

Suomen Pankin kirjasto



0000091044

IVA5a

Kirjasto: alaholvi

SUOMEN PANKKI D

Adjustment of prices to wages

Suomen pankin taloustieteellisen tutkimuslaitoksen julk.
3 1964

ADJUSTMENT OF PRICES TO WAGES

by J.J.Paunio

Institute for Economic Research

Bank of Finland

Series D:3 Mimeographed studies

November 1964

ADJUSTMENT OF PRICES TO WAGES ^{*}

by J.J.Paunio

Is a strong labour organisation able to achieve a general rise in workers' wages, and in what way does such an increase affect prices and employment? The problem though old, is still unsolved. When put in this way the question is so general in nature that no unambiguous answer can be expected. The analysis, which will be entirely theoretical, will be carried out with the aid of a comparatively simple model, in which the assumptions are practically the same as those found in the traditional theory. The model is static and the analysis therefore comparative static.

The analysis is set in a closed economy, where perfect competition prevails in the commodity market. The wage is exogenous in the model. Labour is the only variable factor of production. Thus the production function can be expressed in the following way:

$$(1) \quad q = q(n),$$

where q denotes the volume of output per time unit, and n employment.

If \bar{w} = the (given) wage per unit of labour, and mc = marginal costs, then

$$(2) \quad mc = \frac{\bar{w}}{q'(n)} .$$

As the model is primarily concerned with the commodity market, $q'(n) = q' [q^{-1}(q)] = g(q)$. Assuming that entrepreneurs are profit maximisers, we get the following equation for the price p .

$$(3) \quad p = \frac{\bar{w}}{g(q)} .$$

^{*} This article was originally published in Finnish in *Kansantaloudellinen Aikakauskirja* No. 2, 1964.

National income, y , is

$$(4) \quad y = pq.$$

Labour being by assumption the only variable factor of production wage income y_w , is

$$(5) \quad y_w = \int_0^q \frac{\bar{w}}{g(u)} du.$$

Thus capital income, y_k , is

$$(6) \quad y_k = pq - \int_0^q \frac{\bar{w}}{g(u)} du.$$

This system of equations is illustrated in Fig. 1. When the volume of output is \bar{Q} and the corresponding price \bar{P} , the rectangle $O\bar{Q}A_1\bar{P}$ represents national income, $O\bar{Q}A_1B_1$ below the marginal cost curve MC_1 being wage income and $A_1B_1\bar{P}$ above the same curve, capital income.

When the model has been developed this far, it is easy to give a specific definition of the problem under consideration. A wage rise increases marginal costs [equation (2)] so moving the curve in Fig. 1 upwards to the MC_2 position. Initially, assuming no other changes occur, the wage increase would reduce output to Q_1 . However, the final new equilibrium position, which it is the primary aim of this study to derive, further depends on how demand responds to the wage rise and to that reduction in output and employment which initially follows the wage rise. It should be noted in particular that the model affords possibilities of establishing how the change in income distribution induced by the wage rise affects consumption and investment demand. According to Fig. 1, income distribution at output level Q_1 (capital income $A_2B_2\bar{P}$ and wage income $OQ_1A_2B_2$) obviously differs from that at full employment level \bar{Q} (see previous paragraph).

Full employment is assumed to exist in the initial situation, which means that the marginal cost curve in Fig. 1 becomes vertical at the full

employment output level \bar{Q} .

The analysis of the effect of income distribution can be considerably simplified by setting forth the following proof prior to the final construction of the model. Fig. 2 shows two marginal cost curves, MC_1 and MC_2 , which differ from each other in respect of wage. We will now see that at one level of output in equilibrium situations, where marginal costs are different on account of a difference in wages, the distribution of income between capital and labour (as defined above) is the same. In other words the distribution of income is the same irrespective of the price level; in the following prices P_1 and P_2 will be considered.

