallissa oletan, että sijoittajat havaitsevat ainoastaan toteutuneet tuotot. Pystyäkseen muodostamaan tuottovaatimuksia asuntosijoituksille he joutuvat siten estimoimaan eri lähteistä syntyvien riskien vaikutuksen tuottoihin. Osoitan Yhdysvaltain asuntohintaindeksejä käyttäen, että sijoittajat uskoivat 1990-luvulla ja vielä 2000-luvun alussa kokonaistaloudellisten riskien olevan asuntomarkkinoilla pieniä. Samalla ajanjaksolla asuntohinnat nousivat voimakkaasti monilla alueilla. Hintojen käännyttyä maanlaajuisesti laskuun vuonna 2005 näkemyksiä kokonaistaloudellisista riskeistä nostettiin voimakkaasti. Nämä empiiriset tulokset tukevat käsitystä finanssikriisiin johtaneista syistä.

*Asiasanat:* rahoitusvakaus, luottosyklit, asuntomarkkinat, subprime-kriisi, rahapolitiikka, makrovakauspolitiikka

*JEL-luokittelu:* E32, E44, E52, E61, G11, G12, G21, R31

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This dissertation project has its roots at the European Central Bank in Frankfurt, Germany, where I spent a little more than a year in 2011–2012. At the time, the financial crisis and the sovereign debt crisis were still very much in their acute phase in Europe. The dramatic events of the crisis years sparked an interest to learn more about the causes of such financial instability and the possible policy responses to them.

As I conclude this project, I thank Professor Fabrice Collard, who offered very insightful feedback on my research during the pre-examination of the dissertation, and subsequently kindly agreed to act as my opponent. Likewise, I thank my second pre-examiner, Professor Andrea Caggese, for his comments and suggestions on my research.

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Next, I thank the Department of Economics and the Graduate School of Arts and Sciences at New York University for their hospitality during my re-

# List of Essays

The interaction of monetary and macroprudential policies
*An article based on this essay is forthcoming in the Journal of Money, Credit and Banking*

House prices, lending standards, and the macroeconomy
*Unpublished manuscript*

Learning about systematic risk in the housing market
*Unpublished manuscript*

# Contents

# 1  Introduction

## 1.1  The financial crisis and recession of 2007–2009

The era of relatively steady growth and low macroeconomic volatility in the United States that had started in the mid-1980's, termed the Great Moderation by Stock and Watson (2002), came to an end in 2007. Problems in a small segment of the financial market — the subprime mortgage market — spread to the rest of the market, and subsequently to the whole economy. This caused a severe downturn that has since come to be known as the Great Recession. The turmoil quickly spread to the international financial markets and eventually caused a global recession.

In the United States, the recession spanned the years 2007–2009. The development of selected macroeconomic variables are depicted in Figures 1.1 and 1.2. First, Figure 1.1 shows the evolution of the real gross domestic product (GDP) and real private residential and non-residential fixed investment in the United States over the period 1987–2017 as percentage deviations from their respective long-run trends. Private residential investment peaked in 2005 at a staggering 25 percent above its long-run trend, and abruptly contracted to 25 percent below trend over the crisis period. The GDP and private non-residential investment started to contract in 2007, marking the beginning of the recession.

Second, Figure 1.2 shows the development of the private credit to GDP ratio and the national house price index. House prices peaked in early 2006, after a period of strong and accelerating growth in the early 2000's. Private sector indebtedness, as measured by the credit-to-GDP ratio, peaked two years later in 2007 at almost 170%, after also having grown steadily since the mid-1990's. It then contracted sharply before levelling off at 150% by 2012. These data illustrate that developments in the real estate market led the general busi-
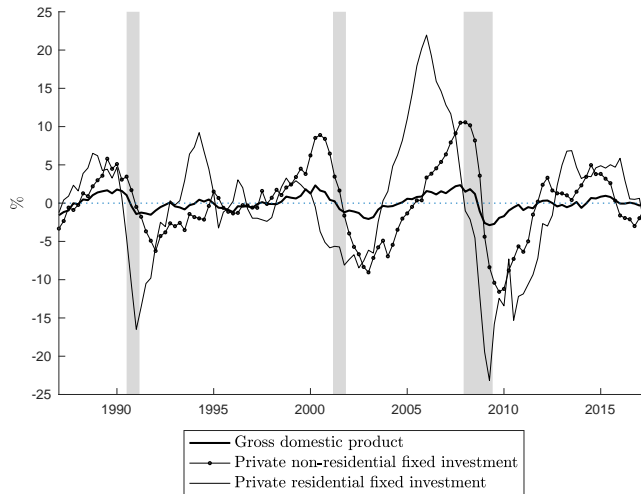
Figure 1.1: Real gross domestic product and real private investment in the United States in 1987–2017

*Note:* Percentage deviations from trend. The data series are quarterly and deflated with the GDP deflator. The trend is extracted with the Hodrick-Prescott filter with smoothing parameter $\lambda = 1600$. Shaded areas indicate National Bureau of Economic Research (NBER) recessions. Source: U.S. Bureau of Economic Analysis, National Income and Product Account tables.

ness cycle, and that the recession was characterised by strong deleveraging of the private sector.

What led to such a dramatic downturn? According to the Financial Crisis Inquiry Commission (2011), an environment of low interest rates and an expanding credit supply, enabled by new financial instruments and increasing securitisation, fuelled the real estate boom of the early 2000's. As real estate prices eventually started to fall and private sector indebtedness was at record levels, the subsequent contraction was made much worse by private sector deleveraging. Risks that were not well understood had built up in the financial system; their unravelling led to a freeze of the financial market. The recession of 2007–2009 was thus tightly linked to the house price and credit cycles.[1]

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[1]The report of the Financial Crisis Inquiry Commission (2011) offers a detailed account of the boom and the subsequent crisis. See e.g. Gorton (2008) and Calomiris (2009) for descriptions of
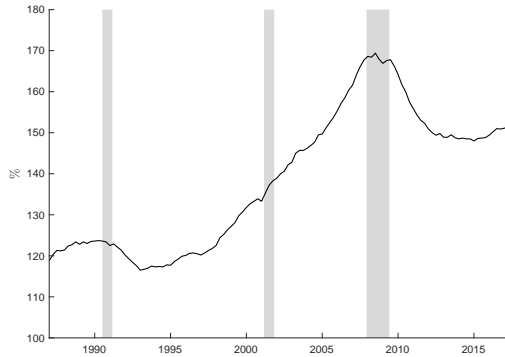
(a) Private credit to GDP ratio

*Note:* The data are quarterly, and computed as the ratio of the end-of-quarter stock of outstanding credit to the quarterly gross domestic product (GDP). The measure of private credit includes all loans to and debt securities issued by the non-financial private sector. Shaded areas indicate National Bureau of Economic Research (NBER) recessions. Source: Bank for International Settlements, Total Credit to Private Non-Financial Sector, Adjusted for Breaks.
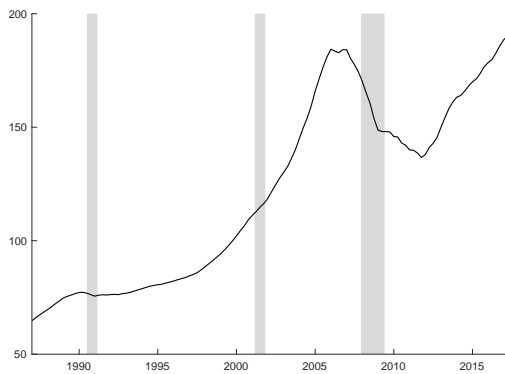


(b) National house price index

*Note:* The data are quarterly. 2000Q1=100. Shaded areas indicate National Bureau of Economic Research (NBER) recessions. Source: Case & Shiller U.S. National Home Price Index.

Figure 1.2: Private credit and house prices in the United States in 1987–2017

In the following sections, I first summarise the macroeconomic research literature on financial frictions and credit cycles. Next, I describe the seminal models of agency problems in corporate finance and the signal extraction model that I apply in the three essays that constitute the main body of this thesis, and link my research to the macro-financial literature. Finally, I summarise in more detail the models and the main findings in each of the essays.

## 1.2 Financial frictions in macroeconomics

The financial crisis seemed to catch many policymakers and economists alike by surprise. However, it has been understood for decades that incomplete information causes significant market failures in financial markets. For example, there may be asymmetric information between lenders and borrowers about the true creditworthiness of the borrower; returns on investment projects may depend on actions of the borrower that are unobserved by financiers; or the true structure of risks and pay-offs to investments may simply be unknown to investors. Compared to a situation of complete information, these features may lead to an excessive amount of debt or risk-taking, to credit rationing, or to a sensitivity of financing to the balance sheet conditions of both lenders and borrowers. At the heart of all of these issues are incentives. Seminal contributions to this literature were made by Stiglitz and Weiss (1981), De Meza and Webb (1987), and Holmström and Tirole (1997). They are discussed in more detail below in Sections 1.3 and 1.4.

These market failures manifested themselves in an evident manner in the crisis. Banks and financiers had taken on excessive risk; financial institutions were very highly levered and hence vulnerable to capital losses; financial instruments had become so complex that their true risks were no longer well understood; risk that was thought to be well-diversified turned out to be anything but — in particular in the market for subprime mortgages in the United States. Examples of such market failures were plentiful both in the United States and in Europe. When losses started to accumulate, first in the subprime market and soon in other parts of the financial system, financial intermediaries sharply cut back lending, not only to households and non-financial firms, but also to each other. This finally led to an international financial market freeze fuelled by mistrust in the balance sheet conditions of the market participants.[2]

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the market for mortgage finance and the subprime crisis.

[2]See Financial Crisis Inquiry Commission (2011).

Banks and financial institutions are vital for the functioning of a modern economy. They intermediate funds between sectors of the economy and pool risks, among other important functions. Both households' and non-financial firms' financing — be it for housing investment or investment into productive capital — depend on the banking sector and more generally on the financial markets. This is also why a financial crisis can have such devastating effects on an economy. If the banking sector's ability to intermediate funds is impaired, it has direct consequences on private consumption and investment. Moreover, the financial sector can amplify shocks to the economy or make their effects more persistent — in other words, exacerbate booms and recessions — precisely because they act as a link between the various sectors of the real economy, while themselves being vulnerable to these shocks.

Starting from the late 1980's and early 1990's, macroeconomic models of financial frictions have been developed to study the potentially disruptive effects of these market failures on the aggregate economy. Seminal general equilibrium models of financial frictions include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Iacoviello (2005). The latter is the first to consider the aggregate impacts of frictions in the housing market and fluctuations in house prices, while the others focus on various frictions in the financing of productive investment.

Since the crisis, the literature on financial frictions in the macroeconomy and on policies that can curb them has proliferated. Influential examples of this more recent literature include, among others, Gerali, Neri, Nessa, and Signoretti (2010), Gertler and Karadi (2011), Brunnermeier and Sannikov (2014), and Cúrdia and Woodford (2016), who study the role of bank capital in business cycle fluctuations. Regarding the housing market — in particular the interactions of house prices, household leverage, and foreclosures with the rest of the economy —, important contributions have been made, for example, by Iacoviello and Neri (2010), Kaplan and Violante (2014), Justiniano, Primiceri, and Tambalotti (2015), and Corbae and Quintin (2015).

This thesis contributes to this growing literature by exploring ways in which various forms of incomplete information and financial frictions in the intermediation of funds can create instability in the credit market. In the macroeconomic literature, the term *financial friction* itself refers to some departure from the assumptions of complete asset markets, complete information among parties in a financial contract, and perfect competition in the financial market. This leads to distortions in the aggregate amount of saving, borrowing, and investment in the economy relative to the efficient outcome.

In this thesis, I focus on specific forms of financial frictions: in the first essay, the friction is caused by a moral hazard problem, and in the second one, by adverse selection. Both of these are agency problems caused by private information of one or more of the parties in a financial contract. The third essay builds on a problem of partial information about the true riskiness of investments, which leads to a signal extraction problem for the investors.

In the first essay, I analyse a New Keynesian general equilibrium model where the investment in physical capital is affected by a particular double moral hazard problem. As a consequence, business cycles in the model are sensitive to the availability of bank capital. This creates a role for a policy that can smooth credit cycles caused by fluctuations in bank capital. In the essay, I explore the interactions of such a policy and conventional monetary policy that focuses on stabilising inflation.

The second and third essays focus on the subprime crisis in the United States from the borrowers' and the financiers' perspectives, respectively. The subprime crisis acted as a trigger for the global financial crisis. In a series of influential empirical studies by Mian and Sufi (2009, 2011) and Mian, Rao, and Sufi (2013), the authors present evidence for a credit supply driven mortgage lending boom. Lenders expanded their supply of mortgage lending and relaxed their lending standards in the run-up to the subprime crisis of 2007–2009. The shift in supply was tightly connected to the expansion of mortgage securitisation since the early 2000's. In hindsight, it was clear that lending standards had been much too loose during the years preceding the crisis, as households who did not have strong income prospects had been encouraged to apply for home loans. As long as house prices would keep increasing, both borrowers and lenders would benefit from these loans.

The second essay concentrates on the incentives of different households to apply for housing loans, and on how these incentives are affected by expected house prices and the interest rate environment. I build an overlapping generations model of housing investment. In the model, lenders are unable to exactly observe the creditworthiness of an individual loan applicant because of private information on income risk of the borrowers. This information asymmetry leads to an undesirably large amount of borrowing, as households who have a high income risk are able to receive loans that they would not if their types were publicly observable.

Finally, the third essay shifts the focus to the perceptions and beliefs of financiers about the riskiness of the borrower pool in the housing market. I depart from the assumption of complete information. Instead, I assume that financiers do not fully observe the state of the economy, and are thus unable

to disentangle aggregate and idiosyncratic risk from each other. Using data on national and regional home price indices in the United States, I estimate an empirical capital asset pricing model with learning to study the extent to which lenders truly believed that the overall risk in the borrower pool was low — or in other words, the extent to which idiosyncratic risks could be diversified away in large portfolios of housing loans.

## 1.3  The Holmström-Tirole double moral hazard

The double moral hazard model of Holmström and Tirole (1997) describes a situation where the capital positions of both the financial intermediary and the borrower matter for the terms of financing of an investment project. I present here the variable investment scale version of the model.

The model has three types of agents: firms, financial intermediaries, and investors. Firms are endowed with investment projects that turn invested funds into final goods. In order to realise a project, a firm needs external funding from an investor and a financial intermediary. Investors and intermediaries are endowed with funds they can invest in projects. The model has two time periods. In the first period, the three parties — a firm, a financial intermediary, and an outside investor — agree on a financial contract to fund a project, and investment decisions are made. In the second period, the returns to investment are realised and observed by all parties, and all financial claims are settled.

It is assumed that all agents are risk neutral and protected by limited liability. All firms are assumed to be identical, i.e. endowed with the same projects. Each firm can choose between three projects: a good one, a bad one, and an ugly one. Each project yields a gross return $R$ per unit invested with probability $p$, and nothing with probability $1 - p$. They differ, however, in their success probability $p$: the good project has a success probability $p = p_H$ (with $0 < p_H < 1$), and the bad and ugly projects both have a success probability $p = p_L < p_H$ (with $0 < p_L < 1$). Denote by $\Delta p = p_H - p_L > 0$ the difference in success probabilities. The projects also differ in terms of the private benefit that accrues to the firm. The good project offers no private benefit, while the bad and ugly projects entail a private benefit of $b$ and $B$ per unit invested in the project, respectively, with $0 < b < B$. It is furthermore assumed that only the good project is economically viable, i.e. its expected excess return is positive:

$$(p_H R - \gamma)I > 0 > (p_L R - \gamma + b)I,$$

7

where $I$ denotes the overall size of the investment project, and $\gamma$ the expected outside return of the funds.

Under these assumptions, a firm may be tempted to choose a non-viable project in order to enjoy a positive private benefit. This is not desirable from the point of view of the investors and the intermediaries, because it leads to a lower success probability for the project. It is assumed that the investors can only verify the realised return on the project, but not which project was chosen.

A financial intermediary can monitor a firm in order to prevent it from choosing the ugly project, but this entails a cost proportional to the size of the project, $cI$, with $c > 0$. This cost is non-veriafiable to the outside investors. This, in turn, creates a second moral hazard problem, as an intermediary may be tempted to forego proper monitoring. In this sense, the outside investors are uninformed, while the intermediaries are informed.

Denote by $A_0$ the initial assets available to a given firm, by $A$ the amount that the firm invests into the project, by $I_m$ the assets available to the intermediary, and by $I_u$ the assets available to the uninformed outside investors. Further, let $R_f$ denote the share of the project's total returns $RI$ going to the firm, $R_m$ to the intermediary, and $R_u$ to the investor. The problem of a firm with initial assets $A_0$ is then:

$$\max p_H R_f + \gamma(A - A_0)$$

$$
\begin{aligned}
\text{s.t.} \quad & A + I_m + I_u \geq I & (1.1) \\
& A \leq A_0 & (1.2) \\
& R_f + R_m + R_u \leq RI & (1.3) \\
& p_H R_u \geq \gamma I & (1.4) \\
& p_H R_m \geq \beta I & (1.5) \\
& p_H R_m - cI \geq p_L R_m & (1.6) \\
& p_H R_f \geq p_L R_f + bI & (1.7)
\end{aligned}
$$

where $\beta$ is the required minimum return on the intermediary's funds.

In this problem, equations (1.1) and (1.2) are feasibility constraints and equation (1.3) the "cake-sharing" constraint. Equation (1.4) is the outside investor's participation constraint: he must get at least his expected outside return in order to be willing to participate. Equations (1.5) and (1.6) are the intermediary's participation and incentive constraints, respectively. It must

8

get at least its market-determined required minimum return $\beta$ to be willing to participate; and it must also be compensated for the monitoring costs in order to be willing to monitor the firm. Finally, equation (1.7) is the firm's incentive constraint: it must be compensated for the loss of the private benefit in order to be willing to choose the good project.

All equations of this optimisation programme can be divided through by the initial assets $A_0$. Define the scaled variables $\tilde{R}_f = \frac{R_f}{A_0}$, $\tilde{R}_m = \frac{R_m}{A_0}$, and $\tilde{R}_u = \frac{R_u}{A_0}$. Holmström and Tirole (1997) show that in equilibrium, all constraints bind. All firm assets are invested into the project, the intermediary will monitor the firm, and the good project will be chosen in equilibrium. The investor and the intermediary will invest up to the point where they get, in expectation, exactly their required minimum returns ($\gamma$ and $\beta$, respectively). The solution to this problem is then:

$$\tilde{R}_f = \frac{b}{\Delta p} \frac{I}{A_0}$$

$$\tilde{R}_m = \frac{c}{\Delta p} \frac{I}{A_0} = \frac{\beta}{p_H} \frac{I_m}{A_0}$$

$$\tilde{R}_u = \left( R - \frac{b+c}{\Delta p} \right) \frac{I}{A_0} = \frac{\gamma}{p_H} \frac{I_u}{A_0}$$

An important property of this solution is that all firms will choose the same optimal investment policy per unit of their initial assets $A_0$. In other words, all firms choose the same capital ratio $\frac{I}{A_0}$ that maximise their expected return, and scale up the investment project in proportion to their initial assets.

This property makes it easy to aggregate across a set of firms of different sizes in terms of their initial assets, and thus makes this formulation of the individual firm's moral hazard problem easy to incorporate into a general equilibrium model. I make use of this property in the model in Chapter 2. The framework also makes a clear distinction between the balance sheets of the borrower, the bank, and the outside investor; a second desirable property, which is lacking in simpler models of moral hazard.

In their article, Holmström and Tirole (1997) show that in this model, a contraction in aggregate intermediary capital will lead to a contraction in aggregate uninformed capital as well, as less resources are available for monitoring. As a consequence of such a credit crunch, total investment $I$ will contract by more than the initial crunch in the intermediary capital, as firms' solvency

ratio, i.e. the ratio of total firm assets to the total amount of assets invested, must increase.

Holmström and Tirole (1997) also point out that the model provides a rationale for countercyclical capital ratios for intermediaries. This is because in a credit crunch, as bank capital is scarcer, the return per unit of capital will be higher and thus a lower capital ratio will be enough to provide the right incentives for the intermediary.

This result has a direct connection to the contemporary discussion on the countercyclical regulation of bank's capital ratios proposed after the financial crisis of 2007–2009 in the international Basel III regulatory framework.[3] However, although this model implies a countercyclicality of capital ratios, it does not, by itself, provide a rationale for the *regulation* of these ratios through government intervention, as the market itself provides enough discipline for the contractual parties through the optimal compensation scheme.

In addition to my research, this double moral hazard approach has also been used in macroeconomics by Chen (2001), Meh and Moran (2010), Faia (2010), Christensen, Meh, and Moran (2011), and Haavio, Ripatti, and Takalo (2016). All of the aforementioned studies deal with the availability of bank capital and its implications for financial stability from a macroeconomic perspective.

## 1.4   Asymmetric information in corporate finance

Besides moral hazard, another agency problem — adverse selection induced by asymmetric information — can lead to loanable funds in the economy being sub-optimally distributed, as compared to a situation of complete information. Whereas moral hazard causes lending to depend on the availability of internal funds of either the borrower or the informed investor, or both as in the model discussed above, asymmetric information between lenders and borrowers can lead to either over-investment or to credit rationing through an inefficient pricing of risk. In these models, borrowers may typically default in equilibrium, and the default probability is sensitive to the borrower's internal funds.

Two seminal models of asymmetric information are of particular interest. The first one is the credit rationing model of Stiglitz and Weiss (1981), and the second one is the over-investment model of De Meza and Webb (1987).[4]

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[3]See the Basel Committee on Banking Supervision (2010).
[4]See House (2006) for a comparison of the properties of these two models.

Stiglitz and Weiss (1981) formulate a model of private types with two sets of agents, banks and firms. All agents are risk neutral. Banks have funds that they can use to finance investment projects. Each firm is endowed with an investment project of identical size $I$ that yields an expected gross return $\varrho$. Furthermore, each firm is endowed with own funds $w < I$, such that it needs external financing to undertake the project. It is assumed that all projects have the same expected return $\varrho$, but have different success probabilities $p$, and hence a different variance of the project return. It is further assumed that banks can observe the expected return, but not the success probability of an individual project. Finally, firms can default on their loans. In order to acquire external financing for the project, a firm therefore also needs to invest its own funds in the project. Denote by $r$ the gross interest paid by the firm on the loan.

It is assumed that banks act competitively and maximise their profits over their pool of loans. Under the assumption of unobserved borrower types, however, they are not price-takers. Instead, banks set the interest rates charged on loans strategically, taking into account the impact that the interest rate has on the composition of the borrower pool.

These particular assumptions lead to a situation where credit rationing can arise in equilibrium. The expected pay-off for a firm on a project is given by $\varrho - pr(I - w)$, which is decreasing in the success probability $p$. The authors show that the equilibrium is characterised by pooling in the credit market, where all firms can get a loan at the same interest rate $r$, and a cut-off strategy for the firms, where only firms with a low enough $p$ apply for loans. From the banks' perspective, these are the riskiest and most undesirable borrowers. This is because the expected return to the bank from an individual loan is $pr(I - w)$, which is increasing in the success probability $p$. This results from the assumed use of standard debt contracts: the lender bears the downside risk if the project fails, while borrowers are protected by limited liability. In this case, as the interest rate increases, the safest borrowers opt out of the credit market first. It is then possible that at the equilibrium interest rate, the demand for loans exceeds the supply of loans. Increasing the interest rate — which would reduce the demand for loans — would result in a lower expected return for the bank, as the least risky firms would be driven out of the market first.

This model describes a situation where asymmetric information leads to too little financing, too high interest rates, and consequently a sub-optimally low level of investment from society's point of view. But this result hinges on the specific assumptions of the model; in particular, on the assumption of

identical expected returns on each project. At the other extreme, asymmetric information may lead to an equilibrium characterised by *too much* financing, too low interest rates, and thus a sub-optimally high level of investment.

This is the case described by De Meza and Webb (1987) in their seminal contribution. Their set-up also features two sets of risk neutral agents, firms and banks. Firms are again endowed with investment projects of equal size $I$ that yield a risky ex-post gross return $R \in \{R^s, R^f\}$, common to all projects. In case of success, with probability $p > 0$, the project yields $R = R^s > 0$, and in case of failure, with probability $1 - p$, it yields $R = R^f = 0$. The firms also have identical initial assets $w < I$. Banks are endowed with funds that they can use to finance these projects. It is assumed that they offer standard debt contracts to the firms, and firms are consequently protected by limited liability. Now, while each firm's distribution of returns has the same support, firms differ in terms of their expected return $\varrho = pR^s$.

It is assumed that banks observe the distribution of projects in the population of firms, but cannot observe the characteristics of an individual firm. Again, as in Stiglitz and Weiss (1981), it is assumed that banks behave competitively and set the interest rates charged on loans strategically to take into account the impact of the interest rate on the loan applicant pool.

These assumptions give rise to a pooling equilibrium in the credit market, again characterised by a cut-off strategy for the borrowers: all firms that enter the credit market receive a loan at an equal interest rate. But now, the equilibrium is characterised by over-investment rather than credit rationing when compared to a situation of complete information. Competition among banks will drive the interest rate down to a zero-profit level so that the market clears. The expected pay-off to a firm that enters the credit market is $p\left(R^s - r(I - w)\right)$. The cut-off type is now the riskiest type, i.e. the one with the lowest probability of success $p$ among all loan applicants. This is due to the fact that, contrary to the Stiglitz and Weiss (1981) model, the firm's expected pay-off on a project is now increasing in $p$ as long as $R^s - r(I - w) \geq 0$.

De Meza and Webb (1987) show that under complete information, each type would be charged an interest rate that reflects their individual probability of default, and the rate charged from the cut-off type would be higher than the one charged in the competitive equilibrium under asymmetric information. The marginal type thus acquires financing that is too cheap from the society's point of view, and selection is towards riskier borrowers. As interest rates rise, the riskiest borrowers first opt out of the credit market.

I apply this model of asymmetric information in the essay in Chapter 3 to explore incentive mechanisms behind the housing boom and the subprime cri-

sis of the 2000's in the United States. The framework allows for a discussion of a market failure that leads to equilibrium over-borrowing by households and to the presence of "subprime" households in the market for housing loans.

The De Meza and Webb (1987) framework of asymmetric information has been used by House (2006) in an over-lapping generations model similar to mine, and more recently by Takalo and Toivanen (2012) and Jokivuolle, Kiema, and Vesala (2014). All of these studies focus on over-investment externalities in the financing of physical investment, not housing.

## 1.5   Partial information and signal extraction

A simple yet powerful departure from the assumption of complete information concerns a situation where agents do not observe the true data generating process of an exogenous process that drives some economic fundamental. I present here a simple univariate example. A more general multivariate presentation is found, for example, in Hamilton (1994, Ch. 13).

Let $z_t$ be a noisy signal about the unobservable data generating process $x_t$:

$$z_t = x_t + \varepsilon_t \tag{1.8}$$

$$x_t = \rho x_{t-1} + u_t. \tag{1.9}$$

Here, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $u_t \sim N(0, \sigma_u^2)$ are both assumed to be independently and identically distributed (i.i.d.) noise. $\rho \in [0, 1)$ is a parameter. In each period $t$, agents observe the signal $z_t$, but not the components $x_t$ and $\varepsilon_t$.

Suppose that economic agents are interested in forming an estimate of $x_t$ conditional on the history of observations $z^t = (z_0, \ldots, z_t)$. This problem is a signal extraction problem, and Equations (1.8) and (1.9) form a univariate state-space system. Assuming that the agents form their beliefs rationally, the solution to this problem is found by Bayesian updating. Given some prior belief about the initial value $x_0$, the agents update their beliefs on $(x_0, x_1, \ldots, x_t)$ in each period as new observations $z_t$ become available by taking advantage of Bayes' rule. This procedure yields a linear estimator of the latent state $x_t$ that is optimal in the sense of minimising the mean square error of the estimator. The Kalman filter methodology provides a powerful framework for forming such estimators in multivariate and non-Gaussian state-space models, as well.

The implications of such signal extraction problems in general equilibrium were first considered by Robert E. Lucas in a series of papers describing his "island model", starting with Lucas (1972). In the Lucas island model, pro-

ducers of goods only observe local prices, but not the general price level in the whole economy in a given time period. The general price level only becomes known with a lag. In order to make supply decisions, producers must figure out the relative contributions of changes in the general price level and changes in relative prices to the price of their own good — the local price —, which they take as given. Production decisions are optimally determined by changes in relative prices. With the notation above, $z_t$ denotes the local price, $x_t$ the general price level, and $\varepsilon_t$ a shock to relative prices. Decisions are thus made under partial information, and producers in this model confound nominal and relative price changes. As a consequence, output becomes sub-optimally volatile, as the only source of uncertainty in the model concerns the relative contributions of $x_t$ and $\varepsilon_t$ to changes in observed prices.

This idea of partial information about economic fundamentals has recently received growing attention both in macroeconomics and in financial economics. In macroeconomics, the asset pricing implications of model uncertainty have been studied by Johannes, Lochstoer, and Mou (2016) and Collin-Dufresne, Johannes, and Lochstoer (2016), among others. In the context of housing markets, Piazzesi, Schneider, and Tuzel (2007) explore asset pricing implications of low-frequency risk in house price fundamentals, and Gelain and Lansing (2014) study the implications of learning on house price dynamics. In finance, Lewellen and Shanken (2002), Adrian and Franzoni (2009), and Trecroci (2014) among others, formulate conditional capital asset pricing models with learning in order to explain cross-sectional variation in asset returns.

In Chapter 4, I apply the conditional capital asset pricing model with Bayesian learning, following Adrian and Franzoni (2009) and Trecroci (2014), to study how the beliefs of investors about returns to housing investment evolved in the run-up and during the subprime crisis. I assume that the true systematic risk of these investments is unobserved, and realised returns provide noisy signals about this risk. Investors may thus confound aggregate and idiosyncratic risk, and are faced with a signal extraction problem of the type described here.

## 1.6   Summary of the essays

This section describes the main modelling assumptions and summarises the key results in the three essays that constitute the main body of this thesis. Each essay is self-contained.

### 1.6.1 Chapter 2: The interaction of monetary and macroprudential policies

In the first essay, I set up and analyse a New Keynesian dynamic stochastic general equilibrium (DSGE) model where the financing of capital investment is affected by a double moral hazard problem of the Holmström and Tirole (1997) type. This friction affects both the demand and supply of credit in the model economy, and thus distorts both investment decisions of firms and saving decisions of households. Besides the friction on credit intermediation, the model is standard to the New Keynesian research literature; in particular, the general price level adjusts sluggishly to shocks due to a Calvo (1983) price-setting friction.[5]

In the model equilibrium, the availability of loans for capital production depends on the balance sheet conditions of both banks and the capital-producing entrepreneurs because of agency costs imposed by the moral hazard problem. The aggregate economy is thus vulnerable to shocks that affect the balance sheets of banks and capital producers.

As the main contribution of the essay, I solve for jointly optimal monetary and macroprudential policies. The monetary policy aims at removing the distortion created by nominal rigidities, and the macroprudential policy is in charge of mitigating the distortion on the level of aggregate investments caused by the financial friction. Specifically, monetary policy targets the inflation rate, and macroprudential policy targets the aggregate leverage of the banking sector, which determines aggregate investment in the economy. This optimal policy problem is set up as a problem of a benevolent social planner who chooses the values of these policy targets so as to maximise social welfare, while respecting the private agents' incentives and optimising behaviour. I further assume that the planner can commit to the chosen policies. This type of optimal policy problem is called a *Ramsey problem* after Ramsey (1927).

I find that the optimal policy can achieve an efficient outcome — in the sense of replicating the equilibrium allocation of resources in an economy without financial and nominal frictions — if the social planner can conduct both monetary and macroprudential policy. Using monetary policy alone is not enough: in this case a policy trade-off between stabilising the inflation rate and the output gap arises. The results provide a justification for a separate macroprudential policy that is conducted in coordination with monetary policy.

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[5]See e.g. Galí (2008) and Woodford (2003) for thorough presentations of the New Keynesian model of business cycles.

This trade-off results from the opposing effects that the nominal and financial frictions have on the price of physical capital. An inflationary shock, for example, requires a deflationary response in order to remove the distortion on relative prices, but this will depress the real price of capital. However, depression of the capital price worsens the incentive problems in the financial market, which distorts the level of investments and creates an output gap. Correcting for this latter distortion would then warrant positive inflation in order to push up the real capital price.

As the second contribution, I systematically compare the performance of simply policy rules to the optimal policy. When policy follows simple rules — in the spirit of the Taylor rule for monetary policy, proposed by Taylor (1993) — the source of fluctuations in the economy is relevant for the choice of the appropriate policy mix. I find that there may be welfare gains from following a monetary policy rule that reacts to assets prices, and in particular, to the real price of capital.

A separate macroprudential policy rule, in the form of countercyclical bank capital regulation, is only useful in counteracting shocks that arise from the financial sector itself; however, it can be counterproductive in mitigating the impacts of shocks that arise from the real sector but affect the financial sector, because it can prevent or slow down the proper adjustment of investments to their efficient level. This suggests that countercyclical capital buffers for banks should rather be used as a discretionary policy tool to make them less vulnerable to financial shocks.

## 1.6.2 Chapter 3: House prices, lending standards, and the macroeconomy

In the second essay, I study the link between house prices, lending standards, and aggregate over-investment in housing. I develop an overlapping generations model of the housing market following House (2006), who studies over-investment in productive capital.

In the model, the market for housing loans is affected by asymmetric information as in the De Meza and Webb (1987) model: borrowers have private types that define their idiosyncratic income risk. Another source of risk are house values, observed by both borrowers and lenders.

The private income risk together with the public house valuation risk determines the borrowers' private equilibrium default probability on housing loans. Selection in equilibrium is towards riskier borrower types: the marginal

borrower is the riskiest one in the borrower pool. He would thus not find it profitable to participate in the credit market under symmetric information, because the cost at which he would be able to borrow would reflect his true private risk, instead of the average risk in the whole borrower pool.

In this context, tight or loose *lending standards* are defined in relation to the first-best allocation of credit that would prevail under symmetric information. In a similar vein, a *subprime* borrower is defined in the model as a household that finds it profitable to apply for a loan and buy a home under asymmetric information, but would not do so under symmetric information as the cost of borrowing would reflect its true private default risk.

In the main contribution of the essay, through a comparative statics exercise, I show that lending standards are loose and the incentives for households to apply for a loan are particularly strong, first, when future house values are expected to be high, which leads to high leverage of borrowers.

However, there are strong non-linearities in the relationship between borrowing incentives and expected house values. When expected future house values are at a very low level, even high-risk households find it worthwhile to borrow in order to finance a house purchase, as housing is cheap. When the expected house values increase, these high-risk borrowers opt out of the market first, as housing becomes more expensive. But as the expected values increase even more, the expected capital gains on housing become so attractive despite risks inherent in high leverage that the high-risk, or subprime, types start to opt back into the market. Correspondingly, household default rates first also decrease, but subsequently increase, as households become more levered.

Second, incentives for borrowing are strong when safe interest rates are low, which implies low costs of borrowing. This result implies that loose monetary policy can have a direct impact on the stability of the housing market through the cost of borrowing and the opportunity cost of housing investment.

These results shed light on incentive mechanisms that can help explain the developments in the U.S. housing market in the early 2000's. The environment was characterised by low interest rates and an expectation of continued increases in house prices. At the same time, households became highly indebted.

Finally, I extend the model to include a strategic complementarity in home-ownership and deadweight costs of default on house prices. The first feature captures, in a simple form, the hype of owning a home that characterises a housing boom. The second feature captures a fire sale externality that char-

acterises a housing bust: as foreclosures increase, the value that debtors can recover by seizing homes and selling them decreases. I demonstrate that these two features can create endogenous cycles in house prices.

### 1.6.3 Chapter 4: Learning about systematic risk in the housing market

In the third essay of this thesis, I explore the perceptions of investors about the systematic risk in the residential housing market in the United States in 1987–2016. The subprime crisis erupted in 2007 after a period of strong increases in residential housing prices in many regions of the United States.

A prominent narrative of the crisis asserts that financiers and policymakers alike believed economic fundamentals to be strong and house prices to keep increasing, which justified a boom in mortgage lending. Because house prices were expected to increase further, it was argued, the systematic — or undiversifiable — risk in investment into the housing market was thought to be negligible.

To systematically study the beliefs of financiers over the years that led to the crisis, I estimate a conditional capital asset pricing model (CCAPM) with Bayesian learning on returns in the residential housing market. I use monthly data on house prices in 17 Metropolitan Statistical Areas (MSAs) from 1987 to 2016 provided by Case & Shiller.

In the model, risk-averse financiers choose portfolio holdings of housing investment to maximise their expected returns. The financiers only care about their wealth, or in other words, about the risk-return trade-off of their investment. I assume that the financiers are unable to disentangle the idiosyncratic and the systematic risk in the portfolio, and there is thus incomplete information about the true risk-return trade-off. I further assume that an individual portfolio is composed of a pool of regional housing investments, and the systematic risk is related to the extent to which the regional market co-moves with the national housing market. The problem of an individual financier in the model, who seeks to price the housing portfolio, is thus a signal extraction problem where the observed returns are noisy signals about the true aggregate state of the housing market.

I show evidence in the data that financiers' perceptions about systematic risk in the housing market were relatively low in the 1990's and the early 2000's, in the period where house prices were booming. At the onset of the subprime crisis, these beliefs were most swiftly updated upward in regions

that had experienced the strongest house price appreciation.

These results lend support to the view that the crisis was fuelled by a loosening of lending standards and a credit boom, as financiers believed house prices to keep rising and investment in the housing market to be safe. In other words, the risk in housing investment was thought to be mostly idiosyncratic and not systematic in nature. Hence it could be diversified away. In hindsight, developments in regional housing markets were correlated, and the United States experienced a dramatic nation-wide fall in house prices — something that was widely thought to be very unlikely as late as in 2005.

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# 2 The interaction of monetary and macroprudential policies[1]

## 2.1 Introduction

The global financial crisis that erupted in 2007 in the United States has highlighted the role of aggregate balance sheet conditions of banks for economic cycles. As a response to the crisis, policymakers have emphasised the importance of macroprudential regulation, as opposed to and in addition of reforms to the regulation and supervision of individual institutions.[2]

Macroprudential policy in general refers to policy measures that aim at mitigating the risks and imbalances of the financial system as a whole, while conventional microprudential banking regulation has focused on single institutions. The mitigation of credit cycles, which can be much more volatile than real output cycles, has been seen as a key policy goal for the new macroprudential framework. This regulatory response stems from the widespread view that a build-up of system-wide risks and imbalances was at the heart of the collapse of the financial system in 2007. In this view, mitigating credit cycles and supporting financial stability are important policy goals in themselves, but they are also essential for the stability of the economy as a whole, as banking crises tend to have long-lasting consequences for real economic

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[1]An article based on this essay is forthcoming in the *Journal of Money, Credit and Banking*. See Silvo (2018).

[2]See, for example, the policy reports by the Bank of England (2009) and the Bank of International Settlements (2011).

activity.

After the crisis, a new policy framework was internationally adopted in the Basel III agreement in 2010 (Basel Committee on Banking Supervision 2010). The new macroprudential policy tools include, among others, counter-cyclical and risk-weighted capital buffers for banks that depend not only on the banks' own balance sheet conditions, but also on aggregate economic and credit conditions.

This essay contributes to the academic and policy discussions on how to set up mandates for monetary and macroprudential regulators, and on the gains of policy coordination between monetary and financial stability mandates. The main contribution of this essay is to solve for the jointly optimal mix of monetary and macroprudential policies in response to various economic and financial disturbances. Very few previous pieces of research on joint monetary and macroprudential policies have solved for the full jointly optimal policy programme, and none have done it in a similar setup as this essay. As a second contribution, this essay also provides a systematic welfare comparison of the optimal policies to simple policy rules.

To do so, I formulate a dynamic stochastic general equilibrium (DSGE) model where banks' balance sheets play a key role in financial intermediation, which in turn has an important effect on economic cycles. These dynamics arise because of informational frictions in credit intermediation caused by an agency problem formulated by Holmström and Tirole (1997).

The key transmission channel from the financial sector to the real economy in this model is the real capital price, which on the one side affects investment decisions in the financial market, and on the other side households' saving behaviour. The fluctuations of this price summarise the effects that the agency cost has on the availability of capital and investment in the economy. It can also be interpreted as Tobin's $q$.

To solve for the optimal policies, I set up a Ramsey policy problem in which a social planner can directly set either one policy variable (the inflation rate), or two policy variables (the inflation rate and the aggregate leverage of the banking sector). I assume that the problem of the planner is constrained in the sense that he cannot remove the agency problem in the economy.

As the main result, I find that the first-best optimal outcome, which corresponds to the frictionless real business cycle (RBC) equilibrium, can be replicated when the social planner can jointly use both a monetary and a macroprudential instrument. On the contrary, using only monetary policy — namely, controlling only inflation — leads to a short-run policy trade-off between stabilising inflation and the output gap. This result is linked to a sem-

25

inal contribution by Clarida, Galí, and Gertler (1999), who discuss a situation where nominal cost-push shocks lead to a similar short-run trade-off in stabilising inflation. The findings in this essay thus broaden our understanding on the sources of potential monetary policy trade-offs.

I also systematically compare the performance of simple rule-based policies to the optimal policies in terms of household welfare. When simple policy rules are used instead of the Ramsey-optimal policies, contrary to the findings in many earlier studies, I find that it may be beneficial for the monetary authority to also react to financial conditions when financial frictions are present in the economy.[3] In particular, a Basel III-type countercyclical capital ratio regulation works well in stabilising the credit cycle. By controlling the aggregate leverage of the banking sector and smoothing out the credit cycle, it can effectively prevent financial shocks from propagating to the real economy.

