

SUOMEN PANKKI
KASSA

A Study of Housing Investment and Housing Market Behaviour

By
Olavi Rantala
Lic.Soc.Sc.

Academic dissertation to be publicly discussed, by permission of the Faculty of
Social Sciences of the University of Helsinki, in Auditorium III, Porthania,
Hallituskatu 11—13, on May 16, 1986, at 10 o'clock a.m.

Helsinki 1986

Olavi Rantala

A Study of Housing Investment and Housing Market Behaviour

Bank of Finland

Helsinki 1986

B:40

Olavi Rantala
A Study of Housing Investment
and Housing Market Behaviour

ISBN 951-686-116-4

Bank of Finland Publications, Series B:40

ISSN 0357-4776

CONTENTS

		page
1	INTRODUCTION	7
2	DEMAND FOR HOUSING IN THE LIFE CYCLE MODEL OF CONSUMER BEHAVIOUR	11
2.1	Introduction	11
2.2	A finite horizon model	14
2.3	An infinite horizon model	18
2.4	The effects of transaction and moving costs on the demand for housing and saving	20
2.5	The effect of progressive income taxation on home-ownership and tenure choice	23
3	DEMAND FOR HOUSING AND PORTFOLIO CHOICE UNDER UNCERTAINTY	27
3.1	Introduction	27
3.2	The effect of asset price uncertainty on consumption and demand for housing	28
3.3	The implications of housing as a necessity for the portfolio distribution	36
3.4	Demand for housing under inflation and income uncertainty	38
3.5	Hedging against inflation risk	48
3.6	The determination of housing prices and rents in a partial equilibrium	49
4	CREDIT AND RENTAL MARKET IMPERFECTIONS AND THE HOUSING INVESTMENT PROCESS	53
4.1	Introduction	53
4.2	Tenure choice in imperfect capital and rental markets	56

4.3	The effect of an expected borrowing constraint on consumption and demand for owner-occupied housing stock	59
4.4	The demand for rental housing and saving for a house purchase	65
4.5	Implications for the household portfolio distribution	69
5	HOUSING MARKET ADJUSTMENT UNDER PERFECT FORESIGHT CONDITIONS	73
5.1	Introduction	73
5.2	Consumer behaviour and price determination on the demand side of the housing market	74
5.3	Flow supply in the housing market	79
5.4	The dynamics of price and quantity adjustment in the housing market	80
5.5	Changes in the steady state housing stock and house prices	84
5.6	Empirical implications	87
6	CONCLUDING REMARKS AND A REFERENCE TO FINNISH HOUSEHOLDS' PORTFOLIO DISTRIBUTIONS	90
	APPENDICES	95
	BIBLIOGRAPHY	113

ACKNOWLEDGEMENTS

While working on this study I have benefited from discussions with numerous people to whom I am greatly indebted. My official examiners Vesa Kanninen and Urho Lempinen made useful suggestions for the writing of this final manuscript. Erkki Koskela has kindly devoted a great deal of time to reading the preliminary drafts. Discussions with members of the research staff of the Bank of Finland Economics and Research Departments have been very useful.

William Moore corrected the English. Pirjo Föhr typed the final manuscript and Tuula Naskali the preliminary drafts. Vuokko Varis drew the figures. Antero Arimo and Marja Hirvensalo organized the printing work. I am grateful for all their help. I should also like to thank the Bank for publishing this study. The financial support of the Yrjö Jahnsson Foundation is gratefully acknowledged. Finally, I should like to thank my wife, Sari, for her understanding and support.

Kerava, March 1986

Olavi Rantala

1 INTRODUCTION

Modern analyses of consumer behaviour are generally based on the hypothesis of intertemporal decision making. The life cycle model assumes that consumption and saving decisions are determined via a planning procedure where consumers maximize the expected utility of future consumption subject to an intertemporal wealth constraint. The important role of saving in the optimal intertemporal allocation process was already considered by Fisher (1930) and in a rigorous mathematical model by Ramsey (1928). However, the present dominant status of the life cycle model in consumer theory results mainly from the studies of Modigliani and Brumberg (1955), Ando and Modigliani (1963), Yaari (1964), Cass and Yaari (1967), Tobin (1967), and Arrow and Kurz (1969).

Life cycle theory has turned out to be a fruitful basis for economic analysis because it can easily be modified to explain durable consumption, dynamic portfolio choice, and labour supply decisions. The life cycle model also gives a framework for a microeconomic analysis of the demand for housing.

Chapter 2 of this study deals with a fairly conventional life cycle model where housing stock represents a durable good. An implication of this model is that the expected appreciation of housing prices may have a positive or a negative effect on housing investments. The possibility of a negative capital gain effect has been overlooked in those studies which have somewhat unfoundedly been based only on the positive effect, and which have assumed that the conventional asset market approach is applicable to the housing market as well (e.g. Sheffrin (1983 p. 170) and Poterba (1984)). In Chapter 2 we also analyze the effects of moving and transaction costs on housing investments and saving. Moreover, we discuss the effects of progressive income taxation on home-ownership and tenure choice.

Housing investment behaviour under uncertainty is a rather unexplored subject in economics. The fact that a considerable proportion of households' wealth is generally invested in houses and other durable goods has not aroused the interest of portfolio theorists in modifying the conventional portfolio theory. Since the pioneering studies of Merton (1969, 1971, 1973) dynamic portfolio theory has generally been confined to deriving rules for optimal portfolio choice between financial assets which yield utility only indirectly, unlike consumer durables.

In Chapter 3 we analyze portfolio models where housing stock is both an asset and a durable good, so that the portfolio distribution depends on housing preferences in addition to the return and risk factors which play the central role in the conventional portfolio theory. The aim of this analysis lies in finding out whether the implications of the standard portfolio model are affected in any way as a result of such a change in consumers' investment opportunities.

The first model in Chapter 3 describes the effects of asset price uncertainty and necessity of housing on the portfolio share of housing property. In the second model the housing prices and consumer prices and consumers' income are stochastic. This model shows that a positive correlation between housing prices and consumer prices generally has a negative impact on the demand for housing services, which diminishes the efficiency of the housing stock to serve as a hedge against inflation risk as compared to the standard portfolio model. The inflation risk also influences the equilibrium rental price of housing services so that the housing property does not necessarily earn a risk premium unlike in the model of Ioannides and McDonald (1982), where the consumer price level was assumed to be constant.

Chapter 4 deals with the effects of borrowing constraints on the housing investment process and households' portfolio distribution. Housing investments may be very sensitive to such imperfections in the capital market because they generally require a considerable amount of external financing.

There seem to be relatively few studies of the effects of capital market imperfections on housing investments. Artle and Varaiya (1978) incorporated a borrowing constraint in a life cycle model where a consumer plans an optimal saving path so as to accumulate a down-payment of an exogenous housing investment target. An analytically similar model has also been presented by Jackman and Sutton (1982). The exogeneity of the housing investment is a questionable assumption but it may perhaps be interpreted as a reflection of the indivisibility of houses. By contrast, Ranney (1981) studied a life cycle model where the housing investment is optimized endogenously subject to a down-payment constraint and the non-negativity of savings. These constraints were, however, assumed to be binding only at the beginning of the planning horizon.

Chapter 4 presents a model which combines some aspects of the aforementioned models. It aims at taking account of the indivisible nature of housing stock while at the same time allowing for its endogeneity. In contrast to Ranney's model this study deals with the effects of an expected future liquidity constraint on the optimal saving for a house-purchase. It also emphasizes the role of borrowing constraints in connection with the indivisible nature of houses in explaining that home-ownership tends to concentrate among wealthy households.

The fourth main theme in this study, and the subject of Chapter 5, is the aggregate behaviour of the housing market. This means an analysis of the short and long run responses of housing prices, new housing production, and housing stock to demand and supply shocks. New housing production is modelled on the basis of profit maximization by the residential constructor firms. The demand for housing is based on households' utility maximization. This is important because for a certain class of preferences the expected appreciation of housing prices may have a negative effect on the demand for housing stock. Therefore the standard asset market approach is applicable to the housing market only under certain preconditions.

The purpose of this study is to analyze housing investments and housing market behaviour mainly from the point of view of owner-occupiers' portfolio decisions. This may be the most apparent difference as compared with earlier Finnish studies such as Loikkanen (1982), which was a search theoretic analysis of demand for rental housing and mobility decisions. The portfolio theoretic approach of this study is also a distinctive feature as compared with the deterministic and more empirically-oriented studies of Kosonen (1984) and Salo (1984).

2 DEMAND FOR HOUSING IN THE LIFE CYCLE MODEL OF CONSUMER BEHAVIOUR

2.1 Introduction

The first issue of this study concerns the determination of consumption and the demand for housing in the continuous-time life cycle model under perfect foresight conditions. In this microeconomic framework we shall study the effects of changes in permanent income, wealth, prices, and interest rate on the demand for non-durable consumer goods and houses.

Moreover, we shall discuss some of the special characteristics which distinguish housing property from other consumer durables. Thus, we shall be analyzing the effects of transaction and moving costs induced by a change of residence. This chapter consists also of a brief analysis of consumers' tenure choice in circumstances where income taxation is progressive and interest costs of housing loans are tax-deductible.

Life cycle theory implies that saving or borrowing serves as a medium to transfer wealth in time in a way which enables the consumer to allocate consumption optimally over the planning horizon. Under conditions of perfect competition the consumer is a price-taker in the goods market and in the capital market. If perfect foresight is assumed, there is only one financial asset in the capital market or the returns on alternative financial assets must be equal in the market equilibrium.

The financial asset serves as a saving device and as a debt instrument which enables the consumer to even out the effects of changes in income flow on consumption. If the consumer has human wealth in terms of discounted expected permanent future earnings, he can freely choose a position with positive financial wealth or a

negative amount of financial wealth by borrowing up to the full value of his human wealth.

The following model is a generalization of the normal life cycle model of non-durable consumption in the sense that the stock of a durable good, the housing stock $H(t)$, is included in consumer's utility function and in his budget and wealth constraints. The material wealth, $W(t)$, at time t is assumed to consist of net financial assets, $A(t)$, and the value of the housing stock, $Q(t)H(t)$, so that

$$(2.1) \quad W(t) = A(t) + Q(t)H(t)$$

The model takes the non-durable consumption as the numéraire good so that $Q(t) = P_h(t)/P(t)$ denotes the relative price of houses where $P_h(t)$ is the absolute price level and $P(t)$ denotes the consumer price level. Correspondingly $W(t) = \tilde{W}(t)/P(t)$ and $A(t) = \tilde{A}(t)/P(t)$ represent the real value of nominal material wealth, $\tilde{W}(t)$, and respectively that of nominal financial assets, $\tilde{A}(t)$, in terms of consumer goods.

The accumulation of material wealth, $W(t)$, and non-durable consumption, $C(t)$, are financed by permanent real wage income $Y = \tilde{Y}(t)/P(t)$, by real interest income, and by the eventual capital gains resulting from changes in the relative price of houses. The change in wealth as expressed in continuous-time by the time derivative $\dot{W}(t) = dW(t)/dt$ is defined in the following equations (for the derivation see Appendix 1a).

$$(2.2a) \quad \dot{W} = \dot{A} + Q\dot{H} + \dot{Q}H = rA + (q-\delta)QH - C + Y$$

$$(2.2b) \quad = rW - RH - C + Y$$

Equation (2.2b) is obtained by substituting $A=W-QH$ for financial assets in (2.2a). The real rate of interest on financial assets, r , is in perfect foresight conditions defined as the difference between the nominal interest rate, i , and the rate of inflation, $p = \dot{P}/P$, so

that $r=i-p$. The relative user cost of housing stock is denoted by $R(t)=(r-q+\delta)Q(t)=(i-p_h+\delta)Q(t)$ where $q=\dot{Q}/Q=p_h-p$ and $p_h=\dot{P}_h/P_h$ and δ denotes the deterioration rate of the housing stock. Thus, the user cost per unit of the housing stock increases due to nominal interest loss, i , and deterioration, δ , and decreases by the appreciation rate of the housing stock, p_h . Variables Y , i , p , p_h and δ are assumed to be exogenous constants for the consumer.

The relative user cost, $R(t)$, is allowed to change in time because the model is purely microeconomic and in this sense comparable to other neoclassical models of durable consumption which study an individual consumer's reactions to changes in relative prices (c.f. Diewert (1974) or Deaton and Muellbauer (1980)). On the other hand, in a steady state equilibrium the relative price of houses is by definition constant, as will be the case in the housing market model of Chapter 5 in this study.

According to the usual custom in life cycle theory it is assumed that the consumer has a strictly concave utility function, which in this case is defined for non-durable consumption and housing stock $U(C(t),H(t))$. Thus it is assumed that the service flow produced by the housing stock is proportional to the stock. It seems natural to impose additional conditions for the marginal utilities so that feasible non-durable consumption and housing stock are bounded strictly positive (c.f. Hu (1980)).¹ The instantaneous utility function is assumed to be discounted by an exponential function $e^{-\rho t}$ where the rate of time preference, ρ , is a positive constant. Alternatively, the rate of time preference could be assumed to be a function of the instantaneous level of utility, as Uzawa (1968) has postulated, but that complication will not be introduced here.

The model could easily be extended to explain labour supply behaviour by including leisure time in the utility function so that wage income would be determined endogenously (c.f. Blinder (1974)

¹ $\lim_{C \rightarrow 0} U_C = \infty$, $\lim_{C \rightarrow \infty} U_C = 0$, $\lim_{H \rightarrow 0} U_H = \infty$, $\lim_{H \rightarrow \infty} U_H = 0$.

or Ghez and Becker (1975)). However, this would not essentially change the implications of the model. Therefore it is assumed that variable Y represents exogenous constant non-capital income.

2.2 A finite horizon model

In the case of a finite and fixed planning horizon the continuous time life cycle model under certainty can be formulated as a problem in which the consumer maximizes utility subject to an intertemporal wealth constraint of the planning horizon. The length of the planning horizon, T , may be interpreted as the remaining lifetime of the consumer. Thus the consumer's objective is to

$$(2.3a) \quad \text{Max}_{C,H} \int_0^T e^{-\rho t} U(C,H) dt$$

subject to

$$(2.3b) \quad \int_0^T e^{-rt} (C+RH) dt = W_0 + \int_0^T e^{-rt} Y dt - e^{-rT} W_T$$

Intertemporal wealth constraint (2.3b) is a definite solution to the flow budget constraint (2.2b) so that W_0 denotes the predetermined initial wealth at time $t=0$ and W_T represents an exogenous bequest at time $t=T$. Alternatively, the model could be closed by specifying a boundary utility function for the bequest in which case the final wealth, $W(T)$, would be determined endogenously via a transversality condition.

Some of the earlier models of the demand for consumer durables were based on static demand functions of the desired stocks of durables. The static demand functions were then completed by the hypotheses of partial adjustment to give an explanation for the empirical observation that the stocks of durables seem to react only gradually for example to changes in income and relative prices. Such models were constructed for instance by Stone and Rowe (1957)

and by Muth (1960). The partial adjustment hypothesis has also been applied in several econometric studies of durable consumption and housing investments.

This model differs essentially from that kind of a behavioral mode by allowing for a discrete jump in the housing stock whenever any change in the exogenous variables takes place. This is, of course, based on the assumption of perfect markets, which means that the marginal costs of housing investments as well as the marginal costs of financing these investments are exogenous for any individual consumer.

On the other hand, it may be the case that for the aggregate household sector the stock supply of some durable goods, such as houses, is momentarily completely inelastic with respect to the market price. Thus, if the housing market clears at the aggregate level, the price level of housing stock jumps discretely as a reaction to unexpected exogenous demand or supply shocks. The resulting change in new housing production then gradually adjusts the stock supply towards a new steady state level in a way which resembles the microtheoretic partial adjustment behaviour. Chapter 5 of this study deals with this macrotheoretic explanation as an alternative for the microtheoretic partial adjustment hypothesis.

In this microeconomic model the initial wealth is predetermined, $W(0)=W_0$, but the portfolio distribution between housing stock and financial assets can be changed immediately after an unexpected change in any exogenous variable because the marginal investment and financing costs are exogenous for the consumer. This allows interpreting of both the non-durable consumption, $C(t)$, and the housing stock, $H(t)$, as the control variables and net wealth, $W(t)$, as the state variable of the optimal control problem. Thus the Hamiltonian, F , with costate variable $\lambda(t)$ can be defined as follows

$$(2.4) \quad F = e^{-\rho t} U(C, H) + \lambda(rW - RH - C + Y)$$

The necessary optimality conditions are (e.g. Kamien and Schwartz (1981))

$$(2.5a) \quad F_C = e^{-\rho t} U_C - \lambda = 0$$

$$(2.5b) \quad F_H = e^{-\rho t} U_H - R\lambda = 0$$

$$(2.5c) \quad F_W = r\lambda = -\dot{\lambda}$$

Conditions (2.5a-b) imply that the marginal rate of substitution between demand for housing services and non-durable consumption equals the relative user cost of the housing stock

$$(2.6) \quad U_H/U_C = R(t) = (r-q+\delta)Q(t)$$

Next we specify the utility function in order to find a closed-form solution for the model. For that purpose we assume the following utility function which will be employed throughout this study

$$(2.7a) \quad U(C,H) = a^{-1}(C-C^*)^{\alpha a}(H-H^*)^{\beta a} \quad \text{if } a < 0, 0 < a < 1 \\ \alpha, \beta > 0, 0 < \gamma = \alpha + \beta < 1$$

$$(2.7b) \quad = \alpha \ln(C-C^*) + \beta \ln(H-H^*) \quad \text{if } a = 0$$

This utility function is a generalization of the familiar Stone - Geary function in the sense that it covers both bounded, $a < 0$, and unbounded, $0 < a < 1$, functional forms. C^* and H^* may be interpreted as subsistence levels which consumption and housing stock must exceed, $C > C^*$ and $H > H^*$. The subsistence levels are assumed to be non-negative, $C^* \geq 0$ and $H^* \geq 0$, to avoid additional non-negativity constraints for consumption and housing stock.

Utility function (2.7a-b) and the expression for the marginal rate of substitution (2.6) give a linear relationship between the non-durable consumption and the housing stock. It implies that the demand for housing is increasingly substituted by non-durable consumption the higher the relative user cost is. Hence, denoting the initial value of the relative user cost by $R_0 = (r-q+\delta)Q_0$

$$(2.8) \quad C(t) = C^* + \alpha \beta^{-1} R_0 e^{qt} (H(t) - H^*)$$

This equation, utility function (2.7a-b), and optimality conditions (2.5a and c) yield the following time path for the housing stock where $H(0)$ denotes the initial stock and $\mu = (r - \rho - (1 - \alpha a)q) / \theta$, where $\theta = 1 - \gamma a > 0$ (see Appendix 1b)

$$(2.9) \quad H(t) = (H(0) - H^*)e^{\mu t} + H^*$$

The demand function of the housing stock can be solved by inserting (2.8) and (2.9) in (2.3b) and integrating which yields

$$(2.10) \quad H(0) = H^* + \frac{\beta \epsilon}{\gamma(1 - e^{-\epsilon T})R_0} (W_0 + W_h - e^{-rT}W_T - W^*)$$

where $\epsilon = (\rho - \gamma ar + \beta aq) / \theta$. W_h denotes human wealth as measured by the present value of permanent income, $W_h = \int_0^T e^{-rt} \gamma dt = \gamma(1 - e^{-rT}) / r$.

W^* represents "subsistence wealth" which is required to finance the present value of subsistence expenditure,

$$W^* = \int_0^T e^{-rt} (C^* + R(t)H^*) dt = C^*(1 - e^{-rT}) / r + R_0 H^* (1 - e^{-(r-q)T}) / (r - q).$$

The consumption function is determined by (2.8) and (2.10)

$$(2.11) \quad C(0) = C^* + \frac{\alpha \epsilon}{\gamma(1 - e^{-\epsilon T})} (W_0 + W_h - e^{-rT}W_T - W^*)$$

Accordingly, this model implies that the demand for housing and consumption are increasing functions of material wealth, W_0 , and human wealth, W_h , but decreasing functions of the bequest, W_T . The relative user cost, R_0 , has a negative effect on the demand for housing stock, but it may have a negative impact on consumption as well. This eventual negative effect comes via W^* if the subsistence housing stock is positive.

In this model the bequest consists of net financial assets and housing property, $W_T = A(T) + Q(T)H(T)$. The housing stock is naturally positive, although no specific constraints were assumed above for either component of the bequest. However, besides positivity of

housing stock, in practice there may be institutional constraints for financial liabilities as well, for example in the sense that in fact only positive wealth can be bequeathed. Such constraints may be quite restrictive as compared to the optimal borrowing path implied by the life cycle model, especially in the infinite horizon case. The only constraint for implicit bequests in the infinite planning horizon model in perfect foresight and perfect capital market conditions comes from the feasibility condition, $W(t) > -Y/r$, which excludes bankruptcy in terms of total material and human wealth (c.f. Arrow and Kurz (1969)).

2.3 An infinite horizon model

Even though consumers' lifetimes are finite, they may under certain conditions be interpreted as having infinite planning horizons. One condition is that each generation includes in its utility function along with its own consumption the utility of the next generation via a bequest motive. Then the utility functions of the consumers of the present generation are compound functions of all successive generations' utilities and consumptions, and consumers effectively have infinite planning horizons. Such a reasoning for an infinite horizon model has been presented for example by Barro (1974). On the other hand, Merton (1971) has shown that a consumer who expects to have an exponentially distributed uncertain lifetime behaves as if he had an infinite planning horizon. It is, of course, easy to criticize the infinite horizon model as being an oversimplistic description of consumer behaviour (c.f. Tobin (1980)).

However, despite the simplifying and restrictive assumptions, the infinite horizon life cycle model has some analytical advantages compared with the finite horizon model which make it worth studying. The infinite horizon model is sometimes easier to solve and it yields simpler behavioral equations than does the finite horizon model. Furthermore, the infinite horizon case presupposes such restrictions on exogenous variables as yield more clearcut comparative static implications than the corresponding finite horizon model. Since

there is no specific reason to confine the analysis to the finite horizon model this study concentrates mainly on the more easily tractable infinite horizon case.

As the previous model is rewritten in the infinite horizon case, the consumer's objective is defined so as to

$$(2.12a) \quad \text{Max}_{C, H} \int_0^{\infty} e^{-\rho t} a^{-1} (C - C^*)^{\alpha a} (H - H^*)^{\beta a} dt$$

s. t.

$$(2.12b) \quad \int_0^{\infty} e^{-rt} (C + RH) dt = W_0 + \int_0^{\infty} e^{-rt} Y dt$$

The present value of permanent income, Y , in intertemporal wealth constraint (2.12b) is well-defined only if $r > 0$. Moreover, for optimal paths (2.8) and (2.9), convergence of integral (2.12b) necessitates that $\rho > \gamma ar - \beta a q$ and also that $r > q$ if $H^* > 0$. Provided that these conditions are fulfilled the sufficiency of the optimal conditions is guaranteed, because then the maximal utility (2.12a) converges and it becomes an increasing and strictly concave function of wealth, as will be seen in the next section.

The optimal demand functions are now readily obtained from (2.10) and (2.11) by letting T go to infinity. The demand function of housing stock is

$$(2.13) \quad H(0) = H^* + \frac{\beta(\rho - \gamma ar + \beta a q)}{\gamma \theta R_0} (W_0 + \frac{Y - C^*}{r} - \frac{R_0 H^*}{r - q})$$

The consumption function is

$$(2.14) \quad C(0) = C^* + \frac{\alpha(\rho - \gamma ar + \beta a q)}{\gamma \theta} (W_0 + \frac{Y - C^*}{r} - \frac{R_0 H^*}{r - q})$$

The comparative static implications of these demand functions are presented in Table 2.1 for bounded, $a < 0$, and unbounded $0 < a < 1$ utility functions. Positive effect is denoted by (+), negative by (-), zero by (0), and ambiguous effect by (?).