If

$$(7:1) \quad mc_1 = \frac{\bar{w}}{q'(n)}$$

and

$$(7:2) \quad mc_2 = \frac{\bar{w} + \Delta w}{q'(n)},$$

then

$$(7:3) \quad mc_2 - mc_1 = \frac{\Delta w}{q'(n)}.$$

If we write

$$(7:4) \quad mc_1 = f(q)$$

and

$$q'(n) = \frac{\bar{w}}{f(q)},$$

then

$$(7:5) \quad mc_2 = f(q) + \frac{\Delta w}{\bar{w}} f(q).$$

Thus the ratio of capital income to wage income at price level P_1 is

$$(7:6) \quad \frac{y_{k1}}{y_{w1}} = \frac{y_1 - \int_0^q f(u) du}{\int_0^q f(u) du} = \frac{y_1}{\int_0^q f(u) du} - 1$$

and at price level P_2

$$(7:7) \quad \frac{y_{k2}}{y_{w2}} = \frac{y_2 - \int_0^q \left[f(u) + \frac{\Delta w}{\bar{w}} f(u) \right] du}{\int_0^q \left[f(u) + \frac{\Delta w}{\bar{w}} f(u) \right] du} = \frac{y_2}{\left(1 + \frac{\Delta w}{\bar{w}}\right) \int_0^q f(u) du} - 1.$$

As $\frac{y_1}{y_2} = \frac{p_1}{p_2}$, and at the same time $\frac{p_2}{p_1} = \frac{\bar{w} + \Delta w}{q'(n)} : \frac{\bar{w}}{q'(n)} = 1 + \frac{\Delta w}{\bar{w}}$,

then by substituting $\frac{p_2}{p_1} \cdot y_1$ for y_2 and $\frac{p_2}{p_1}$ for $\left(1 + \frac{\Delta w}{\bar{w}}\right)$

in equation (7:7) we get

$$(7:8) \quad \frac{y_{k2}}{y_{w2}} = \frac{\frac{p_2}{p_1} y_1}{\frac{p_2}{p_1} \int_0^q f(u) du} - 1 = \frac{y_1}{\int_0^q f(u) du} - 1,$$

or

$$(7:9) \quad \frac{y_{k1}}{y_{w1}} = \frac{y_{k2}}{y_{w2}}.$$

This proof facilitates the analysis in so far as the effect of income distribution upon demand and output can be determined, whatever the size of the wage increase, with the aid of one single marginal cost function.

In Fig. 1, the new volume of output produced by the wage increase is Q_1 .

On the strength of the proof the effect of income distribution at a change from \bar{Q} to Q_1 can be analysed at price \bar{P}_1 (curve MC_1), while price \bar{P} and curve MC_2 can be disregarded.

If we accept the fairly general conception that a decrease in output raises the proportion of income received by labour and reduces that received by capital, the marginal cost function must be expressed so as to conform with this assumption.

The marginal cost function, equation (3) is an aggregate function, which may be thought of as the total of the marginal cost curves of individual firms (see Fig.3). The marginal cost function can be assumed to have the form $f' > 0$ and $f'' > 0$.¹ However, such properties are not sufficient to prove that the marginal cost function really does divide the national income in the way assumed. But if the marginal cost function is explicitly given the form of a parabola, a proof is possible. Thus

$$(8:1) \quad mc = \alpha q^2 + \beta \quad (q \geq 0; \quad mc = 0, \text{ when } q < 0).$$

We now have to prove - provided that $\lambda(q) = \frac{qf(q)}{\int_0^q f(u) du}$ - that [(see (7:4)]

$$(8:2) \quad \lambda'(q) > 0.$$

The left member of this inequality

$$(8:3) \quad \lambda'(q) = \frac{\int_0^q f(u) du \left[qf'(q) + f(q) \right] - \left[f(q) \right]^2 \cdot q}{\left[\int_0^q f(u) du \right]^2}.$$

By incorporating equation (8:1) in the numerator of the above expression, we find the latter to be positive²; in other words, the inequality (8:2) is valid.

The national income is divided as follows:

$$(9:1) \quad y = i + (y-s),$$

which reduces to

$$(9:2) \quad s = i.$$

1. The derivatives of the corresponding marginal productivity function g are $g' < 0$ and $g'' < 0$.

2. By substitution we obtain $(\frac{\alpha}{3} q^3 + \beta q) \cdot 2\alpha q^2 + (\frac{\alpha}{3} q^3 + \beta q)$

$(\alpha q^2 + \beta) - (\alpha q^2 + \beta)^2 \cdot q = \alpha\beta q^2 (> 0).$

In these equations i = investment and s = saving.