In contrast, when the disturbances arise from real supply or demand side shocks, and not from the financial sector, the form of macroprudential policy matters, as it can hinder proper economic adjustment by creating an additional friction on the adjustment of investment. In this case, a unified mandate for the monetary authority, where besides price developments the central bank also pays attention to financial conditions — in particular to the real price of capital — seems to be sufficient.

It is not evident how monetary policy should react to financial imbalances – if at all – and how new macroprudential policy measures should be coordinated with monetary policy over the business cycle. According to the Tinbergen principle, a policymaker should have as many policy instruments as there are policy objectives, and each instrument should be assigned to one objective. In light of this view, then, a separate macroprudential tool should be useful when frictions in credit intermediation are present. The Ramsey-optimal policy analysis in this essay supports this view.

It has to be noted, however, that the policies studied in this essay are ones that aim at stabilising the credit cycle — as measured by deviations of the loans-to-output ratio of the economy from its steady state — by responding to various shocks that affect banks' capital positions. The model is solved through log-linearisation around its efficient steady state, in the sense that it coincides with the steady state of the frictionless (first-best) economy. The solution thus abstracts away from potential asymmetric macro-financial linkages and large financial shocks, as well as from steady-state distortions. There is no

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[3]See for example Bernanke and Gertler (2001), Iacoviello (2005) and Carlstrom and Fuerst (2007).

externality in financing that would require correction ex-ante, but the banks
in the model are vulnerable to macroeconomic and financial conditions, be-
cause their ability to fund investment projects depends on their balance sheet
conditions. This causes undesirable distortions away from the first-best allo-
cation.[4] The results in this essay thus reflect a view of macroprudential policy
as a tool for managing business cycles that arise from dynamic distortions in
the economy, analogously to the traditional view of monetary policy in the
New Keynesian tradition.[5] A similar rationale is given for optimal policies for
example in De Paoli and Paustian (2017).

The remainder of the essay is organised as follows. First, Section 2.2 re-
views earlier research related to my results. Section 2.3 then outlines the the-
oretical framework. Next, Section 2.4 presents the calibration and discusses
the empirical fit of the theoretical model. The main contribution of this essay
is presented in Section 2.5, which discusses the implications of the financial
friction to the aggregate economy, and the Ramsey-optimal policy plans that
can offset the effects of this friction to restore the first-best. Next, Section 2.6
analyses the aggregate dynamics of the model economy under various sim-
ple policy rules and compares these policy regimes to the Ramsey policies.
Section 2.7 discusses the welfare implications of the different policy regimes.
Finally, Section 2.8 concludes.

## 2.2  Related literature

The literature on financial frictions in macroeconomic theory is vast and grow-
ing. Most early studies on financial frictions in a general equilibrium setting
do not model financial intermediation explicitly, abstracting from banks' cap-
ital position altogether. Examples include Kiyotaki and Moore (1997), Carl-
strom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), Iacoviello
(2005), Monacelli (2008), Faia and Monacelli (2007), and Adrian and Shin
(2010).

More recent research does consider the role of credit intermediation in
business cycles. Cúrdia and Woodford (2010b, 2016) formulate a model with
costly intermediation and time varying credit spreads that arise from bor-

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[4]Holmström and Tirole (1997) discuss more extensively the countercyclical bank leverage im-
plied by the agency problem that is also applied in this essay.

[5]The policy analysed in this essay could perhaps more accurately be termed as "financial sta-
bility policy"; to keep in line with previous literature, however, I use the term "macroprudential
policy".

rower default risk, and find that monetary policy should respond to current and future credit spreads. Further building on Cúrdia and Woodford (2010b), Cúrdia and Woodford (2010a, 2011) model balance sheets of both the central bank and private banks. Their results suggest that in a deep enough financial crisis, such unconventional monetary policy measures can be efficient.

Canzoneri, Collard, Dellas, and Diba (2016) suggest, building on Cúrdia and Woodford (2010b, 2016), that financial market frictions can be strongly countercyclical and have amplification effects on business cycles and fiscal multipliers. This finding supports the view that mitigating credit cycles has important consequences for general economic conditions.

Gertler and Karadi (2011) formulate an influential model of financial frictions where a moral hazard problem exists between a bank and its depositors, affecting the supply side of credit. However, in this framework, the balance sheets of the bank and the borrower, on the demand side, are effectively indistinguishable. Gertler and Kiyotaki (2010, 2015) extend the model of Gertler and Karadi (2011) to include interbank credit markets; both Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) find that direct lending by the central bank is an efficient monetary policy tool in mitigating financial turmoil.

Another strand of literature, including this essay, uses the Holmström–Tirole (1997) double moral hazard framework to explicitly model frictions on both the demand and the supply side of credit intermediation. As a consequence, both the lenders' and the borrowers' balance sheets matter for the financing conditions. This approach to modelling agency costs has previously been used by Chen (2001), Meh and Moran (2010), Faia (2010), Christensen, Meh, and Moran (2011), and most recently by Haavio, Ripatti, and Takalo (2016). The latter find a role for public equity injections into banks in order to prevent and mitigate effects financial shocks on credit intermediation.

In a different approach, Gerali, Neri, Nessa, and Signoretti (2010) extend the model of Iacoviello (2005) to include a monopolistically competitive banking sector. In the model, borrowers are constrained by collateral constraints, and banks are constrained by a cost of deviating from an exogenously given capital ratio target. Hence, in this framework both banks' and borrowers' balance sheets also matter, but unlike in the Holmström and Tirole (1997) framework, not as an equilibrium outcome from an optimal contracting problem.

The literature on jointly optimising monetary and macroprudential policies is still scarce. Most authors only focus on rule-based policies. Angelini, Neri, and Panetta (2014) analyse an economy with collateral constraints where a policymaker sets the parameters of simple monetary and macroprudential

policy rules to maximise a quadratic policy objective. Angeloni and Faia (2013) study jointly optimal rule-based monetary policies and capital regulation using a model of bank runs. Christensen et al. (2011) build on Meh and Moran (2010) to investigate optimal rule-based capital ratio regulation and monetary policy.

With regard to jointly optimal Ramsey policies, Collard, Dellas, Diba, and Loisel (2017) study jointly optimal monetary and macroprudential policies in a model where limited liability and deposit insurance cause excess risk-taking in the financial sector. De Paoli and Paustian (2017) analyse both cooperative and non-cooperative optimal monetary and macroprudential policies, both under commitment and discretion, in a New Keynesian economy with always-binding borrowing constraints as in Kiyotaki and Moore (1997). In line with the results in this essay, Collard et al. (2017) and De Paoli and Paustian (2017) also find it desirable to separate the two objectives of financial and price stability to be dealt with two distinct policies.

This essay is closest, in terms of model structure, to Christensen et al. (2011). The findings in this essay are mostly in line with theirs and those of Angelini et al. (2014), who also find modest gains from macroprudential regulation when shocks arise from the supply side, and larger benefits when shocks arise from the financial sector itself. Angeloni and Faia (2013), on the other hand, find that a countercyclical response to financial conditions, either by reacting to bank leverage or asset prices, unambiguously increases welfare. A similar result is obtained by De Paoli and Paustian (2017), who find that a macroprudential policy, in the form of a tax on borrowing, is welfare-improving, no matter the source of fluctuations.

## 2.3 The model

The model presented in this section builds and expands on the recent work by Haavio et al. (2016). It incorporates into an otherwise fairly standard New Keynesian setup a financial sector affected by double moral hazard of the Holmström and Tirole (1997) type, whereby a moral hazard problem exists both between the bank and its depositors, and the bank and its borrowers. This allows for a friction to exist both on the supply and the demand side of credit. This, in turn, creates a distinction between banks' and productive firms' capital as they earn different returns in equilibrium.

### 2.3.1 Structure of the economy

The economy consists of atomistic households, a production sector, a financial sector, and a government. The total mass of households is one. Each household has three types of members with distinct roles: an entrepreneur, a banker, and a worker-consumer[6]. Each banker manages a bank, each entrepreneur undertakes risky projects to produce new capital goods, and each worker supplies labour to firms, consumes final goods, and saves. There is perfect insurance between the family members within a household, so that the model can be described with a representative household.

The production sector is standard to New Keynesian models, except for capital production. Intermediate good firms employ capital and labour to produce goods, which are then bundled into final goods by final good firms. Capital is produced by risk neutral entrepreneurs, who undertake risky projects to do so. Risk neutral banks collect deposits from households and issue loans to entrepreneurs, who need funding for their projects. The banks also monitor the entrepreneurs' projects to guarantee efficient use of the funds.

Finally, the government conducts monetary and macroprudential policy.

### 2.3.2 Households

In each period, part of the economy's entrepreneurs and bankers exit. An entrepreneur survives into the next period with a constant and exogenous probability $\lambda^e \in (0,1)$, and exits with probability $1 - \lambda^e$. A banker's survival probability is, similarly, $\lambda^b \in (0,1)$. New entrepreneurs and bankers are born in every period to replace the exiting ones, such that the shares of entrepreneurs and bankers in the economy remain constant over time. Consequently, the fraction of worker-consumers in the economy also stays constant.

While a banker or an entrepreneur is active, they do not consume; they merely engage in banking or entrepreneurial activities and accumulate net worth. Because they are assumed to be risk-neutral and financially constrained, they have an incentive to delay consumption until they are no longer constrained. The assumption of finite horizons for bankers and entrepreneurs follows Bernanke et al. (1999), and ensures that these agents cannot accumulate enough wealth to become financially unconstrained. When they exit, their net worth is transferred to their household, to be consumed or saved.

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[6]The terms "worker-consumer", "worker" and "depositor" will be used interchangeably to denote the family member who is not an entrepreneur or a banker, depending on the specific context.

The working member of the household consumes, makes saving decisions and portfolio choices, and supplies labour in each period in a standard manner. The household can save into one-period deposits at a bank or into physical capital; the deposits are discussed in detail in Section 2.3.5, where the financial sector is introduced.

The representative household maximises its utility:

$$\max_{\{C_t, D_{t+1}, K_{t+1}, L_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \quad 0 < \beta < 1, \tag{2.1}$$

subject to a budget constraint:

$$P_t C_t + P_t q_t K_{t+1} + D_{t+1} = W_t L_t + P_t \left( r_t^K + (1-\delta)q_t \right) K_t + (1+r_t)D_t + P_t T_t, \tag{2.2}$$

where $C_t$ is real consumption, and $D_t$ is the aggregate amount of nominal deposits, yielding a gross nominal interest rate $1 + r_t$.[7] $L_t$ is labour supply, $K_t$ is the real capital stock, and $T_t$ are real lump sum transfers received by the household, which include profits from the monopolistically competitive firms owned by the household and net returns from banking and entrepreneurial activities.

The final good acts as numeraire, and $P_t$ is its price index. Then, $q_t$ is the real price of capital in terms of the final good. It may differ from unity because of the distortion created by the financial friction, described below; it can also be interpreted as Tobin's $q$. Finally, $W_t$ is the nominal wage rate, and $r_t^K$ is the real rental rate of capital. The parameter $\beta \in (0,1)$ denotes the discount factor of the household, and $\delta \in (0,1)$ is the depreciation rate of capital.

I specify a standard CES utility function for the household:

$$U(C_t, L_t) = Z_t^c \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi L_t^{1+\theta}}{1+\theta}.$$

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[7]Individual deposits are subject to project-level idiosyncratic risk. I assume that the representative household invests into a fully diversified portfolio of deposits, or equivalently into a mutual fund, the return on which is not subject to aggregate risk, following e.g. Carlstrom and Fuerst (1997). I further assume that the deposits are subject to a "cash-in-advance"-like constraint: households choose the aggregate amount of funds to allocate in bank deposits in period $t$, in real terms $\frac{D_t}{P_t}$, already at the end of period $t-1$, although the individual deposits are made in $t$. For this reason, the opportunity cost of funds is $r_t > 0$, even though the financing contract is an inter-period contract. This treatment of deposits follows Faia (2010).

Here $\sigma > 0$ is the risk aversion parameter, $\theta > 0$ is the inverse of the Frisch elasticity of labour substitution, and $\chi > 0$ is the labour disutility coefficient. $Z_t^c$ is an exogenous preference shock, which captures real demand-side disturbances.

This household problem leads to the following standard optimality conditions for labour supply and for deposit and capital holdings:

$$w_t = -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} \tag{2.3}$$

$$1 = \beta E_t \left[ \lambda_{t,t+1} (1 + r_t) \frac{P_t}{P_{t+1}} \right] \tag{2.4}$$

$$q_t = \beta E_t \left[ \lambda_{t,t+1} (r_{t+1}^K + (1 - \delta)q_{t+1}) \right], \tag{2.5}$$

where $\lambda_{t,t+1} = \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)}$ is the marginal rate of intertemporal substitution, and $w_t = \frac{W_t}{P_t}$ is the real wage.

### 2.3.3 Final good production

Final good producers bundle intermediate goods $Y_t(i)$ into final goods $Y_t$ using a standard aggregation technology

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 0. \tag{2.6}$$

There is free entry and exit in the final good sector, and the firms are perfectly competitive.

The maximisation problem of the final good producers, combined with the zero-profit condition, yields the standard expressions for the demand schedules of each intermediate good $Y_t(i)$ and the aggregate price level $P_t$:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \tag{2.7}$$

$$P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}. \tag{2.8}$$

### 2.3.4 Intermediate good production

There is a continuum of intermediate good producers of mass one, indexed by $i$. At the beginning of each period, the intermediate firm $i$ rents capital $K_t(i)$ from the household at price $r_t^K$, and employs labour $L_t(i)$ at a nominal wage rate $W_t$.

Each intermediate firm uses a Cobb-Douglas production technology:

$$Y_t(i) = Z_t K_t(i)^\alpha (L_t(i))^{1-\alpha}, \tag{2.9}$$

where $Z_t$ is an exogenous total factor productivity shock.

Cost minimisation by the intermediate firm yields the standard optimality conditions for the capital and labour demand, given the relative factor prices, and a condition for the real marginal cost $\psi_t$:

$$\frac{r_t^K}{w_t} = \frac{\alpha L_t(i)}{(1-\alpha)K_t(i)}, \tag{2.10}$$

$$\psi_t = \left(\frac{r_t^K}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{(1-\alpha)} Z_t^{-1}. \tag{2.11}$$

Each firm is able to set its price in a staggered manner as in Calvo (1983). In any given period, the constant probability of being able to reset the price is $1 - \omega$, with $0 < \omega < 1$. The profit maximisation problem of the intermediate firm $i$ who is able to reset the price in period $t$ is:

$$\max_{P_t(i)} E_t \left[ \sum_{k=0}^\infty \omega^k Q_{t,t+k} \left( \frac{P_t(i)}{P_{t+k}} - \Psi_{t+k|t} \right) Y_{t+k|t}(i) \right], \tag{2.12}$$

subject to the demand condition:

$$Y_{t+k|t}(i) = \left( \frac{P_t(i)}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}.$$

$Q_{t,t+k} = \beta^k \lambda_{t,t+k} \frac{P_t}{P_{t+k}}$ denotes the stochastic discount factor that is obtained from the household's optimality conditions. $\Psi_t$ denotes the nominal marginal cost.

In equilibrium, all intermediate firms symmetrically choose the price $P_t(i)$

$= P_t^*$. This yields the expression for the optimal price:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \omega^k Q_{t,t+k} \psi_{t+k|t} Y_{t+k|t} P_{t+k}^{\varepsilon+1}}{E_t \sum_{k=0}^{\infty} \omega^k Q_{t,t+k} Y_{t+k|t} P_{t+k}^{\varepsilon}}. \tag{2.13}$$

In this equilibrium, the aggregate price index (2.8) can then be written as:

$$P_t = \left[ \omega P_{t-1}^{1-\epsilon} + (1-\omega)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \tag{2.14}$$

and the gross inflation rate between periods $t$ and $t-1$ as:

$$\pi_t = \left[ \omega + (1-\omega) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \tag{2.15}$$

### 2.3.5 Capital good production

Capital needed in the production of intermediate goods is produced by entrepreneurs. They can acquire external funding for risky investment projects from banks. The banks, on the other hand, invest both their own funds and the deposits of workers into the projects. The details of this three-party financing contract are given in the next section. The financial sector is affected by agency costs created by a double moral hazard problem as formulated by Holmström and Tirole (1997).

#### 2.3.5.1 The financing contract

This section describes the partial equilibrium in the financial market. In what follows, small letters denote individual-level variables, whereas capital letters denote aggregate variables.

The financial sector consists of banks, each managed by a banker, that channel funds from the workers to the entrepreneurs. Workers can choose to deposit their savings at a bank.[8] I assume that the banking system has an aggregate amount of real deposits $\frac{D_t}{P_t}$ available in period $t$, which it can

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[8]To make the financial sector non-trivial, I assume that a worker cannot deposit his savings in the bank managed by the banker in the same household; nor can the banker lend funds to the entrepreneur in his own household.

intermediate to entrepreneurs within that period. To attract deposits, the return on the risky investment has to be high enough for the depositor. In this sense, an individual deposit is not riskless, but rather has to be understood as a short-term investment subject to idiosyncratic risk. At the end of the period, after the returns from the investment project are realised, the banker returns the proceeds from the project to the depositor. Anticipating the equilibrium of the financial sector, the aggregate amount of these proceeds, in real terms, will turn out to be equal to $(1 + r_t)\frac{D_t}{P_t}$, reflected in the household's budget constraint.

An entrepreneur's objective is to maximise her expected profit from the investment project. She can borrow money from the bank in order to lever the return to the project. However, she can choose to neglect the investment project to obtain a private benefit. The depositor nor the banker cannot observe whether the project was neglected or not. If the entrepreneur chooses to neglect the project in favour of her private benefit, the productive investment project is less likely to succeed. This presents the first form of moral hazard in the financial sector and creates a friction to the *demand side* of funds, restricting the ability of the entrepreneur to acquire external funding.

In order to mitigate this moral hazard problem, the banker needs to monitor the entrepreneur. But this has a non-verifiable cost to the banker; because of this, he might want to forgo the monitoring. The worker observes whether the project succeeds or not but cannot verify whether the banker properly monitored the entrepreneur. This is the second form of moral hazard in the financial sector, which creates a friction to the *supply side* of funds. To mitigate this second moral hazard problem, and to be able to attract deposits from the worker, the banker needs to invest some of his own funds to be properly incentivised to monitor the project, i.e., he must have some "skin in the game".

Formally, if $i_t$ is the size of an individual investment project, $n_t$ is the net worth of the entrepreneur, $a_t$ is the net worth of the banker, $\kappa_t$ is the unit cost of monitoring the investment project, and $d_t$ is the deposit of the worker in period $t$, then:

$$i_t - n_t \leq a_t + d_t - \kappa_t i_t \tag{2.16}$$

gives the maximum amount of external funding an entrepreneur can get for her project, given her own net worth.

A successful project turns $i_t$ final goods into $Ri_t$ capital goods with $R > 1$. A failed project yields $R = 0$. The one-period contract specifies how the returns of the project are divided between the worker ($R_t^w$), the banker ($R_t^b$)

35

and the entrepreneur ($R_t^e$):

$$R \geq R_t^w + R_t^b + R_t^e. \tag{2.17}$$

There are two types of projects: "good" and "bad" ones (or non-neglected and neglected ones). The project succeeds with probability $p \in \{p_H, p_L\}$, with $\Delta p = p_H - p_L > 0$ and $1 > p_H > p_L > 0$. If the entrepreneur chooses the good project, the success probability is $p_H$, but there is no private benefit to her. There is also a continuum of bad projects, each with the same success probability $p_L$, but with an associated positive non-verifiable private benefit $b$ with $0 < b \leq \bar{b}$, proportional to the size of the project.

By choosing a monitoring intensity $\kappa_t \geq 0$, the banker can prevent the entrepreneur from choosing any of the bad projects with $b \geq b(\kappa_t)$. I assume $b'(\kappa) \leq 0$, $b''(\kappa) \geq 0$ and $\lim_{\kappa \to \infty} b'(\kappa) = 0$. Because monitoring is costly, it is never possible for the banker to monitor at a level that completely eliminates all bad projects.

In order for the three parties to be willing to participate in the contract, the following incentive and participation constraints must be met:

$$q_t p_H R_t^w i_t \geq (1 + r_t) d_t \tag{2.18}$$

$$q_t p_H R_t^b i_t \geq (1 + r_t^a) a_t \tag{2.19}$$

$$q_t p_H R_t^b i_t - \kappa_t i_t \geq q_t p_L R_t^b i_t \tag{2.20}$$

$$q_t p_H R_t^e i_t \geq q_t p_L R_t^e i_t + b(\kappa_t) i_t \tag{2.21}$$

Equations (2.16)–(2.21) define the financial contract. Equation (2.18) is the participation constraint of the depositor, which tells that the depositor must obtain a gross return at least as high from participating in the project, as she would get on the deposit otherwise; $r_t$ is the net outside return on the deposit, which is equal to the short-term market interest rate. Similarly, equation (2.19) is the participation constraint of the banker, where $r_t^a$ is the market-determined required minimum return on bank capital. The equilibrium rate of return will be described below.

Equations (2.20) and (2.21) are the incentive constraints of the banker and the entrepreneur, respectively. In order for the banker to be willing to monitor the entrepreneur, the return from the good project, net of monitoring cost, must be at least as much than the return from the bad project. The entrepreneur, in turn, must get at least as much from the good project as she would get from the bad project together with the private benefit.

In equilibrium, all constraints bind.[9] It is easy to see why: first, the two resource constraints (2.16) and (2.17) are trivially binding at optimum. Second, the compensations $R_t^e$ and $R_t^b$ must be high enough to properly incentivise the entrepreneur and banker to behave; but by the pie-sharing constraint (2.17), the more is allocated to them, the less is left for the depositor, who is the residual claimant of the project return. Thus, the depositor will not participate unless the minimum possible shares that satisfy the incentive and participation constraints are allocated to the entrepreneur and the banker.

As a consequence, in each period, the entrepreneur and the banker invest their whole net worth, as well as the whole deposit of the worker, into the investment project, the banker always monitors the entrepreneur, and the entrepreneur always undertakes the good project.

In order to guarantee that the good investment project is desirable compared to the bad projects from the household's point of view, I further assume that $q_t p_H R > \max\{1 + r_t, q_t p_L R + \bar{b}\}$. This assumption also guarantees that the project has a positive rate of return and positive pledgeable income.

### 2.3.5.2 Optimal investment and leverage

In this section, I solve for the optimal leverage ratio of the entrepreneur, and the corresponding optimal size $i_t$ of an investment project. From the incentive constraints (2.20) and (2.21), the banker and the entrepreneur must get at least:

$$R_t^b = \frac{\kappa_t}{q_t \Delta p} \tag{2.22}$$

$$R_t^e = \frac{b(\kappa_t)}{q_t \Delta p} \tag{2.23}$$

to be properly incentivised in equilibrium. In other words, the more severe the moral hazard of the entrepreneur at any given monitoring level, the more she must be compensated for undertaking the good project instead of the bad one; and the costlier monitoring is, the more the banker has to be compensated.

The depositor is the residual claimant of the return, who can then get at most:

$$R_t^w = R - R_t^b - R_t^e = R - \frac{b(\kappa_t) + \kappa_t}{q_t \Delta p}. \tag{2.24}$$

Therefore it is in the best interest of the depositor that the project is prop-

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[9]See Holmström and Tirole (1997) for a detailed proof and discussion.

erly monitored to guarantee that the good project is chosen. In equilibrium, the entrepreneur and the banker get the minimum return that satisfies their incentive constraints, and the depositor gets the maximum residual return.

From the participation constraints (2.18) and (2.19) it follows:

$$R_t^w = \frac{(1+r_t)d_t}{q_t p_H i_t} \tag{2.25}$$

$$R_t^b = \frac{(1+r_t^a)a_t}{q_t p_H it}. \tag{2.26}$$

Combining equation (2.22) with (2.26) yields:

$$\frac{a_t}{i_t} = \frac{p_H}{\Delta p} \frac{\kappa_t}{1+r_t^a}. \tag{2.27}$$

Further, combining equation (2.24) with (2.25) yields:

$$\frac{d_t}{i_t} = \frac{q_t p_H R}{1+r_t} - \frac{p_H}{\Delta p} \frac{\kappa_t + b(\kappa_t)}{1+r_t}. \tag{2.28}$$

Equations (2.27) and (2.28) indicate that the greater is the cost of monitoring, $\kappa_t$, the less deposits can be attracted from the worker, as the worker cannot be convinced as easily that the project is properly monitored. The amount of deposits is also decreasing in the severity of the moral hazard, $b(\kappa_t)$. On the other hand, it is increasing in the total expected return of the project, $q_t p_H R$.

Substituting equations (2.27) and (2.28) into the resource constraint (2.16) gives, after some manipulation, the optimal investment as a function of the inverse leverage $g_t$:

$$i_t = \frac{n_t}{g_t}, \tag{2.29}$$

where the inverse leverage $g_t \equiv g(r_t, r_t^a, q_t, \kappa_t)$ is given by:

$$g_t = 1 - \frac{q_t p_H R}{1+r_t} + \frac{p_H}{\Delta p} \frac{b(\kappa_t)}{1+r_t} + \left(1 + \frac{p_H}{\Delta p}\left(\frac{1}{1+r_t} - \frac{1}{1+r_t^a}\right)\right)\kappa_t. \tag{2.30}$$

Here, $\frac{q_t p_H R}{1+r_t} - 1 \equiv \rho_t$ is the net pledgeable income of the project, i.e. maximum net excess return that the entrepreneur can promise to the investors. Equation (2.30) shows that the worse the moral hazard of the entrepreneur, the costlier monitoring, the smaller the net pledgeable income, or the lower

the real value of capital $q_t$ is, the less the entrepreneur can attract external funding, or lever the investment size.

Now, the problem of the entrepreneur is to choose $i_t$ to maximise her expected profit, given her net worth $n_t$ and the inverse leverage $g_t$. As the return is proportional to the investment size, expected profit is maximised when $i_t$ is maximised, or in other words, when the leverage ratio $\frac{1}{g_t}$ is maximised. Given prices, $g_t$ is fully determined by the monitoring intensity $\kappa_t$.

Let $\kappa_t^*$ denote the monitoring intensity that maximises the expected profit of the entrepreneur. Using equations (2.21) and (2.29), and taking as given the prices $q_t$, $r_t$ and $r_t^a$, the entrepreneur's expected profit in terms of $\kappa_t^*$ can be expressed as:

$$q_t p_H R_t^e i_t = \frac{p_H}{\Delta p} \frac{b(\kappa_t^*) n_t}{g(\kappa_t^*)}. \tag{2.31}$$

Thus, the monitoring intensity that maximises the entrepreneur's expected profit is found by solving:

$$\kappa_t^* = \arg\max_{\kappa_t} \frac{b(\kappa_t)}{g(\kappa_t)}. \tag{2.32}$$

In order to solve this problem, I assume the following functional relationship between the monitoring intensity and the size of the private benefit:[10]

$$b(\kappa_t) = \begin{cases} \Gamma \kappa_t^{-\frac{\eta}{1-\eta}} & \text{if } \kappa_t > \underline{\kappa} \\ \bar{b} & \text{if } \kappa_t \leq \underline{\kappa}, \end{cases} \tag{2.33}$$

where $0 < \eta < 1$, $\Gamma > 0$, $\bar{b} > 0$. In other words, there is a lower bound $\underline{\kappa}$ for the efficiency of monitoring under which the maximum private benefit is always feasible. When $\kappa_t > \underline{\kappa}$, the amount of private benefit is a strictly convex function of the monitoring intensity, increasing in $\Gamma$, and decreasing in $\eta$.

This specification of the monitoring technology yields the following interior solution to the problem (2.32):

$$\kappa_t^* = \frac{\eta \rho_t}{1 + \frac{p_H}{\Delta p} \left( \frac{1}{1+r_t} - \frac{1}{1+r_t^a} \right)}, \tag{2.34}$$

---

[10]This form is chosen to capture the assumption that the private benefit is decreasing in the intensity of monitoring, while also yielding tractable analytical results.

which, when substituted into equation (2.30), yields the following equilibrium degree of inverse leverage:

$$g(\kappa_t^*) = \frac{p_H}{\Delta p} \frac{b(\kappa_t^*)}{1 + r_t} - (1 - \eta)\rho_t, \tag{2.35}$$

which in turn determines the equilibrium investment size.

In this model, the endogenous monitoring intensity $\kappa_t^*$ plays a key role in the dynamics of the financial sector. If $\kappa_t^*$ were constant, the private benefit, and thus the incentives of the entrepreneur, would also be time-invariant. As a result, the response of bank lending to any shock that would lead to a reduction in the banker's own capital would be dampened.

In contrast, when $\kappa_t^*$ is endogenous, it reacts to developments in the financial markets. If the banker's own capital deteriorates, it is optimal to reduce the monitoring intensity, and the moral hazard problem is exacerbated. As a consequence, less deposits can be attracted, and less loanable funds are available. Endogenous monitoring therefore amplifies fluctuations in bank lending. This is the key driver behind the financial dynamics of this model, and it is what makes bank capital fundamentally different from entrepreneurial net worth or deposits. The aggregate implications of this mechanism are discussed in more detail in Section 2.6.

Finally, using equations (2.27) and (2.28), the resulting equilibrium leverage of an individual banker is:

$$\gamma_t \equiv \frac{d_t + a_t}{a_t} = 1 + \frac{1 + r_t^a}{1 + r_t} \left( \frac{\Delta p}{p_H} \frac{q_t p_H R}{\kappa_t^*} - \frac{b(\kappa_t^*)}{\kappa_t^*} - 1 \right) > 1. \tag{2.36}$$

Similarly to the findings in Angeloni and Faia (2013) and Christensen et al. (2011), the banker's leverage depends positively on the return on bank capital $(1 + r_t^a)$ and on the uncertainty of the project returns $(\frac{\Delta p}{p_H})$, and negatively on the return on deposits $(1 + r_t)$, on the monitoring cost $(\kappa_t^*)$ and on the private benefit $(b(\kappa_t^*))$.

## 2.3.6 Aggregation

I focus on the symmetric equilibrium where all projects are monitored at the same intensity $\kappa_t^*$ given by equation (2.34), and consequently the capital structure, given by the ratios of own and external funds to total investment $(\frac{n_t}{i_t}, \frac{a_t}{i_t}$

and $\frac{d_t}{i_t}$), is equal across entrepreneurs, bankers and depositors, respectively. The size of the project $i_t$ may, however, vary across entrepreneurs.

The corresponding aggregate ratios are then simply given by:

$$\frac{N_t}{I_t} = \frac{n_t}{i_t}, \quad \frac{A_t}{I_t} = \frac{a_t}{i_t}, \quad \frac{D_t/P_t}{I_t} = \frac{d_t}{i_t}, \tag{2.37}$$

where capital letters denote aggregate amounts.

The equilibrium aggregate investment in the economy is determined by:

$$\frac{N_t}{I_t} = g(\kappa_t^*), \tag{2.38}$$

where $g(\kappa_t^*)$ is given by equation (2.35).

Using the relation (2.27), the equilibrium rate of return to bank capital is given by:

$$1 + r_t^{a*} = \frac{1 + \eta \rho_t \frac{I_t}{A_t}}{(1 + r_t)^{-1} + \frac{\Delta p}{p_H}}. \tag{2.39}$$

In other words, because of the binding participation and incentive constraints, all bank capital yields its minimum required return in equilibrium. Because monitoring is costly, it must be that $1 + r_t^{a*} > 1 + r_t$ in equilibrium.[11]

Next, the laws of motion of the three types of capital are described by the following equations. In equilibrium, the capital stock in the economy evolves according to:

$$K_{t+1} = (1 - \delta)K_t + p_H R I_t, \tag{2.40}$$

where $p_H R I_t$ is the expected amount of new capital produced by the entrepreneurs.

Entrepreneurial and bank net worth are defined to evolve according to:

$$N_{t+1} = \lambda^e (1 + r_t^e) \frac{r_{t+1}^K + (1 - \delta)q_{t+1}}{q_t} N_t \tag{2.41}$$

---

[11]Investing the funds $a_t$ in the open market would yield the market rate $1 + r_t$. Equating the banker's expected profit from the project with the outside return for the funds yields $q_t p_H R_t^b i_t - \kappa_t i_t = (1 + r_t)a_t$. Substituting in the equilibrium conditions (2.22) and (2.27) yields, after some manipulation, the condition $1 + r_t^a = \frac{p_H}{p_L}(1 + r_t)$. As $\frac{p_H}{p_L} > 1$ by assumption, this condition is always satisfied, and $1 + r_t^a > 1 + r_t$ in equilibrium. C.f. the proof in Holmström and Tirole (1997).

$$A_{t+1} = Z_{t+1}^b \lambda^b (1 + r_t^a) \frac{r_{t+1}^K + (1 - \delta)q_{t+1}}{q_t} A_t, \tag{2.42}$$

where $r_{t+1}^K + (1 - \delta)q_{t+1}$ is the marginal value of a unit of capital in period $t + 1$, which is composed of two parts: the rental income at the beginning of the period $r_{t+1}^K$, and the value of undepreciated capital $(1 - \delta)q_{t+1}$ remaining at the end of the period. $\lambda^e$ and $\lambda^b$ are the fractions of entrepreneurs and bankers, respectively, surviving from period $t$ to $t + 1$. The return to entrepreneurial capital is simply defined as $1 + r_t^e \equiv \frac{q_t p_H R_t^e I_t}{N_t}$, which is the ratio of expected profit to net worth.

To introduce a shock arising in the financial market into the model, I let the accumulation of bank capital be affected by an exogenous aggregate shock, $Z_t^b$. I assume $Z_t^b$ is an AR(1) process with a normally distributed i.i.d. innovation term. A negative shock to $Z_t^b$ corresponds to an exogenous and unanticipated decrease in the accumulation of bank capital, or in other words, a sudden erosion of bank capital $A_t$ available at the start of a period $t$, common to the whole banking sector. The shock affects the banks' ability to extend funding to entrepreneurs. It can be interpreted as a shock to the quality of bank capital, similar to the shock analysed for example in Gertler and Karadi (2011).

Finally, by using the demand schedule (2.7) and the production technology (2.9), the aggregate production is:

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha}. \tag{2.43}$$

The aggregate consistency constraint of the economy is:

$$Y_t = (C_t + I_t)s_t, \tag{2.44}$$

where $C_t$ denotes aggregate private consumption and $I_t$ aggregate investment.[12] Finally, $s_t \equiv (1 - \omega) \left( \frac{P_t^*}{P_t} \right)^{-\varepsilon} + \omega \pi_t^\varepsilon s_{t+1} \geq 1$ is the resource cost caused

---

[12]I assume that the monitoring of investment projects does not consume real resources. If it did, the resource constraint would be $Y_t = (C_t + (1 + \kappa_t^*)I_t)s_t$. This assumption is not quantitatively restrictive, as the total monitoring cost $\kappa_t^* I_t$ is very small in equilibrium: in the baseline calibration of the model, $\kappa_t^* I_t$ is approximately 0.027% of total resources. Omitting it, however, greatly facilitates the solution of the analytical steady state of the model, as the steady state of the real part of the economy then only depends on the financial sector indirectly through the asset price $q_t$. This assumption also renders the calibration of the model simpler and allows to nest the standard New Keynesian models without a financial friction within the model; see the discussion in Section 2.4 for details.

| | |
|---|---|
| *The production phase* | |

– The aggregate shocks $Z_t$, $Z_t^c$ and $Z_t^b$ are realised.
– Households rent capital $K_t$ and supply labour $L_t$ to intermediate firms.
– Intermediate and final production take place.

*The investment and consumption phase*

– Entrepreneurs acquire funding for new investment projects. The financing contract is agreed upon, given $N_t$, $A_t$ and $D_t/P_t$.
– Monitoring and realisation of the investment projects take place.
– The outcome of the projects are observed. Returns to investment are distributed according to the contract.
– Entrepreneurs and bankers accumulate net worth $N_{t+1}$ and $A_{t+1}$. Exiting bankers and entrepreneurs transfer their accumulated wealth to their household.
– Consumption and saving decisions of the household $(C_t, D_{t+1}, K_{t+1})$ take place.

Table 2.1: Timing of events

by the price dispersion.

To close the model, a monetary policy for setting the nominal interest rate $r_t$ needs to be specified. I start by analysing the constrained optimal policies under a social planner's solution in Section 2.5. Then, I look at simple policy rules that can be used to approximate the constrained optimum, detailed in Section 2.6.

### 2.3.7 Timing of events

The timing of the events is described in Table 2.1. Each time period is divided into two phases, detailed in the upper and lower panels of the table. All aggregate uncertainty is resolved at the beginning of the period. In the first phase, production decisions take place. In the second phase, then, consumption and investment decisions — including the financing contract — are made.

### 2.3.8 Equilibrium

The competitive equilibrium of the economy is a time path:

$$\left\{ C_t, L_t, K_{t+1}, A_{t+1}, N_{t+1}, D_{t+1}, q_t, r_t^K, r_t, r_t^{a*}, r_t^e, w_t, \psi_t, P_t, P_t^*, \right.$$
$$\left. R_t^b, R_t^e, R_t^w, \kappa_t^* \right\}_{t=0}^{\infty} \tag{2.45}$$

that satisfies the households' problem, the final and intermediate firms' problems, the optimal financing contract, and the aggregate consistency condition. The equilibrium dynamics as well as the deterministic steady state equilibrium of the model economy are summarised in Appendix 2.A.

## 2.4 Calibration and business cycle properties

This section first presents the calibration of the model, and then discusses its business cycle properties by comparing standard business cycle statistics computed from empirical data to key model moments.

### 2.4.1 Calibration

The model nests the standard real business cycle (RBC) and New Keynesian (NK) models when steady state subsidies for employment and investment, which correct for the steady state distortions caused by the nominal and financial frictions, respectively, are assumed to be in place. The steady state of the model is then efficient.[13] By shutting down the financial friction, the standard NK model is obtained, and by shutting down both the financial and nominal frictions, the model corresponds to the standard RBC model.

The calibration of the model largely follows the calibration strategy discussed in Haavio et al. (2016). Assuming that a steady state investment subsidy is in place, the steady state of the New Keynesian macro block is not affected by the parameters of the financial sector. Thus the macro block and the financial block of the model can be calibrated independently, and when the financial friction is shut down, the model nests the standard New Keynesian model.

The macro block is calibrated in a standard fashion to the New Keynesian literature to match a quarterly frequency in U.S. data. The parameter values are summarised in the upper panel of Table 2.2.

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[13]The investment subsidy is derived in Appendix 2.A.2.

| Panel 1: New Keynesian block | | |
|---|---|---|
| Discount factor | $\beta$ | 0.9951 |
| Risk aversion | $\sigma$ | 2 |
| Capital depreciation rate | $\delta$ | 0.025 |
| Elasticity of substitution (mark-up: 10 %) | $\epsilon$ | 11 |
| Capital share | $\alpha$ | 0.33 |
| Frisch elasticity of labour supply | $\theta$ | 0.5 |
| Disutility of labour supply | $\xi$ | 2 |
| Calvo parameter | $\omega$ | 0.8 |
| Persistence of productivity shock | $\rho$ | 0.95 |
| Persistence of preference shock | $\rho_c$ | 0.7 |
| Std. dev. of productivity shock | $\sigma_\epsilon$ | 0.006 |
| Std. dev. of preference shock | $\sigma_c$ | 0.005 |
| Panel 2: Financial block | | |
| Elasticity of monitoring | $\eta$ | 0.2992 |
| Monitoring intensity | $\Gamma$ | 0.0017 |
| Survival rate of entrepreneurs | $\lambda^e$ | 0.9842 |
| Survival rate of bankers | $\lambda^b$ | 0.9507 |
| Success probability of good project | $p_H$ | 0.95 |
| Gross return of investment project: $R = \frac{1}{p_H}$ | $R$ | 1.0526 |
| Probability differential $\Delta p = p_H - p_L$ | $\Delta p$ | 0.0454 |
| Persistence of bank capital depreciation shock | $\rho_b$ | 0 |
| Std. dev. of bank capital depreciation shock | $\sigma_b$ | 0.006 |

Table 2.2: Calibration of the model

The financial block is calibrated to match some steady state characteristics of the model. The entrepreneur and banker survival rates, $\lambda^e$ and $\lambda^b$ respectively, are calibrated to match a steady state excess return on entrepreneurial capital of 4.5% and an excess return on (core) private bank capital of 20% per annum, compared to the short-term market interest rate. These values are consistent with the estimates in Albertazzi and Gambacorta (2009).

The calibration of the monitoring parameters $\eta$ and $\Gamma$ pin down the monitoring cost in the steady state, and consequently the steady state leverage of the entrepreneur, because leverage is fully determined by the monitoring

intensity. On the other hand, bank leverage also depends on the monitoring intensity, as it determines its ability to attract deposits. Hence, these two parameters are the key parameters governing the financial sector dynamics.

The exact cost of monitoring activities in banks is hard to pin down empirically. Banks' overhead costs as a fraction of total assets in the U.S. are estimated to be around 3% by the World Bank (2013). Albertazzi and Gambacorta (2009) document operating expenses between 1.2% to 3.6% of total bank assets in developed countries. However, overhead costs and operating expenses also include costs not directly related to core banking activities. Philippon (2015) estimates that the unit cost of financial intermediation has been stable at 1.5% to 2% of intermediated assets in the U.S. over the past decades. However, besides monitoring costs, these figures also include costs for asset management and liquidity services. To take this into account, the calibration of $\eta$ and $\Gamma$ matches a per annum monitoring cost of 1.2% of total bank assets in steady state, which is slightly below the lower bound of the estimates reported by Philippon (2015).