Table 2.1 Effects of changes in exogenous variables on the demand for housing stock and non-durable consumption

	Y	W_0	Q_0	q	r	
H(0)	+	+	-	?	?	$a < 0, H^* > 0$
	+	+	-	+	?	$0 \leq a < 1, H^* = 0$
C(0)	+	+	-	?	?	$a < 0, H^* > 0$
	+	+	0	+	?	$0 \leq a < 1, H^* = 0$

The demand for housing stock is an increasing function of permanent income, Y , and wealth, W_0 , but a decreasing function of the relative price level of houses, Q_0 . A more important observation, perhaps, is that the expected capital gain, q , may have either positive or negative effect on the demand for the housing stock. The expected continuous appreciation of house prices influences positively the demand for the housing stock by lowering the user cost $R_0 = (r - q + \delta)Q_0$. The negative effect may, however, arise via term $\rho - \gamma ar + \beta a q$ if $a < 0$ or via term $-R_0 H^* / (r - q)$ if $H^* > 0$ (see Appendix 1c). Thus modelling the demand for housing stock should not be based solely on the user cost effect, as seems to have been done for example in the rational expectations model of the housing market presented by Sheffrin (1983 p. 170) and Poterba (1984). A more detailed discussion of this problem is presented in Chapter 5 of this study.

2.4 The effects of transaction and moving costs on the demand for housing and saving

One characteristic of housing investments that distinguishes them from the purchase of consumer goods is the considerable outlay necessitated by a change of residence. The following analysis illustrates the effects of both lump-sum type and proportional transaction and moving costs on the behaviour of a consumer as implied by the previous neoclassical model. For this purpose we shall compare the maximal derived utility with and without the transaction

and moving costs in order to see under what conditions it pays for the consumer to change the location of his residence as a reaction to changes in exogenous variables.

To simplify the exposition, we assume that $C^* = H^* = 0$. This does not affect the main implications of the model. By inserting the optimal paths $C(t)$ and $H(t)$ implied by equations (2.8), (2.9), and (2.13) into the objective function (2.12a) and integrating we end up with the derived utility function

$$(2.15) \quad J = \text{Max}_{C, H} \int_0^{\infty} e^{-\rho t} a^{-1} C^{\alpha} H^{\beta} dt = a^{-1} x R_0^{-\beta \alpha} (W_0 + Y/r)^{\gamma \alpha}$$

where $x = \gamma^{-\gamma \alpha} \theta^{\theta} \alpha^{\alpha} \beta^{\beta} (\rho - \gamma \alpha + \beta \alpha q)^{-\theta} > 0$. Thus the maximal utility is a strictly concave function of wealth. In the following (2.15) represents the derived utility in the case that the consumer does not move and therefore does not pay any transaction or moving costs.

Assume then that any change of residence induces a lump-sum moving cost, say Z . Whether it is worth paying it becomes an additional negative argument in wealth constraint (2.12b) and, consequently, in derived utility function (2.15).

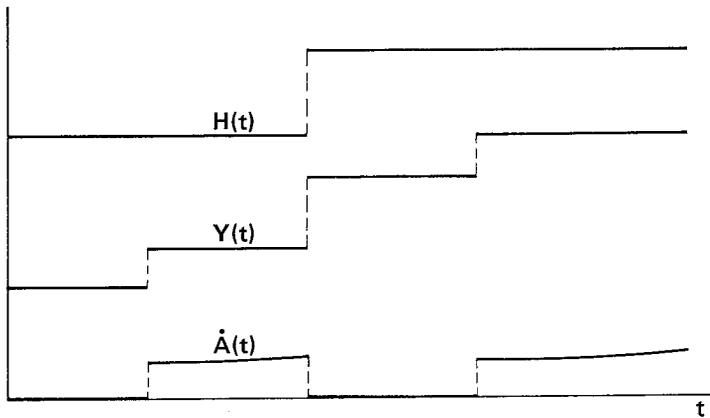
Consider then, for example, the consumer's reaction to an unanticipated increase in permanent income, ΔY , which increases the derived utility. If he would react by changing residence and paying the moving cost his derived utility function would change to form

$$(2.16) \quad J(Z) = a^{-1} x R_0^{-\beta \alpha} (W_0 - Z + (Y + \Delta Y)/r)^{\gamma \alpha}$$

Whether the consumer in fact reacts by moving and increasing his housing stock depends on whether this action leads to an increase in his derived utility so that $J(Z) > J$. By comparing (2.16) to (2.15) we can see that this can only happen if the present value of the increase in permanent income exceeds the moving cost so that $\Delta Y/r > Z$. On the contrary, the consumer does not make any discrete change in his demand for housing if $\Delta Y/r < Z$.

Figure 1.1 illustrates the time paths of the demand for housing, $H(t)$, and financial saving, $\dot{A}(t)$. Moreover, there is an income path, $Y(t)$, which is rising step-wise. However, the increments of permanent income are unanticipated so that only one step is observed at a time. Any change of residence induces a moving cost so that the demand for housing reacts only to sufficiently large unanticipated increases in income, such as the second one in Figure 1.1. The same holds true for non-housing consumption, because it depends positively on the demand for housing as implied by equation (2.8). Thus, contrary to the implication of the standard life cycle model, small increases in permanent income, $\Delta Y \ll rZ$, are all saved by increasing financial assets so that only the saving, $\dot{A}(t)$, jumps upwards as described in Figure 1.1. Moreover, in the finite horizon case the model would imply that such moving costs tend to prevent moving and increase financial saving especially among older people because the present value of any additional income decreases as the planning horizon gets shorter.

Figure 2.1: The reactions of housing stock and saving to increases in permanent income



In general, by comparing the relevant derived utility functions, J and $J(Z)$, we can study the consumer's reactions to changes in all other exogenous variables, W_0 , Q_0 , q , and r . The results are

qualitatively similar to those derived above in the sense that only sufficiently large changes in exogenous variables induce changes in housing stock. Smaller changes in the exogenous variables influence primarily the saving in financial assets and also consumption in the event of a change in the relative user cost, R_0 .

Assume next that buying a new housing unit also induces a proportional transaction cost, for example a stamp tax, denoted by z . If this transaction cost is paid, the effect is comparable to an increase in the housing price level so that the effective unit price becomes $(1+z)Q_0$. This price variable substitutes Q_0 in wealth constraint (2.12b). Thus the derived utility function (2.16) changes to form

$$(2.17) \quad J(Z, z) = a^{-1} x ((1+z)R_0)^{-\beta a} (W_0 - Z + (Y + \Delta Y)/r)^{\gamma a}$$

In this case the derived utility is a decreasing function of both the proportional transaction cost, z , and the lump-sum moving cost, Z . By comparing (2.17) to (2.15) we can see that in this case a sufficient increase in permanent income for a discrete reaction in housing stock to take place is $\Delta Y/r > Z + (W_0 + Y/r)((1+z)^{\beta/\gamma} - 1)$. Thus, raising the stamp tax, z , tends to inactivate the housing market in the sense that fewer households are simultaneously participating in the market and changing residence when permanent income level rises. The effect varies across consumers because it depends on their wealth, income, and preferences.

2.5 The effect of progressive income taxation on home-ownership and tenure choice

We have implicitly assumed that the consumer obtains the services of the housing stock by owner-occupancy. However, the neoclassical model described in sections 2.1-2.3 clearly implies that the consumer is indifferent between owning and renting the housing unit if its user cost equals its rental rate. In fact, the equilibrium rental rate of the housing stock, say \underline{R} , equals the user cost, R ,

under perfect market and perfect foresight conditions, so that
 $\underline{R} = (i - p_h + \delta)Q$.

However, in the real world conditions there may be several reasons for distortions in the perfect indifference property of tenure choice. Systematic differences in home-ownership and tenure choice between households at different income and wealth levels may arise from taxation. Recently this issue has received considerable attention in the literature (e.g. Weiss (1978), King (1980), Englund and Persson (1982), Feldstein (1982), Titman (1982), Hendershott and Hu (1983), and Kau and Keenan (1983)). The following case illustrates briefly the effects of progressive income taxation on home-ownership and tenure choice. In the following the progressivity is for the purposes of simplicity defined in terms of a continuously differentiable tax function, $T(\hat{Y})$, of the taxable income, \hat{Y} , so that the marginal tax rate is positive, $T' = dT/d\hat{Y} > 0$, and, moreover, the marginal tax rate is an increasing function of taxable income, $T'' = d^2T/d\hat{Y}^2 > 0$.

When the effects of the tax system on tenure choice are analyzed it is useful to separate the ownership and supply of the housing stock from the demand for housing services. In the following the former is denoted by H^0 and the latter by H . Thus the consumer's material wealth is $W = A + QH^0$. His taxable income consists of wage income, Y , and of rental return on housing stock, $\underline{R}H^0$, but it is reduced by the tax-deductions of housing property, DH^0 , so that $\hat{Y} = Y + (\underline{R} - D)H^0$. Thus Hamiltonian (2.4) can be rewritten as follows

$$(2.18) \quad F = e^{-\rho t} U(C, H) + \lambda [rW + (\underline{R} - R)H^0 - \underline{R}H - C + Y - T(Y + (\underline{R} - D)H^0)]$$

In this case the consumer's wealth increases among other things by rental return on housing property, $\underline{R}H^0$, and decreases due to the cost of owning the housing stock, RH^0 , as well as due to rental expense on housing services, $\underline{R}H$, and taxes, $T(\hat{Y})$.

The necessary condition for owning housing property is that it yields at least as much as the alternative asset, which in this

model is the financial asset. At the break-even point condition $\partial F/\partial H^0=0$ holds. Applying this condition to (2.18) and solving for the required rental return, \underline{R} , gives

$$(2.19) \quad \underline{R} = (R-T'D)/(1-T')$$

where $R = (i-p_h+\delta)Q$.

In general the ownership of the housing stock tends to concentrate among those households who break even at the lowest level of rental return. The required return, \underline{R} , may depend on income level via the marginal tax rate, T' , because from (2.19) $\partial \underline{R}/\partial Y = T''(R-D)/(1-T')^2$. This implies that the income level has no effect on \underline{R} in the case that all costs are tax-deductible and capital gains are taxed so that $D=R=(i-p_h+\delta)Q$. Otherwise, if $D \neq R$, the progressive tax system is non-neutral in the sense that home-ownership and tenure choice depend on households' income Y .

In practice such a non-neutrality is quite possible because the capital gains are usually exempted from income taxation at least after a fixed minimum holding period. If interest costs are tax-deductible, so that $D=iQ$, the income effect is negative, $\partial \underline{R}/\partial Y < 0$, if $R-D = (-p_h+\delta)Q < 0$. This induces systematic differences in tenure choice between households at different income levels. High-income earners with high marginal tax rates tend to own the housing stock while low-income earners prefer tenancy because they benefit less from the tax-deductibility of interest payments.

The existing Finnish tax laws allow home-owners to deduct interest costs from taxable income only up to a fixed upper limit. On the other hand, it has recently been suggested in Finland that interest costs should be made deductible from taxes instead of from taxable income. Both the present limitation of interest deductions and the suggested reform are meant to restrict the high-income earners' benefits from interest deductions. They are, however, also non-neutral in the sense that they leave the required rental price of housing dependent on owners' marginal tax rate.

In addition to the progressive income taxation there may also be other reasons for systematic differences in tenure choice. Non-linear productivity and non-linear costs of capacity utilization of the housing stock may induce an externality associated with tenancy that works in favour of ownership, as has been shown by Henderson and Ioannides (1982).

This study emphasizes the role of imperfections in the capital market in forcing the less wealthy households towards tenancy. This case is studied in Chapter 4. One further reason for systematic dependence of home-ownership on households' wealth is uncertainty about housing prices. This subject will be treated in the following chapter.

3 DEMAND FOR HOUSING AND PORTFOLIO CHOICE UNDER UNCERTAINTY

3.1 Introduction

Housing stock is generally one of the most important assets in household portfolios. Yet the special implications of housing stock being both an asset and a durable consumption good for the results of portfolio theory have received relatively little attention in the literature.

One of the main themes in this study is consumers' demand for housing under uncertainty. The effects of uncertainty can be studied by combining the previous neoclassical model of demand for housing with the conventional dynamic portfolio theory. The following analysis concerns uncertainty about asset prices, inflation, and income changes. However, including all these random factors in one model is quite a complicated procedure. Therefore the problem is divided into two cases.

In the first of these, the effects of asset price uncertainty are studied in a model which contains a portfolio of risky assets, housing stock, and consumption as the consumer's decision variables. Furthermore, the implications of housing as a necessity for the portfolio distribution are illustrated. In the second case the effects of random inflation and random price and rent of housing stock and of uncertain income changes are studied in a model which assumes identical preferences and rational expectations for consumers. In this case the housing market equilibrium is determined simultaneously with the individual portfolio distributions.

3.2 The effect of asset price uncertainty on consumption and demand for housing

The first topic in this chapter concerns the effect of asset price uncertainty on non-durable consumption and demand for housing. This will be studied by combining the deterministic neoclassical model of demand for housing from Chapter 2 with the conventional dynamic portfolio theory.

Another issue concerns the effect of housing as a necessity on consumers' portfolio structure. Econometric research into the demand for housing on the basis of the linear expenditure system derived from the Stone - Geary utility function has given support to the hypothesis that housing is a necessity. In other words, the income elasticity of the demand for housing seems to be between zero and one. This sort of result has been obtained from time series data by Deaton (1975) among others, and from cross section analysis, for instance by Olsen and Barton (1983).

The static linear expenditure system typically defines income or total expenditure as the scale variable in the commodity demand functions. In a dynamic model, such as the following is, the natural counterpart for income is the consumer's wealth. Furthermore in a dynamic model the demand for housing is two dimensional. If the service flow of a housing unit is obtained by ownership, the housing stock is required for consumption and for investment purposes. Thus, if housing is a necessity, the portfolio share of the housing stock is a decreasing function of the consumer's wealth in a similar way as in the static demand theory, where necessity would imply a decreasing average propensity to spend on housing (c.f. Green (1976)).

In terms of the Stone-Geary utility function the necessity means that there is a lower bound or a subsistence level which the consumer's housing stock must exceed. In other words, there is a kind of indivisibility which limits a usable housing stock to being a large unit as compared with other consumer goods.

The importance of this hypothesis depends, of course, on the value of the subsistence housing stock as compared with the consumer's wealth. This kind of indivisibility may be a particularly relevant property if there are capital market imperfections which prevent some consumers from borrowing enough to be able to exceed the subsistence level. This topic will be treated in Chapter 4. It is nevertheless useful to investigate the meaning of the hypothesis first under perfect capital market conditions.

The next model applies the same assumptions about financial assets as the conventional dynamic portfolio theory. In other words, it is assumed that consumers can choose between a riskless financial asset yielding an instantaneous nominal rate of interest equal to i , and n risky assets with instantaneous rates of return defined by stochastic processes.

It has become a common practice in dynamic portfolio models to specify the uncertainty of the market prices of risky assets by continuous-time stochastic Ito processes (c.f. Merton (1969, 1971, 1973), Fischer (1975), Breeden (1979), Chow (1979), Richard (1979)). In this case the instantaneous rate of return on the j th risky asset is

$$(3.1) \quad dP_{sj}/P_{sj} = g_j dt + \sigma_j dz_j$$

$P_{sj}(t)$ denotes the market price of the j th asset at time t . g_j denotes its expected percentage change and σ_j denotes the standard deviation of the return per unit of time. Increments of the Wiener process, dz_j , are temporally independent and normally distributed with zero mean and variance equal to dt .

In the following we assume that the expected returns and variances are constants, so that the price changes follow geometric Brownian motion and the market price levels are lognormally distributed and non-negative. According to the multiplication rule for Wiener processes the instantaneous covariance per unit time of the returns on two risky assets is $\sigma_{jk} = \sigma_j \sigma_k \rho_{jk}$ where ρ_{jk} is the correlation

per unit time between processes dz_j and dz_k (c.f. Kamien and Schwartz (1981) or Malliaris and Brock (1982)).

The percentage changes in the prices of both non-durable consumption and housing stock are at this stage assumed to be deterministic constants, $p=(dP/dt)/P$ and $p_h=(dP_h/dt)/P_h$ respectively. The relative price is denoted by $Q(t)=P_h(t)/P(t)=Q_0e^{qt}$ where $q=p_h-p$ as in Chapter 2. Thus the relative price is assumed to change continuously over time in a similar way as in the previous deterministic model.

The consumer's wealth, $W(t)$, consists of a riskless asset, housing property, and a portfolio of risky assets.

$$(3.2) \quad W(t) = A(t) + Q(t)H(t) + \sum_{j=1}^n G_j(t)S_j(t)$$

$A(t)$ denotes the value of the riskless asset deflated by consumer prices $P(t)$. $H(t)$ denotes the quantity of the housing stock. $S_j(t)$ is the number of shares of the j th risky asset in the portfolio, and $G_j(t)=P_{S_j}(t)/P(t)$ is the relative price of the j th risky asset.

The consumer's budget constraint is now changed from the deterministic case (2.2a-b) so that wealth increases also by the real capital gains on risky assets.

$$(3.3a) \quad dW = (rA + (q-\delta)QH - C + Y)dt + \sum_{j=1}^n \left(\frac{dP_{S_j}}{P_{S_j}} - \frac{dP}{P} \right) G_j S_j$$

$$(3.3b) \quad = (rW - RH + \sum_{j=1}^n (g_j-i)w_jW - C + Y)dt + \sum_{j=1}^n \sigma_j w_j W dz_j$$

The denotation is kept unchanged so that Y represents the real permanent income, $r=i-p$ is the constant real rate of interest, and $R(t)=(i-p_h+\delta)Q(t)$ is the relative user cost of the housing stock. g_j-i is the expected excess return on the j th risky asset and

$w_j = G_j S_j / W$ denotes the portfolio proportion of the j th risky asset. Equation (3.3b) is obtained by substituting stochastic process (3.1) for dP_{s_j} / P_{s_j} , and $A = W - \sum w_j W$ for the riskless asset in (3.3a).

In the present model there are altogether $n+2$ portfolio components. $n+1$ of these can be chosen independently under the wealth constraint (3.2). Denoting the vector of portfolio shares of risky assets by \underline{w} , the consumer's objective for an infinite planning horizon is defined so as to

$$(3.4) \quad \text{Max}_{\{C, H, \underline{w}\}} E_0 \int_0^{\infty} e^{-\rho t} U(C, H) dt$$

subject to (3.3b), $dR = qRdt$ and $W(0) = W_0$.

E_0 denotes the conditional expectations operator expressing that the maximal utility is conditional on the predetermined wealth W_0 .

The derived utility can be defined as a function of real wealth, $W(t)$, and the relative user cost of housing stock, $R(t)$. It is time dependent because the relative user cost is by assumption changing over time.

$$(3.5) \quad J(W, R, t) = \text{Max}_{\{C, H, \underline{w}\}} E_t \int_t^{\infty} e^{-\rho(\tau-t)} U(C, H) d\tau$$

The consumer's instantaneous utility function of consumption, C , and housing stock, H , is assumed to be the generalized Stone - Geary utility function

$$(3.6a) \quad U(C, H) = a^{-1} (C - C^*)^{\alpha a} (H - H^*)^{\beta a}, \quad a < 0, 0 < a < 1$$

$$\alpha, \beta > 0, 0 < \gamma = \alpha + \beta < 1$$

$$(3.6b) \quad = \alpha \ln(C - C^*) + \beta \ln(H - H^*), \quad a = 0$$

where $C > C^*$, $C > 0$ and $H > H^*$, $H > 0$. Positive subsistence levels $C^* > 0$ and $H^* > 0$ imply that consumption and housing are necessities, while if

$C^* < 0$ and $H^* < 0$ they are luxuries. The signs of these displacement parameters also indicate the consumer's attitude towards risk. Relative risk aversion is decreasing if $C^* + RH^* > 0$, constant if $C^* + RH^* = 0$, and increasing if $C^* + RH^* < 0$. Absolute risk aversion is always decreasing since $a < 1$.¹

In the following analysis we assume that housing is a necessity, $H^* > 0$, and, moreover, that $C^* \geq 0$ such that no additional non-negativity constraint for consumption is needed. In other words, the consumer is assumed to display decreasing relative risk aversion as well as decreasing absolute risk aversion.

In this infinite horizon autonomous problem the exponential time preference factor may be eliminated. Therefore, assuming utility function (3.6a), the Hamilton - Jacobi - Bellman equation can be formulated as follows

$$(3.7) \quad 0 = \phi(C, H, \underline{w}; W, R, t) =$$

$$\begin{aligned} \text{Max}_{\{C, H, \underline{w}\}} & [a^{-1}(C-C^*)^{\alpha a}(H-H^*)^{\beta a} + J_t + (rW - RH + \sum_{j=1}^n (g_j - i)w_j W - C + Y)J_W \\ & + qR J_R + \frac{1}{2} \sum_{j=1}^n \sum_{k=1}^n \sigma_{jk} w_j w_k W^2 J_{WW}] \end{aligned}$$

where the partial derivatives are $J_t = \partial J / \partial t$, $J_W = \partial J / \partial W$, $J_R = \partial J / \partial R$ and $J_{WW} = \partial^2 J / \partial W^2$.

This yields the first-order conditions

$$(3.8a) \quad \phi_C = \alpha(C-C^*)^{\alpha a - 1}(H-H^*)^{\beta a} - J_W = 0$$

¹Define the total expenditure as $E = C + RH$. Then the instantaneous indirect utility function is $\bar{U}(E; R) = a^{-1} \gamma^{\alpha} \alpha^{\alpha a} \beta^{\beta a} R^{-\beta a} (E - C^* - RH^*)^{\gamma a}$. In terms of total expenditure the absolute risk aversion is $A(E) = -\bar{U}''(E) / \bar{U}'(E) = (1 - \gamma a) / (E - C^* - RH^*)$ and the relative risk aversion is $R(E) = EA(E) = (1 - \gamma a)E / (E - C^* - RH^*)$. Therefore $A'(E) = -(1 - \gamma a) / (E - C^* - RH^*)^2 < 0$ and $R'(E) = -(C^* + RH^*) / (E - C^* - RH^*)^2 < 0$ if $C^* + RH^* > 0$ and $R'(E) > 0$ if $C^* + RH^* < 0$.

$$(3.8b) \quad \phi_H = \beta(C-C^*)^{\alpha a} (H-H^*)^{\beta a - 1} - R J_W = 0$$

$$(3.8c) \quad \phi_{W_j} = (g_j - i) W J_W + \sum_{k=1}^n \sigma_{jk} w_k W^2 J_{WW} = 0 \quad (j=1, \dots, n)$$

Condition (3.8c) expresses the optimal portfolio shares of risky assets as a system of linear equations. Hence the demand function of any particular risky asset can easily be solved in the following form, where v_{jk} denotes the elements of the inverted variance-covariance matrix (see Appendix 2).

$$(3.9) \quad G_j S_j = w_j W = -(J_W / J_{WW}) \sum_{k=1}^n v_{jk} (g_k - i)$$

The distribution of the risky portfolio is independent of individual preferences because for any asset the ratio $G_j S_j / \sum G_k S_k$ is independent of $-J_W / J_{WW}$. This implies that the separation property of portfolio choice as well as the capital asset pricing model hold in this case where consumer prices and the price of housing stock are assumed to be deterministic. Moreover, when closed form demand functions for consumer goods and housing are derived, the risky assets may without loss of generality be aggregated as a composite risky asset. The demand function of risky assets is

$$(3.10) \quad GS = \sum_{j=1}^n G_j S_j = -(J_W / J_{WW}) (g - i) / \sigma^2$$

The weighted average expected return on the composite risky asset is $g = \sum \hat{w}_j g_j$, and the variance is $\sigma^2 = \sum \sum \sigma_{jk} \hat{w}_j \hat{w}_k$. The weights $\hat{w}_j = G_j S_j / GS$ and $\hat{w}_k = G_k S_k / GS$ depend on the exogenous returns, risks, and covariances but they are independent of the consumer's preferences.

