Tuomas Välimäki

Central bank tenders: three essays on money market liquidity auctions

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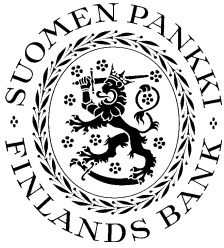   ....

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Abstract

Most OECD central banks implement monetary policy by supplying reserves to the banking sector with the aim of influencing short-term interbank interest rates. To interpret the monetary policy stance accurately, one needs to be familiar with the mechanism for determining the money market equilibrium. The aim of this study is to deepen our understanding of the various effects of different intervention styles on the short-term money market when monetary policy is implemented with an operational framework similar to that of the European Central Bank (ECB).

In the first essay of this study, we model banks' demand for central bank reserves (liquidity) for each day of an n-day reserve maintenance period and analyse liquidity determination under alternative liquidity policy rules that a central bank might apply in fixed rate tenders. It is shown that there is a tradeoff between the central bank's ability to keep a market interest rate close to the tender rate and the stability of liquidity holdings within a maintenance period. The second essay presents a model of a single bank's optimal bidding in the context of fixed rate liquidity tenders. It is shown that banks' bidding crucially depends on the central bank's liquidity policy for tender allotments. This essay also analyses ECB liquidity policy in terms of the model. The final essay models the money market equilibrium and analyses banks' bidding when the central bank uses variable rate tenders. The liquidity supply is fully endogenised by having the central bank minimise a loss function the includes deviations-from-target of interest rate and liquidity. ECB experiences with variable rate tenders are also studied in this essay.

Key words: central bank operational framework, short-term interest rates, money markets, tenders, liquidity policy, bidding

Tiivistelmä

Useimmat OECD-maiden keskuspankit pyrkivät rahapolitiikan käytännön toimeenpanossaan ohjaamaan lyhyimpiä rahamarkkina-korkoja lainaamalla keskuspankkirahaa pankeille. Tästä syystä on tärkeätä ymmärtää, miten lyhyimpien rahamarkkinoiden tasapaino muodostuu, jotta kyetään arvioimaan rahapolitiikan mitoitusta. Tämän tutkimuksen tarkoitus on syventää tietoamme erilaisten interventiotapojen vaikutuksista rahamarkkinoihin, kun rahapolitiikan toimeenpanossa käytetään sellaista välineistöä, jota EKP käyttää.

Tutkimuksen ensimmäisessä esseessä *"Fixed rate tenders and the overnight money market equilibrium"* mallinnetaan rahamarkkinalikviditeetin kysyntä erikseen jokaiselle tietyn varantojenpitoperiodin päivälle sekä analysoidaan keskuspankin likviditeettipolitiikan eri vaihtoehtojen vaikutuksia likviditeetin määräytymiseen, kun operaatiot toteutetaan kiinteäkorkoisina huutokauppoina. Siinä osoitetaan, että keskuspankin on annettava joko rahamarkkinalikviditeetin vaihdella varantojenpitoperiodin sisällä pankkien korkonäkemyksen mukaan tai rahamarkkinakoron poiketa rahapolitiikan ohjauskoron mukaiselta tasolta. Toisessa esseessä *"Bidding in fixed rate tenders"* mallinnetaan yksittäisen pankin optimaalinen tarjoustenteko kiinteäkorkoisissa huuto-kaupoissa. Pankkien tarjousten osoitetaan ratkaisevasti riippuvan keskuspankin valitsemasta politiikasta, kun se päättää huutokaupassa jaettavaksi aiotun likviditeetin määrästä. Tässä yhteydessä tarkastellaan myös EKP:n likviditeetinjakopolitiikkaa käytetyn mallin valossa. Tutkimuksen kolmannessa esseessä *"Variable rate liquidity tenders"* mallinnetaan rahamarkkinoiden tasapaino sekä analysoidaan pankkien tarjoustentekoa keskuspankin toteuttaessa operaationsa vaihtuvakorkoisina huutokauppoina. Likviditeetin määräytyminen on tässä mallissa puhtaasti endogeenista, sillä keskuspankki päättää huutokaupoissa jaettavan likviditeetin määrän minimoimalla tappiofunktiotaan, joka koostuu koron sekä likviditeetin poikkeamista tavoitellusta. Esseessä tarkastellaan myös EKP:n kokemuksia vaihtuvakorkoisista huutokaupoista.

Asiasanat: rahapolitiikan välineet, rahamarkkinakorot, rahamark-kinat, huutokaupat, likviditeettipolitiikka, huutokauppatarjoukset

Foreword

The roots of this thesis can be traced back to 1995 when I joined the Bank of Finland and became involved in the analysis of the effects of reserve averaging on the money markets. My interest in the determination of short-term interest rates under different intervention styles was further enhanced by my experiences as a member of the EMI/ECB Task Force on Monetary Policy Instruments and Procedures in 1997–1998. In September 1999, I joined the Research Department of the Bank of Finland while continuing my studies at the Helsinki School of Economics and working on my licentiate thesis. The first essay of this study stems from that project. After completing the licentiate thesis, I was given the opportunity to broaden the research into doctoral thesis, for which I am deeply grateful to the Bank.

Jouko Vilmunen deserves my warmest thanks for all the effort he has put into my project. His enthusiastic supervision encouraged me to carry on with my chosen line of research. I also want to express my appreciation to the Head of the Research Department, Juha Tarkka, the entire department staff, and the two official licentiate thesis examiners, Professor Seppo Honkapohja and Professor Mikko Puhakka. A very special 'thank you' goes to each of the two preliminary examiners of this dissertation, Professor Giuseppe Bertola and Professor Vesa Kanniainen.

From the remaining long list of people to whom I owe my gratitude let me cite Pentti Pikkarainen, Jarmo Kontulainen and Antti Suvanto, who helped enable me to stay on at the Research Department for three years. Rafael Repullo from CEMFI and Nuno Cassola from the ECB have helped me, for example, by providing opportunities to present my work outside the Bank. Professor Pertti Haaparanta, who has been my official supervisor at the Helsinki School of Economics, certainly deserves my gratitude. Glenn Harma improved my English a great deal in the process of writing the thesis. I also would like to thank Antti Ripatti, who patiently helped me with LaTex codes, as well as all my colleagues who helped to create the pleasant working atmosphere on the fifth floor of Aleksi 36 and Kluuvi.

Finally, I owe a great deal to my dear wife Maria for her patience throughout the lengthy endeavour. And I hope that my daughter Anna, who was born around the start of the project, and my son Olli,

who came into the world just before I completed the research, have
not been missing their dad too much in the process.

25.4.2003
Tuomas Välimäki

Contents

Introduction

Contents

1   Motivation

Monetary policy is nowadays implemented through open market operations in most of the OECD economies. Using these operations, central banks usually supply reserves to the banking sector with the aim of influencing short-term interbank interest rates. Consequently, understanding the mechanism that determines the level, expected value, and volatility of the shortest money market rate (normally the interbank rate on unsecured overnight deposits) is crucial to anyone who wants to be able to correctly interpret the monetary policy stance. The importance of the overnight rate of interest is enhanced by its pivotal role in the term structure of interest rates; it is usually the shortest maturity for which well-organised markets exist.

There is a large amount of academic literature describing the functioning of the market for central bank reserves. However, the analysis in most of this literature is partial. It abstracts from monetary policy; the role of the central bank is exogenous (ie central bank liquidity management is taken as given). Furthermore, this literature focuses heavily on indentifying and explaining the stylised facts of the fed funds market (ie the US market for unsecured overnight deposits). Yet, it has been shown that many behavioural properties identified for the fed funds market do not hold under different institutional setups and alternative methods of intervention (Prati et al 2002). The effect of intervention style on the money market has become increasingly topical since the start of the European Central Bank (ECB).

The aim of this thesis is to contribute to filling this gap by analysing the *equilibrium* outcome for the money market under various alternative liquidity policies when the operational framework of the model central bank resembles that of the ECB. The ECB implements monetary policy by actively managing money market reserves via liquidity tenders, by providing the banks with two standing facilities, and allowing averaging of reserve holdings. Essay 1 presents a model of the determination of equilibrium in the interbank market for overnight liquidity when the central bank applies fixed-rate tenders in its liquidity provision. Essay 2 models the optimal bidding behaviour of a single bank in the context of fixed-rate liquidity tenders, and analyses ECB liquidity policy in terms of the model. The final essay derives an equilibrium model for the money market,

12

and analyses the bidding behaviour of the banks when the central bank applies variable rate tenders. Before summarising the main contribution of this thesis, we review the existing literature on the functioning of money markets and on liquidity tenders.

2   Literature overview

The literature related to the theme that is common to all three essays in this thesis – how the equilibrium for the money market is determined under various liquidity policies that the central bank may apply in liquidity tenders – can be divided into two parts. We first review the literature on the functioning of the money market, and then turn to the rapidly growing literature on central bank liquidity auctions.

2.1   Functioning of the money market

The classic model by Poole (1968) serves as a benchmark model for analysing commercial bank reserve management under uncertainty. Updated versions of this model are still frequently applied in modelling banks' demand for liquidity (eg in Clowse and Dow 1998, Pérez Quirós and Rodríguez Mendizábal 2001, Bartolini, Bertola and Prati 2002). Also, the first essay of this thesis applies, as a starting point, a model that is similar to that of Poole.

Most of the literature on the determination of conditions in the money market is concentrated on identifying and explaining the systematic patterns in the functioning of the market for federal funds. For example, Ho and Saunders (1985) build a micro model of the fed funds market in which the positive spread between the fed funds rate and rates on other short-term money market instruments results from the liquidity benefit of fed funds. They also use this framework to explain the observed tendency of large institutions to be net borrowers in the market. However, as noted by Spindt (1985), their model is not able to explain endogenously the intertemporal behaviour of the market rate. Spindt and Hoffmeister (1988) explain by institutional features (including end-of-day balance accounting,

periodic reserve requirements and line limits on the amount of reserves a bank can borrow from or lend to the market at a given point in time) the increase in volatility of the fed funds rate when the time remaining in the current reserve maintenance period reduces. Hamilton (1996) presents a time-series analysis of daily changes in the fed funds rate between 1 March 1984 and 28 November 1990. Based on that, he conclusively rejects the hypothesis of the rate having followed a martingale within a reserve maintenance period.[1] He also develops a model that explains the identified patterns in the rate as a result of line limits, transaction costs and weekend accounting conventions. Clouse and Dow (1998) build a model of the fed funds market where rate variations result from changes in the supply of reserves, together with a fixed cost associated with discount window borrowing. With this model, they attempt to explain the occasional instances of extremely high rates.

The problem with all these studies is that, first, they do not allow for any explicit role for central bank interventions or the role is very limited at best. Bartolini, Bertola and Prati (2002) address this question by modelling banks' liquidity management jointly with the official intervention policy. However, as their model accounts for the operational framework applied by the Federal Reserve, it is not directly applicable for analysing different institutional setups and intervention conventions (eg those of the ECB). This introduces the second problem with most of the existing literature on reserve markets. It applies intensively institutional setups that are characteristic of the fed funds market, and the findings are not necessarily applicable elsewhere or even to the fed funds market after institutional changes. A recent study by Prati, Bartolini and Bertola (2002) presents an international comparison of short-term money markets in G7 countries as well as in the euro area. They find that many key behavioural features identified for the fed funds market in the previous studies are not robust over institutional details. Hence, they conclude that the operational framework applied, together with the intervention styles of different central banks, play a crucial role in the day-to-day behaviour of the short-term rate. They ask future researchers to address the institutional differences and intervention

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[1] According to the martingale hypothesis, the overnight rate on the days before the final day of reserve maintenance period equal the expected rate for the final day, if reserve holding is based on averaging.

styles as factors explaining differences in the behaviour of interbank markets. The aim of this thesis is to contribute to this analysis.

The factors affecting the demand and supply for euro liquidity are described in Bindseil and Seitz (2001). They also build an econometric model to explain the variability of the spread between the overnight market rate and the official intervention rate. Pérez Quirós and Rodríguez Mendizábal (2001) model the behaviour of the overnight market for central bank funds in a framework similar to the euro area. They claim that the introduction of a deposit facility into a framework that includes a periodic reserve requirement and lending facility stabilises the overnight rate of interest. However, their analysis is again partial; it abstracts from monetary policy, as it lacks central bank interventions.

We turn next to the fast growing literature on the effects of central bank intervention methods on counterparties' bidding behaviour and on the equilibrium of the money markets.

2.2 Central bank liquidity tenders

The intervention procedures of central banks have received much attention, in both the financial markets and the academic literature, since the establishment of the European Central Bank. The active liquidity management of the ECB is conducted through main refinancing operations, in which the ECB provides the market with reserves, using either a fixed rate or variable rate tender procedure. In fixed rate tenders, the ECB pre-announces the rate at which banks can obtain liquidity, whereas in variable rate tenders the banks bid for both the quantity of reserves they want to obtain and the price they are willing to pay. Between January 1999 and June 2000, the ECB conducted its main refinancing operations as fixed rate tenders. During that period, the allotment ratio (ie allotted volume / aggregate bid amount) was on average some 8% and seemed to be on the decline.

This 'overbidding' phenomenon finally induced the ECB to switch the tendering procedure from fixed to variable rate.[2]

The academic literature on ECB interventions starts with Nautz and Oechssler (2000), who model fixed rate tenders as a strategic bidding game between banks, and come to the conclusion that the game does not have an equilibrium, and that the information content of the bids is negligible. Ehrhart (2001) conducts an experimental investigation into fixed rate tenders, where the intervention style of the central bank is similar to that developed in Nautz and Oechssler. He concludes from his experimental results that the fixed rate tender procedure can be 'a regular invitation to the banks to continually raise their bids from auction to auction'. The problem with these two papers is that they abstract from the interbank market, although it is precisely the interest rate of the short term money market that determines banks' incentives to borrow from the central bank, and thus it affects the bidding behaviour. Furthermore, the key results of these models rely on the assumption that the central bank supplies the markets with less liquidity than is actually needed by the banks. However, the incentive for the central bank to apply such a tight liquidity policy is not explained in the papers.

Among the literature on central bank tenders, Ayuso and Repullo (2000) is probably the closest reference to the essays in this thesis. They build a model of fixed and variable rate tenders in which the central bank minimises a loss function that depends on the squared difference between the market rate of interest and the tender rate, and banks' bidding behaviour is determined by this spread. They explain the overbidding phenomenon of fixed rate tenders by a positive spread, which was a result of an asymmetric loss function, ie a loss function that penalises interbank rates below more heavily than above the target rate. They also show that variable rate tenders have multiple equilibria characterised by overbidding. The key results of the paper depend crucially on the asymmetry of central bank preferences. However, such an asymmetry is difficult to justify. Why would a deviation of the market rate below the operational target

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[2]On 8 June 2000, the Governing Council of the ECB decided to switch to the variable rate tender procedure as of 27 June 2000. In an ECB press release, dated 8 June 2000, the new tender mechanism was announced as being '*a response to the severe overbidding problem which has developed in the context of the fixed rate tender procedure*'.

16

create a larger loss to the central bank than a deviation above it, when the target rate is deemed a correct one for the ultimate goal of the monetary policy? Ayuso and Repullo (2001) test whether the banks' overbidding results from the expectations of future tightening of monetary policy or from the existence of a positive spread between short-term money market rates and the main refinancing rate that results from contemporaneous restrictions in the supply of liquidity. They find empirical evidence supporting the latter option. Finally, Bindseil (2002) constructs one- and two-period models of the money markets in which liquidity is supplied via tenders, to analyse the stability of interest rates within the reserve maintenance period, the smoothness of the reserve fulfilment path and bidding costs. The model by Bindseil addresses many of the questions analysed in the essays of this thesis. However, his approach differs considerably from the one we have adopted. Firstly, our essays derive the demand for liquidity from the profit maximisation problem of a single bank, after which we consider various liquidity policy rules for the central bank that determine the intended supply of liquidity, and we finish by analysing bidding behaviour as a function of the spread between the expected market rate of interest and the tender rate. The approach developed in Bindseil takes the martingale hypothesis as given, and forms the equilibrium condition for aggregate bidding on that basis. The differences in treating liquidity uncertainty allow Bindseil to apply the martingale hypothesis. In his two-period model, there is only one liquidity shock in a reserve maintenance period, and it occurs after the final market session of the whole maintenance period. Hence, in his model the martingale hypothesis will naturally hold. In the essays of this thesis, there are autonomous liquidity shocks realised each day before and after the market sessions. Thus, the martingale hypothesis cannot be ex ante taken as given.

Next, we briefly summarise the central features and the contribution of the three essays of this thesis.

3 Summary of the essays and contributions

This section summarises the contents of the three essays of this thesis, and presents their main contribution. Briefly, in the first essay we construct a model of the determination of equilibrium in the interbank market for overnight liquidity where the central bank uses fixed rate tenders in its liquidity provision. The second essay develops a model of the optimal bidding behaviour of a single bank in the context of fixed rate liquidity tenders, and it also analyses ECB liquidity policy in terms of the model. In the final essay, the money market equilibrium and the bidding behaviour of the banks is modelled in the context of variable rate tenders.

3.1 Fixed rate tenders and the overnight money market equilibrium

The first essay develops an equilibrium model of money market conditions in the context of fixed rate liquidity tenders. Based on a single bank's profit maximisation problem in the interbank overnight market for liquidity, we model the demand for liquidity, both in a framework with a daily reserve requirement, and separately for each day of an n-day reserve maintenance period where the averaging provision is applied (ie compliance with the reserve requirement is determined by the average of end-of-day reserve balance over the maintenance period). From the inverse demand functions, we derive the market rate of interest as a function of liquidity. Then we analyse how money market liquidity itself is determined under alternative liquidity policy rules of the central bank. We consider three possible liquidity policy rules for the central bank. First, the central bank may provide the market with all liquidity that is bid for. Secondly, the central bank can scale back the bids it receives in proportion to the bid amounts. For the proportional liquidity allotment procedure, we assume that the central bank targets either liquidity or interest rate.

We show that the expected overnight rate is more tightly in the hands of the central bank, if either the full allotment procedure or pure interest rate targeting policy rule is used vs the liquidity targeting rule. Under either full allotment or proportional allotment with interest rate targeting, the expected overnight rate on a given day equals the expected tender rate (or target rate) for that day, as long as the policy is known to the counterparties and (in case of interest rate targeting) the target is not set below the tender rate. Determination of the expected overnight rate under liquidity targeting policy will depend on the central bank's ability to control the daily supply of liquidity. If banks expect the central bank to increase the tender rate during the following days, the expected overnight rate will respond immediately to these expectations, by rising to the higher expected level. However, if the central bank is expected to cut the tender rate during the ongoing reserve maintenance period, the expected overnight rate will not react to these expectations, but will instead remain at the level of the tender rate. When the banks have static interest rate expectations, the realised overnight rate is expected to equal the tender rate.

With full allotment, the spread between the realised overnight rate and the tender rate will depend entirely on the realised liquidity shock between the tender operation and clearance of the overnight market. Furthermore, it is shown that the volatility of the overnight rate increases when banks deviate from smooth reserves holding, due to expectations of a tender rate change. When the central bank applies liquidity targeting, the effect of a stochastic liquidity shock on the market rate is small. Hence, variations in the realised overnight rate reflects to a greater extent changes in interest rate expectations than stochastic variations in autonomous liquidity factors. However, if banks expect a rate cut in the near future, the daily supply of liquidity is no longer in the hands of the central bank. Thus, the overnight rate will be determined as in case of full allotment, and its volatility will be related to the stochastic liquidity shocks. With the proportional allotment procedure and an interest rate target, the variations in the spread between target rate and overnight rate reflect stochastic errors in the central bank's estimates of the banks' demand for liquidity and developments in the autonomous liquidity factors. As with full allotment, the volatility of the overnight rate under interest rate targeting depends on interest rate expectations, as they affect

the part of the demand curve at which equilibrium will be realised. Thus, volatility is expected to be higher if a rate change is expected than with neutral expectations.

The *stochastic volatility* of the overnight rate should be lower with liquidity targeting than with the other procedures. However, this is not necessarily the case for the total volatility of the rate, at least if the rate expectations vary a great deal over time. Furthermore, stochastic volatility should not be harmful for the conduct of monetary policy, as long as it is fully understood by the counterparties and the public that these variations originate solely from errors in predictions of liquidity developments (and also from errors in estimating the effect of interest rate expectations on the demand for liquidity, in the case of interest rate targeting). The relative degree of volatility under full allotment vs interest rate targeting depends on the relative size of the aggregate estimate error (in forecasting autonomous liquidity factors) made by the banks and the central bank both in estimating liquidity developments and in anticipating the effect of interest rate expectations on the demand for liquidity.

The paper also demonstrates how optimal bidding in the tender operations varies considerably according to which procedure is used by the central bank.

3.2 Bidding in fixed rate tenders: theory and evidence from ECB tenders

The second essay models the optimal bidding behaviour of a single bank in fixed rate liquidity tenders under various liquidity policies applied by the central bank. We show that the bid amount depends crucially on the relation between the central bank's liquidity target and neutral liquidity. Neutral liquidity refers to the amount of reserves at which the expected market rate of interest equals the tender rate. If the liquidity target of the central bank is above the neutral level, the banks bid only for the amount at which money market liquidity is neutral, so that liquidity determination will not be directly in the hands of the central bank. On the other hand, if the target liquidity is at or below the neutral liquidity, the banks

20

overbid (in excess of neutral liquidity), in order to profit from the expected positive spread between market rate and tender rate.

We introduce four potential liquidity policies for the central bank; i) full allotment (the central bank accepts all bids), ii) neutral liquidity policy rule (central bank estimates liquidity at which the expected market rate equals target rate and provides reserves accordingly), iii) restricted liquidity supply (central bank provides less than the neutral liquidity), and iv) liquidity targeting rule (central bank aims at stabilising the amount of liquidity in the money market throughout the reserve maintenance period). The banks bid for neutral liquidity if the central bank applies full allotment or it uses liquidity targeting and the banks expect an interest rate cut in the near future. Overbidding occurs under interest rate targeting and restricted liquidity supply or liquidity targeting and expectations that the central bank will not cut its rates in the near future.

We also show that when liquidity allotted by the central bank in the tender must be covered by collateral, the amount of overbidding will be a positive function of the interest rate spread between expected market rate of interest and tender rate. Thus, the bid ratio (aggregate bids / allotted amount) should behave differently under different liquidity policy rules, as the expected market rate of interest depends on the allotment rule applied by the central bank. With full allotment or liquidity targeting and banks' expectations of a tender rate cut, the expected market rate will equal the tender rate and the central bank will not be rationing the allotted amount. Thus, under these conditions we would not expect to see overbidding by the banks. However, under a neutral liquidity policy, the bid amount will depend on the banks' collateral borrowing capacity, even though the expected market rate of interest will equal the tender rate also in this case. Under restricted liquidity supply, the extent to which banks overbid, depends on the restriction rule of the central bank. For example, if the limited liquidity supply is based on preference asymmetry, the bid amount should reflect the effect of the asymmetry on the expected spread between market rate and tender rate. Finally, with a liquidity-oriented allotment policy, the expected market rate will be a function of the expected future market rate, and for this reason the amount of bids in excess the neutral amount will also be positively correlated with interest rate expectations.

This essay also studies the ECB's liquidity policy and the bidding of its counterparties. According to the data, the overall ECB's liquidity provision could not be considered as restricted. On average, the ECB did provide liquidity in excess of the reserve need arising from the minimum reserve requirements. However, there still seems to have been a significant positive spread between the market rate and the main refinancing rate during the period the ECB applied the fixed rate tenders. This was especially the case in tenders preceding ECB rate hikes. Consequently, even though the ECB's overall liquidity policy was not restrictive, the timing of the liquidity provision seems not to have met the demand of the banks. Moreover, the reaction of the ECB to banks' interest rate expectations was not unambiguous. The ECB appears to have increased its allotment beyond the level indicated by the reserve requirement when there were moderate expectations of tighter future interest rate policy. However, when the expectations were for a big change (ie when the spread between the one-week EURIBOR and the main refinancing rate was more than 25 bps), the ECB seems to have reverted to tighter control of liquidity. This indicates that the liquidity policy applied by the ECB was neither pure interest rate targeting nor pure liquidity targeting, but rather something in between. The ECB seems to have given weight to both holding the market rate close to the main refinancing rate and to stabilising money market liquidity.

The amount of bids submitted in the tenders increased considerably during the period with fixed rate tenders. This seems to have resulted from two factors. Firstly, from the start in January 1999 until the autumn 1999, the banks had either static interest rate expectations or they expected the ECB to cut its rates, whereas from autumn 1999 until the change of the tender procedure in June 2000, the interest rate expectations were either static or indicated an increase in the tender rate. The rate hike expectations of the latter half of the period of fixed rate tenders were clearly reflected in the spread between the one-week EURIBOR and the tender rate. That is, the banks did not assume the ECB would adjust its liquidity supply (fully) to the increase in demand for liquidity resulting from the rate hike expectations. As the amount of liquidity the banks are willing to obtain in a tender is the larger, the wider the spread between market rate and tender rate, each bank aimed at getting a bigger share of the total allotment in many tenders during the second half of

the period of fixed rate tenders than during the first half. Secondly, to obtain a certain allotment in a tender a bank must place a bid that equals the amount it is willing to take times the bid ratio used in the tender. The expectation of the coming bid ratio in a tender seems to have depended positively on the bid ratios of recent tenders. Thus, the aggregate bid amount at a given expected interest rate spread was considerably larger during the latter half of the period. However, the bid amount seems to have grown already during the first half of 1999. According to our model, this indicates that banks expected a restricted liquidity supply during the period, when the ECB was not expected to raise its rates. This could mean either that for some reason the banks prefer frontloading of reserve holdings to stable liquidity or that the banks assumed the liquidity policy of the ECB to have been more restrictive than it actually was at the beginning of Stage Three of EMU.

Finally, the discrepancy inherent in simultaneously controlling the price of a good (here, the level of the market rate of interest) and its quantity (here, stabilising liquidity over a reserve maintenance period) seems to result in ever increasing bid ratios when a rate hike is expected. The remarkable increase in bid ratios that occurred between October 1999 and June 2000 caused the ECB to change the tender procedure to variable rate tenders. With variable rate tenders, expectations of a rate hike are immediately reflected in the tender rate. Thus, the banks' incentive to overbid in the operations is diminished. According to the model of the paper, an alternative method for the ECB to overcome the declining allotment ratios would have been to give up the aim of stabilising liquidity within a reserve maintenance period. This could have been done by either applying the full allotment procedure or moving to stricter interest rate targeting.

3.3   Variable rate tenders

The final essay of this thesis constructs an equilibrium model for the short-term money market, in which the central bank uses variable rate tenders. We assume a two-day reserve maintenance period, in order to keep the model as tractable as possible, while still having the effect of interest rate expectations. The relation between market

rate of interest and liquidity for both days of the reserve maintenance period is derived from a single bank's profit maximisation problem in the interbank market. Here, the central bank chooses its the intended liquidity supply by minimising a quadratic loss function that contains both the deviations of expected market rate from its target rate and differences between liquidity supply and target liquidity. This means that the central bank aims to keep the market rate of interest close to a target level, but it also wants to stabilise liquidity over the reserve maintenance period. The banks are assumed to observe symmetric signals as to the coming market rate while preparing their bids. The ECB may choose to operate with either multiple rate (ie discriminatory price) or single rate (ie uniform price) tenders. However, in the analysis, the main emphasis is on multiple rate tenders, as these have been used by the ECB in its variable rate main refinancing operations.

We show that, in the model, the central bank is able to meet its targets for both the expected interest rate and expected liquidity on the final day, if it does not use the rates of the standing facilities as an independent signalling device. In this case, the distribution of liquidity shocks determines how the interest rate corridor (established by the standing facilities) should be set around the target rate; as long as the shock distribution is symmetric, the central bank targets can be met by setting the corridor such that the target rate is at the mid-point (ie a symmetric corridor). However, if the rates of the standing facilities are set independently of the target rate, the difference between the expected market rate and the target rate depends on the asymmetry of both the shock distribution and interest rate corridor.

Also, according to our model, under static interest rate expectations the central bank will provide to the markets, on the first day of a reserves maintenance period, an amount of liquidity equal to the target liquidity and the expected market rate of interest will equal the level targeted, regardless of the preference weighting parameter, as long as the shock distribution is symmetric. However, if the shock distribution is asymmetric, the equilibrium liquidity supply depends on the preference weighting of the central bank. The higher the relative weight of liquidity deviations, the more the interest rate differs from its target and the closer the equilibrium liquidity is to the reserve requirement.

24

The determination of the equilibrium liquidity supply becomes more complicated when the banks expect the target rate to be changed between the two days of the reserves maintenance period. In such a case, the central bank will provide the more liquidity, the higher the expected future interest rate (at least as long as it pays attention to money market rates). In the model, the expected value of the market rate for the first day will be above the target rate when an interest rate hike is expected. However, the simultaneous effects of the expected interest rate hike and the increasing liquidity supply on the expected market rate for today are not necessarily monotonic.

In the analysis of the banks' bidding behaviour, we showed that when a reserve price (in case of the ECB, the minimum bid rate) is not applied under the multiple rate procedure, the aggregate demand schedule of the banks is flat at the expected market rate, at least up to the amount the central bank is willing to provide. However, the introduction of a minimum bid rate alters the bidding behaviour. It was shown that, when the minimum bid rate is effective, the determination of equilibrium in the money market is similar to the case with fixed rate tenders and central bank accepting all bids submitted. Yet, in this case, the market liquidity will be below and the expected market rate above the level preferred by the central bank.[3] The probability of a minimum bid rate being effective is highest when the central bank's target rate is expected be cut, and it depends inversely on the difference between the current target and the minimum bid rate.

We also explain 'underbidding' as a phenomenon that results from the minimum bid rate becoming effective. Moreover, we show that when the maturities of consecutive tenders overlap, underbidding is enhanced to the extent that the expected market rate for the first period will rise above the prevailing target rate. Thus, in a framework that includes the combination of overlapping tenders and a reserve price, the expected short-term market rate will always rise above the target rate when banks expect the central bank to change its target within the ongoing reserves maintenance period – regardless of the direction of the expected change.

In the analysis of single rate tenders, we assume the central bank scales the supply of liquidity back from the intended level when

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[3]This is a feature that is different from the equilibrium with fixed rate tenders with 100% acceptance.

'collusive seeming' bidding behaviour is detected. This punishes the banks by forcing them to use the marginal lending facility more intensively. Hence, in equilibrium, the banks bid for the liquidity the central bank is willing to submit at the rate expected to be realised with that liquidity supply. The punishment strategy is needed to deter the banks from submitting very steep demand curves, for which any interest rate between minimum bid rate and expected market rate (with the intended liquidity) could be maintained as a symmetric pure strategy equilibrium (as shown in Back and Zender 1993).

We also study the ECB's liquidity provision, and the banks' bidding behaviour in the ECB variable rate tenders. The conclusion is that the ECB seems to have taken account of both interest rate and liquidity considerations in its allotment decisions. However, the effect of interest rate expectations[4] on actual liquidity provision appears to have been so moderate that we see the ECB as a liquidity-oriented central bank. That is, the liquidity provision has closely followed the benchmark allotment, according to which bank reserves are held stable within each reserve maintenance period. Furthermore, there were four cases of obvious underbidding. A closer analysis of these cases indicates that the essential reason for underbidding is the combination of interest rate cut expectations and the minimum bid rate, and this is further enhanced by the overlapping nature of the maturities of consecutive tenders. However, the pronounced widening of the spread between market rate and main refinancing rate that followed some of the underbidding episodes seems to have reflected more the increased probability of the ECB 'punishing' the underbidding via a lower liquidity provision in the subsequent operation than a normal increase in market rate due to a decrease in liquidity.

Finally, we explore the banks bidding in the ECB main refinancing operations. The demand schedules appear to have been fairly flat at rates close to the marginal rates of the allotments. However, we found some evidence that increasing uncertainty as to the coming marginal rate of the allotment affects the bidding so that the demand schedules submitted will be steeper.

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[4]Interest rate expectations are measured as the spread between one-week EURIBOR and minimum bid rate.

3.4   Contributions

One of the main contributions of the first essay is the determination of the inverse demand function for liquidity in a monetary policy operational framework that includes standing facilities and averaging of reserves (with n-day maintenance period), and where the central bank has multiple operations during each reserve maintenance period. The shift of the analysis from daily reserve requirements to periodic requirements changes the model from static to dynamic. The effect of interest rate expectations on the demand for liquidity would be crucial already in a stylised model with two-day reserve maintenance periods. However, we show that when the length of the maintenance period is increased beyond two days to n days, the demand for liquidity is reduced by the effect of current reserve holdings on the cost of the future liquidity uncertainty. We have named this effect the dynamic cost factor. Another novelty of this essay is in the analysis of the effects of various central bank liquidity policies. The inclusion of the different allotment styles in the study raises the level of the analysis from partial towards equilibrium analysis.

The emphasis of the second essay is in exploring the bidding behaviour of the banks in the central bank fixed rate tenders. The main contribution of the study is the identification of the optimal bidding behaviour of a single bank under various liquidity policy rules that the central bank may follow, and the analysis of the effects of collateral requirements on the bidding. Furthermore, the essay analyses the ECB fixed rate tenders and explains the ECB's liquidity policy and the behaviour of its counterparties in terms of the model.

A novelty of the final essay is the derivation of the central bank's liquidity policy from a quadratic loss function including the difference between expected market interest rate and target rate and deviations of money market liquidity from a stable path. This means that, instead of studying various explicitly announced liquidity policy rules, we can analyse a continuum of policies separated by the relative weights assigned to interest rate and liquidity considerations. Furthermore, this essay indicatively identifies the banks' optimal bidding behaviour for the banks in a variable rate liquidity tender. The shift from fixed rate to variable rate tenders moves the analysis more towards the literature on auctions. The essay contributes to the literature by explaining the phenomenon known as underbidding

27

as an outcome of the existence of a reserve price in the tenders. Moreover, we show that an operational framework of the central bank that includes overlapping maturities of the consecutive tenders and a reserve price results in a perverse situation where the expected market rate will increase from the level targeted by the central bank when the target rate is expected to be cut within the same reserves maintenance period. The analysis of bidding behaviour in variable rate tenders will probably be refined considerably in the near future, as a result of recent advances in the theory of multi-unit common value auctions.

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Fixed rate tenders and the overnight money market equilibrium

Contents

Abstract

This study presents a model of the determination of the equilibrium in the interbank market for overnight liquidity when the central bank uses fixed rate tenders in its liquidity provision. We consider here three alternative liquidity policy rules for the central bank. First, the central bank may provide the market with all the liquidity that is bid for. Secondly, the central bank can scale back the bids it receives in proportion to the bid amount. In the proportional liquidity allotment procedure we assume that the central bank uses either a liquidity or an interest rate targeting approach. We show that the expected overnight rate is more tightly in the hands of the central bank if the full allotment procedure or pure interest rate targeting policy rule is used than with the liquidity targeting rule. We also demonstrate how optimal bidding in the tender operations varies considerably according to which procedure is chosen by the central bank.

32

1 Introduction

The overnight rate of interest is probably not the most important rate in monetary policy transmission. However, the importance of the interbank overnight market should not be understated, as it is the market in which the central bank implements monetary policy, and also because overnight is normally the shortest maturity for which there are well organised markets. (Hence, the yield curve can be seen to reflect the expected future values of the overnight rate.) Therefore, understanding the functioning of the monetary policy operational framework (ie the instruments and procedures of the central bank) is essential in order to be able to evaluate the monetary policy stance, and to interpret the reasons for and consequences of variations in conditions in the overnight market. For example, in one framework a change in the overnight rate of interest can be seen as indicating a change in the tightness of monetary policy, whereas in another framework changes in the overnight rate may always originate from stochastic liquidity shocks and thus have no information value at all.

The literature on the overnight markets is heavily concentrated on describing and explaining the stylized facts of the fed funds market, ie the market for interbank overnight reserves in the United States. For example, according to Hamilton (1996) the observed cyclical behaviour of the fed funds rate may result from line limits, transaction costs or weekend accounting conventions. Also Furfine (1998) shows that intra-maintenance period variations in the fed funds rate are a consequence of volatility in daily interbank payment volumes. However, an exception to this fed-funds focus is Perez-Quiros and Rodriguez (2000), which models the behaviour of overnight funds in the euro area using a framework that includes standing facilities. They claim that the introduction of a deposit facility stabilises the overnight rate of interest. However, that paper, like most papers analysing the fed funds market, abstracts from monetary policy. This means that in the standard literature the analysis is only partial; liquidity[1] is exogenous and there is no role for active liquidity management by the central bank. A notable exception is Bartolini, Bertola and Prati (1999), which models the interbank

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[1]In this study, liquidity refers to banks' deposits with the central bank, unless otherwise stated.

money markets by giving an explicit role to central bank intervention. Their model is, however, not very suitable for studying the behaviour of overnight markets in Europe, where the operational framework in most countries in recent years has included standing facilities, and especially as the open market operations have been conducted in the form of *fixed rate tenders*. When using fixed rate tenders, the central bank not only provides the markets with liquidity. It also signals the stance of monetary policy with the interest rate set in the operations. Thus, it is important to understand how the allotment procedure used by the central bank affects both the demand for liquidity in tenders and the amount of liquidity in the overnight markets.

There has recently emerged a growing literature on the operating framework of the European Central Bank, including Ehrhart (2000) and Ayuso and Repullo (2000). The paper by Ehrhart presents an experimental investigation of the banls' bidding behaviour under a fixed rate tender procedure and exogenous (tight) supply of liquidity. Ehrhart claims that fixed rate tenders lead to continuously increasing overbidding in the operations. The model used in Ehrhart's paper, however, abstracts from the interbank overnight market as the place where the value of liquidity is determined. The paper by Repullo and Ayuso shows that if the central bank has an asymmetric loss function that depends on the quadratic difference between interbank rate and target rate of the central bank, fixed rate tenders have a unique equilibrium characterised by extreme overbidding. However, they do not explicitly give the motivation for this kind of asymmetry in preferences, especially as their model abstracts from interest rate expectations.

In this paper we present an equilibrium model of the behaviour of overnight markets, where the central bank manages liquidity via fixed rate tenders. First, we model the determination of the overnight rate of interest as a function of money market liquidity. Then we analyse how money market liquidity itself is determined under various allotment rules that may be used by the central bank. The primary emphasis in the analysis is on comparing the money market equilibrium when the central bank accepts all the bids it receives in the tender (*full allotment procedure*) with the equilibrium when the central bank scales back the bids it receives (*proportional allotment procedure*). The operational framework that the central bank is assumed to use to implement its monetary policy closely resembles

34

that used by the ECB between January 1999 and June 2000. However, for simplicity we assume that the central bank conducts one operation each day, whose maturity is overnight. The consequences of these simplifications are discussed briefly in the conclusions of the paper.

The paper is organised as follows. In section 2 we describe the functioning of the money market and the role of the central bank in liquidity management. Section 3 analyses the determination of the overnight rate, when the operational framework does not include reserve averaging. In section 4 we introduce the dynamics that come with the averaging provision. Finally, section 5 concludes and gives a summary of the main findings of the paper.

2   The money market and the operational framework

By money market we refer to the market where institutions enter into transactions with each other by trading unsecured debt, negotiable debt instruments or collateralised loans. For simplicity, we abstract from the fact that the interest rates of different instruments carry different premia over the risk-free yield curve. Thus, when referring to a market rate of interest for a specific maturity, we assume that such unique risk-free interest rates for all the relevant maturities exist.

The terms 'money market liquidity' and 'bank reserves' are used interchangeably throughout this paper to refer to the balances banks have on their settlement accounts with the central bank. By interbank trading we refer to money market trades between credit institutions that participate the central bank operations. It is worth emphasising that even though interbank trades redistribute money market liquidity among the banks, they do not affect the total amount of liquidity in the market. Only transactions that also involve the central bank can change (aggregate) money market liquidity.

It is assumed here that one day (overnight) is the shortest maturity in the organised interbank market.[2]   Thus, it is also the starting

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[2]There may also be interbank markets for intraday liquidity. However, we are not interested on such markets for the purposes of this study, as intraday trades do not affect the end-of-day balances on the banks' settlement accounts.

point of the yield curve. In overnight trading the value date of the transaction is the trading day (same day settlement) and the maturity date is the following banking day (normally, the maturity of a Friday overnight loan is three days). We assume that normal interbank trading with instruments of longer maturities are settled with a lag of at least one banking day. Hence, the only way a bank can offset its liquidity shocks is by trading in the overnight market. These shocks may stem from unexpected deposit withdrawals, new deposits, or any other unanticipated transaction with same-day settlement.

Central bank objectives and operational framework

We ignore whether the central bank uses monetary targeting, direct inflation targeting or any other procedure as an indicator or intermediate target in achieving price stability (or any other primary goal it might have). We merely assume that the central bank uses a short-term interest rate as a policy rate or operating target.[3] However, the maturity of this rate need not be overnight.

In this study, we want to model the determination of the overnight rate and especially how the determination of its expected value is affected by the operational target of the central bank. The formation of the expected overnight rate is of special interest, as it is the *expected values* of the rate that are the basis for determination of the yield curve, and rates considerably longer than overnight are normally assumed to be important for the transmission of monetary policy. Therefore, as we expect the operational framework to affect the expected values of the overnight rate, we also expect these operational issues to affect the transmission mechanism of monetary policy. For example, it is obvious that transmission of the overnight rate's volatility along the yield curve to longer maturities depends crucially on whether variations in the overnight rate affect the expected values of future overnight rates.[4]

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[3]This is currently the case at least in the euro area, USA and Japan. See eg European Central Bank (2000), Federal Reserve Bank of New York (2000) and Bank of Japan (1999).

[4]See eg Ayuso, Haldane and Restoy (1994) who claim that the differences in the transmission of volatility from the overnight market to longer money markets in Spain, France, UK and Germany resulted primarily from differences in the operational frameworks of these countries.

We assume here that the operational framework of the central bank contains three different (ECB style[5]) instruments that can be used to meet the operational target. These are: *i) open market operations* by which the central bank actively manages money market liquidity, *ii)* the interest rate corridor set by the *standing facilities (marginal lending facility and deposit facility).* The use of the standing facilities can be initiated by banks. Furthermore, the central bank can affect the demand for money market liquidity by *iii)* imposing *reserve requirements.*

Throughout this essay, we assume that active liquidity management (open market operations) is conducted solely by fixed rate money market tenders. The ECB conducted its open market operations in this way between January 1999 and June 2000. One purpose of these tender operations is to provide the banks with refinancing. However, an at least equally important function of these operations is their role in signalling the monetary policy stance of the central bank.[6]

Besides open market operations, liquidity conditions in interbank trading are affected by the standing facilities. The marginal lending rate sets an upper limit (ceiling) for the secured interbank overnight rate. The central bank is always willing to provide additional liquidity at this pre-specified interest rate against eligible collateral. Thus, no bank is willing to pay more than the marginal lending rate for reserves from the interbank market. The lower limit (floor) for the overnight rate is set by the rate of the deposit facility. The banks are allowed to place overnight deposits with the central bank at this pre-specified interest rate. Hence, the interest rates of the standing facilities effectively create a corridor in which the interbank overnight rate of interest may fluctuate (*the interest rate corridor*). The central bank can affect the volatility of the overnight rate eg via the width of the corridor. The central bank may also use the rates for the standing facilities to signal the future stance of monetary policy.

When open market operations are conducted so as not to affect the trading day's interbank overnight liquidity (eg if transactions are

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[5]The operational framework of the ECB is described in detail in ECB publication 'The Single Monetary Policy in Stage Three; General Documentation on ESCB Monetary Policy Instruments and Procedures', September 1998. Below, we refer to this document simply as GD.

[6]See eg GD, page 4, or European Central Bank (2000, page 49).

settled at $T+1$), the supply of overnight liquidity on a given day is fixed as long as the overnight rate stays within the corridor (ie as long as the price of borrowing liquidity from the market is cheaper than the marginal lending rate and the revenue from an interbank loan is above the deposit rate).[7] However, the supply of liquidity is affected by stochastic shocks that cannot be anticipated by the banks or central bank. The size of the liquidity shock for the central bank may be different from the sum of the net shocks the banks face. The shock for an individual bank is the net sum of unexpected flows into and out of its reserve account (more precisely the difference between this amount and its forecast value). The liquidity shock from the central bank's viewpoint is the deviation of net changes in the autonomous liquidity factors from their expected value. This shock might include (depending on the institutional setup of the currency area of the central bank) eg unexpected variations in government balances with the central bank or changes in the amount of currency in circulation.

The third instrument at the disposal of the central bank is the reserve requirement (for the ECB, the minimum reserve requirement). The central bank can require that credit institutions hold a share of their liabilities as minimum reserves with the central bank. These reserves are assumed to be held in banks' the settlement accounts. Averaging provisions may be allowed in the maintenance of the minimum reserves. If averaging is used, compliance with the reserve requirement is determined by the average of the end-of-day balances an institution has on its reserve (settlement) account during the maintenance period.

In addition to these three instruments, the operational framework has an additional crucial feature: overdrafts are forbidden. This means that, if a bank would otherwise end the day with a debit balance on its settlement account, it must cover the negative balance by borrowing from the marginal lending facility. Thus, both the aggregate end-of-day liquidity of the banking sector as a whole and the end-of-day liquidity of a single bank must always be at least zero.

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[7]Same day settlement could be allowed for in our model without qualitatively changing the results.

The evolution of money market liquidity during a banking day

Trading in the overnight market is conducted throughout the day. Thus, in the real world there is no single overnight rate for any specific day. However, we assume that overnight trading is conducted by a Walrasian auctioneer at a certain point of time during the trading day. The clearing rate used by the auctioneer is assumed to equal the volume-weighted average of interest rates applied in interbank overnight trading during that day. Normally such a rate is calculated by the central bank or some other institution as a reference rate for the markets.[8]
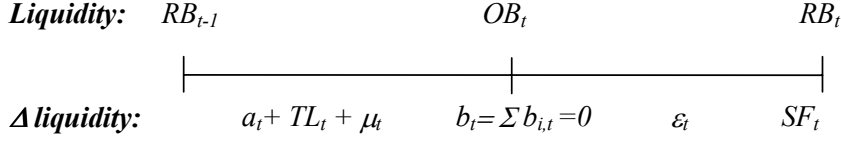
Furthermore, we assume that there are two distinct and independently distributed liquidity shocks for the banks during a banking day.[9] The first shock (μ) is realised before the overnight markets are cleared, and the second one (ε) after the settlement. The expected value for both of these shocks is zero (E[μ]=E[ε]=0). The construction of two separate shocks follows from the fact that the aggregate net shock a bank faces consists of a continuum of small independent shocks occurring throughout the day. When we model the overnight market as being cleared at a single point in the day, μ is the net effect of all shocks before that moment, and ε is the net effect of shocks occurring afterwards. If the overnight market were modelled as being settled at the end of the day, so that there would be only one shock per day, there would not be any uncertainty about the banks' end-of-day reserve balances in the overnight trading. Thus, the overnight rate would equal either the marginal lending rate or the deposit rate (depending on whether there is a shortage or a surplus in the overnight market) on the last day of the reserve maintenance period (absent averaging every day). By the construction with two separate shocks, we ensure that uncertainty regarding each bank's reserve position makes its demand schedule for reserves smoothly downward sloping.

At the beginning of a banking day t, money market liquidity equals the previous banking day's aggregate reserve balances (RB_{t-1} =

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[8]In case of the the Eurosystem, ECB calculates EONIA, which is a volume weighted average interest rate of interbank overnight deposits reported by certain panel banks. In the USA the fed funds fate is the counterpart of EONIA in Europe.

[9]A similar division of the liquidity shock into two parts can be found in Bartolini et al (1998).

Figure 1: **Evolution of money market liquidity during the day**

Liquidity: RB_{t-1}		OB_t	RB_t

$$\vdash \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad | \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad |$$

Δ liquidity:	$a_t + TL_t + \mu_t$	$b_t = \Sigma b_{i,t} = 0$	ε_t	SF_t

$\sum RB_{i,t-1}$). Each bank knows its own balance, and the total amount is known to the central bank. The new and maturing monetary policy transactions, whose value day is t, are also known with certainty by the central bank.

Figure 1 shows the timing of the evolution of money market liquidity on a single banking day. a_t is the sum of expected net changes in autonomous liquidity factors (including maturing central bank operations), TL_t is the amount of liquidity provided to the markets in the open market operation (tendered liquidity), and μ_t is the first liquidity shock of the day. OB_t is the amount of liquidity in the overnight market when it clears ($OB_t = RB_{t-1} + a_t + TL_t + \mu_t$). We denote its expected value by eOB_t (ie $eOB_t = RB_{t-1} + a_t + TL_t$). $b_{i,t}$ is the net amount bank i borrows from the interbank market. The lending of reserves to the market is treated as negative borrowing. The banks' aggregate net borrowing from the markets must equal zero, as in every deal there is a borrower and a lender for the same amount of reserves (ie $b_t = \sum b_{i,t} = 0$). ε_t is the second liquidity shock of the day.

Let RR_i denote bank i's daily reserve requirement, and RR its aggregate counterpart. We can define the minimum required daily balances for the remaining period (RDB) as:

$$RDB_{i,t} = \frac{T * RR_i - \sum_{k=1}^{t-1} RB_{i,k}}{T - (t-1)}, \tag{2.1}$$

where T is the number of days in the maintenance period. Equation 2.1 gives the average amount of reserve balances bank i should have

40

daily (from day t to the end of the on-going maintenance period) in order to hit the reserve requirement exactly. We denote the banking-sector-wide counterpart of $RDB_{i,t}$ by $RDB_t(= \sum RDB_{i,t})$. In a system without reserve averaging, $RDB_{i,t}$ and RDB_t will always equal RR_i and RR respectively.

SF_t denotes the banks' net use of the standing facilities (ie liquidity credits - use of the deposit facility). Apart from the fact that a bank can use the standing facilities at any time, a bank should always acquire liquidity credits (LC) from the marginal lending facility if its end-of-day reserve balances would otherwise be negative or if at the last day of the maintenance period its reserve balances would not be large enough to meet the reserve requirement (ie $LC_{i,t} = \max\left(0, -\left(OB_{i,t} + \varepsilon_{i,t}\right)\right)$ for $t < T$ and $LC_{i,T} = \max\left(0, -\left(OB_{i,T} + \varepsilon_{i,T}\right), -\left(OB_{i,T} + \varepsilon_{i,T} - RDB_{i,T}\right)\right)$.[10] Similarly, a rational bank will always deposit all reserves exceeding the reserve requirement into the deposit facility (ie $DF_{i,t} = \max\left(0, OB_{i,t} + \varepsilon_{i,t} - (T - (t-1)) * RDB_{i,t}\right)$), as otherwise these reserves will earn zero interest.

Finally, the reserve balances (ie the aggregate end-of-day balance) are denoted by RB_t, which is the sum of all factors included in OB_t, the second liquidity shock and the net use of the standing facilities $(RB_t = OB_t + \varepsilon_t + LC_{i,T} - DF_{i,T})$. Now, based on the optimal use of the standing facilities, we know that $RB_t \in [0, (T - (t-1) \, RDB_t)]$, and consequently the required daily balances for the remaining period can never be negative (ie $RDB_t \geq 0$).[11]

Let us add one more definition to the liquidity terminology. The excess reserves of bank i ($ER_{i,t}$) is the amount of reserves it has

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[10]If a bank fails to get its reserve account balances to zero (or on the last day of the reserves maintenance period to the required level), the bank will face penalties that are considerably heavier than the cost of using the marginal lending facility. Thus, a rational bank will always acquire liquidity credits when facing either of these situations.

[11]$RB_t = OB_t + \varepsilon_t + LC_T - DF_T = OB_t + \varepsilon_t + \max\left(0, -(OB_t + \varepsilon_t)\right) - \max\left(0, OB_t + \varepsilon_t - (T - (t-1)) * RDB_t\right)$. Thus, if $OB_t + \varepsilon_t < 0$, then $RB_t = 0$, and if $OB_t + \varepsilon_t > 0$, the maximum value for RB_t is $(T - (t-1)) * RDB_t$. Now, RDB_{t+1} is minimised with the maximum value for RB_t. Therefore,

$$RDB_{t+1}^{\min} = \frac{T*RR - \sum_{k=1}^{t-1} RB_k - (T-(t-1))RDB_t}{T-(t-1)-1} = \frac{T*RR - \sum_{k=1}^{t-1} RB_k - \left(T*RR - \sum_{k=1}^{t-1} RB_k\right)}{T-t} = 0.$$

Thus, RDB_{t+1} is always at least zero. Since we know that $RDB_1 = RR \geq 0$, RDB_t cannot be negative.

in excess of the required daily balances *at the moment of overnight market clearance* ($ER_{i,t} = OB_{i,t} - RDB_{i,t}$), and its expected value will be denoted by eER ($eER_{i,t} = eOB_{i,t} - RDB_{i,t}$).

There are two different factors behind the demand for bank reserves. First, as long as overdrafts are forbidden, the banks cannot have debit balances on their settlement accounts with the central bank at the end of the day. The uncertainty introduced by the assumption of a liquidity shock occurring after the overnight markets have cleared ensures that the demand for reserves depends negatively on the interest rate (otherwise the demand for reserves could be a step function, in which case the overnight rate would equal either the marginal lending rate or the deposit rate, depending on whether liquidity is below or above zero or the reserve requirement).

Another factor behind the demand for reserves is the minimum reserve requirement (if imposed by the central bank). In the absence of averaging, the only change is that the cost minimising end-of-day liquidity for a bank is the required amount instead of zero. With averaging, the story changes completely. In this case the demand for reserves will be similar to the case without reserve requirements only on the last day of the reserve maintenance period. Before the last banking day, an optimizing bank can have reserves in excess of or less than the requirement. Only the average amount of reserves held with the central bank is relevant. Using the averaging provision a bank can optimise on the cost of holding reserves by maintaining them whenever it judges the cost to be lowest for that averaging period.[12] A bank will demand less (more) reserves on a single day if the overnight rate on that day is high (low) relative to the rate it assumes to prevail on the following days during the same period.[13] Thus, an averaging provision will enhance the interest rate elasticity of the demand for bank reserves or, to put it the other way around, changes in the interest rate due to temporary liquidity shocks are smaller with averaging, *ceteris paribus.*

Next we set out a simple model for determination of the overnight rate of interest as a function of money market liquidity. After that,

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[12]This is sometimes referred to as intertemporal arbitrage. However, the word arbitrage may be misleading, as the gain here is uncertain.

[13]If a bank holds more reserves at the beginning of a maintenance period than at the end, it is said to be *frontloading* reserves. In the opposite case the bank is *backloading* reserves.

we model the supply of liquidity as a function of banks' interest rate expectations and the central bank's operational target. To simplify the calculations that follow, we assume the banks to be homogeneous and their mass to sum to unity. The central bank is assumed to operate only via fixed rate liquidity tenders. This is how many European central banks used to operate in the 1990s, and also how the ECB conducted its main refinancing operations during the first 18 months.

3 Model without reserve averaging

3.1 Overnight rate as a function of liquidity

Let us first consider the demand for bank reserves in a system without reserve requirements. This is also the starting point for analysis of the demand for money market liquidity in an operational framework that includes the reserve averaging provision. In modelling the demand for reserves in the overnight market, we follow the classical model introduced by Poole (1968), and frequently used by others (eg Bartolini, Bertola & Prati, 1999). The main difference in our model vs Poole's is the introduction of the rates for the standing facilities.