The leverage of non-financial US firms is estimated to be around 2.3–2.5 by Kalemli-Ozcan et al. (2012). They also find that leverage ratios of financial firms are very heterogeneous in the U.S. and depend on the type of the bank. Large investment banks have leverage ratios in the order of 20, while commercial banks typically have leverage ratios ranging from 10 to 12. The elasticity of monitoring and monitoring intensity are calibrated in such a way as to produce a leverage ratio of around 1.5 for non-financial firms — entrepreneurs, in this model —, and a leverage ratio of 16.5 for banks.

The success probability of the good project and the gross return from the project, $p_H$ and $R$, are normalised such that $p_H R = 1$, which makes the evolution of the aggregate capital accumulation comparable to the standard New Keynesian case. I set $p_H = 0.95$, which implies a net return $R - 1$ on the investment project equal to approximately 5%.

Finally, the financial shock is calibrated to be purely transitory at $\phi_b = 0$. The standard deviance of the shock is calibrated to be the same than that of the productivity shock. The parameters of such a financial shock is hard to pin down empirically, but the calibration is intended to represent the financial shock as a strong but brief event on the financial markets. Given that it takes time for banks hit by the shock to rebuild their capital stock, the purely transitory shock has persistent effects.

### 2.4.2 Business cycle properties

Table 2.3 displays business cycle statistics for both real and financial variables in the data and two versions of the model: one where the financial friction is active, but prices are flexible ("RBC"); and one with both the financial and the nominal friction present ("NK").

The data for the real variables are from the U.S. Bureau of Economic Analysis (BEA) and the U.S. Bureau of Labor Statistics (BLS). The data for the financial and banking variables are collected from the Board of Governors of the Federal Reserve System and the Federal Financial Institutions Examination Council (FFIEC) data publications. All data series are in logs, except the real interest rate and the real return on bank capital. The model variable corresponding to each data series is also shown in the first column of the table.

In the data, output $Y$ is the real GDP; consumption $C$ are real personal consumption expenditures; investment $I$ is real non-residential fixed investment; labour supply $L$ are total non-farm hours; and the real wage $w$ is compensation per hour in the non-farm sector.[14] The variables are quarterly and deflated with the GDP deflator. The real interest rate $rr$ is the one-year Treasury bill rate less the one-year-ahead expected inflation, expressed in quarterly terms. Expected inflation is measured as the median expected price change over twelve months in the University of Michigan Survey of Consumers.

The selection of financial variables poses some challenges due to data availability and an imperfect mapping between the available data and the corresponding variables in the theoretical model. In the data, bank capital and bank assets are measured as the total net worth and total assets of all U.S. banks, and entrepreneurial net worth and assets as the total non-financial corporations' net worth and assets in the U.S. Bank leverage $\gamma$ and entrepreneurial leverage $1/g$ are computed as the ratio of total assets to total net worth of all U.S. banks and non-financial corporations, respectively. The return $r^a$ is measured as the return on average equity of all U.S. banks, expressed in quarterly terms. Loans are the total loans issued by all U.S. banks. All financial variables are deflated with the GDP deflator, or in the case of $r^a$, with the one-year-ahead expected inflation.

In the real sector, the model exhibits behaviour similar to other typical business cycle models. This can be seen from the moments shown in the upper panel of Table 2.3. The model broadly matches the relative standard deviations of consumption, investment, the real wage and the real interest rate, as well as the persistence of most series in the data. The degree of

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[14]The choice of data for the real variables follows Smets and Wouters (2007).

|  | Data | | | | RBC model | | | NK model | | |
|---|---|---|---|---|---|---|---|---|---|---|
|  | $\sigma$ (%) | Rel. $\sigma$ | AR(1) | Corr. | Rel. $\sigma$ | AR(1) | Corr. | Rel. $\sigma$ | AR(1) | Corr. |
| **Real variables** | | | | | | | | | | |
| Output $Y$ | 1.62 | 1 | 0.85 | 1.00 | 1.00 | 0.78 | 1.00 | 1.00 | 0.80 | 1.00 |
| Consumption $C$ | 1.25 | 0.77 | 0.82 | 0.67 | 0.43 | 0.58 | 0.75 | 0.42 | 0.58 | 0.76 |
| Investment $I$ | 4.63 | 2.86 | 0.88 | 0.97 | 2.87 | 0.82 | 0.97 | 2.89 | 0.85 | 0.97 |
| Labour supply $L$ | 1.53 | 0.94 | 0.92 | 0.95 | 0.31 | 0.83 | 0.86 | 0.37 | 0.80 | 0.81 |
| Labour productivity $Y/L$ | 1.17 | 0.72 | 0.73 | 0.95 | 0.76 | 0.67 | 0.98 | 0.74 | 0.65 | 0.96 |
| Real wage $w$ | 0.95 | 0.59 | 0.68 | 0.98 | 0.76 | 0.67 | 0.98 | 0.75 | 0.71 | 0.98 |
| Real interest rate $rr$ | 0.26 | 0.16 | 0.64 | 0.96 | 0.13 | 0.22 | -0.31 | 0.12 | 0.28 | -0.28 |
| **Financial variables** | | | | | | | | | | |
| Bank capital $A$ | 1.94 | 1.20 | 0.73 | | 8.96 | 0.83 | 0.96 | 8.89 | 0.85 | 0.96 |
| Firm net worth $N$ | 3.03 | 1.87 | 0.88 | | 0.33 | 0.92 | 0.74 | 0.38 | 0.93 | 0.80 |
| Bank leverage $\gamma$ | 2.39 | 1.48 | 0.74 | | 5.17 | 0.19 | -0.42 | 4.92 | 0.26 | -0.41 |
| Firm leverage $1/G$ | 0.40 | 0.25 | 0.85 | | 2.73 | 0.81 | 0.96 | 2.68 | 0.82 | 0.96 |
| Return on bank capital $r^a$ | 0.62 | 0.38 | 0.58 | | 5.48 | 0.25 | 0.34 | 5.15 | 0.31 | 0.33 |
| Bank assets $A+D$ | 1.68 | 1.04 | 0.73 | | 8.52 | 0.81 | 0.75 | 8.44 | 0.83 | 0.77 |
| NFC assets $I$ | 3.07 | 1.89 | 0.90 | | 2.87 | 0.82 | 0.97 | 2.89 | 0.85 | 0.97 |
| Loans $I - N$ | 2.72 | 1.68 | 0.92 | | 8.74 | 0.81 | 0.97 | 8.65 | 0.83 | 0.97 |
| Loans-to-output $\frac{I-N}{Y}$ | 2.30 | 1.42 | 0.89 | | 7.79 | 0.81 | 0.96 | 7.69 | 0.83 | 0.96 |

Table 2.3: Business cycle statistics

*Note:* $\sigma$: standard deviation, in %. Rel. $\sigma$: standard deviation of a variable X relative to that of output Y, $\frac{\sigma_X}{\sigma_Y}$. AR(1): first autocorrelation coefficient. Corr.: contemporaneous correlation with output. All data series are quarterly, expressed in real terms, and detrended with the Hodrick-Prescott (H-P) filter with smoothing parameter $\lambda = 1600$. The models are log-linearised around their steady state. A theoretical H-P filter with $\lambda = 1600$ is applied to the model solution, and theoretical moments are then computed from the filtered solution. "RBC model": model with the financial friction, but no nominal friction. "NK model": model with both the financial and the nominal friction, with a standard Taylor rule operational,corresponding to policy *(i)* in Section 2.6.2.

procyclicality of consumption and, to a somewhat lesser extent, of investment and labour supply are also matched. However, in the model, unlike in the data, the real wage is strongly procyclical. This is due to fully flexible wage setting in the model.

The cyclical properties of the financial sector is displayed in the lower panel of Table 2.3. First, entrepreneurial net worth is persistent and procyclical both in the data and in the model. However, entrepreneur leverage is more procyclical and volatile in the model than in the data. Second, bank assets and capital are persistent and procyclical, both in the model and in the data. Aggregate bank leverage is more volatile than output both in the data and the model, but more so in the model. However, it is countercyclical in the model, but virtually acyclical in the data.[15] Third, the return on bank capital is more volatile in the model than in the data. However, its persistence and correlation with output are approximately matched. Finally, loans and the loans-to-output ratio are procyclical and more volatile than output both in the data and the model. The persistence of these series is well matched.

The main caveats of the model are the absence of equity finance and the accumulation of bank and entrepreneurial net worth only through retained earnings. This makes the balance sheet items of both banks and entrepreneurs strongly procyclical and autocorrelated. It also renders bank leverage countercyclical, as bank capital is reduced in recessions. At the same time, lending and the loans-to-output ratio are procyclical, because bank capital is relatively less scarce in expansionary periods. As a consequence, banks are able to attract more deposits and expand lending. This feature of the model matches the observed data.

Second, short-term debt is the only source of external funding for the entrepreneurs. This also implies that there is no portfolio problem in the financing of investment, and consequently no choice over risk. In contrast, the data include a much richer variety of financial activity. Finally, the model includes only three structural shocks. Richer sources of uncertainty could help to match more closely the volatilities of the model variables to the ones observed in the data.

With these caveats in mind, the main contribution of this paper is in the theoretical analysis of optimal policy in the presence of a particular kind of financial friction. The calibration of the model aims at matching the microstructure of the specific financing problem to data as closely as possible; the

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[15]The cyclicality of bank leverage is sensitive to the choice of data and the exact definition of leverage; e.g. Meh and Moran (2010) identify countercyclical bank leverage in their dataset.

model succeeds in replicating steady state leverage ratios, returns on equity, and various spreads, as described in the previous section.

## 2.5 The constrained optimum

This section discusses the implications of the agency problem in financial intermediation, and analyses the constrained optimal solution under both flexible and sticky prices, when the moral hazard problem is nonetheless present. Optimal policy responses of the Ramsey planner are numerically computed and presented through impulse response analysis. The efficient first-best optimum, to which the constrained optimal solutions are compared, is the equilibrium of the standard flexible price real business cycle (RBC) model without any friction in financial intermediation. Subsequently, I compare the results to dynamics under non-optimised simple policy rules.

### 2.5.1 Implications of the financial friction

I first focus on a flexible price economy where the Calvo parameter is set to $\omega = 0$. If $b(\kappa_t) = 0$, i.e., there are no private benefits available to the entrepreneur, and consequently, no need for monitoring ($\kappa_t = 0$), the incentive constraints (2.20) and (2.21) trivially hold; there is no incentive problem.

Then, the entrepreneur and the banker are indifferent between undertaking the good project and not when $R_t^e = R_t^b = 0$. The depositor-worker receives the whole gross return, $R_t^w = R$. In this case, the financial intermediation becomes invisible in the sense that it is as if the worker himself would directly undertake the project, i.e., the representative household becomes a capital producer. This is observationally equal to the standard frictionless RBC model, where the household's savings are directly channelled into productive investment. As a consequence, aggregate investment equals household savings in equilibrium: $I_t = D_t/P_t$. The RBC model is thus nested within the model presented in this essay.

Frictions in the financial market cause aggregate investment to be at a suboptimal level: too low or high compared to the frictionless case. When these frictions are present, aggregate investment $I_t$ depends on the total amount of entrepreneurial and bank capital, $N_t$ and $A_t$, and – through their effect on leverage – on the size of private benefits $b(\kappa_t)$ and the monitoring intensity $\kappa_t$. The more severe the agency problem, the less funds can be channelled

into investment projects. The inefficiency of credit intermediation is exacerbated by the monitoring cost: because of it, less resources are available for productive investment. Both the entrepreneurs and the bankers are thus capital-constrained. Moreover, Haavio et al. (2016) and Silvo (2018) show that, as a consequence of the costly monitoring, bank capital is *scarce* relative to entrepreneurial capital. Increasing the relative availability of bank capital would increase aggregate investment.

### 2.5.2   Constrained optimum in a flexible price economy

This and the following sections present the model dynamics under Ramsey optimal policy plans. The Ramsey policy problem consists of maximising the representative household's lifetime welfare, given by equation (2.1), conditional on the non-linear equilibrium conditions of the private sector. I then approximate the optimality conditions of the Ramsey problem by a first-order Taylor approximation in logs around the deterministic steady state of the model. The full steady state of the model is given in Appendix 2.A.2. The setup and the numerical solution of the Ramsey policy problem are described in technical detail in Appendix 2.B.

I assume that the planner cannot remove the moral hazard problem — i.e cannot make $b(\kappa_t) = 0$ feasible for any value of $\kappa_t$ —, but can set aggregate bank leverage directly to the value that maximises household welfare.[16] The planner's solution is defined as being efficient when it replicates the flexible price perfect competition allocation, which is the first-best outcome.

In the flexible price economy where the financial friction is present, the fluctuation of the real price of capital ($q_t$) away from unity creates a wedge between the first-best and the actual outcome. The value of Tobin's $q$ fixed at unity is a key feature of the frictionless RBC model, which follows from the frictionless adjustment of capital. In order to replicate the first-best, the Ramsey planner thus needs to offset this wedge.

Figure 2.1 shows the response of the economy to a one percent negative productivity shock. The figure illustrates that the constrained optimal solution of the social planner is efficient: the allocation replicates the first-best allocation up to a second-order approximation, denoted by the solid black line in the figure. The solid grey line denotes the response in the economy with financial

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[16]This policy can be thought of as a set of transfers in each period between the entrepreneurs, the bankers and the depositors that reallocates wealth so as to alter the relative scarcity of bank and entrepreneur capital and thus to achieve the optimal level of investment.

Figure 2.1: Effects of a one percent negative total factor productivity shock in the flexible price model

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state. "Frictionless RBC": model with fully flexible prices and no financial friction. "Constrained-optimal RBC": model with fully flexible prices but with the financial friction; the planner sets aggregate bank leverage. "Laissez-faire": model with fully flexible prices but with the financial friction; no policy intervention.

frictions when a social planner intervenes by implementing the Ramsey policy, and the dashed line indicates the response of the economy with financial frictions and no policy intervention. The output gap $X_t$ is defined as the gap between actual output $Y_t$ and the efficient output $Y_t^e$: $X_t = \frac{Y_t}{Y_t^e}$.[17]

The shock increases marginal costs of firms. Capital demand by intermediate firms then decreases, but because of the financial friction, less than in the frictionless RBC economy. Entrepreneurs attempt to continue undertaking investment projects to keep accumulating wealth. Without any policy inter-

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[17]The efficient level of output is a benchmark computed as the output achievable with the resources of the economy in the absence of monopolistic competition, the pricing friction, and the financial friction.
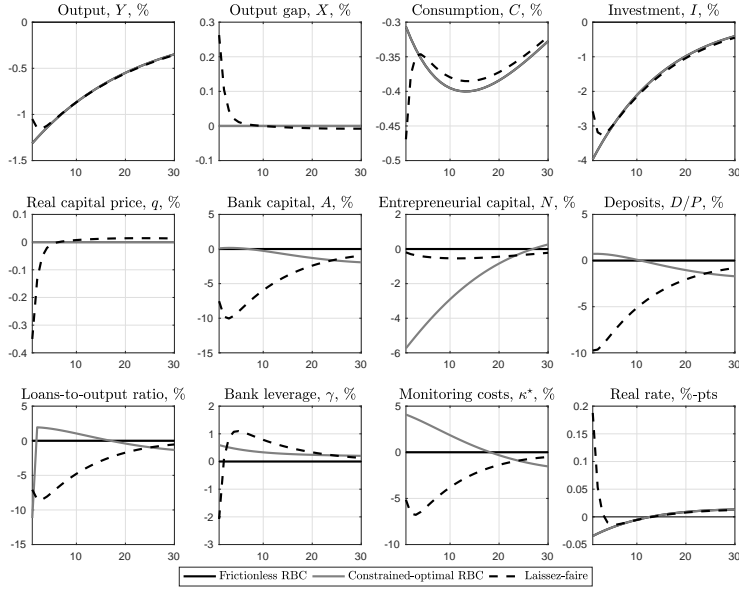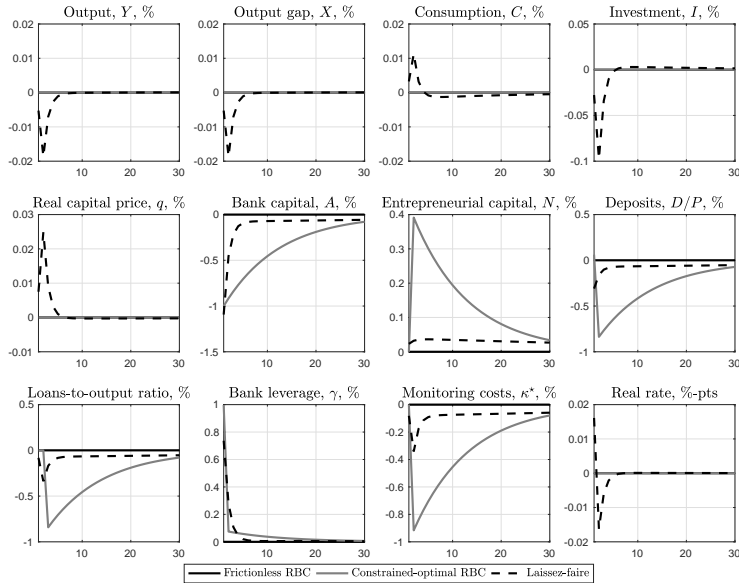
Figure 2.2: Effects of a one percent negative bank capital shock in the flexible price model

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state. "Frictionless RBC": model with fully flexible prices and no financial friction. "Constrained-optimal RBC": model with fully flexible prices, but with the financial friction; the planner sets aggregate bank leverage. "Laissez-faire": model with fully flexible prices, but with the financial friction; no policy intervention.

vention, this results in a positive output gap as investment does not adjust enough, and in a fall of the real capital price $q_t$ as a result of the weakened demand for capital.

Instead, with policy intervention, the social planner sets aggregate bank leverage to the level that maximises household welfare, given the friction in the financial sector. This corresponds to a re-allocation of resources between the three parties of the financial contract in such a way as to keep the real price of capital, $q_t$, fixed at unity. First, the planner transfers resources from the entrepreneur to the banker. This allows the banker to increase the monitoring intensity and attract more deposits. The entrepreneur, however, has less net worth and can undertake only smaller projects. The demand for loanable

funds thus falls, which then decreases the supply of new capital in the economy, driving up the capital price $q_t$. The stabilisation of Tobin's $q$ allows to replicate the first-best outcome in the real sector, and in particular, the efficient level of investment.

An exactly similar logic applies to stabilising the economy after a negative bank capital shock, shown in Figure 2.2. The first-best outcome can again be replicated. In this case, the destruction of bank capital interferes with the bankers' ability to extend loans for new projects, such that the supply of new capital decreases and the capital price $q_t$ increases if there is no policy intervention, as shown by the dashed lines. The social planner transfers resources from the depositor and the banker to the entrepreneur, who can then invest in more capital-producing projects with less external funding from the market. This pushes down the capital price, as the demand for loans is reduced.

Appendix 2.D reports the corresponding results of a shock to the stochastic discount factor. Here, the same conclusion holds: the planner can replicate the efficient allocation by appropriately controlling aggregate bank leverage.

### 2.5.3   Constrained optimum in a sticky price economy

Next, I re-introduce the nominal rigidity into the model. Now, besides the financial friction, the standard New Keynesian frictions – price stickiness and monopolistic competition in intermediate production – also affect the economy.

With all of these frictions at work, the planner has to offset fluctuations caused not only by the agency problem, but also those caused by the nominal rigidity to achieve efficiency of the constrained optimum. I look at two different policy set-ups: first, a situation in which the planner only controls the rate of inflation; and second, a situation in which the planner can jointly set both the inflation rate and the aggregate bank leverage. Results for the preference shock are relegated to Appendix 2.D.

The dashed lines in Figure 2.3 show the response of the economy to a negative productivity shock when the social planner optimally sets the inflation rate. With only one policy variable, the planner cannot fully offset the shock. Following the shock, intermediate good firms' marginal costs increase, which leads to a positive output gap, and now also creates inflationary pressure. At the same time, capital demand decreases. Consequently, $q_t$ drops, as capital supply does not adjust enough due to the financial friction. The planner is not able to stabilise both the output gap and $q_t$, as the two move in opposite directions. On the one hand, offsetting the price distortions would warrant
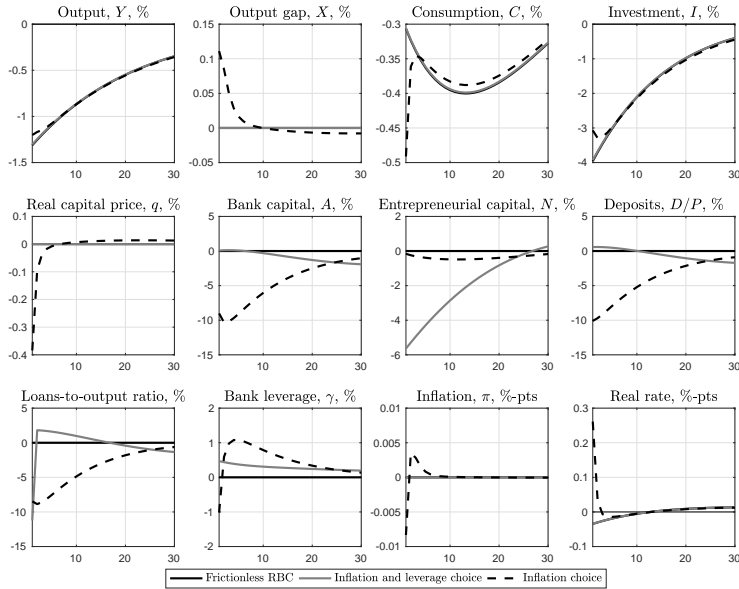
Figure 2.3: Effects of a one percent negative total factor productivity shock in the sticky price model

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state. "Inflation choice": model with sticky prices and the financial friction; social planner sets the inflation rate. "Inflation and leverage choice": model with sticky prices and the financial friction; social planner sets the inflation rate and aggregate bank leverage. "Frictionless RBC": the frictionless flexible price (first-best) benchmark.

setting inflation to zero. On the other hand, in order to offset the financial friction, the planner would need to generate an increase in inflation in order to push up the capital price. As a consequence, the planner finds it optimal to partly accommodate the shock.

It is useful to contrast this result with the standard New Keynesian model, where no financial friction exists: faced with technology shocks, a Ramsey planner can replicate the first-best by fully stabilising inflation, assuming that there is a steady-state employment subsidy in place that offsets the steady state mark-up of the intermediate producers.

In contrast, the solid grey lines in Figures 2.3 show the constrained op-
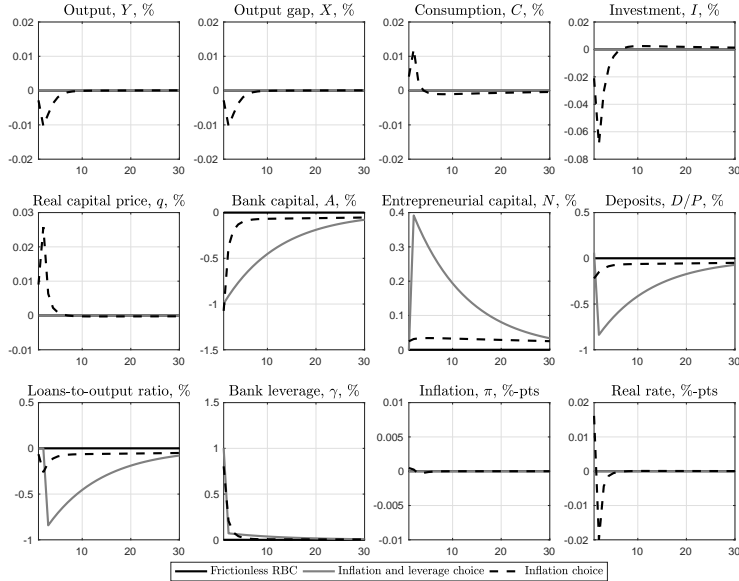
Figure 2.4: Effects of a one percent negative bank capital shock in the New Keynesian model

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state. "Inflation choice": model with sticky prices and the financial friction; social planner sets the inflation rate. "Inflation and leverage choice": model with sticky prices and the financial friction; social planner sets the inflation rate and aggregate bank leverage. "Frictionless RBC": the frictionless flexible price (first-best) benchmark.

timal response when the planner controls two policy variables: the inflation rate and the aggregate bank leverage. Now the first best allocation, shown in solid black lines, can be replicated, and the constrained optimum is efficient. One policy variable can be used to offset the distortion caused by price stickiness by stabilising inflation, and the other to offset the distortion in financial intermediation by stabilising Tobin's $q$, so that the policy trade-off is resolved.

The same applies when the economy is hit by a bank capital shock, as depicted in Figure 2.4. With only one policy variable, the planner cannot fully offset the shock. In both cases, Tobin's $q$ and the output gap move in opposite directions, causing a policy trade-off. This trade-off can only be resolved by

adding a second policy variable that can deal with the wedge in real capital price.

In particular, after a bank capital shock, when the planner only controls the inflation rate, the deterioration of bank capital causes a drop in investment and leads to a negative output gap. At the same time, the decrease in capital stock leads to an increase in the real price of capital and causes households to substitute savings for current consumption, which reinforces the drop in investment. In contrast, when the planner can also control bank leverage, the planner can reallocate resources from the depositors and the banks to the entrepreneurs to stabilise Tobin's $q$, as the ability of banks to finance investment is reduced.

## 2.6 Simple policy rules

The Ramsey policy plans described above represent a constrained optimal solution to setting the monetary and macroprudential policies. The Ramsey policy is, however, a rather abstract policy plan that serves as a benchmark for simpler policies and as a tool to help understand how the frictions in the model economy should be dealt with. In particular, the kind of transfers needed in each period to replicate the first-best allocation are likely very hard to implement in a more realistic setting.

The optimal policy analysis suggests that the policymaker needs two separate policy instruments in order to stabilise the economic fluctuations: one to deal with inflation, and one to deal with the financial friction. A need for separate macroprudential regulation then naturally arises in this context. In this section, I analyse the dynamics of the model economy under various simple policy rules.

### 2.6.1 Policy mandates

As seen in the previous section, the stabilisation of the economy requires in general two distinct policy tools to deal with the two separate frictions.The Ramsey policies assume that the policymaker acts under commitment, observes all variables in the economy, and can set the instrument values to maximise the household's lifetime welfare. The planner, by jointly setting both inflation and bank leverage, can replicate the first best. The responses to both inflation and financial imbalances are perfectly coordinated in the sense that they are jointly optimal and set to maximise the same objective, the household

|       | Policies | Instruments |
|-------|----------|-------------|
| *(i)*   | MP       | Taylor rule |
| *(ii)*  | MP + MR  | Augmented Taylor rule |
| *(iii)* | MP + MR  | Taylor rule + capital ratio regulation |

Table 2.4: Policy regimes under simple rules

*Note:* MP: monetary policy; MR: macroprudential policy. Instruments: policy instruments at use.

welfare. However, in the absence of such a Ramsey planner, it is not immediately clear what the policy targets should be and how the policy mandates should be divided. In a more realistic model, the policymaker uses simple policy rules that react to a few key variables.

The simple policy rule set-ups correspond to three distinct policy regimes: *(i)* a regime where only conventional monetary policy that aims at price stability is conducted; *(ii)* a regime where the monetary authority deals with both price stability and financial stability; and *(iii)* a regime where there are two independent policymakers, one for the monetary policy and one for the macroprudential policy, each with their own policy targets. The policy regimes are summarised in Table 2.4. The first column specifies which policies are at use: monetary policy, or both monetary and macroprudential policies. The second column specifies the policy instruments.

The first regime, on the first row of Table 2.4, consists of a conventional monetary policy rule, which reacts to inflation and output gap. Next, on the second row of Table 2.4, the second regime consists of an augmented monetary policy rule. In addition to inflation and output gap, the monetary authority also reacts to the real price of capital, or Tobin's $q$. This policy regime corresponds to one where the central bank tries to explicitly deal with financial imbalances as well as price stability, but only has one instrument at use. It is a dual mandate, and can be thought of as an approximate counterpart to the Ramsey policy with inflation choice only.

Finally, the third regime is one where the tasks of price and financial stability are divided to two different authorities with different policy targets. Monetary policy follows a standard Taylor rule. In addition, there is a macroprudential policy that aims at stabilising the credit cycle, as measured by the deviation of the loans-to-output ratio from its steady state value, or the "credit gap". This is the primary credit cycle indicator identified in the Basel III framework. The macroprudential policy tool is a countercyclical regulatory

| Regime | Taylor rule $[\phi_\pi \quad \phi_x \quad \phi_q]$ | Capital regulation $\phi_\gamma$ |
|--------|---------------------------------------------------|-----------------------------------|
| *(i)* | $[1.5 \quad 0.5 \quad 0]$ | – |
| *(ii)* | $[1.5 \quad 0.5 \quad 1]$ | – |
| *(iii)* | $[1.5 \quad 0.5 \quad 0]$ | 5 |

Table 2.5: Calibration of policy parameters

*Note:* Regimes: *(i)* standard Taylor rule; *(ii)* augmented Taylor rule; *(iii)* standard Taylor rule and capital ratio regulation. Policy coefficients: $\phi_\pi$: weight on inflation; $\phi_x$: weight on output gap; $\phi_q$: weight on real price of capital; $\phi_\gamma$ weight on banks' capital ratio.

requirement on bank capital as in Basel III.

## 2.6.2 Monetary and macroprudential policy rules

### 2.6.2.1 Monetary policy

First, the central bank sets the nominal short-term interest rate $r_t$ using a Taylor rule of the form:

$$1 + r_t = \frac{1}{\beta}\pi_t^{\phi_\pi} X_t^{\phi_x} q_t^{\phi_q}, \tag{2.46}$$

where $\pi_t$ is the period-to-period gross inflation rate, $X_t = \frac{Y_t}{Y_t^e}$ is the output gap, and $q_t$ is the real price of capital. The calibration of the policy parameters is given in Table 2.5. The "standard'" Taylor rule refers to a rule which only reacts to inflation and output gap and sets $\phi_q$ to zero. The "augmented" Taylor rule refers to a rule which sets $\phi_q$ away from zero. The weights on inflation and output gap are set to 1.5 and 0.5, respectively, which are standard values in the New Keynesian literature.

### 2.6.2.2 Capital regulation

In policy regime *(iii)*, I assume that a financial regulator sets a minimum requirement for the bank capital in order to stabilise the loans-to-output ratio.

In particular, now, the financing contract (2.16)–(2.21) in Section 2.3.5.1 is augmented with the regulatory requirement:

$$\gamma_t a_t \geq a_t + d_t, \tag{2.47}$$

where $\gamma_t$ is the minimum capital ratio requirement, which sets a maximum allowed leverage ratio for the banker. In this case, the banker is not able to optimally choose the monitoring intensity $\kappa_t$ in an unconstrained way. Instead, he must set it to comply with the regulatory requirement $\gamma_t$, which he takes as given. This also creates a constraint on the entrepreneur's leverage. I assume that equation 2.47 binds in steady state and in the vicinity of the steady state.

Analogously to the Taylor rule, I assume that the countercyclical capital ratio regulation follows a policy rule of the form:

$$\gamma_t = \gamma \left(\Xi_t\right)^{-\phi_\gamma},\tag{2.48}$$

where $\Xi_t \equiv \frac{(I_t - N_t)/Y_t}{(I - N)/Y}$ is the deviation of the loans-to-output ratio from its steady state value. This ratio is the "credit gap". $\gamma$ is the steady state value of the capital ratio requirement. I set it such that it coincides with the unconstrained bank leverage in steady state, given by equation (2.36), such that the steady state of the model is not affected by the regulation.[18] Finally, I set the policy coefficient to $\phi_\gamma = 5$, which corresponds to an aggressive policy of stabilising the credit cycle.

### 2.6.3 Dynamics under simple policy rules

Figure 2.5 shows the policy responses to a negative productivity shock. The shock is inflationary and produces a positive output gap, so that monetary policy responds by raising the nominal interest rate.

The adverse shock also affects financial intermediation. There is downward pressure on investment, which decreases lending. The weakened demand for capital pushes down the real price of capital. When a macroprudential policy tool is active, it attempts to address this issue.

The augmented Taylor rule, shown in solid grey lines, attempts to offset the fall in real capital price, $q_t$, by raising the interest rate less than when monetary policy does not react to $q_t$, but in doing so it allows for a greater output gap, as the two move in opposite directions. The fall in real capital price, however, decreases inflationary pressure.

When the capital ratio regulation is active instead, it seeks to stabilise the loans-to-output ratio by initially allowing for a higher bank leverage in order

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[18]This implies that I do not take a stance on what would be an optimal *steady state* or permanent level of capital ratio regulation. In the Basel III framework, the recommended ratio of Tier 1 capital to total assets is 10.5%. In the model, the steady state ratio is 6.0%.

Figure 2.5: Policy response to a one percent adverse productivity shock

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state.

to sustain investment. The over-supply of financing depresses $q_t$ even further and induces an even greater output gap. This is also inflationary. Consequently, the monetary authority must counteract this by raising the interest rate even more. In this case, the two policies work against each other.

In contrast, when the shock arises in the financial sector, the capital ratio regulation can be helpful. Figure 2.6 shows the policy responses to a negative shock to bank capital. The deterioration of bank capital hinders the banks' ability to monitor investment projects. This in turn worsens the moral hazard problem, discourages depositors, and leads to higher requirements of entrepreneurial capital. The decrease in lending translates into decreased investment and output.

However, the shock is also inflationary, because the drop in investment implies an increase in the real price of capital, which in turn encourages households to substitute savings for consumption. The shock also leads to
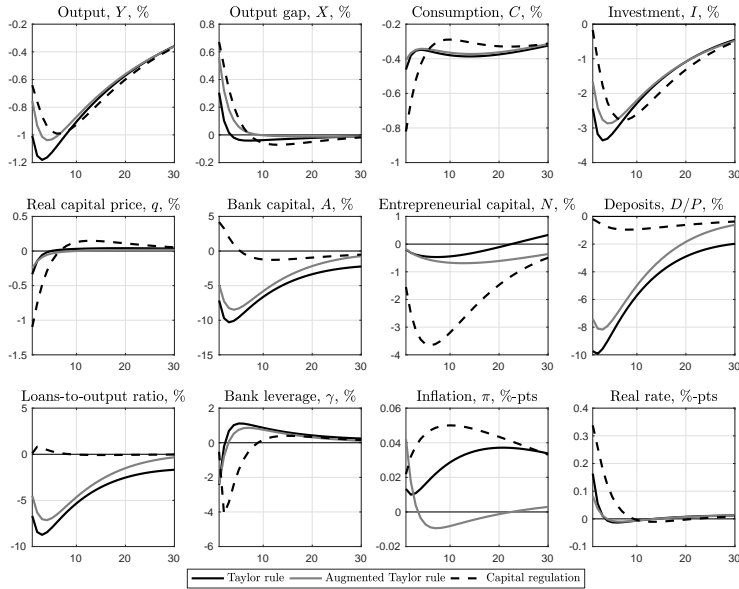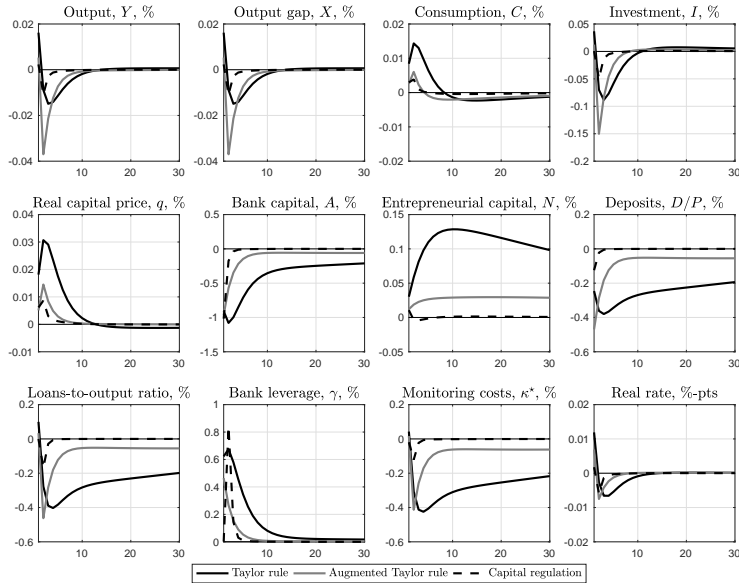
Figure 2.6: Policy response to a one percent adverse bank capital shock

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state.

a persistently lower level of bank capital and lending, because it takes time for banks to re-accumulate capital.

When the capital ratio regulation is active, it allows bank leverage to increase aggressively, which immediately offsets the impact of the bank capital shock and sustains investment. The propagation of the financial shock into the real sector is then mitigated, and the monetary authority needs to respond only mildly. The augmented Taylor rule works in the same way, but is not able to respond as aggressively, such that the shock is partially transmitted to the real economy. The economy nonetheless returns to the steady state faster than under the conventional Taylor rule.

This shows that when there are shocks arising from the financial sector itself, there are benefits to a separate macroprudential tool. However, restricting bank leverage can be counterproductive in stabilising fluctuations caused by shocks arising from outside, but affecting the financial sector. Here, in particu-

lar, I have considered total factor productivity shocks. In this case, the policies — which are not coordinated but instead follow independent mandates — work against each other. The augmented Taylor rule offers a compromise that tries to counterbalance the two policy objectives and that works reasonably well when faced both with real and financial shocks.

In the next section, I quantify the welfare effects of the different policy regimes. While the quantitative differences in welfare under the different policy regimes are small, the exercise provides an ordering of the regimes in terms of household welfare, compared to the first-best outcome.

## 2.7   Welfare analysis

In this section, I provide a welfare comparison of the different policy regimes. I compute the welfare cost associated with each policy as consumption equivalent amounts relative to a benchmark policy. The welfare evaluation follows the strategy described in Schmitt-Grohe and Uribe (2006). The consumption equivalent welfare cost is defined as the fixed fraction of consumption that the household must give up under the benchmark policy regime, in each period, to be indifferent between the benchmark policy and the policy it is being compared to. A positive cost indicates that the household is better off under the benchmark policy. The benchmark model to which the others are compared here is the frictionless RBC model, which yields the first-best allocation. The details of the computation of the consumption-equivalent welfare measure is given in Appendix 2.C.

Table 2.6 shows the welfare properties of the economies relative to the first-best. The second column of Table 2.6 reports the welfare costs of the different policy regimes. In addition, the third, fourth and fifth columns of Table 2.6 display the standard deviations of inflation, output gap and the loans-to-output ratio in percentages, respectively, under the different policy regimes.[19]

First, the welfare costs compared to the first-best allocation are small in absolute terms under any policy regime. For example, the worst-performing policy — the standard Taylor rule —results in a welfare cost of approximately 0.05% of consumption in each period compared to the first-best. This magnitude is in line with many earlier studies comparing welfare effects of different

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[19]The results are not sensitive to the volatility of the bank capital shock. In particular, the welfare ranking remains unchanged even under bank capital shocks that are an order of magnitude more volatile.

| Policy | Welfare cost (%) | $\sigma_\pi$ | $\sigma_x$ | $\sigma_\Xi$ |
|---|---|---|---|---|
| Ramsey 2 var. | 0.000 | 0.00 | 0.00 | 9.2 |
| Ramsey 1 var. | 0.001 | 0.01 | 0.13 | 15 |
| Aug. Taylor rule | 0.004 | 0.04 | 0.42 | 13 |
| Capital regulation | 0.038 | 0.14 | 0.58 | 0.08 |
| Taylor rule | 0.048 | 0.11 | 0.21 | 16 |

Table 2.6: Consumption equivalent welfare costs under different policies relative to the frictionless RBC model

*Note:* The models are in logs and solved with a second-order Taylor approximation around the steady state. A positive welfare cost indicates a welfare loss relative to the first-best allocation. "Ramsey 2 var.": Ramsey policy, inflation rate and banking sector leverage. "Ramsey 1 var.": Ramsey policy, inflation rate only. "Taylor rule": Standard Taylor rule reacting to inflation and output gap. "Aug. Taylor rule": Taylor rule reacting to inflation, output gap and real capital price. "Capital regulation": Standard Taylor rule and capital ratio regulation. $\sigma_\pi$: Standard deviation of inflation (%). $\sigma_x$: Standard deviation of the output gap (%). $\sigma_\Xi$: Standard deviation of the loans-to-output ratio (%).

monetary policy regimes.[20]

The first row shows the welfare properties of the Ramsey policy with both monetary and macroprudential policy. It is optimal to fully stabilise both inflation and the output gap, but not the credit gap. As the efficient allocation is replicated, there is no welfare loss. The second row shows the Ramsey policy under monetary policy only. Now, it is optimal to focus on stabilising inflation, but not the output gap. Thus some deviation from the efficient level of output is traded off for price stability. This is also seen when comparing the third and fifth rows, which display the performance of the augmented Taylor rule and the standard Taylor rule, respectively. The augmented rule results in higher welfare by trading off some output stability in favour of inflation stability.

The rule-based policies do not fall far behind in welfare levels compared to the two Ramsey policies. The best regime is the augmented Taylor rule, shown on the third row of the table, and second comes the policy mix that combines a standard Taylor rule for monetary policy with the capital ratio regulation. The standard Taylor rule alone fares worst. This suggests that when financial frictions are present, some form of macroprudential regulation is desirable.

The welfare evaluation is somewhat in contradiction with earlier results on the conduct of monetary policy in the presence of financial frictions. The con-

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[20]The seminal discussion by Lucas (2003) finds consumption-equivalent welfare gains from eliminating business cycle fluctuations, in general, to be roughly in the order of magnitude of 0.05%.

ventional view in the literature is that the central bank need not concern itself with asset price fluctuations, except insofar as they reflect changes expected inflation.[21] In this model, even when prices are fully flexible, the fluctuations of real asset prices reflect the sub-optimal levels of investment caused by the agency cost, and it is beneficial for the policymaker to react to them. However, it is not optimal to attempt to fully stabilise the credit gap $\Xi_t$.

