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Preface

The liberalization of the financial markets, the re-definition of Finnish monetary aggregates and rapid advances in econometric methodology in the late 1980s together awakened the need in Finland for up-to-date money demand equations. Work on the thesis began under the direction of Antti Suvanto when I worked as a researcher at the Bank of Finland in autumn 1991. The project continued with the aid of the cooperative banks' research foundation and the Yrjö Jahnsson Foundation and was accepted as a thesis for the licentiate degree in May 1993.

With respect to the econometric methodology involved, the primary influence has been the Nordic Multivariate Cointegration Workshop led by Katarina Juselius and Søren Johansen, in which I had the opportunity to participate over the years 1990–1993. Presentations made at the workshop and numerous discussions with the other participants and especially with Professor Juselius were of immense value in formulating the thesis. I am deeply indebted to these people.

Besides my colleagues, Professor Erkki Koskela also commented on the economic interpretation of the findings. Glenn Harma patiently checked the English language, correcting errors and making numerous suggestions for improving my modes of expression. Aila Raekoski handled the thankless task of converting the text from the original LaTeX version into WordPerfect. My sincerest thanks go to her and to all the others who helped me see the project to completion.

Helsinki, April 1994

Antti Ripatti

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1 Introduction

The demand for money plays an important role in macroeconomic models. Traditionally, it ties together the financial and real sectors of the economy. The theory of the demand for money is quite compact, and empirical findings have generally supported it. Since the forecasting failure of U.S. demand for money models in the late 1970s, money demand analysis has been a lively subject among econometricians. Dozens of new methods have been applied to "explain" the forecasting failure.

However, the demand for money has not been a very popular subject among Finnish economists. Credit rationing in Finland has made it difficult to make comparisons with studies of other countries. The application of foreign examples to the Finnish case has failed to adequately treat credit rationing and its implications. The only way of taking credit rationing into consideration in the early studies[1] of money demand in Finland was to use the inflation rate as a substitute for the interest rate.

The Bank of Finland has changed the definitions of the monetary aggregates as the old definitions of M1 and M2 in particular became outdated. The structure of the financial markets had changed a great deal since the 1970s. The changed role of the central bank and financial institutions influenced the components of money. Today, the monetary aggregates are more closely tied to the amount of liquidity than before, and money has greater significance in the economy. The development of the financial sector during the 1980s created a new set of markets – new transaction tasks for money. According to some measures, money has grown three times faster than GDP during 1980s. The amount of monetary transactions has probably changed.

For an econometrician, the demand for money is still an attractive area of application. The theory is compact, and the time series are usually long and of good quality. The number of parameters is limited and there is also a consensus on the desirable features of money demand. It is a good forum in which to apply new methods.

Moreover, time series analysis has improved during the 1980s. Unit root econometrics has influenced macroeconometric modelling to a great extent. Earlier studies require re-evaluation. The notions of cointegration and the error correction mechanism have brought the economic theory and econometric models closer together.

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[1] See Suvanto (1980) for a survey of Finnish money demand studies.

The study is organized in four parts as follows: In part 1, we survey the theoretical models of the demand for money and derive a dynamic framework. Part 2 presents the econometric methods to be used in part 4 of the study[2]. The data and the institutions as well as earlier results and testable hypotheses are presented in part 3.

1.1 On Money

The usual approaches taken in defining what is called money are the transactions approach and the asset approach. In a classical framework money is thought of as a medium of exchange and, simultaneously, as a store of value. The medium-of-exchange role is probably based on the ability of money to reduce transaction costs. A special feature of money as a medium of exchange is that it is indestructible in this role as long as it is 'acceptable'. Then, according to I. Fisher, 'any property right which is generally acceptable in exchange may be called money' (I. Fisher 1922, p. 5). Information also has a role in the medium-of-exchange definition of money. Money is something that distributes information more evenly between individuals and creates an efficient alternative to barter.

If money does not reward the holder with a cash return and it deteriorates at the rate of inflation, all other assets are denominated in money terms. The asset approach to money tries to distinguish money from other non-interest-bearing assets such as consumer goods. Patinkin (1965), for example, solves the problem by including money as an argument in the utility function. The utility of money is based on what will happen if an individual runs out of cash. This utility approach provides a rationale for distinguishing between money and non-money. Money is desired because of its characteristics. Portfolio 'mean-variance' analysis is also used to analyze money demand. Money is then not an argument in the utility function but a part of a portfolio of financial assets. The yield on money is based on its usefulness in transactions.

One motivation for the existence of money is the uncertainty of 'double coincidence of wants'. There is also uncertainty concerning the reputation of a potential trading partner. Overlapping-generations models ignore the medium-of-exchange function of money and

[2] Computation has been done with RATS 3.14, PC-GIVE/FIML 6.1, Gauss 2.0 and SURVO 84C. Katarina Juselius and Henrik Hansen have kindly provided me with a system of Cointegration Analysis of Time Series (CATS).

emphasize instead an asset's use as a store of value. The sidestepping of the medium-of-exchange function of money by these models has serious implications in that they fail to provide an adequate framework for analysing monetary issues.

Empirical definitions of money are usually based on the medium-of-exchange function. The degree of liquidity is usually the basis for separating the different monetary aggregates. For example, time deposits are not included in M1 but are included in M2. Barnett (1980) is a thorough study of monetary aggregates. The commonly used simple-sum aggregation of different money-like assets is valid only when aggregating over 'goods' that are perfect substitutes. In constructing ideal monetary indices, the weights of financial assets in the indices are based on the user cost defined with the help of the interest rate for each asset and the benchmark asset. The aggregation influences the analysis of money demand. According to LaCour (1991) the interest rate elasticity is much smaller in the ideal index number aggregates than in the simple-sum aggregate. This occurs mainly due to the interest rate effect captured by the ideal index numbers themselves.

McCallum and Goodfriend (1988, p. 16) separate money from other assets by means of the following definition:

> assets are part of the money stock if and only if they constitute claims to currency, unrestricted legal claims that can be promptly and cheaply exercised (at par).

Here, money is a *medium of exchange* and *unit of account*.

Demand for money studies investigate how much money (as defined in the study) agents are willing to hold and what factors determine the level of holdings. In this study, we focus on the demand for two alternative commonly used measures of money, M1 and M2. In a small open economy, like Finland, the central bank cannot control the amount of money supplied under a fixed exchange rate regime. It must be ready to buy or sell all amounts of currencies offered or demanded at the fixed rate. The amount of money in the economy is determined by the demand for it. The currency band, which is used in Finland, gives central bank a modicum of control over the money supply, but this possibility cannot be operationalized in practice.

The present poor condition of the banking sector in Finland could be considered as a money supply shock. Credit expansion is no longer automatic, due to the problems of banks' customers. However, the data used here do not cover the most severe phase of the banking crisis.

1.2 On Econometric Modelling

Econometric modelling of macroeconomic time series has undergone a great deal of change during last decade. The growing use of vector autoregression, the unit root literature, the error correction mechanism, as well as cointegration, the structured concept of causality and exogeneity and recent advances in non-linear modelling have all provided an enlargening set of tools and concepts for empirical research. However, these tools and cencepts are very rarely used effectively in practice. The connections between the fields are not usually exploited. In this study we have tried to use the econometric development work that is being done in the Nordic Multivariate Cointegration Workshop[3] in particular and to bring in some other ideas. One side-effort, which eventually uses some of the main results, is to test a very specific rational expectations model for money demand.

The main object of the study has been to derive empirical models for money demand in Finland. Our approach uses the following macroeconometric modelling *strategy*.

First, we do not start with a specific theoretical model of money demand. Rather, we consider the set of rival theories and approaches and try to test the restrictive implications of the theories in a more general framework. In this way, we try to avoid the problems of choosing a wrong theory *ex ante*. However, we must choose the set of theories, the starting-point, by excluding some possibly useful and important factors. Here, the theories, together with economic intuition (whatever that is) suggest a set of economic variables that are closely connected to the problem. Chapters 2 and 3 concentrate on these issues within the context of the demand for money.

The second (and difficult) step is to find empirical counterparts to the theoretical variables. Sometimes that is not possible, as the theory is not very explicit about the measurement of economic phenomena. Moreover, the set of variables might be very sensitive to the initial assumptions of the theoretical models. In practice, the existence of data and the length of time series restrict the set of variables and the questions one can ask of the data. Chapter 8 of the study describes the data and the problems of choosing empirical determinants of the demand for money.

[3] Early versions and pre-estimations of the study have been presented in the workshop. I have benefited from discussions and research projects of the participants — the studies and comments of Katarina Juselius have motivated and greatly improved the study.

I have chosen the vector autoregressive (VAR) model as a starting point. The VAR model has several advantages; including a minimal number of restrictive exogeneity assumptions. In several cases it describes very well the variation in the data. The most important disadvantage of the VAR model is the huge overparametrization and consequent loss of degrees of freedom. Further discussion of the VAR is found in part II of the study.

VAR models allow us to test parameter restrictions with the unrestricted VAR as an alternative hypothesis. These restrictions could be derived from the economic theory. Unfortunately, the theory is usually not very explicit about the restrictions. The statistical and theoretical models differ too much and it is very difficult to formulate the questions to be asked of the data. Ideally, one can "test a theory", i.e. find empirical evidence for or against it.

Once one has found the levels of exogeneity of the variables in the model, the other variables can be conditioned on those variables. This will improve the efficiency of the estimation. But for an economist this is usually not enough. The structural model is normally the main objective of econometric modelling. Starting with the VAR, one has a good chance of avoiding incorrect *a priori* exogeneity assumptions. On the other hand, the information set (mainly variables) is often not large enough for an *economically* very well specified model for each endogenous variable. This problem, of course, comes up very common in modelling simultaneous equations. The technical details involved in this step are discussed in chapter 6.

One way of testing theories is to derive an econometric model directly from the theoretical model, choose the variables and then procede to estimate the model. If the model diagnostics reject the statistical assumptions of the model, the statistical model does not support the theoretical model. The study is then back at step one. If the evidence does not support the theory, one can hardly proceed with the analysis using the same model. This kind of strategy is applied in testing a specific rational expectations model in chapter 7 and sections 11.2 and 12.2.

2 Theoretical Models of the Demand for Money

There are several theoretical approaches to modelling the demand for money. The qualitative results of these approaches are very much alike. The demand for money depends on the price level, income, various interest rates and perhaps inflation. Goldfeld and Sichel (1990) and McCallum and Goodfriend (1987) are good summaries of the different approaches. Laidler's (1985) textbook review ignores the newest models, while Fisher's (1989) book emphasizes the demand system approach. Barro and Fisher (1976) summarize carefully the early literature.

The earlier literature on money did not concern individual choice – money demand *per se*. Instead, the 19th century quantity theory of money tradition, which culminated in Fisher (1911), concentrates on market equilibria by means of the following simple identity

$$MV \equiv PT, \tag{2.1}$$

where M is the quantity of money, V the velocity, P the price level and T the volume of transactions. Velocity, $T/(M/P)$ was thought to be determined by institutional and technological factors and thus to be constant. The Cambridge economists relaxed the constant velocity assumption and emphasized that the velocity would be affected by foregone "investment income", i.e. interest earnings.

Keynes' search for a simple model led him to distinguish three motives for holding money: transactions, precautionary and speculative. Payments that cannot be considered as regular and planned are paid with money that is held for precautionary reasons. The speculative demand for money arises from the cost of holding money rather than other assets.

2.1 Transaction Demand for Money

Baumol (1952) and Tobin (1956) developed an inventory approach to the demand for money. Baumol (1952) abstracts from the speculative and precautionary demand and concentrates on the transaction demand. The transactions are perfectly foreseen and occur in a steady stream.

There are two assets in the economy — money and an interest-bearing asset. The value of transactions in one period is Y, the demand for cash is M and the interest rate r. Each time the agent withdraws cash M he has to pay a "broker's fee" b. Thus, he makes Y/M withdrawals over the course of a year, at a total cost in broker's fees of bY/M. Because he spends the money at a steady rate, his average cash holdings are $M/2$ and thus the cost of holding money is $rM/2$. The total costs are then

$$\frac{bY}{M}+\frac{rM}{2}. \tag{2.2}$$

The first order condition for minimizing total cost with respect to M yields

$$-\frac{bY}{M^2}+\frac{r}{2}=0. \tag{2.3}$$

The transaction demand for money is therefore

$$M=\sqrt{\frac{2bY}{r}}=\sqrt{2}\,b^{½}Y^{½}r^{-½} \tag{2.4}$$

The transaction elasticity is thus 1/2 and the interest elasticity −1/2.

Tobin (1956) found that the interest elasticity was not constant because of teh integer restriction on transactions. The possible outcome was that agents do not hold money at all. Goldfeld and Sichel (1990) found that under the alternative assumption of a proportional broker's fee there may be non-constant interest and income elasticities.

Miller and Orr[1] (1966) presented a stochastic version of Baumol's (1952) inventory approach. There are only two assets in the economy — money and an interest-bearing bond, whose "marginal and average yield is r per dollar per day". Transfers between these two assets may take place at any time at a constant marginal cost of b per transfer. Transfers may be regarded as taking place instantaneously. The net income flow is assumed to be stochastic, following a Bernoulli-distributed random walk without drift. Thus, over a long interval of n

[1] Pesola (1987) simulated the cash management of a firm using the Miller–Orr approach.

days, the observable distribution of changes in the cash balance will have mean $\mu_n = ntm(p-q)$ and variance $\sigma_n^2 = 4ntpqm^2$, where t is the number of transactions per day, p is the probability of an increase in cash balances of m dollars and $q = 1 - p$ is the probability of a decrease in cash balances of m dollars. They also assume that $p = q = 1/2$, so the changes are symmetric. The mean of daily changes is then $\mu_n = 0$ and the variance $\sigma_n^2 = nm^2t = n\sigma^2$.

The firm seeks to minimize the long-run average cost of managing the cash balance by means of the following policy: "The cash level is allowed to flow freely until it reaches either the lower bound, zero, or an upper bound, h, at which times a portfolio transfer will be undertaken to restore the balance to a level of z". The firm minimizes the expected cost with respect to h and z. The expected cost per day of managing the firm's cash balance over a finite planning horizon of T days can be expressed as

$$E(C) = b\frac{E(N)}{T} + rE(M), \tag{2.5}$$

where $E(N)$ is the expected number of portfolio transfers during the planning period; b is cost per transfer; $E(M)$ is the average daily cash balance; and r is the daily rate of interest.

The average time interval between transfers is $D(z,h) = z(h - z)$. Thus the first term on the right-hand side of (2.5) approaches $1/D(z,h)$ for large T. The mean of the second term of (2.5) the long-run average cash balance in terms of z and h, is $(h + z)/3$. Combining the previous results, the problem can be written, defining $Z = h - z$, in the form

$$\min_{Z,z} E(C) = b\frac{m^2t}{zZ} + r\frac{Z+2z}{3}. \tag{2.6}$$

The first order conditions yield the optimal values

$$z^* = \left(\frac{3bm^t}{4r}\right)^{1/3} \tag{2.7}$$

and

$$Z^* = 2z^* \Longleftrightarrow h^* = 3z^*. \tag{2.8}$$

The optimal return point $z^* = (1/3)\ h^*$ lies to the left of the midpoint of the interval $[0,h]$.

The long-run average demand for money by firms can be written as

$$m^* = \frac{4}{3}\left(\frac{3bm^2t}{4r}\right)^{1/3} = \frac{4}{3}\left(\frac{3b}{4r}\sigma^2\right)^{1/3}. \tag{2.9}$$

The variance term, σ^2, is the observable variance of daily cash flows. The empirical contribution of Miller–Orr is that their model yields clear parameter restrictions on the transaction demand for money. The interest rate elasticity is −1/3 and the scale/transaction elasticity is 1/3 if transactions are measured in transaction units (dollars) and 2/3 if transactions are measured by cash flow variance.

2.2 Interest Rates – Result of Portfolio Approach

Tobin (1958) applied portfolio analysis to the demand for money. In his model, the agent allocates his wealth between a riskless asset and a risky asset. The yield of the risky asset is greater than that of the riskless asset. The optimal portfolio for a risk-averse agent is found by maximizing expected utility. The portfolio depends on the agent's wealth and the expected yield and variance of the risky asset, i.e. on the probability distribution of the return on the risky asset. In the multi-asset case, the demand functions depend also on the covariances of the asset returns.

The analysis implies a negative interest rate elasticity, which is the main econometric result of the model, which also provides a rationalization for Keynes' liquidity preference hypothesis.

The problem is that money is not a riskless asset, in real terms. If there exist any riskless assets that have a positive yield (like a savings account), then there is no demand for non-interest bearing cash. The character of the model motivates the use of the transaction approach.

Tobin (1958) also rationalized the Keynesian speculative demand, in which portfolio-holders expect that interest rates will rise. They will hold money and in order to avoid capital losses on bonds.

According to Friedman (1956) money is also a form of wealth. It is an asset that yields a flow of services to the holder. He emphasizes the substitution between money and assets other than bonds. He feels that other forms of wealth, like physical capital, should also be take into consideration. He suggests that the yield on physical capital can be measured by the inflation rate.

2.3 Combining Approaches

Ando and Shell (1975) present a two period consumption-savings model, where agents have three assets: one risky, equity E, and two riskless, money M and savings account S. The riskless assets have yields r_m (on money) and r_s (on savings accounts). The return on equities, $\underline{r_e}$ and inflation, $\underline{\Delta p}$, are stochastic. Agents maximize expected utility.

The model allows for transaction costs in exchanging money for other financial assets. C_i is consumption in period i. The transaction process is described by the transaction cost function $T(M,C_1)$. First period consumption, C_1, does not depend on portfolio decisions. Let W be the amount of initial assets. First period prices are set at one. When the following identities apply

$$e = \frac{E}{W-0.5C_1}$$

$$s = \frac{S}{W-0.5C_1}$$

$$m = \frac{M}{W-0.5C_1},$$

the second period consumption is

$$C_2 = W-C_1+(W-0.5C_1)[e\underline{r_e}+s(r_s-\underline{\Delta p})+m(r_m-\underline{\Delta p})]-T(M,C_1). \tag{2.10}$$

The form of the utility function is $U(C_1,C_2)$. Hence,

$$U^*\{C_1,W-C_1+(W-0.5C_1)[e\underline{r_e}+s(r_s-\underline{\Delta p})+m(r_m-\underline{\Delta p})]-T(M,C_1)\}. \tag{2.11}$$

Note that $e + s + m = 1$. The agent maximizes the expected value of U^* over the distribution $\phi(\underline{r_e},\underline{\Delta p})$ with respect to e, m and s. The problem is then

$$\max_{e,m} V,$$

where

$$V = E\Big\{U\Big[W - C_1 + (W - 0.5C_1)\{e\underline{r_e} + (1 - e - m)(r_s - \underline{\Delta p}) + m(r_m - \underline{\Delta p})\}$$

$$- T\big((W - 0.5C_1)m, C_1\big)\Big]\Big\}.$$

The first order conditions are

$$\frac{\partial V}{\partial e} = E\{(W - 0.5C_1)[\underline{r_e} - (r_s - \underline{\Delta p})]U'\} = 0$$

and

$$\frac{\partial V}{\partial m} = E\{(W - 0.5C_1)[-(r_s - \underline{\Delta p}) + (r_m - \underline{\Delta p}) - T_M(M, C_1)]U'\} = 0.$$

Inflation cancels in the second equation, and thus we obtain

$$(W - 0.5C_1)[r_m - r_s - T_M(M, C_1)]E(U') = 0. \qquad (2.12)$$

T_M stands for the partial derivative of T with respect to $M = m(W - 0.5C_1)$. According to Ando and Shell (1975): "It should be interpreted as the marginal reduction in transaction cost as money holding is increased by one". We can assume that on the left-hand side of (2.12) $W - 0.5C_1 \neq 0$ and $E(U') \neq 0$. Thus equation (2.12) can be simplified as

$$r_s - r_m = T_M(M, C_1). \qquad (2.12)$$

The demand for money is then a function of consumption C_1 and the difference in interest rates $r_s - r_m$. Ando and Shell (1975) emphasize that "... the division of W into E and $M + S$ can be considered almost independently of the demand for M, except in as far as the demand for M determines r_s, and r_s affects the division of W between E and $M + S$". They also derive the exact conditions for this property of the model.

The problem with the Ando–Shell model is that it assumes that C_1 is determined independently of the portfolio decision. Money demand is determined by a yield-transaction cost tradeoff with no allowance for risk. Baba, Hendry and Starr (1992) extend the model on the assumption that there is a capital market imperfection, "characterized as a spread between borrowing and lending rates available to a typical wealth holder" (pp. 27). With this assumption, they can break the $r_s - r_m$ relation[2]. This property of the Ando–Shell model can be misleading, because financial innovations and deregulation have given us a menu of financial assets that combine the portfolio and transaction features.

In their survey for the New Palgrave Dictionary of Economics, McCallum and Goodfriend (1987 and 1988) analyze an economy which has three assets: money, bonds and capital. They examine household behaviour under certainty and uncertainty. Households maximize their intertemporal utility. The utility function includes consumption c and leisure time l as arguments. Thus, it can be written in the following form

$$u(c_t,l_t) + \beta u(c_{t+1},l_{t+1}) + \beta^2 u(c_{t+2},l_{t+2}) + \cdots \tag{2.13}$$

The model includes the following assumptions

- The utility function u is well-behaved, so that unique positive values will be chosen for c_t and l_t.
- The household's production function is first degree homogeneous with capital and labour as inputs.
- Labour is supplied inelastically, implying a production function of the form $y_t = f(k_{t-1})$, where k is stock of capital.
- $f(\cdot)$ is well-behaved.
- Capital is unconsumed output with the same price as the consumption goods and a rate of return of $f'(k_t)$.

Households also have money $m = M/P$, bonds b, with yield R, and capital k. Shopping is done during the shopping time s, which is subtracted from leisure time $l = 1 - s$. Shopping time depends positively on consumption and negatively on money:

[2] In this study, as we see later, we cannot test or utilize the hypothesis $r_s - r_m$, because there is no good measure availble for r_m.

$$s_t = \psi(c_t, m_t), \tag{2.14}$$

$\psi'_c > 0$ and $\psi'_m < 0$. To write the household's budget constraint we define $b_t = B_t/P_t$, v_t as the real value of lump-sum transfers from government and $\pi_t = (P_t - P_{t-1})/P_{t-1}$ as the inflation rate. The budget constraint is

$$f(k_{t-1}) + v_t \geq c_t + k_t - k_{t-1} + m_t - (1+\pi_t)^{-1} m_{t-1} + (1+R_t)^{-1} b_t - (1+\pi_t)^{-1} b_{t-1}. \tag{2.15}$$

Maximizing utility (2.13) under certainty subject to shopping time (2.14) and budget constraint (2.15), the model implies that

$$m_t = \mu(k_{t-1}, m_{t-1}, b_{t-1}, v_t, v_{t+1}, \ldots, R_t, R_{t+1}, \ldots, \pi_t, \pi_{t+1}, \ldots), \tag{2.16}$$

which does not closely resemble the results of the previous models. The model also implies the following more traditional form of the demand for money.

$$M_t/P_t = L(c_t, R_t) \tag{2.17}$$

The only factors affecting money demand are P_t, c_t and R_t. The model applies both the portfolio and the transaction approaches. It can be shown that $L(\cdot)$ is increasing in c_t and decreasing in R_t.

The next step of McCallum and Goodfriend (1988) is to introduce uncertainty into the model. The household is assumed to know the current values of P_t, R_t and v_t, but only the non-degenerate probability distributions of their future values. There is also uncertainty in production. The marginal product of capital $f'(k_t)$ is viewed as random. If the analysis is made more complex, no closed form solution analogous to (2.16) will exist. Fortunately, "according to our model, the relationship of M_t/P_t to the transaction and opportunity-cost variables is invariant to changes in the probability distribution of future variables" (McCallum and Goodfriend 1987, p. 9).

They also vary the model by assuming that money is measured at the start of the period. The resulting feature is that M_{t+1}/P_t is related to R_t, planned c_t and P_t/P_{t-1}. "The fundamental nature of the relationships are, however, the same as above" (McCallum and Goodfriend 1988, pp. 18). An elastic labour supply would imply real wage as an additional argument in (2.17). More generally, one can increase the number of variables in (2.17) by adding other relevant margins of substitution to shopping time requirements. One example might be *inflation*, which enters into the relationship if stocks of commodities held by households are added. Relation (2.17) would be lost if the intertemporal utility function (2.13) were not *time-separable*.

2.4 Demand-systems Approach

Money demand studies have benefited very much from consumer demand studies. To a certain extent, money can be considered as a good with an interest rate as its price. In the demand-systems approach one "derives a set of demand functions for liquid assets from a more general framework in which the representative household maximizes an intertemporal utility function defined over commodities, money, leisure and financial assets, subject to an appropriate intertemporal budget constraint" (Fisher 1989, pp. 80).

In the following example, we postulate a direct utility function at time t of the general form

$$U(x_{1t},\ldots,x_{mt}),$$

where we assume weak separability between monetary assets and consumer goods. The parameters x_{it} are different assets. The utility function is maximized with the following the constraint.

$$\sum_{i=1}^{m} \pi_{it} x_{it} - M_t = 0,$$

where

$$\pi_i = \frac{p_i^*(R_t - r_{it})}{1 + R_t}.$$

p_i^* is the expected price of the asset with yield r_i and R_t is the yield on a benchmark asset. As the next step, we obtain the indirect utility function

$$g(v_1,\ldots,v_n),$$

where

$$v_i = \frac{\pi_i}{m}$$

and m is the quantity of the benchmark asset.

After applying Roy's identity, one can move from estimation of the indirect utility function to estimation of elasticities of substitution among the assets. The estimation usually employs the translog approximation.

2.5 Econometric Implications

In econometric work, one can rarely estimate directly the equations derived from theory. There are several reasons for this. Generally, as we have seen above, the theories are far too simplified compared to the complexity of the real world. They focus on special issues and do not even attempt to answer all questions or to describe the system[3] as a whole. For example, the inventory theoretic approach concentrates on the transaction motive for money demand while ignoring the other issues.

The theories do not specify the functional forms of the relationships they describe, although they sometimes include certain assumptions about derivatives etc. At times, questions concerning dynamics and uncertainty, which are very important in empirical work, are forgotten. In the next chapter, we derive the dynamics of the demand for money.

The theory does provide a lot of useful insight for econometric study. It helps to structure econometric work. Theory provides a set of factors that influence money demand. If a general framework is applied in an econometric study, the restricting hypotheses derived from economic theory can be tested.

According to the theoretical models above, the demand for money depends on the price level, transactions, opportunity cost of holding money and the own-yield on money. Price elasticity is positive – in empirical studies usually equal to one, which means that real money is the interesting variable. Transaction elasticity is considered to be positive, as is the own-yield elasticity. The opportunity cost elasticity should be negative.

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[3] The problem studied is assumed to be orthogonal to the rest of the economy.

$$M = f(\underset{+}{P}, \ \underset{+}{T}, \ \underset{-}{R}, \ \underset{+}{R_M}) \tag{2.18}$$

The model presented by McCallum and Goodfriend (1989) implies most of these stipulations. The Ando–Shell (1975) model clarifies the relationship with respect to the own-yield of money. These models also have solid microfoundations. When modelling reality, households' and firms' demand for money are mixed — not separated. This also disturbs the testing of the hypothesis because some theories describe the money demand of households and some the money demand of firms.

The hypotheses and claims, given by the theories, could hold in the long run. The *testable restrictions* could be the following: The quantity theory of money presents a hypothesis about the velocity of money. An interesting restriction is whether it is stable or not. The empirical counterpart to stability is stationarity. As Goldfeld and Sichel (1990) write, "(Price) Homogeneity of money demand, at least in the long run, is generally presumed to be a feature of any well-specified money demand function". The inventory approaches produce testable restrictions on income and interest rate elasticities. Income elasticity should be 1/2 according to Baumol's model and 1/3 according to the Miller–Orr model. Baumol's model put a restriction on the constant term also: it should be 2. According to Ando and Shell (1975) the interest rate elasticities should be equal with opposite signs. Inflation could be an opportunity cost of money.

3 Dynamic Behaviour of the Demand for Money

Consumer demand studies also provide guidance regarding the dynamic behaviour of the demand for money. Once one has an idea of which factors affect the stability of the static demand for money, it is important to clarify the dynamics involved.

The infinite dynamic loss function is a common tool for analysis of the demand for money[1]. The economic agent is assumed to minimize the discounted sum of future expected losses generated by the difference between actual and desired money balances and the adjustment of money balances. The loss function at time t is usually presented in the following form

$$L_t=\sum_{s=0}^{\infty}\delta^s\Big[\lambda_1(m_{t+s}-m^*_{t+s})^2+(m_{t+s}-m_{t+s-1})^2 \\ +\lambda_2(m_{t+s}-m_{t+s-1})(m^*_{t+s}-m^*_{t+s-1})\Big], \tag{3.1}$$

where m^* is the 'desired' or optimal steady state level of money balances in the case of no adjustment costs and m is planned money balances. At this stage of the study, we do not make any explicit assumptions — other than those of a stochastic nature — as to how the process is determined. The agent suffers losses when the planned and desired levels of the money balances are in the disequilibrium. The second component reflects the adjustment cost of changing money balances. δ is a discount factor ($0 < \delta \leq 1$) and the magnitudes of the cost parameters, $\lambda_1,\lambda_2 > 0$, affect the evolution towards the steady state. Immediate adjustment is non-optimal unless λ_1 approaches infinity. We have normalized on the second term. The agent is assumed to choose a money balance sequence $\{m_t\}_{s=0}^{\infty}$ that minimizes L_t, conditional on information available at time t[2].

[1] See, for example, Kanniainen and Tarkka (1986) and Domowitz and Hakkio (1990). Labour demand is the most common application of the dynamic optimization framework.

[2] The information set I_t consists of all factors dated at time t or earlier.