Proposition 1 *Without the averaging provision, i) bank i chooses its net borrowing in order to balance the probability of ending the day with a debit balance with the relative cost of using the standing facilities, and ii) the overnight rate of interest is the average of the two rates of the standing facilities weighted by the probabilities of a shortfall or excess in the money market.*

Bank i's demand for overnight reserves in the interbank market can be achieved as the first order condition of the bank's profit maximisation problem. The cost of borrowing reserves (income from lending) that bank i faces, is simply the overnight rate of interest at day T (r_T^{on}). The income from borrowing (cost of lending) is the interest rate of the two standing facilities (ie the marginal lending rate and the deposit

43

rate, denoted by r_T^m and r_T^d respectively) weighted by their usage.[14] The maximisation problem becomes:

$$\max_{b_{i,T}} \mathrm{E}(\Pi) = r^m \left[\int_{-\infty}^{-ER_{i,T}-b_{i,T}} (ER_{i,T} + b_{i,T} + \varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right]$$

$$+ r^d \left[\int_{-ER_{i,T}-b_{i,T}}^{\infty} (ER_{i,T} + b_{i,T} + \varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right]$$

$$- r^{on} b_{i,T}, \qquad (3.1)$$

where $f(\varepsilon_T)$ is the density function of the stochastic error term, whose cumulative counterpart we denote by $F(\varepsilon_T)$. The first order condition with respect to interbank borrowing can be derived using Leibniz's formula:

$$(r^m - r^d) F\left(-ER_{i,T} - b_{i,T}^*\right) + (r^d - r^{on}) = 0 \qquad (3.2)$$

or

$$F\left(-ER_{i,T} - b_{i,T}^*\right) = \frac{r^{on} - r^d}{r^m - r^d}, \qquad (3.3)$$

where $F\left(-ER_{i,T} - b_{i,T}^*\right)$ represents the probability of bank i being overdrawn with the optimal borrowing. Equation (3.3) proves the first part of proposition 1.

The right hand side of equation (3.3) gives the location of the overnight market rate within the interest rate corridor set by the standing facilities. The lower the market rate within the corridor, the larger the equilibrium borrowing for bank i. Intuitively this can be interpreted so that when the relative cost of acquiring liquidity credits $(r^m - r^{on})$ decreases compared with the (opportunity) cost of using the deposit facility $(r^{on} - r^d)$, the optimal policy for the bank is to increase its probability of being overdrawn (simultaneously the bank decreases its probability of having to rely on the deposit facility).

If the *cdf* has an inverse function $(F^{-1}(\cdot))$, we can derive the explicit form of bank i's borrowing function:

$$b_{i,T}^*\left(-ER_{i,T}, r^{on}\right) = -ER_{i,T} - F^{-1}\left(\frac{r^{on} - r^d}{r^m - r^d}\right). \qquad (3.4)$$

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[14]For the rest of this section we will drop the time subscripts (T) from the interest rates.

Equation (3.4) shows clearly that optimal net borrowing equals excess reserves (ie the gap between existing reserves and required reserves, $-ER_{i,T}$) and the inverse of the probability of a liquidity shock leaving the bank with negative end-of-day balances, given the location of the market rate within the official corridor. The optimal borrowing naturally decreases with the amount of excess reserves before the clearing of overnight markets and with the interbank overnight rate (the rates of the two standing facilities are taken as given).

Bank i can act as a borrower or lender in the market. However, as long as the overnight market rate stays strictly inside the corridor, money market liquidity is constant, as there will be no transactions with the central bank. As a consequence, aggregate borrowing must be zero. We can get the market-clearing overnight rate of interest from equation (3.3) simply by setting aggregate borrowing at zero (ie $b_{i,T}^* = b_T^* = 0$ and $-ER_{i,T} = -ER_T$):

$$r^{on} = r^m F(-ER_T) + r^d(1 - F(-ER_T)), \qquad (3.5)$$

which completes the proof of proposition 1.

There are three factors determining the overnight rate of interest: i) the interest rate corridor, ie the rates of the standing facilities set by the central bank, ii) the distribution of liquidity shocks after the last open market operation affecting day T's liquidity, and iii) the supply of liquidity relative to liquidity need. In our model the rates of the standing facilities are given prior to the overnight trading.[15] If we assume the distribution of liquidity shocks to be stable, the only varying parameter determining the overnight rate in our model is the supply of (excess) liquidity. Hence, *the key questions facing the central bank are, how tightly it can control the daily supply of liquidity, and what the effects of volatility are on money market liquidity.* The answer to the first question depends on i) the central banks ability to forecast both developments in the autonomous liquidity factors and the banks' aggregate demand for liquidity, and ii) the central bank's ability to provide the markets with the estimated liquidity need. The effect of overnight volatility depends crucially on how the counterparties interpret movements in the overnight rate as reflecting the monetary policy stance. This depends largely on the procedure the central bank uses in choosing the amount to supply.

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[15]In the ECB's framework, a change in the rates of the standing facilities can be effective on the following banking day at the earliest.

To address the question of how the supply of money market liquidity is determined, we model the demand for bank reserves in money market tenders under different liquidity policy rules used by the central bank. The banks' bidding behaviour varies with the central bank's approach to liquidity allotment.

3.2 Determination of money market liquidity

For the case without reserve averaging, we assume that the central bank conducts one liquidity operation each day. We will also assume that the structural deficit in the money market is large enough so that the tender operations will always be liquidity providing.[16] In these liquidity increasing fixed rate tenders the central bank announces the rate of interest at which it stands ready to provide the counterparties with liquidity. After an announcement, each bank may submit a bid to the central bank specifying the amount of liquidity it is willing to borrow at the announced rate. The central bank can accept all the bids it receives in full (*full allotment*) or it can scale the bids down proportionally (*proportional allotment*). Here we assume that the counterparties know in advance whether the central bank is using a full or proportional allotment strategy. If the aggregate bids the central bank receives do not exceed the amount of liquidity the central bank targets to lend, it will provide the markets with all the liquidity bid for, even under the proportional allotment procedure; ie 100% acceptance of bids need not indicate the full allotment approach. Let us next consider these two methods separately.

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[16]By large enough we mean here a probability of nearly one that the whole banking sector will end the day with debit balances, if no liquidity is provided through tender operations. If this were not the case, the central bank might sometimes have to use liquidity draining instead of providing operations. In such a case, the effects of the central bank's liquidity target being above or below the neutral liquidity (see propositions 4 and 5) would be reversed. We maintain the assumption of liquidity deficit merely to limit the number of cases under study. This assumption does not otherwise limit the analysis.

3.2.1 Full allotment

We start by defining some terminology for the bidding strategies of the banks. First, we define the *private value* of a certain amount of expected excess reserves for a bank as a weighted average of the rates of the standing facilities, where the weights are determined by the bank's probability of having to use the standing facilities with this amount of expected excess reserves. A bank has *neutral liquidity* if its probability-weighted cost of relying on the standing facilities equals the tender rate (ie the private value of neutral liquidity equals the tender rate). In *neutral bidding,* a bank bids for the amount that would leave the bank with neutral liquidity. The size of a neutral bid $(TL_{i,T}^{neutral})$ is implicitly given by:

$$r^m * G(RDB_{i,T} - RB_{i,T-1} - a_{i,T} - TL_{i,T}^{neutral}) \qquad (3.6)$$
$$+r^d(1 - G(RDB_{i,T} - RB_{i,T-1} - a_{i,T} - TL_{i,T}^{neutral})) = r^T,$$

where $G(\cdot)$ is the cumulative distribution function of the sum of the two stochastic error terms μ_T and ε_T, ie $G(RDB_{i,T} - RB_{i,T-1} - a_{i,T} - TL_{i,T}^{neutral})$ is the probability of bank i being overdrawn after acquiring $TL_{i,T}^{neutral}$ from the liquidity tender. The expected excess reserves before the realization of μ is $eER_T^{neutral} = RDB_{i,T} - RB_{i,T-1} - a_{i,T} - TL_{i,T}^{neutral}$. We can write equation (3.6) as:

$$r^m G(-eER_{i,T}^{neutral}) + r^d(1 - G(eER_{i,T}^{neutral})) = r^T. \qquad (3.7)$$

Equation (3.7) just states the fact that the private value of neutral expected excess reserves equals the tender rate. We also know that this would be the exact amount a bank would bid for in a fixed rate tender with full allotment liquidity provision, if there were no secondary market for liquidity.[17]

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[17]If there were no interbank market for central bank reserves, the profit maximization problem of bank i at the liquidity auction would be very similar to that decribed in equation (3.1). In this case maximisation would be taken w.r.t $TL_{i,T}$ instead of $b_{i,t}$, and $(ER_{i,T} + \varepsilon_T)$ should be replaced by $(eER_{i,T} + \nu_T)$ and $f(\varepsilon_T)d\varepsilon_T$ by $g(\nu_T)d\nu_T$. Thus, FOC becomes:

$$r^m G\left(-eER_{i,T}^*\right) + r^d\left(1 - G\left(-eER_{i,T}^*\right)\right) = r^T. \qquad (3.8)$$

The optimal expected excess reserves, $-eER_{i,T}^*$, (defined implicitly in equation (3.8)), equals the neutral expected excess reserves in equation (3.7).

Strategic overbidding occurs if a bank bids for more than the neutral bidding strategy implies, in order to profit from the (positive) difference between the tender rate and the banks estimate of the market overnight rate ($r^T < \mathrm{E}[r^{on}]$). Accordingly, *strategic underbidding* occurs when a bank bids for less liquidity than the neutral strategy requires, to profit from the bank's estimation of a negative difference between tender rate and market overnight rate ($r^T > \mathrm{E}[r^{on}]$).

Proposition 2 *Without the averaging provision, the expected value of the overnight rate of interest under full allotment will equal the tender rate, and the aggregated bids must equal the amount given by the neutral bidding strategy.*

From equation (3.5) we know that the overnight rate of interest is a function of (excess) money market liquidity. Thus, besides the central bank's allotment policy, the bidding behaviour of a single bank depends on the bidding strategies of other banks. In equilibrium the bidding strategy of the representative bank must be such that with the equilibrium liquidity the expected overnight rate of interest (ie the price of liquidity at the clearance of the market) will equal the price of liquidity at the tender. This is derived from the fact that, if $\mathrm{E}[r^{on}] > r^T$, every (atomistic) bank maximises profits by increasing its bid up to the maximum level (or placing an infinitely large bid if there is no maximum bid), and selling the extra liquidity in the overnight market. However, in such a case the total liquidity will be infinitely large or at least large enough to bring the overnight rate down to its minimum value (ie $\mathrm{E}[r^{on}] = r^d$), which would contradict the assumption of the expected overnight rate exceeding the tender rate. Also, if $\mathrm{E}[r^{on}] < r^T$, every bank will maximise profits by placing a zero bid (ie not participating the tender), and buying the needed liquidity from the overnight market. However, in such a case the total liquidity in the interbank market would be sufficiently low that the expected value of the overnight rate would rise to the ceiling (ie $\mathrm{E}[r^{on}] = r^m > r^T$). *Therefore, the only possible sustainable equilibrium is such that the difference between the expected overnight rate and the tender rate is zero, $\mathrm{E}[r^{on}] = r^T$.* In such a case, no bank can make positive expected profits by changing its bid. We also know from equation (3.5) that the overnight rate is a decreasing function of money market liquidity, which includes the tendered reserves. Thus,

there can be only one level of expected liquidity that can be sustained as an equilibrium. When all banks are bidding according to the neutral strategy, the overnight rate becomes:

$$r^{on} = r^m F(RDB_T - RB_{T-1} - a_T - TL_T^{neutral} - \mu_T) \qquad (3.9)$$
$$+ r^d (1 - F(RDB_T - RB_{T-1} - a_T - TL_T^{neutral} - \mu_T)).$$

It can be shown that (at the time of the tender operation) the expectation of the cumulative distribution function of the second shock (expectation taken over the distribution of μ) will equal the cumulative distribution function of the sum of the two independent shocks (ie $E_{f_\mu}[F(\cdot)] = G(\cdot)$; in the following we will denote $E_{f_\mu}[F(\cdot)]$ simply by $E[F(\cdot)]$ to simplify the notation).[18] Thus, with neutral bidding, the expected value of the overnight rate is given by:

$$E[r^{on}] = r^m E\left[F(RDB_T - RB_{T-1} - a_T - TL_T^{neutral} - \mu_T)\right]$$
$$+ r^d \left\{1 - E\left[F(RDB_T - RB_{T-1} - a_T - TL_T^{neutral} - \mu_T)\right]\right\}$$
$$= r^m E\left[F(-ER_T^{neutral})\right] + r^d \left\{1 - E\left[F(-ER_T^{neutral})\right]\right\} \qquad (3.10)$$
$$= r^m G(-eER_T^{neutral}) + r^d \left[1 - G(-eER_T^{neutral})\right] = r_T^T,$$

where we label the amount of liquidity at the clearance of the overnight market after the neutral bid that exceeds the required daily balances as neutral excess reserves (ie $ER_T^{neutral} = RB_{T-1} + a_T + TL_T^{neutral} + \mu_T - RDB_T$). Equation (3.10) tells us that the expected value of the overnight rate, when the banks use neutral bidding, equals the tender rate. Thus, in the only sustainable equilibrium

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[18]Let $\nu = \mu + \varepsilon$. It can be shown that $G(\nu) = \int F_\varepsilon(\nu - \mu) f_\mu(\mu) d\mu$, where f_ε, f_μ, F_ε, and F_μ refer to distributions and cumulative distributions of the error terms ε and μ respectively. By the definition of expectation, we have $G(\nu) = E_{f_\mu}[F_\varepsilon(\nu - \mu)]$, where the expectation is taken over the distribution of μ. The proof of this is given in technical appendix A.

the total bids must equal the neutral bidding strategy for the banks $(TL_T^{neutral} = TL_T^*)$.[19]

Furthermore, from equation (3.10) we get the excess reserves after the equilibrium bid $(ER_T^* = RB_{T-1} + a_T + TL_T^* + \mu_T - RDB_T)$:

$$\mathrm{E}\left[F(-ER_T^*)\right] = G(-eER_T^*) = \frac{r_T^T - r_T^d}{r_T^m - r_T^d}, \qquad (3.11)$$

which defines the equilibrium central bank borrowing implicitly as a function of the interest rates applied by the central bank. We can see that *equilibrium bidding will leave the money market with liquidity that equates the probability of it being overdrawn with the location of the tender rate within the interest rate corridor.*

If the cumulative distribution function $G(\cdot)$ has the inverse function $G^{-1}(\cdot)$, we can derive the explicit form for the equilibrium bidding:

$$eER_T^* = -G^{-1}\left(\frac{r_T^T - r_T^d}{r_T^m - r_T^d}\right) \Leftrightarrow TL_T^* = LG_T - G^{-1}\left(\frac{r_T^T - r_T^d}{r_T^m - r_T^d}\right),$$
$$(3.12)$$

where LG_T is the (estimated) liquidity gap between required daily balances and the sum of morning balances and autonomous liquidity factors $(LG_T = RDB_T - RB_{T-1} - a_T)$. In this model the amount of bank reserves demanded at the tender will depend on the liquidity gap, expected overnight rate (ie tender rate), the rates of the two

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[19]Note that the unique equilibrium we have derived here does not contain any information on how the liquidity is distributed among the banks in the tender. From the point of view of a single *atomistic* bank (which takes the total money market liquidity as given), every bid size will lead to zero expected profit, as long as the expected overnight rate equals the tender rate. If one would like to restrict the number of possible distributions of the tendered liquidity, one possibility would be to impose an extra assumption, according to which there is positive probability (possibly infinitesimal) that a bank cannot enter the interbank market on that day. In such a case there would be a unique equilibrium for each individual bank, in which each bank will bid its neutral liquidity. The reason is that as long as the bank can enter the interbank market any bid is equally good for the bank, but in the infinitesimally probable case, where it is not able to enter into transactions with other banks, it is optimal to bid according to the neutral strategy (as with neutral bidding, the private value of the liquidity for which the bank bids, equals the tender rate). A similar result (uniqueness of a single bank's bidding) can also be derived in a model where the banks are not atomistic.

standing facilities, and the distribution of liquidity shocks. The central bank sets the expectation for the market overnight rate of interest indirectly via the tender rate, and the rates of the standing facilities are also announced directly by the central bank. Thus, *if the shock distribution is taken as given, the central bank is able to determine the demand for reserves (and thus the expected money market liquidity) by choosing the location of the tender rate within the interest rate corridor.*

Note that, if the shock distribution is symmetric and the central bank applies a symmetric interest rate corridor (ie $\frac{r_T^T - r_T^d}{r_T^m - r_T^d} = \frac{1}{2}$), the equilibrium expected amount of excess reserves will be zero (as $G(0) = \frac{1}{2}$ for symmetric shock distribution). If the shock distribution is skewed to the left (*right*), the banks will on average have positive (*negative*) excess reserves when the interest rate corridor is symmetric.

The actual overnight rate on a particular day will deviate from its expected value (the tender rate) because of the (net) liquidity shocks occurring between the allotment of the tender operation and the clearing of the interbank overnight markets (μ). However, *the variation in the actual overnight rate does not contain any information on the stance of monetary policy.* It is merely a consequence of the sum of net errors made by the banks in estimating their need for liquidity in tender operations. Hence, *the volatility should not be transmitted to longer-term interest rates* (interest rates that are more important in monetary policy transmission).

We further clarify the determination of the overnight rate and the relevance of the two liquidity shocks by figure 2. In drawing this figure, we have assumed for clarity that both shocks are normally distributed, and the interest rate corridor is symmetric around the tender rate.[20]

In figure 2, S^T is the perfectly elastic supply of tender reserves, and D^T denotes the demand for reserves during the operation (given by equation (3.12), or if $G^{-1}(\cdot)$ does not exist, implicitly given by equation (3.11)). The equilibrium amount of reserves expected to prevail at the clearance of the overnight market (eOB_T) is given by the equality of demand and supply (point a). The expected

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[20]Demand functions in all figures in this paper are based on the assumption of normally distributed liquidity shocks.

Figure 2: **Determination of overnight rate: symmetric corridor**
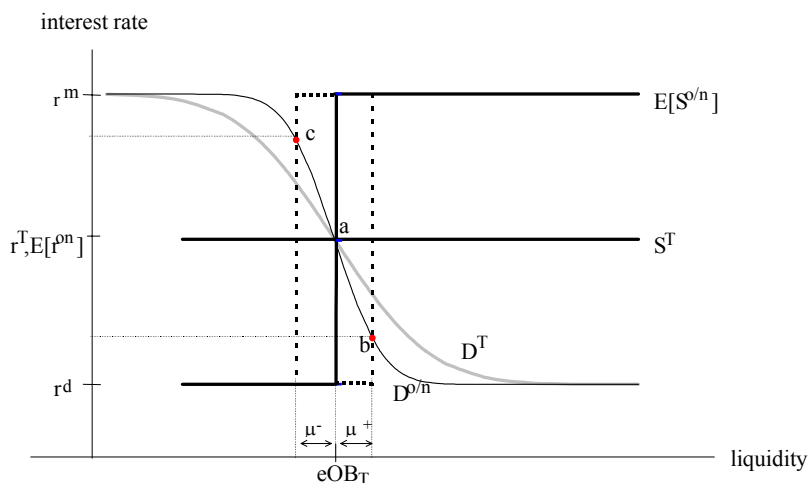
(equilibrium) value of the overnight rate is r^T and, for a symmetric corridor, equilibrium (ie the expected level of) excess reserves is 0.

The equilibrium liquidity gives us the location of the inelastic part of the expected supply of liquidity in the overnight market $(\mathrm{E}\big[S^{o/n}\big])$.[21] The true supplies of liquidity after two alternative realisations of the first shock $(\mu^+, \text{ and } \mu^-)$ are given by the two dashed lines. The demand for liquidity at the clearance of the overnight market is denoted by $D^{o/n}$. With liquidity close to the expected value, the interest rate elasticity of $D^{o/n}$ (based on $F(\cdot)$) is smaller than that of D^T (based on $G(\cdot)$), as the variance of the remaining shock (ε) is smaller than the variance of the total shock $(\mu + \varepsilon)$ (the stochastic error terms μ and ε are independently distributed).[22] This means that the variations in the overnight rate of interest due to a shock of a given size is larger after some of the liquidity uncertainty

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[21] The perfectly elastic parts of overnight supplies are naturally at the levels of the rates of the standing facilities, as the banks can get all the liquidity they want at the marginal lending rate and can deposit liquidity in the central bank at the deposit rate.

[22] Note that distribution $g_{\mu+\varepsilon} \sim N(0, \sigma_{\mu+\varepsilon})$ is derivable from distribution $f_\varepsilon \sim N(0, \sigma_\varepsilon)$ through a mean preserving spread.

has disappeared. If the first shock of the day is positive (μ^+), the realised value of the overnight rate will be lower than its expected value (point b). Similarly, the overnight rate increases up to point c due to a negative liquidity shock.

The volatility of the overnight rate in this setting depends on the timing of the clearance of the interbank market, as μ and ε reflect the share of the aggregate shock occurring before and after the clearance of the market respectively.[23] The earlier the markets clear, the smaller the share of the flow of the daily shocks occurring before the clearance (smaller σ_μ and larger σ_ε), and the closer $D^{o/n}$ is to D^T. Intuitively, early clearing of the interbank market increases the uncertainty of a bank's end-of-day balance at the clearance of the market, which increases the interest rate elasticity of the demand for reserves. Thus, *the volatility of the overnight interest rate is lower in markets that are active already in the mornings compared with markets in which interbank trading merely settles the foreseen liquidity needs of the banks.*

Finally, let us consider the case in which the interest rate corridor is asymmetric. Figure 3 shows the determination of the overnight rate when the tender rate is in the lower part of the corridor.

Here again, the equilibrium at the tender operation gives the expected value of both the liquidity and the overnight rate (point a). The expected value of the overnight rate still equals the tender rate. However, the expected liquidity is now greater than zero, as the relative cost of having to use the deposit facility is lower than the cost of acquiring credit from the marginal lending facility, and thus the banks are willing to increase the probability of using the deposit facility.

Note that even though the expected value of the overnight rate equals the tender rate $(\mathrm{E}\left[r^{o/n}\right] = r^T)$, the overnight rate at the expected liquidity is lower than the tender rate ($r^{o/n}\mid_{\mu=0} < r^T$; see point b in figure 3). This result is obvious, as we know that, having assumed normally distributed error terms, the demand for reserves is convex (concave) at liquidity levels above (below) zero and that the relative curvature of $D^{o/n}$ is higher than that of D^T. This means that we expect to see the overnight rate realised below the tender rate more frequently than above it, if the tender rate is in the lower part

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[23] The aggregate shock consists of a continuum of small independent shocks.

Figure 3: **Determination of overnight rate; asymmetric corridor**

of the interest rate corridor. Also the interest rate variations due to liquidity shocks are not symmetric around the expected value. This again results from the convexity of demand at rates below the middle of the corridor.

Converse effects can be shown for the case where the tender rate is in the upper part of the corridor.

The central bank can affect the amount of excess reserves demanded and the volatility of the interbank overnight rate by choosing both the width of the interest rate corridor and the location of the tender rate within the corridor. These effects should be taken into account if the central bank wants to use the rates of the standing facilities as an independent signalling device.

3.2.2 Proportional allotment

In the case of proportional allotment, the banks know that the central bank has a target for liquidity and that it will try to allot liquidity

54

according to this target regardless of the total amount bid by the banks. Let us define the targeted amount as TL^s. Now, the actual tendered liquidity (TL) will not always be the total amount of bids (TL^d). The amount of liquidity the central bank actually provides to the markets is either the amount targeted by it or the aggregate amount of bids, whichever is the smaller ($TL = \min(TL^d, TL^s)$). Thus, the banks must take into account the behaviour of the central bank as well as the behaviour of the other banks when preparing their bids.

Proposition 3 *If the central bank applies proportional allotment with a neutral liquidity target, the banks will place bids in excess of the neutral strategy, and the expected value of the overnight interest rate will equal the tender rate.*

Let us assume that the banks expect the central bank to target liquidity, such that it will (on average) leave the markets with neutral liquidity (ie liquidity at which the expected overnight rate of interest equals the tender rate, $TL^s = TL^{neutral}$).[24] We also assume that the central bank's estimate of banks' demand for reserves is unbiased in order to have neutral liquidity. If the banks now use the neutral bidding strategy (as under full allotment), the liquidity supplied to the markets will be the smaller of two variables with the same mean: i) the central bank's estimate of tendered reserves needed for neutral liquidity (which is based on the central bank's forecast of autonomous liquidity factors, a^{CB}), and ii) the sum of banks' estimates on their reserve needs for neutral liquidity (ie the aggregate bid, TL^*, which is based on the banks' forecast of the autonomous liquidity factors, $a = \sum_i a_i$). The banks' aggregate estimate of the autonomous liquidity factors need not (and normally does not) coincide daily with that of the central bank, even though both are unbiased estimates

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[24]In section 4.2 (where reserve fulfilment is based on averaging), we divide the proportional allotment procedure into liquidity targeting and interest rate targeting. In liquidity targeting the central bank sets liquidity directly as the target, whereas in interest rate targeting the amount the central bank is willing to lend will be derived indirectly from the banks demand function. Here, both of these two approaches would produce similar results, as the reserve holding is not based on averaging. Thus, the neutral liquidity target we have here can be thought of as a direct liquidity target or to be derived from a neutral interest rate target (where the target rate equals the tender rate).

of the same stochastic variable. Therefore, the overnight rate, with liquidity based on the neutral demand would normally differ from that based on the neutral supply, even though they have the same expected value[25]. Consequently, the expected value of the overnight rate with the liquidity actually tendered ($TL = \min\left(TL^*, TL^{CB}\right)$) would be above its expected value with either of these single liquidity variables ($\mathrm{E}[r^{on}] = \mathrm{E}[r^{on}|_{TL=\min(TL^*, TL^{CB})}] > \mathrm{E}[r^{on}|_{TL=TL^*}] = \mathrm{E}[r^{on}|_{TL=TL^{CB}}] = r^T$), as the overnight rate of interest is a decreasing function of liquidity. In such a case, the representative bank is evidently able to profitably deviate from the neutral bidding strategy. By increasing its bid, a bank will have excess liquidity at the tender rate, and the income from selling this extra liquidity in the money market is expected to be higher than the tender rate. Similarly, underbidding is ruled out as a sustainable equilibrium strategy in this setting, as the expected liquidity would then be smaller than with neutral bidding, and so the incentive to deviate from the overbidding strategy would be even stronger than from neutral bidding. Therefore, *all sustainable equilibria with this kind of proportional allotment procedure must result in overbidding.* If the aggregate amount of bids exceeds the estimated neutral level by sufficiently much ($TL^d >> TL^*$), the tendered liquidity will always equal the central bank's target amount ($TL = \min(TL^d, TL^{CB}) = TL^{CB}$).[26] Hence, the supply of daily liquidity would be determined solely by the target of the central bank. Consequently, the expected value of the interbank overnight rate would equal the tender rate.

The amount of overbidding cannot always be determined uniquely in this setting. For example, total bids amounting to twice the real liquidity need would lead to the same result as total bids amounting to

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[25]That is, if there is positive probability of $\sum_i a_i$ being different from a^{CB}, there is positive probability that $TL^* \neq TL^{CB}$ and thus there will be positive probability of the overnight rate being diffrent with TL^* or TL^{CB}, even though the expected value of the overnight rate, under full allotment, equals that with liquidity targeted by the central bank ($\mathrm{E}[r^{on}|_{TL=TL^*}] = \mathrm{E}[r^{on}|_{TL=TL^{CB}}] = r^T$).

[26]How much the aggregate bids need to exceed the expected neutral level depends on the minimum size of the central bank's estimate of the autonomous liquidity factors ($a^{\min}$), as the target liquidity of the central bank depends inversely on the estimate of the autonomous liquidity factors. If $TL^d \geq TL^{CB}|_{a^{CB}=a^{\min}}$, the aggregate bids will always exceed the amount the central bank is willing to provide to the markets, and consequently there will never be full allotment.

three times the real need, if the real liquidity need multiplied by two is not smaller than the maximum value of the target liquidity of the central bank (ie $2TL^* \geq TL^{CB}|_{a^{CB}=a^{\min}}$). Therefore, any such amount of total bids that is large enough to maintain $TL^d \geq TL^{CB}|_{a^{CB}=a^{\min}}$ would be an equilibrium. Now, from a single bank's point of view any bid would lead to zero profit as long as it can be sure that the central bank can control the liquidity according to its target (ie as long as $p(TL^d \geq TL^{CB}|_{a^{CB}=a^{\min}}) = 1$). However, if there is even the slightest probability that the aggregate bids might be lower than the central bank's target amount, it would be optimal to bid the maximum possible value.[27]

Proposition 4 *The proportional allotment procedure with liquidity target above neutral liquidity results in full allotment.*

If the central bank's strategy were to maintain the expected overnight rate below the tender rate, it would aim at flooding the market with reserves in excess of the neutral liquidity. However, the central bank would not be able to do this, as the banks would not be willing to provide it with large enough bids (ie $TL^d < TL^s$), if the price of liquidity is expected to be lower in the markets than in the tender operation (ie if $E[r^{on}] < r^T$). In this case, the central bank would

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[27]If the central bank did not limit the bid size in any way, the optimal bid would in principle be infinite. However, in practice the size of a bid would still be limited at least to some numerical value. Furthermore, the bid size could also be limited by the central bank (eg the ECB requires the banks to be able to cover the amount of reserves they are allotted by adequate collateral) or by market imperfections (eg the banks usually have limited credit lines that are needed to distribute the liquidity in the overnight market). If there were such collateral requirements or credit lines, the maximum bid would be limited by the probability of being allotted more reserves than would be optimal, taking these limitations into account. Thus, the optimal bid of a single bank would depend (partly) on its expectation of the allotment ratio (ie allotted liquidity/aggregate bids) in the tender: the lower the expected allotment ratio, the lower the probability of reaching these limits and thus the more one will bid in the tender. As in this setup there is no natural focal point for expectations of the allotment ratio, the bank could use the allotment ratio of the previous tender (or average of such ratios in the past few tenders) as such a point while preparing its bid. If this is the case, the bank's optimal bid will increase from tender to tender (as the focal point diminishes continuously), until the allotment ratio reaches such a low level that the bank would be sure of the central bank having control of the allotted liquidity (ie $p\,(\text{aggregate bids} \geq \text{central bank's target liquidity}) = 1$).

not be able to control the quantity of money market liquidity, and consequently, this strategy would produce an outcome identical to the full allotment case.

Proposition 5 *If the central bank applies the proportional allotment procedure with liquidity target below the neutral liquidity, the successful bidders will have positive expected profit and therefore, bidding becomes infinite.*

If the central bank wants to squeeze the markets in order to keep the overnight rate of interest (its expected value, to be exact) above the tender rate, it provides the markets with liquidity below the neutral level. This kind of liquidity policy, when expected by the counterparties, raises the incentive for overbidding, as the income from selling the extra liquidity to the markets is increased compared with the neutral situation. Thus, the central bank is able to steer both the overnight rate and the money market liquidity with this kind of policy. However, the rationale for using fixed rate tenders in this vein could very well be questioned. If the central bank behaves in this manner, it actually uses the supply of liquidity as its policy variable instead of the tender rate. Then, the whole process of implementing monetary policy would be more transparent to the public if variable rate tenders were used. Furthermore, it is hard to rationalise the transfer of profit to the successful bidders caused by the use of fixed rate tenders with liquidity supply rationed below the neutral demand. This procedure would benefit those who can make the largest bids (above their neutral liquidity demand). Hence, this procedure would eventually lead to infinitely large bids, if the bid size is not somehow rationed.

Figures 4 and 5 clarify the determination of overnight rate under the proportional allotment procedure with neutral liquidity target, both for symmetric interest rate corridor and asymmetric corridor.[28] The only differences between these two figures and those for the full allotment procedure (figures 2 and 3) are in the demand for and supply of liquidity at the tender operation. Now, D^T is an arbitrary point at the level of the tender rate and at huge liquidity (relative to the

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[28] In figure 4, as in the rest of the figures in the eesay, we assume liquidity shocks to be distributed normally.

58

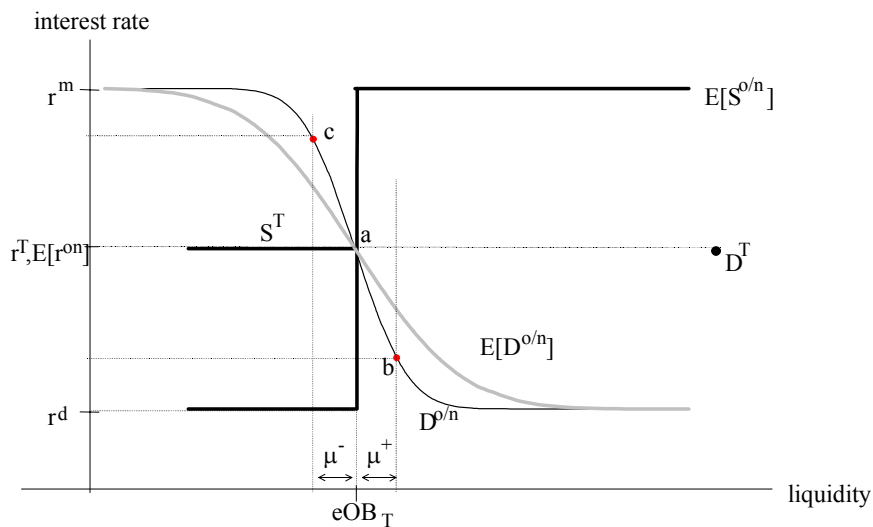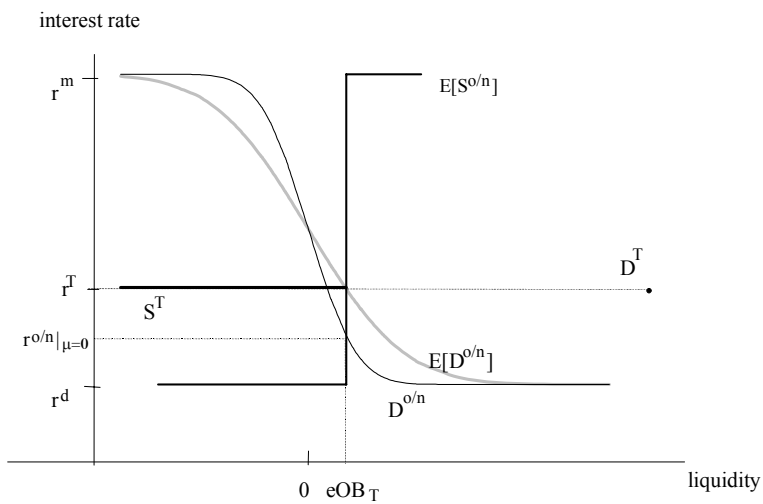Figure 4: **Determination of overnight rate; symmetric corridor**

Figure 5: **Determination of overnight rate; asymmetric corridor**

59

true need). The supply is again perfectly elastic, but only up to the amount targeted by the central bank. This target amount is given by the central bank's expectation of the demand for liquidity at the tender rate (ie $\mathrm{E}\left[S^{o/n}\right]$ at r^T). Thus, $\mathrm{E}\left[D^{o/n}\right]$ is similar to D^T. The difference between them is that D^T is based on $G\left(\cdot\right)$, which is the banks' expectations over $F\left(\cdot\right)$, and $\mathrm{E}\left[D^{o/n}\right]$ is based on the central banks expectation of $F\left(\cdot\right)$.

3.3   Comparision of the allotment techniques

We next summarise the findings from the two previous sections, and try to answer the question: how do these two equilibria with different allotment mechanisms (full allotment and proportional allotment with neutral liquidity target) differ from each other?

- *The demand for and supply of liquidity*

  The demand for liquidity at the clearance of the overnight market does not depend on the allotment procedure; as it is a function only of prevailing money market liquidity, interest rates of the standing facilities, and the distribution of liquidity shocks. The shape of liquidity supply for the overnight market is also independent of the approach used in allotting the liquidity. The supply is perfectly inelastic at the level of overnight balances from deposit rate to marginal lending rate, and the supply is perfectly elastic at the rates of the standing facilities.

  However, the demand and supply at the tender operation differ according to allotment procedure. If the proportional allotment method is used, the supply is perfectly elastic only up to the central bank's liquidity target; with full allotment, the supply is perfectly elastic without limits. The demand for liquidity is arbitrarily large relative to the real need under proportional allotment with neutral liquidity target. In the case of full allotment (or proportional allotment with tender rate in the upper part of the corridor), *equilibrium demand* is determined by the probability-weighted cost of using the standing facilities.

  Due to the differences in demand for and supply of liquidity at the tender, the location of the inelastic part of the supply

60

of overnight liquidity at the clearance of the market may differ according to allotment procedure.

- *Level and volatility of the overnight rate*

  With both procedures, the expected value of the overnight rate will equal the tender rate. If the interest rate corridor is symmetric, the relative volatility of these approaches depends on the size of the first liquidity shock of the day. If the central bank can estimate the behaviour of total liquidity better than the corresponding aggregated estimates of the banks (ie if $\mu^{CB} < \sum \mu_i$), the volatility of the overnight rate of interest is smaller under the proportional allotment method. However, this is not necessarily the case, particularly if the central bank publishes its estimate before the tender operation. In the case of an asymmetric interest rate corridor, the volatility depends again on the relative accuracy of the liquidity estimates. However, now it also depends on the relative accuracy of the estimates of the cumulative distributions. It is however not obvious that $\mathrm{E}^{\mathrm{CB}}\left[F\left(\cdot\right)\right]$ will be a more accurate estimate of $F\left(\cdot\right)$, than $G\left(\cdot\right)$. Hence, without further assumptions one can not say whether the volatility of the overnight rate is greater with full allotment.

- *Signalling monetary policy and transmission of volatility*

  In the case of full allotment, the expected value of the overnight rate for a specific day is always the value expected to be used in the tender operation affecting that day's liquidity. Thus, the yield curve (based on future values of the overnight rate) should reflect only the expectations as to the behaviour of the tender rate. These expectations should not be related to the overnight rate realised on a specific day, as their deviation from the tender rate is produced merely by banks' forecast errors. Thus, *the signals given by the tender operations are unambiguous, and the volatility of the overnight rate should not be transmitted to longer periods.*

  The same reasoning applies to the case with proportional allotment, as long as the strategy used in choosing the level targeted by the central bank is known to the public (or at least to the counterparties). If the target must be

read from the past behaviour of the central bank (ie past allotment decisions), variations in realisations overnight might be interpreted as changes in the monetary policy stance. Thus, in such a case it would not be certain that the volatility would not be transmitted to longer maturities. This, harmful, transmission could be avoided *either* explicitly by the central bank announcing the allotment policy *or* by it making the liquidity policy implicitly public through publishing the liquidity forecast, on which it bases its liquidity allotment decision.

- *Using the interest rate corridor as an independent signalling device*

From the previous analysis, it is clear that a symmetric interest rate corridor is simplest for the central bank to operate with as long as liquidity shocks are expected to be symmetrically distributed. This results from the fact that if the corridor is symmetric, the demand for liquidity at the tender will equal the demand for liquidity at the clearance of the market. Also, the variation of the overnight rate around the tender rate is symmetric with a symmetric corridor. However, the central bank might like to give monetary policy signals independently of the tender rate via the rates of the standing facilities. For example, having a tender rate in the lower part of the corridor could indicate that the central bank anticipates that its next tender rate change will be upwards.

Using the corridor independently is rather complicated, in conjunction with the proportional allotment method. If the tender rate lies in the upper part of the corridor, the central bank is not able to meet its target liquidity, and consequently the procedure will in fact be similar to full allotment. Also, if the tender rate is in the lower part of the corridor, the central bank must adjust its liquidity target up from 0 (or the level of the reserve requirement), to keep the target amount neutral.[29] Estimating the new target liquidity (after a change in the tender

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[29]The target amount of the central bank will differ from zero if the shock distribution is asymmetric. The amount would be positive (negative) if the distribution is skewed to the left (right).

rate's location within the corridor) can be difficult, especially if the shock distributions are not constant over time.

The use of the rates of the standing facilities as an independent policy instrument is perhaps not so difficult with the full allotment procedure. However, in this case the central bank must keep in mind that the asymmetry of the corridor affects the demand for excess reserves and hence also the cost of the setup to the banks.

4   Model with averaging

If fulfilment of the reserve requirement is judged by the average value of reserve holdings during a reserve maintenance period, instead of a daily requirement, the demand for daily reserves changes dramatically. As in the section 3, we assume here that the central bank conducts liquidity operations daily. We also assume that a liquidity operation will mature on the day when the following operation is settled. Thus, the maturity of the liquidity provided is overnight unless we relax the assumption of the frequency of the operations. Furthermore, we will continue to assume that the structural liquidity deficit of the money market enlarged by the reserve requirement is large enough for the probability of the banking sector ending the day with debit balances to be close to one if no liquidity is provided through the (liquidity providing) tender operations.[30]

On the final day of a reserves maintenance period, there is no room left for averaging the reserve holdings; each bank needs to hold (at least) $T * RR_i - \sum_{k=1}^{t-1} RB_{i,k}$ in order to meet the reserve requirement. Thus, determination of the overnight rate during the final day of the reserves maintenance period is identical to the case without averaging

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[30]If this were not the case, the central bank should use liquidty draining instead of providing operations under some circumstances. In such a case, the central bank's loss of control over liquidty , that we are about to see to stem from expectations of an interest rate cut under liquidity targeting, would instead result from expectations of a rate hike. We assume the liquidity deficit to be large enough merely to limit the number of cases under study. This assumption does not otherwise limit the analysis.

provision described in the section 3. Thus, the $\mathrm{E}[r_T^{on}]$ will equal the probability-weighted average of the expected values of the marginal lending rate and the deposit rate at the end of the period.

$$\mathrm{E}\left[r_T^{on}\right] = \mathrm{E}\left[r_T^m\right] G(-eER_T) + \mathrm{E}\left[r_T^d\right](1 - G(-eER_T)), \qquad (4.1)$$

where the expected excess reserves (eER_T) equals the money market liquidity at the time of overnight markets clearing less the minimum required daily balances ($eOB_T - RDB_T$). As in the case without reserve averaging, we divide the following analysis of the demand for liquidity according to which approach the central bank uses in its allotment decisions.

4.1   Full allotment

4.1.1   Penultimate day (T-1)

As we saw in the section 3, banks are willing to bid for neutral liquidity in the last operation of the maintenance period (affecting day T reserves), when a full allotment procedure is used by the central bank. Thus, the expected value (at the last tender) of the last day's overnight rate will equal the tender rate ($\mathrm{E}_T\left[r_T^{on}\right] = r_T^T$). At the interbank market clearance on $T-1$, the banks know that their liquidity holdings on that day do not affect the last day's overnight rate, as the situation in the overnight market will be neutralised in the last tender operation. Thus, the expected value of the last day's overnight rate equals the expected value of the last tender rate, if the expectations are taken at T-1 or earlier ($\mathrm{E}_{T-1}\left[r_T^{on}\right] = \mathrm{E}_{T-1}\left[r_T^T\right]$). Consequently, *in the case of pure averaging*[31], the cost of borrowing (income from lending) an extra unit of liquidity from the interbank markets on day T-1 would be r_{T-1}^{on}, and the expected income from (cost of) it would simply be $\mathrm{E}\left[r_T^T\right]$ (resulting from the ability to avoid a unit of borrowing on the next day). Thus, the market-clearing overnight rate on T-1 would be the interest rate expected to be used in the last operation affecting this maintenance period's

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[31]By pure averaging we refer to a system where the end-of-day balance of a credit institution is not limited by any regulations other than the reserve requirement.

liquidity (ie $r_{T-1}^{on} = \mathrm{E}_{T-1}\left[r_T^T\right]$) for any level of reserves. Hence, no bank would be willing to borrow (lend) at rates above (below) the tender rate. *However, central banks do not normally allow for pure averaging.* Averaging provisions do not normally allow overdrafts. We will maintain here the assumption that overdrafts are forbidden, ie if a bank would otherwise end a day with a debit balance, it must cover the deficit by a liquidity credit from the marginal lending facility.

Proposition 6 *In equilibrium, the expected change in the overnight rate between the two final days of the maintenance period equals the probability weighted average of the spreads between tender rate expected for the final day vs each of the current interest rates of the standing facilities, where the weights reflect the probabilities of using each of the standing facilities already at the end of T-1.*

The cost of borrowing liquidity from the interbank market at T-1 is r_{T-1}^{on} times the borrowed amount. At the penultimate day, the expected income consists of three parts: *i)* the amount of marginal credit expected to be avoided through the interbank borrowing times the marginal lending rate[32], *ii)* the amount of liquidity expected to be deposited into the deposit facility (ie the amount of reserves exceeding the requirement for the whole maintenance period) times the deposit rate, and *iii)* the amount of reserve deposits the bank is expected to have by the end of the day times the expected price of tomorrow's liquidity ($\mathrm{E}\left[r_T^T\right]$), as the reserves held today reduce the need to borrow liquidity tomorrow in order to fulfil the reserve requirement.

The profit maximisation problem of a bank operating in the overnight market at the penultimate day of the maintenance period is given in appendix B, which also shows how equation (4.2), which describes the determination of the overnight rate, is derived from the first order condition (w.r.t. the interbank borrowing) of the profit maximisation problem:

$$
\begin{aligned}
r_{T-1}^{on} = \mathrm{E}_{T-1}\left[r_T^T\right] & \{1 - F(-OB_{T-1}) \qquad\qquad\qquad (4.2) \\
& - [1 - F(2RDB_{T-1} - OB_{T-1})]\} \\
& + r_{T-1}^m F(-OB_{T-1}) + r_{T-1}^d\left[1 - F(2RDB_{T-1} - OB_{T-1})\right].
\end{aligned}
$$

---

[32] $F(-OB_{T-1})$ is the probability that the liquidity shock, ε_{T-1}, is less than $-OB_{T-1}$.

We can also write equation (4.2) as the expected change in the overnight rate of interest (ie the difference between today's overnight rate and the expected rate to be used in the final tender operation):

$$r_{T-1}^{on} - E_{T-1}\left[r_T^T\right] = \left[\left(r_{T-1}^m - E_{T-1}\left[r_T^T\right]\right) F(-OB_{T-1})\right] \quad (4.3)$$
$$+ \left(r_{T-1}^d - E_{T-1}\left[r_T^T\right]\right)\left[1 - F(2RDB_{T-1} - OB_{T-1})\right],$$

where the expected change in the overnight rate between the two final days of the maintenance period equals the probability-weighted average of the spreads between the expected tender rate vs each of the current rates of the standing facilities.

If the interest rate corridor is symmetric[33] and the banks do not anticipate a change in the tender rate, the overnight rate at T-1 will be below that expected for the last day only if the probability of being overdrawn at T-1 is smaller than the probability of fulfilling the reserve requirement for the whole period at T-1.[34] If we assume the liquidity shocks to be distributed symmetrically, we see that the overnight rate is expected to decrease between T-1 and T, as long as the amount of liquidity traded in the overnight market at T-1 is less than the minimum required daily balances to be held at T-1 and T (if $OB_{T-1} < RDB_{T-1}$, then $r_{T-1}^{on} > E_{T-1}\left[r_T^T\right] = E_{T-1}\left[r_T^{on}\right]$). Similarly, with a symmetric corridor and symmetric shock distributions, the overnight rate is expected to increase during the two last days, if the liquidity at the overnight clearance at T-1 is greater than the minimum required daily balances.

From equation (4.3) we know that the overnight rate of interest on the penultimate day of the maintenance period is an increasing function in all central bank rates (expected value of the last tender rate, current deposit rate and current marginal lending rate; $\frac{\partial r_{T-1}^{on}}{\partial E_{T-1}[r_T^T]}, \frac{\partial r_{T-1}^{on}}{\partial r_{T-1}^m}, \frac{\partial r_{T-1}^{on}}{\partial r_{T-1}^d} > 0$), and a decreasing function in both current reserve holdings (money market liquidity at the time of clearing) and the minimum required daily balances ($\frac{\partial r_{T-1}^{on}}{\partial OB_{T-1}}, \frac{\partial r_{T-1}^{on}}{\partial RDB_{T-1}} < 0$). The RDB itself is increasing in the reserve requirement and decreasing in the past reserve holdings ($\frac{\partial RDB_{T-1}}{\partial RR} > 0, \frac{\partial RDB_{T-1}}{\partial \Sigma_{j=1}^{T-2} RB_j} < 0$).

---

[33] By symmetric interest rate corridor we refer to the situation where the tender rate is the mid-point of the interest rate corridor (ie $r_t^T = \frac{r_t^m + r_t^d}{2}$).

[34] With a symmetric corridor and constant tender rate $\left|r_{T-1}^m - E_{T-1}\left[r_T^T\right]\right| = \left|r_{T-1}^d - E_{T-1}\left[r_T^T\right]\right|$. Thus, we must have $F(-OB_{T-1}) < 1 - F(2 * RDB_{T-1} - OB_{T-1})$; otherwise the RHS of the equation (4.3) would not be negative.

66

Bidding behaviour and the determination of equilibrium liquidity

Proposition 7 *Under full allotment, equilibrium bidding at T-1 is such that it balances the expected change in the price of liquidity with the probability-weighted average of the spreads between the tender rate expected for the final day vs each of current rates of the standing facilities.*

The demand for liquidity in the penultimate tender (affecting the liquidity on T-1) will depend on the expected value of the overnight rate for that day. We know that the price of overnight liquidity at T-1 is a decreasing function of the overnight balances on that day. Thus, with reasoning similar to the case with no averaging (see section 3.2.1), *the banks will in equilibrium be bidding for liquidity until the expected value of today's overnight rate equals today's tender rate* ($E_{T-1}\left[r_{T-1}^{on}\right] = r_{T-1}^T$).[35] Let us denote the expected change in the tender rate between the last two days of the period by Δr^T (ie $\Delta r^T = E_{T-1}\left[r_T^T\right] - r_{T-1}^T$). Based on equation (4.3) and the facts that in equilibrium $E_{T-1}\left[r_{T-1}^{on}\right] = r_{T-1}^T$ and $E\left[F\left(-OB_{T-1}^*\right)\right] = G\left(-eOB_{T-1}^*\right)$ we obtain:

$$
\begin{aligned}
E_{T-1}\ \left[r_{T-1}^{on}\right] - E_{T-1}\left[r_T^T\right] &= r_{T-1}^T - E_{T-1}\left[r_T^T\right] = -\Delta r^T = \\
&\left(r_{T-1}^m - E_{T-1}\left[r_T^T\right]\right) G(-eOB_{T-1}^*) \\
&+ \left(r_{T-1}^d - E_{T-1}\left[r_T^T\right]\right)\left[1 - G(2RDB_{T-1} - eOB_{T-1}^*)\right],
\end{aligned} \quad (4.4)
$$

where eOB_{T-1}^* denotes the expected overnight balances at the clearance of the overnight market with equilibrium bidding (ie $eOB_{T-1}^* = RB_{T-2} + a_{t-1} + TL_{T-1}^*$).

Let us divide the analysis of equilibrium liquidity according to the banks' interest rate expectations:

Neutral interest rate expectations

By neutral interest rate expectations we refer to the situation where the banks do not anticipate a change in the tender rate, ie $E_{T-1}\left[r_T^T\right] = r_{T-1}^T$. Let us denote this rate simply by r^T.

---

[35]If this were not the case, the banks could make a profit by increasing their bids if $E_{T-1}\left[r_{T-1}^{on}\right] > r^T$ or by lowering their bids if $E_{T-1}\left[r_{T-1}^{on}\right] < r^T$. Hence, in equilibrium the overnight rate is expected to remain constant at the level of the tender rate during the two last days of each reserve maintenance period.

Proposition 8 *At T-1, optimal bidding under full allotment is a function of the tender rate's location within the interest rate corridor and the probability of having to rely on the standing facilities.*

Equation (4.4) can, under neutral interest rate expectations, be reduced to:

$$
(r^m_{T-1} - r^T) \ G(-eOB^*_{T-1}) = \tag{4.5}
$$
$$
\left(r^T - r^d_{T-1}\right) \left[1 - G(2RDB_{T-1} - eOB^*_{T-1})\right].
$$

Equation (4.5) implicitly defines the optimal bidding to be such that it balances the probability-weighted average of the spreads between the tender rate vs each of the rates of the standing facilities. The LHS of equation (4.5) is positive and monotonically decreasing in liquidity, and the RHS is positive, but monotonically increasing with liquidity. Thus, there always exists a level of liquidity that satisfies the equilibrium condition.

Let us assume for a moment that the interest rate corridor is symmetric and that liquidity shocks are distributed symmetrically. In this case, we know by equation (3.10) that the equilibrium liquidity for tomorrow (at T) is RDB_T.[36] By the symmetry assumptions, equation (4.5) reduces further to $G(-eOB^*_{T-1}) = G(-2RDB_{T-1} + eOB^*_{T-1})$,[37] from which we see that $eOB^*_{T-1} = RDB_{T-1}$. That is, the banks are expected to hold reserves according to their minimum required daily balances at T-1. Thus, with $RB_{T-1} = eOB^*_{T-1}$, RDB_T equals RDB_{T-1}, whereas the expected value of RDB_T will be higher than RDB_{T-1}, at least if eOB^*_{T-1} is very low (ie if $F(-OB_{T-1})$ is significantly above zero).[38] This means that the mean value of money market liquidity is slightly higher on the last day of the maintenance period than the equilibrium liquidity for the previous

---

[36] Under these symmetry assumptions, equation (3.10) reduces to $(r^m - r^T_T)[G(-eER^*_T) - 1 + G(-eER^*_T)] = 0 \Rightarrow G(-eER^*_T) = \frac{1}{2} \Rightarrow eER^*_T = 0 \Rightarrow eOB^*_T = RDB_T$.

[37] Recalling that for a symmetric shock distribution $G(-x) = 1 - G(x)$.

[38] This holds, because for the last day:

$$
RDB_T = \begin{cases} 2RDB_{T-1} - OB_{T-1} - \varepsilon_{T-1}, \text{ if } \varepsilon_{T-1} > -OB_{T-1} \\ 2RDB_{T-1}, \text{ if } \varepsilon_{T-1} < -OB_{T-1} \end{cases}, \quad \text{thus}
$$

$\text{E}[RDB_T] = (2RDB_{T-1} - eOB_{T-1})[1 - F(eOB_{T-1})] + 2RDB_{T-1} * F(-eOB_{T-1})$, which reduces in this symmetric case to $\text{E}[RDB_T] = RDB_{T-1}[1 + F(-RDB_{T-1})]$. More generally

$\text{E}[RDB_t] = \frac{(T-t+1)RDB_{t-1} - OB_{t-1}}{T-t}[1 - F(-OB_t)] + \frac{(T-t+1)RDB_{t-1}}{T-t}F(-OB_t)$.

day, if the required daily balance at T-1 is relatively low ($eOB_{T-1} = RDB_{T-1} \leq \mathrm{E}[RDB_T] = \mathrm{E}[OB_T]$).

If the tender rate is in the upper part of the corridor ($(r^m - r^T) < (r^T - r^d)$), the equilibrium liquidity must leave the probability of being overdrawn higher than the probability of being forced to use the deposit facility (ie $G(-eOB^*_{T-1}) > 1 - G(2RDB_{T-1} - eOB^*_{T-1})$). In this case, the overnight liquidity is expected to increase more during the last two days than with a symmetric corridor, as $G(-eOB^*_{T-1}) > 1 - G(2RDB_{T-1} - eOB^*_{T-1}) \Rightarrow G(eOB^*_{T-1}) < G(2RDB_{T-1} - eOB^*_{T-1}) \Rightarrow eOB^*_{T-1} < RDB_{T-1}$ and thus $\mathrm{E}[OB^*_T] = \mathrm{E}[RDB_T] > RDB_{T-1} > eOB^*_{T-1}$.