Finally, it should be noted that this analysis compares different economies, corresponding to each different policy rule, around a common steady state. The assumption of an investment subsidy plays an important role here. Without it, the steady state allocation of the economy would not be efficient, and a steady state capital ratio requirement to correct for this could create sizable welfare gains. An augmented Taylor rule that reacts to asset price movements cannot affect such a steady state distortion. Moreover, when the countercyclical capital ratio requirement is only occasionally binding, the significance of steady state regulation becomes even greater. These issues are left for future research.

## 2.8 Conclusions

This essay investigates the policy implications of jointly setting monetary and macroprudential policies when there are financial frictions in the economy. The framework is otherwise a standard New Keynesian one, with an additional real friction arising from agency costs in financial intermediation. The model nests the standard New Keynesian model when this frictions is shut down.

The main contribution of this essay is to solve for the jointly Ramsey-optimal monetary and macroprudential policies. The key finding is that with both a nominal and a financial friction, a social planner needs to control both the inflation rate and the aggregate bank leverage in order to replicate the first-best allocation. The constrained optimum is then efficient. When the Ramsey planner can only set the inflation rate, he cannot fully stabilise the economy, and a short-run policy trade-off between stabilising inflation and output gap emerges.

This trade-off results from the opposite effects that the nominal and the financial frictions have on the real price of physical capital: an inflationary shock requires a deflationary policy response in order to offset the relative

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[21]See for example Bernanke and Gertler (2001), Iacoviello (2005) and Carlstrom and Fuerst (2007).

price distortions, but this will depress the real price of capital. However, a depression of the capital price worsens the incentive problems in the financial market, which distorts the level of investment and worsens the output gap. Correcting for this latter distortion and closing the output gap would warrant an inflationary policy response to push up the real capital price.

I also analyse simple rule-based policies and compare them to the benchmark provided by the Ramsey-optimal policies. In particular, I analyse Taylor-type monetary policy rules and a Basel III-type countercyclical capital ratio regulation that reacts to the deviation of the credit-to-output ratio from its steady state. I find that if there are important fluctuations arising from financial shocks, the availability of the countercyclical capital ratio regulation enhances the effectiveness of policy in stabilising the economy. The macroprudential policy supports credit intermediation, and leaves the monetary policy to deal with inflation.

However, when shocks other than financial ones – such as technology or demand shocks – are the main drivers of business cycles, conventional interest-rate-based monetary policy is enough to deal with cyclical fluctuations, in particular if the monetary authority also reacts to financial market conditions, i.e. to the real price of capital.

A separate tool for regulating the credit cycle is thus beneficial in mitigating financial shocks and their propagation to the real economy, but it may be more desirable to use it in a discretionary manner to counter financial shocks specifically, rather than as an active business cycle policy. In this sense, an active Basel III-type countercyclical capital ratio regulation that reacts to all fluctuations in the credit cycle, no matter their source, may be counterproductive. This suggests that there are gains from properly coordinating the use of macroprudential policy with conventional monetary policy.

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## 2.A Summary of the model

### 2.A.1 Dynamic equilibrium conditions

This section summarises the dynamic model equations.

#### 2.A.1.1 The macro block

The household's optimality conditions are:

$$w_t = \frac{\chi L_t^\theta}{U_C(C_t, L_t)} \qquad \text{(Labour supply)}$$

$$1 = \beta E_t \left[ \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} \frac{1 + r_t}{\pi_{t+1}} \right] \qquad \text{(Bond Euler eq.)}$$

$$q_t = \beta E_t \left[ \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} \left( r_{t+1}^K + (1 - \delta) q_{t+1} \right) \right] \qquad \text{(Capital Euler eq.)}$$

where $U_C(C_t, L_t) = Z_t^c \left[ \frac{1}{1-b}(C_t - bC_{t-1}) \right]^{-\sigma}$ and $\log Z_t^c = \phi_c \log Z_{t-1}^c + \epsilon_t^c$, $\epsilon_t^c \sim N(0, \sigma_c^2)$, i.i.d.

The symmetric equilibrium conditions of the intermediate production sector are:

$$\frac{r_t^K}{w_t} = \frac{\alpha L_t(j)}{(1 - \alpha) K_t(j)} \qquad \text{(Relative factor price)}$$

$$\psi_t = \left( \frac{r_t^K}{\alpha} \right)^\alpha \left( \frac{w_t}{1 - \alpha} \right)^{(1-\alpha)} Z_t^{-1} \qquad \text{(Real marginal cost)}$$

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \omega^k Q_{t,t+k} \psi_{t+k|t} Y_{t+k|t} \pi_{t+k}^{\varepsilon+1}}{E_t \sum_{k=0}^{\infty} \omega^k Q_{t,t+k} Y_{t+k|t} \pi_{t+k}^{\varepsilon}} \qquad \text{(Optimal pricing)}$$

where $Q_{t,t+k} \equiv \beta^k \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} \frac{P_t}{P_{t+k}}$ and $\log Z_t = \phi \log Z_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, \sigma^2)$, i.i.d.

Furthermore, the numerator of the optimal pricing decision can be reformulated recursively as:

$$\text{Num}_t = \psi_t Y_t + \omega (1 + r_t)^{-1} E_t \pi_{t+1}^{\varepsilon+1} \text{Num}_{t+1},$$

and the denominator as:

$$\text{Denom}_t = Y_t + \omega(1 + r_t)^{-1}E_t\,\pi_{t+1}^{\varepsilon}\,\text{Denom}_{t+1},$$

which allows expressing the optimal price relative to the aggregate price level recursively (for computational convenience) as:

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1}\frac{\text{Num}_t}{\text{Denom}_t}.$$

The aggregate dynamic equilibrium conditions are:

$$Y_t = Z_t K_t^{\alpha}(L_t)^{1-\alpha} \qquad\qquad \text{(Aggregate production)}$$

$$Y_t = (C_t + I_t)s_t \qquad\qquad \text{(Aggregate consistency constraint)}$$

$$s_t = (1 - \omega)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \omega\pi_t^{\varepsilon}s_{t-1} \qquad \text{(Correction for price dispersion)}$$

$$K_{t+1} = p_H R I_t + (1 - \delta)K_t \qquad\qquad \text{(Capital accumulation)}$$

$$P_t = \left[\omega P_{t-1}^{1-\varepsilon} + (1 - \omega)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad\qquad \text{(Aggregate price level)}$$

$$1 = \omega\pi_t^{\varepsilon-1} + (1 - \omega)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon} \qquad\qquad \text{(Price dispersion)}$$

### 2.A.1.2 The financial block

The aggregate equilibrium in the financial sector is described by the following equations.

$$\kappa_t^* = \frac{\eta \rho_t}{1 + \frac{p_H}{\Delta p} \left( \frac{1}{1+r_t} - \frac{1}{1+r_t^a} \right)} \qquad \text{(Optimal monitoring)}$$

$$b(\kappa_t^*) = \Gamma(\kappa_t^*)^{-\frac{\eta}{1-\eta}} \qquad \text{(Private benefit)}$$

$$\rho_t = \frac{q_t p_H R}{1+r_t} - 1 \qquad \text{(Net pledgeable income)}$$

$$g_t = \frac{p_H}{\Delta p} \frac{b(\kappa_t^*)}{1+r_t} - (1-\eta)\rho_t \qquad \text{(Entrepreneur's inverse leverage ratio)}$$

$$R_t^e = \frac{b(\kappa_t^*)}{q_t \Delta p} \qquad \text{(Entrepreneur's return share)}$$

$$R_t^b = \frac{\kappa_t^*}{q_t \Delta p} \qquad \text{(Banker's return share)}$$

$$R_t^w = R - R_t^e - R_t^b \qquad \text{(Worker's return share)}$$

$$1 + r_t^a = \frac{p_H}{\Delta p} \kappa_t^* \frac{I_t}{A_t} \qquad \text{(Return on bank capital)}$$

$$1 + r_t^e = \frac{p_H}{\Delta p} b(\kappa_t^*) \frac{I_t}{N_t} \qquad \text{(Return on entrepreneurial capital)}$$

$$I_t = \frac{N_t}{g_t} \qquad \text{(Investment size)}$$

$$\gamma_t = 1 + \frac{1+r_t^a}{1+r_t} \left( \frac{\Delta p}{p_H} \frac{q_t p_H R}{\kappa_t^*} - \frac{b(\kappa_t^*)}{\kappa_t^*} - 1 \right) \qquad \text{(Banker's leverage ratio)}$$

$$\frac{D_t}{P_t} = (1 + \kappa_t^*)I_t - N_t - A_t \qquad \text{(Aggregate deposits)}$$

$$N_{t+1} = \lambda^e (1 + r_t^e) \frac{r_{t+1}^K + (1-\delta)q_{t+1}}{q_t} N_t$$
$$\text{(Accumulation of entrepreneur's net worth)}$$

$$A_{t+1} = Z_{t+1}^b \lambda^b (1 + r_t^a) \frac{r_{t+1}^K + (1-\delta_t)q_{t+1}}{q_t} A_t$$
$$\text{(Accumulation of banker's net worth)}$$

where $\log Z_t^b = \phi_b \log Z_{t-1}^b + \epsilon_t^b$, $\epsilon_t^b \sim N(0, \sigma_b^2)$, i.i.d.

### 2.A.1.3 Government policy

Finally, the model is closed by the Taylor rule for the nominal interest rate, and the rule for the macroprudential policy.

$$1 + r_t = \frac{1}{\beta} \pi_t^{\phi_\pi} X_t^{\phi_x} q_t^{\phi_q} \qquad \text{(Taylor rule)}$$

$$\gamma_t = \gamma (\Xi_t)^{-\phi_\gamma} \qquad \text{(Capital ratio regulation)}$$

Here, $X_t \equiv \frac{Y_t}{Y_t^e}$ is the output gap and $\Xi_t \equiv \frac{(I_t - N_t)/Y_t}{(I - N)/Y}$ is the deviation of the loans-to-output ratio from its steady state value, or the "credit gap". $\gamma$ is the steady state banker leverage ratio.

## 2.A.2 Deterministic steady state

The deterministic steady state of the model is as follows. It is assumed that a steady state employment subsidy is in place such that $\mu = 1$, so that the steady state is not distorted by the monopolistic competition.

### 2.A.2.1 The macro block

The steady state of the macro block of the model is:

$$1 + r = \frac{1}{\beta}$$

$$P^* = P = 1$$

$$\pi = 1$$

$$s = 1$$

$$\psi = 1$$

$$q = \frac{1 + \rho}{\beta p_H R^*}$$

$$r^K = q(r + \delta)$$

$$w = (1 - \alpha) \left( \frac{r^K}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}$$

$$K = \left[ \left( \frac{r^K}{\alpha} \right)^{\frac{\theta+\alpha}{\alpha-1}} \frac{1-\alpha}{\chi} \left( \frac{r^K}{\alpha} - \frac{\delta}{p_H R} \right)^{-\sigma} \right]^{\frac{1}{\theta+\sigma}}$$

$$L = K \left( \frac{r^K}{\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$Y = \frac{r^K K}{\alpha}$$

$$I = \frac{\delta K}{p_H R}$$

$$C = Y - I$$

$$Z = Z^c = Z^b = 1$$

The steady state is non-distorted and identical to the efficient steady state — where prices are assumed flexible and there is no financial friction in investment — when $q = 1$. The efficient steady state can thus be replicated by imposing a steady state investment subsidy on the gross return of the investment project, $R$. Denote this subsidy by $1 + \varsigma$. Then, the subsidy needed to replicate the efficient steady state is:

$$q = \frac{1+\rho}{\beta p_H R (1+\varsigma)} = 1 \quad \Leftrightarrow \quad \varsigma = \frac{1}{\beta} - 1 + \frac{\rho}{\beta} = r + \frac{\rho}{1+r},$$

where $R(1+\varsigma) \equiv R^*$ is the subsidised return on investment. I assume that such a subsidy is in place.

### 2.A.2.2 The financial block

The steady state of the financial block of the model is:

$$1 + r^a = \frac{\beta}{\lambda^b}$$

$$1 + r^e = \frac{\beta}{\lambda^e}$$

$$R^b = \frac{\kappa^*}{q\Delta p}$$

$$R^e = \frac{b(\kappa^*)}{q\Delta p}$$

$$R^w = R - R^e - R^b$$

$$\kappa^* = \left(\Gamma\frac{\eta}{1-\eta}\right)^{1-\eta} \left(\frac{\beta - \frac{\lambda^e}{\beta}}{\beta - \frac{\lambda^b}{\beta} + \frac{\Delta p}{p_H}}\right)^{1-\eta}$$

$$b(\kappa^*) = \Gamma(\kappa^*)^{-\frac{\eta}{1-\eta}}$$

$$\rho = \frac{\kappa^*}{\eta}\left[1 + \frac{p_H}{\Delta p}\left(\beta - \frac{\lambda^b}{\beta}\right)\right]$$

$$g = \beta\frac{p_H}{\Delta p}b(\kappa^*) - (1-\eta)\rho$$

$$\gamma = 1 + \frac{\beta}{\lambda^b}\left[\frac{\Delta p}{p_H}\frac{1+\rho}{\kappa^*} - \beta\left(\frac{b(\kappa^*)}{\kappa^*} + 1\right)\right]$$

$$A = \frac{\lambda^b}{\beta}\frac{p_H}{\Delta p}\kappa^* I$$

$$N = \frac{\lambda^e}{\beta}\frac{p_H}{\Delta p}b(\kappa^*)I$$

$$D = (1+\kappa^*)I - A - N$$

## 2.B   The Ramsey problem

The problem of the Ramsey planner can be formulated as follows. Let $y_t$ be a vector containing the $n$ endogenous variables of the economy, including the planner's choice variables, and $u_t$ the vector of exogenous variables. The agents in the economy optimise taking the planner's choice variables (the policy variables) as given. The equilibrium of the private economy is described by the $m$ first-order conditions and transition equations:

$$E_t[f(y_{t-1}, y_t, y_{t+1}, u_t)] = 0.$$

This leaves $n - m$ choice variables for the planner.

The Ramsey planner chooses the values of these policy variables in each

76

period to maximise household welfare, subject to the private economy's equilibrium conditions:

$$\max_{\{y_\tau\}_{\tau=t}^{\infty}} E_\tau \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(C_\tau, L_\tau)$$

$$\text{s.t.} \quad E_\tau[f(y_{\tau-1}, y_\tau, y_{\tau+1}, u_\tau)] = 0 \quad \forall\, \tau \in \{\ldots, t-1, t, t+1, \ldots\}.$$

In Section 2.5.3, the Ramsey policy problem is solved and analysed numerically using the Dynare 4.5 software package, which is run in MATLAB. The Dynare routine first numerically computes the first order conditions of the planner problem above, subject to the true nonlinear equilibrium conditions of the private economy. The latter consist of the first order conditions of the private sector, the laws of motion for stock variables, and the aggregate resource constraint. The Ramsey optimality conditions will thus correctly include welfare-relevant second order terms of the private sector equilibrium conditions. The routine then linearises these planner's first order conditions around the steady state of the model by using a first-order Taylor approximation. The solution is computed under the assumption of timeless perspective, following Woodford (2003), to avoid the time inconsistency problem associated with Ramsey policies under commitment.

## 2.C The welfare cost

The welfare cost of the different policy measures are computed as consumption equivalent costs relative to a benchmark policy as described in Schmitt-Grohe and Uribe (2006).

Define the welfare of the household under the benchmark allocation (denoted by $R$), conditional on the state of the economy at time zero, as:

$$V_0^R = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^R, L_t^R),$$

where $C_t^R$ and $L_t^r$ denote the plans for consumption and hours worked under the benchmark policy regime.

Similarly, define the conditional welfare under an alternative policy plan (denoted by $A$) as:

$$V_0^A = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^A, L_t^A).$$

Assume that at time zero, all variables are equal to their steady-state values. Since the steady state of the model is undistorted and unaffected by the different policy regimes, the initial state of the economy is the same for the benchmark and the alternative policies.

Next, denote by $x$ the consumption-equivalent conditional welfare cost of the alternative policy regime, relative to the benchmark regime. Formally, the cost $x$ is implicitly defined by:

$$V_0^A = E_0 \sum_{t=0}^{\infty} \beta^t U((1-x)C_t^R, L_t^R).$$

Using the CES functional form for periodic utility and solving for $x$ yields:

$$V_0^A = E_0 \sum_{t=0}^{\infty} \beta^t \left[ Z_t^c \frac{((1-x)C_t^R)^{1-\sigma}}{1-\sigma} - \frac{\chi(L_t^R)^{1+\theta}}{1+\theta} \right]$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[ ((1-x)^{1-\sigma} - 1)Z_t^c \frac{(C_t^R)^{1-\sigma}}{1-\sigma} \right] + V_0^R$$

$$\Leftrightarrow \quad x = 1 - \left[ 1 + \frac{V_0^A - V_0^R}{E_0 \sum_{t=0}^{\infty} \beta^t Z_t^c \frac{(C_t^R)^{1-\sigma}}{1-\sigma}} \right]^{\frac{1}{1-\sigma}}.$$

Note that when $V_0^A = V_0^R$, the measure equals zero.

Simulating the allocations — solved by taking a second-order Taylor approximation around the steady state to ensure a correct welfare ranking of the alternative policies — for a long enough time horizon $T$ and repeating the simulation $N$ times, for $N$ large enough, yields an estimate of the conditional expectations $V_0^R$ and $V_0^A$. This allows to numerically estimate the cost $x$.

## 2.D    Impulse responses to the preference shock
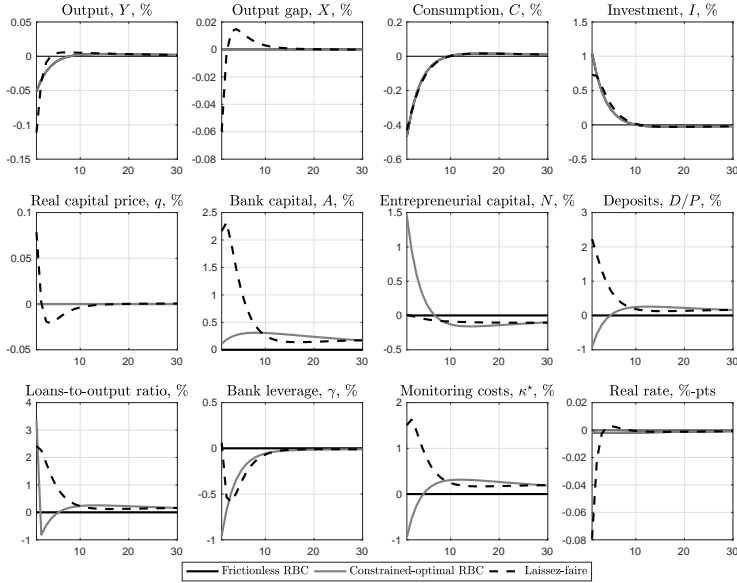


Figure 2.7: Effects of a one percent negative preference shock under Ramsey policy in the flexible price model

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state. "Frictionless RBC": model with fully flexible prices and no financial friction. "Constrained-optimal RBC": model with fully flexible prices but with the financial friction; the planner sets aggregate bank leverage. "Laissez-faire": model with fully flexible prices but with the financial friction; no policy intervention.

Figure 2.8: Effects of a one percent negative preference shock under Ramsey policy in the sticky price model

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state. "Inflation choice": model with sticky prices and the financial friction; social planner sets the inflation rate. "Inflation and leverage choice": model with sticky prices and the financial friction; social planner sets the inflation rate and aggregate bank leverage. "Frictionless RBC": the frictionless flexible price (first-best) benchmark.
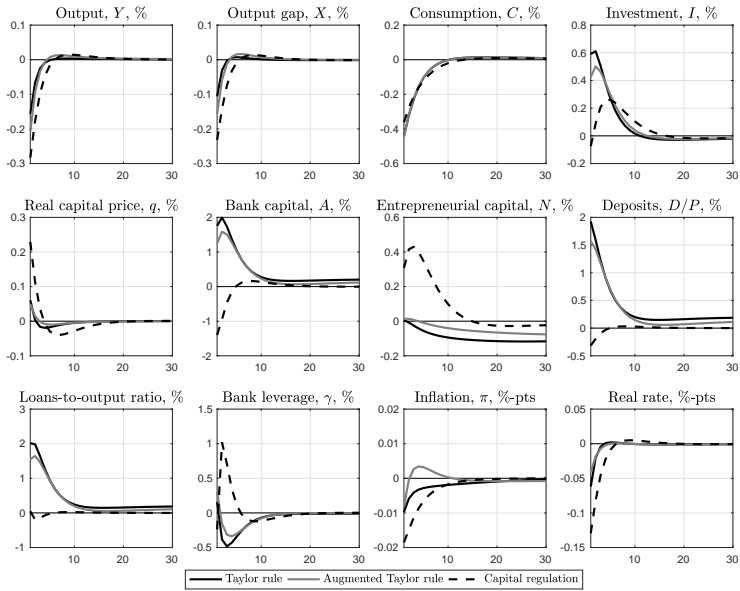
Figure 2.9: Policy response to a one percent negative preference shock under simple policy rules

*Note:* Impulse responses are reported as percentage (%) or percentage-point (%-pts) deviations from steady state.

# 3 House prices, lending standards, and the macroeconomy

## 3.1 Introduction

The housing boom of the early 2000's in the U.S. culminated in the subprime crisis of 2007–2009. The episode demonstrated the need to understand the dynamics of house prices, housing debt, and their links to the aggregate economy.

This essay offers a demand-side explanation of the housing boom and bust by concentrating on loan demand and changes in the composition of the borrower pool over the house price cycle. It presents a theoretical model of the housing market with over-investment in equilibrium, and explores the incentives of households to take on debt over the house price cycle. I show that asymmetric information coupled with deadweight costs of default can create endogenous cycles in house prices. I also show that lending standards are loose and the incentives for less-than-creditworthy borrowers to apply for a loan are particularly strong, first, when future house values are expected to be high, which leads to high leverage of borrowers; and second, when safe interest rates are low, which implies low costs of borrowing. However, there are strong non-linearities in the relationship between borrowing incentives and economic fundamentals. This analysis helps to explain the behaviour of households during the housing boom of the early 2000s in the U.S. in an environment characterised by low interest rates and increasing house prices.

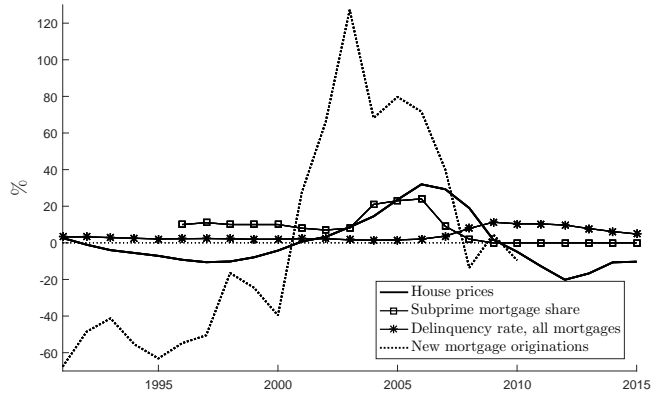The data on housing markets in the U.S. have a few salient features. Fig-

Figure 3.1: Mortgage lending, delinquencies, and the house price cycle

*Note:* House prices: %-deviation from a linear trend; source: S&P Case-Shiller U.S. National Home Price index. Subprime mortgages: % share of total mortgage originations; source: Inside Mortgage Finance, collected from the Financial crisis inquiry report (2011) for years 1996–2008 and Zibel and Andriotis (2015) for years 2009–2014. Delinquency rate: % share of all single-family residential mortgages; source: Board of Governors of the Federal Reserve System. New mortgage originations: total volume of new single-family residential mortgage originations, 1990–2010, %-deviation from sample mean; source: U.S. Federal Housing Finance Agency.

ure 3.1 shows the co-movement of new residential mortgage originations, the mortgage delinquency rate, and subprime mortgage lending over the house price cycle in the United States in 1990–2015. Starting from 1998, house price appreciation picked up and peaked in 2006 at 30% above trend before falling dramatically. This dramatic boom-bust episode in house prices was, however, accompanied and even overshadowed by a surge in mortgage originations. They grew from almost 70% below the sample mean in 1990 to 120% above the mean in 2003. The most impressive growth in originations coincides with the period from 2000 to 2003, at the onset of the housing boom.

At the same time, the share of subprime mortgage origination was relatively stable at around 10% of all mortgage originations until 2004, after which it rapidly expanded, peaked at 24% in 2006 together with house prices. It then collapsed virtually to zero from 2009 onward. Meanwhile, the delinquency rate was stable at slightly above 2% until 2006. As house prices started to fall, delinquencies on mortgages started increasing and reached 11% in 2009. They have remained elevated since then. The expansion of subprime lending

is related to a loosening of lending standards that followed the increased securitisation of subprime loans, as argued e.g. by the Financial Crisis Inquiry Commission (2011).

In this essay, I analyse the incentives of potential borrowers to enter the loan market and subsequently to potentially default on the loan by using a model of asymmetric information in the credit market. I model the housing market in a setting where the intermediation of loans is inefficient because of an adverse selection problem, modelled after the classic framework of De Meza and Webb (1987, 1990). I assume fundamental house values are exogenous and heterogeneous, but endogenise the sales price of housing. In the model, all households are subject to two shocks: a shock to house value, and an income shock. The former is observable by all agents, whereas the latter is private information to the borrower. The households also face two fundamental choices. First, they must decide whether to buy or rent a home; they may need to borrow to finance housing purchases, but their future income is uncertain at the time of purchase. Second, after income uncertainty is resolved, they can choose to default on their existing debt. In this model, default acts as a form of insurance against adverse house value and income shocks by allowing partial consumption smoothing, but carries a deadweight cost to the household.

In order to talk about *over*-borrowing, there has to be a market failure that allows households to take on too much debt from a social perspective. Similarly, a loosening or tightening of *lending standards* can be modelled as a deviation from the first-best in lenders' optimal decisions in terms of to whom, and at what price, loans should be granted.[1] In the model presented in this essay, borrowers differ in their default risk in a way that is unobservable to lenders. This particular information structure leads to over-borrowing in equilibrium, and subprime borrowers are defined as the set of borrowers who do get a loan under asymmetric information, but would not if their types were publicly observable.

The main contributions of this essay are two-fold. The first set of results comes from a theoretical comparative statics exercise that explores participation and default incentives in the cross-section of households. The second set focuses on incentives over time to explain a boom-bust cycle in the housing market.

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[1]The term "lending standards" is used here to describe the behaviour of lenders under asymmetric information, relative to their actions under symmetric information. The term is used in a similar way in a host of literature concerning informational asymmetries in the credit market; see e.g. Dell'Ariccia and Marquez (2006) and the references therein.

First, I show that there is a non-linear relationship between expected house values and participation in the credit market in the static model. This non-linearity is created by a trade-off between the cost of borrowing and expected capital gains. When future house values, and thus prices, are expected to be low, participation is high. As expected house values rise, the types with the lowest expected income first opt out of the market because housing becomes more expensive. Eventually however, the expected capital gains become so attractive that the risky types enter the market again. Because the borrowers in this environment are more levered, default becomes optimal if they are hit by an adverse combination of income and house value shocks.

On the other hand, the relationship between safe interest rates and credit market participation is monotone: lower interest rates attract more borrowers as outside returns and borrowing costs are both low. An environment of low interest rates and high expected house values thus render the economy vulnerable to adverse house value and income shocks as household become highly levered. I also demonstrate that tightening default legislation that treats all borrowers equally is not an effective way to curb over-borrowing and reduce household leverage. An effective policy must target incentives of different types directly in order to improve the robustness of household balance sheets.

Second, I show that the model is capable of accommodating a deterministic cycle in house prices, and that these dynamic patterns match those observed in the U.S. data in terms of relative magnitudes and the timing of the cycles. The endogenous cycle is created by a strategic complementarity in the households' preference for housing consumption together with a deadweight cost of defaults. These elements capture, first, the 'hype' surrounding homeownership during the boom phase, and subsequently, the foreclosure wave that induces the bust phase.

The remainder of the essay is organised as follows. First, Section 3.2 reviews related research contributions. Section 3.3 then describes the model economy. Next, Section 3.4 solves for the credit market equilibrium under symmetric and asymmetric information, and Section 3.5 describes the timing of events and characterises the equilibrium of the aggregate economy. The main results are discussed in Section 3.6, which explores the link between house prices, interest rates, and the selection into the housing market through a comparative statics exercise, and Section 3.7, which extends the baseline model to accommodate an endogenous boom-bust cycle in house prices. Finally, Section 3.8 concludes.

## 3.2   Related literature

The results in this essay complement the evidence in a series of influential empirical studies by Mian and Sufi (2009, 2011) and Mian et al. (2013). The authors find evidence for a credit supply driven mortgage lending boom, where lenders expanded their supply of mortgage lending and relaxed their lending standards in the run-up to the subprime crisis of 2007–2009. The shift in supply was tightly connected to the expansion of mortgage securitisation since the early 2000's.

In a theoretical setup, Justiniano et al. (2015) find that the housing boom and bust in the United States was more likely driven by shocks to house prices, not an exogenous loosening of credit conditions. On the other hand, Favilukis et al. (2017) and Gelain et al. (2018), among others, show that a relaxation of financial constraints — namely collateral requirements — can lead to a large house price boom.

I focus on an analysis of credit demand and borrower behaviour as a potential driver for housing booms that is complementary to the supply-side explanations, such as house price and credit supply shocks. Using loan-level data, Dell'Ariccia et al. (2012) provide empirical evidence that the relaxation of lending standards prior to the subprime crisis was partly explained by an increase in credit demand, when controlling for supply-side factors such as house prices and securitisation. The theoretical results in this essay are consistent with this evidence.

This essay is also linked to a growing body of literature on the interactions between house prices prices and aggregate consumption, including Iacoviello (2005), Kiyotaki et al. (2011), Iacoviello and Neri (2010), Iacoviello and Pavan (2013), Kaplan and Violante (2014), Gorea and Midrigan (2017), and Guerrieri and Iacoviello (2017). However, these studies do not explain or motivate why some borrowers are credit-constrained. In contrast to them, my focus is on formulating explicitly the incentives for credit market participation and default.

This research also connects to recent theoretical models of the housing market where mortgage borrowers can strategically default, such as Corbae and Quintin (2015), Guler (2015), Arslan et al. (2015), Elenev et al. (2016), and Garriga et al. (2016). These studies are more quantitative in nature than this essay, but their findings are in line with the ones presented here: households have the strongest incentives to default when they both face an adverse income shock and their home equity is negative. However, these studies do not

consider private information, with the exception of Guler (2015).[2]

A recent strand of literature has attempted to explain inefficiencies in aggregate investment by exploiting an adverse selection framework. Eisfeldt (2004), Morris and Shin (2012), Bigio (2015), House (2006), Takalo and Toivanen (2012), and Jokivuolle et al. (2014) analyse investment in entrepreneurial projects, or the financing of capital production, in general equilibrium when the allocation of finance is affected by asymmetric information on project quality. Similarly to this essay, the latter three studies also apply the De Meza and Webb (1987, 1990) framework. On the credit supply side, Dell'Ariccia and Marquez (2006) find that increasing asymmetric information across lenders can lead to a tightening of lending standards, as lenders screen loan applicants better.

The model presented in this essay builds upon the general equilibrium framework of House (2006), who focuses on risk-neutral entrepreneurs who invest in capital investment projects. To my knowledge, the De Meza and Webb (1987) adverse selection framework has previously not been applied to model inefficient credit intermediation in the mortgage market in a macroeconomic setting, although the theory lends itself quite naturally to this context. This essay also adds to the literature by considering two sources of uncertainty jointly, instead of only one.

Finally, this essay connects to an old literature in macroeconomics and macro-finance that focuses on deterministic and endogenous cycles in investment. Examples include Suarez and Sussman (1997), Azariadis and Smith (1998), Matsuyama (2007), Favara (2012), Beaudry et al. (2015), Matsuyama et al. (2016), and Azariadis et al. (2016). All rely on some combination of strategic complementarities and costs of default to generate an endogenous cycle. In a slightly different set-up, Gorton and He (2008) model a repeated strategic game of borrower screening between banks, where changes in the private information of lenders lead to an endogenous credit cycle.

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[2]Contrary to this essay, however, the author argues that adverse selection has lead to credit rationing rather than over-investment in the housing market, and that the run-up to the subprime crisis can be explained with an increase in information symmetry and thus an increase in efficiency of credit intermediation caused by technological advances, rather than an exacerbation of a market failure. In contrast, I argue that information *asymmetry* can explain the run-up to the crisis. For instance, new financial innovations that were thought to pool the risks in individual subprime loans, such as mortgage backed securities, are an example of increasing opaqueness of the credit market.

## 3.3 The model

In this section, I describe in detail the economic environment of the model. First, I describe the stochastic structure of the economy and the information sets of each agent. I then formulate the problems of each type of agent. The subsequent sections solve for the equilibrium in the credit and housing markets.

### 3.3.1 Description of the economy

The economy consists of three types of agents: consumers, lenders, and real estate agents. The consumers have a finite lifetime, and they consume housing services and other consumption goods and receive an endowment income. The lenders extend loans to consumers in a perfectly competitive credit market. The real estate agents buy housing from consumers exiting the economy as well as from lenders in possession of foreclosed housing, refurbish them, and sell them to newborn consumers.

There is a continuum of mass one of households that consist of risk averse consumers. Each consumer only lives for two periods. In each period, a new generation of consumers enters as the previous one exits, so that the total mass of consumers stays constant. Each consumer receives an exogenous income in both periods of her life. Income in the first period of life is certain and identical across all consumers, but the income in the second period is subject to idiosyncratic risk. In the first period, the consumer must make a tenure choice of either buying or renting a unit of housing. In order to buy a house, she may need a loan from a lender. In the second period, she consumes housing services and other consumption goods. Similar to the income, the value of housing in the second period is also subject to idiosyncratic risk. I also assume that debt is not perfectly enforceable, so that a borrower can default on a loan in the second period instead of repayment.

There is also a continuum of mass one of lenders, who are risk neutral. They maximise the expected profit on their lending activity. They have access to an infinitely elastic supply of funds.[3] The lenders grant loans to borrowers, collect loan repayments, and consume their profits. The credit market is perfectly competitive and anonymous, and the loans are one-period loans.

Finally, there is a continuum of mass one of perfectly competitive real estate agents, who are also risk neutral and who maximise their expected profit.

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[3]For example, through the international financial market.

They buy the housing stock from each exiting generation of households, as well as used housing held by lenders, refurbish it at no cost, and sell or rent it to the entering generation.

### 3.3.2 The credit market

There is a perfectly competitive credit market where households can apply for a loan $l_0$ from a continuum of atomistic lenders. A lender observes the realisation of the house value $q_1$ and the income $y_1$ after the loan contract has been agreed upon, but before the loan repayment is scheduled to be made. If there is no default, the lender collects the loan repayment $(1 + r_0)l_0$, where $r_0$ is the offered interest rate. In the event of default, the lender seizes the house, worth $q_1 h$, and also has recourse to a fraction $0 < \xi < 1$ of the borrower's income, $y_1$.

The credit market equilibrium is described in Section 3.4.

### 3.3.3 The consumer problem

Consider a consumer who lives for two periods, $t = 0, 1$. A consumer who wishes to become a home-owner buys housing $h$ in $t = 0$ at a unit price $q_0$. At the time of the purchase, the value of the house in the next period, $q_1$, is uncertain. It can be $q_1 = q_H$ with probability $\phi$, or $q_1 = q_L$ with probability $1 - \phi$, with $q_H > q_0 > q_L \geq 0$. As a benchmark, I assume $q_0 h > y_0$, so that a prospective home buyer will need a loan to finance the purchase.[4] If the individual chooses to rent instead, she can earn the safe market rate $1 + \bar{r}$ on a safe deposit $a_0$, while paying a rent $s_1$ per unit of housing in the second period.

Similarly to the house value, the endowment of the consumer in period $t = 1$ is uncertain, and can take on the values $y_1 \in \{y_H, y_L\}$ with $y_H > y_0 > y_L > 0$. The probability of receiving $y_H$ or $y_L$ are $\pi$ and $1 - \pi$, respectively. The combination of these two sources of uncertainty may trigger a default by homeowners who have debt. The joint distribution of $y_1$ and $q_1$ is shown in Table 3.1.

I assume that there is a fixed aggregate stock of housing $\bar{h}$. Each individual must occupy a housing unit of equal size $h$, which provides a flow of housing services; each unit is ex-ante identical. Since income $y_0$ is equal across

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[4]This need not be the case. A situation where buyers are able to self-finance the home purchase is discussed in Section 3.6.2.

| $q_1 , y_1$ | $y_H$ | $y_L$ |
|---|---|---|
| $q_H$ | $\pi\phi$ | $(1-\pi)\phi$ |
| $q_L$ | $\pi(1-\phi)$ | $(1-\pi)(1-\phi)$ |

Table 3.1: Joint distribution of endowment $y_1$ and house value $q_1$ in $t = 1$

households as well, the assumption of fixed $h$ means that, importantly, the ratio of housing to income $\frac{h}{y_0}$ — or equivalently leverage — is constant across borrowers.[5] I abstract from the choice over how much housing to acquire.

The consumers differ by their probability of realising a high endowment $y_H$: there is a continuum of types $\pi \in [0,1]$. This implies that different consumers also face a different risk of default if they have debt. Otherwise, they have the same preferences, the same support for the income distribution $y_1 \in \{y_H, y_L\}$, and the same stochastic process and support for the housing value shock on $q_1$.[6]

The type $\pi$ is private information observed only by the consumer herself, and not by other agents in the economy. Lenders and real estate agents know the distribution of $\pi$, denoted by $F(\pi)$, which is time-invariant.

I assume that consumers only value consumption in the second period of their life. Then, anticipating the equilibrium, in the first period there is no consumption; the endowment received in $t = 0$ is fully invested either into housing $h$ or a safe deposit $a_0$. The budget constraints of a homeowner and a tenant in the first period are then:

$$\begin{aligned} \text{Home-owner:} \quad & q_0 h = y_0 + l_0 \\ \text{Tenant:} \quad & a_0 = y_0 \end{aligned}$$

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[5]The strict assumptions of identical $y_0$ and $h$ are made for convenience, but it is the equal leverage ratio that is crucial. This assumption is important in equilibrium, because it prevents borrowers from signaling their types to lenders by choosing different house sizes and consequently different leverage. The implications of this assumption are discussed in more detail in Section 3.4.4.

[6]Note that the parameter $\pi$ defines both the expected income and the income risk for each type. This is a key assumption of the De Meza and Webb (1987) model. It stands in contrast with the Stiglitz and Weiss (1981) model, where each type has the same expected income, but faces a different income risk; i.e "riskier" types' realised incomes are mean-preserving spreads of "safer" types' incomes. These distributional assumptions define whether the credit market equilibrium exhibits under- or over-investment. See House (2006) for a thorough discussion on the implications of these assumptions.

At the beginning of the second period, the income $y_1$ is observed, and the home-owner with a home loan makes the decision of whether or not to default. If there is no default, she pays back the loan principal and interest and consumes the rest of her wealth. However, if she chooses to default on the loan, the lender seizes the house and a fraction $\xi y_1$ of the income, and the borrower must convert into a tenant, paying rent $s_1$ on housing. A defaulting borrower also faces a deadweight cost of default $\kappa \geq 0$. It can be interpreted as capturing various real costs associated with default, such as legal costs or loss of access to credit markets. A tenant consumes her income endowment and her savings. The price of the consumption good acts as numeraire and is normalised to unity.

The ex-post budget constraints of the household in the second period of their life, $t = 1$, are:

Home-owner:
*No default*     $c_1 + (1 + r_0)l_0 = y_1 + q_1 h$
*Default*        $c_1 + s_1 h = (1 - \xi)y_1 - \kappa$
Tenant:        $c_1 + s_1 h = y_1 + (1 + \bar{r})a_0$

The consumer derives utility from consuming housing services and other goods in the second period of her life, captured by the utility function $u(c) + \chi_i v(h)$, where $i = H, R$ designates a home-owner ($H$) or tenant ($R$). The utility function is separable in housing services and other goods, and I assume $u(\cdot)$ increasing and concave: $u'(\cdot) > 0, u''(\cdot) < 0$. Utility from housing services satisfies $v(\cdot) > 0$ and $v(0) = 0$. In addition, home-owners enjoy a utility premium on housing services: $\chi_H$ is normalised to 1 for home-owners, and $\chi_R = \chi < 1$ for tenants.[7]

I denote by $p(\pi)$ is the individual ex-ante probability of not defaulting in $t = 1$, given the home-owner's type $\pi$. It is an equilibrium object, for which the expression is derived in Section 3.4.1.

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[7]Since the housing choice is a binary one (own/rent), the separability assumption is not crucial, but it is analytically convenient. For the same reason, the shape of $v$ is not important as long as an owner enjoys a utility premium relative to a tenant. The assumption of a utility premium on owning is a common one made in the literature on housing investment. It is made to compensate for the riskiness of owning in order ensure at least some households want to own rather than rent. See e.g. Iacoviello and Pavan (2013).

The value function of a consumer of type $\pi$ in $t_0$ who becomes a home-owner is:

$$V_H(\pi) = p(\pi)E\left[u(c_{nd}(y_1, q_1)) + v(h)|\pi\right] +$$
$$(1 - p(\pi))E\left[u(c_d(y_1)) + \chi v(h)|\pi\right] \tag{3.1}$$
$$\text{s.t.} \quad q_0 h = y_0 + l_0 \tag{3.2}$$
$$c_{nd}(y_1, q_1) = y_1 + q_1 h - (1 + r_0)l_0 \tag{3.3}$$
$$c_d(y_1) = (1 - \xi)y_1 - \kappa - s_1 h \tag{3.4}$$

where $c_{nd}$ denotes period 1 consumption conditional on no default, and $c_d$ period 1 consumption conditional on default.