The first order condition (excluding the transversality condition) can be obtained by differentiating[3] L with respect to x_{t+s}. The result generates a second order difference equation of the form

$$\begin{aligned} &\delta m_{t+s+1} - (1+\delta+\lambda_1)m_{t+s} + m_{t+s-1} = \\ &\qquad \delta\lambda_2 m^*_{t+s+1} - (\delta\lambda_2 + \lambda_2 + \lambda_1)m^*_{t+s} + \lambda_2 m^*_{t+s-1}, \\ &\qquad s \geq 0, \quad \text{given } m_{t-1}. \end{aligned} \tag{3.2}$$

When we substitute x_{t+s} for $m_{t+s} - \lambda_2 m^*_{t+s}$ we obtain

$$\delta x_{t+s+1} - (1+\delta+\lambda_1)x_{t+s} + m_{t+s-1} = -\lambda_1(1-\lambda_2)x^*_{t+s}.$$

This can be written in terms of the lag operator L as

$$(\delta L^{-1} - (1+\delta+\lambda_1) + L)x_{t+s} = -\lambda_1(1-\lambda_2)x^*_{t+s}.$$

Factoring the left hand side, we obtain

$$-\frac{1}{\mu_1}(1-\mu_1 L)(1-\delta\mu_1 L^{-1})x_{t+s} = -\lambda_1(1-\lambda_2)x^*_{t+s}. \tag{3.3}$$

The roots of the characteristic equation, $\delta\mu^2 - (1+\delta+\lambda_1)\mu + 1 = 0$, are positive and lie on either side of unity. μ_1 in (3.3) represents the stable root. We may then describe the optimal policy at t as

[3] The first order condition of dynamic programming problems of the form

$$\begin{aligned} &\sup_{\{x_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}) \\ &\text{s.t. } x_{t+1} \in \Gamma(x_t), \quad t = 0,1,2,\ldots \\ &\text{given } \; x_0 \in X \end{aligned}$$

is

$$0 = F_{x_{t+1}}(x^*_t, x^*_{t+1}) + \beta F_{x_{t+1}}(x^*_{t+1}, x^*_{t+2}), \quad t = 0,1,2,\ldots$$

See, for example, Lucas and Stokey (1989). So in our case, when $L_t = \sum_{s=0}^{\infty} \delta^s l_t$, we write the f.o.c as $\dfrac{\partial l_t}{\partial m_{t+s}} + \delta \dfrac{\partial l_{t+1}}{\partial m_{t+s}} = 0.$

$$x_t = \mu_1 x_{t-1} - \mu_1 \lambda_1 (\lambda_2 - 1) \sum_{s=0}^{\infty} (\delta\mu_1)^s m_{t+s}^*.$$

m_{t+s}^* can be replaced by its expectation formed at time t. From the characteristic equation, we know that $\mu_1\lambda_1 = (1-\mu_1)(1-\delta\mu_1)$ and thus we can obtain

$$\Delta m_t = \lambda_2 \Delta m_t^* + (1-\mu_1)\left\{\lambda_2 m_{t-1}^* - m_{t-1} + (1-\delta\mu_1)(1-\lambda_2)\sum_{s=0}^{\infty}(\delta\mu_1)^s m_{t+s}^*\right\} \quad (3.4)$$

The partial adjustment model can be achieved by setting $\lambda_2 = 0$. Equation (3.4) incorporates forward-looking behaviour. The adaptive adjusting mechanism is generated through the adjustment costs. One can, of course, criticize the *quadratic* adjustment cost approach and the underpinnings of the objective. The approach could be extended, as in Nickell (1985 and 1986), toward greater disaggregation. The different components of money holdings might have different adjustment costs. This issue is taken up later in the discussion of the empirical variables. In some cases, the adjustment costs of money holdings may not be symmetric.

To estimate (3.4) we must specify the sequence of the future expected target variables m_{t+s}^*. The cost parameter, λ_2, can be set to zero without destroying the error correction form (Nickell, 1985). Making an assumption about the equation of motion of m_t^*, one can reach the error correction form presented, for example, in Engle and Granger (1987). Nickell (1985) gives an example (No. 3) where m_{t+s}^* follows a kth order autoregressive scheme with a unit root and drift. We may then write the equation defining m_{t+s}^* in the form

$$m_{t+s}^* = \gamma(s)g + m_{t-1}^* + \sum_{i=0}^{k-2} \gamma_i(s)\Delta m_{t-i}^*.$$

When we substitute this into (3.4) we obtain

The model is in the error correction form even if we set $\lambda_2 = 0$. The lagged difference of m_t can be obtained if m_{t+s}^* also has a moving average error. In the error correction form, the money changes are determined by the deviation between the desired and actual levels of money holdings (error correction term) in the previous period and the (lagged) changes of the determinants of desired money holdings.

An alternative approach to m^* is to avoid specifying it and to concentrate on the direct estimation of the first order condition (3.2). In the case of $\lambda_2 = 0$, the equation (3.2) reduces to the form

$$
\begin{aligned}
\Delta m_t = {} & (1-\mu_1)(1-\lambda_2)(1-\delta\mu_1)\left(\sum_{s=0}^{\infty}(\delta\mu_1)^s\gamma(s)\right)g \\
& +\left\{\lambda_2+(1-\mu_1)(1-\lambda_2)(1-\delta\mu_1)\left(\sum_{s=0}^{\infty}(\delta\mu_1)^s\gamma_0(s)\right)\right\}\Delta m_t^* \\
& +\sum_{i=1}^{k-2}\left\{(1-\lambda_2)\sum_{s=0}^{\infty}(\delta\mu_1)^s\gamma_i(s)\right\}\Delta m_{t-i}^* \\
& +(1-\mu_1)(m_{t-1}^*-m_{t-1})
\end{aligned}
$$

$$
\Delta m_t = \delta\Delta m_{t+1} - \lambda_1(m_t - m_t^*),
$$

where the first term of the right-hand side can be replaced by its expectation conditioned on the present information, delta $\delta E(\Delta m_{t+1}|I_t)$.

The intertemporal loss function approach gives us theoretical insight into the dynamics of money demand. In the application above, we have made some implicit assumptions about the exogeneity of the factors affecting money demand. The one and only endogenous variable is money itself. All other variables are assumed to be exogenous. The model should be extended in a one way or another to accept the endogeneity of the whole system, when the equation of motion of m_t^* consists of some of the determinants of m^*.

4 Foundations of VAR

Instead of applying the results of the previous chapter, one can consider the forward-looking loss function as the economic reasoning behind the error correction mechanism. In macroecometric applications, the error correction approach is often well-suited to the data. In this chapter I follow the usual procedure for deriving the linear vector autoregressive (VAR) model. The VAR model makes it possible to introduce a minimal number of *a priori* restrictions on the econometric model. Thus, we avoid making any strict exogenous assumption by modelling each variable.[1] The disadvantage is that the number of parameters increases rapidly as the system is expanded, with the resultant loss of degrees of freedom. Another consideration is that the set of variables is chosen according to the specific economic problem at hand — here, money demand. Hence, we know beforehand that some equations are probably misspecified, at least in the economic sense. This can cause problems in estimating the structural equations. However, this problem cannot be avoided in any econometric procedure.

One of the main concerns in using VAR, however, is to interpret it as a reduced form of a dynamic structural econometric model (DSEM) — the traditional simultaneous equation model. The focus here is on VAR with error correction, i.e. some variables are assumed to be non-stationary and to cointegrate. An analytical description of the stationary case is presented in Monfort and Rabemananjara (1990); Hendry and Mizon (1990) describe the main features of the case of cointegration, and Juselius (1992) extends the analysis.

Hendry and Richard (1983) and Juselius (1992), for example, derive linear vector autoregression from the following assumptions. They first assume that the observed data are realizations of a sequence of p dimensional random vectors $Z_t = \{z_1,...,z_t\}'$ from the joint density function $D(z_1,...,z_T|Z_0,\theta)$. Parameter vector θ is finite dimensional, $\theta \in \Theta$, Z_0 is a matrix of initial conditions. Since the realizations are sequential, process $D(\cdot)$ can be factored as

$$D(Z_t|Z_0,\theta) = \prod_{t=1}^{T} D(z_t|Z_{t-1},Z_0,\theta).$$

[1] One should keep in mind that even in a simple regression model each variable is "modelled" as a sum of products. In a VAR model each variable is modelled explicitly.

The next step on the way to the VAR model, is to assume the *normality* (4.1), *independence* (4.1), *homoscedasticity* (4.3), *linearity* (4.2) and *truncation* (4.2) of the conditional process:

$$(z_t|Z_{t-1},Z_0,\theta) \sim N(\mu_t,\Sigma) \quad t=1,...,T, \tag{4.1}$$

where

$$\mu_t = E(z_t|Z_{t-1},Z_0,\theta) = \sum_{i=1}^{k} \Pi_i z_{t-i}, \tag{4.2}$$

$$E[(z_t-\mu_t)(z_t-\mu_t)'|Z_{t-1},Z_0,\theta] = \Sigma. \tag{4.3}$$

When we combine (4.1) and (4.2), we obtain $\varepsilon_t = z_t - E(z_t|Z_{t-1},Z_0,\theta) = z_t - \mu_t$ which is white noise relative to Z_{t-1} and Z_0. The predictions are thus *unbiased.* We finally arrive at the VAR model:

$$z_t = \sum_{i=1}^{k} A_i z_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim NID_p(0,\Sigma). \tag{4.4}$$

Since we are also interested in the non-stationary case, we assume that the roots λ_i of

$$\left| I - \sum_{i=1}^{k} A\,{}^i\Lambda_i \right| = 0$$

satisfy $|\lambda_i| > 1$, except for $p - r$ of them which are equal to 1. Thus the process z_t is non-explosive.

Appendix: Some Definitions

In the following simple example I repeat the definitions of the properties of trending variables[2]. The stochastic process X_t

$$X_t = \rho X_{t-1} + \varepsilon_t, \quad t = 1,...,T$$

where $\varepsilon_t \sim NID(0,\sigma^2)$, can be decomposed into the (stochastic) initial value and impulse responses

$$X_t = X_0 \rho^t + \sum_{i=0}^{t} \rho^i \varepsilon_{t-i}.$$

The process X_t is *stationary*, if $-1<\rho<1$ and $X_0 \sim N(0,\frac{\sigma^2}{1-\rho^2})$.

If $\rho = 1$,

$$X_t = X_{t-1} + \varepsilon_t \tag{4.5}$$

$$\Delta X_t = \varepsilon_t \tag{4.6}$$

$$X_t = X_0 + \sum_{i=1}^{t} \varepsilon_i \tag{4.7}$$

and the process is *non-stationary*. A *trending* process is a non-stationary process that becomes stationarity after differencing. The vector process $V_t = [Y_t\ X_t]$, where

$$X_t = X_{t-1} + \varepsilon_{1t} \tag{4.8}$$

$$Y_t = \rho Y_{t-1} + \varepsilon_{2t}, \quad -1<\rho<1 \tag{4.9}$$

is non-stationary, but ΔV_t is stationary. The process V_t is said to be trending.

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[2] The example is adapted from the Søren Johansen's lectures in Helsinki 1991.

We define $Z_t = \begin{bmatrix} Y_t + X_t \\ Y_t - X_t \end{bmatrix}$, where X_t and Y_t are defined above. The parts Z_{1t} and Z_{2t} are trending or *integrated.* They have a *common trend* X_t and they are *cointegrated,* because $Z_{1t} + Z_{2t}$ is stationary. The linear combination [1 1]Z_t is stationary and the vector [1 1]' is called a *cointegration vector.* The *degree of cointegration* or *cointegration rank* is one.

An *error correction mechanism,* ECM, is introduced in the following

$$\begin{aligned} \Delta Z_{1t} &= \alpha_1 (Z_{1,t-1} - \beta_1 Z_{2,t-1}) + \varepsilon_{1t} \\ \Delta Z_{2t} &= \alpha_2 (Z_{1,t-1} - \beta_1 Z_{2,t-1}) + \varepsilon_{2t}. \end{aligned} \tag{4.10}$$

When the vector $Z_t = [Z_{1t}\ Z_{2t}]$ is post-multiplied by the vector [1 $-\beta_1$]' we obtain

$$\Delta(Z_1 - \beta_1 Z_2)_t = (\alpha_1 - \beta_1 \alpha_2)(Z_1 - \beta_1 Z_2)_{t-1} + (\alpha_1 \varepsilon_1 - \alpha_2 \varepsilon_2)_t.$$

The process $Z_1 - \beta_1 Z_2$ is stationary if $-1 < 1 + \alpha_1 - \beta_1 \alpha_2 < 1$. If vector process (4.10) is multiplied by the vector [α_2 $-\alpha_1$], the result is

$$\Delta(\alpha_2 Z_1 - \alpha_1 Z_2)_t = (\alpha_1 \varepsilon_1 - \alpha_2 \varepsilon_2)_t.$$

Here, $\alpha_2 Z_1 - \alpha_1 Z_2$ is a random walk. Process Z_t is a trending process, but its components are cointegrated by the vector [1 $-\beta_1$]'.

Process Z_t has different representations. The error correction form is

$$\Delta Z_t = \begin{bmatrix} \alpha_1 & -\alpha_1 \beta_1 \\ \alpha_2 & -\alpha_2 \beta_1 \end{bmatrix} Z_{t-1} + \varepsilon_t. \tag{4.11}$$

The vector autoregressive (VAR) form is

$$Z_t = \begin{bmatrix} 1 + \alpha_1 & -\alpha_1 \beta_1 \\ \alpha_2 & 1 - \alpha_2 \beta_1 \end{bmatrix} Z_{t-1} + \varepsilon_t.$$

5 Johansen's VAR model

Voluminous empirical studies have not rejected the unit root hypothesis for behaviour for macroeconomic time series. It is generally useful to model macroeconomic times series as trending variables even if in the autoregressive representation the roots of the variables are near unity.[1]

In this chapter, I present the FIML estimation within the VAR of the cointegration relations, the stable combination of the variables and ways to test the long run structural hypothesis. The presentation is based on the papers of Søren Johansen (1988, 1992) and Katarina Juselius (see, e.g., Johansen and Juselius, 1990).

First we assume — as in 4.4 above — that z_t is a p dimensional VAR(k)-process

$$A(B)z_t = \varepsilon_t, \quad t=1,...,T, \tag{5.1}$$

where $A(B) = I_p + A_1 B + ... + A_k B^k$ is a matrix polynomial, $Bz_t = z_{t-1}$ and $\varepsilon_t \sim NID(\mathbf{0},\Sigma)$. The number of equations is p. The residuals are independently and normally distributed. The residual covariance matrix, Σ, does not need to be diagonal. One can add deterministic variables, such as the constant μ and l centred seasonal dummies[2] D_t, to the equation. When the lags are written explicitly, we obtain

$$z_t = A_1 z_{t-1} + ... + A_k z_{t-k} + \mu + \psi D_t + \varepsilon_t, \quad t=1,...,T. \tag{5.2}$$

The VAR model can be written as an ECM, as in equation (4.11), thusly:

$$\Gamma(B)(1-B)z_t = \Gamma_k z_{t-k} + \varepsilon_t = -\alpha(\beta' z_{t-k}) + \varepsilon_t, \quad t=...,T,$$

where $\Gamma(B) = \sum_{i=1}^{p-1} \Gamma_i B^i$, $\Gamma_k = \Pi = -\alpha\beta'$, α and β are $p \times r$ dimensional matrices and $\Gamma_i = -I_p + A_1 + A_2 + ... + A_i$, $i=1,...,p-1$. The lag polynomial $\Gamma(B)$ is like the lag polynomial $A(B)$ in the previous equation. The kth coefficient Γ_k is called the Π matrix, which in the case of cointegration has a reduced rank, (rank(Π) $< p$). Any reduced rank matrix can be presented as a product of two matrices. Thus, Π is

---

[1] See Haldrup and Hylleberg (1991) for the near unit root analysis and Cochrane (1991) for the criticism of unit root tests.

[2] Centering here means that the sum of seasonal dummies is zero.

partitioned as α and β'. Matrix β is called the cointegration matrix. In cointegration, α is called the matrix of loadings of the equilibrium errors of the linear combinations defined by β.

The rank of the matrix Π determines the number of cointegration vectors. Two special cases might occur. When all p components of z_t are stationary, matrix Π has the full rank p. Thus, there are as many cointegration vectors (each containing unity in one component and zero in other components, e.g. $[1\ 0\ 0\ 0]'$) as there are variables in z_t. When no cointegration exists between the variables in z_t, the matrix Π has zero rank. Then all variables are integrated order 1, but there is no cointegration between them. In such a case, the simple VAR model in differences is the proper framework for empirical analysis.

Changing notation $(1 - B) = \Delta$, the model can be written as

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + ... + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + \mu + \Psi D_t + \varepsilon_t, \quad t = 1,...,T. \tag{5.3}$$

The original VAR model (5.2) can be expressed with the levels set to the lag $t - 1$. This parametrization is usually more attractive in terms of the economics. It is usually more natural to assume that the adjustment runs toward the next period than toward the kth period.

$$\Delta z_t = \Pi z_{t-1} + \Gamma_1 \Delta z_{t-1} + ... + \Gamma_{k-1} \Delta z_{t-k+1} + \mu + \Psi D_t + \varepsilon_t, \quad t = 1,...,T. \tag{5.4}$$

Here $\Pi = -I_1 + \sum_{i=1}^{k} A_i$ and $\Gamma_i = -(\sum_{j=i+1}^{k-1} A_j)$.

The connection between these two forms can easily be seen in the following example with two lags ($k = 2$).

$$z_t = \Pi_1 z_{t-1} + \Pi_2 z_{t-2} + \varepsilon_t \tag{5.5}$$

$$z_t - z_{t-1} = (\Pi_1 - I) z_{t-1} + \Pi_2 z_{t-2} + \varepsilon_t$$

$$= (\Pi_1 - I)(z_{t-1} - z_{t-2}) + (\Pi_1 + \Pi_2 - I) z_{t-2} + \varepsilon_t$$

$$\Delta z_t = (\Pi_1 - I) \Delta z_{t-1} + \Pi z_{t-2} + \varepsilon_t \tag{5.6}$$

$$\Delta z_t = (\Pi_1 + \Pi_2 - I) z_{t-1} + \Pi_2 (z_{t-2} - z_{t-1}) + \varepsilon_t$$

$$\Delta z_t = \Pi z_{t-1} - \Pi_2 \Delta z_{t-1} + \varepsilon_t$$

$$\Delta z_t = \Pi z_{t-1} + \Gamma \Delta z_{t-1} + \varepsilon_t. \tag{5.7}$$

Equations (5.5), (5.6) and (5.7) are different forms of the same process.

5.1 ML Estimator for Cointegration Vectors

To find the maximum likelihood estimators of the parameters of model (5.4), we introduce the following notation

$$Z_{0t} = \Delta z_t \tag{5.8}$$

$$Z_{1t} = [\Delta z_{t-1} \cdots \Delta z_{t-k+1} D_t 1]' \tag{5.9}$$

$$Z_{kt} = z_{t-1}. \tag{5.10}$$

The first vector includes the dependent difference term; the second includes all the lagged differences of the variables on the right hand side, as well as the deterministic variables (constant, seasonal dummies, etc.). The lagged level terms are in the third vector. Thus model (5.4) can be written as follows:

$$Z_{0t} = \Gamma Z_{1t} + \Pi Z_{kt} + \varepsilon_t, \quad t=1,...,T,$$

where $\Gamma=[\Gamma_1 \cdots \Gamma_{k-1}\ \Psi\ \mu]$ has dimension $p \times (p(k-1)+l+1)$ and vector Z_{1t} has dimension $p(k-1)+l+1$. The term $p(k-1)$ is the number of lagged differences, l is the number of centred seasonal dummies and *1* is constant. The moment matrix of the vectors Z_{it} is

$$M_{ij} = T^{-1}\sum_{t=1}^{T} Z_{it}Z_{jt}', \ (i,j=0,1,k).$$

Regressing Z_{0t} on Z_{1t} and Z_{kt} on Z_{1t}, we obtain the following residuals

$$R_{0t} = Z_{0t} - M_{01}M_{11}^{-1}Z_{1t},$$

$$R_{1t} = Z_{kt} - M_{k1}M_{11}^{-1}Z_{1t}.$$

The matrices Γ and Π are estimated by least squares.

Johansen (1988 and 1992) has proved that the ML estimators of α and β can be obtained by solving the following eigenvalue problem

$$|\lambda S_{11} - S_{10}S_{00}^{-1}S_{01}| = 0 \iff \tag{5.11}$$

$$|\lambda I - S_{11}^{-1/2}S_{10}S_{00}^{-1}S_{01}S_{11}^{-1/2}| = 0.$$

where

$$S_{ij} = T^{-1}\sum_{t=1}^{T} R_{it}R_{jt} \quad i,j=0,1.$$

S_{11} represents the residual moment matrix from the least squares regression of Z_{kt} on Z_{1t}, S_{00} is the residual moment matrix from the least squares regression of Z_{0t} on Z_{1t} and S_{01} is the cross-product moment matrix.

The solution of the eigenvalue problem above generates eigenvalues $\hat{\lambda}_1 > ... > \hat{\lambda}_p$ the eigenvectors $\hat{V} = [\hat{v}_1 \cdots \hat{v}_p]$, which are usually normalized $\hat{V}'S_{11}\hat{V} = I_p$. The estimators of the cointegration vectors β are $\hat{\beta} = [\hat{v}_1 \cdots \hat{v}_r]$, the eigenvectors corresponding to the r largest eigenvalues. Johansen (1988, 233—236 and 1992) shows that the ML estimator of the space spanned by β is the space spanned by r canonical variates reflecting the r largest squared canonical correlations between residuals R_{0t} and R_{1t}.

It is important to note that we can estimate only the space spanned by β, not the individual cointegration vectors. We can give economic interpretation to the cointegration vectors only after identification. Before that they are a statistical property of the data.

Johansen (1988) derives two tests for the number of cointegration vectors. Technically speaking, we are testing the rank of Π. The null hypothesis can be formulated as

$$H_0 : \text{rank}(\Pi) \leq \text{r or } \Pi = \alpha\beta', \tag{5.12}$$

where α and β are the $p \times r$ matrices described on page 34. The likelihood ratio test for the hypothesis above is

$$-2\ln(Q) = -T\sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i^2),$$

where $\hat{\lambda}_{r+1}^2,...,\hat{\lambda}_p^2$ are the $p - r$ smallest squared canonical correlations — eigenvalues. The alternative hypothesis is now that the number of cointegration vectors is larger than r. This test is called the *trace test.*

The other test, which is called *maximum eigenvalue test*, λ_{max}, tests the null hypothesis that the cointegration rank is $r - 1$ against the alternative that the rank is r.

$$H_0 : \text{rank}(\Pi) = r-1$$

The test statistic is the following:

$$-2\ln(Q) = -T\ln(1-\hat{\lambda}_r^2).$$

The distributions of the test statistics are non-standard. They are sort of "multivariate extensions of the Dickey-Fuller distribution". Asymptotic critical values of these test statistics are simulated by Osterwald–Lenum (1990) for $p = 12$ dimensional systems. There has been only some minor testing of the small sample properties of these tests. This[3] indicates that the asymptotics cannot be reached very well if the number of observations is much less than one hundred.

The determination of the cointegration rank involves a sequence of tests. Johansen (1991a) shows how to avoid the problems involved in such tests. He recommends using the trace test starting from the hypothesis $r = 0$ and continuing until the null is not rejected.

5.2 Identification of the Cointegration Vectors

As was emphasized in the preceding section, we only estimate the space spanned by β. Thus, the long run parameters β and α are computed under the following restrictions (as Juselius 1992, p. 8, points out):

- stationarity, $\beta' z_t \sim I(0)$,
- orthogonality of β_j, $\beta' S_{11} \beta = I$, which is, from the economic point of view, an arbitrary way to normalize β,
- ordering given by the maximal canonical correlation between the stationary and the non-stationary part of the model, with given lagged differences.

These restrictions guarantee that the model is identified in a statistical sense. From the economic standpoint the model is not identified —

[3] The simulations unfortunately have not yet been published.

cointegration vectors can have many different economic interpretations. We can, for example, give a "money demand" interpretation to several cointegration vectors. To obtain the economic interpretation, we can relax the last two restrictions.

The usual way to interpret β is to partition z_t to two components such that $\beta=[I_r\ \tilde{\beta}']'$. So the identity part reflects "endogenous" variables y_t to be explained by "exogenous" variables x_t. What is usually overlooked is that the process x_t should have the property[4]

$$\tilde{\beta}'x_t \sim I(1),\ \tilde{\beta}\neq 0. \tag{5.13}$$

From (5.13) it follows that x_t contains all the non-stationarity (common trends) of z_t. Thus, one should keep in mind the above property and avoid mechanical adoption of the partition.

The identification problem is similar to the identification of simultaneous equation models. Johansen (1992) gives the formal presentation of identification of cointegration vectors, which can be applied to any system of linear equations. For each cointegration vector we formulate restrictions, H_i ($p\times(p-k_i)$ matrix), such that each Φ_i, satisfies

$$\Phi_i = H_i\varphi_i$$

The same restrictions can be formulated with the help of R_i ($p\times k_i$ matrix). The connection between these two is defined as $H_i=R_{i\perp}$ (so $H_i'R_i=0$); thus,

$$R'_{i\perp}\Phi_i=0.$$

The restricted β-space has the form

$$\Phi=(H_1\varphi_1,\ldots,H_r\varphi_r)$$

and the necessary and sufficient condition for the first equation to be identified is

$$\operatorname{rank}(R'_1\Phi_2,\ldots,R'_1\Phi_r)=r-1.$$

He also formulates the theorem (Johansen, 1992, p. 4):

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[4] See Saikkonen (1992), who also derives the test statistics for the hypothesis.

Theorem 1 *The restrictions* $H_1,\ldots,H_r$ *identify the first equation iff* *rank* $(R'_1 H_{i_1},\ldots,R'_1 H_{i_k}) \geq k,$
for any set $1 < i_1 < \cdots < i_k \leq r$ *and* $k = 1,\ldots,r-1.$

Thus the system is not identified if, for example, $R'_1 H_i = 0$ for $i = 2,\ldots,r.$ Summarizing:

> "Hence it is not possible by taking linear combinations of, for instance, $\Phi_2,\ldots,\Phi_r$ to construct a vector which is restricted in the same way as Φ_1 and in this sense could be confused with the equation defined by Φ_1" (Johansen, 1992, p. 3).

5.3 Granger Representation Theorem and Moving Average Representation

The Granger representation theorem[5] guarantees the invertibility of the VAR model to the vector moving average model in the case of cointegration. Stationarity is normally needed for the invertibility. Johansen (1991b) formulates the theorem as follows: If z_t follows the process described by equation (5.4) and

$$\Pi = \alpha\beta',$$

where α and β are $p \times r$ matrices with rank of r and

$$\alpha'_\perp \Psi \beta_\perp,$$

where $\Psi = A_1 + 2A_2 + \cdots + kA_k$ has a full rank of $p - r$ and $\alpha_\perp$ is orthogonal to the matrix α, then z_t has a moving average of the form

$$z_t = C(B)(\varepsilon_t + \mu + \Phi D_t).$$

Henceforth, we let

[5] The carefull reader will recognize that this is a multivariate extension of the Beveridge-Nelson decomposition.

$$C = C(1) = \beta_\perp(\alpha'_\perp \Psi \beta_\perp)^{-1}\alpha'_\perp.$$

When $C_1(B) = (C(B) - C(1))/(1-B)$ and $C(B) = C + (1-B)C_1(B)$ the process z_t has an moving average form

$$z_t = z_0 + C\sum_{i=1}^{t} \varepsilon_i + \tau t + C(B)\Phi\sum_{i=1}^{t} D_i + C_1(B)\varepsilon_t. \tag{5.14}$$

The matrix C tells how the cumulative residuals of the each equation (random walk term $\sum_{i=1}^{t} \varepsilon_i$ in (5.14)) of the VAR system affect the levels of the variables, i.e. how the cumulative economic shocks[6] are spread over the system. The non-orthogonality of the residuals creates difficulties in interpretation. Differences between residual variances also cause a problem in scaling the components in C.

Stock and Watson (1988) call $\alpha'_\perp\sum_{i=1}^{t} \varepsilon_i + \alpha'_\perp\mu t$ the *common trends* in the system. The common trends are the linear combinations of the innovations that form the random walk component of the system[7]. They are the driving force of the system.

5.4 The Role of the Constant

In a difference equation of the form of (5.4), the constant has an important role. If it is non-zero, the level process has a linear trend. The constant can be restricted to the cointegration relation and thus avoid the existence of linear trend. If the model has a linear trend, the test statistic for the rank of Π has a different asymptotical distribution.

The constant term of the difference model can be partitioned

$$\mu = \alpha\beta_0 + \alpha_\perp\gamma,$$

where $\beta_0 = (\alpha'\alpha)^{-1}\alpha'\mu$ is an $r \times 1$ vector, which describes the "constant terms" within the cointegration vectors. $\alpha_\perp$ is a $p \times (p-r)$ matrix with full rank. $\gamma = (\alpha'_\perp\alpha_\perp)^{-1}\alpha'_\perp\mu$ is a $(p-r)\times 1$ vector that describes the

[6] Here we assume that cumulative shocks can also be interpreted as economic shocks. Then, of course, the model must be the real data-generating process. However, the residuals — shocks — are, in practice, model-specific.

[7] See, for example, Paruolo (1992) and Englund *et al.* (1992) for further discussion of common trends and their interpretation.

slopes of the linear trends. If the models do not have linear trend, $\alpha_{\perp}\gamma=0$ and $\mu=\alpha\beta_0$ and one can write $\alpha\beta' z_{t-1}+\mu$ from (5.4) as

$$\alpha\beta' z_{t-1}+\mu=\alpha\beta' z_{t-1}+\alpha\beta_0=\alpha\tilde{\beta}' z_{t-1},$$

where $\tilde{z}_t=[z'_t\ 1]'$ abd $\tilde{\beta}=[\beta'\ \beta_0]'$. (Johansen, 1991c and Juselius, 1991a)

To test the cointegration rank and the existence of linear trend one can use the following approach suggested by Johansen (1991a). In each null hypothesis T is the trace test statistic in the case of linear trend and T^* in the absence of linear trend, both having the subscript r. In the two (p = 2) dimensional case, the test procedure is the following:

$$\begin{array}{cccc}
\text{no trend} & \text{trend} & & \\
H_0^* \subset & H_0 & T_0^* & T_0 \\
 & \cap & & \\
H_1^* \subset & H_1 & T_1^* & T_1 \\
 & \cap & & \\
H_2^* \subset & H_2 & T_2^* & T_2
\end{array}$$

The null hypothesis is rejected only if all the previous hypotheses have been rejected. (For example, C_r is the critical value and $\bar{\alpha}$ the significance level.)

$$\{\hat{r}=1,\ \text{no trend}\}=\{T_0^*>C_0^*, T_0>C_0, T_1^*\leq C_1^*\}$$
$$\{\hat{r}=1,\ \text{trend}\}=T_0^*>C_0^*, T_0>C_0, T_1^*>C_1^*, T_1\leq C_1\}$$

The probability of a correct decision approaches $1-\bar{\alpha}$ asymptotically. (Johansen, 1991d)

5.5 Testing Structural Hypotheses

The cointegration vectors can be identified by means of restrictions, which are usually interesting from an economics standpoint. An example of such a hypothesis might involve a cointegration vector that is known *a priori*. This could be a hypothesis of stationarity. The cointegration space would be in the form

$$\begin{array}{cc} p & \begin{bmatrix} 0 & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & * & \dots & * \\ 1 & * & \dots & * \\ 0 & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & * & \dots & * \end{bmatrix}. \\ & r \end{array}$$

Another common situation might involve a variable that does not belong to the cointegration space at all, restricting some components of β to zero.

$$\begin{array}{cc} p & \begin{bmatrix} * & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & * \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ & r \end{array}$$

One can also test restrictions on relations between components of the same vector, but not different vectors, i.e. cross-equation restrictions.

Johansen (1988 and 1990) has formulated different types of restrictions as follows:

$$\mathrm{H}_1 : \beta = H_1\varphi, \quad H_1\ (p\times s),\ \varphi\ (s\times r),\ r\leq s\leq p \tag{5.15}$$

$$\mathrm{H}_2 : \beta = (H_2,\psi), \quad H_2\ (p\times r_1),\ \psi\ (p\times r_2),\ r=r_1+r_2 \tag{5.16}$$

$$\begin{aligned} \mathrm{H}_3 : \beta &= (H_3\varphi,\psi), \\ &\quad H_3\ (p\times s),\ \varphi\ (s\times r_1),\ \psi\ (p\times r_2),\ r_1\leq s\leq p,\ r=r_1+r_2. \end{aligned} \tag{5.17}$$

The alternative hypothesis of $\mathrm{H}_1-\mathrm{H}_3$ is formalized in equation (5.12) — no restriction exists. In the first type of hypothesis (H_1) the same

$(p - s)$ restrictions are placed on all r cointegration vectors, by means of design matrix H_1. The hypothesis can be stated as $\mathrm{sp}(H_1) \subset \mathrm{sp}(\beta)$. According to Johansen (1988, 239) the test statistic is asymptotically χ^2 distributed with $r(p-s)$ degrees of freedom. One can easily apply the test to the exclusion hypothesis (excluding a variable from the cointegration relations) or to the homogeneity hypothesis (similar to price homogeneity in demand for money studies).