Similarly, if $(r^m - r^T) > (r^T - r^d)$, then $(G(-eOB^*_{T-1}) < (1 - G(2RDB_{T-1} - eOB^*_{T-1}))$, and the direction of the evolution of liquidity on the last two days depends on the magnitude of the asymmetry, as well as on the size of RDB_{T-1}.

Expectations of an increase in interest rates

Proposition 9 *Under full allotment, the expected overnight liquidity is greater when an interest rate hike is expected than with neutral expectations, but the interest rate expectations do not affect the expected value of the overnight market rate.*

If the banks anticipate an increase in the tender rate during the remainder of the period ($r^T_{T-1} < \mathrm{E}_{T-1}[r^T_T]$ ie $\Delta r^T > 0$), the demand for liquidity in the penultimate tender will increase considerably, as the banks perceive the price of today's central bank liquidity to be cheap compared with that of tomorrow's. To get an equilibrium in the tender at T-1, the banks place bids again in order to equate the expected value of today's overnight rate with today's tender rate ($\mathrm{E}_{T-1}[r^{on}_{T-1}] = r^T_{T-1}$). However, at the moment of overnight trading, r^{on}_{T-1} is a function of r^T_T instead of r^T_{T-1} (see equation (4.3)). Thus, the expected overnight liquidity must now be larger than in the case of neutral expectations, as the RHS of equation (4.4) must be negative.

Equation (4.4) tells us that the banks will bid for liquidity until the spreads between each of current rates of the standing facilities vs the expected tender weighted by the probabilities by which these facilities are expected to be used, equals the negative of the expected difference between the rates of the two remaining tenders. With expectations

of increased interest rates, the expected difference between the two tender rates is positive ($\Delta r^T = E_{T-1}\left[r_T^T\right] - r_{T-1}^T > 0$). Thus, in order to get a negative value on the RHS of equation (4.4), the probability of using the marginal lending facility must be lower than with neutral interest rate expectations (ie $G(-eOB_{T-1}^{*,\text{increasing exp.}}) < G(eOB_{T-1}^{*,\text{neutral exp.}})$). This means that the equilibrium liquidity at T-1 will be larger with expectations of increases than with neutral expectations ($eOB_{T-1}^{*,\text{increasing exp.}} > eOB_{T-1}^{*,\text{neutral exp.}}$). As the liquidity at T-1 is larger with expectations of increases, the $RDB_T^{\text{increasing exp.}} < RDB_T^{\text{neutral exp.}}$, and consequently $eOB_T^{*,\text{increasing exp.}} < eOB_T^{*,\text{neutral exp.}}$. Therefore, the expectation of an increase in the tender rate between the two last days of the maintenance period does not carry over to the market overnight interest rate, but it is transmitted to the equilibrium overnight liquidity, as stated in proposition 9.
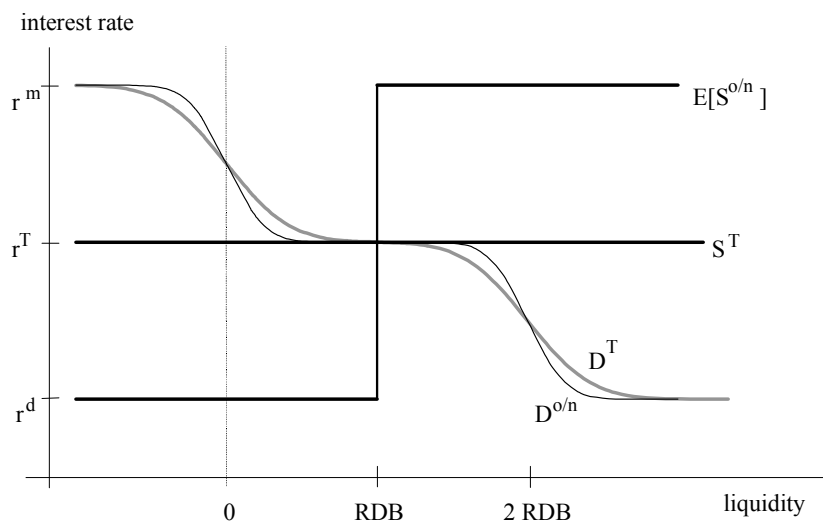
Expectations of a decrease in interest rates

Proposition 10 *Under full allotment, the expected overnight liquidity is smaller when an interest rate cut is expected than with neutral expectations, but the expectation does not affect the expected value of the overnight market rate.*

Under full allotment, the expected overnight liquidity is greater when an interest rate hike is expected than with neutral expectations. However, the interest rate expectation does not affect the expected value of the overnight market rate. Following the approach above, with expectations of decreases in interest rates during the remainder of the maintenance period ($r_{T-1}^T > E_{T-1}\left[r_T^T\right]$) the RHS of equation (4.4) must be positive in equilibrium. To have this, the banks should bid for less liquidity than with neutral expectations ($eOB_{T-1}^{*,\text{decreasing exp.}} < eOB_{T-1}^{*,\text{neutral exp.}}$). As in the case with expectations of increases, the overnight rate does not react to the expected fall in the tender rate. The expectations are reflected merely in the amount of overnight liquidity in the money market.

To sum up, if the central bank uses full allotment, the overnight rate at T-1 will equal the tender rate affecting the liquidity at T-1, whatever expectations the banks have of the tender rate for the last day. However, the equilibrium liquidity depends on

70

Figure 6: **Determination of overnight rate at T-1: symmetric corridor and neutral interest rate expectations**


interest rate expectations, ie $eOB_{T-1}^{*,\text{increasing exp.}} > eOB_{T-1}^{*,\text{neutral exp.}} > eOB_{T-1}^{*,\text{decreasing exp.}}$.

Figure 6 shows the determination of the overnight rate on the penultimate day of the maintenance period and the effect of the averaging provision. Once again D^T and S^T denote the demand and supply in the tender. The vertical part of the expected overnight supply is at RDB_{T-1}, which is the level of liquidity demanded at r^T. The demand for reserves at the clearance of the overnight market is now very elastic at liquidity levels close to the equilibrium. Thus, stochastic liquidity shocks do not affect the overnight rate of interest as much as in the case without the averaging provision.

However, even though the interest rate elasticity of the demand for liquidity increases with the averaging provision, we are not able to state that the volatility of the overnight rate of interest decreases with it. Figure 7 shows the case where banks are expecting a rise in the tender rate ($r_{T-1}^T < r_T^T$). The part of demand curve $D^{o/n}$ that seems to be most elastic is still around the minimum required daily balances for the rest of the period (RDB_{T-1}). However, the equilibrium liquidity

71

Figure 7: **Determination of overnight rate at T-1; increasing interest rate expectations**
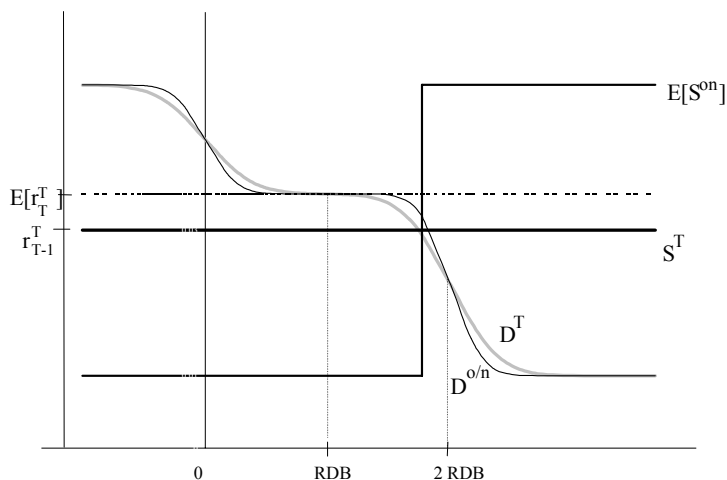
provided to the market is now well above this level. Thus, we are not able to say unambiguously whether the demand for liquidity is now more or less elastic than in the case without averaging provision. We may conclude that, depending on the banks' expectations, the averaging provision may lower the interest rate variability. However, the banks' interest rate expectations under the averaging provision will lead to variations in the equilibrium liquidity and consequently also to variations in the volatility of the overnight rate.

4.1.2   Earlier days $(1,2,...,T\text{-}3,T\text{-}2)$

We now move on to analyse the situation on the days prior to the last two days of the reserves maintenance period. On the penultimate day, the banks already had the luxury of averaging; as long as RDB_{T-1} was positive, the probability of having to rely on either of the standing facilities was less than one. The amount of liquidity the banks held on that day did not affect liquidity conditions on the following day, as the situation was neutralised in the last tender operation. The analysis of the situation prior to the last two days becomes more

72

complicated, as the liquidity held at t ($t=1,2,...,T$-2) affects the cost of holding reserves (ie the probability of having to use the standing facilities) and consequently the demand for liquidity in the tenders held at $t+1,...,T$-1. The channel of this effect is the RDBs of the following days.

The cost of borrowing (income from lending) reserves in the overnight market at t ($t = 1, 2, ..., T - 2$) is $r_t^{on} * b_{i,t}$. The income from liquidity bought (cost of liquidity sold) is again a mixture of several components: *i)* the marginal lending rate times the expected decrease in the amount of marginal lending taken at t, *ii)* the deposit rate times the expected amount to be placed in the deposit facility today, and *iii)* the expected decrease in the cost of borrowing liquidity either from the central bank or from the markets later during the same maintenance period. With full allotment, the banks know that the equilibrium *ex ante* price of overnight liquidity at t,...T equals the tender rate for that day.[39] For simplicity we assume here that the central bank will not change the tender rate more that once during the remainder of the maintenance period. This assumption should not be too restrictive, as eg in case of the ECB the maximum number of main refinancing operations during one maintenance period is five (so, at the first operation there are only three or four operations where the tender rate could be changed). We also assume that the banks are unaware of the timing of the possible change. Thus, if a rate change is expected, the banks expect it to be effective already in the next operation.[40] The expected value of the future tender rate is denoted by $\mathrm{E}_t \left[r_f^T \right]$.

Besides these three factors, which were used also in the determination of the overnight rate at $T - 1$, we now have a fourth component affecting the overnight rate at $1, 2, ..., T - 3, T - 2$. *iv)* An increase in reserve balances held at t lowers the minimum required daily balances for the remaining period of the following days ($\partial RDB_j / \partial RB_t < 0; j = t + 1, ..., T - 1$). The cost of liquidity uncertainty the banks face during the rest of the period

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[39]Otherwise, a bank could make a profit by changing its bid in the tender, as we have seen before.

[40]If the banks could be certain that the expected change will not occur in the next operation but could be effective in the following one, the front- or backloading of reserves (resulting from the expectations) that this model suggests would be divided between this operation and the next one.

is a decreasing function of RDB_j, as the probability of having to rely on the standing facilities on a particular day is a decreasing function of the required daily balances for that day, and a liquidity shock can be neutralised in the following tender operation only if it does not force the banks to use standing facilities on that day $\left(\frac{\partial \text{cost of uncertainty at } j}{\partial RDB_j} = \frac{\partial \text{cost of uncertainty}}{\partial \text{prob. of using s.f.}} \frac{\partial \text{ prob. of using s.f.}}{\partial RDB_j} < 0\right).$[41] The cost of uncertainty on the last day of the maintenance period depends, as we saw earlier, on the rates of the standing facilities and on the distribution of liquidity shocks. However, on day j this is true only if $eRDB_j = 0$ (ie if the reserve requirement has already been fulfilled for the whole period, the banks no longer have the averaging possibility). Otherwise, we have to take into account that borrowing reserves has an extra effect on the maximisation problem by affecting the probability of being forced to use the standing facilities (through the RDB_j's). Henceforth, we will call this fourth determinant in the profit maximisation problem *the dynamic cost factor (dcf)*.

Proposition 11 *At day t, the overnight rate is the tender rate expected to prevail over the rest of the maintenance period, plus the probability-weighted cost of having to rely on marginal lending today, minus the sum of the probability-weighted cost of having to rely on the deposit facility today (at t) and the increase in the cost of future uncertainty associated with the extra borrowing.*

The profit maximisation problem of a bank operating in the interbank overnight market is explicitly given in appendix C. The first order condition for the profit maximisation problem with respect to b_t gives us (after aggregation over the unitary mass of banks) the overnight rate of interest at t as a function of liquidity:

---

[41]The probability of using the deposit facility on day t is $\left(1 - F\left[(T - j + 1)RDB_j - OB_j^*\right]\right)$. Thus, the probability decreases when RDB_j increases, *ceteris paribus*. If the probability of using the deposit facility decreased and the probability of using the marginal lending facility were unchanged, the overnight rate for that day would decrease. Then, the banks would be lowering their demand for liquidity to restore the equilibrium between overnight rate and tender rate. Thus, OB_j^* would increase, which would lower the probability of being overdrawn and increase the probability of having to use the deposit facility. When balance is restored to the market, the probability of using either of the standing facilities is smaller than before the increase in RDB_j.

$$r_t^{on} = \mathrm{E}_t\left[r_f^T\right] + \left(r_t^m - \mathrm{E}_t\left[r_f^T\right]\right) F(-OB_t) \tag{4.6}$$
$$+ \left(r_t^d - \mathrm{E}_t\left[r_f^T\right]\right)\left\{1 - F\left[(T - t + 1)RDB_t - OB_t\right]\right\}$$
$$+ \sum_{j=t+1}^{T-1}\left(\mathrm{E}\left[r_j^m - r_f^T\right]\frac{\partial eOB_j^*}{\partial b_t}G(-eOB_j^*)\right.$$
$$+ \mathrm{E}\left[r_j^d - r_f^T\right]\left[-(T - j + 1)\frac{\partial eRDB_j}{\partial b_t} + \frac{\partial eOB_j^*}{\partial b_t}\right]$$
$$\left.\times\left\{1 - G\left[(T - j + 1)eRDB_j - eOB_j^*\right]\right\}\right),$$

where $G(-eOB_j^*)$ and $(1 - G((T - j + 1)RDB_j - eOB_j^*))$ are the probabilities of having to use the two standing facilities at j. These probabilities are affected by interbank lending today, as lending today lowers both $eRDB_j$ and eOB_j^*.

Noting that $\frac{\partial eOB_j^*}{\partial b_t} = \frac{\partial eOB_j^*}{\partial eRDB_j}\frac{\partial eRDB_j}{\partial b_t}$ and taking the partial derivative $\frac{\partial eRDB_j}{\partial b_t}$, we can analyze equation (4.6) further to see explicitly the dcf as a function of the probability of not having to rely on the standing facilities today:

$$r_t^{on} = \mathrm{E}_t\left[r_f^T\right] + \left(r_t^m - \mathrm{E}_t\left[r_f^T\right]\right) F(-OB_t) \tag{4.7}$$
$$+ \left(r_t^d - \mathrm{E}_t\left[r_f^T\right]\right)\left\{1 - F\left[(T - t + 1)RDB_t - OB_t\right]\right\}$$
$$+ \left\{F\left[(T - t + 1)RDB_t - OB_t\right] - F(-OB_t)\right\}$$
$$\times \sum_{j=t+1}^{T-1}\left(\mathrm{E}\left[r_j^m - r_f^T\right]\frac{\partial eOB_j^*}{\partial eRDB_j}G(-eOB_j^*)\left(\frac{-1}{T - j + 1}\right)\right.$$
$$+ \mathrm{E}\left[r_j^d - r_f^T\right]\left[1 - \left(\frac{1}{T - j + 1}\right)\frac{\partial eOB_j^*}{\partial eRDB_j}\right]$$
$$\left.\times\left\{1 - G\left[(T - j + 1)eRDB_j - eOB_j^*\right]\right\}\right)\Lambda_j,$$

where we have used the definition $\frac{\partial eRBD_{i,j}}{\partial b_t} = \frac{-[F(IB_t) - F(-OB_t)]}{T - j + 1}\Lambda_j$, in which $\Lambda_j = 1 + \sum_{k=t+1}^{j-1}\frac{\partial eOB_k^*}{\partial eRDB_k}\frac{-1}{T - k + 1}\left(1 + \sum_{l=t+1}^{k-1}\frac{\partial eOB_l^*}{\partial eRDB_l}\frac{-1}{T - l + 1} \times ... \times \left\{1 + \frac{\partial eOB_{t+2}^*}{\partial eRDB_{t+2}}\frac{-1}{T - t - 1}\left[1 + \frac{\partial eOB_{t+1}^*}{\partial eRDB_{t+1}}\left(\frac{-1}{T - t}\right)\right]\right\}\right).$[42] Now, equations (4.6) and (4.7) prove proposition 11.

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[42] See appendix C for the derivation of equation (4.7).

Proposition 12 *When the central bank applies full allotment, the banks bid for liquidity so that the expected change in the tender rate equals the probability-weighted costs of using the standing facilities today plus the dynamic cost factor.*

We know that in equilibrium (under full allotment) the banks bid for liquidity until $E_t\left[r_t^{on}\right] = r_t^{T}$.[43] Thus, the equilibrium condition for the money market at t $(t = 1, 2, ..., T-1)$ is:

$$
\begin{aligned}
E_t\left[r_t^{on}\right] - E_t\left[r_f^T\right] = r_t^T - E_t\left[r_f^T\right] &= \left(r_t^m - E_t\left[r_f^T\right]\right) G(-eOB_t^*) \quad (4.8) \\
&+ \left(r_t^d - E_t\left[r_f^T\right]\right) \left\{1 - G\left[(T-t+1)RDB_t - eOB_t^*\right]\right\} \\
&+ \sum_{j=t+1}^{T-1} \left(E_t\left[r_j^m - r_f^T\right] \frac{\partial eOB_j^*}{\partial eRDB_j} \frac{\partial eRDB_j}{\partial b_t} G(-eOB_j^*) \right. \\
&+ E_t\left[r_j^d - r_f^T\right] \left(\frac{\partial eOB_j^*}{\partial eRDB_j} - (T-j+1) \right) \frac{\partial eRDB_j}{\partial b_t} \\
&\left. \times \left\{1 - G\left[(T-j+1)eRDB_j - eOB_j^*\right]\right\} \right)
\end{aligned}
$$

or

$$
\begin{aligned}
E\left[r_t^{on}\right] - E_t\left[r_f^T\right] = r_t^T - E_t\left[r_f^T\right] &= \left(r_t^m - E_t\left[r_f^T\right]\right) G(-eOB_t^*) \quad (4.9) \\
&+ \left(r_t^d - E_t\left[r_f^T\right]\right) \left\{1 - G\left[(T-t+1)RDB_t - eOB_t^*\right]\right\} \\
&+ \left\{G\left[(T-t+1)RDB_t - eOB_t^*\right] - G(-OB_t^*)\right\} \\
&\times \sum_{j=t+1}^{T-1} \left(E\left[r_j^m - r_f^T\right] \frac{\partial eOB_j^*}{\partial eRDB_j} G(-eOB_j^*) \left(\frac{-1}{T-j+1} \right) \right. \\
&+ E\left[r_j^d - r_f^T\right] \left(1 - \left(\frac{1}{T-j+1} \right) \frac{\partial eOB_j^*}{\partial eRDB_j} \right) \\
&\left. \times \left\{1 - G\left[(T-j+1)eRDB_j - eOB_j^*\right]\right\} \right) \Lambda_j;
\end{aligned}
$$

both of which implicitly define the banks' optimal bidding at t (given $TL_t^* = eOB_t^* - RB_{t-1} - a_t$).

As the optimal bidding at t is a function of future optimal bidding (implicitly given by eOB_j^*), the equilibrium liquidity, eOB_t^*, must be calculated recursively using backward induction. This means that we must first solve OB_T^* as a function of RDB_T (which is known at T),

---

[43] Again, if this were not the case, banks could make positive profits by changing their bidding behaviour.

and use this to solve for eOB^*_{T-1} as a function of RDB_{T-1} and so on. Thus, while deciding on its bid at t, a bank must calculate the optimal path of reserve holdings for all days remaining in the current maintenance period.

If the banks have fulfilled their reserve requirement for the whole maintenance period already before t ($RDB_t = RDB_{t+1} = ... = RDB_T = 0$), the extra borrowing no longer affects the future uncertainty, as the dynamic cost factor becomes zero. Thus, the rest of the period will be similar to the case without averaging, and the equilibrium bidding is defined simply by:

$$
\begin{aligned}
r_t^T - E_t\left[r_f^T\right] &= \left(r_t^m - \mathrm{E}_t\left[r_f^T\right]\right) G(-eOB_t^*) \\
&\quad + \left(r_t^d - \mathrm{E}_t\left[r_f^T\right]\right) \left[1 - G(-eOB_t^*)\right].
\end{aligned}
$$

To see the effect of the averaging provision, we are interested in cases where RDB_t is strictly positive. If $RDB_t > 0$, the dynamic cost factor is negative (we know that $\frac{\partial RDB_j}{\partial RB_t} < 0$ and $\frac{\partial eOB_j^*}{\partial RDB_j} > 0$). That is, the dynamic cost factor always encourages the banks to reduce the cost of future liquidity uncertainty by postponing the holding of reserves.

Three-day maintenance period as an example

To get an intuitive grasp of the optimal borrowing determined by equation (4.8), let us consider the very simplest case in which the dcf is present. Assume $T=3$ (or equivalently $t = T - 2$) and that the liquidity shocks are normally distributed ($\mu_t \sim \mathrm{N}(0, \sigma_\mu^2)$ and $\varepsilon_t \sim \mathrm{N}(0, \sigma_\varepsilon^2) \Rightarrow \nu_t \sim \mathrm{N}(0, \sigma_\nu^2)$). The equilibrium equation at $t=1$ becomes:

$$
\begin{aligned}
r_1^T - \mathrm{E}_1\left[r_2^T\right] &= (r_1^m - \mathrm{E}\left[r_f^T\right])N(-eOB_1^*) \qquad\qquad (4.10) \\
&\quad + \left(r_1^d - \mathrm{E}\left[r_f^T\right]\right)\left[1 - N(3RDB_1 - eOB_1^*)\right] \\
&\quad + \left[N(3RDB_1 - eOB_1^*) - N(-eOB_1^*)\right] \\
&\quad \times \left\{ \mathrm{E}\left[r_2^m - r_f^T\right]\left(-\frac{1}{2}\right)\frac{\partial eOB_2^*}{\partial eRDB_2}N(-eOB_2^*) \right. \\
&\quad \left. + \mathrm{E}\left[(r_2^d - r_f^T)\right]\left(1 - \frac{1}{2}\frac{\partial eOB_2^*}{\partial eRDB_2}\right)\left[1 - N(2eRDB_2 - eOB_2^*)\right] \right\},
\end{aligned}
$$

where $N(\cdot)$ is the cumulative distribution function of the normally distributed aggregate shock. The dynamic cost factor (ie the third term on the RHS) is always negative. Under neutral or decreasing interest rate expectations, the LHS of the equation is non-negative (ie $r_1^T - E_1\left[r_2^T\right] \geq 0$). Hence, with such expectations the banks should aim at liquidity that will leave the probability-weighted cost of using the marginal lending facility lower than the probability-weighted cost of using the deposit facility today.

In case of neutral interest rate expectations ($E\left[r_f^T\right] = r_1^T$, which we denote as r^T) and symmetric interest rate corridor, we know that $eOB_2^* = eRDB_2$ (thus $\frac{\partial eOB_2^*}{\partial eRDB_2} = 1$). We also know that $G(-OB) = 1 - G(OB)$ for symmetric shock distributions. Thus, $1 - N(2eRDB_2 - eOB_2^*) = N(-eOB_2^*)$, and we can write equation (4.10) as:

$$0 = (r_1^m - r^T)N(-eOB_1^*) + \left(r_1^d - r^T\right)\left(1 - N(3RDB_1 - eOB_1^*)\right)$$

$$+ \left[N(3RDB_1 - eOB_1^*) - N(-eOB_1^*)\right](-\frac{1}{2})E\left(r_2^m - r_2^d\right)N(-eRDB_2).$$

This can be further reduced under the symmetric interest rate corridor (ie $r^m - r^T = -(r^d - r^T) = 0.5(r^m - r^d)$) to:

$$N(-eOB_1^*) - N(-3RDB_1 + eOB_1^*) = \tag{4.11}$$
$$N(-\frac{3RDB_1 - eOB_1^*}{2})\left[N(3RDB_1 - eOB_1^*) - N(-eOB_1^*)\right].$$

Equation (4.11) says that with equilibrium bidding, the difference between the probabilities of overdrawing and being forced to use the deposit facility today will equal the probability of overdrawing tomorrow after not being forced to use the standing facilities today. We could easily solve equation (4.11) for the equilibrium liquidity (hence also for the equilibrium bidding) if we knew the variances of the shock distributions. In table 1, we have calculated the equilibrium overnight balances for different variances in the shock distribution, as well as for three different interest rate expectations. Here we have assumed that the reserve requirement is 100 units. We also assume that when the banks expect the central bank to change its tender rate, they expect it to do so by 0.25 %-points between the first and the second tenders ($3\% \rightarrow 3.25\%$ or $3\% \rightarrow 2.75\%$). Furthermore,we assume that the corridor is expected to be symmetric during the remaining period (assumed width, 4%).

78

Table 1. **Equilibrium liquidity vs uncertainty**

| $\sigma_{\mu+\varepsilon} \setminus eOB_1 *$	neutral exp.	incr. exp.	decr. exp.
10	94	276	12
20	100	252	24
50	101	181	60

Table 1 illustrates the fact that the equilibrium liquidity is a function of both interest rate expectations and the distribution of the liquidity shocks (when the standard deviation of the shocks is normally distributed). If banks are expecting the central bank rates to be constant, the equilibrium bidding will leave the market with the less reserves, the smaller their volatility. Intuitively this means that the more certain they can be as to their end-of-day balances, the more they can backload their reserve holdings, and thus lower the cost of future uncertainty (ie probability of having to rely on the standing facilities in the future).

The equilibrium liquidity is, however, affected much more by interest rate expectations than by the volatility. If the banks expect a rate rise (cut) they will try to front- (back-) load the reserves. *The lower the volatility of the liquidity, the more the front- or backloading*. This again is natural, as the more certain you are about the evolution of reserves, the greater the incentive to take advantage of the expected difference between today's and expected future values of the overnight rate.

Figures 8, 9 and 10 demonstrate the determination of the overnight rate on the first day of a three-day maintenance period. When drawing these figures, we have assumed that the reserve requirement is 100/day (ie also $RDB_1 = 100$), and that both daily liquidity shocks are normally distributed with zero mean and standard deviations of 10 and 20. In figures 9 and 10 the tender rate is expected to be changed by 25 basis points (bps) from the starting value of 3%. The thicker and lighter curves illustrate the demand for liquidity at the tenders, whereas the demand at the overnight market clearance is given by the thinner darker curves.

From figure 8 we see that the first shock of the day must be very large compared with the total liquidity, to make the overnight rate deviate significantly from the tender rate. The equilibrium liquidity is at the level of the required daily balances for the remaining maintenance period. The interest rate elasticity of the demand for

Figure 8: **Determination of overnight rate on the first day of a 3-day maintenance period: neutral interest rate expectations**
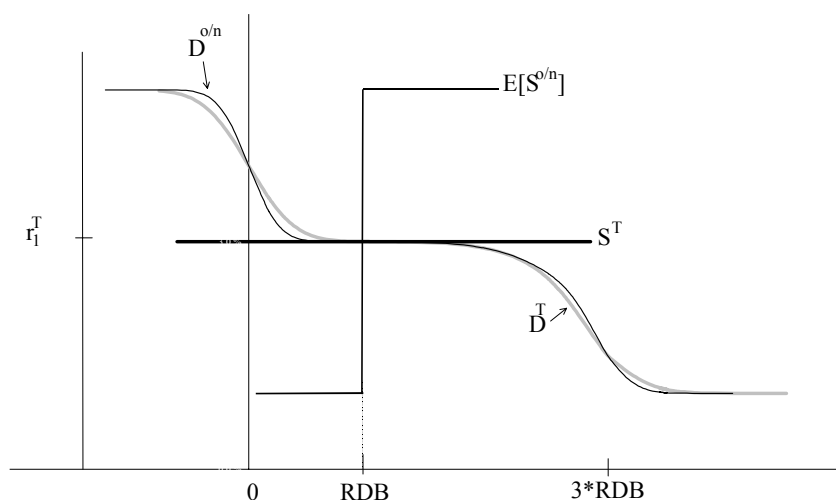

liquidity seems to be large at liquidity levels from around 0.5RDB up to 2RDB.

Figure 9 shows the case where the banks expect central bank rates to be decreased by 25 bps. The expectations will strongly affect the demand for reserves. The banks will try to postpone reserve holding to the second day, when it will be cheaper. The interest rate elasticity of the demand is much less at the equilibrium liquidity than it was in case of neutral expectations. Now, the value of the overnight rate expected at the tender (ie the tender rate) is higher than the overnight rate would be if the first shock equals its expected value (ie if $\mu = 0$). This again results from the combination of the convexity of the demand at these low levels of liquidity and the fact that some of the uncertainty has faded away between the tender operation and clearance of the overnight market.

Figure 10 illustrates the opposite case, where the banks expect the central bank to increase its rates. In this case the banks will frontload liquidity, as its price is expected to be higher tomorrow. We see from the figure that the difference between the expected overnight rate and the overnight rate at the expected liquidity is smaller in this case than

80

Figure 9: **Determination of overnight rate on the first day of a 3-day maintenance period: decreasing interest rate expectations**
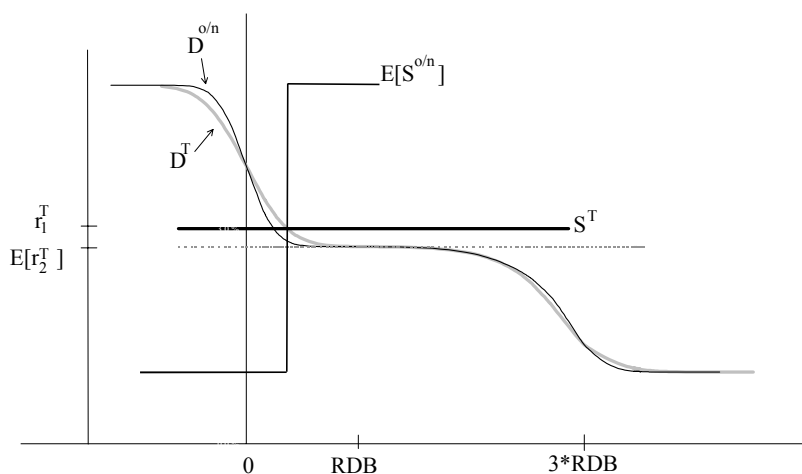
if a rate cut were expected. This result comes from the fact that the dynamic cost factor is larger at high levels of liquidity. Hence, the demand functions are not symmetric around their inflection points. Due to the *dcf,* the demand function is more convex at low liquidity levels than concave at high liquidity levels.

The effect the liquidity volatility has on the equilibrium liquidity is illustrated by figure 11. It shows us how the equilibrium liquidity decreases from $eOB' = 266$ to $eOB = 232$, as the standard deviation of the liquidity shock is doubled from 20 to 40 (the darker demand curve is based on the higher standard deviation).[44] Thus, the magnitude of frontloading (with increasing expectations) clearly depends on liquidity volatility. Similar effects could be illustrated for neutral and decreasing interest rate expectations.

The reserve requirement defines directly the minimum daily balances for the remaining period at the first day of the reserve

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[44]The demand curves here are based on normally distributed liquidity shocks. The amount of reserve requirement is 100, and the tender rate is expected to be raised from 3% to 3.5%.

Figure 10: **Determination of overnight rate on the first day of a 3-day maintenance period: increasing interest rate expectations**
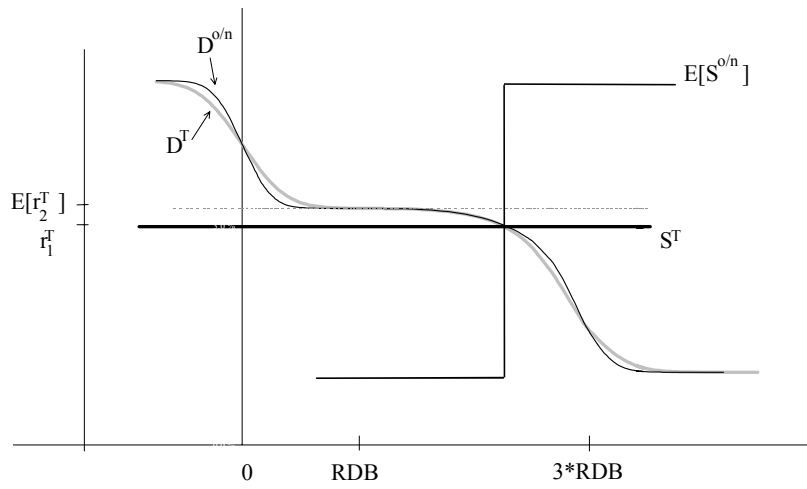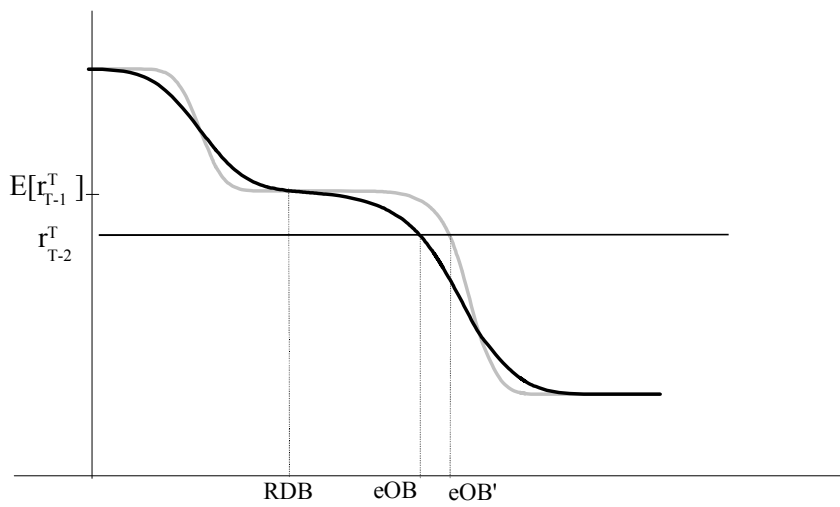

Figure 11: **The effect of an increase in the volatility of liquidity shocks on the equilibrium liquidity**

maintenance period (ie $RBD_1 = RR$). As the equilibrium liquidity eOB_1^* is a function of the liquidity uncertainty, we notice that eOB_1^*/RBD_1 is decreasing in RR (at least as long as the shock distribution is independent of the reserve requirement). Thus, whether the banks with neutral interest rate expectations will be front- or backloading reserves at the beginning of the maintenance period depends on the size of the reserve requirement compared to the liquidity volatility. If the equilibrium liquidity at $t = 1$ is larger than the reserve requirement ($eOB_1^*/RBD_1 > 1$, ie the banks are frontloading reserves at the beginning of the period), we expect RDB_2 to be smaller than RDB_1. Therefore, $eOB_2^*/eRBD_2$ will be larger than eOB_2^*/RR. Thus, if $eOB_1^*/RBD_1 > 1$, we expect the overnight balances to decrease on the following days (as $\frac{\partial eOB_j^*}{\partial eRBD_j} > 0$), however, we do not expect the frontloading of the reserves to disappear (as $\frac{\partial\left(eOB_j^*/eRBD_j\right)}{\partial eRBD_j} < 0$). Similarly, if the initial reserve requirement is low ($eOB_1^*/RBD_1 < 1$), we expect the equilibrium liquidity to increase as time passes, but we do not expect the backloading of reserves to disappear (if the interest rate expectations do not change).

Figure 12 illustrates the effect the size of the reserve requirement has on the equilibrium liquidity. The darker demand curve is based on a reserve requirement of 100, whereas the lighter is based on that of 200. Both demand curves assume the standard deviation of liquidity shocks to be 25, and the tender rate to be raised from 3% to 3.5%. Here, the $eOB = 2.6RDB$ while $eOB' = 2.8RDB'$, which illustrates us the fact that eOB_t/RDB_t is increasing in RR.

To sum up the findings of this section we may conclude that, if the monetary policy framework includes the averaging provision for reserve holdings, and if the central bank uses a full allotment procedure in liquidity provision, the following will hold:

1. The expected value of the overnight rate of interest will equal the tender rate expected for that day.

2. The timing of reserve holdings within the maintenance period depends firstly, on banks' interest rate expectations, but also on the distribution of liquidity shocks and the size of the reserve requirement. The central bank can affect the timing of reserve holding and the banks' possibility of doing intraperiod arbitrage

Figure 12: **The effect of the size of reserve requirement on the equilibrium liquidity**
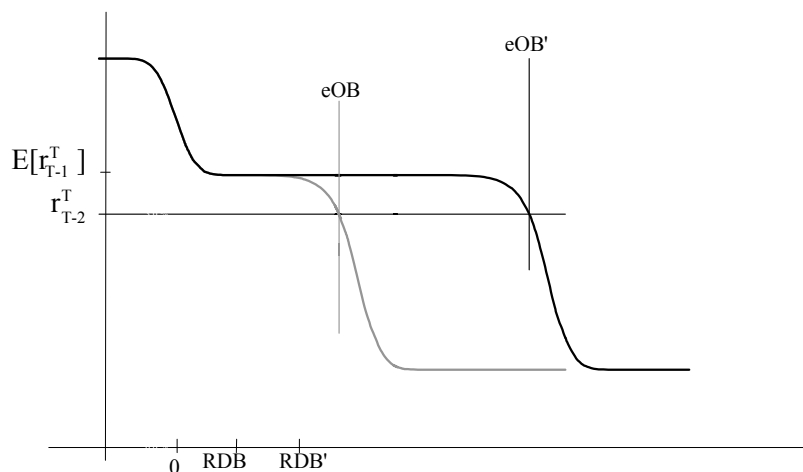
by choosing the width of the interest rate corridor and the position of the tender rate within the corridor.

3. The volatility of the overnight rate depends on interest rate expectations. If the banks have neutral expectations, the interest rate elasticity of the demand for liquidity is expected to be very large, ie the stochastic liquidity shocks are not expected to swing the overnight rate far from the tender rate. However, the demand will become less elastic as the equilibrium liquidity changess (due to expected changes in central bank rates) towards zero or fulfillment of the whole requirement.

4. The value of the overnight rate at the expected overnight liquidity might be geared towards the expected new tender rate, as some of the liquidity uncertainty vanishes between the tender operation and the clearance of the overnight market. The size of this effect depends on the amount of uncertainty resolved before the clearance (ie on the magnitude of the difference between demand for reserves at the tender and at the interbank market

84

clearance). The earlier the market clears, the smaller this effect will be. Thus, if the interbank market is active throughout the day, the volatility of the overnight rate is expected to be smaller. And this effect will be more evident if a rate cut is expected.

5. Variations in the demand for liquidity and in the volatility of the interbank overnight rate of interest are largely the result of changes in the demand for reserves due to expected movements in the central bank rates. This volatility could be avoided by timing the changes in official interest rates. If the central bank chose to adjust its rates only at the first tender operation of each maintenance period, the speculative demand for reserves would vanish, and the overnight rate would be very stable around the tender rate (at least before the last day of the maintenance period).

Some qualifications on the model
Expectations of changes in central bank rates will produce pronounced variations in equilibrium liquidity in the model presented above. The effects of rate changes are likely to be much more moderate if we introduce market imperfection into the model. For example, if the banks faced collateral requirements for central bank lending and line limits in interbank dealing, the banks' incentive to deviate from a path of steady reserve holdings could be diminished substantially. A similar effect would obtain if the banks were risk averse in the sense of not being interested solely in maximising expected profits and the volatility of profits were also included in the utility function. This kind of risk aversion could reduce banks' willingness to speculate on (uncertain) future rate changes by front- or backloading reserves.

4.2   Proportional allotment

Besides the full allotment procedure, there are several alternative rules that the central bank can use for liquidity allotment in fixed rate tenders. Here, we will concentrate on two simple policy rules, to keep the analysis manageable. According to the first rule, central bank tries to minimise variations in money market liquidity (*liquidity targeting*). In this approach the central bank could at t provide the

markets with liquidity that *either* brings the expected required daily balances for the remaining period up to the reserve requirement (ie with targeted liquidity eOB_t^{CB}, $RDB_{t+1} = RR$) *or* it could provide the markets simply with RDB_t (with targeted liquidity, $RDB_{t+1} = RDB_t$). That is, the amount of liquidity allotted by the central bank should minimise the variations in liquidity, either for the whole period or for the rest of the period. The difference between these two policies is very small. Here, we will assume that *in liquidity targeting the central bank aims at always providing the markets with liquidity so that $eOB_t^{CB} = RDB_t$.*[45]

The alternative policy rule studied here is that the central bank tries to provide the market with liquidity that will keep the (expected) overnight rate as close to a target value (set by the central bank itself) as possible (*interest rate targeting*). This target value (r_t^{targeted}) may or may not equal the tender rate.

The demand for overnight balances at the clearance of the market is a function of money market liquidity, the distribution of shocks and current and expected future central bank rates. It does not depend on the approach used in allotting liquidity in the tender operation. Thus, if we substitute the expected value of the future overnight rate for the expected value of the future tender rate in equations (3.5), (4.2) and (4.7), we get the equations for determining the overnight rate at T, $T-2$ and $T-j$ ($j = 2, ..., T-1$) respectively. The substitution is necessary, since the expected overnight rate at a given date is not necessarily equal to the expected tender rate if the proportional allotment procedure is used. We saw already in section 3 that proportional allotment reduces to full allotment if the demand for liquidity at the (fixed rate) tender does not exceed the amount the central bank is willing to provide to the markets. Thus, we are now mainly interested in cases where the expected value for the overnight rate equals or exceeds the tender rate. In such a case the banks will be increasing their bids from the optimal level under full allotment (ie they will be overbidding) in order to profit from the expected difference between the price of liquidity in the tender operation and that in the interbank market.

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[45] Thus, we assume that the central bank minimises liquidity variations for the rest of the period (by this procedure $eOB_t = eOB_{t+1} = ... = eOB_T$) and does not try to counter the effect of previous liquidity shocks in new operations.

4.2.1 Liquidity targeting

The expected value of the overnight rate of interest on the last day of the maintenance period is given by $\mathrm{E}_T\left[r_T^{on}\right] = r_T^m G\left(-eER_T\right) + r_T^d\left[1 - G\left(-eER_T\right)\right]$. With *liquidity targeting,* we know (by definition) that $eER_T = eOB_T^{CB} - RDB_T = 0$, as long as the central bank is able to allot liquidity according to its target. *To limit the number of cases we need to study, we assume henceforth that the shock distributions are symmetric* (unless otherwise mentioned). Hence, $G\left(0\right) = 0.5$ and the expected value for the last day's overnight rate will equal the mid-point of the interest rate corridor $\left(\frac{r_T^m + r_T^d}{2}\right)$, if the banks place enough bids in the tender.

In section 3.2.2 we saw that the banks overbid[46], if the expected value of the last day's overnight rate is not below the tender rate. Thus, the central bank will receive enough bids and consequently is able to control the daily supply of liquidity if the last day's tender rate is not in the upper part of the corridor $\left(r_T^T \leq \frac{r_T^m + r_T^d}{2}\right)$. If the rate is in the upper part $\left(r_T^T > \frac{r_T^m + r_T^d}{2}\right)$, the banks could make positive profits by lowering their bids below the liquidity targeted by the central bank. In this case, the central bank would not receive enough bids relative to its target, and the equilibrium would be determined as in the case of full allotment. Therefore, the expected value of the last day's overnight rate is the higher of the tender rate or mid-point of the corridor:

$$\mathrm{E}_T\left[r_T^{on}\right] = \max\left(\frac{r_T^m + r_T^d}{2}, r^T\right). \qquad (4.12)$$

Henceforth, we assume that the central bank uses a symmetric interest rate corridor $\left(r_t^T = \frac{r_t^m + r_t^d}{2}\right)$ while following a liquidity targeting policy. Hence, the expected overnight rate on the last day of the maintenance period will naturally equal the tender rate.[47] The reason for assuming a symmetric corridor with liquidity targeting is based on the following facts. First, the central bank would not be able to meet its target if the

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[46]That is, the actual bid amount is greater than the optimal bid under full allotment. The optimal bid under full allotment is referred to also as the real liquidity demand of the bank.

[47]If the shock distribution were not symmetric, the interest rate corridor would also be asymmetric, for $r_T^m G\left(0\right) + r_T^d\left[1 - G\left(0\right)\right] = r^T$ to hold.

tender rate were in the upper part of the corridor (as we saw above). Secondly, if the tender rate were in the lower part of the corridor, i) the stance of current monetary policy would be determined by the mid-point of the corridor instead of the fixed tender rate, and ii) as we will later see, this kind of situation would lead to infinite bidding by the banks and to windfall gains to successful bidders.

Penultimate day (T-1)

Proposition 13 *When the central bank applies liquidity targeting, the overnight rate on the penultimate day will equal the probability-weighted average of the central bank rates (expected tender rate and current rates of the standing facilities).*

With liquidity targeting, the expected amount of overnight balances of the banks at T-1 is RDB_{T-1}, as long as the central bank has control over daily money market liquidity (ie as long as the demand for liquidity in the tender exceeds the amount of reserves the central bank is willing to provide to the markets). Otherwise, the expected overnight balances will equal the optimal balances under the full allotment procedure (eOB^*_{T-1}). Let us define $z_t = \min(eOB^*_t, RDB_t)$, where z_t is the expected overnight balances at clearance of the market under proportional allotment with liquidity targeting. The following will hold for the overnight rate at the penultimate day of the maintenance period:[48]

$$
\begin{aligned}
r^{on}_{T-1} = \mathrm{E}_{T-1}\left[r^T_T\right] &\left[F(2RDB_{T-1} - z_{T-1} + \mu_{T-1})\right. \quad (4.13) \\
&\left. - F(-z_{T-1} + \mu_{T-1})\right] + r^m_{T-1}F(-z_{T-1} + \mu_{T-1}) \\
&+ r^d_{T-1}\left[1 - F(2RDB_{T-1} - z_{T-1} + \mu_{T-1})\right].
\end{aligned}
$$

Note that we can use $\mathrm{E}_{T-1}\left[r^T_T\right]$ as the price of borrowing tomorrow, as $\mathrm{E}_{T-1}\left[r^{on}_T\right] = \mathrm{E}_{T-1}\left[r^T_T\right]$, with either liquidity targeting or full allotment.

Proposition 14 *With liquidity targeting, the expected value of the overnight rate on the penultimate day equals the current tender rate if the banks have neutral or decreasing interest rate expectations, and is very close to the expected tender rate for the last operation if a rate hike is expected.*

---

[48] For the determination of equation (4.13), see from appendix B how equation (4.2) is derived from the profit maximization problem of a single bank.

The expected value of the overnight rate at $T - 1$ is now given by:

$$
\begin{aligned}
E_{T-1}\left[r_{T-1}^{on}\right] = {} & E_{T-1}\left[r_T^T\right] + \left(r_{T-1}^m - E_{T-1}\left[r_T^T\right]\right) G(-z_{T-1}) \\
& + \left(r_{T-1}^d - E_{T-1}\left[r_T^T\right]\right) \left[1 - G(2RDB_{T-1} - z_{T-1})\right]. \quad (4.14)
\end{aligned}
$$

From section 4.1 we know that the relation between eOB_{T-1}^* and RDB_{T-1} depends on the interest rate expectations of the banks and the symmetry of the corridor around the tender rate. Here, we have assumed the interest rate corridor to be symmetric. Thus, according to subsection 4.1, we would expect that: i) $eOB_{T-1}^* > RDB_{T-1}$ if the overnight rate is expected to increase during the last two days of the maintenance period, ii) $eOB_{T-1}^* = RDB_{T-1}$ if the rate is expected to remain constant, or iii) $eOB_{T-1}^* < RDB_{T-1}$ if the banks expect the overnight rate to decrease.[49] Therefore, we expect the central bank to be in a position to allot the targeted amount as long as the banks do not expect the overnight rate to decrease between T-1 and T.

Let us assume for a moment that the central bank does have control over the daily liquidity supply (ie $eOB_{T-1}^* \geq RDB_{T-1} \Rightarrow z_{T-1} = RDB_{T-1}$). The expected overnight rate at T-1 will be given by:

$$
\begin{aligned}
E_{T-1}\left[r_{T-1}^{on}\right] = {} & E_{T-1}\left[r_T^T\right] + \left(r_{T-1}^m - E_{T-1}\left[r_T^T\right]\right) G(-RDB_{T-1}) \\
& + \left(r_{T-1}^d - E_{T-1}\left[r_T^T\right]\right) \left[1 - G(RDB_{T-1})\right], \quad (4.15)
\end{aligned}
$$

which can also be written as the difference between the expected overnight rate for today and that of the following banking day:

$$
E_{T-1}\left[r_{T-1}^{on}\right] - E_{T-1}\left[r_T^T\right] = \left(\frac{r_{T-1}^m + r_{T-1}^d}{2} - E_{T-1}\left[r_T^T\right]\right) 2G(-RDB_{T-1}).
$$
$$
(4.16)
$$

The expected value of the overnight rate at T-1 will equal the tender rate expected to be used in the last tender (which we have assumed to be in the middle of the interest rate corridor on the last day) only if the banks have neutral expectations as to central bank rates (in that case $r_{T-1}^T = \frac{r_{T-1}^m + r_{T-1}^d}{2} = \frac{r_T^m + r_T^d}{2} = E_{T-1}\left[r_T^T\right]$). This results from the fact that, if the banks are expecting the central bank to increase its

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[49]If the interest rate corridor is not symmetric, with neutral interest rate expectations, we would expect that $eOB_{T-1}^* > RDB_{T-1}$ if the tender rate is in the upper part of the corridor, or $eOB_{T-1}^* < RDB_{T-1}$ if it is in the lower part.

rates, the term $\frac{r^m_{T-1}+r^d_{T-1}}{2}-\mathrm{E}_{T-1}\left[r^T_T\right]$ will become negative (RHS<0), and consequently the expected overnight rate must be lower than the expected tender rate for the last operation.[50] Similarly, if the central bank is expected to cut its rates, the overnight rate is expected to decrease during the last two days of the period. This means that our assumption of the central bank being in control of the daily liquidity would be correct and that the overnight rate would indeed be given by equation (4.15), if the central bank rates are expected to remain constant or to be raised.[51] However, if a rate cut is expected, the liquidity allotment is not determined by the central bank's target. In such a case, the equilibrium liquidity and the expected overnight rate would be (similarly to the case with full allotment) $z_{T-1}=eOB^*_{T-1}$ and $\mathrm{E}_{T-1}\left[r^{on}_{T-1}\right]=\mathrm{E}_{T-1}\left[r^T_{T-1}\right]$.[52]

If we modify equation (4.16) slightly, we see that when the central bank can control the liquidity, the expected overnight rate will be between today's tender rate and that expected for tomorrow $(r^T_{T-1}\leq\mathrm{E}_{T-1}\left[r^{on}_{T-1}\right]<r^T_T)$.[53] We also see that this rate approaches asymptotically the expected tender rate for T, as RDB_{T-1} increases

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[50]The expected value of the overnight rate would equal today's tender rate if $RDB_{T-1}=0$. However, it is extremely unlikely that the liquidity shocks could bring RDB_{T-1} down to zero if the central bank uses liquidity targeting.

[51]We have just shown that $\mathrm{E}_{T-1}\left[r^{on}_{T-1}\right]<\mathrm{E}_{T-1}\left[r^{on}_T\right]$ if the banks expect the central bank either to keep its rates constant or to increase them. With reasoning similar to section 3.2.2, we see that in this case the banks will be overbidding $(eOB^*_{T-1}<eOB^{actual}_{T-1})$. This will reinforce the fact that the central bank has control over the expected liquidity if expectations are either neutral or increasing.

[52]If the interest rate corridor were asymmetric, the central bank would be able to control the supply of daily liquidity as long as i) the probability-weighted average of the mid-point of the corridor is not less than the current tender rate $(\mathrm{E}\left[r^{mid}_T\right]\left[1-2G\left(-RDB_{T-1}\right)\right]+r^{mid}_{T-1}2G\left(-RDB_{T-1}\right)\geq r^T_{T-1})$ when the tender rate is kept in the lower part of the corridor, or ii) the probability-weighted average of the expected future tender rate and the current mid-point is not less than the current tender rate $(\mathrm{E}\left[r^T_T\right]\left[1-2G\left(-RDB_{T-1}\right)\right]+r^{mid}_{T-1}2G\left(-RDB_{T-1}\right)\geq r^T_{T-1})$ when the tender rate is kept in the lower part of the corridor.

[53]By using the fact that, with symmetric interest rate corridor, $r^T_{T-1}=\frac{r^m_{T-1}+r^d_{T-1}}{2}$, we can write equation (4.16) as:

$$\mathrm{E}_{T-1}\left[r^{on}_{T-1}\right]-r^T_{T-1}=\left(\mathrm{E}_{T-1}\left[r^T_T\right]-r^T_{T-1}\right)\left[1-2G(-RDB_{T-1})\right]. \qquad (4.17)$$

For expectations of increased interest rate, the RHS of equation (4.17) must be non-negative, as $2G\left(-RDB_{T-1}\right)\leq1$ (assuming symmetric shock distribution, we have $G\left(0\right)=0.5$ and $RDB_{T-1}\geq0$).

Figure 13: **Determination of overnight rate at** *T-1*: **proportional allotment with liquidity targeting and increasing interest rate expectations**
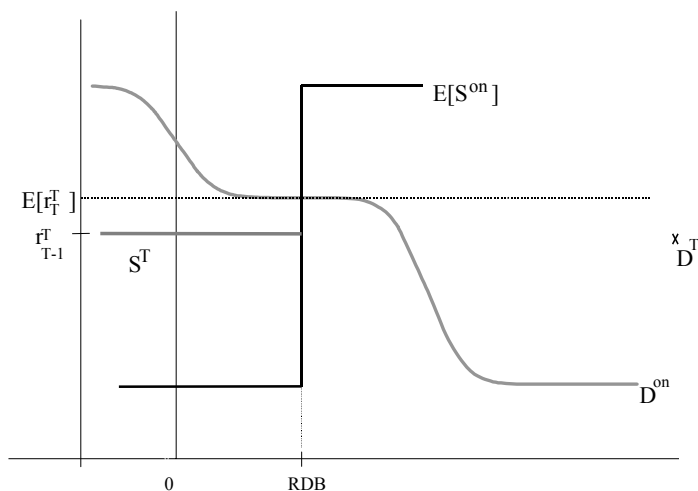
(ie as the probability of overdrawing diminishes). We know that, under liquidity targeting, the expected value for RDB_{T-1} is the average reserve requirement (RR). If the aim of averaging the reserve requirement is to increase the interest rate elasticity of the demand for reserves, we might expect that the central bank sets RR well above the average size of a stochastic liquidity shock. Thus, we expect most of the interest rate expectations to be absorbed by the overnight rate already at $T-1$, as stated in proposition 14. For example, if the standard deviation of *normally distributed* liquidity shocks is 25% of the average liquidity (ie the reserve requirement), the expected value of the overnight rate at $T-1$ will absorb more than 99% of the expected change in the tender rate.

Figure 13 illustrates the determination of the overnight rate when the central bank uses a proportional allotment procedure with liquidity targeting and the banks expect a rate rise. The figure shows that the expected overnight rate at $T-1$ will be very close to the expected tender rate for tomorrow. We also see that the money market equilibrium is expected to be found from the highly elastic

part of the demand curve. Thus, the variations in the overnight rate reflect changes in interest rate expectations rather than the effect of stochastic liquidity shocks.