Substituting the period 0 budget constraint (3.2) into the no-default budget constraint (3.3) and rearranging yields the intertemporal budget constraint

$$c_{nd}(y_1, q_1) = y_1 + (1 + r_0)y_0 + \Delta q h - r_0 q_0 h \tag{3.5}$$

where $\Delta q h \equiv (q_1 - q_0)h$ equals the capital gain on the house. The term $r_0 q_0$ is the user cost of housing borne in the second period.

Similarly, the value function of a consumer who becomes a tenant is:

$$V_R(\pi) = E\left[u(c_r(y_1)) + \chi v(h)|\pi\right] \tag{3.6}$$
$$= \pi u(c_r(y_H)) + (1 - \pi)u(c_r(y_L)) + \chi v(h)$$
$$\text{s.t.} \quad c_r(y_1) = y_1 + (1 + \bar{r})y_0 - s_1 h \tag{3.7}$$

An individual chooses to buy a house and become a home-owner if and only if $V_H(\pi) \geq V_R(\pi)$. The trade-off that the individual faces is between the risky capital gain on housing and the utility premium on housing services as a home-owner, versus the less risky consumption granted by the safe return on savings as a tenant.

The value function of a household of type $\pi$ in the first period is thus:

$$V(\pi) = \max\left\{V_H(\pi), V_R(\pi)\right\}. \tag{3.8}$$

### 3.3.4 The real estate market

There is also a competitive real estate market, where a continuum of atomistic real estate agents act. In each period, a representative real estate agent buys housing both from non-defaulting exiting home-owners and from lenders

who have seized the houses of defaulting home-owners. The real estate agents refurbishes the housing stock at no cost, and sells it to the new, entering generation.

I assume that the whole housing stock, both owner-occupied and rental housing, are subject to the same distribution of value shocks. Then, in any given period, a fraction $\phi$ ends up as high value ($q_H$), and a fraction $1 - \phi$ as low value ($q_L$).

Competition drives the profits of the real estate agents to zero, so that the sales price of the refurbished housing to a generation entering in period $t$ is:

$$q_t = \phi q_H + (1 - \phi)q_L. \tag{3.9}$$

The real estate agents also rent part of the housing stock to the defaulting homeowners whose houses have been foreclosed, and households who have chosen to rent, in any given period $t$ at a rental rate of $s_t$, and bear the user cost $r_{t-1}q_{t-1}$ per unit of housing. Perfect competition then drives the rental rate down to equal the user cost of housing:

$$s_t = r_{t-1}q_{t-1}. \tag{3.10}$$

### 3.3.5 Timing

In every period $t$, a new generation enters. A generation indexed by its entry period $t = \tau$. Each generation lives for two periods. The timing within the two periods of a generation entering in $t = 0$ is outlined in Table 3.2.

## 3.4 Credit market equilibrium

The credit market intermediates funds to consumers who wish to become homeowners but who cannot self-finance their housing investment. Under the assumptions made in Section 3.3, all consumers receive an identical endowment in the first period of their life, so that all (or none) of those who wish to buy a house must borrow.

In this section, I characterise the credit market equilibrium by focusing on a Perfect Bayesian equilibrium. I derive first the optimal default decision of the borrower, given the loan amount and repayment, and next the terms of the loan contract offered by the lender, given the ex-ante expectation of default in the borrower pool. The equilibrium in the loan market in the first

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*Period 0*

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– The generation $\tau = 0$ is born and receives the first-period endowment $y_0$.

– The preceding generation $\tau = -1$ sells their housing stock to the real estate agents, consume, and exit.

– The new generation observes the house price $q_0$ and the income $y_0$, and make its housing choice. The cutoff type $\hat{\pi}_0$ is determined.

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*Period 1*

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– The house values $q_1 \in q_H, q_L$ as well as the income realisations $y_1 \in y_H, y_L$ are realised and observed by all agents.

– The homeowners make their optimal default choice.

– The successful homeowners sell their housing to the real estate agents at the price $q_1$ and consume $c^{nd}(y_1, q_1)$.

– The renters consume $c^r(y_1)$ and the foreclosed homeowners consume $c^d(y_1)$.

– The generation $\tau = 0$ exits while a new one enters.

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Table 3.2: Timing of events

period of a given generation is then characterised by the terms of the credit contract and a set of borrowers who accept this contract, given the ex-ante default probabilities. I analyse the equilibrium both under symmetric and asymmetric information in order to highlight the externality that causes over-investment.

### 3.4.1 Optimal default decision

In the second period, a home-owner will choose not to default if and only if:

$$u(c_{nd}(y_1, q_1)) + v(h) \geq u(c_d(y_1)) + \chi v(h) \tag{3.11}$$

for a given realisation $(y_1, q_1)$. Then, the ex-ante probability of no default conditional on the borrower type $\pi$, denoted by $p(\pi)$, is:

$$p(\pi) = \Pr\left\{u(c_{nd}(y_1, q_1)) + v(h) \geq u(c_d(y_1)) + \chi v(h) \mid \pi\right\} \tag{3.12}$$

$$= \Pr\left\{(1 - \chi)v(h) \geq u(c_d(y_1)) - u(c_{nd}(y_1, q_1)) \mid \pi\right\}. \tag{3.13}$$

In other words, the homeowner will not default if the utility premium from owner occupied housing relative to tenant occupied housing is greater than the utility in terms of consumption insurance acquired by defaulting.

Because $u'(c) > 0$ by assumption and $\frac{\partial c_{nd}(y_1,q_1)}{\partial y_1} = 1 > \frac{\partial c_d(y_1)}{\partial y_1} = 1 - \xi$ for $0 < \xi < 1$, the right-hand side of the inequality in expression (3.13) is decreasing in $y_1$. Thus the probability of no default, $p(\pi)$, is increasing with the probability of high income $\pi$. Correspondingly, the conditional default probability is $1 - p(\pi)$, which is decreasing in $\pi$.

### 3.4.2 Credit market equilibrium under symmetric information

As a benchmark, assume first that borrower types $\pi$, and thus also individual ex-ante default probabilities $p(\pi)$, are publicly observable. Then, a lender will offer a loan contract $\mathcal{C}_\pi$ characterised by an interest rate $r(\pi)$ to each type $\pi$. The symmetric information equilibrium thus provides the first-best benchmark for the credit market, since each consumer faces individual loan terms that reflect their true riskiness.

Because borrowers choose when to default strategically, the state $(y_1, q_1)$ under which default occurs, and thus the lender's return in the default state, are also endogenous and depend on the type. I define a *default set* $D(\pi)$ as the set of states of nature $(y_1, q_1)$ in which a borrower of type $\pi$ finds it ex-post optimal to default:

$$D(\pi) \equiv \{(y_1, q_1) \mid u(c_d^{FB}(y_1; \pi)) + \chi v(h) > u(c_{nd}^{FB}(y_1, q_1; \pi)) + v(h)\}.$$

Next, I define a lender's expected gross recovery rate, in case of default, on a loan given to a $\pi$-type as:

$$d(\pi) = E\left[\frac{q_1 h + \xi y_1}{l_0} \mid (y_1, q_1) \in D(\pi)\right]. \tag{3.14}$$

In the first period, each potential borrower will demand a loan of equal size, $l_0 = q_0 h - y_0$, because they do not value consumption in $t = 0$. The expected return to a lender on such a loan is then:

$$E\Pi(\pi) \equiv [p(\pi)(1 + r(\pi)) + (1 - p(\pi)) d(\pi)] l_0. \tag{3.15}$$

Perfect competition ensures that the expected return on each individual

loan is equal to the opportunity cost of the funds, $1 + \bar{r}$. The equilibirum rate offered to each $\pi$-type can then be solved from the zero-profit condition:

$$[p(\pi)(1 + r(\pi)) + (1 - p(\pi))d(\pi)]\,l_0 = (1 + \bar{r})l_0$$

$$\Leftrightarrow \quad 1 + r(\pi) = \frac{1 + \bar{r}}{p(\pi)} - \frac{1 - p(\pi)}{p(\pi)}d(\pi). \tag{3.16}$$

Given the offered rate $r(\pi)$, a borrower accepts the loan and becomes a homeowner if $V_H(\pi) \geq V_R(\pi)$, as given by equations (3.1) and (3.6); otherwise, she becomes a tenant. High-$\pi$ borrowers enjoy lower interest rates on their loans. At the limit, the borrower with $\pi = 1$ faces no income risk, but only a risk of the house losing its value.

Denote by the superscript *FB* (for "first-best") variables under the symmetric information equilibrium. Given the rate $r(\pi)$ and using the budget constraints (3.4) and (3.5), the first-best consumption of a type-$\pi$ homeowner in the no-default (*nd*) and default (*d*) states, respectively, are:

$$c_{nd}^{FB}(y_1, q_1; \pi) = (1 + r(\pi))\,y_0 + y_1 + \Delta q h - r(\pi)q_0 h \tag{3.17}$$

$$c_d^{FB}(y_1; \pi) = (1 - \xi)y_1 - \kappa - s_1^{FB}h, \tag{3.18}$$

and the consumption of a tenant is:

$$c_r^{FB}(y_1; \pi) = y_1 + (1 + \bar{r})y_0 - s_1^{FB}h. \tag{3.19}$$

In the above, I assume that the rental rate under symmetric information is $s_t^{FB} = s_t = r_{t-1}^* q_{t-1}$, where $r_t^*$ is the equilibrium pooling rate under asymmetric information. This assumption ensures that the value of renting is equal under symmetric and asymmetric information for all types. In other words, the opportunity cost of owning is kept constant when comparing the two economies.

**Proposition 1** *The credit market equilibrium under symmetric information is characterised by a set of loan contracts $\mathcal{C}_\pi$, characterised by an individual interest rate $r(\pi)$, and a cut-off type $\hat{\pi}^{FB} \in [0, 1]$ such that all individuals with $\pi \geq \hat{\pi}^{FB}$ accept $\mathcal{C}_\pi$ and become home-owners, and all individuals with $\pi < \hat{\pi}^{FB}$ become tenants, and the lenders break even in expectation for every type $\pi$ individually.*

**Proof** In Appendix 3.A.

The first-best, or efficient, cutoff type $\hat{\pi}^{FB}$ is just indifferent between buying and renting, and is characterised by:

$$V_H^{FB}(\hat{\pi}^{FB}) = V_R^{FB}(\hat{\pi}^{FB})$$

$$\Leftrightarrow \quad E\left[p(\hat{\pi}^{FB})\left(u(c_{nd}^{FB}(\hat{\pi}^{FB})) + v(h)\right) + \right.$$

$$\left. (1 - p(\hat{\pi}^{FB}))\left(u(c_d^{FB}) + \chi v(h)\right) \mid \hat{\pi}^{FB}\right] =$$

$$E\left[u(c_r) + \chi v(h) \mid \hat{\pi}^{FB}\right]$$

$$\Leftrightarrow \quad p(\hat{\pi}^{FB}) = \frac{E\left[u(c_r^{FB}) - u(c_d^{FB}) \mid \hat{\pi}^{FB}\right]}{E\left[u(c_{nd}^{FB}(\hat{\pi}^{FB})) - u(c_d^{FB}) + (1 - \chi)v(h) \mid \hat{\pi}^{FB}\right]}. \quad (3.20)$$

The credit market equilibrium under symmetric information is efficient and establishes a first-best benchmark. There is efficient risk-sharing because the cost of borrowing of each type correctly reflects their true default risk. The implications of this feature are further discussed in Section 3.6.

### 3.4.3 Credit market equilibrium under asymmetric information

Next, I return to the assumption that a borrower's type is unobservable to the lender. I focus on a particular Perfect Bayesian equilibrium in pure strategies characterised by pooling of borrowers.

Again, all borrowers demand a loan of equal size, $l_0 = q_0 h - y_0$, because consumption in $t = 0$ is not valued by any consumer and hence the whole endowment is either saved into the safe asset or offered as down payment on a house. However, now low-$\pi$ borrowers have the incentive to mimic high-$\pi$ borrowers in order to get a loan with favourable terms. This is due to the consumption insurance given by limited liability in case of default. Because all consumers have an identical initial income $y_0$ and all demand an equal sized loan, the high types are not able to signal their type to the lenders. There exists now an equilibrium in which the lender offers a common interest rate $r_0$ to all potential borrowers, such that he makes in expectation a non-negative expected profit on the pool of loans.

In this case, because the loan interest rate is the same for all types, the ex-post optimality of default does not depend on type. All consumers then find it optimal to default in the same states, and their default sets do not depend on type. I therefore assume that the lender's expected recovery rate on a loan

is the same across all individuals; I denote this expected recovery rate on the whole loan pool by $\bar{d}$.

The equilibrium interest rate is determined by the break-even condition of the lenders. The expected return of a lender who charges an interest rate $r_0$, given the pool of loan applicants characterised by the cutoff $\hat{\pi}$, is given by:

$$E\Pi \equiv \left( E\left[ p(\pi) \mid \pi \geq \hat{\pi} \right] (1 + r_0) + E\left[ (1 - p(\pi)) \mid \pi \geq \hat{\pi} \right] \bar{d} \right) l_0. \tag{3.21}$$

In equilibrium, the competition in the credit market drives the return on the loan down to equal the market rate, or the opportunity cost of the funds. The equilibrium interest rate offered to all loan applicants, denoted by $r^*$, can again be solved from the zero-profit condition:

$$E\Pi = (1 + \bar{r})l_0$$

$$\Leftrightarrow \quad 1 + r_0 = \frac{1 + \bar{r}}{E\left[ p(\pi) \mid \pi \geq \hat{\pi} \right]} - \frac{E\left[ (1 - p(\pi)) \mid \pi \geq \hat{\pi} \right]}{E\left[ p(\pi) \mid \pi \geq \hat{\pi} \right]} \bar{d}$$

$$\equiv 1 + r^*. \tag{3.22}$$

Given the offered rate $r_0 = r^*$, an individual accepts the contract $\mathcal{C}$ if $V_H(\pi; r^*) \geq V_R(\pi)$. The cutoff type $\hat{\pi}$ is implicitly determined by:

$$V_H(\hat{\pi}; r^*) = V_R(\hat{\pi})$$

$$\Leftrightarrow \quad p(\hat{\pi}) = \frac{E[u(c_r(y_1)) - u(c_d(y_1))|\hat{\pi}]}{E[u(c_{nd}(y_1, q_1, r^*)) - u(c_d(y_1)) + (1 - \chi)v(h)|\hat{\pi}]}, \tag{3.23}$$

where $c_{nd}, c_d$ and $c_r$ are given by equations (3.5), (3.4) and (3.7), respectively.

**Proposition 2** *The credit market equilibrium under asymmetric information is characterised by a pooling contract $\mathcal{C}$, characterised by a common interest rate $r^*$, and a cut-off type $\hat{\pi} \in [0, 1]$ such that all individuals with $\pi \geq \hat{\pi}$ accept $\mathcal{C}$ and become home-owners, all individuals with $\pi < \hat{\pi}$ become tenants, and the lenders break even in expectation on the whole pool of loans given $\hat{\pi}$.*

**Proof** In Appendix 3.A.

**Proposition 3** *In equilibrium, $\hat{\pi} \leq \hat{\pi}^{FB} \forall \hat{\pi}$ when the ratio of expected recovery rates $\frac{d(\hat{\pi})}{\bar{d}}$ is small enough.*

**Proof** In Appendix 3.A. The exact condition for $\frac{d(\hat{\pi})}{\bar{d}}$ is also given in the Appendix.

Proposition 3 implies that there is over-borrowing in equilibrium. This result is discussed in more detail in Section 3.6.1. The intuition for the condition on $\frac{d(\hat{\pi})}{d}$ is the following. The greater the expected recovery rate in case of default, the lower the interest rate charged on the loan is. The condition then guarantees that $r(\hat{\pi}) \geq r^*$, i.e. that the marginal borrower would pay a higher interest on the loan if her type were publicly observable. In this case, she would no longer be indifferent on entering the credit market, and would become a tenant instead. I assume that this is the case in all of the subsequent analysis.

### 3.4.4 Robustness of the credit market equilibrium

The pooling equilibrium described in this section rests on very strict assumptions. Three assumptions, in particular, are crucial: first, the fixed size of housing; second, the absence of consumption in the first period; and third, perfect competition in the credit market. The first two assumptions prevent any signaling of private types by borrowers in the first period. The third one places a bound on the interest rates that a lender can offer. The assumption of equal initial endowments, on the other hand, could be relaxed to also allow for idiosyncratic income in the first period, as long as this income is not correlated with type (as in the second period). Because of these assumptions, lenders cannot infer anything about the loan applicants' types when offering the loan contracts, and pooling arises in equilibrium.

The pooling equilibrium also crucially depends on the set of available loan instruments and on market incompleteness. Here, I have assumed that only standard debt contracts are available.[8] Then, under the assumptions above, no signaling by applying for loans for houses of different size or entailing different down payments is possible. But other types of contracts could allow for the separation of types in equilibrium.

The possibility of default offers partial consumption insurance if the individual is faced with adverse shocks, i.e. some state contingency. The absence of complete markets makes the loan contract lucrative for the low types. If lenders were allowed to offer state-contingent debt contracts, with higher interest rates but lower repayment ratios in case of adverse shocks, some types

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[8]De Meza and Webb (1999) argue that the standard debt contract can be optimal under certain conditions in a model of private types. In particular, assume that only the debt repayment is *verifiable*, and not the wealth of the borrower, when the debt is redeemed. However, in case of default, assume that the borrower's wealth can be also verified at no cost. Under such circumstances the standard debt contract is the optimal contract.

with a high income risk would find it optimal to self-select into these contracts instead.

Another possibility would be to allow for unsecured debt contracts. Then, a lender could offer contracts with a higher interest rate but no collateral alongside the collateralised debt contract. Again, this would offer better insurance for the lower types and could result in a separating equilibrium.

Finally, as noted for example by Tirole (2006, Ch. 6.5), it could be optimal for a lender to merely pay a lump sum compensation for the worst types to stay out of the credit market.

However, none of these features — state-contingent debt contracts, unsecured debt, or lenders offering to pay potential applicants in order for them *not* to apply — are observed in actual markets for housing credit. I have chosen to make very strict assumptions in order to sustain a pooling equilibrium, because it offers a convenient way to define subprime borrowing in a theoretical context.

Although pooling is a very strong and not necessarily realistic equilibrium outcome, *some* pooling of applicants seems likely to have been present in the U.S. housing market prior to the financial crisis. Indeed, Demyanyk and Van Hemert (2011) provide empirical evidence from loan-level data that the mortgage interest spread between prime and subprime borrowers narrowed significantly over the boom years, even after controlling for borrower and loan characteristics. At the same time, controlling for these same characteristics, they show that the quality of loans deteriorated. This should have been reflected in an *increase* of the spread, when in fact the opposite was observed.

Finally, the assumption of perfectly competitive credit markets is crucial in sustaining the pooling equilibrium. Competition drives down the interest rate in such a way that no lender is able to deviate from the equilibrium by offering contracts with higher interest rates in an attempt to screen loan applicants. It is in essence the combination of private types and perfect competition that drives the selection of subprime types in the market and creates the over-investment externality.[9]

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[9]This result links to a debate on the relationship between bank competition and financial stability. On the one hand, Ruckes (2004) and Dell'Ariccia and Marquez (2006) argue that an increase in competition may induce lenders to lower their lending standards in order to gain market shares. On the other hand, Gorton and He (2008) show that strategic bank competition can cause endogenous fluctuations in lending standards when screening applicants is costly. More competition then leads to tighter lending standards. Dell'Ariccia et al. (2012) find that lending standards are tighter in the subprime loan market when there is more competition.

## 3.5 Parametrisation and model solution

In this section, I first characterise the equilibrium of the overlapping generations model, and then outline the solution method of the model. The equilibrium is a fixed-point problem, and I solve for it numerically; I discuss here the particular assumptions on functional forms and parameter values made in the computation.

### 3.5.1 Equilibrium and aggregation

The equilibrium is an allocation $\{c_{t+1}^\tau, \hat{\pi}_t, h_t, r_t^*, q_t, s_t\}_{t=0}^\infty$ such that given the interest rate $r_t^*$, the cut-off $\hat{\pi}_t$ satisfies the condition (3.23) and the household consumption plan is given by $c_{t+1}^\tau = \{c^{nd}, c^d, c^r\}$, defined by equations (3.5), (3.4) and (3.7); the loan interest rate $r_t^*$ satisfies (3.22) given $\hat{\pi}_t$; the house sales price $q_t$ satisfies (3.9); the rental rate $s_t$ satisfies (3.10); and the following conditions hold:

$$Y^\tau = C^\tau + H^\tau \tag{3.24}$$

$$\bar{h} = \int_0^1 h \, d\pi, \tag{3.25}$$

where equation (3.24) is the aggregate consistency condition, and (3.25) is the housing market clearing condition; $\bar{h}$ denotes the aggregate housing stock. $C^\tau$ is the aggregate consumption of goods, $H^\tau$ is the aggregate consumption of housing services, and $Y^\tau$ is the aggregate income of generation $\tau$, born in period $\tau = t$. They are defined as:

$$C^\tau = \int_0^{\hat{\pi}_\tau} c_{t+1}^r \, d\pi + (1 - \gamma_\tau) \int_{\hat{\pi}_\tau}^1 c_{t+1}^d \, d\pi + \gamma_\tau \int_{\hat{\pi}_\tau}^1 c_{t+1}^{nd} \, d\pi \tag{3.26}$$

$$H^\tau = \int_0^{\hat{\pi}_\tau} s_{t+1}^\tau \, d\pi + (1 - \gamma_\tau) \int_{\hat{\pi}_\tau}^1 s_{t+1}^\tau \, d\pi + \gamma_\tau \int_{\hat{\pi}_\tau}^1 r_t^* q_t \, d\pi \tag{3.27}$$

$$Y^\tau = \int_0^{\hat{\pi}_\tau} [(1 + \bar{r})y_t + y_{t+1}] \, d\pi + (1 - \gamma_\tau) \int_{\hat{\pi}_\tau}^1 (y_{t+1} - \kappa) \, d\pi$$

$$+ \gamma_\tau \int_{\hat{\pi}_\tau}^1 [(1 + r)y_t + y_{t+1} + \Delta q_{t+1}] \, d\pi \tag{3.28}$$

where $\hat{\pi}_\tau$ denotes the cutoff type in generation $\tau$, and $\gamma_\tau$ the ex-post fraction of non-defaulting home-owners in generation $\tau$ in the second period of

their life. By the law of large numbers, $\gamma_\tau \to E[p(\pi)|\pi \geq \hat{\pi}]$.

### 3.5.2 Functional forms

I assume log utility $u(\cdot) = v(\cdot) = \log(\cdot)$ and a uniform distribution of types with $\pi \sim Uniform(0,1)$. Under the latter assumption, $f(\pi) = 1$ and $F(\pi) = \pi$ denote the p.d.f. and c.d.f. of $\pi$, respectively. These simple functional forms allow for an analytical solution of the equilibrium default probability and the equilibrium interest rate as well as a simple characterisation of the cut-off type.

In particular, the optimal default choice (3.11) is given by the condition:

$$u(c_d(y_1)) + \chi v(h) > u(c_{nd}(y_1,q_1)) + v(h)$$
$$\Leftrightarrow \quad \log(c_d(y_1)) + \chi \log(h) > \log(c_{nd}(y_1,q_1)) + \log(h)$$

By substituting in the budget constraints (3.4) and (3.3), this condition can be solved for:

$$y_1 + \Phi q_1 h < \Phi\left((1+r^*)l_0 - \frac{\kappa}{h^{1-\chi}}\right)$$

where $\Phi \equiv \left(1 - \frac{1-\xi}{h^{1-\chi}}\right)^{-1} > 0$.

Therefore, the ex-ante optimal default probability of a type $\pi$ is given by

$$\Pr\{\text{'default'} \mid \pi\} = \Pr\left\{y_1 + \Phi q_1 h < \Phi\left((1+r^*)l_0 - \frac{\kappa}{h^{1-\chi}}\right) \mid \pi\right\}$$
$$\equiv 1 - p(\pi).$$

The borrower thus optimally defaults if and only if the weighted sum of $y_1$ and $q_1$ realises a low enough value. In other words, default is optimal in the joint event of both $y_1$ and $q_1$ realising a low enough value. In particular, I assume that in the benchmark parametrisation, default is optimal when $(y_1, q_1) = (y_L, q_L)$.

The ex-ante default probability of a borrower from a lender's point of view, given the borrower cut-off type, is then:

$$E\left[(1-\phi)(1-\pi)|\pi \geq \hat{\pi}\right] = 1 - \phi - (1-\phi)E\left[\pi|\pi \geq \hat{\pi}\right]$$

| Parameter | Value |
|---|---|
| Safe interest rate $\bar{R}$ | 1.04 |
| Utility from owner-occ. h $v_H(h)$ | 0.2 |
| Lender appropriation rate $\xi$ | 0.288 |
| High house value prob. $\phi$ | 0.915 |
| Default DWL cost $\kappa$ | 2.08 |
| $q_0/y_0$ | 2.94 |
| $q_H/q_0$ | 1.07 |
| $q_L/q_0$ | 0.24 |
| $y_H/y_0$ | 1.43 |
| $y_L/y_0$ | 0.71 |

Table 3.3: Baseline parameter calibration

$$= 1 - \phi - (1 - \phi)\frac{\int_{\hat{\pi}}^1 \pi f(\pi)\,\mathrm{d}\pi}{1 - F(\hat{\pi})}$$

$$= \frac{1}{2}(1 - \phi)(1 - \hat{\pi}),$$

and correspondingly the no-default probability is $E[1 - (1 - \phi)(1 - \pi)|\pi \geq \hat{\pi}] = \frac{1}{2}(1 + \phi + (1 - \phi)\hat{\pi})$. Finally, the equilibrium pooling interest rate (3.22) in this case is:

$$1 + r^* = \frac{1 + \bar{r}}{\frac{1}{2}(1 + \phi + (1 - \phi)\hat{\pi})} - \frac{(1 - \phi)(1 - \hat{\pi})}{1 + \phi + (1 - \phi)\hat{\pi}}\frac{q_L h + \xi y_L}{l_0}, \qquad (3.29)$$

### 3.5.3 Numerical calibration and solution

The equilibrium of the model is found as a solution to a fixed point problem in $\pi$, given by equation (3.23), together with the zero-profit condition (3.22) that determines the equilibrium interest rate $r^*$. In general, no analytic solution for this problem exists. I therefore solve for the equilibrium allocation numerically by using an iteration algorithm. The solution algorithm is described in Appendix 3.B.

The parameter values used in the numerical model solution and simulation are given in Table 3.3. They are chosen to mach some key long-run housing market statistics from the U.S. data, shown in Table 3.4, alongside with the corresponding statistics generated by the model under the calibration given

| Variable | Data | Model |
|---|---|---|
| Safe interest rate | 1.04 | 1.04 |
| Homeownership rate | 0.66 | 0.66 |
| Average recovery rate | 0.5 | 0.48 |
| Delinquency rate | 0.023 | 0.028 |
| Loan-to-value ratio | 0.7 | 0.66 |
| House prices peak-to-trend | 1.29 | 1.07 |
| House price trough-to-trend | 0.72 | 0.24 |
| Real personal inc. 1975-2015 | 1.33 | 1.43 |
| Income loss at unempl. | 0.76 | 0.71 |
| Loan spread | 162 bp | 162 bp |
| Household indebtedness ratio | 1.30 | 1.94 |
| Rent-to-income ratio | 0.38 | 0.20 |
| Default only when $(y_L, q_L)$ | ✓ | ✓ |

Table 3.4: Data moments and model counterparts

in Table 3.3. The data are annual and include data series on house prices, the homeownership rate, the mortgage delinquency rate and real personal income.

In the data, the changes in house prices are computed as percentage deviations of the Case-Shiller national home price index from its long-run trend in 2006 (the peak-to-trend ratio) and in 2012 (the trough-to-trend ratio). The trend is computed as a linear trend from the sample over 1975–2014. In 2006, at the peak of the house price cycle, house prices were 29% above the long-run trend, and in 2012, in the trough, they were 28% below trend.

The homeownership rate in the U.S. was 66% on average over the sample period 1991–2015. The delinquency rate is the average delinquency rate on all single-family residential mortgages over 1991–2007, *before* the foreclosure crisis; it was on average 2.3% over this period. I have omitted the crisis period from the sample period in order to capture the long-run average delinquency rate without this rather unusual episode. Real income growth is measured as the growth of the real median personal income growth over 1975–2015, which was 33%.

The model matches all of these long-run averages quite well, with the exception of house prices. In the model, the relative prices $q_H/q_0$ and $q_L/q_0$ are crucial for the default incentives and the homeownership decisions of households. With the parametrisation given here, a low house value realisation is

relatively unlikely, but the low house value is much lower than in the data; the high house value is also parametrised to be lower than the peak observed in the data. These assumptions are needed in order to discourage enough low types from entering the credit market and to match the homeownership and delinquency rates.

According to Ospina and Uhlig (2016), the average loan-to-value ratio at origination over 2006–2012 was 0.7. The model approximately matches this value. Hayre and Saraf (2008) estimate that losses in the event of default range from 35% to 60%. I choose parameter values that produce a recovery rate of 0.48, which is well in the range of these estimates, and also consistent with the value 0.5 used in Corbae and Quintin (2015).

In line with the results in Corbae and Quintin (2015), who also find that borrowers defaulted on mortgage loans during the U.S. foreclosure crisis only when they were sufficiently underwater on their home equity *and* suffered a negative income shock, I choose parameters such that default occurs in equilibrium only when $y_1 = y_L$ and $q_1 = q_L$, i.e. when the home-owner suffers both an adverse income and an adverse house value shock at the same time. In other states of the world, the borrower has no incentive to default. In this case, the default probability of a type $\pi$ is $1 - p(\pi) = (1 - \phi)(1 - \pi)$, which is the joint probability of the event $(y_1, q_1) = (y_L, q_L)$.

Finally, based on figures reported in Kawano and LaLumia (2017), annual income loss at unemployment is at around 76% in the data. I choose the income parameters such that the ratio $\frac{y_L}{y_0}$ roughly replicates this figure.

Given the stylised structure and timing assumptions of the model, it is hard to pin down realistic values for the interest rates and spreads. The safe gross interest rate is set to 1.04. This is a rather low value, but ensures the existence of a pooling equilibrium.[10] The spread between 30-year fixed rate mortgages and the 30-year Treasury bond, on a per annum basis, has been on average 162bp over the sample 1977–2002, before the financial crisis and the boom. The model is consistent with this figure. Household indebtedness, measured as the ratio of total household debt to disposable income, was around 130% at the peak of the boom in 2006. The model produces a ratio somewhat higher than this peak observed in the data. Finally, national ac-

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[10]The debt is modelled as a one-period loan in the model, which is unrealistic for a mortgage. On the other hand, in the model, each generation of households only lives for two periods. This timing is hard to reconcile with typical mortgage maturities and interest rates in the data. This horizon for the household can, however, be interpreted as a period over which the borrower holds a given mortgage, or a time span over which the mortgage is not refinanced, rather than a whole lifetime.

count data on household consumption expenditures shows a share of around 40% going towards rents, whereas the model produces a somewhat lower figure.

Taken together, the figures in Tables 3.3 and 3.4 show that the model, while simple, can replicate some of the long-run averages in the U.S. housing market data. In the next section, I look at some comparative statistics to gauge further how the selection into homeownership depends especially on the safe interest rate $\bar{r}$, the housing value risk $\phi$, and the penalty on income at default $\xi$.

## 3.6 Equilibrium characteristics

In this section, I describe some equilibrium features of the model: over-investment in housing, the source of this inefficiency, and the links between default rates, house prices, and interest rates.

### 3.6.1 The over-investment externality and subprime borrowing

The key feature of models of the credit market with a De Meza and Webb (1987) type of information structure is that there is over-investment compared to the first-best whenever there is a non-zero possibility of borrower default.

The marginal type $\hat{\pi}$ is the riskiest type in the loan pool, meaning that the loan pool is riskier than it would be under symmetric information. The set of borrowers of borrowers that are able to get a loan under asymmetric information, but would not do so under symmetric information can be called *subprime* borrowers. They are borrowers whose loans are not socially optimal. In particular, under the assumption of uniform distribution of types, this set is equal to the share $\hat{\pi}^{FB} - \hat{\pi}$. Similarly, under uniformly distributed types, the homeownership rate is defined as $1 - \hat{\pi}$.

This is illustrated in Figure 3.2, which shows the value functions of a tenant, $V_R(\pi)$, and a home-owner, $V_H(\pi)$, as a function of type $\pi$, together with the value function of a homeowner under symmetric information, $V_H^{FB}(\pi)$. The value function of a tenant is by assumption on the rental rate equal under both symmetric and asymmetric information. All value functions are increasing in the type: a high type has, on expectation, a higher consumption in the second period because of the lower income risk.

Two notable features of the equilibrium are shown in the figure. First, there is a discontinuity in the homeowner's value function under symmetric
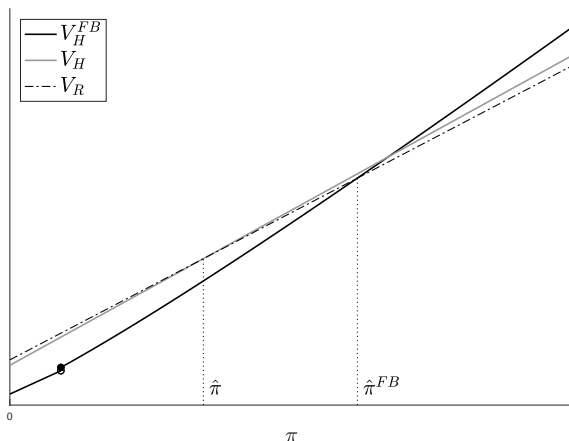
Figure 3.2: Value functions and cut-offs under symmetric and asymmetric information

information, $V_H^{FB}(\pi)$. This is because the lowest types, left of the jump, default whenever they face the low house value shock $q_1 = q_L$, regardless of their income $y_1$.[11] This high default probability is reflected in a very high interest rate offered to them, which makes borrowing very expensive.

This discontinuity is absent in the corresponding value function under asymmetric information, $V_H(\pi)$. This reflects the externality associated with entry into the credit market. If an agent of type $\pi$ enters the market and takes a loan, it yields her an expected marginal utility of consumption $u'(c_{nd})$ with probability of $p(\pi)$, i.e. if she does not default. This expectation is the higher, the higher the type. The cost of entry, however, does not depend on type: the pooling interest rate $1 + r^*$ is the same for all entrants.

However, by entering the market, the marginal borrower makes the pool marginally riskier, and thus increases the interest rate faced by all other agents in the credit market as well. It is this change in the interest rate $r^*$ induced by entry, which affects the consumption available in the no-default state for every borrower, that the marginal borrower does not internalise under asymmetric information. In the symmetric information case, the interest rate $1 + r(\pi)$ correctly reflects each type's riskiness, such that the externality disappears.

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[11]The jump corresponds to a switch from a larger default set $D(\pi)$ to a smaller one.

The second important feature is *cross-subsidisation* induced by credit market pooling. The highest types are better off under symmetric information, as their cost of borrowing is lower than the pooling rate. Conversely, there is a set of types that would not enter the credit market under symmetric information, but do so under asymmetric information. The low types are better off under asymmetric information, and their borrowing is being subsidised by the high types.

### 3.6.2 Comparative statics

To gauge how the the over-borrowing externality depends on the various parameters of the model, I perform a comparative statics exercise. I change three key parameters of the model one by one while keeping all other parameters fixed. The parameters of interest are the probability of a high house value realisation $\phi$, the safe interest rate $1 + \bar{r}$, and the lender appropriation rate $\xi$.

The extent of the market failure is quantified in Figures 3.3, 3.4, and 3.5, respectively. They show how the marginal types $\hat{\pi}$ and $\hat{\pi}^{FB}$, the ex-ante default probability of default in the loan pool observed by the lender, $E[1 - p(\pi)|\pi \geq \hat{\pi}]$, and the equilibrium pooling loan interest rate $r^*$ change as the three key model parameters change.

As Panel (a) of Figure 3.3 shows, the model exhibits a non-linear (inverse U-shape) relationship between the probability of a high house value realisation $\phi$ and the marginal type, all other parameter being fixed at their benchmark values. Strikingly, the homeownership rate is 100% (i.e. the marginal type is $\hat{\pi} = 0$) both when a high house value is very unlikely and very likely.

At the limit, when the high house value $y_H$ will never occur, i.e. when $\phi = 0$, the purchase price of the house is equal to the low value: $q_0 = q_L$. Then, buying a house is very cheap, and there is no price risk involved. As a consequence, owning is very attractive to all types, even those with low expected income. Even as $\phi$ becomes positive, for low values, $q_0 = \phi q_H + (1 - \phi)q_L$ remains low, and buyers are able to self finance (i.e. $q_0 h \leq y_0$). The threshold under which households are able to self-finance the purchase is marked by a vertical dashed line.

As $\phi$ increases, the purchase price $q_0$ and loan amounts also increase. At first, the worst types who have the lowest expected income start opting out and renting instead. However, default remains non-optimal for all types in the borrower pool, because the borrowers are not very levered and the cost of default exceeds the cost of servicing the loan.
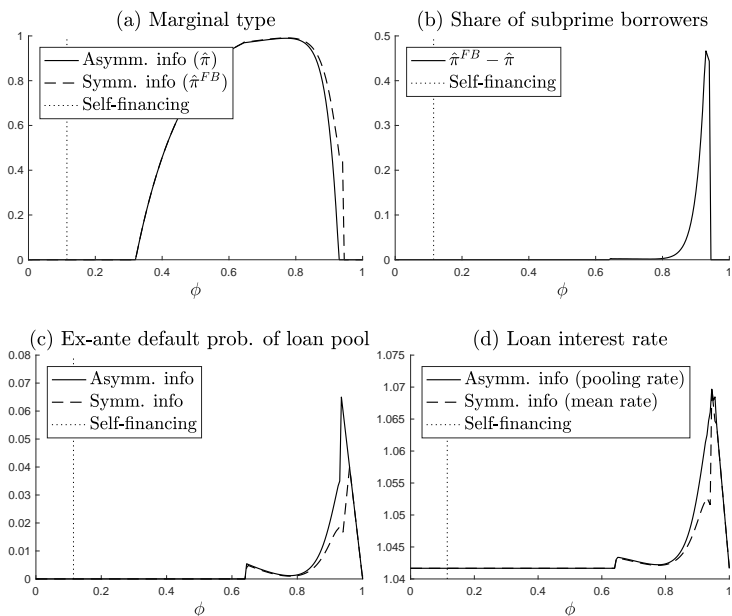
Figure 3.3: Equilibrium cut-off type, subprime borrowing, default probability of the loan pool, and loan interest rate as a function of the house value risk $\phi$

Eventually, as $q_0$ keeps rising, default becomes optimal for some types. There is a discontinuous jump of the ex-ante default probability of the borrower pool away from zero. At this point, as defaults become a non-zero probability event, the over-investment externality kicks in, and the asymmetric information equilibrium allocation no longer replicates the efficient one. As the probability of the high house value and therefore the purchase price $q_0$ keep rising, the worst types keep opting out of the credit market, and thus the borrower pool becomes safer. Although defaults still happen, the pool becomes less risky.

However, as the high house value becomes likely enough, riskier types with worse income expectations are drawn back to the credit market. Consequently, the ex-ante default rate sharply increases. It is in this region of increasing market participation that the over-investment externality is at its

worst. For a given value of $\phi$, the vertical distance of the two curves measures the amount of subprime borrowing. This difference between the two curves in Panel (a) is plotted in Panel (b). It can also be interpreted to measure the looseness of lending standards relative to first-best: it shows that lending standards are loose exactly when high future house values are relatively likely. The loan interest rates, depicted in Panel (d), depend on the default probabilities, shown in Panel (c).

For very high values of $\phi$, in the range 0.95–0.99, homeowners default whenever they are hit by a low house value under asymmetric information as well, regardless of their income realisation, because they are so highly levered. Then the ex-ante default probability is no longer dependent on the composition of the borrower pool. In this region, the inefficiency thus disappears.

These results are consistent with empirical evidence presented by Mian and Sufi (2011), who study the impact of exogenous house price increases on household borrowing. They show that low credit quality households increased their borrowing by much more than high credit quality households in U.S. regions where housing supply was inelastic. These regions experienced the most significant house price increases in 2002–2006. Prior to 2002, however, borrowing by low credit quality households did not increase although house prices were already increasing. The authors also show that in regions with inelastic housing supply, although the increase in house prices started already around 2000, the total indebtedness of all households (the total debt-to-income ratio) actually decreased slightly from 2000 until 2002, and started increasing only in 2003. This suggests that an exogenous increase in house prices first led to a slight decrease of household indebtedness, and subsequently in a strong increase, with low quality borrowers increasing their borrowing the most.

Figure 3.4 shows a similar exercise of comparative statics for the safe interest rate $1 + \bar{r}$, all other parameter being fixed at their benchmark values. The baseline value is $1 + \bar{r} = 1.044$.