Optimality conditions (3.8a-b) yield the same linear marginal rate of substitution relationship between consumption and housing stock

that was derived above in (2.8). Moreover, we obtain the following equations between consumption, demand for housing, and the marginal derived utility of wealth (see Appendix 2)

$$(3.11) \quad C = C^* + \frac{1-\beta a}{\alpha} \frac{\beta a}{\theta} R - \frac{\beta a}{\theta} J_W - \frac{1}{\theta}$$

$$(3.12) \quad H = H^* + \frac{\alpha a}{\alpha} \frac{1-\alpha a}{\beta} \frac{\alpha a-1}{\theta} R - \frac{1}{\theta} J_W$$

where the auxiliary parameter is $\theta=1-\gamma a > 0$.

When the closed-form demand functions are solved the derived utility function must first be solved from optimality equation (3.7). In this model the derived utility function is

$$(3.13) \quad J = a^{-1} x R^{-\beta a} (W+W_h - W^*)^{\gamma a}$$

where the human wealth is $W_h = Y/r$ and the "subsistence wealth" is $W^* = C^*/r + RH^*/(r-q)$ (see Appendix 2). These variables are well-defined only if $r > 0$ and $r > q$, as in the deterministic model of Chapter 2. x is a constant which must be positive so that the derived utility (3.13) becomes a strictly concave function of wealth. These restrictions are also necessary for the convergence of the expected utility (3.5) and they constitute sufficient conditions for an optimum (c.f. Merton (1969)).

Given the solution for the derived utility function (3.13) the demand functions are determined by (3.10), (3.11), and (3.12). The consumption function is

$$(3.14) \quad C = C^* + \left(\frac{\alpha(\rho - \gamma ar + \beta a q)}{\gamma \theta} - \frac{\alpha a (g-i)^2}{2\theta \sigma^2} \right) (W + \frac{Y-C^*}{r} - \frac{RH^*}{r-q})$$

The demand for housing stock is

$$(3.15) \quad H = H^* + \left(\frac{\beta(\rho - \gamma ar + \beta a q)}{\gamma \theta R} - \frac{\beta a (g-i)^2}{2\theta^2 \sigma^2 R} \right) \left(W + \frac{Y - C^*}{r} - \frac{RH^*}{r-q} \right)$$

When these demand functions are compared with the corresponding deterministic demand functions, (2.13) and (2.14), we can see that uncertainty brings additional terms into the coefficients of disposable wealth which now depend on the market price of risk $(g-i)/\sigma$. The effects of asset price risk, σ^2 , and expected excess return on the risky portfolio, $g-i$, depend on the degree of relative risk aversion $\theta=1-\gamma a$. High degrees of relative risk aversion, $a < 0$, imply a negative elasticity of consumption and demand for housing with respect to σ^2 and a positive elasticity with respect to $g-i$, while low degrees of relative risk aversion, $0 < a < 1$, imply the opposite effects. In the event that the utility function is logarithmic, $a=0$, the asset price risk and excess return have no impact on consumption and demand for housing stock. The effects of other exogenous variables, Y , W , Q , q , and r are similar to those of the deterministic case that were presented above in Table 2.1.

The demand function of the composite risky asset is obtained from (3.10) by utilizing (3.13)

$$(3.16) \quad GS = \frac{g-i}{\theta \sigma^2} \left(W + \frac{Y - C^*}{r} - \frac{RH^*}{r-q} \right)$$

Accordingly, the demand for risky assets is an increasing function of wealth, W , and income, Y , and the excess return, $g-i$, but a decreasing function of the risk, σ^2 . Moreover, the expected appreciation of housing prices, q , has a negative impact on the demand for risky assets if housing is a necessity so that $H^* > 0$. The effect of real interest rate, r , is generally ambiguous.

Finally, we may note that the model is structurally fairly similar to the previous deterministic model. Therefore the earlier conclusions about consumers' tenure choice remain valid here as well. Moreover, the effects of transaction and moving costs are principally similar in

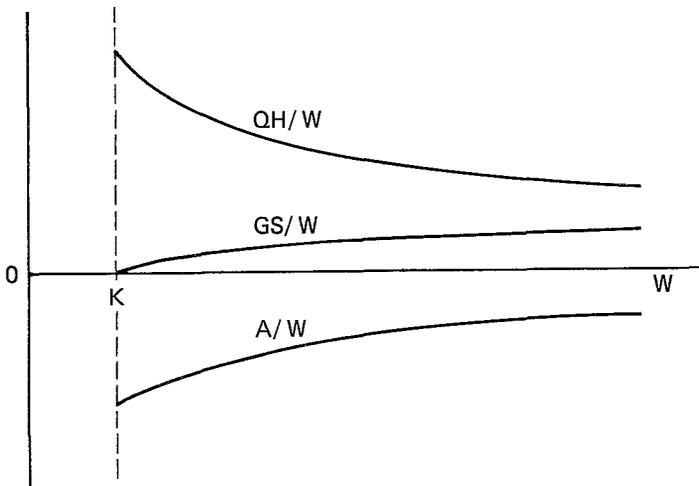
the two models. The main difference in this respect is that in the portfolio model the risky assets may also serve as a "buffer stock" against small changes in permanent income or other exogenous variables.

3.3 The implications of housing as a necessity for the portfolio distribution

As was mentioned above, econometric studies suggest that housing is a necessity in terms of the Stone - Geary utility function. This result is intuitively clear since the shelter produced by housing is naturally one of the basic needs of consumers. Next the implications of this property will be illustrated from the point of view of households' portfolio distribution. The previous model is suitable for this purpose.

Take the demand functions (3.15) and (3.16) of that model. Rewrite equation (3.15) in the form $QH = QH^* + h(W - K)$ where $K = W^* - W_h$ and assume that $QH^* > hK > 0$. The portfolio share of the owner-occupied housing stock, $QH/W = h + (QH^* - hK)/W$, is then a decreasing function of net wealth W . On the other hand, equation (3.16) shows that the portfolio proportion of risky assets GS/W is an increasing function of the consumer's wealth. These portfolio shares are demonstrated in Figure 3.1.

Figure 3.1: Portfolio distribution as a function of consumer's wealth



The portfolio share of the riskless asset A/W is drawn as a negative but increasing function of wealth in Figure 3.1. In this case the consumer invests more than his material wealth in housing stock and risky assets and finances the rest of the investments by borrowing.

The hypothetical portfolio shares drawn in Figure 3.1 qualitatively coincide with the corresponding empirical distributions implied by the Finnish cross-section data as depicted below in Figure 6.1 (p. 92), but only for households above the average household net wealth level. The average portfolio proportion of housing stock of these households seems to be a decreasing function of net wealth while the average portfolio share of securities increases as a function of wealth in the same way as the portfolio proportion of risky assets in Figure 3.1. The portfolio proportion of riskless assets of wealthier households seems to be negative but increasing as in Figure 3.1 if measured by the difference between bank deposits and loans.

On the other hand, the Finnish data indicate that among the households with wealth below the average level the portfolio proportion of housing property is an increasing function of wealth, and as a mirror image of that the housing loans also increase in relation to net wealth. In other words, taking the whole household sector into consideration there seems to be a kind of kinked shape in the portfolio share of housing stock and loans, as can be seen in Figure 6.1.

One reason for this phenomenon may be that consumers cannot borrow by means of the collateral of their human capital, the present value of their wage and transfer income, to the extent presupposed by the previously analyzed neoclassical theory with the assumption of a perfect capital market. This kind of loan market imperfection affects in the first place the less wealthy households and hinders their housing investments. A more detailed discussion of capital market imperfections will be presented in Chapter 4 of this study.

3.4 Demand for housing under inflation and income uncertainty

The next issue concerns the demand for housing and rental market equilibrium under uncertainty about housing and consumer prices and consumers' income flow. The effects of expected returns and risks on the portfolio share of housing stock are studied. Furthermore, the determination of the market price of housing stock and the rental price of housing services is analyzed in a rational expectations equilibrium state.

There seem to be relatively few studies about the demand for housing and the equilibrium rental rate under housing price uncertainty. The few exceptions are the discrete time two-period case analyzed by Henderson and Ioannides (1982) and the continuous time model constructed by Ioannides and McDonald (1982). These models assumed random housing prices but did not take into account the possibility of inflation uncertainty induced by unpredictable changes in consumer prices.

The inflation uncertainty may, however, considerably influence the portfolio share of housing property and the determination of the rental price of housing services. The following analysis shows that a conventional portfolio model where housing stock is a pure asset implies that sufficiently risk averse households may optimally hedge against inflation risk by investing in housing property. But housing stock is also used for consumption purposes. A positive correlation between the rate of inflation and changes in housing prices generally tends to reduce the demand for housing services. This negative demand effect works against the eventual positive supply effect so that hedging against inflation risk is a less likely motive for housing investments than the conventional portfolio theory implies. This was already noted in Rantala (1983), although with more restrictive assumptions than those applied below.

Inflation risk also influences the equilibrium rental price of housing services. The financial assets may become relatively more risky than housing property in real terms, so that the housing

stock does not necessarily earn a risk premium, in contrast to the situation in the model of Ioannides and McDonald (1982), where the consumer price level was assumed constant.

However, the assumption of random housing market prices and consumer prices complicates the analysis and necessitates simplifying assumptions elsewhere to make the model tractable. Hence it will be assumed that consumers' portfolios consist of only two assets, housing stock and a financial asset. For simplicity it will also be assumed that changes in income are proportional to wealth and that all consumers have constant relative risk aversion utility functions.

Besides, the assumption of continuous trading of housing stock and consumer goods in complete markets where prices are formed as continuous functions of time may in fact be too simplistic from an empirical point of a view. Nevertheless, modelling the rate of inflation as an Ito process is by no means exceptional, as can be seen from studies made for example by Fischer (1975), Gertler and Grinols (1982), and Poncet (1983). One of the advantages of this specification is that only two parameters, mean and standard deviation, are needed to completely characterize the random inflation process.

One of the central assumptions made in the following model is that all consumers are perfectly identical except possibly for their wealth. In other words, all consumers' preferences are similar and the changes in their income flows are equal in proportion to their wealth. Furthermore, they have rational expectations and they are perfectly aware of the equilibrium solution of the housing market given the values of exogenous and predetermined variables.

Since we are interested only in the equilibrium state of the market of housing services we may assume that the rent-price ratio of the housing stock is constant. Therefore the changes in both the market price of a unit of housing stock $P_h(t)$ and the corresponding rental rate $\tilde{R}(t)$ may be defined to follow the same Ito process

$$(3.17) \quad dP_h/P_h = d\tilde{R}/\tilde{R} = p_h dt + \sigma_h dz_h$$

where p_h is the expected percentage change in the price per unit time, σ_h denotes its standard deviation per unit time, and z_h is a Wiener process. This specification satisfies the assumption that the rent-price ratio, which is determined in the housing market equilibrium, is a constant, say $V = \tilde{R}(t)/P_h(t)$.

The Ito process for consumer prices $P(t)$ is defined as

$$(3.18) \quad dP/P = p dt + \sigma_p dz_p$$

where p is the expected rate of inflation per unit time, σ_p is the standard deviation of inflation per unit time, and z_p is a Wiener process.

The changes in nominal income $\tilde{Y}(t)$ are assumed to be proportional to nominal net wealth $\tilde{W}(t)$ and are specified as the following Ito process

$$(3.19) \quad d\tilde{Y} = \tilde{y}\tilde{W}dt + \sigma_y\tilde{W}dz_y$$

where \tilde{y} and σ_y are, respectively, the mean and standard deviation per unit time of the change in income in proportion to wealth, and where z_y is a Wiener process. This specification was also applied by Ioannides and McDonald (1982).

Consumers' portfolios are simplified compared with the previous model by assuming that they contain only two marketable assets. The wealth consists of financial asset $\tilde{A}(t)$ and housing property $P_h(t)H^0(t)$.

$$(3.20) \quad \tilde{W}(t) = \tilde{A}(t) + P_h(t)H^0(t)$$

The financial asset is assumed to yield riskless nominal interest rate i per unit time. A nominally riskless asset has also been introduced in other models of portfolio choice under inflation

uncertainty (e.g. Fischer (1975), Gertler and Grinols (1982), and Poncet (1983)). The instantaneous depreciation rate δ of the housing stock is also assumed to be deterministic in this model.

The representative consumer's budget constraint is defined as

$$(3.21a) \quad d\tilde{W} = (i\tilde{A} + \tilde{R}H^0 - \delta P_h H^0 - \tilde{R}H - PC)dt + d\tilde{Y} + \frac{dP_h}{P_h} P_h H^0$$

$$(3.21b) \quad = ((i+\tilde{y})\tilde{W} + (p_h - \delta - i + V)w\tilde{W} - \tilde{R}H - PC)dt + \tilde{W}(\sigma_h w dz_h + \sigma_y dz_y)$$

where $H(t)$ denotes the consumption of housing services which in principle is separable from the investment demand for the housing stock $H^0(t)$. $C(t)$ denotes the composite consumption of other non-durable consumer goods and services. Wealth constraint (3.20) has been utilized in equation (3.21b) to substitute for the financial asset. Furthermore, $w = P_h(t)H^0(t)/\tilde{W}(t)$ denotes the portfolio share of the housing stock. The accumulation equation implies that wealth increases due to income, capital gains, and rental income from housing property and decreases by expenditure on housing and other consumption.

It is convenient to solve the model in terms of relative prices by defining non-housing consumption as the numéraire good. The wealth constraint thus modified is

$$(3.22) \quad W(t) = \tilde{W}(t)/P(t) = A(t) + Q(t)H^0(t)$$

where $A(t) = \tilde{A}(t)/P(t)$ and $Q(t) = P_h(t)/P(t)$.

The deflated budget constraint can be derived from definitions (3.18) and (3.21b) by Ito's stochastic differentiation rule (see Appendix 3a). Thus

$$(3.23) \quad dW = ((r+y)W + (r_h + V)wW - RH - C)dt + W(\sigma_h w dz_h + \sigma_y dz_y - \sigma_p dz_p)$$

$r = i - p + \sigma_p^2$ may be defined as the expected real rate of interest under inflation uncertainty (c.f. Fischer (1975)). $y = \tilde{y} - \sigma_{yp}$ is the expected real change in income and $r_h = p_h - \sigma_{hp} - \delta - i$ denotes the expected real rate of appreciation of the housing stock less the unit interest cost. σ_{yp} denotes the instantaneous covariance per unit time between processes $\sigma_y dz_y$ and $\sigma_p dz_p$, and σ_{hp} is the instantaneous covariance per unit time between $\sigma_h dz_h$ and $\sigma_p dz_p$. The presence of the standard deviation of the rate of inflation in the expected real returns is basically due to the non-linear effect which the random consumer price level has via the denominator in the definition of real wealth (3.22).

The percentage change in relative housing prices, $Q(t)$, and relative rental rate, $R(t) = \tilde{R}(t)/P(t)$, is

$$(3.24) \quad dQ/Q = dR/R = qdt + \sigma_h dz_h - \sigma_p dz_p$$

where $q = p_h - p + \sigma_p^2 - \sigma_{hp}$ is the expected percentage change in the relative price (see Appendix 3a).

The representative consumer's objective for an infinite planning horizon is defined so as to

$$(3.25) \quad \text{Max}_{C, H, w} E_0 \int_0^{\infty} e^{-\rho t} U(C, H) dt$$

subject to (3.23), (3.24) and $W(0) = W_0$.

The corresponding derived utility function of real wealth and relative rental rate is in this case independent of explicit time because we assume for simplicity the constant relative risk aversion case of instantaneous utility function (3.6a-b) where $C^* = H^* = 0$. Thus

$$(3.26) \quad J(W,R) = \text{Max}_{C,H,w} E_t \int_t^{\infty} e^{-\rho(\tau-t)} a^{-1} C^{\alpha} H^{\beta} d\tau$$

The next step is to define the consumer's objective in terms of the Hamilton - Jacobi - Bellman optimality equation

$$(3.27) \quad 0 = \phi(C,H,w;W,R) =$$

$$\text{Max}_{C,H,w} [a^{-1} C^{\alpha} H^{\beta} - \rho J + (r+y)W + (r_h + V)w - RH - C] J_W$$

$$+ qR J_R + \frac{1}{2} (\sigma_h^2 W^2 + 2(\sigma_{hy} - \sigma_{hp})w + \sigma_y^2 + \sigma_p^2 - 2\sigma_{yp}) W^2 J_{WW}$$

$$+ \frac{1}{2} (\sigma_h^2 + \sigma_p^2 - 2\sigma_{hp}) R^2 J_{RR} + ((\sigma_h^2 - \sigma_{hp})w + \sigma_{hy} - \sigma_{hp} - \sigma_{yp} + \sigma_p^2) WR J_{WR}]$$

This gives the first-order conditions

$$(3.28a) \quad \phi_C = \alpha C^{\alpha-1} H^{\beta} - J_W = 0$$

$$(3.28b) \quad \phi_H = \beta C^{\alpha} H^{\beta-1} - R J_W = 0$$

$$(3.28c) \quad \phi_w = (r_h + V)W J_W + (\sigma_h^2 W + \sigma_{hy} - \sigma_{hp}) W^2 J_{WW} + (\sigma_h^2 - \sigma_{hp}) WR J_{WR} = 0$$

Optimality conditions (3.27) and (3.28a-c) yield the following consumption function, demand function of housing services, and portfolio share of housing stock where $\gamma = \alpha + \beta$ and $\theta = 1 - \gamma\alpha$ (see Appendix 3b).

$$(3.29) \quad C = \left(\frac{\alpha \varepsilon}{\gamma \theta} - \frac{\alpha a}{2} \sigma_h^2 W^2 \right) W = cW$$

$$(3.30) \quad H = \left(\frac{\beta \varepsilon}{\gamma \theta} - \frac{\beta a}{2} \sigma_h^2 W^2 \right) R^{-1} W = hR^{-1} W$$

$$(3.31) \quad w = \frac{QH^0}{W} = \frac{p_h^{-\delta-i+V-\beta a \sigma_h^2 - \alpha a \sigma_{hp} - \theta \sigma_{hy}}}{\theta \sigma_h^2}$$

The auxiliary parameter ϵ is the following function of exogenous constants

$$(3.32) \quad \epsilon = \rho - \gamma a(i + \tilde{y}) + \alpha a p + \beta a p_h - \frac{\alpha a(1 + \alpha a)}{2} \sigma_p^2 - \frac{\beta a(1 + \beta a)}{2} \sigma_h^2 + \frac{\gamma \theta a}{2} \sigma_y^2 \\ - \alpha \beta a^2 \sigma_{hp} + \beta \gamma a^2 \sigma_{hy} + \alpha \gamma a^2 \sigma_{yp}$$

The portfolio share of the housing stock, $w = QH^0/W$, and the rent-price ratio $V = R/Q = \tilde{R}/P_h$ are, however, still undetermined in the sense that they depend on each other, as can be seen from equation (3.31). In order to solve these endogenous variables separately as functions of purely exogenous factors the market equilibrium of housing services must be solved first.

As was mentioned above, all individual consumers are assumed to be perfectly identical except possibly for their wealth. This presumption was the basis for assuming that consumers expect the rental market to be in equilibrium and thus the rent-price ratio to be a constant V . But it means also that all consumers have equal marginal propensities to spend on housing, h , as well as equal portfolio shares of housing stock, w . Both of these endogenous factors are independent of wealth and depend only on the exogenous parameters due to the linear homogeneity of the demand functions with respect to wealth. In other words, the individual values of h and w hold at the aggregate level as well.

Aggregate demand equals aggregate supply in the market equilibrium of housing services. Suppose that the household sector consists of N consumers. Let H_j denote the demand for housing of the j th consumer, and let H_j^0 denote his supply of housing services, and let W_j be his net wealth, where $j=1, \dots, N$. Thus the aggregate demand for housing services is $\sum H_j$, the aggregate supply is $\sum H_j^0$ and the aggregate wealth of the household sector is $\sum W_j$. Then the market equilibrium of housing services is determined by the following system of equations.

$$(3.33a) \quad R\Sigma H_j = h\Sigma W_j$$

$$(3.33b) \quad Q\Sigma H_j^0 = w\Sigma W_j$$

$$(3.33c) \quad \Sigma H_j = \Sigma H_j^0$$

These conditions yield an equilibrium relationship between the portfolio share of the housing stock and the rent-price ratio $V=R/Q$

$$(3.34) \quad w = \frac{Q}{R} h = \frac{1}{V} \left(\frac{\beta \varepsilon}{\gamma \theta} - \frac{\beta a}{2} \sigma_h^2 w^2 \right)$$

On the other hand, the rent-price ratio can be solved from equation (3.31)

$$(3.35) \quad V = \frac{R}{Q} = i - p_h + \delta + (\beta a + \theta w) \sigma_h^2 + \alpha a \sigma_{hp} + \theta \sigma_{hy}$$

Substituting this expression for V in equation (3.34) yields finally the following quadratic equation for the optimal portfolio share of housing stock, $w=QH^0/W$, as an implicit function G of the exogenous variables

$$(3.36) \quad G = \mu \sigma_h^2 w^2 + (i - p_h + \delta + \beta a \sigma_h^2 + \alpha a \sigma_{hp} + \theta \sigma_{hy}) w - \frac{\beta \varepsilon}{\gamma \theta} = 0$$

where ε is defined in (3.32) and the auxiliary parameters are $\mu=1-\alpha a-\beta a/2>0$, $0<\gamma=\alpha+\beta\leq 1$ and $\theta=1-\gamma a>0$. The easiest way for comparative static analysis of the optimal portfolio share is to apply implicit function differentiation to equation (3.36). For example, $\partial w/\partial \sigma_h^2 = -(\partial G/\partial \sigma_h^2)/(\partial G/\partial w)$. The feasible optimal portfolio share of housing stock must naturally be the positive root $w>0$ of equation (3.36) for which $\partial G/\partial w>0$.

The signs of the partial derivatives of the optimal portfolio distribution with respect to the exogenous variables are collected in Table 3.1 for different degrees of relative risk aversion, $1-\gamma a$.

Risk aversion may be classified as being high if $a < 0$ and low if $0 < a < 1$.

For the sake of comparison it may be useful to present the signs of the effects of changes in the exogenous variables also in the case that the housing stock is a pure risky asset and does not yield directly any positive marginal utility. This case is obtained from equation (3.36) simply by setting $\beta = 0$ and $\alpha = 1$, which means that $H = U_H = 0$ and corresponds to an assumption that consumers have constant relative risk aversion utility functions defined only for the non-housing consumption $U(C) = a^{-1} C^a$. Accordingly, the representative consumer's housing stock is

$$(3.37) \quad \left. \begin{matrix} H^0 \\ | \\ H=0 \end{matrix} \right\} = \frac{p_h - \delta - i - a \sigma_{hp} - (1-a) \sigma_{hy}}{(1-a) \sigma_h^2} W$$

This is a quite normal portfolio model in the case of inflation uncertainty. The comparative static results of this reference case are also presented in Table 3.1 for different degrees of relative risk aversion, $1-a$.