Earlier days (1, ... ,T-3, T-2)

In the case of full allotment, there was a considerable difference in the determination of money market equilibrium as between the penultimate day of the maintenance period and the days before that. This difference occurs because, in the early part of the period, the banks must take into account the effect of their liquidity holdings on the required daily balances for the remaining period on the following days whereas, on the penultimate day, this dynamic cost factor is absent. Also here the *dcf* affects the demand for reserves on the earlier days. However, it will affect the amount of liquidity provided to the markets only if the central bank is not able to control the (daily) amount of reserves to be allotted to the markets. As on the penultimate day of the maintenance period, the banks will be overbidding ($eOB_t^{actual} > eOB_t^*$) at t, if the expected overnight rate is not lower with the required minimum daily balances than with the equilibrium liquidity under full allotment (ie overbidding occurs if $E[r_t^{on}|_{eOB_t=RDB_t}] \geq E[r_t^{on}|_{eOB_t=eOB_t^*}] = r_t^T$). Thus, control over the daily liquidity supply is in the hands of the central bank, as long as the demand for reserves in a tender under full allotment would be at least equal to the RDB_t (as the overnight rate is a monotonically increasing function of liquidity). If control over the supply of reserves is in the hands of the central bank, the overnight rate at t (obtained once again as the first-order condition of the banks' profit maximizing problem) will be:[54]

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[54]For the derivation of equation (4.18) see from appendix C how equation (4.6) was derived as first-order condition of the profit maximisation problem of a bank. Note that here we have to substitute eOB_j^* for $eRDB_j$, as the equilibrium liquidity is determined by the central bank target instead of the optimal demand of the banks. Thus, we also replace $\frac{\partial eOB_j^*}{\partial b_t}$ by $\frac{\partial eRDB_j}{\partial b_t}$.

$$r_t^{on} = \mathrm{E}_t\left[r_f^{on}\right] + \left(r_t^m - \mathrm{E}_t\left[r_f^{on}\right]\right) F(-OB_t) \qquad (4.18)$$
$$+ \left(r_t^d - \mathrm{E}_t\left[r_f^{on}\right]\right)\{1 - F\left[(T - t + 1)RDB_t - OB_t\right]\}$$
$$+ \sum_{j=t+1}^{T-1} \left(\mathrm{E}\left[r_j^m - r_f^{on}\right] G(-eRDB_j) + \mathrm{E}\left[r_j^d - r_f^{on}\right](-T + j)\right.$$
$$\times \left.\{1 - G\left[(T - j)eRDB_j\right]\}\right) \frac{\partial eRDB_j}{\partial b_t},$$

where $\mathrm{E}_t\left[r_f^{on}\right]$ is the expected overnight rate for the remaining days within the same period.

Proposition 15 *With liquidity targeting, the expected value of the overnight rate and the equilibrium liquidity (at t) will depend on the central bank's ability to control the liquidity, which itself depends on interest rate expectations and the level of the reserve requirement. When a rate cut is expected, the control of liquidity will be in the hands of the banks, and this approach will be identical to full allotment. When the central bank is expected to increase the tender rate, it can control the liquidity, and the expected overnight rate will react to expectations by rising immediately close to the expected new level of the tender rate.*

The expected value of the overnight rate will be given by (note that $eOB_t = RDB_t$):

$$\mathrm{E}_t\left[r_t^{on}\right] = \max\left[r_t^T, \mathrm{E}_t\left[r_f^{on}\right] + \left(r_t^m - \mathrm{E}_t\left[r_f^{on}\right]\right) G(-RDB_t)\right.$$
$$+ \left(r_t^d - \mathrm{E}_t\left[r_f^{on}\right]\right)\left[1 - G((T - t)RDB_t)\right] \qquad (4.19)$$
$$+ \sum_{j=t+1}^{T-1} \left(\mathrm{E}\left[r_j^m - r_f^{on}\right] G(-eRDB_j) + \mathrm{E}\left[r_j^d - r_f^{on}\right]\right.$$
$$\left.\times(-T + j)\{1 - G\left[(T - j)eRDB_j\right]\}\right) \frac{\partial eRDB_j}{\partial b_t}\right].$$

That is, the expected value of the overnight rate is given by taking the expected value of equation (4.18), as long as this produces a rate that is not lower than the tender rate. Otherwise the expected overnight rate will equal the tender rate, as in the case of full allotment. Equation (4.18) shows that, if the central bank can allot

liquidity according to its target, the expected overnight rate at t will equal the sum of the expected future overnight rate, current marginal lending rate and the deposit rate, each weighted by its probability of occurrence,[55] plus the dynamic cost factor. If the central bank has control over the expected liquidity, the liquidity targeting rule indicates that $eRDB_{t+i} = eRDB_{t+2} = ... = eRDB_{T-1}$. Thus, $\frac{\partial eRDB_{t+1}}{\partial b_t} = -\frac{1}{T-t}\left\{G\left[(T-t+1)RDB_t - OB_t\right] - G(-OB_t)\right\}$, and equation (4.19) can be further modified as:

$$
\begin{aligned}
\mathrm{E}_t\left[r_t^{on}\right] \;=\; & \max\left[r_t^T, \mathrm{E}_t\left[r_f^{on}\right] + \left(r_t^m - \mathrm{E}_t\left[r_f^{on}\right]\right)G(-RDB_t) \right. \quad (4.20)\\
& + \left(r_t^d - \mathrm{E}_t\left[r_f^{on}\right]\right)\left\{1 - G\left[(T-t)RDB_t\right]\right\} \\
& + \left\{G\left[(T-t+1)RDB_t - OB_t\right] - G(-OB_t)\right\}\left(-\frac{1}{T-t}\right) \\
& \times \sum_{j=t+1}^{T-1}\left(\mathrm{E}\left[r_j^m - r_f^{on}\right]G(-RDB_{t+1}) + \mathrm{E}\left[r_j^d - r_f^{on}\right]\right. \\
& \left.\left. \times\,(-T+j)\left\{1 - G\left[(T-j)RDB_{t+1}\right]\right\}\right)\right].
\end{aligned}
$$

For example, at $T-2$ the expected overnight rate will be determined as:

$$
\begin{aligned}
\mathrm{E}\left[r_{T-2}^{on}\right] = & \max\left\{r_t^T, \mathrm{E}_{T-2}\left[r_{T-1}^{on}\right] + \left(r_{T-2}^m - \mathrm{E}_{T-2}\left[r_{T-1}^{on}\right]\right)\right. \\
& \times G(-RDB_{T-2}) + \left(r_{T-2}^d - \mathrm{E}_{T-2}\left[r_{T-1}^{on}\right]\right)G(-2RDB_{T-2}) \\
& -0.5\mathrm{E}\left[r_{T-1}^m - r_{T-1}^d\right]G(-eRDB_{T-1}) \quad (4.21) \\
& \left. \times\left[G\left(2RDB_{T-2}\right) - G\left(-RDB_{T-2}\right)\right]\right\},
\end{aligned}
$$

which defines $\mathrm{E}\left[r_{T-2}^{on}\right]$ to be the higher of the current tender rate and the probability-weighted average of the rates of the standing facilities and expected future overnight rate, less the dynamic cost factor. In this case the *dcf* is half the width of the interest rate corridor, weighted by the probability of being overdrawn tomorrow if it was not necessary to rely on the standing facilities today.

The key motive for the averaging provision is probably the effect it has on the interest rate elasticity of the demand for reserves. Thus, we may assume that the requirement will be

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[55] That is, the lowest expected overnight rate weighted by the probability of not using the standing facilities, marginal lending rate weighted by the probability of being overdrawn, and the deposit rate weighted by the probability of fulfiling the whole reserve requirement.

large compared with the standard deviation of liquidity. If so, the probability of having to rely on either of the two standing facilities will be relatively low and liquidity will match the required daily balances for the remaining period. Also, the effect of the *dcf* will be minimal in this case. Therefore, in case of increasing interest rate expectations, the expected value of the overnight rate for today (at the liquidity targeted by the central bank) would be very close to the value of the overnight rate expected to prevail in the remaining days of the maintenance period $(\mathrm{E}[r_{T-2}^{on}] \simeq \mathrm{E}[r_{T-1}^{T}] \Rightarrow \mathrm{E}[r_{T-2}^{on}] \simeq \mathrm{E}[r_{T-1}^{on}] \simeq \mathrm{E}[r_{T-1}^{T}]$ etc). Consequently, the expected value of today's overnight rate would exceed the current tender rate, and the central bank would indeed be able to control the supply of overnight liquidity. Similarly, if the current tender rate were higher than the expected mid-point on the last day of the period (ie a rate cut is expected), the expected overnight rate, with liquidity at the level targeted by the central bank, would fall below the current tender rate. Hence, the demand for reserves in the tender operation would not be high enough for the central bank to be able to allot liquidity according to its target, and again determination of the money market equilibrium would follow the case of full allotment.

Note that, when the banks have neutral interest rate expectations, the optimal amount of liquidity the banks will bid for under the full allotment procedure is an increasing function of the liquidity volatility, as seen in section 4.1.2. If the volatility of liquidity is high compared to the equilibrium liquidity, the central bank will get enough bids to control the daily supply of reserves, and the expected overnight rate could increase to slightly above the tender rate. However, if the central bank has set the reserve requirement high relative to the volatility, we might expect the banks to be willing to backload their reserve holdings, and consequently the equilibrium would be determined as in the case of full allotment. Therefore, in choosing the size of the reserve requirement, the central bank must take into account that a higher requirement will increase the interest rate elasticity of the demand for reserves but might also reduce the central bank's ability to control the daily supply of liquidity. *Thus, we expect that there is an upper limit for the reserve requirement the central bank can apply with liquidity targeting. The central bank might like to increase the interest rate elasticity of the demand for liquidity by*

increasing its reserve requirement, but only up to the point where it will still be in control of the daily supply of liquidity under neutral interest rate expectations.

We have seen that the expected value of the overnight rate will be close to the expected mid-point of the interest rate corridor throughout the maintenance period, if the central bank can control the expected supply of money market liquidity. The central bank has control over the expected supply of money market liquidity if interest rates are expected to be raised or with neutral interest rate expectations and high enough uncertainty about liquidity. If a rate cut is expected, the expected overnight rate will equal the current tender rate. Furthermore, as long as a rate cut is not anticipated by the banks, the equilibrium overnight rate is expected to be realised near the required daily balances for the remaining period, where the demand for reserves has a relatively high interest rate elasticity. Consequently, variations in the overnight rate will largely reflect near-term changes in expectations of the central bank rates. However, if a rate cut is expected, the banks will be backloading their reserve holdings and the equilibrium overnight rate will be found on the less elastic part of the demand curve (as in the case of full allotment). Thus, with decreasing interest rate expectations, the volatility of the overnight rate will reflect the stochastic variations in money market liquidity.

4.2.2   Interest rate targeting

The determination of money market equilibrium under proportional allotment with interest rate targeting has many features in common not only with liquidity targeting but also with full allotment. The main difference between interest rate targeting and liquidity targeting is that, under interest rate targeting, the amount of liquidity the central bank aims to provide to the market (eOB_t^{CB}) is implicitly derived from the central bank's interest rate target. That is, the central bank is willing to provide the market with reserve balances that will bring the expected value of the overnight rate to the level of the target $\left(E\left[r_t^{on}|_{eOB_t=eOB_t^{CB}}\right] = r_t^{\text{targeted}} \right)$.

96

Proposition 16 *If the target of the central bank is below the tender rate, interest rate targeting will result in full allotment.*

The central bank will not have control over the daily supply of liquidity if the expected value of the target is lower than the expected overnight rate under full allotment (ie if $\mathrm{E}\left[r_t^{\text{targeted}}\right] < r_t^T$). If this is the case, the banks could make positive expected profits by lowering their bids from eOB_t^{CB} to eOB_t^*.[56] Thus, the expected money market liquidity will be at the level chosen by the central bank (eOB_t^{CB}) only if $r_t^{\text{targeted}} = \mathrm{E}\left[r_t^{on}|_{eOB_t=eOB_t^{CB}}\right] \geq \mathrm{E}\left[r_t^{on}|_{eOB_t=eOB_t^*}\right] = r_t^T$.[57] Otherwise, the expected equilibrium liquidity under interest rate targeting will equal the equilibrium liquidity under full allotment (eOB_t^*).

To distinguish the properties typical of proportional allotment with interest rate targeting, we henceforth focus on cases where the target of the central bank is set (and is expected to be set) at least at the level of the tender rate, ie $r_t^{\text{targeted}} \geq r_t^T$.

Proposition 17 *When the central bank applies interest rate targeting and the target is not below the tender rate, the banks will bid for more reserves than under full allotment. That is, the central bank can control the expected liquidity, and the expected overnight rate will equal the central bank's target.*

Now, in this case we know that $eOB_t^{CB} \leq eOB_t^*$. If the target rate equals the tender rate and banks bid according to the neutral strategy, money market liquidity at clearance of the market will be the minimum of the values of two variables with the same

---

[56]The gain from a lower bid would be the price of liquidity at the tender (r_t^T), and the expected cost would be the price of liquidity at the interbank market ($\mathrm{E}\left[r_t^{\text{targeted}}\right]$). Thus, bidding for more than eOB_t^* would create expected losses for the banks, and therefore, if the target rate of the central bank is below the tender rate, the equilibrium for the representative bank is to bid according to its neutral strategy, as in the case with the full allotment procedure.

[57]$r_t^{\text{targeted}} \geq r_t^T \Leftrightarrow \mathrm{E}\left[r_t^{on}|_{eOB_t=eOB_t^{CB}}\right] \geq \mathrm{E}\left[r_t^{on}|_{eOB_t=eOB_t^*}\right] \Rightarrow OB_t^{CB} \leq eOB_t^*$, as r_t^{on} is monotonically decreasing with eOB_t. Based on our earlier argument we know that, if the liquidity the central bank wants to allot to the market is not less than the liquidity bid for under full allotment, the banks will be strategically overbidding (ie $OB_t^{CB} \leq eOB_t^* \Rightarrow TL^s < TL^* < TL^{actual}$).

mean, ie $\min(OB_t^{CB}, OB_t^*)$[58]. Thus, the overnight rate at t will be $r_t^{on}\left(\min\left(OB_t^{CB}, OB_t^*\right)\right)$, which will be higher than $r_t^{on}\left(eOB_t^{CB}\right) = r_t^{on}\left(eOB_t^*\right) = r_t^T$. That is, the expected value of the overnight rate will be higher than the tender rate if banks bid according to the neutral strategy. Therefore, the banks have an incentive to bid for more liquidity than they would demand under full allotment. In fact the equilibrium bidding strategy would be such that the bid amount would exceed the maximum value the central bank, with neutral strategy, can be expected to provide to the markets $(TL^{\text{bid amount}} \geq TL^{CB,\max})$.[59] With this kind of overbidding, the central bank will always get bids for more reserves than it is willing to provide to the markets, and consequently the expected value for the overnight rate at t will equal the expected value of the central bank's target.

On the last day of the maintenance period, the overnight rate is again given by $r_T^{on} = r_T^m F(-ER_T) + r_T^d [1 - F(-ER_T)]$. Thus, the amount of liquidity to be allotted by the central bank at T will be implicitly given by:

$$
\begin{aligned}
\mathrm{E}\quad & [r_T^{on}] = r_T^m H(-eOB_T^{CB} + RDB_T) \\
+\quad & r_T^d\left[1 - H(-eOB_T^{CB} + RDB_T)\right] = r_T^{\text{targeted}},
\end{aligned}
$$

where $H = \mathrm{E}_{f_\mu}^{CB}\left[F(-eER_T^{CB})\right]$.[60] If the inverse of the expected cumulative distribution function $H^{-1}(\cdot)$ exists, the money market liquidity aimed at by the central bank in making its decision on the allotment will be given by:

$$
eOB_T^{CB} = RDB_T - H^{-1}\left(\frac{r_T^{\text{targeted}} - r_T^d}{r_T^m - r_T^d}\right).
$$

---

[58]Even though eOB_t^{CB} and eOB_t^* on average have the same value they can differ from each other on a daily basis, as the expectation over the development in autonomous liquidity factors may differ as between the central bank and the banks (a_t^{CB} may be different from a_t).

[59]For the 76 ECB main refinancing operations that were conducted as fixed rate tenders, the allotment ratios were less than 10% of the total bid amount in 62 operations and below 5% in 29 operations. The ratios for tenders conducted in 2000 reflect even more dramatic overbidding; the allotment ratio was below 10% in every tender and below 5% in all but three of 24 tenders.

[60]In $\mathrm{E}_{f_\mu}^{CB}[\cdot]$ the central bank's expectations are taken over the shock distribution f_μ.

The difference between the target and realised overnight rates results mainly from the stochastic liquidity shock emerging between the tender operation and clearance of the market. However, part of the difference might stem from the central bank's inability to estimate the real cumulative distribution of shocks (ie $H(\cdot) \neq F(\cdot)$). Also, the symmetry of the corridor around the central bank's target affects the distribution of overnight rate realisations (ie $H(\cdot)$, like $G(\cdot)$, has a wider distribution than $F(\cdot)$).

Penultimate day (T-1)

For the penultimate day of the maintenance period, the overnight rate expected to prevail on the last day again enters the equation determining the current rate. Thus, the central bank's allotment decision, eOB_{T-1}^{CB}, is implicitly given by:

$$
\begin{aligned}
r_{T-1}^{\text{targeted}} \;=\;\; & \mathrm{E}^{CB}\left[\mathrm{E}_{T-1}^{banks}\left[r_T^{\text{targeted}}\right]\right]\left[H(2RDB_{T-1} - eOB_{T-1}^{CB})\right.\\
& \left. - H(-eOB_{T-1}^{CB})\right] + \left(r_{T-1}^m\right)H(-eOB_{T-1}^{CB})\\
& + r_{T-1}^d\left[1 - H(2RDB_{T-1} - eOB_{T-1}^{CB})\right],
\end{aligned} \qquad (4.22)
$$

where $\mathrm{E}^{CB}\left[\mathrm{E}^{banks}\left[r_T^{\text{targeted}}\right]\right]$ is the central bank's expectation for the last day's target rate anticipated by the banks at T-1. Equation (4.22) says, that the central bank should provide the markets with liquidity such that the probability-weighted sum of the target rate the central bank anticipates the banks will expect for the last day and the rates of the standing facilities will equal today's target rate. The effect of an expected change in the interest rate target on the amount of liquidity to be allotted can be clearly seen by modifying equation (4.22) slightly:

$$
\begin{aligned}
& r_{T-1}^{\text{targeted}} - \mathrm{E}^{CB}\left[\mathrm{E}^{banks}\left[r_T^{\text{targeted}}\right]\right] = \qquad\qquad\qquad (4.23)\\
& \left\{\left(r_{T-1}^m - \mathrm{E}^{CB}\left[\mathrm{E}^{banks}\left[r_T^{\text{targeted}}\right]\right]\right)H(-eOB_{T-1}^{CB})\right\}\\
& + \left(r_{T-1}^d - \mathrm{E}^{CB}\left[\mathrm{E}^{banks}\left[r_T^{\text{targeted}}\right]\right]\right)\left[1 - H(2RDB_{T-1} - eOB_{T-1}^{CB})\right].
\end{aligned}
$$

From (4.23) we see that the central bank allots liquidity in order to balance the expected change in the target rate with the probability-weighted cost of using the standing facilities. The planned allotment

is a decreasing function of the expected target (as long as RDB_{T-1} is strictly positive, ie the probability of using the standing facilities is less than one[61]) and a decreasing function of the location of the target rate within the interest rate corridor (ie the lower the target rate within the corridor, the higher the relative cost of overdrawing, which is to be compensated by the lower probability of overdrawing associated with greater liquidity).

With neutral interest rate expectations and a symmetric interest rate corridor, equation (4.23) says that the liquidity provided by the central bank will equal the required daily balances for the remaining period $(eOB_{T-1}^{CB} = RDB_{T-1})$.[62] If the target rate is in the upper part of the corridor (ie if the cost of acquiring liquidity credit is lower than the opportunity cost of having to use the deposit facility), the liquidity provided by the central bank should leave the markets with a higher probability of overdrawing than of exceeding the reserve requirement (ie $eOB_{T-1}^{CB} > RDB_{T-1} \Rightarrow H(-eOB_{T-1}^{CB}) < 1 - H(2RDB_{T-1} - eOB_{T-1}^{CB})$). Similarly, if the target rate is in the lower part of the corridor, the markets should be provided with liquidity that will make overdrawing less probable than exceeding the requirement $(eOB_{T-1}^{CB} < RDB_{T-1})$.

If the banks were anticipated to expect the central bank to increase the target rate, the central bank would aim at providing enough liquidity to equate the negative of the expected rate increase with the probability-weighted difference between the rates of the standing facilities and the expected target rate, ie the central bank should provide the market with extra liquidity in order to make the RHS negative enough. Similarly, under decreasing anticipated expectations the central bank would offer the banks so little liquidity as to make the probability of overdrawing higher than the probability of exceeding the reserve requirement.

---

[61]If the probability of using the standing facilities were 1, we would have $RDB_{T-1} = 0 \Rightarrow H(-eOB_{T-1}^{CB}) = H(2RDB_{T-1} - eOB_{T-1}^{CB}) \Rightarrow r_{T-1}^{\text{targeted}} = r_{T-1}^m H(-eOB_{T-1}^{CB}) + r_{T-1}^d (1 - H(-eOB_{T-1}^{CB}))$, and the expected overnight rate for today would be independent of the expected target rate for tomorrow.

[62]Under neutral interest rate expectations $(r_{T-1}^{\text{targeted}} - E^{CB}\left[E_{T-1}^{banks}\left[r_T^{\text{targeted}}\right]\right] = 0)$ and a symmetric corridor $(r_{T-1}^m - E^{CB}\left[E^{banks}\left[r_T^{\text{targeted}}\right]\right] = -\left(r_{T-1}^d - E^{CB}\left[E^{banks}\left[r_T^{\text{targeted}}\right]\right]\right))$, equation (4.23) can be written as $H(-eOB_{T-1}^{CB}) = 1 - H(2RDB_{T-1} - eOB_{T-1}^{CB})$. Thus, $eOB_{T-1}^{CB} = RDB_{T-1}$.

Earlier days (1, ... ,T-3, T-2)

Proposition 18 *With interest rate targeting, the differences between the overnight rate and the central bank's target rate result from stochastic liquidity shocks and the central bank's inability to estimate the effect of interest rate expectations on the demand for liquidity.*

In the earlier days of the maintenance period, the overnight liquidity targeted by the central bank is implicitly given by:[63]

$$
\begin{aligned}
r_t^{\text{targeted}} - \mathrm{E}^{CB}\left[\mathrm{E}_t^{banks}\left[r_f^{\text{targeted}}\right]\right] &= \left(r_t^m - \mathrm{E}^{CB}\left[\mathrm{E}_t^{banks}\left[r_f^{\text{targeted}}\right]\right]\right) \\
&\times H(-eOB_t^{CB}) + \left(r_t^d - \mathrm{E}^{CB}\left[\mathrm{E}_t^{banks}\left[r_f^{\text{targeted}}\right]\right]\right) \\
&\left\{1 - H([T - t + 1]RDB_t - eOB_t^{CB}]\right\} \\
&\sum_{j=t+1}^{T-1}\left(\mathrm{E}^{CB}\left[\mathrm{E}_t^{banks}\left[r_j^m - r_f^{\text{targeted}}\right]\right]\right. \\
&\frac{\partial eOB_j^{CB}}{\partial eRDB_j}\frac{\partial eRDB_j}{\partial b_t}H(-eOB_j^{CB}) + \mathrm{E}^{CB}\left[\mathrm{E}_t^{banks}\left[r_j^d - r_f^{\text{targeted}}\right]\right] \\
&\left(-(T - j + 1)\frac{\partial eRDB_j}{\partial b_t} + \frac{\partial eOB_j^{CB}}{\partial b_t}\right) \\
&\left.\times\left\{1 - H\left[(T - j + 1)eRDB_j - eOB_j^{CB}\right]\right\}\right).
\end{aligned}
\tag{4.24}
$$

This equation determining the expected overnight rate at t under the proportional allotment procedure with interest rate targeting closely resembles the one determining the expected rate under full allotment, the more so if the central bank always equates the target rate to the tender rate. Thus, any difference between the amount of liquidity the central bank plans to provide to the markets and equilibrium liquidity under full allotment must stem from a difference between $\mathrm{E}^{CB}\left[\mathrm{E}_t^{banks}\left[r_f^{\text{targeted}}\right]\right]$ and $\mathrm{E}_t\left[r_f^T\right]$ or between $H(\cdot)$ and $G(\cdot)$. Consequently, the actual overnight rate under proportional allotment

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[63]For the derivation of equation (4.24) see appendix C. For how the overnight rate at t is determined from the bank's profit maximisation problem, see equation see (4.6). Recall from above that, with interest rate targeting, the central bank tries to equate the overnight rate to its target value $(\mathrm{E}\left[r_t^{on}|_{eOB_t=eOB_t^{CB}}\right] = r_t^{\text{targeted}})$.

with interest rate targeting will differ from the overnight target rate because of a stochastic liquidity shock or the central bank's inability to correctly estimate banks' expectations or the shock distributions. Also, the path of reserve holdings under this approach resembles that under full allotment. When an interest rate cut (hike) is expected, the central bank should considerably backload (frontload) reserve holdings to keep the expected overnight rate on target. In addition to interest rate expectations, the current and future probabilities of being forced to use the standing facilities will affect the planned allotment.

With rational expectations and symmetric information, the central bank should be able to work out the banks' expectations of central bank interest rates (ie $\mathrm{E}^{CB}\left[\mathrm{E}_t^{banks}\left[r_f^{\text{targeted}}\right]\right]$ reduces to $\mathrm{E}\left[r_f^{\text{targeted}}\right]$). In such a case, the only difference between equations (4.8) and (4.24) is in the cumulative distribution functions $G\left(\cdot\right)$ and $F\left(\cdot\right)$ (assuming $r^{\text{targeted}} = r^T$). Still, the central bank's task of determining the amount of liquidity to be allotted can be very troublesome when there are expectations of interest rate changes. The difficulties might arise from the fact that it could be very difficult to estimate the tail probabilities of $F\left(\cdot\right)$ (ie $H\left(-RDB\right)$ could be a much better estimator of $F\left(-RDB\right)$ than $H\left(-3RDB\right)$ is of $F\left(-3RDB\right)$). However, the central bank's estimate of the liquidity needed in the interbank market should not be a systematically biased estimator of the true liquidity needed, even at these high or low levels of reserves.

We show in figures 14 and 15 the effect of the central bank having biased estimates of banks' expectations of interest rate developments. To avoid insisting on the central bank being able to reproduce the expectations of private banks, one can consider these figures as examples of the effects of difficulty in estimating tail probabilities of the cumulative shock distribution. Figure 14 illustrates the determination of the expected supply of overnight liquidity when the central bank's estimate is lower than the true value of banks' expectations of the coming increase in the tender rate. We see that, because of the erroneous estimate of interest rate expectations, the estimated demand for liquidity will fall below the banks' true demand at the current tender rate. Consequently, the realised overnight rate is very likely to be above the tender rate. Similarly, if the central bank's estimate overstates the true expected increase in the target

102

Figure 14: **Effect of an underestimate (by the central bank) of a rate hike expected by the banks**
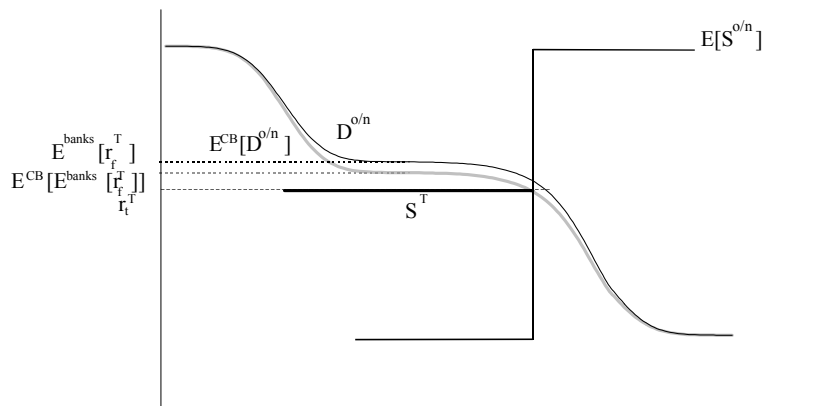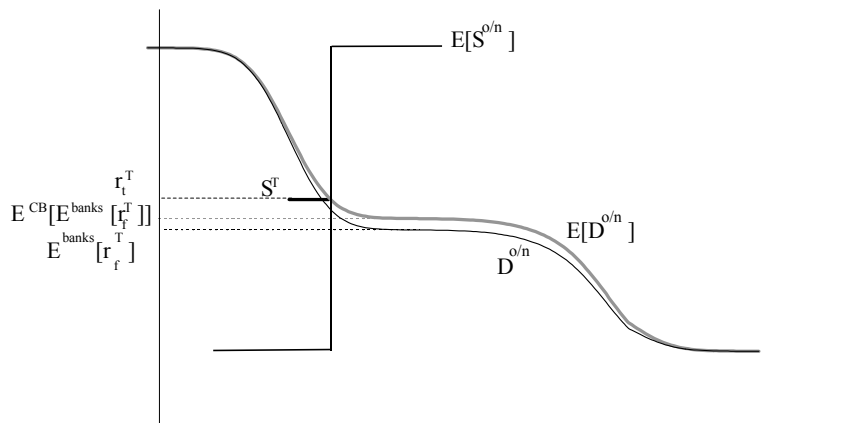

Figure 15: **Effect of an underestimate (by the central bank) of a rate cut expected by the banks**

rate, the expected supply would be too large relative to the demand for liquidity at the overnight market. Hence, in this case the realised overnight rate would most likely be below the tender rate. Note that the central bank is able to allot 'too much' liquidity here, as the banks are overbidding at equilibrium (ie bidding for more than their true demand).

Figure 15 illustrates the similar effects (of a biased estimate of interest rate expectations), when a rate cut is expected. In the figure, the central bank underestimates the rate cut expected by the banks. Thus, the supply of liquidity is too high relative to the actual demand. Because of this incorrect estimate, the overnight rate is expected to fall below the tender rate. Similarly, an underestimate would lead to a lack of overnight liquidity, and the overnight rate would most likely rise above the tender rate.

5   Summary and conclusions

In this essay we have built a model of the determination of equilibrium in the overnight money market when the central bank steers the market with an interest rate corridor and open market operations in the form of fixed rate tender operations. The demand for overnight reserves at a given price is shown to be a function of the expected future tender rate, current and expected rates of the standing facilities and the distribution of liquidity shocks. The supply of reserves is not exogenous in this model. It depends on both the liquidity policy on which the central bank bases its allotment decisions and the banks' aggregate demand for reserves in the tender operation (with the given policy). Three alternative liquidity policies are considered in the paper. First, in a full allotment procedure, the central bank provides the banks with all the reserves they bid for in the tender. Second, in the proportional allotment procedure, the central bank scales the bids back in proportion to the individual bids. As regards the proportional allotment procedure, we study two different policy rules; in liquidity targeting, the central bank aims at holding money market liquidity constant throughout the remainder of the maintenance period, whereas in interest rate targeting it tries

to provide the market with liquidity that would bring the overnight rate to the targeted level.

The determination of the money market equilibrium when the central bank does not allow reserve averaging was studied in section 3. We saw that with either full allotment or proportional allotment with interest rate targeting, the expected value for the overnight rate will equal the tender rate.[64] The volatility of the rate was seen to depend on the distribution of shocks. Thus, the relative volatility of the overnight rate as between these two approaches depends on the relative accuracy of estimates of the shock distributions made by the banking sector as a whole vs that made by the central bank.

In the case of full allotment, variations in the overnight rate entirely reflect the stochastic and temporary liquidity shocks (ie the banks' forecasting errors). Thus, these variations do not affect the expected values of future overnight rates and consequently are not transmitted along the yield curve to longer maturities. Therefore, the signals given by the central bank rates should be unambiguous. The same is true with the proportional allotment procedure, as long as the policy is known to the public. However, if the targeted policy must be read from the past behaviour of the central bank, variations in the realised overnight rate of interest could be (mis)interpreted as changes in the monetary policy stance. In such a case, one can not be certain that overnight volatility is not transmitted to longer maturities.

The symmetric interest rate corridor is the simplest one for the central bank to operate (at least as long as the shock distribution can be expected to be symmetric). However, if full allotment is used, the rates of the standing facilities can be used as independent policy rates. The asymmetry of the corridor would certainly affect the demand for excess reserves, but it would not affect the expected tender rate. With the proportional allotment procedure, independent signalling with the corridor is very complicated because, if the tender rate were in the upper part of the corridor, the central bank would not receive bids for as much liquidity as it wanted to provide to the market and

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[64]If averaging is not used, proportional allotment with interest rate targeting is similar to liquidity targeting.

consequently the determination of the overnight rate would be similar to the case of full allotment.[65]

The central bank can affect the volatility of the interbank overnight rate by choosing the width of the interest rate corridor. It was also shown that the stochastic overnight volatility depends on the timing of the interbank market. We saw that volatility is lower in a market that is already active in the morning, compared to one in which interbank trading takes place merely to deal with banks' foreseen liquidity needs.

The analysis of money market equilibrium under reserve averaging is contained in section 4. The demand for reserves is not similar on different days of the reserve maintenance period. On the last day of the period, the demand is similar to that for the case without averaging, ie the demand for bank reserves at a given overnight rate depends on current central bank rates and the distribution of liquidity shocks. However, already on the penultimate day, interest rate expectations enter the demand function. The higher the expected tender rate for the following day, the more reserves the banks will demand today, at a given overnight rate. On days prior to the penultimate day, the demand for reserves is also affected by the dynamic cost factor, ie holding more reserves today will increase the future cost of liquidity uncertainty associated with the probability of having to rely on the standing facilities.

The main characteristics of the money market equilibrium can be summarised as three different liquidity policy rules.

1. Expected level of the overnight rate
In the case of full allotment the expected overnight rate equals the tender rate for that day. Thus, the expected overnight rate for a given

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[65]This effect is also part of the reason why we stayed with *neutral* interest rate targeting (ie target rate equals tender rate) in analysing proportional liquidity allotment procedures when the central bank does not allow averaging. If the target rate were below the tender rate, the banks would not bid enough in the tender, and this procedure would no longer be a proportional allotment procedure. If the target rate were above the tender rate, there would be enormous excess bidding and monetary policy would be implemented by choosing the liquidity instead of setting the interest rate (which we implicitly assume for fixed rate tenders). Also this procedure would lead to the unjustified transfer of expected profits to successful bidders.

day in the future will equal the tender rate expected to prevail on that day.

Under the proportional allotment procedure with interest rate targeting, the expected value of the overnight rate will (by definition) equal the target rate of the central bank, as long as the policy is known to the counterparties and the target is not set lower than the tender rate.[66] Therefore, in this case the expected overnight rate for a given future day will equal the expected target rate for that day.

Determination of the expected overnight rate under liquidity targeting policy will depend on the central bank's ability to control the daily supply of liquidity. If the banks expect the central bank to increase the tender rate during the following days, the expected overnight rate will immediately respond to these expectations, by *rising to the new higher level* expected (or to a level very close to it). However, if the central bank is expected to cut the tender rate within the remainder of the current reserve maintenance period, the expected overnight rate will not react to these expectations, but it will stay at the level of the current tender rate, as under full allotment procedure. In the case where the banks have neutral interest rate expectations, the overnight rate is expected to actualise at the level of the (constant) tender rate.

The fact that under liquidity targeting the expected overnight rate will differ from the tender rate, if a rate hike is expected, means that the central bank does not have complete control over the (expected) price of bank reserves. Thus, it might well be the case that the monetary policy signals given through the tender rate are not as unambiguous as with either full allotment or interest rate targeting. Also, the fact that the overnight rate will react only to expectations of a rate increase whereas an expected rate cut would be reflected in the equilibrium liquidity may well lead the counterparties and the public to (falsely) assume that the central bank has asymmetric preferences over the deviations of the overnight rate from the tender

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[66]If the target were lower than the tender rate, this approach would be similar to the case of full allotment, and consequently the expected rate would equal the tender rate instead of the target. Thus, we expect the central bank to set the target rate at least to equal the tender rate.

(or target) rate, even though the asymmetry lies in the structure of the operational framework rather than in the preferences.[67]

2. Expected overnight liquidity

The expected equilibrium liquidity under full allotment will depend on the required daily balances for the remaining period, the interest rate expectations of the banks, and on distribution of liquidity shocks. The higher the tender rate is expected to be during the rest of the maintenance period, the more liquidity banks are willing to hold today at the given tender rate. Hence, we expect the banks to frontload reserve holdings if a rate hike is expected and to backload if they anticipate a rate cut. The lower the liquidity volatility, the more largely banks are willing to substitute their interest rate view for steady reserve keeping. That is, the higher the probability of being forced to use the standing facilities with a given level of liquidity at the overnight interbank trading, the less banks will attempt 'intertemporal arbitrage' (to front- or backload reserve holdings).

Under liquidity targeting, the central bank is able to control the daily supply of liquidity as long as the expected future overnight rate with the target liquidity is not lower than the expected future tender rate. Thus, with increasing or neutral expectations (and high enough volatility), the expected overnight liquidity will be at the target level of the central bank, ie at the level of required daily balances for the remaining period. However, if a rate cut is expected (and possibly with neutral liquidity and very low stochastic liquidity shocks), the expected supply of overnight liquidity will equal that under full allotment (ie the banks will backload reserve holdings).

With interest rate targeting, the equilibrium liquidity depends on the difference between target rate and tender rate. In (the very unlikely) case where the target is set below the tender rate, this whole approach reduces to full allotment. If the target is set at the level of the tender rate or higher, the expected liquidity will also be determined as in the case of full allotment. However, in

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[67]This becomes more evident if one considers the case where the money markets operate on a liquidity surplus rather than deficit. In such a case the tenders would be liquidity draining instead of liquidity providing operations. Therefore, the central bank would not be in control of the daily liquidity supply, if a rate increase were expected. Thus, the expected overnight rate would equal the current tender rate with increasing interest rate expectations, but it would drop to the expected future level if a rate cut were expected.

this case the equilibrium liquidity will be a decreasing function of the *central bank's expectations of the rate the banks anticipate* to prevail during the remainder of the period, and on the central bank's expectation of the distribution of the second liquidity shock for each day. Consequently, there will be frontloading of reserve holdings if rate increase expectations are anticipated by the central bank, and backloading if rate cut expectations are anticipated.

3. Volatility of the overnight rate

With full allotment the spread between overnight rate and tender rate will depend entirely on the liquidity shock that is realised between the tender operation and clearance of the overnight market. The interest rate elasticity of the demand for overnight reserves will increase as the liquidity approaches either zero or the amount of reserves that would fulfil the requirement for the whole period. Thus, the volatility of the overnight rate increases when the banks want to front- or backload reserves, due to interest rate expectations. We also expect the variations to be asymmetric when the banks expect a change in the rate. The reason for the asymmetry lies in the shape of the demand curve at the equilibrium liquidity; it is expected to be convex if a rate cut is expected (due to the backloading of reserves) or concave if a rate rise is expected (as there will be frontloading of reserves).

The interest rate expectation also affects interest rate volatility during the remainder of the maintenance period (however, not on the two last days of the maintenance period), as the reserve holdings at t affect the required daily balances for the remaining period (RDB) on the following days. For example, if the banks expect the tender rate to be increased, they will hold more reserves today, which will decrease the RDB for the following days. Thus, the probability of a bank having to use either of the standing facilities on the following days increases, which will lower the interest rate elasticity for these days. Similarly, if a rate cut is expected, today's volatility will increase, but volatility for the following days will diminish.

When the central bank has a target for liquidity and it is expected to be in control of the daily supply of liquidity, the expected overnight liquidity at t will naturally be the target liquidity and the expected overnight rate for that day will be close to the tender rate expected to prevail in the future. In this case the interest rate elasticity of the demand for liquidity should be high, as the probability of being forced

to use the standing facilities at the targeted liquidity is low. Hence, the variations in the realised overnight rate should reflect to greater extent variations in the interest rate expectations than stochastic variations in autonomous liquidity factors. However, if the banks expect a rate cut in the near future, the daily supply of liquidity is no longer in the hands of the central bank. Thus, the overnight rate will be determined as in case of full allotment, and its volatility will be related to the stochastic liquidity shocks.

In the proportional allotment procedure with interest rate targeting, the variations in the spread between target rate and overnight rate reflect the central bank's stochastic errors in estimating both the banks' demand for liquidity and the changes in autonomous liquidity factors. As in the case of full allotment, the volatility of the overnight rate under interest rate targeting depends on interest rate expectations, as they affect the part of the demand curve at which the equilibrium will be realised. Thus, volatility is expected to be higher if a rate change is expected than with neutral expectations.

We have seen that the *stochastic volatility* of the overnight rate should be lower with liquidity targeting than with the other procedures, if an interest rate change is expected to occur in the near future. However, this need not be the case for total volatility of the rate, at least if the rate expectations vary a great deal over time. Furthermore, the stochastic volatility should not be harmful to the conduct of monetary policy, as long as it is totally understood by the counterparties and the public that these variations originate solely from prediction errors regarding changes in liquidity (and also from errors in estimating the effect of interest rate expectations on the demand for liquidity, in the case of interest rate targeting). This should not cause difficulties in the full allotment case, as the counterparties are always aware that liquidity errors are due to their own errors. However, the central bank must pay a great deal of attention to making its goals understood and believed by the counterparties, if it uses interest rate targeting. One possible method of doing this is to explicitly announce the target of the central bank.

The relative degrees of volatility under full allotment and interest rate targeting depends on the relative size of the aggregate estimate error (in forecasting autonomous liquidity factors) made by the banks and the central bank's error in estimating changes in liquidity and in anticipating the effect of interest rate expectations on the demand

for liquidity. The central bank might have superior knowledge of how the autonomous liquidity factors change as compared to the banks. However, this need not to be the case, at least if the central bank publishes its liquidity forecast prior to each tender operation. Furthermore, estimation of the effect of interest rate expectations on the demand for reserves will be a tough task for the central bank.

4. Bidding behavior of the banks

Under the full allotment procedure, the banking sector as a whole will bid according to the neutral strategy, ie overbidding can never be sustained as an equilibrium strategy in full allotment. Assuming *either* an infinitesimal probability of a bank being unable to participate in the interbank market on any particular day *or* by introducing a fixed cost of entering the market, this result can be extended to the level of single banks. In such a case every bank will bid according to its true liquidity need, and the allocation of liquidity in the tender will follow the expected true liquidity demand by the banks.

Bidding behaviour under liquidity targeting will depend on the banks' interest rate expectations. If the banks expect the tender rate to be increased, they will overbid to profit from the expected difference between tender rate and overnight rate. In such a case the profits of a bank will depend directly on the amount of liquidity it is allotted. Thus, a bank's optimal bid would eventually be infinitely large, if the size of a bid is not somehow limited. The case where the banks have neutral expectations (and the true demand for liquidity, ie demand under full allotment, is not less than the RDB) will also produce overbidding equilibria. If the banks bid according to a neutral strategy, the expected value of the overnight rate will in this case rise above the tender rate. Thus, by overbidding, the banks ensure that the supply of liquidity is determined by the central bank. However, when the true demand is close to the RDB, the overbidding will not lead to positive expected profits but will merely end the opportunity for such profits. If a rate cut is expected (or with neutral expectations and equilibrium bidding below the RDB), the banks will be bidding according to their real demand, as in the case of full allotment.

Under interest rate targeting, the banks will place bids in excess of their true demand for liquidity to either profit from the expected difference between overnight rate and target rate (if the target is above

the tender rate) or prevent such opportunity (if the target rate equals the tender rate).

The bidding behaviour of the banks is very different under each of these procedures, as we have seen. There is no overbidding under full allotment (at least at the aggregate level), ie all banks bid according to their true demand for reserves. In interest rate targeting, multiple overbidding equilibria exist. These equilibria are characterised only by the fact that the aggregate bids should amount to more than the central bank can be expected to be willing to provide. The problem here is that the allocation of liquidity is somehow arbitrary, unless the number of equilibria can be reduced.[68] One way for the central bank to limit this number would be to signal the most probable allotment ratio to the banks (ie the expected percentage of the bids to be accepted). For example, if the central bank told the counterparties, that the most probable allotment ratio is $1/k$, and that it will deviate from the announced ratio only to make use of the superior knowledge it has of the evolution of the autonomous liquidity factors, there would be a unique equilibrium where each bank bids k times its real demand. With liquidity targeting, the overbidding would be similar to that under interest rate targeting, if the overnight rate were expected to be realised at the level of the tender rate. However, if the expected overnight rate were above the tender rate, there would not be equilibrium bidding at all (as the optimal bid of a single bank would be infinite if the amount were not limited) or, if the bid amount was limited the equilibrium bidding would consist of every bank placing the maximum bid.

We would like to conclude by stating that in light of our model the full allotment procedure seems to be a very market-oriented liquidity policy rule, where the monetary policy stance is uniquely determined by the tender rate, and the banks bid according to their true liquidity demand. The stochastic volatility of the liquidity and the overnight rate of interest is lowest with the liquidity targeting policy rule. However, the transparency of monetary policy signalling with this kind of a liquidity policy is not always good. The interest rate targeting procedure shares many of the good qualities of the

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[68]However, even if the allocation of reserves does not meet the true demand for them, this should not be a problem as long as the overnight market is efficient and banks are interested only in the expected profits.

full allotment procedure, at least if the policy is made explicit and is believed by the counterparties. Also, the stochastic liquidity volatility (of the full allotment) could probably be limited under this procedure in some cases (eg if the central bank has superior knowledge of the evolution of autonomous liquidity factors and it can properly estimate the effect of interest rate expectations on liquidity demand). However, the drawback of this policy seems to be the multiplicity of bidding equilibria. Furthermore, estimation of the true demand for liquidity might be an extremely difficult task if a rate change were expected.

Finally, one should notice that most of the differences in the formation of expected overnight rate and in its volatility arise from the expected changes in central bank rates during the remainder of the current reserve maintenance period. Thus, the equilibria under all these procedures would be very similar if the central bank changed the tender rate (or the target rate) only in the first operation of each maintenance period. In such a case, there would never be speculative front- or backloading of reserves in order to profit from intraperiod arbitrage opportunities.

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114

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A   Proof of equation (3.10)

The proof of equation (3.10) and the relationship between $G\left(\cdot\right)$ and $E[F\left(\cdot\right)]$.[69]

Let $F_\mu\left(\cdot\right)$, $F_\varepsilon\left(\cdot\right)$ and $G\left(\cdot\right)$ be the cumulative distribution functions of independent stochastic variables μ, ε and ν ($\nu = \mu + \varepsilon$) respectively.

From the definition of a distribution function we have:

$$G(s) = P\{\nu \leq s\}. \tag{A.1}$$

It can be shown that the distribution function of the sum ν has the representation

$$G(s) = \int F_\varepsilon(s - \mu) dF_\mu(\mu) \tag{A.2}$$

or, assuming continuous distributions with density functions f_ε and f_μ respectively,

$$G(s) = \int f_\varepsilon(s - \mu) f_\mu(\mu) d\mu. \tag{A.3}$$

But note that by the definition of an expectation, we have

$$G(s) = \mathrm{E}_{f_\mu}[f_\varepsilon(s - \mu)], \tag{A.4}$$

ie the distribution function of the sum can be written as an expected value, where expectation is taken over the distribution of μ.

This result can be proved using the convolution theorem in association with characteristic functions: the characteristic function (chf) of a distribution (with density f), $\psi_f(t)$, say, is defined by

$$\psi_f(t) = \mathrm{E}\left(e^{itx}\right) = \int e^{itx} f\left(x\right) dx. \tag{A.5}$$

For independent random variables x and y, the characteristic function of the distribution of their sum, $\nu = \mu + \varepsilon$, is the product of the individual characteristic functions:

$$\psi_g(t) = \mathrm{E}\left(e^{its}\right) = \mathrm{E}\left(e^{itx}\right)\mathrm{E}\left(e^{ity}\right) = \psi_{f_x}(t)\psi_{f_y}(t). \tag{A.6}$$

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[69]I am indebted to Jouko Vilmunen for the derivation of this appendix.

On the other hand, we know that the chf of the convolution

$$(f * h)(\nu) = \int f(\nu - x) h(x) \, dx \tag{A.7}$$

equals the product of the chf:s of the distributions with densities f and h respectively, ie

$$\psi_{f*h}(t) = \psi_f(t)\psi_h(t). \tag{A.8}$$

Since the chf determines the distribution uniquely, we must consequently have

$$\psi_{f_x * f_y}(t) = \psi_g(t) \tag{A.9}$$

so that the density function of the distribution of the sum ν, g must be the convolution of the densities

$$g(u) = \int f_y(u - x) f_x(x) \, dx. \tag{A.10}$$

Integrating both sides and changing the order of integration on the RHS we obtain:

$$G(s) = \int \left[\int^s f_y(u - x) \, du \right] f_x(x) \, dx = \int F_y(s - x) f_x(x) \, dx. \tag{A.11}$$

Applying the above results to equation (3.7), we obtain:

$$
\begin{aligned}
r^T &= r^m G(-eER_{i,T}^{neutral}) + r^d \left[1 - G(eER_{i,T}^{neutral}) \right] \tag{A.12} \\
&= r^m \mathrm{E}_{f_\mu} \left[F_\varepsilon \left(-eER_{i,T}^{neutral} - \mu_T \right) \right] \\
&\quad + r^d \left\{ 1 - \mathrm{E}_{f_\mu} \left[F_\varepsilon \left(eER_{i,T}^{neutral} + \mu_T \right) \right] \right\} \\
&= r^m \mathrm{E}_{f_\mu} \left[F_\varepsilon \left(-ER_{i,T}^{neutral} \right) \right] + r^d \left\{ 1 - \mathrm{E}_{f_\mu} \left[F_\varepsilon \left(ER_{i,T}^{neutral} \right) \right] \right\} \\
&= \mathrm{E}_{f_\mu} \left[r^m \mathrm{E}_{f_\mu} \left[F_\varepsilon \left(-ER_{i,T}^{neutral} \right) \right] + r^d \left\{ 1 - \mathrm{E}_{f_\mu} \left[F_\varepsilon \left(ER_{i,T}^{neutral} \right) \right] \right\} \right] \\
&= \mathrm{E}_{f_\mu} \left[r^{on} \right].
\end{aligned}
$$

That is, the expected overnight rate, where the expectation is taken w.r.t. the distribution of the shock μ ($\mathrm{E}_{f_\mu}[\cdot]$), equals the tender rate.

B   Profit maximisation problem at T-1

The overnight rate of interest at T-1 as a function of liquidity is determined by the first-order condition of the banks' profit maximisation problem:

$$\max_{b_{i,T-1}} \mathrm{E}\left[\Pi_{T-1}\right] = \tag{B.1}$$

$$r_{T-1}^m \int_{-\infty}^{-OB_{i,T-1}-b_{i,T-1}} (OB_{i,T-1} + b_{i,T-1} + \varepsilon_{T-1}) f(\varepsilon_{T-1}) d\varepsilon_{T-1}$$

$$+\mathrm{E}_{T-1}\left[r_T^T\right] \int_{-OB_{i,T-1}-b_{i,T-1}}^{\infty} (OB_{i,T-1} + b_{i,T-1} + \varepsilon_{T-1}) f(\varepsilon_{T-1}) d\varepsilon_{T-1}$$

$$+ \int_{IB_{i,T-1}-b_{i,T-1}}^{\infty} (-IB_{i,T-1} + b_{i,T-1} + \varepsilon_{T-1}) f(\varepsilon_{T-1}) d\varepsilon_{T-1}$$

$$\times (r_{T-1}^d - \mathrm{E}_{T-1}\left[r_T^T\right]) - r_{T-1}^{on} * b_{i,T-1},$$

where the difference between the amount of reserves needed to fulfill the reserve requirement for the whole remainder of the maintenance period and the overnight balances before the interbank lending at $T-1$ is denoted by $IB_{i,T-1}$ (ie $IB_{i,T-1} = 2RDB_{i,T-1} - OB_{i,T-1}$).

Using the Leibniz's rule, we get the FOC:

$$\frac{\partial \mathrm{E}\left[\Pi_{T-1}\right]}{\partial b_{i,T-1}} = r_{T-1}^m F\left(-OB_{i,T-1} - b_{i,T-1}\right)$$

$$+r_{T-1}^d \left(1 - F\left(2RDB_{i,T-1} - OB_{i,T-1} - b_{i,T-1}\right)\right)$$
$$+\mathrm{E}_{T-1}\left[r_T^T\right] \{1 - F\left(-OB_{i,T-1} - b_{i,T-1}\right)$$
$$- \left[1 - F(2RDB_{i,T-1} - OB_{i,T-1} - b_{i,T-1})\right]\}$$
$$- r_{T-1}^{on} = 0,$$

which yields:

$$r_{T-1}^{on} = \mathrm{E}_{T-1}\left[r_T^T\right] \{1 - F(-OB_{T-1})$$
$$- \left[1 - F(2RDB_{T-1} - OB_{T-1})\right]\}$$
$$+ r_{T-1}^m F(-OB_{T-1}) + r_{T-1}^d \left[1 - F(2RDB_{T-1} - OB_{T-1})\right]$$

C   Profit maximisation problem at t

Bank i's profit maximisation problem at the interbank market at t is:

$$
\max_{b_{i,t}} \mathrm{E}\left[\Pi_t\right] = r_t^m \int_{-\infty}^{-OB_{i,t}-b_{i,t}} (OB_{i,t} + b_{i,t} + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \tag{C.1}
$$

$$
+\mathrm{E}\left[r_f^T\right] \int_{-OB_{i,t}-b_{i,t}}^{\infty} (OB_{i,t} + b_{i,t} + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t
$$

$$
+ \left(r_t^d - \mathrm{E}\left[r_f^T\right]\right) \int_{IB_{i,t}-b_{i,t}}^{\infty} (-IB_{i,t} + b_{i,t} + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t
$$

$$
+ \sum_{j=t+1}^{T-1} \left(\mathrm{E}_t\left[r_j^m - r_f^T\right] \left[\int_{-\infty}^{-eOB_{i,j}^*} (eOB_{i,j}^* + \nu_j) g(\nu_j) d\nu_j \right.\right.
$$

$$
\left. - \int_{-\infty}^{-eOB_{i,j}^{*\prime}} (eOB_{i,j}^{*\prime} + \nu_j) g(\nu_j) d(\nu_j) \right]
$$

$$
+\mathrm{E}_t\left[r_j^d - r_f^T\right] \left[\int_{JB_{i,j}^*}^{\infty} (-JB_{i,j}^* + \nu_j) g(\nu_j) d\nu_j - \right.
$$

$$
\left.\left. \int_{JB_{i,j}^{*\prime}}^{\infty} (-JB_{i,j}^{*\prime} + \nu_j) g(\nu_j) d\nu_j \right]\right) - r_t^{on} b_{i,t},
$$

where $\nu_j = \mu_j + \varepsilon_j$, $IB_{i,t} = (T - t + 1)RBD_{i,t} - OB_{i,t}$, $JB_{i,j} = (T-j+1)eRBD_{i,j} - eOB_{i,j}^*$, $JB_{i,j}^{*\prime} = (T-j+1)eRBD_{i,j}^{\prime} - eOB_{i,j}^{*\prime}$ and the equilibrium liquidity at j ($eOB_{i,j}^*$) is a function of the expected required daily balances for the remaining period at j ($eRBD_{i,j}$), and $eRBD_{i,j}$ itself depends on the expected equilibrium reserve holdings up to $j-1$:

$$eRBD_{i,j} =$$

$$\left\{ (T - t + 1)\, RDB_t - \left[\int_{-OB_{i,t}-b_{i,t}}^{\infty} (OB_{i,t} + b_{i,t} + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right.\right.$$

$$\left.\left. - \int_{IB_{i,t}-b_{i,t}}^{\infty} (IB_{i,t} + b_{i,t} + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right] - eOB^*_{t+1} - ... - eOB^*_{j-1} \right\}$$

$$\times \frac{1}{(T - j + 1)}.$$

$eOB^{*\prime}_{i,j}$ is the expected equilibrium overnight balances, assuming the bank did not participate in the interbank overnight market at t. Thus, $eOB^{*\prime}_{i,j}$ is a function of $eRBD'_{i,j}$ (the expected required daily balances for the remaining period at j, without participation in the overnight market at t);

$$eRBD'_{i,j} = \left\{ (T - t + 1)\, RDB_t - \right.$$

$$\left[\int_{-OB_{i,t}-b_{i,t}}^{\infty} (OB_{i,t} + b_{i,t} + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right.$$

$$\left. - \int_{IB_{i,t}-b_{i,t}}^{\infty} (IB_{i,t} + b_{i,t} + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right]$$

$$\left. - eOB^{*\prime}_{t+1} - ... - eOB^{*\prime}_{j-1} \right\} \frac{1}{(T - j + 1)}.$$

Consequently, $eOB^*_{i,j}$ and $eRBD_{i,j}$ are functions of b_t, while $eOB^{*\prime}_{i,j}$ and $eRBD'_{i,j}$ are not.