The marginal type is increasing in the safe interest rate, as depicted in Panel (a) of the figure. When $\bar{r}$ is low, the cost of lending as well as the outside return for savings is low as well; therefore purchasing a house is very attractive. As the safe interest rate increases, borrowing becomes more expensive and the outside return better; this leads to the worst types opting out of the credit market. The inefficiency is worst for relatively low values of $\bar{r}$, as shown in Panel (b). In the symmetric information economy, each agent's cost of funding reflects their true riskiness through a type-dependent margin over the safe rate, which increases faster for low types than when they face the pooling interest rate.
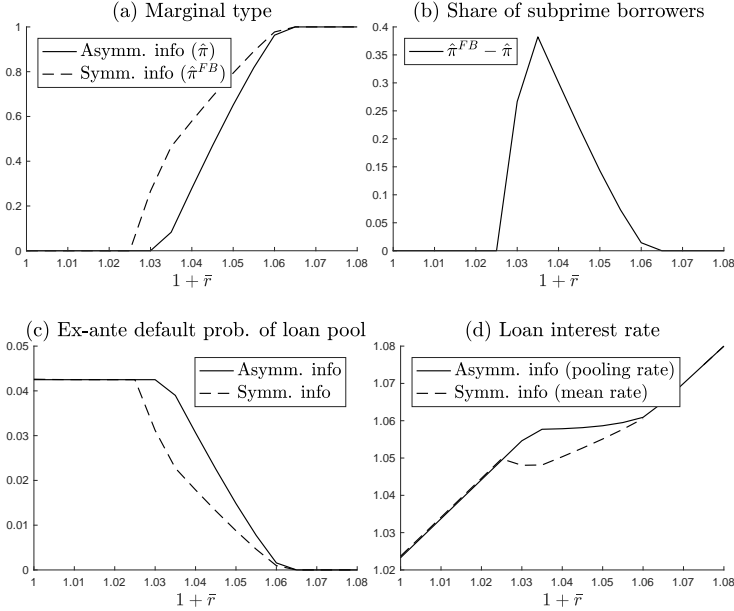
Figure 3.4: Equilibrium cut-off type, subprime borrowing, default probability of the loan pool, and loan interest rate as a function of the safe interest rate $1 + \bar{r}$

Finally, Figure 3.5 shows a comparative statics exercise with respect to the lender appropriation rate $\xi$. When $\xi = 0$, the lender cannot seize any of the second-period income of the borrower in case of default, and when $\xi = 1$, he can seize all of it, in addition to the house. The baseline value is $\xi = 0.288$. For values below $\xi = 0.25$, no equilibrium exists given the rest of the parameter values in the benchmark calibration; thus only the range $\xi \in [0.25, 1]$ is reported in the figure.

This exercise shows that stricter regulation in case of default – as $\xi$ increases – mitigates incentives for default, inPanel (c). At around $\xi = 0.45$, no households wants to take out a loan and risk losing such a share of their income in case of default. But this also means that the default regulation is so strict that the homeownership rate goes down to zero, as shown in Panel (a)

(a) Marginal type
Asymm. info ($\hat{\pi}$)
Symm. info ($\hat{\pi}^{FB}$)

(b) Share of subprime borrowers
$\hat{\pi}^{FB} - \hat{\pi}$

(c) Ex-ante default prob. of loan pool
Asymm. info
Symm. info

(d) Loan interest rate
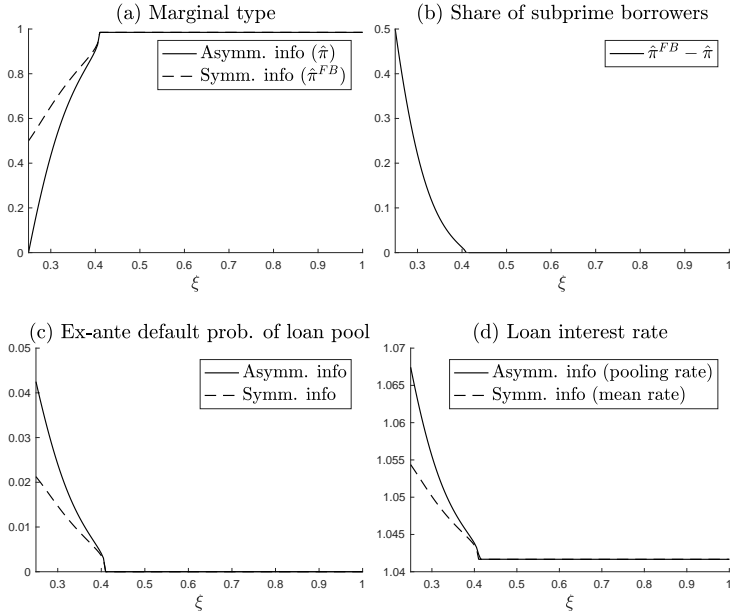Asymm. info (pooling rate)
Symm. info (mean rate)

Figure 3.5: Equilibrium cut-off type, subprime borrowing, default probability of the loan pool, and loan interest rate as a function of the lender appropriation rate $\zeta$

of the figure. It also affects equally all borrowers, regardless of their default risk.[12]

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[12]De Meza and Webb (1990) suggest that a 100% tax on capital gains, with full offset of losses, would restore an efficient allocation. This type of scheme would in essence eliminate the risk, but also the gain, from the risky investment in housing. All types would then be indifferent between housing and the safe asset, and there would be no incentive to default. With such a tax scheme, the government would effectively pool together the idiosyncratic risks and provide full consumption insurance for the households. However, this type of policy on housing gains and losses does not seem feasible in practice.

### 3.6.3 Implications for monetary policy and financial regulation

The characteristics of the equilibrium described in this section have some poignant policy implications. The comparative statics exercises highlight that the over-investment externality is particularly severe first, when house values are relatively high on expectation, which is reflected in high purchase prices; and second, when interest rates are relatively low. These are exactly the type of conditions that prevailed in the U.S. during the housing boom. Under these conditions, households have strong incentives to become very highly levered, making the economy vulnerable to adverse income or house price shocks through an increased likelihood of default.

In terms of policy, first, there is a direct link between the over-investment externality and the conduct of monetary policy. A low interest rate regime directly fuels households' incentives to take on debt, which is not surprising. Notably, the effective Federal Funds rate was low, at around 1%, in 2002–2003 at the height of the housing boom.

Second, the cross-subsidisation of high-risk borrowers is at the heart of the externality. In the U.S., as well as many other countries, promoting home-ownership by supporting cheap and accessible mortgage borrowing has been a political priority for decades. This policy has promoted lenient lending standards and made borrowing persistently too cheap for many households (Financial Crisis Inquiry Commission 2011).

Finally, during the housing boom years of the early 2000s, it was not uncommon for lenders to offer low "teaser rates" on new adjustable-rate mortgage loans (ARMs), further increasing the attractiveness of these loans in the eyes of many households. When the interest rates on the loans increased after the teaser period, many borrowers found themselves unable to service their debt. This suggests that they would not, or perhaps should not, have borrowed on terms that truly reflected their credit risk.[13]

There is also scope for macroprudential policy that curbs households' incentives to borrow. Removing favourable tax treatment of housing loans, imposing loan-to-value or loan-to-income constraints and making lenders responsible for properly testing the creditworthiness of potential borrowers are examples of policies that directly affect the incentives to borrow, and thus render the economy as a whole more robust to adverse shocks.

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[13]Financial Crisis Inquiry Commission (2011) finds that so-called "option ARM" loans, which even allowed for negative amortisation, accounted for nearly 10% of mortgage originations in 2005 and 2006.

## 3.7 An endogenous boom-bust cycle

The housing market is an asset market that is prone to experiencing boom-bust episodes. First, housing supply (new housing starts) typically adjusts only slowly to demand shocks due to, for example, zoning and time-to-build issues. Second, the market is rather opaque and illiquid compared to many other asset markets, because of high transaction costs and the inability to sell short. This can lead to persistent deviations from equilibrium prices and to long-lasting cycles.

This point is made by Crowe et al. (2013), who document 51 real estate boom-bust episodes in a sample of 40 countries over 2003–2009. Cerutti et al. (2017) analyse a sample of 53 countries over the years 1970–2012, and find that in most years, between 20 to 60% of the countries were experiencing a house price boom; most countries in their sample experienced at least one boom episode over this period. More than half of the housing boom episodes in their sample coincided with a credit boom, and almost 85% of all housing booms in the sample ended in recessions. In summary, house price boom-bust episodes are globally a rather common phenomenon, and house price cycles are tightly linked with credit cycles.

The model I have discussed so far is essentially static in nature. Because there is no aggregate uncertainty, the expected value of housing is the same for each entering generation, and there are no aggregate cycles. The problem of each generation is identical to any other. In this section, I modify the model to allow for an endogenous aggregate boom-bust cycle in credit and house prices.

Two additional ingredients are needed. The first one is a positive demand externality in housing: households derive more utility from owning a house when many others own a house as well. As I show, the existence of such an externality in housing consumption also leads to strategic demand complementarity in homeownership. It is a reduced-form way to capture a hype surrounding homeownership, reflecting a housing boom in a simple manner.

One potential source for such positive spillovers is a neighbourhood externality stemming from owner-occupied (as opposed to rental) housing consumption: owner-occupied houses tend to be better maintained and thus more pleasant for neighbours. Second, there may be positive spillovers through social and political action. Unlike tenants, homeowners have a financial stake in making their neighbourhood a more attractive place to live and are thus more active in their communities, making it more attractive for others as well. Glaeser and Shapiro (2003) find some empirical evidence for these positive

114

spillovers from owner-occupied housing consumption and homeownership.

The second ingredient is a deadweight cost of default on the price of fore-closed housing. So far, the only deadweight cost in the model ($\kappa$) is incurred on the income of the defaulting household. This is an idiosyncratic cost that the defaulting household alone bears. It captures personal costs of default; for example legal costs, costs of moving, loss of access to the credit market after default, or loss of reputation after default. On top of this idiosyncratic dead-weight cost, I now impose a cost of default on the aggregate housing stock, which depends on the aggregate default rate in the economy.

Several reasons for such a cost exist. First, it can be thought of capturing the fact that foreclosed housing often sells at a big discount relative to mar-ket prices. Sellers of foreclosed housing often need to liquidate the property quickly and have an incentive to accept a low price because of the illiquid nature of the housing market. Mian et al. (2013) find a strong negative effect of foreclosures on house prices in a zip code-level analysis. They estimate that the increase in foreclosures accounted for 33% of the decline in house prices in 2007–2009 in the U.S. They argue that this was due to a fire sale effect: the market was flooded with foreclosed properties when the economy was already weak, which forced sellers to accept low prices.

Finally, there may be negative spillovers to foreclosed housing similar to the positive externalities from homeownership discussed above. Campbell (2013) reviews empirical evidence on the existence of such physical foreclo-sure externalities. He argues that foreclosures have a negative spillover effect on prices of nearby housing, as the occupants of foreclosed housing may be-come negligent, and a high rate of foreclosures may lead to social problems in the neighbourhood. Campbell et al. (2011) estimate an average foreclosure discount of 27% in a sample of individual house transactions in Massachusetts over the period 1987–2009. Clauretie and Daneshvary (2009) summarise em-pirical research on the overall size of the foreclosure discount, and find that it has often been estimated to be 22–24% of average sales prices in the samples studied, consistent with Campbell et al. (2011).

### 3.7.1 Strategic complementarity in homeownership

I modify the value function of a homeowner in equation (3.1) by adding a demand externality that depends on the aggregate homeownership rate $1-$

$\hat{\pi}$.[14] Specifically, the value function is now:

$$V_H(\pi; \hat{\pi}) = p(\pi) E\left[u(c_{nd}(y_1, q_1)) + v(h) + g(1 - \hat{\pi}) \mid \pi\right] + \\ (1 - p(\pi)) E\left[u(c_d(y_1)) + \chi v(h) \mid \pi\right] \tag{3.30}$$

I assume that $g : [0, 1] \to \mathbb{R}$ is a strictly strictly increasing function of the homeownership rate $1 - \hat{\pi}$ with $g(0) = 0$ and $0 < g'(1 - \hat{\pi}) < 1 \quad \forall \, \hat{\pi} \in [0, 1]$.

The positive demand externality also implies strategic demand complementarity in homeownership. To see this, recall that the optimal action of a given household is defined by the cut-off strategy, given by equation (3.23). This action depends on the expected pay-offs of homeownership relative to renting, and consequently now also depends on the actions of others through the aggregate homeownership rate $1 - \hat{\pi}$.

The assumption of a slope of the function $g$ below unity implies weak strategic complementarity. Cooper and John (1988) and more recently Beaudry et al. (2015) show that a slope above one implies strong complementarity, which is a sufficient condition for equilibrium multiplicity. Beaudry et al. (2015) also show that a slope of the demand complementarity function below unity guarantees a unique equilibrium, but can accommodate a deterministic cycle around this equilibrium.

A further assumption is that a tenant does not enjoy this demand externality. A tenant's value function is thus unchanged from the baseline formulation (3.6), reproduced here for convenience:

$$V_R(\pi) = E\left[u(c_r(y_1)) + \chi v(h) \mid \pi\right]. \tag{3.31}$$

The household's value function is then:

$$V(\pi; \hat{\pi}) = \max\left\{V_H(\pi; \hat{\pi}), V_R(\pi)\right\}. \tag{3.32}$$

The ex-post default choice and the ex-ante no-default probability $p(\pi; \hat{\pi})$ are modified accordingly and now also depend on the aggregate homeownership rate. A borrower of type $\pi$ defaults in the second period if and only if:

$$u(c_d(y_1)) + \chi v(h) > u(c_{nd}(y_1, q_1)) + v(h) + g(1 - \hat{\pi}).$$

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[14]This definition of the homeownership rate hinges on the continued assumption of uniformly distributed types.

## 3.7.2 Deadweight cost of default

The second extension to the baseline model is the inclusion of a deadweight cost of default on the house price. I now assume that each housing unit seized and sold by the lender after a default suffers a cost $1 - \psi$ proportional to the market price $q_1$, with $0 < \psi < 1$. That is, a house $h$ that is foreclosed only yields $\psi q_1 h$ to the lender who sells it to a real estate agent, and the real estate agent also receives $\psi q_1 h$. The share $(1 - \psi) q_1 h$ is lost. This represents a discount on the value of the housing stock that is foreclosed. A homeowner who does not default on his loan is not subject to this cost, and is able to sell his house at the market price $q_1 h$.

I denote by $\mathfrak{S}$ the set of possible states of the world $(y_1, q_1)$ and by $S$ the number of different states in $\mathfrak{S}$. In particular, $\mathfrak{S} = \{(y_H, q_H), (y_L, q_H), (y_H, q_L), (y_L, q_L)\}$ and $S = 4$.

The value of the aggregate stock of housing of an exiting generation in period $t$, denoted by $\bar{q}_t$, is then:

$$\bar{q}_t = f(\bar{\pi}_t, \phi) \left[ (\mathbb{I} - \mathbb{1}_{d,t}) \, \mathbf{q} + \mathbb{1}_{d,t} \, \psi \mathbf{q} \right] \tag{3.33}$$

where $\mathbb{1}_d$ in a $S \times S$ diagonal matrix with elements $i_{ss} = 1$ if there is default in state $s \in \mathfrak{S}$ and $i_{ss} = 0$ if there is not.

Furthermore, $f(\bar{\pi}, \phi) = \text{vec}([\bar{\pi} \quad 1 - \bar{\pi}]'[\phi \quad 1 - \phi])'$ $(1 \times S)$ is the joint distribution vector of the average income type on the market $\bar{\pi}$ and the house value shock $\phi$, where $\bar{\pi} \equiv E[\pi \,|\, \pi \geq \hat{\pi}]$. Finally, $\mathbf{q} = [\mathbf{q_H} \quad \mathbf{q_L}]'$ is a $S \times 1$ vector of house values.

This expression picks up the states in which default happens and discounts the house values by $\psi$ in those states, while keeping them unaffected in the no-default states. It then produces the expected value of the aggregate housing stock.

## 3.7.3 Parametrisation of the extended model

The behaviour of the model and the dynamics of the house price cycle are sensitive to the parameter values. The length, magnitude and frequency of the deterministic cycle can vary significantly. I show below a specific example that demonstrates that the model can, qualitatively, replicate a similar boom-bust pattern in house prices, default rate, and new lending to the data in Figure 3.1.

| Parameter | Value | Model moment | Static | Boom-bust |
|---|---|---|---|---|
| Demand complementarity $\alpha$ | 0.7 | - | - | - |
| DWL cost on housing $\psi$ | 0 | - | - | - |
| Safe interest rate $\bar{r}$ | 1.04 | Safe interest rate | 1.04 | 1.04 |
| Utility from owner-occ. h $v_H(h)$ | 0.2 | Homeownership rate | 0.66 | 0.75 |
| Lender appropriation rate $\xi$ | 0.288 | Average lender recovery | 0.5 | 0.1 |
| High house value prob. $\phi$ | 0.915 | Delinquency rate | 0.03 | 0.032 |
| Default DWL cost $\kappa$ | 2.08 | Default only when $(y_L, q_L)$ | ✓ | ✓ |
| $q_0/y_0$ | 2.94 | Loan-to-value ratio | 0.66 | 0.66 |
| $q_H/q_0$ | 1.07 | House p. peak-to-trend | 1.07 | 1.08 |
| $q_L/q_0$ | 0.24 | House p. trough-to-trend | 0.24 | 0.25 |
| $y_H/y_0$ | 1.43 | Rel. income growth | 1.43 | 1.43 |
| $y_L/y_0$ | 0.71 | Rel. income loss | 0.71 | 0.71 |

Table 3.5: Boom-bust parameter calibration

*Note:* "Static" refers to the targeted equilibrium values under the baseline calibration, replicated from Table 3.3. "Boom-bust" refers to the equilibrium steady-state values under the boom-bust calibration, around which the system oscillates.

In the numerical computations, I use the functional form

$$g(1 - \hat{\pi}) = \alpha \log(1 + (1 - \hat{\pi}))$$

for the demand externality function. Here $\alpha > 0$ is a parameter that governs the strength of the complementarity. Setting $0 < \alpha < 1$ guarantees that $0 < g'(\cdot) < 1$ everywhere in the domain $[0, 1]$.

The parametrisation mostly follows the baseline presented in Table 3.3 in Section 3.5.3. The demand complementarity parameter $\alpha$ and the haircut on house values $\psi$ govern the behaviour on the cycle, and are chosen to qualitatively match the cyclical patterns observed in Figure 3.1. The value of $\alpha$ is set to 0.7, such that the externality is strong enough to generate a cycle but low enough to rule out equilibrium multiplicity. The discount parameter $\psi = 0$ is set to an extreme value; it is needed to generate a big enough bust in the house price. The discounts observed empirically in general range from 22% to 27% (Clauretie and Daneshvary 2009; Campbell et al. 2011). Finally, the demand complementarity strengthens the incentives for buying a house. Because of this, the resulting homeownership rate is higher than in the static model.
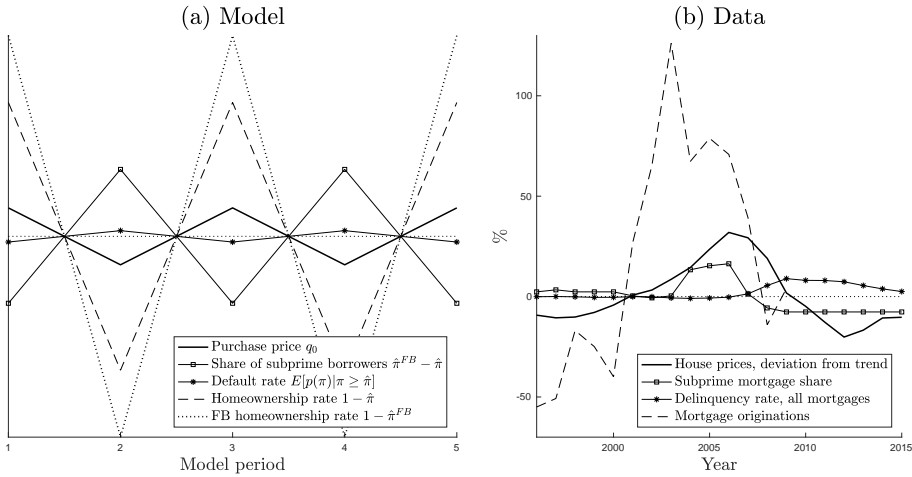
(a) Model             (b) Data

Figure 3.6: Boom-bust model dynamics and the data

*Note:* Panel (a): Model dynamics, deviations from mean. Panel (b): Data, percentage deviations from mean or trend.

### 3.7.4   Boom-bust dynamics

This section presents and discusses the dynamics of the model under the parametrisation in Table 3.5, and compares these dynamics to the patterns observed in data.

Panel (a) of Figure 3.6 plots the dynamics of the extended model. With the given parametrisation, the model exhibits a deterministic cycle of two periods around its equilibrium. All variables are expressed in deviations from their mean values. Panel (b) replicates the data from Figure 3.1; here, for ease of comparison with the model counterparts, all series are either de-meaned or de-trended by removing a linear trend.

The model dynamics work as follows. Assume, first, a situation where the price of housing $q_0$ and the homeownership rate $1 - \hat{\pi}$ are both high, as in period 1 of Panel (a). Because there are a lot of risky households in the market, the default rate increases in the next period, and homeownership decreases. Consequently, the price $q_0$ is depressed because of the deadweight loss caused by the increase in defaults. But the fall in the house price attracts more new buyers in the following period, and the cycle starts again.

| Ratio of standard errors | Data | Model |
|---|---|---|
| Loan originations to house prices | 4.1 | 4.7 |
| Subprime loan originations to all loan originations | 0.13 | 0.50 |
| Default rate to loan originations | 0.05 | 0.04 |
| Default rate to house prices | 0.23 | 0.20 |

Table 3.6: Relative volatilities of the series in Figure 3.6

*Note:* Relative volatility is measured as the ratio of standard errors. "Model": series in Panel (a), "Data": series in Panel (b) of Figure 3.6, respectively.

Here, the first-best homeownership is more sensitive to the changes in the house price, which makes the share of subprime borrowers move counter-cyclically with respect to house prices. However, the homeownership rate is always at a lower level under symmetric information than under asymmetric information.

Panel (b) of Figure 3.6 shows the developments in the housing market in the U.S. in 1990–2015. In the data, house prices peak in 2006. New mortgage originations peak slightly earlier, in 2003, and remain elevated through 2006. Subsequently, new originations fall together with house prices. Likewise, the share of subprime borrowing remains stable until 2003, starts to rapidly increase in 2004 and peaks with house prices in 2006, after which it collapses together with house prices. The delinquency rate only picks up later, in 2009, as house prices fall.

The model largely exhibits these same patterns. When house prices are rising, homeownership increases; in the context of the model, this translates directly into new loan originations. House prices peak as housing demand peaks. Finally, as in the data, defaults peak after house prices, as the increase in borrowing translates into increased defaults with a delay.

In terms of relative magnitudes, the model also performs well. The relative volatilities of the series in Figure 3.6 are presented in Table 3.6. All ratios are of the same order of magnitude both in the data and the model. Most notably, new loan originations are more volatile than house prices. In the model, there are only two assets available to the household — the safe deposit and the risky investment in housing. Moreover, the diversification of endowments into both assets is not optimal. Therefore small changes in the price of the risky assets induce large fluctuations in the households' investment incentives, and consequently, the homeownership rate as well as the share of subprime

borrowers. On the other hand, as only a small fraction of borrowers default in any given period, only a small fraction of the housing stock is subject to a haircut. There is no aggregate shock that would lead to large swings in house prices.

In the data, the changes in the homeownership rate are small (not depicted in Figure 3.6), but the volume of new residential mortgage loan originations grows sevenfold in absolute terms, or from 70% below mean to 120% above mean over the boom period. Beside loans for new purchases, this amount also includes refinancing and home equity loans, which are strictly speaking absent in the model. Nonetheless, mortgages for new purchases amount to a substantial share of total mortgage originations.[15]

It has to be noted, however, that the fluctuations in the theoretical model are very small in absolute terms. They are weak oscillations around the model equilibrium. The model is not intended to be a quantitative model of business cycles; in order to work as such, it would need more powerful mechanisms that induce aggregate fluctuations. Here, all uncertainty is idiosyncratic, and the fluctuations are created by externalities stemming from the demand complementarity on the one hand, and the default discount on the other hand.

Finally, endogenising housing supply in the model would dampen the effects of the incentive mechanism. A perfectly inelastic housing supply coupled with a perfectly elastic credit supply imply that credit demand by households alone drives the boom–bust dynamics in the model. However, as long as housing supply is not very elastic, and credit on the other hand is easily accessible, the incentive mechanisms described in this essay are at work. These conditions are empirically plausible in many regions of the United States; Mian and Sufi (2011) provide empirical evidence on strong house price growth and a marked increase in household indebtedness in areas where housing supply was inelastic in 2002–2006.

## 3.8 Conclusions

This essay outlines a model of the housing market where housing finance is affected by an adverse selection problem. The selection is towards less creditworthy borrowers. The aim of the essay is to shed light on the specific incentives that can cause equilibrium over-investment and endogenous fluctuations

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[15]See e.g. the Mortgage Bankers Association (2016) for more detailed information on mortgage origination by purpose.

in lending standards, captured by the endogenous share of subprime borrowers in the market. The market failure leads to households being too levered compared to the first-best credit allocation, which renders the aggregate economy fragile. To my knowledge, this essay is the first to apply the De Meza and Webb (1987) adverse selection framework to study housing investment.

The model succeeds in replicating some key long-run averages in the U.S. housing market before the subprime crisis, such as the default rate, homeownership rate, and loan-to-income ratio. The dynamic model can also replicate the patterns empirically observed in the U.S. during the housing boom and the foreclosure crisis, in 1990–2015, in terms of relative magnitudes and the timing of the cycles. The demand-side mechanism discussed in this essay is complementary to supply–side explanations of the subprime crisis, such as credit or housing supply shocks.

Using comparative statics, I show that the over-borrowing externality is the most severe, first, when future house values are expected to be high; and second, when safe interest rates are relatively low, which implies low costs of borrowing and a low opportunity cost on housing investment. Under these circumstances, the incentives of subprime borrowers to enter the credit market are the strongest. These are also exactly the circumstances which prevailed in the U.S. prior to the subprime crisis in the early 2000s; the results are consistent with a substantial market failure in housing finance in the years prior to the crisis. They also demonstrate that there are strong and important links between the conduct of monetary policy and financial stability, as interest rates directly affect incentives for credit market participation.

In terms of policy, I find that a tightening of foreclosure regulation, which affects all defaulting borrowers equally, is not an effective way to curb this problem. Removing the over-borrowing externality calls for measures that affect incentives directly. Effective policies could include removing favourable tax treatment of housing loans and owner-occupied housing, imposing loan-to-value or loan-to-income constraints on lending, and regulatory action designed to make lenders properly test the creditworthiness of potential borrowers.

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## 3.A Proofs of propositions

### Proof of Proposition 1

The homeowner consumption in the no-default state $c_{nd}^{FB}$ is increasing in $\pi$: $\frac{\partial c_{nd}^{FB}}{\partial \pi} = -\frac{\partial r(\pi)}{\partial \pi} l \geq 0$, as:

$$\frac{\partial r(\pi)}{\partial \pi} = \underbrace{\left[ \frac{E\left[q_1 h + \xi y_1 \mid \pi\right]}{l_0} - (1 + \bar{r}) \right]}_{<0} \underbrace{\frac{p'(\pi)}{p(\pi)^2}}_{>0} < 0.$$

The value $V_R^{FB}(\pi) = E\left[u(c_r^{FB}) + \chi v(h) | \pi\right]$ continuous and differentiable by the assumptions made on $u(\cdot)$. It is linearly increasing in type $\pi$:

$$\frac{\partial V_R^{FB}(\pi)}{\partial \pi} = u(c_r^{FB}(y_H)) - u(c_r^{FB}(y_L)) > 0$$

$$\frac{\partial^2 V_R^{FB}(\pi)}{\partial \pi^2} = 0.$$

The value

$$V_H^{FB}(\pi) = E\left[ p(\hat{\pi}^{FB}) \left( u(c_{nd}^{FB}(\hat{\pi}^{FB})) + v(h) \right) + \right.$$
$$\left. (1 - p(\hat{\pi}^{FB})) \left( u(c_d^{FB}) + \chi v(h) \right) \mid \hat{\pi}^{FB} \right]$$

is not necessarily continuous and differentiable everywhere: it can exhibit discontinuous jumps. To see this, define *default sets* $D(\pi)$ as the set of states of nature $s \in \mathfrak{S}$ in which a borrower of type $\pi$ ex-post finds it optimal to default:

$$D(\pi) \equiv \{s \in \mathfrak{S} \mid u(c_d^{FB}(y_1^s; \pi)) + \chi v(h) > u(c_{nd}^{FB}(y_1^s, q_1^s; \pi)) + v(h)\},$$

where $y_1^s$ and $q_1^s$ denote the realisation of income $y_1$ and house value $q_1$ in state $s$. The default probability $p(\pi)$ is decreasing in $\pi$, which implies that $\pi > \pi' \implies D(\pi) \subseteq D(\pi')$. In other words higher types cannot default in strictly more states than lower types. Finally, when the set of states $\mathfrak{S}$ is a discrete set, either $D(\pi) = D(\pi') \forall \pi, \pi'$, or $D(\pi) \subset D(\pi')$ for some $\pi = \pi' + \epsilon$ for an arbitrarily small $\epsilon > 0$. In the former case, all types default in exactly the same states, and $V_H^{FB}(\pi)$ is continuous and differentiable in $\pi$. In the latter case, there are discontinuous jumps at types where the switch

from a default set $D(\pi')$ to $D(\pi) \subset D(\pi')$ happens, because ex-ante welfare $V$ is always strictly higher the less likely a default is for a given type.

However, because utility $u(\cdot)$ is increasing in consumption by assumption, consumption in the no-default state is increasing in $\pi$, consumption in the default state is constant in the type $\pi$, and the no-default probability is increasing in $\pi$, it follows that expected utility is increasing in $\pi$. This directly implies that $V_H^{FB}(\pi)$ is always increasing in $\pi$, also at any possible discontinuity points.

Then, there exists a parametrisation (specific functional forms and a set of parameter values) for which the value functions $V_R^{FB}(\pi)$ and $V_H^{FB}(\pi)$ intersect at most once in the interval $\pi \in [0, 1]$. If no intersection exists in this interval, if $V_R^{FB}(\pi) < V_H^{FB}(\pi) \, \forall \pi$, define $\hat{\pi}^{FB} = 0$, and if $V_R^{FB}(\pi) > V_H^{FB}(\pi) \, \forall \pi$, define $\hat{\pi}^{FB} = 1$. I assume that the parametrisation is such that I can rule out cases where the value functions could intersect more than once. □

## Proof of Proposition 2

Similarly to the symmetric information case, the value of a tenant $V_R(\pi)$ is linearly increasing in $\pi$:

$$\frac{\partial V_H(\pi)}{\partial \pi} = u(c^r(y_H)) - u(c^r(y_L)) > 0$$

The value of a home-owner $V_H(\pi)$ is increasing in $\pi$ following the same argument as $V_H^{FB}(\pi)$, presented in the proof of Proposition 1 above. Then, there exists a parametrisation (specific functional forms and a set of parameter values) for $V_R(\pi)$ and $V_H(\pi)$ such that they intersect at most once in the interval $\pi \in [0, 1]$. If no intersection exists in this interval and $V_R(\pi) < V_H(\pi) \, \forall \pi$, define $\hat{\pi} = 0$, and if $V_R(\pi) > V_H(\pi) \, \forall \pi$, define $\hat{\pi} = 1$. □

## Proof of Proposition 3.

**Lemma 1** $r(\hat{\pi}) \geq r^*$, where $r(\hat{\pi})$ is the interest rate that the marginal type $\hat{\pi}$ would pay under symmetric information, and $r^*$ is the equilibrium pooling rate under asymmetric information.

**Proof.** I denote $\bar{p} \equiv E[p(\pi) \mid \pi \geq \hat{\pi}]$ and $\bar{d} \equiv E[d(\pi) \mid \pi \geq \hat{\pi}]$. From the

equilibrium conditions for $r(\pi)$ and $r^*$, we have:

$$1 + r(\hat{\pi}) \geq 1 + r^*$$

$$\Leftrightarrow \quad \frac{1 + \bar{r}}{p(\hat{\pi})} - \frac{1 - p(\hat{\pi})}{p(\hat{\pi})} d(\hat{\pi}) \geq \frac{1 + \bar{r}}{\bar{p}} - \frac{1 - \bar{p}}{\bar{p}} \bar{d}. \tag{3.34}$$

Case 1. $d(\pi) = \bar{d}$.
Substituting $d(\pi) = \bar{d}$ into equation (3.34):

$$\frac{1 + \bar{r}}{p(\hat{\pi})} - \frac{1 - p(\hat{\pi})}{p(\hat{\pi})} \bar{d} \geq \frac{1 + \bar{r}}{\bar{p}} - \frac{1 - \bar{p}}{\bar{p}} \bar{d}$$

$$\Leftrightarrow \quad \underbrace{\frac{1 + \bar{r}}{\bar{d}}}_{\geq 1} \geq \underbrace{\frac{\frac{1 - p(\hat{\pi})}{p(\hat{\pi})} - \frac{1 - \bar{p}}{\bar{p}}}{\frac{1}{p(\hat{\pi})} - \frac{1}{\bar{p}}}}_{=1}$$

$$\Rightarrow \quad r(\hat{\pi}) \geq r^* \quad \forall \, \hat{\pi}.$$

Case 2. $d(\pi) > \bar{d}$.
Solving from equation (3.34), we have the condition:

$$\frac{d(\hat{\pi})}{\bar{d}} \leq \left( 1 - \frac{p(\hat{\pi})}{\bar{p}} \right) (1 - p(\hat{\pi})) \frac{1 + \bar{r}}{\bar{d}} + \frac{p(\hat{\pi})}{\bar{p}} \frac{1 - \bar{p}}{1 - p(\hat{\pi})}. \tag{3.35}$$

Case 3. $d(\pi) < \bar{d}$.
When the condition (3.35) above holds, $r(\hat{\pi}) \geq r^*$ also when $d(\pi) < \bar{d}$.

Therefore the interest rate charged on the marginal type under symmetric information $r(\hat{\pi})$ is greater than the pooling rate $r^*$ whenever condition (3.35) holds. $\square$

**Proposition 3** *In equilibrium, $\hat{\pi} \leq \hat{\pi}^{FB} \, \forall \, \hat{\pi}$ when condition* (3.35) *holds.*

**Proof.** By Lemma 1, the marginal borrower $\hat{\pi}$ would pay a (weakly) higher interest rate if her type were publicly observable. The homeowner's value function is decreasing in $r_0$:

$$\frac{\partial u(c_{nd})}{\partial r_0} = y_0 - q_0 h \underbrace{< 0}_{\text{by assumption}} \quad \Rightarrow \quad \frac{\partial V_H}{\partial r_0} = p(\pi) \frac{\partial u(c_{nd})}{\partial r_0} < 0$$

Therefore the marginal type would be worse off under symmetric information and would not enter the credit market when $r(\hat{\pi}) > r^*$. When $r(\hat{\pi}) = r^*$, she would be indifferent on entering the credit market, and would also be the marginal type under symmetric information. This implies $\hat{\pi} \geq \hat{\pi}^{FB}$. $\square$

## 3.B   Numerical solution algorithm

The numerical method for finding the equilibrium is an iteration algorithm based on guessing and iterating on an interest rate until convergence. The algorithm searches for an equilibrium through backward induction, starting from solving the second-period problem of the consumer given the guess for the interest rate, and given this solution, solving the first-period problems of the consumer and the lender. The algorithm proceeds as follows.

1. Guess for an equilibrium interest rate $r^*$.

2. Compute consumption profiles in all different states in $t = 1$ and the utilities $u(c)$ from these consumption profiles given the guess for $r^*$.

3. Compute the ex-post default decision in $t = 1$ of each type $\pi$ for all realisations $(y_1, q_1)$.

4. Given the ex-post default choices, compute the ex-ante private default probabilities, expected values $V_H$ and $V_R$, and tenure choices in $t = 0$.

5. Find the cut-off type $\hat{\pi}$ who is indifferent between the expected values $V_H$ and $V_R$ given the guess $r^*$.

6. Given the guess $r^*$ and the borrower pool defined by $\hat{\pi}$ implied by the guess, compute the lenders' expected profit.

7. If the lenders' expected excess profit is not equal to zero, adjust the guess $r^*$ and iterate until convergence.

# 4 Learning about systematic risk in the housing market

## 4.1 Introduction

> [U]nquestionably, housing prices are up quite a bit; I think it's
> important to note that fundamentals are also very strong. We've got
> a growing economy, jobs, incomes. We've got very low mortgage
> rates. We've got demographics supporting housing growth. —
> We've never had a decline in house prices on a nationwide basis.

*Ben Bernanke, July 1st 2005, CNBC.*

Did investors mis-estimate the amount on undiversifiable risk in the U.S.
real estate market during the run-up to the subprime crisis that erupted in
2007? This essay argues that uncertainty about the true undiversifiable risk
in residential investment contributed to the real estate boom. Investors in the
real estate market were confident during the housing boom years in the late
1990's and early 2000's, which was justified by the seemingly strong economic
fundamentals. As the subprime crisis unfolded, investors had to adjust their
beliefs on undiversifiable risk very strongly upward, in particular in regions
where the boom had been strong. I show evidence from the U.S. data that
supports this view.

I apply a conditional capital-asset pricing model (CAPM) with Bayesian
learning to estimate a time-varying measure of undiversifiable, or systematic,
risk in the U.S. residential housing market.[1] I assume that the underlying sys-

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[1]In the finance literature, the terms "undiversifiable" and "systematic" are often used inter-
changeably to refer to risk that cannot be diversified away. The term "aggregate risk" is also

tematic risk of the real estate portfolio — its market beta — is unobservable and time-varying. This means that investors cannot disentangle idiosyncratic and systematic risk in a specific portfolio, and must instead attempt to estimate them by using observable portfolio returns. I estimate the model to U.S. data by using excess returns on investment in residential housing computed from regional house price indices.

The results indicate that the market beta[2] of the residential housing portfolio was relatively low in most regions in the U.S. especially in the early 2000's, which coincides with the years of strongest house price appreciation. Starting from 2006, when house prices peaked, these patterns were reversed: investors swiftly revised their beliefs about systematic risk upward. Idiosyncratic uncertainty about excess returns also increased at the onset of the crisis.

A prominent narrative of the housing boom and the subprime crisis in the U.S. attributes the expansion in mortgage lending to a loosening of credit standards, enabled by widespread securitisation of mortgages and a liberalisation of the financial market. This narrative is advocated, for example, in the influential series of studies by Mian and Sufi (2009, 2011) and Mian et al. (2013). Mortgage-backed securities offered good returns with seemingly low risk. This expansion in credit supply fuelled demand for housing and contributed, in hindsight, to a bubble in house prices in many geographical areas. At the same time, increasing house prices coupled with the loosening of lending standards attracted worse and worse borrowers, resulting in a build-up of risk in the borrower pool, and eventually, the subprime crisis.

What, then, can explain this deterioration of credit standards on the lender side? Mortgage-backed securities were deemed safe and often held AAA ratings, because they pooled together mortgages in a way that diversified away credit risk that was believed to be inherently idiosyncratic. A widespread collapse of house prices was believed to be unlikely.[3] This essay argues that this kind of belief in low systematic risk may have contributed to the observed increase in credit supply, even if lenders understood that the quality of the borrower pool was deteriorating.

In this essay, I gauge the beliefs of institutional investors in the housing market, who invest in large portfolios of residential housing — or securities whose returns depend on the development in residential house prices — in

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sometimes used for undiversifiable risk.

[2]The market beta measures the extent of co-movement of a given asset's expected return with that of the expected return on the market portfolio; it defines the risk premium demanded by investors on that asset. It is defined formally in Section 4.2.

[3]See the report by the Financial Crisis Inquiry Commission (2011, Ch. 7).

a given geographical area, and who care only about the risk/return trade-off of their investment. Such a portfolio can also be regarded as relatively liquid, unlike investment into housing by a single household, on the borrower side.[4] In other words, I am interested in the beliefs of the financier side of the residential housing market, which guide the *supply* of funding for housing investment.

I assume that investors are rational, but uncertain about the true riskiness of the housing market portfolio. They form beliefs on the housing market beta, which defines the risk premium demanded on the housing portfolio, through Bayesian updating. Potential mis-pricing of risk is only evident ex-post. In the model, the CAPM pricing equation emerges as an equilibrium condition. It is the uncertainty induced by learning that leads to mis-pricing that is ex-post observable to the econometrician. This point is also made explicitly by Lewellen and Shanken (2002).

This essay is related to a large literature on the conditional CAPM in finance and financial economics. The original static CAPM, introduced by Sharpe (1964) and Lintner (1965) and expanded on by Black et al. (1972), is the workhorse model of asset pricing in finance. Its empirical shortcomings are famously documented by Fama and French (1992, 1993, 2004).

A substantial share of research has focused on the conditional version of the CAPM, which postulates that the pricing equation holds only conditional on the information set of the investor in a given period. In this model, the relationship between the expected return on the market portfolio and the expected return on an asset — the asset's beta — is no longer constant, but time-varying. The conditional CAPM has been developed, among others, by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001). It has had more success in explaining cross-sectional variation in stock returns than the static CAPM.

The conditional CAPM relates naturally to models of learning. This essay is tightly linked to this branch of the CAPM literature. Pastor and Veronesi (2009) provide an overview of models of learning in finance. Lewellen and Shanken (2002), Adrian and Franzoni (2009), and Trecroci (2014) formulate versions of the conditional CAPM with Bayesian learning where investors are assumed to update their beliefs on the time-varying parameters of the CAPM according to observations on asset returns. Huang and Hueng (2008) model and estimate a conditional CAPM with adaptive learning. The focus in all of

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[4]Households who purchase a house likely also have preferences over aspects of the house other than the risk/return profile associated with its expected price development, such as location and amenities.

these studies is in comparing the performance of the CAPM with learning in explaining cross-sectional variation in asset returns against other conditional versions of the CAPM.

This essay is methodologically closest to those of Adrian and Franzoni (2009) and Trecroci (2014). I adapt the model formulated by Trecroci (2014) and apply it to the U.S. real estate market to explain the development of investors' beliefs during the housing boom and the subprime crisis. The setup is similar to that of Adrian and Franzoni (2009), but unlike this essay and Trecroci (2014), they assume the existence of conditioning variables on the latent aggregate state and a mean-reverting process for it. I also generalise and expand on the setup of both Adrian and Franzoni (2009) and Trecroci (2014) by allowing heteroskedasticity in forecast errors through a flexible GARCH specification for idiosyncratic uncertainty and by estimating the full set of structural parameters.