Applying the generally accepted assumption that the marginal propensity of capitalists to save is greater than that of workers because of the higher per capita earnings of the former, the saving function can be written¹

$$(10) \quad s = \eta_k y_k + \eta_w y_w - \mu, \quad (\eta_k > \eta_w),$$

where η = the marginal propensity to save, and μ = consumption when $y = 0$.

In accordance with post-Keynesian thinking² on the concept of marginal efficiency, investment can be assumed to be a function of the rate of interest, in addition we assume it to be linear

$$(11:1) \quad i = \frac{\bar{e} - a}{-\delta},$$

where \bar{e} = marginal efficiency, and a and δ parameters. The nature of the interdependence can be founded on KALECKI's principle about "increasing marginal risk"³ as applied to macro behaviour. Given the interest rate investment can also be ascertained. Fig. 4 shows the assumed function \bar{e} , the value of which is a , when $i = 0$. At interest rate \bar{e} , investment is \bar{i} .

As investment in this model is dependent upon so subjective a factor as the willingness to invest, it seems evident that it should also be influenced by the current cyclical situation, in particular by the current level of profits, and perhaps also by the degree of utilisation of the capital stock. These factors are assumed to exert their influence by

-
1. $s_w = \eta_w y_w - \mu_w$ and $s_k = \eta_k y_k - \mu_k$; for the per capita saving functions it is probable that $\mu_w < \mu_k$.
 2. See BJÖRN THALBERG A Keynesian Model Extended by Explicit Demand and Supply Functions for Investment Goods, Stockholm 1962, p. 11.
 3. M. KALECKI Essays in the Theory of Economic Fluctuations, London 1939, p. 98-102; see also THALBERG, p. 12.

shifting the function Φ nearer to, or farther from, the origin (see Fig.4). The dependence of investment on these factors can be expressed as follows:

$$(11:2) \quad a = \psi(q) R_k,$$

where function ψ denotes the effect of the degree of capacity utilisation, and R_k denotes profits. It is assumed that all profits are received by the capitalists, or according to equation (6)

$$(11:3) \quad R_k = \frac{pq - \int_0^q \frac{\bar{w}}{g(u)} du}{\frac{\bar{w}}{g(q)} \cdot K},$$

where K = the capital stock at constant prices, and the denominator thus shows the value of the capital stock at current prices.

The less the utilisation of capital stock, the less will entrepreneurs be willing to invest, and vice versa. In this context, capacity should not be taken to mean that level of output which corresponds to the maximum technical utilisation, but that volume of output which cannot be exceeded without a rise in the average total costs of firms. Output in excess of that volume would probably make entrepreneurs consider an extension of the existing capital stock profitable. As there was initially full employment of labour, that is to say a kind of technical capacity limit set by the labour force, it cannot simply be assumed that full employment level of output corresponds to level of output capacity. Since this point is not of great importance in the present context we can start from

$$(11:4) \quad q_{opt} \leq \bar{q},$$

where q_{opt} = productive capacity and \bar{q} = output at full employment of labour. In equation (11:2) the degree of capacity utilisation affects the $\psi(q)$ of the function (11:2). This is illustrated by the following function.

$$(11:5) \quad \bar{y} = \bar{y} + \psi(q - q_{opt})$$

provided that $\bar{y} > 0$.

By incorporating equations (11:3) and (11:5) in (11:2) and also in (11:1) as written $\frac{a - \bar{y}}{\delta}$, and taking the rate of interest as given, we obtain the investment function

$$(11:6) \quad i = \frac{\left[\bar{y} + \psi(q - q_{opt}) \right] \left(pq - \int_0^q \frac{\bar{w}}{g(u)} du \right)}{\frac{\bar{w}}{g(q)} \cdot \delta K} - \frac{\bar{y}}{\delta},$$

which is valid when investment is non-negative. A notable feature of this investment function is that investment falls to zero while q is still positive.