The second hypothesis, H_2, means that one knows *a priori* r_1 of the r cointegration vectors. These known vectors are presented in design matrix H_2. The other r_2 relations (ψ) are estimated freely. The hypothesis is $\mathrm{sp}(H_2) \subset \mathrm{sp}(\beta)$. The test statistic (Johansen and Juselius 1990b, 20) is asymptotically $\chi^2((p-r)r_1)$ distributed. This method can be applied e.g. to the stationarity tests.

According to the third hypothesis, H_3, the first r_1 cointegration vectors (φ) are restricted by means of the design matrix H_3. The rest of the cointegration vectors (ψ) are estimated freely. The hypothesis is $\dim(\mathrm{sp}(\beta) \cap \mathrm{sp}(H_3)) \geq r_1$. The test statistic (Johansen and Juselius 1990b, 23) is asymptotically $\chi^2((p-s-r_2)r_1)$ distributed. These restrictions are like the those above, but they are applied to the subspace of β. This makes the test easy to apply to economic restrictions. The third type of hypothesis can be extended, for example, so that each cointegration vector has a set of restrictions of its own. The alternative in these cases is as above, formalized in equation (5.12). The hypothesis could be written as $(H_1\varphi_1,...,H_r\varphi_r)$. This is a very attractive way of testing for (over)identification of the cointegration space.

5.6 Weak Exogeneity and Partial Analysis

A variable is weakly exogenous[8] with respect to a parameter if the inference can be conditioned on the variable of interest without losing any information (Mills 1990, 290). Johansen (1990) proposes a way to test weak exogeneity of z_t with respect to long run parameters α and β. The hypothesis is stated as

[8] The definitions of the classes of exogeneity, as presented in Engle, Hendry and Richard (1983), are given in the appendix of this study.

$H_4: \alpha = A\psi, \quad A \ (p \times m).$

It is easier to carry out the test procedure using the matrix $B\ (p \times (p-m))$, which has the property $B'A = 0$. Then the hypothesis H_4 can be written in the form $B'\alpha = 0$. One can apply the test, for example, to the exogeneity assumptions, to test whether a row of α is zero. According to Johansen and Juselius (1990b, 200) the test statistic is $\chi^2(r(p-m))$ distributed.

Johansen (1990, 4—12) also proves that if $b'\alpha = 0$, then $b'\Delta z_t$ is weakly exogenous with respect to parameters α and β. In regard to weak exogeneity, no efficiency loss is suffered in estimating β. If some variables are weakly exogenous with respect to long run parameters, one can condition the model to those weakly exogenous variables. For example, if we define $z_t = [y_t \ x_t]'$, where x_t is the vector of weakly exogenous variables, the model (with two lags) would be as follows:

$$\Delta y_t = A\Delta x_t + \Gamma_1^* \Delta z_{t-1} + \alpha^* \beta' z_{t-2} + \mu + \Psi D_t + \varepsilon_t, \quad t = 1,...,T. \tag{5.18}$$

So, instead of modelling x explicitly, we condition the system on it and thus add the present changes, Δx_t, to the equation. However, this exogeneity with respect to long run parameters α and β does not guarantee weak exogeneity with respect to short run parameters Γ. However, these possibilities help us to use quite a large set of variables, when we, in the case of weak exogeneity, can analyze only partial systems.

Johansen and Juselius (1990a) derive the test statistics for the hypothesis $H_5 = H_1 \cap H_4$. The hypothesis is a joint hypothesis.

$H_5 : \beta = H_1\varphi, \ \alpha = A\varphi.$

The alternative hypothesis is the one which is formalized in equation (5.12), the unrestricted case. When the restrictions have been ordered linearly any one of the previous hypotheses can serve as an alternative hypothesis if it has fewer restrictions (Johansen and Juselius 1989, 39).

5.7 The $I(2)$ model

If the processes z_t and Δz_t are non-stationary and $\Delta^2 z_t$ is stationary, z_t should be differenced twice to get stationarity[9]. The process is integrated order two, $z_t \sim I(2)$. The model (5.1) can be formulated as

$$\Delta^2 z_t = \Pi z_{t-1} + \Gamma \Delta z_{t-2} + \sum_{i=1}^{k-2} \Phi_i \Delta^2 z_{t-i} + \varepsilon_t, \quad t=1,\dots,T. \tag{5.19}$$

The lag of the first two terms in the right hand side can be parametrized to any lag.

Matrix $\Pi = \alpha\beta'$ has a reduced rank, $p \times r$, as in the $I(1)$ case. The matrix $\alpha'_\perp \Gamma \beta_\perp = \varphi\eta$ also has a reduced rank, so that both φ and η are $(p-r)\times s$ matrices[10]. Then the moving average form is

$$z_t = C_2 \sum_{j=1}^{t} \sum_{i=1}^{j} \varepsilon_i + C_1 \sum_{i=1}^{t} \varepsilon_i + C_2(B)\varepsilon_t + C_1 \mu t + \frac{1}{2} C_2 \mu t^2.$$

Johansen (1991d) shows that the space spanned by z_t can be partitioned into the r stationary direction βs and $p - r$ non-stationary direction $\beta_\perp s$. The matrix $\beta_\perp$ can also be partitioned into the two directions $[\beta^1_\perp \ \beta^2_\perp]$, where $\beta^1_\perp = \beta_\perp(\beta'_\perp\beta_\perp)^{-1}\eta$ has dimension $p \times s_1$ and $\beta^2_\perp = \beta_\perp \eta_\perp$ has dimension $p \times s_2$, and $s_1 + s_2 = p - r$.

The process $\beta' z_t$ can achieve stationarity only with a suitable linear combination of a differenced process Δz_t. The linear combination $\beta^1_\perp z_t$ is stationary when differenced once and $\beta^2_\perp z_t$ when differenced twice (see also Juselius 1991b). It follows that:

[9] The process has two unit roots.

[10] Compare to the $I(1)$ case when $\alpha'_\perp \Gamma \beta_\perp$ has a full rank. See also the Granger Representation Theorem.

$\beta' z_t \sim I(1)$
$\beta_\perp^{1\prime} z_t \sim I(1)$
$\beta_\perp^{2\prime} z_t \sim I(2)$
$\beta_\perp^{1\prime} \Delta z_t \sim I(0)$
$\beta_\perp^{2\prime} \Delta^2 z_t \sim I(0)$
$\{\beta' z_t + \omega \beta_\perp^{2\prime} \Delta z_t\} \sim I(0),$

where $\omega = (\alpha'\alpha)^{-1}\alpha'\Gamma\beta_\perp^2(\beta_\perp^{2\prime}\beta_\perp^2)^{-1}$. We also know that $\beta R_{1t} \sim I(0)$, where R_{1t} is z_t corrected by Δz_{t-1}[11].

If $r > s_2$, β can be partitioned as $\beta = [\beta^0 \ \beta^1]$. The dimension of the matrix β^0 is then $p \times (r - s_2)$ and that of β^1 is $p \times s_1$. Matrix $\beta^0 = \beta\xi$, where ξ is orthogonal to ω. β^0 describes the direction for which the process $\beta^0 z_t$ is stationary itself. β^1 describes the direction for which the process $\beta^{1\prime} z_t$ is stationary only in a suitable linear combination of the Δz_t. Hence both sp(β) and sp($\beta_\perp$) define non-stationary directions. The difference is that $\beta' z_t$ can be made stationary only in a suitable linear combination of Δz_t, whereas $\beta_\perp^{1\prime} z_t$ and $\beta_\perp^{2\prime} z_t$ can be made stationary by simple differencing. In conclusion, one can refer to β as a stationary space and $\beta_\perp$ as a non-stationary space. (Juselius, 1991b)

The test of $I(2)$ness is usually done simultaneously with the test of rank(Π). With a test procedure one can define r, s_1, s_2.

[11] One can utilize this property when investigating the $I(2)$ness of the data.

6 From VAR to Structural Models

6.1 Identification

Model (5.4) is repeated here.

$$\Delta z_t = \alpha\beta' z_{t-1} + \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1}\Delta z_{t-k+1} + \mu + \Psi D_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0,\Sigma). \tag{6.1}$$

This is usually referred as the *reduced form* of the model. It is a statistical model of the data. The parameters are $\theta = \{\alpha,\beta,\Gamma_1,\ldots,\Gamma_{k-1}, \mu,\Psi,\Sigma\}$. They can be partitioned into short run parameters $\theta_s = \{\alpha,\Gamma_1, \ldots,\Gamma_{k-1},\mu,\Psi,\Sigma\}$ and long run parameters $\theta_L = \{\beta\}$.

The *structural form* can, for instance, be written as

$$B_0 \Delta z_t = B_k \beta' z_{t-1} + B_1 \Delta z_{t-1} + \ldots + B_{k-1}\Delta z_{t-k+1} + b + \Xi D_t + u_t, \quad u_t \sim NID(0,\Omega). \tag{6.2}$$

The structural parameters $\lambda = \{B_0,\ldots,B_k,\beta,\mu,\Xi,\Omega\}$ can also be partitioned into short run parameters $\lambda_s = \{B_0,\ldots,B_k,b,\Xi,\Omega\}$ and long run parameters $\lambda_L = \{\beta\}$. The parameters of the reduced form and the structural form are connected through the equations

$$\begin{aligned}
\Gamma_i &= B_0^{-1} B_i, \quad i = 1,\ldots,k-1 \\
\alpha &= B_0^{-1} B_k \\
\mu &= B_0^{-1} b \\
\varepsilon &= B_0^{-1} u_t \\
\Sigma &= B_0^{-1} \Omega B_0^{-1\prime}
\end{aligned}$$

Model (6.2) can be written more briefly[1] as

$$B_0 \Delta z_t = \tilde{B} f_t + u_t, \quad u_t \sim NID(0,\Omega), \tag{6.3}$$

where $\tilde{B} = [B_k \; B_1 \; \ldots \; B_{k-1}]$ and $f_t = [\beta' z_{t-1} \; \Delta z_{t-1} \; \ldots \; \Delta z_{t-k+1}]'$. B should be non-singular and the diagonal elements are generally assumed to be unity.

---

[1] For simplicity, the constant term μ, the seasonal dummies and the other deterministic variables D_t are left out.

To derive uniquely the short run structural parameters λ_S from the short run reduced form parameters θ_S, one has to impose at least $p(p-1)$ restrictions on the structural short run parameters. The long run parameters are the same in the reduced and structural forms. This is a very important point, as it means that the long run structure can be investigated separately from the short run structure and thus simplifies the analysis. The super-consistency of β makes[2] it possible to handle $\hat{e}_{t-1}=\hat{\beta}'z_{t-1}$ as any other stationary regressor, when deriving the asymptotic results. The tests of structural hypothesis of the long run parameters have been discussed in Chapter 4. As Hendry *et al.* (1988) state, the focus of analysis should be on the reduced form. Stability of the parameters of the reduced form model should be carefully tested because all that follows in the analysis of the model is based on the stability of the parameters.

6.2 Reducing VAR

Hendry and Mizon (1990) classify model (6.1) as a closed model, i.e. one that is unrestricted and conditioned only on the lags, initial values and deterministic variables (such as constant or seasonal dummies). The open model is the system in which some variables are treated as valid conditioning, or exogenous, variables. According to Clements and Mizon (1990) a natural sequence to proceed from (6.1) is: dynamic simplification, Granger non-causality, exogeneity and, finally, structural economic hypothesis.

Consider the present model (6.1) in the form

$$\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \alpha \hat{e}_{t-1} + \varepsilon_t, \tag{6.4}$$

where $\hat{e}_t = \hat{\beta}' z_t$. The constant and the deterministic variables are excluded in order to simplify the computation. We also partition vector z_t into y_t, which describes the basic variables of interest, $y_t \in R^{p_1}$ and the determinants of $y_t, x_t \in R^{p_2}$ $(p_1+p_2=p)$, (the possible candidates for exogenous variables):

---

[2] See Engle and Granger (1987), who examine a similar situation with $B_0=I_p$.

$$z_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}, \quad t = 1,...,T.$$

We can write model (6.4) as

$$\begin{bmatrix} \Delta y_t \\ \Delta x_t \end{bmatrix} = \sum_{i=1}^{k-1} \begin{bmatrix} \Gamma_{yyi} & \Gamma_{yxi} \\ \Gamma_{xyi} & \Gamma_{xxi} \end{bmatrix} \begin{bmatrix} \Delta y_{t-i} \\ \Delta x_{t-i} \end{bmatrix} + \begin{bmatrix} \alpha_y \\ \alpha_x \end{bmatrix} \hat{e}_{t-1} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix}. \tag{6.5}$$

We can now test the hypothesis of *strong exogeneity* of the parameters determining y_t[3], according to which

$$H_0 : \Gamma_{xyi} = 0 \text{ and } \alpha_x = 0, \quad i = 1,...,k-1. \tag{6.6}$$

Hypothesis (6.6) can be tested using the Wald test. What follows is a general result (see, e.g. Saikkonen, 1991). We also restrict ourselves to the case where $\text{rank}(\alpha_x) \geq r$. Johansen (1989) and Boswijk (1991b) derive the LR and LM test statistics for the hypothesis $\alpha_x = 0$.

Assuming that H_0 applies, we can derive the conditional model. First Σ is partitioned as was z_t and then (6.5) is left-multiplied by the matrix

$$\begin{bmatrix} I_{p_1} & -\Sigma_{yx}\Sigma_{xx}^{-1} \\ 0 & I_{p_2} \end{bmatrix} = \begin{bmatrix} I_{p_1} & -\Upsilon \\ 0 & I_{p_2} \end{bmatrix}.$$

Thus, we obtain

$$\begin{bmatrix} \Delta y_t - \Upsilon \Delta x_t \\ \Delta x_t \end{bmatrix} = \sum_{i=1}^{k-1} \begin{bmatrix} \Gamma_{yyi} & \Gamma_{yxi} - \Upsilon \Gamma_{xxi} \\ 0 & \Gamma_{xxi} \end{bmatrix} \begin{bmatrix} \Delta y_{t-i} \\ \Delta x_{t-i} \end{bmatrix} + \begin{bmatrix} \alpha_y \\ 0 \end{bmatrix} \hat{e}_{t-1} + \begin{bmatrix} \varepsilon_{yt} - \Upsilon \varepsilon_{xt} \\ \varepsilon_{xt} \end{bmatrix} \tag{6.7}$$

Designating $\eta_t = \varepsilon_{yt} - \Upsilon \varepsilon_{xt}$, we obtain

[3] See the definitions in the appendix to Chapter 4.

$$E(\eta_t\varepsilon_{xt}') = E(\varepsilon_{yt}\varepsilon_{xt}') - \Upsilon E(\varepsilon_{xt}\varepsilon_{xt}') = \Sigma_{xy} - \Sigma_{yx}\Sigma_{xx}^{-1}\Sigma_{xx} = 0.$$

Because

$$\varepsilon_t \sim NID(0,\Sigma),$$

$$\begin{bmatrix}\eta_t\\ \varepsilon_{xt}\end{bmatrix} \sim NID\left(\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}\tilde{\Sigma} & 0\\ 0 & \Sigma_{xx}\end{bmatrix}\right),$$

where $\tilde{\Sigma} = \Sigma_{yy} - \Sigma_{yx}\Sigma_{xx}^{-1}\Sigma_{xy}$. Noting that $\{\eta_t\}\perp\{\varepsilon_{xt}\}$, we should point out that $\{\eta_t\}\perp\{\Delta x_t\}$ (because $\Delta x_t = \sum_{i=1}^{k-1}\Gamma_{xxi}\Delta x_{t-i} + \varepsilon_{xt}$). From equation (6.7) we thus obtain

$$\Delta y_t = \sum_{i=1}^{k-1}\Gamma_{xxi}\Delta y_{t-i} + \Upsilon\Delta x_t + \sum_{i=1}^{k-1}\tilde{\Gamma}_i\Delta x_{t-i} + \alpha_y\hat{e}_{t-1} + \eta_t, \tag{6.8}$$

where $\tilde{\Gamma}_i = \Gamma_{yxi} - \Upsilon\Gamma_{xxi} (i = 1,\ldots,k-1)$.

The structural form can be derived from this conditional VAR model as at the end of the previous section.

$$C_0\Delta y_t = \sum_{i=1}^{k-1} C_i\Delta y_{t-i} + C_k\hat{e}_{t-1} + \sum_{i=0}^{k-1}\tilde{C}_i\Delta x_{t-i} + \nu, \quad \nu_t \sim NID(0,\Lambda),$$

where

$$\begin{aligned}
\Gamma_i &= C_0^{-1}C_i, \quad i = 1,\ldots,k-1\\
\alpha_y &= C_0^{-1}C_k\\
\tilde{\Gamma} &= C_0^{-1}\tilde{C}_i, \quad i = 1,\ldots,k-1\\
\Upsilon &= C_0^{-1}\tilde{C}_0\\
\mathrm{diag}[\tilde{\Sigma}\ \Sigma_{xx}] &= C_0^{-1}\Lambda C_0^{-1\prime}.
\end{aligned}$$

This can be written briefly as

$$C_0\Delta y_t = Dw_t + \nu_t, \quad \nu_t \sim NID(0,\Lambda), \tag{6.9}$$

where $D=[C_1 \dots C_k \ \tilde{C}_0 \dots \tilde{C}_{k-1}]$ and $w_t=[\Delta y_{t-1} \dots \Delta y_{t-k+1} \ \hat{e}_{t-1} \ \Delta x_t \dots \Delta x_{t-k+1}]'$. As above, C_0 should be non-singular and generally the diagonal elements are assumed to be unity.

The model is now in the traditional simultaneous equation form with stationary components. The traditional results and diagnostics apply (see e.g. Hendry *et al.*, 1988).

Hendry and Mizon (1990) and Mizon and Clements (1990) both emphasize the VAR as an alternative hypothesis when testing encompassing or over-identifying restrictions. They apply the notion of encompassing to the DSEM. The idea is to use VAR as a benchmark such that the DSEM encompasses the VAR. They refer to model (6.3) as a closed system and (6.9) as an open system. They use the encompassing idea in comparing rival DSEMs.

The aim of this chapter has been to present the connections between the unrestricted VAR model and the traditional simultaneous equation model in the case of non-stationary variables. The starting point was the unrestricted VAR, and the modelling procedure starting with the testing of Granger non-causality and weak exogeneity and ended with the over-identification restriction. The reason for this approach is to avoid possible misspecification due to incorrect identifying restrictions, which has been the main reason for the increased use of VAR models. Also, it was desired to maintain the easy interpretability of the structural models. The restrictions and the conditioning should always suit the data.

7 Testing Rational Expectations

The first order condition (3.2) of the loss function could be estimated directly to get estimates of the parameters of interest. To find an equation to be estimated, we exclude the last term from the dynamic loss function, i.e. $\lambda_2 = 0$. The condition (3.2) can thus be written in the form

$$\Delta m_t = \delta \Delta m_{t+1} - \lambda_1 (m_t - m_t^*) \tag{7.1}$$

To make the first term on the right hand side more applicable, we replace it with its expectation $\Delta m_{t+1} + \eta_{t+1} = E(\Delta m_{t+1} | I_t) = E_t \Delta m_{t+1}$. To make the problem stochastic we add the (forecasting) error term, which must fullfill the orthogonality condition and $E(\eta_{t+1}) = 0$ if expectations are rational.

The next step is to introduce the forcing variables $m_t^* = \tilde{\beta}' \tilde{X}_t + \varepsilon_t$ of the Euler equation[1]. Thus, the following equation would be estimated:

$$\Delta m_t = \delta E_t \Delta m_{t+1} - \lambda_1 (m_t - \tilde{\beta}' \tilde{X}_t) + \nu_t, \quad \nu_t = \delta \eta_{t+1} + \lambda_1 \varepsilon_t. \tag{7.2}$$

The error correction term can be written in the form

$$\begin{bmatrix} 1 & -\tilde{\beta}' \end{bmatrix} \begin{bmatrix} m_t \\ \tilde{X}_t \end{bmatrix} = \beta' X_t = e_t. \tag{7.3}$$

A serious problem concerning (7.2) araises from the fact that the conditioning variables I_t are correlated with the error term ν_t. Learning, among other things[2], might be one reason for this. $E(\Delta m_t | I_t)$ could be approximated with the instruments, which are the past values of Δm_t and the target error term e_t. The number of the past values (lag length, k) could be determined empirically. Heteroscedasticity and serial-correlation-consistent standard errors could be achieved by the Hansen's (1982) Generalized Method of Moments estimator (GMM).

The model (3.4) can be reparametrized as

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[1] As these f.o.c. are called.

[2] See, for example, Wallius (1992), which gives reasons for residual autocorrelation in the rational expectations case.

$$\Delta m_t = (\mu_1 - 1)(m_{t-1} - m^*_{t-1}) + (1 - \mu_1)\sum_{s=0}^{\infty} (\delta\mu_1)^s \Delta m^*_{t+s}, \tag{7.4}$$

when $\lambda_2 = 0$. Conditioning the future realization of m^*_{t+s}, $s > 0$ on the present information I_t and using the definition of forcing variables, one obtains

$$\begin{aligned}\Delta m_t = &(\mu_1 - 1)(m_{t-1} - \beta'\tilde{X}_{t-1}) + (1 - \mu_1)\sum_{s=0}^{\infty} (\delta\mu_1)^s \beta' E_t \Delta\tilde{X}_{t+s} \\ &+ (1 - \mu_1)(1 - \delta\mu_1)\varepsilon_t\end{aligned} \tag{7.5}$$

Campbell and Shiller (1987) propose a way to test the rational expectations restrictions of the model (7.5). Their idea is to set up a general VAR model for the stationary variables Δm_t and the target error e_t. They are related as $\Delta e_t = \Delta m_t - \beta'\Delta\tilde{X}_t$. Using this relation and the VAR model, the future values of $\beta'\Delta\tilde{X}_t$ of (7.5) can be calculated.

Consider estimating a VAR presentation for Δm_t and e_t:

$$\begin{bmatrix} \Delta m_t \\ e_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta m_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \tag{7.6}$$

where *a(L)*, *b(L)*, *c(L)* and *d(L)* are lag polynomials of order *k*. Campbell and Shiller (1987) derive the non-linear restriction for the parameters of the VAR representation and the Wald test of the restriction.

Engsted and Haldrup (1991) apply the method to the demand for labour in Denmark[3]. Following their modification, I write the model in the shorter form as $Z_t = AZ_{t-1} + u_t$, where $Z_t = [\Delta m_t, \ldots, \Delta m_{t-k+1}, e_t, \ldots, e_{t-k+1}]$ and A is the matrix of VAR parameters. To get Δm_t and e_t from the VAR model, we define $2p + 1$ vectors g and h as

$$\begin{aligned} \Delta m_t &= g'Z_t \\ e_t &= h'Z_t. \end{aligned}$$

[3] The idea of joining Johansen's method and the dynamic loss function with forcing variables is originally their's. My study relies heavily on their modification and I am gratefull that the early version of their study was made available to me.

The expectation term in (7.5) can be obtained by conditioning on the information set H_t, which consists of all current and lagged values of Δm_t and e_t. Then $E(Z_{t+j}|H_t) = A^j Z_t$ and the expectation term is

$$\sum_{s=0}^{\infty} (\delta\mu_1)^s \beta' E_t \Delta\tilde{X}_{t+s} = (g-h)'(I-\delta\mu_1 A)^{-1} Z_t + h'(I-\delta\mu_1 A)^{-1} Z_{t-1}. \tag{7.7}$$

When we insert this into the equation (7.5) and condition on H_t, we obtain

$$\begin{aligned} E(\Delta m_t|H_t) &= \Delta m_t^* \\ &= (1-\mu_1)(g-h)'(I-\delta\mu_1 A)^{-1} Z_t + (\mu_1 - 1)h'(I-(I-\delta\mu_1 A)^{-1}) Z_{t-1}, \end{aligned} \tag{7.8}$$

which is the theoretical level of Δm_t when the rational expectations restrictions are satisfied.

For practical testing of model (7.5), they suggest to comparing — testing the differences — the theoretical Δm_t^* and the realization of Δm_t. "If the present value model is true, these variables should differ only because of sampling error" (Campbell and Shiller, 1987, pp. 1069). Their idea is that under rational expectations the desired change of the level of money should be the same as the theoretical level, $\Delta m_t^* = \Delta m_t$.

The estimate of β can be found using Johansen's cointegration analysis. The adjustment $1-\mu_1$ can also be estimated using Johansen's approach. The loadings α can be interpreted as the desired adjustment only if the forcing variables are weakly exogenous with respect to α and β. Otherwise, the adjustment cost should be estimated from (7.5) using the instrumental variable (IV) technique. Some results for δ can be achieved by estimating (7.2) with GMM.

8 From Theoretical Variables to Empirical Counterparts

The theoretical models of the demand for money clearly suggest a set of variables acting as regressors for money demand. But it is generally not easy to find empirical counterparts for these theoretical variables. The period of credit rationing in Finland makes the choices even more difficult. If the empirical counterpart of a theoretical variable is not good enough, it is impossible to make any inference about the theories of the demand for money. The source for all the data used in this study is the Bank of Finland's data base.

The estimation period is roughly the 1980s, depending on the model. According to the results of Ripatti (1992a) and (1992b), there might be structural change in the quarterly model for M1 in the early 1980s. Another critical point is 1991 when the interest withholding tax was introduced.

The models are based on monthly data. There are two reasons for this. First, high frequency data is needed to capture the adjustment behaviour of agents. Secondly, according to the simulation results of Osterwald-Lenum (1991) much more than 50 observations are needed to "reach" the infinity of the asymptotic results. One might well raise the point that monthly data is too frequent for modelling the slow adjustment involved here. My own experience — based on the results of Ripatti (1992a) and (1992b) — is that a month is usually not too short a period for analysing the main features of money demand.

8.1 Monetary Aggregates

The Bank of Finland changed the definitions of the monetary aggregates in the beginning of 1991. The narrowest money, M1, includes currency and cheque and other transaction accounts which can be considered quite liquid. The former definition of M1 also included time deposits, which are not really very liquid. Time deposits belong to M2 according to the new definition. Some high yield investment and deposit accounts and home savings premium deposit accounts are also included in M2. M3 is defined as M2 plus certificates of deposits held by the public. M3 did not exist in the former definitions of money (Jokinen, 1991).

Table 8.1 **New monetary aggregates, 31 Nov. 1990, bill. FIM**

		Cash in circulation	13.5
	−	Cash in the banks	4.4
=		Cash held by the public	9.1
	+	Cheque and savings accounts in banks	29.0
	+	Other transaction accounts in banks	90.0
=	M1	Narrow money	128.1
	+	Other FIM deposits in banks (e.g. time deposits)	129.6
=	M2	Broad money	257.7
	+	The Certificates of Deposit held by the public	33.7
=	M3		291.4

The logarithm of M1 is plotted in figure 8.1. The typical seasonal feature of M1 is the increased amount of cash held by the public during the summer vacation period. Tax rebates in December also cause a seasonal increase in M1. The spike at the beginning of 1990 was caused by the bank employees' strike. The spike at the end of 1990 resulted from the discontinuance of two-year special taxfree savings accounts. Money from these accounts was transferred to the normal transaction and savings accounts. The new withholding tax on deposit accounts has also caused major changes in the amount of M1.

The aggregates M2 and M3 are plotted in figures 8.2 and 8.3. The jump in December 1988 in figure 8.2 has the same background as the drop of M1 in December 1990: the special taxfree 24-month savings account. Capital gains taxes were increased at the beginning of 1989. Real M1 and M2 are presented in the figures 8.4 and 8.5. The dramatic decline of M1 since 1991 is a result of the introduction of the withholding tax.

The new M1 and M2 aggregates are available as monthly series from January 1980. The aggregate M3 is available from the beginning of 1983. They are published frequently in the Bank of Finland Bulletin. One should note that the market for CDs was established in early 1987, so the effective estimation period for M3 starts only from 1988. These 36 observations contain too little information for reliable statistical inference.

We have attempted to capture the deregulation of financial markets with credit expansion, that is, the sum of markka and foreign currency credits, deflated by the consumer price index (figure 8.6). The expansion of foreign currency credit alone is also given (figure 8.7.)