Again applying the Leibniz rule and aggregating over the unitary mass, yields the FOC:

$$\text{E}_t\left[r_f^T\right] + \left(r_t^m - \text{E}_t\left[r_f^T\right]\right) F(-OB_t) \tag{C.2}$$
$$+ \left(r_t^d - \text{E}_t\left[r_f^T\right]\right) \{[1 - F(T-t+1)RDB_t - OB_t]\}$$
$$+ \sum_{j=t+1}^{T-1} \left(\text{E}\left[r_j^m - r_f^T\right] \frac{\partial eOB_j^*}{\partial b_t} G(-eOB_j^*)\right.$$
$$+ \text{E}\left[r_j^d - r_f^T\right]\left(-(T-j+1)\frac{\partial eRDB_j}{\partial b_t} + \frac{\partial eOB_j^*}{\partial b_t}\right)$$
$$\left.\times \left\{1 - G\left[(T-j+1)eRDB_j - eOB_j^*\right]\right\}\right) - r_t^{on} = 0$$

The equilibrium condition for the money market $\left(\text{E}_t\left[r_t^{on}\right] = r_t^T\right)$ is thus (note that $\frac{\partial eOB_j^*}{\partial b_t} = \frac{\partial eOB_j^*}{\partial eRDB_j} \frac{\partial eRDB_j}{\partial b_t}$):

$$\text{E}_t\left[r_t^{on}\right] - \text{E}_t\left[r_f^T\right] = r_t^T - \text{E}_t\left[r_f^T\right] = \left(r_t^m - \text{E}_t\left[r_f^T\right]\right) G(-eOB_t^*)$$

$$\tag{C.3}$$

$$+ \left(r_t^d - \text{E}_t\left[r_f^T\right]\right) \{1 - G\left[(T-t+1)RDB_t - eOB_t^*\right]\}$$
$$+ \sum_{j=t+1}^{T-1} \left(\text{E}_t\left[r_j^m - r_f^T\right] \frac{\partial eOB_j^*}{\partial eRDB_j} \frac{\partial eRDB_j}{\partial b_t} G(-eOB_j^*)\right.$$
$$+ \text{E}_t\left[r_j^d - r_f^T\right]\left(\frac{\partial eOB_j^*}{\partial eRDB_j} - (T-j+1)\right) \frac{\partial eRDB_j}{\partial b_t}$$
$$\left.\times \left\{1 - G\left[(T-j+1)eRDB_j - eOB_j^*\right]\right\}\right).$$

We can analyse equations (C.3) and (C.2) a bit further by taking the partial derivative $\frac{\partial eRDB_j}{\partial b_t}$:

$$\frac{\partial eRBD_j}{\partial b_t}$$
$$= \frac{-1}{T-j+1}\left\{[F(IB_t) - F(-OB_t)] + \sum_{k=t+1}^{j-1} \frac{\partial eOB_k^*}{\partial eRDB_k} \frac{\partial eRDB_k}{\partial b_t}\right\}.$$

Going a step further we obtain:

$$
\frac{\partial eRBD_{i,j}}{\partial b_t} = \frac{-1}{T-j+1} \Bigg(\left[F(IB_t) - F(-OB_t) \right] \qquad\qquad (C.4)
$$

$$
+ \sum_{k=t+1}^{j-1} \frac{\partial eOB_k^*}{\partial eRDB_k} \frac{-1}{T-k+1}
$$

$$
\times \Bigg\{ \left[F(IB_t) - F(-OB_t) \right] + \sum_{l=t+1}^{k-1} \frac{\partial eOB_l^*}{\partial eRDB_l} \frac{\partial eRDB_l}{\partial b_t} \Bigg\} \Bigg)
$$

repeating the partial derivatives in succession, equation C.4 becomes:

$$
\frac{\partial eRBD_{i,j}}{\partial b_t} = \frac{-\left[F(IB_t) - F(-OB_t) \right]}{T-j+1}
$$

$$
\times \Bigg(1 + \sum_{k=t+1}^{j-1} \frac{\partial eOB_k^*}{\partial eRDB_k} \frac{-1}{T-k+1}
$$

$$
\times \Bigg\{ 1 + \sum_{l=t+1}^{k-1} \frac{\partial eOB_l^*}{\partial eRDB_l} \frac{-1}{T-l+1}
$$

$$
\times \Bigg[1 + \sum_{m=t+1}^{l-1} \frac{\partial eOB_m^*}{\partial eRDB_m} \frac{-1}{T-m+1}
$$

$$
\times \Bigg(... \times \Bigg\{ 1 + \frac{\partial eOB_{t+2}^*}{\partial eRDB_{t+2}} \frac{-1}{T-t-1}
$$

$$
\times \Bigg[1 + \frac{\partial eOB_{t+1}^*}{\partial eRDB_{t+1}} \left(\frac{-1}{T-t} \right) \Bigg] \Bigg\} \Bigg) \Bigg] \Bigg\} \Bigg)
$$

Clearly the equilibrium bidding at t depends on the optimal path of reserve holdings during the rest of the maintenance period. Thus, one must solve the optimal bidding recursively by first deriving OB_{T-1}^* as a function of RDB_{T-1}, and using that information to calculate OB_{T-2}^* as a function of RDB_{T-2}, and so forth.

Bidding in fixed rate tenders: theory and evidence from ECB tenders

Contents

Abstract

This paper presents a model of the optimal bidding behaviour of a single bank in the context of fixed rate liquidity tenders. Banks' bidding is shown to depend crucially on the central bank liquidity policy as regards tender allotments. The paper also analyses ECB liquidity policy in terms of the model. The ECB, while applying fixed rate tenders, appears to have been attempting to stabilise the market interest rate at a level close to the main refinancing rate. However, this aim was at least partially overridden by that of stabilising total money market liquidity over the course of the reserve maintenance period – even more so when banks were expecting the ECB to raise the main refinancing rate in the near future. The banks' aggregate bids increased considerably during the period of fixed rate tenders. This 'overbidding' was seen to result mainly from profit opportunities associated with a positive spread between market interest rate and main refinancing rate. The positive spread resulted from the combination of expectations of an interest rate hike and liquidity-oriented allotment policy.

124

1   Introduction

Discussion concerning the European Central Bank's (ECB) operational framework during the first 18 months of operation was focused on banks' behaviour in the main refinancing operations (MRO). These operations were conducted as fixed rate tenders, where the ECB announced the rate at which the banks could obtain liquidity (*main refinancing rate*). After the announcements, the banks informed the ECB of the amount of reserves they were willing to borrow at the given rate. Finally, the ECB decided on the amount of liquidity to be provided to the markets. If the aggregate bid was larger than the allotment, each bank received an equal proportion of its bid[1]. The banks seemed to place bids far above the amount needed to fulfill reserve requirements. Thus, the allotment ratios (ie liquidity allotted / aggregate bid) in the ECB weekly tenders averaged 8% and varied from 100% (7 April 1999) to the low point of 0.87% (31 May 2000). Furthermore, the allotment ratio seemed to decline especially during the first half of 2000. This development was considered a sign of severe overbidding, and eventually led to revision of the ECB's tender procedure.[2]

The bidding of the banks in the ECB main refinancing operations has been given a variety of explanations in some recent papers. For example, Nautz and Oechssler (2000) and Ehrhart (2000) each build a simple model of banks' bidding in the fixed rate tenders. They both claim that the overbidding phenomenon is an optimal response when the central bank is supplying liquidity less than the banks demand for. However, these papers do not pay attention to central bank incentives to act as proposed but merely assume that the money market liquidity desired by the central bank is less than the amount that is optimal to the banks (at the given rate). Furthermore, these papers abstract from the interbank market for bank reserves. However, it is precisely the money market rates that largely determine bidding behaviour

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[1]For example, if the aggregate bid were 1 500 units and the ECB allotted 1 000 units, a bank that bid for 300 units would have received 200 units.

[2]On 8 June 2000 the Governing Council of the ECB decided to swich to the variable tender procedure as of 27 June 2000. In an ECB press release (8 June 2000) the new tender mechanism was announced to be '*a response to the severe overbidding problem which has developed in the context of the fixed rate tender procedure*'.

in tender operations, and these rates are strongly affected by the amount of reserves provided to the banks in the tenders. Ayuso and Repullo (2000) construct a model where banks' bidding is determined by the difference between the target rate of the central bank and the expected money market rate. They propose that overbidding in the ECB fixed rate tenders resulted from an asymmetric preference function of the ECB. In their model the central bank provides the markets with such liquidity as will on average keep the overnight rate above the tender rate, as it has a loss function that penalises more heavily interbank rates below than above the target. However, the paper does not consider the ECB's motive for such an asymmetric loss function. The rationale behind the proposed asymmetricity is not at all trivial. The more so, if we consider that the ECB has stated that '*The ECB tended to orient its allotment decisions towards ensuring an average interbank overnight rate close to the tender rate*' (ECB 2000b). Finally, Ayuso and Repullo (2001) tests whether the overbidding of the banks resulted from the expectations of a future tightening of the monetary policy or from the existence of a positive spread between short-term money market rates and the main refinancing rate due to contemporaneous restriction of the supply of liquidity. They find empirical evidence supporting the latter option.

In this essay we propose an alternative explanation for the evolution of bids in the ECB main refinancing operations. We show that it can be optimal to bid in excess one's neutral demand for liquidity even if the central bank has symmetric preferences over interest rate variations in the interbank market. The incentive to 'overbid' is enhanced if the central bank pays attention to deviations of liquidity from the level indicated by the reserve requirement. For example, when banks expect the central bank to increase its policy rates during the remainder of the current reserves maintenance period, it is optimal for them to hold more reserves now (at current rates) than after the rate change has occurred. A liquidity-oriented central bank might want to curb such frontloading of the reserves. The difference between this kind of liquidity-oriented policy and the asymmetric preferences rationale suggested by Ayuso and Repullo is that, with a central bank interested in stable liquidity, the spread between the expected overnight rate and the tender rate should be affected by interest rate expectations, whereas with asymmetric preferences the spread should only reflect the expected asymmetry in preferences.

126

We show that the liquidity orientation of the central bank survives remarkably well in light of the empirical evidence we have from the ECB fixed rate tenders and is in line with information the ECB has published on its liquidity policy.

The essay is organised as follows. In section 2 we model the optimal bidding strategy for a single bank. Section 3 describes some of the liquidity policies the central bank may choose to follow. In section 4 we consider what kind of paths the bidding will take under the various liquidity policies, and we also introduce the effects that arise with the collateral requirement. Section 5 reviews the evidence from the first 18 months of ECB operations and section 6 concludes.

2 Model of optimal bidding

The money market consists of a central bank (monopoly supplier of liquidity) and n homogeneous banks that demand liquidity in order to fulfill reserve requirements and avoid having to use the standing facilities. The model money market liquidity consists of the net sum of autonomous liquidity factors[3] and the amount of reserves provided to the market in the tender operations. Let us denote the estimated amount of autonomous liquidity factors either by a^{CB} or a^{banks}, depending on who makes the forecast ($a^{banks} = \sum_{i=1}^{n} a_{\text{bank } i}$), liquidity provided through tenders by q ($q = \sum_{i=1}^{n} q_i$), and liquidity shock (ie the forecast error of the autonomous liquidity factors) by either ε^{CB} or ε^{banks}. We will divide the shock a single bank faces (ε_i) to two zero mean parts μ/n and ξ_i, where μ/n is bank i's share of the shock into the aggregate money market liquidity and ξ_i is a liquidity distribution shock[4]. Furthermore, we'll assume that the aggregate liquidity shock is independent of the distribution shock (ie $\mu \perp \xi_i$,

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[3]The autonomous liquidity factors are the balance sheet items of the central bank that are not affected by monetary policy operations. The most important autonomous factors affecting euro area liquidity are net government deposits with the Eurosystem, banknotes and items in course of settlement. See ECB (2000c, 40–41) for a more detailed presentation of these factors.

[4]A distribution shock merely transfers liquidity from one bank to another. Thus, the distribution shocks must sum up to zero (a positive shock to bank i must always be accompanied by a negative shock of identical size to the rest of the banks, ie $\xi_i = -\xi_{-i}$, where ξ_{-i} denotes $\sum_{j=1}^{n} \xi_j - \xi_i$).

thus $\varepsilon_i = \mu/n + \xi_i$ and $\varepsilon = \sum_{i=1}^{n} \varepsilon_i = \mu$, as $\sum_{i=1}^{n} \xi_i = 0$). Note that, even though the estimation of the autonomous liquidity factors made by the central bank need not equal that of the banks, we will always have $a^{CB} + \varepsilon^{CB} = a^{banks} + \varepsilon^{banks} =$ ex post amount of autonomous liquidity factors. To ease the notation, we will drop the superscripts of both a and ε whenever they are not necessary. The amount of liquidity allotted in a tender can't exceed the supply or demand at the given price. Therefore, it will equal the minimum of the total amount of liquidity bid for by the banks in the tender ($\sum_{i=1}^{n} b_i = b$) and the amount the central bank is willing to provide to the markets (denoted by c). Total ex post money market liquidity is given by:

$$l = a + \varepsilon + q = a + \varepsilon + \min(b, c). \tag{2.1}$$

It can be shown that in a system with marginal lending facility, deposit facility and reserve averaging, the interbank rate of interest at relevant maturity[5] is a monotonically decreasing function of money market liquidity and rates of the standing facilities (r^{SF}), and it is increasing in the expected future central bank rates (r^{ef}). That is, $r = r\left(l, r^{SF}, r^{ef}\right)$, where $\frac{dr}{dl} < 0$, $\frac{dr}{dr^{SF}} < 0 \wedge \frac{dr}{dr^{ef}} > 0$.[6] The expected market rate of interest at the given central bank rates (both current and expected future rates) is given by:

$$\mathrm{E}\left[r\left(l|r^{SF}, r^{ef}\right)\right] = \int_{\varepsilon^{\min}}^{\varepsilon^{\max}} r\left(a + \varepsilon + \min[b, c]|r^{SF}, r^{ef}\right) f\left(\varepsilon\right) d\left(\varepsilon\right),$$

where $f\left(\varepsilon\right)$ is the probability density function of the aggregate liquidity shock.[7]

We define the neutral amount of tendered reserves at the given central bank rates (both current and expected future rates) and

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[5]The relevant maturity of the comparable market rate of interest is the same as that of the tender operation. Note that if the tender operation is collateralized, the comparable rate must also be collateralized. The maturity of the ECB weekly tenders is two weeks, and the liquidity obtained in these operations must be covered by adequate collateral. We will return to the questions that araise from collateral requirements in section 4.

[6]A detailed discussion of the specific functional forms of the demand for overnight liquidity can be found eg in Välimäki (2001).

[7]Note that, if the distribution of liquidity shocks of the banks deviates from that of the central bank, the overnight rate expected by the banks need not coincide with that of the central bank.

128

autonomous liquidity factor estimate $(q^{neutral}|a, r^{CB})$[8] to be such that with it the expected market rate of interest will equal the tender rate[9]:

$$\mathrm{E}\left[r\left(a + \varepsilon + q^{neutral}|r^{CB}\right)\right] \tag{2.2}$$

$$= \int_{\varepsilon^{min}}^{\varepsilon^{max}} r\left(a + \varepsilon + q^{neutral}|r^{CB}\right) f\left(\varepsilon\right) d\left(\varepsilon\right) = r^{T}.$$

As the market rate of interest is decreasing with liquidity, the expected value of the rate will be above (below) the tender rate if $\min(b, c) < q^{neutral}$ ($\min(b, c) > q^{neutral}$).

Let us next consider the bidding of a single, risk-neutral bank for three cases: i) $\min(b, c) < q^{neutral}$, ii) $\min(b, c) = q^{neutral}$ and iii) $\min(b, c) > q^{neutral}$.

The amount of liquidity allotted to bank i in a tender operation is either the amount it bid for (if $c > b$) or the bid amount scaled back by the allotment ratio c/b (if $c < b$):

$$q_i = \min\left(b_i, \frac{c}{b}b_i\right) \tag{2.3}$$

Thus, the expected amount to be received from the tender is given by:

$$\mathrm{E}\left(q_i\right) = b_i \mathrm{E}\left[\min\left(1, \frac{c}{b}\right)\right].$$

That is, the expected amount of reserves to be allotted to bank i will i) equal the bid amount if it is certain, that the total amount of bids will be lower than the central bank's target (ie if $p(c > b) = 1$), ii) equal the expected proportion $\mathrm{E}[c/b]$ of the bid amount if it is certain that the banking sector as a whole will demand more reserves than the central bank aims to provide (ie if $p(c > b) = 0$), and iii) be smaller than b_i if the bank cannot be sure whether the bid will be scaled back (ie if $p(\frac{c}{b} > 1) \in (0, 1)$).

Let us denote the private value of a specific amount of reserves for bank i by $r_i^{pv}(x)$ (ie $r_i^{pv}(x)$ is the value of x units of liquidity to bank i when it does not participate the interbank market). The private value is decreasing in liquidity. Also, let l_i^T and l_i^m denote the amount of liquidity (with given current and expected future

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[8] r^{CB} denotes the vector of current and future central bank rates.

[9] The main refinancing rate is the tender rate applied by the ECB.

central bank rates) at which the private value of liquidity would equal the tender rate and market rate respectively (ie $r_i^{pv}\left(l_i^T|r^{CB}\right) = r^T$ and $r_i^{pv}\left(l_i^m|r^{CB}\right) = r$). Finally, let q_i^T and q_i^m denote the amount of liquidity that must be allotted to bank i, in order for the expected private value of liquidity to equal the tender rate or the expected market rate respectively (ie q_i^T and q_i^m are implicitly given by $\int_{\varepsilon_i^{\min}}^{\varepsilon_i^{\max}} r^{pv}\left(a_i + \varepsilon_i + q_i^T|r^{CB}\right) f\left(\varepsilon_i\right) d\left(\varepsilon_i\right) = r^T$ and $\int_{\varepsilon_i^{\min}}^{\varepsilon_i^{\max}} r^{pv}\left(a_i + \varepsilon_i + q_i^m|r^{CB}\right) f\left(\varepsilon_i\right) d\left(\varepsilon_i\right) =$ E$[r]$).

Bank i can obtain the liquidity it desires either from the central bank tender operation or from the interbank market. We start the analysis of a single banks behaviour first by considering the bank's profit maximizing problem in the interbank market after the liquidity shock has occurred. The problem of the risk-neutral bank i is:

$$\max_{s_i} \Pi = \int_{a_i + \mu/n + \xi_i + q_i}^{a_i + \mu/n + \xi_i + q_i + s_i} r_i^{pv}\left(x\right) dx - s_i r, \qquad (2.4)$$

where s_i is the net amount borrowed from the market. The first term on the RHS of equation (2.4) is the change in the private value of traded liquidity, and the second term is the direct cost of borrowing it from the market.[10] The FOC for the problem is:

$$\frac{\partial\left(\cdot\right)}{\partial s_i} = -r + r_i^{pv}\left(a_i + \mu/n + \xi_i + q_i + s_i^*\right) = 0 \qquad (2.5)$$
$$\Rightarrow\ r_i^{pv}\left(a_i + \mu/n + \xi_i + q_i + s_i^*\right) = r.$$

Equation (2.5) tells us that, with equilibrium borrowing, bank i adjusts its private value of liquidity to the level of the market rate. The explicit borrowing function is given by:

$$s_i^* = r_i^{pv^{-1}}\left(r\right) - a_i - \mu/n - \xi_i - q_i \qquad (2.6)$$

Now, positive interbank borrowing by bank i must be met by negative borrowing (of the same magnitude) by the rest of the banks (ie $s_i = -s_{-i}$)[11], and consequently the aggregate interbank borrowing must sum to zero. Thus, the following holds for aggregated amounts:

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[10]Net lending to the interbank market is naturally denoted by negative borrowing.

[11]Throughout the paper, we denote the aggregate value of any variable excl. the value for bank i by subscript $-i$, ie $x_{-i} = \sum_{j=1}^n x_j - x_i$.

130

$$\sum_{i=1}^{n} r_i^{pv^{-1}} (r) = a + \mu + q,$$

and as the banks are homogeneous, we can derive the following equation for the market rate of interest:

$$r\left(a + q + \mu\right) = r_i^{pv}\left(\frac{a + \mu + q}{n}\right). \tag{2.7}$$

Inserting equation (2.7) into equation (2.6) gives the optimal interbank borrowing of bank i:

$$s_i^* = \left(\frac{a + q}{n}\right) - a_i - \xi_i - q_i. \tag{2.8}$$

That is, in the interbank market, the banks equate their differences in amounts of liquidity held. The differences in liquidity held before the interbank market operations result from either the (distributive) liquidity shock or differences in banks' bidding behaviour.

At the central bank tender, the banks are assumed to bid so as to maximise their expected profits. The cost of the liquidity acquired is naturally the tender rate times the amount allotted to the bank, while the expected income from the allotment is the expected change in the market value of the quantity traded in the interbank market plus the expected change in the private value of the amount held by the bank. The bidding strategy of bank i must be based on maximizing the following equation:

$$
\begin{aligned}
\max_{q_i} \mathrm{E}\left[\Pi_i\right] \;=\; & \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \Big\{ s_i\left(r\left(q_{-i}, \mu\right), \mu, \xi_i\right) r\left(q_{-i}, \mu\right) \\
& - s_i^*\left(r\left(q, \mu\right), \mu, \xi_i, q_i\right) r\left(q, \mu\right) \\
& + \int_{r_i^{pv^{-1}}(r(q_{-i},\mu))}^{r_i^{pv^{-1}}(r(q,\mu))} r_i^{pv}\left(x\right) dx \Big\} f\left(\xi_i, \mu\right) d\xi_i d\mu - q_i r^T \\
\text{s.t. } s_i^* \;=\; & r_i^{pv^{-1}}(r) - a_i - \mu/n - \xi_i - q_i \text{ and } q_i \geq 0.
\end{aligned}
$$

We divide the analysis of optimal bidding in a tender into two parts according to the relative size of the aggregate bid vs the central bank's

target amount (b vs c). We first consider the case in which the central bank uses the full allotment strategy, after which the case where the central bank scales the excess bids down according to its target.

Case 1:   Full allotment

With full allotment, the central bank always provides the banks with all the liquidity bid for (ie $c = b \Rightarrow \frac{c}{b} = 1$). In this case, the profit maximizing problem of bank i at the tender is:

$$
\max_{b_i} \mathrm{E}\left[\Pi_i\right] = \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ s_i|_{b_i=0} r\left(b_{-i}, \mu\right) - s_i^*|_{b_i=b_i} r\left(b, \mu\right) \right. \quad (2.9)
$$

$$
\left. + \int_{r_i^{pv^{-1}}(r(b_{-i},\mu))}^{r_i^{pv^{-1}}(r(b,\mu))} r_i^{pv}\left(x\right) dx \right\} f\left(\xi_i, \mu\right) d\xi_i d\mu - b_i r^T
$$

$$
\text{s.t. } s_i^* = r_i^{pv^{-1}}\left(r\right) - a_i - \mu/n - \xi_i - b_i \text{ and } b_i \geq 0, \qquad (2.10)
$$

which can be transformed into the following Kuhn-Tucker formulation:

$$
L(b_i, \nu) = \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ \left(r_i^{pv^{-1}}\left(r\left(b_{-i}, \mu\right)\right) - a_i - \mu/n - \xi_i \right) \right.
$$

$$
\times r\left(b_{-i}, \mu\right) - \left(r_i^{pv^{-1}}\left(r\left(b, \mu\right)\right) - a_i - \mu/n - \xi_i - b_i \right) r\left(b, \mu\right)
$$

$$
\left. + \int_{r_i^{pv^{-1}}(r(b_{-i},\mu))}^{r_i^{pv^{-1}}(r(b,\mu))} r_i^{pv}\left(x\right) dx \right\} f\left(\xi_i, \mu\right) d\xi_i d\mu - b_i r^T + \nu b_i. \quad (2.11)
$$

The first order conditions corresponding to the Lagrangian are:

$$
\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ r\left(b, \mu\right) - \left[r_i^{pv^{-1}}\left(r\left(b, \mu\right)\right) - a_i - \mu/n - \xi_i - b_i \right] \right.
$$

$$
\left. \times \frac{\partial r\left(b^*, \mu\right)}{\partial b_i} \right\} f\left(\xi_i, \mu\right) d\xi_i d\mu - r^T + \nu = 0 \qquad\qquad (2.12)
$$

$$
\nu b_i^* = 0 \qquad\qquad (2.13)
$$

$$
\nu \geq 0 \qquad\qquad (2.14)
$$

Equation (2.12) implicitly defines the optimal bid for bank i, and when the optimum bid is positive the condition can be reduced to:

$$
\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} s_i^* \frac{\partial r\left(b^*, \mu\right)}{\partial b_i} f\left(\xi_i, \mu\right) d\xi_i d\mu = \mathrm{E}\left[r\left(b^*, \mu\right)\right] - r^T. \quad (2.15)
$$

132

That is, with optimal central bank borrowing bank i *either* equates the expected change in the value of interbank borrowing to the expected difference between the market rate of interest and the tender rate *or* does not participate the tender.

Let's next consider the conditions under which it is optimal for bank i to bid such an amount that it will bring the aggregate liquidity to its neutral level (ie when $b_i^* = q^{neutral} - b_{-i} \Leftrightarrow b_i^* + b_{-i} = b^* = q^{neutral}$). Now, $b_i = q^{neutral} - b_{-i}$ is the optimal bid, if the following holds:

$$\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \Big[r_i^{pv^{-1}} \left(r \left(q^{neutral}, \mu \right) \right) - a_i - \mu/n - \xi_i$$

$$- \left(q^{neutral} - b_{-i} \right) \Big] \frac{\partial r \left(q^{neutral}, \mu \right)}{\partial b_i} f \left(\xi_i, \mu \right) d\xi_i d\mu = 0. \ (2.16)$$

Equation (2.16) holds only if $b_{-i} = q_{-i}^T$, in which case $b_i^* = q^{neutral} - q_{-i}^T = q_i^T$.[12] That is, *it is optimal for bank i to bid the aggregate reserves up to the neutral level, when the aggregate bid of the rest of the banks is neutral, and it is precisely the neutral demand of bank i that is needed to close the gap.* This incentive applies to all banks. Thus, *every bank bidding for its neutral liquidity is an equilibrium solution for the profit maximization problem.*

If the aggregate bid of the other banks is less than their neutral demand would be (ie $q_{-i}^T < q^{neutral} - q_i^T$), it will be optimal for bank i to bid for more than the neutral demand, but still less than that needed to bring the aggregate bid up to the neutral level (ie $q_i^T < b_i^* < q^{neutral} - q_{-i}^T$). Intuitively, by this kind of bidding, bank i increases its probability of being a lender in the interbank market with the

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[12]This comes from the fact that equation (2.16) can be rewritten as:

$$\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ \frac{a + q^{neutral}}{n} - \left[a_i + \left(q^{neutral} - b_{-i} \right) \right] \right\}$$

$$\frac{\partial r \left(q^{neutral}, \mu \right)}{\partial b_i} f \left(\xi_i, \mu \right) d\xi_i d\mu = 0,$$

ie the liquidity of bank i after the tender must equal its share of total neutral liquidity. Its easy to see that this holds only when the aggregate bid of all other (than i) banks is their neutral (aggregate) bid, ie we must have $\frac{a + q^{neutral}}{n} = a_i + q_i^T = a_i + \left(q^{neutral} - b_{-i} \right)$, and hence, it is optimal for bank i to bid the liquidity up to the neutral level only if $b_{-i} = q^{neutral} - q_i^T = q_{-i}^T$.

expected value of the market rate being above the tender rate. As this incentive applies to all banks, it is not feasible to assume that the rest of the banks bid less than their neutral demand (ie the rest of the banks could increase their profits by increasing their bids). Consequently, we do not expect this kind of profit opportunity to become available to bank i. Similarly, if the aggregate bid of the other banks is larger than their neutral demand (ie $q_{-i}^T > q^{neutral} - q_i^T$), it is optimal for bank i to place a smaller bid than the neutral demand would imply, but to bid for more than the amount which would take the aggregate liquidity down to the neutral level (ie $q_i^T > b_i^* > q^{neutral} - q_{-i}^T$). With this kind of bid, bank i will increase its probability of being a borrower in the interbank market while the expected value of the market rate will be below the tender rate. Again the incentive applies for all banks, and thus we do not find feasible the presumption that the rest of the banks bid for more liquidity than neutral demand implies (the rest of the banks could in this case increase their expected profits by lowering their bids). Consequently, under full allotment, *the equilibrium where every bank bids for its neutral liquidity is unique.*

Case 2: Proportional allotment, ie $c < b \Rightarrow q_i = \frac{c}{b} b_i$

As the liquidity is determined (at least partly) by the preferences of the central bank, the expected value of the market rate of interest depends also on the central bank's target liquidity, c. When $\min(c, b) < q^{neutral} \Rightarrow \mathrm{E}[r] > r^T$, $\min(c, b) > q^{neutral} \Rightarrow \mathrm{E}[r] < r^T$, and $\min(c, b) = q^{neutral} \Rightarrow \mathrm{E}[r] = r^T$. We analyse separately the cases where the aggregate bid of other banks exceeds the central bank's target amount (ie $b_{-i} > c$) and where the bids of the rest of the banks is not large enough for the target to be fulfilled (ie $b_{-i} < c$).

We study first the case where $b_{-i} \geq c$. The market liquidity and hence the market rate of interest will depend only on the central bank's target (ie $\frac{\partial r(q, \varepsilon)}{\partial b_i} = 0$, as $\frac{\partial q}{\partial b_i} = 0$). Thus, bank i will choose bid so as to maximise the expected profit, which is simply the allotted amount of liquidity times the expected difference between the market rate of interest and the tender rate:

$$
\begin{aligned}
\max_{b_i} \mathrm{E}\left[\Pi_i\right] &= b_i \frac{c}{b} \left(\int_{\varepsilon^{\min}}^{\varepsilon^{\max}} r\left(c, \varepsilon\right) f\left(\varepsilon\right) d\varepsilon - r^T \right) \\
\text{s.t. } b_i &\geq 0.
\end{aligned}
\tag{2.17}
$$

We can formulate the following Lagrangian:

$$L\left(b_i, \nu\right) = b_i \frac{c}{b} \left(\int_{\varepsilon^{\min}}^{\varepsilon^{\max}} r\left(c, \varepsilon\right) f\left(\varepsilon\right) d\varepsilon - r^T \right) + \nu b_i,$$

from which we can derive the following FOCs:

$$\left(\frac{c}{b} - b_i c b^{-2} \right) \left(\mathrm{E}\left[r\left(c\right)\right] - r^T \right) + \nu = 0 \qquad (2.18)$$

$$\nu b_i = 0 \text{ and } \nu \geq 0. \qquad (2.19)$$

Clearly the optimal bid depends directly on the difference between the expected overnight rate with the central bank's liquidity target and the tender rate. The optimal bid is:

$$b_i^* = \begin{cases} b_i^{\max} & \text{, if } \mathrm{E}\left[r\left(c\right)\right] > r^T \\ \left[0, b_i^{\max}\right] & \text{, if } \mathrm{E}\left[r\left(c\right)\right] = r^T \\ 0 & \text{, if } \mathrm{E}\left[r\left(c\right)\right] < r^T \end{cases} \qquad (2.20)$$

When $\mathrm{E}[r\left(c\right)] > r^T$, the optimal bid would be the maximum bid a bank can place in the tender ($b_i^* = b_i^{\max}$). As this applies to all banks, the presumption that $b_{-i} > c$ is feasible.[13] However, if the target liquidity of the central bank is large enough to push the expected value of the market rate below the tender rate (ie $\mathrm{E}[r\left(c\right)] < r^T$), it would be optimal for bank i not to participate in the tender (or to place a zero bid). As this applies to all banks, the presumption $b_{-i} > c$ would not be feasible, and the central bank would not be in control of the liquidity. The case in which the market rate would equal the tender rate with the liquidity targeted by the central bank is, however, not as straight forward. The optimal bid for a single bank is anything from zero up to the maximum it can bid, as long as the bank can be certain that the banking sectorwise aggregate bid will at least equal the target level of the central bank (ie $p\left(b^* \geq c\right) = 1$). Otherwise, the expected liquidity would be below neutral liquidity (ie $\mathrm{E}[\min\left(c, b\right)] < c$). Consequently, the expected value of the market rate of interest would increase above the tender rate (ie $\mathrm{E}[r\left(c\right)] = r^T < \mathrm{E}[r\left(\min\left(c, b\right)\right)]$) and therefore the optimal bid would be $b_i^{\max}$.

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[13]With the natural implicit assumption that the possible limit for the bids is high enough, ie $b_i^{\max} > c$.

Consider next the second possibility, ie the case in which the aggregate bid of the other banks is below the target of the central bank ($b_{-i} < c$). In this case the determination of the optimal bid for bank i is the same as in the case of full allotment, as long as the aggregate bid of all banks remains below the target amount of the central bank (ie $b_i^* + b_{-i} < c$). Consequently, there will be a unique equilibrium in which all banks bid their neutral demand ($b_i^* = q_i^T$), if $q^{neutral} \leq c$. However, if $b_{-i} < c \leq q^{neutral}$, bank i's bid has an effect on the expected market liquidity, but the expected market rate of interest will be above the tender rate regardless of the size of b_i. Now, it can be shown that it is optimal for bank i to get as large a portion of the liquidity allotted to the market as possible, and sell the liquidity in excess of its own need to the market with positive expected profits. The rest of the banks could also increase their expected profits by increasing their bids. Thus, the presumption that $b_{-i} < c$ would not be feasible. Furthermore, with $q^{neutral} = c$, the optimal bid is $b_i^{\max}$ if $p(b \geq c) < 1$. Thus the presumption that $b_{-i} < c$ can hold only if $q^{neutral} \leq c$.

Now, by combining the two cases above, we may conclude that the optimal bid of bank i depends on the central bank's liquidity policy. Under the full allotment procedure, the equilibrium bid is q_i^T, for which the expected money market liquidity will always equal the neutral demand of the banks, and the expected market rate of interest will be at the level of the tender rate. When the central bank applies the proportional allotment procedure, the optimal bid depends on the difference between the target liquidity of the central bank and the neutral liquidity; when $c \leq q^{neutral}$ the optimal bid is $b_i^{\max}$,[14] and the central bank chooses the expected money market liquidity, and the expected liquidity is chosen by the banks at $q^{neutral}$ when $c > q^{neutral}$. That is, with fixed rate tenders, the central bank is able to raise the expected value of the market rate of interest (above the tender rate) by constraining the liquidity supply, but it cannot lower the expected rate below the tender rate.

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[14]We will analyse the maximum bid in section 4.

3 Liquidity policy of the central bank

Based on the analysis above, we expect the banks' optimal bid to depend on the difference between the central bank's liquidity target and the neutral liquidity. To understand why a particular path in the evolution of bids occurs, we must analyse what kind of liquidity policy the central bank applies. The alternative liquidity policy rules considered here are: full allotment, interest rate targeting (neutral liquidity policy), restricted liquidity supply, and liquidity targeting.

1. *Full allotment*

    The simplest procedure for the central bank to follow is the *full allotment* policy. With full allotment, the central bank always provides the market with all the liquidity bid for by the banks (ie $c = b \Rightarrow c/b = 1$). Under full allotment, we know that the equilibrium amount the banks bid for equals $q^{neutral}$, and consequently the expected market rate of interest will equal the current tender rate.

2. *Interest rate targeting rule (neutral liquidity policy)*

    In *interest rate targeting*, the central bank estimates the amount of liquidity demanded by the banking sector that will take the market rate to the level of the tender rate (ie $c = c^{irt} = q^{neutral}$; thus, we also call this procedure the neutral liquidity policy rule). Consequently, the expected market rate of interest equals the tender rate also with this procedure. From before, we know that the equilibrium bidding depends on wether the banks can expect the central bank to always be in position to control the liquidity. If the answer is yes, the optimal bid of a single bank i will be anything from zero up to the maximum amount the bank is able to bid for. However, when the bank is not able to count on $p\left(c^{irt} \leq b\right) = 1$, the optimal bid is the maximum bid it can place without facing any extra costs.[15]

    What could motivate the central bank to choose interest rate targeting over full allotment? Now, we have

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[15] The determination of the maximum bid is analyzed in section 4.

$$\mathrm{E}\left[r|l = a^{banks} + d^T + \varepsilon^{banks}\right] \ = \mathrm{E}\left[r|l = a^{CB} + c^{irt} + \varepsilon^{CB}\right] \ = r^T.$$
That is, the expected market rate equals the tender rate, with either procedure. However, if either of these two parties (banks or central bank) possesses private information on the evolution of autonomous factors or the functional form of the market rate as a function of liquidity, the probability of the amount of neutral liquidity demanded by the banks equalling the neutral liquidity estimated by the central bank is below one (ie $p\left(q^{T,CB} = q^{T,banks}\right) < 1$). Now, if the central bank has superior knowledge on the development of the autonomous factors, it might be able to contain the stochastic volatility of the market rate by controlling the expected money market liquidity.[16] Thus, basically the selection between interest rate targeting policy and the full allotment procedure is one between restraining the *stochastic* volatility of the market rate vs having the banks bid for more than their neutral demand amount.[17]

3. *Restricted liquidity supply*

As the third option, we consider a policy rule according to which the central bank provides the markets with less liquidity than is needed to keep the expected market rate of interest at the level of the tender rate (ie $c^{rls} < \mathrm{E}^{CB}\left[q^{neutral}\right]$). This *restricted liquidity supply* could be rationalised eg by asymmetric preferences of the central bank, as suggested by Ayuso and Repullo (2000). According to the asymmetric preferences argument, the central bank prefers deviations of the market rate above the policy rate (here the tender rate) to deviations below the policy rate. Consequently, the true interest

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[16]Note that the central bank can restrict the overnight volatility only if its superior knowledge of autonomous factors is great enough to compensate for the potentially inferior knowledge it has of the effect of interest rate expectations on the demand for liquidity. Another way by which the central bank might restrict the stochastic volatility of the overnight rate (with full allotment) would naturally be to make its private information public, and thus increase the accuracy of the banks' liquidity forecasts.

[17]Note that even if the stochastic volatility with full allotment were higher than with interest rate targeting, this volatility will not be transmitted to longer-term interest rates, as the expected value of the future overnight rate is not affected by this volatility. Consequently, the potential excess volatility resulting from full allotment does not interfere the signalling or transmission of monetary policy.

rate target of the central bank is above the policy rate. This can be achieved only by constraining the liquidity supply below the neutral liquidity level.

4. *Liquidity targeting rule*

The last option we consider here is *liquidity targeting*. When applying this procedure, the central bank is not only interested in the expected market rate of interest, but it will also pay attention to the level of liquidity in the money market. According to this rule, the central bank wants to provide the markets with liquidity, that will on average equal the amount needed to fulfill the reserve requirement *and* be as stable as possible throughout the remaining maintenance period. This means that even though the central bank allows the banks to average, it will try to prevent the banking sector as a whole from speculating on interest rate developments during the rest of the reserve maintenance period by timing the reserve holdings. Therefore, the bidding behaviour of the banks depends largely on the expectations of future interest rates during the rest of the period.[18] We denote the target liquidity of the liquidity targeting central bank by $c^{liquidity}$. Now, when the banks do not expect the central bank to change its interest rates during the rest of the current reserve maintenance period, we assume $q^{neutral}$ to be very close to $c^{liquidity}$ (ie there is no incentive for banks to either front- or backload reserve holdings). However, this not need be the case, especially if the reserve requirement is small relative to the standard deviation of liquidity shocks.[19] If the banks expect the tender rate to be raised in the near future, we assume that $q^{neutral}|_{r^{ef}>r^T} > q^{neutral}|_{r^{ef}=r^T} \approx c^{liquidity}$, as the banks would like to postpone their reserve holdings until the rate change has taken place. Consequently, the expected market rate will raise above the tender rate, and the banks will have an incentive to overbid in the tender operations. When the

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[18]Note that the market rate of interest with given liquidity is the higher, the higher the expected future value of central bank rates $\left(\frac{\partial r(l|r^{ef})}{\partial r^{ef}} > 0 \right)$. Consequently, the higher the expected future rates, the lower the neutral bid for bank i (ie $\frac{\partial q_i^T}{\partial r^{ef}} > 0$).

[19]See Välimäki (2001) for a detailed discussion of this issue.

139

banks expect the central bank to cut its rates in the near future, we assume that $q^{neutral}|_{r^{ef}<r^T} < q^{neutral}|_{r^{ef}=r^T} \approx c^{liquidity}$, as the banks would like to hold more reserves before the rate change occurs. Consequently, the banks will not bid enough for the central bank to be able to allot the target amount. Thus, with expectations of a rate cut, we expect liquidity targeting to operate like the full allotment rule.

Let us next try to analyse more closely the bidding behaviour of the banks with each of these liquidity policies, and also introduce the effect of collateral requirements on bids.

4   Collateral requirements, maximum bids and bid ratios

The dynamic path of the bid ratio (aggregate bids / allotted amount) will be different under each of the four liquidity policies described above. First, when the central bank uses the *full allotment* procedure, the bid ratio is naturally unity. However, the path under the other liquidity policies is largely affected by determination of the size of the maximum bid and/or the expected path of the tender rate during the rest of the maintenance period.

According to the *General documentation on Eurosystem monetary policy instruments and procedures* (ECB 2000d), the ECB may impose a maximum bid limit in order to prevent disproportionately large bids. However, the ECB did not explicitly announce any such limit while conducting fixed rate tender operations between January 1999 and June 2000. Furthermore, *the ECB requires counterparties to be in a position to cover the amount of liquidity they are allotted by a sufficient amount of eligible collateral.* If a counterparty is not able to provide the ECB with the required collateral, it may impose penalties on the counterparties. These sanctions may take the form of financial penalties or suspension of the counterparty from subsequent tender operations for a given period.[20]

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[20]A detailed description of the ECB's sanctioning regime in case of non-compliance with counterparty obligations, see ECB (2000 d, annex 6).

The profit for bank i from the liquidity it is allotted and which it will sell at the overnight market is simply the traded amount times the difference between tender rate and (comparable) market rate, as long as the bank has collateral to cover the whole amount it receives from the tender. The cost of acquiring liquidity in excess of the possessed collateral is basically defined by the sanctions regime. However, the banks are allowed to borrow the collateral they need from the market. The collateral cost of allotment q_i is given by:

$$\int_{q_{-i}}^{q_{-i}+q_i} h(x)\, dx,$$

where $h(x)$ is the marginal cost of an additional unit of collateral. Denote the amount of collateral bank i has without any extra cost by k_i; there will be a cost of acquiring extra collateral when the total allotment exceeds k. Furthermore, bank i must submit the collateral to the central bank before receiving the liquidity. We assume that, due to credit lines, each bank faces a limit (denoted by z_i) on collateral borrowing. Hence, if the allotment for bank i is larger than the limit (ie if $q_i > z_i$), the bank will fail to comply with the tender rules and will be sanctioned by $(q_i - z_i)\, r^{sanction}$. For the rest of the section, we assume that the limit for borrowing is always higher than the neutral liquidity for the banks (ie $z_i > q_i^T$). Therefore, the credit lines will reduce the banks incentive to bid only when it is optimal for the banks to 'overbid' (ie to bid in excess of the neutral demand for liquidity).

Under *full allotment,* if k is large enough to always cover the neutral demand of the banks (ie $q^{neutral} < k$), there is no extra cost to the bank due to the collateral requirement, and naturally the collateral requirement does not affect the equilibrium bidding. When k is below $q^{neutral}$, the collateral requirement will affect the equilibrium bidding. In this case the banks will continue to place bids such that the expected secured market rate of interest equals the tender rate. However, the equilibrium liquidity in this case is reduced to the level at which the private value of liquidity to the banks will be the sum of market rate of interest and the marginal cost of collateral.[21] Thus, scarcity of collateral reduces the equilibrium liquidity, but it will not

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[21]The proof for this is presented in appendix A.

move the expected value of (collateralised) market rate of interest away from the tender rate. Now, if the marginal cost of collateral increases with the allotted amount, we expect the collateralisation to reduce the banks' incentive to frontload reserve holdings when a rate hike is expected.

Based on section 2, we know that under *interest rate targeting* the optimal bid for bank i is anything from 0 to $b_i^{\max}$, if it can be certain that the banking-sector-wide aggregate bid is larger than c^{irt}. Otherwise, it is optimal for the bank to bid $b_i^{\max}$. The certainty is achieved only by supplying a bid that is greater than $c^{irt} - b_{-i}$. As both c^{irt} and b_{-i} are unknown at the time the bid must be placed, full certainty is achieved only by bidding at least the maximum amount the central bank's neutral target ($b_i^* = [c^{itr,\max}, b_i^{\max}]$). If this amount is not feasible (ie if $b_i^{\max} < c^{itr,\max}$), the optimal bid is $b_i^{\max}$.

Now, the question is, what defines the maximum bid with the ECB tender rules. As long as the interest-rate-targeting central bank is in control of liquidity ($b \geq c^{irt}$), bank i cannot have an expected profit between the market rate of interest and the tender rate.[22] Therefore, the bank should make a bid such that it will not face any extra costs from the actual allotment. Now, by bidding $b_i \leq z_i$, the maximum allotment for bank i is z_i ($q_i \leq k_i$), and it will under all circumstances avoid being short of collateral. If $z_i > c^{itr}$, bank i can be sure, that the control of liquidity is in the hands of the central bank by placing a bid in excess of the central bank's target ($c^{itr} \leq b_i \leq z_i$); hence, the equilibrium bid is anything from c^{irt} to z_i. However, if $z_i < c^{itr}$, the equilibrium bid of bank i depends on the probability at which the aggregate bid of the banks will be higher than the target amount of the central bank ($p(b > c^{irt})$). If this probability is close to unity, it is very unlikely that there will be a positive expected spread between market interest rate and tender rate. Thus, in such a case we expect bank i to bid z_i, as that is the maximum bid with zero probability of failing to meet the collateral requirement. Consequently, the aggregate bid will be $\sum_i z_i$, which would leave the central bank control of the expected

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[22]Note that the market rate of interest that the central bank targets must be the collateralized rate. If the central bank's target was set at the unsecured rate and $k < c^{irt}$, the collateralized market rate would be below the tender rate. Hence, the banks would behave as under full allotment and the central bank would not get enough bids to allot liquidity according to the target.

liquidity (ie $p\left(\sum_i z_i > c^{irt}\right) = 1$). In this case, we expect the bid ratio to be $\frac{z}{c^{itr}}$.

Under a *restricted liquidity policy*, the optimal bid for bank i is always $b_i^{\max}$. Now, unlike in the case of liquidity targeting, bank i will have an expected profit by trying to get as large a share of allotted liquidity as possible, as there is a positive expected spread between market rate and tender rate. If bank i's borrowing limit is greater than the total allotted volume (ie $z_i > c^{rls}$), the optimal bid is infinite or it bounded only by the requirement that the bid be a numerical value. Thus, we are interested here in how the maximum bid is determined when bank i would fail to comply by the collateral requirement after being allotted a large proportion of the total allotment (ie $z_i < c^{irt}$). The expected income for bank i from the tender is the expected market rate of interest multiplied by the amount allotted to the bank, while the expected cost is the tender rate multiplied by the allotted amount plus the expected cost from the non-compliance with the tender rules. When the bank estimates the allotment it receives with a given bid, it must make some assumptions on the bidding behaviour of other banks and the total amount the central bank will provide to the market. We denote the subjective probability density function over the bids of the other banks by $g\left(b_i\right)$. Now, the optimal bid for bank i is the outcome of the following maximization problem:

$$
\max_{b_i} \left\{ \left(\mathrm{E}\left[r|_{q=c^{rls}}\right] - r^T\right) \int_{\frac{c^{rls}-z_i}{z_i}b_i}^{b_{-i}^{\max}} \frac{c^{rls}}{b_{-i}+b_i} b_i g\left(b_{-i}\right) db_{-i} \right.
$$

$$
- r^{sanction} \int_0^{\frac{c^{rls}-z_i}{z_i}b_i} \left(\frac{c^{rls}}{b_{-i}+b_i}b_i - z_i\right) g\left(b_{-i}\right) db_{-i}
$$

$$
\left. - S \int_0^{\frac{c^{rls}-z_i}{z_i}b_i} g\left(b_{-i}\right) db_{-i} \right\}, \tag{4.1}
$$

where $\int_0^{b_{-i}^{\max}} \frac{c^{rls}}{b_{-i}+b_i} b_i g\left(b_{-i}\right) db_{-i}$ is the expected allotment to bank i, $\frac{c^{rls}-z_i}{z_i}b_i$ is the minimum value of b_{-i} for bank i not to fail to comply with the tender rules, $r^{sanction}$ is the penalty rate that the central bank applies for the amount of bid that is not covered with collateral,

and S denotes the fixed cost arising from non-compliance[23]. Now, differentiating equation (4.1) w.r.t. b_i yields the following FOC:

$$
\left(\mathrm{E}\left[r|_{q=c^{rls}}\right] - r^T\right) \left[\int_{\frac{c^{rls}-z_i}{z_i}b_i^*}^{b_{-i}^{\max}} \frac{c^{rls}b_{-i}}{b^2} g\left(b_{-i}\right) db_{-i} \right. \tag{4.2}
$$

$$
\left. - \left(c^{rls} - z_i\right) g\left(q_i^*\right)\right] = r^{sanction} \left[\int_0^{\frac{c^{rls}-z_i}{z_i}b_i^*} \frac{c^{rls}b_{-i}}{b^2} g\left(b_{-i}\right) db_{-i}\right]
$$

$$
+ S \frac{c^{rls} - z_i}{z_i} g\left(q_i^*\right)
$$

Equation (4.2) implicitly defines the optimal bid of bank i as a function of both the expected interest rate spread and the expected bids of the other banks. Although the economic intuition behind the FOC might be obscure, we can derive the following conclusions from it. The wider the expected (positive) interest rate spread, the larger the optimal bid. That is, the higher the expected profit from overbidding, the higher the expected cost the bank is willing to face from the possibility of failing the tender. Similarly, bank i's optimal bid grows as the expectation of the aggregate bid of the other banks increases. That is, the possibility of non-compliance with tender rules with a given bid diminishes when the rest of the banks bid for more liquidity; thus, bank i can increase its own bid to balance the expected gains with the expected losses. Furthermore, raising the sanctions (either penalty rate or fixed cost of failure) will naturally reduce the optimal bid.

With constant expected interest rate spread, the evolution of the bid amount, will depend mostly on the method of forming expectations of the aggregate bid of the rest of the banks, on which basis the bank also forms expectations of the forthcoming allotment ratio. Now, as $\frac{c^{rls}b_{-i}}{(b_{-i}+b_i)^2}$ is a convex function w.r.t. b_{-i}, we know by

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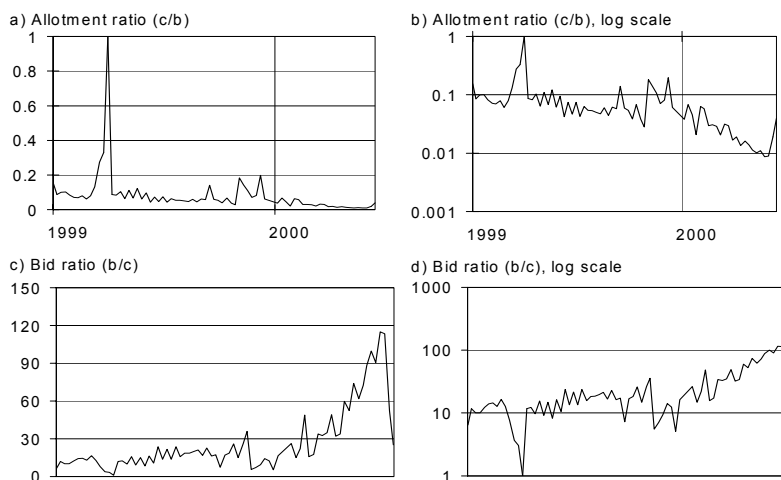[23]There is no fixed cost mentioned in the ECB rules for non-compliance with counterparty obligations; however, there is likely to be some sort of implicit reputational cost from the failure to cover the bid amount with eligible collateral. For example, the fact that the fed funds rate is sometimes below the discount rate is usually explained in the literature by the implicit cost related to the use of discount window. Furthermore, eg when discussing the behaviour of the treasurer in the main refinancing operations of the ECB, Vergara (2000, p. 17) mentions the bank's willingness to protect its reputation vis-à-vis the central bank as the major constraint for overbidding.

Jensen's inequality that the optimal bid of bank i increases with the accuracy of its subjective PDF of the bids placed by the other banks. Now, we might expect the uncertainty over the bidding behaviour of the other banks to be greatest at the first operation. Thus, in the first operation, the allotment for each bank is likely to be below the ex post optimal amount (which is naturally z_i). The realization of the bid ratio gives the bank new information about the bidding behaviour of the other bidders, and based on this information bank i can make a larger bid than it otherwise would have been able to make. However, the bank will expect the other banks also to behave in the same manner (ie to increase their bids), which itself leads to a further increase in the optimal bid. This train of reasoning will lead us to expect the optimal bids to increase over time. Thus, the bid ratio is likely to increase from tender to tender if the expected interest rate spread is constant and the ratio is expected to fluctuate more widely when the expected spread changes.[24]

Finally, under a *liquidity oriented allotment policy*, the bidding behaviour was shown to depend on the expected future tender rate. From above, we know that when a rate cut is expected this policy will work like full allotment, ie we expect a bid ratio of unity. When the central bank is not expected to change its rate, this policy should be similar to interest rate targeting. When a rate increase is expected, there will be a positive expected spread between market rate and tender rate, in which case the optimal bid will be given by equation (4.2). Therefore, the development of the bid ratio in time depends largely on expectations of the forthcoming tender rate. We would

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[24]Note that the spread need not be constant even if the asymmetry of the preferences is constant. Assume that the central bank would like the expected spread to be positive but as small as possible, subject to the requirement that more than 60% of the realisations of the spread should be positive. It can be shown that the difference between expected median of the market rate and its mean value may depend on the expectations over the development of the future tender rate. For example, with normally distributed shocks we expect that the market rate will be more often above its expected value than below it when an increase in the tender rate is expected. If a rate cut is expected, we expect the opposite to be true. Thus, in this case, we would expect the spread between the market rate and the tender rate to be smaller when an increase is expected than when a rate cut is expected. Also, the amount of liquidity to be allotted is larger when an increase is expected, which could also reduce the rate of growth of the bid ratio relative to the case when a rate cut is expected.