Let us reformulate the model by introducing an explicit marginal cost function into the equations. The complete model is as follows:

$$(I) \quad q = q(n)$$

$$(II) \quad mc = \alpha q^2 + \beta$$

$$(III) \quad p = mc$$

$$(IV) \quad y = (\alpha q^2 + \beta) \cdot q$$

$$(V) \quad y_w = \int_0^q (\alpha u^2 + \beta) du$$

$$(VI) \quad y_k = (\alpha q^2 + \beta) \cdot q - \int_0^q (\alpha u^2 + \beta) du$$

$$(VII) \quad s = \eta_k (\alpha q^2 + \beta) \cdot q - (\eta_k - \eta_w) \int_0^q (\alpha u^2 + \beta) du - \mu$$

$$(VIII) \quad i = \frac{\left[\bar{y} + \psi(q - q_{opt}) \right] \left[(\alpha q^2 + \beta) \cdot q - \int_0^q (\alpha u^2 + \beta) du \right]}{\delta \cdot K \cdot (\alpha q^2 + \beta)} - \frac{\bar{y}}{\delta}$$

$$(IX) \quad s = i$$

The model consists of a micro part [equations (I)-(III)] with (III) as the equilibrium equation, and a macro part [equations (IV)-(IX)] with (IX) as the equilibrium equation. With regard to the macro equations (VII) and (VIII) it should be noted that although investment and saving were assumed to be functions of nominal variables, the output volume alone appears as an explanatory variable.

It should be recalled that this model was designed for analysing the effect of an autonomous wage increase upon prices and output. But on the strength of the proof presented in (7:1)-(7:9) we need not solve the whole equilibrium system of nine unknowns and nine equations simultaneously. As the effect of the wage increase, although appearing in the model as a shift of the marginal cost function [equation (II)], can be disregarded in connection with the functions i and s , we can split the model in a simple, yet meaningful way. The volume of output (several possible equilibria) is determined by the macro equations (VII) and (VIII) and the equilibrium equation (IX) so that we must now find out how the initial reduction in output caused by the wage-rise, as derived from the micro equations (II) and (III), affects the final level of output through the functions i and s . In other words, we are concerned with the form and position of these functions in the initial situation and at lower output levels.

By derivation of the saving function, we obtain

$$(12:1) \quad \frac{ds}{dq} = (2\eta_k + \eta_w)\alpha q^2 + \eta_w \beta$$

and

$$(12:2) \quad \frac{d^2s}{dq^2} = (4\alpha\eta_k + 2\alpha\eta_w) q.$$

It is evident at once that both derivatives are positive.

By derivation of the investment function¹ we get

$$(12:3) \quad \frac{di}{dq} = \frac{2\alpha}{3\delta K} \frac{2\alpha\psi q^5 + (\bar{v} - \psi q_{opt})\alpha q^4 + 4\beta\psi q^3 + 3(\bar{v} - \psi q_{opt})\beta q^2}{(\alpha q^2 + \beta)^2}$$

and

$$(12:4) \quad \frac{d^2i}{dq^2} = \frac{2\alpha}{3\delta K(\alpha q^2 + \beta)^4} \left[2\alpha^3\psi q^8 + 8\alpha^2\beta\psi q^6 - 2(\bar{v} - \psi q_{opt})\alpha^2\beta q^5 + 18\alpha\beta^2\psi q^4 + 4(\bar{v} - \psi q_{opt})\alpha\beta^2 q^3 + 12\beta^3\psi q^2 + 6(\bar{v} - \psi q_{opt})\beta^3 q \right]$$

It is obvious from these equations that the sign of the expression $(\bar{v} - \psi q_{opt})$ has a decisive influence on the signs of derivatives from the investment function. Let us therefore elaborate function \bar{v} .

If $\bar{v} = 1$ and excess utilisation of capacity is assumed not to give rise to expectations of profits more than double what they are at present, then

$$(13:1) \quad -1 \leq \psi(q - q_{opt}) \leq 1.$$

If $\psi(\bar{q} - q_{opt}) = 1$, then (\bar{q} = full employment output)

$$(13:2) \quad \psi = \frac{1}{\bar{q} - q_{opt}},$$

and

$$(13:3) \quad \bar{v} - \psi q_{opt} = 1 - \frac{q_{opt}}{\bar{q} - q_{opt}}.$$

As it is reasonable to assume that $\bar{q} < 2 q_{opt}$, we may conclude that

$$(13:4) \quad \bar{v} - \psi q_{opt} < 0.$$

1. $i = \frac{2\alpha}{3\delta K} \cdot \frac{(\bar{v} - \psi q_{opt})q^3 + \psi q^4}{\alpha q^2 + \beta} - \frac{\bar{q}}{\delta}.$