Figure 8.1 **Monthly M1**

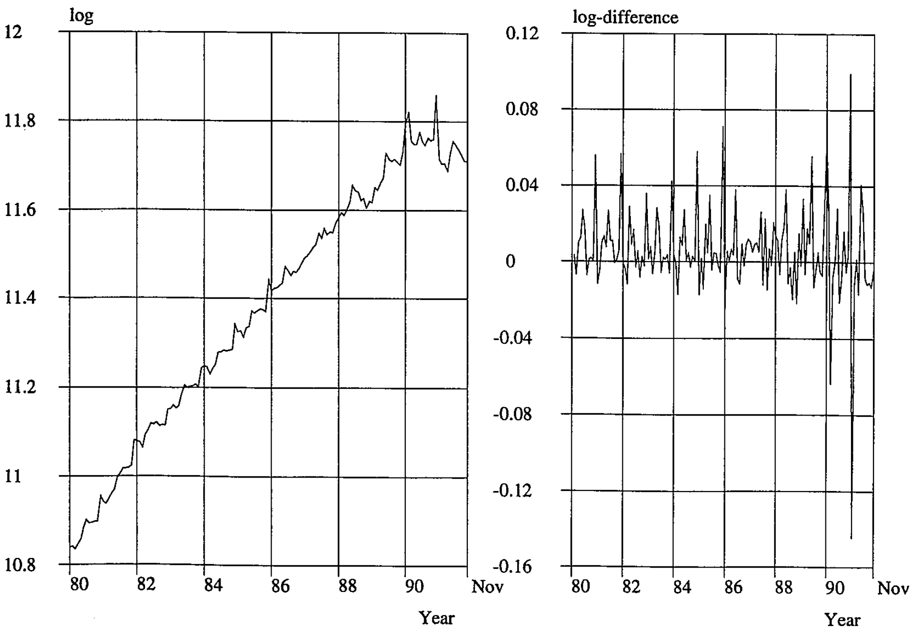


Figure 8.2 **Monthly M2**

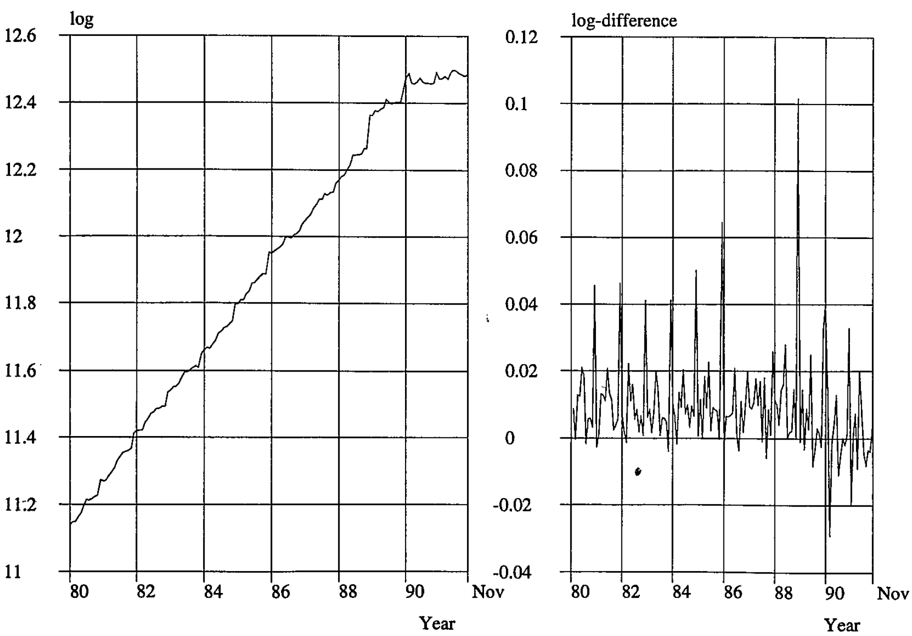


Figure 8.3 **Monthly M3**

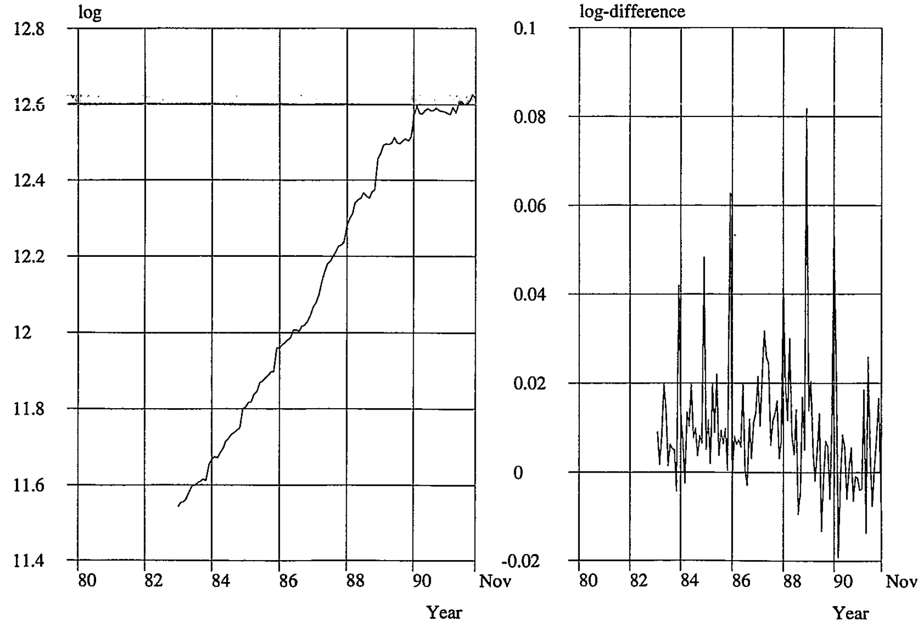


Figure 8.4 **Monthly real M1**

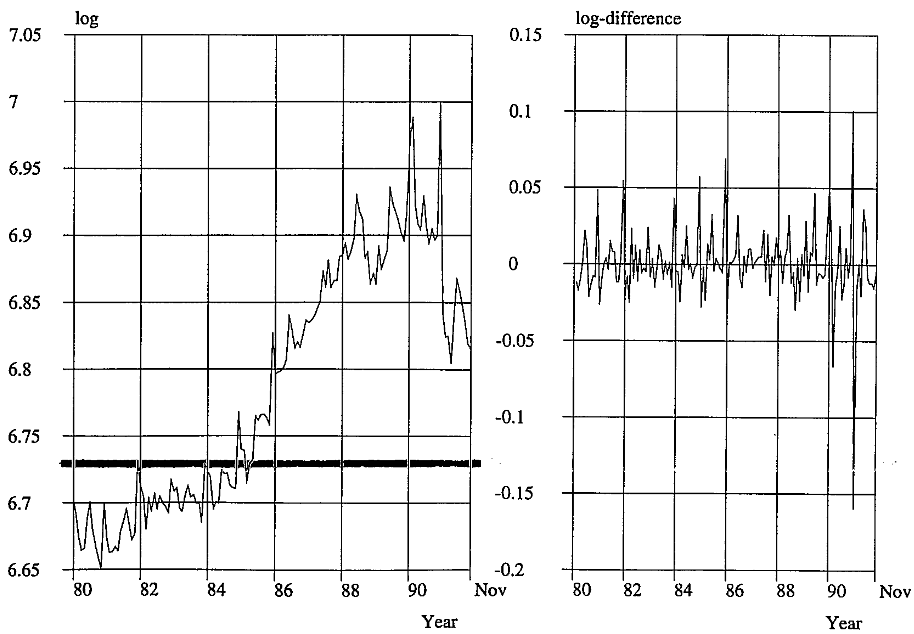


Figure 8.5 **Monthly real M2**

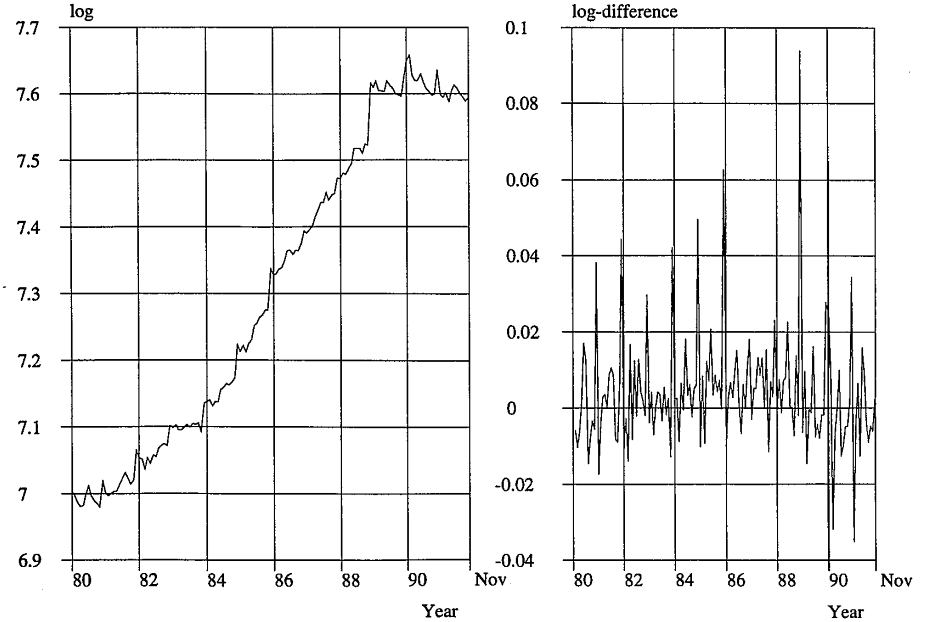


Figure 8.6 **Credits, 1985 prices**

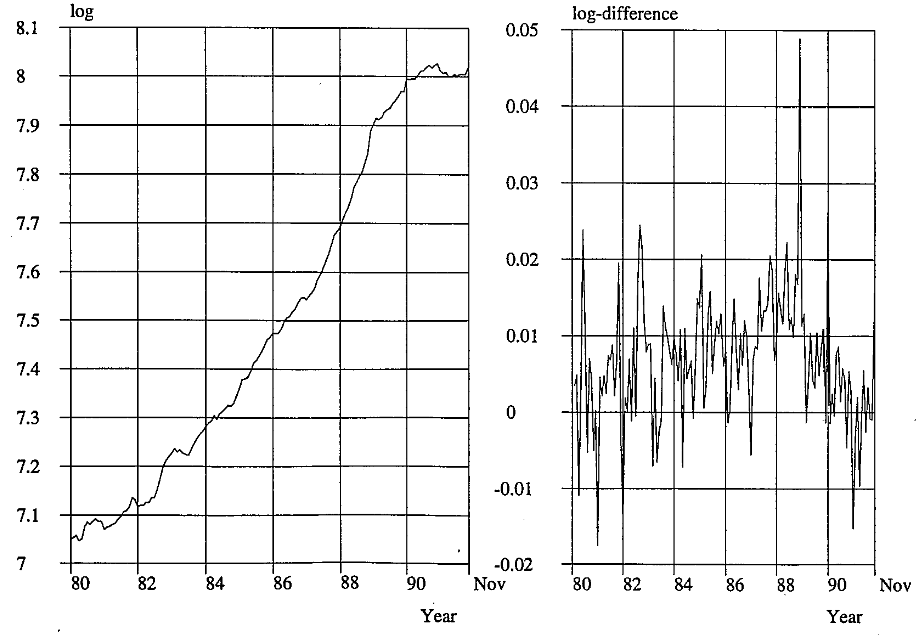
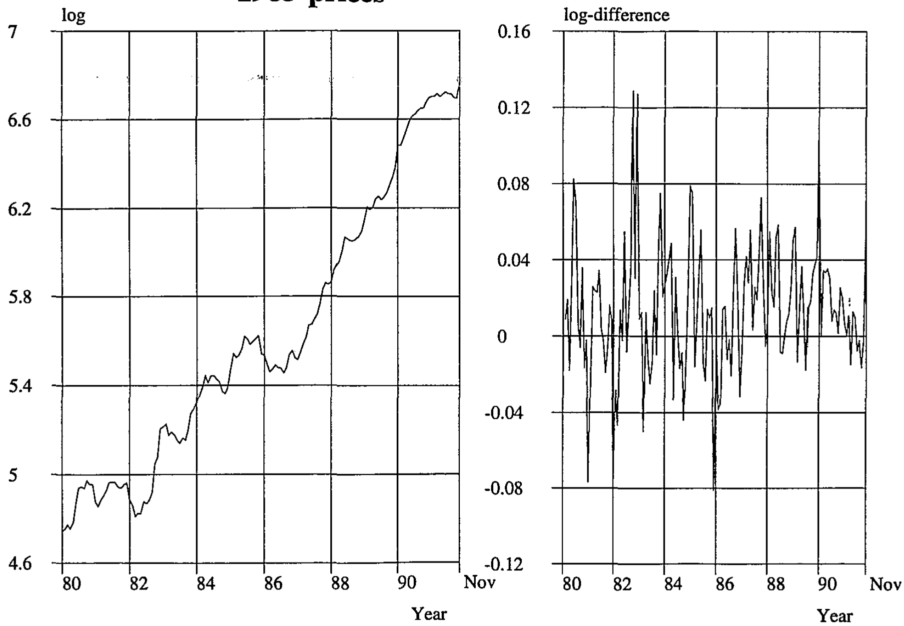


Figure 8.7 **Foreign currency-denominated credits, 1985 prices**



8.2 Velocity of Money

The velocity of money is defined by the identity of the quantity theory of money, $V \equiv \frac{PT}{M} \equiv \frac{T}{M/P}$, i.e. the volume of transactions scaled by real money. The velocities are plotted in figures 8.8 — 8.10. M2 and M3 velocities are clearly downward sloping and unstable. The growth of money has been greater than the growth of transactions (measured by GDP). The only exception is the apparent stability of M1 velocity.

Figure 8.8 **Velocity of monthly M1**

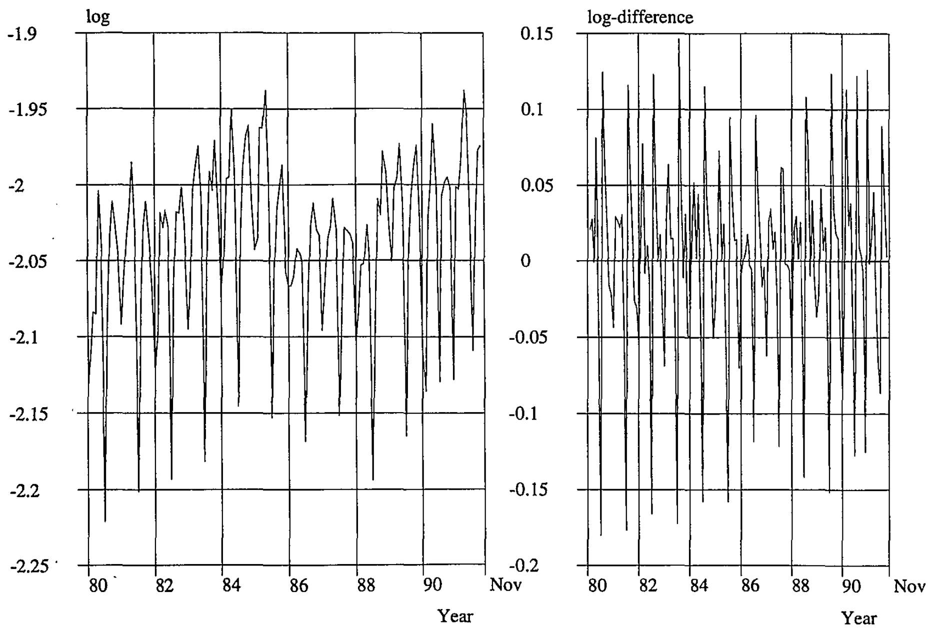


Figure 8.9 **Velocity of monthly M2**

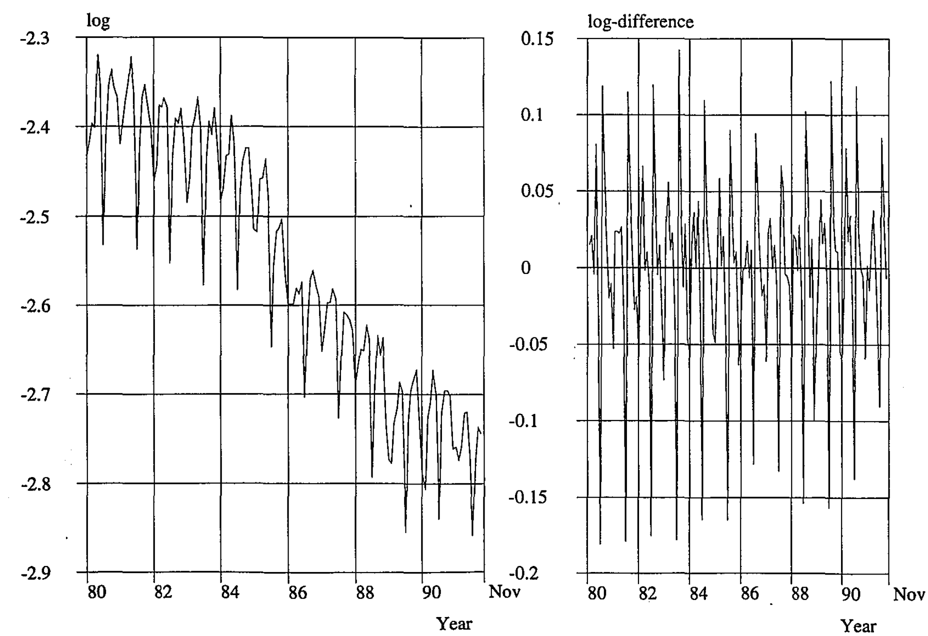
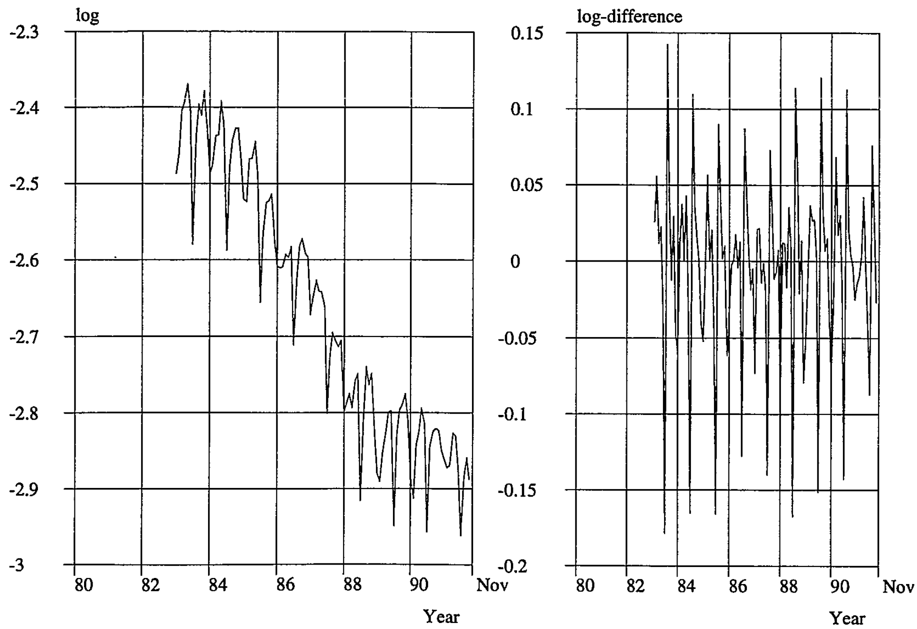


Figure 8.10 **Velocity of monthly M3**



8.3 Scale Variables

The only two possible scale variables in the monthly models are the monthly GDP volume indicator and interbank clearing activity. The former series (figure 8.11) is a combined index of various indicators such as retail sales, industrial production, production of electricity, railway traffic, etc. It is dominated by a strong seasonal pattern, which may be artificial. As an indicator, it involves several problems. It diverges from SNA-based GDP volume figures. During the go-go years of the late 1980s, it showed smaller annual growth than the SNA-based index; recently, the reverse has been true.

Another interesting scale variable is interbank clearing transactions[1]. These are cheques and other transfers between banking groups. It does not measure transactions within a banking group (as e.g. savings banks). It can be supposed that it measures at least monetary transactions quite well. However, according to Ripatti (1992b) the statistical properties of the model for narrow money with interbank clearing as the scale variable do not at all support the demand for money interpretation.

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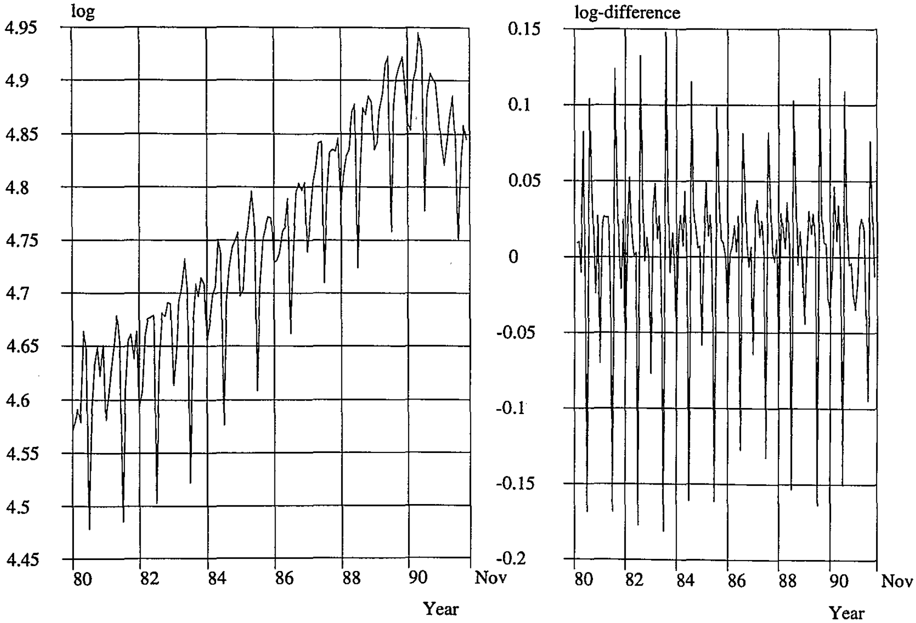
[1] Thanks to Juha Tarkka for the idea.

Figure 8.11 **Volume Index of GDP**



8.4 Price Level

In studies of the demand for money, the price level is usually measured by the implicit GNP price deflator, the consumer price index or the wholesale price index. In this sutdy, we use the consumer price index. Its advantage is that it includes the prices of imported goods. The problem is that the way housing expenditures is measured has changed during the estimation period. The index is shown in figure 8.12.

8.5 Own Yield of Money and Opportunity Costs

The own yield of money is a very problematic variable in the Finnish case. The interest yield of savings and transfer accounts has been very low, and it has been closely connected to the Bank of Finland's base rate[2]. So it has not changed much. Interest income, when linked to the base rate, has usually been taxfree.

[2] The Bank of Finland's base rate is changed very rarely (latest change was at the end of 1989).

Figure 8.12 **Consumer Price Index**

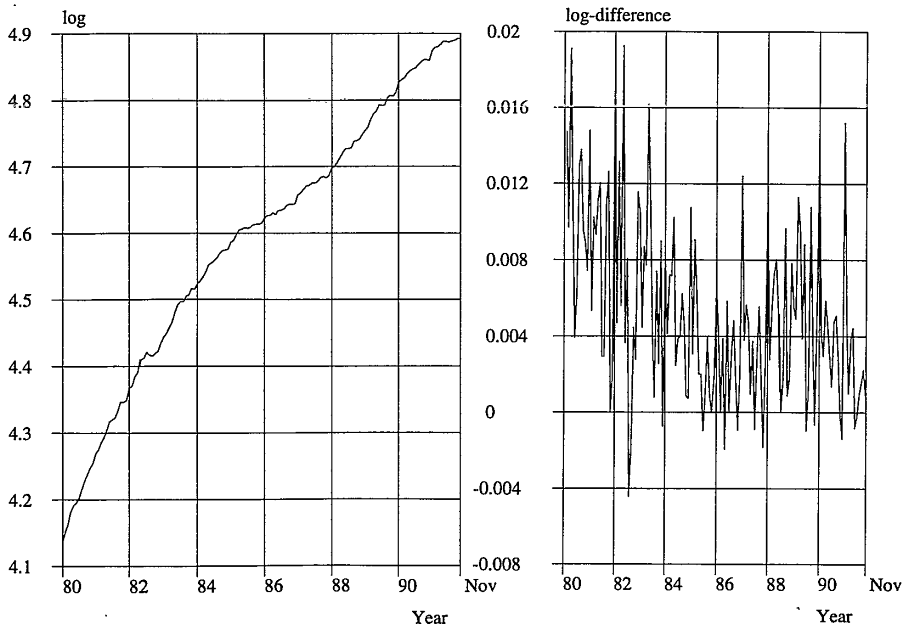


Figure 8.13 **Three-month Eurorate for FIM**

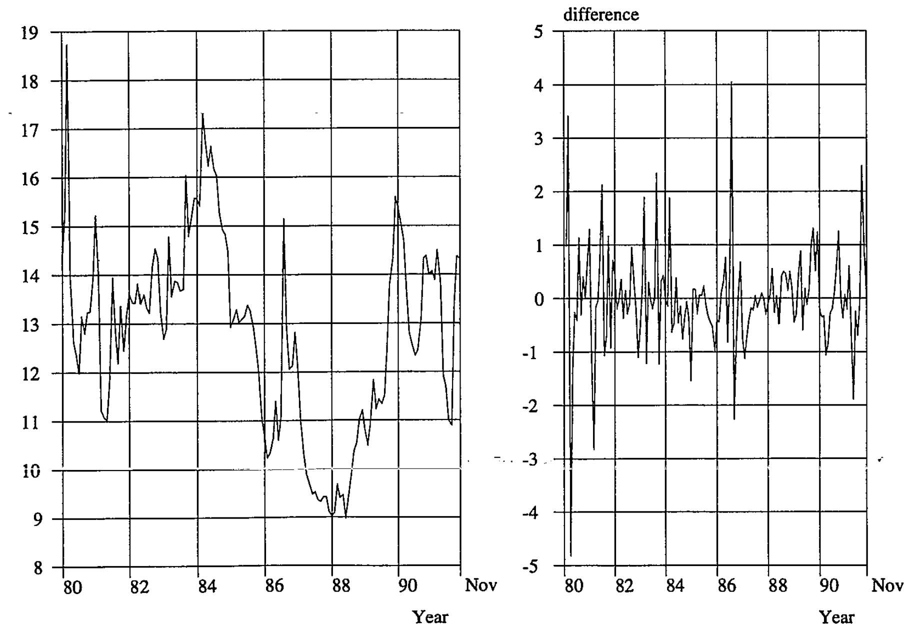
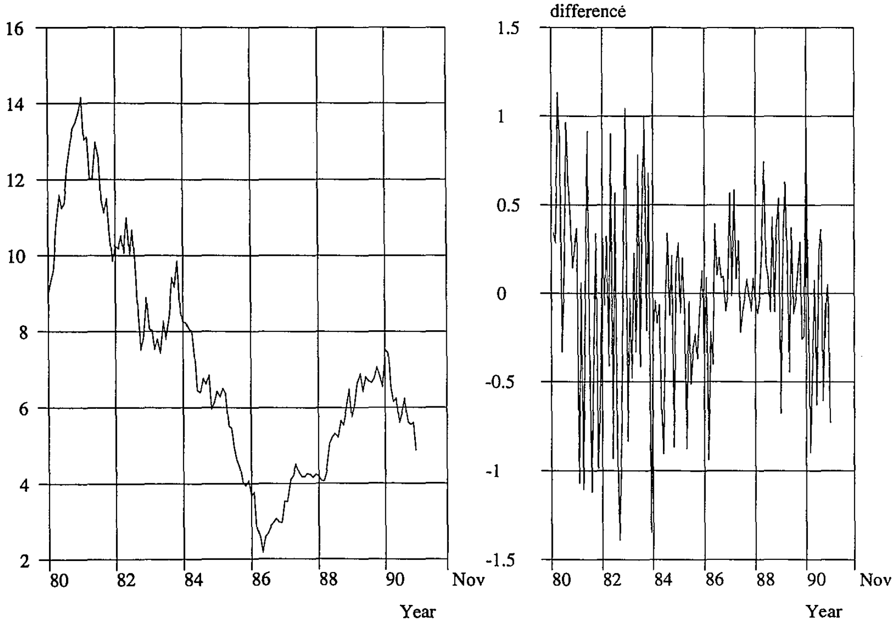


Figure 8.14 **Effective yield of bonds**



In the final models of M1 and M2, I have not used any variable for the own yield of money. Generally, interest on savings and transactions accounts is paid only on the minimum balance for the month. Moreover, good statistical data is not available. The opportunity cost of M1 and M2 is measured by the three-month Eurorate for FIM (see figure 8.13).

The average savings rate is dominated by changes in the base rate. The other variations are caused mainly by changes in the weights of different accounts. We have tried to use the average savings rate in the models for nominal M2 but have left it out of the models for real M2. It is not a stochastic variable because it is closely linked to the base rate. One possible opportunity cost of money is the inflation rate (1-year log-change of the CPI), given in figure 8.14.

Several exogenous shocks also occurred in the period studied. They are modelled by dummy variables.

- Devaluation speculation in August 1986, extremely high interest rates (> 30 %) (DSPEC)

  Affected only short-term interest rates

- Increase of capital gains tax in January 1989

  Affected the monetary aggregates M1 and M2 in December 1988

- The beginning, January 1989 (TAX), and end, January 1991 (FREETAX) of special taxfree 24-month time deposit.

  Affected the monetary aggregates M1 and M2

- Strike of bank employees in February 1990 (STRIKE)

  Affected money by increasing it; the interest rate was frozen

- The introducion of the interest withholding tax on bank accounts (WITHDTAX) at the beginning of 1991. A 15 % withholding tax on bank accounts created real competition between banks. The accounts included in M1 became interest bearing.

- Changes in the discount rate (IBR)

  The administrative rate of Bank of Finland.

- "Uncovered interest parity"

  $$UIP_t = \text{FIM Eurorate}_{t-1} - \text{currency basket rate}_{t-1} - \Delta\text{currency index}_t/\text{currency index}_{t-1}. \qquad (8.1)$$

  Measures the currency substitution effect. It is set to zero before 1982.

Money, price level and scale variables are in logarithmic form. Interest rates and the inflation rate are divided by one hundred[3]. The following form of money demand is usually applied:

$$M_t = P_t^{\beta_1} Y_t^{\beta_2} \exp\{\beta_3 I_t^{own} + \beta_4 I_t\}. \qquad (8.2)$$

The elasticities of price and scale variables are directly β_1 and β_2. Interest rate elasticities are $\eta_i = \beta_i I_t$. Equation (8.2) is identified as a demand for money equation if $\beta_1 = 1$ (price homogeneity), $\beta_2 > 0$ (positive scale elasticity), $\beta_3 \geq 0$ (own yield of money is positive) and $\beta_4 < 0$ (elasticity of opportunity costs is negative)[4]. There can be several opportunity costs of money in the equation. When identifying loadings, money should not be weakly exogenous with respect to the long run parameters. The interesting question is whether the transaction elasticity is close to unity or not.

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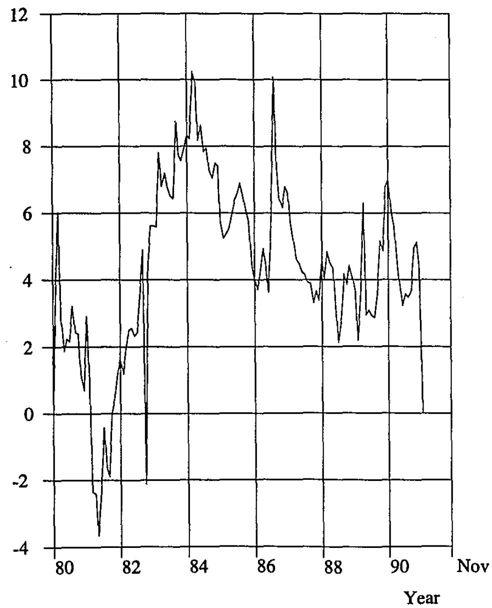
[3] This is the usual, probably never well tested, form of estimating the demand for money. The (1/100) level of interest rates is approximately log(1 + *i*).

[4] See, e.g. Hendry & Ericsson 1991a, 838.

Figure 8.15 **Inflation**



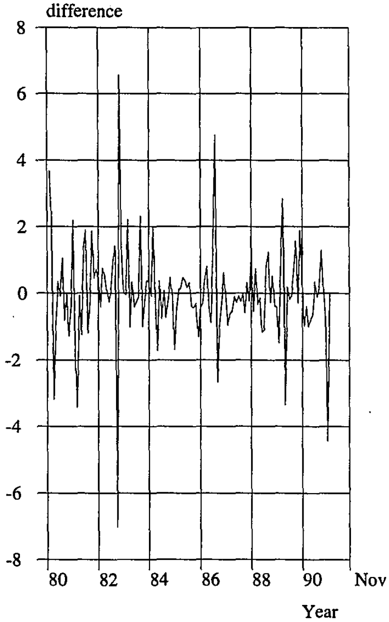
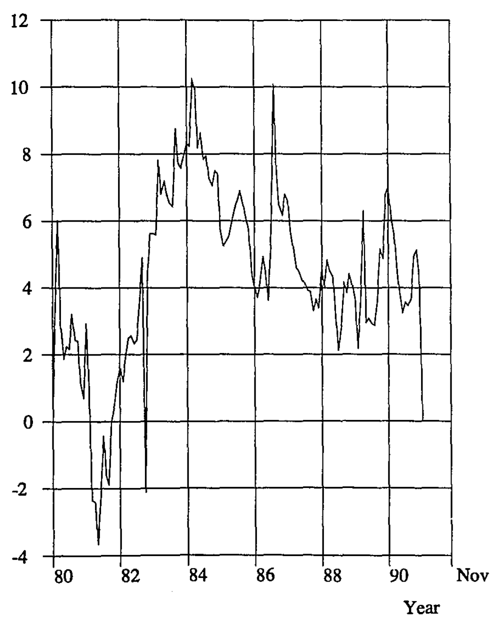
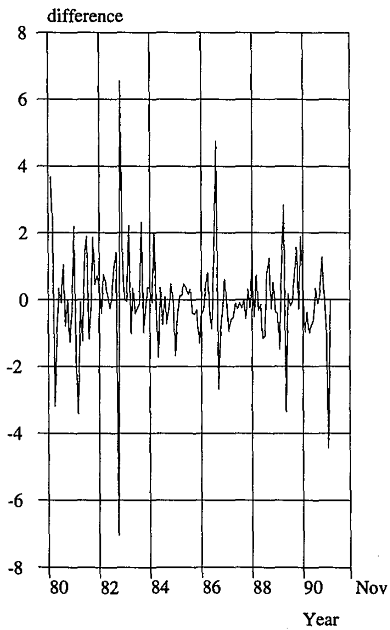


Figure 8.16 **Uncovered interest parity**





9 Some Earlier Results

In this chapter I review some results of earlier demand for money studies in Finland and some foreign studies, which have used econometric methods and model specification similar to those used in this study. An interesting question related to the earlier money demand studies for Finland is whether some of the long-run elasticities lie in the same cointegration space as those found here. The interesting questions in connection with the foreign studies are more on the methodology side and in comparisons of the effects of financial deregulation.