Figure 1: **Allotment and bid ratios in ECB fixed rate tenders**

a) Allotment ratio (c/b)

b) Allotment ratio (c/b), log scale

c) Bid ratio (b/c)

d) Bid ratio (b/c), log scale

expect the ratio to increase the faster, the higher the expected market
rate is relative to the current tender rate, and to collapse to unity
when a rate cut is expected.

We next look at the data from the ECB tenders to analyse the ECB's
liquidity policy and banks' bidding behaviour in these operations, in
light of the model built in sections 2–4.

5   Experience with ECB tenders

The allotment and bid ratios (ie c/b or b/c) from the ECB fixed
rate tenders during 7 Jan 1999 – 21 Jun 2000 are given in figure 1.
These charts clearly show that the ECB did not use the full allotment
procedure in its FRTs. Furthermore, the bid ratio seems to increase
(allotment ratio seems to decrease) over time. However, it is not
clear whether the growth of the bid ratio accelerated over time. We
cannot determine what motivated the banks in their bidding simply
by analysing the realised bidding behaviour. As we saw in section 4,

the increasing bid ratio could be a result of various different liquidity policy rules used by the central bank. Thus, we must analyse these ratios together with the available data on interest rate and liquidity.

We next attempt to assess what were the key factors affecting ECB liquidity provision, ie what kind of liquidity policy rule the ECB seemed to have followed. After that we will turn to analyse how the banks saw the ECB liquidity policy being driven (ie what could have caused the bid ratio to increase so much).

5.1 Liquidity provision of the ECB

5.1.1 EONIA spread

Figure 2 illustrates the overnight spread (ie EONIA[25] – main refinancing rate[26]) from the start of Stage Three until 23 Jun 2000. The figure draws attention to at least two separate features. First, there are regular spikes (upward and downward) in the spread. These spikes reflect the increased volatility of the overnight rate that is associated with ends of reserve maintenance periods, due to greater liquidity uncertainty in the last days of the period.[27] Another key feature is that the (average) spread seems to increase time. The average spread for the whole time period is 6.8 basis points (bps): while it is only 1.9 bps for the first half and 12 bps for the second half. The same figures are 10.0, 5.8 and 14.3 bps respectively, if we remove

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[25]EONIA (Euro Overnight Index Average) is a measure of the effective interest rate prevailing in the euro interbank overnight market. It is calculated as a weighted average of interest rates on unsecured overnight contracts on deposits in euro, as reported by a panel of contributing banks. (ECB, 2001)

[26]The *main refinancing rate* is the rate applied in ECB fixed rate tenders. Thus, if we use term *tender rate* in the empirical part of this paper, we refer to the main refinancing rate.

[27]The increase in the overnight volatility at the end of a reserve maintenance period is a typical feature of reserves averaging. This increase results from the fact that the interest rate elasticity of the demand for reserve balances increases as the banks' ability to average liquidity shocks diminishes toward the end of an averaging period (on the last day of the maintenance period there is no averaging possibility at all). A more strict statistical analysis of the days of the maintenance period can be found in Perez-Quiros (2000).

Figure 2: **Overnight spread, 4 Jan 1999–23 Jun 2000**
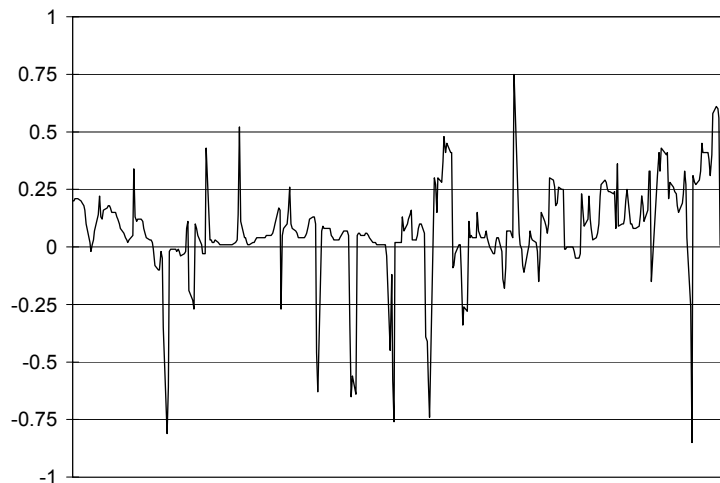
the end of each maintenance period.[28] Furthermore, the difference in the size of the average overnight spread between the subperiods (5 Jan–30 Sep 1999 and 1 Oct 1999–23 Jun 2000) is so large that we can reject the hypothesis of it resulting from stochastic variations of the spread (the null-hypothesis of the two average spreads equalling each other is rejected at all conventional confidence levels). Thus, there has been a shift (or several shifts) in market conditions toward tighter supply of liquidity relative to demand. Consequently, we expect to find a change (or several changes) in *either* ECB liquidity policy *or* liquidity demand conditions (or both) during the 18 months period in question.

Does a positive average overnight spread indicate that the ECB has provided the markets with liquidity that was on average below the natural demand described in chapter 2? Before considering the question, we should note, that there are at least two flaws in using the EONIA as a 'comparable market rate of interest' in the analysis. First, the maturity of EONIA is overnight whereas the maturity of

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[28]here, we have not included the spreads from the settlement day of the last tender operation until the end of each maintenance period. We have also left out the spread from 30 Dec 1999 due to the millennium effect on the EONIA. The spread was 75 bps on that day.

tendered liquidity is two weeks. The bias from the maturity difference is probably not so drastic with the interest rate as low as it was in the period analysed. Furthermore, this problem could be avoided by using effective interest rates. However, the second flaw might be more drastic. The EONIA is calculated from unsecured interbank deposits, whereas the tenders are fully collateralised operations. Thus, it may very well be the case that the EONIA should (on average) be a few basis points above the tender rate, even with neutral liquidity. Henceforth, we will call this difference between the two rates *the natural spread*. Furthermore, the natural spread need not be constant over time. Consequently, we must be very careful in drawing conclusions based on the EONIA spread as to the tightness of the ECB liquidity policy.

Now, the hypothesis of the EONIA spread being zero is rejected at every reasonable confidence level for the whole period as well as for the second half of the period. However, the hypothesis cannot be rejected, even at 10% significance level, for the first part of the period when the ends of periods are included. Still, it will be rejected even at the 1% significance level if the ends of each reserve maintenance periods are removed from the sample. This result means that either the ECB did not use interest rate targeting as its policy rule in liquidity allotment decisions or there exists a positive natural spread between the two rates. However, the spread for the second half of the period seems to be so far above zero (or the spread during the first subsample), that it probably cannot be explained by the risk premium associated with these unsecured overnight deposits.[29] That is, we reject the idea of the ECB applying a pure interest rate targeting rule (as defined in section 3) at least for the latter subsample. Still, we would not feel very comfortable in saying that the neutral liquidity policy rule should be rejected also for the first subsample.

The evolution of the overnight spread can be illustrated also by average EONIA calculated from the five days following each tender

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[29]The natural spread between the EONIA and the tender rate should be at least 8bps, for the null hypothesis (EONIA – tender rate = natural spread) not to be rejected even at the 1% significance level. Furthermore, the parameter estimate for allotment size is statistically insignificant when we regress the interest rate spread against the amount allotted in each tender. Hence, we do not expect the wide spread between unsecured market rate of interest and main refinancing rate to originate from scarcity of collateral.

Figure 3: **Average EONIA spreads and changes in main refinancing rate**

1= 5-day avg EONIA spread; 2= end of MP; 3= change in main refinancing rate
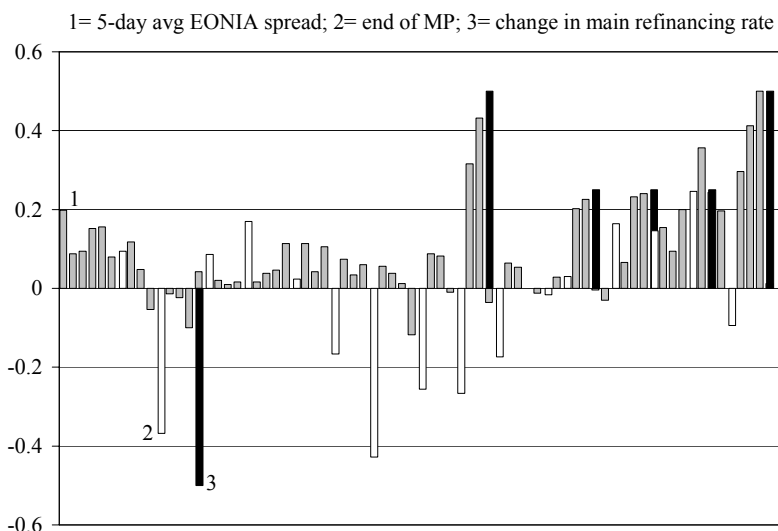
operation (days on which tender operations aim at affecting liquidity). This is done in figure 3. The grey bars in figure 3 show the five-day EONIA spreads, the white bars represent the average EONIA spreads following the last tender operation of each maintenance period, and the black bars show the changes in the tender rate.

The data behind figure 3 show, that on average the spread was positive and increasing over time. The 18 months of fixed rate tenders can again be divided into two subperiods. The first nine months are characterised by neutral or decreasing interest rate expectations (in this subsample there was only one rate cut and no increases), whereas the second half of the period is characterised by neutral or increasing interest rate expectations (there were five rate increases and no cuts during this subsample).

If the liquidity policy of the ECB was restrictive (ie liquidity provided was less than neutral liquidity), both these subperiods should (according to the model described in section 2) display positive spreads between the tender rate and a comparable market rate of interest. As regards the second period, it seems fair to conclude that on average the amount of liquidity provided to the market was

150

smaller than the neutral demand would have required; the natural spread between EONIA and the main refinancing rate should have been some 6.5 bps for the neutral liquidity policy not to be rejected at the 10% significance level. Furthermore, if the ends of reserve maintenance periods are omitted, the natural spread should have been 11 bps for the same significance level. Thus, we are again willing to reject the idea that the ECB used pure interest rate targeting (neutral liquidity policy rule), at least during the second half of the 18 months in question. However, these figures do not reject (with the same acceptance rules as above) the interest rate targeting hypothesis for the first part of the period. Still, the 10% significance level would require the natural spread to be some 3 bps for the neutral liquidity policy not to be rejected, if the ends of the maintenance periods are omitted. Furthermore, the overnight spreads differ so much as to between the two subsamples, that we can by all reasonable significance levels reject the assumption of the difference being a result of stochastic variations in liquidity. Hence, there must have been a change in the supply of liquidity relative to the demand. However, without analyzing the liquidity data, we cannot say whether this change in the relative liquidity supply results from a change in the liquidity policy rule used by the ECB (eg from interest rate targeting to a asymmetric preferences rule à la Ayuso-Repullo) or from increased demand for liquidity under a liquidity targeting policy rule.

One thing suggesting that a liquidity targeting rule might be behind the increase in the average overnight spread is that the EONIA spread tends to increase significantly before increases in the main refinancing rate.[30] This feature is quite apparent in figure 4, which shows the EONIA and the main refinancing rate as levels instead of as a spread. We expect the demand for liquidity to depend on interest rate expectations; as indicated in section 2, the banks try to profit from the averaging provision by frontloading reserve holdings when a rate increase is expected. Consequently, if the central bank does not increase the liquidity supply according to the increased demand (eg if the central bank uses liquidity targeting) the EONIA spread

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[30]The average figure for the spread after the two operations before the interest rate increase was 32 bps, whereas it was only 3 bps after other operations. If we omit the final operation of each reserve maintenance period the corresponding averages are 32 bps and 6 bps.

Figure 4: **EONIA and main refinancing rate**
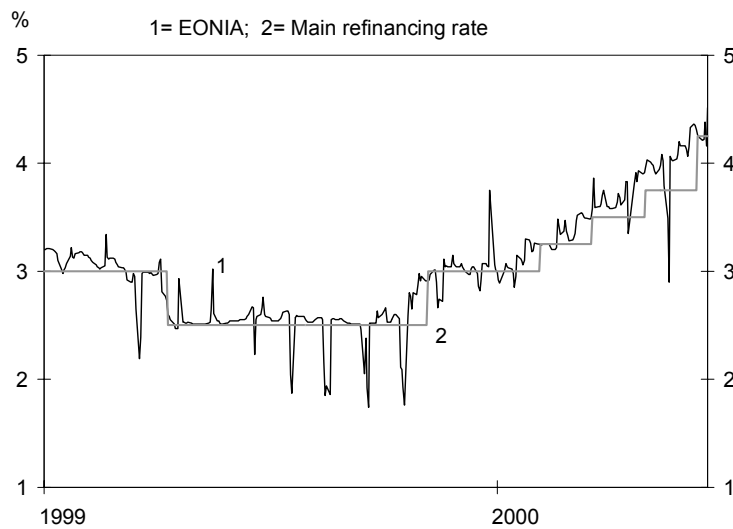
will react (increase) to these interest rate expectations. Thus, the behaviour we have seen in the EONIA spread could very well be a result of liquidity targeting.

We next review the liquidity data from the period in which the ECB used fixed rate tenders.

5.1.2 Liquidity provision

Bindseil and Seitz (2001, p.11) summarises the logic of ECB liquidity management as: '*The ECB attempts to provide liquidity through its open market operations in a way that, after taking into account the effects of autonomous liquidity factors, counterparties can fulfil their reserve requirement*'. This indicates that the ECB uses the reserve requirement as its benchmark for liquidity provision during the whole reserve maintenance period. However, this does not say anything about the timing of liquidity provision. That is, this statement does not necessarily mean that the ECB attempted to hold liquidity stable at the level of the requirement *within* the maintenance period. Furthermore, according to the ECB's Annual Report 1999,

152

Figure 5: **The use of standing facilities at the final day of each maintenance period**
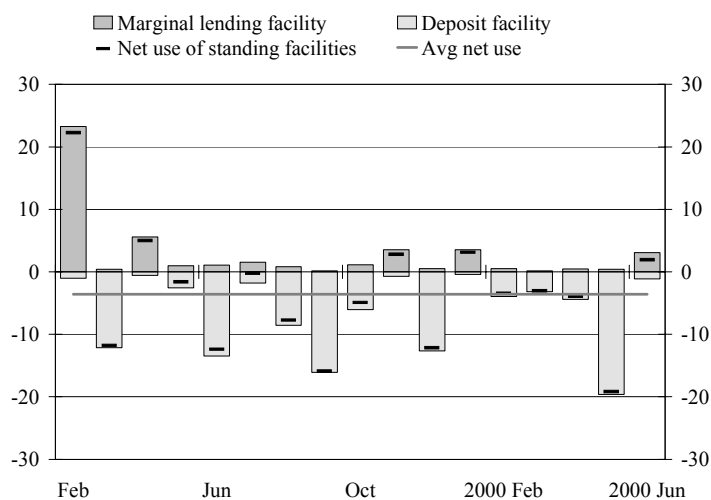
'*The ECB tended to orient its allotment decisions towards ensuring an average interbank overnight close to the tender rate*'. Thus, we expect that ECB liquidity policy would aim at controlling the price of liquidity, in addition to providing the liquidity required by the reserve requirement, at least during 1999. Let us next analyse the evidence of total liquidity provision during a reserve maintenance period. After that, we will examine the timing of reserve holdings.

The ECB seems to have provided the markets with at least a fair amount of liquidity (relative to the reserve requirement) over the whole reserve maintenance period. This is illustrated in figure 5, which shows the use of standing facilities at the end of each reserve maintenance period. Net use averaged -EUR 3.6 billion. That is, on average the amount of reserves deposited (on the final day of the reserve maintenance period) in the deposit facility (EUR 6.4 billion) was EUR 3.6 billions larger than the amount of liquidity credits acquired through the marginal lending facility (EUR 2.8 billion). This 'loose' total liquidity provision is also shown in the end-of-period spikes of the EONIA spread. The average spread calculated from the last banking day of each reserve maintenance period is -15 bps.

Figure 6: **Reserve balances in relation to reserve requirements**
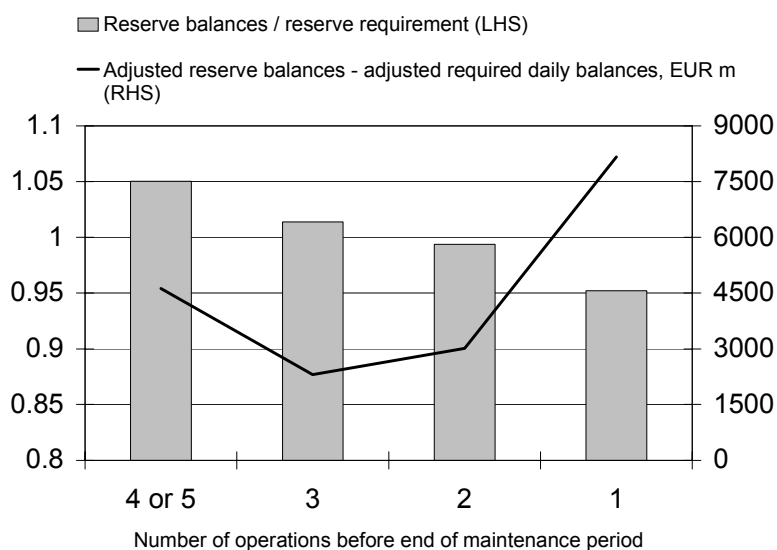
If the ECB provides the markets with liquidity in excess of what the banks need to meet their reserve requirements, the positive average EONIA spread (in excess of the neutral spread) must come from the banks' willingness to hold reserves earlier (during the maintenance period) than the central bank is willing to provide to them.

Figure 6 illustrates the timing of reserve holding during reserve maintenance periods. On average, the ECB did allow for some frontloading of reserves, by providing the banks with more liquidity in the early days of maintenance periods than in the later days. The level of reserve balances after the first operation (or first two operations when there were five operations in a maintenance period) was some 5% above the reserve requirement. After that, the amount of reserves in the market gradually declined from one operation to another. This decline did not mean that the level of reserves relative to the amount needed to fulfil requirement declined, as the need naturally declines when there have been reserves (in excess the requirement) during the early days of the maintenance period. In figure 6 this is illustrated by the curve, which indicates the amount of reserves in the market

154

in excess of the required daily balances.[31] We see that the ECB did (on average) provide the markets with more reserves than needed in order to fulfil reserve requirement in all operations during the reserve maintenance period. However, as the average EONIA spread still was above its natural level (at least during the second half of the time period), the banks on average wanted to frontload reserve holding by more than the ECB allowed.

We next study more closely the factors affecting the reserve provision of the ECB. We conduct a simple OLS-regression to measure the relative importance of i) *the banks' liquidity need arising from the reserve requirement* and ii) *the banks' interest rate expectations in the ECB's decision* on the amount of liquidity to be allotted. The regression equation is of the following form:

$$\text{average liquidity supply} = b_1 RDB + b_2 spread + b_3 spread^2$$

The liquidity variable to be explained is the average amount of reserves on the five banking days following the tender operation.[32] The observations for the last operation of each maintenance period are omitted from the regression. In this way we take into account that there is only one weekly operation, and that interest rate expectations affect mainly the demand for tender liquidity when there is still at least one operation remaining in the same maintenance period. The explanatory variables are the required daily balances for the remaining period (RDB) and the one-week EURIBOR spread (one-week EURIBOR – main refinancing rate). With the former, we measure the demand for liquidity resulting from the reserve

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[31]Required daily balances is the amount of liquidity that, if held daily (on average) until the end of the maintenance period, would just meet the reserve requirement (ie there would be no need for marginal lending or using the deposit facility). That is,

$$RDB_t = \frac{T \times RR - \sum_{j=1}^{t-1} RB_j}{T - (t-1)},$$

where T is the number of days in the maintenance period, RR the reserve requirement and RB_j the reserve balances held on day j.

[32]Note that this is the ex post money market liquidity ($l = a^{CB} + q + \varepsilon^{CB}$). It is not the amount of liquidity the ECB attempted to allot the markets, as it also contains expected autonomous factors and liquidity shocks. The reason for using this (publicly announced) ex post liquidity measure is simply that we did not have the figures for desired liquidity supply or liquidity shocks.

requirement. Thus, we expect b_1 to be close to one. The EURIBOR spread is used as an indicator of the banks' expectations of the average EONIA spread up to the following tender.[33] We allow the liquidity effect of interest rate expectations to be nonlinear by adding the square of the spread to the equation. This formulation should capture the possible concavity of the effect. The response of the central bank is not expected to be linear, as the effect of expectations on the demand for reserves is expected to be nonlinear. Furthermore, we expect the effect of interest rate expectations to be insignificant when the central bank is applying *pure* liquidity targeting and positive with interest rate targeting. However, it should be noted that, if the banks expect the central bank to follow pure interest rate targeting, there should not be much variability in the expected value of the EONIA spread. Furthermore, as the effect of interest rate expectations on the demand for liquidity is monotonically increasing, the estimated effect on the supply of reserves should also be monotonically increasing (over the relevant range of the EURIBOR spread) if the central bank applies pure interest rate targeting. The regression results are given in table 1.

Table 1: **Determinants of the supply of liquidity**

Dependent variable: average liquidity supply (EUR bn)			
Variable	Coefficient	Stand dev	t-probability
RDB	1.008	0.013	0.000
Euribor-spread	63.37	20.03	0.003
Euribor-spread^2	-185.8	49.92	0.001
Adj. R^2	0.63		
n	50		

The regression is based on a sample of 50 observations.[34] The parameter estimates are clearly statistically significant at the 1%

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[33] We will return the question of the appropriateness of the one-week EURIBOR as an indicator of the expected EONIA in the following section.

[34] There were 68 main refinancing operations between 24 Feb 1999 (the start of the first normal-length maintenance period) and 21 Jun 2000, of which 16 were the last operations of reserve maintenance periods. We have also excluded the one operation in which the allotment ratio was 100%, as the ECB was unable to determine the allotted amount in that tender. The operation settled on 30 Dec 1999 was also excluded due to the special circumstances of the millennium change.

significance level, for both required daily balances and the two interest rate variables.

The required daily balances seems to be the starting point for the ECB liquidity provision. This amount is adjusted by the interest rate expectations of the banks. The estimated effect of the EURIBOR spread on liquidity provision was concave. An increase in interest rate expectations raised the liquidity supply when the EURIBOR spread was less than 17 bps. At this level, the effect of interest rate expectations reaches its peak value of EUR 5.5 billion (ie approximately 5% of average liquidity). The effect of strong interest rate expectations on liquidity provision vanishes when the spread reaches 35 bps. This result would suggest that the ECB did not use pure liquidity or pure interest rate targeting as its guiding liquidity policy rule. The regressors explain 64% of the variations in liquidity supplied. Thus, one third of the liquidity variations result from stochastic shocks and other variables not included in the regression equation.

The effect of interest rate expectations on the liquidity supplied is illustrated in figure 7, where the EURIBOR spread is depicted on the horizontal axis. The diamonds in the figure are interest rate – liquidity observation pairs, where the liquidity measure is the difference between (average) liquidity supplied and estimated liquidity provision stemming from required daily balances ($1/5 \sum_{1=t}^{t+5} RB_i - 1.007572 RDB_t$).

The estimated reaction curve for the ECB shows that it did increase the supply of liquidity when the EURIBOR spread was positive and below some 34 bps. Hence, we can reject the hypothesis of ECB using a pure liquidity targeting policy. However, the reaction curve is far from being monotonically increasing over the range of interest rate observations. Thus, there seems to have been some kind of a loss function for the ECB, which included targets for both liquidity (required daily balances) and interest rate (tender rate). The ECB did smooth interest rate deviations from the target by supplying extra liquidity (compared to the liquidity target) when the EONIA was expected to be above the tender rate. However, with very strong interest rate expectations, the need for extra liquidity (to bring the EONIA closer to the tender rate) seems to have been so great that the ECB reverted back to stricter liquidity orientation in its allotment policy.

157

Figure 7: **Effect of the interest rate expectations on liquidity**
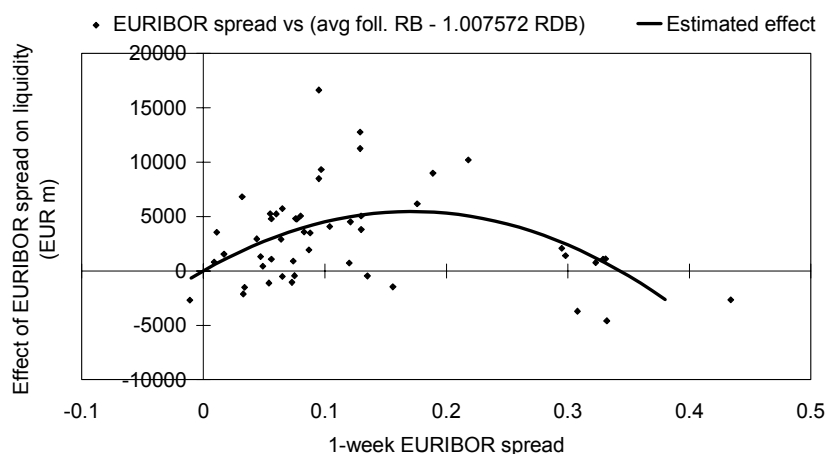
The estimated concavity of the liquidity effect (from interest rate expectations) depends largely on the observations with strong interest rate expectations (EURIBOR spread at some 30 bps or more). If we excluded the (eight) observations with EURIBOR spread greater than 25 bps from the regression, the maximum liquidity effect would increase to some EUR 9.3 billion (EURIBOR spread 32 bps). Furthermore, if we omit the observations with strong interest rate expectations, a linear response to the expectations fits the data better than the parabolic form. In this case, the estimated liquidity effect is an increase of EUR 0.40 billion per bp increase in the EURIBOR spread.[35] The estimation for the eight observations with strong interest rate expectations showed that very wide EURIBOR spread did not have a significant effect on the liquidity provided by the ECB.

In this section we have seen that on average the ECB's liquidity supply over the whole of each reserve maintenance period has not been restrictive. Also, the liquidity supply of the ECB before the ends of reserve maintenance periods was mainly driven by the liquidity need arising from the reserve requirement. However, the ECB seemed to allow for some frontloading of reserves when interest rate expectations

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[35] The esimation with linear interest rate effect and exculding observations with strong expectations yields the following result:

average liquidity supplied $= 1.012 RDB + 40068 Euribor \ spread.$

158

Figure 8: **1-week spread (1-week EURIBOR – main refinancing rate) on days of main refinancing operations**
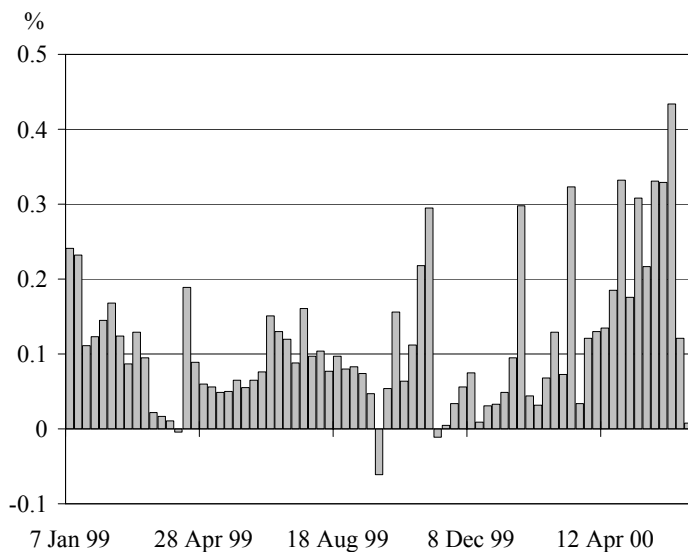
were not too strong but, when the expectations rose to a very high level, the ECB returned to simple liquidity targeting. That is the ECB did not use liquidity targeting in its purest form. It did pay some attention to keeping the EONIA close to the tender rate, but it did not allow the banks to speculate on interest rate changes by notably adjusting the timing of reserve holdings.

Let us turn next into the banks' perception of ECB liquidity policy.

5.2 Banks' perception of ECB liquidity policy in light of the data

According to the expectations hypothesis, we might use the one-week EURIBOR as an indicator of banks' expectations of the EONIA for the following week. Figure 8 illustrates the one-week EURIBOR spread (ie one-week EURIBOR – main refinancing rate) with the same settlement days as for the main refinancing operations.

159

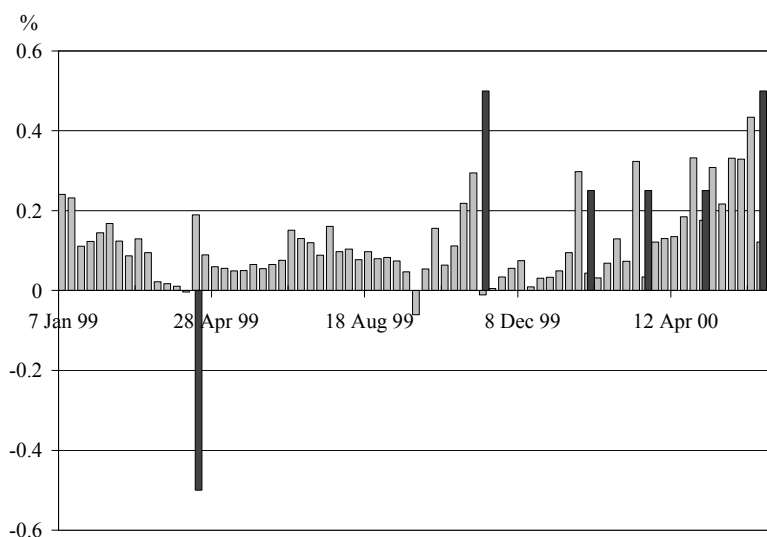Figure 9: **1-week EURIBOR spread and changes in main refinancing rate**

Figure 8 shows that most of the time the spread was (significantly) above zero. We must be careful not to compare apples with oranges in drawing conclusions on the significance of this spread. We saw earlier that there might be a natural positive spread between the EONIA and the main refinancing rate. Furthermore, there is no reason that the spread should not be wider for a one-week deposit than an overnight deposit. Thus, there may be a positive natural spread between the one-week EURIBOR and the main refinancing rate. Consequently, a (small) positive average spread between the one-week EURIBOR and the main refinancing rate need not indicate that the banks' assume the central bank to apply a restrictive liquidity policy. Furthermore, this natural spread need not be constant over time.

When we take into account the ECB's rate changes, we notice that the spread reacted in advance to policy rate changes (see figure 9). The average EURIBOR spread was 26 bps on the two tenders before each tender rate increase, whereas the average spread on other days of main refinancing operations was 9 bps. This indicates that the banks did not expect the ECB to have been using a pure interest rate targeting policy. The banks wanted to frontload their reserve holdings

160

Figure 10: **EURIBOR spread and bid volumes**
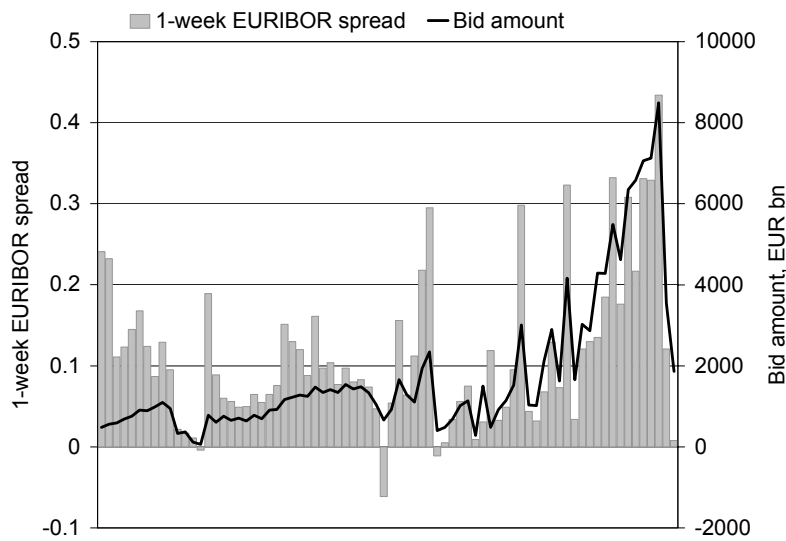
when an increase in the price of central bank reserves was expected, and they did not expect the ECB to fully adjust the liquidity supply for the increased demand. That is, the banks expected the ECB to conduct a liquidity oriented policy that would result in a (unusually high) positive spread between overnight rate and tender rate when an increase in the tender rate was expected.

The bid amount is represented along with the banks' interest rate expectations (one-week EURIBOR spread) in figure 10. This figure indicates that there is a close connection between interest rate expectations and the bidding behaviour – a phenomenon that we would expect to find when a liquidity-oriented policy is applied (or expected to be applied). Furthermore, the figure suggests that something must have restricted the rate of increase of the bids. The figure clearly illustrates, that the bid amount is not a function of the interest rate spread alone. For example, the bid amount at the tender settled on 3 Nov 1999 is almost three times that at the one settled at 10 May 2000, even though the EURIBOR spread is some 30bps at both operations. Most probably the element restricting bid size has been the possibility of non-compliance with the tender rules that originates from the collateral requirement, ie there seems to be an

161

upper limit for a bank's ability to cover the allotted amount with eligible collateral. The banks seem to have been able to bid the more boldly (at a given interest rate spread), the higher the bids in recent operations. This is just the reaction we would expect to see if the restricting element in bidding is a limit on the possibility of borrowing collateral from the market, and the banks use past bid sizes as a benchmark when they form expectations of bidding behaviour for the current tender.

We analysed the bidding strategy of the banks when (according to the model built in section 2) the optimal bid is the maximum bid by explaining aggregate bids at t by the average of bid ratios applied in the four most recent tenders, the one-week EURIBOR spread, and a trend component. According to the model, the banks should bid their neutral demand when the expected spread is negative. Consequently, we excluded the three observations with negative interest rate spread from the sample.

Now, in accordance with section 4, the optimal allotment for a bank depends positively on the expected spread between market rate and the tender rate. As the actual allotment to a bank is the bid it places in the tender multiplied with the allotment ratio, the optimal bid (without uncertainty) would be the bid ratio times the optimal allotment. However, in preparing their bids, the banks are unaware of the bids of the other banks (as well as of the amount of liquidity to be allotted), and so bid size is expected to increase with the product of the expected bid ratio and the expected interest rate spread. Its almost impossible to measure the banks' subjective probability density function for the bid ratio. Thus, we simply used the average bid ratio from the four previous tenders as an indicator of the expectation of the coming bid ratio.

Furthermore, we do not expect the interest rate spread and expected allotment to be independent. When the interest rate spread increases, the rest of the banks are likely to increase their bids, which bank i should take into account in deciding its bid. Thus, we included the product of the interest rate spread and average of past bid ratios in the set of explanatory variables. The functional form of this product term need not be linear. To explore the potential non-linearity, we used figure 11, which is a scatter plot with the aggregate bid amount on the vertical axis and the product of the average of past bid ratios and the interest rate spread on the horizontal axis. This suggested

162

Figure 11: **Bid amount vs product of average past bid ratio and interest rate spread**
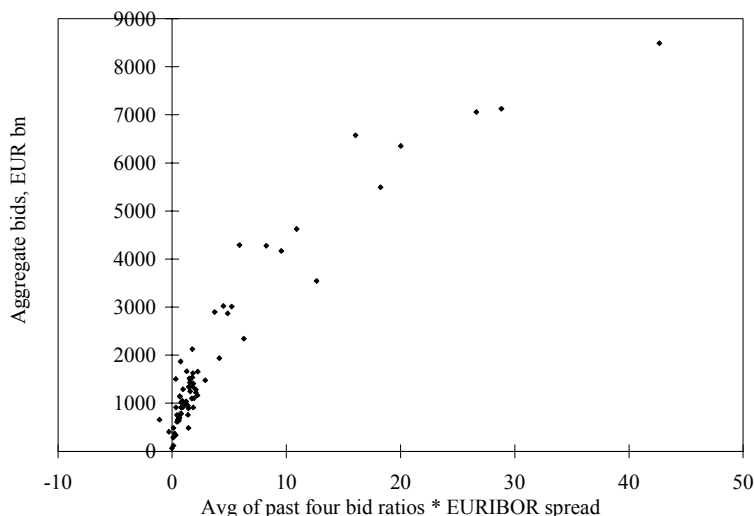
that the effect of the product of expected bid ratio and interest rate spread is of the second order, which indicated that, besides the direct product term, we should include its square in order to capture the non-linearities of the term's effect on bidding behavior. However, the effect of the product term need not be independent of the level of the individual factors within the term. Thus, we included in the estimation equation *both* the interaction of the product term and the interest rate spread *and* the interaction of the product term and the past bid ratio. Finally, we also introduced a trend to capture both the potential effect of the banks expecting the bids to steadily increase in time and/or to allow the limits for borrowing collateral to increase in time.

The estimated OLS regression equation took the following form[36]:

$$b_t = \beta_1 t + \left(\beta_2 + \beta_3 p_t + \beta_4 w_t + \beta_5 p_t w_t \right) p_t w_t,$$

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[36]Note that besides this formulation, we estimated a similar equation that contained in its set of explanators the direct interest rate spread (w) and the past average bid ratio (p). However, neither of the parameter estimates received a statistically significant value.

where b_t is the aggregate bid of the banks at t, p is the average
of the four previous bid ratios ($p = \frac{\sum_{i=t-4}^{t-1}\left(\frac{b_i}{c_i}\right)}{4}$), and w is the one-
week EURIBOR spread ($w = r^{\text{one-week } EURIBOR} - r^T$). The estimation
results are given in table 2.

Table 2.

Dependent variable: *Bids*[1]			
Variable	Coefficient	Std. Error[2]	t-probability
Trend	12.80	2.370	0.000
pw	787.1	89.93	0.000
pw^2	-1,308	274.6	0.000
p^2w	-5.012	0.8528	0.000
$(pw)^2$	10.54	2.557	0.000
adj R^2	0.974	n	69
DW	2.16		

[1] In EUR bn

[2] White heteroskedasticity-consistent standard errors

All parameter estimates are highly significant in the regression. The
variables also seem to explain fairly well the variations in the bid
amount. The accuracy of the estimation results can be evaluated
using figures 12 and 13, which present two illustrations of realised bid
amounts and fitted values.

According to the estimation, there was a positive trend in the
evolution of bids. The aggregate bid size tended to increase by some
EUR 13 billion from tender to tender. This trend growth might result
from the banks' increasing capacity to borrow collateral. However,
it could also result from the method the banks use in forming their
expectations of the coming bid ratio (ie the banks might have expected
the bid ratio to increase just slightly from tender to tender).

The estimation also shows that the product of average recent
bid ratios and expected interest rate spread has a very significant
direct impact on the bid amount. However, this effect depends on its
components p and w. The negative parameter estimates for pw^2, and
p^2w could be interpreted as resulting from the increase in uncertainty
concerning the coming bid ratio associated with a wider interest rate
spread and higher values of past bid ratios. Figure 14 illustrates bid
amount (in excess of the trend value) as a function of the expected

164

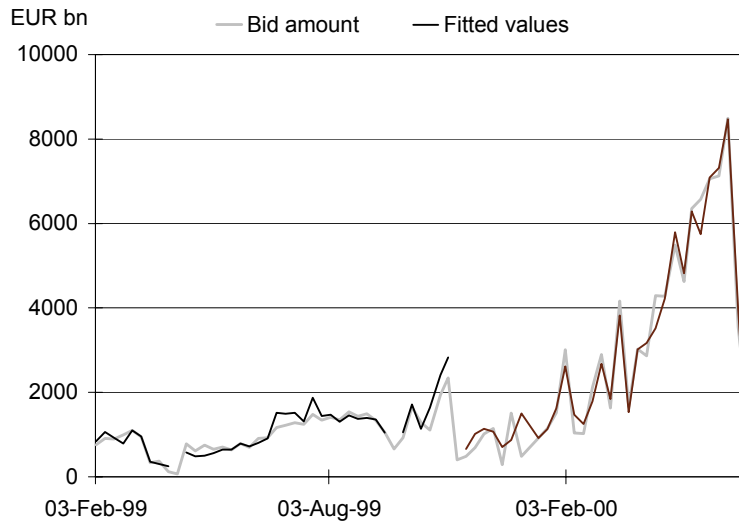Figure 12: **Bid amount and fitted values**

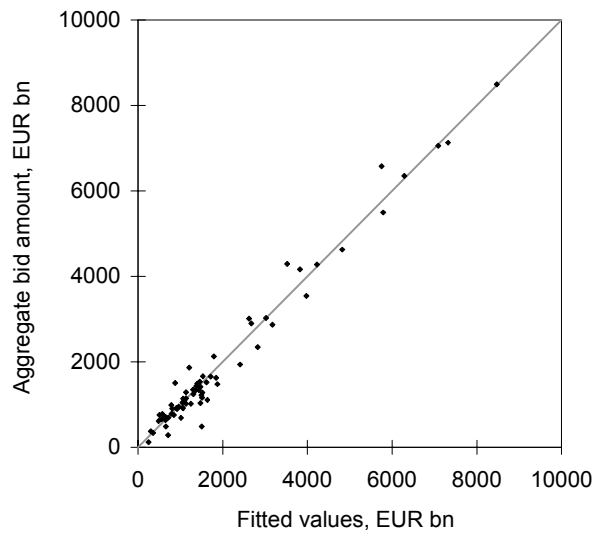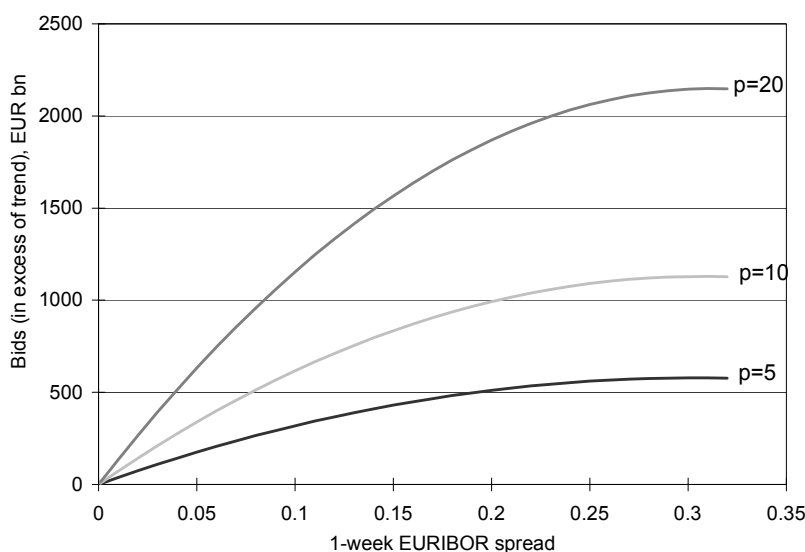Figure 13: **Scatter plot of bid volume and fitted values**

165

Figure 14: **Bid amount as a function of expected interest rate with different values for past average bid ratios**

interest rate spread for different values of past average bid ratios. The picture shows by how much (according to the parameter estimates) the aggregate bid of the banks would have been above the trend bid when the past bid ratio took the value of 5, 10 or 20. We can see that eg when the expected interest rate spread doubled from 10 to 20 bps, the bid amount increased from 617 to 993 (ie some 60%). Also, the interest spread at 10 bps would (with these parameters) have led to a bid of EUR 1 154 billion above the trend value when the past average bid ratio was 20 (instead of EUR 617 bn when the past bid ratio was 10).

To get an idea as to how the bid ratio evolves with such parameters, figure 15 shows the interest rate spread that leads to a bid ratio equal to the past average ratio, when the allotted amount equals its average value (EUR 68 billion). This spread depends on the trend value in the bidding. Thus, we calculated the equilibrium paths for four different points in time ($t = 10, 20, 30$ and 40). The figure shows that the higher the past bid ratio, the higher the interest rate spread needed to be for the bid ratio to remain at the level of

166

Figure 15: **Interest rates needed for static bid ratios**
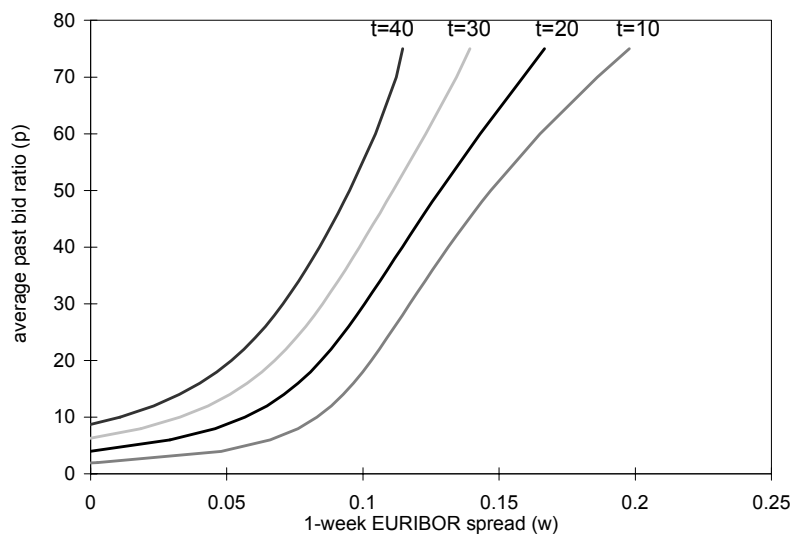
the past average. For example, with $t = 30$, the interest rate spread would have needed to be some 6.8 bps for $b_t/c_t = p_t = 20$, and 8.5 bps for $b_t/c_t = p_t = 30$. Furthermore, when the trend component in bidding increases over time, the interest rate spread needed for $b_t/c_t = p_t$ (with given p_t) diminished. For example, the spread needed for $b_{20}/68 = p_{20} = 30$ was 10.1 bps, while it was only 8.5 bps for $b_{30}/68 = p_{30} = 30$. Note however that over time the effect of the trend component on the bid ratio increases. For example, the effect of the trend on the bid is 128 at $t = 10$, while it is 384 at $t = 30$. Thus, with $c = 68$, the effect on the bid ratio is 1.9 and 5.6 at $t = 10$ and $t = 30$.

Finally, in order to analyse how well the banks' estimated the ECB's liquidity policy, figure 16 illustrates the difference between the one-week EURIBOR with settlement on settlement days of the tenders and the average of the EONIA rates from the date until the next settlement day.[37] The solid lines in the figure illustrate the

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[37] For example, for the tender operation settled at 3 Mar 1999 we use the one-week EURIBOR quoted on 1 Mar 1999 and the five EONIAs in 3–9 Mar, such that the Friday quotation is weighted by three, due its being in effect over the weekend.

Figure 16: **Spread between 1-week Euribor and average of following 5 EONIAs**
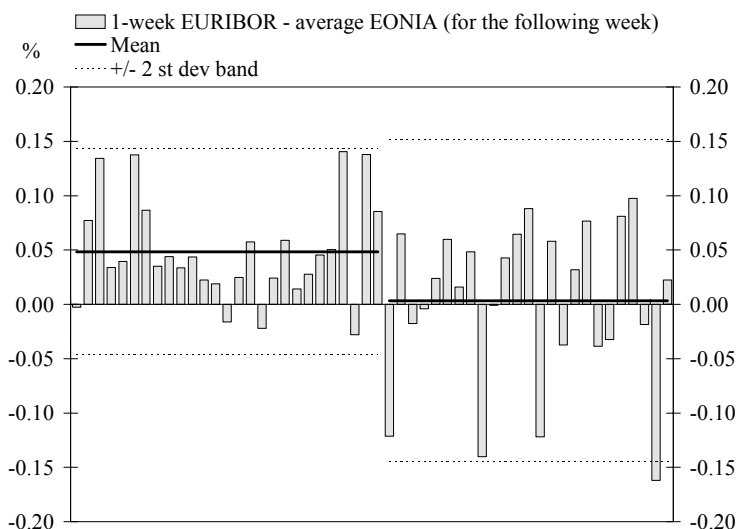
mean spreads for two subperiods. The break point dividing the total sample is at 20 Oct 1999, which is two operations before the first tender rate increase made by the ECB. The dashed lines give the two-standard-deviation bands for variations in the spread. During the first subperiod, the spread between the one-week EURIBOR spread and the average of the following EONIAs was statistically significantly above zero, whereas we cannot reject the null hypothesis for the spread being zero for the second subperiod. Furthermore, the difference between the two mean values is statistically significant at the 5% level. If the neutral spread between these two rates were stable throughout the total sample period, it would seem that the liquidity policy of the ECB was not as tight as the banks expected it to be during the first subperiod, while the banks seem not to have such bias during the second subperiod. The bias might have disappeared due to a tighter liquidity policy of the ECB under the interest rate hike expectations (ie during the second subperiod) than under the neutral expectations. Another possible explanation might be found from the banks' learning process concerning the liquidity policy of the ECB.

168

6 Summary and conclusions

In this paper we have constructed a model that describes the optimal bid of a single bank in money market tenders under various liquidity policies applied by the central bank. We saw that the bid amount depends crucially on the relation between the central bank's liquidity target and the neutral liquidity of the banks. With neutral money market liquidity, the private value of the liquidity for the banks equals the tender rate. When the amount of liquidity targeted by the central bank is above the neutral liquidity level, the banks will place bids such that money market liquidity will be neutral. If the target liquidity is at or below the neutral level, the banks will overbid, ie they will bid in excess of the neutral liquidity.

In section 3 we introduced four potential liquidity policies for the central bank: full allotment, neutral liquidity policy rule, restricted liquidity supply and liquidity targeting. The banks will bid for neutral liquidity, if the central bank applies full allotment or uses liquidity targeting (ie the central bank aims at stable liquidity conditions in the money market) and the banks expect a interest rate cut in the near future. Overbidding will occur under interest rate targeting (at least when the target rate is not below the tender rate), restricted liquidity supply (eg due to asymmetric central bank preferences as per Ayuso and Repullo 2000) or under liquidity targeting when the central bank is expected not to cut its rates in the near future.

In section 4 we saw that, when the liquidity allotted by the central bank in the tender needs to be covered with collateral, the amount of overbidding will be a function of the interest rate spread between expected market rate of interest and tender rate. Thus, the bid ratio (ie the aggregate bids / allotted amount) should behave differently under various liquidity policy rules, as the expected market rate of interest depends on the allotment decision rule applied by the central bank. With full allotment or when the banks expect the tender rate to be cut under liquidity targeting, the expected market rate will be at the level of the tender rate and the central bank will not be rationing the allotted amount. Thus, under these conditions we do not expect to see overbidding by the banks. However, under a neutral liquidity policy the bid amount will depend on the banks' collateral borrowing capacity, even though the expected market rate of interest will equal the tender rate also in this case. Under restricted liquidity supply,

the extent to which the banks will overbid, depends on the restriction rule of the central bank. For example, if the limited liquidity supply is based on preference asymmetry, the bid amount should reflect the effect of the asymmetry on the expected spread between market rate and tender rate. Finally, with liquidity-oriented allotment policy, the expected market rate will be a function of the expected future market rate, and for this reason the amount of bids in excess the neutral amount will also be positively correlated with interest rate expectations.

Section 5 studied the liquidity policy of the ECB and bidding of the banks against the model derived in the preceding sections. We showed that overall the liquidity provision of the ECB could not be considered as being restricted. On average, the ECB provide the markets with liquidity that was quite abundant compared to the reserve need based on the reserve requirement. Thus, we are not convinced by the argument that the ECB had asymmetric preferences over the sign of interest rate differences between market rate of interest and tender rate. However, there still seems to have been a significant positive spread between market rate and main refinancing rate, especially in tenders preceding tender rate increases by the ECB. Consequently, even though the overall liquidity policy of the ECB was not restrictive, the timing of the liquidity provision seems not to have met the demand of the banks. Furthermore, we saw that the reaction of the ECB to the banks' interest rate expectations was not unambiguous. The ECB increased its allotment from the level indicated by the reserve requirements when there was moderate expectations of tighter future interest rate policy. However, when the expectations were quite pronounced (ie when the spread between one-week EURIBOR and main refinancing rate was above 25 bps), the ECB seems to have reverted to tighter control of liquidity (ie the allotted amount seems to have been based solely on reserve requirements). This indicates that the liquidity policy applied by the ECB did not fall under pure interest rate targeting or pure liquidity targeting but was something in between. That is, the ECB put weight on both holding the market rate close to the main refinancing rate and trying to stabilise liquidity. When interest rate expectations became strong, the increase in the neutral amount of liquidity seems to have been so large from the viewpoint of stabilising liquidity that in such cases the ECB reverted to pure liquidity targeting policy. However, as all the cases of strong

170

interest rate increase expectations occurred in the second half of the period, we could not rule out the possibility of the ECB having applied liquidity policy based on interest rate targeting until the autumn of 1999.

The aggregate bid of the banks increased considerably during the period of fixed rate tenders. This was seen to result from two factors. First, during the period from the start of January 1999 until September 1999, the environment was characterised by neutral or falling interest rate expectations, whereas during the period from October 1999 until the change of the tender procedure in June 2000, interest rate expectations were either neutral or an increase in the tender rate was expected. These expectations (of a rate hike) were reflected in the spread between the one-week EURIBOR and the tender rate. That is, the banks did not assume that the ECB would adjust its liquidity supply (fully) to the increase in demand for liquidity with a rate hike expectations. Because the amount of liquidity the banks are willing to receive from the tender is the larger, the wider the spread between market rate and tender rate, each bank, with fixed rate tenders, was willing to take a bigger share of the total allotment in many tenders during the second half of the period than during the first half. Secondly, to get a certain allotment from a tender, a bank must place a bid that is the amount the bank is willing to take times the bid ratio to be used in the tender. The expectation of the coming bid ratio in a tender was seen to depend positively on the bid ratios of the recent tenders. Thus, the aggregate bid at a given expected interest rate spread was considerably larger during the latter half of the period. However, the bid amount was seen to grow already during the first half of 1999. According to our model, this indicates the banks expect a restricted liquidity supply in any the period in which the ECB was not expected to raise its rates. This could mean either that for some reason the banks prefer frontloading of the reserve holdings to stable liquidity or that the banks assumed the liquidity policy of the ECB to have been more restricted than it really was at the beginning of Stage Three of the EMU.

Finally, the inconsistency of simultaneously targeting the level of the market rate of interest and trying to hold liquidity stable within the reserve maintenance period leads to ever increasing bid ratios when a rate hike is expected. The remarkable increase in bid ratios (decline in allotment ratios) that occurred between October 1999 and

June 2000 led the ECB to change the tender procedure to variable rate tenders. With variable rate tenders, expectations of a rate hike will be immediately reflected in the tender rate. Thus, the banks' incentive to overbid in the operations is diminished. According to the model presented here, alternative methods for the ECB to overcome the declining allotment ratios would have been to give up the aim of stabilizing liquidity within a reserve maintenance period. This could have been done either by applying the full allotment procedure or by moving to interest rate targeting in a stricter from.

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A Full allotment with collateralisation

The problem of risk neutral bank i in the collateralised interbank market is:

$$\max_{s_i} \Pi = \int_{a_i+\mu/n+\xi_i+b_i}^{a_i+\mu/n+\xi_i+b_i+s_i} r_i^{pv}(x)\,dx - s_i r - \int_{b_i}^{b_i+s_i} h_i(x)\,dx, \quad \text{(A.1)}$$

The FOC is:

$$r_i^{pv}(a_i + \mu/n + \xi_i + b_i + s_i^*) - r - h_i(b_i + s_i^*) = 0 \quad \text{(A.2)}$$
$$\Rightarrow \quad r_i^{pv}(a_i + \mu/n + \xi_i + b_i + s_i^*) = r + h_i(b_i + s_i^*).$$

That is, the private value of liquidity after optimal interbank borrowing equals the sum of the (collateralised) market rate of interest and the marginal cost of collateral.