Table 3.1: Effects of changes in exogenous variables on the portfolio share of housing stock as a durable good ($0 < \beta < 1$) and as an asset ($\beta = 0$)

	p_h	i	p	\tilde{y}	σ_h^2	σ_p^2	σ_y^2	σ_{hp}	σ_{hy}	σ_{yp}	
$w = QH^0/W$?	?	-	+	?	?	-	?	?	+	$a < 0$
$0 < \beta < 1$	+	-	0	0	-	0	0	0	-	0	$a = 0$
	+	-	+	-	-	-	+	-	?	+	$0 < a < 1$
$w = QH^0/W$	+	-	0	0	-	0	0	+	-	0	$a < 0$
$\beta = 0$	+	-	0	0	-	0	0	0	-	0	$a = 0$
	+	-	0	0	-	0	0	-	-	0	$0 < a < 1$

Most of the differences between the two cases are quite obvious. If the housing stock is a pure asset and $\beta=0$ the portfolio distribution is totally independent of the mean rates of inflation, p , and income changes, \tilde{y} , as well as of the standard deviations, σ_p and σ_y . This is a normal portfolio theoretic result. But it holds also in the event that housing stock is demanded for consumption purposes, $0 < \beta < 1$, if consumers' utility functions are logarithmic, $a=0$. Otherwise, if $0 < \beta < 1$ and $a \neq 0$, all exogenous factors including the parameters of the inflation and income processes that affect the marginal propensity to spend on housing h in demand function (3.30) via parameter ϵ defined in (3.32) also have an impact on the optimal portfolio share of the housing stock w .

When $0 < \beta < 1$, the capital gain and interest rate effects are generally ambiguous for high degrees of relative risk aversion, $a < 0$, in a similar way as they were ambiguous in the deterministic model of section 2.3. However, at lower levels of relative risk aversion, $0 < a < 1$, these effects are unambiguous, so that the optimal portfolio share of the housing stock depends positively on the expected rate of appreciation of housing prices, p_h , but negatively on the opportunity cost, i , and the risk associated with housing prices, σ_h^2 .

When $0 < \beta < 1$ and $a \neq 0$ the consumption of housing services depends on the means and standard deviations of inflation and income changes, as can be seen from equations (3.30) and (3.32). The directions of the effects depend on the degree of relative risk aversion, but for a given level of risk aversion inflation and income changes have opposite effects on the portfolio distribution via means, p and \tilde{y} , and standard deviations, σ_p and σ_y . Furthermore, a positive covariance between inflation and income changes, $\sigma_{yp} > 0$, increases the demand for housing services unless $a=0$.

3.5 Hedging against inflation risk

As was mentioned above, one of the issues in this chapter is consumers' hedging against inflation risk by investing in housing property. The behavioral mode is clearcut if housing stock is a pure asset and $\beta=0$. Then the hedging effect is positive if the covariance between inflation and changes in the price of housing stock is positive, $\sigma_{hp} > 0$, and if consumers' relative risk aversion is sufficiently high, $a < 0$.

But the fact that the housing stock is also required for consumption purposes, and $0 < \beta < 1$, brings about changes in consumers' behaviour. A positive covariance between changes in the two prices, $P_h(t)$ and $P(t)$, has in general a negative impact on the consumption of housing services, for as can be seen from equations (3.30) and (3.32) the marginal propensity to spend on housing, h , depends negatively on σ_{hp} unless $a=0$. Hence for high degrees of relative risk aversion, $a < 0$, the total impact on the portfolio share of the housing stock of a positive covariance between the rate of inflation and changes in the price of housing stock remains in general ambiguous, since it depends positively on the hedging effect and negatively on the consumption effect. In fact the positive hedging effect dominates the negative consumption effect, or relative price effect, only if $a < 0$ and $w > -\beta^2 a \gamma^{-1} \theta^{-1} > 0$. On the other hand, for low degrees of relative risk aversion, $0 < a < 1$, a positive covariance between inflation and changes in the price of housing stock has an unambiguously negative effect on the optimal portfolio share of housing property.

The different effects of covariance term σ_{hp} can perhaps be clarified if we artificially separate the supply and demand effects from each other. The pure supply side effect can be seen from equation (3.37). Thus the covariance, σ_{hp} , has a positive hedging effect on H^0 if the relative risk aversion, $1-a$, exceeds unity so that $a < 0$.

The pure demand effect can be seen clearly if we eliminate the supply side by setting $w=H^0=0$ in equation (3.30). In this case the representative consumer's demand for housing services is

$$(3.38) \quad H \Big|_{H^0=0} = \frac{\beta \epsilon}{\gamma \theta R} W$$

Now the covariance σ_{hp} influences the consumer only via term ϵ which is defined in equation (3.32). This shows that the demand for housing services depends negatively on σ_{hp} except when the utility function is logarithmic and $a=0$, in which case the covariance has no impact on the housing stock.

Now that the comparative analysis of the optimal portfolio share of the housing stock has been presented, a similar type of procedure could be applied to the equilibrium rent-price ratio as well. However, a closer inspection of equation (3.35) and Table 3.1 reveals that effects of many exogenous variables on the rent-price ratio $V=R/Q$ remain ambiguous given the assumptions of the model. Therefore more specific assumptions are needed to clarify the relationship between the rent-price ratio and the exogenous variables.

3.6 The determination of housing prices and rents in a partial equilibrium

The previous analysis was performed on the assumption that the aggregate supply of housing stock equals the aggregate demand for housing services. This assumption was sufficient for determining the optimal portfolio share of the housing stock and the corresponding level of rental rate of housing services given the market price of houses and other exogenous factors.

However, considering the simultaneous behaviour of the household sector as an aggregate it may quite rightfully be assumed that the instantaneous aggregate supply of housing stock is completely inelastic with respect to changes in the market price. Changes in the stock supply of houses take time because both residential construction and the depreciation of the existing stock are time-consuming processes.

Next we shall consider a simple partial equilibrium model where the aggregate supply of housing stock is a constant, say H_0 . Hence in the market equilibrium

$$(3.39) \quad H_0 = \Sigma H_j^0 = \Sigma H_j$$

The additional assumption leads to the following linear relationships between the market price of housing stock, $P_h(t)$, the corresponding rental price, $\tilde{R}(t)$, and the aggregate wealth of the household sector, $\Sigma \tilde{W}_j(t)$

$$(3.40a) \quad P_h = \frac{w}{H_0} \Sigma \tilde{W}_j$$

$$(3.40b) \quad \tilde{R} = \frac{h}{H_0} \Sigma \tilde{W}_j$$

Since the relative changes in wealth of all consumers are equal and w , h , and H_0 are constants, the percentage changes in P_h and \tilde{R} equal the percentage changes in wealth for the household sector and for individual consumers. Hence deleting for brevity the subscripts denoting the individual consumers

$$(3.41a) \quad dP_h/P_h = d\tilde{R}/\tilde{R} = p_h dt + \sigma_h dz_h =$$

$$(3.41b) \quad d\tilde{W}/\tilde{W} = (i+y+(p_h-\delta-i+v)w-\tilde{R}H/\tilde{W}-PC/\tilde{W})dt + \sigma_h w dz_h + \sigma_y dz_y$$

$$(3.41c) \quad = (i+y+(p_h-\delta-i)w-c)dt + \sigma_h w dz_h + \sigma_y dz_y$$

Equation (3.41b) is the representative consumer's budget constraint (3.21b) for the optimal portfolio share of the housing stock, w , optimal demand for housing services, H , and optimal non-housing consumption, C . Equation (3.41c) has been obtained by utilizing definitions (3.29) and (3.30) for c and h and equilibrium condition (3.34).

The equivalence conditions (3.41a-c) imply that in a rational

expectations equilibrium the drifts of Ito processes $dP_h/P_h = d\tilde{R}/\tilde{R}$ and $d\tilde{W}/\tilde{W}$ are equal. On the other hand, the stochastic terms must also be equal so that $\sigma_h dz_h = \sigma_h w dz_h + \sigma_y dz_y$.

Denote the instantaneous correlations per time unit between processes dz_h , dz_y , and dz_p by ρ_{hy} , ρ_{hp} , and ρ_{yp} respectively. Then equations (3.41a-c) yield the following equilibrium conditions

$$(3.42a) \quad p_h = i + (\tilde{y} - c - \delta w)/(1-w)$$

$$(3.42b) \quad \sigma_h = \sigma_y/(1-w)$$

$$(3.42c) \quad \rho_{hy} = 1$$

$$(3.42d) \quad \rho_{hp} = \rho_{yp}$$

Introducing the supply constraint for the housing stock gives rise to an additional equation (3.42a) which determines the expected percentage change p_h per time unit of the price of housing stock and rental rate in (3.41a) as a function of i , \tilde{y} , and δ and the exogenous variables which determine c and w . Condition (3.42b) says that the standard deviation per time unit of percentage changes in the price and rent of housing stock is proportional to the standard deviation of changes in income. This implies that, according to condition (3.42c), percentage changes in the price and rent of the housing stock are perfectly correlated with the relative changes in wealth which are generated by the stochastic income flow. This in turn implies, according to condition (3.42d), that the instantaneous correlation between changes in housing prices and consumer prices equals the correlation between inflation and income changes.

Substituting (3.42b-c) for σ_y and ρ_{hy} in $\sigma_{hy} = \sigma_h \sigma_y \rho_{hy}$ and inserting this in (3.35) yields an equilibrium condition for the rental price of housing services, $\tilde{R}(t)$, as a function of the price of housing stock, $P_h(t)$, and the mean, p_h , and the standard deviation, σ_h , of percentage changes in housing prices (see Appendix 3c).

$$(3.43) \quad \tilde{R}(t) = (i - p_h + \delta + (1 - \alpha)\sigma_h^2 + \alpha\sigma_{hp})P_h(t)$$

Accordingly, there is a risk premium term, $(1-\alpha)\sigma_h^2 + \alpha\sigma_{hp}$, in the equilibrium rent-price ratio $V = \tilde{R}/P_h = R/Q$. In the case of a zero covariance between changes in the price of housing stock and inflation, $\sigma_{hp} = 0$, the housing stock earns a premium since $(1-\alpha)\sigma_h^2 > 0$. This conclusion was made also by Ioannides and McDonald (1982) from a model where consumer prices were assumed constant.

But this result does not generally hold if the rate of inflation is uncertain. Equation (3.43) implies that the risk premium term may well be zero or even negative for sufficiently negative values of risk aversion parameter α or covariance σ_{hp} . In the present model the risk premium is positive only if $\rho_{yp} > -(1-\alpha)\sigma_y / (\alpha(1-w)\sigma_p)$ (see Appendix 3c). In other words, the risk premium is positive only for sufficiently high correlation between changes in income and inflation.

Summarizing the results we may conclude that housing stock, or any durable good, has special implications for consumers' portfolio distributions. The portfolio share of housing stock depends always on consumers' preferences because it is demanded for both consumption and investment purposes, unlike pure assets. Moreover, a positive covariance between random changes in the price and rent of housing stock and consumer prices has a negative impact on the demand for housing services which diminishes the efficiency of housing stock to serve as a hedge against inflation risk.

4 CREDIT AND RENTAL MARKET IMPERFECTIONS AND THE HOUSING INVESTMENT PROCESS

4.1 Introduction

All the previous analysis has rested on the assumption that capital markets are perfect in the sense that consumers may borrow up to the limit of the collateral of their material and human capital. However, under practical loan market conditions lenders may not have such confidence in borrowers' potential earnings as to accept the total present value of future income as a guarantee for loans. The purpose of this chapter is to analyze the effects of this type of capital market imperfection from the point of view of consumption and housing investments.

A liquidity constraint may be a particularly relevant factor in the housing investment process, since housing stock is generally the single most important asset in household portfolios. Thus if credit constraints prevent housing investments of consumers with small means, the ownership of housing property tends to concentrate in the higher wealth classes. This is obviously revealed in the quite different portfolio structures of households at different wealth levels.

There is a large body of literature about the rationality of credit rationing, as for example the survey of Baltensperger (1978) shows. The behaviour of the supply side of the loan market, however, falls outside the scope of this study. Instead, it is simply assumed that the loan market is imperfect in the sense that mainly physical assets qualify as a collateral for borrowing.

Moreover, imperfection of the rental market of housing services will be introduced into the analysis. An important presumption in the following model is also the indivisibility of houses. This will be modelled simply by means of a Stone-Geary type of utility

function which sets a lower bound or a subsistence level on the size of the consumer's housing stock. This does not, of course, correspond to the usual concept of indivisibility, but it suffices to characterize that a usable housing stock is typically a large unit compared with other consumer goods.

This kind of "indivisibility" of housing stock is a crucial property in imperfect capital markets in the sense that those consumers who cannot finance sufficient housing investments that would exceed their subsistence housing stocks must choose tenancy. This gives rise to systematic differences in tenure choice as an alternative to the effect induced by the non-neutrality of the tax system described in section 2.5.

The effects of capital market imperfections on consumers' intertemporal behaviour have been examined in several studies. Two main types of loan market imperfections have been analyzed in the literature.

First, the imperfection may be revealed in connection with interest rate determination. Lenders may set interest rates depending on the size of the borrower's loan. This mode of lender behaviour has been studied in the context of a life cycle model by Appelbaum and Harris (1979). On the other hand, financial intermediaries generally differentiate between interest rates on loans and deposits. This has been taken into account in the studies of Tobin and Dolde (1971), Flemming (1973), Watkins (1979), Hu (1980), and Shah (1981).

The second main type of capital market imperfection is formed of quantity constraints either on the flow of borrowing or on the stock of consumers' loans. The effects of such liquidity constraints have been analyzed for instance by Heller and Starr (1979). In some cases the borrowing constraint is assumed to depend on consumers' income. Such models have been presented by Russel (1974), Wu (1974), Koskela (1978), and Shah (1982).

There are relatively few studies about the effects of capital market imperfections on housing investments. Artle and Varaiya (1978)

constructed a life cycle model where a consumer has an exogenous housing investment target which can be achieved only by saving a required down-payment for a mortgage loan. They analyzed several characteristics of the optimal life cycle consumption and saving paths. Specifically, they showed that consumption may be discontinuous at the time of the house purchase even though the consumer's income flow is continuous. An analytically similar model has been constructed later by Jackman and Sutton (1982). They also studied a case where a consumer saves optimally to finance a profitable but illiquid and exogenous investment such as a housing unit. Furthermore, they showed that interest rate policy has asymmetric effects in the sense that an increase in interest rates has a larger net impact on consumption than a fall.

A critical characteristic in these studies is the assumed exogeneity of the housing investment. Why would the demand for housing be independent of consumer's wealth and income? Any particular housing unit may quite rightfully be regarded as an indivisible good but nevertheless there normally is a continuum of different sizes of houses in the market for households to choose. Thus there is no reason why consumers would not optimize the size of their housing investments even if there are imperfections in the capital market.

Proceeding from the neoclassical model of demand for consumer durables allows the housing investment to be determined endogenously via consumer's utility maximizing behaviour, as Ranney (1981) has shown. She assumed various forms of capital market imperfections, such as the non-negativity of savings, a down-payment constraint, and a differential between mortgage interest rate and interest rate on savings. She studied housing investment behaviour under a binding liquidity constraint and in perfect loan market circumstances. The analysis was, however, based on the simplifying assumption that the non-negativity constraint on saving is binding at most in the beginning of the planning horizon and thus has no effect on the dynamic properties of life cycle consumption, demand for housing and saving.

The following model is in fact a kind of combination of the aforementioned models. It aims at taking account of the indivisibility of the housing stock while at the same time allowing for its endogeneity by proceeding from the neoclassical theory of demand for consumer durables, unlike Artle and Varaiya or Jackman and Sutton. In contrast with Ranney's model this study is concerned with the effects of an expected future liquidity constraint on the optimal saving for a house-purchase. It also emphasizes the role of borrowing constraints in connection with the indivisible nature of houses in explaining that home-ownership tends to concentrate among wealthy households.

4.2 Tenure choice in imperfect capital and rental markets

Most of the analysis in the previous chapters of this study has been based on the assumption that capital and rental markets are perfect. Individual consumers' tenure choice depends on the rental cost of housing compared with the user cost of the same housing stock. These costs equal in the rental market equilibrium under perfect foresight conditions if the tax-system is neutral, whereas the owners of the housing stock may earn a risk premium if house prices are random. The following case illustrates briefly the effects of capital market imperfections on tenure choice and rental rate determination.

Capital markets are assumed to be imperfect in the sense that there is a borrowing constraint on net financial assets $A(t) \geq \underline{A}$. The consumer's demand for housing stock for investment purposes is denoted by $H^0(t)$ and his consumption of housing services is $H(t)$. The user cost of housing stock under perfect market and perfect foresight conditions is $R(t) = (i - p_h + \delta)Q(t)$ where i denotes the nominal rate of interest and p_h and δ respectively denote the percentage change in the price and the depreciation rate of the housing stock. $Q(t) = P_h(t)/P(t)$ is the relative price of houses and consumer goods. The rental price of the housing stock in relation to consumer prices is denoted by $\underline{R}(t)$.

The consumer's objective for an infinite planning horizon is defined as so as to

$$(4.1a) \quad \text{Max}_{C, H, H^0} \int_0^{\infty} e^{-\rho t} U(C, H) dt$$

s. t.

$$(4.1b) \quad \dot{W} = rW + \underline{R}H^0 - \underline{R}H - C + Y$$

$$(4.1c) \quad A = W - QH^0 > \underline{A}$$

$$(4.1d) \quad W(0) = W_0$$

The consumer optimizes his consumption, C , demand for housing services, H , and investment in housing stock, H^0 , in order to maximize discounted utility over the planning horizon. Accumulation equation (4.1b) implies that wealth increases as a result of rental income from housing property, $\underline{R}H^0$, and of other real income, and decreases due to the cost of ownership of the housing stock, $\underline{R}H^0$, and rental expense on housing services, $\underline{R}H$, as well as consumption expenditure. Taking account of the borrowing constraint expressed in condition (4.1c) the Hamiltonian may be written as follows

$$(4.2) \quad F = e^{-\rho t} U(C, H) + \lambda (rW + \underline{R}H^0 - \underline{R}H - C + Y) + \eta (W - QH^0 - \underline{A})$$

Assume that the borrowing constraint is binding continuously, so that $\eta > 0$ and $W = QH^0 + \underline{A}$. The break-even point of home-ownership, $\partial F / \partial H^0 = 0$, gives the required rental return on housing stock

$$(4.3) \quad \underline{R} = R + (\eta/\lambda)Q = (i - p_h + \delta + \eta/\lambda)Q$$

Accordingly, the perfect capital market equilibrium condition, $\underline{R}(t) = R(t) = (i - p_h + \delta)Q(t)$, no longer holds. Instead, equilibrium in the rental market now requires that the shadow cost of the effective borrowing constraint be compensated in the rental price of housing services as condition (4.3) implies. This would presumably take place in a perfect rental market if all consumers were identical in every respect. Under these conditions the shadow

cost of ownership of the housing stock would be compensated in the rental market equilibrium by a return equal to $(\eta/\lambda)Q$ in excess of the perfect capital market rental price R .

The specified liquidity constraint influences a borrowing consumer more strongly the larger his expected income, Y , is in relation to his present wealth, W , because it implies that future earnings do not fully qualify as a collateral for borrowing. Thus, if consumers have identical preferences but different income and wealth, the required return on home-ownership is highest among those consumers who expect large income in relation to their present wealth and they therefore choose tenancy. In these circumstances the ownership of the housing property tends to concentrate among the wealthier households, which are less severely constrained in the credit market.

However, despite the fact that borrowing constraints in general favour tenancy, there may be institutional imperfections in the rental market which ultimately tend to force even the liquidity-constrained households towards owner-occupancy. Distortions affecting the rental market equilibrium may result for instance from public rent controls applied to landlords and from housing allowances granted to tenants. The former tend to suppress the supply of rental housing stock while the latter increase the demand for rental housing services. Such a contradictory policy may give rise to an excess demand for rental housing services, a situation where tenants are subject to quantity constraints in the sense that they cannot find vacant dwellings at the prevailing level of rents. This may induce either a shadow cost associated with the binding availability constraint of rental dwellings or an increase in the unregulated rental rate if there is an uncontrolled segment in the rental market (c.f. Fallis and Smith (1984)).

Such indications of imperfections in the rental market of housing have been in evidence for example in Finland. These are also in the background of the analysis below, which assumes that the rental price of housing is so high compared with the user cost that consumers generally prefer owner-occupancy of the housing stock.

4.3 The effect of an expected borrowing constraint on consumption and demand for owner-occupied housing stock

The next issue concerns the effects of the borrowing constraint $A(t) \geq \underline{A}$ on consumption and housing investment behaviour. The rental market for dwellings is assumed to be imperfect in the sense that the rental cost, for some reason, exceeds the user cost of a liquidity-constrained consumer, so that he prefers owner-occupancy and $H^0(t) = H(t)$ in model (4.1a-d).

A strictly positive lower bound is assumed for the usable housing unit by specifying a Stone-Geary type of utility function for the consumer

$$(4.4) \quad U(C, H) = a^{-1} (C - C^*)^{\alpha a} (H - H^*)^{\beta a}$$

where the parameters are restricted as stated in (3.6a-b). The subsistence level of housing stock is assumed to be positive $H^* > 0$, and the subsistence level of non-durable composite consumption is assumed to be non-negative $C^* \geq 0$. Given the specification of the utility function the Hamiltonian is

$$(4.5) \quad F = e^{-\rho t} a^{-1} (C - C^*)^{\alpha a} (H - H^*)^{\beta a} + \lambda (rW - RH - C + Y) + \eta (W - QH - \underline{A})$$

The necessary conditions are

$$(4.6a) \quad F_C = e^{-\rho t} \alpha (C - C^*)^{\alpha a - 1} (H - H^*)^{\beta a} - \lambda = 0$$

$$(4.6b) \quad F_H = e^{-\rho t} \beta (C - C^*)^{\alpha a} (H - H^*)^{\beta a - 1} - R\lambda - \eta Q = 0$$

$$(4.6c) \quad F_W = r\lambda + \eta = -\dot{\lambda}$$

$$(4.6d) \quad \eta > 0, \quad \eta (W - QH - \underline{A}) = 0$$

These are different from the perfect capital market optimality conditions (2.5a-c), owing to the possibility of a liquidity-constrained period. Condition (4.6d) implies

that the shadow cost of borrowing becomes positive, $\eta > 0$, in a liquidity-constrained period.

This section analyzes the case where the consumer has sufficient wealth to invest in a housing unit which exceeds the subsistence level $H(0) > H^*$. The situation in which this is not possible is described in section 4.4.

The first question to be analyzed is, of course, under what circumstances the borrowing constraint may become effective. Therefore the change in consumer's net financial assets, $\dot{A}(t)$, is first solved assuming that there is no borrowing constraint in order to see whether the consumer, for a given set of exogenous parameter values, tends to increase his financial wealth or borrow more. A borrowing constraint is, of course, more important for a borrower than for a saver.

In an unconstrained situation, $\eta = 0$, there is a linear relationship between composite consumption and demand for housing according to conditions (4.6a-b)

$$(4.7) \quad C(t) = C^* + \alpha\beta^{-1}R(t)(H(t) - H^*)$$

For simplicity of exposition the following model assumes that the relative user cost of housing is a constant $R(t) = (r + \delta)Q_0$, where the real rate of interest is $r = i - p$. In this case optimality conditions (4.6a-c) yield the following time path for the housing stock (c.f. Appendix 1b)

$$(4.8) \quad H(t) = (H(0) - H^*)e^{\frac{r - \rho}{\theta}t} + H^*$$

where $\theta = 1 - \gamma\alpha > 0$ and $0 < \gamma = \alpha + \beta < 1$.

Consumption and demand for housing are optimized subject to the intertemporal wealth constraint

$$(4.9) \quad \int_0^{\infty} e^{-rt} (C(t) + RH(t)) dt = W_0 + \frac{Y}{r}$$

The following optimal demand for housing is obtained in the event of there being no borrowing constraint

$$(4.10) \quad H(0) = H^* + \frac{\beta(\rho - \gamma ar)}{\gamma \theta R} \left(W_0 + \frac{Y - C^* - RH^*}{r} \right)$$

where $W_0 + (Y - C^* - RH^*)/r > 0$ by the requirement of the consumer's solvency. Therefore $\rho > \gamma ar$, which will be assumed subsequently.