9.1 Results Using Finnish Data

The demand for money has not been a very popular research topic in Finland during the 1980s. Solttila and Johansson (1987) and Mikkola (1989) have worked on it. The demand for money has also been estimated within large macroeconometric models (see Tarkka and Willman, 1990). Some studies have concentrated on the comparison of several countries and have thus been restricted to single specifications (see e.g. Kanniainen and Tarkka, 1986). Suvanto (1980) is a complete survey of the earlier studies.

In Suvanto (1980) M1 is defined as currency held by the public plus demand deposits of the public. M2 is M1 plus time deposits. The monetary aggregates are thus roughly comparable to the present ones. Some differences arise in the definitions of unused overdraft facilities, indexed deposits etc. The estimation periods for the different studies cover the years since 1910.

In the models for M1 the long run income elasticities have varied between 0.7 and 0.85. For M2, they have been somewhat higher — around 1.6. This contradicts some of the results of Ripatti (1992b), but is probably consistent with those of Ripatti (1991). The interest rate elasticities have generally been impossible to determine directly. There has not been any market-based interest rate for the estimation periods for the earlier studies. The inflation rate elasticity has varied between –0.01 and –0.06 for M1 and –0.06 and –0.10 for M2. The comparable semi-elasticities are around 5—10 times larger.

Kanniainen and Tarkka (1986) have the shock-absorption feature in their model for five economies. Their theoretical model is also derived from the forward-looking loss function. The important difference is that they focus on money supply shocks[1]. In their testable model the money supply is determined by domestic credit expansion, surplus on current account and government net borrowing abroad. They estimate models for real money. The scale elasticity for broad money is quite low (0.17), interest rate elasticity, which is not significant, is also quite low (–0.024), but inflation elasticity is higher (–0.99). The results are estimated for the period 1960–1982. For the last 10 years the results have been qualitatively the same.

Solttila and Johansson (1987) use monthly data in their estimation of the demand for money in Finland. Their aim is to compare the estimation results of different time periods. They conclude that the adjustment has become faster since the 1970s and that interest rate elasticities have increased. They use a different definition of narrow money (M1), which is comparable to the present definition of M1. The scale elasticities are 0.32 in the short run and 0.71 in the long run. The interest rate semi-elasticity is –1.0 in the short run and –2.1 in the long run.

Mikkola (1989) is the first demand for money study in Finland that considers the unit root problem. She estimates the error correction model with Engle and Granger's (1987) two-step method. The estimation period is a year longer than in Solttila – Johansson (1987). The long run scale elasticity is 0.66 and interest rate elasticity –0.31. She also tried to pick up the effect of increased stock exchange turnover but did not find that it increased the transactions demand for money. The money measure was M1, which probably did not include transaction accounts that are now included in M1.

Tarkka and Willman (1989) present the money demand function in their financial sector of the BOF4-model. They estimate demand for broad money, which is roughly the present M2. The functional form is quite extraordinary:

$$\frac{M_t}{Y_{t-1}} = \beta_0 - \beta_1[(1-\text{tax})I_t - I_t^D] + \alpha_2\frac{\Delta Y_t}{Y_{t-1}} + \alpha_3\frac{M_{t-1}}{Y_{t-1}},$$

where Y is nominal GDP, I is the short-term market rate of interest, I^D is the tax-free time deposit rate, tax is the capital market tax rate calculated from the interest rate differential between taxable and taxfree bonds. The long run elasticity of *nominal* GDP is 1, interest rate semi-elasticity is –1.24 and the own-yield elasticity is 1.54.

[1] Such an interpretation can be given to some estimation results of Ripatti (1992b).

Johansen and Juselius (1990) also estimated long run elasticities of money demand in Finland. They found that M1 and GDP are cointegrated ($\beta = (1\ \ -1)'$) and thus the velocity is stationary. The interest rates and inflation were found to be stationary.

Table 9.1 **Long run elasticities from the earlier studies using Finnish data**

Measure for real money	Scale elasticity	Interest rate semi-elasticity	Inflation semi-elasticity	Estimation period	Study
M1	0.7 – 0.85		–0.1 – 0.6	1910–1970	Suvanto (1980)
M2	1.6		–0.6 – 1.0	1910–1970	Suvanto (1980)
M2	0.17	–0.2	5	1960–1982	Kanniainen – Tarkka (1986)
M1	0.7	–2		1983–1985	Solttila – Johansson (1987)
M1	0.66	–0.31		1983–1986	Mikkola (1989)
M2	1	–1.24		1985–1989	Tarkka – Willman (1990)
M1	1			1960–1986	Johansen – Juselius (1990)

9.2 Other Studies Using Similar Modelling

Judd and Scadding (1982) is a good summary of pre-1980s empirically-oriented US demand-for-money studies. Goldfeld and Sichel (1990) complement the estimation results of the 1980s. Since then, the Johansen method of estimating cointegrated systems has been a major contribution to the demand-for-money studies. It has been applied in several studies[2]. I do not review all the results here, but the Nordic studies are quite interesting because of the similarities of the economies.

Johansen and Juselius (1990) is a representative application of the method. The results have been extended in Juselius (1991b). She focuses on M1 and the interpretation of the unrestricted VAR with error correction terms. The price and scale homogeneities are accepted. The own-yield and the interest semi-elasticities are quite high (roughly 4).

[2] The list is long: Johansen and Juselius (1990a) for Denmark and Finland, Juselius (1989) and (1991b) for Denmark, Muscatelli and Papi (1990) for Italy, Hendry and Ericsson (1991) for UK and USA, Boughton (1991) for large industrial countries, Hoffman and Rasche (1991).

Inflation does not enter the cointegration relation because it can be modelled as stationary. The long run money demand relation is found to be stable over the period of financial deregulation. The adjustment toward equilibrium is not very fast (size of error correction coefficient –0.29). The long run interest rate (bond rate) is found to be determined outside the system and to be weakly exogenous for long run parameters. An interesting conclusion of the analysis of system and causal chain model is that monetary shocks do not affect Danish inflation.

Bårdsen (1991) analyzes Norwegian data. His aim is also to examine whether financial deregulation and changes in monetary policy have affected the demand for money. Like Juselius (1991b), he uses quarterly data since the 1960s and applies cointegration methods. He carefully checks the parameter constancy of the model. He finds a conditional model with constant parameters. Prices, real expenditures and interest rates are super-exogenous for the financial structural shocks met by Norway. The own-yield of money – measured by the interest rate on demand deposits – was found to play a crucial role.

Hendry has recently[3] estimated the demand for money in the U.K. and U.S. As representative of those studies, I repeat some of the results of Hendry and Ericsson (1991a) here. In terms of econometrics they deal with the following issues: "non-constancy of conventional equations", "exogeneity or endogeneity of money", "the invertibility of existing models", "long-run and short-run determinants" and "causal links". They summarize their model of U.K. money demand as follows:

> The model of U.K. money demand remains constant over the 1980s when the opportunity cost is adjusted to account for financial innovation. Correct dynamic specification and inclusion of the relevant interest rates are central to obtaining a congruent empirical model. Prices, incomes, and interest rates are super exogenous for the parameters of the conditional money–demand equation. That refutes the Lucas critique for changes in the parameters of expectations processes, and precludes either inverting the money-demand equation to obtain a constant model of inflation regarding money growth or interpreting the error-correction model as derived from a forward-looking expectations-based theory models. However, we can provide a forward-looking interpretation of the error-correction model using data–based predictors.

[3] Hendry and Ericsson (1991a) and (1991b) and Baba, Hendry and Starr (1992).

Regarding U.S. money demand they argue that

> The 'missing money'[4] appears to be due to misspecified dynamics and omitted interest-rate volatility, not financial innovation; the underlying M1 demand function remained constant in spite of the Fed's New Operating Procedures; and the very large increases[5] in M1 witnessed in the mid- to late-1980s can be seen as lagged adjustment to falling interest rates and inflation and the introduction of interest-bearing checking accounts.

Baba, Hendry and Starr (1992) repeat the above analysis concentrating more on cointegration and bond volatility. They argue that the constancy of their model is based on the bond volatility[6].

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[4] The problem of money demand in U.S. was that the models tended to 'overpredict' the demand for money in the late 1970s.

[5] This has been another problem with the U.S. money demand equations.

[6] Bond volatility is measured by the "moving standard deviation of holding period yields on long-term bonds". Also the own-yield of M1 is learning adjusted for new interest bearing accounts. Muscatelli and Papi (1990) use similar learning adjustment behaviour for financial innovations.

10 Testable Hypotheses

Economic theories can usually be formalized as econometric hypotheses regarding long run parameters. We consider the following parametrization of the long run structure

$$m_t = \beta_1 p_t + \beta_2 y_t + \beta_3 I_t^{own} + \beta_4 I_t + \beta_5 \Delta_{12} P_t, \tag{10.1}$$

where the lower case letters represent variables in logarithmic form. The others are divided by hundred.

Price homogeneity (in the long run) is probably the most important hypothesis. It is a minimum requirement of the money demand function. It can be expressed as

$$H_p : \quad \beta_1 = 1. \tag{10.2}$$

Another, desirable property of money demand is scale homogeneity. It can be expressed in the same way, but in general it should apply with price homogeneity. So,

$$H_{py} : \quad \beta_1 = \beta_2 = 1. \tag{10.3}$$

These hypotheses are not based on any theory of money demand, but are rather in the empirical tradition of demand-for-money studies. The scale-homogeneity restriction depends on the measurement of the scale variable. Other possible theory based scale elasticities are those found by Baumol (1952), $\beta_2 = 0.5$, and Miller – Orr (1966), $\beta_2 = 0.33$ if transactions are measured by unit size of transaction or $\beta_2 = 0.67$ if transactions are measured by variance of cash flow.

The stable velocity of money implied by the quantity theory of money can be expressed as a parameter restriction

$$H_{velo} : \quad \beta_1 = \beta_2 = 1 \wedge \beta_3 = \beta_4 = \beta_5 = 0. \tag{10.4}$$

Thus it contains 5 restrictions on equation (10.1). Ando and Shell (1975) argue that the difference of the own-yield and opportunity yield of money, i.e. the net opportunity cost of money, applies here. This can be expressed as the following parameter restriction

$$H_{net} : \quad \beta_3 = -\beta_4. \tag{10.5}$$

This restriction is not applicable if the own-yield of money is zero. These restrictions should be tested with both the M1 and M2 models.

Another interesting class of hypotheses concerns whether the earlier estimation results for the demand for money are within the cointegration space. These hypotheses can be derived from table 10.1). Other hypotheses are discussed with the estimation results.

Table 10.1 **Earlier results as cointegration vectors**

Measure for real money $\beta_1 = 1$	Scale elasticity β_2	Interest rate semi-elasticity β_4	Inflation semi-elasticity β_5	Estimation period	Name of hypothesis
M1	0.8	0	–0.6	1910–1970	H_FS
M2	1.6	0	–1.0	1910–1970	H_FS
M2	0.17	–0.2	5	1960–1982	H_FKT
M1	0.7	–2	0	1983–1985	H_FSJ
M1	0.66	–0.31	0	1983–1986	H_FM
M2	1	–1.24	0	1985–1989	H_FTW
M1	1	0	0	1960–1986	H_FJJ

The implicit assumption in every equation is that $\beta_3 = 0$.

11 Models for Narrow Money

Narrow money (M1) includes cash in circulation and savings, transaction and checkable deposits (see table (8.1). Foreign currency deposits are not included. Graphs of the log-level and differences of nominal and real money are presented in figures 8.1 and 8.4.

The estimation period for the monthly data is January 1980 – December 1990. The other eleven months are used to test the forecasting performance of the model. The introduction of the interest withholding tax in January 1991 is a good benchmark for the estimated model.

The organization of chapter follows the strategy for modelling the demand for money. We start with the unrestricted VAR model in difference form. As was emphasized in chapter 6, we can concentrate on the analysis of the long run structure first and then carry out the short run analysis for fixed long run parameters. The steps of the strategy are as follows:

1. The long run relationships are first estimated for nominal money. The key issues are whether the variables in the system are $I(1)$ and whether prices are homogeneous (unit price elasticity). If prices are homogeneous, we can impose this restriction on the data and continue the analysis in real terms.

2. In the analysis of real money, several structural hypotheses of long run structure are tested and a final long run structural form is obtained.

3. We take a side step to test the rational expectations hypothesis in the context described in chapter 7. We use the estimates of long run parameters from step 2.

4. The final and most important step is to test weak exogeneity and Granger non-causality and to obtain the structural model of money demand. In this step the long run structure is already fixed and we concentrate on the short run dynamics and error correction effects of the cointegration relationship. The econometric methodology for this step is presented in chapter 6.

We also have to discuss intermittently what the different results mean and how they can be interpreted within the money demand framework, that is, from a data-analytical point of view. At each step it is important that the statistical model we analyze fits well with the data, i.e. the information set.

11.1 Long Run Relations

The first step is to estimate the unrestricted VAR model described in equation (5.4) on page 34:

$$\Delta z_t = \alpha\beta' z_{t-1} + \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1}\Delta z_{t-k+1} + \mu + \Psi D_t + \varepsilon_t, \quad t = 1,\ldots,T.$$

The chosen lag length of the model, k, is the minimum length at which the residual autocorrelation and non-normality disappear. This is not a formal method but is motivated by the fact that the longer the lag length the less the degrees of freedom. Tests of lag length are not very strong with few degrees of freedom. Some exogenous shocks are picked up by dummies, D_t, among the lagged differenced variables. These shocks were discussed at the end of chapter 8. The vector of deterministic variables also includes 11 centred seasonal dummies.

11.1.1 Nominal Balances

The model for nominal money is estimated with the constant term μ. Thus the linear trend is allowed. The shortest possible lag length is four, i.e. $k = 4$. The vector of system variables z_t includes the following:

m logarithm of monetary aggregate M1,
p logarithm of consumer price index, 1985 = 100,
y logarithm of GDP volume index, 1980 = 100,
I three month money market rate divided by 100.

Inflation was also tested as a system variable, but it turned out to be non-significant[1].

First, we should determine the cointegration rank. The first block in table 11.1 summarizes estimated eigenvalues (first row), maximum eigenvalue test statistics for different null hypotheses (second row) and comparable 90 % fractiles (third row) and, finally, the trace test statistics and fractiles. Following Johansen (1991a) we start the sequence of trace tests (fourth row, first column containing numbers) from the hypothesis $r = 0$. We reject the null hypothesis $r = 0$, since $66.108 \geq 43.95$. Then we continue to the next hypothesis one column

[1] The χ^2 test for the exclusion was never rejected.

to the right. The null hypothesis is $r \leq 1$. In this case it is near the borderline, $26.969 \geq 26.78$, but we reject again. We cannot reject the null $r \leq 2$, since $7.956 \leq 13.33$. So we conclude that the cointegration rank is two ($r = 2$). The maximum eigenvalue test statistics, λ_{max}, yields the same conclusion. However, the *sequence* of λ_{max} tests of the comparable hypotheses can yield an erroneous conclusion.

The test of cointegration rank is equivalent to testing whether the set of eigenvalues is non-zero. Also, the sorted eigenvalues can give us some insight concerning the cointegration rank. If there is a clear break in the magnitude of eigenvalues, one might suspect that the rest of the eigenvalues are zero. Here we see (table 11.1) that there is a drop after the second eigenvalue, the third being very close to zero. When the cointegration rank is two, only the first and second cointegration vectors are of interest.

Table 11.1 **Initial results of cointegration analysis of M1**

Testing rank of Π				
H_0	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
λ	0.2634	0.1380	0.0498	0.0111
λ_{max}	39.139	19.013	6.533	1.424
90 % fractiles	24.73	18.60	12.07	2.69
trace test	66.108	26.969	7.956	1.424
90 % fractiles	43.95	26.78	13.33	2.69

β matrix				
Variable	β_1	β_2	β_3	β_4
m	0.1426	1.0000	0.0869	−0.3884
p	−0.8860	−0.7112	−0.0492	1.0000
y	1.0000	−1.4320	0.0333	−0.8752
I	0.9862	0.4489	1.0000	0.3511

α matrix				
Equation	β_1	β_2	β_3	β_4
Δm	−0.0208	−0.1131	−0.0777	0.0220
Δp	0.0531	−0.0170	−0.0136	−0.0026
Δy	0.0114	0.2191	−0.0913	0.0033
ΔI	−0.0487	−0.0329	−0.0501	−0.0178

Π matrix				
Equation	m	p	y	I
Δm	−0.131	0.125	0.119	−0.141
Δp	−0.010	−0.037	0.079	0.030
Δy	0.212	−0.158	−0.308	0.019
ΔI	−0.037	0.051	0.012	−0.119

The first cointegration vector is the first column in the β matrix, and so on. Because each component in a cointegration vector corresponds to a variable, the symbols of the variables are listed in the first label column of the matrix. The label row ranks the cointegration vectors. Since each cointegration vector determines a stationary linear combination of the variables, one can scale the cointegration vectors in an arbitrary way[2]. The scaling of each vector has been done for ease of interpretation.

When interpreting the cointegration vector, one can observe that $\beta' z_t = e$, i.e. $\beta' z_t$ is "something stationary" or "zero". So, in interpreting the first cointegration vector, one can proceed as if in the long run the following holds:

$$0.1426m - 0.8860p + y + 0.9862I = e \quad \Longleftrightarrow$$
$$y = -0.1426m + 0.8860p - 0.9862I - e,$$

where e is "something stationary". The economy is in equilibrium in the long run and the equilibrium is determined by the cointegration vector. The interpretation of α is then quite straightforward. The α coefficients in the first column of the α matrix in table 11.1 show to what extent the equilibrium error, e, in the first cointegration vector is corrected in each equation. The equations are referred to in the first label column. The corresponding cointegration vectors are symbolized as β_i, $i = 1,...,r$ in the first label row. One should note that the αs vary when the cointegration vectors are rescaled, because it is Π which is fixed, e.g. $\Pi = c\alpha(1/c)\beta'$.

The interpretation of the first vector is not clear. The first component of the vector is very close to zero (compared to the others; unfortunately we do not have standard errors here). Thus, this appears to be a kind of price or aggregate demand equation. The second cointegration vector (second column in β matrix) certainly has the properties of money demand. Price and income elasticities are quite close to unity and the interest rate elasticity is of reasonable magnitude compared to earlier results with Finnish data (see table 9.1 on page 70). The loadings, i.e. α matrix (now in reduced form), also support the picture given by the cointegration vectors. Only the speed of the adjustment of GDP in response to excess money is surprising. The reason for the rapid adjustment might lie on the wrong interpretation of the second cointegration vector. I also estimated the same model

[2] One can divide a stationary variable by any scalar and the resulting variable is stationary.

with a linear trend in the cointegration space, thus modelling GDP growth and again found rapid adjustment of GDP. This phenomen also shows up in Bårdsen (1991).

The above results are, of course, based on normal, independent residuals. The residual diagnostics[3] (table 11.2) are not alarming. There is some extra kurtosis in the price and interest rate change equations. Autocorrelation is not a problem here either.

Table 11.2 **Residual analysis**

Covariance/Correlation matrix				
Equation	m	p	y	I
m	0.0001	0.0308	−0.0914	−0.1312
p	0.0308	0.0000	−0.0356	0.1397
y	−0.0914	−0.0356	0.0001	−0.0016
I	−0.1312	0.1397	−0.0016	0.0000

Residual diagnostics						
Eq.	B–P.Q(32)/2 8	ARCH(4)	skew.	kurt.	Norm. $\chi^2(2)$	R^2
Δm	1.3771	2.0509	0.1358	0.3358	0.9949	0.7924
Δp	0.6741	18.8863	0.0693	1.2537	8.4854	0.6494
Δy	0.9515	7.3651	−0.4645	0.7331	7.4698	0.9706
ΔI	1.4432	4.4805	0.0387	2.3927	30.5658	0.3912

Autocorrelation $2/\sqrt{T}=0.17678$								
Equation	1 lag	2 lag	3 lag	4 lag	5 lag	6 lag	7 lag	8 lag
Δm	−0.170	−0.070	0.009	0.063	−0.076	−0.176	−0.037	0.131
Δp	0.024	0.030	−0.070	−0.117	0.015	−0.041	0.057	−0.131
Δy	0.012	−0.011	−0.087	−0.159	0.111	0.106	0.086	0.051
ΔI	−0.127	0.065	0.004	−0.026	−0.140	0.248	−0.266	−0.005

The stationarity of the variables can be tested by checking whether any linear combination in which all but one are restricted to zero, e.g. [1 0 0 0]', is stationary. This is done to each variable in table 11.3 with each possible choice of *r*. The stationarity test statistics (table 11.3) show clearly that none of the variables is stationary conditioned on this information.

[3] The first column gives results for the Box–Pierce autocorrelation test, the second for fourth order conditional heteroscedasticity ($\sim\chi^2_4$), the next two for the third and fourth moments (kurtosis-3) of the process, which should be zero for Gaussian variables, the fifth column gives Jarque–Bera (1980) normality test statistics ($\sim\chi^2_2$) based on the two previous moments and the last one gives the multiple correlation results.

Table 11.3 **Stationarity testing**

	LR test, $\chi^2(p-r)$			
df	m	p	y	I
3	23.30	20.13	26.20	26.36
2	15.74	14.58	15.37	15.37
1	3.34	2.68	3.50	2.93

Another important question is whether any of the variables is integrated second order. The test statistics are presented in table 11.4. The next to last column of the table of test statistics is an ordinary trace test on the rank of Π; comparable critical values are given in the last column. First one tests the ordinary cointegration rank using these two columns (as we have already done in table 11.1). Then one chooses the row corresponding to the r chosen and starts testing the rank of $\alpha'_{\perp}\Gamma\beta_{\perp}$ (see equation 5.19, page 45).The symbol s is used for this rank. The testing sequence is started from the leftmost non-zero test statistic $T(r,s)$. The null hypothesis is $s = 0$. Critical values are now tabulated as the last row of table. The number of $I(2)$ variables is $p - r - s$. The tests are computed for each possible r to check the robustness of the result.

Table 11.4 indicates that there is no $I(2)$-ness in the data. If one assumes one $I(2)$ common trend, the stationary β space[4] is like the second cointegration vector of the $I(1)$ case and the non-stationary β space is like the first cointegration vector.

Table 11.4 **Testing $I(2)$-ness**

$p-r$	r	$T(r,s)$				$T(r)$	95 %
4	0	210.1484	122.6849	63.4530	21.6353	66.1083	47.210
3	1		121.8074	65.7375	12.9865	26.9694	29.680
2	2			67.2631	14.9724	7.9561	15.410
1	3				9.8123	1.4236	3.7620
95 % fractiles		47.210	29.680	15.410	3.7620		

The interesting economic hypothesis presented in the previous chapter can be tested within the cointegration framework. The stable velocity of money can be accepted with the p value of 0.05. So the acceptance is clearly on the borderline. The interest rate seems weakly exogenous

[4] I.e. linear combination of the variables which converts the system from $I(2)$ to $I(0)$.

for long run parameters (p value 0.07). The joint hypothesis of the price equation (or aggregate demand) interpretation of the first vector and price and GDP homogeneity of the second vector is clearly accepted (see table 11.5).

Those non-stationary forces, which drive the system, are called *common trends*. Being non-stationary, they are stochastic trend components, which are common for the system. Matrix $\alpha_\perp$ determines these forces. The difficulties concerning identification, as in β, apply for common trends too. In the system, there are always $p - r$ common trends. In table 11.6 two common trends of the present system are presented as columns. Because no standard error can be computed, we do not have any information about the scale.

The exogeneity of the interest rate hypothesis is supported by the second common trend vector (table 11.6). The interest rate component of the vector has a very high coefficient (allthough we do not know the standard errors of the components). According to this interpretation, the interest rate could be determined outside the system and could also be a driving force of the system.

Table 11.5 **Testing structural hypothesis**

	2 linear restriction(s) on 2 β vectors LR test, $\chi^2_2 = 0.18$, p value 0.92	
	Eigenvalues, λ	
	0.262	0.139
	β matrix	
Variable	β_1	β_2
m	0.00000	1.00000
p	1.00000	−1.00000
y	−1.51898	−1.00000
I	−1.00000	0.73869
	α matrix	
Equation	β_1	β_2
Δm	−0.019	−0.113
Δp	−0.046	−0.009
Δy	0.054	0.221
ΔI	0.025	−0.039

Table 11.6 **Common trends**

	$\alpha_\perp$ (= alpha-orthogonal)	
Variable		
m	–69.0315	64.7510
p	–90.8619	–71.9328
y	–55.0419	12.3319
I	–82.6296	–103.3478

11.1.2 Real Balances

Following the price homogeneity (i.e. unit price elasticity) result of the previous chapter, one can impose the restriction on the data and continue the analysis with real money. The consequence of imposing price homogeneity on the data (computing $m - p$) is that the price homogeneity restriction is also imposed on the short run coefficients. Our tests have concerned only the long run. The coefficient of current prices does not exist in the model. Instead, the lagged coefficients of both money and prices are present. In the money equation, these coefficients are not of opposite sign and equal size. Thus, we could violate the model with a restriction that is not consistent with the data.

Table 11.7 gives test statistics for the existence of linear trend. We test jointly the cointegration rank and the presence of linear trend. The testing procedure is described in section 5.4. The results suggest that the cointegration rank is one and that there is a linear trend in the data[5]. The model would also be economically acceptable without a linear trend.

Table 11.7 **Testing for presence of linear trend in the M1 model**

	no linear trend			linear trend		
r	λ_i	trace test	critical value (90 %)	λ_i	trace test	critical value (90 %)
0	0.139	38.23	32.00	0.139	26.16	26.79
1	0.131	19.02	17.85	0.052	7.06	13.33
2	0.008	1.06	7.53	0.001	0.17	2.69

[5] To be precise, they suggest that there is linear trend but no cointegration in the data. Because the rejection is so close to the *asymptotic* borderline, we reject the null and continue testing.

Table 11.8 **Residual analysis**

Covariance/Correlation matrix			
Equation	$(m-p)$	y	I
$(m-p)$	0.0001	−0.0651	−0.1577
y	−0.0651	0.0001	0.0008
I	−0.1577	0.0008	0.0000

System variables statistics		
Variable	mean	variance
$(m-p)$	0.002625	0.000442
y	0.002289	0.004072
I	0.000007	0.000070

Residual diagnostics						
Eq.	B–P.Q(32)/28	ARCH(4)	Skew.	Kurt.	Norm. $\chi^2(2)$	R^2
$\Delta(m-p)$	1.3298	1.5427	0.1272	0.1878	0.5334	0.7860
Δy	1.0081	7.8631	−0.3662	0.8928	7.1129	0.9691
ΔI	1.4157	3.6902	−0.0997	2.3627	29.9836	0.3658

Autocorrelation $2/\sqrt{T}$ = 0.17678								
Equation	1 lag	2 lag	3 lag	4 lag	5 lag	6 lag	7 lag	8 lag
$(m-p)$	−0.142	−0.076	−0.027	0.065	−0.016	−0.188	−0.056	0.158
Δy	−0.002	−0.032	−0.135	−0.214	0.092	0.106	0.105	0.068
ΔI	−0.083	0.030	0.030	−0.024	−0.077	0.300	−0.227	−0.013

The residual diagnostics are good (table 11.8). No autocorrelation exists. Extra kurtosis disturbs the residuals in the interest rate change equation. According to simulations done by Eitrheim (1991) and Osterwald–Lenum (1991), the extra kurtosis has little effect on β and so it is not a serious problem. The stationary tests clearly reject stationarity for every variable (table 11.9).

Table 11.9 **Stationarity testing**

LR test, $\chi(p-r)$			
dgf	$(m-p)$	y	I
2	18.28	17.14	12.24
1	6.44	6.68	1.50

The test statistics for weak exogeneity (table 11.10) reveal that the interest rate is weakly exogenous for long run parameters within this context. The short run dynamics support that interpretation too. The dual presentation of the system — moving average (MA) presentation — reveals the same phenomenon (table 11.11). The residual variance of the ΔI equation is quite small compared to the variance of the other variables. Despite this, the impact of the cumulative impulses in the interest rate equation badly distributes the other equations. Their impact is the largest in every equation. This is a consequence of the fact that the interest rate is determined outside the system. Again, the common trends ($\alpha_\perp$) tell the same story. The coefficients of I in the common trends are very high, indicating that the interest rate could be the driving force of the system.

Table 11.10 **Testing weak-exogeneity w.r.t. α and β**

LR test, $\chi^2(r)$			
Dgf	$(m-p)$	y	I
1	7.93	5.45	1.98
2	9.23	11.65	4.32

Table 11.11 **MA representation and decomposition of trend**

Impact-matrix $C(1)$ for the MA representation			
Equation	$\Sigma\varepsilon_{(m-p),t}$	$\Sigma\varepsilon_{y,t}$	$\Sigma\varepsilon_{I,t}$
$(m-p)$	−1.4486	2.4761	11.9571
y	0.8979	0.4794	−1.0474
I	2.7072	−2.4042	−15.3210

Constant and linear trends in the levels		
Equation	constant	linear trend
$(m-p)$	0.35508	0.01291
y	−0.38590	0.00392
I	0.12401	−0.01097

$\alpha_\perp$ (= alfa-orthogonal)		
Variable		
$(m-p)$	42.7624	62.7751
y	67.7710	24.9185
I	92.1027	−100.3980

The differences between the exclusion test statistics and the weak exogeneity test statistics[6] are given below the βs and αs for interpretational purposes (table 11.12). According to the test statistics, the interest rate might be excluded from the cointegration relation. The coefficients of the vectors are somewhat confusing. According to the loadings, the error correction mechanism works: "excess money"[7] is adjusted in the money equation. But it is adjusted in the GDP change equation too. I think there is some danger that aggregate demand is mixed with money demand. Or, money, as measured by M1, leads GDP. This explanation sounds quite natural, as money should exist before transactions.