Equation (A.2) can be rewritten as:

$$s_i^* = r_i^{pv^{-1}}(r + h_i(b_i + s_i^*)) - (a_i + \mu/n + \xi_i + b_i). \quad \text{(A.3)}$$

Aggregating over the whole banking sector, yields the following equation:

$$\sum_{i=1}^{n} r_i^{pv^{-1}}(r + h_i(b_i + s_i^*)) = a + \mu + b, \quad \text{(A.4)}$$

from which we can derive the sum of market rate of interest and marginal cost of collateral as:

$$r + h_i(b_i + s_i^*) = r_i^{pv}\left(\frac{a + \mu + b}{n}\right). \quad \text{(A.5)}$$

Substituting equation (A.5) back into equation (A.3) yields:

$$s_i^* = \frac{a + b}{n} - (a_i + \xi_i + b_i),$$

which is identical to equation (2.8) in section 2.

The bank's maximisation problem at the tender becomes:

$$
\max_{b_i} \mathrm{E}\left[\Pi_i\right] = \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ s_i|_{b_i=0} \left[r\left(b_{-i}, \mu\right)\right] - s_i^* \left[r\left(b, \mu\right)\right] \right. \tag{A.6}
$$

$$
+ \int_{a_i+\mu/n+\xi_i+s_i^*|_{b_i=0}}^{a_i+\mu/n+\xi_i+b_i+s_i^*} r_i^{pv}\left(x\right) dx \Bigg\} f\left(\xi_i, \mu\right) d\xi_i d\mu
$$

$$
- \int_{s_i|_{b_i=0}}^{b_i+s_i^*} h\left(x\right) dx - b_i r^T
$$

$$
\text{s.t. } r_i^{pv}\left(a_i + \mu/n + \xi_i + b_i + s_i^*\right)
$$
$$
- r - h\left(b_i + s_i^*\right) = 0 \text{ and. } b_i \geq 0 \tag{A.7}
$$

from which we can derive the following Lagrangian:

$$
L = \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ s_i|_{b_i=0} \left[r\left(b_{-i}, \mu\right)\right] - s_i^* \left[r\left(b, \mu\right)\right] \right.
$$

$$
+ \int_{a_i+\varepsilon_i+s_i^*|_{b_i=0}}^{a_i+\varepsilon_i+b_i+s_i^*} r_i^{pv}\left(x\right) dx \Bigg\} f\left(\xi_i, \mu\right) d\xi_i d\mu - \int_{v_i|_{b_i=0}}^{b_i+v_i^*} h\left(x\right) dx - b_i r^T
$$

$$
- \lambda\left[r_i^{pv}\left(a_i + \mu/n + \xi_i + b_i + s_i^*\right) - r - h\left(b_i + s_i^*\right)\right] - \nu b_i.
$$

The FOCs for the maximization problem are:

$$
\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ -s_i^* \frac{\partial r\left(b, \mu\right)}{\partial b_i} - r\left(b, \mu\right) \frac{\partial s_i^*}{\partial b_i} \right. \tag{A.8}
$$

$$
+ r_i^{pv}\left(r_i^{pv^{-1}}\left[r\left(b, \mu\right) + h\left(b_i + s_i^*\right)\right]\right)\left(1 + \frac{\partial s_i^*}{\partial b_i}\right) \Bigg\} f\left(\xi_i, \mu\right) d\xi_i d\mu
$$

$$
- h\left(b_i + s_i^*\right)\left(1 + \frac{\partial s_i^*}{\partial b_i}\right) - r^T = 0,
$$

$$
r_i^{pv}\left(a_i + \mu/n + \xi_i + b_i + s_i^*\right) - r - h\left(b_i + s_i^*\right) = 0 \text{ and} \tag{A.9}
$$

$$
b_i \geq 0, \tag{A.10}
$$

which can also be represented as:

$$
\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} s_i^* \frac{\partial r\left(b, \mu\right)}{\partial b_i} f\left(\xi_i, \mu\right) d\xi_i d\mu = \tag{A.11}
$$

$$
\mathrm{E}\left[r\left(b, \mu\right)\right] - r^T,
$$

$$
r_i^{pv}\left(a_i + \mu/n + \xi_i + b_i^* + s_i^*\right) - r - h\left(b_i^* + s_i^*\right) = 0, \tag{A.12}
$$

$$
\text{and } b_i \geq 0, \tag{A.13}
$$

176

where equation (A.11) is similar to equation (2.15).

Now, we have seen that at the equilibrium all banks will be bidding for neutral liquidity under full allotment, even if we introduce collateral cost into the model. However, the neutral amount of liquidity (the amount that takes the market rate of interest to the level of the tender rate) is lower if borrowing is costly due to collateral requirements. This is obvious since if there are no collateral costs, $r_i^{pv}\left(q_i^{T,\,\text{no coll. req.}}\right) = r^T$, whereas under a costly collateral requirement, $r_i^{pv}\left(q_i^{T,\,\text{costly coll.}}\right) - h_i\left(q_i^{T,\,\text{costly coll.}} + s_i^*\right) = r^T$. Thus, the neutral liquidity decreases due to the cost of collateral $(q_i^{T,\,\text{no coll. req.}} > q_i^{T,\,\text{costly coll.}})$.

Variable rate liquidity tenders

Contents

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Abstract

This paper constructs an equilibrium model for the short end of the money market, in which the central bank provides liquidity via variable rate tenders. The relation between market rate of interest and liquidity is derived from a single bank's profit maximisation problem in the interbank market, and the CB determines its liquidity provision by minimising a quadratic loss function that contains both deviations of expected market rate from CB target rate and differences between liquidity supply and target liquidity. We model equilibrium bidding behaviour in the tenders and explain the underbidding phenomenon resulting from the minimum bid rate. We also show that, when maturities of consecutive operations overlap, the expected market interest rate will rise above the CB's target whenever a target rate change (hike or cut) is expected to occur in the same reserve maintenance period. Finally, we review the data from the ECB variable rate tenders and find that the ECB has been fairly liquidity oriented in its allotment decisions.

1 Introduction

The operational framework of a central bank has significant effects on the behaviour of the money market rates. Eg Prati, Bartolini and Bertola (2002) study the interbank overnight markets of the G7 countries and euro area and find that the day-to-day behaviour of short-term interest rates are more likely to reflect institutional arrangements than market frictions. Hence, it is far from being clear that models describing the federal funds market can be applied to the case of euro money markets, as the operating procedures applied by the European Central Bank (ECB) differ in many respect from those applied by the Fed.[1] The purpose of this paper is to analyse the determination of the short-term money market equilibrium in an operational framework similar to the one applied by the ECB. Probably the most significant change in the operating procedures of the ECB during the first three years of operation, was the adoption of variable rate tenders in the main refinancing operations (weekly liquidity auctions) in place of the fixed rate tenders that were applied until June 2000. In this paper we concentrate in the case where the MROs are executed as variable rate tenders. The analysis of fixed rate tenders can be found in Välimäki (2001 and 2002).

According to the ECB the shift to variable rate tenders was '*a response to the severe overbidding problem which has developed in the context of the fixed rate tender procedure*'.[2] Välimäki (2001) shows that overbidding is an equilibrium feature in fixed rate liquidity tenders when the central bank applies proportional allotment procedure. Furthermore, Välimäki (2002) claims that the sharp increase of the bid amount vs the amount of reserves that actually

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[1]For example, contrary to the Fed the ECB restricts the fluctuation of the overnight rate by an interest rate corridor created by the standing facilities. Also, the ECB steers liquidity mostly by weekly tenders, whereas the Fed operates almost daily on the markets; the reserve requirements in the Eurosystem are fully remunerated whereas the Fed does not remunerate the reserve holdings; the reserve requirements of the ECB are large compared to the volatility of the liquidity, whereas the effectiviness of the reserve requirements as a stabilator of liquidity demand in US is some times questioned. For further details on the operational framework of the ECB see '*The Single Monetary Policy in Stage Three; General Documentation on ESCB Monetary Policy Instruments and Procedures*' (ECB, 2000).

[2]ECB press release dated 8 June 2000.

was provided to the market in the ECB fixed rate tenders resulted from a combination of expectations of an interest rate hike and the ECB's liquidity-oriented allotment policy. Basically the problems the ECB experienced with fixed rate tenders were consequences of the goal of smooth reserve holding during a reserve maintenance period while applying a fixed price. If a monopoly supplier of a good has problems in fixing both price and quantity, the natural solution is to let the market decide one of these. A shift to variable rate tenders is one way of letting the price adjust to demand, while a change from proportional allotment to full allotment[3] would basically mean that the adjustment is done on the quantity side. The ECB adopted variable rate tenders on 28 June 2000. However, the auction format that the ECB introduced is not a pure variable rate tender, as the ECB also introduced a minimum bid rate, which is the lower limit for bids in the tender. *Therefore, when a rate cut is expected by the banks, an ECB tender format now functions very much like a fixed rate tender.*

The closest reference to this paper is Ayuso and Repullo (2000). They analyse both the fixed and variable rate tenders of the ECB, and show that variable rate tenders also have multiple equilibria characterised by varying degrees of overbidding. However, by publishing the intended allotment volume, an equilibrium without overbidding can be obtained. In their two-period model there is one liquidity tender, in which the central bank minimises a loss function that depends on the squared difference between the interbank rate and the central bank's target rate. The expected market rate of interest will differ from the central bank's target rate when the loss function penalises more heavily market rates below the rate. In our model there is one operation on each day of a two-day a maintenance period. Thus, *the expected central bank rates for the second period will affect the amount of liquidity demanded already at the first period.* Furthermore, our model central bank has a loss function that penalises *both* differences between the market rate of interest and the central bank's target rate *and* deviations of money market liquidity from the steady path of reserve holding. Therefore, *expectations of second-period rates will affect the expected first-period market rate of interest.*

---

[3]Full allotment refers to the case in which the central bank accepts in full all bids placed in a fixed rate tender, ie the bids are not scaled down.

Also, the effect of a minimum bid rate in our model is different from that in the Ayuso and Repullo model.

Bindseil (2002) also analyses the ECB open market operations. His approach, however, differs considerably from that of this essay. Bindseil takes the martingale hypothesis as given and on that basis forms the equilibrium condition for aggregate, whereas the hypothesis does not necessarily hold with the micro foundations developed here. Furthermore, Bindseil focuses on the case where the CB's allotment decision is based on a rigid liquidity target, while we allow for a richer set of possible liquidity policies. Finally, a recent paper by Nyborg et al (2002) analyses empirically the bidding in the ECB's main refinancing operations using microdata from the 53 first auctions following the switch to the variable rate procedure.

The standard literature on multiple unit auctions is not directly applicable to central bank liquidity tenders, as in those auctions the seller does not usually maximise its revenue from the auction. However, in deciding on the auction format, the central bank should not be immune to the lessons that can be learned eg from Back and Zender (1993 and 2001), who show that a sealed bid uniform-price auction may lead to equilibria in which the price actually paid in the auction (here the tender rate) is considerably below the true value (here, the corresponding market rate of interest).

The rest of the paper is organised as follows. First, we briefly review the main features of the operational framework the ECB applies. In section 2 we model the demand for liquidity in the interbank money market for both days of a two-day reserve maintenance period. We can use these demand functions to derive the equations that determine the market rate of interest as a function of liquidity. In section 3 we model central bank behaviour in its allotment decisions. In the model the central bank minimises a quadratic loss function in which deviations in both liquidity and expected interest rate from target levels can be taken into account. Section 4 describes the banks' bidding in these tenders. Also the effects of minimum bid rate and overlapping maturities of consecutive tenders are analysed. Section 5 briefly reviews the experience with the ECB variable rate main refinancing operations and section 6 summarises and draws some conclusions.

1.1 ECB operational framework in brief

To put it briefly, the ECB uses three different types of monetary policy instruments: i) active liquidity management is conducted via open market operations, ii) the banks are provided with standing facilities, and iii) all credit institutions are subject to reserve requirements.

First, the ECB conducts main refinancing operations (MRO) once a week. The role of the MROs is to provide liquidity to the banks and to signal the monetary policy stance of the Eurosystem. These operations are liquidity providing tenders with two-weeks maturity. They can be executed in the form of fixed rate (FRT) or variable rate tenders (VRT). In a FRT the interest rate is specified by the ECB in the tender announcement, while in a VRT the counterparties of the ECB bid in terms of both the amount of reserves they want to obtain and the interest rate at which they wish to enter into transactions. A VRT can be conducted using the multiple or single rate procedure[4]. The ECB arranges the bids in descending order (in terms of bid rate) and accepts the highest bids until the amount of liquidity to be provided to the market is allotted. The lowest rate at which bids are accepted is called the marginal rate. The ECB may restrict the supply of liquidity at the marginal rate. If this is the case, the ECB applies pro rata rationing for these bids. In the multiple rate procedure, the allotment interest rate for each accepted bid is the interest rate offered at the given bid, while in the single rate procedure the marginal rate of the allotment is applied to all accepted bids.

In addition to the MROs, the ECB monthly conducts a longer-term refinancing operation. However, as the ECB does not use these to signal monetary policy stance or actively manage liquidity conditions, we exclude these operations from our analysis.[5] Furthermore, the ECB may execute irregular operations to fine tune liquidity conditions. These operations have been extremely rare during the first three years of Stage Three of EMU.[6]

---

[4]The terms multiple and single rate procedures are applied by the ECB. In auctions literature these procedures are also known as discriminatory price (or American auction) and single price (or Dutch) auction respectively. Note that outside the financial literature the term 'Dutch auction' has a different meaning.

[5]*'In these operations, the Eurosystem does not, as a rule, intend to send signals to the market and therefore normally acts as a rate taker.'* (ECB 2000, p. 15)

[6]Only four fine tuning operations were conducted between 1 Jan 1999 and 31 Dec 2001.

184

The two standing facilities the ECB offers to the banks are the *marginal lending facility* and the *deposit facility*. The banks may obtain overnight reserves from the ECB through the marginal lending facility, the rate for which is pre-specified by the ECB. This *marginal lending rate* (r^m) provides a ceiling for the interbank overnight rate of interest. Furthermore, banks can place overnight deposits with the ECB's deposit facility at the pre-specified interest rate. This *deposit rate* (r^d) provides a floor for the market overnight rate.

In addition to open market operations and standing facilities, the ECB requires that banks hold deposits with the Eurosystem. However, the ECB applies an averaging provision to banks' reserve holdings. That is, compliance with the reserve requirement is determined by the average of end-of-day balances of the reserve accounts with the Eurosystem. The reserve maintenance period is one month (from 24th of the calendar month to the 23rd of the following month).

Finally, overdrafts are forbidden in the ECB framework. Consequently, the end-of-day debit balances on the banks' reserve accounts held with the central bank must be covered by lending from the marginal lending facility.

2   Interbank market

The evolution of liquidity during a day is assumed to be as follows. The reserve balance of a bank at the beginning of the day is the end-of-day balance of the previous banking day (RB_{t-1}). The bank estimates the effect of changes in autonomous liquidity factors (a_t: assumed here to include the effect of maturing central bank operations). The forecast error of this estimate is the liquidity shock the bank encounters. We assume that part of the shock (μ_i) is realised before settlement of the overnight market, which is assumed to occur at a given moment of the day, while the rest of the shock (ε_i) is realised after the overnight market is closed. The amount of reserves the bank receives (before the interbank overnight trading) at the tender is denoted by q_i. Thus, bank i's liquidity at the overnight market (l_i) equals $RB_{t-1,i} + a_i + q_i + \mu_i$. The net borrowing of bank i from the interbank market is denoted by b_i. Therefore, its end-of-day balance

is $RB_{i,t} = l_i + b_i + \varepsilon_i$, unless the bank has to use the standing facilities. If the cumulative reserve holdings of bank i are larger than the reserve requirement for the whole period, it will place the excess reserves in the deposit facility. Furthermore, the bank must obtain reserves from the marginal lending facility if its end-of-day balances would otherwise be negative or otherwise fail to comply with the reserve requirement (relevant only on the last day of the reserve maintenance period).

Reserve balances held on different evenings of the same reserves maintenance period are perfect substitutes for each other as regards the reserve requirement. Also, all units of liquidity are identical irrespective of whether borrowed from the central bank or the interbank market. Since trading in the interbank market takes place after the tender, we assume that the banks seek liquidity from the interbank market in order to comply with the reserve requirement at minimum cost, whereas the demand at the central bank tender depends solely on the expected profit opportunity between the price of liquidity at the tender and the expected value of it in the interbank market. The expected market rate depends on the amount of liquidity provided to the market at the tender, while the bidding of the banks depends on expectations of the market rate. We will approach this problem by first modelling the demand for liquidity at the interbank market as a function of total money market liquidity. Based on the demand functions, we derive the market rate of interest as a function of money market liquidity. After that, we model the central bank's intended liquidity supply so that it takes into account this relation as a constraint on the loss function it minimises.

The cost of obtaining reserves from the market is the overnight rate of interest; while the return on reserves depends on the second liquidity shock. The yield on reserves borrowed from the interbank market is the marginal lending rate (r^m) for the amount of reserves that is a substitute for acquiring liquidity from the marginal lending facility, ie if the bank's reserve balance after the second liquidity shock is negative (or below the required reserves at the end of the last day of the maintenance period). The yield on balances in excess of the requirement for the whole maintenance period is the deposit rate. The expected value of positive balances (that are below the amount that would fulfil the reserve requirement for the whole period) is the expected value of the reserves on the following day(s), as the reserve balances for today and tomorrow are substitutes for each other.

The length of the reserve maintenance period must be at least two days, in order for interest rate expectations to affect the demand for euro. Hence, we will develop a model of the demand for reserves in a two-day maintenance period in order to keep the model as tractable as possible while still maintaining the effect that arises from interest rate expectations. Demand functions for longer maintenance periods can be found in Välimäki (2001). We begin the modelling from the second day of the period (when averaging is no longer possible), as the maximization problem for the first day (with the averaging possibility) must be solved recursively using the result for the following day.

2.1 Final day (no averaging)

The profit maximization problem of a risk-neutral atomistic bank in the interbank market on the final day of the reserves maintenance period is the following:

$$
\max_{b_{i,2}} \mathrm{E}(\Pi) \;=\; r_2^m \left[\int_{-\infty}^{-l_{i,2}+rdb_{i,2}-b_{i,2}} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right]
$$

$$
+ r_2^d \left[\int_{-l_{i,2}+rdb_{i,2}-b_{i,2}}^{\infty} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right]
$$

$$
- r_2 b_{i,2}, \tag{2.1}
$$

where $b_{i,2}$ is bank i's net borrowing from the interbank market, $f(\varepsilon_2)$ is the *pdf* of the second shock of the day, and $rdb_{i,2}$ is the amount of reserves with which the bank would exactly meet its reserve requirement[7].

---

[7]The required daily balance for the rest of the maintenance period ($rdb_{i,2}$) is calculated from the reserve requirement per day (R), and the amount of reserves already held as reserve balances within the current reserves maintenance period:

$$
rdb_{i,t} = \frac{T * R_i - \sum_{k=1}^{t-1} RB_{i,k}}{T - (t-1)},
$$

where T is the number of days in a maintenance period. Thus, on the first day of a two-day maintenance period, we have $rdb_{i,1} = R_i$; on the second day, $rdb_{i,2} = 2R_i - RB_{i,1}$.

The first-order condition with respect to interbank borrowing is:

$$(r_2^m - r_2^d)F\left(-l_{i,2} + rdb_{i,2} - b_{i,2}^*\right) + (r_2^d - r_2) = 0, \qquad (2.2)$$

where $F\left(\cdot\right)$ is the *cdf* of ε_2. $F\left(-l_{i,2} + rdb_{i,2} - b_{i,2}^*\right)$ gives the probability of bank i being forced to use the marginal lending facility under optimal borrowing. We can rewrite equation (2.2) as:

$$F\left(-l_{i,2} + rdb_{i,2} - b_{i,2}^*\right) = \frac{r_2 - r_2^d}{r_2^m - r_2^d}, \qquad (2.3)$$

which relates the probability of using the marginal lending facility to the location of the market rate of interest within the interest rate corridor set by the standing facilities.

If the cumulative distribution function has an inverse function $(F^{-1}\left(\cdot\right))$, we can derive the explicit form of bank i's borrowing function:

$$b_{i,2}^*\left(-l_{i,2}, r_2\right) = -l_{i,2} + rdb_{i,2} - F^{-1}\left(\frac{r_2 - r_2^d}{r_2^m - r_2^d}\right). \qquad (2.4)$$

Bank i can act as a borrower or lender in the market. However, as long as the overnight market rate stays strictly inside the corridor, money market liquidity is constant, as there will be no transactions with the central bank. Therefore, aggregate borrowing must be zero $(\sum_{i=1}^{n} b_i = 0)$. We can get the market-clearing rate of interest from equation (2.3) simply by setting the aggregate borrowing to zero and aggregating over the unit measure of banks (i.e. $b_{i,2}^* = b_2^* = 0$ and $-l_{i,2} = -l_2$):

$$r_2 = r_2^m F(rdb_2 - l_2) + r_2^d(1 - F(rdb_2 - l_2)). \qquad (2.5)$$

Equation (2.5) gives the market rate of interest as a probability weighted average of the two rates of the standing facilities. The higher the rates of the standing facilities, the higher the overnight rate; and the more liquidity there is (relative to required daily balances) in the market, the lower the market rate of interest. The required daily balance on the final day of the maintenance period is simply the requirement for the whole period less the amount of reserves held on the first day of the maintenance period ($rdb_2 = 2R - z_1$, where

z_1 denotes $l_1 + \varepsilon_1 + sf_1$, with sf_1 denoting possible use of standing facilities at end of day 1). Thus, we can write equation (2.5) as:

$$r_2 = r_2^m F(2R - z_1 - l_2) + r_2^d (1 - F(2R - z_1 - l_2)). \qquad (2.6)$$

Furthermore, the expected value of the market rate of interest on the second day is given by:

$$\mathrm{E}\,[r_2] = r_2^m G(rdb_2 - l_2 - \mu_2) + r_2^d(1 - G(rdb_2 - l_2 - \mu_2)), \qquad (2.7)$$

where the expectations are taken over the distribution f_μ, and $G\,(\cdot)$ is the *cdf* of the sum of the two liquidity shocks, μ_2 and ε_2. The proof for this can be found in Välimäki (2001). Equation (2.7) says that the expected market rate with a given allotment equals the probability-weighted average of the standing facility rates, where the probability weights take into account both of the shocks and the amount allotted in the tender.

2.2   First day (averaging)

On the first day of the two-day reserve maintenance period the amount of reserves (prior to use of standing facilities) bank i holds is divided between the reserve balance and use of the standing facilities as follows: for $l_{i,1} < 0$, the bank must acquire $-l_{i,1}$ units of liquidity from the marginal lending facility; if $l_{i,1} > 2R$, the bank will have a reserve balance of $2R$ and $l_{i,1} - 2R$ units must be placed in the deposit facility; if $0 < l_{i,1} < 2R$, all the liquidity will be held as reserve deposits. The bank's profit maximization problem takes the following form:

$$
\max_{b_{i,1}} \mathrm{E}(\Pi) \;=\; r_1^m \left[\int_{-\infty}^{-l_{i,1}-b_{i,1}} (l_{i,1} + b_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right]
$$

$$
+ \mathrm{E}_1\left[r_2\right] \left[\int_{-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1} + b_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right]
$$

$$
+ \left(r_1^d - \mathrm{E}_1\left[r_2\right] \right)
$$

$$
\times \left[\int_{2R_{i,1}-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1} + b_{i,1} - 2R_{i,1} + \varepsilon_1) f(\varepsilon_1) d\varepsilon_1 \right]
$$

$$
- r_1 b_{i,1}, \tag{2.8}
$$

from which we get the FOC for the profit maximizing problem of interbank borrowing:

$$
r_1^m F(-l_{i,1} - b_{i,1}) + \mathrm{E}_1\left[r_2\right] \left[F(2R_{i,1} - l_{i,1} - b_{i,1}) - F(-l_{i,1} - b_{i,11})\right]
$$
$$
+ \left(r_1^d \right) \left[1 - F(2R_{i,1} - l_{i,1} - b_{i,1})\right] - r_1 b_{i,1} = 0. \tag{2.9}
$$

From equation (2.9) we can derive (after aggregation) the market rate of interest as a function of liquidity:

$$
r_1 = \mathrm{E}_1\left[r_2\right] \left\{F(2R - l_1) - F(-l_1)\right\}
$$
$$
+ r_1^m F(-l_1) + r_1^d \left[1 - F(2R - l_1)\right]. \tag{2.10}
$$

The market rate of interest on the first day of the maintenance period will be a probability-weighted average of the standing facility rates and the market rate of interest expected to prevail on the following day of the maintenance period. As in the case of the second day, the expected value of the market rate on the first day of the reserve maintenance period is given by:

$$
\mathrm{E}\left[r_1\right] \;=\; \mathrm{E}_1\left[r_2\right] \left\{G(2R - l_1 - \mu_1) - G(-l_1 - \mu_1)\right\} \tag{2.11}
$$
$$
+ r_1^m G(-l_1 - \mu_1) + r_1^d \left[1 - G(2R - l_1 - \mu_1)\right],
$$

Next we turn to the analysis of central bank behaviour in the tenders.

190

3 Central bank behaviour: supply of liquidity

In deciding its liquidity allotment, the model central bank aims at keeping the expected value of the market rate of interest as close as possible to a target derived from the ultimate goal of the central bank (eg maintaining price stability). However, the central bank might also like to stabilise the reserve holdings within a reserve maintenance period. For example, in ECB (2000) it is stated that in the final allotment decision the considerations of the ECB relate to *the smoothness of the reserve fulfillment path and the level of the interest rates.* Hence, here the model central bank minimises a loss function that consists of two parts: the expected difference between market rate and target rate and the deviations of expected liquidity from the steady path of reserve holding. The precise functional form of the central bank's loss function is usually not announced. Here, the central bank is assumed to minimise a quadratic loss function that consists of the weighted sum of squared percentage deviations of expected market rate of interest and expected liquidity from their target values. The loss function takes the following form:

$$
L_t \;=\; \frac{1}{2}\left(1-\lambda_t\right)\left(\frac{\mathrm{E}\left[r_t\right]-\overline{r}_t}{\overline{r}_t}\right)^2 + \frac{1}{2}\lambda_t\left(\frac{\mathrm{E}\left[m_t+\eta_t\right]-\overline{m}_t}{\overline{m}_t}\right)^2 \tag{3.1}
$$
$$
s.t.\; r_t = r_t\left(m_t+\eta_t\right)
$$

where m_t is the central bank's estimate of liquidity in the overnight market with a given supply of liquidity (ie $m_t = RB_{t-1} + a_t^{CB} + q_t$, where a^{CB} is the central bank's estimate over the autonomous liquidity factors), η_t is a zero-mean liquidity shock (ie the estimation error for the autonomous liquidity factors), $\overline{m}_t$ and $\overline{r}_t$ are the central bank's targets for liquidity and interest rate respectively, λ_t measures the relative weight of the preferences over the two objectives, and finally the minimisation is subjected to the inverse demand function $r_t = r_t\left(m_t+\eta_t\right)$, which gives the market rate of interest as a function of money market liquidity. We can substitute the restriction directly into the loss function. Hence, the optimisation problem can be written as:

$$\min_{m_t} L_t = \frac{1}{2}(1-\lambda_t)\left(\frac{\mathrm{E}\left[r\left(m_t+\eta_t\right)\right]-\overline{r}_t}{\overline{r}_t}\right)^2 + \frac{1}{2}\lambda_t\left(\frac{m_t-\overline{m}_t}{\overline{m}_t}\right)^2,$$

$$(3.2)$$

for which the first order condition is:

$$\frac{1-\lambda_t}{\overline{r}_t{}^2}\left\{\mathrm{E}\left[r_t\left(m_t^*+\eta_t\right)\right]-\overline{r}_t\right\}\frac{\partial \mathrm{E}\left[r_t\left(m_t^*+\eta_t\right)\right]}{\partial m_t} + \frac{\lambda_t}{\overline{m}_t^2}\left(m_t^*-\overline{m}_t\right)=0.$$

$$(3.3)$$

The FOC implicitly determines the optimal liquidity supply, ie the amount of money market liquidity the central bank plans to supply in the tender operation $\left(q_t^* = m_1^* - RB_{t-1} - a_t^{CB}\right)$. From equation (3.3) we see that the expected interest rate will be above (below) the target rate if the optimal liquidity is above its target.[8] Consequently, if the expected market rate of interest with liquidity at the target level is above the target rate (ie $\mathrm{E}[r\left(\overline{m}\right)] > \overline{r}$), both the optimal liquidity and the expected interest rate will be above their target values (ie $m^* > \overline{m}$, and $\mathrm{E}[r\left(m^*\right)] > \overline{r}$). Similarly, if $\mathrm{E}[r\left(\overline{m}\right)] < \overline{r}$ ($\mathrm{E}[r\left(\overline{m}\right)] = \overline{r}$), then $m^* < \overline{m}$ and $\mathrm{E}[r\left(m^*\right)] < \overline{r}$ ($m^* = \overline{m}$ and $\mathrm{E}[r\left(m^*\right)] = \overline{r}$).

To illustrate the effect of a change in the demand for liquidity on the equilibrium liquidity and market rate of interest, figure 1 shows three different demand curves for money market liquidity (the thick curves), and an indifference curve based on the central bank's minimization problem (the thin curve). The (inverse) demand curves are derived using equation (2.10) with different values for the expected future interest rate. The higher the expected interest rate for the second period, the greater the demand -at a given market rate- for the first period. Only if the banks' demand for liquidity passes through $(\overline{m}, \overline{r})$ will the expected interest rate and the expected liquidity be at

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[8]This results from the following:

$$\mathrm{sign}\left[\underbrace{\frac{1-\lambda}{\overline{r}^2}}_{>0}\left\{\mathrm{E}\left[r\left(l^*+\mu\right)\right]-\overline{r}\right\}\underbrace{\frac{\partial \mathrm{E}\left[r\left(l^*+\mu\right)\right]}{\partial l}}_{<0}\right] = \mathrm{sign}\left[\underbrace{-\frac{\lambda}{\overline{l}^2}\left(l^*-\overline{l}\right)}_{<0}\right]$$

$$\Rightarrow \quad \mathrm{sign}\left[\mathrm{E}\left[r\left(l^*+\mu\right)\right]-\overline{r}\right] = \mathrm{sign}\left[l^*-\overline{l}\right]$$

Figure 1: **Money market equilibrium under different demand conditions.**
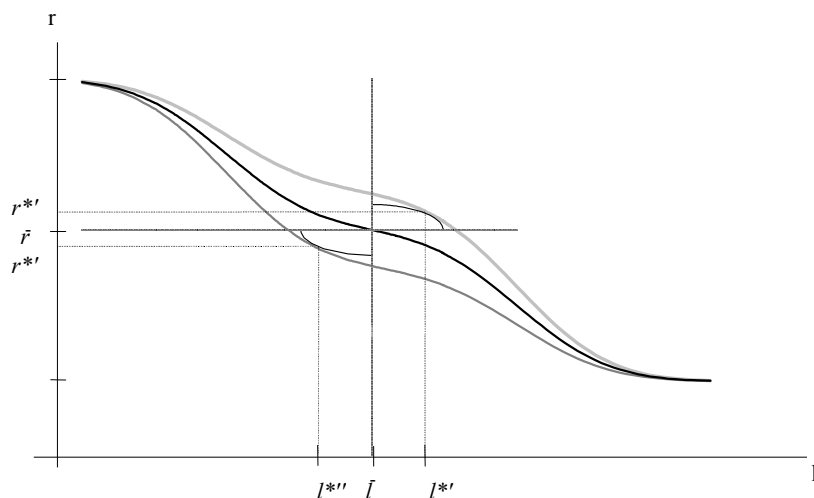
their target levels. Also, if the inverse demand curve at $\overline{m}$ is above (below) $\overline{r}$, both equilibrium liquidity and the expected market rate will be higher (lower) than the target values are.

Let us assume that the central bank chooses its liquidity target $(\overline{m}_t)$ such that it aims first to supply the market with the required reserves, and secondly, to hold the market liquidity as stable as possible throughout the rest of the maintenance period. Hence, the liquidity target will always equal rdb_t, which on the first day of a two-period maintenance period is simply the required reserves $(\overline{m}_1 = R)$ and, on the second day, it is the sum of the daily reserve requirements minus the reserves held during the first day $(\overline{m}_2 = 2R - l_1 - \varepsilon_1 - sf_1)$. Furthermore, we assume that the target rate $\overline{r}$ is derived from the ultimate goal of the central bank, and is thus exogenous to the central bank's liquidity management.[9] Next, we study the optimal liquidity supply and expected market rate of interest in the two-day model.

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[9]For example, in case of the Eurosystem, the coverning council chooces the level of interest rate that is deemed appropriate in light of the goal of price stability. This kind of a target rate is deemed exogenously given to the liquidity managers of the Eurosystem.

3.1 Final day (no averaging)

Inserting equation (2.7) into the minimisation problem of equation (3.2) gives the central bank's loss function for the final day of the reserve maintenance period[10]:

$$
\min_{m_2} L_2 \;=\; \frac{1}{2}(1-\lambda_2)\left(\frac{r_2^d + \left(r_2^m - r_2^d\right)G(2R - z_1 - m_2) - \overline{r}_t}{r_t}\right)^2
$$
$$
+\frac{1}{2}\lambda_2\left(\frac{m_2 - \overline{m}_2}{\overline{m}_2}\right)^2,
$$

the FOC for which is:

$$
\frac{1-\lambda_2}{\overline{r}_2{}^2}\left\{r_2^d + \left(r_2^m - r_2^d\right)G(2R - z_1 - m_2^*) - \overline{r}_2\right\} \tag{3.4}
$$
$$
\times\quad \left(r_2^m - r_2^d\right)\left(-g(2R - z_1 - m_2^*)\right) + \frac{\lambda_2}{\overline{m}_2^2}\left(m_2^* + \eta_2 - \overline{m}_2\right) = 0.
$$

Now, by inserting the liquidity target of the central bank ($\overline{m}_2 = 2R - z_1$), we can rewrite equation (3.4) as:

$$
\frac{1-\lambda_2}{\overline{r}_2{}^2}\left\{r_2^d - \overline{r}_2 + \left(r_2^m - r_2^d\right)G(2R - z_1 - m_2^*)\right\} \tag{3.5}
$$
$$
\times\quad \left(r_2^m - r_2^d\right)\left(-g(2R - z_1 - m_2^*)\right) = \frac{\lambda_2}{\left(2R - z_1\right)^2}\left(2R - z_1 - m_2^*\right),
$$

which implicitly defines liquidity supplied as a function of the position of the interest rate target within the corridor, the reserves shortfall for the whole period, and the distribution of shocks.

 The sign of the RHS of equation (3.5) depends on the equilibrium liquidity vs the required daily balances:

$$
RHS \begin{cases} > 0, \text{ if } m_2^* < 2R - z_1 \\ = 0, \text{ if } m_2^* = 2R - z_1 \\ < 0, \text{ if } m_2^* > 2R - z_1 \end{cases},
$$

while the sign of the LHS is given by the sign of: $\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) - G(2R - z_1 - m_2^*)$. This means that the

---

[10]Note that we have substituted m_2 for l_2 of equation (2.7), what matters here is the central bank's (rather than banks') liquidity estimate.

sign of the LHS depends on the location of the target rate within the corridor vs the probability of being forced to use the marginal lending facility to meet the reserve requirement with the equilibrium liquidity.

Assume for now that the shock distribution is symmetric. If the target rate of the central bank is the mid-point of the corridor (ie $\overline{r}_2 = 0.5\left(r_2^m + r_2^d\right)$), the equilibrium supply of liquidity is simply the target liquidity of the central bank regardless of the weights given to the two objectives (ie for all λ_2).[11] However, if the target rate lies in the upper half of the corridor (ie $\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) > 0.5$), the equilibrium supply of liquidity must be below the target (ie $m_t^* < 2R - z_1$) unless the weight given to interest rate in the objective function of the central bank is zero (ie as long as $\lambda_2 > 0$). In this case, the expected interest rate will be between the target rate and the mid-point ($\overline{r}_t \geq r_t^* \geq r_t^{mid}$, where the strict inequalities hold when both objectives have positive weights, ie $0 < \lambda_2 < 1$). Similarly, when the target rate is in the lower half of the corridor, the equilibrium liquidity must be greater than the target amount, and the expected interest rate will again be between the target rate and the mid-point of the corridor ($\overline{r}_t \leq r_t^* \leq r_t^{mid}$; strict inequalities when $0 < \lambda_2 < 1$).

If the shock distribution is left skewed (right skewed), then $G\left(0\right) < 0.5$ ($G\left(0\right) > 0.5$). In this case, the central bank is not able to meet its targets for interest rate and liquidity with an interest rate corridor that is symmetric about the target rate. Hence, to have $r_t^* = \overline{r}_t$ and $m_2^* = \overline{m}_2$, the rates of the standing facilities should be set so as to locate the target rate in the lower (upper) half of the corridor. More specifically, the corridor should be set so that $\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) = G(0)$, which would produce $m_2^* = 2R - z_1$ and $\mathrm{E}[r_2] = r_2^d + \left(r_2^m - r_2^d\right)G(0) = \overline{r}_2$, ie the equilibrium liquidity and the expected market rate would equal their target values.

When the central bank wants to use the standing facilities as an independent signalling device, the equilibrium liquidity or expected interest rate does not necessarily equal the corresponding target of the central bank. In such a case the relative deviations from targets are determined by the preference-weighting parameter, λ_2. The

---

[11]When the target rate is the mid-point of the interest rate corridor $\left(\overline{r}_2 - r_2^d\right)/\left(r_2^m - r_2^d\right) = 0.5$. With a symmetric shock distribution, $G(2R - z_1 - m_2^*) = 0.5$ for $m_2^* = 2R - z_1$, in which case both RHS and LHS are equal to zero.

lower the value of λ_2, the more weight attached to the interest rate deviations. An extreme case is naturally $\lambda_2 = 0$, when the central bank is interested only in the interest rate, and the liquidity supply is determined simply by setting $r_2^d + \left(r_2^m - r_2^d\right) G(2R - z_1 - m_2^*) = \overline{r}_2$ (i.e. $G(2R - z_1 - m_2^*) = \left(\overline{r}_2 - r_2^d\right) / \left(r_2^m - r_2^d\right)$) and the expected market rate of interest will equal the target. At the other extreme ($\lambda_2 = 1$) the central bank cares only about providing the market with liquidity that will on average equal the reserve requirement. In this case, the central bank will provide the markets with $m_2^* = \overline{m}_2 = rdb_2$, and $\mathrm{E}[r_2] = r_2^d + \left(r_2^m - r_2^d\right) G(0)$. Hence, the relation between the expected market rate of interest and the target will depend on the asymmetry of both the shock distribution and the interest rate corridor.

For the rest of the essay, we assume as a benchmark case that the central bank sets the rates of the standing facilities in order to meet the target for interest rate and the liquidity or, if the rates of the standing facilities are used as independent tools for signalling the monetary policy stance, the weight given to liquidity considerations on the *final day* of the reserves maintenance period is zero. The case in which the interest rate corridor is determined independently and there is a independent target also for liquidity is discussed only briefly.

3.2   First day (averaging)

We can rewrite equation (2.11) describing the expected market rate of interest for the first day of the maintenance period as:

$$
\begin{aligned}
\mathrm{E}\left[r_1\right] \;=\;& \mathrm{E}_1\left[r_2\right] + \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right) G(-m_1) \\
& + \left(r_1^d - \mathrm{E}_1\left[r_2\right]\right)\left[1 - G(2R - m_1)\right].
\end{aligned}
\tag{3.6}
$$

That is, the expected value of the first-day market rate is the sum of the expected value for the final day and the probability-weighted spreads between current rates of the standing facilities and the expected final-day overnight rate. Note that here the relevant liquidity estimate is m_1 instead of l_1 from equation (2.11).

By inserting equation (3.6) and the central bank's target for liquidity ($\overline{m}_1 = R$) into the loss function, we obtain the following minimisation problem for the central bank:

$$\min_{m_1} L_1 \;=\; \frac{1}{2}\frac{1-\lambda_1}{(\bar{r}_1)^2}\left\{ \mathrm{E}_1\left[r_2\right] + \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right)G(-m_1)\right.$$

$$\left. + \left(r_1^d - \mathrm{E}_1\left[r_2\right]\right)\left[1 - G(2R - m_1)\right] - \bar{r}_1\right\}^2$$

$$+ \frac{1}{2}\frac{\lambda_1}{R^2}\left(\mathrm{E}\left[m_1 + \eta_1\right] - R\right)^2,$$

for which the FOC is:

$$\frac{1-\lambda_1}{(\bar{r}_1)^2}\left\{ \mathrm{E}_1\left[r_2\right] + \left(r_1^m - \mathrm{E}_1\left[r_2^*\right]\right)G(-m_1^*)\right. \tag{3.7}$$

$$+ \left(r_1^d - \mathrm{E}_1\left[r_2\right]\right)\left[1 - G(2R - m_1^*)\right] - \bar{r}_1\right\}$$

$$\times \left[\left(r_1^d - \mathrm{E}_1\left[r_2\right]\right)g(2R - m_1^*) - \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right)g(-m_1^*)\right]$$

$$+ \frac{\lambda_1}{R^2}\left(m_1^* - R\right) = 0.$$

Equation (3.7) implicitly defines the optimal liquidity supply for the first day of the reserve maintenance period as a function of current and expected future central bank rates, the central bank preference-weighting parameter, the liquidity shock distributions, and the reserve requirement. Let us next study the optimal liquidity supply in two parts. First, we assume interest rate expectations to be static, and later we analyse the effect of a change in interest rate expectations.

3.2.1 Static interest rate expectations

When the interest rate corridor is not used as an independent signalling device, the expected market rate for the final day of the maintenance period equals the target rate expected to prevail during that day (ie $\mathrm{E}_1\left[r_2\right] = \mathrm{E}_1\left[\bar{r}_2\right]$). With static expectations for the central bank target rate ($\mathrm{E}_1\left[\bar{r}_2\right] = \bar{r}_1 \equiv \bar{r}$), we can write the FOC of equation (3.7) as:

$$\frac{1-\lambda_1}{(\bar{r})^2}\left\{\left(r_1^m - \bar{r}\right)G(-m_1^*) + \left(r_1^d - \bar{r}\right)\left[1 - G(2R - m_1^*)\right]\right\} \tag{3.8}$$

$$\times \quad \left[\left(r_1^d - \bar{r}\right)g(2R - m_1^*) - \left(r_1^m - \bar{r}\right)g(-m_1^*)\right] = \frac{\lambda_1}{R^2}\left(R - m_1^*\right).$$

If the shock distribution is symmetric, the expected final-day market rate of interest equals the mid-point of the interest rate corridor expected for that day, which under static expectations is the mid-point of today's corridor (ie $E_1[r_2^*] = E_1[\bar{r}_2] = E[r_2^{mid}] = r_1^{mid} \equiv r^{mid}$). Hence, for symmetric shocks equation (3.8) can be further reduced to:

$$
\begin{aligned}
& \frac{1 - \lambda_1}{(r^{mid})^2} \left(r_1^d - r^{mid}\right)^2 \{1 - G(-m_1) - G(2R - m_1)\} \\
\times\ & [g(2R - m_1) + g(-m_1)] \\
=\ & \frac{\lambda_1}{R^2} \left(R - m_1^*\right).
\end{aligned}
\tag{3.9}
$$

The RHS is decreasing in m_1^*, and is positive for $m_1^* < R$, zero for $m_1^* = R$ and negative for $m_1^* > R$. The sign for the LHS is given by:

$$
\text{sign}\left\{G(m_1 - 2R) - G(-m_1)\right\} = \begin{cases} -,\ \text{if } m_1^* < R \\ 0,\ \text{if } m_1^* = R \\ +,\ \text{if } m_1^* > R. \end{cases}
$$

The FOC is fulfilled if and only if $m_1^* = R$. Thus, we may conclude that, *under static interest rate expectations, the central bank will provide the markets with liquidity that equals the target liquidity, and the expected market rate of interest will be at the level targeted by the central bank, regardless of the preferences-weighting parameter if the shock distribution is symmetric.*[12]

For asymmetric shock distributions, the sign of the LHS of equation (3.8) is:

$$
\text{sign } LHS\ (3.8) =
$$
$$
\begin{cases} +,\ \text{if } \left(r_1^m - \bar{r}\right) G(-m_1^*) < \left(\bar{r} - r_1^d\right) [1 - G(2R - m_1^*)] \\ 0,\ \text{if } \left(r_1^m - \bar{r}\right) G(-m_1^*) = \left(\bar{r} - r_1^d\right) [1 - G(2R - m_1^*)] \\ -,\ \text{if } \left(r_1^m - \bar{r}\right) G(-m_1^*) > \left(\bar{r} - r_1^d\right) [1 - G(2R - m_1^*)]. \end{cases}
$$

That is, the sign of the LHS depends on the size of the probability-weighted expected cost of marginal lending vs the probability-weighted expected cost of having to use the deposit facility. Now,

---

[12]

$$
\begin{aligned}
E[r_1^{on}] &= r^{mid} + \left(r_1^m - r^{mid}\right) G(-m_1^*) + \left(r_1^d - r^{mid}\right) [1 - G(2R - m_1^*)] \\
&= r^{mid} + \left(r_1^m - r^{mid}\right) G(-R) + \left(r_1^d - r^{mid}\right) G(-R) = r^{mid}. \quad (3.10)
\end{aligned}
$$

the central bank will provide liquidity exactly according to the target (ie $m_1^* = R$) only if $[1 - G(R)] = G(-R)$. With asymmetric shock distribution, this is not necessarily the case. Hence, the liquidity provision will depend on the asymmetry of the distribution and the size of the reserve requirement. For example, if the probability of overdrawing with $m_1 = R$ is larger (smaller) than the probability of having reserves in excess of the reserve requirement $(G(-R)/[1 - G(R)] > 1 \ (G(-R)/[1 - G(R)] < 1))$, the expected interest rate with such a liquidity policy will be above (below) the target rate, and so the optimal liquidity provision will be $m_1^* > R$ $(m_1^* < R)$. In this case, the liquidity supply will depend also on the preference-weighting of the central bank. The higher the λ_1, the more the interest rate will differ from its target and the closer the equilibrium liquidity will be to the reserve requirement.

3.2.2 Change in central bank rates expected

The derivation of the optimal liquidity supply becomes a bit more complicated when the banks expect the central bank to change its target rate within the reserve maintenance period, as in such a case the expected change in the rate affects the demand for liquidity before the change actually occurs. By the envelope theorem, we know that the change in optimal liquidity supply when the expected future interest changes is given by:

$$\frac{dm_1(\mathrm{E}_1\left[r_2\right])}{d\mathrm{E}_1\left[r_2\right]} = -\frac{\partial^2 L_1 / \partial m_1 \partial \mathrm{E}_1\left[r_2\right]}{\partial^2 L_1 / \partial m_1^2}.$$

As the central bank minimises L_1, we know that the denominator on the right hand side is positive, due to the second order condition for minimization. Thus, we have:

$$\mathrm{sign}\frac{dm_1(\mathrm{E}_1\left[r_2\right])}{d\mathrm{E}_1\left[r_2\right]} = \mathrm{sign} - \frac{\partial^2 L_1}{\partial m_1 \partial \mathrm{E}_1\left[r_2\right]}.$$

That is, the sign of the derivative of the optimal liquidity w.r.t. the expected future interest rate is the opposite of the sign of the second cross-partial of the loss function w.r.t. m_1 and $\mathrm{E}_1\left[r_2\right]$. For the first day's minimization problem we have:

$$\frac{\partial^2 L_1}{\partial m_1 \partial \mathrm{E}_1\left[r_2\right]} = \frac{1 - \lambda_1}{\left(\overline{r}_1\right)^2} \left\{ \left[G(2R - m_1) - G(-m_1)\right] \right. \tag{3.11}$$

$$\times \left[\left(r_1^d - \mathrm{E}_1\left[r_2\right]\right) g(2R - m_1) - \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right) g(-m_1)\right]$$

$$+ \left\{\mathrm{E}_1\left[r_2\right] - \overline{r}_1 + \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right) G(-m_1)\right.$$

$$\left. + \left(r_1^d - \mathrm{E}_1\left[r_2\right]\right)\left[1 - G(2R - m_1)\right]\right\} \left[g(-m_1) - g(2R - m_1)\right]\right\}.$$

Inserting the FOC into equation (3.11), we can derive the sign of the central bank's optimal liquidity response to an increase in the expectations of the future interest rate from:

$$\mathrm{sign} - \frac{1 - \lambda_1}{\left(\overline{r}_1\right)^2} \left[G(2R - m_1) - G(-m_1)\right] \tag{3.12}$$

$$\times \left[\left(r_1^d - \mathrm{E}_1\left[r_2\right]\right) g(2R - m_1) - \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right) g(-m_1)\right]$$

$$- \frac{\lambda_1}{R^2}\left(R - m_1^*\right)$$

$$\times \frac{\left[g(-m_1) - g(2R - m_1)\right]}{\left[\left(r_1^d - \mathrm{E}_1\left[r_2\right]\right) g(2R - m_1^*) - \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right) g(-m_1^*)\right]}.$$

which is positive (at least when the shock distribution is symmetric and single peaked) as long as there is any weight given to the interest rate considerations (ie $\lambda < 1$).[13] Therefore, we may conclude that *if the central bank pays any attention to money market interest rates, it will provide the more liquidity, the higher the expected future interest rate.*

Now, we know that the expected value of the market rate for today will be above the target rate when an interest rate hike is expected. However, the simultaneous effect of both the expected interest rate hike and the increasing liquidity supply on the expected market rate for today is not necessarily monotonic. That is, when $m_1^* > \overline{m}_1$, $\mathrm{E}_1\left[r_1\right] > \overline{r}_1$. However, if $m_1^* > \widehat{m}_1 > \overline{m}_1$, we cannot conclude that $\mathrm{E}_1\left[r_1\right]$ is above $\widehat{r}_1$, for $\widehat{r}_1 = \mathrm{E}_1\left[r_1\left(\widehat{m}_1 + \eta_1\right)\right] > \overline{r}_1$. The effect of a

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[13]This results from the following: $(1 - \lambda_1)\left(\overline{r}_1\right)^2\left[G(2R - m_1) - G(-m_1)\right] > 0$ and $\left[\left(r_1^d - \mathrm{E}_1\left[r_2\right]\right) g(2R - m_1) - \left(r_1^m - \mathrm{E}_1\left[r_2\right]\right) g(-m_1)\right] < 0$, as long as the expected future rate is within the interest rate corridor. If initially $m_1^* > R$, $(R - m_1^*) < 0$, with a symmetric single peaked shock distribution, then $\left[g(-m_1) - g(2R - m_1)\right] < 0$, whereas if initially $m_1^* < R$ $(m_1^* = R)$ then $(R - m_1^*) > 0$ $((R - m_1^*) = 0)$ and $\left[g(-m_1) - g(2R - m_1)\right] > 0$.

200

change in the expected future rate on the expected rate for today is given by:

$$\frac{\partial E_1 [r_1]}{\partial E_1 [r_2]} = \{G(2R - m_1) - G(-m_1)\} \tag{3.13}$$

$$- \left[\left(E_1 [r_2] - r_1^d \right) g(2R - m_1) + (r_1^m - E_1 [r_2]) g(-m_1) \right] \frac{\partial m_1}{\partial E_1 [r_2]},$$

where the first term on the RHS is the probability of not having to rely on the standing facilities today with m_1 units of liquidity, and the second term gives the effect of the change in the probability of being forced to use the standing facilities on the expected cost of using them. From above, we know that $\partial m_1 / \partial E_1 [r_2] > 0$. Thus, the expected interest rate for today will increase due to an increase in the expected rate for tomorrow, if $\{G(2R - m_1) - G(-m_1)\} > \left[\left(E_1 [r_2] - r_1^d \right) g(2R - m_1) + (r_1^m - E_1 [r_2]) g(-m_1) \right] \partial m_1 / \partial E_1 [r_2]$. Because the functional form of (3.13) is tedious, we are satisfied with the fact that the higher the preferences-weighting parameter (ie the more weight given to liquidity considerations), the more likely it is that an increase in the expected future rate will lead to a positive change in the expected market rate for today. This comes from the fact that $\partial^2 m_1 / \partial E_1 [r_2] \partial \lambda_1 < 0$.

Let us next examine graphically a few examples to get an idea of how the effect of expectations of future rates on the expected rate for today depends on the distribution of shocks (hence the interest rate elasticity of the demand for liquidity) and on the size of the expected change.

Figure 2 shows a set of inverse demand functions for liquidity at different levels of uncertainty as to the development of liquidity. All the demand functions are calculated assuming normally distributed zero-mean shocks, reserve requirement (ie target liquidity) of 2000, standard deviations of (250; 500; 1000), and expectations of a 25 basis point interest rate hike[14]. The curvatures of the inverse demand functions decrease as the shock distributions become wider. *The more the liquidity uncertainty, the more likely the central bank will react to higher interest rate expectations by letting both the interest rate and the liquidity differ from target.* Figure 3 illustrates this

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[14]We used the following rate assumptions in drawing the figure: target rate 3%, deposit rate 2%, marginal lending rate 4% and expected final-day rate 3.25%.

Figure 2: **Effect of uncertainty on the demand for liquidity.**
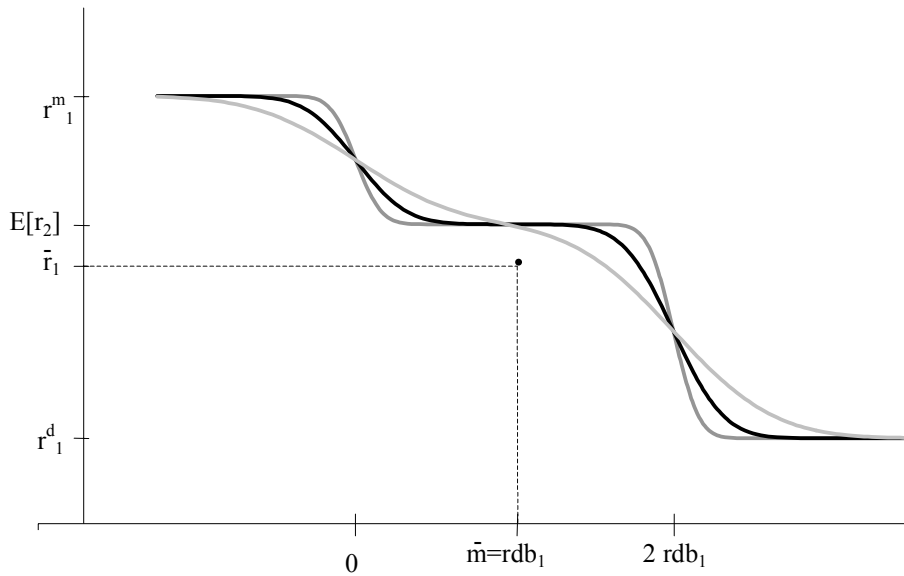
effect where the standard deviation of each liquidity shock is 50%
of the reserve requirement. The equilibrium levels for the interest
rate and liquidity naturally depend crucially on the central bank's
preferences-weighting parameter. However, it is quite obvious that,
when the inverse demand function is nearly linear, they both increase
with the expectations. The more accurately the evolution of money
market liquidity is estimated, the greater the interest rate elasticity
of the demand for liquidity, as long as liquidity itself is close to the
target level. Thus, with little liquidity uncertainty, it is likely that
the equilibrium interest rate-liquidity point will lie close to either of
the targets, ie the central bank will let the expectations be reflected
mainly in either the liquidity or the market rate. This is illustrated
with figure 4, where the standard deviation of the liquidity shock
is 10% of the reserve requirement. Furthermore, with reasonably
little liquidity uncertainty, it is possible that the optimal reaction
of a central bank minimizing the quadratic differences is to jump
from tight control of liquidity into tight control of interest rates when
expectations of an interest rate hike reach a sufficiently high level.