For the subsequent analysis it is useful to define the consumer's total expenditure as a sum of his consumption and investment. Since investment flow is by definition $I(t) = \dot{H}(t) + \delta H(t)$, the time path of expenditure is in terms of the initial housing stock

$$(4.11a) \quad E(t) = C(t) + QI(t)$$

$$(4.11b) \quad = \psi Q(H(0) - H^*) e^{\frac{r - \rho}{\theta} t} + E^*$$

where $\psi = \alpha(r + \delta) / \beta + \delta + (r - \rho) / \theta$ and subsistence expenditure is $E^* = C^* + \delta QH^*$. In the following we assume that $\psi > 0$ and that the real rate of interest is relatively low, $0 < r < \rho$, so that the expenditure path is declining over time as depicted below in Figure 4.1.

The change in the consumer's financial assets depends on the difference between income and total expenditure. Financial saving is the following function of wealth and the exogenous variables

$$(4.12a) \quad \dot{A}(t) = rA(t) - E(t) + Y$$

$$(4.12b) \quad = s(W(t) + \frac{Y - C^* - RH^*}{r})$$

where the propensity to save out of wealth is

$$s = r - \beta(\rho - \gamma ar)(r + \psi) / (\gamma \theta(r + \delta)) \quad (\text{see Appendix 4a}).$$

Let us now introduce the borrowing constraint, $A(t) \geq \underline{A}$, into the analysis and assume first that $s > 0$, so that saving, $\dot{A}(t) > 0$, is optimal. If the borrowing constraint is effective at all, it must be binding in the beginning of the planning horizon, $t=0$, because the consumer plans to diminish his liabilities continuously thereafter. Therefore the borrowing constraint is binding only for a sufficiently large demand for initial housing stock, $H(0)$, associated with a sufficiently large initial borrowing, so that $A(0) = W_0 - QH(0) = \underline{A}$. This case basically corresponds to the model analyzed by Ranney (1981).

Suppose next that $s=0$, so that the consumer neither saves nor borrows. Then the liquidity constraint is either binding continuously, $A(t) = \underline{A}$, or always ineffective, $A(t) > \underline{A}$. In the former case the consumer's total expenditure equals always his income, $E(t) = Y + r\underline{A}$, while in the latter case his expenditure follows the unconstrained path (4.11b).

The third case, $s < 0$, in which the consumer is a borrower, $\dot{A}(t) < 0$, forms the basis for the subsequent analysis. In this case the consumer's liabilities ultimately approach a finite limit, $A(t) \rightarrow -(Y - E^*)/r$, if $0 < r < \rho$ and if only the requirement for solvency restricts his borrowing (see Appendix 4b). Therefore, if the borrowing constraint is expected to become binding at all, it must be tighter than the asymptote of debts in an unconstrained optimum, so that $A(t) \geq \underline{A} > -(Y - E^*)/r$.

If the credit limit is tight enough, there is a finite time point, say $t = \tau$, when the borrowing constraint becomes binding, so that $\eta(\tau) > 0$ and $W(\tau) = \underline{A} + QH(\tau)$. In the unconstrained period, from $t=0$ to $t=\tau$, consumption and demand for housing are optimized subject to total disposable wealth less the present value of wealth at the end of the period

$$(4.13) \quad \int_0^{\tau} e^{-rt} (C(t) + RH(t)) dt = W_0 + \int_0^{\tau} e^{-rt} Y dt - e^{-r\tau} W(\tau)$$

where

$$(4.14) \quad W(\tau) = \underline{A} + QH(\tau) = \underline{A} + Q(H(0) - H^*)e^{\frac{r-\rho}{\theta}\tau} + QH^*$$

Inserting (4.7), (4.8), and (4.14) in (4.13) and integrating yields the demand function of housing stock under an expected borrowing constraint

$$(4.15) \quad H(0) = H^* + \frac{W_0 + \frac{Y - C^* - RH^*}{r} (1 - e^{-r\tau}) - QH^*e^{-r\tau} - \underline{A}e^{-r\tau}}{\frac{\gamma R}{\beta \epsilon} (1 - e^{-\epsilon\tau}) + Qe^{-\epsilon\tau}}$$

where $\epsilon = (\rho - \gamma ar) / \theta > 0$. Given the solution for $H(0)$, equation (4.7) then determines the consumption function $C(0)$, and wealth constraint (4.1c) in turn determines the demand for net financial assets $A(0) = W_0 - QH(0)$.

However, the demand functions are not yet completely specified, because the point in time, $t = \tau$, when the borrowing constraint becomes binding is endogenous. It can be determined from the condition that at that time borrowing ceases, $\dot{A}(\tau) = 0$, so that total consumption and investment expenditure equals total income $E(\tau) = Y + r\underline{A}$. Inserting this condition in equation (4.11b) yields the following solution

$$(4.16) \quad \tau = \frac{\theta}{r - \rho} \ln \frac{Y + r\underline{A} - E^*}{\theta Q(H(0) - H^*)}$$

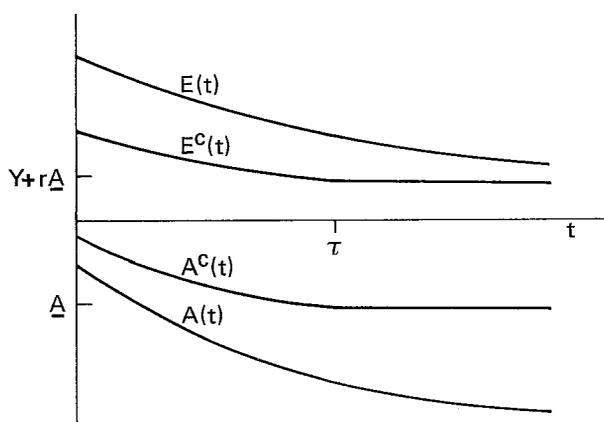
Endogenous variables $H(0)$ and τ are determined simultaneously by equations (4.15) and (4.16). These equations imply that $\partial \tau / \partial \underline{A} < 0$ provided that $(\partial H(0) / \partial \tau)(\partial \tau / \partial H(0)) < 1$ and $0 < r < \rho$ (see Appendix 4c). In other words, if the borrowing constraint is expected to become more restrictive, $d\underline{A} > 0$, it also becomes binding sooner. This outcome seems intuitively acceptable, for it also implies that $\partial H(0) / \partial \underline{A} < 0$, and according to (4.7) that $\partial C(0) / \partial \underline{A} < 0$. This means that the more restrictive the borrowing constraint is, the smaller is the demand for housing and consumption. Moreover, the wealth effect is positive in both demand functions, $\partial H(0) / \partial W_0 > 0$ and $\partial C(0) / \partial W_0 > 0$. Thus the model differs in these respects essentially from the models

of Artle and Varaiya (1978) or Jackman and Sutton (1982), where the housing investment cannot react to changes in borrowing constraint, wealth, income, and prices because it has been assumed exogenous.

An important implication of this model is that unlike in the perfect capital market case described by equation (4.10), the permanent income, Y , and the relative housing price level, Q , may have ambiguous effects on the demand for housing stock, $H(0)$. This result is understandable because an increase in the permanent income or a decrease in housing prices increases the consumer's willingness to invest in housing stock according to equation (4.15), but according to equation (4.16), the borrowing constraint then becomes binding sooner if $0 < r < \rho$.

The dynamics of total consumption and investment expenditure, $E(t)$, and net financial assets, $A(t)$, can be seen in Figure 4.1. $E(t)$ and $A(t)$ describe the paths of an unconstrained optimum, while $E^C(t)$ and $A^C(t)$ refer to the case where the borrowing constraint is expected to become binding at $t = \tau$. Thereafter $A^C(t) = \underline{A}$ and total expenditure equals permanent income less interest expense on the debt, $E^C(t) = Y + r\underline{A}$.

Figure 4.1: Time paths of total expenditure and net financial assets



4.4 The demand for rental housing and saving for a house purchase

Thus far we have assumed that, in spite of the liquidity constraint, the consumer has a sufficient amount of wealth to buy a home immediately, and that he in fact carries out the investment on account of the lower cost associated with owner-occupancy. However, if he has a relatively small amount of wealth at his disposal, he may be in a position where, even with substantial future earnings Y , his current wealth, W_0 , is insufficient for a housing investment that would exceed the subsistence level. This happens in the former model whenever $H(0) < H^*$ in demand function (4.15).

In this situation the consumer is forced to acquire the housing services from the rental market at a higher cost than he would have to pay as an owner-occupier. Even so, he may start saving, and after having accumulated a sufficient amount of wealth he can buy a home at a later date, say at time point $t=T$. Thereafter the housing cost decreases and the consumer stays as an owner-occupier. This two-stage planning procedure is formulated in the following model.

The consumer's preferences are described by the same Stone-Geary utility function (4.4) in both sub-periods. The rental price of housing is denoted by \underline{R} and the user cost by $R=(r+\delta)Q$. The consumer's saving problem as a tenant and investment problem as an owner-occupier is defined as follows

$$(4.17a) \quad J(W(0), 0) = \text{Max}_{C, H, T} \left[\int_0^T e^{-\rho t} U(C, H) dt + \int_T^{\infty} e^{-\rho t} U(C, H) dt \right]$$

$$= \text{Max}_{C, H, T} \left[\int_0^T e^{-\rho t} U(C, H) dt + J(W(T), T) \right]$$

s. t.

$$(4.17b) \quad W(0) = A_0 \quad 0 \leq t < T$$

$$(4.17c) \quad W(t) = A(t) \geq \underline{A} \quad "$$

$$(4.17d) \quad \dot{W} = rW - \underline{R}H - C + Y \quad "$$

$$(4.17e) \quad W(t) = A(t) + QH(t) \geq \underline{A} + QH(t) \quad t \geq T$$

$$(4.17f) \quad \dot{W} = rW - RH - C + Y \quad "$$

The investment problem, from time $t=T$ onwards, is solved similarly as in section 4.3. The saving problem is solved conditionally on the derived utility from the owner-period, $J(W(T), T)$, so that both the accumulated wealth, $W(T)$, and the length of the saving period, T , are optimized via transversality conditions.

The Hamiltonian for the saving period is defined as

$$(4.18) \quad F = e^{-\rho t} \alpha^{-1} (C - C^*)^{\alpha a} (H - H^*)^{\beta a} + \lambda (rW - \underline{R}H - C + Y) + \eta (W - \underline{A})$$

The necessary conditions are

$$(4.19a) \quad F_C = e^{-\rho t} \alpha (C - C^*)^{\alpha a - 1} (H - H^*)^{\beta a} - \lambda = 0$$

$$(4.19b) \quad F_H = e^{-\rho t} \beta (C - C^*)^{\alpha a} (H - H^*)^{\beta a - 1} - \underline{R}\lambda = 0$$

$$(4.19c) \quad F_W = r\lambda + \dot{\eta} = -\dot{\lambda}$$

$$(4.19d) \quad \eta \geq 0, \quad \eta (W - \underline{A}) = 0$$

$$(4.19e) \quad \lambda(T) = J_W(W(T), T)$$

$$(4.19f) \quad F(T) = -J_t(W(T), T)$$

Consumption is larger in relation to the demand for housing in the saving period than in the investment period, since rental cost exceeds user cost. There is again a linear relationship between consumption and demand for housing according to conditions (4.19a-b)

$$(4.20) \quad C(t) = C^* + \alpha \beta^{-1} \underline{R} (H(t) - H^*)$$

If the borrowing constraint is not binding, so that $\eta=0$, demand for housing changes in time as stated in equation (4.8). Expenditure items are optimized subject to an intertemporal wealth constraint where the current total wealth is decreased by the present value of wealth at the end of the saving period

$$(4.21) \quad \int_0^T e^{-rt} (C(t) + \underline{RH}(t)) dt = W_0 + \int_0^T e^{-rt} Y dt - e^{-rT} W(T)$$

Inserting (4.20) and (4.8) in (4.21) and integrating yields the demand for rental housing at the beginning of the saving period

$$(4.22) \quad H(0) = H^* + \frac{W_0 + \frac{Y - C^* - \underline{RH}^*}{r} (1 - e^{-rT}) - e^{-rT} W(T)}{\frac{\gamma R}{\beta \varepsilon} (1 - e^{-\varepsilon T})}$$

This demand function as well as the corresponding consumption function determined by (4.20) display normal properties. Wealth, W_0 , and permanent income, Y , have positive effects in both demand functions. Rental price, \underline{R} , has a negative impact on the demand for housing. Moreover, saving more intensively to accumulate wealth, $W(T)$, necessitates a decrease in housing and consumption expenditure, but saving for a longer time, T , allows larger expenditure because the same saving target can be achieved with less effort.

The length of the saving period and optimal wealth at the end of the period are, however, endogenous variables in this model. They are determined by the transversality conditions. Optimal wealth is determined by condition (4.19e) and optimal saving time by condition (4.19f) (c.f. Kamien and Schwartz (1981 p. 148)).

The derived utility in the saving period is a function of the initial real wealth, W_0 , permanent real income, Y , relative rental price of housing, \underline{R} , and the real rate of interest, r . On the other hand, the derived utility in the investment period is a function of real income, relative user cost of housing stock, $R = (r + \delta)Q$, and the real rate of interest.

Transversality condition (4.19e) implies that the marginal derived utility of wealth at the end of the saving period, the left-hand partial derivative J_W^- , equals the marginal derived utility of wealth at the beginning of the investment period, the right-hand

partial derivative J_W^+ . Transversality condition (4.19f) in turn implies that left- and right-hand partial time derivatives, J_t^- and J_t^+ , and the Hamiltonians, F^- and F^+ , are equal at the optimal switch-point in time, T . These two conditions form a two-equation system (4.23a-b) which determines simultaneously the two endogenous variables, $W(T)$ and T , in terms of the exogenous variables.

$$(4.23a) \quad J_W^-(W(T), T; W_0, Y, \underline{R}, r) = J_W^+(W(T), T; Y, R, r)$$

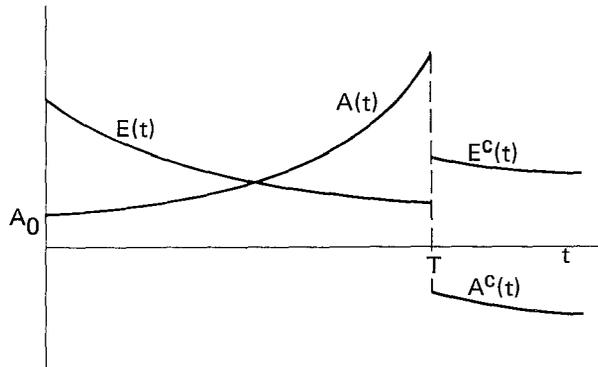
$$(4.23b) \quad J_t^-(W(T), T; W_0, Y, \underline{R}, r) = J_t^+(W(T), T; Y, R, r)$$

Although the derived utility functions, J^- and J^+ , as well as the implicit functions (4.23a-b) can be solved, they do not yield clearcut comparative static implications. The extreme cases, where T goes either to zero or to infinity, can be described more easily.

If wealth increases or house prices decrease sufficiently, the consumer needs no saving period before the housing investment. Of course, the house can be bought immediately also if the borrowing constraint is reduced sufficiently. In this case the model reduces to the form studied in section 4.3. On the other hand, if the difference between rental price and user cost of housing decreases sufficiently, there comes a point, say $\underline{R} = R + (\eta/\lambda)Q$ as in the model in section 4.2, where the consumer becomes indifferent in his tenure choice, so that he may optimally stay as a tenant all the time.

The case in which T is positive and finite is illustrated in Figure 4.2. The time path of total consumption and housing expenditure is denoted by $E(t) = C(t) + \underline{R}H(t)$, and the time path of financial assets by $A(t)$. The consumer initially has wealth A_0 which is, however, insufficient for a housing investment. He saves by choosing a sufficiently low expenditure path, which decreases over time, as in Figure 4.2, if the real interest rate is lower than the rate of time preference, $0 < r < \rho$. He accumulates financial savings until at $t=T$ he can afford to buy a housing unit that exceeds the subsistence level, $H(T) > H^*$. Thereafter he stays as an owner-occupier and behaves according to the model described in section 4.3.

Figure 4.2: Expenditure and asset paths during the housing investment process



The credit market imperfection was modelled above simply by means of a constant borrowing constraint $A(t) \geq \underline{A}$. A more conventional but also somewhat more complicated way to model the housing loan market would be to assume that there is a down-payment constraint which is proportional to the housing investment, say $A(T) = dQH(T)$, where $0 < d < 1$. The rest of the investment would be financed by a housing loan, equal to $(1-d)QH(T)$, which would induce a continuous annuity over the loan period. The main implication from assuming such a less restrictive down-payment constraint seems to be that the housing investment can be made sooner, since it requires a smaller amount of wealth.

4.5 Implications for the household portfolio distribution

The demand function of owner-occupied housing stock in the case of an expected borrowing constraint was derived in (4.15). This equation can be rewritten in the form

$$(4.24a) \quad QH = QH^* + h(W-K)$$

where

$$(4.24b) \quad K = (\underline{A} + QH^*)e^{-r\tau} - \frac{Y - C^* - RH^*}{r} (1 - e^{-r\tau})$$

and

$$(4.24c) \quad h = \left(\frac{\gamma(r+\delta)}{\beta\epsilon} (1 - e^{-\epsilon\tau}) + e^{-\epsilon\tau} \right)^{-1}$$

Since $h > 0$, the demand function (4.24a) is well-defined only if the consumer's wealth is sufficiently large, $W > K$, so that $H > H^*$. Otherwise he must live as a tenant even if the rental cost exceeds the user cost of housing.

Equation (4.24b) shows that the critical level of wealth, K , is an increasing function of both the borrowing constraint, \underline{A} , and the subsistence housing stock, H^* . In the following it is assumed that $K > 0$, which means that even for a consumer endowed with positive wealth there are two tenancy regimes between which he makes a choice, depending on whether $W \leq K$ or $W > K$.

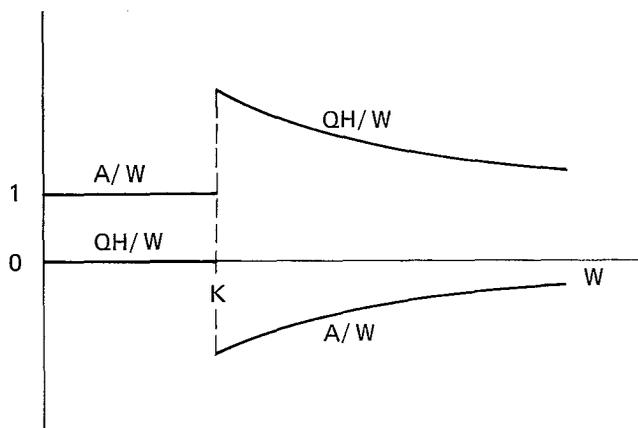
The portfolio shares of owner-occupied housing stock, QH/W , and net financial assets, A/W , are

$$(4.25a) \quad \frac{QH}{W} = h + \frac{QH^* - hK}{W}$$

$$(4.25b) \quad \frac{A}{W} = 1 - h - \frac{QH^* - hK}{W}$$

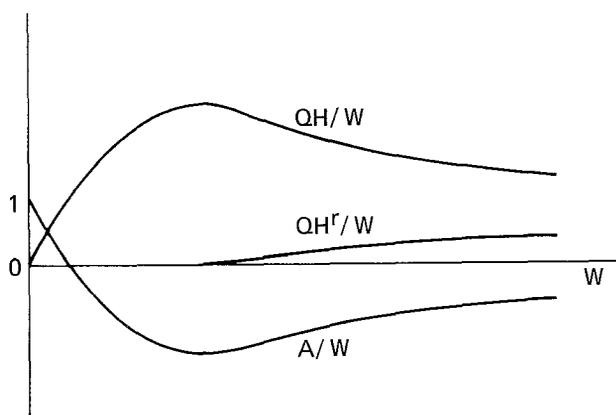
Geometrically the portfolio shares are described by rectangular hyperbolas. If $QH^* > hK$, the portfolio share of the housing stock is a decreasing function of wealth and the portfolio share of net financial assets is an increasing function of wealth in a similar fashion to the way in which they were depicted above in Figure 3.1., which described the portfolio distribution in perfect capital markets. In the present model equations (4.25a-b) are defined only for $W > K$ as depicted in Figure 4.3. Below the critical level of wealth the consumer cannot own any housing property, $QH/W = 0$. He may, however, live as a tenant and the portfolio share of his net financial assets is unity, $A/W = 1$.

Figure 4.3: Portfolio distribution as a function of consumer's wealth



The critical wealth level, of course, varies among consumers because they differ with respect to the subsistence level of housing, other preferences, earnings, and borrowing constraints. Thus, if there is some dispersion in K , the aggregate portfolio shares may look something like those depicted in Figure 4.4.

Figure 4.4: Aggregate portfolio distribution



The average portfolio proportion of owner-occupied housing stock, QH/W , is rising at lower levels of net wealth because an increasing number of households exceed the critical wealth level where they become home-owners. Home-ownership becomes dominant at higher wealth levels but at the same time owner-occupied housing stock decreases in proportion to net wealth as implied by equation (4.25a). The stock of rental houses, H^r , is allocated mainly among the wealthier households if they suffer less from the borrowing constraints. The portfolio distribution outlined in Figure 4.4 is, of course, only tentative.

Summarizing the results of this chapter, we may conclude that credit and rental market imperfections, in connection with the conventional Stone-Geary utility function, may give rise to systematic differences in tenure choice and portfolio structure between households at different wealth levels. In these circumstances home-ownership tends to concentrate among wealthy households. The generally applied progressive tax system reinforces this tendency, but it may remain valid even if the tax system is neutral.

The model described a situation in which the representative consumer is basically a dissaver who wants to borrow up to the limit of his material wealth and present value of his future earnings but cannot do so because of the borrowing constraint. Nevertheless, despite his general willingness to borrow, if owner-occupancy is cheaper than tenancy but the housing investment cannot be financed immediately, saving for a later house-purchase may turn out to be profitable. In this sense the model gives a partial explanation for the empirical observation that households' financial saving seems to continue relatively uninterruptedly even in periods of a very low, or occasionally even negative, real rate of interest.

5 HOUSING MARKET ADJUSTMENT UNDER PERFECT FORESIGHT CONDITIONS

5.1 Introduction

Thus far we have studied mainly the demand side of the housing market from the point of view of an individual consumer. The supply of new housing units will now be introduced into the analysis. This chapter studies the price and quantity reactions in the housing market induced by exogenous demand or supply shocks. Consumers are assumed to have perfect foresight in the sense that the actual change in house prices is equal to what was anticipated.

Examples of models considering perfect foresight type of rational expectations can be found in many fields of economics.

Macroeconomic examples include some of the monetary models of inflation and the theories of exchange rate determination in the case of rational expectations.

The assumption of perfect foresight has also been applied to microeconomic partial equilibrium models, for example to explain corporate investment behaviour (c.f. Begg (1982)). Sheffrin (1983) and Poterba (1984) present rational expectations models of the housing market. In these models the changes in house prices are determined from the relationship between the rental demand price and the user cost of housing stock. The changes in the housing stock are composed of new production and depreciation of the existing stock. The demand and supply functions are, however, not explicitly derived from any microeconomic models.

The following analysis of housing market behaviour is more specific, because it derives the supply behaviour on the basis of profit maximization by the residential constructors. The demand for housing is based on households' utility maximization. This is important, because for a certain class of preferences the expected

capital gain may have a negative effect on the demand for the housing stock (see Appendix 1c). In that case the conventional asset market approach gets into difficulties. The model no longer determines a unique adjustment path for the housing market.