This essay also connects to a growing literature on the implications of model uncertainty and learning on macroeconomic outcomes.[5] Since the financial crisis of 2007–2009, in an effort to understand time-varying risk, much work has been put into exploring the implications of model uncertainty and the evolution of beliefs on the dynamics of macroeconomic aggregates and asset prices. Recent contributions include Johannes et al. (2016), Collin-Dufresne et al. (2016), and Luzzetti and Neumuller (2016).

In the context of housing markets, Piazzesi et al. (2007) explore asset-pricing implications of low-frequency risk in house price fundamentals. Gelain and Lansing (2014) as well as Granziera and Kozicki (2015) study the implications of rational and non-rational learning on house price dynamics and in particular on generating bubbles. My focus is instead on the evolution of investor beliefs, implied by learning, taking the house price developments as given.

The remainder of the essay is structured as follows. Section 4.2 outlines the theoretical model of asset pricing with learning and derives the laws of motion for investor beliefs. Section 4.3 discusses the empirical strategy and the data. Section 4.4 presents and discusses the estimation results. Finally, Section 4.5 concludes.

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[5]See Hansen (2014) and the references therein for a general discussion of model uncertainty and asset pricing.

## 4.2 The model and methodology

In this section, I outline the conditional CAPM and the underlying assumptions. The model with learning results in a Bayesian signal extraction problem for an individual financier who attempts to determine the appropriate risk premium on a given asset. The optimal solution to this problem can be found by applying the Kalman filter, which gives the equilibrium laws of motion for the beliefs of investors about the latent state of the economy, as new observations become available. In addition, the Kalman smoother yields the corresponding ex-post estimates of the latent state that use the full sample of observations, against which the ex-ante beliefs can be evaluated. Finally, I discuss how the structural parameters of the model can be estimated using maximum likelihood methods.

### 4.2.1 The conditional CAPM with learning

The formulation of the conditional CAPM in this section is standard and follows, for example, that found in Lettau and Ludvigson (2001). The learning process is modelled similarly to Adrian and Franzoni (2009) and Trecroci (2014).

#### 4.2.1.1 Derivation of the conditional CAPM

The conditional CAPM requires a rather small amount of theoretical structure. I assume that investors in financial markets are risk-averse and rational, but have incomplete information on the data-generating process of the economy. The investors are assumed to have preferences over the riskiness of their portfolios. I denote the common information set of all investors in period $t$ by $\mathcal{I}_t$. The problem of an individual investor is then to choose the optimal portfolio subject to the risk-return trade-off. The optimal portfolio is mean-variance efficient, or in other words, yields the desired mean return with the smallest possible variance.[6]

I assume the existence of a safe asset $f$ whose gross return is $\rho_t^f$, and further that investors can borrow and lend unconstrained at this risk-free rate. Furthermore, there is a set of risky assets, indexed by $i$. In the context of housing markets, here, each risky asset corresponds to a specific regional portfolio

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[6]This results is standard in the CAPM literature.

of investment into residential housing.[7] In each period $t$, investors observe the risk-free gross return, $\rho_t^f$, gross returns on each risky asset $i$, $\rho_t^i$, and gross returns on the value-weighted market portfolio, $\rho_t^M$, such that $\{\rho_t^f, \rho_t^i, \rho_t^M\} \in \mathcal{I}_t$.

Next, I define the excess returns on asset $i$ as $R_t^i \equiv \rho_t^i - \rho_t^f$, and on the value-weighted market portfolio, correspondingly, as $R_t^M \equiv \rho_t^M - \rho_t^f$. Finally, I assume that the financial market is perfectly competitive and presents no arbitrage opportunities.

The well-known result by Ross (1976) states that in the absence of arbitrage, there exists a strictly positive pricing kernel $M_{t+1}$ such that the following holds for excess returns on any asset $i$:

$$E_t[M_{t+1}R_{t+1}^i] = 0, \tag{4.1}$$

where $E_t[\cdot] \equiv E[\cdot|\mathcal{I}_t]$ denotes the investors' expectation conditional on information at time $t$.

Under these assumptions, the market return is itself a pricing kernel. As a consequence, equation (4.1) holds for $M_{t+1} = R_{t+1}^M$ and $R_{t+1}^i = R_{t+1}^M$, which then implies:

$$E_t[R_{t+1}^M] = -\frac{cov_t(R_{t+1}^M, R_{t+1}^M)}{E_t[R_{t+1}^M]} = -\frac{var_t(R_{t+1}^M)}{E_t[R_{t+1}^M]}, \tag{4.2}$$

where $E_t(\cdot) \equiv E(\cdot|\mathcal{I}_t)$, $var_t(\cdot) \equiv var(\cdot|\mathcal{I}_t)$, and $cov_t(\cdot) \equiv cov(\cdot|\mathcal{I}_t)$ denote the conditional expectation, variance, and covariance, respectively.

Then, substituting $M_{t+1} = R_{t+1}^M$ into equation (4.1), the following holds for any asset $i$ conditional on period $t$ information:

$$
\begin{aligned}
E_t[R_{t+1}^i] &= -\frac{cov_t(R_{t+1}^i, R_{t+1}^M)}{E_t[R_{t+1}^M]} \\
&= -\frac{cov_t(R_{t+1}^i, R_{t+1}^M)}{var_t(R_{t+1}^M)} \frac{var_t(R_{t+1}^M)}{E_t[R_{t+1}^M]} \\
&\equiv E_t[\beta_{t+1}^i] E_t[\eta_{t+1}], \tag{4.3}
\end{aligned}
$$

where $E_t[\beta_{t+1}^i] \equiv \frac{cov_t(R_{t+1}^i, R_{t+1}^M)}{var_t(R_{t+1}^M)}$ is asset $i$'s risk loading on the market portfo-

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[7]This is the Sharpe-Lintner formulation of the CAPM (see Sharpe (1964)).

lio, or its *market beta*, and $E_t[\eta_{t+1}] \equiv -\frac{var_t(R^M_{t+1})}{E_t[R^M_{t+1}]}$ is the expected market risk premium. Equation (4.3) is the conditional CAPM equation.

Together with equation (4.2), (4.3) implies that:

$$E_t[R^i_{t+1}] = E_t[\beta^i_{t+1}]E_t[R^M_{t+1}]. \tag{4.4}$$

Equation (4.4) establishes an equilibrium relationship between expected excess returns of an asset $i$ and the market portfolio in the CAPM.

Following Lettau and Ludvigson (2001) and Adrian and Franzoni (2009), I make the standard assumption that the realised excess return $R^i_{t+1}$ depends linearly on the pricing kernel. The linear factor model together with equation (4.4) implies that the realised return $R^i_{t+1}$ depends linearly on $R^M_{t+1}$:

$$R^i_{t+1} = \alpha^i_{t+1} + \beta^i_{t+1}R^M_{t+1} + \varepsilon^i_{t+1}, \tag{4.5}$$

$$\varepsilon^i_{t+1}|\mathcal{I}_t \sim N(0, \sigma^2_{i,t+1}), \quad \sigma^2_{i,t+1} = a^i_0 + a^i_1(\varepsilon^i_t)^2 + a^i_2\sigma^2_{i,t} \tag{4.6}$$

where $\varepsilon^i_{t+1}$ are i.i.d. innovations. I assume that the variance of this innovation, $\sigma^2_{i,t+1}$, is time-varying and follows a deterministic GARCH(1,1) process with parameters $a^i_0$, $a^i_1$ and $a^i_2$, where $a^i_1 + a^i_2 < 1$.

This structure allows for a flexible specification of heteroskedasticity in the innovations. It makes the model unconditionally non-Gaussian. Importantly, however, given the information set $\mathcal{I}_t$, the innovation $\varepsilon^i_{t+1}$ and thus the model are also conditionally Gaussian.

Finally, I assume $E_t[\alpha^i_{t+1}] = 0 \, \forall t$, or in other words, that the CAPM equation (4.4) holds in expectation.

### 4.2.1.2  Learning in the conditional CAPM

So far, the exposition of the model has been based on the workhorse conditional CAPM. In this section, I discuss how to incorporate incomplete information and learning into the model framework in a tractable fashion.

The crucial assumption is that investors do not observe either shocks to the pricing kernel $M_{t+1}$ or to the individual assets' returns. Aggregate shocks to the pricing kernel could reflect, for example, shocks to future cash flows on the market portfolio or shocks to the market discount rate that affect either the variance of the market return or its covariance with the return on asset $i$

in equation (4.2).[8]

Because investors cannot disentangle systematic from idiosyncratic risk, they cannot perfectly deduce asset $i$'s risk loading on the market return ($\beta^i_{t+1}$) nor Jensen's alpha ($\alpha^i_{t+1}$).[9] In particular, they only observe realised returns and attempt to estimate the true risk loading of each asset $i$. I assume that they update their beliefs on this risk loading by using Bayesian updating. The information set of a given investors at time $t$ investing in a given portfolio $i$ is then given by $\mathcal{I}_t = \{R^i_t, R^M_t\}$. They thus have only partial information about the true state of the economy at time $t$. This uncertainty will never be resolved.

Then, from the point of view of an investor, the time-varying coefficients $\alpha^i_{t+1}$ and $\beta^i_{t+1}$ are modelled as random variables. Following Adrian and Franzoni (2009) and Trecroci (2014), I assume that the time-varying risk loading $\beta^i_{t+1}$ and the time-varying intercept $\alpha^i_{t+1}$ follow autoregressive processes characterised by:

$$\beta^i_{t+1} = (1 - f^i)\bar{\beta}^i + f^i \beta^i_t + u^i_{t+1} \tag{4.7}$$

$$\alpha^i_{t+1} = (1 - g^i)\bar{\alpha}^i + g^i \beta^i_t + v^i_{t+1} \tag{4.8}$$

where $f^i$ and $g^i$ are constant autoregressive coefficients, $\bar{\beta}^i$ and $\bar{\alpha}^i$ are the long-run stable means of $\beta^i_{t+1}$ and $\alpha^i_{t+1}$, respectively, and $(u^i_{t+1}, v^i_{t+1}) \sim N(0, \Sigma^i_v)$ are jointly normally distributed i.i.d. innovations, uncorrelated with $\varepsilon^i_{t+1}$ $\forall t$. I assume that all innovations are serially uncorrelated.

Furthermore, I assume that the values of the time-varying coefficients $\alpha^i_{t+1}$, $\beta^i_{t+1}$, as well as $\bar{\alpha}^i$ $\bar{\beta}^i$, the aggregate shocks $v^i_{t+1}$ and $u^i_{t+1}$ and the idiosyncratic shock $\varepsilon^i_{t+1}$ are all unobserved to the investors in all periods $t$. The autoregressive structures of $\alpha^i_{t+1}$ and $\beta^i_{t+1}$, the conditional normality of the innovations $v^i_{t+1}$, $u^i_{t+1}$ and $\varepsilon^i_{t+1}$, and the GARCH structure of $\varepsilon^i_{t+1}$ are known to the investors. I also assume that the structural parameter values $g^i$, $f^i$, $\Sigma^i_v$, $a^i_0$, $a^i_1$, $a^i_2$ are all time-invariant and known.

The learning CAPM presented here assumes that all investors believe that the asset returns follow the CAPM pricing equation (4.4). This condition is an equilibrium pricing condition, but each investor takes it as given as a price-taker. They also have identical information sets, such that their beliefs on the

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[8]See Campbell and Vuolteenaho (2004) for a discussion.

[9]"Jensen's alpha", after Black et al. (1972), can be used as a measure of asset mispricing in the CAPM framework.

underlying aggregate state ($\alpha_{t+1}^i$ and $\beta_{t+1}^i$) are identical. Thus the equilibrium condition given by equation (4.4) must continue to also hold under partial information.

Of course, from the point of view of the econometrician, there might very well be deviations from this equilibrium condition. It is well known from the finance literature that the conditional CAPM does not explain the cross-section of asset returns very well.[10] However, this is not a concern here. I am interested in gauging the beliefs of investors, conditional on their information sets at a given point in time, under the assumption that they believe the CAPM pricing condition to hold. Lewellen and Shanken (2002) point out that, even when investors are rational and follow the CAPM pricing condition, incomplete information and learning can give rise to predictability in asset returns ex-post, and thus an apparent failure of the CAPM empirically.

Finally, I have assumed that the realisations of shocks are unobserved, but that the investors do know the structural parameters of the model. Therefore, there is no learning about the deep parameters or about the model itself, but only about latent systematic risk. Incorporating parameter or model learning would add another layer of learning into the model, increasing investors' uncertainty about the latent state. While this assumption has implications for the results in quantitative terms, it would likely not affect the overall conclusions. This issue is left for future research.

### 4.2.2 Optimal Bayesian updating

Based on the assumptions laid out in the previous section, the problem of an individual financier is a signal extraction problem: realised returns are noisy signals on the underlying state, which he must try to estimate in order to be able to price the housing portfolio. The optimal linear solution to such a signal extraction problem under Bayesian learning is found by applying the Kalman filter. This gives the optimal solution to the investor's signal extraction problem in the form of equilibrium laws of motion for the latent state.

Under the current assumptions — in particular under conditional normality of the model — the Kalman filter gives the best sequential linear projection of the latent state $E_t[\alpha_{t+1}^i]$ and $E_t[\beta_{t+1}^i]$.[11] In addition, the Kalman smoother

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[10] For a general discussion on the performance of the conditional CAPM, see for example Lettau and Ludvigson (2001) and Fama and French (2004).

[11] This is a standard result that follows from the properties of normally distributed signals and

provides an algorithm that can be used for an *ex-post* evaluation of these beliefs, given the full set of observations up to a final period $T$. This section follows the assumptions and derivation of the Kalman filter and smoother presented in Harvey (1989) and Hamilton (1994, Ch. 13).

### 4.2.2.1 State-space representation of the learning CAPM

The dynamic system to be estimated is given by equations (4.4), (4.7) and (4.8) as well as equations for the unknown time-invariant state means:

$$R^i_{t+1} = \alpha^i_{t+1} + \beta^i_{t+1} R^M_{t+1} + \varepsilon^i_{t+1}$$
$$\bar{\alpha}^i = \bar{\alpha}^i \quad \forall\, t$$
$$\bar{\beta}^i = \bar{\beta}^i \quad \forall\, t$$
$$\alpha^i_{t+1} = (1 - g^i)\bar{\alpha}^i + g^i \alpha^i_t + v^i_{t+1}$$
$$\beta^i_{t+1} = (1 - f^i)\bar{\beta}^i + f^i \beta^i_t + u^i_{t+1}$$

This system can be written in state-space form as:

$$R^i_t = H'_t \zeta^i_t + \varepsilon^i_t \tag{4.9}$$
$$\zeta^i_{t+1} = F^i \zeta^i_t + v^i_{t+1}, \tag{4.10}$$

where (4.9) is the observation equation and (4.10) is the state equation. In this notation, I define $\zeta^i_t$ to be the unobserved state vector, $H_t$ the vector of time-varying coefficients on the state, $v^i_{t+1}$ the vector of state innovations, and $F^i$ the autoregressive constant coefficient matrix on the state vector:

$$\zeta^i_t = \begin{bmatrix} \alpha^i & \bar{\beta}^i & \alpha^i_t & \beta^i_t \end{bmatrix}'$$
$$H_t = \begin{bmatrix} 0 & 0 & 1 & R^M_t \end{bmatrix}'$$
$$v^i_{t+1} = \begin{bmatrix} 0 & 0 & v^i_{t+1} & u^i_{t+1} \end{bmatrix}'$$

---

Bayesian updating. Even under non-Gaussian signals, the Kalman filter provides the minimum mean square linear estimator of the latent state conditional on the information set. See Hamilton (1994, Ch. 13) for a formal proof.

$$F^i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 - g^i & 0 & g^i & 0 \\ 0 & 1 - f^i & 0 & f^i \end{bmatrix}$$

Starting from some prior values, the Kalman filter then produces sequentially updated linear estimates of $E_t[\zeta_{t+1}^i]$. The relevant information set in period $t$ is $\mathcal{I}_t = \{R_t^i, H_t\}$.

I introduce the following standard notation for the Kalman filter estimates. First, let $\zeta_{t+1|t}^i = \hat{E}[\zeta_{t+1}^i | \mathcal{I}_t]$ denote the *a priori* linear projection of $\zeta_{t+1}^i$ on $\mathcal{I}_t$, or the best linear least-squares estimate of $E_t[\zeta_{t+1}^i]$. Further, $\zeta_{t+1|t+1}^i = \hat{E}[\zeta_{t+1}^i | \mathcal{I}_{t+1}]$ denotes the corresponding *a posteriori* linear projection of $\zeta_{t+1}^i$ on $\mathcal{I}_{t+1}$.

Second, I define the *a priori* forecast error as $\tilde{\zeta}_{t+1|t}^i = \zeta_{t+1}^i - \zeta_{t+1|t}^i$, and the *a posteriori* projection error as $\tilde{\zeta}_{t+1|t+1}^i = \zeta_{t+1}^i - \zeta_{t+1|t+1}^i$. The corresponding mean squared errors are defined as $P_{t+1|t}^i = E[\tilde{\zeta}_{t+1|t}^i \tilde{\zeta}_{t+1|t}^{i\prime}]$ and $P_{t+1|t+1}^i = E[\tilde{\zeta}_{t+1|t+1}^i \tilde{\zeta}_{t+1|t+1}^{i\prime}]$.

Finally, I denote the forecast of $R_{t+1}^i$, or the linear projection of $R_{t+1}^i$ on $\mathcal{I}_t$, by $R_{t+1|t}^i$, and the conditional forecast error of $R_{t+1}^i$ by $\tilde{R}_{t+1}^i = R_{t+1}^i - R_{t+1|t}^i$.

Equipped with this notation, the recursive Kalman filter projections can be written as:

$$R_{t|t-1}^i = H_t \zeta_{t|t-1}^i \tag{4.11}$$

$$\zeta_{t|t}^i = \zeta_{t|t-1}^i + \kappa_t^i \tilde{R}_t^i \tag{4.12}$$

$$\zeta_{t+1|t}^i = F^i \zeta_{t|t}^i \tag{4.13}$$

$$P_{t|t}^i = P_{t|t-1}^i - \kappa_t^i (H_t' P_{t|t-1}^i H_t + \sigma_{i,t}^2) \kappa_t^{i\prime} \tag{4.14}$$

$$P_{t+1|t}^i = F^i P_{t|t}^i F^{i\prime} + \Sigma_\nu^i \tag{4.15}$$

where

$$\kappa_t^i \equiv P_{t|t-1}^i H_t (H_t' P_{t|t-1}^i H_t + \sigma_{i,t}^2)^{-1}$$

is the Kalman gain.[12]

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[12]The derivation of these projections is straightforward but somewhat tedious; a standard presentation can be found e.g. in Hamilton (1994, Ch. 13.2).

Starting from some priors $\zeta_{0|0}^i$ and $P_{0|0}^i$, this recursion gives the updated beliefs on the unobserved state, given the observations in $\mathcal{I}_t$, in each period $t = 1, 2, ..., T$. Of particular interest are the estimates of the time-varying coefficients $\beta_{t+1|t}^i$ and $\alpha_{t+1|t}^i$. They give the evolution of the investors' beliefs on the CAPM relationship (4.4) over time, as they observe new data on asset returns.

Contrary to the investors in the model, the econometrician observes the full sample of observations up to a final period $T$. This allows for an ex-post evaluation of the ex-ante beliefs of the investors by applying the Kalman smoother. The fixed interval smoothing algorithm gives the best sequential linear least-squares estimate of $E[\zeta_t^i|\mathcal{I}_T]$, given the information set $\mathcal{I}_T$ in the final period $T$, denoted by $\zeta_{t|T}^i$:

$$\zeta_{t|T}^i = \zeta_{t|t} + P_t^*(\zeta_{t+1|T}^i - F^i \zeta_{t|t})$$

$$P_{t|T} = P_{t|t} + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*'}$$

where $P_t^* \equiv P_{t|t}F^{i'}P_{t+1|t}^{-1}$. The recursion now starts from the final period $T$ and runs backward.

### 4.2.2.2 Maximum likelihood estimation of the filter parameters

The GARCH structure in the innovation term $\varepsilon_{t+1}^i$ renders the model unconditionally non-Gaussian, but conditionally Gaussian, as discussed in Section 4.2. This conveniently allows for a standard derivation of the Kalman filter as well as estimation of the filter parameters by maximum likelihood.

The likelihood function $L$, or the joint density of the observations $\mathbf{R^i} \equiv (R_1^i, \ldots, R_T^i)$, can be written as:

$$L(\mathbf{R^i}; \theta^i) = \prod_{t=1}^{T} p(R_t^i|\mathcal{I}_{t-1}; \theta^i),$$

where $p(R_t^i|\mathcal{I}_{t-1}; \theta)$ denotes the density of $R_t^i$ conditional on $\mathcal{I}_{t-1}$ and the structural parameters $\theta^i$. The first two moments of this conditional density are $E_{t-1}[R_t^i] = R_{t|t-1}^i$ and $var_{t-1}(R_t^i) = var_{t-1}(\tilde{R}_t^i)$, where $\tilde{R}_t^i$ is the forecast error $R_t^i - R_{t|t-1}^i$. These quantities are a direct by-product of the Kalman filtering procedure, such that the construction of the likelihood function is

straightforward.

In the conditionally Gaussian model described above in Section 4.2.2.1, the log-likelihood function depends only on the conditional mean and variance of $R_t^i$ and can be written as:

$$\log L(\mathbf{R^i}; \theta^i) = -\frac{T}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log(\Sigma_t^i) - \frac{1}{2}\sum_{t=1}^{T}(\tilde{R}_t^i)^2(\Sigma_t^i)^{-1}, \qquad (4.16)$$

where I denote the variance of the forecast error by $\Sigma_t^i \equiv var_{t-1}(\tilde{R}_t^i) = H_t' P_{t|t-1}^i H_t + \sigma_{i,t}^2$.

This log-likelihood is an implicit function of the structural parameters $\theta^i$. To estimate $\theta^i$, the log-likelihood can be maximised by using a numerical optimisation routine after the Kalman filter estimates of the latent state have been constructed.

The parameter vector $\theta^i \equiv (f^i, g^i, \sigma_{u,i}^2, \sigma_{v,i}^2, \sigma_{uv,i}, a_0^i, a_1^i, a_2^i)$ collects all structural parameters of the state-space model (4.9)–(4.10). The covariance matrix of the state innovations $\Sigma_v^i = \begin{bmatrix} \sigma_{u,i}^2 & \sigma_{uv,i} \\ \sigma_{uv,i} & \sigma_{v,i}^2 \end{bmatrix}$ has three parameters to estimate; there is therefore eight structural parameters in total to be estimated in the model. I discuss the parameters estimates in Section 4.3.3.

## 4.3 The data and empirical strategy

In this section, I first describe the data used in the estimation and present some descriptive statistics. Next, I discuss the estimation strategy, and in particular, the priors used in the estimation of the Kalman projections (4.11)–(4.15). The estimation results are presented in Section 4.4.

### 4.3.1 Description of the data

To gauge the beliefs of investors in the residential housing market during the housing boom of the late 1990's to early 2000's and the financial crisis of the late 2000's in the U.S., I focus on regional markets in different parts of the U.S. A portfolio of investment into residential housing in a given region then represents an asset $i$ in the conditional CAPM.

To measure returns on these regional portfolios, I use monthly regional Case & Shiller U.S. Home Price indices from 17 metropolitan statistical areas
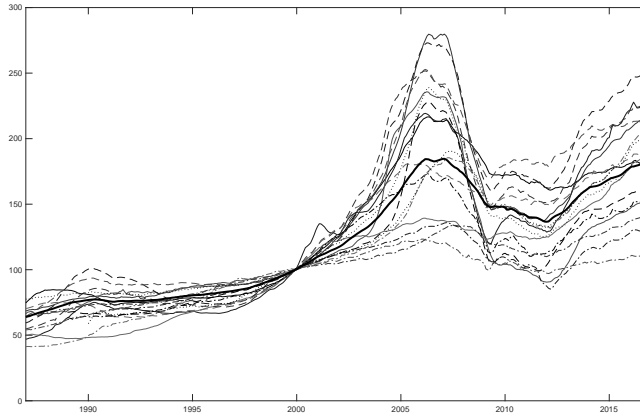
Figure 4.1: Regional Case-Shiller home price indices in the U.S. in 1987–2016

*Note:* January 2000 = 100. Thick solid black line: Case-Shiller National Home Price index. Thin lines: regional home price indices from 17 metropolitan statistical areas.

(MSAs), which cover the years 1987–2016, published by Standard & Poor's. Monthly returns are computed from each series as month-on-month percentage changes in the price index.[13]

As the market portfolio used to price the risk in the regional housing markets, I use the monthly Case & Shiller U.S. National Home Price index, which pools together the development in residential house prices in the whole U.S. It is constructed as a weighted sum of the regional series and published by Standard & Poor's. Similarly to the regional indices, I compute the return on this market portfolio as month-on-month percentage changes in the index.

To compute excess returns in each regional market $i$, denoted by $R_t^i$ in the model, I subtract the one-month Treasury bill return from the returns computed from the house price indices.

Figure 4.1 shows the residential house price development in the 17 differ-

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[13]Ideally, data on mortgage-backed security (MBS) yields would be used in the estimation. Unfortunately, there is no index on MBS prices or yields that would cover the pre-crisis years that I am aware of. An early MBS index (the ABX index), launched by the market analytics company IHS Markit, starts from January 2006, only a few months before house prices peaked. Although the house price appreciation computed from the house price indices does not necessarily directly map to returns on MBS, it is a measure of returns on investment in residential housing, the asset class underlying MBS.
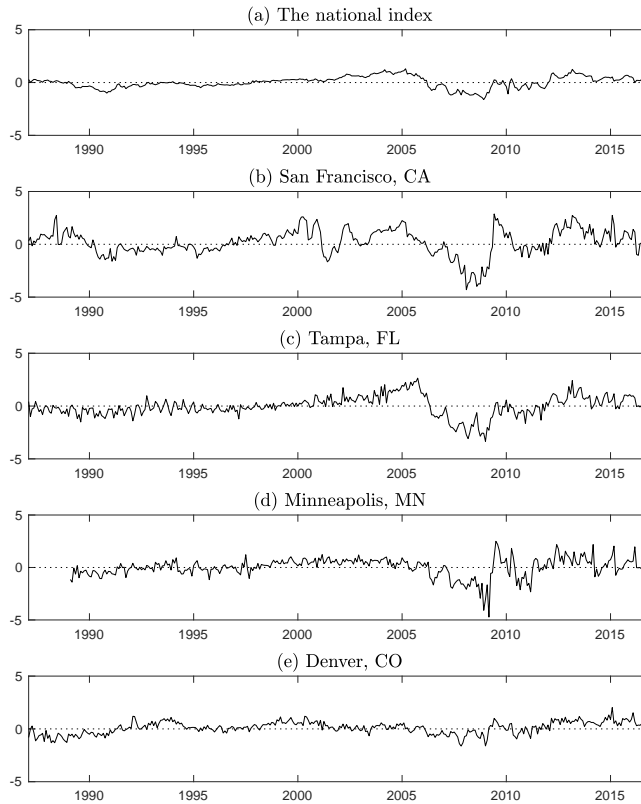
Figure 4.2: Excess returns on the national and selected regional Case-Shiller home price indices in the U.S. in 1987–2016

ent MSAs, as well as the aggregate national house price index (plotted with a thick black solid line), from 1987 to 2016. January 2000 is the base period of all indices. Three features stand out in the data. First, price developments in the various regions were rather homogeneous prior to 2000. Second, after 2000, the price patterns diverge considerably, with some regions (such as Las Vegas, Nevada; Tampa, Florida; and San Francisco, California) experiencing large booms and busts, while in others prices stayed rather stable (such as Denver, Colorado; and Cleveland, Ohio). Finally, cross-sectional price dispersion has

remained remarkably high even after the subprime crisis.

Figure 4.2 displays the monthly excess returns on the national home price index — i.e. the excess return on the market portfolio — as well as monthly excess returns on four regional indices, which will be discussed in more detail in Section 4.4. The figure shows that excess returns on the national index are relatively close to zero and not very volatile over the whole sample; the average excess return on the national index is 4 basis points in the sample, with a standard deviation of 0.5%.

The regional indices are more volatile. Their standard deviations are 1.2% for San Francisco; 1.0% for Tampa; 0.9% for Minneapolis; and 0.6% for Denver. San Francisco and Tampa, which experienced strong house price booms in the 2000's prior to the subprime crisis, exhibit positive excess returns all through this period, while Minneapolis does not. All three, however, suffer big depreciations in house prices from 2006 onwards. In Denver the price index, and consequently excess returns, stay relatively stable through the crisis period.

### 4.3.2 Setup of the Kalman filter

I run the Kalman filter for each region $i$ separately. The implicit assumption is then that excess returns in a given region are only correlated with returns in other regions through the returns on the national price index, and not directly. For MSAs geographically removed from each other, this assumption is reasonable. However, from some large states (notably California and Florida), more than one metropolitan area is represented in the data sample. A possible extension to the framework presented here would be to jointly estimate the latent risk loadings in metropolitan areas in these states.

The estimation proceeds in three steps. First, I estimate the structural parameters of the model for each region by maximum likelihood constructed with the Kalman filter. In the estimation of the parameters, I use the full data series for each region. In the second step, for each region, I re-run the Kalman filter with the estimated parameter values to extract an estimate of the ex-ante beliefs on the latent state $\zeta^i_t$, given by the filtered estimate $\zeta^i_{t|t-1}$. In the final step, I run this ex-ante estimate through the Kalman smoother to extract the ex-post estimate of the latent state, given by the smoothed estimate $\zeta^i_{t|T}$.

The *ex-ante* estimate $\zeta^i_{t|t-1}$, which uses observations only up to a given period $t-1$, is the estimate of *investor* beliefs in period $t \leq T$ about the true risk loading of each regional housing market on the national housing portfolio in that period. The *ex-post* estimate $\zeta^i_{t|T}$ gives the *econometrician*'s best estimate

of the true risk loading in period $t \leq T$, given the full sample of observations up to period $T$. This ex-post estimate therefore allows to evaluate whether investors, at the time, under- or overestimated the systematic risk in housing investment in a given region $i$.

In what follows, I drop the superscript $i$ from variables that refer to the risky asset or portfolio; here, it refers to the regional housing portfolio that is being compared to the overall market portfolio.

### 4.3.3 Structural parameter estimates

The maximum likelihood estimates of the structural parameters of the Kalman filter are summarised in Table 4.1. All MSA region parameters are estimated independently. Standard errors are shown in parentheses. The parameters are estimated rather precisely due to the relatively long monthly data series.

Notably, most regions display considerable persistence in the autoregressive process for the latent state. In almost all regions, either $f$, $g$, or both are estimated to be very close to unity, or sometimes even equal to unity, up to the numerical precision used in the estimation. These results reflect those discussed in Trecroci (2014). In the context of stock market returns, he argues that the time-varying coefficients $\alpha_{t+1}$ and $\beta_{t+1}$ are likely to be very persistent. He does not estimate them, but rather assumes them to be random walks and sets $f = g = 1$, whereas I have estimated them from the data. The autoregressive coefficients $f$ and $g$ have very important implications for the behaviour of the beliefs on the latent state.

The second notable feature is that all states exhibit statistically significant heteroskedasticity in the idiosyncratic innovation $\varepsilon_t$: all or part of the GARCH coefficients $a_0, a_1, a_2$ significantly differ from zero for all MSAs. This is another feature common to many data series on financial returns.

To start off the Kalman filter recursion, I use a diffuse prior with $\zeta_{0|0} = \mathbf{0}$ ($4 \times 1$) and $P_{0|0} = \kappa \mathbb{I}_4$, where $\kappa$ is a large scalar value. In the estimation, I use $\kappa = 50$. The data samples are large enough that neither the structural parameter estimates nor the estimates of the latent state $\zeta_t$ are sensitive to the choice of priors.

I find that in all cases, the null hypothesis of normally distributed forecast errors $\tilde{R}_t^i$ cannot be rejected at the 5% significance level, which suggests that the state-space model is not misspecified. This is not the case when the model is estimated under the assumption of homoskedastic idiosyncratic innovations. Thus the conditionally Gaussian GARCH structure for the variance of

| Region | $N$ | Parameter | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $f$ | $g$ | $\sigma_u^2$ | $\sigma_v^2$ | $\sigma_{uv}$ | $a_0$ | $a_1$ | $a_2$ |
| Atlanta | 263 | 1.00 | 0.33 | 0.02 | 0.01 | 0.00 | 0.00 | 0.35 | 0.54 |
| | | (0.00) | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) | (0.01) |
| Boston | 359 | 0.78 | 0.83 | 0.06 | 0.02 | 0.00 | 0.01 | 0.27 | 0.23 |
| | | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) |
| Charlotte | 359 | 0.98 | 1.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.06 | 0.52 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.02) |
| Cleveland | 359 | 0.88 | 0.77 | 0.02 | 0.08 | -0.03 | 0.03 | 0.36 | 0.38 |
| | | (0.00) | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) | (0.01) |
| Denver | 359 | 0.64 | 0.97 | 0.01 | 0.04 | -0.01 | 0.00 | 0.05 | 0.85 |
| | | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Las Vegas | 359 | 0.89 | 0.47 | 0.07 | 0.19 | -0.06 | 0.00 | 0.33 | 0.40 |
| | | (0.00) | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) |
| Los Angeles | 359 | 1.00 | 0.94 | 0.07 | 0.02 | -0.01 | 0.01 | 0.14 | 0.04 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) |
| Miami | 359 | 0.89 | 0.94 | 0.04 | 0.11 | -0.03 | 0.00 | 0.16 | 0.62 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) |
| Minneapolis | 311 | 0.97 | 0.58 | 0.06 | 0.03 | -0.04 | 0.00 | 0.06 | 0.89 |
| | | (0.00) | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| New York | 359 | 0.95 | 0.81 | 0.04 | 0.02 | -0.01 | 0.00 | 0.03 | 0.91 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Phoenix | 311 | 0.41 | 0.98 | 0.06 | 0.02 | -0.03 | 0.00 | 0.10 | 0.49 |
| | | (0.02) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) | (0.03) |
| Portland | 359 | 0.91 | 0.96 | 0.02 | 0.05 | -0.02 | 0.01 | 0.17 | 0.61 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| San Diego | 359 | 0.89 | 0.96 | 0.04 | 0.08 | 0.02 | 0.01 | 0.32 | 0.31 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) |
| San Francisco | 359 | 0.87 | 0.86 | 0.12 | 0.06 | 0.00 | 0.00 | 0.20 | 0.45 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) |
| Washington | 359 | 0.89 | 0.96 | 0.01 | 0.03 | 0.00 | 0.02 | 0.25 | 0.46 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.01) | (0.01) |
| Seattle | 289 | 0.66 | 0.91 | 0.02 | 0.03 | -0.02 | 0.00 | 0.03 | 0.94 |
| | | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Tampa | 359 | 0.87 | 0.94 | 0.00 | 0.05 | 0.00 | 0.00 | 0.03 | 0.95 |
| | | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |

Table 4.1: Maximum likelihood estimates of structural parameters

*Note:* Estimated individually for each MSA region. Standard errors in parentheses. $N$: number of monthly observations. Parameters: $f$, $g$: AR(1) coefficients for $\beta_t$ and $\alpha_t$, respectively. $\sigma_u^2$, $\sigma_v^2$, $\sigma_{uv}$: elements of the state innovation covariance matrix $\Sigma_v$. $a_0$, $a_1$, $a_2$: GARCH(1,1) coefficients of the idiosyncratic innovation $\varepsilon_t$.

the idiosyncratic innovation $\varepsilon_t^i$ is crucial for a good fit of the model to data.

## 4.4  Estimation results

In this section, I describe and discuss the estimation results for selected MSA regions: San Francisco, CA; Tampa, FL; Minneapolis, MN; and Denver, DN. These regions are chosen to display the various different patterns observed in the data as well as the latent state estimates. These data patterns are displayed in Figure 4.2 and discussed in Section 4.3.1.

For the sake of space, the Kalman filter estimates for the majority of regions are relegated to Appendix 4.A. All results reported here as well as in the appendix omit a burn period of 50 months (13% of observations) from the start of the sample to mitigate the potential effect of the choice of priors on the estimates.

Out of the regions discussed here, San Francisco and Tampa experienced very strong booms and subsequent busts in residential house prices. Minneapolis, on the other hand, followed rather closely the evolution of the national index. Finally, in Denver, residential house prices did not experience a significant boom-bust episode.

Before discussing the time-varying Kalman filter estimates, Table 4.2 shows naive OLS estimates from regressions of each regional excess return series on the national excess returns. The OLS regressions are estimated individually for each region. These estimates correspond to a static CAPM model, where the estimation equation is:

$$R_t^i = \alpha^i + \beta^i R_t^M + \epsilon_t^i. \tag{4.17}$$

The results in Table 4.2 are ordered by the $R^2$ statistic of each regression. It measures how well movements in national excess returns explain movements in the regional excess returns.

Two main observations emerge from Table 4.2. First, a high $R^2$ coincides with high estimates of the regional housing market betas $\beta$, which are above unity and statistically significant for cities such as Miami and Tampa in Florida, San Francisco and Los Angeles in California, and Las Vegas in Nevada. These are places that also experienced the strongest booms and busts in the residential housing markets. On the other hand, the national excess returns have no explanatory power for Atlanta and Phoenix; these areas have estimated housing market betas that do not statistically significantly differ

150

| | (1) Miami | (2) Washington | (3) Tampa | (4) Los Angeles | (5) New York | (6) San Diego |
|---|---|---|---|---|---|---|
| $\alpha$ | 0.006 | 0.034 | -0.061* | 0.086* | -0.045* | 0.088* |
| | (0.031) | (0.023) | (0.027) | (0.035) | (0.022) | (0.038) |
| $\beta$ | 1.694*** | 1.324*** | 1.484*** | 1.689*** | 1.020*** | 1.525*** |
| | (0.074) | (0.048) | (0.057) | (0.068) | (0.045) | (0.082) |
| $R^2$ | 0.74 | 0.72 | 0.72 | 0.67 | 0.64 | 0.58 |
| $N$ | 359 | 359 | 359 | 359 | 359 | 359 |
| | (7) San Francisco | (8) Las Vegas | (9) Boston | (10) Portland | (11) Charlotte | (12) Denver |
| $\alpha$ | -0.130** | -0.084 | -0.009 | 0.166*** | -0.053* | 0.090*** |
| | (0.043) | (0.048) | (0.027) | (0.031) | (0.020) | (0.026) |
| $\beta$ | 1.673*** | 1.746*** | 0.903*** | 0.747*** | 0.495*** | 0.555*** |
| | (0.085) | (0.123) | (0.051) | (0.056) | (0.043) | (0.044) |
| $R^2$ | 0.57 | 0.53 | 0.49 | 0.33 | 0.33 | 0.27 |
| $N$ | 359 | 359 | 359 | 359 | 359 | 359 |
| | (13) Cleveland | (14) Minneapolis | (15) Seattle | (16) Phoenix | (17) Atlanta | |
| $\alpha$ | -0.073* | 0.009 | 0.172*** | 0.032 | -0.064 | |
| | (0.029) | (0.049) | (0.049) | (0.077) | (0.041) | |
| $\beta$ | 0.511*** | 0.450*** | -0.209** | 0.146 | -0.036 | |
| | (0.064) | (0.088) | (0.077) | (0.145) | (0.072) | |
| $R^2$ | 0.23 | 0.08 | 0.02 | 0.00 | 0.00 | |
| $N$ | 359 | 335 | 289 | 311 | 263 | |

Table 4.2: Naive OLS estimates of regional housing market betas

*Note:* Estimated individually for each MSA region on the full samples. Standard errors in parentheses. *N*: number of monthly observations. Significance levels: 0.001 ***, 0.01 **, 0.05 *.

from zero. They also did not experience a housing market boom prior to the crisis.

Second, the estimated Jensen's alphas ($\alpha$) do not statistically significantly differ from zero for seven out of seventeen regions. In the regions where it differs from zero, there is no clear pattern on the sign and magnitude of the coefficient. This suggests that there was no widespread and systematic mispricing of the regional housing market portfolios over the sample period. The time-varying estimates for Jensen's alpha will be discussed in more detail below.

These naive CAPM estimates assume that the relationship between the excess returns stays stable over the whole period. However, as will be seen from the Kalman filter estimates, this is not the case: investor beliefs about the coefficients fluctuate quite a lot over the sample period, when a conditional learning CAPM model is estimated. However, these static estimates should correspond to the final estimates of the long-run means of the latent states, $\bar{\alpha}$ and $\bar{\beta}$, at the end of the sample period. In the Kalman filter estimations discussed below, the final estimates of the long-run means do not statistically significantly differ from these static coefficients in regions for which a stationary autoregressive process for the latent state is estimated.[14]

Figure 4.3 shows the estimates on the latent state in San Francisco. Panel (a) shows the beliefs on the time-varying intercept $\alpha_{t+1}$, and Panel (b) the beliefs on the time-varying risk loading $\beta_{t+1}$. In both panels, the grey solid line with bullets shows the prediction on the state in period $t + 1$ given information up to period $t$, updated each period as new observations are made. This corresponds to the ex-ante belief of investors at the time. The black solid line shows the smoothed estimated formed by using the whole sample, which represents the econometrician's ex-post estimate of the latent state. The gray dashed line shows the ex-post estimate of the time-invariant mean of these latent processes. The gray shaded areas represent 80% confidence intervals for the smoothed estimates, computed from the root mean squared errors (RMSE), or the square roots of the elements of $P_{t|T}$.[15] Finally, Panel (c) shows the estimated time-varying variance of the idiosyncratic innovation $\varepsilon_{t+1}$, given information up to period $t$.