On the basis of the inequality (13:4) we obtain

$$(14:1) \quad \frac{di}{dq} \gtrless 0,$$

when

$$(14:2) \quad \frac{2\alpha\psi q^5 + 4\beta\psi q^3}{\alpha q^4 + 3\beta q^2} \gtrless \left| \bar{j} - \psi q_{opt} \right|.$$

It is obvious that when q rises above zero, $\frac{di}{dq}$ may be negative, but

as $i < 0$, when $q = 0$, then in the relevant range

$$(14:3) \quad \frac{di}{dq} > 0,$$

because the left member¹ of the inequality is evidently an increasing function of q .

From the inequality (13:4) we also find that

$$(14:4) \quad \frac{d^2i}{dq^2} \gtrless 0$$

dependent on whether

$$(14:5) \quad \frac{2\alpha^3\psi q^8 + 8\alpha^2\beta\psi q^6 + 18\alpha\beta^2\psi q^4 + 12\beta^3\psi q^2}{4\alpha\beta^2q^3 + 6\beta^3q - 2\alpha\beta q^5} \gtrless \psi q_{opt} - \bar{j}.$$

As for the second derivative, when q is greater than zero, $\frac{d^2i}{dq^2}$ may be positive. But as sufficiently high values for q make the left member of the inequality (14:5) negative, which changes the direction of the inequality, we can only conclude that when investment is positive, $\frac{d^2i}{dq^2}$ is either positive or first negative and subsequently, from some particular value of q positive.

The immediate reduction in output due to a wage increase is obtained by means of the equation

$$(15:1) \quad \bar{p} = (\alpha q^2 + \beta) \left(1 + \frac{\Delta w}{\bar{w}}\right),$$

where \bar{p} stands for the initial price level. Let us now consider this

1. This can be written $\frac{(2\alpha q^2 + 4\beta)\psi}{\alpha q^2 + 3\beta} q$.

output volume q^p and the initial output volume as given by the i and s functions.

1. Assume that in the initial situation $\frac{ds}{dq} > \frac{di}{dq}$ and $\frac{d^2s}{dq^2} > \frac{d^2i}{dq^2}$.

Let q^m denote the new equilibrium level of output (see Fig. 5a and 5b).

If $q^p > q^m$, then \bar{q} is still the equilibrium position. If $q^p = q^m$, q^m is the new equilibrium, and if $q^p < q^m$, production is cumulatively declining.

2. Assume that $\frac{ds}{dq} = \frac{di}{dq}$. When $\frac{d^2s}{dq^2} > \frac{d^2i}{dq^2}$ (Fig. 6a and 6b), output declines cumulatively. When $\frac{d^2i}{dq^2} > \frac{d^2s}{dq^2}$, \bar{q} still represents equilibrium (Fig. 6c; though not necessarily if $\frac{d^2i}{dq^2}$ becomes negative).

3. Assume that $\frac{ds}{dq} < \frac{di}{dq}$. In this case q^m represents the new equilibrium only if $\frac{d^2i}{dq^2} > \frac{d^2s}{dq^2}$ (Fig. 7). Otherwise output is cumulatively declining.

On the basis of this specification it will be found that if stability prevails in the initial situation, and the wage increase is moderate ($q^p \leq q^m$), then full employment is maintained and the price level rises according to the equation

$$(15:2) \quad p = (\alpha \bar{q}^2 + \beta) \left(1 + \frac{\Delta w}{\bar{w}}\right)$$

and $\frac{p}{\bar{p}} = \frac{\bar{w} + \Delta w}{\bar{w}}$. Income distribution remains unchanged as was proved

by equations (7:1)-(7:9).

In cases of non-stability¹, the model only shows that the price level falls, but it does not enable us to determine by how much.

In some situations output may settle at a lower level (when stability

1. Including non-stability produced by a very strong wage increase.

changes into non-stability, or vice versa). In such cases the price level is

$$(15:3) \quad p = (\alpha (q^m)^2 + \beta) \left(1 + \frac{\Delta w}{\bar{w}}\right).$$

Here $p \gtrless \bar{p}$ dependent upon the extent of the output reduction. The corresponding income distribution can be calculated by means of equations (V) and (VI).