5.2 Consumer behaviour and price determination on the demand side of the housing market

The demand side of the housing market is assumed to consist of utility maximizing consumers. The framework specified in Chapter 4 is used to illustrate the role of credit controls in stabilizing the housing market. However, an important modification will be made in that model because we shall now consider the aggregate behaviour of the household sector. The aggregate supply of housing stock is assumed to be a predetermined constant, at least momentarily. This assumption seems to be justified due to the fact that changes in the stock supply of houses take time because both residential construction and the depreciation of the existing stock are time-consuming processes. On the other hand, the market price of the housing stock is determined endogenously in the following model.

Consumers are assumed to have similar preferences so that they react similarly to changes in the exogenous variables. The representative consumer's problem is defined so as to

$$(5.1a) \quad \text{Max}_{C,H} \int_0^{\infty} e^{-\rho t} U(C,H) dt$$

s.t.

$$(5.1b) \quad \dot{W}(t) = rW(t) - R(t)H(t) - C(t) + Y$$

$$(5.1c) \quad W(t) = A(t) + Q(t)H(t) \geq \underline{A} + Q(t)H(t)$$

$$(5.1d) \quad W(0) = W_0$$

The relative user cost of housing stock is allowed to change over time according to the formula $R(t) = (r - q + \delta)Q(0)e^{qt}$ where the percentage change in relative house prices is $q = \dot{Q}/Q$. The real rate of interest is denoted by $r = i - p$ and the rate of depreciation is δ as before.

The Hamiltonian may be defined as

$$(5.2) \quad F = e^{-\rho t} U(C, H) + \lambda (rW - RH - C + Y) + \eta (W - QH - \underline{A})$$

The necessary conditions are

$$(5.3a) \quad F_C = e^{-\rho t} U_C - \lambda = 0$$

$$(5.3b) \quad F_H = e^{-\rho t} U_H - R\lambda - Q\eta = 0$$

$$(5.3c) \quad F_W = r\lambda + \eta = -\dot{\lambda}$$

$$(5.3d) \quad \eta \geq 0, \quad \eta (W - QH - \underline{A}) = 0$$

Assume that the borrowing constraint is expected to become binding at some future date $t = \tau$. Then consumption and the demand for housing are optimized subject to the intertemporal wealth constraint

$$(5.4) \quad \int_0^{\tau} e^{-rt} (C(t) + R(t)H(t)) dt = W_0 + \int_0^{\tau} e^{-rt} Y dt - e^{-r\tau} (\underline{A} + Q(\tau)H(\tau))$$

If consumers' preferences are specified by utility function $U(C, H) = a^{-1} C^{\alpha} H^{\beta a}$, necessary conditions (5.3a-c) and constraint (5.4) yield the following demand function of housing stock under an expected borrowing constraint

$$(5.5a) \quad QH = h(W + \frac{Y}{r}(1 - e^{-r\tau}) - e^{-r\tau} \underline{A})$$

where

$$(5.5b) \quad h = \left(\frac{\gamma(r - q + \delta)}{\beta \epsilon} (1 - e^{-r\tau}) + e^{-\epsilon\tau} \right)^{-1}$$

where $\epsilon = (\rho - \gamma a r + \beta a q) / \theta > 0$, $\theta = 1 - \gamma a > 0$, and $0 < \gamma = \alpha + \beta < 1$. The time point, $t = \tau$, when the borrowing constraint is expected to become binding is determined endogenously via the condition that total income equals total expenditure as in the model in section 4.3.

If the aggregate supply of housing stock is predetermined for the household sector, the relative market price, Q , jumps discretely if some exogenous variable changes so that consumers must revalue their housing property. Equation (5.5a) implies that an expected borrowing constraint, \underline{A} , has a decreasing effect on individual consumers' demand for housing. On the other hand, at the aggregate level with the inelastic stock supply, H , this effect turns to house prices, Q . Thus monetary policy may stabilize the housing prices and residential construction via the demand side of the housing market either by controlling the interest rate or by influencing the borrowing constraints.

The main issue here is to characterize the price path, $Q(t)$, that maintains the housing market equilibrium continuously in time. We use two different ways in modelling the pricing behaviour. The first approach is analogous to that applied in the stochastic partial equilibrium model of section 3.6. In this case it is for simplicity assumed that $\tau = \infty$ which means that the capital market is perfect. Therefore demand function (5.5a-b) reduces to form

$$(5.6) \quad QH = \frac{\beta(\rho - \gamma a r + \beta a q)}{\gamma \theta (r - q + \delta)} \left(W + \frac{Y}{r} \right)$$

We may assume that, given the exogenous variables, consumers plan to maintain the portfolio balance continuously so that equilibrium condition (5.6) holds momentarily and also for the future changes in housing stock and relative house prices. This assumption was applied in section 3.6. Differentiating (5.6) with respect to time gives

$$(5.7) \quad \dot{Q}H + Q\dot{H} = \frac{\beta(\rho - \gamma a r + \beta a q)}{\gamma \theta (r - q + \delta)} \dot{W}$$

To take a simple example, suppose that the housing stock is expected to grow at a constant percentage rate, g , so that

$$(5.8) \quad \dot{H} = gH$$

This equation determines \dot{H} in (5.7). Thus, inserting (5.6) in (5.7) and in the accumulation equation (5.1b), together with the optimal consumption function, gives (see Appendix 5a)

$$(5.9) \quad q = \dot{Q}/Q = \frac{r - \rho - \theta g}{1 - \alpha a}$$

This perfect foresight pricing formula implies that the percentage change in the relative housing price level is positively related to the real rate of interest, r . On the other hand, the rate of change in house prices depends negatively on g , which reflects the fact that in a rational expectations equilibrium consumers anticipate the capital loss that results from an upward trend in the housing stock. This means also that the initial housing price level, $Q(0)$, jumps downwards as a reaction to an expected increase in the growth rate of the housing stock, g , which can be seen by substituting q in equation (5.6) by the pricing formula (5.9) (see Appendix 5a).

The second approach in deriving the price path is to consider only the momentary equilibrium where the predetermined aggregate stock supply equals the current aggregate demand but the portfolio balance is not required to hold continuously in the sense of equation (5.7). This type of perfect foresight housing market model has been presented by Sheffrin (1983) and Poterba (1984), who derive a differential equation for house prices from a relationship between the rental demand price and the user cost of the housing stock. However, such pricing behaviour is not derived from any explicit model of consumer behaviour.

Basically, the following model assumes utility maximizing consumers who optimize consumption and demand for housing as stated in

problem (5.1a-d). Thus the aggregate demand function of the housing stock (5.10) consists of the predetermined and exogenous variables of equations (5.5a-b)

$$(5.10) \quad H(t) = H^d(Q(t), \dot{Q}(t); W, Y, r, A)$$

$(-)$ $(?)$ $(+)$ $(+)$ $(-)$ $(-)$

It is assumed that the housing price level, $Q(t)$, has a decreasing effect on the demand for housing stock. Moreover, the demand for housing is assumed to be an increasing function of the aggregate wealth, W , and income, Y , of the household sector, but a decreasing function of the tightness in the financial market as indicated by the real interest rate, r , and the borrowing constraint, A . Thus it is assumed that some consumers may be in a position where they have to take into account liquidity constraints when planning future consumption and demand for housing. The important thing is that the expected rate of appreciation of housing prices, $\dot{Q}(t)$, may have either a positive or a negative effect on the demand for housing stock. This may be the case even if the capital market is perfect (see Appendix 1c).

Thus assuming that the aggregate stock supply is momentarily fixed and that the housing market clears, equation (5.10) implicitly defines two alternative differential equations for house prices (see Appendix 5b).

$$(5.11a) \quad \dot{Q}(t) = \dot{Q}(Q(t), H(t); W, Y, r, A)$$

$(+)$ $(+)$ $(-)$ $(-)$ $(+)$ $(+)$

$$(5.11b) \quad \dot{Q}(t) = \dot{Q}(Q(t), H(t); W, Y, r, A)$$

$(-)$ $(-)$ $(+)$ $(+)$ $(-)$ $(-)$

The first equation describes the determination of price expectations in the case that the capital gain effect is positive and the second equation where the effect is negative. It should be noted that although perfect foresight has been assumed above, it is not essential for the implications of the model. This can be seen by substituting a multiplicative price expectation function $\dot{Q}^e = E\dot{Q}$

for \dot{Q} in (5.10) and assuming imperfect foresight, $0 < E < 1$, which does not change the qualitative properties of equations (5.11a-b).

5.3 Flow supply in the housing market

We shall next characterize the properties of the production side of the housing market. Changes in the stock supply of houses consist of new production, $I(t)$, and depreciation of the existing stock, $\delta H(t)$, so that

$$(5.12) \quad \dot{H}(t) = I(t) - \delta H(t)$$

The flow supply of new houses, $I(t)$, may be modelled on the basis of profit maximizing behaviour of the constructor firms. Since we are considering only the partial equilibrium in the housing market, we may assume that the capital stock used in residential construction, say K , is an exogenous constant. Moreover, the firms may be assumed to take the wage rate as an exogenous constant determined in the labour market. In the following the real wage, w , is defined in terms of consumer prices, P , so that the nominal wage rate is wP .

Housing production is optimized by adjusting the demand for labour. Profit maximizing firms optimize the demand for labour in order to equate marginal productivity and real wage cost in terms of housing prices $P_h(t) = PQ(t)$, where $Q(t)$ denotes the relative market price of houses. For example, in the case of the Cobb-Douglas production function $I = K^\mu N^{1-\mu}$, $0 < \mu < 1$, the optimal production is $I = ((1-\mu)Q/w)^{(1-\mu)/\mu} K$ (see Appendix 5c). Hence the new housing production is an increasing function of the relative market price of houses, $Q(t)$, and the disposable capital stock, K , but a decreasing function of the real wage rate, w .

$$(5.13) \quad I(t) = I(Q(t); K, w)$$

(+ (+)(-)

5.4 The dynamics of price and quantity adjustment in the housing market

The housing market is in a steady state when simultaneously $\dot{H}=0$ and $\dot{Q}=0$. In the following the stationary housing stock is denoted by \bar{H} and the stationary relative price level by \bar{Q} . Either one of the price equations (5.11a-b) and the flow supply equation (5.12) form a pair of approximately linear differential equations in the neighbourhood of the steady state

$$(5.14a) \quad \dot{Q} = \dot{Q}_Q(Q-\bar{Q}) + \dot{Q}_H(H-\bar{H})$$

$$(5.14b) \quad \dot{H} = I_Q(Q-\bar{Q}) - \delta(H-\bar{H})$$

Partial derivatives \dot{Q}_Q and \dot{Q}_H may be positive or negative according to (5.11a-b), but $I_Q > 0$ according to (5.13). The determinant of the system is

$$(5.15) \quad D = -\delta\dot{Q}_Q - I_Q\dot{Q}_H$$

The characteristic roots of the pair of differential equations (5.14a-b) are

$$(5.16a) \quad a = -(\delta - \dot{Q}_Q + ((\delta + \dot{Q}_Q)^2 + 4I_Q\dot{Q}_H)^{1/2})/2$$

$$(5.16b) \quad b = -(\delta - \dot{Q}_Q - ((\delta + \dot{Q}_Q)^2 + 4I_Q\dot{Q}_H)^{1/2})/2$$

Basically the steady state may be either a saddle point or a stable equilibrium. It is a saddle point, in which case $a < 0$ and $b > 0$, if the expected capital gain has an increasing effect on the demand for housing stock so that $\dot{Q}_Q > 0$ and $\dot{Q}_H > 0$ according to (5.11a). On the other hand, the steady state equilibrium is stable if $\dot{Q}_Q < 0$ and $\dot{Q}_H < 0$ according to (5.11b).

The general solutions of the time paths of house prices and housing stock are

$$(5.17a) \quad Q(t) = (Q(0) - \bar{Q} - V_1)e^{at} + V_1e^{bt} + \bar{Q}$$

$$(5.17b) \quad H(t) = (H_0 - \bar{H} - V_2)e^{at} + V_2e^{bt} + \bar{H}$$

where V_1 and V_2 are constants and H_0 is the initial supply of housing stock. In the stable case the model does not determine a unique adjustment path for the housing market. Whatever the constants $Q(0)$, V_1 , and V_2 may be the market approaches a steady state. In the saddle path case, where $a < 0$ and $b > 0$, the only possibility for the housing market to approach a steady state is to select the conditionally stable paths which are determined by setting the coefficients of the unstable factor, e^{bt} , in (5.17a-b) to zero, $V_1 = V_2 = 0$. These conditionally stable paths correspond to the following "partial adjustment" rules where the coefficient of adjustment is the negative characteristic root, $a < 0$

$$(5.18a) \quad \dot{Q}(t) = a(Q(t) - \bar{Q})$$

$$(5.18b) \quad \dot{H}(t) = a(H(t) - \bar{H})$$

The coefficient of adjustment depends on the properties of both the flow supply function and the stock demand function of houses, as can be seen from equation (5.16a). Market adjustment along the conditionally stable saddle path gives a rationale for partial adjustment behaviour. This is an alternative explanation for the conventional microeconomic hypothesis which assumes that changes for example in the stock of a durable good induce increasing marginal costs which motivate the gradual adjustment behaviour (c.f. Deaton and Mue1lbauer (1980)).

Figures 5.1 and 5.2 illustrate the adjustment processes of the housing market when the steady state is a saddle point. In both figures the left hand side describes the new housing production, I , as an increasing function of the relative housing price level, Q . The right hand side describes the price and quantity adjustment of the housing stock. The steady state of the housing market is at the point where curves $\dot{H}=0$ and $\dot{Q}=0$ intersect. The $\dot{H}=0$ locus is the set

of pairs of relative house prices and housing stocks along which the housing stock remains constant. It is upward sloping because $dQ/dH = \delta/I_0 > 0$ when $\dot{H}=0$ in (5.12). The $\dot{Q}=0$ locus is the set of pairs of prices and housing stocks for which the relative house prices remain constant so that there are no anticipated capital gains or losses. This locus is downward sloping because $dQ/dH = -\dot{Q}_H/\dot{Q}_Q < 0$ when $\dot{Q}=0$ in (5.11a).

Figure 5.1 illustrates the market adjustment after an exogenous increase in the demand for housing. The housing market is assumed to be initially in a steady state where housing stock is H_0 , house prices are Q_0 , and new housing production is I_0 . An increase in the demand for housing shifts the $\dot{Q}=0$ locus upwards so that the steady state shifts to points \bar{H} , \bar{Q} , and \bar{I} . Because the stock supply is momentarily completely inelastic, the market price must jump to $Q(0)$ for the housing market to attain the unique stable arm that leads towards the new steady state, \bar{H}, \bar{Q} . The increase in the housing stock is achieved via a jump in production initially from I_0 to $I(0)$. After the initial jump the price level and production gradually decrease towards the steady state levels. Thus the market adjustment is characterized by the overshooting of initial house prices and housing production as compared with the final equilibrium levels, so that $Q(0) > \bar{Q}$ and $I(0) > \bar{I}$.

Figure 5.2 describes a situation where an exogenous increase in new housing production takes place. This shifts both the $\dot{H}=0$ locus and line I downwards and the steady state moves from points H_0, Q_0, I_0 to points $\bar{H}, \bar{Q}, \bar{I}$. Because the housing stock remains initially at H_0 , house prices must jump downwards to $Q(0)$ to attain the stable arm which ensures convergence to the long run equilibrium. Again the housing stock is increased via an overshooting in new production as compared to the final steady state level.

Figure 5.1: An increase in the demand for housing

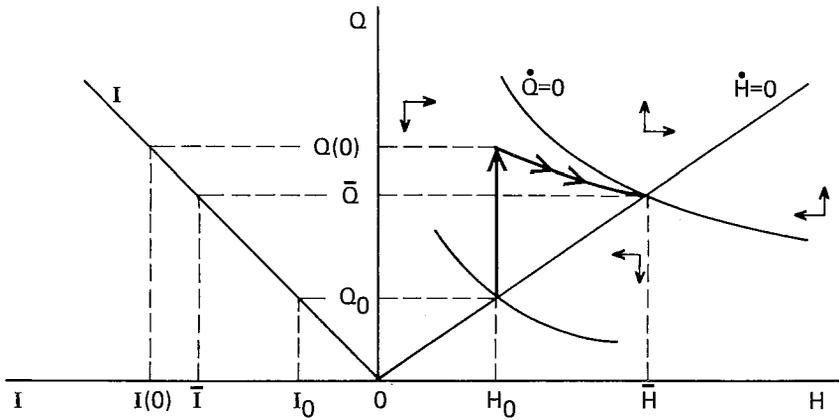
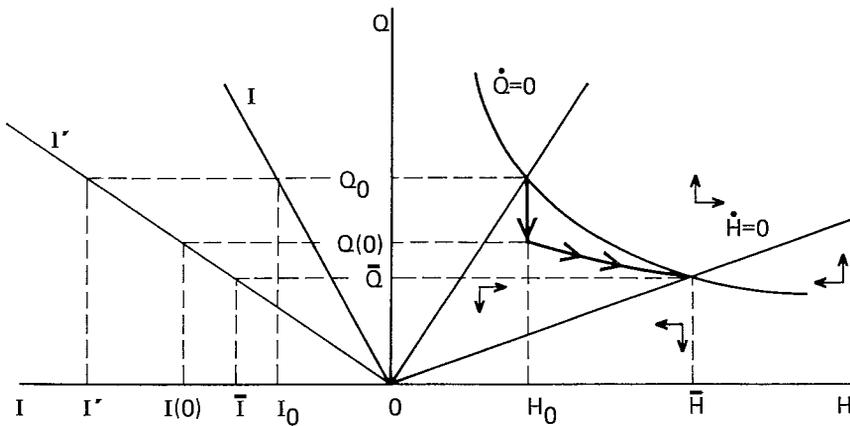


Figure 5.2: An increase in new housing production



The size of the initial overshooting of the price level on the saddle path, at $t=0$, can be computed by means of the linearized model (5.14a) and equation (5.18a) by eliminating $\dot{Q}(0)$ which gives

$$(5.19) \quad Q(0) - \bar{Q} = (\dot{Q}_H / (a - \dot{Q}_Q))(H_0 - \bar{H})$$

Since the coefficient on the right-hand side of the equation is negative, the initial price level overshoots the steady state level, $Q(0) > \bar{Q}$, if a change in some exogenous variable induces an increase in the steady state housing stock, so that $\bar{H} > H_0$. New housing production overshoots also because it is positively related to the market price level. On the other hand, house prices and residential construction undershoot if $\bar{H} < H_0$.

5.5 Changes in the steady state housing stock and house prices

This section describes the effects of changes in exogenous variables on the steady state housing stock and market price level when the steady state equilibrium is a saddle point. The changes in exogenous variables, which are analyzed here, should be interpreted as unexpected shocks. On the other hand, anticipated changes in exogenous variables have already affected demand or supply decisions so that the current level of house prices and residential construction already reflects market participants' expectations. In other words, anticipated changes in exogenous variables do not induce any new reactions apart from those that took place when the new information unexpectedly became available for the first time.

Denote the vector of the exogenous arguments in the demand function (5.10), W , Y , r , and \underline{A} , by X , and the exogenous variables of the derived production function (5.13), K and w , by Z . Then the relationships between the steady state house prices, the housing stock, and the exogenous variables may be defined by the following pair of implicit functions

$$(5.20a) \quad \dot{Q}(\bar{Q}(X,Z), \bar{H}(X,Z); X) = 0$$

$$(5.20b) \quad \dot{H}(\bar{Q}(X,Z), \bar{H}(X,Z); Z) = 0$$

The long run effects of a change in the demand for housing are given by rules

$$(5.21a) \quad \partial \bar{Q} / \partial X = \delta \dot{Q}_X / D$$

$$(5.21b) \quad \partial \bar{H} / \partial X = I_Q \dot{Q}_X / D$$

Thus a demand shock shifts both the steady state price level and the housing stock in the same direction because both $\delta > 0$ and $I_Q > 0$. Moreover, positive demand effects are reflected in the instantaneous overshooting of house prices. This situation was described in Figure 5.1 and the corresponding time paths of housing stock, house prices, and new housing production are depicted in Figure 5.3. House prices and housing investment first jump upwards and then gradually fall to the final equilibrium levels, \bar{Q} and $\bar{I} = \delta \bar{H}$. On the other hand, housing stock grows smoothly from the initial level to the new steady state level, \bar{H} .

The long run effects of a change in new housing production are

$$(5.22a) \quad \partial \bar{Q} / \partial Z = \dot{Q}_H I_Z / D$$

$$(5.22b) \quad \partial \bar{H} / \partial Z = -\dot{Q}_Q I_Z / D$$

Accordingly, an exogenous supply impulse has opposite effects on steady state house prices and housing stock because $\dot{Q}_H > 0$ and $\dot{Q}_Q > 0$. Figure 5.4 illustrates a case where new housing production grows exogenously, for example due to a decrease in employers' social security contributions which reduces labour costs. This leads first to an instantaneous downward jump in house prices. Prices then continue falling smoothly on the saddle path to the new equilibrium level while the housing stock gradually increases.

Figure 5.3: Reactions to an increase in the demand for housing

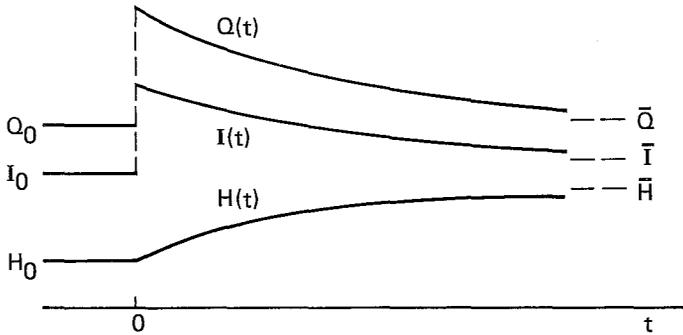
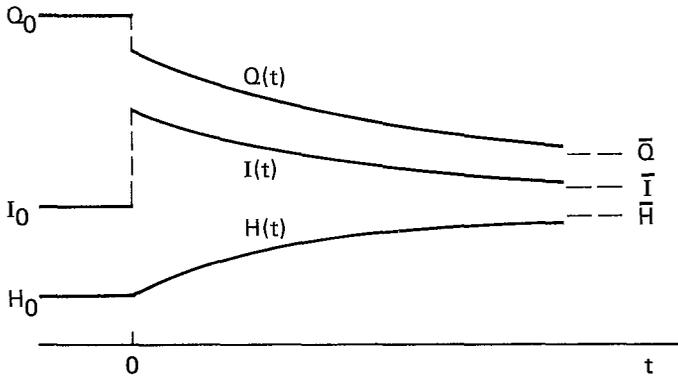


Figure 5.4: Reactions to an increase in new housing production



The instantaneous price effects, \dot{Q}_x , are defined in equation (5.11a) and the supply effects, I_z , in equation (5.13). The determinant, D , is negative according to (5.15) since we assume that the steady state is a saddle point. Thus we may compute the effects of changes in exogenous variables on the steady state house prices, \bar{Q} , and housing stock, \bar{H} . These results are presented in Table 5.1. The steady state level of new housing production is determined by the steady state housing stock, so that $\bar{I} = \delta \bar{H}$.

Table 5.1: Effects of changes in exogenous variables on steady state house prices, housing stock, and housing investments

	W	Y	r	<u>A</u>	K	w
\bar{Q}	+	+	-	-	-	+
\bar{H}	+	+	-	-	+	-
\bar{I}	+	+	-	-	+	-

The model implies that an increase in wealth, W , and permanent income, Y , has an expansionary effect on the housing market in the long run, whereas real interest rate, r , and borrowing constraint, \underline{A} , have contractive effects. On the supply side, an increase in the capital stock, K , and a decrease in wage rate, w , have positive effects on the steady state housing stock and new housing production but a reducing impact on house prices.