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[14]This does not hold for the regions for which a unit root process for the latent state is estimated; in this case, the observed signals do not reveal any information about the long-run means, and as a consequence, the filtered estimates of the long run mean do not converge to the OLS estimates.

[15]The RMSE captures both estimation uncertainty and fundamental uncertainty, stemming from the structural innovations to the latent state. The confidence interval depicted in the figures is therefore not a conventional measure of estimation uncertainty only.
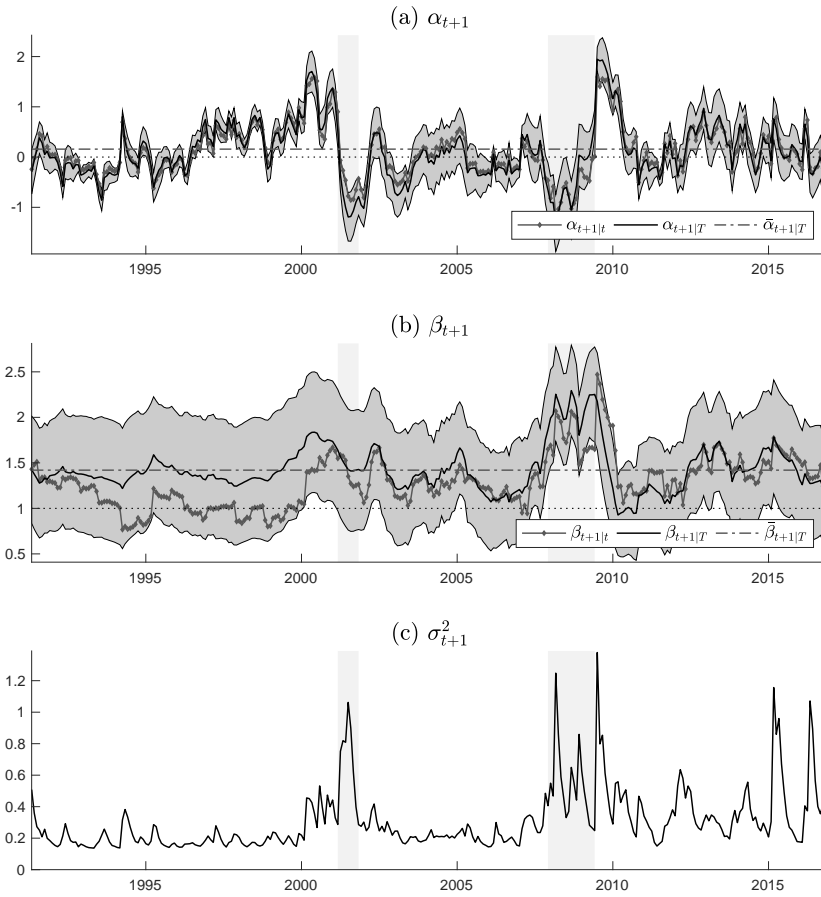
Figure 4.3: Ex-ante (filtered) and ex-post (smoothed) beliefs on the latent state, San Francisco, CA

*Note:* Panel (a): estimates of $\alpha_{t+1}$ and its mean $\bar{\alpha}$. Panel (b): estimates of $\beta_{t+1}$ and its mean $\bar{\beta}$. Panel (c): estimated forecast error variance $\sigma_t^2$ for the observed regional house price return. Estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates $\alpha_{t+1|T}$ and $\beta_{t+1|T}$ computed from the RMSE. The light shaded bars represent NBER recessions.
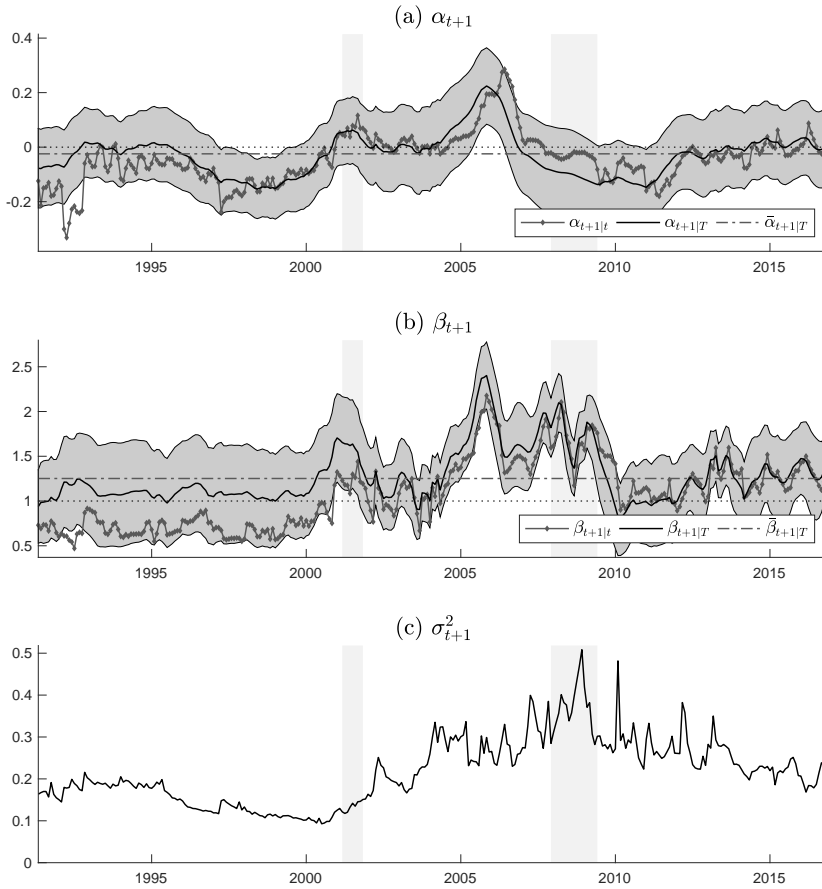
Figure 4.4: Ex-ante (filtered) and ex-post (smoothed) beliefs on the latent state, Tampa, FL

*Note:* Panel (a): estimates of $\alpha_{t+1}$ and its mean $\bar{\alpha}$. Panel (b): estimates of $\beta_{t+1}$ and its mean $\bar{\beta}$. Panel (c): estimated forecast error variance $\sigma_t^2$ for the observed regional house price return. Estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates $\alpha_{t+1|T}$ and $\beta_{t+1|T}$ computed from the RMSE. The light shaded bars represent NBER recessions.

Panel (b) of Figure 4.3 shows, first, that the ex-ante belief about the systematic risk loading $\beta_{t+1|t}$ is below the ex-post estimate $\beta_{t+1|T}$ in the 1990's and early 2000's, although it is within the 80% confidence interval of the latter: there is no evidence of a systematic mis-estimation of undiversifiable risk, as evaluated ex-post. The ex-ante belief is revised upwards from below unity in the late 1990's to above two during the financial crisis. Both the ex-ante and the ex-post beliefs are above the estimated ex-post long run mean $\bar{\beta}_{t+1|T}$ during this period.

On the other hand, the ex-ante estimate $\alpha_{t+1|t}$ is very close to the ex-post estimate $\alpha_{t+1|T}$, in Panel (a), and fluctuates around zero. $\alpha_{t+1|t}$ can be interpreted as an estimate of Jensen's alpha, or the extent of systematic mispricing of the asset. A Jensen's alpha below (above) zero implies that the asset systematically under-performs (over-performs) the overall market in terms of its average expected return.[16] Here, there is no evidence of such mispricing. However, as the pricing model here is simple and univariate, no strong interpretation on the estimated value of $\alpha_{t+1}$, and its deviations away from zero, should be made.

Finally, the idiosyncratic uncertainty has peaked before both recessions in the sample period, as seen from Panel (c).

Figure 4.4 shows a similar pattern for Tampa, in Florida. California and Florida were both among the states that experienced the biggest booms and busts in house prices. Here, there is some evidence that the ex-ante belief about $\beta_{t+1}$ is significantly below the ex-post estimate in the 1990's and early 2000's. The beliefs about the systematic risk loading are also strongly updated upward in 2005, as house prices peak. On the other hand, estimates for $\alpha_{t+1}$ do not significantly differ from zero, suggesting that there was no mispricing of the housing portfolio in terms of Jensen's alpha. The idiosyncratic uncertainty depicted in Panel (c) increases in volatility, but also in level, in the run up to the crisis starting from 2000. This suggests a greater uncertainty about future returns to residential housing, at the time, even as house prices were still rising.

Figure 4.5 shows the estimation results for Minneapolis. The pattern is again similar to the ones discussed before, although Minneapolis experienced a much milder appreciation in house prices than San Francisco or Tampa. The pattern in house prices in the Minneapolis region followed quite closely the national average. Here, ex-ante beliefs about $\beta_{t+1}$ are significantly below the ex-post estimates in the 1990's. Starting from 2000, both ex-ante and ex-

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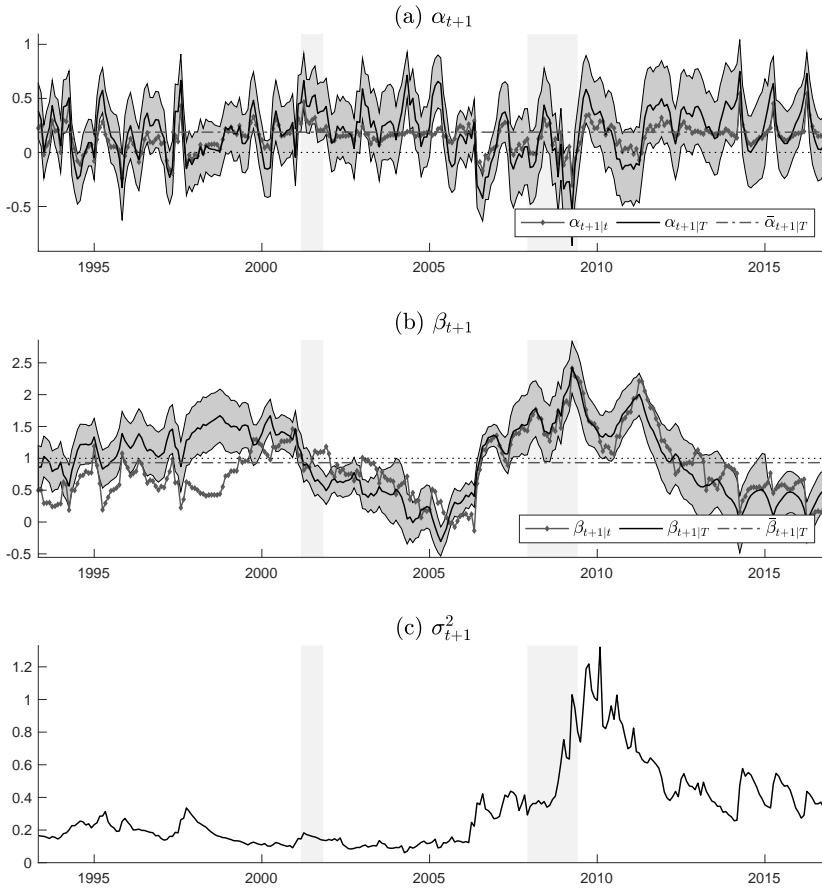
[16]See Black et al. (1972) for a detailed discussion.

Figure 4.5: Ex-ante (filtered) and ex-post (smoothed) beliefs on the latent state, Minneapolis, MN

*Note:* Panel (a): estimates of $\alpha_{t+1}$ and its mean $\bar{\alpha}$. Panel (b): estimates of $\beta_{t+1}$ and its mean $\bar{\beta}$. Panel (c): estimated forecast error variance $\sigma_t^2$ for the observed regional house price return. Estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates $\alpha_{t+1|T}$ and $\beta_{t+1|T}$ computed from the RMSE. The light shaded bars represent NBER recessions.
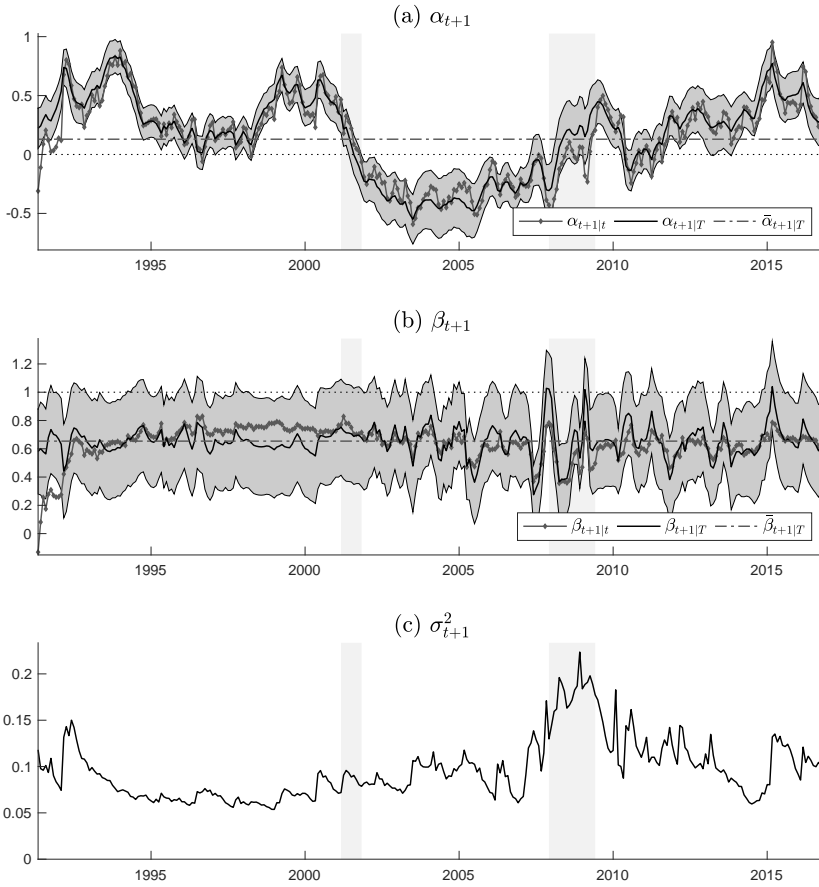
Figure 4.6: Ex-ante (filtered) and ex-post (smoothed) beliefs on the latent state, Denver, CO

*Note:* Panel (a): estimates of $\alpha_{t+1}$ and its mean $\bar{\alpha}$. Panel (b): estimates of $\beta_{t+1}$ and its mean $\bar{\beta}$. Panel (c): estimated forecast error variance $\sigma_t^2$ for the observed regional house price return. Estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates $\alpha_{t+1|T}$ and $\beta_{t+1|T}$ computed from the RMSE. The light shaded bars represent NBER recessions.

post estimates are updated downward before the crisis, reaching a level below zero in 2005–2006, significantly below the estimate of the long-run mean. This suggests that investors believed residential housing investment to be very safe, in fact hedging against aggregate risk in the national housing market at the time. Up until 2005, idiosyncratic uncertainty, depicted in Panel (c), is also regarded very low. These beliefs are swiftly updated upward starting in 2006.

Finally, Figure 4.6 shows the estimates for Denver, in Colorado, where house price appreciation was more subdued than the national average during the run-up to the crisis, and where residential house prices never dropped much even during the crisis. Here, as seen in Panel (b), the ex-ante belief about $\beta_{t+1}$ never diverges much from the ex-post estimate. The ex-post estimate broadly stays below one, suggesting low systematic risk. The idiosyncratic uncertainty in Panel (c) also stays relatively low through the whole sample. Interestingly, in Panel (a), beliefs about $\alpha_{t+1}$ are very low during the run up to the crisis, both ex-ante and ex-post, suggesting a view that the housing market in Denver was under-performing relative to the national market during the years 2001–2006. This is reflected in a negative Jensen's alpha.

Appendix 4.A shows the estimates for the rest of the MSA regions. The patterns are heterogeneous, but most share a few general common features already discussed in this section. In particular, most regions show, first, a general revision of beliefs about the systematic risk loading upward during the run-up to the crisis, starting from the late 1990's and the early 2000's; and second, a marked increase in idiosyncratic uncertainty at the onset of the subprime crisis, starting from 2006.

## 4.5 Conclusions

In this essay, by using time series on residential house price appreciation in different MSA regions in the United states and excess returns computed from these series, I study investor beliefs about undiversifiable risk in the residential housing market. I focus on the beliefs of institutional investors who are able to invest in large regional housing portfolios and only care about the risk/return trade-off of their investment, and therefore on the supply side of financing in the housing market.

Based on this data, I present evidence on a broad pattern in investor beliefs. First, in the early 1990's, beliefs about undiversifiable risk were at low levels in most regional housing markets. Second, these beliefs were slowly updated upward in the run-up to the crisis. Regardless, in many regions, in-

vestors seemed to believe that systematic risk was relatively low even during the boom years in the early and mid-2000's. At the onset of the subprime crisis, these beliefs were swiftly updated upward most prominently in regions that had experienced the biggest booms. In regions where house prices had stayed more stable, fluctuations in beliefs were much more subdued. Evaluated ex-post, there is no evidence of a broad mis-estimation of systematic risk, however: in most cases, ex-post estimates are similar to the ex-ante predictions. In some regions, however, the ex-ante and ex-post estimates on the latent systematic risk significantly diverge.

The main contribution of this essay to the literature is to empirically identify these belief patterns in U.S. data. I model the relationship between excess returns in regional housing market portfolios and the national portfolio by using the conditional CAPM, with the additional assumption that investors do not observe the underlying state of the economy that governs the risk loading of the regional portfolio on the national market. Instead, they attempt to learn the state by observing realised returns on these portfolios. As a result, although investors believe the conditional CAPM relationship to hold in equilibrium, there may be ex-post mispricing of assets and portfolios because of uncertainty about the true state. Using this model, I estimate the implied laws of motions of investor beliefs with the Kalman filter. The results are robust to the specific parametric assumptions and priors.

The empirical results lend support to the prominent narratives of the financial crisis of 2007–2009. First, although in hindsight there was a clear boom-bust pattern in the real estate market, it would have been difficult to see this ex-ante: the ex-post estimates on the true systematic risk in most cases do not point to an elevated risk before the crisis. In the context of the model, the investors believed the unobservable state of the economy to be favourable. This is consistent with beliefs of most investors and policy-makers in the early 2000's: economic fundamentals were indeed believed to be very strong, and house prices were believed to keep rising. This likely contributed to the loosening of lending standards in housing finance.

This observation is also consistent with the boom in mortgage securitisation that started in the 1990's. Mortgage-backed securities, which pool together large amounts of individual mortgages, were believed to be very safe, because they were thought to be very well diversified. Of course, it later turned out that they were not, but instead, the default probabilities of the individual mortgages were quite correlated with each other.

As a final note, the results presented in this essay are empirical and based on the conditional CAPM, which is a reduced form relationship between the

risk premia of different assets. However, the results suggest that partial information and investor beliefs may have important implications on macroeconomic dynamics. In particular, asset price bubbles may very well be driven by genuine but misguided beliefs on robust fundamentals. This opens up many possibilities for future research.

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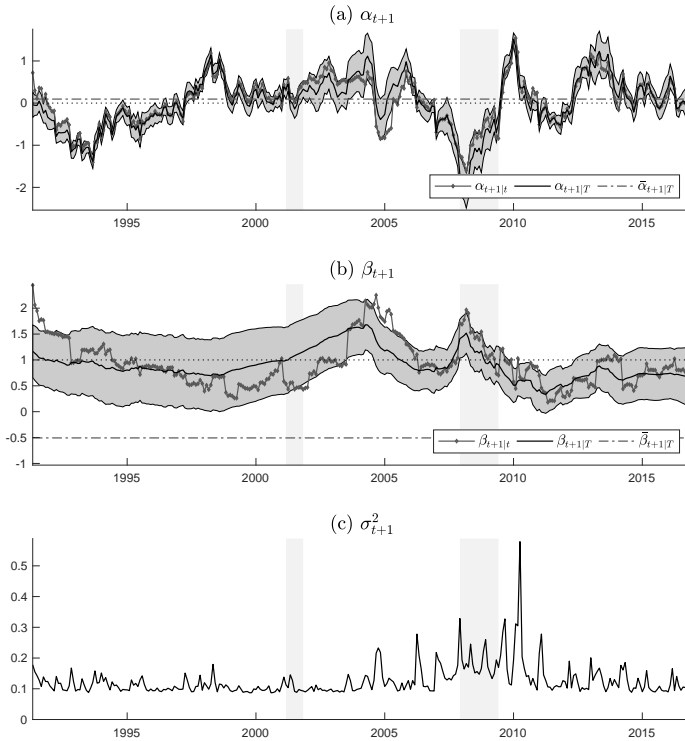
## 4.A   Kalman filter estimates of the latent state
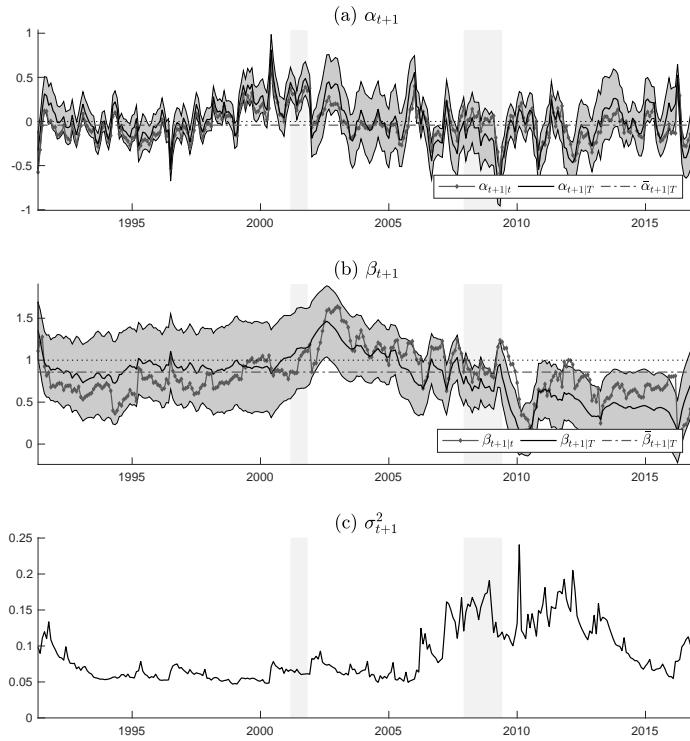


Figure 4.7: Predicted and smoothed latent coefficients in New York, NY
*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
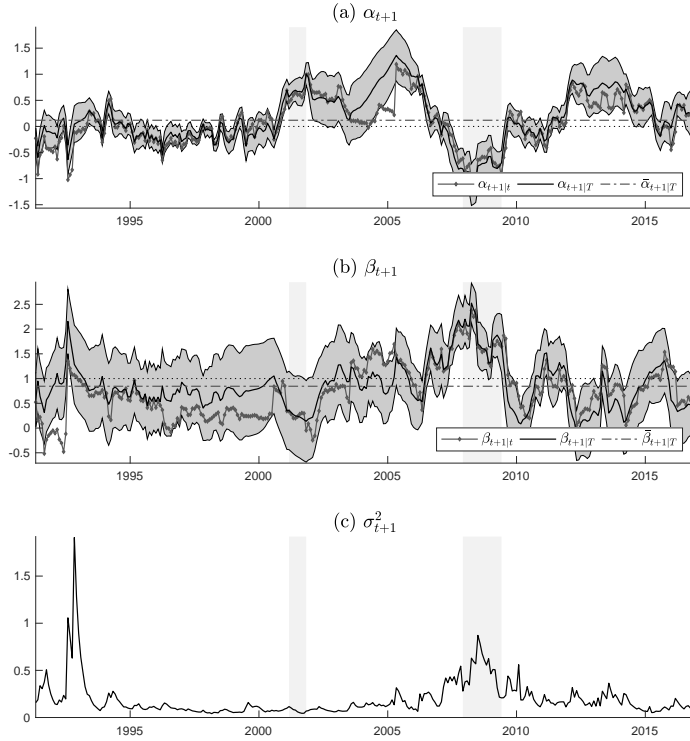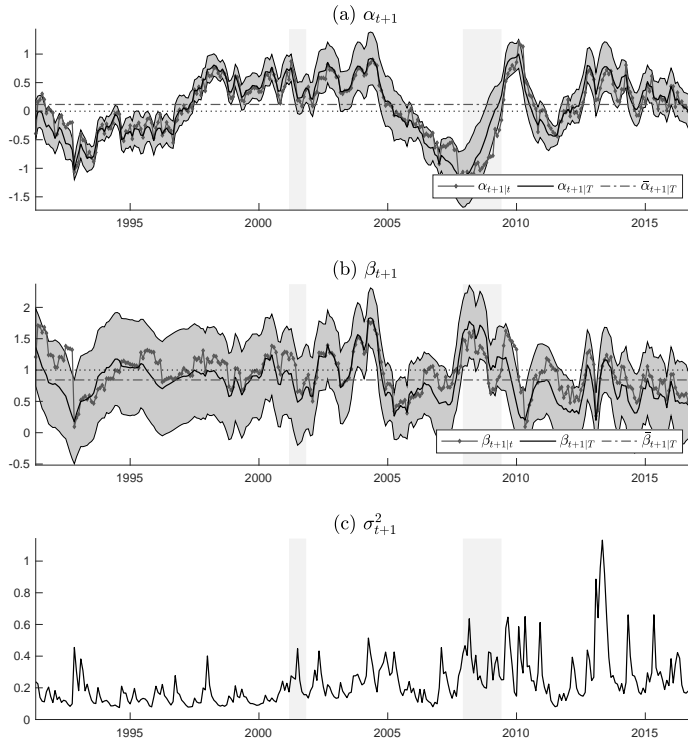
Figure 4.8: Predicted and smoothed latent coefficients in Miami, FL

*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
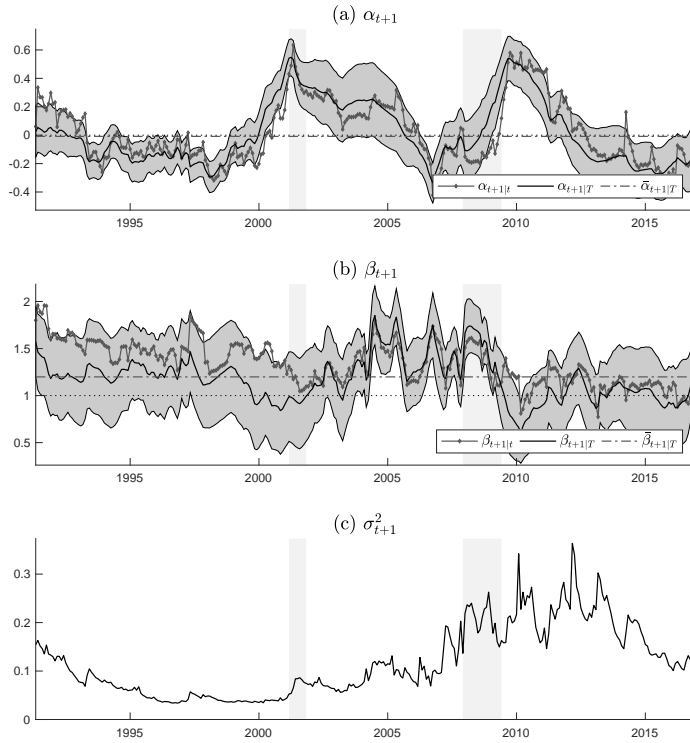
165
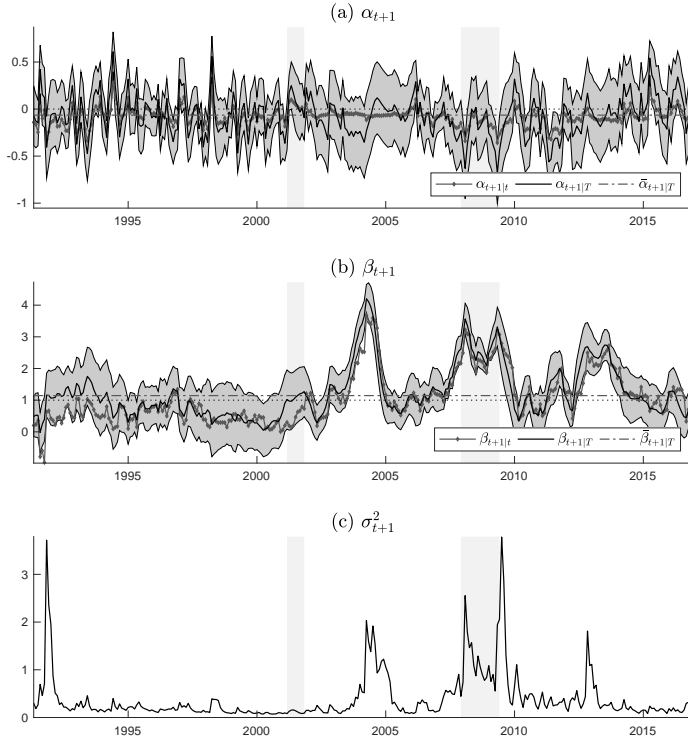
Figure 4.9: Predicted and smoothed latent coefficients in Las Vegas, NV
*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
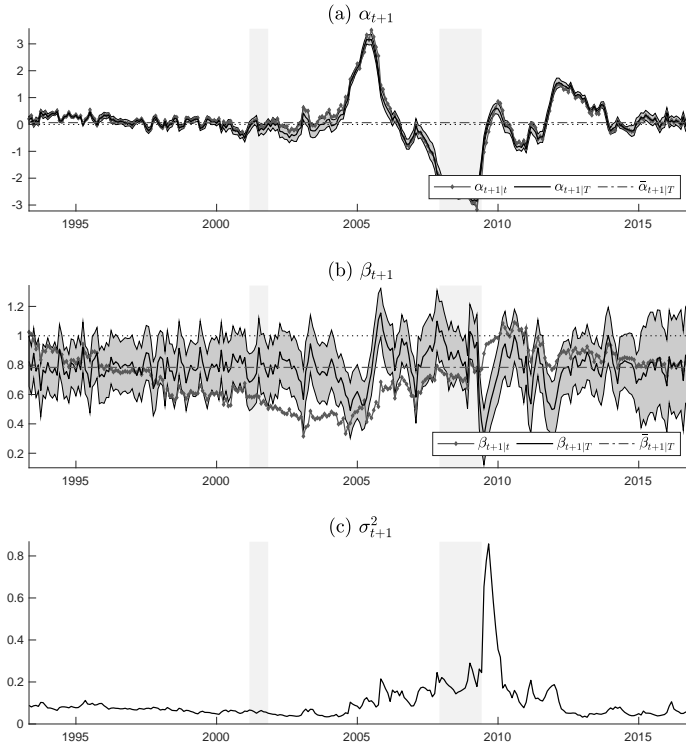
Figure 4.10: Predicted and smoothed latent coefficients in Phoenix, AZ
*Note:* estimated on the sample 1989:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
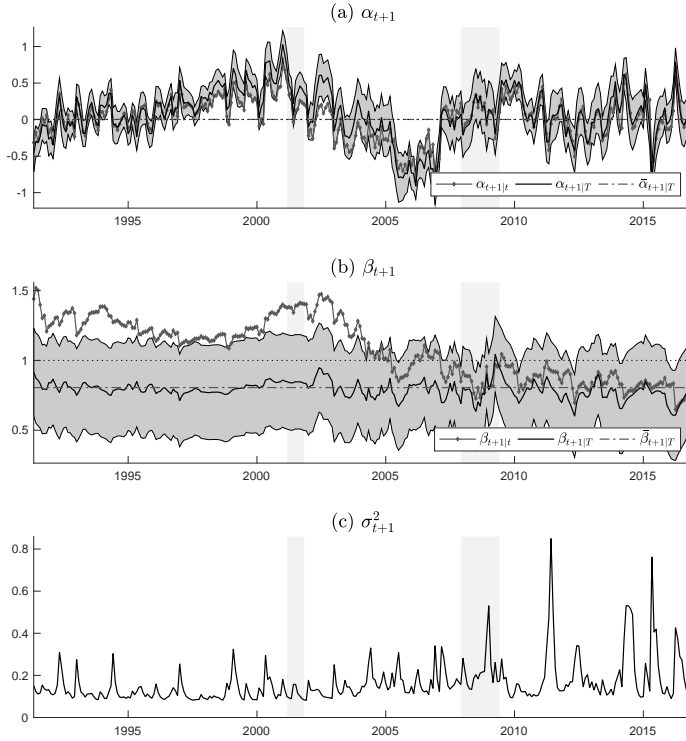
Figure 4.11: Predicted and smoothed latent coefficients in Los Angeles, CA
*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.

Figure 4.12: Predicted and smoothed latent coefficients in San Diego, CA

*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
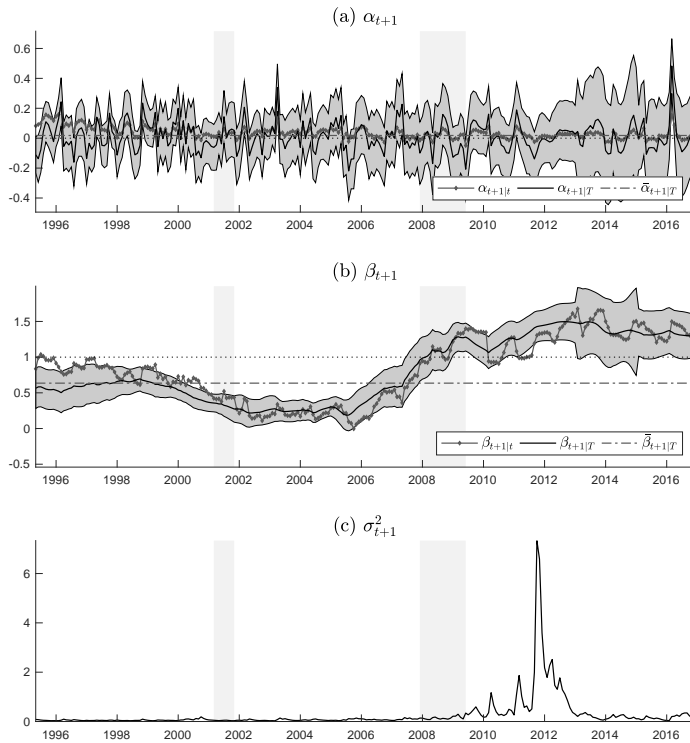
Figure 4.13: Predicted and smoothed latent coefficients in Washington, DC
*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.

Figure 4.14: Predicted and smoothed latent coefficients in Boston, MA
*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.

Figure 4.15: Predicted and smoothed latent coefficients in Atlanta, GA

*Note:* estimated on the sample 1991:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
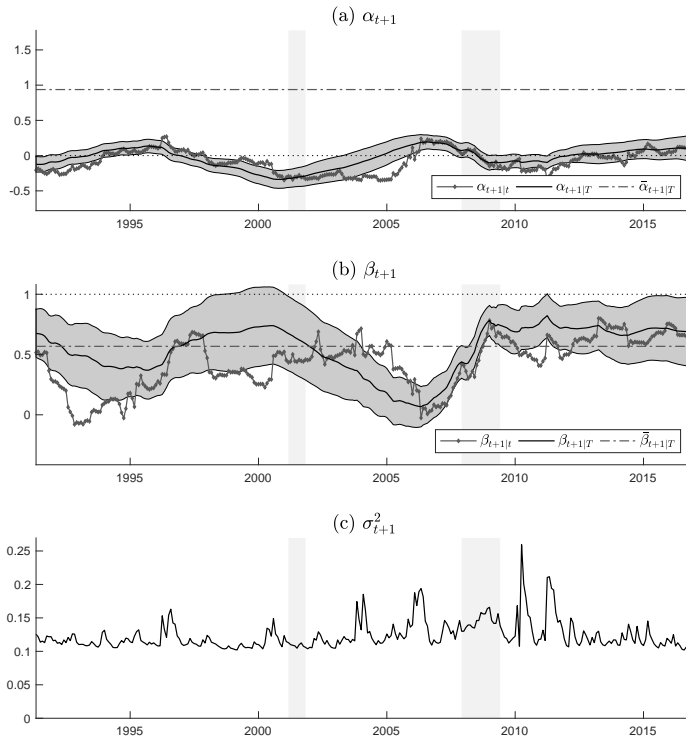
Figure 4.16: Predicted and smoothed latent coefficients in Charlotte, NC
*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.

(a) $\alpha_{t+1}$



(b) $\beta_{t+1}$
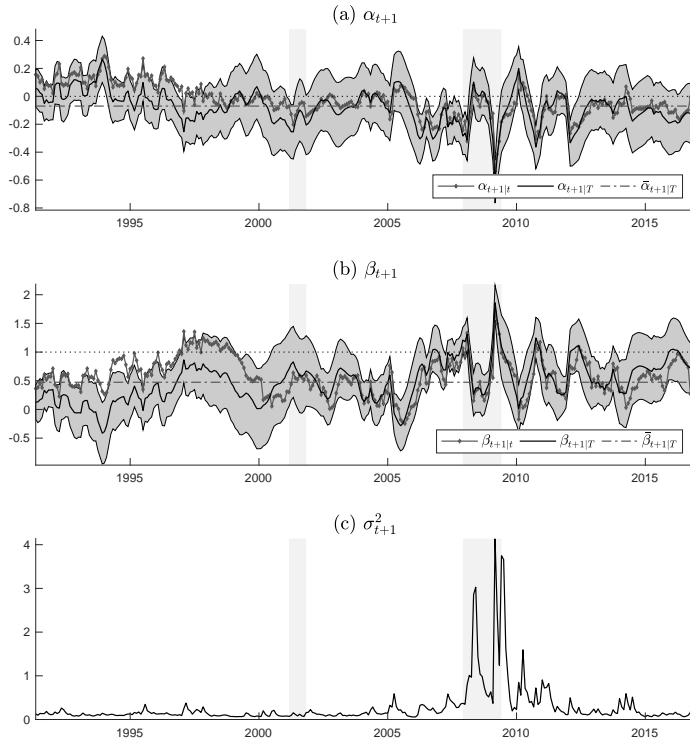


(c) $\sigma^2_{t+1}$



Figure 4.17: Predicted and smoothed latent coefficients in Cleveland, OH

*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
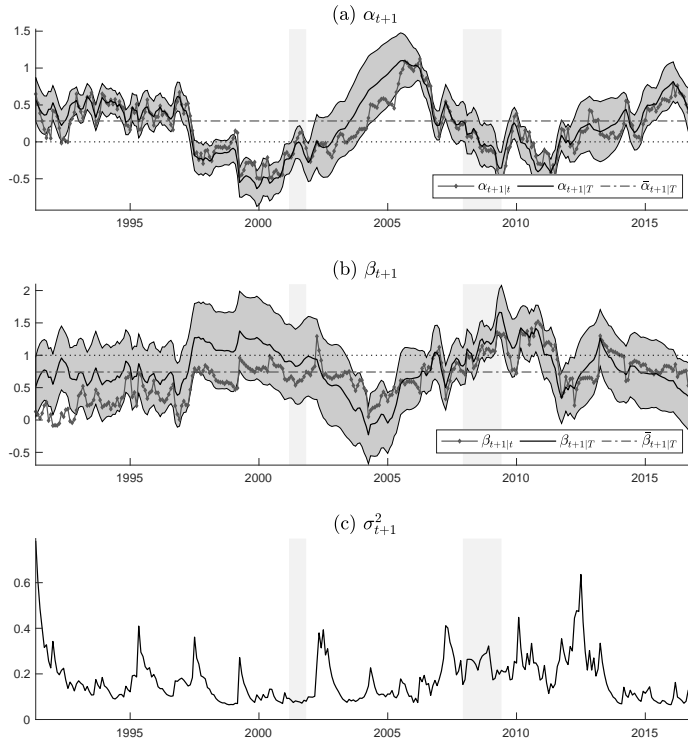
Figure 4.18: Predicted and smoothed latent coefficients in Portland, OR
*Note:* estimated on the sample 1987:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.
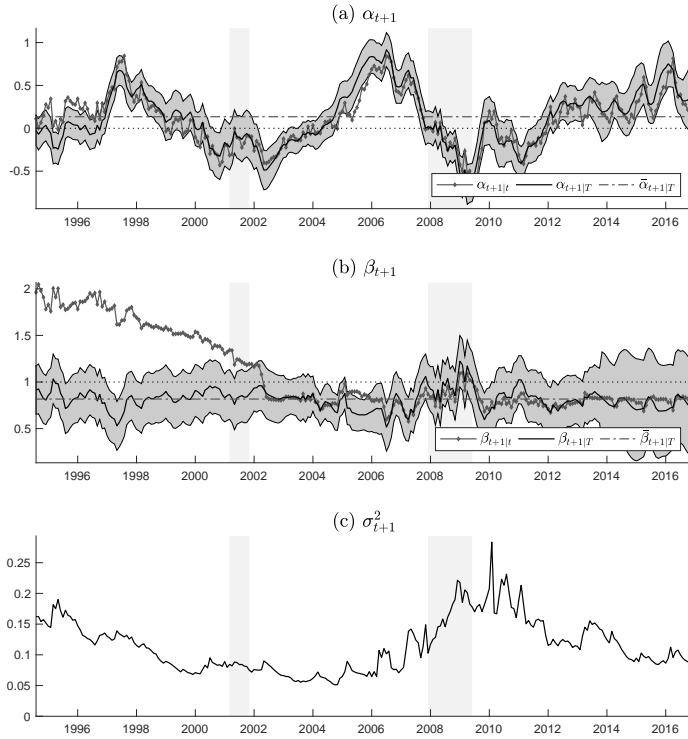
Figure 4.19: Predicted and smoothed latent coefficients in Seattle, WA
*Note:* estimated on the sample 1990:2–2016:12. The gray shaded areas denote the 80% confidence interval for the ex-post (smoothed) estimates computed from the RMSE. The light shaded bars represent NBER recessions.

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