The investment and saving functions were constructed in such a way that when output decreases, the change in income distribution partly reduces the willingness to invest, and partly increases the propensity to consume, in addition to which the increase in non-utilisation of capacity also weakens the willingness to invest. It is probable that consumers react more rapidly to changes in their income than investors do to changes in the cyclical situation because investment plans cannot be altered so quickly. For this reason I am inclined to regard the stable conditions represented in Fig. 5a, 5b and 6c as being most probable.

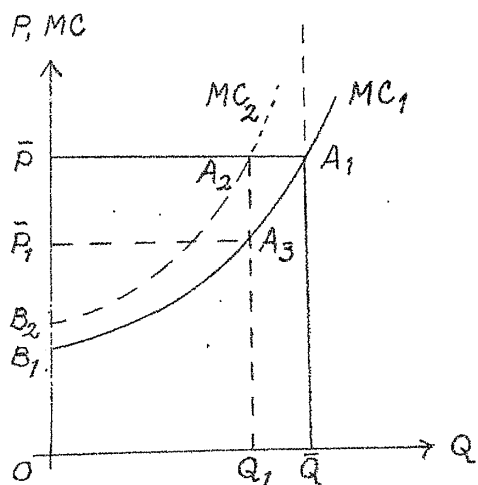
Thus the question put at the beginning of this study can be answered, within the framework of the model, as follows:

1. A wage increase will produce a rise in workers' nominal wages, but their real incomes will remain unchanged, because prices will rise in the same proportion as wages¹. The total demand curve will be vertical like curve D, in Fig. 8 (which relates to Fig. 1).

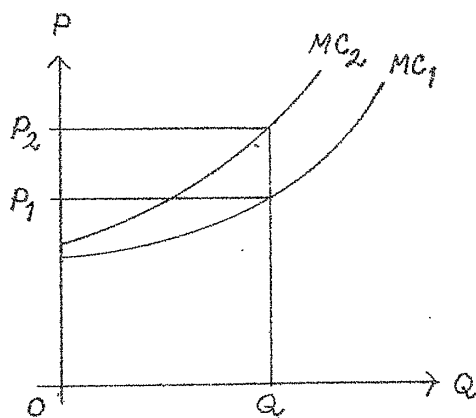
2. The more sensitive investors are to cyclical changes, and the greater the wage increases, the more probable it is that output and employment fall off and that prices also fall eventually. Fig. 8 shows some alternative demand curves, D_2 , D_3 and D_4 , the latter being purely hypothetical.

1. A final result compatible with the full-cost principle, and arrived at by means of marginal analysis.

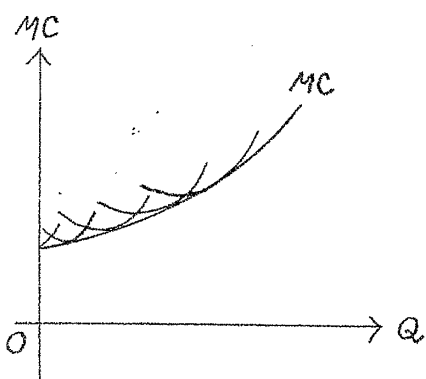
1.



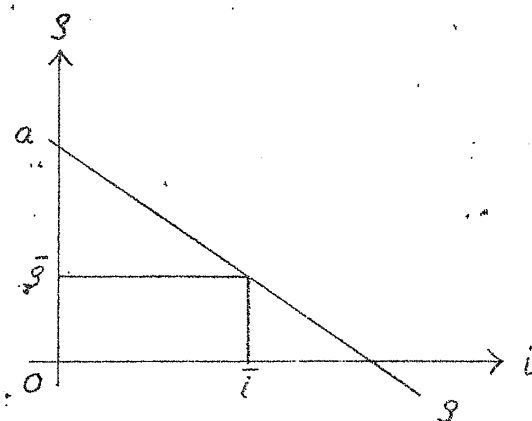
2.



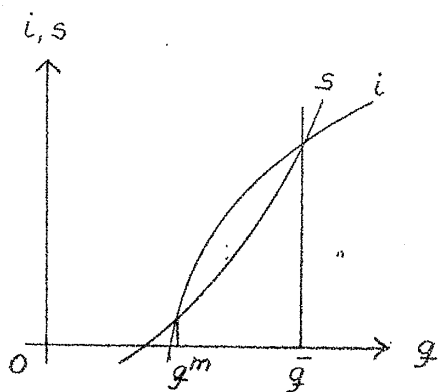
3.