Table 11.12 **"Significance" of β and α**

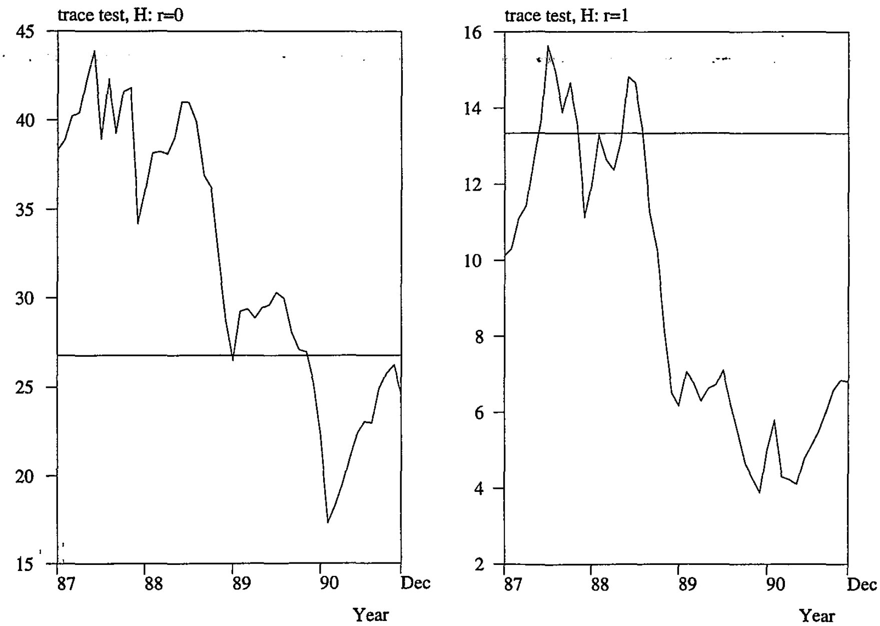
β matrix, (test stat. of exclusion, approx. $\chi^2(1)$)			
Variable	β_1	β_2	β_3
$(m - p)$	1.0000	–0.2065	1.0000
	(11.46)	(5.37)	(10.77)
y	–0.9317	0.2954	–0.2146
	(10.77)	(6.66)	(4.13)
I	0.8441	1.0000	1.2043
	(4.13)	(6.20)	(1.00)

α matrix (test stat. of weak exogeneity, approx. $\chi^2(1)$)			
Equation	β_1	β_2	β_3
$\Delta(m - p)$	–0.1423	–0.0385	–0.0042
	(7.93)	(1.30)	(5.45)
Δy	0.1575	–0.1208	–0.0017
	(5.45)	(6.20)	(1.98)
ΔI	–0.0499	–0.0541	0.0032
	(1.98)	(2.35)	(0.00)

[6] This means that the exclusion (zero restriction) test statistics have been computed for every variable with every possible value of r. Then the value of the test statistic is subtracted from the value of the test statistic for $r - 1$. This gives the new test statistics in parentheses under the coefficients of β. They are approximately χ^2_1 distributed. The test statistics below α are computed in the same way. In this case the hypothesis was weak exogeneity. They are also approximately χ^2_1 distributed. The problem, which can also be seen from this example, is that the computed test statistics are not independent and thus not very reliable. However, one can use them as a tool for interpreting cointegration relations and to "scale" the coefficients of the β vectors in some manner and get a rough picture of the properties of the data.

[7] The presence of excess money means that the amount of money in the economy is larger than the "desired" level of money — the level set by the cointegration relation.

Figure 11.1 **Recursive trace tests for $H_0 : r = 0$ and $H_0 : r \leq 1$**

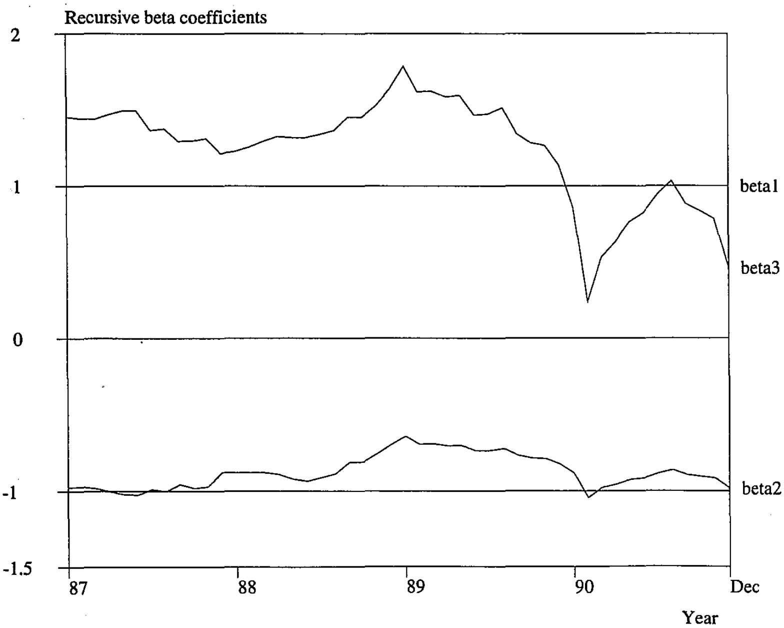


The recursive analysis reveals how the system is affected by the possible structural changes. The unstable parameters indicate the presence of the problem of omitted variables. The system is estimated recursively from 1987 to the end of 1990. The parameters of interest at this stage are those in the cointegration space[8]. Within the space, the money demand relation is what we are interested in. The exogenous shocks have been captured by dummies. The recursive trace tests are presented in figure 11.1. Both of them show that there is a drop in the value of the test statistics at the end of 1988. The dummy for 24-month taxfree time deposits was not sufficient to model the total disturbance from the shock. The shock affected the system. Another important and not totally modelled shock is the bank employees' strike in February 1990. The value of the trace tests drops in the strike period but recovers quickly thereafter. The trace test statistics for $H_0 : r = 1$ dropped below the critical value because of the strike shock. The small (zero) eigenvalues disturb the recursive trace test results[9] and thus cloud the picture given by the recursive analysis of the system.

[8] It cannot be over-emphasized that we are estimating only the cointegration space, *not* the individual vectors. See section 4.3.

[9] The trace test is computed from the $p - r$ smallest eigenvalues.

Figure 11.2 **Recursive β_1, money demand relation**



The recursive β is not affected by the first shock as were the trace tests (figure 11.2). The scale elasticity (beta2) is very stable during the recursive period. The most dramatic changes are in the interest elasticity (beta3). Before the strike shock it is much greater than unity, but it drops during the shock period. This might involve a measurement problem. During the strike, the interest rate was frozen at its pre-strike level. The amount of money increased very much but agents were not able to adjust their balances. Recursive estimates of the loadings, α, support this interpretation (figure 11.3). The adjustment coefficient in the money change equation (alpha1) dropped[10] during the strike. Otherwise it has been fairly stable.

There are several structural hypotheses to be tested. To analyze money demand in a single equation framework, as in the beginning of the study, GDP and the interest rate should be weakly exogenous for long run parameters. The data does not support this hypothesis; it is clearly rejected (p value < 0.001). But weak exogeneity of the interest rate is accepted (p value 0.16). The velocity hypothesis is also accepted, allthough it is compatible with the aggregate demand

[10] This *a priori* knowledge is difficult into incorporate to the statistical model. The "inseparability" of $\alpha\beta'$ makes it impossible to use the technique of $\alpha(1 - \text{STRIKE}_t)\beta'$, where STRIKE could be a dummy which is unity during the strike period.

interpretation (p value 0.11). The scale homogeneity is also accepted (p value 0.29). The joint hypothesis of scale homogeneity and weak exogeneity of interest rate is accepted (p value 0.27). The hypothesis H_{FS} (table 10.1) is rejected (p value < 0.001); H_{FSJ} is also clearly rejected (p value 0.02). Mikkola's (1989) error correction model H_{FM} is also clearly rejected (p value < 0.001). This raises some doubts about Engle and Granger's two-step method. H_{FJJ}, the velocity hypothesis, is clearly accepted. The interpretation problem remains.

The recursive estimation shows that the interest rate semi-elasticity dropped in 1990, most likely because of the strike shock. The elasticity was 1.5 before 1990. When tested with the whole data set the value is accepted (p value 0.16). The estimated space[11] is presented in table 11.13. The value of the joint test of the restricted β with weak exogeneity of interest rates is on the borderline (p value 0.05); with weak exogeneity of both GDP and the interest rate, the null is rejected (p value 0.03). The restricted equilibrium error (ECM), $\beta' z_t$, is presented in figure 11.4. The right-hand graph, which gives the equilibrium error conditioned on the short-run dynamics, seasonal and other dummies and trend, $\beta' R_{1t}$, clearly indicates an excess of money during the bank employee strike. The left-hand graph is naturally non-stationary, because it has seasonal variation. The right-hand figure, however, is more revealing because the short run variation has been removed.

Table 11.13 **Testing the structural hypothesis**

1 linear restriction on 1 β vector	
LR test, $\chi^2(1) = 1.94$, p value 0.16	
Eigenvalues, λ	
	.1255
β matrix	
Variable	β_1
$(m - p)$	1.0000
y	−.8777
I	1.5000
α matrix	
Equation	β_1
$\Delta(m - p)$	-.1179
Δy	.0863
ΔI	−.0525

[11] When the estimation period is extended to the full sample period, the scale elasticity is 0.856.

Figure 11.3 **Recursive α_1, loadings of money demand relation**

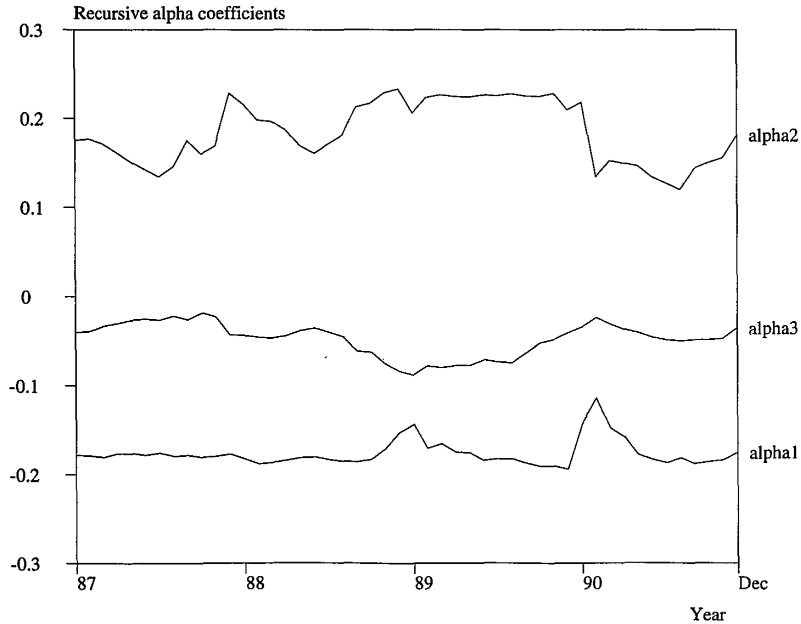
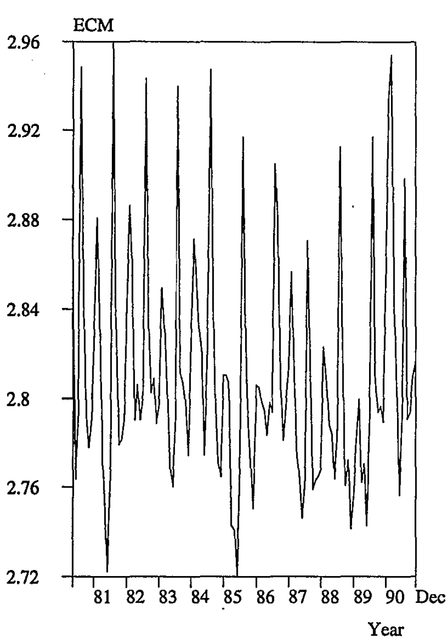
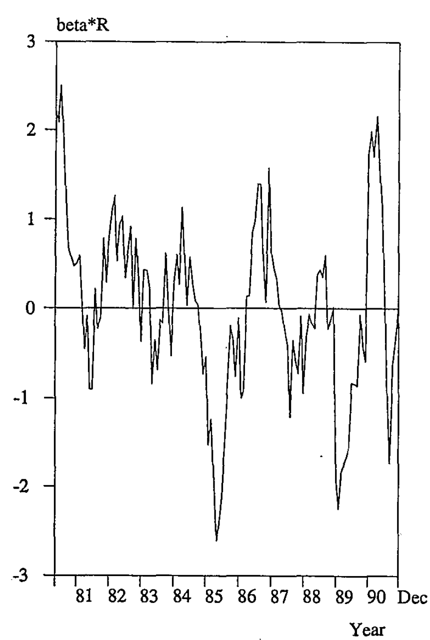


Figure 11.4 **"Excess money" (equilibrium error) and conditional equilibrium error**





11.2 Testing the Rational Expectations Hypothesis

To test the rational expectations hypothesis I have estimated directly the Euler equation (7.2). To estimate the expectations term, $E_t\Delta m_{t+1}$, I used lagged Δm_t and e_t as instruments and to correct the standard errors of estimates, I used the correction recommended by Hansen (1982) and Hansen and Hodrick (1980).

The lag length of the instruments is 12. The constant, seasonal dummies and shock dummies presented in the beginning of this part, are used in estimating both the expectation and the Euler equation. The estimated results are reported in table 11.14.

Table 11.14 **Direct estimation of Euler equations**

Constant	δ	λ_1
–0.1935	–0.1035	–0.06764
(0.06776)	(0.09604)	(0.02251)
	Presetting $\delta = 0.95$	
–0.4435		–0.1523
(0.03664)		(0.01274)

Autocorrelation-corrected standard errors in parenthesis.

The estimation results are poor. The discount factor δ, which should lie between 0 and 1 is negative. It does not differ significantly from zero. The adjustment term has a wrong sign and it differs significantly from zero. The discount factor is set at 0.95, which means the interest rate is about five per cent. That improves the result, but the adjustment term still has a wrong sign. The rational expectations hypothesis is not supported by the data.

An alternative approach is that of Campbell and Shiller (1987), following the modifications proposed by Engsted and Haldrup (1991). The test method is presented in chapter 7. First, we use the VAR model to get matrix A in equation (7.8). The adjustment coefficients can be obtained from Johansen's VAR model when GDP and the interest rate are weakly exogenous, so $(\mu_1 - 1) = -0.1282$. The discount factor gets the value $\delta = 0.95$, as before. The constant and the shock and seasonal dummies are included in both the VAR model and the model in which we test the hypothesis $\Delta m_t = \Delta m_t^*$. So the model is

$$\Delta m_t = \omega_0 + \omega_1 \Delta m_t^* + D_t + \varepsilon_t. \tag{11.1}$$

We use the F test for the hypothesis $H_0 : \omega_0 = \omega_1 = 0$.

The estimation result for model (11.1) is presented in table 11.15. The F test of the hypothesis rejects the null of zero coefficients (p value 0.04). Thus, the Campbell and Shiller (1987) approach provides only minor support for the rational expectations hypothesis and for the forward-looking loss function approach.

Table 11.15 **Test of rational expections**

Variable	coefficient	standard error	t-statistic
Constant	0.190	0.0839	2.261
Δm^*	0.360	0.161	2.238
IBR	0.333	0.492	0.676
UIP	0.0240	0.04870	0.494
DSPEC	−0.0194	0.00863	−2.255
TAX	−0.0343	0.01170	−2.920
STRIKE	0.0571	0.00830	6.886
FREETAX	0.0638	0.01170	5.458
cs1	−0.0510	0.00519	−9.827
cs2	−0.0288	0.00572	−5.034
cs3	−0.0510	0.00517	−9.869
cs4	−0.0361	0.00521	−6.942
cs5	−0.0387	0.00519	−7.447
cs6	−0.0167	0.00533	−3.140
cs7	−0.0475	0.00512	−9.273
cs8	−0.0213	0.00932	−2.289
cs9	−0.0546	0.00592	−9.233
cs10	−0.0444	0.00511	−8.688
cs11	−0.0446	0.00505	−8.825

11.3 Structural Model for Real Balances

The basis for the structural modelling is the unrestricted VAR model estimated for the long run analysis. We now continue the analysis with fixed, estimated $\hat{\beta}$. The model is presented in equation (5.4) on page 34, but now β is considered to be fixed.

$$\Delta z_t = \alpha\beta' z_{t-1} + \Gamma_1 \Delta z_{t-1} + ... + \Gamma_{k-1} \Delta z_{t-k+1} + \mu + \Psi D_t + \varepsilon_t, \quad t = 1,...,T.$$

Hereafter, β = [1 − 1 −0.856 1.5]′ is called the error correction term *ECM*. Interest rate semielasticity is set at 1.5.

$$ECM_t = (m-p)_t - 0.856 y_t + 1.5 I_t \tag{11.2}$$

The first step in the structural modelling is to condition some of the variables.

One result from the long run analysis is that the interest rate is a driving force (common trend) of the system and thus determined outside the system. The natural way to reduce the dimensionality of the system is to condition on the interest rate. That does not affect the estimation of the system if the interest rate is weakly exogenous for the parameters of interest. The parameters of interest at this stage of the study are the short run and long run structural parameters of the model. The hypothesis for weak exogeneity of interest rate for the short run, and the long run parameters are tested using an F test.[12] The test result is convincing. The null is clearly not rejected (p value 0.293). Thus the following analysis can be conditioned on current and past interest rates.

The conditional model was estimated recursively using OLS. The forecasting analysis indicated failure in forecasting the development of money change at the end of 1990 and beginning of 1991. An extra dummy variable (WITHDTAX) was added to capture the turbulence at the end of 1990 and beginning of 1991. At that time, the taxation of bank accounts changed so that the withholding tax became an optional form of taxation. The tax rate was 15 per cent. The estimation period is extended to September 1991.

The structural model for narrow money, estimated by FIML, is quite simple. The standard errors are given in parenthesis below the point estimates.

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[12] What we really test is whether all the lagged differences of real money and GDP indicator and the α coefficient of the *ECM* in the interest rate equation are zero. This joint test can be an F test. The idea is formalized on page xx.

$$\Delta(m-p)_t = \underset{(.11861)}{-.357}\ \Delta I_t \ \underset{(.01183)}{-.033}\ TAX_t \ \underset{(.00843)}{+.056}\ \mathrm{STRIKE}_t$$
$$\underset{(.03422)}{-.140}\ ECM_{t-1} \ \underset{(.00846)}{+.098}\ \mathrm{WITHDTAX}_t \qquad (11.3)$$

$$\sigma = .011189$$

$$\Delta y_t = \underset{(.04121)}{.106}\ ECM_{t-1} \ \underset{(.13024)}{-.391}\ \Delta I_{t-1} \qquad (11.4)$$
$$\underset{(.08224)}{-.459}\ \Delta y_{t-1} \ \underset{(.08049)}{-.233}\ \Delta y_{t-2}$$
$$\sigma = .013162$$

The model is essentially the reduced form model with restricted parameters. The reason for the lack of a simultaneous effect is probably that a month is too short a period for GDP to reflect changes in the money stock. The construction of the error correction term is presented in equation (11.2). The model encompasses[13] the VAR. The LR test for over-identifying restrictions has a p value of 0.90. The seasonal coefficients and constant, with standard errors, are presented in table 11.16.

Table 11.16 **Constant and structural seasonal coefficients for M1**

Variable	$\Delta(m-p)$		y	
	coeff.	σ(S.E.)	coeff.	σ(S.E.)
Constant	.4409	0.0990	–.3042	.1190
cs1	–.0484	0.005336	–.0616	.005855
cs2	–.0258	0.005318	–.0241	.007230
cs3	–.0391	0.005208	.0161	.007251
cs4	–.0327	0.004979	.0298	.006420
cs5	–.0318	0.004903	.0370	.006263
cs6	–.0143	0.004988	.005169	.006053
cs7	–.0411	0.004863	–.1551	.005805
cs8	–.0180	0.006587	0.0184	.0135
cs9	–.0365	0.005054	0.0474	.0132
cs10	–.0372	0.004960	0.0455	.0119
cs11	–.0393	0.004962	0.0101	.006394

[13] According to Hendry (1989): "... a test of over-identifying restrictions is equivalent to a test of the restricted reduced form parsimoniously encompassing the unrestricted reduced form. If this hypothesis is not rejected, the model constitutes a valid reduction of the system and as such is more interpretable, more robust and (being lower dimensional) allows more powerful tests for mis-specification." *Encompassing* here means that the smaller model explains all features of the data that are explained by the VAR model.

The correlation matrix of the reduced form residuals shows that there is hardly any simultaneity in the residuals either (table 11.17). The residuals (see figure 11.15) easily pass the normality test and are not autocorrelated.

Table 11.17 **Residual analysis**

Reduced form residual correlation matrix		
Equation	Δ(m−p)	Δy
$\Delta(m-p)$	1.0000	
Δy	.0629	1.0000

		Residual diagnostics			
Equation	Mean	Std.dev.	Skew.	kurt.	Norm. $\chi^2(2)$
$\Delta(m-p)$	−.000001	1.003670	−.031930	.106530	.085
Δy	.000001	1.003670	−.171257	.576793	2.494

				Autocorrelation $2/\sqrt{T} = 0.17678$				
Equation	1 lag	2 lag	3 lag	4 lag	5 lag	6 lag	7 lag	8 lag
$\Delta(m-p)$	−.0970	.0094	−.1593	.1243	−.0805	.0000	.0003	.1610
Δy	.0183	.0075	−.0735	−.0916	.1007	.1872	.0831	.0930

The nice picture of residuals is complemented by the nice fit. Figure 11.6, together with the structural equations, reveal some key features of the monetary transmission mechanism in Finland. First of all, the demand for money does not depend on any lagged differences. So, money changes adjust fully, not partially, to changes in the explanatory factors and in the long run money demand relationship. The interest rate changes have an immediate effect on the demand for money; other effects come with a lag. The demand for money is affected by tax changes involving savings and time deposit accounts in banks. Thus the model is not very suitable for analysing *tax* policy, because taxes are not separately modelled. The stable, desired, level of money, henceforth referred to as excess money, has a marked effect on the change in money. The short run interest rate semi-elasticities are much smaller than the long run elasticities; both have negative signs. The reason for this might be that in the short run agents do not have possibilities or do not care to adjust their balances very much to changes in the interest rate. The long run elasticity is much higher, because in the long run money balances are adjusted completely, after changes are considered to be permanent.

Figure 11.5 **Residuals**

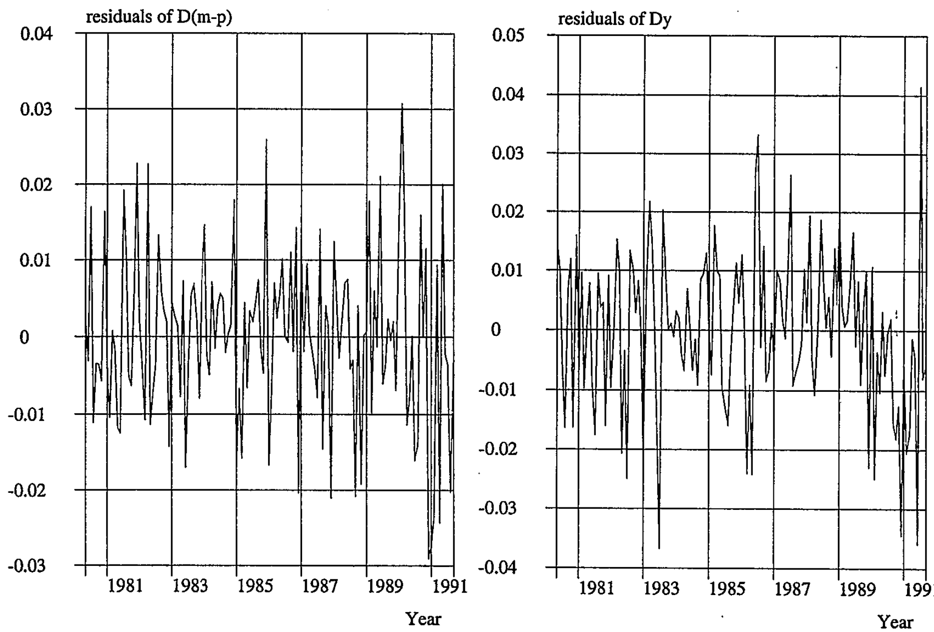
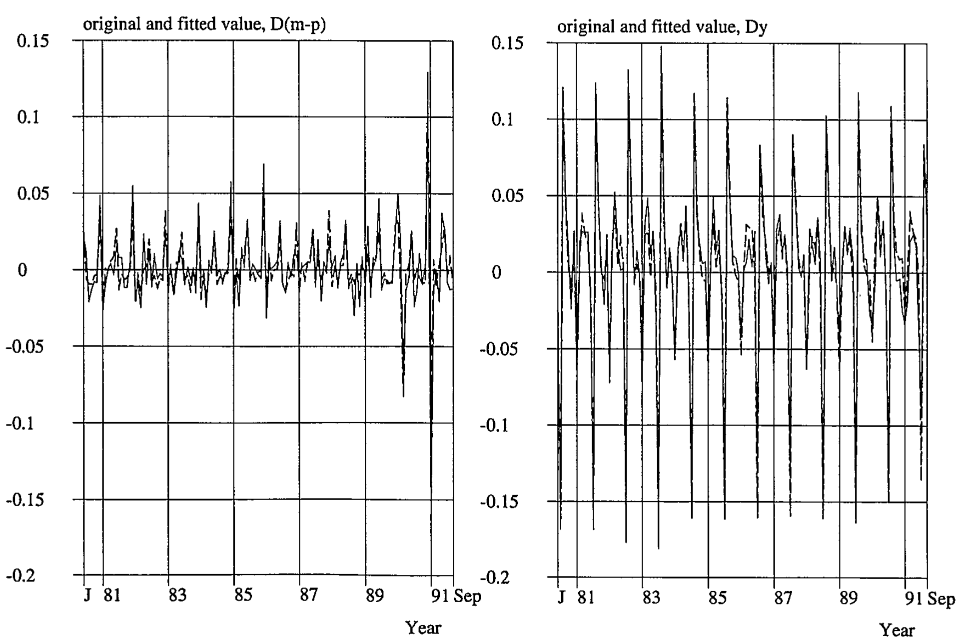


Figure 11.6 **Current and fitted values**



The GDP change is mostly affected by its own lagged values. Interest rate changes have some effect too, but not immediately. They occur within a month. One should note that the model for GDP changes is not satisfactory in the economic sense; the information set was originally chosen for purposes of analysing money demand. However, the model is data consistent; it describes the systematic variation present in the data. A great deal of the variation in the GDP indicator is due to its seasonal pattern.

One of the most interesting features of the model is that excess money seems to have a real effect. This seems quite natural, as transactions cannot be carried out without money. Policy affecting the demand for money will have positive effects on GDP growth. During the model period – the period of fixed exchange rates and free capital movements – money was endogenous. Money demand was controlled only by changes in capital gains taxation.

An interesting outcome of the nominal analysis was that excess money did not have any inflation effect. This result was not verified by any test, but is indicated in table 11.5 on page 81. The α coefficient of the second cointegration vector in the inflation (Δp) equation is negative (–0.009), implying that excess money would lower the price level.

12 Models for Broad Money

Broad money (M2) consists of M1 and time deposits. The main trends in M2 were described in chapter 8. The estimation period is January 1980 to September 1991. In some cases the last 12 observations are saved for forecasting analysis.

12.1 Long Run Relations

The long run analysis follows the same strategy as the analysis of M1. The larger size of the system complicates the analysis. Several cointegration relations are found. $I(2)$-ness causes some difficulties. The choice of variables is not obvious, so several specifications are tried.

12.1.1 Nominal Balances

The model for M2 is like the model for M1 augmented with credit stock, which measures the financial deregulation of the eighties. It also measures the wealth effect of money demand by representing housing price developments[1]. The predetermined variables, constant and shock and seasonal dummies are as in the unrestricted model of nominal M1. The lag length, k, of the system is four. The vector of system variables z_t includes the following:

m logarithm of monetary aggregate M2,
p logarithm of consumer price index, 1985 = 100,
y logarithm of GDP volume index, 1980 = 100,
I three month money market rate divided by 100,
cr logarithm of banks' real lending (credit), deflated by the CPI.

Some other specifications are presented in the last section of the chapter. We first estimated the unrestricted VAR as in the previous chapter.

[1] See Takala and Pere (1992) describing the co-movement of banks' real lending and real house prices.

The residual performance of the system is not quite satisfactory (table 1.2). Some non-normality in the form of excess kurtosis exists in almost every equation. Fortunately, autocorrelation is not a problem.

Table 12.1 **Residual analysis**

Residual diagnostics						
Equation	B–P.Q(34)/30	ARCH(4)	Skewn.	Kurto.	Norm. $\chi^2(2)$	R^2
Δm	1.1092	1.0519	0.3326	1.4088	13.9558	0.8750
Δp	0.9447	25.6718	0.0802	2.1180	25.9425	0.6835
Δy	1.0437	6.7152	-0.0553	0.9167	4.9021	0.9657
ΔI	1.6033	4.6993	0.1269	2.4112	33.8009	0.3916
Δcr	1.4574	7.1599	-0.1067	0.2429	0.6012	0.5587

Autocorrelation $2/\sqrt{T}=0.17025$								
Equation	1 lag	2 lag	3 lag	4 lag	5 lag	6 lag	7 lag	8 lag
Δm	-0.100	-0.157	0.050	0.049	-0.013	-0.079	-0.105	0.103
Δp	-0.052	0.081	-0.071	-0.129	0.052	-0.106	0.022	-0.152
Δy	0.007	-0.052	-0.149	-0.154	-0.007	0.075	0.062	0.028
ΔI	-0.060	0.014	0.020	-0.055	-0.122	0.273	-0.209	0.025
Δcr	0.008	0.020	-0.059	0.031	0.098	-0.078	-0.070	-0.117

To perform the $I(2)$ analysis, we first specify the cointegration rank. According to the sequence of trace tests (next-to-last column in table of test statistics 12.2) the cointegration rank is three, i.e. $r = 3$. Using the row where $r = 3$ and starting the testing from the leftmost non-zero test statistics, we test the hypothesis $s = 0$, which is rejected. The next hypothesis, $s \leq 1$, is not rejected. So, the system has one ($p - r - s = 5 - 3 - 1 = 1$) $I(2)$ common trend (table 12.2). The $I(2)$ system has two linear combinations which bring the $I(2)$ variables down to I(0). We call them stationary linear combinations. The second vector of the stationary linear combination of the system variables can be interpreted as the money demand. The first one could be a sort of "aggregate demand". The driving $I(2)$ common trend is dominated by real money and credit describing the driving force of the banking sector[2]. The common trend has both sides of the banks' balance sheet in real terms. It could be described by the equation $m - p - cr =$ stochastic trend.

[2] Overly strong conclusions should not be drawn, because we do not know the standard errors of the estimates.