The effects of permanent income and wage rate were computed separately in Table 5.1. A simultaneous increase in permanent income and wage rate raises the steady state level of house prices but the effect on housing stock is generally ambiguous because the demand for housing increases while the new housing production decreases.

5.6 Empirical implications

In the case of saddle path stability the model implies that changes in the housing stock follow partial adjustment rule (5.18b) which resembles the conventional formula applied in many econometric studies of the demand for housing. The partial adjustment hypothesis is usually based on the assumption of increasing adjustment costs, but here it follows from the optimal adjustment of the housing market along the saddle path.

The previous analysis suggests that aggregate housing investments may be interpreted either as the flow supply of new housing units or as an adjustment process of the demand for the housing stock. Thus two alternative housing investment functions may be defined

$$(5.23a) \quad I(t) = I(Q(t); Z(t))$$

$$(5.23b) \quad = -a\bar{H}(X(t), Z(t)) + (a+\delta)H(t)$$

The first equation is the derived production function (5.13) where the arguments are house prices, $Q(t)$, and vector $Z(t)$ which represents the exogenous supply variables. There is some econometric evidence for the validity of such a supply function in Finland (c.f. Rantala (1982)). Equation (5.23b) is derived by adding depreciation of the housing stock to the partial adjustment formula (5.18b) where the coefficient of adjustment is $a < 0$. The latter equation implies that housing investments are a function of both the current housing stock, $H(t)$, and the exogenous variables $X(t)$ and $Z(t)$, which influence the demand for housing and new housing production respectively. Moreover, this equation implies that if expectations are rational, housing investments may in principle be explained empirically without observations about the current price level, $Q(t)$, because price information only reflects the relationship between the expected values of exogenous variables and the predetermined housing stock.

The model has some interesting empirical implications. The long term effects of changes in exogenous variables seem intuitively acceptable. Moreover, the instantaneous jumps in house prices and new housing production may give an explanation for the volatility of house prices and residential construction which have been observed for example in Finland. The model implies that the volatility may be induced by both demand and supply shocks. On the other hand, economic policy can stabilize the housing market by controlling the interest rate or borrowing constraints, or via tax systems that influence either households' disposable income or total labour costs in the residential construction business.

The above-described possible non-uniqueness of the adjustment paths, however, raises some doubts about the applicability of the standard approach in modelling the pricing behaviour in the housing market. Thus further research in this area is evidently needed.

6 CONCLUDING REMARKS AND A REFERENCE TO FINNISH HOUSEHOLDS' PORTFOLIO DISTRIBUTIONS

Chapter 2 of this study described the determination of the demand for housing in the life cycle model of consumer behaviour. In Chapter 3 the housing stock was incorporated in the dynamic portfolio model. The consumer's preferences were specified by the Stone - Geary utility function, which is familiar from the static demand theory where it is used to derive the well-known linear expenditure system.

The dynamic portfolio theory implies that consumers' wealth is the proper scale variable in the commodity demand functions, instead of income or total expenditure as implied by the conventional demand analysis. Thus, if housing is generally regarded as a necessity as some of the econometric studies of the linear expenditure system have shown, the owner-occupied housing stock should display a decreasing portfolio proportion as households' net wealth increases. Conversely, the net financial assets should increase in proportion to net wealth. Such a theoretical portfolio distribution was depicted in Figure 3.1.

Figure 6.1 indicates that the hypothetical portfolio shares in fact seem to coincide with the Finnish portfolio distribution, but only for those households whose wealth exceeds the mean wealth level.¹

¹KOSONEN and SUONIEMI (1982, Appendix 4, table II) present a table of various asset stocks and households' net wealth at the beginning of 1979 in proportion to disposable income for five income brackets. These estimates have been used to compute the portfolio shares presented in Figure 6.1. Because the original data have not been available to the present author, so that only the asset-income ratios could be utilized here, the portfolio share estimates may be somewhat biased if households' income and wealth do not correlate perfectly. A comparison with an earlier cross-section sample studied by HÄMÄLÄINEN (1974, 1981) shows that households' portfolio distributions have been relatively stable in the sense that the overall view of the asset proportions at the beginning of 1969 is fairly similar to the distribution ten years later presented in Figure 6.1.

Among these wealthier households the average portfolio proportion of owner-occupied housing stock seems to be a decreasing function of wealth as implied by the necessity of housing. On the other hand, the Finnish cross-section sample described in Figure 6.1 indicates that among the households with wealth below the mean level the portfolio proportion of owner-occupied housing stock is an increasing function of wealth and, correspondingly, the housing loans also increase in relation to net wealth.² The rental housing stock is included in other assets in Figure 6.1. The increasing portfolio share of this item may indicate that the rental housing stock is mainly owned by the wealthier households.

Systematic differences in tenure choice between households at different income levels may result from progressive income taxation as was described in section 2.5. However, there may also be other reasons for differences in the tenure mode. In Chapter 4 of this study we emphasized the role of credit market imperfections in the allocation process, where the housing stock tends to concentrate among wealthier households.

If expected permanent income does not fully qualify as a collateral for borrowing, households in narrow circumstances may not be able to finance a housing stock which would exceed the subsistence level implied by a utility function of the Stone-Geary type. Generally, the effect of borrowing constraints on consumers' housing investments is the more restrictive the less wealth they have currently at their disposal compared with their expected future earnings. In these circumstances both the owner-occupied and the rental housing stock tend to concentrate among the wealthier households who are less severely constrained in the credit market.

²The "loans" item in Figure 6.1 includes only housing loans and other personal loans. The debts of private enterprises are included under "other assets, net".

Figure 6.1: Finnish households' portfolio distributions in 1979, asset stocks in proportion to net wealth, per cent

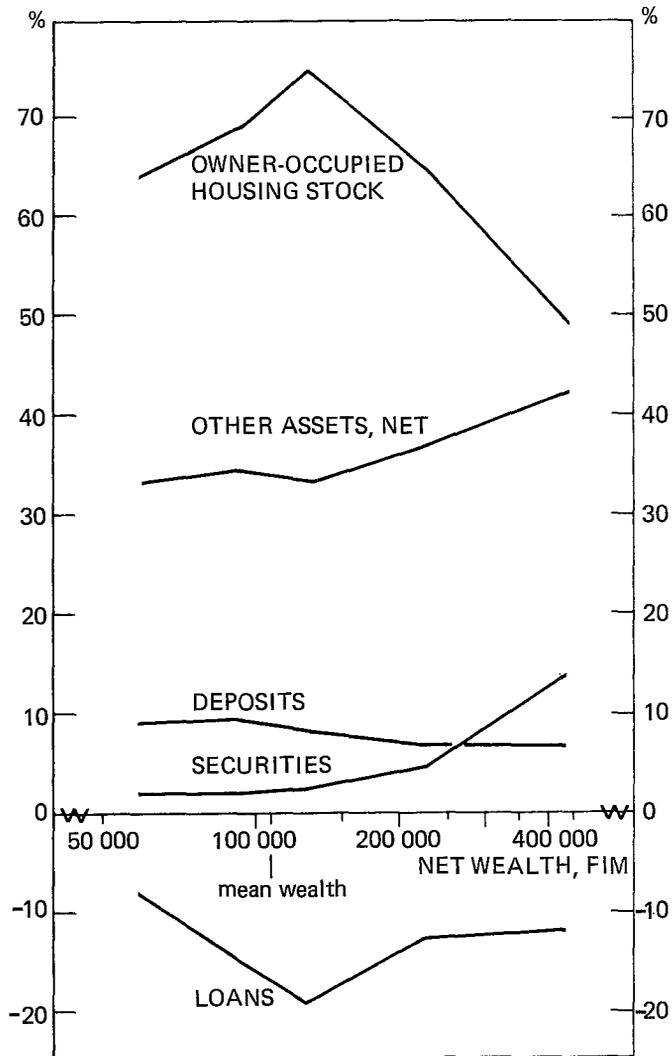


Figure 4.4 illustrated tentatively the aggregate portfolio shares of housing stock and net financial assets as functions of households' net wealth in imperfect capital markets. The humped shape in the portfolio proportion of owner-occupied housing stock was basically induced by borrowing constraints and the necessity of housing. The fact that the empirical portfolio distributions presented in Figure 6.1 and the theoretical distributions depicted in Figure 4.4 display qualitatively similar properties implies that credit market imperfections may have had a considerable effect on Finnish households' portfolio compositions, particularly on housing investments which typically require a large amount of external financing and are therefore sensitive to the availability of housing loans.

APPENDICES

APPENDIX 1a

Using the symbols given in the text the nominal wealth is defined as follows

$$(1) \quad \tilde{W}(t) = \tilde{A}(t) + P_h(t)H(t)$$

Deleting time indices, (t), for the sake of brevity, the nominal budget constraint is defined as follows

$$(2) \quad \dot{\tilde{W}} = \dot{\tilde{A}} + P_h \dot{H} + \dot{P}_h H = i\tilde{A} + (p_h - \delta)P_h H - PC + \tilde{Y}$$

Dividing both sides of (2) by P gives

$$(3) \quad \dot{\tilde{A}}/P + Q\dot{H} + (\dot{P}_h/P)H = iA + (p_h - \delta)QH - C + Y$$

But $\dot{\tilde{A}}/P = \dot{A} + pA$ from $\dot{A} = d(\tilde{A}/P)/dt = \dot{\tilde{A}}/P - pA$

and $\dot{P}_h/P = \dot{Q} + pQ$ from $\dot{Q} = d(P_h/P)/dt = \dot{P}_h/P - pQ$

Thus (3) becomes

$$(4) \quad \dot{A} + pA + Q\dot{H} + \dot{Q}H + pQH = iA + (p_h - \delta)QH - C + Y$$

This means that

$$(5) \quad \dot{W} = \dot{A} + Q\dot{H} + \dot{Q}H = (i-p)A + (p_h - p - \delta)QH - C + Y$$

which corresponds to equation (2.2a) in the text.

APPENDIX 1b

Equations (2.5a), (2.7a), and (2.8) yield

$$(1) \quad \lambda = e^{-\rho t} U_C = e^{-\rho t} \alpha (C - C^*)^{\alpha-1} (H - H^*)^{\beta\alpha} \\ = V e^{-\pi t} (H - H^*)^{-\theta}$$

where $V = \alpha^{\alpha a} \beta^{1-\alpha a} R_0^{\alpha a-1}$, $\pi = \rho + (1-\alpha a)q$ and $\theta = 1 - (\alpha + \beta)a$.

Thus

$$(2) \quad \dot{\lambda} = (-\pi - \theta \dot{H} (H - H^*)^{-1}) V e^{-\pi t} (H - H^*)^{-\theta}$$

Inserting (1) and (2) in (2.5c) yields

$$(3) \quad \dot{H} = \mu (H - H^*)$$

which has the definite solution (2.9) where $\mu = (r - \rho - (1 - \alpha a)q) / \theta$.

APPENDIX 1c

Demand function (2.13) can be rewritten as follows

$$(1) \quad H(0) = H^* + h\hat{W}/Q_0$$

where

$$(2) \quad \hat{W} = W_0 + (Y-C^*)/r - (r-q+\delta)Q_0H^*/(r-q) > 0$$

and

$$(3) \quad h = \frac{\beta(\rho-\gamma ar+\beta aq)}{\gamma\theta(r-q+\delta)} = \frac{\beta(\rho-\alpha ar+\beta a\delta)}{\gamma\theta(r-q+\delta)} - \frac{\beta^2 a}{\gamma\theta} > 0$$

Thus (3) yields

$$(4) \quad \partial h/\partial q = \frac{\beta(\rho-\alpha ar+\beta a\delta)}{\gamma\theta(r-q+\delta)^2} = (h + \frac{\beta^2 a}{\gamma\theta})/(r-q+\delta)$$

Hence

$$(5) \quad \partial H(0)/\partial q = (\hat{W}/Q_0)(\partial h/\partial q) - \delta H^*/(r-q)^2$$

is positive if $0 \leq a < 1$ and $H^* = 0$ but it may be negative for some $a < 0$ or $H^* > 0$.

APPENDIX 2

Optimality condition (3.8c) can be written in the following matrix form

$$(1) \quad (\underline{g}-i\underline{1})\underline{w}J_W + \underline{\Omega}\underline{w}W^2J_{WW} = 0$$

where $\underline{g} = (g_1 \dots g_n)'$, $\underline{1} = (1 \dots 1)'$, $\underline{w} = (w_1 \dots w_n)'$ and Ω denotes the matrix of covariances σ_{jk} .

Solving for $\underline{w}W$ from (1) gives

$$(2) \quad \underline{w}W = -(J_W/J_{WW})\Omega^{-1}(\underline{g}-i\underline{1})$$

Denoting the elements of the inverted variance-covariance matrix Ω^{-1} by v_{jk} , the demand for the j th risky asset can be expressed as follows

$$(3) \quad G_j S_j = w_j W = -(J_W/J_{WW}) \sum_{k=1}^n v_{jk} (g_k - i)$$

The demand for the composite risky asset is

$$(4) \quad \begin{aligned} GS &= \sum_{j=1}^n G_j S_j = -(J_W/J_{WW}) \sum_{j=1}^n \sum_{k=1}^n v_{jk} (g_k - i) \\ &= -(J_W/J_{WW})(g-i)/\sigma^2 \end{aligned}$$

The weighted average expected return on the composite risky asset is

$$(5) \quad g = \sum_{j=1}^n \hat{w}_j g_j$$

and the variance is

$$(6) \quad \sigma^2 = \sum_{j=1}^n \sum_{k=1}^n \hat{w}_j \hat{w}_k \sigma_{jk}$$

where the weights are determined by (3) and (4) so that $\hat{w}_j = G_j S_j / GS$ and $\hat{w}_k = G_k S_k / GS$.

For consumption and the demand for housing optimality conditions (3.8a-b) yield

$$(7) \quad C = C^* + \alpha \beta^{-1} R (H - H^*)$$

and conversely

$$(8) \quad H = H^* + \alpha^{-1} \beta R^{-1} (C - C^*)$$

Condition (3.8b) yields

$$(9) \quad C = C^* + (\beta^{-1} (H - H^*)^{1 - \beta a} R J_W)^{\frac{1}{\alpha a}}$$

Substituting from (8) for H and solving for C gives (3.11). Then (8) and (3.11) yield (3.12).

When the solution for the Hamilton - Jacobi - Bellman equation (3.7) is sought it is useful to aggregate the risky assets as a composite asset. Therefore equation (3.7) is rewritten as if there were only one risky asset with portfolio share $w = GS/W$, expected return g , and variance of return σ^2

$$(10) \quad 0 = a^{-1} (C - C^*)^{\alpha a} (H - H^*)^{\beta a} J_t + (rW - RH + (g - i)wW - C + Y) J_W \\ + qR J_R + \frac{1}{2} \sigma^2 (wW)^2 J_{WW}$$

The optimal C, H and $w = GS/W$ are given in (3.11), (3.12), and (3.10) where g is defined in (5) and σ^2 in (6). Inserting those C, H and w in (10) and collecting terms yields

$$(11) \quad 0 = a^{-1} \theta \alpha^{\frac{\alpha a}{\theta}} \beta^{\frac{\beta a}{\theta}} R - \frac{\beta a}{\theta} J_W - \frac{\gamma a}{\theta} J_t + J_t$$

$$+ r(W+(Y-C^*-RH^*)/r)J_W + qR J_R - \frac{1}{2} \left(\frac{g-i}{\sigma}\right)^2 J_W^2 / J_{WW}$$

The derived utility function $J(W,R,t)$ is probably of the same form as in the deterministic model of Chapter 2. Therefore we apply the following trial solution

$$(12) \quad J(W,R,t) = a^{-1} x R^{-\beta a} \hat{W}^{\gamma a}$$

where $\hat{W}=W+Z$. x is an undetermined constant and $Z(t)$ is an unknown function of time t .

The needed partial derivatives are

$$(13a) \quad J_W = \gamma x R^{-\beta a} \hat{W}^{\gamma a - 1}$$

$$(13b) \quad J_{WW} = -\gamma \theta x R^{-\beta a} \hat{W}^{\gamma a - 2}$$

$$(13c) \quad J_R = -\beta x R^{-\beta a - 1} \hat{W}^{\gamma a}$$

$$(13d) \quad J_t = \gamma x R^{-\beta a} \hat{W}^{\gamma a - 1} \dot{Z}$$

where $0 < \gamma = \alpha + \beta < 1$ and $\theta = 1 - \gamma a > 0$.

It is useful to solve first $Z(t)$. Therefore we first look at those two terms of (11) which are relevant for the solution. By inserting J_t from (13d) and J_W from (13a) and collecting terms we obtain

$$(14) \quad J_t + r(W+(Y-C^*-RH^*)/r)J_W$$

$$= \gamma r x R^{-\beta a} (W+Z)^{\gamma a - 1} (W+\dot{Z}/r + (Y-C^*-R_0 e^{qt} H^*)/r)$$

where the user cost is $R(t) = R_0 e^{qt}$.

The exponent of $\hat{W}=W+Z$ must be γa , as can be seen below in (17). This condition gives the following differential equation for $Z(t)$ in (14)

$$(15) \quad W + \dot{Z}/r + (Y - C^* - R_0 e^{qt} H^*)/r = W + Z$$

Provided that $r > q$ the stable solution is

$$(16) \quad Z(t) = Y/r - C^*/r - R(t)H^*/(r-q)$$

Given this solution for $\hat{W}=W+Z$ and the partial derivatives (13a-d) equation (11) can be written as follows

$$(17) \quad 0 = a^{-1} x R^{-\beta a} \hat{W}^{\gamma a} \left\{ \theta \alpha^{\frac{\alpha a}{\theta}} \beta^{\frac{\beta a}{\theta}} \gamma - \frac{\gamma a}{\theta} x - \frac{1}{\theta} - \rho \right. \\ \left. + \gamma a r - \beta a q + \frac{\gamma a}{2\theta} \left(\frac{g-i}{\sigma} \right)^2 \right\}$$

This equation gives the solution for $x > 0$

$$(18) \quad x = \gamma^{-\gamma a} \theta^{\theta} \alpha^{\alpha a} \beta^{\beta a} \left\{ \rho - \gamma a r + \beta a q - \frac{\gamma a}{2\theta} \left(\frac{g-i}{\sigma} \right)^2 \right\}^{-\theta}$$

APPENDIX 3a

The change in real wealth, $W(t)$, is obtained by applying Ito's stochastic differentiation rule to $W(t)=\tilde{W}(t)/P(t)$ which gives in this case

$$(1) \quad dW = d\tilde{W}/P - W(dP/P) + W(dP/P)^2 - (d\tilde{W}/P)(dP/P) \\ = (d\tilde{W}/P)(1-dP/P) - W(dP/P - (dP/P)^2)$$

$d\tilde{W}$ is defined in (3.21b) and dP/P in (3.18). Moreover, according to the multiplication rule for Wiener processes (e.g. Malliaris and Brock (1982))

$$(2) \quad (d\tilde{W}/P)(dP/P) = W(\sigma_{hp}wdt + \sigma_{yp}dt)$$

and

$$(3) \quad (dP/P)^2 = \sigma_p^2 dt$$

Thus, by inserting (2), (3), (3.21b), and (3.18) in (1) and collecting terms we obtain

$$(4) \quad dW = ((i-p+\sigma_p^2+\tilde{y}-\sigma_{yp})W + (p_h-\sigma_{hp}-\delta-i+V)wW - RH - C)dt \\ + W(\sigma_h wdz_h + \sigma_y dz_y - \sigma_p dz_p)$$

which corresponds to equation (3.23).

The percentage change in relative housing prices, $Q(t)=P_h(t)/P(t)$, and relative rental rate, $R(t)=\tilde{R}(t)/P(t)$, is

$$(5) \quad dQ/Q = dR/R = (dP_h/P_h)(1-dP/P) - dP/P + (dP/P)^2$$

By inserting (3.17), (3.18), and (3) in (5) and collecting terms we obtain

$$(6) \quad dQ/Q = dR/R = (p_h-p+\sigma_p^2-\sigma_{hp})dt + \sigma_h dz_h - \sigma_p dz_p$$

which corresponds to equation (3.24).

APPENDIX 3b

When the demand functions are solved it is useful to rewrite the Hamilton - Jacobi - Bellman equation (3.27) as follows

$$(1) \quad 0 = \phi(C, H, w; W, R) =$$

$$\begin{aligned} \text{Max}_{C, H, w} \quad & [a^{-1} C^{\alpha} H^{\beta} a - \rho J + ((r+y)W - RH - C)J_W + qR J_R \\ & + k_1 W^2 J_{WW} + k_2 R^2 J_{RR} + k_3 WR J_{WR} + K] \end{aligned}$$

where

$$(2a) \quad k_1 = (\sigma_y^2 + \sigma_p^2 - 2\sigma_{yp})/2$$

$$(2b) \quad k_2 = (\sigma_h^2 + \sigma_p^2 - 2\sigma_{hp})/2$$

$$(2c) \quad k_3 = \sigma_{hy} - \sigma_{hp} - \sigma_{yp} + \sigma_p^2$$

Moreover

$$(3) \quad K = K_1 W - K_2 W^2/2$$

where

$$(4a) \quad K_1 = (r_h + v)W J_W + (\sigma_{hy} - \sigma_{hp})W^2 J_{WW} + (\sigma_h^2 - \sigma_{hp}^2)WR J_{WR}$$

and

$$(4b) \quad K_2 = -\sigma_h^2 W^2 J_{WW}$$

The trial solution for the derived utility function of real wealth and relative rental price of housing is in this case of the following form

$$(5) \quad J(W, R) = a^{-1} x R^{-\beta} a_W \gamma a$$

where x is an undetermined constant which must be positive so that the derived utility becomes a strictly concave function of wealth.

The needed partial derivatives are

$$(6a) \quad J_W = \gamma x R^{-\beta a} W^{\gamma a - 1}$$

$$(6b) \quad J_{WW} = -\gamma \theta x R^{-\beta a} W^{\gamma a - 2}$$

$$(6c) \quad J_{WR} = -\gamma \beta a x R^{-\beta a - 1} W^{\gamma a - 1}$$

$$(6d) \quad J_R = -\beta x R^{-\beta a - 1} W^{\gamma a}$$

$$(6e) \quad J_{RR} = \beta(1 + \beta a) x R^{-\beta a - 2} W^{\gamma a}$$

where $\gamma = \alpha + \beta$ and $\theta = 1 - \gamma a$.