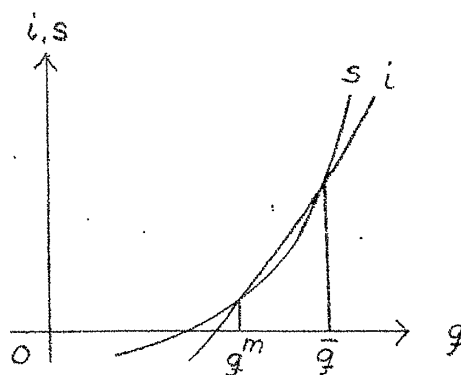
4.



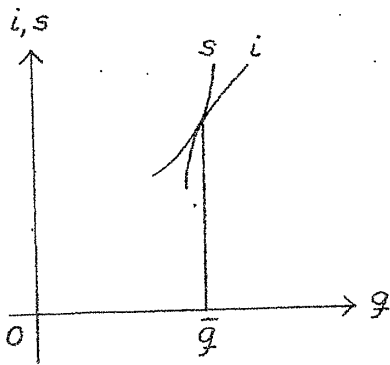
5 a.



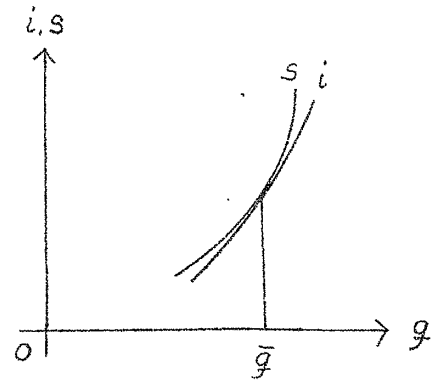
5 b.



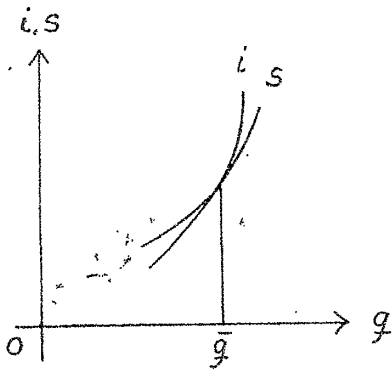
6a.



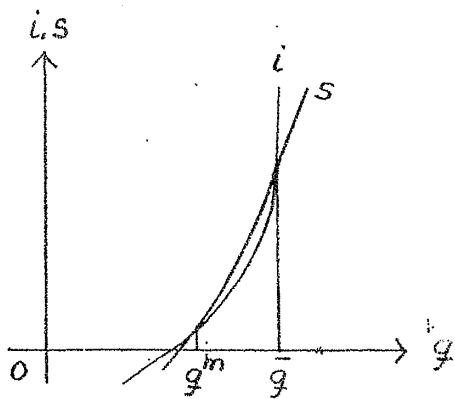
6b.



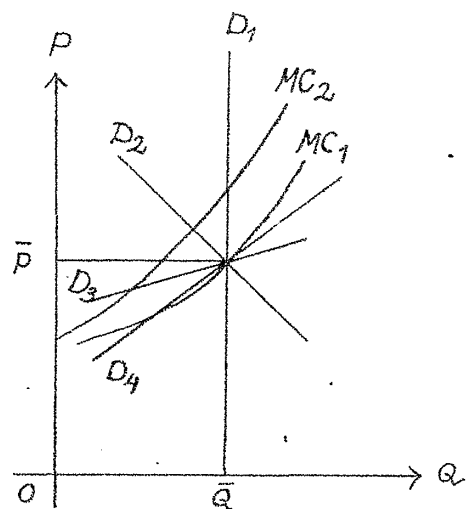
6c.



7.



8.



IVA5a 1964 13895.3

Suomen

Suomen pankin taloustieteellisen
tutkimuslaitoksen julkaisuja ; D
Paunio, J.J.

Adjustment of prices to wages.

1996-02-28

SUOMEN PANKIN
KIRJASTO