Figure 3: **Effect of interest rate hike expectations on equilibrium liquidity supply, with high liquidity uncertainty.**
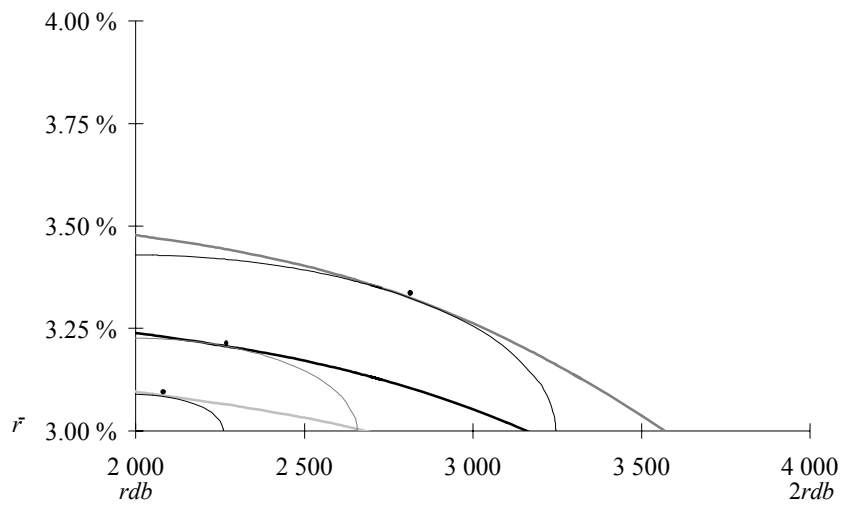

Figure 4: **Effect of interest rate hike expectations on equilibrium liquidity supply, with low liquidity uncertainty.**
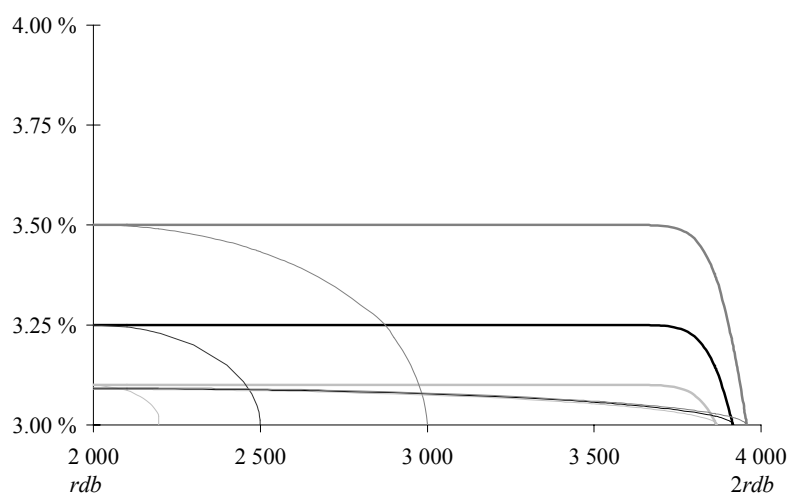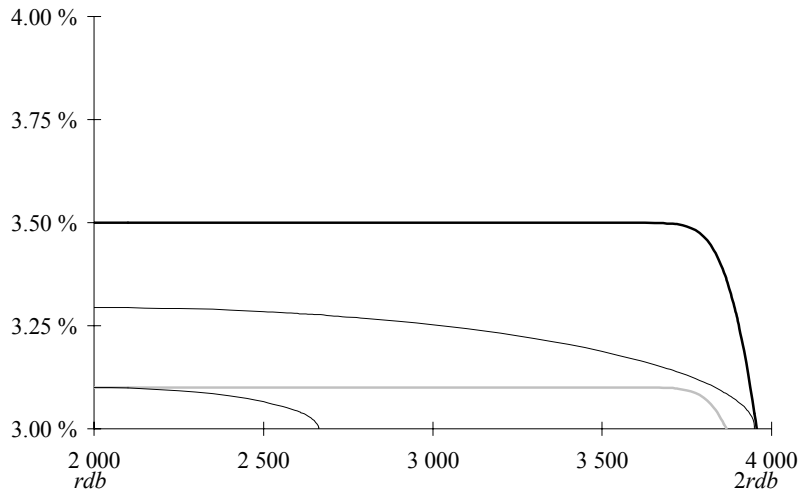
Figure 5: **Jump in the reaction function**

This case is illustrated in figure 5. Next we analyse the bidding behaviour of the banks in the tenders.

4   Bidding in tenders

There are n homogeneous banks eligible to participate in the tenders. Each of the banks can place up to ten bids in each tender. A bid consists of a quantity-interest rate pair, in which the specified interest rate is the rate at which the bidder wants to transact, and the quantity is the amount in which it wants to transact. The bids of bank i are arranged in descending order, such that the bid amount with the highest bid rate $r_{i,1}^T$ is denoted by $b_{i,r_{i,1}^T}$, with the second highest rate $r_{i,2}^T$ by $b_{i,r_{i,2}^T}$, and so on (ie $r_{i,1}^T > r_{i,2}^T > ... > r_{i,10}^T$).

The monopoly supplier of liquidity (central bank) aims at supplying m^* (defined in previous chapter) units of liquidity to the market, regardless of the shape of the demand schedule of the banks; ie in contrast to most auctions, here the seller is not trying to maximise its income from the tender. However, we assume that if collusion (or

collusion-like behaviour) between bidders is detected by the central bank, it will reduce the supply below m^*, which is costly enough to the banks to deter collusive behaviour.

The ex post value of a unit of liquidity is its secondary market price and is common to all banks. As shown above, this price is a decreasing function of the liquidity, and it always lies within the interest rate corridor set by the rates of the standing facilities. Therefore, absent a reserve price[15], there will always be enough bids to enable the central bank to provide liquidity according to m^*.[16] The realised market rate of interest is a random variable, due to the liquidity shock η. However, the expected market rate of interest is common to all banks, as *we assume that either there is an explicitly announced central bank target for the market rate (in which case $E[r] = r^{\text{target}}$) or the banks receive a common signal on the forthcoming market rate* (for the euro area, the quotations on the two-week EONIA swap rate could serve as such a signal). Hence, $\text{E}_i\left[r\left(m^* + \eta\right)\right] = \text{E}[r\left(m^* + \eta\right)]$ for all banks, as long as a reserve price is not used or is ineffective. We return later to the case where there is a binding reserve price for the bids.

The analysis of the bidding behaviour is divided into two parts. We begin with the multiple rate procedure (also known as *American or discriminatory auction*), in which the allotment rate for an accepted bid is the bid rate (ie the rate specified in the given bid). After that, we consider the single rate procedure (also known as *Dutch or uniform price auction*), where the marginal rate[17] of the allotment is applied to all accepted bids. The ECB has used multiple rate auctions in all of its main refinancing operations conducted in the form of variable rate tenders and also in most of its longer-term refinancing operations. The Dutch auction procedure has been applied so far only in the first two longer-term refinancing operations.

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[15]The reserve price is the lowest price at which the counterparties are allowed to submit bids in auctions.

[16]This follows from the fact that a bank can make a positive profit by borrowing liquidity from the auction and lending it back to the central bank if the price of liquidity at the tender is below the deposit rate.

[17]The *marginal rate of allotment* is a term used by the ECB for the lowest rate at which bids are accepted. In auction literature this kind of rate is usually refered to as the stop-out price.

4.1 Multiple rate auctions

The profit for bank i from participating in a variable rate tender, in which the multiple rate procedure is applied, is simply the allotment volume weighted sum of the differences between expected market rate and bid rate. It is given by:

$$\Pi \;=\; \sum_{j=1}^{10} q_{i,r_{i,j}^T}\left(\mathrm{E}\left[r\left(m^* + \eta\right)\right] - r_{i,j}^T\right) \tag{4.1}$$

$$s.t. \; q_{i,r_{i,j}^T} \;=\; \begin{cases} b_{i,r_{i,j}^T}, \text{ if } B_{r_{i,j}^T} \le m^* \\ \frac{m^* - B_{r_{j-1}^T}}{b_{r_{i,j}^T}} b_{i,r_{i,j}^T}, \text{ if } B_{r_{j-1}^T} < m^* < B_{r_{i,j}^T} \\ 0, \text{ if } B_{r_{j-1}^T} \ge m^* \end{cases},$$

where $b_{r_{i,j}^T}$ is the aggregate bid at $r_{i,j}^T$, $B_{r_{i,j}^T}$ denotes the cumulative bid amount of all banks at interest rate equal to or above $r_{i,j}^T$, and $B_{r_{j-1}^T}$ denotes the cumulative bid amount at all rates higher than $r_{i,j}^T$. The highest rate at which $B_{r_{i,j}^T}$ exceeds the amount the central bank is willing to provide to the market is called the marginal rate of the allotment (ie $r^{\mathrm{marginal}} \equiv r_j^T$ that satisfies $B_{r_{j-1}^T} < m^* < B_{r_j^T}$)[18]. The net profit from bid $b_{i,r_{i,j}^T}$ is the difference between the bid rate and the forthcoming market rate of interest times the amount of liquidity actually provided to the bank at the given rate. The bank will not be allotted any liquidity for a bids at rates below the marginal rate, whereas bids at rates above the marginal rate are accepted in full (ie the allotted amount here equals the bid amount). Furthermore, for bids at the marginal rate, the central bank will use pro rata rationing in the allotment procedure. This means that bids at the marginal rate are accepted only partially, such that the share of the total allotment for each bid at the marginal rate equals its fraction of the total bids at the rate.

It's easy to see that optimal bid is zero at all rates exceeding the expected market rate of interest (ie $b_{i,r_{i,j}^T}^* = 0$ when $r_{i,j}^T > \mathrm{E}[r\left(m^*\right)]$), as otherwise the bid would yield negative expected profits. Also, bids at rates below the marginal rate yield zero profit, as there will not be

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[18]Note that we have omitted the bank index i from subscripts of rates that apply to all banks.

any residual liquidity supply at these rates. *Hence, positive expected profit can be made only with bids at rates between the marginal rate and the expected market rate of interest.*

Suppose now that the marginal rate of the allotment is lower than the expected market rate of interest (ie $r^{\mathrm{marginal}} < \mathrm{E}[r\,(m^* + \eta)]$). Consider bank i placing a bid for the entire quantity of the allotment at a rate infinitesimally above the marginal rate ($b_{i,r^{\mathrm{marginal}}+\in} = m^*$, where $r^{\mathrm{marginal}} + \in < \mathrm{E}[r\,(m^* + \eta)]$). The allotment for bank i, with this bid, must be at least as large as it would be with a bid at the marginal rate. Thus, a bank behaving in this manner would be making positive profit. However, this incentive applies to all banks. Thus, the optimal policy for another bank would be to place a bid large enough to satisfy the whole supply at a rate that is infinitesimally higher than that of bank i's, and so forth until the marginal rate reaches the expected market rate of interest. Thus, in equilibrium, all banks place large enough bids for the central bank to be able to provide liquidity according to its target (m^*) at a price equal to the expected market rate, ie $B_{\mathrm{E}[r(m^*+\eta)]} \geq m^*$ and $r^T_{i,j} = \mathrm{E}[r\,(m^* + \eta)]$ for all i and j. *The aggregate demand schedule the central bank faces is flat at $E[r\,(m^* + \eta)]$, at least up to m^*, and the bids at rates below the expected market rate are ineffective and so can be ignored.*

The analysis of this section applies to the pure case where the banks' bidding is not restricted by a reserve price. The next section will deal with the case where the central bank explicitly states the minimum rate for bids to be accepted.

4.1.1   Minimum bid rate

The ECB has applied a reserve price for bids in its main refinancing operations conducted as variable rate tenders. This *minimum bid rate* (r^{minimum}) is the minimum interest rate at which counterparties may place bids in the tenders. The purpose of this section is to analyse the effect of the minimum bid rate on bidding behaviour.

From the section above, we know that bids are never placed at higher rates than the expected market rate of interest. Assume first that the expected market rate of interest with the liquidity provided by the central bank is below the minimum bid rate (ie

$E\left[r\left(m^{\text{provided by the CB}}+\mu\right)\right] < r^{\text{minimum}}$). Now, it would be profitable for bank i not to participate in the tender (even at the minimum bid rate) and to buy the needed liquidity from the interbank market. As this applies to all banks, the central bank would not be able to provide the market with liquidity in excess of the amount z implicitly given by $E[r\left(z+\mu\right)] = r^{\text{minimum}}$. Therefore, *the expected market rate of interest will always be at or above the minimum bid rate.* As a consequence, the maximum value for the minimum bid rate should be the interest rate targeted by the central bank.[19]

Even if the minimum bid rate is set at or below the central bank's target rate, the reserve price can either be restrictive or it might as well be inefficient. Assume first that the equilibrium liquidity estimated by the central bank (m^*) is such that the expected market rate of interest is at or above the level of the minimum bid rate, ie $E[r\left(m^*+\mu\right)] \geq r^{\text{minimum}}$. The reserve price set for the liquidity will not be restrictive because, as shown in the above section, the banks will bid for at least m^* at the expected market rate. Now assume that the expected market rate with the estimated equilibrium liquidity is below the minimum bid rate (ie $E[r\left(m^*+\mu\right)] < r^{\text{minimum}}$). In this case, if the central bank is able to provide the market with m^*, it will be profitable for all banks individually not to participate in the tender, but to borrow the liquidity needed from the interbank market. As a consequence, the central bank would not be able to provide the estimated equilibrium liquidity to the market. This situation is similar to that of fixed rate tenders, with the central bank accepting in full all bids (also referred to as full allotment).[20] Here the counterpart for the fixed tender rate would be the minimum bid rate that is fixed and made public in the tender announcement. Välimäki (2001 and 2002) show that in fixed rate tenders with full allotment procedure, the banks will bid for liquidity such that the expected market rate of interest will equal the tender rate (here the minimum bid rate). Therefore, we expect that the bidding in variable rate tenders, where

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[19] Assume the contrary: $r^{\text{minimum}} > \overline{r}$. The expected market rate and expected liquidity will be above those of the levels targetted by the central bank (ie $E[r] > \overline{r}$ and $m > \overline{m}$). It is possible that this does not change the optimal reaction of the central bank when a rate hike is expected. However, this can never be optimal when interest rate expectations are neutral or when a rate cut is expected.

[20] For further information on the fixed rate tenders and banks' behaviour therein, see Välimäki (2001 and 2002).

$\mathrm{E}[r\,(m^* + \mu)] < r^{\mathrm{minimum}}$, is such that the aggregate bid amount is implicitly given by $\mathrm{E}[r\,(b + \mu)] = r^{\mathrm{minimum}}$ and the central bank will accept all bids. This means that *when the minimum bid rate is effective, the market liquidity will be below and the expected market rate above the level preferred by the central bank.*

When are we likely to see the minimum bid rate become effective? We know that if banks expect the central bank to lower its target rate for operations in the remaining reserve maintenance period, the estimated equilibrium rate will be below the current target rate (ie $\mathrm{E}[r\,(m^* + \eta)] < \overline{r}$) as long as the central bank gives positive weight to market liquidity. Thus, when the interest rate is expected to be cut, the smaller the difference between target rate and minimum bid rate, the more likely the equilibrium (estimated by the central bank) rate will be below the minimum bid rate. However, with expectations of a higher interest rate, the minimum bid rate should never become effective, as in such case $\mathrm{E}[r\,(m^* + \eta)] > \overline{r} \geq r^{\mathrm{minimum}}$.

Now, that the implications of a minimum bid rate have been set out, we turn to a phenomenon that is closely related to the case in which the minimum bid rate becomes an effective constraint on bidding.

4.1.2   Underbidding

When the ECB switched from fixed to variable rate tenders, it stated in a press release dated 8 June 2000 that '*For the purpose of signalling the monetary policy stance, the minimum bid rate is designed to play the role performed, until now, by the rate in fixed rate tenders*'. If this is interpreted as $\overline{r_t} = r_t^{\mathrm{minimum}}$, we expect (according to the analysis in the section above) the minimum bid rate to become effective whenever the banks expect the rate to be cut during the rest of the current reserve maintenance period.

In connection with ECB variable rate tenders, the term 'underbidding' has been used several times by market players and the financial press (Bindseil, 2002). According to Bindseil, underbidding refers to a lack of bids in a fixed rate tender such that the central bank cannot allot the liquidity actually needed by the banks to fulfill smoothly their reserve requirements. Now, even though Bindseil applies the term to fixed rate tenders, this is also a feature of variable

rate tenders with minimum bid rate. This is apparent since Bindseil himself refers to four cases of underbidding during the period in which the ECB has been using variable rate tenders.

If underbidding is understood as a phenomenon in which the central bank does not allot so as to smooth out reserve holdings, it need not be limited to cases in which the minimum bid rate is binding. If interpreted this way, there would be underbidding whenever $m^* < \bar{l}$, even though in this case the deviations from smooth reserve holdings would be intentional (as long as $b \geq m^*$). If the central bank is purely liquidity oriented (in the model of the previous section, $\lambda_1 = 1$) underbidding is closely related to a binding minimum bid rate, as in such a case underbidding occurs when the banks' optimal bid is lower than the amount desired by the central bank (ie $b < m^* = \bar{l}$). Furthermore, we will here use the term underbidding to refer only to cases in which the banks' aggregate bid is lower than the equilibrium liquidity estimated by the central bank (ie $b < m^*$). This is also the situation when the reserve price for liquidity is effective.

Is underbidding a problem for the central bank? To answer this question, we must examine the motives of a central bank for incorporating a reserve price for bidding in its operational framework. If a liquidity-oriented central bank uses the minimum bid rate as a signalling device for monetary policy stance, underbidding is of course problematic, since underbidding means that the central bank is not in control of the level of money market liquidity. In this case, underbidding and the subsequent loss of control over liquidity is the price the central bank pays for using the minimum bid rate as a policy signalling device.

We assume for now that the central bank's loss function depends on the banks' interest rate expectations, such that the central bank is purely liquidity-oriented ($\lambda_1 = 1$) when interest rates are expected to be raised in the near future and purely interest rate-oriented ($\lambda_1 = 0$) when rates are expected to be lowered. We know from above that if the minimum bid rate equals the central bank's target rate, and the banks expect a rate cut, the banks will restrict their bids to the amount at which the expected market rate equals the minimum bid rate, which is also the central bank's target. Moreover, in this framework, the banks will bid enough to enable the central bank to control the liquidity, when the rate is expected to be raised. If the central bank had this kind of asymmetric reaction function vis-à-vis

to interest rate expectations, the so-called underbidding should not present a major problem.

The underbidding could be somewhat more problematic in the ECB's framework than in the model framework used here. The difference between the two is that the maturity of ECB weekly main refinancing operations is two weeks whereas the maturity of operations in the model of the previous section equals the frequency of the operations. The overlapping nature of ECB operations adds an extra incentive for banks to lower their bids. We illustrate this with a simple example.

Underbidding and overlapping operations

Let's assume that in its liquidity provision the central bank uses variable rate tenders with minimum bid rate set at the level of its interest rate target ($r_t^{\mathrm{minimum}} = \overline{r}_t$). Furthermore, assume that there are two operations left in the current reserve maintenance period and that the target rate (as well as the minimum bid rate) of the central bank is expected to be cut between the two operations. The central bank is expected to allot liquidity according to its interest rate target in the last operation; thus, the expected market rate for the last period (ie from settlement of the last operation until the end of the maintenance period) equals the expected target rate for it.

If the maturity of an operation is one period (ie to the settlement of the next operation), the amount of liquidity the banks bid for (at the minimum bid rate) in the first operation is just the amount at which the expected market rate of interest equals the minimum bid rate. Thus, no positive profits can be made by shifting lending between central bank operations and the interbank market.

Now assume, by contrast, that also the first operation matures at the end of the maintenance period, ie the maturity of the first operation is two periods. In order not to have a profit opportunity in shifting lending between central bank operations and interbank market, the average (effective) market rate for the two subperiods must equal the rate at which liquidity is borrowed from the central bank for the two periods (ie the bid amount in the first operation is implicitly given by $0.5(\mathrm{E}[r\,(b_1 + \mu_1)] + \mathrm{E}[\overline{r}_2]) = r_1^{\mathrm{minimum}}$). If this were not the case, it would be profitable for all banks individually to increase the bid amount when $\mathrm{E}[r\,(b_1 + \mu_1)] < 2r_1^{\mathrm{minimum}} - \mathrm{E}[\overline{r}_2]$

211

or decrease it when $\mathrm{E}[r(b_1 + \mu_1)] < 2r_1^{\mathrm{minimum}} - \mathrm{E}[\bar{r}_2]$. This would occur until the equality was established again. When the central bank is expected to lower its target rate, we have $\mathrm{E}[r(b_1 + \mu_1)] = 2r_1^{\mathrm{minimum}} - \mathrm{E}[\bar{r}_2] > r_1^{\mathrm{minimum}}$; ie the expected market rate of interest is higher than the minimum bid rate. This means that the overlapping nature of central bank operations increases the underbidding; the money market liquidity will be even further below the estimated equilibrium liquidity. The overlap of the maturities of consecutive operations, together with the reserve price for bidding, leads to the perverse situation in which the expected market rate for the first period increases above the target rate when the target rate is expected to be lowered in the following operation. Furthermore, we have seen that if the central bank places any weight on liquidity considerations, the expected market rate for the first period will increase above the target when the target is expected to be raised in the following operation. *Thus, the expected market rate for the first period increases above the target rate whenever the central bank is expected to change its target, regardless of direction*

Next we analyse the other procedure available to the central bank in applying variable rate tenders – namely the single rate auction.

4.2   Single rate auctions

The profit for bank i from participating in a single rate variable rate tender is given by:

$$\Pi = \sum_{j=1}^{10} q_{i,r_{i,j}^T} \left(\mathrm{E}\left[r\left(m^* + \eta\right)\right] - r^{\mathrm{marginal}} \right) \qquad (4.2)$$

$$s.t. \ q_{i,r_{i,j}^T} = \begin{cases} b_{i,r_{i,j}^T}, \ \text{if } B_{r_{i,j}^T} \le m^* \\ \frac{m^* - B_{j-1}}{b_{r_{i,j}^T}} b_{i,j}, \ \text{if } B_{r_{j-1}^T} < m^* < B_{r_{i,j}^T} \\ 0, \ \text{if } B_{r_{j-1}^T} \ge m^* \end{cases}.$$

The only difference in profit maximisation problems for multiple and single rate tenders is that in the former the cost of liquidity acquired is the bid rate whereas in the latter it is the marginal rate for the whole allotted amount. Clearly the equilibrium outcome for the case with

212

multiple rate tenders constitutes an equilibrium also in this case. A single bank cannot make positive profit by bidding at rates above or below the expected market rate, when the aggregate bid of the other banks at the expected market rate is at least m^*, as lower bids will be disregarded, and bids at rates above this level will provide zero profit (or negative profit if the bid is large enough to raise the marginal rate of the allotment). However, with the single rate procedure, there are plenty of other potential bidding equilibria.

Back and Zender (1993) analyse auctions for divisible goods and show that, for any price that is between the reserve price applied in the auction and the value of the (divisible) good being auctioned, there is a symmetric pure-strategy equilibrium in which the seller receives exactly that price. In this setting, this means that the banks should be able to maintain a bidding strategy in which the marginal rate of the tender is below the expected market rate, by placing very steep demand curves in which the inframarginal bid rates are relatively high, as these bids are never marginal (ie they do not affect the marginal rate) and are thus costless for the banks to submit. However, in a later paper Back and Zender (2001) show that if the seller has the option to cancel part of the supply after observing the bids, it will eliminate many of the 'collusive seeming' equilibria of the auction. Furthermore, in equilibrium the seller will always sell the full amount.

To take into account the potentially adverse effect of collusive equilibria under single rate auctions, we assume the central bank to cut back the intended supply form m^* if it detects collusive behaviour by the banks. This punishes the banks immediately by raising the marginal rate above the expected market rate of interest. The banks know that the central bank will punish collusive bidding, which eliminates the equilibria with demand schedules that are steeper than the true inverse demand functions would suggest. *In this case, the outcome of the single rate auction procedure will resemble that of the multiple rate procedure, ie the central bank receives bids for at least m^* units of liquidity at the expected market rate of interest.* However, the demand schedule up to m^* need not to be flat in this case, as the inframarginal bidding is costless. Thus, with single rate tenders, each bank placing a bid that reflects its true demand curve for liquidity in the interbank market is also an equilibrium solution.

The effects of the minimum bid rate and the underbidding phenomenon are similar whether the tenders are single or multiple rate tenders.

After having developed a model of the central bank liquidity supply and banks' behaviour in the liquidity tenders, we will in the next section try to evaluate the experience with ECB variable rate tenders in light of the model.

5 Experience with ECB variable rate tenders

The ECB applied the multiple rate procedure in all 92 main refinancing operations conducted as variable rate tenders between 23 June 2000 and 30 March 2002. A minimum bid rate was applied in each of these operations. We assume here that the minimum bid rate was the ECB's short-term operational target rate, as it stated in a press release (8 June 2000) following the decision to change the tender procedure from fixed to variable rate that the *minimum bid rate will take the role that previously the tender rate had in signalling the monetary policy stance.* In this section we first present a preliminary study on the liquidity provision of the ECB and then take the first step in the analysis of the banks' bidding behaviour in the ECB variable rate tenders.

5.1 Liquidity provision of the ECB

The liquidity management of the ECB is comprehensively described in the May 2002 issue of the ECB monthly Bulletin. It states that the baseline for ECB liquidity provision is the so-called benchmark allotment, which basically consists of smooth fulfilling of reserve requirements, taking into account banking sector liquidity needs arising from autonomous liquidity factors and the reserve requirement (ECB 2002). This means that when liquidity is provided according to the benchmark allotment rule the banks' reserve holdings (money market liquidity) are expected to be stable over the course of a reserve

214

maintenance period. However, there is a natural exception to this rule. The analysis of the previous section suggests that, with this kind of an operational framework, the ECB should face underbidding when the banks expect it to cut its target (ie here the minimum bid rate) within the current reserves maintenance period. Accordingly, in the four main refinancing operations (settled on 14 February, 11 April, 10 October and 7 November 2001), the allotted amount was not de facto decided by the central bank. In these tenders the bid amount was apparently less than that of the benchmark allotment and the pro rata rationing was not used.

The equation for calculating the benchmark allotment for the main refinancing operations is given in the annex to the ECB monthly bulletin article mentioned above. To illustrate the ECB liquidity allotment policy, we estimated such benchmark allotments for the 92 MROs between 23 June 2000 and end-March 2002 and regressed the excess supply of liquidity (ie actual-benchmark allotment) in 88 of these tenders against a constant, the benchmark liquidity and the spread between the one-week EURIBOR and the minimum bid rate (henceforth, EURIBOR spread). We omitted the four tenders in which underbidding was obvious from this simple OLS-regression, as in these cases the decision over the allotment volume was not in the hands of the ECB. The regression equation took the following form:

$$
\begin{aligned}
excess\ supply\ &=\ a + b_1\ benchmark\ allotment \\
&+ b_2\ EURIBOR\ spread + error\ term
\end{aligned}
$$

The ex ante expectation for a liquidity-oriented central bank (ie a central bank keen on stabilising liquidity holdings and not so concerned about the interest rate variability) is that the parameter estimates for both explanatory variables should be statistically insignificant, as the amount of liquidity supplied in excess of the benchmark allotment should be determined only by changes in forecasted autonomous liquidity factors. On the contrary, if the central bank is not purely liquidity oriented in its liquidity decisions, we expect the banks' expectations concerning the evolution of central bank rate(s) to affect the liquidity provision, because these expectations will affect the banks' demand for liquidity in the interbank market. A significant positive parameter estimate for the EURIBOR spread, is taken as an indication of central bank concern about the interest rate deviations. There might also be a natural

215

spread between the one-week EURIBOR and the target rate. This natural spread (if constant) makes a negative contribution to the constant a.

The benchmark level of liquidity is also included as an explanatory variable, as it could affect the liquidity decision if the central bank is willing to stabilise the amount of liquidity to be provided in the two overlapping tenders. The central bank might not want the difference in sizes of the two overlapping MROs to be too large; this kind of bias for equality in the amount provided in each operation could be reflected in a negative parameter estimate for the benchmark liquidity in the regression and as a positive contribution to the constant.

The parameter estimates, White Heteroskedasticity-consistent standard errors and the associated probabilities are given in table 1.

Table 1   **The excess liquidity supply**

Dependent variable: Excess supply (EUR bn)			
Variable	Coefficient	Std. error	Probability
Constant	3.911	1.991	0.053
Benchmark	-0.063	0.027	0.024
EURIBOR spread	0.094	0.033	0.006
Adj. R^2	0.163		
n	88		

The parameter estimates for both the benchmark liquidity and the EURIBOR spread are significantly different from zero. This suggests that, besides considering the banks' benchmark need for liquidity, the ECB, in deciding on liquidity allotments has tried to smooth the difference in allotted volumes of consecutive operations and given positive weight to market expectations of the evolution of interest rates. However, the regression suggests that a 20 basis-point increase in the EURIBOR spread would be countered by the ECB only by allotting an extra EUR 1.8 bn to the markets. This is a relatively small amount compared to the average (benchmark) liquidity of more than EUR 120 bn. Furthermore, when the liquidity is close to the benchmark level, we expect the interest rate elasticity of liquidity to be at its lowest; thus, this extra allotment is not expected to have a large impact on realised interest rates, unless it affects counterparties' expectations of the central bank's future interest rate

216

policy. Moreover, the exogenous variables in the regression explain only some 20% of the total variation of excess supply. Hence, changes in the estimated effect on autonomous liquidity factors in the period between publishing of the estimate and liquidity allotment decision seems to be responsible for most of the differences between actual and benchmark allotment. Consequently, despite the statistically significant values for both explanatory variables in the regression, we are not willing to reject the idea of the ECB being a liquidity oriented central bank; the weight given to interest rate considerations might be positive, but it's effect on the liquidity provision is relatively small. Yet, how the ECB has reacted to underbidding needs to be further explored in order to get a full picture of the ECB's liquidity policy.
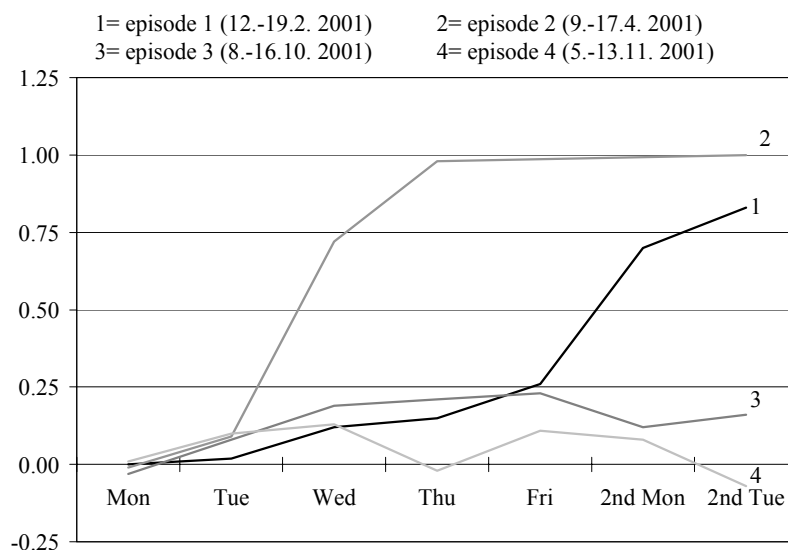
5.1.1 Underbidding episodes

There were four tenders in which the banks bid for an amount that was clearly below the benchmark allotment for smooth reserve holding between the switch to variable rate tenders with minimum bid rate (in June 2000) and March 2002. These tenders were settled on 14 February, 11 April, 10 October and 7 November 2001. The reason for this so-called underbidding in each case was that the banks expected the ECB to cut its interest rates before the next main refinancing operation (in the same reserve maintenance period). The expectations were fulfilled only in the last underbidding episode, as the ECB cut its main refinancing rate and the rates of the standing facilities in the operation that was settled on 14 November 2001.

The amounts (EUR bn, as reported in Bindseil 2002) of liquidity actually allotted in these tenders (vs benchmark) were: 65 (88), 25 (53), 60 (79) and 38 (66). According to the analysis of the previous chapter, we expect the bid amount to be so low that the shortest money market rate will be above the main refinancing rate (due to the overlapping nature of the weekly ECB main refinancing operations with two weeks maturity) if the ECB applies the benchmark allotment rule in the last tender of the reserve maintenance period. However, if the ECB were expected to punish the underbidding behaviour by supplying less liquidity than what the benchmark would suggest in the consecutive operation, the incentive to underbid would be diminished.

According to our calculations, the actual liquidity provision (vs the benchmark) in the operations following those where underbidding occurred were 155 (182.5), 172 (176), 82 (86) and 116 (119). That is, in the first underbidding episode, the ECB clearly provided too little liquidity (in the first operation after the underbidding) for the banks to fulfill reserve requirements without using the marginal lending facility. Similar policies seem to have been applied also in the second and third episodes, although to a lesser extent. During these incidences, the difference between actual and benchmark allotment was only EUR 4 billion, which probably could have resulted from a change in the forecast of the autonomous liquidity factors. However, as the banks' net recourses to standing facilities before the end of the maintenance period were EUR 61 and 25 billion (as reported in Bindseil 2002), we assume the ECB to have intentionally provided liquidity below the benchmark allotment. In the last episode, the actual liquidity provision in the first operation following the underbidding also was below the benchmark allotment, but in this case there was still one operation left in the same reserve maintenance period, and the net recourse to standing facilities was negative. Hence, in our view, the ECB applied benchmark liquidity provision in the last overbidding episode.

Figure 6 shows the evolution of EONIA during the week following the four cases of underbidding. At the penultimate main refinancing operation of the reserve maintenance period ending 23 April 2001, the banks expected the ECB to cut its rates in the following operation (episode 1). Thus, they wanted to postpone the holding of reserves until the expected rate cut would have taken place and the price of reserves would have come down. On the aggregate level, the banks can do this kind of backloading only by bidding for less liquidity than a smooth reserve holding path would suggest, ie by underbidding. Due to the underbidding, EONIA increased from the level of the main refinancing rate (MRR) to 20-25 basis points above it. This is just what the analysis of the previous section would indicate to happen if the maturities of two consecutive operations overlap. On the day of the announcement of the following operation, the spread between EONIA and MRR rose to 70 bps. It reached 83 bps on allotment day of the second operation, when it was clear that the ECB did not supply enough liquidity for the banks to fulfill their

218

Figure 6: **Evolution of the EONIA after the four underbidding episodes.**

1= episode 1 (12.-19.2. 2001)   2= episode 2 (9.-17.4. 2001)
3= episode 3 (8.-16.10. 2001)   4= episode 4 (5.-13.11. 2001)

reserve requirements without recourse to the marginal lending facility (ie according to the benchmark rule).

With the banks having experienced tight liquidity provision following the first underbidding episode, the EONIA spread increased up to 72 bps already on the settlement day of the operation in which underbidding took place for the second time (episode 2). The spread widened even further on the following days, so that the EONIA nearly equalled the marginal lending rate already on the day after the settlement. EONIA rose again considerably above the *MRR* during the week following the third underbidding episode in October 2001, but this time the spread never exceeded 23 bps. Thus, the rate seems to have somehow followed the path we would expect with rate cut expectations and overlapping operations. Hence, it seems likely the banks did not expect the ECB to punish underbidding severely this time. In the last episode, the EONIA spread again behaved as the model would suggest; it remained below 13 bps until the next tender.

We find the bidding in the underbidding episodes to reflect rational behaviour on the part of the banks that expect the central bank to cut

its rates at the subsequent operation (in the same reserve maintenance period). The rise of the very short-market rate above the tender rate after an operation with underbidding can be a result simply of the overlapping nature of the main refinancing operations. However, *the substantial widening of the spread between market rate and MRR* (eg up to 70 bps or more, as in the second episode) *reflects more the increasing probability of the central bank 'punishing' the underbidding by providing less-than-benchmark liquidity, than the normal increase in market rate due to a lower level of liquidity.* This interpretation is in contrast with the explanation for the positive spread given in Bindseil (2002). Bindseil suggests the spread results from the banks' lack of ability to bring aggregate bids in line with liquidity needs. If the spread between market rate and tender rate is the result of a bank's inability to estimate precisely the bid volume of other banks, and hence to make a correct bid reflecting its own liquidity demand, the aggregate liquidity should stochastically vary around the equilibrium level. This means that in one underbidding episode the probability of too-little liquidity provided should be approximately the same as the probability of too much.

Our claim is that the *basic reason for underbidding is the combination of interest rate cut expectations and the minimum bid rate. The incentive to bid according to the smooth path for reserve holding is further enhanced by the overlap of the maturities of consecutive tenders.* These features of the operational framework result in a positive spread between a market rate with maturity shorter than two weeks and the two-week-maturity main refinancing rate. The size of this spread reflects the size and probability of the expected rate cut, as long as the central bank is expected to follow the benchmark strategy in subsequent tenders. However, if the central bank is expected to 'punish' underbidding by restricted liquidity provision in the coming tender(s), the incentive to underbid will be reduced. But it will be deterred totally only if the punishment is expected to be large enough to compensate for the effect of the expected rate cut. Whether or not the demand for liquidity is more stochastic due to the banks' inability to coordinate bids will affect only the volatility of the short-term market rates. Hence, the possible coordination problems is never the initial reason for underbidding.

There were three rate cuts that were not preceded by underbidding during the period under examination. These took place in the

tenders settled on 15 May, 5 September and 19 September 2001. We suggest that the ECB's tight liquidity provision after the two first underbidding episodes deterred the banks from underbidding before the May rate cut. The same may apply also to the first rate cut in September 2001. Yet, some banks seem to have underbid in the operation prior to the cut, as the bid amount totalled only EUR 72.9 billion, while the allotted amount was EUR 70 billion. Hence, in this operation the bid ratio (ie aggregate amount of bids / allotted amount) was only 1.04, whereas on average the bid ratio was 2.0 in the tenders without underbidding. The third rate cut was implemented in the aftermath of the 11 September attack. Hence, it really could not be foreseen by the banks (during the last operation before the cut), and consequently underbidding related to rate cut expectations did not occur. Furthermore, during that time the demand for liquidity most likely exceeded that of more normal times.

We will focus next on the bidding behaviour in the tenders not marked by underbidding.

5.2   Bidding in ECB main refinancing operations

In this section we will examine the banks' bidding behaviour in ECB variable rate main refinancing operations.[21] According to the analysis of section 4.1, we expect the banks to display demand schedules that are flat at the expected market rate up to the amount indicated by the benchmark allotment rule, as the ECB seems to follow the benchmark rule quite closely. Consequently, the marginal rate of the allotment is expected to equal the comparable market rate of interest.

ECB (2002) reports the spreads vs the marginal rate of certain market rates for the period from 27 June 2000 to 12 June 2001. The average spread between the marginal rate and the two-week general collateral repo rate was –0.6 basis points, while it was –3.4 bps against the two-week EONIA swap rate. These figures indicate the marginal rate of the allotment to have been fairly close to the comparable

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[21]A detailed description of bidding behaviour in ECB open market operations from January 1999 until mid-June 2001 can be found in ECB (2001), and Nyborg et al (2002) analyse bidding and performance in the first 53 variable rate main refinancing operations based on a dataset that includes all bids submitted in these tenders.

market rate. The cumulative bid amount at rates above the marginal rate divided by total allotment volume averaged 0.71, while the figure was 1.25 when bids at the marginal rate are taken into account. Therefore, the ECB would (on average) have had to have scaled the allotment volume back by 29% to increase the marginal rate by one basis point or increase it by 25% to cut the rate by one bp. To change the marginal rate by two basis, points the allotment volume would have had to be reduced by 65% or increased by 55%. These figures, together with the fact that the bid volume at the marginal rate was some 54% of allotted amount, indicate that the aggregate bid was fairly large at rates around the marginal rate; ie the demand curve was relatively flat. Furthermore, the average spread between the weighted average rate of the accepted bids and the marginal rate was 1.4 basis points in the period from 27 June 2001 to the end of March 2002. This also reflects the flatness of the demand schedule in these tenders. However, this spread seems to have depended positively on the spread between eg the one-week EURIBOR and the minimum bid rate. This relation is illustrated in figure 7, which plots the spread between the weighted average of the rates on accepted bids and the marginal rate against the spread between the one-week EURIBOR and the minimum bid rate. We interpret this dependence as an indication of uncertainty as to the coming marginal rate being increased when the spread between market rate and minimum bid rate increased, and hence (at least some of) the banks seem to have responded to this uncertainty by supplying bids at higher rates or at different prices.

A positive spread between average accepted rate and marginal rate can be rationalised by assuming two types of banks participating in the central bank tenders. Furthermore, it can be shown that in such a case the spread might be increasing in interest rate uncertainty. In the case of the euro money market, there is a group of small banks that obtain refinancing only through central bank operations, ie they participate the central bank main refinancing operations, but do not actively trade liquidity in the interbank market. Henceforth, we refer to these as 'small banks', whereas the banks that participate in both central bank tenders and interbank market will be called 'large banks'. It is shown in appendix A that it is optimal for a small bank to submit bids at rate higher than the expected marginal rate. In this case the increasing cost of central bank liquidity is compensated by the reduction in uncertainty (about the amount of liquidity provided

222

Figure 7: **Effect of interest rate expectations on the spread between weighted average and marginal rate**
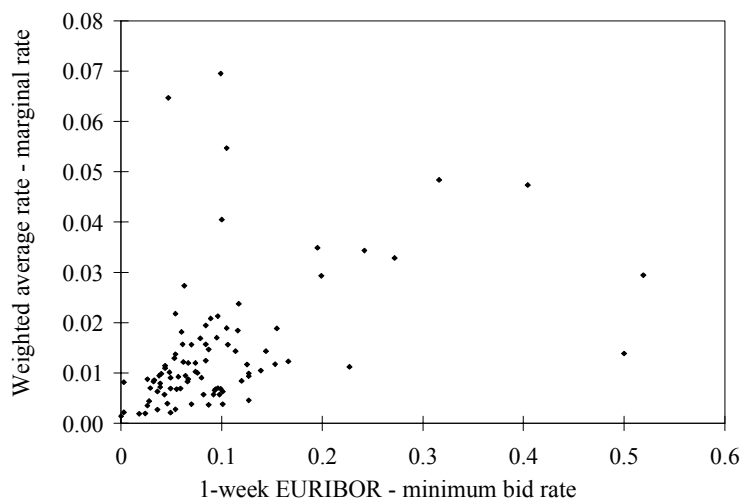
to the bank) that is associated with bidding at the marginal rate. Moreover, if small banks have inferior information on the forthcoming market rate (and thus also on the marginal rate of the allotment) they are expected to bid at increasingly higher rates (relative to the expected marginal rate) as the uncertainty increases. Normally, greater uncertainty is associated with more intense interest rate hike expectations. Consequently, when there are counterparties that do not trade in the interbank market, the aggregate demand schedule is (weakly) downward sloping instead of flat. However, the assumption of heterogeneous banks does not otherwise alter the results we derived for heterogeneous banks; the marginal rate is still determined by the bids of large competitive counterparties. From money market view point, the central bank reserves provided to small institutions should not be taken into account in the money market liquidity.

Another explanation for the positive spread between the weighted average of rates on accepted bids and the marginal rate of the allotment would ensue by assuming the banks to be risk averse and possessing asymmetric information about the coming marginal rate. Wang and Zender (2002), in a recent paper on multi-unit auctions,

show that the equilibrium bid schedules for risk averse bidders who have private information are downward sloping, ie demand reduction is present in those cases to enable banks to avoid the winner's curse. However, in a recent paper, Nyborg et al (2002) analyse bidding in the ECB main refinancing operations using microdata. They find that private information and the winner's curse are not important in these auctions.

6 Summary and conclusion

In this paper we have constructed an equilibrium model for the short-end of the money markets when the central bank provides liquidity through variable rate tenders. The length of the reserve maintenance period in the model is assumed to be two days, in order to keep the model as tractable as possible while retaining the effect of interest rate expectations. The relation between market rate of interest and liquidity for both days of the reserves maintenance period is derived from a single bank's profit maximisation problem in the interbank market. The central bank decides on the intended liquidity supply by minimising a quadratic loss function that contains both the deviations of expected market rate from the central bank's target rate and differences between liquidity supply and target liquidity. This means that the central bank aims at holding the market rate of interest close to a target value, but it also tries to stabilise liquidity over the reserve maintenance period. The banks are assumed to observe symmetric signals as to the coming market rate, while preparing their bids. In the section on banks' bidding behaviour, we analysed bidding under both multiple rate and single rate procedures.

We show that the central bank can meet its targets for both the expected interest rate and expected liquidity on the final day if it does not use the standing facilities as an independent signalling device. In this case, the location of the interest rate corridor (set by the standing facilities) around the target rate depends on the distribution of liquidity shocks; as long as the shock distribution is symmetric, the central bank targets can be met by setting the corridor such that the target rate is at the mid-point (ie a symmetric corridor). However, if the rates of the standing facilities are set independently of the target

224

rate, the difference between expected market rate and target rate will depend on the asymmetry of both the shock distribution and interest rate corridor.

We also showed that on the first day of the reserve maintenance period, under static interest rate expectations, the central bank will provide the markets with liquidity equal to target liquidity, and the expected market rate of interest will be at the level targeted by the central bank, regardless of the preferences-weighting parameter, when the shock distribution is symmetric. However, if the shock distribution was asymmetric, the liquidity supply will depend on the preference weighting of the central bank. The higher the relative weight on liquidity deviations, the more the interest rate will differ from target and the closer the equilibrium liquidity will be to the reserve requirement.

Determination of the optimal liquidity supply becomes more complicated when the banks expect the target rate to be changed between the two days of the reserve maintenance period. In such a case, as long as the central bank pays any attention to money market interest rates, it will provide the more liquidity, the higher the expected future interest rate. We show that the expected value of the market rate for the first day will be above the target rate when an interest rate hike is expected. However, the simultaneous effect of both the expected interest rate hike and the increasing liquidity supply on the expected market rate for today is not necessarily monotonic.

In the analysis of bidding behaviour, we showed that under a multiple rate procedure, without a reserve price (for the ECB, the minimum bid rate), the aggregate demand schedule the banks display is flat, at the expected market rate at least up to the amount the central bank is aiming to provide. The introduction of a minimum bid rate alters the bidding behaviour; it was shown that when the minimum bid rate is effective, equilibrium in the money market is determined as in the case of fixed rate tenders with the central bank accepting all bids submitted. However, in this case the market liquidity will be below, and the expected market rate above, the level preferred by the central bank.[22] The minimum bid rate is likely to become effective when the target rate is expected be cut; the more so,

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[22]This is a feature that is different from the equilibrium with fixed rate tenders and 100% acceptance.

the smaller the difference between current target and minimum bid rate.

It was also shown that 'underbidding' results from the minimum bid rate becoming effective. Moreover, we show that when the maturities of consecutive tenders overlap, underbidding is encouraged to the extent that the expected market rate for the first period rises above the prevailing target rate. Thus, in a framework that includes both overlapping tenders and a reserve price, the short-term market rate will rise above the target level whenever the banks expect the central bank to change its target during the ongoing reserve maintenance period – whether upward or downward.

In our analysis of single rate tenders, we assumed the central bank scales the supply of liquidity back from the intended level, if 'collusive seeming' bidding behaviour is detected. This kind of behaviour would punish the banks by forcing them into greater usage of the marginal lending facility. Hence, in equilibrium, the banks bid for the amount of liquidity the central bank is willing to provide at the rate expected to be realised with that liquidity supply. The punishment strategy is needed to deter the banks from submitting very steep demand curves, for which any interest rate between minimum bid rate and expected market rate (with the intended liquidity) could be maintained as a symmetric pure strategy equilibrium (as shown by Back and Zender 1993).

We also studied the liquidity supply of the ECB and bidding behaviour of banks in the ECB variable rate tenders. We found that the ECB seems to have put some weight on both interest rate and liquidity considerations in its allotment decisions. However, the effect of interest rate expectations[23] on actual liquidity provision was found to be so modest that we are willing to view the ECB as a highly liquidity-oriented central bank. That is, liquidity provision closely followed the benchmark allotment, according to which bank reserves are held stable within each reserve maintenance period. Furthermore, four cases of obvious underbidding were found. Based on a closer analysis of these cases, we claimed that the basic reason for underbidding was the combination of interest rate cut expectations and the minimum bid rate, which was further enhanced by the overlapping nature of the maturities of consecutive tenders. However,

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[23]Interest rate expectations were measured as the spread between the one-week EURIBOR and the minimum bid rate.

the notable widening of the spread between market rate and main refinancing rate, following some of the underbidding episodes, seems to have reflected to a greater extent the increasing probability of the ECB 'punishing' the underbidding via lower liquidity provision in the subsequent operation than the normal increase in market rate due to a decrease in liquidity.

Finally, we analysed the banks' bidding in the ECB main refinancing operations, and found the demand schedules to have been fairly flat at rates close to the marginal rate of the allotments. We found some evidence that uncertainty as to the coming marginal rate of the allotment affects the bidding so that the demand schedules displayed are steeper.

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A   Heterogeneous banks

In this section, we deviate from the basic setup by having some banks that do not participate in the interbank market. In the case of the euro money market, there is a group of small banks that acquire their refinancing solely through central bank operations, ie they participate in the central bank main refinancing operations, but do not actively trade liquidity in the interbank market. Henceforth, we refer to these as 'small banks', whereas banks that participate in both central bank tenders and interbank market will be called 'large banks'. Small banks bid for liquidity solely to minimise the cost of holding reserves; they do not aim at making profits on price differences between auctioned liquidity and interbank liquidity. Thus, in a variable rate tender a small bank bids to maximise the following:

$$\max_{b_i} \Pi = pv_i\left(q_i\right) - \sum_{j=1}^{10} q_{i,j,t} r_{i,j}^T$$

$$s.t.\ q_{i,j} = \begin{cases} b_{i,j}, \text{ if } B_{r_{i,j}^T} \leq m^* \\ \frac{m^* - B_{j-1}}{B_j} b_{i,j}, \text{ if } B_{j-1} < m^* < B_{r_{i,j}^T} \\ 0, \text{ if } B_{j-1} \geq m^* \end{cases} ,$$

where $pv_i\left(q_i\right)$ denotes the private value of q_i units of liquidity to bank i. The functional form depends on whether averaging is still possible, ie whether or not it is the final day of the reserves maintenance period or not.[24] The private value increases in liquidity, but its marginal

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[24]The private value of the liquidity held by bank i at the last day of the reserve maintenance period is:

$$pv_i\left(q_i\right) = r_2^m \left[\int_{-\infty}^{-l_{i,2}+rdb_{i,2}-b_{i,2}} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right]$$

$$+ r_2^d \left[\int_{-l_{i,2}+rdb_{i,2}-b_{i,2}}^{\infty} (l_{i,2} - rdb_{i,2} + b_{i,2} + \varepsilon_2) f(\varepsilon_2) d\varepsilon_2 \right], \text{ (A.1)}$$

and for the first day of the period:

growth decreases with liquidity.[25] Thus, the optimal bid for bank i must be such that with it $\partial pv_i/\partial q_i = r^T_{i,j}$, ie the marginal increase in private value equals the bid rate. According to the model derived in the previous sections, the marginal rate of the allotment should equal the expected market rate of interest (ie $r^{\mathrm{marginal}} = E[r(m^*)]$). Therefore, while preparing its bid *in a multiple rate auction*, the bank has to consider that increasing the bid rate from r^{marginal} is costly. However, the optimal cumulative aggregate bid, $B^*_{r^{\mathrm{marginal}}}$, is not unique; in order for bank i to make an optimal bid at the marginal rate, it should be able to estimate the proportion for which pro rata rationing is used at the marginal rate (ie the optimal bid would be

$$
\begin{aligned}
pv_i(q_i) \;=\; & r^m_1 \left[\int_{-\infty}^{-l_{i,1}-b_{i,1}} (l_{i,1}+b_{i,1}+\varepsilon_1)f(\varepsilon_1)d\varepsilon_1 \right] \\
& + E_1[r_2]\left[\int_{-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1}+b_{i,1}+\varepsilon_1)f(\varepsilon_1)d\varepsilon_1 \right] \\
& + \left(r^d_1 - E_1[r_2]\right)\left[\int_{2R_{i,1}-l_{i,1}-b_{i,1}}^{\infty} (l_{i,1}+b_{i,1}-2R_{i,1}+\varepsilon_1)f(\varepsilon_1)d\varepsilon_1 \right].
\end{aligned}
$$

[25] The change in private value when liquidity changes in the last day is given by:

$$
\partial pv_i/\partial q_i = r^d_2 + (r^m_2 - r^d_2)F\left(-l_{i,2}+rdb_{i,2}-q_{i,2}\right). \tag{A.2}
$$

Thus,

$$
\partial pv^2_i/\partial^2 q_i = -(r^m_2 - r^d_2)f\left(-l_{i,2}+rdb_{i,2}-q_{i,2}\right). \tag{A.3}
$$

For the first period:

$$
\begin{aligned}
\partial pv_i/\partial q_i \;=\; & E_1[r_2] + (r^m_1 - E_1[r_2])F(-l_{i,1}-q_{i,1}) \\
& + \left(r^d_1 - E_1[r_2]\right)\left[1 - F(2R_{i,1}-l_{i,1}-q_{i,1})\right].
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\partial pv_i/\partial q_i \;=\; & E_1[r_2] + (r^m_1 - E_1[r_2])F(-l_{i,1}-q_{i,1}) \\
& + \left(r^d_1 - E_1[r_2]\right)\left[1 - F(2R_{i,1}-l_{i,1}-q_{i,1})\right]
\end{aligned}
$$

231

implicitly given by $\left(\partial pv_i/\partial q_i\right)\left(m^* - B_{j-1}\right)/B_j = r^{\mathrm{marginal}}$). Errors made in estimating the allotment ratio are costly for bank i, as they shift the amount of liquidity it receives away from the optimal level. As a consequence, we expect the cost of placing a bid at a rate slightly above the marginal rate to be profitable for the small banks, as the increase in the cost of central bank liquidity is well compensated for by the reduction in uncertainty associated with bidding at the marginal rate. If we assume that, in addition to (or as a consequence of) not participating in the interbank market, small banks have inferior information on the forthcoming market rate of interest (and thus also on the marginal rate of the allotment), we expect these banks to increase the spread between bid rate and expected marginal rate whenever the central bank is expected to increase its target in the near future. This is due to the fact that when the target rate is expected to be changed the uncertainty related to the marginal rate increases. Thus, to avoid bidding at rates below the marginal rate, the small banks are willing to bear higher costs for acquiring liquidity from the central bank. Consequently, if the banks are heterogeneous, the demand schedule is not expected to be flat, the more so when the heterogeneous banks also possess asymmetric information.

Note that this problem for small banks arises only in the multiple rate auction format. The inframarginal bid at rates above the marginal rate are not costly, and so in a single rate auction, the small banks can present the central bank with a demand curve that reflects their true demand for liquidity arising from the reserve requirement. Therefore, they are allotted the liquidity that is optimal for them at the marginal rate.

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