Table 12.2 **Testing $I(2)$-ness**

p–r	r			$T(r,s)$			$T(r)$	95 %
5	0	268.3107	168.7329	95.2021	51.8961	19.1320	112.5142	68.524
4	1		169.7178	95.3519	33.5066	.5397	71.4339	47.210
3	2			70.7057	22.7851	1.0042	38.1583	29.680
2	3				39.3876	.6107	13.7734	15.410
1	4					.8189	1.2538	3.7620
95%		68.524	47.210	29.680	15.410	3.7620		

Stationary linear combinations		
m	1.00	−1.00
p	−.55	1.05
y	−.74	1.03
I	−.26	−0.87
cr	−.69	0.35

	$\alpha_\perp^2$
m	.015
p	−.011
y	−.002
I	−.008
cr	−.016

One can do the normal $I(1)$ analysis too. Then one obtains different types of "cointegration vectors", by definition, than in the $I(2)$ analysis. When performing the normal $I(1)$ analysis in the presence of $I(2)$-ness, the cointegration relations represent linear combinations which bring the $I(2)$ variables down to $I(1)$. With this exception, the interpretations are as in the $I(1)$ analysis. The normal $I(1)$ results are presented in table 12.3. The interpretation of the cointegration vectors is not very clear. The third cointegration vector has the properties of the demand for money. The second one has the "aggregate demand" properties. For more carefull analysis, the cointegration space should be restricted. Price homogeneity seems to be present in two out of three vectors. One interesting point is that the interest rate could be excluded from the system.

When comparing the estimated linear combinations in the $I(2)$ and $I(1)$ analyses, the common feature is that price homogeneity seems to be present in those vectors which can be interpreted as demand for money. We do not have the statistical tools to explicitely test such a structural hypothesis in the $I(2)$ system. Despite the lack of exact statistical evidence, we continue the analysis of M2 in real terms.

Table 12.3 **"Significance" of β and α**

	β matrix (test stat. of exclusion, approx. $\chi^2(1)$)				
Variable	β_1	β_2	β_3	β_4	β_5
m	–.3161	–1.4117	1.0000	1.0000	1.0000
	(.14)	(3.61)	(10.65)	(6.41)	(1.85)
p	1.0000	.8203	–.9708	-1.2451	-1.2270
	(1.85)	(1.82)	(9.35)	(4.68)	(.72)
y	–.6997	1.0000	-1.1991	2.2619	–.2651
	(.72)	(1.73)	(8.76)	(11.00)	(.99)
I	.7444	.3926	1.0539	3.3894	–.9176
	(.99)	(.31)	(5.26)	(9.21)	(.11)
cr	.1611	.9700	–.3359	-1.0952	–.4166
	(.11)	(6.34)	(7.53)	(8.72)	(1.00)

	α matrix (test stat. of weak exogeneity, approx. $\chi^2(1)$)				
Equation	β1	β_2	β_3	β_4	β_5
Δm	–.0444	–.0617	–.0363	–.0028	–.0164
	(2.73)	(7.48)	(9.24)	(.47)	(3.73)
Δp	–.0317	.0372	.0022	–.0052	–.0013
	(3.73)	(8.88)	(9.99)	(10.96)	(1.61)
Δy	–.0668	–.0785	.2632	–.0162	.0207
	(1.61)	(1.40)	(11.14)	(7.98)	(.47)
ΔI	.0158	–.0086	–.0908	–.0332	.0147
	(.47)	(.07)	(3.35)	(8.24)	(.48)
Δcr	–.0187	–.0697	–.0279	.0257	.0105
	(.48)	(7.16)	(2.09)	(8.80)	(.00)

12.1.2 Real Balances

The model for real M2 is further augmented with the inflation rate, which is assumed to represent the own-yield of fixed property. The tests for linear trend and cointegration rank reveal that there is no linear trend in the model and the cointegration rank is two (table 12.4).

The residuals are fine (table 12.5) except for non-normality in the interest rate equation and conditional heteroscedasticity in the GDP equation. Also, the lower multiple correlations could indicate that the interest and inflation rates are determined outside the system.

Table 12.4 **Testing for presence of linear trend in the model for M2**

	no linear trend			linear trend		
H_0	λ_i	trace test	critical value (90 %)	λ_i	trace test	critical value (90 %)
$r = 0$	0.226	86.33	71.86	0.216	80.04	64.84
$r \leq 1$	0.169	50.81	29.64	0.164	46.45	43.95
$r \leq 2$	0.105	25.15	32.00	0.095	21.68	26.79
$r \leq 3$	0.067	9.92	17.85	0.055	7.94	13.32
$r \leq 4$	0.002	0.29	7.53	0.001	0.18	2.68

Table 12.5 **Residual analysis**

Residual diagnostics						
Eq.	B–P.Q(34)/30	ARCH(4)	Skewn.	Kurt.	Norm. $\chi^2(2)$	R^2
$\Delta(m-p)$	0.9272	1.4714	0.1781	0.9077	5.4670	0.8487
y	0.9226	9.0388	-0.0671	0.8220	3.9887	0.9666
ΔI	1.5767	5.3032	0.1309	2.1069	25.9187	0.3931
$\Delta\Delta_{12}P$	1.6722	17.0210	-0.3414	0.6375	5.016	0.17596
Δcr	1.7385	7.2299	-0.1280	0.2457	0.723	0.56438

Autocorrelation $2/\sqrt{T} = 0.17025$								
Equation	1 lag	2 lag	3 lag	4 lag	5 lag	6 lag	7 lag	8 lag
$\Delta(m-p)$	-0.077	-0.118	0.019	0.099	0.036	-0.101	-0.041	0.113
Δy	0.016	-0.050	-0.149	-0.149	-0.020	0.081	0.022	0.051
ΔI	-0.049	0.013	0.012	-0.072	-0.101	0.267	-0.165	0.003
$\Delta\Delta_{12}P$	-0.036	0.023	-0.059	0.030	0.083	-0.035	0.040	-0.205
Δcr	0.007	0.048	-0.076	0.042	0.096	-0.072	-0.055	-0.162

Table 12.6 **Stationarity testing**

	LR test, $\chi^2(p-r)$				
dgf	$(m-p)$	y	I	$\Delta_{12}P$	cr
4	.00	29.50	18.67	31.11	30.37
3	20.73	20.57	10.72	22.34	20.69
2	11.35	11.30	4.50	14.19	11.36
1	7.33	7.39	2.92	8.67	7.29

Table 12.7 **Testing exclusion**

	LR test, $\chi^2(r)$				
dgf	$(m-p)$	y	I	$\Delta_{12}P$	cr
1	3.83	.43	1.20	.38	6.22
2	11.84	9.16	4.55	.97	10.76
3	15.94	13.48	8.43	1.57	16.04
4	20.46	20.44	17.62	8.69	20.40

Table 12.8 **Testing weak-exogeneity with respect to α and β**

	LR test, $\chi^2(r)$				
Dgf	$(m-p)$	y	I	$\Delta_{12}P$	cr
1	7.04	2.19	.02	1.72	6.38
2	12.52	12.62	2.34	2.12	9.10
3	12.73	16.85	5.35	3.05	14.63
4	21.79	23.65	12.14	5.69	23.03

According to the stationarity tests, none of the variables is stationary (table 12.6). Only the interest rate has some hint of stationarity. The interest and inflation rates are also on the border of being excluded from the system (table 12.7). We do not exclude them. The same two variables are also indicated for weak exogeneity (table 12.8). The joint test for weak exogeneity of the interest and the inflation rate with respect to long run parameters (table 12.9) is not rejected (p value 0.25).

The second cointegration vector has the properties of money demand; the first one could represent the cointegration of money and credit. The joint test of the money-credit relation and the GDP homogeneity of money demand (table 12.10) is not rejected (p value 0.22). The stationarity of the velocity of money is clearly rejected.

Table 12.9 **Testing the structural hypothesis**

	2 linear restrictions on α, LR test, $\chi^2(4) = 5.44$, p value 0.25	
	Eigenvalues, λ	
	.2171	.1472
	β matrix	
Variable	β_1	β_2
m	1.0000	1.0000
y	−.0854	−1.4017
I	−.6567	.4805
$\Delta_{12}P$	−.2688	.1681
cr	−.6836	−.2853
Constant	−1.8852	1.3067
	α matrix	
Equation	β_1	β_2
$\Delta(m - p)$	.1101	−.0256
Δy	.0927	.2527
ΔI	.0000	.0000
$\Delta\Delta_{12}$	.0000	.0000
Δcr	.0970	−.0460

Table 12.10 **Testing the structural hypothesis**

	4 linear restrictions on 2 β vectors, LR test, $\chi^2(2) = 3.06$, p value 0.22	
	Eigenvalues, λ	
	.2096	.1697
	β matrix	
Variable	β_1	β_2
$(m-p)$	−1.4584	1.0000
y	.0000	−1.0000
I	.0000	.7261
$\Delta_{12}P$	.0000	.1735
cr	1.0000	−.3883
Constant	3.3377	.1698
	α matrix	
Equation	β_1	β_2
$\Delta(m-p)$	−.0830	−.0619
Δy	−.0547	.3234
ΔI	.0105	−.0905
$\Delta\Delta_{12}P$	−.0103	.0206
Δcr	−.0936	−.0553

The impact matrix $C(1)$ in the moving average representation reveals the exogeneity feature of the interest and inflation rates (table 12.11). The interest rate cumulative residuals dominate in the money, interest, price and credit equations and, along with the cumulative price residuals, in the GDP equation also. The above reasoning is valid only if the variances of the impulses of interest rate and inflation are about the same size as the variance of the other impulses. So the coefficients should be scaled according to their variances. Correlation between residuals would also disturb the analysis.

The cumulative price residuals play an important role in the interest and GDP equations. The third common trend vector is dominated by inflation. The nominal growth could be a driving force of the system. The real money — credit relation (the second common trend vector) also seems to be a candidate for a common trend.

Table 12.11 **MA representation and decomposition of trend**

The impact matrix $C(1)$ for the MA representation					
Equation	$\sum \varepsilon_{(m-p),t}$	$\sum \varepsilon_{y,t}$	$\sum \varepsilon_{I,t}$	$\sum \varepsilon_{\Delta_{12}P,t}$	$\sum \varepsilon_{cr,t}$
$(m-p)$	1.8147	−.3095	.0052	−1.365	−.5782
y	−1.4535	1.0556	2.9383	−4.3718	1.7364
I	−1.6285	.9315	3.1303	−2.4490	1.5510
$\Delta_{12}P$	−.1193	−.2049	−1.0564	−.8090	.7722
cr	5.6252	−2.0425	−7.9508	3.2510	−5.5758

$\alpha_{\perp}$ (= alpha-orthogonal)			
Equation			
$(m-p)$	43.3190	−118.4148	3.0945
y	18.8300	5.6765	33.7947
I	90.3521	64.9892	69.9447
$\Delta_{12}P$	71.8191	62.9104	−184.8210
cr	−115.3965	135.5560	.9610

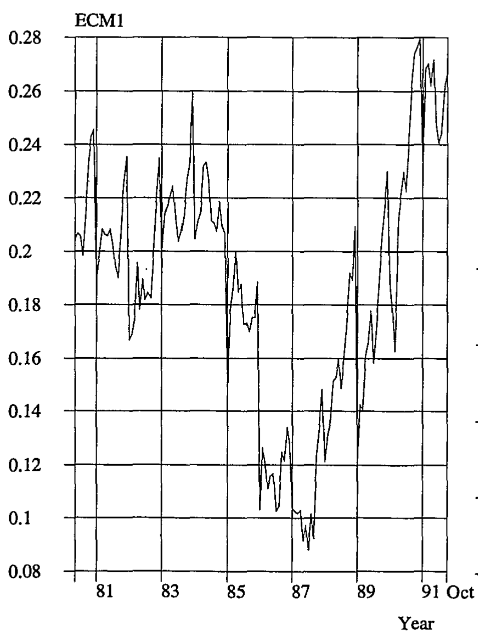
In the restricted β-space (table 12.10 and figure12.1[3]) the money demand vector differs from historical estimates of the scale elasticity of money demand. The usual scale elasticity has been about 1.6 in Finland, but the financial deregulation correction (credit expansion) captures some of the effect of the scale variable. The interest rate

[3] As in the analysis of M1, one should present here the relation βR_{1t}, which represents the cointegration relation corrected by short run dynamics and deterministic variables — such as constant, seasonal dummies, shock dummies. However, in spite of the non-stationary seasonal variation, these figures are easier to interpret.

elasticity has the expected size, slightly less than one. The first cointegration vector shows the cointegration relation of both sides of the bank's balance sheet. The other hypotheses derived from the former studies are clearly rejected. The early period for the first error term (left side of figure 12.1) describes tight credit policy before the deregulation of central bank financing.

The recursive trace tests show that the special taxfree 24-month time deposit accounts affected the β-space. The estimates, however recovered from the shock quite rapidly (figure 12.2). The recursive trace test is disturbed by the $p - r$ eigenvalues, which are in fact zero but are computed and used in the test statistic. So, the unstable picture given by the recursive trace test is not in fact alarming because it only reflects the eigenvalue problem. Thus, the recursive analysis cannot make effective use of the recursive trace tests. The loadings are estimated recursively for fixed β. So β is not estimated recursively. The recursive estimation of the loadings in the money equation reveals the diminishing absolute value of the loadings (figure 12.3). This increasing exogeneity feature could be explained by tax incentives for time deposits, which are not modelled here. The backward recursion seems quite unstable too (figure 12.4). Therefore, there is a danger that the M2 system is not stable enough for forecasting purposes.

Figure 12.1 **The equilibrium errors of the restricted cointegration relations**



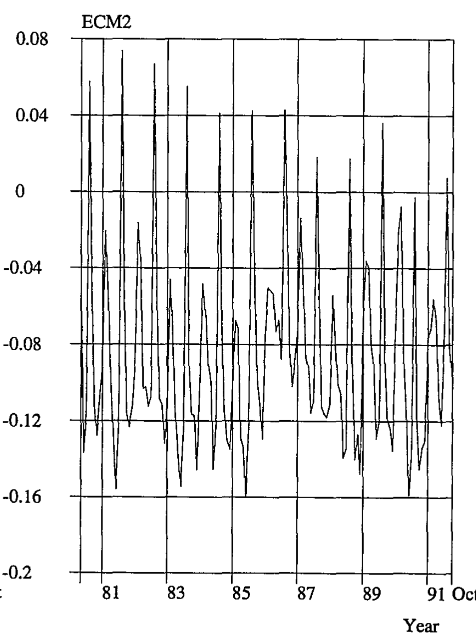


Figure 12.2 **Recursive trace tests**

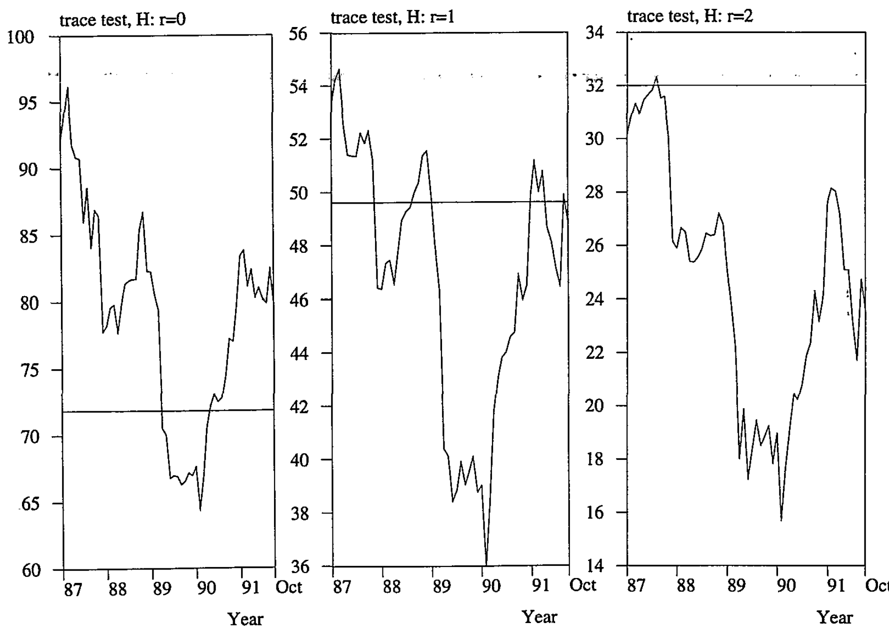


Figure 12.3 **Recursive estimation of loadings in the money equation**

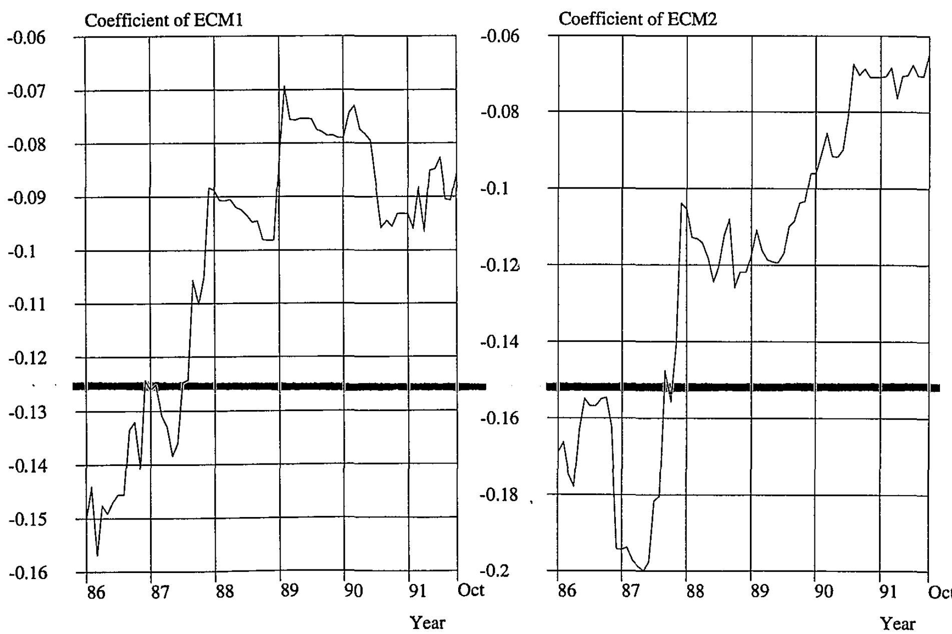
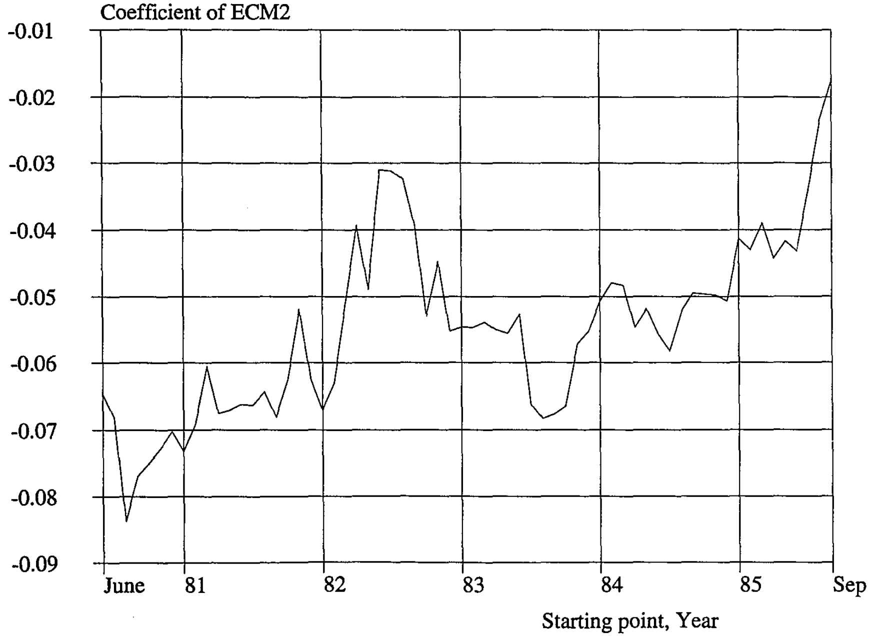


Figure 12.4 **Backwards recursive estimation of loadings in the money equation**



12.1.3 Some Alternative Models

The choice of model is not always easy. In practice, one cannot rely only on information criteria. Economic interpretability is an important criterion. Usually, one tends to choose a model that satisfies *a priori* expectations. Unfortunately, these expectations are hardly ever formulated explicitly and tested properly.

The desired, though unstated, property of the model presented above is that the scale elasticity should be close to unity. That is the reason for including credit in the equation. So it is quite natural that one of the cointegration vectors reflects both sides of the banks' combined balance sheet.

According to the results of Juselius (1991b and 1991c), scale homogeneity is an achievable property. However, in the earlier studies of Finnish M2, scale elasticity has been around 1.6. The money demand studies of Juselius also suggest that the linear trend could be included in the model to capture the unmodelled variables in the aggregate demand cointegration vector. Some models which satisfy these conditions and fit the full sample data well are presented in this section.

Model excluding credits

The model with linear trend in the cointegration relation and no credit variable is estimated with four lags and the same set of non-stochastic variables as above. The estimation period is January 1980 to October 1991. The cointegration rank is tested to be two. The estimated unrestricted cointegration space is presented in table 12.12.

Table 12.12 **Cointegration space, model with linear trend in cointegration space**

	Eigenvalues, λ	
	.2042	.1487
	β matrix	
Variable	β_1	β_2
$(m - p)$	−1.8010	1.0000
y	1.0000	−1.2437
I	3.8794	1.0225
$\Delta_{12}^2 P$	2.9008	0.0948
Trend	0.0100	−0.0027

	α matrix	
Equation	β_1	β_2
$\Delta(m - p)$	−.0184	−.0808
Δy	−.0370	.1236
ΔI	−.0088	−.0419
$\Delta\Delta_{12}P$	−.0218	.0319

The second cointegration vector has the properties of money demand. The linear trend does not differ from zero (see table 12.13). The first cointegration vector has the main properties of aggregate demand, where the linear trend represents technological change and missing factors. Inflation has a diminishing effect on aggregate demand, which does not correspond with intuition. The loading matrix can be interpretated as above. One should note that only the cointegration space is well defined at this stage.

Inflation could be excluded from the cointegration space. The p value for the exclusion test is 0.26. Serious problems arise, because the interest rate is close to trend stationary: the p value for the null of trend stationarity is 0.08. To identify the money demand relation in the cointegration space, the interest rate elasticity is restricted to unity and linear trend is excluded. The former restriction has no *a priori* justification and is thus arbitrary. The restricted cointegration space and the loadings are presented in table 12.13. Unfortunately, this attractive model is far from being stable; even the signs of the elasticity estimates in the cointegration relations change when the estimation period is shortened. To continue the analysis with such a model would only be fooling oneself.

Table 12.13 **Restricted cointegration space; model with linear trend in cointegration space**

	Eigenvalues, λ	
	.190	.194
	β matrix	
Variable	β_1	β_2
$(m - p)$	1.0000	−1.6791
y	−2.1361	1.0000
I	1.0000	2.2579
$\Delta_{12}P$	0.8174	1.9052
Trend	0.0000	0.0087

	α matrix	
Equation	β_1	β_2
$\Delta(m - p)$	−.082	−.008
Δy	.090	−.069
ΔI	−.024	−.005
$\Delta\Delta_{12}P$	.010	−.034

Model with own-yield of money

Since 1991, transactions accounts have been interest bearing. The introduction of the interest withholding tax has led to a rise in the interest rate on bank accounts. Competition among banks has also been a factor in the rising yield on bank accounts. The yield of bank accounts has since attained importance in the money demand context. The average interest rate on bank accounts does not exist for the period before 1984. But because of its close connection with the Bank of Finland's base rate, it can be artificially generated for the early 1980s. The series is presented in figure 12.5. Despite the disadvantage in using such a series and the interpretational difficulties, I have estimated a model using the series. The bond yield (see figure 12.6) is

Figure 12.5 **Own-yield of M2, prior to October 1984, generated**

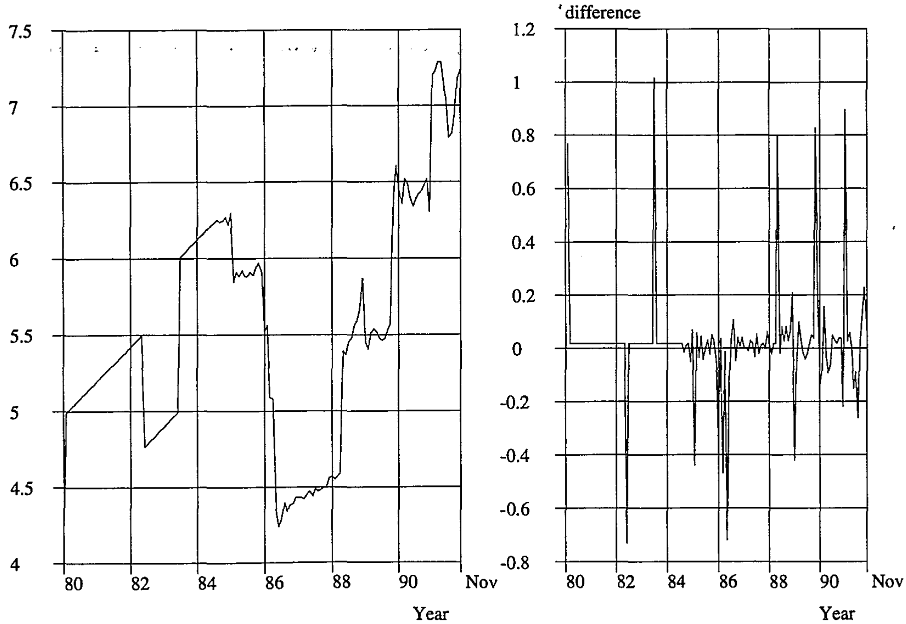
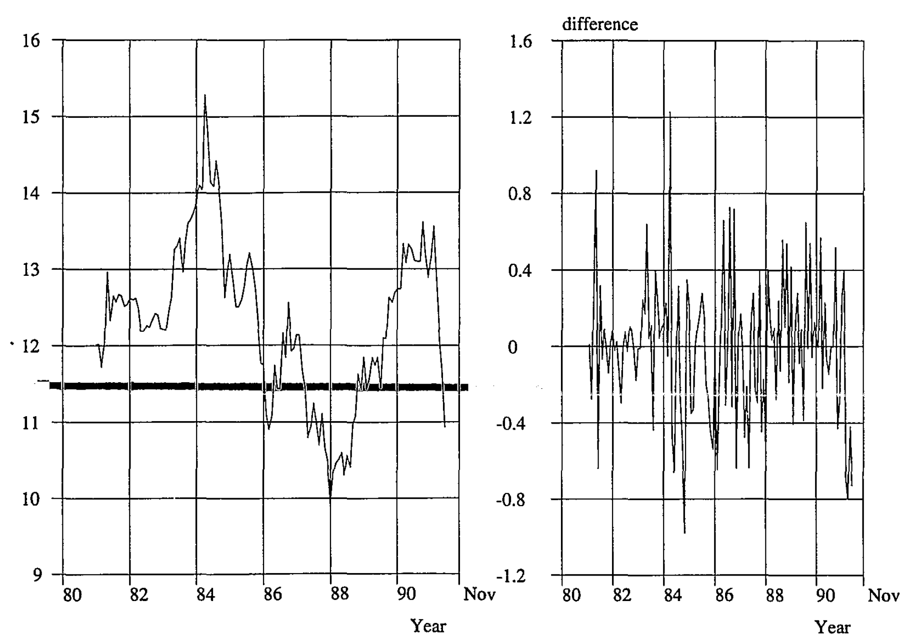


Figure 12.6 **Bond rate**



added as a measure of the opportunity cost of money. A consequence of this is that the bond rate displaces the short run interest rate in the cointegration relation.

The estimation period and predetermined variables are the same as above. Again, the change in estimation period dramatically affects to the results. Thus, one might well be suspicious of these results. The volatility of the bond rate was also included in the model. The exclusion of bond rate volatility (computed as a 12-month moving variation coefficient) was tested and found to be clearly excludable from the cointegration relation[4].

The initial set of variables consisted of real money (M2), GDP, average own-yield of money and, as measures of the opportunity costs of money, inflation, the short rate and the bond rate. The constant was restricted in the cointegration relations and the predetermined variables are as above. The cointegration rank was tested to be four in the first step. The system was reduced when the short rate and inflation did not enter the cointegration relation. They were excluded in the second step and the rank of the reduced system is two. Because of the deterministic nature of the own-yield of money the model was conditioned on it.

The model fits the data quite well (table 12.14). The only problem is the non-normality of residuals for the money equation. This excess kurtosis is caused by the peak in December 1985, for which there is no special reason. The second cointegration vector in an unrestricted cointegration space has the properties of the demand for money. The interpretation of the first (stronger) cointegration vector is not so straightforward. The own-yield of money seems to dominate it (table 12.15). The natural consequence is that the cointegration space is not identified in the economic sense.

[4] Koskela and Virén (1986) have found it to be a significant variable determining US money demand.

Table 12.14 **Residual analysis**

Equation	mean	variance	max	min	R^2
ΔM	0.0000	0.0000	0.0264	−0.0136	0.8532
ΔY	0.0000	0.0001	0.0302	−0.0306	0.9686
ΔIb	0.0000	0.0000	0.0102	−0.0071	0.3801

Residual diagnostics						
Eq.	B–P.Q(31)/27	ARCH(4)	Skew	Ex.Kurt	Norm.$\chi^2(2)$	R^2
ΔM	0.8779	0.8383	0.6650	2.2902	36.5298	0.8532
ΔY	0.7786	8.9290	−0.3336	0.7013	4.8799	0.9686
Ib	1.0068	0.4002	0.2601	0.7807	4.5836	0.3801

Autocorrelation $2/\sqrt{T} = 0.17889$								
Equation	1 lags	2 lags	3 lags	4 lags	5 lags	6 lags	7 lags	8 lags
ΔM	−0.068	−0.058	0.024	0.125	0.040	−0.073	−0.103	0.043
ΔY	0.058	−0.038	−0.103	−0.018	0.015	0.112	0.086	0.035
ΔIb	0.032	0.018	0.010	0.147	0.118	−0.077	−0.029	−0.041

Table 12.15 **Non-restricted cointegration space**

	Eigenvalues, λ	
	0.2904	0.1812
	β matrix	
Variable	β_1	β_2
M	−0.2102	−0.5588
Y	0.2237	1.0000
Ib	1.0000	−1.3349
Id	4.3332	4.4074
Constant	0.5210	−0.4243

	α matrix	
Equation	β_1	β_2
ΔM	−0.1581	0.1654
Y	−0.4410	−0.4117
Ib	−0.1469	0.0255

The hypothesis that money is not present in the first cointegration vector and that the deposit and bond rates enter the cointegration with identical coefficients is not rejected (p value 0.17). The consequence is that the non-restricted cointegration vector – money demand – has very large interest rate elasticities. The system is not economically identified (table 12.16). For example, if one takes a linear combination $\beta^* = \beta_1 + \beta_2$, the new vector satisfies the criteria of money demand.

Neither GDP, bond rate nor own-yield is stationary. None of the variables is weakly exogenous on the long run parameters.

The system can be identified by imposing a new restriction on the money demand vector. For example, when GDP semi-elasticity is restricted to two or to 1.8 (table 12.17), which easily pass the test, the system is identified. The restrictions, however, are arbitrary.