Optimality conditions (3.28a-b) imply that

$$(7) \quad C = \alpha \beta^{-1} R H$$

and, conversely, that

$$(8) \quad H = \alpha^{-1} \beta R^{-1} C$$

Condition (3.28b) gives

$$(9) \quad C = (\beta^{-1} H^{1 - \beta a} R J_W)^{\frac{1}{\alpha a}}$$

Substituting from (8) for H and solving for C gives

$$(10) \quad C = \alpha^{\frac{1 - \beta a}{\theta}} \beta^{\frac{\beta a}{\theta}} R^{-\frac{\beta a}{\theta}} J_W^{-\frac{1}{\theta}}$$

Substituting from (6a) for J_W gives

$$(11) \quad C = \gamma \frac{1}{\theta} \frac{1-\beta a}{\alpha} \frac{\beta a}{\beta} \frac{1}{x} - \frac{1}{\theta} W$$

Thus from (8)

$$(12) \quad H = \gamma \frac{1}{\theta} \frac{\alpha a}{\alpha} \frac{1-\alpha a}{\beta} \frac{1}{x} - \frac{1}{\theta} R^{-1} W$$

The optimal portfolio share of housing property is determined by condition $\partial \phi / \partial w = \partial K / \partial w = 0$. Thus from (3)

$$(13) \quad w = QH^0/W = K_1/K_2$$

Hence at the optimum we have $K_1 = K_2 w$ so that from (3)

$$(14) \quad K = K_1 w - K_2 w^2 / 2 = K_2 w^2 / 2$$

Thus, using (4b) and (6b) gives

$$(15) \quad K = \frac{\gamma \theta x}{2} \sigma_h^2 w^2 R^{-\beta a} W^{\gamma a}$$

Substituting from (11), (12), and (15) for C, H, and K and from (6a-e) for the partial derivatives in equation (1) and collecting terms gives

$$(16) \quad 0 = a^{-1} x R^{-\beta a} W^{\gamma a} \left\{ \theta \alpha \frac{\alpha a}{\theta} \frac{\beta a}{\beta} \frac{1}{\gamma} - \frac{\gamma a}{\theta} \frac{1}{x} - \frac{1}{\theta} - \varepsilon + \frac{\gamma \theta a}{2} \sigma_h^2 w^2 \right\}$$

where

$$(17) \quad \varepsilon = \rho - \gamma a(r + y) + \beta a q + \gamma \theta a k_1 - \beta a(1 + \beta a) k_2 + \gamma \beta a^2 k_3$$

This gives equation (3.32) after substituting from (2a-c) for k_1, k_2 , and k_3 and using definitions $r = i - p + \sigma_p^2$, $y = \tilde{y} - \sigma_{yp}$, and $q = p_h - p + \sigma_p^2 - \sigma_{hp}$.

Equation (16) gives the solution for the constant $x > 0$

$$(18) \quad x = \gamma^{-\gamma a} \theta^{\theta} \alpha^{\alpha} \beta^{\beta a} \left(\varepsilon - \frac{\gamma \theta a}{2} \sigma_h^2 w^2 \right)^{-\theta}$$

Given the solution for x , equations (11) and (12) determine the consumption function (3.29) and the demand for housing services (3.30). The portfolio share of housing property (3.31) is determined by equation (13) where K_1 and K_2 are determined in (4a-b) by using (6a-e) and definition $r_h = p_h - \sigma_{hp} - \delta - i$.

APPENDIX 3c

Substituting equilibrium conditions (3.42b-c) for σ_y and ρ_{hy} gives

$$(1) \quad \sigma_{hy} = \sigma_h \sigma_y \rho_{hy} = (1-w) \sigma_h^2$$

Substituting this for σ_{hy} in (3.35) gives the following equilibrium rent-price ratio

$$(2) \quad V = R/Q = \tilde{R}/P_h = i - p_h + \delta + (1-\alpha a) \sigma_h^2 + \alpha a \sigma_{hp}$$

which corresponds to equation (3.43).

Using (3.42b and d) the risk premium term can be written in the following form

$$(3) \quad (1-\alpha a) \sigma_h^2 + \alpha a \sigma_{hp} = (1-\alpha a) \sigma_y^2 / (1-w)^2 + \alpha a \sigma_y \sigma_p \rho_{yp} / (1-w)$$

The risk premium is positive if $\rho_{yp} > - (1-\alpha a) \sigma_y / (\alpha a (1-w) \sigma_p)$.

APPENDIX 4a

Substituting $A=W-QH$ for financial assets and $E=\psi Q(H-H^*)+E^*$ for expenditure in (4.12a) gives

$$(1) \quad \begin{aligned} \dot{A} &= rW-rQH-\psi Q(H-H^*)-C^*-\delta QH^*+Y \\ &= rW+Y-C^*-(r+\delta)QH^*-(r+\psi)Q(H-H^*) \end{aligned}$$

The demand for housing stock, H , is determined by equation (4.10). Inserting this in (1) gives the saving function

$$(2) \quad \dot{A} = \left(r - \frac{\beta(\rho-\gamma ar)(r+\psi)}{\gamma\theta(r+\delta)} \right) \left(W + \frac{Y-C^*-RH^*}{r} \right)$$

which corresponds to equation (4.12b).

APPENDIX 4b

Budget constraint and the time paths of housing stock (4.8) and consumption (4.7) imply that if the borrowing constraint is not binding

$$(1) \quad \begin{aligned} \dot{W} &= rW - RH - C + Y \\ &= rW - \frac{\gamma R}{\beta} (H(0) - H^*) e^{\frac{r-\rho}{\theta} t} + Y - C^* - RH^* \end{aligned}$$

This gives the general solution for the time path of net wealth

$$(2) \quad W(t) = \frac{\gamma \theta}{\beta(\rho - \gamma ar)} R(H(0) - H^*) e^{\frac{r-\rho}{\theta} t} - \frac{Y - C^* - RH^*}{r} + V e^{rt}$$

But the constant of integration must be zero, $V=0$, as implied by equation (4.10). Thus the time path of net financial assets is

$$(3) \quad \begin{aligned} A(t) &= W(t) - QH(t) \\ &= \left(\frac{\gamma \theta (r + \delta)}{\beta(\rho - \gamma ar)} - 1 \right) Q(H(0) - H^*) e^{\frac{r-\rho}{\theta} t} - \frac{Y - E^*}{r} \end{aligned}$$

Thus ultimately $A(t)$ approaches $-(Y - E^*)/r$ if $0 < r < \rho$.

APPENDIX 4c

Equations (4.15) and (4.16) imply the following relationships between the endogenous and exogenous variables

$$(1) \quad H = f(\tau; A, W, Y, Q, r) \\ (+)(-)(+)(+)(-)(?)$$

$$(2) \quad \tau = g(H; A, W, Y, Q, r) \\ (+)(-)(0)(-)(+)(?)$$

Provided that the determinant is positive

$$(3) \quad D = 1 - f_{\tau}g_H > 0$$

the effect of the borrowing constraint on the length of the liquidity-unconstrained period is negative

$$(4) \quad \partial\tau/\partial A = (g_A + f_A g_H)/D < 0$$

In this case the borrowing constraint has a negative impact on the demand for housing stock

$$(5) \quad \partial H/\partial A = (f_A + f_{\tau}g_A)/D < 0$$

Moreover, the wealth effect is positive

$$(6) \quad \partial H/\partial W = f_W/D > 0$$

The effects of permanent income, Y, and relative housing prices, Q, are

$$(7) \quad \partial H/\partial Y = (f_Y + f_{\tau}g_Y)/D$$

$$(8) \quad \partial H/\partial Q = (f_Q + f_{\tau}g_Q)/D$$

These effects are generally ambiguous for as implied by (1) and (2) the numerators in (7) and (8) may be positive or negative, even if $D > 0$.

APPENDIX 5a

Rewrite demand function (5.6) in form

$$(1) \quad QH = h(W+Y/r)$$

where

$$(2) \quad h = \frac{\beta(\rho - \gamma ar + \beta aq)}{\gamma \theta (r - q + \delta)}$$

The consumption function is

$$(3) \quad C = \alpha \beta^{-1} RH = \alpha \beta^{-1} (r - q + \delta) h (W + Y/r)$$

Inserting (5.8), (5.1b), (1), (2), and (3) in (5.7) gives

$$\begin{aligned} (4) \quad \dot{Q}H + Q\dot{H} &= (q+g)QH = (q+g)h(W+Y/r) \\ &= h\dot{W} = h(rW - (r-q+\delta)QH - C + Y) \\ &= (r - \gamma(r-q+\delta)h/\beta)h(W+Y/r) \end{aligned}$$

Thus

$$(5) \quad q + g = r - \gamma(r-q+\delta)h/\beta$$

Solving for $q = \dot{Q}/Q$ finally gives equation (5.9). Substituting equilibrium condition (5.9) in demand function (5.6) gives

$$(6) \quad QH = \frac{\beta(\rho - \alpha ar - \theta g)}{\gamma(\rho - \alpha ar + (1 - \alpha a)\delta + \theta g)} (W + Y/r)$$

APPENDIX 5b

Equation (5.10) can be rewritten as an implicit function, G , which implies zero excess demand for the existing housing stock, H , i.e. the fixed stock supply

$$G = H^d(Q, \dot{Q}; W, Y, r, \underline{A}) - H = 0$$

Thus, for example, $\partial \dot{Q} / \partial Q = -(\partial G / \partial Q) / (\partial G / \partial \dot{Q}) = -(\partial H^d / \partial Q) / (\partial H^d / \partial \dot{Q}) > 0$ if $\partial H^d / \partial \dot{Q} > 0$ in equation (5.10). On the other hand, $\partial \dot{Q} / \partial Q < 0$ if $\partial H^d / \partial \dot{Q} < 0$. The signs of the other partial derivatives in equations (5.11a-b) can be derived analogously.

APPENDIX 5c

It is assumed that firms optimize the demand for labour, N , so as to maximize the profit from housing production, $I = I(K, N)$, subject to the fixed capital stock, K

$$(1) \quad \text{Max}_N [PQI(K, N) - wPN]$$

If the production function is $I = K^\mu N^{1-\mu}$, where $0 < \mu < 1$, the necessary optimality condition implies that

$$(2) \quad N = ((1-\mu)Q/w)^{1/\mu} K$$

This gives the optimal production

$$(3) \quad I = K^\mu N^{1-\mu} = ((1-\mu)Q/w)^{(1-\mu)/\mu} K$$

BIBLIOGRAPHY

ANDO, A. and MODIGLIANI, F. (1963) "The "Life-Cycle" Hypothesis of Saving: Aggregate Implications and Tests", American Economic Review, Vol. 53, No. 1, March, pp. 55-84.

APPELBAUM, E. and HARRIS, R. (1978) "Imperfect Capital Markets and Life-Cycle Saving", Canadian Journal of Economics, May, pp. 319-324.

ARROW, K.J. and KURZ, M. (1969) "Optimal Consumer Allocation over an Infinite Horizon", Journal of Economic Theory, Vol. 1, pp. 68-91.

ARTLE, R. and VARAIYA, P. (1978) "Life Cycle Consumption and Homeownership", Journal of Economic Theory, Vol. 18, pp. 38-58.

BALTENSPERGER, E. (1978) "Credit Rationing, Issues and Questions", Journal of Money, Credit and Banking, Vol. 10, No. 2, May, pp. 170-183.

BARRO, R.J. (1974) "Are Government Bonds Net Wealth?" Journal of Political Economy, Vol. 82, No. 6, November/December, pp. 1095-1117.

BEGG, D.K.H. (1982) "The Rational Expectations Revolution in Macroeconomics, Theories and Evidence", Philip Allan, Oxford.

BLINDER, A.S. (1974) "Towards an Economic Theory of Income Distribution", MIT Press, Cambridge.

BREEDEN, D.T. (1979) "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities", Journal of Financial Economics, Vol. 7, No. 3, September, pp. 265-296.

CASS, D. and YAARI, M.E. (1967) "Individual Saving, Aggregate Capital Accumulation and Efficient Growth" in Shell, K., ed.: "Essays on the Theory of Optimal Economic Growth", MIT Press, Cambridge.

CHOW, G.C. (1979) "Optimum Control of Stochastic Differential Equation Systems", Journal of Economic Dynamics and Control, Vol. 1, pp. 143-175.

DEATON, A. (1975) "Models and Projections of Demand in Post-War Britain", Cambridge Studies in Applied Econometrics:1, Chapman and Hall, London.

DEATON, A. and MUELLBAUER, J. (1980) "Economics and Consumer Behaviour", Cambridge University Press, Cambridge.

DI EWERT, W.E. (1974) "Intertemporal Consumer Theory and the Demand for Durables", Econometrica, Vol. 42, No. 3, May, pp. 497-516.

ENGLUND, P. and PERSSON, M. (1982) "Housing Prices and Tenure Choice with Asymmetric Taxes and Progressivity", Journal of Public Economics, Vol. 19, No. 3, December, pp. 271-290.

- FALLIS, G. and SMITH, L.B. (1984) "Uncontrolled Prices in a Controlled Market: The Case of Rent Controls", American Economic Review, Vol. 74, No. 1, March, pp. 193-200.
- FELDSTEIN, M. (1982) "Inflation, Tax Rules and the Accumulation of Residential and Nonresidential Capital", Scandinavian Journal of Economics, 84 (2), pp. 293-311.
- FISCHER, S. (1975) "The Demand for Index Bonds", Journal of Political Economy, Vol. 83, No. 3, June, pp. 509-534.
- FISHER; I. (1930) "The Theory of Interest", Yale University Press, New Haven.
- FLEMMING, J.S. (1973) "The Consumption Function when Capital Markets are Imperfect: The Permanent Income Hypothesis Reconsidered", Oxford Economic Papers, Vol. 25, No. 2, July, pp. 160-172.
- GERTLER, M. and GRINOLS, E. (1982) "Monetary Randomness and Investment", Journal of Monetary Economics, Vol. 10, No. 2, September, pp 239-258.
- GHEZ, G.R. and BECKER, G.S. (1975) "The Allocation of Time and Goods over the Life Cycle", NBER, New York.
- GREEN, H.A.J. (1976) "Consumer Theory", Macmillan Press, London.
- HELLER, W.P. and STARR, R.M. (1979) "Capital Market Imperfection, the Consumption Function and the Effectiveness of Fiscal Policy", Quarterly Journal of Economics, Vol. 93, No. 3, August, pp. 455-463.
- HENDERSHOTT, P.H. and HU, S.C. (1983) "The Allocation of Capital Between Residential and Nonresidential Uses: Taxes, Inflation and Capital Market Constraints", Journal of Finance, Vol. 38, No. 3, June, pp. 795-812.
- HENDERSON, J.V. and IOANNIDES, Y.M. (1983) "A Model of Housing Tenure Choice", American Economic Review, Vol. 73, No. 1, March, pp. 98-113.
- HU, S.C. (1980) "Imperfect Capital Markets, Demand for Durables, and the Consumer Lifetime Allocation Process", Econometrica, Vol. 48, No. 3, April, pp. 577-594.
- HÄMÄLÄINEN, S. (1974) "Palkansaaajatalouksien säästämisestä, Suomen Pankin säästämistiedustelu vuodelta 1969", Suomen Pankki A:37, Helsinki.
- HÄMÄLÄINEN, S. (1981) "Suomalaisten palkansaaajatalouksien säästämisikäyttyminen, Poikkileikkausanalyysi säästämiseen vaikuttavista tekijöistä", Suomen Pankki B:36, Helsinki.
- IOANNIDES, Y.M. and McDONALD, R.L. (1982) "Optimal Asset Holdings and Housing Market Equilibrium under Rational Expectations", Boston University. A paper presented at the European Meeting of the Econometric Society in Pisa, August 1983.

JACKMAN, R. and SUTTON, J. (1982) "Imperfect Capital Markets and the Monetarist Black Box: Liquidity Constraints, Inflation and the Asymmetric Effects of Interest Rate Policy", Economic Journal, Vol. 92, No. 365, March, pp. 108-128.

KAMIEN, M.I. and SCHWARTZ, N.L. (1981) "Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management", Elsevier North Holland, New York.

KAU, J.B. and KEENAN, D. (1983) "Inflation, Taxes and Housing: A Theoretical Analysis", Journal of Public Economics, Vol. 21, No. 1, pp. 93-104.

KING, M.A. (1980) "An Econometric Model of Tenure Choice and Demand for Housing as a Joint Decision", Journal of Public Economics, Vol. 14, No. 2, October, pp. 137-159.

KOSKELA, E. (1978) "On the Theory of Consumption Function with Borrowing Constraints", Schweizerische Zeitschrift für Volkswirtschaft und Statistik, Vol. 114, No. 4, Dezember, pp. 739-752.

KOSONEN, K. (1984) "Säästäminen, asunto ja velka: poikkileikkaus-analyysi suomalaisten kotitalouksien säästämisikäyttäytymisestä", kansantaloustieteen lisensiaattityö, Helsingin yliopisto.

KOSONEN, K. and SUONIEMI, I. (1982) "Säästämistutkimus 1979", Säästöpankkien tutkimussäätiö, Helsinki.

LOIKKANEN, H.A. (1982) "Housing Demand and Intra-Urban Mobility Decisions: A Search Approach", Societas Scientiarum Fennica, Helsinki.

MALLIARIS, A.G. and BROCK, W.A. (1982) "Stochastic Methods in Economics and Finance", North-Holland, Amsterdam.

MERTON, R.C. (1969) "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case", Review of Economics and Statistics, Vol. 51, No. 3, August, pp. 247-257.

MERTON, R.C. (1971) "Optimum Consumption and Portfolio Rules in a Continuous-Time Model", Journal of Economic Theory, Vol. 3, pp. 373-413.

MERTON, R.C. (1973) "An Intertemporal Capital Asset Pricing Model", Econometrica, Vol. 41, No. 5, September, pp. 867-887.

MODIGLIANI, F. and BRUMBERG, R. (1955) "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data" in Kurihara, K.K., ed.: "Post Keynesian Economics", Allen & Unwin, London.

MUTH, R.F. (1960) "The Demand for Non-Farm Housing" in Harberger, A.C., ed.: "The Demand for Durable Goods", The University of Chicago Press, Chicago.

OLSEN, E.O. and BARTON, D.M. (1983) "The Benefits and Costs of Public Housing in New York City", Journal of Public Economics, Vol. 20, No. 3, April, pp. 299-332.

PUBLICATIONS OF THE BANK OF FINLAND

Series B (ISSN 0357-4776)

(Nos. 1 - 31, Publications of the Bank of Finland
Institute for Economic Research, ISSN 0081-9484)

1. VALTER LINDBERG National Income of Finland in 1926 - 1938. 1943. 185 p. In Finnish, summary in German.
2. MATTI LEPPÖ Der private und der öffentliche Anteil am Volkseinkommen. 1943. 104 p. In German.
3. T. JUNNILA The Property Tax as a Supplementary Tax on Funded Income. 1945. 183 p. In Finnish, summary in English.
4. MIKKO TAMMINEN The Fluctuations in Residential Building and Their Causes in the Towns of Finland during the Time of Independence. 1945. 281 p. + appendix. In Finnish, summary in English.
5. T. JUNNILA - G. MODEEN Taxation of Physical Persons in Finland in 1938 and 1945. 1945. 82 p. In Finnish.
6. HEIKKI VALVANNE Taxation of Corporations in Finland in 1938 - 1945. 1947. 105 p. In Finnish.
7. YNGVAR HEIKEL Development of the Industry of Finland in 1937 - 1944. A Research on the Basis of the Balances of the Industrial Companies. 1947. 158 p. In Swedish, summary in English.
8. T. JUNNILA Inflation. I. Inflation, Its History, and How It Is Explained by the Theory of the Value of Money. The Inflation in Finland in 1939 - 1946. 1947. 304 p. In Finnish.
9. MIKKO TAMMINEN Foreign Exchange Rates and Currency Policy. I. 1948. 218 p. In Finnish.
10. HEIKKI VALVANNE State Income and Expenditure and Turnover on Cash Account. A Research Plan and Its Application on the Years 1945 - 1947. 1949. 117 p. In Finnish.
11. K.O. ALHO The Rise and Development of Modern Finnish Industry in 1860 - 1914. 1949. 240 p. In Finnish.
12. REINO ROSSI The Interest Rate Policy of the Bank of Finland in 1914 - 1938. 1951. 327 p. In Finnish, summary in English.
13. HEIMER BJÖRKQVIST The Introduction of the Gold Standard in Finland in 1877 - 1878. 1953. 478 p. In Swedish, summary in English.

14. OLE BÄCKMAN Clearing and Payments Agreements in Finnish Foreign Trade. 1954. 92 p. In Finnish.
15. NILS MEINANDER The Effect of the Rate of Interest. 1955. 310 p. In Swedish, summary in English.
16. VEIKKO HALME Expörts as a Factor in the Trade Cycles of Finland in 1870 - 1939. 1955. 365 p. In Finnish, summary in English.
17. REINO ROSSI The Finnish Credit System and the Lending Capacity of the Banks. 1956. 191 p. In Finnish.
18. HEIKKI VALVANNE Budget Balance in the Macroeconomic Theory of Budgetary Policy. 1956. 194 p. In Finnish, summary in English.
19. HEIMER BJÖRKQVIST Price Movements and the Value of Money in Finland during the Gold Standard in 1878 - 1913. A Structural and Business Cycle Analysis. 1958. XII+391 p. In Swedish, summary in English.
20. J.J. PAUNIO A Study in the Theory of Open Inflation. 1959. 154 p. In Finnish and English.
21. AHTI KARJALAINEN The Relation of Central Banking to Fiscal Policy in Finland in 1811 - 1953. 1959. 183 p. In Finnish, summary in English.
22. PENTTI VIITA Factor Cost Prices in Finnish Agriculture and Industry Compared with International Market Prices in 1953 - 1958. 1959. 155 p. In Finnish, summary in English.
23. JAAKKO LASSILA National Accounting Systems. 1960. 92 p. In Finnish.
24. TIMO HELELÄ A Study on the Wage Function. 1963. 186 p. In Finnish, summary in English.
25. JAAKKO LASSILA The Behaviour of Commercial Banks and Credit Expansion in Institutionally Underdeveloped Financial Markets. 1966. 172 p. In Finnish, summary in English.
26. LAURI KORPELAINEN The Demand for Household Furniture and Equipment in Finland, 1948 - 1964. 1967. 139 p. In Finnish, summary in English.
27. HENRI J. VARTIAINEN The Growth in Finnish Government Revenue due to Built-in Flexibility and Changes in Tax Rates, 1950 - 1964. 1968. 216 p. In Finnish, summary in English.
28. PERTTI KUKKONEN Analysis of Seasonal and Other Short-term Variations with Applications to Finnish Economic Time Series. 1968. 136 p. In English.
29. MARKKU PUNTILA The Assets and Liabilities of the Banking Institutions in Finnish Economic Development 1948 - 1964. 1969. 116 p. In Finnish, summary in English.

30. J.J. PAUNIO A Theoretical Analysis of Growth and Cycles. 1969. 80 p. In English.
31. AHTI MOLANDER A Study of Prices, Wages and Employment in Finland, 1957 - 1966. 1969. 119 p. In English.
32. KARI NARS Foreign Exchange Strategies of the Firm. A Study of the Behaviour of a Sample of Finnish Companies under Exchange Rate Uncertainty 1970 - 77. 1979. 214 p. In Swedish, with summary in English (ISBN 951-686-054-0), and in Finnish (ISBN 951-686-063-X).
33. SIXTEN KORKMAN Exchange Rate Policy, Employment and External Balance. 1980. 133 p. In English. (ISBN 951-686-057-5)
34. PETER NYBERG Emigration, Economic Growth, and Stability. A Theoretical Inquiry into Causes and Effects of Emigration in the Medium Term. 1980. 135 p. In Swedish, summary in English. (ISBN 951-686-058-3)
35. HANNU HALTTUNEN Exchange Rate Flexibility and Macroeconomic Policy in Finland. 1980. 189 p. In English. (ISBN 951-686-064-8)
36. SIRKKA HÄMÄLÄINEN The Savings Behaviour of Finnish Households. A Cross-section Analysis of Factors Affecting the Rate of Saving. 1981. 171 p. + appendices. In Finnish, summary in English. (ISBN 951-686-074-5)
37. URHO LEMPINEN Optimizing Agents, Exogenous Shocks and Adjustments in the Economy: Real and Nominal Fluctuations in Economies with a Wage Rigidity. 1984. 271 p. In English. (ISBN 951-686-100-8)
38. HEIKKI KOSKENKYLÄ Investment Behaviour and Market Imperfections with an Application to the Finnish Corporate Sector. 1985. 279 p. + appendices. In English. (ISBN 951-686-110-5).
39. ESKO AURIKKO Studies of Exchange Rate Policies and Disequilibria in the Finnish Economy. 1986. 153 p. In English. (ISBN 951-686-115-6).
40. OLAVI RANTALA A Study of Housing Investment and Housing Market Behaviour. 1986. 117 p. In English. (ISBN 951-686-116-4)