Table 12.16 **Structural hypothesis test**

	2 linear restrictions on 1 β vector, LR test, $\chi^2(1) = 1.91$, p value 0.17	
	Eigenvalues, λ	
	0.2632	0.1994
	β matrix	
Variable	β_1	β_2
M	0.0000	1.0000
Y	−0.0490	−1.8570
Ib	1.0000	5.6991
Id	1.0000	−5.9549
Constant	0.1871	0.8976
	α matrix	
Equation	β_1	β_2
ΔM	−0.0891	−0.0586
ΔY	−1.7475	0.3233
Ib	−0.3076	0.0169

Table 12.17 **Structural hypothesis test**

	LR test, $\chi^2(1) = 1.89$, p value 0.17	
	Eigenvalues, λ	
	0.2631	0.1995
	β matrix	
Variable	β_1	β_2
M	0.0000	1.0000
Y	−0.0499	−1.8000
Ib	1.0000	4.4233
Id	1.0000	−7.2379
Constant	0.1926	0.6905
	α matrix	
Equation	β_1	β_2
ΔM	−0.1674	−0.0579
ΔY	−1.3305	0.3232
ΔIb	−0.2870	0.0168

When the end of the estimation period is changed from November 1991 to December 1990, the results change substantially. The main consequence is that the bond rate is clearly stationary. This feature makes it impossible to model it as part of a stable money demand. This fragility of the estimation results is a consequence of mis-specification or changing structure, both of which put into question the concept of a stable demand for money. For this reason, we return to the original model.

12.2 Testing the Rational Expectations Hypothesis

The presence of two cointegration vectors is puzzling in the context of testing the rational expectations hypothesis. One way to avoid the problem is, agaist all principles of econometrics, to delete the first cointegration vector. Another violation of the principles of econometrics is to assume weak exogeneity for the variables other than money. The rest of the analysis uses the format for analysing the rationality of M1 demand.

Direct estimation of the Euler equations yields poor estimates, as presented in table 12.18. The constant, seasonal dummies and exogenous shock dummies have been used to capture the non-stochastic variation in the data in the estimation of expectations and the Euler equation.

Table 12.18 **Direct estimation of the Euler equations**

Constant	δ	λ_1
−0.00892	−0.0874	−0.10472
(0.00724)	(0.05546)	(0.01535)
	Presetting $\delta = 0.95$	
0.002742		−0.1599
(0.008009)		(0.02497)

Autocorrelation-corrected standard errors are in the parenthesis below the coefficients.

As in the analysis of narrow money, the results are poor. The discount factor, δ, does not differ significantly from zero. The adjustment is significant, but it has a wrong sign. Adjustment costs cannot be negative. Next, the discount factor was fixed at $\delta = 0.95$. The results are still poor, with the wrong sign for the adjustment cost parameter. According to the above results, the rational expectations hypothesis is not consistent with the data.

The modified Campbell and Shiller approach was applied next. The adjustment coefficient was taken directly from the loadings matrix of the long run analysis, $(\mu_1 - 1) = -0.06$. The discount factor has the fixed value, as before. As in the model for narrow money, the constant and shock and seasonal dummies are included in the analysis. The results are presented in table 12.19. The small t values for the constant and the change in the desired money level indicate, as does like the F-test (p value 0.394), that this approach does not support the rational expectations hypothesis either.

Table 12.19 **Test of rational expectations**

Variable	coefficient	standarderror	t-statistic
Constant	0.00476	0.00670	0.71
Δm^*	0.2725683	0.276	0.989
IBR	−0.0405	0.0774	−0.52
UIP	0.0291	0.0325	0.89
DSPEC	−0.0104	0.00585	−1.77
TAX	0.0556	0.00813	6.83
STRIKE	0.0302	0.00569	5.31
FREETAX	−0.00306	0.00806	−0.37
WITHDTAX	−0.0276	0.00809	−3.40
cs1	−0.0438	0.00356	−12.28
cs2	−0.0310	0.00440	−7.05
cs3	−0.0395	0.00364	−10.83
cs4	−0.0341	0.00347	−9.80
cs5	−0.0336	0.00345	−9.73
cs6	−0.0247	0.00360	−6.84
cs7	−0.0381	0.00345	−11.05
cs8	−0.0286	0.00767	−3.72
cs9	−0.0427	0.00362	−11.80
cs10	−0.0372	0.00355	−10.48
cs11	−0.0375	0.00347	−10.81

12.3 Structural Model for Real Balances

As in the model for narrow money, the unrestricted VAR is an essential starting point for the structural modelling of broad money. Our primary interest is in the effect of excess money in the money and GDP equations. We use the estimated β space from table 12.10. The error correction terms, i.e. the banks' combined balance sheet and excess money, are as follows:

$$ECM_{CR,t} = 3.3377 - 1.4584(m-p)_t + cr_t \tag{12.1}$$

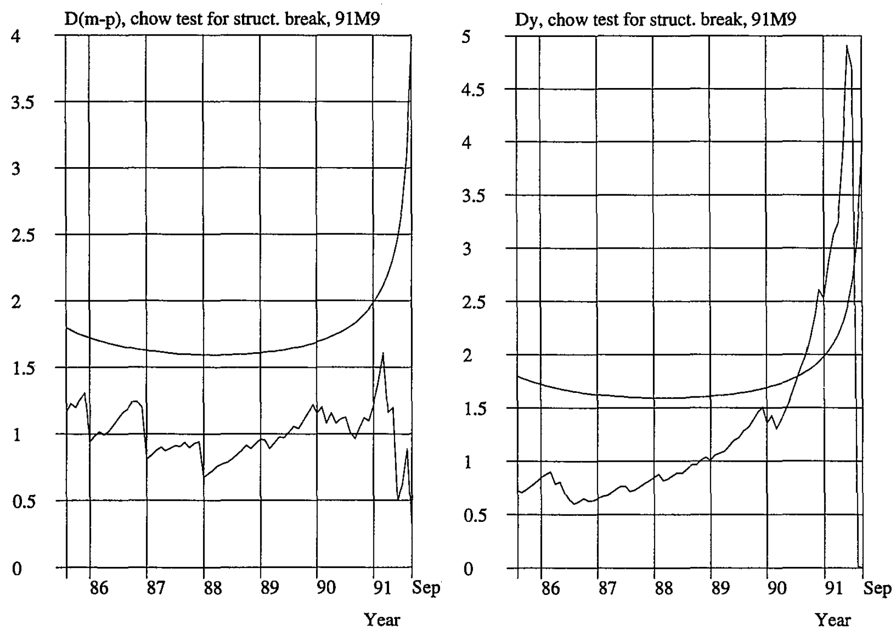
$$ECM_{M,t} = 0.1698 + (m-p)_t - y_t + 0.7261I + 0.17\Delta_{12}P_t - 0.3883cr_t. \tag{12.2}$$

In the analysis of the long run structure, the interest rate and inflation rate were found to be weakly exogenous for the long run parameters. They may be weakly exogenous for short run parameters. The moving average analysis pointed in the same direction, but without considering the variances of the components there is liitle more that can be concluded. Both the interest and inflation rates are determined outside the system. The weak exogeneity of these variables for both short and long run parameters was tested. The results of separate F tests were very convincing. The weak exogeneity restriction for the interest rate was not rejected (p value 0.601). The p value for weak exogeneity of the inflation rate was even larger, 0.889. The results are so convincing that there is no need for joint testing of weak exogeneity for those variables.

Another candidate for a weakly exogenous variable is the credit variable. The test statistic, though, does not support this (p value 0.013). The important point is that we are not very keen on the equation, because the other important factors are not included in the information set. Because the final model is estimated using FIML, we want to avoid the possible misspecification effects of the credit equation on the estimates of the other two equations. In the following, we condition the models on credit.

The conditional, unrestricted open reduced form model was estimated recursively using OLS. The recursive Chow test, with September 1991 as a fixed point, shows that there are no structural breaks in the money change equation (figure 12.7), but the decline of GDP indicates a possible structural break in the GDP equation. So, the recursive results are reasonable, at least in the money equation, which is the equation of interest. The whole sample period January 1990 to September 1991 was used in the estimation.

Figure 12.7 **Fixed point (1991M9) Chow tests**



The structural models are as follows:

$$
\begin{array}{lllllllll}
\Delta(m-p)_t & = & -0.126 & \Delta I_t & +0.414 & \Delta cr_t & -0.293 & \Delta\Delta_{12}P_t & \\
 & & (.064) & & (0.085) & & (0.11) & & \\
 & & -0.031 & ECM_{CR,t-1} & -0.090 & \Delta(m-p)_{t-1} & & & \\
 & & (0.019) & & (0.060) & & & & \\
 & & +0.041 & \mathrm{TAX}_t & +0.029 & \mathrm{STRIKE}_t & & & (12.3) \\
 & & (.0075) & & (.0049) & & & & \\
 & & +0.159 & \mathrm{IBR}_t & -0.017 & \mathrm{WITHDTAX}_t & & & \\
 & & (.088) & & (.0066) & & & &
\end{array}
$$

$\sigma = 0.006102$

$$
\begin{array}{llllllll}
\Delta y_t & = & -0.355 & \Delta I_{t-1} & +0.268 & ECM_{M,t-1} & +0.238 & \Delta cr_{t-2} \\
 & & (0.12) & & (0.059) & & (0.13) & \\
 & & -0.387 & \Delta y_{t-1} & -0.213 & \Delta y_{t-2} & +0.181 & \Delta(m-p)_t \qquad (12.4) \\
 & & (0.081) & & (0.076) & & (0.14) &
\end{array}
$$

$\sigma = 0.01244.$

The model has reduced form or restricted VAR-model features, except for the last (simultaneous) term in the GDP equation. It was significant in the 2SLS estimation but not in the reported FIML estimation. The residual diagnostics are presented in table 12.20. The simultaneity of the model is captured mainly by the coeffient estimates, not the covariace of the residuals. The residuals pass the normality test, but slight autocorrelation might exist in the money change equation. The reason for the rather high normality test statistics in the money change equation can be seen in figure 12.8. The peak in December 1985 is not fully captured by the seasonal dummies presented in table 12.21. The estimated structural model encompasses the VAR model (the p value for the overidentifying restrictions is 0.205), not as clearly as in the M1 model but still quite convincingly.

Table 12.20 **Residual analysis**

Reduced form residual correlation matrix		
Equation	$\Delta(m-p)$	Δy
$\Delta(m-p)$	1.0000	
Δy	–.0402	1.0000

Residual diagnostics					
Equation	Mean	Std.dev.	Skew.	kurt.	Norm. $\chi^2(2)$
$\Delta(m-p)$	–.000000	1.003670	–.310260	.757256	5.112
Δy	.000000	1.003670	–.253673	.302653	1.902

Autocorrelation $2/\sqrt{T}=0.17678$								
Equation	1 lag	2 lag	3 lag	4 lag	5 lag	6 lag	7 lag	8 lag
$\Delta(m-p)$	–.1929	–.0081	–.0609	.1958	–.0496	.0245	–.0274	.1017
Δy	–.0034	–.0460	–.1127	–.1506	.0286	.1458	.0265	.0463

Figure 12.8 **Residuals**

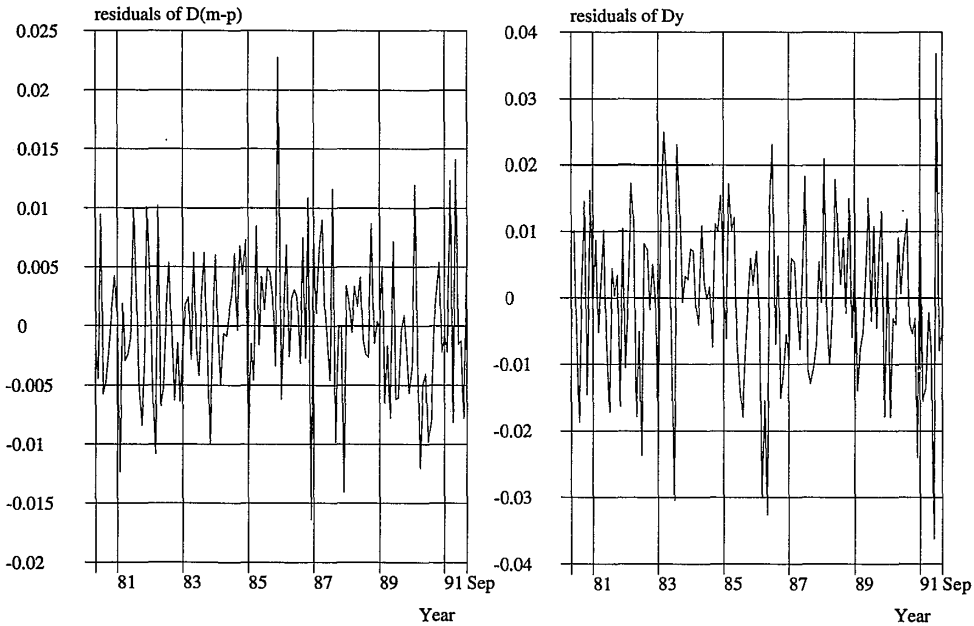


Table 12.21 **Constant and structural seasonals for M2**

Variable	$\Delta(m-p)$		Δy	
	coeff.	σ	coeff.	σ
Constant	.0282	0.005948	.0312	0.0101
cs1	–.0416	0.003693	–.0663	0.0090
cs2	–.0365	0.002809	–.0350	0.0089
cs3	–.0401	0.002760	.0059	0.0102
cs4	–.0351	0.002753	.0208	0.0087
cs5	–.0346	0.002654	.0323	0.0083
cs6	–.0260	0.002662	.0045	0.0071
cs7	–.0366	0.002735	–.1549	0.0081
cs8	–.0381	0.002681	.0007	0.0141
cs9	–.0424	0.002654	.0343	0.0148
cs10	–.0379	0.002723	.0393	0.0130
cs11	–.0395	0.002709	.0087	0.0086

Figure 12.9 **Equilibrium errors for the restricted cointegration relations**

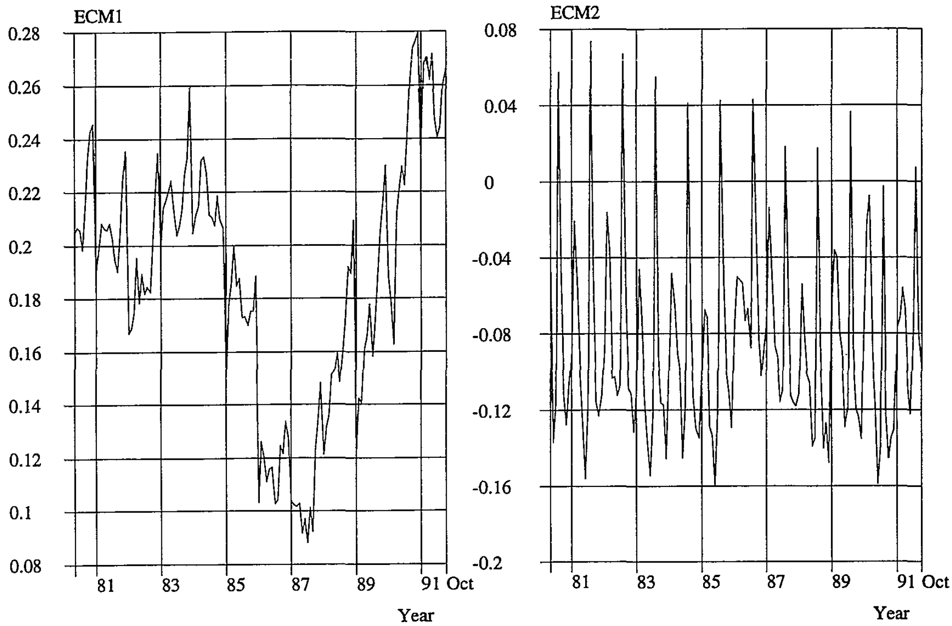
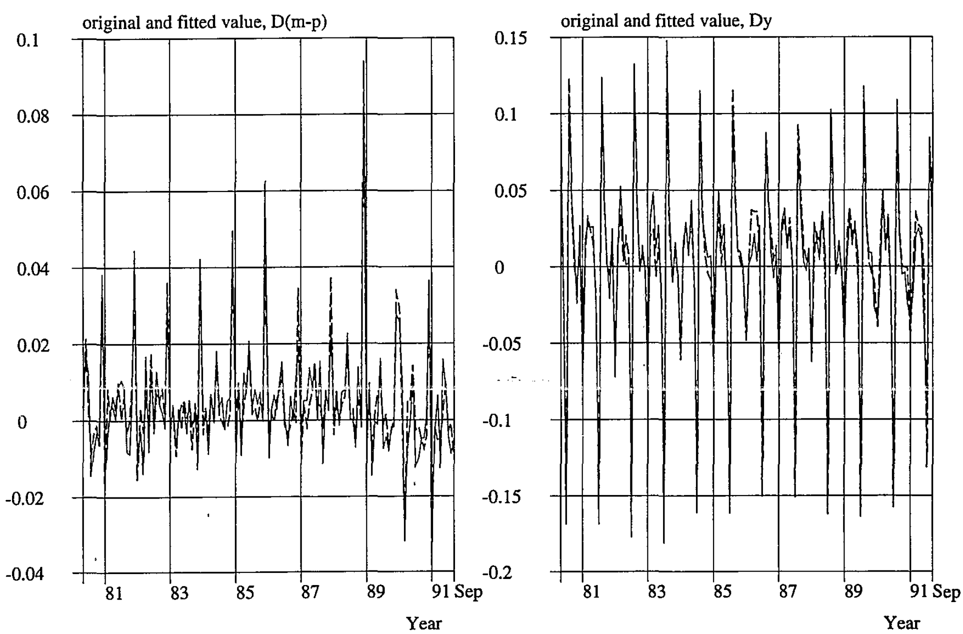


Figure 12.10 **Current and fitted values**



The key message given by the models is clear. Excess money has positive real effects in the long run. The mechanism works as follows: Money is measured by M2, with time deposits as the major component. Time deposits are not very liquid. Thus, excess money cannot be adjusted in the money equation. However, the excess money accelerates GDP growth. Because of the illiquidity of time deposits, the adjustment is quite strong. The effect is present in the short run dynamics also. We must in any case emphasize that, according to these FIML estimates, the short run effect does not differ significantly from zero.

The interest rate change has a clear simultaneous negative effect on money change. It is not very strong, being weaker than the interest rate effect in the long run. It is not surprising that a positive change in credit causes a strong immediate positive effect on money change. The inflation change has a negative effect on money change, stronger than the effect of a change in the interest rate. The interesting point is that the short run effect is stronger than the long run effect. For the long run, inflation could be excluded from the excess money relation. Tax shocks have effects on money change. This indicates that the model should be augmented by a tax variable to make it useful for tax policy simulation. Another implication is that there is no use testing the strong exogeneity hypothesis. It is rejected because of the significant shock dummies. The level of the Bank of Finland's base rate (IBR) has an important positive effect on money change. However, one should be carefull in drawing any conclusions as to the direction of causality in this case. Probably, it is the reverse of our hypothesis.

The GDP change equation is strongly affected by the excess money cointegration relation. Other important factors are the own lagged values. This might be an indication of omitted factors affecting GDP. The interest rate affects GDP only when lagged once. The real sector of the economy does not react within a month. It is quite natural that the effect is negative. Another important implication is a positive impact for credit. The change in credit affects GDP change within two months. The effect has a postive sign. One important implication of this is that a credit crunch also has a quick impact on production.

13 Discussion

I have derived the optimal demand for money behaviour using the intertemporal expected loss approach. Agents minimize the sum of their expected future quadratic losses. Losses are generated by the deviation from the desired level of money balances. Following the examples of Nickell (1985) it is possible to find a model that can be written in the error correction form. Unfortunately, the model is restricted to the univariate case. Thus, only one "endogenous" variable is allowed.

The desired level of money is determined by the forcing variables. To restrict the set of forcing variables, I have cited some of the main theories of the demand for money. The choice of variables is based on these theories. Theoretical models based on different assumptions yield results that are nearly identical. The demand for money is determined by prices, income, interest rates and probably by the inflation rate. Empirical researchers have interpreted the theoretical models quite freely, using a variety of measures for these basic variables and have supplemented the set with various measures of risk.

Econometric issues are very important in modelling the demand for money. Money demand applications have been an empirical benchmark for several new econometric methods. I have concentrated in the structural issues. For error correction models, the structure can be implemented in both the long run and short run parts of the models. As Granger (1983) emphasizes, the notion of cointegration is the time series analyst's answer to the use of the error correction mechanism. Cointegration is always a multivariate problem. I have used the vector autoregressive approach in this study. Cointegration analysis, and hence the multivariate error correction model, can also be applied within this framework. I have used the FIML approach developed by Johansen (1988, 1991).

Economic theories usually derive an equilibrium relation between economic variables. The cointegration approach allows us to test these theories, provided they yield empirical consequences. The notion of cointegration liberates us from short run disturbances. We can test a theory against an alternative, which should contain as few restrictions as possible. The unrestricted VAR is a natural choice for this alternative. I have explicitely written down the empirical restrictions yielded by theories determining the forcing variables. I also tested some of them as structural hypotheses for the long run. Some of them are rejected so obviously that it is not worthwhile to test them. Some testable hypotheses concerning money demand are the price and income homogeneity of money, Baumol's and Miller and Orr's restrictions on income elasticity and the stability of money velocity.

The cointegration vectors determine the equilibrium relation of the variables. I have also investigated how the error in equilibrium (a shock) affects the system's endogenous variables. To do that, I set out a structural model of the endogenous variables. The endogeneity/exogeneity partitioning of the data was based on the statistical properties of the data rather than the (sometimes) misleading economic theory. Unfortunately, this could not be done mechanically due to the lack of information in the data set. The data-contradicting conditioning of the variable, discussed above, was seen to be harmless. Finally, data-consistent structural restrictions have been imposed on the model. The testing of residuals for the final model clearly indicates that the statistical model fits the data quite well.

I took a sidetrack from the VAR approach to the structural model. By changing some assumptions, the loss function approach yields a rational expectations model for money demand. The results of the analysis of the long run structure can be utilized in testing the rational expectations hypothesis if joint weak exogeneity of variables other than money is present in the data. The estimation results indicated clearly that the rational expectations hypothesis as presented above was not consistent with the data.

I repeat here the main estimates of the models. The structural model for narrow money is

$$
\begin{aligned}
\Delta(m-p)_t &= -.357\ \Delta I_t - .140\ ECM_{t-1} \\
&\quad -.033\ \mathrm{TAX}_{\mathrm{t}} + .056\ \mathrm{STRIKE}_{\mathrm{t}} + .098\ \mathrm{WITHDTAX}_{\mathrm{t}} \\
\Delta y_t &= .106\ ECM_{t-1} - .391\ \Delta I_{t-1} - .459\ \Delta y_{t-1} - .233\ \Delta y_{t-2} \\
ECM_t &= (m-p)_t - 0.856\ y_t + 1.5;I_t
\end{aligned}
$$

and for broad money:

$$
\begin{aligned}
\Delta(m-p)_t &= -0.126\ \Delta I_t + 0.414\ \Delta cr_t - 0.293 \Delta\Delta_{12} P_t \\
&\quad -0.031\ ECM_{CR,t-1} - 0.090\ \Delta(m-p)_{t-1} \\
&\quad +0.041\ \mathrm{TAX}_{\mathrm{t}} + 0.029\ \mathrm{STRIKE}_{\mathrm{t}} + 0.159\ \mathrm{IBR}_{\mathrm{t}} - 0.017\ \mathrm{WITHDTAX}_{\mathrm{t}} \\
\Delta y_t &= -0.355\ \Delta I_{t-1} + 0.268\ ECM_{M,t-1} + 0.238\ \Delta cr_{t-2} \\
&\quad -0.387\ \Delta y_{t-1} - 0.213\ \Delta y_{t-2} + 0.181\ \Delta(m-p)_t \\
ECM_{CR,t} &= 3.3377 - 1.4584\,(m-p)_t + cr_t \\
ECM_{M,t} &= 0.1698 + (m-p)_t - y_t + 0.7261\ I + 0.17\ \Delta_{12} P_t - 0.3883\ cr_t.
\end{aligned}
$$

The interest rate change effect is stronger in the equation for narrow money. Narrow money is more liquid than broad money and can thus react more quickly to interest rate changes. The models are not very suitable for policy simulation because they are strongly affected by the dummies, which capture, for example, tax changes of money aggregates. Taxes, of course, are policy instruments. Consequently, the testing of super-exogeneity is not usefull here. The system has already failed in policy analysis, which is the prime motivation for testing super-exogeneity. The dynamic structure, however, makes the models quite useful for forecasting. Forecasting analysis showed that the forecast performance of the system was fairly good except for periods when policy changes occurred (e.g. the WITHDTAX period).

In some countries foreign currency is a substitute for domestic currency for transactions purposes. In this case part of the demand for money is channelled into foreign currencies. Higher foreign interest rates or expected devaluation might attract domestic investors too. This phenomenon is referred to as currency substitution. In regard to Finnish M1 and M2, we found no evidence of currency substitution in the 1980s. There are several possible reasons for this. First, the final barriers to capital movements were abolished only in the late 1980s, prior to which households were not able to substitute other currencies for the markka. Second, the money measures M1 and M2 are quite narrow and are based mainly on the transactions motive for holding money. So, they do not adjust to the rapid changes in interest rates or speculative factors. Foreign currency in accounts or in cash do not serve these liquidity needs as do the accounts included in M1. Third, devaluation risk was covered by higher domestic interest rates during this period. Fourth, the aggregate M2 is composed largely of time deposits, which are not highly liquid and are therefore difficult to adjust to rapid speculative movements without loosing interest income.

The long run interest rate elasticities are systematically higher than the short run elasticities. "Long run" here means equilibrium. So, the long run elasticities reflect equilibrium behaviour. A natural reason for the lower short run interest rate elasticities might be the adjustment (transaction) costs. Economic agents are not so willing to adjust their money portfolio in the short run as in the long run. When they see that an interest rate shock is sufficiently permanent, they are ready to react. To quantify the importance of interest rates, one can calculate, for example, that a one percentage point change in short rates would cause a 0.004 percentage point change in M1 immediately and a 0.015 percentage point change in the long run. At the present level of M1 these translate to approximately FIM 500 million and FIM 2000 million, respectively.

Another important feature of the interest rate behaviour of money demand is that interest rates are clearly exogenous when money is measured by M1 or M2. The use of M3 as a measure of money might result in endogenous interest rates[1]. Theories of a small open economy under fixed exchange rates emphasize that interest rates are determined by foreign interest rates plus the expected rate of devaluation, not the amount of money in the economy. This feature is very strongly present in Finnish money demand data.

The monetary policy implications of the models are quite interesting. The nominal models reveal that the excess money does not affect the inflation rate even though the price level itself affects money immediately. Money has some real effects through the error correction terms. If, for example, the money level exceeds the desired level, the error correction term is positive and has a positive effect on GDP change. Excess money balances cause some GDP growth, which will reach equilibrium within a year. For broad money, the effect is much stronger than for narrow money. Those who have their money in time deposit accounts suffer from the illiquidity of their money holdings.

Some estimation results indicated that money did not drive prices in either the M1 or M2 models. The results seem natural, since there was no room for extra monetary growth because of the endogeneity of money. Tax changes were anticipated to a great extent and did not have an inflation effect.

The construction of GDP data itself does not explain the result, which also obtains in the quarterly models. It also exists in foreign studies[2]. A simple and natural explanation would be that money balances, especially M1, are held for expected transactions. One needs money to buy things. Money leads transactions. In that sense money should have real effects.

In summary: the monetary aggregates M1 and M2 do not have much meaning under a fixed exchange rate regime. Their levels are basically determined by transactions and interest rates. The aggregates do not influence prices or interest rates. The precautionary motive for holding money might cause real effects for money. Empirical models of the demand for money, however, can be very informative during the present floating exchange rate regime. They could determine a baseline level of money holdings and thus give some indication of the state of monetary policy.

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[1] M3 is defined as M2 plus certificates of deposit.

[2] Such effects can be seen in models for every Nordic country. Because of a suggestion by Gunnar Bårdsen, the investment indicator was added to the GDP equation (M1 model), but it did not change the results at all.

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Appendix A

Some Definitions

Engle *et al.* (1983) define different forms of causality and exogenity as follows. We first partition vector z_t into y_t, which describes the basic variables of interest ($y_t \in R^p$), and x_t ($x_t \in R^q$), the determinants of y_t. Then

$$z_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}, \quad t = 1,...,T.$$

One should notice that $Y_t = \{y_1,...,y_t\}$ and analogously for X_t. What follows is also conditioned throughout on X_0

Definition 1 Y_{t-1} *does not* Granger cause x_t *with respect to* Z_{t-1} *iff*

$$D(x_t|Z_{t-1},\theta) = D(x_t|X_{t-1},Y_0,\theta)$$

Let the following

$$B^* z_t + \sum_{i=1}^{k} C^*(i) z_{t-i} = \varepsilon_t \tag{A.1}$$

describe a set of structural relationships. B^* and C^* are matrix functions of θ and ε_t is the corresponding "disturbance". Then Engle *et al.* (1983) define *predeterminedness* and *strict exogeneity* as follows.

Definition 2 x_t *is* predetermined *in (A.1), iff*

$$x_t \perp \varepsilon_{t+i} \quad \textit{for all } i \geq 0.$$

Definition 3 x_t is strictly exogenous *in (A.1), iff*

$$x_t \perp \varepsilon_{t+i} \quad \textit{for all } i.$$

Next we specify parameters of interest as $\psi = f(\theta)$. We are also interested in estimating a model for y_t, given the determinants x_t, but we are not interested in specifying a model for x_t. Thus we want to partition the distribution function into a *sequential cut*:

Definition 4 $[(y_t|x_t;\lambda_1),(x_t;\lambda_2)]$ *operates a* sequential cut *on* $D(z_t|Z_{t-1},\lambda)$ *iff*

$$D(z_t|Z_{t-1},\lambda)=D(y_t|x_t,Z_{t-1},\lambda_1)D(x_t|Z_{t-1},\lambda_2)$$

where λ_1 *and* λ_2 *are variation free, i.e.*

$$(\lambda_1,\lambda_2)\in\Lambda_1\times\Lambda_2.$$

λ_1 and λ_2 are *not* variation free, for example, when there exist cross equation restrictions.

Definition 5 x_t *is* weakly exogenous *over the sample period for* ψ *iff there exists a reparameterization with* $\lambda = (\lambda_1, \lambda_2)$ *such that*

(i) ψ is a function of λ_1,

(ii) $[(y_t|x_t;\lambda_1),(x_t;\lambda_2)]$ *operates a sequential cut*

Definition 6 x_t *is* strongly exogenous *over the sample period for* ψ *iff it is weakly exogenous for* ψ *and in addition*

(iii) *y does not Granger cause x.*

When (ii) holds, the joint likelihood function $L^0(\lambda;Z_T)$ can be factored as

$$L^0(\lambda;Z_T)=L_1^0(\lambda_1;Z_T)L_2^0(\lambda_2;Z_T).$$

The two factors can be analyzed independently of each other. So, all sample information concerning the parameters λ_1 are in the first factor.

The strong exogeneity makes it possible to use the model for forecasting purposes.

To be able to give the conditions for using the estimated model for policy analysis, we define *structural invariance* and *super exogeneity.*

Definition 7 A *parameter is* invariant *for a class of interventions if it remains constant under these interventions. A model is invariant for such interventions if all its parameters are thus invariant.*

Definition 8 *A conditional model is* structurally invariant *if all its parameters are invariant for any change in the distribution of the conditioning variables.*

Definition 9 x_t *is* super exogenous *for* ψ *if* x_t *is weakly exogenous for* ψ *and the conditional model* $D(y_t|x_t, Z_{t-1}, \lambda_1)$ *is structurally invariant.*

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